Essays on the Effects of Government Expenditure Stimulus

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Declaration

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Disclaimer

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Contents

Abstract

This thesis studies implications of using the expednitur side of fiscal policy using recent advances in macroeconomic and macroeconometric modelling, tailered for policy analysis. The first chapter investigates the relationship between government expenditure multipliers and the phase of the business cycle. The second chapter executes an optimal public investment policy analysis and investigates the relationship between monetary policy stance and the size of the public consumption multiplier. The third chapter estimates dependence of public consumption and investment expenditure multipliers on a set of macroeconomic factors in a panel setting.

In the first chapter, we investigate the higher-nonlinearity of the relationship between the size of the government expenditure multiplier and the phase of the business cycle. We relax a common in relevant literature assumption that various recessionary periods have similar quantitative implications on the multiplier size. By focusing on the US during the post-World War II period, we estimate historical multipliers that vary along the timeline. We use a time-varying parameter vector autoregression model (TVP-VAR) and prepare the time series using the linear projection method. The first chapter shows that TVP-VAR models can successfully estimate government expenditure multipliers that depend on the business cycle phase. We conclude that government expenditure multipliers are counter-cyclical before the late 1980s and pro-cyclical afterwards. The potency of the discretionary government expenditure in stimulating output declines after the 1980s due to a decrease in the non-defence public consumption multiplier that is more sensitive to the monetary policy stance.

The second chapter constructs a New Keynesian Dynamic Stochastic Gen-

eral Equilibrium (DSGE) model designed to evaluate welfare effects of monetary and public investment policies, as well as to study the relationship between the size of the public consumption multiplier and the formulation of the monetary policy. The optimal policy analysis shows that U.S. historical monetary policy rule did not yield highest possible household welfare. Additionally, we show that an optimised simple public investment rule has only a modest response to past public debt and does not respond to output fluctuations at all. The second part of the analysis seeks to determine the formulation of monetary policy that prompts the highest multiplier values. We show that active inflation targeting and output gap stabilisation policies of the monetary authority diminish the size of public consumption multiplier at all horizons. A short-range of output growth gap targeting policies can effectively increase the multiplier in the long-run.

The third chapter applies the Local Projection method, that controls for the hitherto unnoticed bias and includes government consumption and investment shocks simultaneously, on the sample of 107 emerging and developing economies, making use of the vintage IMF WEO dataset. The necessity to include all the relevant shocks is discussed, concluding that failure to control for relevant policy shocks may result in biased multipliers. Empirical analysis delivers a positive government investment multiplier that is significant even five years after the original fiscal injection and a government consumption multiplier that is only significantly negative in the long-term. Additionally, a set of state-dependent multipliers are computed. Economic slack is associated with higher multiplier values, for both unanticipated government consumption and investment. Rising levels of government debt reduce the potency of government investment stimuli and higher openness to trade magnifies negative output implications of government consumption. Larger public capital stock and size of the public sector tend to diminish the effects of government investment stimulus.

Chapter 1

Introduction

This thesis collects three papers studying the effects of government expenditure on economic activity in the context of modern macroeconomic and macroeconometric modelling frameworks. The main objective is to evaluate the effects of government expenditure on key macroeconomic variables, which contributes to the current academic literature and provides guidance for government policies.

Understanding the implications of fiscal policy interventions in a range of economic settings is imperative to sound macroeconomic policy action. A given fiscal intervention can produce a wide range of outcomes depending on the circumstances in which the fiscal shock takes place. After the outburst of the global financial crisis, policy-oriented academic research concentrated on identification of such circumstances and evaluation of their effects on the potency of fiscal stimulus. The revived interest in the analysis of fiscal policy was driven by an understanding that conventional monetary policy did not constitute an ultimate macroeconomic policy toolkit, that can be effectively used under any circumstances.

Academics and policymakers embarked on a mission to develop guidelines for the use of fiscal policy, focusing on when it was appropriate to use it and when it was not. Such analyses relied on more sophisticated modelling techniques, such as state-dependent macroeconometric models (Auerbach and Gorodnichenko, 2012, 2013a, 2013b, Bachman and Sims, 2012, Bernardini and Peersman, 2018, Ramey and Zubairy, 2018, etc.) and large detailed Dynamic Stochastic General Equilibrium (DSGE) models with “all the bells and whistles” (Christiano, Eichenbaum and Rebello, 2011, Erceg and Linde, 2014, Drautzburg and Uhlig, 2015, Linde and Trabandt, 2019, etc.). Such modelling approaches are often sensitive to the underlying econometric assumptions or choice of features included in DSGE models. This thesis contributes to the literature by challenging common assumptions and extending the models to account for important factors commonly disregarded in recent publications.

The first chapter investigates the relationship between the size of the government expenditure multiplier and the phase of the business cycle. This chapter challenges a common assumption that all post-WWII U.S. recessions are qualitatively similar, or, in other words, are part of the same macroeconometric regime. In the Keynesian school of economic thought, government expenditure is believed to deliver a higher impact on economic growth and unemployment at times when the gross domestic product contracts. Due to a lack of a convincing structural explanation of such a mechanism in the current general equilibrium framework, it is difficult to believe that the mere fact of a recession occurring will always imply a given change in the government expenditure multiplier value. Indeed, it is the change in the propagation of the fiscal injection in the underlying economic system that would drive the value of the multiplier to change during recessions, and economic systems tend to evolve over time.

The U.S. economy experienced various structural changes throughout the post-WWII period—more developed and efficient financial markets, varying monetary policy regimes and rising reliance on the global markets could alter the way fiscal shocks propagate the economy. It is highly unlikely for the fiscal multiplier to change during every recession in the exact same way. State-dependent macroeconometric models, due to the complicated nature of their estimation, are often limited to the set of factors an econometrician is willing to consider. Failure to accommodate third factors or regimes may result in biased conclusions and misleading policy advice.

In contrast to the relevant literature, the first chapter produces evidence of state-dependent government expenditure multiplier without explicitly relying on a pre-defined business cycle indicator. A Time-Varying Parameter Vector Autoregressive model produces a set of time-varying fiscal multipliers that are, essentially, allowed to be driven by an infinite number of unobserved and undefined regimes. Evaluating these multiplier series, we show that the relationship of the fiscal multiplier and the phase of the business cycle is more complicated than that considered in relevant research. Government expenditure multiplier can be pro-cyclical as well as counter-cyclical. This change in state-dependent behaviour seems to be driven by the change in the monetary policy regime.

The second chapter formulates the welfare-maximising simple rules for monetary and public investment policies by constructing a New Keynesian DSGE model that incorporates features crucial to current policy analysis. What should be the goal of the macroeconomic policy? What policy instruments should be used and how? These questions are integral to modern policymaking. Academic literature has long emphasised the usefulness of constructing DSGE models in analysing welfare implications of different formulations of simple policy rules. Indeed, simple rules often perform well in mimicking a more complicated fully optimal Ramsey policy in stabilising the business cycles and boosting household welfare.

Essential features of the policy-oriented DSGE models, such as the presence of non-Ricardian households, relative price and wage distortions, real price and wage rigidities, and distortionary taxes are often omitted entirely or partially from the optimal policy debate. Furthermore, the relevant literature long disregarded the role of public investment in the stabilisation of the business cycle. The second chapter aims to fill this research gap. We investigate if the U.S. historical systematic monetary policy rule delivered the highest possible population welfare in a New Keynesian DSGE model that accommodates all the above-mentioned distortionary features. Additionally, we are interested in comparing the welfare implications of a simple public investment rule that targets debt and output fluctuations and a public investment policy aiming to stimulate future output.

The final chapter of this thesis evaluates the sensitivity of government expenditure multipliers to a set of widely considered in macroeconomic literature factors using a panel of 107 emerging and developing economies. Understanding of how effective is a fiscal stimulus under a given economic profile of a country is crucial for international organisations and local governments in producing sound economic advice and successful market interventions. Factors such as the phase of the business cycle, involvement in international trade and size of the public sector, among many others, are widely considered as capable of altering the outcomes of government expenditure stimulus and, therefore, need to be analysed. Despite the great importance of generating such a fiscal policy framework, the fiscal multiplier literature on emerging and developing economies is scarce.

The lack of quarterly data for a large set of emerging markets created a long-lasting obstacle for conventional identification via multivariate macroeconometric models. Recent developments in econometric modelling deliver a solution to this problem—the impact of the fiscal injections can be evaluated by feeding narrative or pre-defined shocks into the Jorda (2005) Local Projection method. This chapter constructs such a panel of unanticipated government expenditure shocks using the data vintages of the World Economic Outlook database of the International Monetary Fund.

We estimate fiscal multipliers for a large sample of developing economies using annual data. The third chapter produces a set of public consumption and investment multipliers that are allowed to depend on the characteristics of the underlying economy. This chapter considers the level of economic slack, openness to trade, size of the public sector and debt dynamics as economic characteristics capable of affecting the outcome of the government intervention. Finally, we study how failure to include all relevant policy shocks or to account for certain biases can produce misleading results if multipliers are estimated using the Local Projection method.

Chapter 2

Are Government Expenditure Multipliers Indeed Counter-cyclical? A Case of the United States.

2.1 Introduction

Among the standard tools of countering economic recessions, policymakers often consider discretionary public expenditure, especially when the monetary policy fails to help the economy overcome recessionary pressures for various reasons. The major argument for the use of discretionary spending rests on the idea that fiscal policy's potency increases during adverse economic conditions. In this regard, two contrasting views have been predominantly discussed in the recent empirical literature—Auerbach and Gorodnichenko (2012) show that public expenditure multipliers are counter-cyclical in the U.S., while Ramey and Zubairy (2018) contest this view by showing a lack of significant difference between multiplier values and the phases of a business cycle. These prominent views implicitly assume that the relationship between the multiplier's size and the stages of the business cycle remains stable over time. It is imperative to challenge such modelling assumptions in cases where a strong theoretical foundation of an economic relationship is yet to be developed.[1] If the state-dependent nature of the multiplier changes over the course of the U.S. history, then some of the implications of Auerbach and Gorodnichenko (2012) and Ramey and Zubairy (2018) may become misleading.

The behaviour of the fiscal multiplier in response to a recession may be subject to the presence of third factors, which aggregated time series models often fail to incorporate due to their limited capability.[2] Such third factors can facilitate higher non-linearity of the relationship that, if not explicitly estimated by the researcher, can remain unanalysed. For example, Ramey and Zubairy (2018) focus on the average effect of a recession on the output response to a discretionary government expenditure shock. If the fiscal multiplier declines during some recessions and rises during the other, then their study's setup may fail to uncover any statistically significant difference between recessions and expansions. If this is the case, then a more flexible model would be required to determine the true fiscal multiplier's dynamics.

This study shows that the implementation of a modelling approach that allows the relationship between the multiplier and the business cycle phase to change over time provides multiplier estimates that do not consistently move in the same direction during each recession. Moreover, based on our results, we can divide the post-World War II (WWII) period into two subperiods—the period between 1949 and late 1980s, wherein the fiscal expenditure multiplier is counter-cyclical, as in Auerbach and Gorodnichenko (2012), and the subsequent pro-cyclical period. This offers a scope for a monetary explanation since the pattern of the fiscal multiplier's dynamics changes around the implementation of the inflation-targeting policy by Paul Volcker's Federal Reserve. Since the size of the fiscal multiplier can be affected through the expected inflation channel of the fiscal transmission mechanism and the implementation of inflation-targeting policies can change the way market agents

[1]Although Keynesian reasoning produces higher multipliers during economic recessions, schools following the neoclassical synthesis have not developed a model capable of replicating this relationship using the dynamic stochastic general equilibrium framework.

[2]Even though regime-switching models are often used to capture such non-linearities, they are limited by the number of regimes an econometrician chooses to focus.

form expectations about future inflation, we believe that there is a scope for future research in this direction.

To the best of our knowledge, this is the first study to present government expenditure multipliers that depend on the phase of a business cycle, using a time-varying parameter vector autoregression (TVP-VAR) framework.[3] Methodologically, our contribution consists of combining a version of the TVP-VAR model proposed by Belmonte, Koop, and Korobolis (2014), an approach that governs the degree to which model parameters are allowed to vary in time; an identification strategy combining short-term zero- and sign-restrictions; and a novel method to detrend the data. The choice of the model allows us to follow the reasoning of Blanchard and Perotti (2002) in extending the order of the VAR lag polynomial to include four lags.[4] Importantly, we estimate the model on stationary series in levels, and thus avoid the rescaling bias pointed out in Ramey and Zubairy (2018). We compute the stationary time series using Hamilton's (2018) linear projection method.

The paper proceeds as follows. Section 2.2 presents a brief overview of the literature that applies the state-dependent and TVP-VAR models to estimate the effects of government expenditure on output. Section 2.3 presents the methodology, and Section 2.4 discusses the identification strategy and data. Section 2.5 presents the time-varying government expenditure multipliers and discusses dependence on the stage of the business cycle. Section 2.6 estimates multipliers in a framework, which acknowledges that policy actions can be anticipated by market agents. Section 2.7 computes multipliers for the components of government expenditure and elaborates why the shift in the multiplier behaviour after the 1980s may be a consequence of inflation-

[3] The model class is believed to be incapable of producing multipliers that depend on the phase of the business cycle (Ramey 2011a; Auerbach and Gorodnichenko 2012). Results of Kirchner, Cimadomo, and Hauptmeier (2010), Pereira and Lopes (2014), and Berg (2015) support this notion. Our results prove that the failure to uncover such a relationship is mostly driven by their modelling choices.

[4] Existing contributions estimate TVP-VAR models of order two. A working paper by Iiboshi and Iwata (2017) that also extends the lag order to 4 and applies identification similar to ours emerged when this study was under development. Authors detrend variables prior to estimation, assuming linear and quadratic trends, and do not focus on the state-dependent nature of the multiplier.

targeting policies. Conclusion and Appendices follow in Section 3.6.

2.2 Literature overview

The theoretical and empirical literature on fiscal multiplier is extensive. As we mainly focus on the empirical part of the debate, the reader is referred to Ramey (2011a) for a more detailed overview of the theoretical contributions. The empirical part of the literature can be broadly divided into three model classes.[5]

First, models disregarding the state-dependent nature of the multiplier (e.g. Ramey and Shapiro 1998; Blanchard and Perotti 2002; Mountford and Uhlig 2005; Ramey 2011b; Barro and Redlick 2011) dominated the debate before the early 2010s. These models focused on the estimation of an average multiplier value, which proved to be misleading in certain applications.[6] Since we are interested in analysing the state-dependent nature of the multiplier, we will refrain from focusing on this model class further.

The second category focused on the state-dependent nature of the fiscal policy's impact. This branch of literature is pioneered by the smooth transition VAR approach of Auerbach and Gorodnichenko (2012). The baseline analysis estimates a set of multipliers for various types of spending, allowing the output response to depend on the business cycle stage. The study shows the U.S. government expenditure multipliers to be as low as -0.33 and as high as 2.24 during expansions and recessions, respectively. Using a non-linear VAR setup, Bachmann and Sims (2012) support higher multipliers during the U.S. economic recessions. Some of the more recent contributions adopt modifications of Jorda's (2005) local projection method for calculating multipliers that depend on an economic state. Using a local projection instrumental variable (LP-IV) model, Ramey and Zubairy (2018) conclude that dependence of the government expenditure multiplier on a business cycle

[5]In this literature overview, we disregard modelling approaches used before Lucas's (1976) and Sim's (1980) critiques.

[6]For example, policy recommendations based on an average multiplier value can potentially harm an economy if it is experiencing a recession.

stage does not hold for various alternative models, shocks, and state specifications.[7] Bernardini and Peersman (2018) extend the model to accommodate more than two states simultaneously. Similar to Ramey and Zubairy (2018), the estimation results offer mixed evidence. Both studies produce state-dependent multipliers in some model specifications. All the studies in this branch of literature have one feature in common—the relationship between the value of the multiplier and the business cycle stage is assumed to have the same sign for the entire underlying sample of years. These models aim at computing an average change in the multiplier value once the economy rolls into a recession. Such a setup can produce misleading results if the relationship is conditional on the presence of third factors, omitted from the model. Similarly, if the nature of the relationship does not remain steady throughout the entire time sample, such averaged estimates can be misleading.

The third class of models capable of capturing such a non-linearity focused on the use of time-varying parameter (TVP) models. This class of models first appeared in the studies by Canova (1993), Sims (1993), and Cogley and Sargent (2003); the modelling framework is further modified by Primiceri (2005), who set up a model with minimum restrictions and both time-varying lag polynomial and time-varying variance-covariance matrix. Primiceri (Ibid.) argues that the methodology captures various non-linearities across time, without depending on excessive structural modelling assumptions, as a specific number of states or by their definition. Such models allow for endogenous changes in the effects of interest without imposing the assumption that these effects remain qualitatively similar. This property makes the TVP framework suitable to determine whether the relationship between the multiplier value and the business cycle stage remained the same over the course of the U.S. history.[8] Although this method gained popu-

[7]The study by Auerbach and Gorodnichenko (2013) first applied the LP method to a fiscal multiplier debate. Nonetheless, the study by Ramey and Zubairy (2018) first used the method on the U.S. data.

[8]Even though it is hard to disentangle the effect of any single state, these models facilitate the estimation of historical multiplier values and assess the effects of the states ex-post. In other words, if the state affects the multiplier value, we expect to observe such a change in the absence of explicit modelling choices or ad hoc definitions of the state.

larity in the macroeconomic literature in the last 20 years, it was recently introduced to the fiscal debate and is still in its infancy.

There are three primary studies on the time-varying government expenditure multiplier—Kirchner, Cimadomo, and Hauptmeier (2010); Pereira and Lopes (2014), and Berg (2015). However, they failed to uncover a relationship between the government expenditure multiplier and the stage of the business cycle.[9] Kirchner, Cimadomo, and Hauptmeier (2010), which, to the best of our knowledge, is the first study to introduce the Bayesian TVP-VAR analysis into the fiscal debate, estimated the model for the European Union (EU). The study concludes that short-run fiscal multipliers increased until the late 1980s, reaching values above unity, and subsequently decreased to 0.5 by the end of the sample. Since the 1980s, the long-run multiplier experienced an even sharper decline. The study did not analyse the dependence of the multiplier value on the stage of the business cycle. Pereira and Lopes (2014) focus on computing time-varying effects of the fiscal policy for the U.S.[10] The study concludes a small degree of time variation in the output response to a discretionary government expenditure shock. Furthermore, the study focuses on estimating elasticities and does not extend the analysis to incorporate multiplier calculation. Berg (2015) also concludes that the potency of fiscal policy declined by the end of the sample in the case of Germany. The estimated multiplier does not demonstrate any dependence on the stage of the business cycle.

Based on results of Kirchner, Cimadomo, and Hauptmeier (2010) and Pereira and Lopes (2014), Ramey (2011a) and Auerbach and Gorodichenko (2012) infer that existing applications of the TVP-VAR methodology have failed to uncover the state-dependent nature of the fiscal multiplier. Unlike this view, our results provide a basis to question whether the failure to uncover the state-dependent nature of the expenditure multiplier is driven by

[9] Eisenstat, Chan, and Strachan (2016) also estimate a TVP-VECM following Blanchard and Perotti (2002), but their contribution mainly lies in modifying the model of Belmonte, Koop, and Korobilis (2014) with alternative prior definition. Their analysis of the U.S. fiscal policy is purely illustrative of their approach.

[10] Unlike our approach, the study uses a non-recursive identification scheme and removes a simple linear trend.

the modelling choices adopted in related studies. Extending the lag polynomial to the fourth order and detrending data using Hamilton's (2018) linear projection method, we present state-dependent fiscal multipliers in a TVP-VAR framework. In this regard, this study fills two important gaps in the literature. First, to the best of our knowledge, it is the first study that succeeds in estimating government expenditure multipliers that depend on the business cycle stage. Second, our approach allows us to investigate whether the state-dependent nature of the public expenditure multiplier remains the same throughout the post-WWII period in the U.S. Therefore, we evaluate the correctness of the prevailing set of modelling strategies analysing the state-dependence of the fiscal multiplier.

2.3 Methodology

This study uses the TVP-VAR model to calculate a series of time-varying fiscal multipliers. Building on the spirit of Fruhwirth-Schnatter and Wagner (2010) and Belmonte, Koop, and Korobolis (2014), the model implements a decision mechanism controlling the degree of the model parameters' time-variation. Following Eisenstat, Chan, and Strachan (2016), this study uses a Tobit prior on the indicators governing the degree of time-variation of the VAR parameters and a Lasso prior on their respective variance. This approach boosts the performance of the stochastic model specification search for large TVP-VAR models while adding the appealing feature of shrinking the model to a more stable time-invariant VAR. Particularly, the latter feature is useful since it allows us to extend the size of our model successfully to include all four lags of the fiscal variables. Although the methodology allows for both a time-varying lag polynomial and a time-varying variance-covariance matrix, it penalises over-parameterisation. Allowing the model parameters to follow random walks ensures a great degree of non-linearity in the parameter transition along the timeline.[11]The following system of equa-

[11]The VAR's time-varying parameters adhere to random walk laws of motion. This modelling choice can be considered a strong assumption. Nonetheless, estimating a time-invariant version of the model on a rolling sample of 100 observations proves it to be a

tions describes the econometric model:

$$Y_t = X_t\beta_t + \Sigma_t u_t, \qquad u_t \sim N(0, I_n) \tag{2.3.1}$$

$$\beta_t = \beta_{t-1} + v_t \qquad v_t \sim N(0, \Omega) \tag{2.3.2}$$

$$log(\sigma_t) = log(\sigma_{t-1}) + \theta_t \qquad \theta_t \sim N(0, W) \tag{2.3.3}$$

where Y_t is an $n \times 1$ vector of observed endogenous variables, X_t is an $n \times m$ matrix of observations on explanatory variables (both endogenous and exogenous variable vectors, their lags, and some contemporaneous elements of Y_t), u_t denotes structural shocks, and Σ_t is a diagonal matrix containing standard deviations of the structural shocks. β_t is a vector containing all coefficients of X_t, and $log(\sigma_t)$ is a vector containing logs of all the non-zero elements of a diagonal matrix Σ_t. Ω and W are the variance-covariance matrices of the disturbances from the parameter laws of motion. At this point, it is crucial to acknowledge that $\Omega = \tilde{\Omega}^{\frac{1}{2}}\Phi\Phi'\tilde{\Omega}^{\frac{1}{2}\prime}$, where $\tilde{\Omega}^{\frac{1}{2}} = diag(\omega_1, \ldots, \omega_m)$ contains the indicators used to access the degree of time-variance and Φ is a lower-unitriangular matrix. The reader is referred to Appendix 2.11, Primiceri (2005), and Eisenstat, Chan, and Strachan (2016) for a more detailed explanation of the setup.

The set of equations 2.3.1, 2.3.2, and 2.3.3 pins down the problem under analysis, which is solved using Bayesian techniques.[12] As the joint posterior density is unknown, parameters of interest are sampled iteratively from conditional densities using the Gibb's sampler. Each set of the presented results is based on 150,000 iterations of the Gibb's sampler; the procedure is sensitive to initial values and is subject to the autocorrelation of the sampled draws. Therefore, in each case, a burn-in period of 100,000 is eliminated; subsequently, every 25th draw of the remainder is used to approximate the posterior density function. As in Primiceri (2005), $Var([u_t \;\; v_t \;\; \theta_t]')$ matrix

reasonable assumption. All the resulting autoregressive coefficients follow either a random walk or an AR(1) process with a coefficient close to 1.

[12]The Bayesian treatment of the problem, as argued in Cogley and Sargent (2005), allows the treatment of coefficients as random variables and alienates the method from the Lucas (1976) critique.

is assumed to be diagonal.

The Bayesian estimation also requires a set of prior distributions. Considering data limitations and to preserve comparability between different subsample estimations, we deviate from the commonly accepted routine of constructing priors based on the pre-sample estimation. Conversely, we assume uninformative priors, as in Eisenstat, Chan, and Strachan (2016).[13]

The estimation gives a set of VAR parameters for every period t under analysis, which is later used to compute impulse response functions (IRFs). Given that the calculated posterior distributions are ergodic, the IRFs, being functions of the estimated parameters, are calculated separately for every parameter draw. Accordingly, the joint set of IRFs represent the IRF distribution. In the same manner, posterior distributions of the multipliers are obtained; this approach presents a straightforward way of significance assessment. The analysis produces two multiplier types:

- Cumulative Multiplier:

$$K_t^{sum} = \frac{\Sigma_{h=0}^{H} f_{t+h}^{y}}{\Sigma_{h=0}^{H} f_{t+h}^{g}} \tag{2.3.4}$$

- Impact Multiplier:

$$K_t^{imp} = f_t^y \tag{2.3.5}$$

where f_{t+h}^y refers to the output response at horizon h to a fiscal shock taking place at time t. f_{t+h}^g refers to the fiscal variable counterpart.

The cumulative multiplier (2.3.4) is assumed to be equal to the integral of the output response divided by the integral of the government expenditure response—they comprise responses to a discretionary government expenditure shock. As argued in Ramey and Zubairy (2018), this definition of the multiplier tends to provide lower multiplier values than alternative definitions.

[13]This choice comes at the cost of broader confidence bands. Nonetheless, as can be seen in Section 2.5, we conclude that the multiplier series are subject to two chronologically ordered regimes. By using uninformative priors, we avoid imposing a prior distribution constructed using data from the first regime on the posterior distribution of the model coefficients in the second one.

2.4 Identification Strategy and Data

This study proposes an identification scheme that combines sign restrictions with short-term zero restrictions, following Rubio-Ramirez, Waggoner, and Zha (2010). The actual implementation of the routine follows the strategy outlined by Binning (2013). Short-term zero restrictions are based on the reasoning of Blanchard and Perotti (2002); additionally, the implementation of sign restrictions evades an important pitfall of Cholesky factorisation in VAR models including tax revenues as an endogenous variable.[14] The proposed identification strategy can be summed up by:

$$Z_t = \begin{array}{c} \\ G_0 \\ NT_0 \\ Y_0 \end{array} \begin{array}{c} \begin{array}{ccc} \varepsilon_t^G & \varepsilon_t^{NT} & \varepsilon_t^Y \end{array} \\ \begin{pmatrix} + & 0 & 0 \\ \times & + & \times \\ \times & - & + \end{pmatrix} \end{array} \tag{2.4.1}$$

where ε_t^G refers to the structural government expenditure shock, ε_t^T to the structural tax shock, and ε_t^Y to the structural output shock.

Identification scheme (2.6) combines the assumption of the lagged discretionary fiscal policy response, as in Blanchard and Perotti (2002), and assigns a minimum amount of sign restrictions necessary for identification. Under the modelling choice, the output shock is allowed to have an immediate positive effect on net taxes, and a net taxes shock lowers the output contemporaneously. In other words, the latter assumption draws an equivalence between an increase in both net taxes as well as marginal tax rates, which only holds if the fiscal policy stance remains at the uphill of the Laffer curve.[15] Appendix 2.10.1 presents a more detailed explanation of the identification strategy.

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[14] Appendix 2.10 provides an elaborate explanation of how such a pitfall can arise in a fiscal VAR framework.

[15] This assumption choice is supported by results of Trabandt and Uhlig (2011) for the 1995–2007 period; for this period, authors conclude that, on an average, both labour and capital tax rates for the U.S. were below levels, prompting maximum tax revenues. Blinder (1981) also supports the argument that the U.S. tax burden, in a broad sense, is unlikely to be on the downhill of the Laffer curve.

The TVP-VAR model includes a joint measure of government investment and consumption (G_t) (referred to as government expenditure in the remainder of the study), tax revenue net of transfer payments (NT_t), and GDP (Y_t). All variables are entered in real per capita terms to control for the effects of population growth and the nominal effects of inflation. There are two major approaches to data preparation in the fiscal VAR literature. In the first approach, Mountford and Uhlig (2005); Kirchner, Cimadomo, and Hauptmeier (2010); Auerbach and Gorodnichenko (2012); and Bachman and Sims (2012) make use of data in levels to preserve the long-run relationships between government expenditure, taxes, and output.[16] The second approach, used by Blanchard and Perotti (2002), Pereira and Lopes (2014), and Berg (2015), emphasises the need to estimate the multivariate model on stationary data, and thus removes the trend prior to estimation or explicitly controls for the trend in the model.

On the one hand, GDP, government expenditure, and net taxes are non-stationary time series, and hence estimations in levels can produce spurious results. On the other hand, detrending data can eliminate important information contained in the series and affect our results. Therefore, the primary objective in data preparation is to develop a stationary transformation that will not remove the part of variation that we want to analyse. This study introduces the Hamilton (2018) linear projection routine as an instrument to obtain such a stationary series for the fiscal VAR analysis. Hamilton's (2018) procedure produces stationary cyclical components of macroeconomic series centred around zero. Similar to the Hodrick and Prescott's (1997) filter, it produces stationary series in levels.[17] Since the resulting stationary series enter the model in levels, estimated multipliers are not subject to the rescaling bias outlined by Ramey and Zubairy (2018). Therefore, the TVP-VAR

[16] Auerbach and Gorodnichenko (2012) do mention that, in the presence of non-stationarity, a properly constructed VECM will be superior to their modelling strategy. Nonetheless, it is not straightforward to justify a specific cointegration vector and performing the analysis in first differences does not produce results in line with the theory or overall empirical consensus on the topic.

[17] Unlike the Hodrick and Prescott's (1997) filter, it does not create artificial correlations that are not present in the true data generation process (Hamilton, 2018).

model estimated on cyclical components via Hamilton's (2018) linear projection method directly produces multipliers, not elasticities. Further details on the linear projection method are provided in Appendix 2.9.1. Figure 2.4.1 presents the stationary series along with the recession dates identified by the National Bureau of Economic Research (NBER).

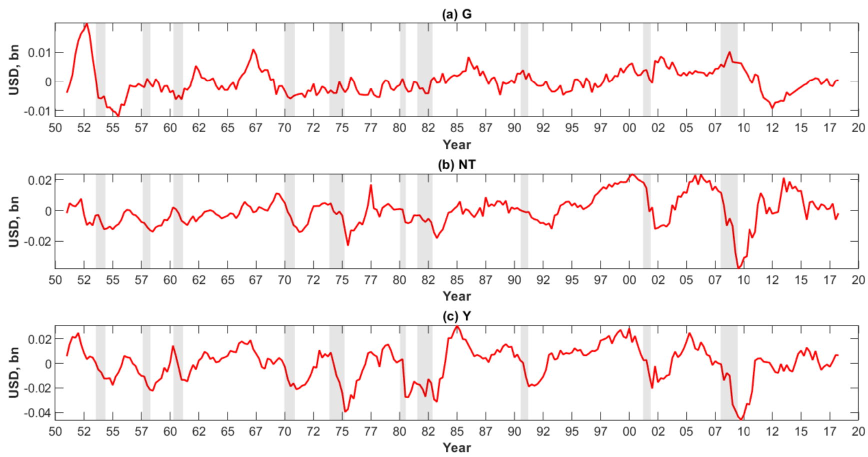
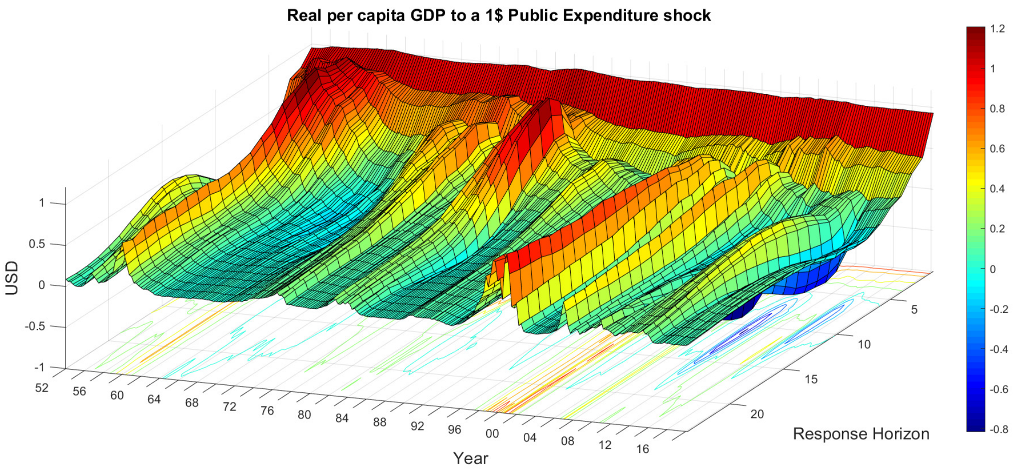


Figure 2.4.1: Stationary Transformations of the real per capita government expensiture (a) teaxes net of transfers (b) GDP (c) over NBER recession dtaes: Obtained via Hamilton's linear projection method.

Data are obtained from the Bureau of Economic Analysis and the Federal Reserve Economic Database. Fiscal variables and GDP are taken from the latest release of the national income and product accounts' tables. Net taxes follow the definition of Blanchard and Perotti (2002); detailed definitions are presented in Appendix 2.9. The time series enter the model in levels of real per capita terms, detrended via the Hamilton (2018) linear projection technique. We run the baseline TVP-VAR model on the 1948Q1–2018Q2 sample, assuming four lags and no intercept terms.

2.5 Government expenditure during economic slack

Estimation of the baseline model provides several notable results. First, the TVP-VAR framework estimates state-dependent IRFs. Second, we observe a loss in the potency of discretionary government expenditure as a tool for stimulating economic activity at the end of President Ronald Reagan's term. Third, the state-dependent relationship experiences a structural break during the 1980s: NBER recessions are characterised by local peaks in the multiplier values before the late 1980s, and the relationship is inverted in the subsequent period. Additionally, we show that, until 1990, interest rate spreads perform well in predicting future shifts in the multiplier value. These results highlight the need to make a careful consideration of the fiscal policy's relationship with the business cycle.



Note: Output response is measured in real U.S. dollars.

Figure 2.5.1: Median output response to an U.S.$1 government expenditure shock as function of time: obtained via a mixture of sign and zero restrictions.

Figure 2.5.1 shows that the time-varying responses of output to a government expenditure shock exhibit state-dependent behaviour.[18] Our approach

[18]In order to maintain clarity, this section focuses on responses of output and resulting multipliers. Similar to Pereira and Lopes (2014), we focus on the median IRFs to minimise

provides a substantial degree of time variation along the post-WWII timeline in the U.S. Unlike Pereira and Lopes (2014), our estimates depend on the stage of the business cycle. We also observe a significant change in both the shapes and magnitudes of output responses before and after the late 1980s: negative output responses to discretionary government expenditure shocks emerge during and after the global financial crisis.

Most of the heterogeneity in the IRFs arises in the distant horizons. The output response on impact remains close to unity for the entire sample of years. Concerning the immediate impact, the government expenditure shocks have a simple Keynesian accounting effect: an extra U.S.$1 worth of government expenditure increases the aggregate demand by roughly the same amount in the same quarter. The various crowding in and crowding out effects occur in the medium- to long-term. Having said that, we want to see if these shifts in the long-term effects are conditional on the stage of the business cycle.

Multiplier	Average	Min		Max	
		date	value	date	value
Impact	0.97***	2017Q4	0.93***	1960Q4	1.03***
Sum (1-year)	0.89**	2011Q1	0.65*	1958Q1	1.38**
Sum (2-year)	0.83	2014Q4	0.05	1982Q4	2.23*
Sum (4-year)	0.78	2009Q2	-0.64	1958Q1	2.29*
Sum (5-year)	0.77	2014Q4	-0.33	1958Q1	2.35*
* - p<0.32, ** - p<0.1, *** - p<0.05					

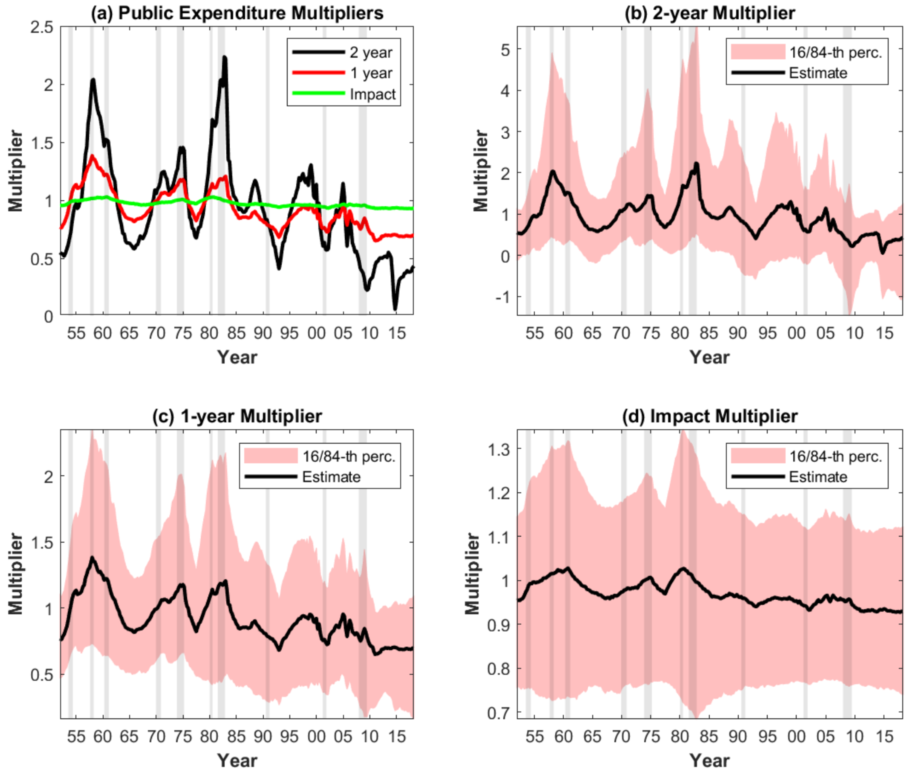
Table 2.5.1: Descriptive statistics for the estimated multiplier series.

Focusing on the multiplier measures, we can see that they do indeed depend on the stage of the business cycle. Table 2.5.1 computes the average, minimum, and maximum values for every multiplier type estimated using the baseline model. The maximum multiplier value is always observed during a recession (as defined by NBER). On the one hand, all these maximum multipliers occur during recessions before the late 1980s. On the other hand,

the effect of occasionally unstable draws. The remaining IRFs can be found in Appendix 2.12

the 4-year cumulative multiplier falls to its minimum value during the global financial crisis. Plotting the estimated multipliers over time sheds further light on how this relationship transformed.

Figure 2.5.2 presents the estimated path of the impact and cumulative government expenditure multipliers; the choice of the confidence bands follows the general pattern in the TVP-VAR literature. All the three-multiplier series demonstrate a pronounced decrease after the 1980s. The 1-year cumulative multiplier almost exclusively falls below the impact value after 1985. Focusing on the 2-year multiplier, it is evident that more long-term crowding out takes place at the same time; discretionary government expenditure seems to be especially harmful in the aftermath of the global financial crisis. In line with Kirchner, Cimadomo, and Hauptmeier (2010); Pereira and Lopes (2014); and Berg (2015), we conclude that the potential of government expenditure in stimulating output fell sharply after the 1980s.



Note: Median multiplier values are presented in figure (a). The rest of the figures contain multipliers along with respective confidence bands, for 2-year (b) and 1-year (c) cumulative multipliers as well as the impact multiplier (d). Confidence bands are in red, calculated as 16th and 84th percentiles of the posterior multiplier distributions.

Figure 2.5.2: Public expenditure multipliers over NBER recession dates.

The baseline model successfully estimates the multiplier series that are higher during recessions; however, the estimated relationship has certain limitations. First, the multiplier is only higher during recessions before the late 1980s. The 2-year cumulative multiplier series demonstrate spikes around the recessions of 1957–58, 1974–75, and 1981–82. Second, after the 1980s, the relationship changes its direction. During the two most recent U.S. recessions, the 2-year cumulative government expenditure multiplier witnesses a decline. This evidence supports our hypothesis that the mixed results presented by Ramey and Zubairy (2018) may not be attributed to the fact that fiscal policy does not robustly depend on the stage of the business cycle but to the fact that such a relationship may not be constant over time. We conclude that the post-WWII period in the U.S. can be divided into two parts. The first part stretches from the beginning of our sample to the late 1980s, while the second proceeds until the modern-day. The relationship between the stage of the business cycle and the fiscal multiplier exhibits opposite signs in these two periods.

We further confirm that the estimated multiplier values depend on the stage of the business cycle before the late 1980s, using a forward-looking indicator of economic slack. Spreads between short-term and long-term interest rates have been widely considered an early-stage predictor of the U.S. recessions. Such spread represents a mix of market sentiments regarding the future path of the economy. We seek to determine if the spreads between the federal funds rate and yields on bonds of different maturities can predict future multiplier values. We obtain five interest rate spreads from the federal reserve economic database—(a) 10-year treasury constant maturity minus federal funds rate, (b) 5-year treasury constant maturity minus federal funds rate, (c) 1-year treasury constant maturity minus federal funds rate, (d) 6-month treasury bill minus federal funds rate, and (e) 3-month treasury bill minus federal funds rate.

We project the multiplier value estimated in our baseline estimation on the eight lags of these spreads. Given that the TVP-VAR coefficients are modelled as random walks, we estimate the linear regression in first differences. A modelling choice distinguishes our study from both Kirchner, Cimadomo, and Hauptmaier (2010) and Berg (2015) that perform a similar ex post estimation in levels. We can see in Figure 2.5.3 that lags of yield spreads perform well in predicting the change in the 2-year cumulative multiplier before 1990. Nonetheless, this relationship seems negligible in the subsequent period. The fit improves if we limit estimation to observations before 1990.[19] Based on these results, we further support the idea that the expenditure multiplier is counter-cyclical before the late 1980s. The relation-

[19]This result is robust to the case of the multiplier calculated using unanticipated discretionary shocks considered in the next section.

ship between the multiplier and the stage of the business cycle experiences a structural break around 1990. We consider how an interaction with the monetary policy may have caused this structural break in Section 2.7.2.

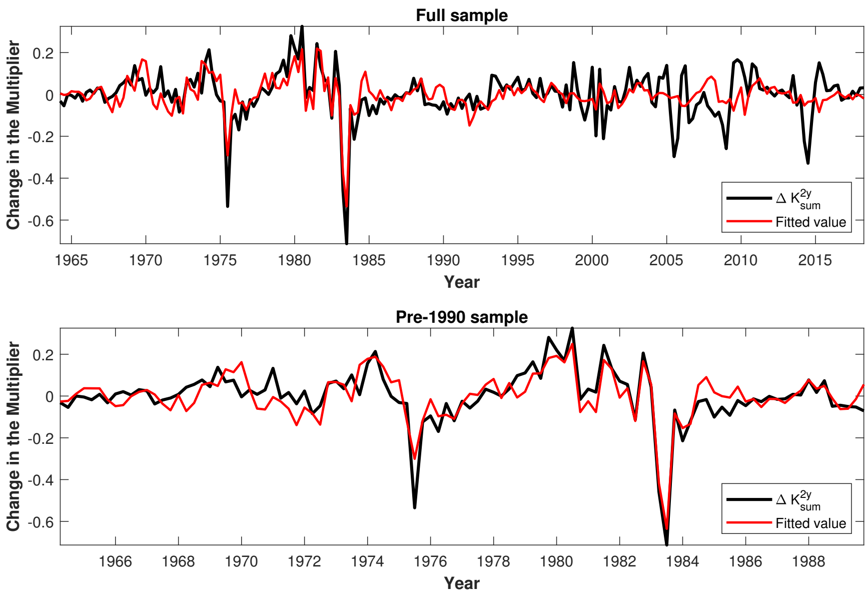


Figure 2.5.3: Predictability of changes in the 2-year cumulative multiplier value by the interest rate spreads.

2.6 Structural shocks and policy anticipation

The results in the previous section should be interpreted with caution. As has been widely discussed in the recent literature, the information set available to an econometrician may be smaller than that available to a representative market agent. If that is the case, then the structural shocks estimated by a TVP-VAR model on such limited information sets should be anticipated by market agents in advance. Ramey (2011) provides evidence that shocks identified using conventional VAR methods do not consider surprise discretionary expenditures; the author concludes that professional forecasts of government expenditure and war dates Granger-cause VAR structural shocks. Mertens and Ravn (2010) investigate the econometric side of the issue, arguing that if structural shocks are anticipated by the market agents, then the moving average representation of the VAR may have non-fundamental roots—roots inside the unit circle. Structural shocks identified by such models will represent a linear combination of the true past and future structural shocks, leading to biased IRFs.

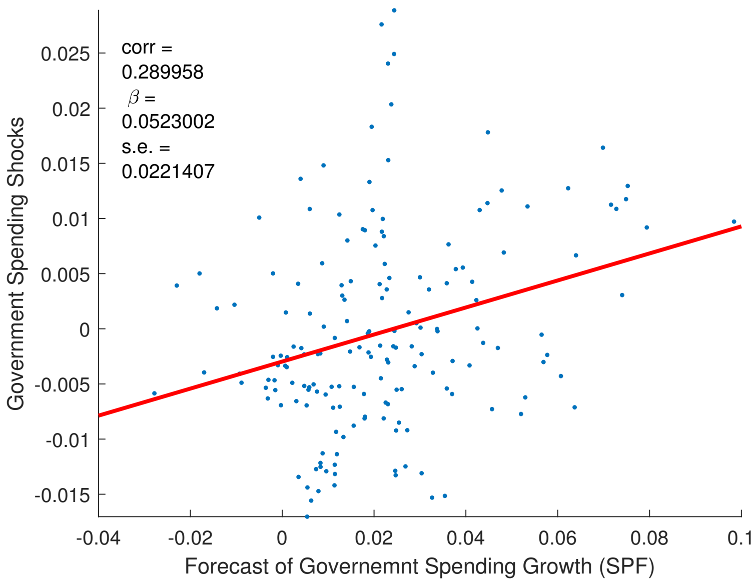


Figure 2.6.1: Forecastibility of VAR shocks to government spending.

Figure 2.6.1 presents the scatter plot of the structural shocks identified by our baseline approach and the forecasts of government expenditure growth obtained from the Survey of Professional Forecasters. Based on the estimated slope coefficient, we can conclude that shocks identified by our model are anticipated by professional forecasters, at least in the previous quarter. Nonetheless, according to the R^2 value of 0.0408, forecasters can only anticipate a small fraction of the variation in our structural shocks.[20] In light of the above, we want to investigate how the estimated multiplier series will change once we account for the timing issue.

[20]Mertens and Ravn (2010) argue that the severity of the anticipation issue in estimating conventional VARs decreases with the decreasing ability of the market agent to anticipate future shocks.

Two major approaches allow accounting for the shock anticipation issue. The first approach uses a narrative shock series, which are unanticipated by construction. In the government expenditure multiplier literature, prominent examples include the Ramey-Shapiro military build-up dates (Ramey and Shapiro, 1998) and military news of Ramey (2011b), and Ramey and Zubairy (2018). The second approach uses professional forecasts to purge the anticipated component out of the shock. Examples include the extensions of Auerbach and Gorodnichenko's (2012) ST-VAR model, expectations-augmented TVP-VAR of Berg (2015), unanticipated shock measures of Auerbach and Gorodnichenko (2013); Abiad, Furceri, and Topalova (2016); Arizala et al. (2017), Furceri and Li (2017), and Furceri et al. (2018). The first approach provides substantial advantages while incurring substantial costs. It is difficult to come across or construct a good narrative shock measure, and these shocks are often of very specific nature. By way of example, Ramey-Shapiro dates or the military news of Ramey and Zubairy comprise measures of military spending shocks. Multipliers calculated for the unanticipated discretionary military expenditure may not necessarily be a good measure of potency for other types of government expenditures or the overall government expenditure. In this section, we follow the approach of Auerbach and Gorodnichenko (2012) and Berg (2015) for extending our TVP-VAR model to include professional forecasts.

Our approach is different from Auerbach and Gorodnichenko (2012) and Berg (2015) in that we do not investigate the dynamic effects of expectations in the context of our model. Therefore, we do not include professional forecasts in the endogenous vector of variables. Conversely, we aim at only purging our structural shocks from the anticipated component, and therefore control only for the current period forecast made in the previous quarter. This modelling choice is a result of a necessary compromise between maintaining the validity of the model, in light of the arguments of Mertens and Ravn (2010) and Ramey (2011), and the additional computational burden owing to an extension in the size of the TVP-VAR model. As discussed in Auerbach and Gorodnichenko (2012), the Survey of Professional Forecasters was subjected to numerous revisions; hence, we follow the authors by using forecasted growth rates instead of levels. Given the limited sample of observations and the high parametrisation of our TVP-VAR, we choose to exclude forecasts of government revenues, owing to the unavailability of measure for most of our sample.[21] We also choose to exclude forecasts for the output as we believe that market agents have a limited ability to anticipate output shocks compared to discretionary fiscal policy shocks. Thus, the validity of our model will not be affected by such an omission.

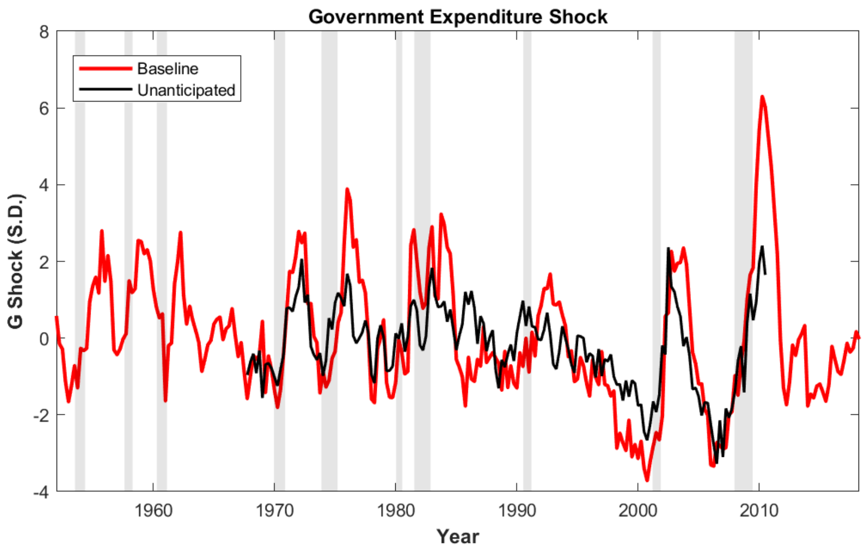


Figure 2.6.2: Government expenditure shocks identified by the baseline model (red) and the extension that controls for the professional forecasts (black), over NBER recession dates.

In order to calculate government expenditure multipliers to an unanticipated government expenditure shock, we extend our baseline model to include a forecast of government expenditure growth made in the previous quarter ($\Delta G^F_{t|t-1}$) as an exogenous control.[22] In doing so, we purge our government expenditure shocks from the anticipated component and arrive at

[21]The longest forecast series for government revenues are available from the University of Michigan's research seminar on quantitative economics model, and these only start at 1982.

[22]We do not include lags of this exogenous control; this is because all the information contained in those lags, which may be useful in predicting our current structural shock, would have been incorporated in the latest forecast.

a model specification that satisfies the criticism of both Ramey (2011) and Mertens and Ravn (2010). At this point, it should be mentioned that the inclusion of forecasts in the TVP-VAR will lower the significance of our results. We expect most of the predicting power of our endogenous lag polynomial to be mirrored in the professional forecast. We estimate this extension on the same sample as in Auerbach and Gorodnichenko (2012)—1966Q4—2010Q3. This approach presents structural shocks that cannot be predicted by professional forecasts. Figure 2.6.2 presents the structural government expenditure shocks from our baseline along with shocks from the model extension controlling for professional forecasts.

Multiplier	Average	Min		Max	
		date	value	date	value
Impact	1.23***	1984Q2	1.09*	1975Q2	1.37***
Sum (1-year)	0.36	1987Q1	0.02	1975Q1	0.80
Sum (2-year)	0.29	2009Q1	-0.57	1975Q1	0.95
Sum (4-year)	0.36	2008Q4	-0.57	1974Q4	0.85
Sum (5-year)	0.32	2007Q2	-0.39	1975Q1	0.81
* - p<0.32, ** - p<0.1, *** - p<0.05					

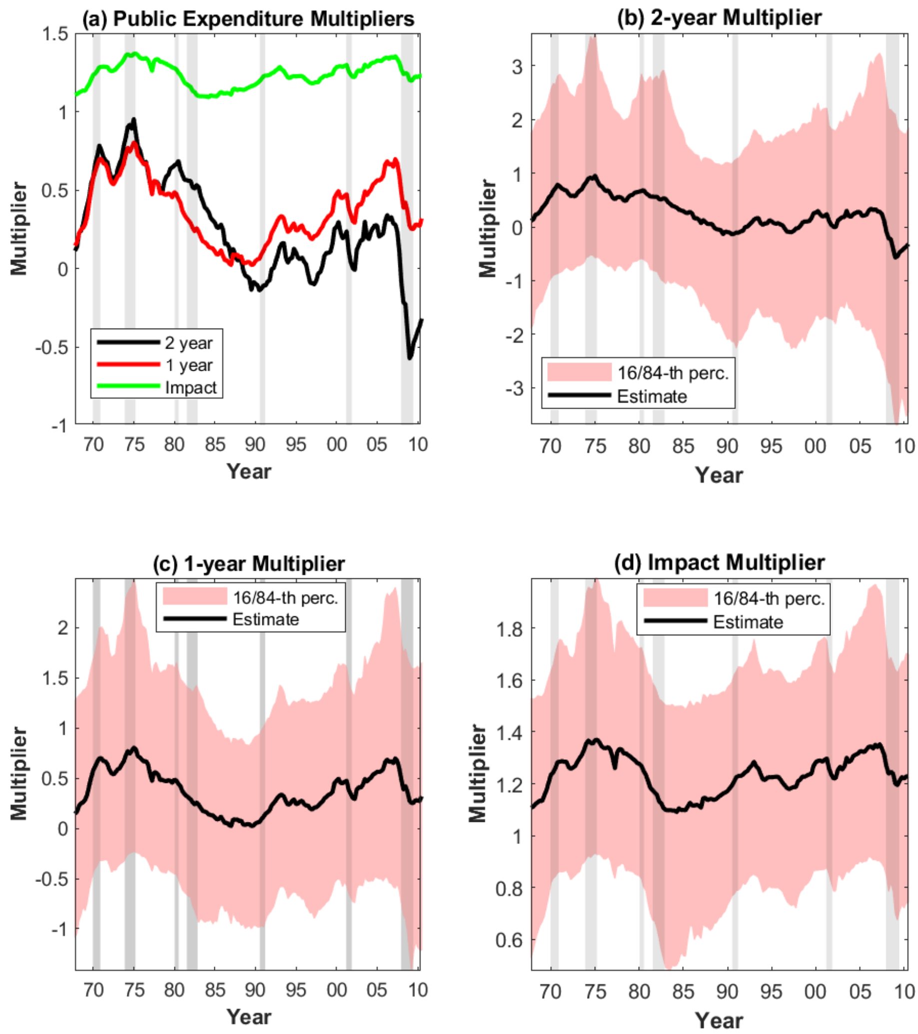
Table 2.6.1: Descriptive statistics for the estimated multiplier series: The case of Unanticipated discretionary shocks.

Controlling for professional forecasts alters our baseline results in the following three ways. First, there is an increase in impact multipliers; an unanticipated U.S.$1 discretionary government expenditure shock increases output by U.S.$1.23, on an average. Second, all cumulative multipliers are lower than those in our baseline.[23] Third, as is evident from Table 2.6.1, maximum and minimum values of the estimated multipliers shift to different dates. Nonetheless, our main result holds. The largest multiplier values are observed during the first part of the sample (before the late 1980s), and

[23]Since the cumulative multipliers is a function of the estimated lag polynomial, it is hard to identify what drives such a decrease. On the one hand, the inclusion of professional forecasts could alter the coefficients of the endogenous lag polynomial, since both of them capture similar effects. On the other hand, unanticipated discretionary shocks can, in principle, result in higher crowding out.

the values of the 2-year, 4-year, and 5-year cumulative multipliers reach the lowest level around the global financial crisis.

Figure 2.6.3 presents the dynamics of the estimated multipliers calculated using this extension. The shift in the state-dependent relationship is also evident in the case of the extension model. The 1-year and 2-year multiplier series exhibit local peaks coinciding with recessions before the late 1980s; however, these peaks are substantially less pronounced than those in the baseline specification. After the 1980s, during the last two U.S. recessions, a prominent decline can be observed. We can conclude that controlling for policy anticipation does not provide sufficient basis to challenge the result of the baseline setup—the relationship between the stage of the business cycle and government expenditure multiplier is not constant over time.



Note: Median multiplier values are presented in the figure (a). The rest of the figures contains multipliers along with
respective confidence bands, for 2-year (b) and 1-year (c) cumulative multipliers as well as the impact multiplier (d). Confidence bands are in red, calculated as 16th and 84th percentiles of the posterior multiplier distributions.

Figure 2.6.3: Unanticipated public expenditure multipliers over NBER recession dates.

2.7 Determinants of the multiplier's dynamics

The expenditure multiplier's behaviour observed in the previous sections can be driven by a variety of factors. In this section, we consider two factors—the composition of government expenditure and the interaction with the monetary policy. We conclude that the observed overall decrease in the multiplier value can be attributed to a change in the non-defence consumption expenditure's impact. Furthermore, we draw a connection between inflation expectations, inflation targeting, and the expenditure multiplier.

2.7.1 Composition of government expenditure

One of the most apparent reasons for shifts in the multiplier value is the change in the composition of government expenditure. Although the share of government expenditure in the post-WWII U.S. GDP remained approximately constant, the same may not hold true with regards to its composition.

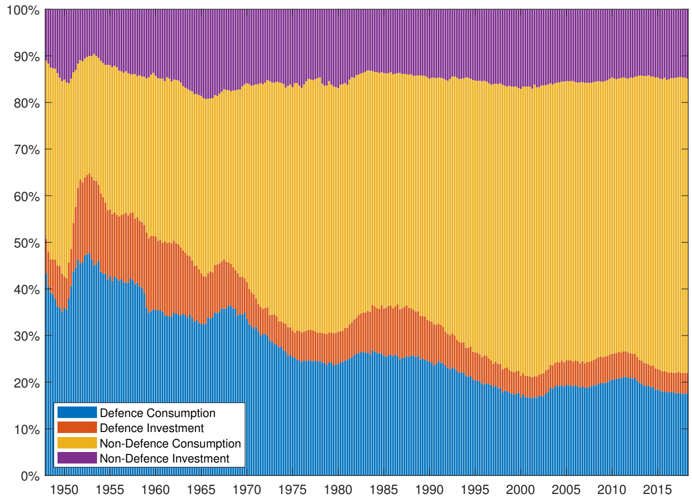


Figure 2.7.1: The composition of the measure of U.S. government expenditure, defined as in Blanchard and Perotti (2002).

Figure 2.7.1 presents how the composition of the government expenditure changed over time. The military component fell from approximately 50% in the 1950s to just above 20% in 2018. Most of this decrease was compensated by an increase in non-defence public consumption. The share of non-defence consumption has been increasing since the end of the Korean War, reaching more than half of the total expenditure by the end of the Vietnam War. It further increased during the terms of George Bush Sr. and Bill Clinton, reaching nearly 60% in the post-Clinton period. In this section, we compare multiplier estimates for two breakdowns—public investment versus public consumption and defence versus non-defence expenditure.

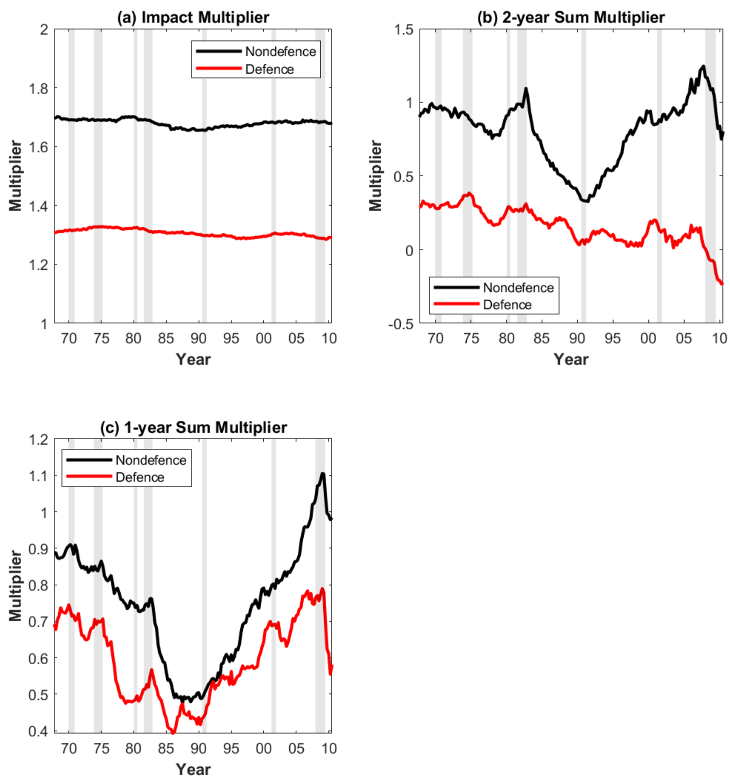


Figure 2.7.2: Impact (a), 2-year (b), and 1-year (c) cumulative multipliers for unanticipated defence and non-defence expenditure, over NBER recession dates.

We follow Ilzetski, Medoza, and Vegh (2013) approach by estimating the effects of separate components of government expenditure in isolation; we control for policy shocks' anticipation in our analysis of the components. Appendix 2.12 provides the full set of relevant IRFs as well as details on the significance of the estimates.

Comparing defence and civilian expenditure, we observe that non-defence spending has a higher capacity for stimulating real economic activity. As can be seen in Figure 2.7.2, civilian expenditure is associated with higher multipliers during the entire post-WWII period. Both spending types lead to sufficient crowding out in the long term; the crowding out is more pronounced in case of the defence expenditure. Since the investment to consumption ratios in both cases remain approximately the same, we tend to attribute the difference between the multipliers to the difference in economic implications of the two types of expenditure.[24] Indeed, it can be assumed that the construction of roads, railways, or schools has a qualitatively different effect on the economy when compared to building a new aircraft carrier or a military jet.

However, public investment and consumption multipliers significantly differ in size and evolution over time, as can be seen in Figure 2.7.3. Government investment has a higher effect on the output on impact: an extra U.S.$1 worth of investments in the public stock of capital increases the GDP by approximately U.S.$3 within the same quarter. Nevertheless, the consumption multiplier remains between zero and unity for the entire period under analysis. The picture drastically changes once we consider longer horizons. Public investment remains higher than unity and steadily increases almost until the global financial crisis hits the economy.[25] The long-term consumption multiplier demonstrates volatile behaviour. In case of the 2-year cumulative multiplier, we observe values higher than unity before the early 1980s, after

[24]Between 1967 and 2010, investment expenditure contributed, on an average, 22.8% and 21.3% to the defence and non-defence expenditures.

[25]The subsequent fall during the global financial crisis may be caused by lower public investment efficiency. Abiad, Furceri, and Topalova (2016) show that public investment shocks in advanced economies can have a low effect on output if investment efficiency is low.

which the multiplier falls substantially and reaches its lowest value during the recession of the early 1990s. It remains negative for almost the entire remainder of the sample.

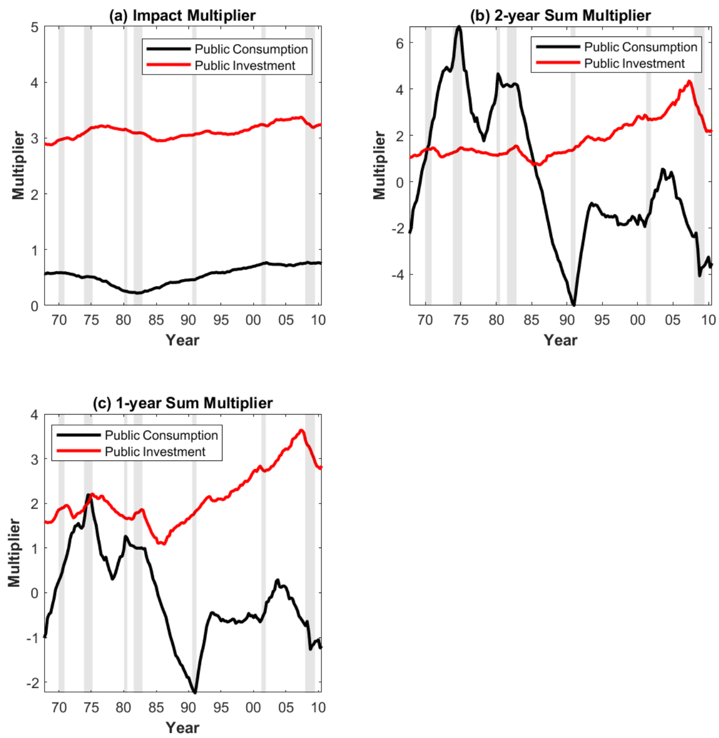


Figure 2.7.3: Impact (a), 2-year (b), and 1-year (c) cumulative multipliers for unanticipated consumption and investment expenditure. over NBER recession dates.

We believe that the fall observed in the baseline multiplier estimates emerges from a shift in output implications of discretionary non-defence consumption expenditure. This is deducted on the basis of the fall in consumption multiplier estimates as well as the pronounced dip in the non-defence expenditure multiplier before the 1980–1990 period. The fact that non-defence consumption has been rising as a share of total government expenditure only supports this idea.

2.7.2 Fiscal-Monetary nexus

Although the increasing proportion of non-defence consumption expenditure can lead to diminishing returns on additional U.S. dollar spent on non-defence consumption, this fact on its own can hardly explain the shift in the relationship between the multiplier value and the stage of the business cycle observed after the late 1980s. The Reagan period was characterised by a gradual fall in interest rates and a decline in the price level. The strong commitment of Paul Volcker's Federal Reserve to inflation targeting, which took place in the 1980s, may provide a possible explanation.

Figure 2.7.4: 2-year cumulative unanticipated government expenditure multiplier and personal consumption expenditure inflation rate.

As can be seen in Figure 2.7.4, the post-1980 period did not experience severe inflationary episodes. The inflation rate gradually decreased to the proximity of 1.5-3%, and remained there for the remainder of the sample. The forward guidance policy was extremely efficient in anchoring long-run inflation expectations in that period. Interestingly, we do not observe severe inflation hikes around recessions after the 1980s, although we observe them before this period. The monetary-fiscal nexus often emphasises the role of the central bank in managing inflation expectations. By committing to maintaining stable inflation, the Federal Reserve can reduce the uncertainty associated with future price fluctuations and potentially eliminate any feedback between the size of fiscal policy's impact and inflation.

Anchored inflation expectations can be related to the failure of interest rate spreads in predicting future multiplier values after 1990, as discussed in Section 2.5. Long-term interest rates depend on the future expected inflation and, therefore, on the expected path of the monetary policy. A change in the monetary policy regime can lead to a change in the decision-making process of market agents, captured by the interest rate spreads. Although the fundamental shift in how the Federal Reserve treated inflation is believed to have taken place in the late 1979, when Paul Volcker became the chairman of the Federal Reserve, explicit inflation target was not discussed in the Federal Open Market Commission meetings until around 1994 (Shapiro and Wilson, 2019). In the early 1990s, the Federal Reserve decided to switch from the vague goal of 'price stability' to an explicit inflation target. This period coincides with the time interest rate spreads lose the ability to predict multiplier fluctuations.

The two ways in which the introduction of inflation-targeting can affect the potency of the fiscal policy are by restricting the expected inflation channel of fiscal policy and by neutralising the government's ability to benefit from an invisible inflation tax. The expected inflation channel allows a government expenditure shock to increase inflation expectations if the monetary authority remains passive. Dupor and Li (2015) construct a sticky-price model that produces a large government expenditure multiplier if expected inflation response to a government expenditure shock is high. Such a response is only possible if monetary policy remains passive in targeting inflation. Alternatively, episodes of high inflation can benefit the fiscal authority if the discretionary expenditure is debt-financed. Higher inflation can diminish the real costs of borrowing since there will be a decline in the real bond values to be repaid. In this way, severe inflationary episodes can partially eliminate the debt burden raised by the government through an invisible inflation tax.

2.8 Conclusion

The recent empirical contributions tend to question whether the government expenditure multiplier depends on the stage of the business cycle. This study provides evidence that the highly non-linear nature of this relationship leads to a failure of its discovery. We find that fiscal multipliers were higher during recessions, but only before the late 1980s. In the subsequent period, we observe lower multiplier values during recessions, as identified by the NBER. This result suggests that econometric set-ups that aim at capturing the average difference in the multiplier value between the stages of the business cycle may produce misleading results. Our results are robust to controlling for the anticipation of fiscal policy shocks.

In line with previous contributions, we conclude that discretionary government expenditure became less potent in stimulating the output after the 1980s, coinciding with the start of the great moderation and the introduction of explicit inflation-targeting in the US. We show that this fall emerges from the shift in the non-defence government consumption multiplier. We further elaborate on the possibility that the commitment to active and transparent inflation-targeting may have played a crucial role in the behaviour of the multiplier observed after 1980s; further research is needed to assess the correctness of this hypothesis.

We believe that the evidence provided in this paper highlights the necessity to identify a monetary explanation for the shift in the multiplier's behaviour after the late 1980s. Further research is needed to shed light on how inflation expectations and explicit inflation-targeting can be connected to the state-dependent nature of the multiplier.

Appendix

2.9 Data

Data is obtained from the Bureau of Economic Analysis (BEA) NIPA tables and Federal Reserve Economic Database (FRED). Real per capita measures of GDP, public expenditure and taxes are calculated by dividing the nominal aggregated measures by the GDP deflator and civilian noninstitutional population above16 years of age. Variables obtained from the BEA NIPA tables are coded in the following format: $TA.B.CLX$, where $TA.B.C$ corresponds to the NIPA table number and LX to the line number in it. Otherwise, variable names are consistent with indices assigned to them in the FRED Database.

1. Real per capita GDP
   Notation: Y_t
   Sample: 1948Q1-2018Q2
   Formula: $\frac{T1.1.5L1}{T1.1.4L1 \times CNP16OV}$
   Source: BEA, FRED

2. Real per capita government consumption
   Notation: GC_t
   Sample: 1948Q1-2018Q2
   Formula: $\frac{T3.9.5L2}{T1.1.4L1 \times CNP16OV}$
   Source: BEA, FRED

3. Real per capita government investment
   Notation: GI_t
   Sample: 1948Q1-2018Q2
   Formula: $\frac{T3.9.5L3}{T1.1.4L1 \times CNP16OV}$
   Source: BEA, FRED

4. Real per capita net taxes
   Notation: NT_t
   Sample: 1948Q1-2018Q2

Formula: $\frac{T3.1L2+T3.1L7+T3.1L10+T3.1L16-T3.1L30-T3.1L27-T3.1L22}{T1.1.4L1\times CNP16OV}$
Source: BEA, FRED

5. Forecast of Public Expenditure Growth Rate
Notation: $\Delta G^F_{t|t-1}$
Sample: 1966Q4 - 2010Q3
Formula: N/A
Source: Auerbach and Gorodnichenko (2012)

6. NBER recession dates
Notation: N/A
Sample: 1948Q1 - 2018Q2
Formula: $USRECQ$
Source: FRED

7. 10-Year to FFR spread
Notation: N/A
Sample: 1962Q1 - 2018Q2
Formula: $GS10 - DFF$
Source: FRED

8. 5-Year to FFR spread
Notation: N/A
Sample: 1962Q1 - 2018Q2
Formula: $DGS5 - DFF$
Source: FRED

9. 1-Year to FFR spread
Notation: N/A
Sample: 1962Q1 - 2018Q2
Formula: $WGS1YR - DFF$
Source: FRED

10. 6-Months to FFR spread
Notation: N/A
Sample: 1962Q1 - 2018Q2

Formula: $TB6MS - DFF$
Source: FRED

11. 3-Months to FFR spread
Notation: N/A
Sample: 1962Q1 - 2018Q2
Formula: $TB3MS - DFF$
Source: FRED

2.9.1 Hamilton (2018) Linear Projection Method

This study introduces Hamilton's (2018) detrending procedure to the fiscal multiplier debate. The procedure makes use of a linear projection model, similar in spirit to direct forecasting or Jorda's (2005) local projection method:

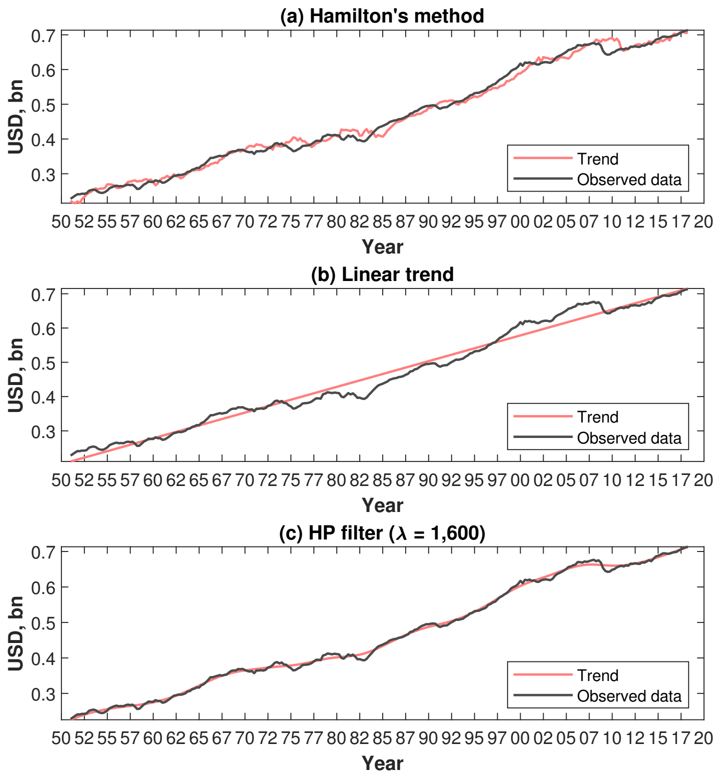
$$y_{t+h} = B(L)y_t + v_{t+h}, \qquad v_{t+h} \sim i.i.d.N(0, \sigma^2)$$

where $B(L)$ is the lag polynomial of the variable being detrended (y_t), h is the prediction horizon and v_{t+h} is an i.i.d. error term. In the case of the quarterly data, Hamilton (2018) recommends to set $h = 8$ and estimate the model with a lag polynomial of order four. Resulting series of residuals ($\hat{v_{t+h}}$) would then represent a stationary zero-mean cyclical component. In essence, the method identifies the cyclical component as the forecast error that is due to macroeconomic developments taking place along those eight quarters.

Hamilton's (Ibid) method has several advantages over alternative detrending procedures. First, it produces a non-linear trend estimate without the necessity to guess the functional form of such non-linearity. Second, due to the peculiarities of the method, it allows the trend to be influenced by macroeconomic events taking place in the past; as can be seen in Figure 2.9.1, the trend estimate experiences a pronounced dip in the aftermath of the global financial crisis.[26] Third, as argued by Hamilton (Ibid), the method does not produce spurious correlations between the resulting cyclical com-

[26] One would be surprised if an event such a the global financial crisis had not affected the potential output or shifted the economy to a new growth path.

ponents and other macroeconomic series, as in the case of the Hodrick and Prescott (HP) filter. Fourth, it produces stationary series in levels.



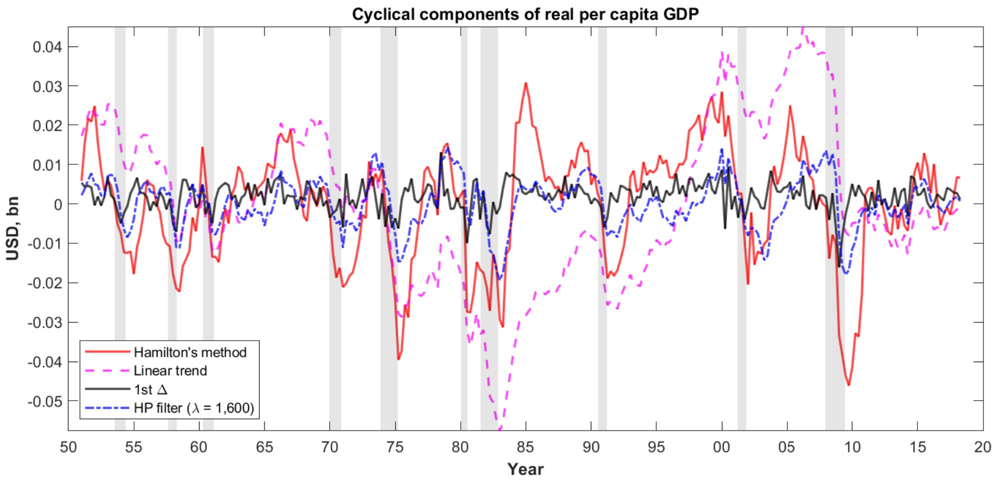
Note: Observed U.S. real per capita public expenditure (Investment+Consumption) along with trends produced by the Hamilton's linear projection technique (a), linear trend estimation (a) and the Hodrick-Prescott filter at $\lambda = 1,600$ (c).

Figure 2.9.1: Detrending procedures and resulting trends.

Estimating the TVP-VAR on such data allows us to interpret resulting IRFs as multipliers, not elasticities. Therefore, without the need to resort to the use of growth rates, this approach allows us to avoid the rescaling bias described in Ramey and Zubairy (2018). Finally, the method allows us to preserve a larger share of low-frequency variation in the target series.

Figure 2.9.2 presents the cyclical components of the real per capita GDP derived using Hamilton's method, HP filter and by removing a simple linear trend; to support the analysis we also plot first differences of the data and recession dates identified by NBER along with the cyclical components. First differences lack a significant share of the lower-frequency variation that contains crucial information necessary for correct fiscal policy evaluation.[27]

Removal of the linear trend produces a cyclical component that does not revert to its mean for long periods of time; TVP-VAR models can be sensitive to the use of such time series. The cyclical component obtained using Hamilton's method delivers a compromise. The method preserves the mean-reverting nature of the series and allows for a sufficient share of low-frequency variation.



Note: Figure presents first differences of GDP, cyclical components obtained using the HP filter and Hamilton's method, and GDP without a linear time trend. Shaded areas are recessions defined by the NBER.

Figure 2.9.2: Transformations of GDP and low frequency variation.

2.10 Identification

Cholesky decomposition, used in Auerbach and Gorodnichenko (2012) and Ramey (2011b), although being the least cumbersome identification approach,

[27]This is precisely the reasoning used by Auerbach and Gorodnichenko (2012) to justify estimation of their ST-VAR in levels.

is misleading in its application to a fiscal VAR where tax revenues are used instead of marginal tax rates. Although the resulting shock series are assumed to be independent, the immediate effect of the shock ordered first on the variable ordered last will contain immediate effects of shocks ordered in between. It is, therefore, crucial to ensure that shocks ordered in between are identified correctly.

Imposing a lower-unitriangular structure on the contemporaneous relations in $[G_t \;\; T_t \;\; Y_t]$ assumes that only innovations in T_t can contemporaneously affect Y_t, and not the other way around. Although such an approach seems to be justified in the case of discretionary government expenditure or marginal tax rates, it is not clear why output shocks cannot affect tax revenues in the same quarter. Let us consider a generic SVAR model with three endogenous variables:

$$Y_t = A_0 Y_t + B(L) Y_t + \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0, \Theta) \tag{2.10.1}$$

where Θ is diagonal. The model in 2.10.1 has a reduced form representation:

$$Y_t = A(L) Y_t + u_t, \qquad u_t \sim \mathcal{N}(0, \Omega)$$

where Ω is a full symmetric matrix, that can be decomposed in a product of lower-unitriangular and diagonal matrices.

$$\Omega = P'P = C\Sigma\Sigma'C'$$

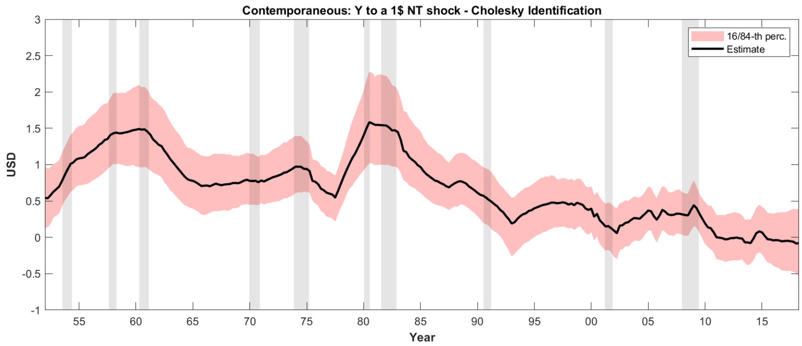
where the lower unitriangular matrix C contains the immediate responses of endogenous variables to the structural shocks. It is well known that the following relationship holds in this set-up:

$$C = (I_3 - A_0)^{-1}$$

Alternatively we can represent elements of C as functions of elements of A_0:

$$A_0 = \begin{bmatrix} 0 & 0 & 0 \\ \alpha_{21} & 0 & 0 \\ \alpha_{31} & \alpha_{32} & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 \\ \alpha_{21} & 1 & 0 \\ \alpha_{31} + \alpha_{21}\alpha_{32} & \alpha_{32} & 1 \end{bmatrix}$$

Now let us return to the Auerbach and Gorodnichenko (2012) case; the immediate output response to a tax shock is given by α_{32}. It is equivalent to the coefficient of current net taxes in the output equation of (2.10.1). Therefore, it is impossible to constitute the direction of causality between output and taxes, as the coefficient can represent effects in both directions. The full set of restrictions imposed by Cholesky ordering is not sufficient to identify the tax shock. Restricting the reverse channel, discussed above, results in the α_{32} coefficient representing a mix between the two effects. That is precisely why Auerbach and Gorodnichenko (2012) had to rely on the elasticity measure defined by Blanchard and Perotti (2002) instead of depending solely on Cholesky in calculating IRFs to a tax shock, as they did with government expenditure shock. Figure 2.10.1 presents the time-varying response of output to a tax shock, the equivalent of α_{32} in our setup, identified via Cholesky decomposition; it is clear that the shock is not identified correctly. The immediate response captures the positive effect of output on the tax base, instead of the negative effect of the tax rate on output. Moreover, as it is evident from the matrix C above, α_{32} enters the calculation of the response of output to a government expenditure shock. Thus, one should not rely on the output responses identified using Cholesky in the Auerbach and Gorodnichenko (2012) setup, even if only output responses to a government expenditure shock are being investigated.



Note: Obtained using Cholesky decomposition.

Figure 2.10.1: Immediate output response to a tax shock over time.

Sign restrictions (e.g. Mountford and Uhlig 2009; Canova and Pappa 2007) or "narrative" (IV/Proxy) identification (e.g. Mertens and Ravn 2014; Stock and Watson 2012; Stock and Watson 2018; Montiel Olea, Stock and Watson 2012; Mumtaz, Pinter and Theodoridis, 2018) should be preferred to a simple Cholesky decomposition in case of a fiscal VAR with tax revenues ordered after government expenditure.

2.10.1 Identification via the mixture of short-term zero and sign restrictions

Our choice of the identification strategy, depicted in (2.4.1), constitutes an alternative way to solve the rebus discussed in Blanchard and Perotti (2002) and Perotti (2007). Following the authors, let us assume that the reduced form shocks identified by our TVP-VAR are linear functions that can be expressed by the system of equations below:

$$\begin{aligned} u_t^G &= \alpha_t^{gy} u_t^Y + \beta_t^{gt} \varepsilon_t^T + \varepsilon_t^G \\ u_t^T &= \alpha_t^{ty} u_t^Y + \beta_t^{tg} \varepsilon_t^G + \varepsilon_t^T \\ u_t^Y &= \alpha_t^{yg} u_t^G + \alpha_t^{yt} u_t^T + \varepsilon_t^Y \end{aligned} \tag{2.10.2}$$

where u_t^G, u_t^T, and u_t^Y are reduced form shocks and ε_t^G, ε_t^T, and ε_t^Y are structural shocks. α_t^{gy} and α_t^{ty} capture the automatic response of the fiscal variables to changes in output (the automatic stabiliser effects) and the systematic discretionary response of fiscal variables to changes in output. We are interested in estimating the IRFs to the random discretionary shocks, in our case we focus on ε_t^G. In order to solve the above system of equations we need to impose a set of assumptions.

1. Following Blanchard and Perotti (2002) we assume $\alpha_t^{gy} = 0$. Such an assumption implies no automatic nor systematic discretionary responses of government expenditure to developments in output. The absence of a systematic discretionary response is a consequence of the policy implementation lag; the policy-maker will need at least a quarter to come up and execute a discretionary government expenditure package in response to a surprise recession. Since our set-up rests on Blanchard and Perotti (Ibid) we rely on their results on the role of automatic stabilisers; authors were not able to identify any automatic feedback from economic activity to government purchases.

2. Another restriction inspired by Blanchard and Perotti (Ibid) is that $\beta^{gt} = 0$. Authors argued that either β_t^{gt} or β_t^{tg} should be set to zero; since the correlation between government expenditure and net taxes is low, both restrictions produced similar results.

3. α_t^{ty} is positive. Allowing α_t^{ty} to be non-zero we imply that output shocks can affect net taxes through the tax base. Since we set it to be positive, we believe that a positive shock to output will expand the tax base and vice verse. Blanchard and Perotti (Ibid) directly estimate the coefficient as a function of two elasticities—elasticity of taxes to their respective tax bases and elasticity of the tax bases to GDP. Average value of α_t^{ty}, estimated on various sub-samples, remained positive. Authors acknowledged that they focused on an average value of α_t^{ty}, while in reality, it should vary over time; our approach. additionally, allows accounting for this fact.

4. α_t^{yt} is negative. Blanchard and Perotti (Ibid) estimate a time-invariant coefficient directly for two cases—deterministic and stochastic trends.

In both cases, the coefficient is negative and equals to -0.868 and -0.876, respectively.

Given the above-mentoned assumptions, we can show that the system in (2.10.2) can be represented as:

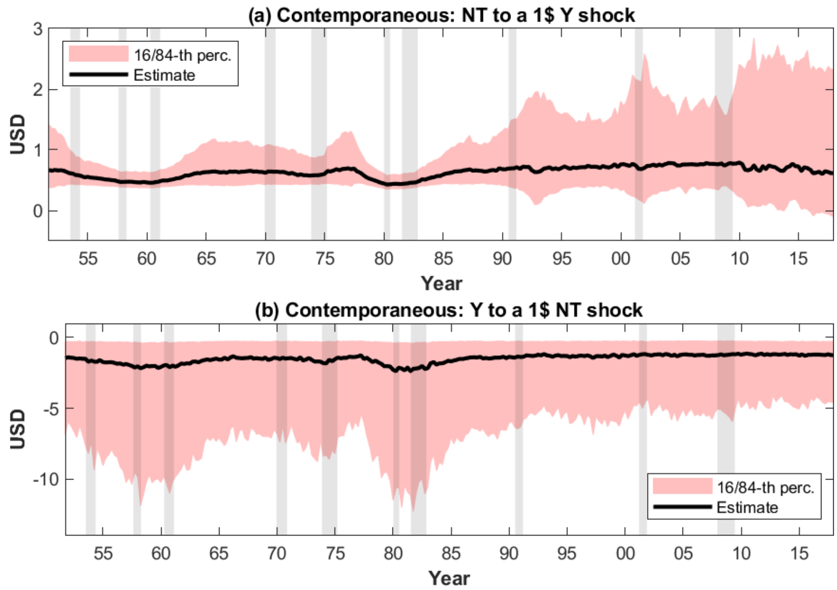
$$
\begin{aligned}
u_t^G &= \varepsilon_t^G \\
u_t^T &= \frac{\alpha_t^{ty}\alpha_t^{yg} + \beta_t^{tg}}{1 - \alpha_t^{ty}\alpha_t^{yt}}\varepsilon_t^G + \frac{1}{1 - \alpha_t^{ty}\alpha_t^{yt}}\varepsilon_t^T + \frac{\alpha_t^{ty}}{1 - \alpha_t^{ty}\alpha_t^{yt}}\varepsilon_t^Y \\
u_t^Y &= \frac{\alpha_t^{yt}\beta_t^{tg} + \alpha_t^{yg}}{1 - \alpha_t^{ty}\alpha_t^{yt}}\varepsilon_t^G + \frac{\alpha_t^{yt}}{1 - \alpha_t^{ty}\alpha_t^{yt}}\varepsilon_t^T + \frac{1}{1 - \alpha_t^{ty}\alpha_t^{yt}}\varepsilon_t^Y
\end{aligned}
$$

For simplicity, let us use the following matrix notation:

$$
\begin{bmatrix} u_t^G \\ u_t^T \\ u_t^Y \end{bmatrix} = \begin{bmatrix} c_t^{11} & 0 & 0 \\ c_t^{21} & c_t^{22} & c_t^{23} \\ c_t^{31} & c_t^{32} & c_t^{33} \end{bmatrix} \begin{bmatrix} \varepsilon_t^G \\ \varepsilon_t^T \\ \varepsilon_t^Y \end{bmatrix}
$$

If α_{ty} is positive and α_{yt} is negative, then c_{22} and c_{33} are both positive time-varying coefficients. These assumptions also imply that c_{32} is negative and c_{23} is positive. It is not necessary to impose the latter assumption, since imposing the former already results in c_{23} being positive; we show this in Figure 2.10.2 below. Finally, c_{11} is positive by definition. Our set of assumptions is not sufficient to identify signs of c_{21} and c_{31}.

This set of assumptions leaves us with an underidentified system and a partial understanding of coefficients' signs. We solve the system using a mixture of sign and short-term zero restrictions listed in (2.6). To obtain this solution, we follow the technique described in Binning (2013). A technical outline of the procedure is presented in the next section



Note: Immediate responses of net taxes to an output shock (a) and output to a net tax shock (b) as functions of time. Obtained using the baseline identification strategy.

Figure 2.10.2: Identification of the two-way channel between taxes and output.

2.10.2 Binning (2014) technique

The Binning technique executes the Rubio-Ramirez (2010) procedure to generate the impact IRF matrix Z via a mixture of sign and zero restrictions. In a time-invariant VAR setting, the identification strategy generates a candidate impact matrix by applying a QR decomposition on a randomly sampled impact matrix. If the candidate matrix satisfies the identifying sign restrictions, we search for a rotation matrix that would impose the necessary zero restrictions. Conditional on the existence of such a rotation matrix, a draw of the impact matrix Z is generated. The following sampling procedure is then repeated 1000 times to generate a distribution of the impact matrix:

1. Apply Choleski factorization on the estimated variance covarience matrix: $CC' = \Omega$;

2. Randomly draw a matrix of size equivalent to Ω from a normal distribution;

3. QR decomposition of this matrix is taken to produce a randomly drawn orthogonal matrix Q^*;

4. Generate a candidate Z^* by post-multiplying C by Q^*;

5. Check whether elements of Z satisfy the specified sign-restrictions;

6. Find a rotation matrix P, such that would impose necessary zero restrictions and not violate $PP' = I$;

7. Calculate final draw of the impact matrix $Z = PZ^*$, keep in mind that $ZZ' = \Omega$

In a time-varying parameter setting, such as the one considered in this chapter, we apply this procedure consequently on every time period of every Ω_t draw produced by Gibb's sampler. Having obtained the distribution of the time-varying impact matrix Z_t, we use its time-specific median for the multiplier calculation.

2.11 Model setup

The methodology used in this paper follows the setup of Eisenstat, Chan and Strachan (2016). The model in (2.3.1)-(2.3.3) can be transformed into:

$$
\begin{aligned}
Y_t &= X_t\alpha + X_t\Phi\tilde{\Omega}^{\frac{1}{2}}\gamma_t + \Sigma_t u_t, && u_t \sim \mathcal{N}(0, I) \\
\gamma_t &= \gamma_{t-1} + v_t^* && v_t^* \sim \mathcal{N}(0, I) \\
log(\sigma_t) &= log(\sigma_{t-1}) + \theta_t && \theta_t \sim \mathcal{N}(0, W)
\end{aligned}
$$

where α contains coefficients from a time-invariant version of the VAR, $\tilde{\Omega}^{\frac{1}{2}}$ and Φ are obtained from a factorization of the variance covariance matrix Ω and $\gamma_{j,t} = (\beta_{j,t} - \alpha_j)/\omega_j$ for $j = 1, \ldots, m$. The above model can be broken down into two separate state space representation models and estimated recursively using the Gibb's sampler. γ_t and σ_t are estimated via following state-space representation models:

- Model 1

$$\begin{aligned} \tilde{Y}_t &= W_t \gamma_t + \varepsilon_t \\ \gamma_t &= \gamma_{t-1} + v_t^* \end{aligned}$$

- Model 2

$$\begin{aligned} \varepsilon_t^{**} &= 2 \times log(\sigma_t) + log(u_t u_t') \\ log(\sigma_t) &= log(\sigma_{t-1}) + \theta_t \end{aligned}$$

where $\tilde{Y}_t = Y_t - X_t \alpha$ and $W_t = X_t \tilde{\Omega}^{\frac{1}{2}} \Phi$. Model 1 is a linear Gaussian state space representation model, thus, it can be solved using the Chan and Jeleznyakov (2009) approach. Model 2, on the other hand, is a linear but non-Gaussian state space representation model. It is solved via the Kim et al. (1998) approach, which uses the fact that $log(u_t u_t')$ has a χ_1^2 distribution, which can be approximated by the mixture of log-normals.

The variance covariance matrix W from the state equation is sampled from $\mathcal{IW}(\bar{W}^{-1}, \bar{T})$:

$$\begin{aligned} \bar{Q} &= \underline{\mathrm{W}} + \Sigma_{t=1}^{T} \theta_t \theta_t' \\ \bar{T} &= \underline{\mathrm{T}} + T \end{aligned}$$

Parameters α and Φ are estimated using simple linear regression techniques. In case of Φ, equation (1) is further rearranged into:

$$Y_t^* = Z_t \phi + e_t$$

where ϕ contains all the non-zero off-diagonal elements of Φ, Y_t^* and Z_t are given by:

$$Y_t^* = Y_t - X_t\alpha - X_t\tilde{\Omega}^{\frac{1}{2}}\gamma_t$$
$$Z_t = X_t\tilde{\Omega}^{\frac{1}{2}}F_t$$

, and F_t is defined as:

$$F_t = \begin{pmatrix} 0 & 0 & \dots & 0 \\ \gamma'_{1,t} & 0 & \dots & 0 \\ 0 & \gamma'_{[1,\dots,2],t} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & \gamma'_{[1,\dots,m-1],t} \end{pmatrix}$$

ω is obtained by defining:

$$Y = \begin{pmatrix} Y_1 \\ \vdots \\ Y_T \end{pmatrix}, \quad X = \begin{pmatrix} X_1 \\ \vdots \\ X_T \end{pmatrix}, \quad \gamma = \begin{pmatrix} \gamma_1 \\ \vdots \\ \gamma_T \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_T \end{pmatrix}$$

and re-arranging (12) into:

$$v_j = g_j\omega_j + \varepsilon$$

where $v_j = Y - X\alpha - G_{\backslash j}\omega_{\backslash j}$, g_j denotes the j-th column of G, $G_{\backslash j}$ denotes a $Tn \times (m-1)$ matrix obtained by deleting the j-th column from G, $\omega_{\backslash j}$ is ω with the j-th row removed and G is $G_t = X_t diag(\gamma_t)$ stacked in a similar way as above:

$$G = \begin{pmatrix} G_1 \\ \vdots \\ G_T \end{pmatrix}$$

ω is then sampled from a conditional posterior density that follows a 2-component mixture of truncated normals:

$$p(\omega_j|Y, \alpha, \gamma, \omega_{\backslash j}, \Sigma, \tau, \lambda) = \hat{\pi}_j \phi_{(-\infty,o)}(\omega_j|\mu_j, \tau_j^2) + (1-\hat{\pi}_j)\phi_{(0,\infty)}(\omega_j|\hat{\mu}_j, \hat{\tau}_j^2)$$

τ_j^2 and λ are sampled in the same manner as in Belmonte, Koop and Korobolis (2014):

$$(\tau_j^{-2}|\lambda, \omega_j) \sim IG\left(\sqrt{\frac{\lambda^2}{(\omega_j - \mu_j)^2}}, \lambda^2\right)$$

$$(\lambda^2|\tau) \sim G\left(\lambda_{01} + m, \lambda_{02} + \frac{1}{2}\Sigma_{j=1}^m \tau_j^2\right)$$

For further peculiarities, please refer to Eisenstat, Chan and Strachan (2016), Fruhwirth-Schnatter and Wagner (2010) and Belmonte, Koop, and Korobolis (2014).

Overall, the Gibb's sampler procedure takes the following from:

1. Draw α from $p(\alpha|Y^T, \gamma^T, \Sigma^T, W, \omega, \tau, \lambda, \Phi)$;
2. Draw γ^T from $p(\gamma^T|Y^T, \alpha, \Sigma^T, W, \omega, \tau, \lambda, \Phi)$;
3. Draw Σ^T from $p(\Sigma^T|Y^T, \alpha, \gamma^T, \Sigma^T, W, \omega, \tau, \lambda, \Phi)$;
4. Draw W from $p(W|Y^T, \alpha, \gamma^T, \Sigma^T, W, \omega, \tau, \lambda, \Phi)$;
5. Draw ω from $p(\omega|Y^T, \alpha, \gamma^T, \Sigma^T, W, \omega, \tau, \lambda, \Phi)$;
6. Draw τ from $p(\tau|Y^T, \alpha, \gamma^T, \Sigma^T, W, \omega, \tau, \lambda, \Phi)$;
7. Draw λ from $p(\omega|Y^T, \alpha, \gamma^T, \Sigma^T, W, \omega, \tau, \lambda, \Phi)$;
8. Draw Φ from $p(\Phi|Y^T, \alpha, \gamma^T, \Sigma^T, W, \omega, \tau, \lambda, \Phi)$;
9. Move to the first step;

The methodology in use requires specification of prior distributions. Standard independent priors are assumed for:

$$
\begin{aligned}
&\alpha_0 \sim \mathcal{N}(0, I_m)\beta_0 \sim \mathcal{N}(0, I_m) \\
&\Sigma_0 \sim \mathcal{N}(0, I_n)
\end{aligned}
$$

A Tobit prior for ω:

$$
\begin{aligned}
\omega_j^* &\sim \mathcal{N}(0, \tau_j^2) \\
\omega_j &= \begin{cases} 0 & \text{if } \omega_j^* \leq 0 \\ \omega_j^* & \text{if } \omega_j^* > 0 \end{cases}
\end{aligned}
$$

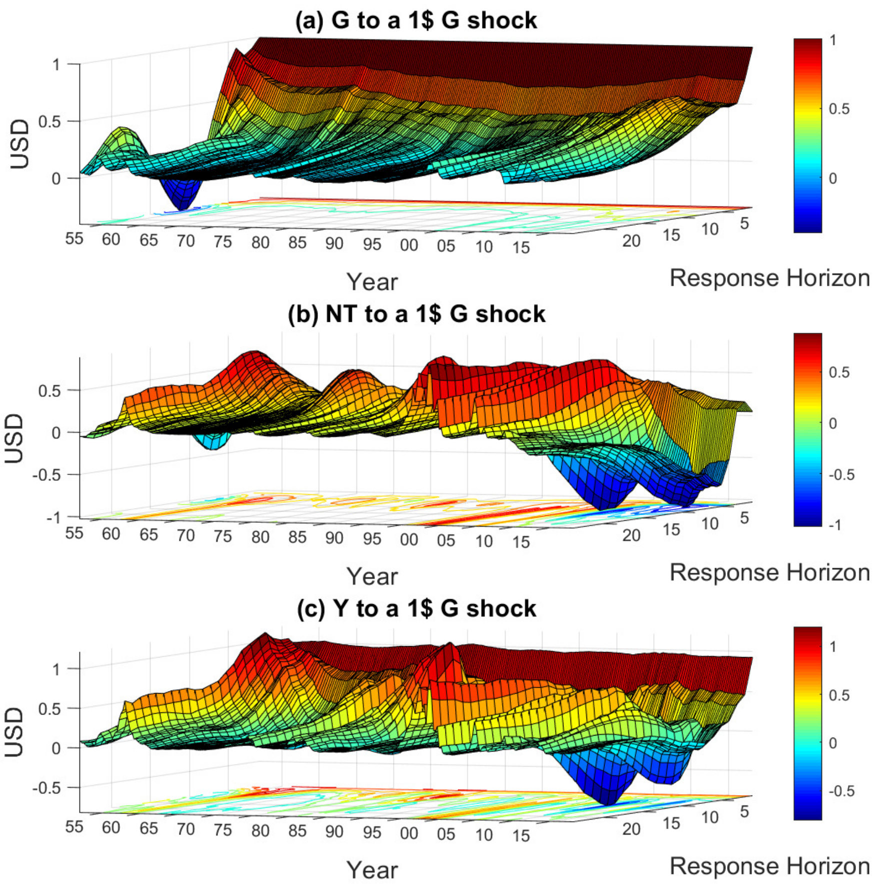
A Lasso prior for tau_j^2:

$$
\begin{aligned}
\tau_j^2 &\sim \mathcal{E}\left(\frac{\lambda^2}{2}\right) \\
\lambda^2 &\sim \mathcal{G}(0.1, 0.1)
\end{aligned}
$$

$$
W \sim \mathcal{IW}(n + 11, 0.01^2(n + 11 - n - 1)I_n)
$$

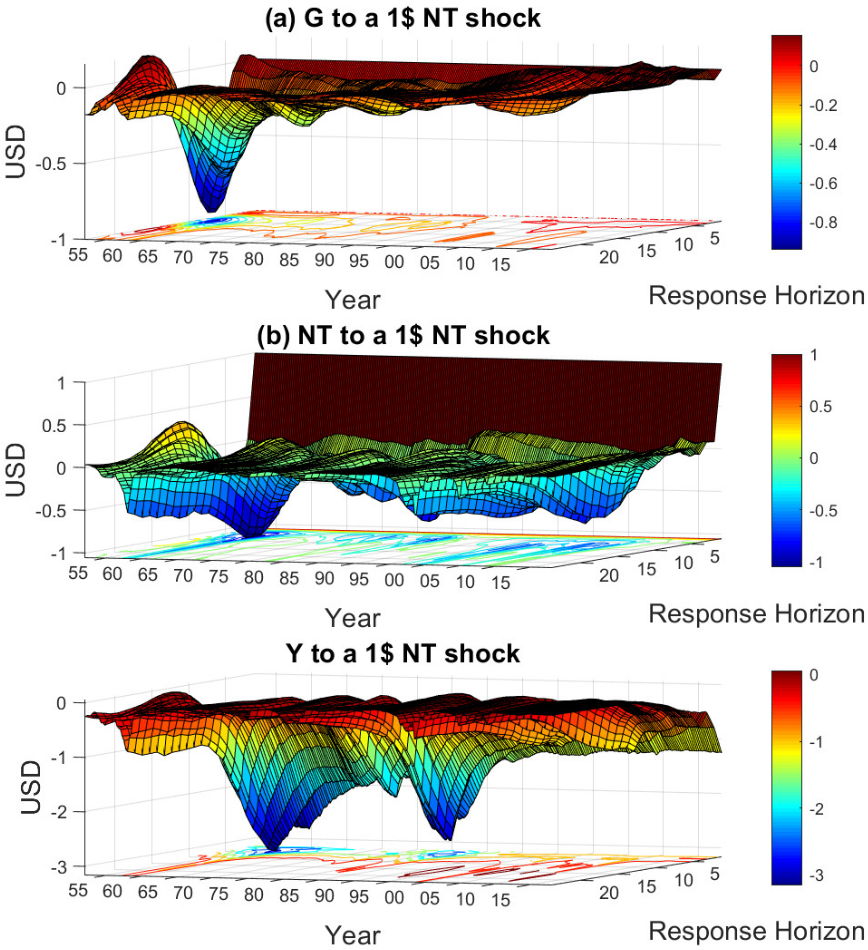
2.12 Time-Varying Impulse Response Functions

2.12.1 Baseline Specification‘



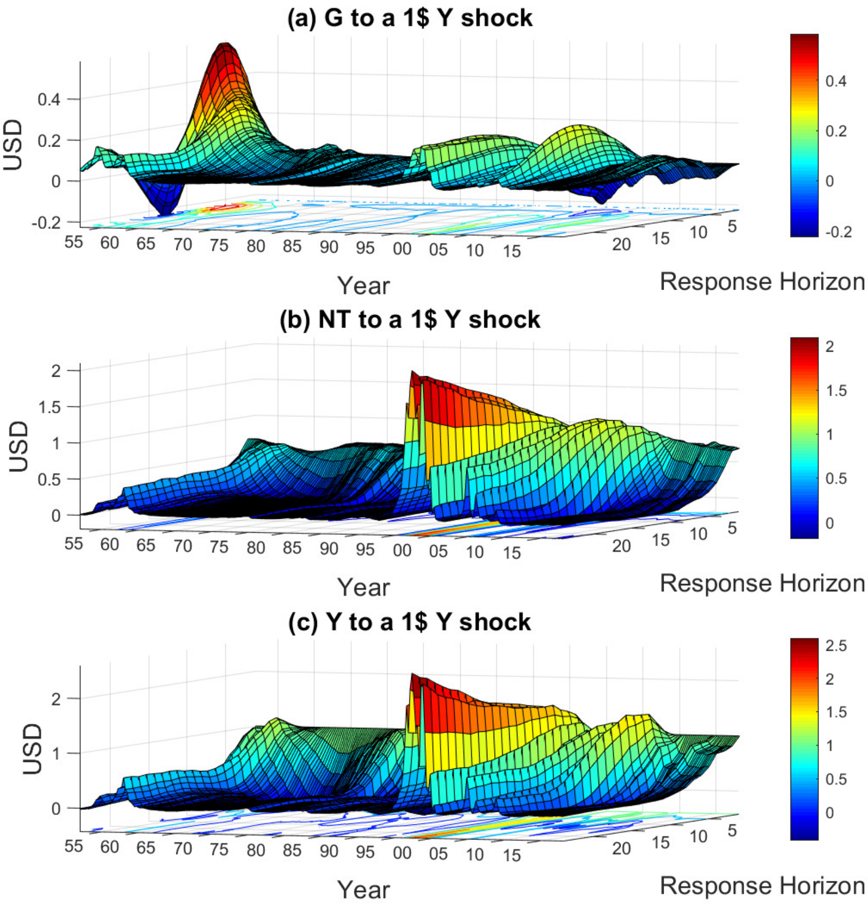
Note: Government expenditure response (a), net taxes' response (b), and output responses (c) to a U.S.$1 government expenditure shock. Responses are measured in real U.S. dollars.

Figure 2.12.1: Baseline estimation. Median responses to discretionary government expenditure shocks as functions of time.



Note: Government expenditure response (a), net taxes' response (b), and output responses (c) to a U.S.$1 tax shock. Responses are measured in real U.S. dollars.

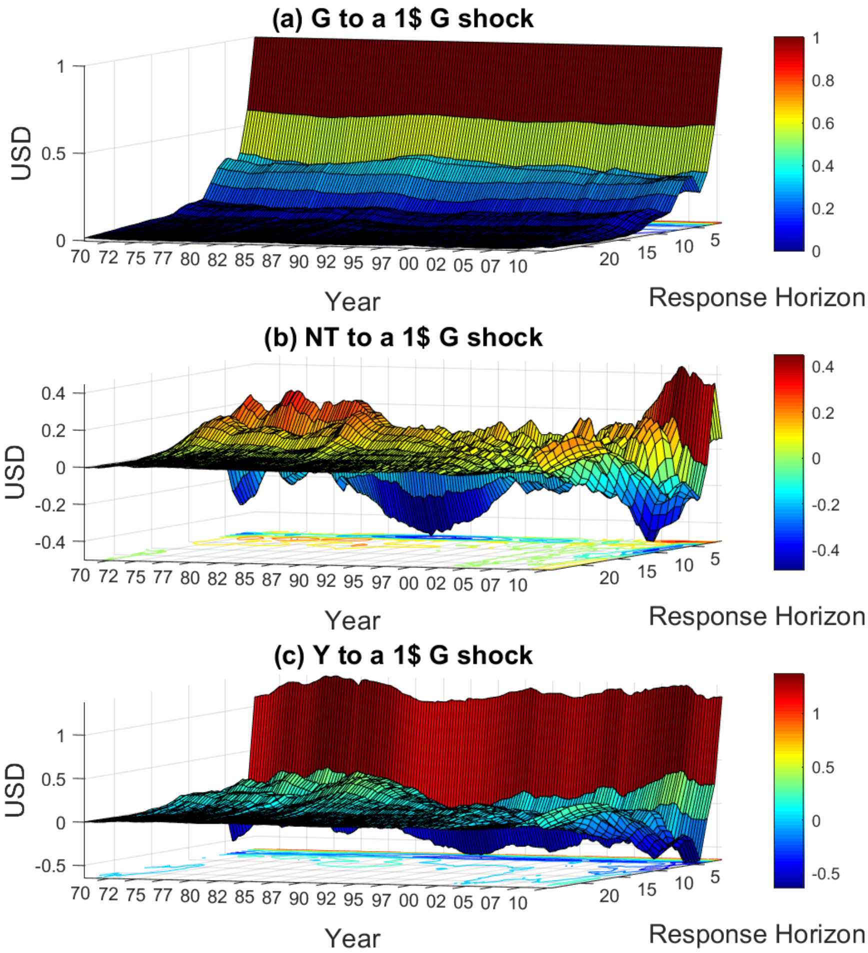
Figure 2.12.2: Baseline estimation. Median responses to tax shocks as functions of time.



Note: Government expenditure response (a), net taxes' response (b), and output responses (c) to a U.S.$1 output shock. Responses are measured in real U.S. dollars.

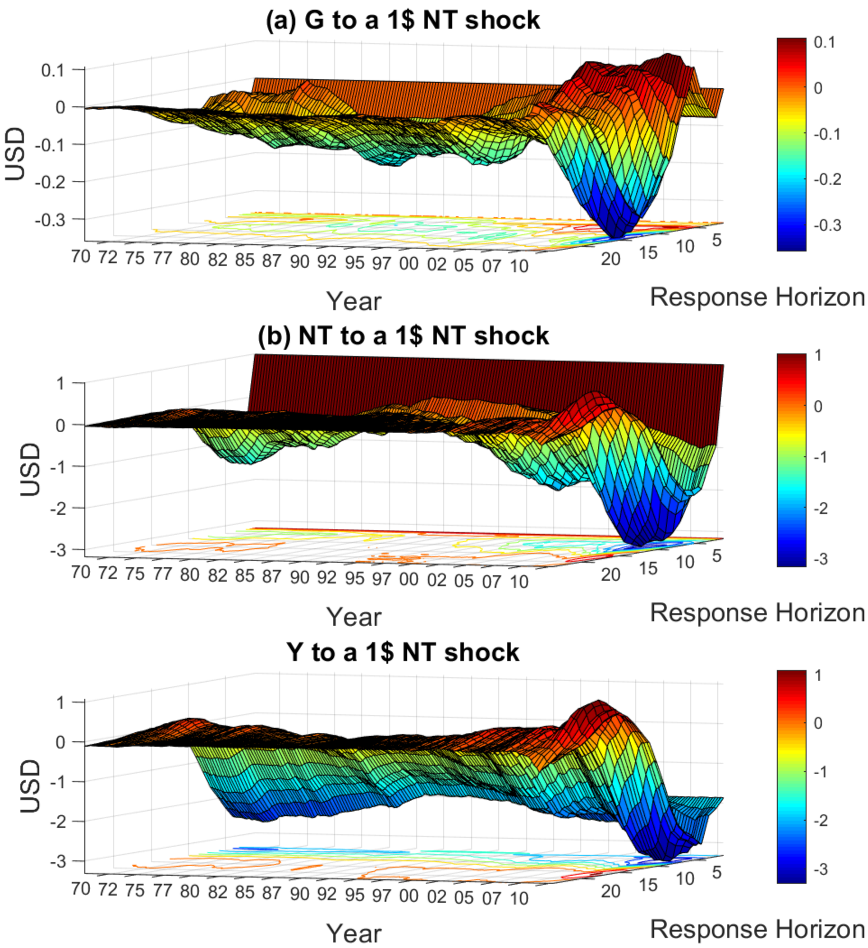
Figure 2.12.3: Baseline estimation. Median responses to output shocks as functions of time.

2.12.2 Unanticipated Government Expenditure Shock Specification



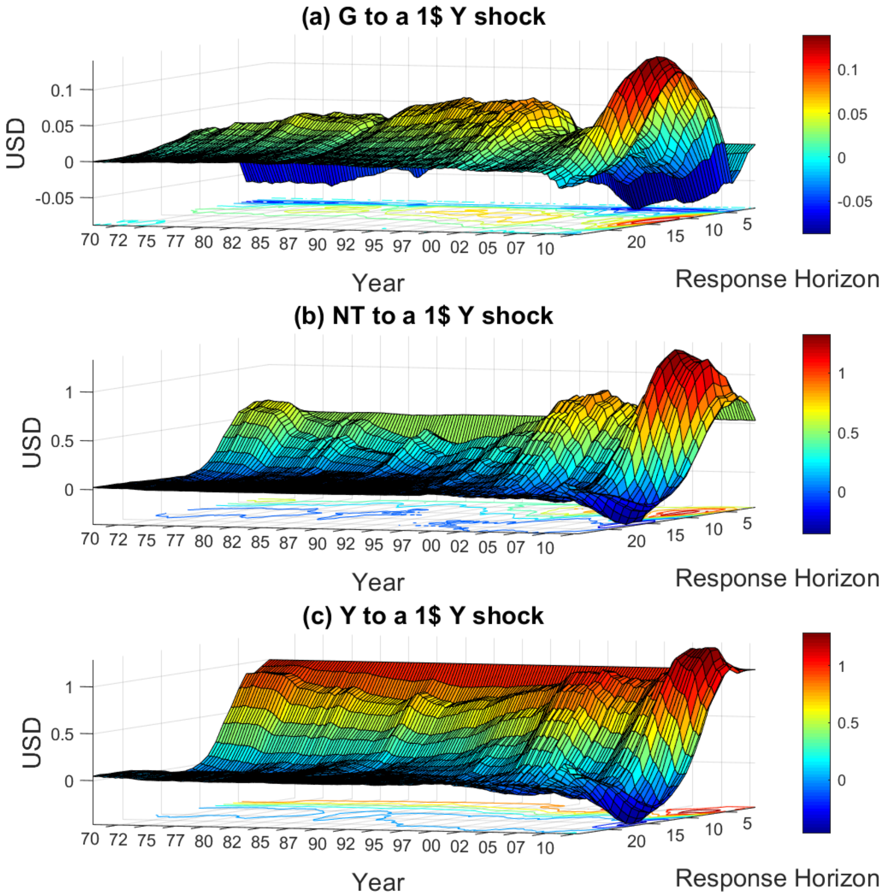
Note: Government expenditure response (a), net taxes' response (b), and output responses (c) to a U.S.$1 unanticipated government expenditure shock. Responses are measured in real U.S. dollars.

Figure 2.12.4: Unanticipated policy shocks. Median responses to government expenditure shocks as functions of time.



Note: Government expenditure response (a), net taxes' response (b), and output responses (c) to a U.S.$1 tax shock. Responses are measured in real U.S. dollars.

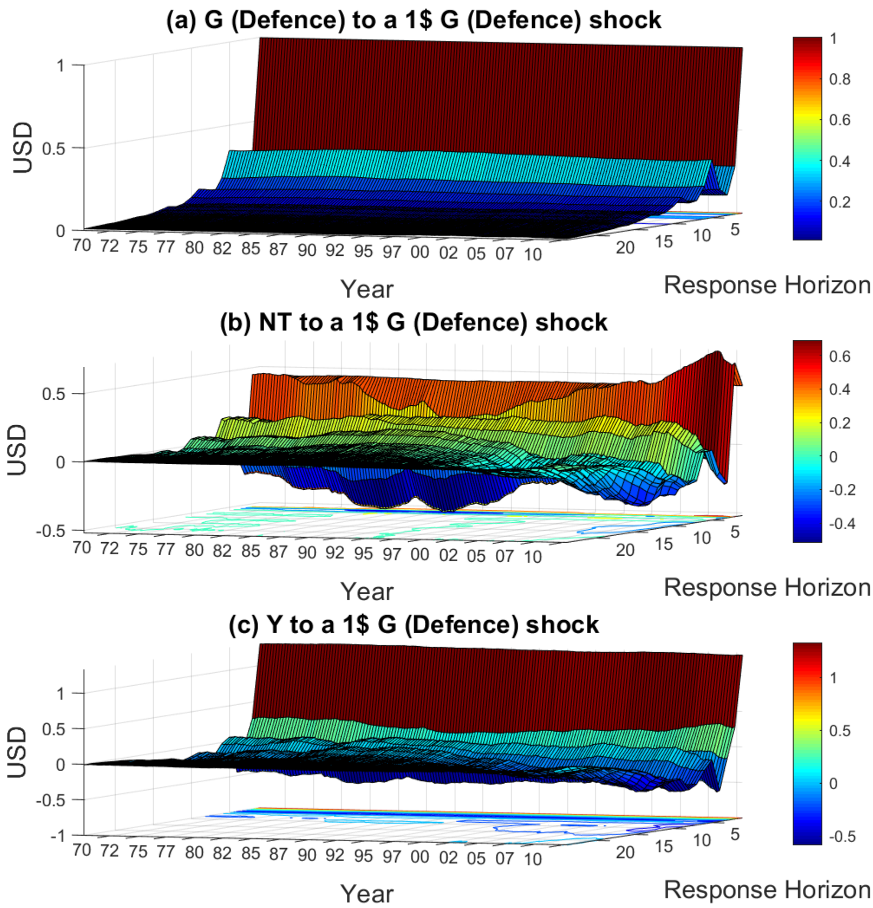
Figure 2.12.5: Unanticipated policy shocks. Median responses to tax shocks as functions of time.



Note: Government expenditure response (a), net taxes' response (b), and output responses (c) to a U.S.$1 output shock. Responses are measured in real U.S. dollars.

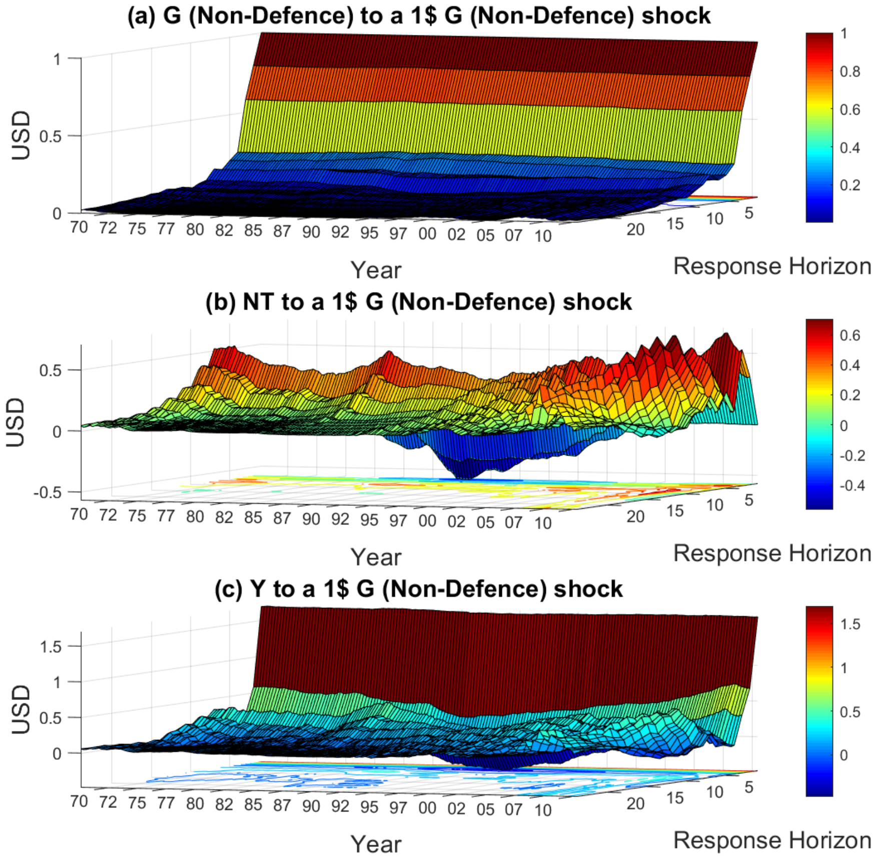
Figure 2.12.6: Unanticipated policy shocks. Median responses to output shocks as functions of time.

2.12.3 Components of Government Expenditure



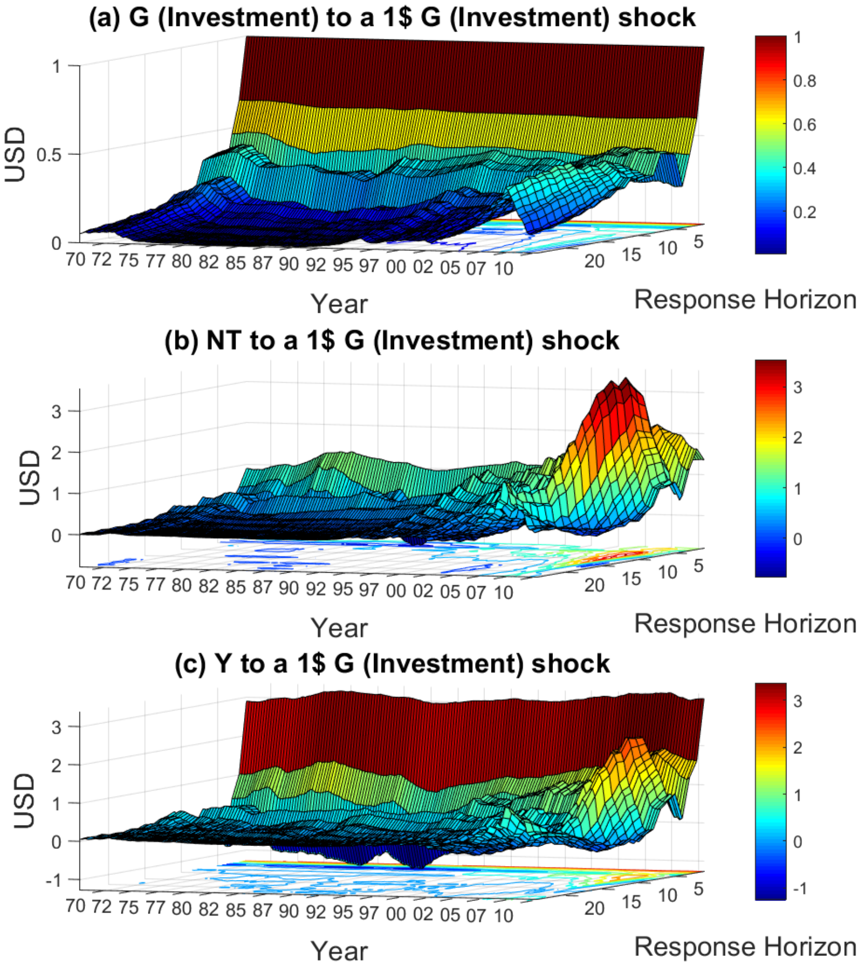
Note: Government expenditure response (a), net taxes' response (b), and output responses (c) to a U.S.$1 output shock. Responses are measured in real U.S. dollars.

Figure 2.12.7: Defence Expenditure. Median responses to discretionary government expenditure shocks as functions of time.



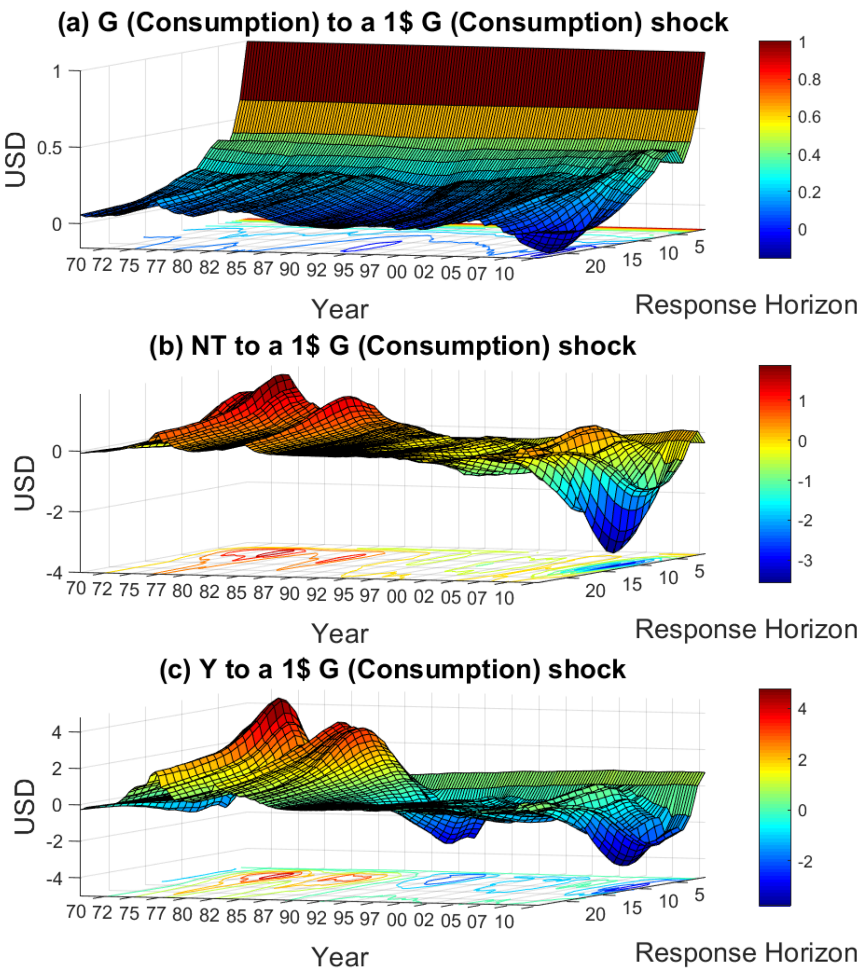
Note: Government expenditure response (a), net taxes' response (b), and output responses (c) to a U.S.$1 output shock. Responses are measured in real U.S. dollars.

Figure 2.12.8: Non-defence Expenditure. Median responses to discretionary government expenditure shocks as functions of time.



Note: Government expenditure response (a), net taxes' response (b), and output responses (c) to a U.S.$1 output shock. Responses are measured in real U.S. dollars.

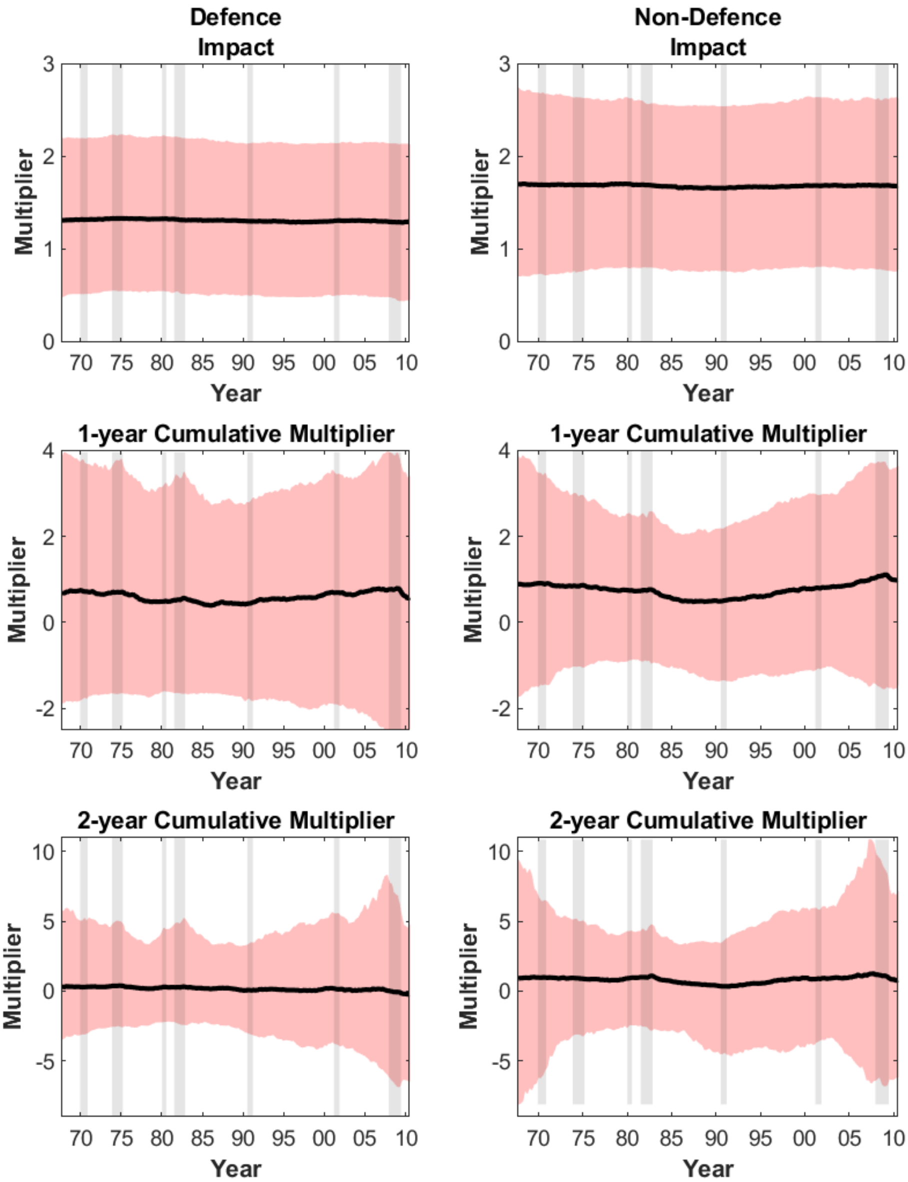
Figure 2.12.9: Public Investment Expenditure. Median responses to discretionary government expenditure shocks as functions of time.



Note: Government expenditure response (a), net taxes' response (b), and output responses (c) to a U.S.$1 output shock. Responses are measured in real U.S. dollars.

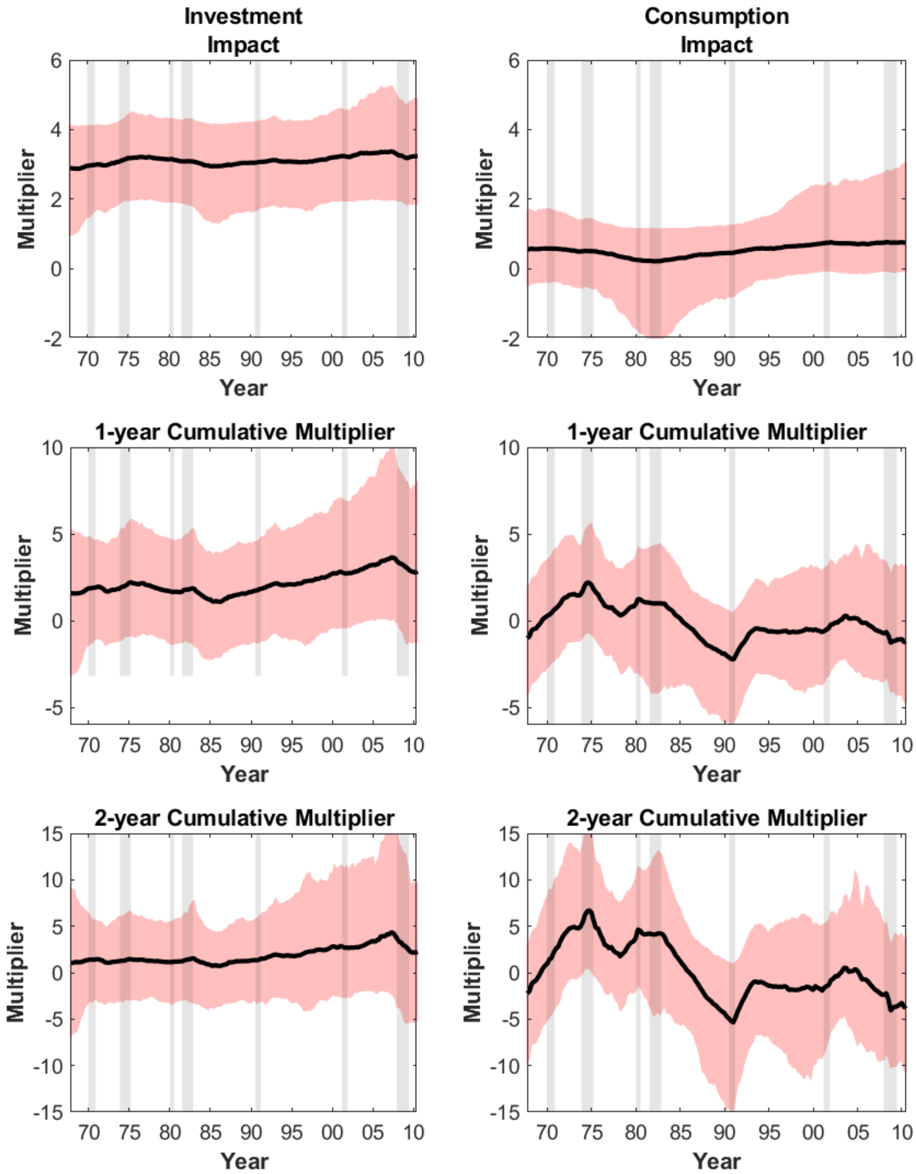
Figure 2.12.10: Public Consumption Expenditure. Median responses to discretionary government expenditure shocks as functions of time.

2.13 Time-varying Multipliers for Components of Government Expenditure



Note: Multipliers along with respective confidence bands, for defence (left) and non-defence (right) government expenditure. Confidence bands are in red, calculated as 5th and 95th percentiles of the posterior multiplier distributions.

Figure 2.13.1: Unanticipated defence and non-defence expenditure multipliers over NBER recession dates.



Note: Multipliers along with respective confidence bands, for government investment (left) and consumption (right) expenditure. Confidence bands are in red, calculated as 5th and 95th percentiles of the posterior multiplier distributions.

Figure 2.13.2: Unanticipated investment and consumption expenditure multipliers over NBER recession dates.

Chapter 3

Simple Optimised Rules, Multipliers and Welfare

3.1 Introduction

Recently, there has been a pronounced interest in analysing the interaction of monetary and fiscal policies in the New Keynesian general equilibrium framework. The relevant literature often emphasised the crucial role of deviating from conventional simplifying assumptions in favour of more complicated models characterised by various distortionary features, presence of non-Ricardian households, abnormal monetary policy regimes and a sophisticated fiscal toolkit. Thus, Leeper, Walker and Yang (2010), Christiano, Eichenbaum and Rebello (2011), Erceg and Linde (2014), among many others, emphasise the importance of considering such factors in evaluating the potency of government expenditure in stimulating output. Drautzburg and Uhlig (2015) extend the Smets and Wouters (2007) model to evaluate fiscal multipliers for components of government expenditure in a model that is both rich enough to mimic macroeconomic fluctuations observed in the empirical literature and includes most of the factors driving the size of fiscal multipliers outlined in the relevant literature. The optimal policy literature, on the other hand, often lacks such non-trivial modelling choices and rarely considers optimal policy analysis using components of government expenditure. This gap in research is especially evident in the case of the optimised simple rules. Initiated by Kollman (2003) and Schmitt-Grohe and Uribe (2007), the branch of literature considered a setting that allowed to evaluate optimal policy in the context of inefficient equilibria using higher-order approximations of the entire Dynamic Stochastic General Equilibrium (DSGE) model. Due to the complexity of the analysis, this line of research does not accommodate the whole spectrum of modelling features that are often considered imperative in empirical policy analysis using linearised DSGE models, such as Smets and Wouters (2007) or Drautzburg and Uhlig (2015). Cantore et al. (2019) constructed the richest New Keynesian setting in this line of research, yet, did not include many of such features. Keeping in mind the research gap outlined above, we seek to complement this branch of literature in the two following ways.

In the first part of the analysis, we seek to identify the welfare-maximising implementable monetary and fiscal policy rules. In contrast to relevant literature, our model allows focusing on the interaction of public investment and monetary policies, as opposed to the overall government expenditure often considered in the debate. We construct the analysis on the foundation of the Drautzburg and Uhlig (2015) model that allows us to isolate productive public investment from total government expenditure and study its implications on household welfare. We extend Schmitt-Grohe and Uribe (2007) modelling approach to a richer setting where a larger number of factors drive business cycles fluctuations. Our model is similar to Cantore et al. (2019), but, unlike it, we introduce nominal price and wage rigidities as Calvo (1983) prices and contracts and, therefore, allow for price and wage dispersions. Additionally, we increase the number of shocks affecting the evolution of the economy by including public-investment-specific shocks and shocks to the wedges on private capital markets and government borrowing. We also accommodate both intertemporally-optimising and rule-of-thumb households.

In contrast to relevant literature, we allow for quasi-kinked demand curves on intermediate goods and labour markets. A convention in the welfare analysis of optimal simple monetary and fiscal rules is to use constant elasticity of substitution (CES) demand curves via the Dixit-Stiglitz (1977) aggregator.

The use of non-CES demand curves, obtained via the Kimball (1995) aggregator, is often overlooked in the literature on optimal policy since, under a linear-quadratic approach of Woodford (2003), the choice of the aggregator does not play a significant role. On the other hand, welfare analysis using a second-order approximation of the entire model can, in principle, be sensitive to the choice of the aggregator. In other words, obtaining the second-order approximation of the structural equations results in different non-linear relative demand functions under Kimball and Dixit-Stiglitz aggregators. Recent contributions of Levin, Lopez-Salido and Yun (2007) and Linde and Trabandt (2018) emphasise the importance of using Kimball (1995) aggregator in optimal policy analysis and evaluation of fiscal multipliers in the context of non-linear models. Therefore, we are also interested in evaluating the implications of allowing for quasi-kinked demand curves in the analysis of simple optimised rules at the second order.

In the second part of the paper, we seek to determine the formulation of the simple monetary policy rule that yields a highest possible value of the public consumption multiplier (referred to as the multiplier-maximising formulation in the rest of the chapter). In the previous chapter, we have delivered evidence of a possible link between the size of the government expenditure multiplier and the monetary policy objective even at times when interest rates are not in a binding ZLB. We, therefore, are naturally interested in investigating the relationship between a systematic monetary policy formulation and the potency of fiscal stimulus. The monetary authority pursuing stabilisation policies can effectively diminish the size of the public consumption multiplier. An additional exogenous government consumption stimulus increases aggregate demand and results in higher inflation and widening output gap. The systematic response of the monetary authority, governed by a simple Taylor-type rule, will then seek to increase the policy rate in response to these developments, bringing the sticky-price output closer to its flexible-price counterpart. Since the long-run response of flexible-price output to a government consumption shock is negative, a strong enough monetary response to inflation or output gap will imply negative long-run multipliers, often presented in the relevant literature.

The degree of policy rate adjustment in response to inflationary pressures and the widening output gap in New Keynesian models with nominal rigidities is determined by the parametrisation of the simple monetary policy rule. By changing the values of the associated parameters, we can alter the size of the policy rate response and, therefore, adjust the strength of the monetary policy feedback mechanism. This feature of the model allows us to study the implications of the specification of monetary policy rules on the real effects of government consumption.

This chapter is structured as follows. Section 3.2 describes the derivation of non-linear equilibrium conditions on the goods and labour markets under Kimball and Dixit-Stiglitz aggregators. Section 3.3 outlines the calibration of model parameters. Section 3.4 describes the computation of optimised simple policy rules and carries out welfare analysis. Section 3.5 describes the approach to finding multiplier-maximising formulation of monetary policy and presents relevant results. Conclusion and appendices follow.

3.2 Model

We base our analysis on the New Keynesian model constructed in Drautburg and Uhlig (2015). The model is set up in a non-linear form as we are interested in the welfare analysis using a second-order approximation of the model equations. This choice has an attractive feature of allowing for quasi-kinked relative demand curves for intermediate goods and labour.

Quasi-kinked demand curves obtained using the Kimball (1995) technology are becoming central to the macroeconomic policy analysis. Levin, Lopez-Salido and Yun (2007) highlight importance of introducing such real rigidities in the optimal policy analysis and Linde and Trabandt (2017) argue that the positive effects of fiscal policy can be substantially overvalued in the presence of a linearised relative demand curve. Allowing for a non-CES demand curve implies that the elasticity of demand for intermediate goods is an increasing function of its relative price. Therefore, a small increase in the relative price that the intermediate producer sets will imply a substantial loss in quantity demanded. In the presence of nominal rigidities, such quasi-kinked demand curves can create a setup in which adjusting firms are hesitant to deviate away from the price of the non-adjusting firms. Thus, the quasi-kinked demand curve introduces the so-called strategic complementarity in the price-setting behaviour of intermediate producers. The non-CES demand for labour produces similar implications for the union's wage-setting problem.

Quasi-kinked demand curves are often used in linearised New Keynesian models but are widely omitted from the literature on optimal policy analysis. Unless all structural equations are approximated at order higher than one, the presence of the quasi-kinked demand curve will only imply a lower slope of the Philips curve than that under conventional Dixit-Stiglitz aggregator (given the same degree of nominal price stickiness). Therefore, it is crucial to obtain a second-order approximation of the entire non-linear model, to allow the inclusion of the Kimball aggregator to affect outcomes of our welfare analysis.

The Drautzburg and Uhlig (2015) model allows us to evaluate welfare implications of different policy formulations under a rich set of frictions, both real and nominal, an extensive government sector and two types of households supplying labour through unions. The economy consists of three blocks: Firms, Households and Government. Production sector consists of monopolistic differentiated intermediate inputs producers and a perfectly competitive final goods production market. Final goods producer uses intermediate producers' output as inputs in their production. Households are divided into two groups: Ricardian and rule-of-thumb. Both households supply their labour to unions that operate under monopolistic competition. Unions then provide labour to labour packers that aggregate it into homogenous packages in a perfectly competitive market. Government executes monetary policy using a Taylor-type rule and controls an elaborate set of fiscal policy instruments. The public sector generates revenues through distortionary labour, income and consumption taxes, although only labour tax rate is time-varying. Revenues are spent on investments in public capital, public consumption and constant transfer payment to households. Additionally, the government can maintain a fiscal deficit by issuing government bonds.

We modify the non-linear setup presented in Drautzburg and Uhlig (2015) to explicitly accommodate non-linear quasi-kinked demand curves on the goods and labour markets and relative distortions in prices and wages. Below we present the derivation of the goods and labour market equilibrium conditions, the part of the model which is set up differently than in Drautzburg and Uhlig (2015). Derivation of the rest of the model equations follows Drautzburg and Uhlig (2015) and is, therefore, left to the Appendix.

3.2.1 Goods market

Following Smets and Wouters (2007), the goods market is populated by two types of producers. First, there is a unit mass of intermediate producers indexed $i \in [0, 1]$, where each one is a monopolistic supplier of good i. Intermediate producers acquire factor inputs form the household sector and use them to produce intermediate goods that are later supplied to the representative final good producer. Second, the representative final producer operates in a perfectly competitive market, using intermediate goods as inputs in the production of final goods via the Kimball production (aggregation) technology.

3.2.1.1 Final good producers

A representative final goods producer seeks to maximise profit by altering its level of production (Y_t) and the quantity of intermediate inputs ($Y_t(i)$) demanded:

$$max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \tag{3.2.1}$$

subject to a Kimball (1995) production technology:

$$\int_0^1 G\left(\frac{Y_t(i)}{Y_t}; \epsilon_t^{\lambda,p}\right) di = 1$$

where P_t is the price of the final good, $P_t(i)$ is the price of intermediate input i and $\tilde{\epsilon}_t^{\lambda,p}$ is the markup shock that alters elasticity of substitution. Since we are interested in the higher order approximations of the model, the Kimball aggregator ($G\left(\frac{Y_t(i)}{Y_t}; \tilde{\epsilon}_t^{\lambda,p}\right)$) is assumed to have an explicit functional form:

$$G\left(\frac{Y_t(i)}{Y_t}; \epsilon_t^{\lambda,p}\right) = \frac{\phi_t^p}{1+\psi^p}\left[(1+\psi^p)\frac{Y_t(i)}{Y_t} - \psi^p\right]^{\frac{1}{\phi_t^p}} - \left[\frac{\phi_t^p}{1+\psi^p} - 1\right]$$

where:

$$\phi_t^p = \frac{(1+\lambda_t^p)(1+\psi^p)}{1+(1+\lambda_t^p)\psi^p}$$
$$\lambda_t^p = \lambda^p \epsilon_t^{\lambda,p}$$

and ψ^p governs the curvature of the relative demand and $\lambda^p \geq 0$ is the net markup.

Using the first-order conditions of the final producers optimisation problem (3.2.1), we can solve for the quasi-kinked relative demand on the goods market:

$$\widetilde{Y}_t(i) = \frac{1}{1+\psi^p}\left[\widetilde{P}_t(i)^{-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)}\left(\Lambda_t^p\right)^{\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} + \psi^p\right] \tag{3.2.2}$$

where $\widetilde{Y}_t(i) = \frac{Y_t(i)}{Y_t}$ is the relative quantity demanded at a relative price $\widetilde{P}_t(i) = \frac{P_t(i)}{P_t}$, that the intermediate producer i sets. Λ_t^p is the Lagrange multiplier in (3.2.1). Using $\int_0^1 G\left(G'^{-1}\left(\frac{\widetilde{P}_t(i)Y_tP_t}{\Lambda_t^p}\right)\right)di = 1$, we can show that Λ_t^p is a function of one of the three measures of relative price dispersion (derived further below):

$$\Lambda_t^p = \left(\int_0^1 \widetilde{P}_t(i)^{1-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)}di\right)^{\frac{1}{1-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)}} \tag{3.2.3}$$

Finally, using the zero-profit condition $Y_t = \int_0^1 \widetilde{P}_t(i)Y_t(i)di$, we can derive the aggregate price index:

$$1 = \frac{1}{1+\psi^p}\Lambda_t^p + \frac{\psi^p}{1+\psi^p}\int_0^1 \widetilde{P}_t(i)di \tag{3.2.4}$$

which yields an alternative definition of Λ_t^p:

$$\Lambda_t^p = 1 + \psi^p - \psi^p \int_0^1 \widetilde{P}_t(i)di \tag{3.2.5}$$

It is crucial to acknowledge at this point, that setting $\psi^p = 0$ delivers a simple Dixit-Stiglitz (1977) aggregator case:

$$\widetilde{Y}_t(i) = \widetilde{P}_t(i)^{-\frac{1+\lambda_t^p}{\lambda_t^p}}$$
$$\Lambda_t^p = 1$$

3.2.1.2 Intermediate good producers

Intermediate producers rent capital directly form the Ricardian households and hire labour from the labour market. They benefit from public capital that enters the production function as an externality to an individual intermediate producer. The production technolgy of the intermediate firms is given by:

$$Y_t(i) = \tilde{\epsilon}_t^a \left(\frac{K_{t-1}^g}{\int_0^1 Y_t(i)di + \phi\mu^t} \right)^{\frac{\zeta}{1-\zeta}} K_t(i)^\alpha \left[\mu^t n_t(i)\right]^{1-\alpha} - \mu^t \phi \tag{3.2.6}$$

where $Y_t(i)$ is the output of an intermediate firm i net of fixed costs, $K_t(i)$ and $n_t(i)$ represent capital services rented and labour hired by firm i in order to produce output, K_t^g is public capital, $\phi_t\mu^t$ is the fixed costs growing at the rate of labor augmenting technical progress and $\tilde{\epsilon}_t^a$ is the stationary Total factor productivity (TFP) process. Public capital increases total factor productivity subject to congestion effect as total production increases. Intermediate firms seek to minimise costs associated with production. They choose labor ($n_t(i)$) and capital ($K_t(i)$) to produce goods while taking wages (W_t) and price of capital (R_t^k) as given. The cost minimisation problem is given by:

$$min_{K_t(i),n_t(i)} W_t n_t(i) + R_t^k K_t(i)$$

subject to the production technology (3.2.6). The above optimisation problem results in the following optimal capital-to-labour ratio:

$$\frac{K_t}{n_t} = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k}$$

and marginal costs:

$$MC_t = (1-\alpha)^{\alpha-1}\alpha^{-\alpha}\frac{W_t^{1-\alpha}\left(R_t^k\right)^{\alpha}\mu^{-(1-\alpha)}}{\tilde{\epsilon}_t^a\left(\frac{K_{t-1}^g}{\int_0^1 Y_t(i)di+\phi\mu^t}\right)^{\frac{\zeta}{1-\zeta}}} \tag{3.2.7}$$

where MC_t represents nominal marginal costs. Since marginal costs and optimal capital-to-labour ratio do not depend on firm-specific decisions, they are equal across all intermediate producers. Finally, nominal firm-specific profits are given by:

$$\Pi_t^p(i)\mu^t P_t = P_t(i)Y_t(i) - W_t n_t - R_t^k K_t$$

where $\Pi_t^p(i)$ is the real de-trended profit of firm i.

3.2.1.3 Philips curve

Given Calvo-type price friction, it is assumed that only a proportion $(1-\zeta_p)$ of intermediate firms are allowed to change prices in period t. The rest of the firms index their prices by a weighted average of past and steady-state gross inflation $\left[\pi_{t-1}^{\iota^p}\bar{\pi}^{1-\iota^p}\right]$. At a given time, each intermediate firm which gets to change its price attempts to maximise profits given the probability that it can be stuck with the newly chosen price $(P_t^*(i))$ for a while. Thus, the problem of the intermediate firm can be defined as:

$$P_t^*(i) = max_{P^*t(i)}\mathbb{E}_t\sum_{s=0}^{\infty}\zeta_p^s\frac{\bar{\beta}^s\xi_{t+s}P_t}{\xi_t P_{t+s}}\left[P_t^*(i)\chi_{t,t+s}^p - MC_{t+s}(i)\right]Y_{t+s}(i) \tag{3.2.8}$$

subject to (3.2.2). where $\frac{\bar{\beta}^s \xi_{t+s}}{\xi_t}$is the stochastic discount factor of Ricardian households and $\chi^p_{t,t+s}$ is defined such that:

$$\chi^p_{t,t+s} = \begin{cases} 1 & s = 0 \\ \Pi^s_{l=0} \pi^{\iota^p}_{t+l-1} \bar{\pi}^{1-\iota^p} & s = 1, \ldots, \infty \end{cases}$$

So, in the absence of further price adjustments prices evolve as $P_{t+s}(i) = \chi^p_{t,t+s} P^*_t(i)$. The solution to (3.2.8) is given by:

$$0 = \mathbb{E}_t \sum_{s=0}^{\infty} \left(\bar{\beta}\zeta_p\right)^s \frac{\xi_{t+s}}{\xi_t} \frac{1 + \psi^p + \psi^p \lambda^p_{t+s}}{\lambda^p_{t+s}(1+\psi^p)} \left(\frac{\chi^p_{t,t+s}}{\pi_{t,t+s}}\right)^{1 - \frac{1+\lambda^p_{t+s}}{\lambda^p_{t+s}}(1+\psi^p)}$$
$$\left(\Lambda^p_{t+s}\right)^{\frac{1+\lambda^p_{t+s}}{\lambda^p_{t+s}}(1+\psi^p)} Y_{t+s} \widetilde{P}^*_t(i)$$
$$-\mathbb{E}_t \sum_{s=0}^{\infty} \left(\bar{\beta}\zeta_p\right)^s \frac{\xi_{t+s}}{\xi_t} \left(\frac{\chi^p_{t,t+s}}{\pi_{t,t+s}}\right)^{-\frac{1+\lambda^p_{t+s}}{\lambda^p_{t+s}}(1+\psi^p)} \left(\Lambda^p_{t+s}\right)^{\frac{1+\lambda^p_{t+s}}{\lambda^p_{t+s}}(1+\psi^p)} Y_{t+s}$$
$$\frac{1+\lambda^p_{t+s}}{\lambda^p_{t+s}} \frac{MC_{t+s}}{P_{t+s}}$$
$$- \mathbb{E}_t \sum_{s=0}^{\infty} \left(\bar{\beta}\zeta_p\right)^s \frac{\xi_{t+s}}{\xi_t} \frac{\psi^p}{1+\psi^p} \frac{\chi^p_{t,t+s}}{\pi_{t,t+s}} Y_{t+s} \left(\widetilde{P}^*_t(i)\right)^{1+\frac{1+\lambda^p_{t+s}}{\lambda^p_{t+s}}(1+\psi^p)} \tag{3.2.9}$$

where we used the fact that $\frac{\chi^p_{t,t+s}}{P_{t+s}} P^*_t(i) = \frac{\chi^p_{t,t+s}}{\pi_{t,t+s}} \widetilde{P}^*_t(i)$. Using the fact that $\frac{\chi^p_{t,t+1}}{\pi_{t,t+1}} = \frac{\bar{\pi}}{\pi_{t+1}}$, the solution expressed in (3.2.9) can be represented in a recursive form:

$$\widetilde{P}_t^* = \frac{Z_{2,t}}{Z_{1,t}} + \frac{Z_{3,t}}{Z_{1,t}} \left(\widetilde{P}_t^*\right)^{1+\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} \tag{3.2.10}$$

$$Z_{1,t} = y_t \left(\Lambda_t^p\right)^{\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} \frac{1+\psi^p+\psi^p\lambda_t^p}{\lambda_t^p(1+\psi^p)} + \mu\zeta^p\bar{\beta}\frac{\xi_{t+1}}{\xi_t} \mathbb{E}_t\left[\left(\frac{\pi_{t-1}^{\iota^p}\bar{\pi}^{1-\iota^p}}{\pi_{t+1}}\right)^{1-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} Z_{1,t+1}\right] \tag{3.2.11}$$

$$Z_{2,t} = y_t \frac{1+\lambda_t^p}{\lambda_t^p} mc_t \left(\Lambda_t^p\right)^{\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} + \mu\zeta^p\bar{\beta}\frac{\xi_{t+1}}{\xi_t} \mathbb{E}_t\left[\left(\frac{\pi_{t-1}^{\iota^p}\bar{\pi}^{1-\iota^p}}{\pi_{t+1}}\right)^{-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} Z_{2,t+1}\right] \tag{3.2.12}$$

$$Z_{3,t} = y_t \frac{\psi^p}{1+\psi^p} + \mu\zeta^p\bar{\beta}\frac{\xi_{t+1}}{\xi_t} \mathbb{E}_t\left[\frac{\pi_{t-1}^{\iota^p}\bar{\pi}^{1-\iota^p}}{\pi_{t+1}} Z_{3,t+1}\right]$$

where $mc_t = \frac{MC_t}{P_t}$ is the real marginal cost and $y_t = \frac{Y_t}{\mu^t}$ is the de-treded output. Since the optimal relative price of an individual intermediate producer (that gets a chance to update it) does not depend on the firm-specific factors, all of the firms will choose the same new optimal relative price, allowing to remove subscript i from $\widetilde{P}_t^*(i)$ in (3.2.10). The Dixit-Stiglitz case is given by assuming $\psi^p = 0$ and $\Lambda_t^p = 1$ in (3.2.9):

$$\widetilde{P}_t^* = \frac{Z_{2,t}}{Z_{1,t}}$$

$$Z_{1,t} = y_t \frac{1}{\lambda_t^p} + \mu\zeta^p\bar{\beta}\frac{\xi_{t+1}}{\xi_t} \mathbb{E}_t\left[\left(\frac{\pi_{t-1}^{\iota^p}\bar{\pi}^{1-\iota^p}}{\pi_{t+1}}\right)^{1-\frac{1+\lambda_t^p}{\lambda_t^p}} Z_{1,t+1}\right]$$

$$Z_{2,t} = y_t \frac{1+\lambda_t^p}{\lambda_t^p} mc_t + \mu\zeta^p\bar{\beta}\frac{\xi_{t+1}}{\xi_t} \mathbb{E}_t\left[\left(\frac{\pi_{t-1}^{\iota^p}\bar{\pi}^{1-\iota^p}}{\pi_{t+1}}\right)^{-\frac{1+\lambda_t^p}{\lambda_t^p}} Z_{2,t+1}\right]$$

3.2.1.4 Aggregation and relative price dispersion

Staggered prices setup generates relative price distortions. By integrating the production function (3.2.6) and substituting in the quasi-kinked relative demand (3.2.2), we can show that higher relative price distortions tend to diminish aggregate output:

$$\Delta_t^p y_t = \int_0^1 y_t(i)di = \epsilon_t^a \left(\frac{k_{t-1}^g}{\mu}\right)^{\zeta} k_t^{\alpha(1-\zeta)} n_t^{(1-\alpha)(1-\zeta)} - \phi$$

where $y_t(i) = Y_t(i)\mu^{-t}$ is the de-trended firm-specific output, $k_t^g = K_t^g \mu^{-t}$ and $k_t = K_t\mu^{-t}$represent de-trended public and private capital, respectively, and Δ_t^p is the overall measure of relative price distortion, defined as:

$$\Delta_t^p = \frac{1}{1+\psi^p}\int_0^1 [\widetilde{P}_t(i)^{-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} \left(\int_0^1 \widetilde{P}_t(i)^{1-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} di\right)^{\frac{\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)}{1-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)}} + \psi^p]di \tag{3.2.13}$$

Following Levin, Lopez-Salido and Yun (2007), overall relative price distortion in (3.2.13) can be represented as a non-linear function of two different measures of price dispersion:

$$\Delta_t^p = \frac{1}{1+\psi^p}\Delta_{1,t}^p \left(\Delta_{2,t}^p\right)^{\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} + \frac{\psi^p}{1+\psi^p}$$

where:

$$\begin{aligned} \Delta_{1,t}^p &= \int_0^1 \widetilde{P}_t(i)^{-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} di \\ \Delta_{2,t}^p &= \left(\int_0^1 \widetilde{P}_t(i)^{1-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} di\right)^{\frac{1}{1-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)}} \end{aligned} \tag{3.2.14}$$

Finally, using (3.2.3) and (3.2.5) and defining the third measure of price dispersion:

$$\Delta_{3,t}^{p} = \int_0^1 \widetilde{P}_t(i)di$$

we can show that the second measure of price dispersion $[\Delta_{2,t}^{p}]$ equals to the Lagrange multiplier of the final producer's optimisation problem $[\Lambda_t^p]$:

$$\Lambda_t^p = \Delta_{2,t}^{p} = \left(\int_0^1 \widetilde{P}_t(i)^{1-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} di\right)^{\frac{1}{1-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)}} = 1 + \psi^p - \psi^p \Delta_{3,t}^{p}$$

The price dispersion set-up, under Calvo-type price frictions, can be finalised by representing the three measures of price dispersion as:

$$\Delta_{1,t}^{p} = (1-\zeta_p)\,\widetilde{P}_t^*(i)^{-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} + \zeta_p \left(\frac{\pi_{t-1}^{\iota^p}\bar{\pi}^{1-\iota^p}}{\pi_t}\right)^{-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} \Delta_{1,t-1}^{p}$$

$$\begin{aligned}
\left(\Delta_{2,t}^{p}\right)^{1-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} &= (1-\zeta_p)\,\widetilde{P}_t^*(i)^{1-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} \\
&\quad + \zeta_p \left(\frac{\pi_{t-1}^{\iota^p}\bar{\pi}^{1-\iota^p}}{\pi_t}\Delta_{2,t-1}^{p}\right)^{1-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)}
\end{aligned}$$

$$\Delta_{3,t}^{p} = (1-\zeta_p)\widetilde{P}_t^*(i) + \zeta_p \left(\frac{\pi_{t-1}^{\iota^p}\bar{\pi}^{1-\iota^p}}{\pi_t}\right) \Delta_{3,t-1}^{p}$$

In the conventional Dixit-Stiglitz case, price dispersion reduces to:

$$\Delta_t^p = (1-\zeta_p)\,\widetilde{P}_t^*(i)^{-\frac{1+\lambda_t^p}{\lambda_t^p}} + \zeta_p \left(\frac{\pi_{t-1}^{\iota^p}\bar{\pi}^{1-\iota^p}}{\pi_t}\right)^{-\frac{1+\lambda_t^p}{\lambda_t^p}} \Delta_{1,t-1}^{p}$$

3.2.2 Labour market

The setup of the labour market is analogous to the goods market. Households supply homogenous labour to unions that differentiate it into heterogeneous packages indexed $l \in [0,1]$ and supply them to the labour packers that aggregate these varieties subject to a Kimball (1995) packing technology. Unions operate under monopolistic competition, while labour packers are perfectly competitive. Since the share of Ricardian households is higher than that of the rule-of-thumb households, the median voter rule applies. Unions operate for the benefit of the intertemporally optimising consumer. Rule-of-thumb households simply match the labour demand of the Ricardian households.

3.2.2.1 Labour packers

Labour packers buy differentiated labour from unions, pack them and supply to the producers of the intermediate goods. A representative labour packer seeks to maximise profit:

$$max_{n_t,n_t(l)} W_t n_t - \int_0^1 W_t(l) n_t(l) dl$$

subject to the 'packing' technology:

$$\int_0^1 H\left(\frac{n_t(l)}{n_t}; \epsilon_t^{\lambda,w}\right) dl = 1$$

where $H\left(\frac{n_t(l)}{n_t}; \epsilon_t^{\lambda,w}\right)$ is the Kimball aggregator, similar to $G\left(\frac{Y_t(i)}{Y_t}; \epsilon_t^{\lambda,p}\right)$ in the case of final producer. Functional form of the $H\left(\frac{n_t(l)}{n_t}; \epsilon_t^{\lambda,w}\right)$ is given by:

$$H\left(\frac{n_t(l)}{n_t}; \epsilon_t^{\lambda,w}\right) = \frac{\phi_t^w}{1+\psi^w}\left[(1+\psi^w)\frac{n_t(l)}{n_t} - \psi^w\right]^{\frac{1}{\phi_t^w}} - \left[\frac{\phi_t^w}{1+\psi^w} - 1\right]$$

where:

$$\phi_t^w = \frac{(1+\lambda_t^w)(1+\psi^w)}{1+(1+\lambda_t^w)\psi^w}$$
$$\lambda_t^w = \lambda^p \epsilon_t^{\lambda,w}$$

Keeping in mind that the labour packer's problem is analogous to the one of final good producer, the solution can be expressed as follows:

$$\tilde{n}_t(l) = \frac{1}{1+\psi^w}\left[\tilde{W}_t(l)^{-\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)}(\Lambda_t^w)^{\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)} + \psi^w\right] \tag{3.2.15}$$

$$\Lambda_t^w = 1 + \psi^w - \psi^w \int_0^1 \widetilde{W}_t(l)dl$$

while the aggregate wage index is given by:

$$1 = \frac{1}{1+\psi^w}(\Lambda_t^w)^{\frac{1}{1-\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)}} + \frac{\psi^w}{1+\psi^w}\int_0^1 \tilde{W}_t(l)dl$$

Assuming $\psi^w = 0$ we arrive at a standard Dixit-Stiglitz aggregator:

$$\tilde{n}_t(l) = \tilde{W}_t(l)^{-\frac{1+\lambda_t^w}{\lambda_t^w}}$$

$$\Lambda_t^w = 1$$

3.2.2.2 Labour unions

Unions maximise the expected present discounted value of future after-tax wage income earned over the cost of supplying labour. The cost of supplying labour equals to the marginal rate of substitution (MRS) between consumption and labour of a representative Ricardian household, or to put it simple, unions treat after-tax wages paid to the final household $[(1-\tau_t^n)W_t^h]$ as costs. Labour unions distribute these profits among Ricardian and rule-of-thumb households. A given union chooses the new optimal wage $[W_t^*(l)]$ keeping in mind that it will not be able to re-optimise later with a probability ζ_w. If the union is not able to reset its wage, it indexes existing wage by steady-state inflation and productivity growth. The union's problem is specified as follows:

$$W_t^*(l) = max_{W_t^*(l)} \mathbb{E}_t \Sigma_{s=0}^{\infty} \left(\zeta_w \bar{\beta}\right)^s \tag{3.2.16}$$
$$\frac{\xi_{t+s}}{\xi_t} \left[\left(1-\tau_{t+s}^n\right) W_t^*(l) \chi_{t,t+s}^w - (1-\tau_{t+s}^n) W_{t+s}^h\right] n_{t+s}(l)$$

subject to the relative demand for labour (3.2.15). $\chi_{t,t+s}^w$is defined as:

$$\chi_{t,t+s}^w = \begin{cases} 1 & s=0 \\ \Pi_{l=0}^s \mu \pi_{t+l-1}^{\iota^w} \bar{\pi}^{1-\iota^w} & s=1,\ldots,\infty \end{cases}$$

So, in the absence of further wage adjustments wages evolve as $W_{t+s}(l) = \chi_{t,t+s}^w W_t^*(l)$. The solution to (3.2.16) is then given by:

$$\begin{aligned} 0 = \mathbb{E}_t \Sigma_{s=0}^{\infty} \left(\zeta_w \bar{\beta}\right)^s \frac{\xi_{t+s}}{\xi_t} (1-\tau_{t+s}^n) n_{t+s} \left(\frac{1+\psi^w(1+\lambda_{t+s}^w)}{(1+\psi^w)\lambda_{t+s}^w}\right) \\ \left(\Lambda_{t+s}^w \pi_{t,t+s}^w\right)^{\frac{1+\lambda_{t+s}^w}{\lambda_{t+s}^w}(1+\psi^w)} \left(\chi_{t,t+s}^w\right)^{1-\frac{1+\lambda_{t+s}^w}{\lambda_{t+s}^w}(1+\psi^w)} \frac{\widetilde{W}_t^*(l)}{\pi_{t,t+s}} \\ - \frac{P_t}{W_t} \mathbb{E}_t \Sigma_{s=0}^{\infty} \left(\zeta_w \bar{\beta}\right)^s \frac{\xi_{t+s}}{\xi_t} n_{t+s} MRS_{t+s} \left(\frac{1+\lambda_{t+s}^w}{\lambda_{t+s}^w}\right) \left(\frac{\Lambda_{t+s}^w}{\chi_{t,t+s}^w} \pi_{t,t+s}^w\right)^{\frac{1+\lambda_{t+s}^w}{\lambda_{t+s}^w}(1+\psi^w)} \\ - \mathbb{E}_t \Sigma_{s=0}^{\infty} \left(\zeta_w \bar{\beta}\right)^s \frac{\xi_{t+s}}{\xi_t} (1-\tau_{t+s}^n) n_{t+s} \chi_{t,t+s}^w \frac{\psi^w}{1+\psi^w} \frac{\left(\widetilde{W}_t^*(l)\right)^{\frac{1+\lambda_{t+s}^w}{\lambda_{t+s}^w}(1+\psi^w)+1}}{\pi_{t,t+s}} \end{aligned}$$

where $\pi^w_{t,t+s} \equiv \frac{W_{t+1}}{W_t} \times \frac{W_{t+2}}{W_{t+1}} \times \cdots \times \frac{W_{t+s}}{W_{t+s-1}} = \frac{W_{t+s}}{W_t}$. The recursive-form solution to (3.2.16) is given by:

$$
\begin{aligned}
\widetilde{W}^*_t(l) &= \frac{1}{w_t}\frac{H_{2,t}}{H_{1,t}} + \left(\widetilde{W}^*_t(l)\right)^{\frac{1+\lambda^w_t}{\lambda^w_t}(1+\psi^w)+1}\frac{H_{3,t}}{H_{1,t}} \\
H_{1,t} &= (1-\tau^n_t)n_t\left(\frac{1+\psi^w+\psi^w\lambda^w_t}{(1+\psi^w)\lambda^w_t}\right)(\Lambda^w_t)^{\frac{1+\lambda^w_t}{\lambda^w_t}(1+\psi^w)} \\
&\quad + \mu\zeta^w\bar{\beta}\frac{\xi_{t+1}}{\xi_t}\frac{w_{t+1}}{w_t}\mathbb{E}_t\left[\left(\frac{w_t}{w_{t+1}}\frac{\pi^{\iota^w}_{t-1}\bar{\pi}^{1-\iota^w}}{\pi_{t+1}}\right)^{1-\frac{1+\lambda^w_t}{\lambda^w_t}(1+\psi^w)}H_{1,t+1}\right] \\
H_{2,t} &= \left(\frac{1+\lambda^w_t}{\lambda^w_t}\right)n_t\frac{MRS_t}{\mu^t}(\Lambda^w_t)^{\frac{1+\lambda^w_t}{\lambda^w_t}(1+\psi^w)} \\
&\quad + \mu\zeta^w\bar{\beta}\frac{\xi_{t+1}}{\xi_t}\mathbb{E}_t\left[\left(\frac{w_t}{w_{t+1}}\frac{\pi^{\iota^w}_{t-1}\bar{\pi}^{1-\iota^w}}{\pi_{t+1}}\right)^{-\frac{1+\lambda^w_t}{\lambda^w_t}(1+\psi^w)}H_{2,t+1}\right] \\
H_{3,t} &= (1-\tau^n_t)n_t\frac{\psi^w}{1+\psi^w} + \mu\zeta^w\bar{\beta}\frac{\xi_{t+1}}{\xi_t}\frac{\pi^{\iota^w}_{t-1}\bar{\pi}^{1-\iota^w}}{\pi_{t+1}}\mathbb{E}_t\left[H_{3,t+1}\right]
\end{aligned}
$$

Where $\frac{MRS_t}{\mu^t} = (1+\tau^c_t)\left(n^{RA}_t\right)^\nu\left[c^{RA}_t - \left(\frac{h}{\mu}\right)c^{RA}_{t-1}\right]$ from the household FOCs in the Appendix. Under the Dixit-Stiglitz aggregator, the non-linear Phillips curve reduces to:

$$
\begin{aligned}
\widetilde{W}^*_t(l) &= \frac{1}{w_t}\frac{H_{2,t}}{H_{1,t}} \\
H_{1,t} &= (1-\tau^n_t)n_t\frac{1}{\lambda^w_t} + \mu\zeta^w\bar{\beta}\frac{\xi_{t+1}}{\xi_t}\mathbb{E}_t\left[\frac{w_{t+1}}{w_t}\left(\frac{w_t}{w_{t+1}}\frac{\pi^{\iota^w}_{t-1}\bar{\pi}^{1-\iota^w}}{\pi_{t+1}}\right)^{1-\frac{1+\lambda^w_t}{\lambda^w_t}}H_{1,t+1}\right] \\
H_{2,t} &= (1-\tau^n_t)n_t\frac{MRS_t}{\mu^t}\left(\frac{1+\lambda^w_t}{\lambda^w_t}\right) \\
&\quad + \mu\zeta^w\bar{\beta}\frac{\xi_{t+1}}{\xi_t}\mathbb{E}_t\left[\left(\frac{w_t}{w_{t+1}}\frac{\pi^{\iota^w}_{t-1}\bar{\pi}^{1-\iota^w}}{\pi_{t+1}}\right)^{-\frac{1+\lambda^w_t}{\lambda^w_t}}H_{2,t+1}\right]
\end{aligned}
$$

3.2.2.3 Wage dispersion

Wage dispersion derivation follows the same steps as price dispersion. Relative wage distortion is given by:

$$
\begin{aligned}
\Delta_t^w &= \frac{1}{1+\psi^w}\Delta_{1,t}^w \left(\Delta_{2,t}^w\right)^{\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)} + \frac{\psi^w}{1+\psi^w} \\
\Delta_{1,t}^w &= (1-\zeta_w)\,\widetilde{W}_t^{*\,-\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)} \\
&\quad + \zeta_w \left(\frac{w_{t-1}}{w_t}\frac{\pi_{t-1}^{\iota^w}\bar{\pi}^{1-\iota^w}}{\pi_t}\right)^{-\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)} \Delta_{1,t-1}^w \\
\left(\Delta_{2,t}^w\right)^{1-\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)} &= (1-\zeta_w)\,\widetilde{W}_t^{*\,1-\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)} \\
&\quad + \zeta_w \left(\frac{w_{t-1}}{w_t}\frac{\pi_{t-1}^{\iota^w}\bar{\pi}^{1-\iota^w}}{\pi_t}\Delta_{2,t-1}^w\right)^{1-\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)} \\
\Delta_{3,t}^w &= (1-\zeta_w)\widetilde{W}_t^* + \zeta_w \left(\frac{w_{t-1}}{w_t}\frac{\pi_{t-1}^{\iota^w}\bar{\pi}^{1-\iota^w}}{\pi_t}\right)\Delta_{3,t-1}^w \\
\Delta_{2,t}^w &= \Lambda_t^w
\end{aligned}
$$

Setting $\psi^w = 0$, results in a standard Dixit-Stiglitz case:

$$
\Delta_t^w = (1-\zeta_w)\,\widetilde{W}_t^{*\,-\frac{1+\lambda_t^w}{\lambda_t^w}} + \zeta_w \left(\frac{w_{t-1}}{w_t}\frac{\pi_{t-1}^{\iota^w}\bar{\pi}^{1-\iota^w}}{\pi_t}\right)^{-\frac{1+\lambda_t^w}{\lambda_t^w}} \Delta_{1,t-1}^w
$$

3.3 Calibration

The model is calibrated based on the posterior mean of model parameters estimated using Bayesian methods in Drautzburg and Uhlig (2015), with one exception. Drautzburg and Uhlig (2015) assume a low value of the Kimball curvature [$\psi^p = -3.333$], which results in a rather high estimated value of the steady-state net markup on the goods market [$\lambda^p = 0.94$]. Following Linde and Trabandt (2018), we assume a more plausible value of the steady-state markup [$\lambda^p = 0.16$]. In order to preserve comparability with the Drautzburg and Uhlig (2015) estimates, we compensate by increasing the curvature of the quasi-kinked demand [$\psi^p = -5.57$]. Our combination of parameters produces an exact same slope of the linearised Philips curve as estimated in the Drautzburg and Uhlig (2015) since it is given by:

$$\kappa = \frac{(1-\zeta^p\bar{\beta}\mu)(1-\zeta^p)}{(1+\iota^p\bar{\beta\mu})\zeta^p}\frac{1}{1-(1+\lambda^p)\psi^p} \tag{3.3.1}$$

Tables 3.3.1 and 3.3.2 present the parameter values used to run our analysis. Any deviations from these are expliciteley specified in the text that follows.

Parameter		Value
Discount factor	β	0.999
Risk aversion	σ	1.17
Habit	h	0.85
Inverse labour supply elasticity	ν	2.16
Capital utilisation	ψ_z	0.43
Elasticity of private investment adjustment costs	ϕ_x	2.255
Trend	μ	1.0048
Capital share	α	0.24
Goods demand curvature	ψ^p	-5.6
Goods markup	λ^p	0.16
Clavo prices	ζ_p	0.81
Price indexation	ιp	0.28
Labour demand curvature	ψ^w	-3.33
Wage markup	λ^w	0.5
Calvo wages	ζ_w	0.83
Wage indexation	ιw	0.41
Elasticity of public investment adjustment costs	ϕ_x^g	3.555
Speed of budget balance adjustment	ψ_τ	0.03375
Interest rate smoothing	ρ_R	0.92
Monetary policy repsonse to inflation	ρ_π	1.63
Monetary policy response to long-run output gap	ρ_y	0.13
Monetary policy response to short-run output gap	$\rho_{\Delta y}$	0.2
Wage markup	λ^w	0.5
Share of RoT households	φ	0.25
Capital depreciation rate	δ	0.0145
Government-spending+net-exports-to-GDP	$\frac{g}{y}$	0.153
Government-investment-to-GDP	$\frac{x^g}{y}$	0.04
Government-spending+net-exports-to-GDP	$\frac{g}{y}$	0.153
Government-investment-to-GDP	$\frac{x^g}{y}$	0.04
Debt-to-GDP	$\frac{b}{y}$	4×0.63
Consumption tax	τ^c	0.05
Capital tax	τ^k	0.36

Table 3.3.1: Parameter values used to run the analysis.

Parameter		Value
Persistence of wage markup shocks	$\rho_{\lambda,w}$	0.97
Moving-average parameter of wage markup shocks	$\theta_{\lambda,w}$	0.92
Standard deviation of wage markup shocks	$\sigma^2_{\lambda,w}$	0.23
Persistence of price markup shocks	$\rho_{\lambda,p}$	0.91
Moving-average parameter of price markup shocks	$\theta_{\lambda,p}$	0.96
Standard deviation of price markup shocks	$\sigma^2_{\lambda,p}$	0.32
Persistence of technology shocks	ρ_a	0.95
Standard deviation of technology shocks	σ^2_a	0.47
Persistence of monetary policy shocks	ρ_r	0.22
Standard deviation of monetary policy shocks	σ^2_r	0.22
Persistence of tax shocks	ρ_τ	0.98
Standard deviation of tax shocks	σ^2_τ	1.44
Persistence of bond-FFR wedge shocks	ρ_b	0.67
Standard deviation of bond-FFR wedge shocks	σ^2_b	0.95
Persistence of private-public-bond wedge shocks	ρ_k	0.91
Standard deviation of private-public bond wedge shocks	σ^2_k	0.08
Persistence of private-investment-specific shocks	ρ_x	0.56
Standard deviation of private-investment-specific wedge shocks	σ^2_x	1.25
Persistence of public-investment-specific shocks	$\rho_{x,g}$	0.97
Standard deviation of public-investment-specific wedge shocks	$\sigma^2_{x,g}$	0.79
Persistence of public consumption shocks	ρ_g	0.98
Covariance of technology shock and public consumption	σ_{ga}	0.3
Standard deviation of public consumption shocks	σ^2_g	0.36

Table 3.3.2: Parameter values used to run the analysis (Cont.).

3.4 Welfare analysis using optimised simple rules

The conventional approach to optimal policy analysis often considers relative welfare implications of switching from a fully optimal Ramsey policy to alternative policy specifications, such as discretionary policy or optimised simple Taylor-type rules. In contrast to the convention, we seek to determine whether the U.S. Federal Reserve followed a welfare-maximising simple monetary policy rule and if not – to what degree would household welfare improve if the policymaker were to switch to a welfare-maximising formulation of a simple rule? The historical monetary policy formulation is represented by a Taylor-type rule, estimated in Drautzburg and Uhlig (2015) on a U.S. data from 1948Q2 to 2008Q4.

$$log\left(\frac{R_t^{FFR}}{R^{\bar{F}FR}}\right) = 0.92log\left(\frac{R_{t-1}^{FFR}}{R^{\bar{F}FR}}\right) + 0.13log\left(\frac{\pi_t}{\bar{\pi}}\right) + 0.01log\left(\frac{y_{t-1}}{y_{t-1}^f}\right) + 0.2\left[log\left(\frac{y_t}{y_t^f}\right) - log\left(\frac{y_{t-1}}{y_{t-1}^f}\right)\right] \quad (3.4.1)$$

Although the stance of the U.S. monetary policy has experienced significant shifts during this period, Bayesian estimation of the simple monetary policy rule delivers an adequate measure of the average rule according to which nominal interest rate was adjusted.[1]

Keeping in mind the level of household welfare implied by the historical rule, we search for parametrisation of a simple implementable rule that maximises household welfare. A simple implementable rule constitutes a more realistic instrument for the welfare maximisation problem at hand. Since the true output gap is not readily available, results obtained by maximising welfare using a full Taylor-type rule, such as the one expressed in (3.4.1),

[1] In such a framework, any significant deviation from the estimated average rule will be accommodated by the monetary policy shock, that is absent in the monetary rule formulation (3.4.1).

will tend to produce welfare levels that are unattainable for a policymaker in the realistic setting. We compare the two cases by calculating the welfare gain of switching from the historical to the optimal formulation of the simple monetary policy rule, measured in per cent permanent consumption change.

We then focus on the welfare implications of using a simple implementable rule in which public investment responds to debt and output fluctuations. Conditional on the historical formulation of the monetary policy rule, we seek to identify a rule for public investment that maximises household welfare. In this case, we compare the resulting household welfare with the level obtained under the default public investment mechanism. In the default Dtrautzburg and Uhlig (2015) setting, public investment is chosen optimally to maximise the discounted future stream of output net of public investment. In other words, we seek to evaluate welfare gain associated with a public investment policy that contributes to debt sustainability versus a policy that purely aims at stimulating output.

Finally, we maximise household welfare by adjusting both simple rules simultaneously. In this case, both nominal interest rate and public investment level are determined by implementable rules, but, in contrast to the previous part of the analysis, the monetary policy rule is not fixed at the historical formulation.

Since there are two types of households, the objective of the policymaker is to maximise the population-weighted (or overall) welfare. We present the welfare gain separately for Ricardian and rule-of-thumb households. Drautzburg and Uhlig (2015) note that rule-of-thumb households discount future more than Ricardian households; therefore, we repeat our analysis, assuming a set of different values of the rule-of-thumb discount parameter.

Finally, we evaluate the sensitivity of the optimised simple monetary policy rule results to the choice of an aggregator. As discussed in Linde and Trabandt (2018), implications of Dixit-Stiglitz (1997) and Kimball (1997) aggregators of intermediate goods tend to differ if the DSGE models are set-up non-linearly. Indeed, the difference between the Dixit-Stiglitz and Kimball aggregators in a linearised model sums up to an additional term that reduces the slope of the Philips curve. On the other hand, at the second-order approximation, the real rigidity arising under Kimball aggregator diminishes the responsiveness of price and wage dispersion to exogenous shocks. Keeping in mind, that the convention in optimal policy analysis is to use the Dixit-Stiglitz aggregator, we seek to determine if the use of Dixit-Stiglitz aggregator would imply different parameter values of our optimised simple implementable rules.

3.4.1 Simple implementable rules

We perform the analysis using an implementable policy rule as in Cantore et. at (2019). The monetary policy rule is characterised by:

$$\log\left(\frac{R_t}{\bar{R}}\right) = \rho_R log\left(\frac{R_{t-1}}{\bar{R}}\right) + \rho_\pi\left(\frac{\pi_t}{\bar{\pi}}\right) + \rho_y\left(\frac{y_t}{\bar{y}}\right) \tag{3.4.2}$$

The monetary authority sets the current interest rate according to a linear function in current inflation rate and sticky-price output, both in deviations from their respective steady-states, subject to some degree of persistence in the interest rate. We seek to identify the welfare-maximising formulation of the rule by searching through a grid of the rule's parameters, as proposed by Schmitt-Grohe and Uribe (2007). To support the procedure, we introduce a set of feasibility constraints on the baseline rule: $0 < \rho_R, \rho_y < 1$ and $0 \leq \rho_\pi < 5$.

In the second part of the analysis, we replace the default public investment setup with an implementable public investment rule. In this case, public investment reacts to past public debt and public investment expenditure, as well as current output, all in deviations from respective steady-states:

$$log\left(\frac{x_t^g}{\bar{x}^g}\right) = \rho_g log\left(\frac{x_{t-1}^g}{\bar{x}^g}\right) - \rho_d log\left(\frac{b_{t-1}}{\bar{b}}\right) - \rho_{gy} log\left(\frac{y_{t-1}}{\bar{y}}\right) \tag{3.4.3}$$

Since the grid search procedure becomes computationally demanding, we need to introduce an additional set of feasibility constraints: $0 < \rho_g, \rho_d, \rho_{gy} < 1$.

3.4.2 Welfare measure

To evaluate welfare losses associated with different monetary and fiscal policy formulations, we calculate unconditional expectations of life-time welfare using a second-order approximation of the entire DSGE model. This choice allows us to perform welfare analysis by approximating the model around the deterministic distorted equilibrium, in contrast to approximating it around the efficient allocation of resources (Woodford, 2003). Since we have two types of households in the model, we can disentangle the welfare implication of Ricardian and rule-of-thumb households. We define the non-linear life-time welfare functions of the two types of households as follows:

1. Ricardian household

$$\mathcal{W}_t^{RA} = \left[\frac{1}{1-\sigma}\left(c_t^{RA} - \frac{h}{\mu}c_{t-1}^{RA}\right)^{1-\sigma}\right] exp\left[\frac{\sigma-1}{1+\nu}\left(n_t\Delta_t^w\right)^{1+\nu}\right] + \beta\mu^{1-\sigma}\mathbb{E}_t\left[\mathcal{W}_{t+1}^{RA}\right]$$

2. Rule-of-thumb household

$$\mathcal{W}_t^{RoT} = \left[\frac{1}{1-\sigma}\left(c_t^{RoT} - \frac{h}{\mu}c_{t-1}^{RoT}\right)^{1-\sigma}\right] exp\left[\frac{\sigma-1}{1+\nu}\left(n_t\Delta_t^w\right)^{1+\nu}\right] + \beta\mu^{1-\sigma}\mathbb{E}_t\left[\mathcal{W}_{t+1}^{RoT}\right]$$

where $\beta^{RA} \neq \beta^{RoT}$ since non-Ricardian households are assumed to be more impatient than the intertemporally-optimising households. Following Drautzburg and Uhlig (2015), we evaluate the sensitivity of the welfare analysis to the choice of the RoT discount parameter. We consider three values of β^{RoT} in our analysis, $\frac{1}{\beta^{RoT}} \in [\frac{1}{\beta^{RA}}, \frac{1}{\beta^{RA}} + 0.06/4, \frac{1}{\beta^{RA}} + 0.3/4]$. Since comparing resulting welfare values directly in utils has obvious limitations, we follow the general suite of complementing the analysis using consumption units. We, therefore, are interested in identifying a compensating increase in permanent consumption that households need to remain under the historical formulation of the policy and not to be subject to alternative policy scenarios. To measure welfare losses in units of consumption, we need to solve the following equation for λ:

$$\mathbb{E}_t\left[\mathcal{W}_t^H(\lambda)\right] = \mathbb{E}_t\left[\mathcal{W}_t^B\right]$$

where λ is the increase in permanent consumption a household under historical policy formulation needs to be indifferent to the prospect of being subject to alternative policy.$\mathcal{W}_t^B$ is the lifetime welfare under alternative policy (policy B) and, since λ is a constant that shows up in each period, $\mathcal{W}_t^H(\lambda)$ is characterised by:

$$\mathbb{E}_t\left[\mathcal{W}_t^H(\lambda)\right] = (1+\lambda)^{(1-\sigma)}\mathbb{E}_t\left[\mathcal{W}_t^H\right]$$

where $\mathcal{W}^H$ is the life-time welfare under historical policy formulation. Therefore, knowing values of $\mathbb{E}\left[\mathcal{W}_t^H\right]$ and $\mathbb{E}\left[\mathcal{W}_t^B\right]$ we have a closed form solution for λ:

$$\lambda = \left(\frac{\mathbb{E}\left[\mathcal{W}_t^B\right]}{\mathbb{E}\left[\mathcal{W}_t^H\right]}\right)^{\frac{1}{1-\sigma}} - 1$$

The policy maker seeks to maximise the population-weighted welfare, that is given by a weighted average of welfare measures for two types of households:

$$\mathcal{W}_t^{TOTAL} = (1-\varphi)\mathcal{W}_t^{RA} + \varphi\mathcal{W}_t^{RoT}$$

3.4.3 Results

Table 3.4.1 contains the welfare gain for the various policy scenarios—monetary policy rule under default public investment mechanism (M), public investment rule under the historical formulation of the monetary policy rule (F-HM), and result for optimised public investment and monetary policy rules (M-F). Simple rules are optimised for three different values of the RoT discount parameters—$\frac{1}{\beta^{RoT}} \in [\frac{1}{\beta^{RA}}, \frac{1}{\beta^{RA}} + 0.06/4, \frac{1}{\beta^{RA}} + 0.3/4]$. We present welfare gains separately for both Ricardian and RoT households in consumption-equivalent terms.

First, the historical monetary policy rule in (3.4.1) does not constitute an optimised simple rule, as population-weighted welfare can be improved by deviating from it. Optimised simple monetary policy rule yields a welfare gain of 0.56% for Ricardian and 0.77% for RoT households, compared to the historical policy formulation. This result is mostly driven by a larger response to inflation under the optimised implementable rule.

Similar to Schmitt-Grohe and Uribe (2007), the optimised simple implementable monetary policy rule is characterised by an inflation response parameter $[\rho_\pi]$ at the upper bound of the considered rage, regardless of the considered policy scenario. The optimised response to output fluctuations is higher than that determined in Schmit-Grohe and Uribe (2007), especially in the case when RoT household discount future at the same rate as Ricardian households.

As we show later on in this section (Table 3.4.2), this difference in results arises due to the presence of wage stickiness in our model. In the absence of nominal wage rigidities, as in the model constructed by Schmitt-Grohe and Uribe (2007), our results support their argument that optimised monetary policy rule is characterised by an absent response to output fluctuations.

Second, optimised simple public investment rule does not perform the role of automatic stabiliser, as the response to sticky-price output $[\rho_{gy}]$ equals to zero, the lower-bound value of a considered range. Under the optimised simple fiscal rule public investment only reacts to debt subject to a high degree of persistency in investment expenditure. Although this rule facilitates debt sustainability, as the fiscal authority will reduce investment in public capital at times of rising debt, it, nevertheless, constitutes a passive fiscal policy rule. A 1% increase in real de-trended debt will imply only a 0.28-0.32% fall in public investment, depending on the chosen RoT discount factor.

Surprisingly, such a debt-stabilising simple rule can deliver welfare levels comparable to the public investment policy aiming to maximise output. Switching from the output-maximising public investment policy to a simple implementable rule a-la Cantore et al. (2019) results in a 0.03% increase in Ricardian and 0.004% decrease in RoT welfare. Overall, population welfare rises, but by a negligible amount.

Third, under the optimised combination of both rules monetary policy is active, moreover, it represents a price level rule if RoT households are assumed to discount future at the same rate as Ricardian households. The optimised public investment rule is similar to the case when the monetary policy rule is fixed at historical formulation but is characterised by a lower reaction to past debt and lower persistence of public investment. It is still sub-optimal for public investment to respond to deviations of sticky-price output. Overall, under optimised simple monetary policy rule, there seems to be less scope for the public investment as an instrument of improving household welfare.

Rule	ρ_r	ρ_π	ρ_y	ρ_g	ρ_d	ρ_{gy}	RA	ROT
							$\lambda \times 100$	$\lambda \times 100$
$\frac{1}{\beta^{ROT}} = \frac{1}{\beta^{RA}} + 0.3/4$								
M	0.7428	5	0.042	n/a	n/a	n/a	0.56	0.77
M-F	0.8008	5	0.064	0.8418	0.1951	0	0.51	0.83
F-HM	fixed at hist. values			0.9191	0.3183	0	0.03	-0.005
$\frac{1}{\beta^{ROT}} = \frac{1}{\beta^{RA}} + 0.06/4$								
M	0.7925	5	0.061	n/a	n/a	n/a	0.55	0.81
M-F	0.8506	5	0.087	0.8442	0.2011	0	0.51	0.87
F-HM	fixed at hist. values			0.9198	0.3099	0	0.03	-0.004
$\frac{1}{\beta^{ROT}} = \frac{1}{\beta^{RA}}$								
M	0.9591	5	0.136	n/a	n/a	n/a	0.54	0.91
M-F	1	5	0.166	0.8436	0.2403	0	0.49	0.98
F-HM	fixed at hist. values			0.9209	0.2774	0	0.03	-0.002

Table 3.4.1: Optimal policy results for alternative scenarios.

Results discussed above are obtained using a second-order approximation of solutions to the intermediate producer's and labour union's problems. As we have shown in Section 3.2, our model makes use of the Kimball (1995) production (packing) technology for intermediate goods (labour packages). The results of our optimised simple rules analysis are sensitive to the choice of the aggregator and presence of staggered contracts. Table 3.4.2 below presents the optimised simple monetary policy rules under two different aggregators, as well as assuming flexible wages.

Following Linde and Trabandt (2018), we preserve comparability between the two types of aggregators by increasing the share of non-adjusting firms and unions in the Dixit-Stiglitz case. The slope of the linearised Philips curve depends on the curvature of the relative demand. If we assume no real rigidities in prices and wages [$\psi^w = \psi^p = 0$], the slopes of the respective linearised Philips curves will be steeper unless we compensate by increasing the severeness of nominal rigidities. Using $\zeta^w = 0.869$ and $\zeta^p = 0.856$ in case of the Dixit-Stiglitz aggregator, we eliminate the difference between implications of the two aggregators in a linearised model. Making sure that the linearised model is not affected by choice of the aggregator, we can focus on the differences arising at higher orders.[2]

The use of the CES demand curve results in a higher reaction to sticky-price output in the optimised simple monetary policy rule. Due to the effect of strategic complementarities, deviations of price (wage) dispersion are of lower magnitude in case of the Kimball aggregator, as firms (unions) that have a chance to set a new price (wage) tend to remain close to the aggregate price (wage) level. Therefore, the distortionary effects of exogenous shocks on the aggregate output, that transmit through relative price and wage distortions, are of a lower magnitude under quasi-kinked demand curves. The monetary authority does not need to react to sticky-price output under the Kimball aggregator to the extent it needs to react in the Dixit-Stiglitz case.

We also show that under the assumption of flexible wages, our results are consistent with Schmitt-Grohe and Uribe (2007). Optimised reaction to the

[2]Drautzburg and Uhlig (2015) estimate the slopes of the Phillips curve to be $\kappa^p = 0.0201$ and $\kappa^w = 0.0104$ for inflation and wages respectively. Since the respective slopes are given by $\kappa^p = \frac{(1-\zeta^p\bar{\beta}\mu)(1-\zeta^p)}{(1+\iota^p\bar{\beta}\mu)\zeta^p}\frac{1}{1-(1+\lambda^p)\psi^p}$ and $\kappa^w = \frac{(1-\zeta^w\bar{\beta}\mu)(1-\zeta^w)}{(1+\bar{\beta}\mu)\zeta^w}\frac{1}{1-(1+\lambda^w)\psi^w}$, our choice of the stickiness parameters results in exact values of κ^p and κ^w if $\psi^w = \psi^p = 0$.

Rule	ρ_r	ρ_π	ρ_y
Kimball			
Sticky prices and wages ($\zeta^p, \zeta^w \neq 0$)	0.7428	5	0.042
Sticky prices and flexible wages ($\zeta^p \neq 0$,$\zeta^w = 0$)	0.8276	5	0
Dixit-Stiglitz			
Sticky prices and wages ($\zeta^p, \zeta^w \neq 0$)	0.7947	5	0.3090
Sticky prices and flexible wages ($\zeta^p \neq 0$,$\zeta^w = 0$)	0.8476	5	0

Table 3.4.2: Optimal policy results for alternative aggregators.

deviations in output is zero if we follow the authors in assuming that only prices are sticky. This result is impervious to the choice of the aggregator. If we allow for sticky wages, our results are in line with Adjemian, Paries and Moyen (2007) that show a non-zero optimised reaction to the output gap in a non-linear Smets and Wouters (2007) model using the Dixit-Stiglitz aggregator.

3.5 Fiscal multiplier and monetary policy stance

As the importance of central banks in managing economic activity was increasing over the course of the last four decades, their toolkit was expanding to meet the challenge. Even more so in recent years, when central banks took on the responsibility for managing financial stability. This development raised the importance of studying interactions of various policy arms in the pursuit of producing a reliable schematics for the policy transmission mechanisms. Nevertheless, an even older question of interaction between monetary and fiscal policies, as the two most prominent tools of economic regulation, remains of great interest. The debate concerning the size of government expenditure impact on economic growth often avoids analysing implications of the commitment of the monetary authority to a certain monetary policy rule.

The general equilibrium framework tends to deliver multipliers in the -2.5 to 2.3 range, depending on various assumption and variety of scenarios.

Thus, purely Neoclassical models tend to emphasise the role of shock duration, and it's financing (Baxter and King, 1993). New Keynesian literature seeks to identify scenarios under which the government expenditure multiplier is greater than one (Ramey, 2011). Some of such assumptions include a higher share of rule-of-thumb households, substantial price stickiness or a binding Zero-Lower Bound and the composition of the stimulus package (Leeper, Walker and Yang, 2010, Christiano, Eichenbaum and Rebello, 2011 and Erceg and Linde, 2014, Drautzburg and Uhlig, 2015, etc.). Following the logic of Christiano, Eichenbaum and Rebelo (2011), the government expenditure multiplier at period t can be represented using the linearised aggregate resource constraint (equation 3.10.17 in the Appendix 3.10):

$$\frac{dY}{dG} = \frac{\hat{y}_t}{\hat{g}_t} = \frac{1}{\bar{y}} \frac{(1-\varphi)\bar{c}^{RA}\hat{c}_t^{RA} + \varphi\bar{c}^{RoT}\hat{c}_t^{RoT} + \bar{x}\hat{x}_t + \bar{x^g}\hat{x}_t^g + \bar{r^k}\bar{k}\hat{u}_t}{\hat{g}_t} + 1 \quad (3.5.1)$$

where $\hat{y}_t$ and $\hat{g}_t$ represent output and government consumption, respectively, both in deviations from steady-state output. The change in private consumption exenditure is a weighted average of changes in consumtpion of Ricardian $[\hat{c}_t^{RA}]$ and rule-of-thumb $[\hat{c}^{ROT}]$ households. Private and public investment are denoted as $\hat{x}_t$ and $\hat{x}_t^g$ respectively and $\hat{u}_t$ stands for the capacity utilisation in deviations from steady-state.

Equation (3.5.1) implies that the necessary condition for the multiplier to be higher than one can be expressed as: $\frac{\bar{c}}{\bar{y}}\hat{c}_t + \frac{\bar{x}}{\bar{y}}\hat{x}_t + \frac{\bar{x^g}}{\bar{y}}\hat{x}_t^g + \frac{\bar{r^k}\bar{k}}{\bar{y}}\hat{u}_t > 0$. In other words, the overall change in private expenditure, public capital and return on capital induced by non-steady-state capacity utilisation, all measured in output units, should be positive. Baxter and King (1993) show that a temporary persistent government purchases shock crowds out private consumption and investment of Ricardian households through a negative wealth effect. As rational agents anticipate an increase in tax rates, private consumption and investment fall. An immediate adjustment in the nominal interest rate, necessary to clear the public bond market under lower household income follows. Since public investment is chosen optimally, a higher interest rate implies a lower shadow price of public capital. Under those circumstances, the fiscal authority cuts public investment in order to compensate for the increase in public consumption. Capacity utilisation and consumption of rule-of-thumb households are the only positive terms in the numerator of (3.5.1). Lower private consumption of Ricardian households incentivises them to supply more labour. An increase in labour translates into higher consumption of the RoT households, as they consume all their income immediately. An increase in labour also pushes the marginal product of capital up, as intermediate producers face increasing quantities of labour per unit of capital. Facing both higher interest rate and return on capital, Ricardian households choose to respond to the higher demand for effective capital by increasing utilisation of existing physical capital stock.

The negative consequences of rising nominal rates can be partially eliminated in the presence of sticky prices. Since prices are sticky, firms will respond to an increase in aggregate demand by increasing production. As a consequence, marginal costs and inflation rise. Higher inflation partially offsets the increase in the real interest rate. Since the real interest rate increase in the sticky-price economy is lower than that observed in a flexible-price economy, the fall in Ricardian consumption and investment is more muted. The fall in public investment is smaller as well, as higher inflation tends to increase the shadow price of public capital. The increase in capacity utilisation is greater in the sticky-price economy since there is a more pronounced increase in the marginal costs.

Therefore, the negative private expenditure and public investment implications of an exogenous government consumption shock can be offset if the inflation rate is allowed to rise. This idea brings us to a crucial role of the monetary authority in managing inflation expectations. If the monetary policy is actively targeting inflation, it will respond to the rising inflation in the sticky-price economy by raising the nominal interest rate. As shown by Christinao, Eichenbaum and Rebelo (2011), in the presence of active monetary policy, expressed as a simple Taylor-type rule that satisfies the Taylor principle, a higher response of the monetary authority to the change in inflation diminishes the size of the multiplier. Authors also show that a higher response to deviations of output from its steady-state is decreasing the size of public consumption multiplier.

In this section, we extend the analysis of Christiano, Eichenbaum and Rebello (2011) in evaluating the role of the monetary authority's Taylor-type rule formulation to a more complicated structural model and a monetary policy rule that accommodates measures of output and growth gap.[3]

3.5.1 Framework for evaluating the monetary policy rule's role

We seek to identify the multiplier-maximising formulation of the simple monetary policy rule, that is we search for the parameters of the monetary policy rule that produce the highest value of the government consumption multiplier. In this part of the analysis, we consider a full Taylor-type rule:

$$log\left(\frac{R_t^{FFR}}{R^{\bar{FFR}}}\right) = \rho_R log\left(\frac{R_{t-1}^{FFR}}{R^{\bar{FFR}}}\right) + \rho_\pi log\left(\frac{\pi_t}{\bar{\pi}}\right) + \rho_y log\left(\frac{y_{t-1}}{y_{t-1}^f}\right) + \rho_{\Delta y}\left[log\left(\frac{y_t}{y_t^f}\right) - log\left(\frac{y_{t-1}}{y_{t-1}^f}\right)\right]$$

This experiment is similar to a hypothetical case in which fiscal policy is given a primary role in the management of the economy, while the monetary policy arm is assigned a secondary role, tasked to support the fiscal stimulus in achieving its target. We also assume, probably unrealistically, that the monetary authority observes the true output gap. We calculate a simple version of the multiplier that is consistent with the majority of empirical studies:

$$K_H = \frac{\sum_{s=1}^{H} \widehat{y_s}}{\sum_{s=1}^{H} \widehat{g_s}} \times \frac{\bar{y}}{\bar{g}} \tag{3.5.2}$$

where $K_H$is the cumulative H-horizon multiplier, $\widehat{y_s}$is the per cent deviation

[3]Christiano, Eichanbaum and Rebello (2011) consider a simple implementable version of the Taylor-type rule in which the interest rate is allowed to respond to deviations of sticky-price output from its steady-state.

of output from its steady-state value at date s due to a pubic consumption shock at $s = 1$ and $\widehat{g_s}$ is the respective per cent deviation of public consumption expenditure. Since we use a log-linearised model, we need to rescale elasticities into multipliers using a ratio of steady-state output-to-government expenditure.[4] We replicate the analysis by calculating present value multipliers, that are more often used in the general equilibrium framework, in the Appendix 3.9. In order to determine the multiplier maximising formulation of the monetary policy rule, we use a parameter grid search procedure, similar to the one used in the previous section. The set of feasibility constraints on the parameters of the Taylor-type rule is given by: $0 < \rho_R < 1$, $0 < \rho_y, \rho_{\Delta y} < 3$ and $0 \leq \rho_\pi < 5$.

3.5.2 Results

Table 3.5.1 presents the multiplier-maximising policy formulations for multipliers at different horizons, along with the multipliers resulting from the historical policy formulation. As is evident from the table, both short-run and long-run multipliers can be increased by deviating from the historical formulation of the monetary policy in the post-WWII U.S. The historical formulation of the monetary policy seeks primarily to minimise output gaps as well as deviations of inflation from the steady-state. Such a formulation, as will be shown later in this section, has negative implications on the size of the government purchases' impact on output. If the fiscal policy takes on a primary role, and monetary policy only supports the fiscal stimulus, the government consumption multiplier increases. The multiplier-maximising formulation of monetary policy rule implies the minimum possible weight on stabilisation of the long-run output gap and inflation. The resulting parameter values $\rho_R = \rho_y = 0$ and $\rho_\pi = 1.0001$ represent values at the lower-bound of parameter ranges considered in our analysis while satisfying the Taylor-principle.

[4] Since various studies assume different steady-state values for these endogenous variables, we may be subject to a similar bias outlined in Ramey and Zubairy (2018). Nonetheless, we are interested in the maximum multiplier values, so multiplication of the elasticities by the same constant will not affect our results.

	Multiplier-maximising		Historical	
	$[\rho_r, \rho_\pi, \rho_y, \rho_{\Delta y}]$	multiplier	$[\rho_r, \rho_\pi, \rho_y, \rho_{\Delta y}]$	multiplier
1-year	[0, 1.0001, 0, 0.1858]	1.216	[0.92,0.13,0.01,0.2]	0.710
2-year	[0, 1.0001, 0, 0.2992]	1.098	[0.90,0.13,0.01,0.2]	0.414
4-year	[0, 1.0001, 0, 0.4398]	0.945	[0.90,0.13,0.01,0.2]	0.078
5-year	[0, 1.0001, 0, 0.4954]	0.894	[0.90,0.13,0.01,0.2]	-0.026
∞	[0, 1.0001, 0, 1.2235]	1.013	[0.90,0.13,0.01,0.2]	-0.603

Table 3.5.1: Multiplier-maximising and historical monetary policy rule.

We complement the results of Christiano, Eichenbaum and Rebello (2011) in three ways. First, we confirm the authors' conclusion that a muted response to inflation leads to a higher multiplier value using a more complicated model with public and private capital, as well as varying capacity utilisation. Second, we show that a higher response to the output gap has similar consequences on the multiplier value as a higher response to deviations of output from its steady-state, discussed by the authors. Third, unlike authors, we show that a lower interest rate persistence implies a higher value of the multiplier. Inclusion of capacity utilisation in our model produces different implications of nominal interest rate persistency.

As can be clearly seen from (3.5.1), higher capacity utilisation increases the value of the public consumption multiplier. The increase in the capacity utilisation results from the abrupt increase in the labour supply and, consequently, a higher marginal product of capital. A slower adjustment of the interest rate would imply a slower buildup of the labour supply, minimising the effect of rising capacity utilisation on the multiplier value.

Another aspect of the multiplier-maximising formulation of the monetary policy rule is that the optimised coefficient on the short-run output gap is positive and increasing with the multiplier horizon. Indeed, since the short-run output gap is the difference in the growth rates between sticky and flexible output, that is the difference between the slopes of the output response in sticky- and flexible-price economies at any given horizon, the short-run output gap $[\Delta y - \Delta y^f]$ becomes negative in the long-run, while it is modestly positive in the short-run.

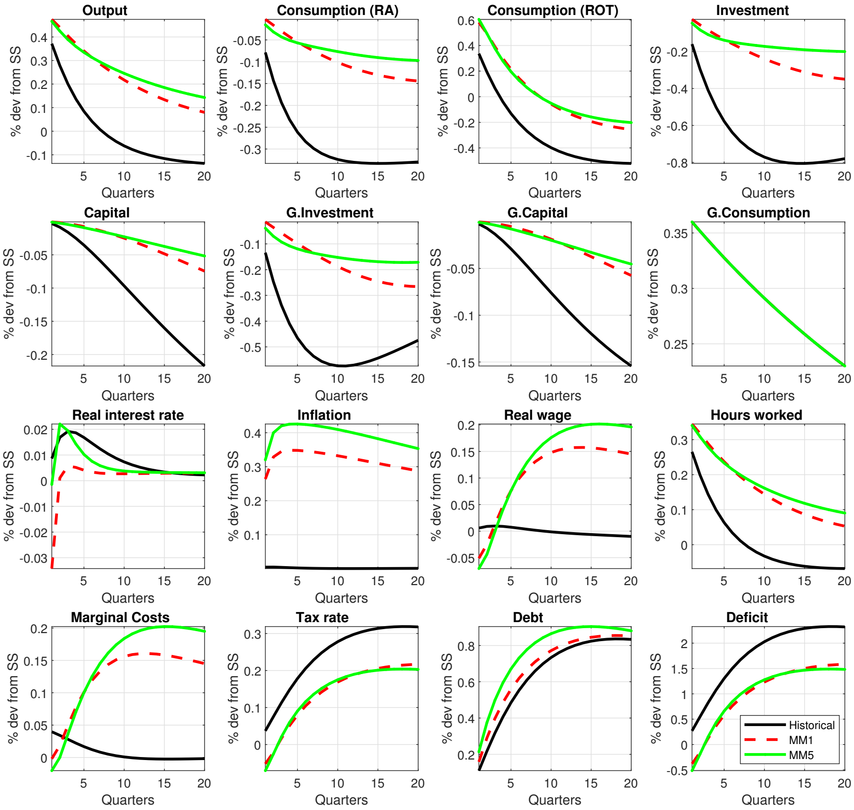


Figure 3.5.1: Short-run responses to a government consumption shock for historical (black), one-year-multiplier-maximising (red) and five-year-multiplier-maximising (green) formulation of monetary policy.

As can be seen in Figure 3.5.1, there is a tradeoff between higher output response immediately and higher output response in the long-run. The value of parameter $\rho_{\Delta y}$ governs this tradeoff; Figure 3.5.1 compares the resulting impulse response functions between the formulations of the monetary policy rule that maximise the one-year multiplier ($MM1$), the five-year multiplier ($MM5$) as well as the historical formulation of monetary policy. $MM1$ formulation results in higher output, consumption, private investment and public investment responses during the first four quarters after the shock. By increasing the $\rho_{\Delta y}$ parameter from 0.186 to 0.495, that is by switching from $MM1$ to $MM5$ formulation of the monetary policy rule, we can already observe higher responses of the same variables in the more distant horizons.

By extending the analysis to more distant horizons, we support this observation. Figure 3.5.2 compares the resulting responses of endogenous variables between the historical formulation, $MM5$ and the formulation of the monetary policy rule that maximises the long-run multiplier (MM_{LR}); the long-run multiplier is defined by (3.5.2) as $t \to \infty$. The long-run benefit of increasing $\rho_{\Delta y}$from 0.495 to 1.224 outweighs the short-term costs associated with such an increase. In the long-term sticky-price output response to a government consumption shock is higher in the second case since the monetary authority will be decreaseing interest rates in response to a negative growth gap.

It is worth mentioning that all multiplier-maximising formulations of monetary policy result in a smaller increase in tax rates and debt required to finance the additional government expenditure. One needs to be aware of the limitations of this policy formulation, as the additional support provided through conventional monetary policy can in principle lead to the economy overheating since it would aim at preserving the output response at the same long-run level once the flexible output converges back to its steady-state. In order to avoid such a pitfall, the monetary authority needs to accommodate the long-run output gap in the policy formulation ($\rho_y > 0$), which will inevitably lead to substantially lower multipliers.

We now seek to understand how deviating from our optimised parameter values will affect the size of the multiplier. We can differentiate between two components of the policy rule. First, indicators responsiveness to which necessarily brings lower output gaps and price stability (ρ_π and ρ_y). Active inflation targeting and larger reaction to long-term output gap tend to diminish the size of the multiplier, as can be seen in Figure 3.5.3. Higher values of these parameters imply that the monetary authority will react sharper to the widening output gap and associated inflation by raising the policy rate. If the increase in the nominal interest rate is large enough, the monetary

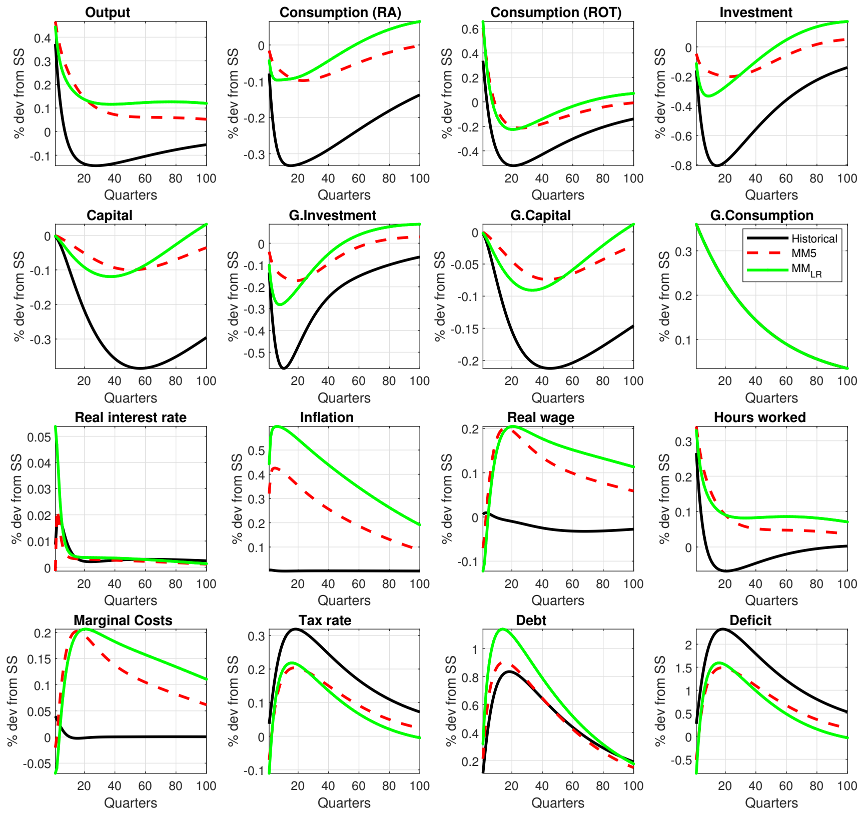
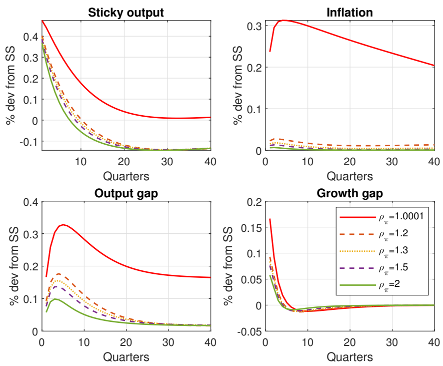
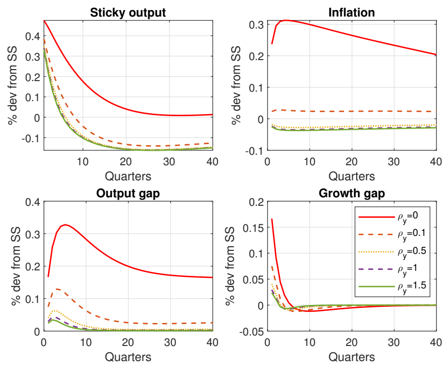


Figure 3.5.2: Long-run responses to a government consumption shock for historical (black), five-year-multiplier-maximising (red) and long-run-multiplier-maximising (green) formulation of monetary policy.

authority will effectively diminish the fall in the real interest rate brought by the public consumption shock. The higher real interest rate will result in lower private expenditure and output in the sticky-price economy, bring it closer to the flexible-price case in which the response of output is already negative four quarters after the shock. Therefore, commitment to price stability or output gap targeting in normal times can eliminate the real positive effects of government expenditure shocks entirely.



Reaction to inflation, assuming $\rho_R = \rho_y = \rho_{\Delta y} = 0$.



Reaction to long-run output gap, assuming $\rho_R = \rho_{\Delta y} = 0$ and $\rho_\pi = 1.1$.

Figure 3.5.3: Long-run responses for different values of ρ_π and ρ_y.

In contrast, the monetary policy's reaction to the growth gap does not necessarily eliminate the output gap, if the associated coefficient of the Taylor-type rule is of modest size. In this case, the monetary policy rule aims at minimising the difference in the slopes between the responses of sticky-price output and its flexible-price counterpart. On impact, government consumption shock increases output in the flexible-price economy, followed by a sharp decrease in the response in the short-term. The output then gradually increases to reach the steady-state level maintained before the shock took place. Therefore, by responding to the growth gap, monetary authority policy will shortly seek to increase the policy rate to diminish the positive growth gap until the growth gap turns negative. It will eventually switch to decreasing rates in the long-term, when the growth gap is negative, as output responses in two economies converge to their joint steady-state from opposite sides. Therefore, if $\rho_{\Delta y}$ is not large enough, the monetary authority will support a long-term positive sticky-price output response. If $\rho_{\Delta y}$ is large, or if the monetary authority will respond to the long-run output gap measure, it can effectively bring the sticky-price response into the negative space fast enough to eliminate this peculiar effect.

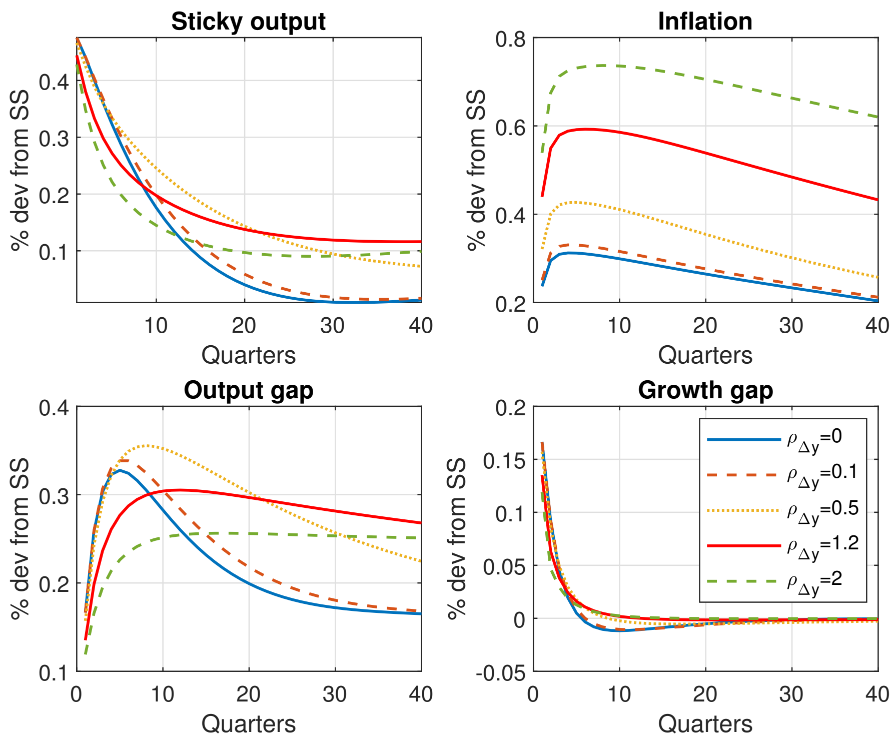


Figure 3.5.4: Long-run responses for different values of $\rho_{\Delta y}$,assuming $\rho_\pi = 1.1$, $\rho_y = 0$, $\rho_R = 0$.

3.6 Conclusion

As the debate concerning fiscal-monetary policy interaction has demonstrated, monetary policy often takes on the primary role in business cycle stabilisation through active inflation targeting. We extend the analysis to a model that is richer in both structural and distortionary features and focus on the role of public investment policy in maximising household welfare. Since we obtain the second-order approximation of the entire Dynamic Stochastic General

Equilibrium model, we can evaluate the sensitivity of the optimal policy results to the choice of a non-CES relative demand curve. We analyse stabilisation performance using the measure of population-weighted household welfare, that is a weighted average of Ricardian and rule-of-thumb household welfare. First, we show that the historical Taylor-type monetary policy rule does not constitute a welfare-maximising simple rule. Higher reaction to inflation throughout the post-WWII U.S. would produce higher population welfare. We also show that optimal reaction to sticky-price output is non-zero if wages are assumed to be sticky and that such an optimal reaction is higher in case of a conventional CES demand than it is under a quasi-kinked demand curve on the labour market. Second, an optimised simple public investment rule that targets debt can deliver similar welfare levels as public investment policy that maximises future output. Third, optimised simple public investment rule is characterised by a modest reaction to past debt and zero reaction to sticky-price output. Welfare-inducing public investment rule does not take on the role of an automatic stabiliser. Finally, the results of optimal policy analysis using implementable rules are sensitive to the choice of the relative demand curve (CES vs non-CES) if the entire model is approximated at the second-order.

The second part of the analysis evaluates the sensitivity of the public consumption expenditure multiplier to changing the formulation of the Taylor-type monetary policy rule. First, we support arguments of Christiano, Eichenbaum and Rebelo (2011) that a more active inflation targeting diminishes the size of public consumption multiplier. Second, even an insignificant non-zero reaction to the output gap has substantial negative implications on the size of the fiscal multiplier. Third, in contrast to the authors, we show that in a model with capacity utilisation, the size of the multiplier is decreasing in the degree of nominal interest rate persistence. Finally, we show that a range of positive coefficients on the growth gap implies larger multipliers values, with the growth gap coefficient governing the trade-off between short-run and long-run output stimulation. The long-run public consumption multiplier benefits form the monetary authority responding stronger to the growth gap.

Appendix

3.7 Symmetric equilibrium

Households

$$1 = \frac{\mathbb{E}_t\left[\exp\left(\frac{\sigma-1}{1+\nu}\left[\left(\Delta^w_{t+1}n_{t+1}\right)^{1+\nu} - \left(\Delta^w_t n_t\right)^{1+\nu}\right]\right)\left(\frac{c^{RA}_{t+1}-\left(\frac{h}{\mu}\right)c^{RA}_t}{c^{RA}_t-\left(\frac{h}{\mu}\right)c^{RA}_{t-1}}\right)^{-\sigma}\right]}{\mathbb{E}_t\left[\frac{\xi_{t+1}}{\xi_t}\frac{\left(1+\tau^c_{t+1}\right)}{\left(1+\tau^c_t\right)}\right]} \tag{3.7.1}$$

$$\mathbb{E}_t\left[\frac{\xi_{t+1}}{\xi_t}\right] = \mathbb{E}_t\left[\frac{\pi_{t+1}}{\bar{\beta}R^{gov}_t}\right] \tag{3.7.2}$$

$$Q_t = \bar{\beta}\mathbb{E}_t[\frac{\xi_{t+1}}{\xi_t}(q^k_t\left(\left(1-\tau^k_{t+1}\right)\left(r^k_{t+1}u_{t+1} - a(u_{t+1})\right) + \delta\tau^k_{t+1}\right) + (1-\delta)Q_{t+1})] \tag{3.7.3}$$

$$1 = Q_t q^x_t\left(1 - S\left(\frac{x_t\mu}{x_{t-1}}\right) - S'\left(\frac{x_t\mu}{x_{t-1}}\right)\left(\frac{x_t\mu}{x_{t-1}}\right)\right) + \bar{\beta}\mathbb{E}_t\left[\frac{\xi_{t+1}}{\xi_t}Q_{t+1}q^x_{t+1}S'\left(\frac{x_{t+1}\mu}{x_t}\right)\left(\frac{x_{t+1}\mu}{x_t}\right)^2\right] \tag{3.7.4}$$

$$r^k_t = a'\left(u_t\right) \tag{3.7.5}$$

$$k^p_t = \frac{1-\delta}{\mu}k^p_{t-1} + q^x_t\left[1 - S\left(\frac{x_t\mu}{x_{t-1}}\right)\right]x_t \tag{3.7.6}$$

$$k_t = \frac{k^p_{t-1}}{\mu}u_t \tag{3.7.7}$$

$$c^{RoT}_t = \frac{s^{RoT}_t + (1-\tau^n_t)w_t n_t + \Pi^p_t}{(1+\tau^c_t)} \tag{3.7.8}$$

$$c_t = (1-\varphi)c^{RA}_t + \varphi c^{RoT}_t \tag{3.7.9}$$

$$\alpha(u_t) = \bar{r^k}\left(u_t - 1\right) + \frac{\bar{r^k}}{2}\frac{\psi_z}{1-\psi_z}\left(u_t - 1\right)^2 \tag{3.7.10}$$

$$S\left(\frac{x_t\mu}{x_{t-1}}\right) = \phi_x\left(\frac{x_t\mu}{x_{t-1}} - \mu\right)^2 \tag{3.7.11}$$

Wage setting

$$\widetilde{w}_t^*(l) = \frac{H_{2,t}}{H_{1,t}} + \frac{H_{3,t}}{H_{1,t}} \tag{3.7.12}$$

$$\begin{aligned} H_{1,t} &= (1-\tau_t^n)n_t \left(\frac{1+\psi^w+\psi^w\lambda_t^w}{(1+\psi^w)\lambda_t^w}\right)\left(\Delta_{2,t}^w\right)^{\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)} \\ &+ \mu\zeta^w\bar{\beta}\frac{\xi_{t+1}}{\xi_t}\mathbb{E}_t\left[\pi_{t+1}\left(\frac{\pi_{t-1}^{\iota^w}\bar{\pi}^{1-\iota^w}}{\pi_{t+1}}\right)^{1-\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)}\left(\frac{w_{t+1}}{w_t}\right)^{\frac{1+\lambda_{t+s}^w}{\lambda_{t+s}^w}(1+\psi^w)} H_{1,t+1}\right] \end{aligned} \tag{3.7.13}$$

$$\begin{aligned} H_{2,t} &= (1-\tau_t^n)n_t \frac{w_t^h}{w_t}\left(\frac{1+\lambda_t^w}{\lambda_t^w}\right)\left(\Delta_{2,t}^w\right)^{\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)} \\ &+ \mu\zeta^w\bar{\beta}\frac{\xi_{t+1}}{\xi_t}\mathbb{E}_t\left[\pi_{t+1}\left(\frac{\pi_{t-1}^{\iota^w}\bar{\pi}^{1-\iota^w}}{\pi_{t+1}}\right)^{-\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)}\left(\frac{w_{t+1}}{w_t}\right)^{\frac{1+\lambda_{t+s}^w}{\lambda_{t+s}^w}(1+\psi^w)+1} H_{2,t+1}\right] \end{aligned} \tag{3.7.14}$$

$$H_{3,t} = (1-\tau_t^n)n_t \frac{\psi^w}{1+\psi^w} + \mu\zeta^w\bar{\beta}\frac{\xi_{t+1}}{\xi_t}\pi_{t-1}^{\iota^w}\bar{\pi}^{1-\iota^w}\mathbb{E}_t\left[H_{3,t+1}\right] \tag{3.7.15}$$

Production and market clearing

$$\frac{k_t}{n_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k} \tag{3.7.16}$$

$$mc_t = (1-\alpha)^{\alpha-1}\alpha^{-\alpha}\frac{w_t^{1-\alpha}\left(r_t^k\right)^{\alpha}}{\left(\epsilon_t^a\right)^{\frac{1}{1-\zeta}}\left(\frac{k_{t-1}^g}{\mu\left(y_t\Delta_t^p+\phi\right)}\right)^{\frac{\zeta}{1-\zeta}}} \tag{3.7.17}$$

$$y_t = \frac{\epsilon_t^a\left(\frac{k_{t-1}^g}{\mu}\right)^{\zeta}k_t^{\alpha(1-\zeta)}\left(n_t\right)^{(1-\alpha)(1-\zeta)}-\phi}{\Delta_t^p} \tag{3.7.18}$$

$$\widetilde{P}_t^* = \frac{Z_{2,t}}{Z_{1,t}} + \frac{Z_{3,t}}{Z_{1,t}}\left(\widetilde{P}_t^*\right)^{1+\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} \tag{3.7.19}$$

$$\begin{aligned} Z_{1,t} &= y_t\left(\Delta_{2,t}^p\right)^{\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)}\frac{1+\psi^p+\psi^p\lambda_t^p}{\lambda_t^p(1+\psi^p)} \\ &\quad + \mu\zeta^p\bar{\beta}\frac{\xi_{t+1}}{\xi_t}\mathbb{E}_t\left[\left(\frac{\pi_{t-1}^{\iota^p}\bar{\pi}^{1-\iota^p}}{\pi_{t+1}}\right)^{1-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)}Z_{1,t+1}\right] \end{aligned} \tag{3.7.20}$$

$$\begin{aligned} Z_{2,t} &= y_t\frac{1+\lambda_t^p}{\lambda_t^p}mc_t\left(\Delta_{2,t}^p\right)^{\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} \\ &\quad + \mu\zeta^p\bar{\beta}\frac{\xi_{t+1}}{\xi_t}\left[\mathbb{E}_t\left(\frac{\pi_{t-1}^{\iota^p}\bar{\pi}^{1-\iota^p}}{\pi_{t+1}}\right)^{-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)}Z_{2,t+1}\right] \end{aligned} \tag{3.7.21}$$

$$Z_{3,t} = y_t\frac{\psi^p}{1+\psi^p} + \mu\zeta^p\bar{\beta}\frac{\xi_{t+1}}{\xi_t}\mathbb{E}_t\left[\frac{\pi_{t-1}^{\iota^p}\bar{\pi}^{1-\iota^p}}{\pi_{t+1}}Z_{3,t+1}\right] \tag{3.7.22}$$

$$y_t = c_t + x_t + \bar{y}g_t + x_t^g + a(u_t)\mu k_{t-1}^p \tag{3.7.23}$$

Price dispersion

$$\Delta_t^p = \frac{1}{1+\psi^p}\Delta_{1,t}^p \left(\Delta_{2,t}^p\right)^{\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} + \frac{\psi^p}{1+\psi^p} \tag{3.7.24}$$

$$\Delta_{1,t}^p = (1-\zeta_p)\,\widetilde{P}_t^*(i)^{-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} + \zeta_p \left(\frac{\pi_{t-1}^{\iota^p}\bar{\pi}^{1-\iota^p}}{\pi_t}\right)^{-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} \Delta_{1,t-1}^p \tag{3.7.25}$$

$$\left(\Delta_{2,t}^p\right)^{1-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} = (1-\zeta_p)\,\widetilde{P}_t^*(i)^{1-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} + \zeta_p \left(\frac{\pi_{t-1}^{\iota^p}\bar{\pi}^{1-\iota^p}}{\pi_t}\Delta_{2,t-1}^p\right)^{1-\frac{1+\lambda_t^p}{\lambda_t^p}(1+\psi^p)} \tag{3.7.26}$$

$$\Delta_{3,t}^p = (1-\zeta_p)\widetilde{P}_t^*(i) + \zeta_p \left(\frac{\pi_{t-1}^{\iota^p}\bar{\pi}^{1-\iota^p}}{\pi_t}\right)\Delta_{3,t-1}^p \tag{3.7.27}$$

$$\Delta_{2,t}^p = 1+\psi^p - \psi^p\Delta_{3,t}^p \tag{3.7.28}$$

Wage dispersion

$$\Delta_t^w = \frac{1}{1+\psi^w}\Delta_{1,t}^w \left(\Delta_{2,t}^w\right)^{\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)} + \frac{\psi^w}{1+\psi^w} \tag{3.7.29}$$

$$\Delta_{1,t}^w = (1-\zeta_w)\,\widetilde{w}_t^{*-\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)} + \zeta_w \left(\frac{w_{t-1}}{w_t}\frac{\pi_{t-1}^{\iota^w}\bar{\pi}^{1-\iota^w}}{\pi_t}\right)^{-\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)} \Delta_{1,t-1}^w \tag{3.7.30}$$

$$\left(\Delta_{2,t}^w\right)^{1-\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)} = (1-\zeta_w)\,\widetilde{w}_t^{*1-\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)} + \zeta_w \left(\frac{w_{t-1}}{w_t}\frac{\pi_{t-1}^{\iota^w}\bar{\pi}^{1-\iota^w}}{\pi_t}\Delta_{2,t-1}^p\right)^{1-\frac{1+\lambda_t^w}{\lambda_t^w}(1+\psi^w)} \tag{3.7.31}$$

$$\Delta_{3,t}^w = (1-\zeta_w)\widetilde{w}_t^* + \zeta_w \left(\frac{w_{t-1}}{w_t}\frac{\pi_{t-1}^{\iota^w}\bar{\pi}^{1-\iota^w}}{\pi_t}\right)\Delta_{3,t-1}^w \tag{3.7.32}$$

$$\Delta_{2,t}^w = 1+\psi^w - \psi^w\Delta_{3,t}^w \tag{3.7.33}$$

Government

$$\begin{aligned} d_t &= \bar{y}g_t + x_t^g + \bar{s}^{endo} + \frac{b_{t-1}}{\mu\pi_t} - \bar{\tau^c}c_t - \bar{\tau^n}w_t n_t \\ &- \bar{\tau^k}k_t r_t^k + \bar{\tau^k}\left[\alpha(u_t)+\delta\right]\frac{k_{t-1}^p}{\mu} \end{aligned} \tag{3.7.34}$$

$$\begin{aligned} \bar{y}g_t + x_t^g &= \frac{b_t}{R_t^{GOV}} + \tau_t^c c_t + \tau_t^n n_t w_t + \tau_t^k k_t r_t^k - s_t^{endo} - \frac{b_{t-1}}{\mu\pi_t} \\ &- \tau_t^k\left[\alpha(u_t)+\delta\right]\frac{k_{t-1}^p}{\mu} \end{aligned} \tag{3.7.35}$$

$$Default : Q_t^g = \bar{\beta}\mathbb{E}_t\left(\frac{\xi_{t+1}}{\xi_t}r_t^g + \frac{\xi_{t+1}}{\xi_t}(1-\delta)Q_{t+1}^g\right) \tag{3.7.36}$$

$$\begin{aligned} 1 &= Q_t^g q_t^{x,g}\left[1 - S_g\left(\frac{x_t^g\mu}{x_{t-1}^g}\right) - S_g'\left(\frac{x_t^g\mu}{x_{t-1}^g}\right)\left(\frac{x_t^g\mu}{x_{t-1}^g}\right)\right] \\ &+ \bar{\beta}\mathbb{E}_t\left[Q_{t+1}^g\frac{\xi_{t+1}}{\xi_t}q_{t+1}^{x,g}S_g'\left(\frac{x_{t+1}^g\mu}{x_t^g}\right)\left(\frac{x_{t+1}^g\mu}{x_t^g}\right)^2\right] \end{aligned} \tag{3.7.37}$$

$$k_t^g = \frac{(1-\delta)}{\mu}k_{t-1}^g + q_t^{x,g}\left[1 - \phi_x^g\left(\frac{x_t^g\mu}{x_{t-1}^g} - \mu\right)^2\right]x_t^g \tag{3.7.38}$$

$$S_g\left(\frac{X_{t+s}^g}{X_{t+s-1}^g}\right) = \phi_x^g\left(\frac{X_{t+s}^g}{X_{t+s-1}^g} - \mu\right)^2 \tag{3.7.39}$$

$$r_t^g = \zeta\frac{\mu\left(y_{t+1} + \frac{\phi}{\Delta_{t+1}^p}\right)}{k_t^g} \tag{3.7.40}$$

$$Rule : log\left(\frac{x_t^g}{\bar{x}^g}\right) = \rho_g log\left(\frac{x_{t-1}^g}{\bar{x}^g}\right) - \rho_d log\left(\frac{b_{t-1}}{\bar{b}}\right) - \rho_y log\left(\frac{y_t}{\bar{y}}\right) \tag{3.7.41}$$

$$Labour \quad tax : \left(\tau_t^n - \bar{\tau^n}\right)w_t n_t + \left(\epsilon_t^\tau - \bar{\epsilon^\tau}\right) = \psi_\tau(d_t - \bar{d}) \tag{3.7.42}$$

$$\tau^c = \bar{\tau^c} \tag{3.7.43}$$

$$\tau^k = \bar{\tau^k} \tag{3.7.44}$$

Monetary policy

$$Full : log\left(\frac{R_t}{\bar{R}}\right) = \rho_R log\left(\frac{R_{t-1}}{\bar{R}}\right) + \rho_\pi log\left(\frac{\pi_t}{\bar{\pi}}\right) + \rho_{\Delta y}\left[log\left(\frac{y_t}{y_t^f}\right) - log\left(\frac{y_{t-1}}{y_{t-1}^f}\right)\right] \tag{3.7.45}$$

$$Implementable : log\left(\frac{R_t}{\bar{R}}\right) = \rho_R log\left(\frac{R_{t-1}}{\bar{R}}\right) + \rho_\pi \left(\frac{\pi_t}{\bar{\pi}}\right) + \rho_{y^*}\left(\frac{y_t}{\bar{y}}\right) \tag{3.7.46}$$

3.8 Steady state

Deterministic steady state of the model can be derived by disregarding stochastic component of the model and making use of the fact that in the steady-state endogenous variables do not change over time. i.e the steady-state $y = y_t$ for $t = 0, 1, \ldots, \infty$. Predetermined steady states: $\bar{\tau^c} = 0.05$, $\bar{\tau^k} = 0.36$, $\bar{\tau^n} = 0.28$. $\bar{g} = 0.153$ (where $g_t = \frac{G_t}{\mu^t y}$), $\frac{\bar{x^g}}{\bar{y}} = 0.04$, $\frac{b}{y} = 4 \times 0.63$, $\bar{\lambda_w} = 0.05$, $\bar{\pi} = 1.007869$. Keeping in mind the above, the steady state of the model can be written recursively:

$$
\begin{aligned}
\epsilon^w &= 1 + \frac{1}{\lambda^w} \\
Q &= 1 \\
u &= 1 \\
\bar{\beta} &= \beta \mu^{-\sigma} \\
R &= \frac{\pi}{\bar{\beta}} \\
R^{GOV} &= R \\
r^g &= \bar{\beta}^{-1} - (1 - \delta) \\
\frac{x}{k^p} &= 1 - \frac{1 - \delta}{\mu} \\
r^k &= \frac{\bar{\beta}^{-1} - 1 + \delta(1 - \tau^k)}{1 - \tau^k} \\
Q^g &= 1 \\
\frac{k^g}{y} &= \frac{x^g}{y} \frac{\mu}{\mu - (1 - \delta)} \\
\frac{y}{y + \phi} &= \left[\left(\frac{y}{\phi} \right)^{-1} + 1 \right]^{-1} \\
\lambda^p &= \frac{y + \phi}{y} - 1 \\
mc &= \frac{1}{1 + \lambda^p}
\end{aligned}
$$

$$\zeta = \frac{x^g}{y} r^g \frac{y}{y+\phi} \frac{1}{\mu - (1-\delta)}$$

$$w = \frac{\alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha)}{\mu^{\frac{\zeta}{(1-\zeta)(1-\alpha)}}} \frac{\left(\frac{k^g}{y}\right)^{\frac{\zeta}{(1-\zeta)(1-\alpha)}} mc^{\frac{1}{(1-\zeta)(1-\alpha)}}}{(r^k)^{\frac{\alpha}{1-\alpha}}}$$

$$w^h = \frac{w}{1+\lambda^w}$$

$$\frac{k}{n} = \frac{\alpha}{1-\alpha} \frac{w}{r^k}$$

$$\frac{k}{y} = (mc)^{\frac{-1}{1-\zeta}} \left(\frac{k^g}{y} \frac{1}{\mu}\right)^{-\frac{\zeta}{1-\zeta}} \left(\frac{k}{n}\right)^{1-\alpha}$$

$$\frac{x}{k} = \frac{x}{k^p} \mu$$

$$\frac{x}{y} = \frac{x}{k} \frac{k}{y}$$

$$\frac{c}{y} = 1 - \frac{x}{y} - \frac{x^g}{y} - g$$

$$\frac{k^p}{y} = \frac{x}{y} \left(\frac{x}{k^p}\right)^{-1}$$

$$\frac{n}{y} = \frac{1 - r^k \frac{k}{y}}{w}$$

$$\frac{s}{y} = \left(\frac{1}{R^{GOV}} - \frac{1}{\mu\pi}\right) \frac{b}{y} + \tau^c \frac{c}{y} + \tau^n w \frac{n}{y} + \tau^k \frac{k}{y} r^k - \tau^k \frac{\delta}{\mu} \frac{k^p}{y} - g - \frac{x^g}{y}$$

$$\frac{s^{RoT}}{y} = \frac{s}{y}$$

$$\frac{c^{RoT}}{y} = \frac{\frac{s}{y} + (1-\tau^n) w \frac{n}{y}}{1+\tau^c}$$

$$\frac{c^{RA}}{y} = \frac{\frac{c}{y} - \varphi \frac{c^{RoT}}{y}}{1-\varphi}$$

$$n = \left[\frac{(1-\tau^n) w \frac{n}{y} \left(\frac{c^{RA}}{y}\right)^{-1}}{(1+\tau^c)(1+\lambda^w)(1-\frac{h}{\mu})}\right]^{\frac{1}{1+\nu}}$$

$$y = n \left(\frac{n}{y}\right)^{-1}$$

$$x^g = \frac{x^g}{y} y$$

$$b = \frac{b}{y} y$$

$$
\begin{aligned}
k^g &= \frac{k^g}{y} y \\
k &= \frac{k}{y} y \\
x &= \frac{x}{y} y \\
k^p &= x \left(\frac{x}{k^p} \right)^{-1} \\
c &= \frac{c}{y} y \\
c^{RoT} &= \frac{c^{RoT}}{y} y \\
c^{RA} &= \frac{c^{RA}}{y} y \\
s &= \frac{s}{y} y \\
d &= yg + x^g + s^{endo} + \frac{b}{\mu\pi} - \tau^c c - \tau^n wn - \left(\frac{\delta}{\mu} - r^k \right) \tau^k k^p \\
s^{RoT} &= s \\
\tilde{P}^* &= 1 \\
\widetilde{Y} &= 1 \\
\Delta_1^p &= 1 \\
\Delta_2^p &= 1 \\
\Delta_3^p &= 1 \\
\Delta^p &= 1 \\
Z_1 &= \frac{y \frac{1+\psi^p+\psi^p\lambda^p}{(1+\psi^p)\lambda^p}}{1-\zeta_p \bar{\beta}\mu} \\
Z_2 &= \frac{y \frac{1+\lambda^p}{\lambda^p} mc}{1-\zeta_p \bar{\beta}\mu} \\
Z_3 &= \frac{y \frac{\psi^p}{1+\psi^p}}{1-\zeta_p \bar{\beta}\mu} \\
\tilde{w}^* &= 1 \\
\widetilde{n} &= 1 \\
\Delta_1^w &= 1
\end{aligned}
$$

$$
\begin{aligned}
\Delta_2^w &= 1 \\
\Delta_3^w &= 1 \\
\Delta^w &= 1 \\
H_1 &= \frac{(1-\tau^n)n\left(\frac{1+\psi^w+\psi^w\lambda^w}{(1+\psi^w)\lambda^w}\right)}{1-\zeta_w\bar{\beta}\mu} \\
H_2 &= \frac{(1-\tau^n)n\frac{w^h}{w}\left(\frac{1+\lambda^w}{\lambda^w}\right)}{1-\zeta_p\bar{\beta}\mu} \\
H_3 &= \frac{(1-\tau^n)n\frac{\psi^w}{1+\psi^w}}{1-\zeta_p\bar{\beta}\mu}
\end{aligned}
$$

3.9 Robustness check. Present Value multiplier.

This robustness check replicates the analysis of Section 3.5 using the present value multiplier that is consistent with other papers analysing fiscal policy using the general equilibrium framework. The present value multiplier is calculated as follows:

$$
K_H^{PV} = \frac{\sum_{s=1}^{H}\left[\mu^s \prod_{j=1}^{s}\left(R_j^{gov}\right)^{-1}\right]\hat{y}_s}{\sum_{s=1}^{H}\left[\mu^s \prod_{j=1}^{s}\left(R_j^{gov}\right)^{-1}\right]\hat{g}_s} \times \frac{\bar{y}}{\bar{g}}
$$

where K_H^{PV} is the cumulative H-horizon present value multiplier, R_j^{gov} is the return on government bonds and μ is the growth trend.

As can be seen from Table 3.9.1, the use of the present value multiplier results in a lower optimised reaction to the growth gap. As was shown in Section 3.5, a greater reaction to the growth gap increases the response of sticky-price output in the long-run. Under the present value multiplier, such a long-run positive effect of the combination of fiscal and monetary policy measures is discounted and contributes less to the resulting multiplier value. Therefore, we observe a multiplier-maximising reaction to the growth gap

	Multiplier-maximising		Historical	
	Rule $[\rho_r,\rho_\pi,\rho_y,\rho_{\Delta y}]$	multiplier	Rule $[\rho_r,\rho_\pi,\rho_y,\rho_{\Delta y}]$	multiplier
1-year	[0, 1.0001, 0, 0.1613]	1.266	[0.92,0.13,0.01,0.2]	0.691
2-year	[0, 1.0001, 0, 0.2396]	1.234	[0.90,0.13,0.01,0.2]	0.427
4-year	[0, 1.0001, 0, 0.2827]	1.221	[0.90,0.13,0.01,0.2]	0.143
5-year	[0, 1.0001, 0, 0.2871]	1.220	[0.90,0.13,0.01,0.2]	0.062
∞	[0, 1.0001, 0, 0.2895]	1.219	[0.90,0.13,0.01,0.2]	-0.231

Table 3.9.1: Multiplier-maximising monetary policy rule: NPV cumulative multiplier.

that is closer to the historical monetary policy rule.

3.10 Non-linear model derivation

In this section we present derivation of the Drautzburg and Uhlig (2015) model. Section 3.2 provides derivation of the equilibrium conditions on the goods and labour markets. This section presents the Household and Government sectors.

3.10.1 Households

There is a unit mass of households, indexed by $j \in [0,1]$, where only a fraction $1-\varphi$ of households are intertemporally optimising. The φ proportion of households do not have access to borrowing or saving. Household welfare (life-time utility) is expressed by the stream of discounted future utilities:

$$\begin{aligned}\mathcal{W}_t &= \mathbb{E}_t \Sigma_{s=0}^{\infty} \beta^s U_{t+s}\left(C_{t+s}(j), C_{t+s-1}, n_{t+s}(j)\right) \\ &= \mathbb{E}_t \Sigma_{s=0}^{\infty} \beta^s \left[\frac{1}{1-\sigma}\left(C_{t+s}(j) - hC_{t+s-1}\right)^{1-\sigma}\right] \exp\left[\frac{\sigma-1}{1+\nu} n_{t+s}(j)^{1+\nu}\right]\end{aligned} \tag{3.10.1}$$

where $h \in [0, 1]$ is the coefficient governing the strength of external habit[5], σ is the inverse of the intertemporal elasticity of substitution, ν is the inverse of the labour supply elasticity. Households discount future utility gains by $\beta \in (0, 1)$. Aggregate consumption is given by:

$$C_t = \int_0^1 C_t(j)\Lambda(dj) = \int_0^{1-\varphi} C_t^{RA}\Lambda(dj) + \int_{1-\varphi}^1 C_t^{RoT}\Lambda(dj) = (1-\varphi)C_t^{RA} + \varphi C_t^{RoT}$$

while aggregate transfers are given by:

$$S_t = \int_0^1 S_t(j)\Lambda(dj) = \int_0^{1-\varphi} S_t^{RA}\Lambda(dj) + \int_{1-\varphi}^1 S_t^{RoT}\Lambda(dj) = (1-\varphi)S_t^{RA} + \varphi S_t^{RoT}$$

3.10.1.1 Intertemporally optimising households

The intertemporally optimising households choose consumption $[C_{t+s}(j)]$, investment in physical capital $[X_{t+s}(j)]$, physical capital $[K_{t+s}^p(j)]$, capacity utilization $[u_{t+s}(j)]$, nominal government bond holdings $\left[B_{t+s}^n(j)\right]$ and labor supply $[n_{t+s}(j)]$ in order to maximise (3.10.1), subject to the sequence of budget constraints (3.10.2) and the law of motion for physical capital (3.10.1). Ricardian households take prices $[P_{t+s}]$, nominal returns on government bonds $\left[q_{t+s}^b R_{t+s}\right]$, the nominal rental rate of capital $\left[R_{t+s}^k\right]$ and nominal wages $[W_{t+s}]$ as given. The budget constraint for a period $t+s$ is given by:

$$\begin{aligned}
&\left(1+\tau_{t+s}^c\right) C_{t+s}(j) + X_{t+s}(j) + \frac{B_{t+s}^n(j)}{R_{t+s}^{gov} P_{t+s}} \leq \\
&\quad S_{t+s} + \frac{B_{t+s-1}^n(j)}{P_{t+s}} + \left(1-\tau_{t+s}^n\right) \frac{W_{t+s}^h n_{t+s}(j) + \lambda_{w,t+s} n_{t+s} W_{t+s}^h}{P_{t+s}} \\
&\quad\quad + \left[\left(1-\tau_{t+s}^k\right)\left(\frac{R_{t+s}^k u_{t+s}(j)}{P_{t+s}} - \alpha\left(u_{t+s}(j)\right)\right) + \delta\tau_{t+s}^k\right] \\
&\quad\quad\quad \left[\left(1-\omega_{t+s-1}^k\right) K_{t+s-1}^p(j) + \omega_{t+s-1}^k K_{t+s-1}^p\right] + \Pi_{t+s}^p \mu^{t+s} \qquad (3.10.2)
\end{aligned}$$

[5]Habbits in the model are external relative to the household, but internal relative to the household type.

where τ^c_{t+s},τ^k_{t+s} and τ^n_{t+s} represent taxes on consumption, capital income and labor income respectively. $\Pi^w_{t+s}\mu^{t+s}$ are real non-de-trended union profits, which are taken as given by the households. S_{t+s} is the nominal lump-sum subsidy provided by the government. $\alpha(u_{t+s}(j))$ is a strictly increasing and strictly convex function of varying capacity utilization, whose first derivative in case of a unit capacity utilization is normalised to be $\alpha'(1) = \bar{r}^k$. Given the fact that at unit capacity there is no additional cost $[\alpha(1) = 0]$, the functional form of $\alpha(u_{t+s}(j))$ is chosen to be:

$$\alpha(u_{t+s}(j)) = \bar{r^k}\left(u_{t+s}(j) - 1\right) + \frac{\bar{r^k}}{2}\frac{\psi_z}{1-\psi_z}\left(u_{t+s}(j) - 1\right)^2$$

There is also financial friction represented by the wedge between returns on private and government bonds $\left[\omega^k_{t+s} \neq 0\right]$. If $\omega^k_{t+s} > 0$ household receive less than one dollar for a dollar of after-tax capital income; this represents agency costs. The law of motion for physical capital has the following form:

$$K^p_{t+s}(j) = (1-\delta)K^p_{t+s-1}(j) + q^x_{t+s}\left[1 - S\left(\frac{X_{t+s}(j)}{X_{t+s-1}(j)}\right)\right]X_{t+s}(j)$$

where δ is the physical capital depreciation rate, q^x_{t+s} captures the relative price of investment varying over time. New investments are subject to adjustment costs; adjustment costs result in a hump-shaped capital stock adjustments to an exogenous shock. Adjustment costs are represented by $1 - S\left(\frac{X_{t+s}(j)}{X_{t+s-1}(j)}\right)$,such that $S(\mu) = S'(\mu) = 0$ and $S'' > 0$. Keeping this in mind, the functional form of $S\left(\frac{X_{t+s}(j)}{X_{t+s-1}(j)}\right)$ is given by:

$$S\left(\frac{X_{t+s}(j)}{X_{t+s-1}(j)}\right) = \phi_x\left(\frac{X_{t+s}(j)}{X_{t+s-1}(j)} - \mu\right)^2$$

where $\phi_x > 0$. The effective capital stock is related to physical capital stock as follows:

$$K_{t+s}(j) = K^p_{t+s-1}(j)u_{t+s}(j)$$

At the symmetric equilibrium, the Ricardian household's optimality conditions are:

$$\xi_t\left(1+\tau_t^c\right)=\exp\left(\frac{\sigma-1}{1+\nu}\left(n_t^{RA}\right)^{1+\nu}\right)\left[c_t^{RA}-\left(\frac{h}{\mu}\right)c_{t-1}^{RA}\right]^{-\sigma} \tag{3.10.3}$$

$$\xi_t\left(1-\tau_t^n\right)w_t^h=\exp\left(\frac{\sigma-1}{1+\nu}\left(n_t^{RA}\right)^{1+\nu}\right)\left(n_t^{RA}\right)^{\nu}\left[c_t^{RA}-\left(\frac{h}{\mu}\right)c_{t-1}^{RA}\right]^{1-\sigma} \tag{3.10.4}$$

$$\xi_t=\bar{\beta}R_t^{gov}\mathbb{E}_t\left(\frac{\xi_{t+1}}{\pi_{t+1}}\right) \tag{3.10.5}$$

$$Q_t=\bar{\beta}\mathbb{E}_t[\frac{\xi_{t+1}}{\xi_t} \left(q_t^k\left(\left(1-\tau_{t+1}^k\right)\left(r_{t+1}^k u_{t+1}^{RA}-\alpha(u_{t+1}^{RA})\right)+\delta\tau_{t+1}^k\right)+(1-\delta)Q_{t+1}\right)] \tag{3.10.6}$$

$$1=Q_t q_t^x\left(1-S\left(\frac{x_t^{RA}\mu}{x_{t-1}^{RA}}\right)-S'\left(\frac{x_t^{RA}\mu}{x_{t-1}^{RA}}\right)\left(\frac{x_t^{RA}\mu}{x_{t-1}^{RA}}\right)\right)+ \\ +\bar{\beta}\mathbb{E}_t\left[\frac{\xi_{t+1}}{\xi_t}Q_{t+1}q_{t+1}^x S'\left(\frac{x_{t+1}^{RA}\mu}{x_t^{RA}}\right)\left(\frac{x_{t+1}^{RA}\mu}{x_t^{RA}}\right)^2\right] \tag{3.10.7}$$

$$r_t^k=\alpha'(u_t^{RA}) \tag{3.10.8}$$

where we use real detrended variables, defined as:

$$\xi_t\equiv\Xi_t\mu^{t\sigma}, c_t^{RA}\equiv\frac{C_t^{RA}}{\mu^t}, w_t^h\equiv\frac{W_t^h}{P_t\mu^t}, \bar{\beta}\equiv\beta\mu^{-\sigma}, Q_t\equiv\frac{\Xi_t^k}{\Xi_t}, r_t^k=\frac{R_t^k}{P_t}, x_t^{RA}=\frac{X_t^{RA}}{\mu^t}$$

Combining (3.10.3) and (3.10.4) we obtain the static optimality condition:

$$\frac{U_{n,t}}{U_{C,t}}=\frac{(1-\tau_t^n)}{(1+\tau_t^c)}w_t^h=\left(n_t^{RA}\right)^{\nu}\left[c_t^{RA}-\left(\frac{h}{\mu}\right)c_{t-1}^{RA}\right] \tag{3.10.9}$$

and the Euler equation for consumption:

$$\frac{U_{C,t+1}}{U_{C,t}} = \mathbb{E}_t \left[\frac{\xi_{t+1}}{\xi_t} \frac{\left(1+\tau^c_{t+1}\right)}{(1+\tau^c_t)} \right]$$
$$= \mathbb{E}_t \left[\exp\left(\frac{\sigma-1}{1+\nu} \left[\left(\Delta^w_{t+1} n_{t+1}\right)^{1+\nu} - \left(\Delta^w_t n_t\right)^{1+\nu} \right] \right) \left(\frac{c^{RA}_{t+1} - \left(\frac{h}{\mu}\right) c^{RA}_t}{c^{RA}_t - \left(\frac{h}{\mu}\right) c^{RA}_{t-1}} \right)^{-\sigma} \right] \tag{3.10.10}$$

Finally, it is worth noting these two relations. The bond premium is defined as

$$R^{GOV}_t = (1+\omega^b_t) R_t = q^b_t R_t$$

3.10.1.2 Non-Ricardian households

Rule-of-thumb households are not interested in the accumulation of capital. Their fraction φ remains constant over time. They face a static budget constraint and are assumed to supply the same amount of labour as the Ricardian households. The budget constraint for the rule-of-thumb households is given by:

$$(1+\tau^c_{t+s}) C^{RoT}(j) \leq S^{RoT} + (1-\tau^n_{t+s}) \frac{W^h_{t+s} n^{RoT}_{t+s}(j) + \lambda_{w,t+s} W^h_{t+s} n_{t+s}}{P_{t+s}} + \Pi^p_{t+s} \mu^{t+s} \tag{3.10.11}$$

Since the Rule-of-thumb consumers do not have access to capital and consume all the labour income, subsidies and production profit at each time period, given the amount of labour supplied, consumption can be solved through the budget constraint. Aggregating over all rule-of-thumb households delivers the budget constraint in its final form:

$$c^{RoT}_t = \frac{s^{RoT}_t + (1-\tau^n_t) w_t n_t + \Pi^p_t}{(1+\tau^c_t)}$$

3.10.2 Government

The government can use monetary and fiscal policy to influence fluctuations in model variables that arise from stochastic exogenous shocks. Government generates revenue by taxing capital $[\tau^k]$, consumption $[\tau^c]$ and labour $[\tau_t^n]$ and spends it on public investment $[X_t^g]$, public consumption $[G_t]$ and transfers $[\bar{S}]$. Only the labour tax rate is allowed to respond to a rising fiscal deficit, while the rest are assumed to remain at steady-state values. In the baseline case, public investment is chosen optimally to maximise the present value of future output net of public investment.

3.10.2.1 Public investment

In the optimal public investment case, the fiscal authority seeks to maximise the stream of future discounted output net of government investment:

$$max_{K_t^g, X_t^g} \mathbb{E}_t \Sigma_{s=0}^{\infty} \beta^s \frac{\Xi_{t+s}}{\Xi_t} [Y_{t+s} - X_{t+s}^g]$$

subject to the government budget constraint:

$$G_{t+s} + X_{t+s}^g + \bar{S} + \frac{B_{t+s-1}^n}{P_{t+s}} \leq$$
$$\frac{B_{t+s}^n}{R_{t+s}^{GOV} P_{t+s}} + \tau_{t+s}^c C_{t+s} + \tau_{t+s}^n n_{t+s} \frac{W_{t+s}}{P_{t+s}} + \tau_{t+s}^k \left[u_{t+s} \frac{R_{t+s}^k}{P_{t+s}} - \alpha(u_{t+s}) - \delta \right] K_{t+s-1}^p$$

and the law of motion of public capital:

$$K_{t+s}^g = (1-\delta) K_{t+s-1}^g + q_{t+s}^{x,g} \left[1 - S_g \left(\frac{X_{t+s}^g}{X_{t+s-1}^g} \right) \right] X_{t+s}^g \qquad (3.10.12)$$

where investment adjustment costs follow the same functional form as their private counterpart:

$$S_g \left(\frac{X_{t+s}^g}{X_{t+s-1}^g} \right) = \phi_x^g \left(\frac{X_{t+s}^g}{X_{t+s-1}^g} - \mu \right)^2$$

Resulting equilibrium conditions, expressed in de-trended real variables, are presented below:

$$Q_t^g = \bar{\beta}\mathbb{E}_t \left(\frac{\xi_{t+1}}{\xi_t} r_t^g + \frac{\xi_{t+1}}{\xi_t} (1-\delta) Q_{t+1}^g \right) \tag{3.10.13}$$

$$1 = Q_t^g q_t^{x,g} \left[1 - S_g \left(\frac{x_t^g \mu}{x_{t-1}^g} \right) - S_g' \left(\frac{x_t^g \mu}{x_{t-1}^g} \right) \left(\frac{x_t^g \mu}{x_{t-1}^g} \right) \right] + \bar{\beta}\mathbb{E}_t \left[Q_{t+1}^g \frac{\xi_{t+1}}{\xi_t} q_{t+1}^{x,g} S_g' \left(\frac{x_{t+1}^g \mu}{x_t^g} \right) \left(\frac{x_{t+1}^g \mu}{x_t^g} \right)^2 \right] \tag{3.10.14}$$

where Q_t^g is the Lagrange multiplier on (3.10.12), $x_t^g \equiv X_t^g \mu^{-t}$ and r_t^g is the implied rental rate of public capital defined as:

$$r_t^g \equiv \zeta \frac{\mu \left(y_{t+1} + \frac{\phi}{\Delta_{t+1}^p} \right)}{k_t^g}$$

The set-up is completed by the inclusion of the law of motion of public capital.

$$k_t^g = \frac{(1-\delta)}{\mu} k_{t-1}^g + q_t^{x,g} \left[1 - \phi_x^g \left(\frac{x_t^g \mu}{x_{t-1}^g} - \mu \right)^2 \right] x_t^g$$

The government budget constraint includes transfers in steady-state to minimise the ability of the government to compensate inefficiencies resulting from imperfect competition in a dynamic setting.

3.10.2.2 Revenues and deficit

The deficit at period t prior to new debt and changes in the tax rates is defined as:

$$d_t = \bar{y} g_t + x_t^g + \bar{s}^{endo} + \frac{b_{t-1}}{\mu \pi_t} - \bar{\tau^c} c_t - \bar{\tau^n} w_t n_t - \bar{\tau^k} k_t r_t^k + \bar{\tau^k} \left[\alpha(u_t) + \delta \right] \frac{k_{t-1}^p}{\mu}$$

Finally, th etax adjustment rule is given by:

$$
\begin{aligned}
(\tau_t^n - \bar{\tau^n})\, w_t n_t + (\epsilon_t^\tau - \bar{\epsilon^\tau}) &= \psi_\tau (d_t - \bar{d}) \\
\tau_t^c &= \bar{\tau^c} \qquad (3.10.15) \\
\tau_t^k &= \bar{\tau^k}
\end{aligned}
$$

3.10.2.3 Monetary policy

The monetary authority sets the short-term nominal interest rate according to a Taylor-type rule. The baseline rule is specified as in Smets and Wouters (2007):

$$
\begin{aligned}
log\left(\frac{R_t^{FFR}}{R^{\bar{FFR}}}\right) &= \rho_R log\left(\frac{R_{t-1}^{FFR}}{R^{\bar{FFR}}}\right) + \rho_\pi log\left(\frac{\pi_t}{\bar{\pi}}\right) + \rho_y log\left(\frac{y_{t-1}}{y_{t-1}^f}\right) \\
&+ \rho_{\Delta y}\left[log\left(\frac{y_t}{y_t^f}\right) - log\left(\frac{y_{t-1}}{y_{t-1}^f}\right)\right] + log(\epsilon_t^r) \quad (3.10.16)
\end{aligned}
$$

Interest rate is persistent over time (governed by ρ_r)and can respond to inflation and output gap (long-run output gap) and growth gap (short-run output gap) ;$\rho_\pi \geq 0$ governs the strength of response to changes in the price level, while $\rho_y \geq$0 governs the reaction to the output gap and $\rho_{\Delta y} \geq 0$ governs the reaction to the growth gap.

3.10.3 Market clearing conditions

In equilibrium, all markets clear. Labour and capital demanded by the intermediate goods producers equal to labour supplied by the unions and capital provided by the Ricardian households, respectively. Additionally, according to the goods market-clearing condition, production is used for private consumption and investment, public consumption and investment, and is subject to variations in capacity utilisation. The model is closed by the following identities:

$$c_t + x_t + \bar{y}g_t + x_t^g = y_t - a(u_t)\mu k_{t-1}^p \tag{3.10.17}$$

$$\int_0^1 n_t(i)di = n_t = \int_0^1 \frac{w_t(l)}{w_t} n_t(l)dl$$

$$\int_0^1 k_t(i)di = k_t = \int_0^{1-\varphi} k_t(j)dj$$

3.10.4 Flexible economy

In the flexible economy, intermediate firms and labour unions can freely re-optimise [$\zeta^p = \zeta^w = 0$]. Since prices and wages are not sticky, there is no price and wage dispersion [$\Delta_t^{p,f} = \Delta_t^{w,f} = 1$]. Markups are constant at the steady-state level and are not subject to stochastic shocks [$\epsilon_t^{\lambda,p} = \epsilon_t^{\lambda,w} = 0$]. Price of intermediate goods and wage set by the union are a constant markup over marginal costs and wage received by the household, respectively:

$$P_t^f = P_t^f(i) = [1 + \lambda^w]\bar{mc}_t^f = 1 \tag{3.10.18}$$

$$w_t^f = w_t^f(l) = \left[1 + \bar{\lambda_w}\right] w_t^{h,f} \tag{3.10.19}$$

(3.10.18) implies that marginal costs are constant at steady-state level:

$$mc_t^f = \bar{mc}$$

Inflation is positive and assumed to be equal to its steady-state value:

$$\pi_t^f = \bar{\pi}$$

3.10.5 Exogenous processes

All shocks are assumed to be log-normally distributed and independent apart from one exception - government spending shock is correlated with technology shock. Shocks follow AR(1) processes except for price and wage markup shocks that follow ARMA(1,1).

1. Wage markup shock:

$$log(\tilde{\epsilon}_t^{\lambda,w}) = \rho_{\lambda,w} log(\tilde{\epsilon}_{t-1}^{\lambda,w}) + u_t^{\lambda,w} - \theta_{\lambda,w} u_{t-1}^{\lambda,w} \qquad u_t^{\lambda,w} \sim i.i.d.N(0, \sigma_{\lambda,w}^2)$$

2. Price markup shock:

$$log(\tilde{\epsilon}_t^{\lambda,w}) = \rho_{\lambda,w} log(\tilde{\epsilon}_{t-1}^{\lambda,w}) + u_t^{\lambda,w} - \theta_{\lambda,w} u_{t-1}^{\lambda,w}, \qquad u_t^{\lambda,w} \sim i.i.d.N(0, \sigma_{\lambda,w}^2)$$

3. Technology shock:

$$log(\epsilon_t^a) = \rho_a log(\epsilon_{t-1}^a) + u_t^a, \qquad u_t^a \sim i.i.d.N(0, \sigma_a^2)$$

4. Monetary policy shock

$$log(\epsilon_t^r) = \rho_r log(\epsilon_{t-1}^r) + u_t^r, \qquad u_t^r \sim i.i.d.N(0, \sigma_r^2)$$

5. Tax shock

$$log(\epsilon_t^\tau) = \rho_\tau log(\epsilon_{t-1}^\tau) + u_t^\tau, \qquad u^\tau \sim i.i.d.N(0, \sigma_\tau^2)$$

6. Bond premium shock

$$log(1 + \omega_t^b) = log(q_t^b) = \rho_b log(q_{t-1}^b) + u_t^b, \qquad u_t^b \sim i.i.d.N(0, \sigma_b^2)$$

7. Wedge on private capital

$$log(1 - \omega_t^k) = log(q_t^k) = \rho_k log(q_{t-1}^k) + u_t^k, \qquad u_t^k \sim i.i.d.N(0, \sigma_k^2)$$

8. Investment-specific shock

$$log(q_t^x) = \rho_x log(q_{t-1}^x) + u_t^x, \qquad u_t^x \sim i.i.d.N(0, \sigma_x^2)$$

9. Public-investment-specific shock

$$log(q_t^{x,g}) = \rho_{x,g} log(q_{t-1}^{x,g}) + u_t^{x,g}, \qquad u_t^{x,g} \sim i.i.d.N(0, \sigma_{x,g}^2)$$

10. Government consumption

$$\begin{aligned} log(g_t) = (1-\rho_g) log(\bar{g}) + \rho_g log(g_{t-1}) \\ + \sigma_{ga} u_t^a + u_t^g, \qquad u_t^g \sim i.i.d.N(0, \sigma_g^2) \end{aligned}$$

Chapter 4

Fiscal Multipliers in Emerging Markets.

4.1 Introduction

Data limitations are among the largest impediments to any research focusing on fiscal multipliers. Lack of quarterly data for many developing economies does not allow incorporating conventional identification strategies in the debate on the size of fiscal multipliers. Except Ilzetzki et al. (2013), who use a quarterly dataset of 20 high-income and 24 developing economies, there is almost no research published on the topic of state-dependent fiscal multipliers using structural vector autoregressive (SVAR) identification. Furceri and Li (2018) use Jorda's (2005) Local Projection (LP) methodology, combined with the ' unanticipated fiscal stimulus' definition of the shock, which allows authors to analyse government investment potency in emerging and developing economies. Such an approach allows them to extend the analysis to annual data frequency and overcome the major pitfalls emphasised in the literature: Jorda's (2005) critique of the impulse response functions (IRF) calculated via SVAR models and Ramey's (2011) critique of timing in conventional SVAR identification strategies.

However, recent literature emphasises additional concerns regarding both multiplier calculations and the use of the conventional LP method. Building on the analysis of Hall (2009), Ramey and Zubairy (2018) raised the issue of a re-scaling bias in multiplier calculations and suggested an alternative approach for estimating fiscal multipliers directly, by dividing shocks with the lagged values of the response variable. As claimed by the authors, such a shock definition would deliver comparable results if multipliers were to be estimated on different data samples, and additionally provide a test for state-dependency that most of the recent literature on US multipliers did not pass. Another interesting argument was made by Tseulings and Zubanov (2014), who argued that the conventional approach to estimating the LP model may deliver biased estimates. Using Monte Carlo simulations, the authors showed evidence of the importance of including future realisations of the shock variable in order to counter the hitherto unnoticed bias in the IRFs. The methodology of Furceri and Li (2018) fails to accommodate these important points raised in the literature, thereby creating a valid basis to extend their analysis.

In addition to implementing measures to counter the concerns mentioned above, this study raises further issues related to the multiplier calculation via the LP method. The impact of various policy shocks should not be estimated in isolation. Incorporating all the observed shocks in a single LP estimation helps to avoid any bias emerging from the fact that discretionary shocks may not be independent. When governments plan to implement discretionary fiscal stimulus, they probably make use of all the weapons in their fiscal-policy arsenal. If the target is to stimulate gross domestic product (GDP) and minimise the adverse effects of the recession, policy-makers will not stop at an expansionary government consumption policy, and will alter government investment, transfer, and taxation policies as well. Coordinating with the central bank, governments can also deliver an additional boost to the economic activity via monetary policy. It is worth mentioning that central bank independence does not imply that policy measures, resulting from implemented policy rules, should not be correlated. The analysis published by Romer and Romer (2016) is but one example. By analysing the Federal Reserve (Fed) records on policy discussions, the authors concluded that the monetary policy authority was actively countering the effects of temporary and permanent transfer payment increases in the United States (US). Following this logic, shocks can not only be correlated simultaneously with each other, but also with future realisation of other shocks in the Tseulings and Zubanov (2014) setting.

This study applies the LP method, which controls the hitherto unnoticed bias and includes government consumption and investment shocks simultaneously, on a sample of 107 emerging and developing economies, using the International Monetary Fund's (IMF) vintage World Economic Outlook (WEO) database . Incorporating the critique of Ramey and Zubairy (2018), we estimate the fiscal multipliers that are allowed to change depending on the state of the economy and other important characteristics of the economies under scope, such as level of economic slack, debt burden, openness to trade, and size of the government sector. We show that failure to incorporate these crucial modelling choices results in underestimation of the positive effect of public investment and overestimation of the negative effect of public consumption on economic growth.

This study extends the literature in three important ways. First , to the best of our knowledge, this is the first research work to analyse the impact of government consumption expenditure in emerging and developing economies, using the LP framework. Second, this study introduces a correction of the LP methodology, pointed out by Tseulings and Zubanov (2014) in the fiscal policy debate. Finally, due to concerns over bias, this study estimates government investment and government consumption multipliers jointly, rather than via isolated estimations. The study is structured as follows. Section 2 presents a brief overview of the literature on the use of the LP method in estimating state-dependent multipliers. Section 3 describes the modelling approach as well as data used for estimation. Section 4 presents the multiplier estimates, including state-dependent multipliers identified for a variety of economic regimes. The Conclusion and Appendices follow.

4.2 Literature overview

Although there seems to be a discernible increase in interest regarding the effects of the fiscal policy on output and other macroeconomic aggregates since the 2008 global financial crisis and the resulting zero-lower bound, the empirical literature on emerging and developing economies appears to be rather limited. Ilzetski et al. (2013) were the first to use a conventional SVAR approach in comparing the impact of fiscal expenditure between developing countries and high-income economies, and to analyse the difference in multiplier values between various economic conditions, such as openness to trade, exchange rate regime, and level of public debt. Since then, the literature on fiscal multiplier has reverted to Jorda's (2005) LP method to estimate fiscal multipliers that do not depend on the underlying multivariate system. Such an approach allows the researcher to estimate impulse responses directly, that is, in the direct forecasting fashion, and eliminates the higher-horizon bias in the IRF estimation. The LP method does not assume any specification of an underlying multivariate system; the approach allows for considerable non-linearity in responses, as these are not governed by analytically derived formulas. Auerbach and Gorodnichenko (2013a) investigated the spill-over effects of an unanticipated fiscal expenditure shock on the output of major trading partners in the Organisation for Economic Co-operation and Development (OECD), introducing the approach that has been followed by many studies. They investigated whether the level of economic slack in the recipient country alters the spill-over effect of a fiscal injection originating in its trading partner. The impact of alternative economic states were analysed by Abiad et al. (2016) for the case of government investment in advanced economies. The authors concluded that there were significant differences in the fiscal policy's potency between the various levels of investment efficiency and economic slack. Both accommodative monetary policies and mode of financing of the fiscal injection also mattered.

Furceri and Li (2017) introduced the LP method to the debate on the impact of fiscal expenditure in emerging and developing economies, by analysing the outcome of government investment spending in 103 developing economies.

Following Auerbach and Gorodnichenkpo (2012), the authors accounted for the shock timing issues raised by Ramey (2011) by calculating a measure of the unanticipated fiscal expenditure shocks using professional forecasts. Typically, market agents have larger information sets compared to the ones that an average econometrician has at hand. As argued by Ramey (2011), they would probably anticipate shocks identified by a simple SVAR methodology and, thus, would be able to adjust to such shocks in advance. Using a measure of professional forecasts allows econometricians to widen this information set without explicitly modelling for a large number of explanatory variables. The approach chosen by Furceri and Li (2017) had other interesting properties. For instance, correct identification of a strictly exogenous fiscal shock has been a major concern in the multiplier literature, as discretionary policy measures are not easily distinguishable from the automatic stabiliser effects. The majority of studies on the topic identify discretionary fiscal policy by assuming that fiscal authorities are not able to react to a business cycle shock in less than a quarter, or construct a narrative measure of the fiscal shock series. Abiad et al. (2016) and Furceri and Li (2018) argued that the unanticipated measure of fiscal stimuli is better suited to solve potential endogeneity issues, given the data at an annual frequency. By definition, the configured model does not capture the fiscal policy responses to anticipated changes in the state of the economy, as they will be accounted for in the fiscal variable projection (professional forecast).

Nonetheless, the approach of Furceri and Li (2017) does not account for some important developments in the literature. First, extreme flexibility of the LP model, as argued by Ramey and Zubairy (2018), helps to overcome the re-scaling bias that is so common in the SVAR literature. Impulse responses of an SVAR model deliver elasticities, not multipliers. In order to obtain multipliers, elasticities should be rescaled by a ratio of GDP to a fiscal variable of interest. Besides the time-varying parameter models, multipliers estimated using this approach will depend heavily on the sample under analysis. Similar elasticities estimated by different studies may deliver substantially different multiplier values, due to different GDP-to-fiscal-variable ratios that are used to obtain the results. The LP method allows to directly estimate cumulative multipliers via dividing shocks by the lagged response variable before the estimation. Secondly, the LP method may be subject to hitherto unnoticed bias, as argued by Tseulings and Zubanov (2014). Based on a sample of 99 countries, the authors showed that future shock realisations could be relevant to the higher horizons of the IRF estimation.[1]

4.3 Empirical Strategy and Data

We apply the LP methodology on a sample of 107 emerging and developing economies. The time span ranges from as early as 1996 to as late as 2016. Data were obtained from the IMF WEO database and include: a) forecast and observed GDP, b) forecast and observed government consumption expenditure, and c) forecast and observed government investment spending. Additional data were used to define the economic states (regimes) during the latter stages of the analysis, which can be found in the Appendix 4.6.

The general LP approach was first used in the context of fiscal multiplier estimation by Auerbach and Gorodnichencko (2013a, b). We modify the method in three important ways. First, estimating multipliers separately for various types of fiscal spending may result in biased multiplier estimates, as unanticipated fiscal shocks are probably still endogenously related to each other. For example, failing to include all the measures jointly may result in the multiplier for a surprise increase in government consumption to partially containing the effect of a simultaneous rise in government investment. Following this logic, we include both government investment and consumption. The model would further benefit from the inclusion of tax shocks, yet, data on tax shocks are limited and of low quality. Second, incorporating a general critique of the LP method in Teulings and Zubanov (2014), we extend the model to include forward-looking measures of fiscal shocks. This inclusion solves the hitherto unnoticed bias problem. Third, we estimate multipliers

[1]Estimating a distant response to a shock of interest may result in some realisations of the latter being omitted from the estimation for some observations, as they will fall betwen the horizon of the response (dependent variable) and the timing of the shock variable (independent variable). A simple LP estimation will fail to account for its effect on the response variable.

directly by dividing fiscal shocks by the lagged response variable. In line with the original Jorda (2005) study, we also include residuals from the previous horizon estimations. Jorda claimed that the mapping of the VAR to the LP equation results in residuals being correlated along the response horizons and including them into the estimation could improve the technique.

For the first part of the analysis, the baseline specification (4.3.1) of the LP model is used to compare fiscal multipliers between government consumption and government investment expenditure. We use a panel version of the LP method, as the dataset only contains 20 yearly observations per country at most.[2] The baseline model can be described as follows:

$$\Delta X_{i,t+h} = \beta^{I,h} FS^I_{i,t} + \beta^{C,h} FS^C_{i,t} + \sum_{j=0}^{h-1} \theta^h_{3,j} FS^I_{i,t+h-j} + \sum_{j=0}^{h-1} \theta^h_{4,j} FS^C_{i,t+h-j} + \alpha^h_i + \gamma^h_t + \hat{\epsilon}^{h-1}_{i,t} + \epsilon^h_{i,t}, \quad \epsilon^h_{i,t} \sim N(0,\sigma^2) \quad (4.3.1)$$

where $\Delta X_{i,t+h} = ln(X_{i,t+h}) - ln(X_{i,t-1})$ is the cumulative growth rate of a response variable, the subscript i denotes the countries, and the subscript t the years. $FS^I_{i,t}$ and $FS^C_{i,t}$ are the measures of unanticipated government investment and government consumption shocks, respectively. The coefficient before these shocks delivers cumulative multipliers $\beta^{I,h}$ and $\beta^{C,h}$ to a unit fiscal injection. $\sum_{j=0}^{h-1} FS^I_{i,t+h-j}$ and $\sum_{j=0}^{h-1} FS^C_{i,t+h-j}$ are the fiscal policy shock leads up to the horizon of estimation; these terms enter the estimation at horizons higher than zero ($h > 0$). This study considers three response variables: real GDP, real private consumption, and real private investment.[3] For $h = 0$, the equation estimates the contemporaneous effect of fiscal shocks on response variables. The effect for each horizon $h = 1, 2, \ldots, H$ is estimated in separate equations. The specification includes country (α^h_i) and year fixed-effects (γ^h_t), and the unanticipated government investment and government consumption shocks, respectively: $FS^I_{i,t}$ and $FS^C_{i,t}$. To identify the unantic-

[2]In view of the data limitations, we seek to estimate an average multiplier value for emerging and developing economies.

[3]Real measures were obtained using GDP deflators for the respective countries

ipated shocks, each annual observation of government spending-to-GDP is compared to the WEO forecast for the ratio made in the October vintage of the corresponding year. Thus, the gap between realisation and forecast is reduced to a single quarter. The forecast error in the fiscal variable is then computed by rescaling the forecast error of the ratio with the observed value of the GDP.[4] These forecast errors enter the model as ratios over the level of the response variable at $t-1$. Following Hall (2009) this approach enables us to estimate multipliers directly -- instead of first estimating elasticities and obtaining multipliers via ex-post rescaling.

$$FS_{i,t}^{\{I,C\}} = \frac{FE_{i,Q4}^{\{I,C\}} - FE_{i,Q4|Q3}^{\{I,C\}}}{X_{i,t-1}} \tag{4.3.2}$$

The right-hand side of the equation includes future realisations of unanticipated fiscal shocks. It is worth mentioning that, if these shocks are correlated, then failure to include all the relevant shocks in the model equation can deliver biased multiplier estimates. The correlation between unanticipated shocks may be further boosted if they originate from the same forecaster. Consider the investment multiplier formula, given the use of the within category estimator:

$$\beta_{FE}^{I} \approx \frac{\sum_{t=1}^{T}\sum_{i=1}^{N}\left(FS_{i,t}^{I} - FS_{i}^{I}\right)\left(\Delta Y_{i,t+h} - \Delta Y_{i}\right)}{\sum_{t=1}^{T}\sum_{i=1}^{N}\left(FS_{i,t}^{I} - FS_{i}^{I}\right)^{2}} - \Omega \tag{4.3.3}$$

where the first term on the right is a simple fixed effects estimator, and Ω is the effect of the government investment shock on other shock of interest. We can disregard Ω if we are certain that the shocks under analysis are not cor-

[4]This choice is driven by the numerous revisions in the IMF WEO vintage database, which does not allow to compute the target shocks directly. An alternative way to specify shocks would be to follow Furceri and Li (2018) and define shocks as forecast errors in the growth rates of the fiscal variables. Nonetheless, this approach does not allow us to estimate the multipliers directly and to avoid re-scaling bias, as argued by Ramey and Zubairy (2018). An important issue to consider is that our definition of shocks potentially includes forecast errors in the value of GDP, which can drive our results since we use government spending-to-GDP ratio forecasts. Adding the GDP forecast error as an extra control shows that this modelling choice does not drive our results. Further details can be found in the Appendix 4.11

related; however, we demonstrate in Appendix 4.9 that this is not true. As can be seen from (4.3.3), the government investment multiplier is the effect of the government investment shock on part of the output that was not affected by the rest of shocks included in the model. This study, therefore, includes both fiscal shocks in the LP equation, rather than estimating their effects in isolation. Shocks exceeding 15% of GDP in magnitude are excluded from the estimation because such extreme shocks values, in our experience, represent data collection mistakes, such as failure to update forecasts to currency denomination events. Appendix 4.10 discusses trimming issues further.

In the second part of the analysis, multipliers are estimated for different economic scenarios. As the main concern of the second part is to determine the efficacy of fiscal stimuli differ across different states of the economy , the baseline specification is etended:

$$\begin{aligned}\Delta X_{i,t+h} = {} & \beta_0^{I,h} FS_{i,t}^{I} + \beta_0^{C,h} FS_{i,t}^{C} + \sum_{j=0}^{h-1} \theta_{0,j}^{I,h} FS_{i,t+h-j}^{I} + \sum_{j=0}^{h-1} \theta_{0,j}^{C,h} FS_{i,t+h-j}^{C} \\ & + \phi_{i,t} \left[\beta_1^{I,h} FS_{i,t}^{I} + \beta_1^{C,h} FS_{i,t}^{C} + \sum_{j=0}^{h-1} \theta_{1,j}^{I,h} FS_{i,t+h-j}^{I} + \sum_{j=0}^{h-1} \theta_{1,j}^{C,h} FS_{i,t+h-j}^{C} \right] \\ & + \alpha_i^h + \gamma_t^h + \hat{\varepsilon}_{i,t}^{h-1} + \varepsilon_{i,t}^h, \quad \varepsilon_{i,t}^h \sim N(0,\sigma^2) \qquad (4.3.4)\end{aligned}$$

where all the prior variable definitions hold and $\phi_{i,t}$ is a binary indicator of a state/regime.

The use of a continuous indicator, shaped in Auerbach and Gorodnichenko (2012) fashion, has been widely used in the relevant literature. Although it is reasonable to use a continuous indicator in the presence of quarterly data, it is not the best choice when data frequency is annual. An LP model incorporating a continuous state indicator would usually pre-multiply explanatory variables with a lagged value of the state (Auerbach and Gorodnichenko, 2013a) or by the current value of the state (Furceri and Li, 2018). Annual data constitutes two major issues when applying the Auerbach and Gorodnichenko (2013a) approach. First, using the lagged value of the state does not make much sense with annual data, as the impact of a given economic state has higher probability of affecting a certain relationship in the next quarter, not so much in the next year. If the recession had ended in November 1982, then fiscal policy would not be in recession mode in 1983 as there were no recessions in 1983. Pre-multiplying shocks by the current value of the continuous indicator, as in Abiad et al. (2016) and Furceri and Li (2017), can lead to spurious results. The continuous state indicator can be correlated with the response variable; thus, the product of shock and the indicator can capture a relationship that is not necessarily present in the true data generating process. For example, Auerbach and Gorodnichenko (2012) define economic slack simply as a transformation of the seven-quarter moving average of GDP growth. In this case, the indicator and the response variable (GDP) are similar in nature and can be correlated. In order to avoid such spurious relationships, we follow Ramey and Zubairy (2018) and Bernardini and Peersman (2018) and define the indicator variable as a dummy,[5] then create an interaction term between the contemporaneous dummy value and the fiscal shocks.

The model, as often observed in related literature, assumes that a shock at zero horizon would not have an immediate effect on the state of the economy. This extension delivers a pair of multipliers, one for each of the defined states. Such states incorporated into this analysis include a) economic slack, b) debt burden, c) openness to trade, and d) size of the government. All the specifications mentioned above will be used to compute cumulative fiscal multipliers for five annual horizons ($H = 5$).

[5]Using a dummy variable instead of a continuous indicator, essentially, is equivalent to a sample split.

4.4 Results

4.4.1 Government Expenditure in Emerging and Developing Economies

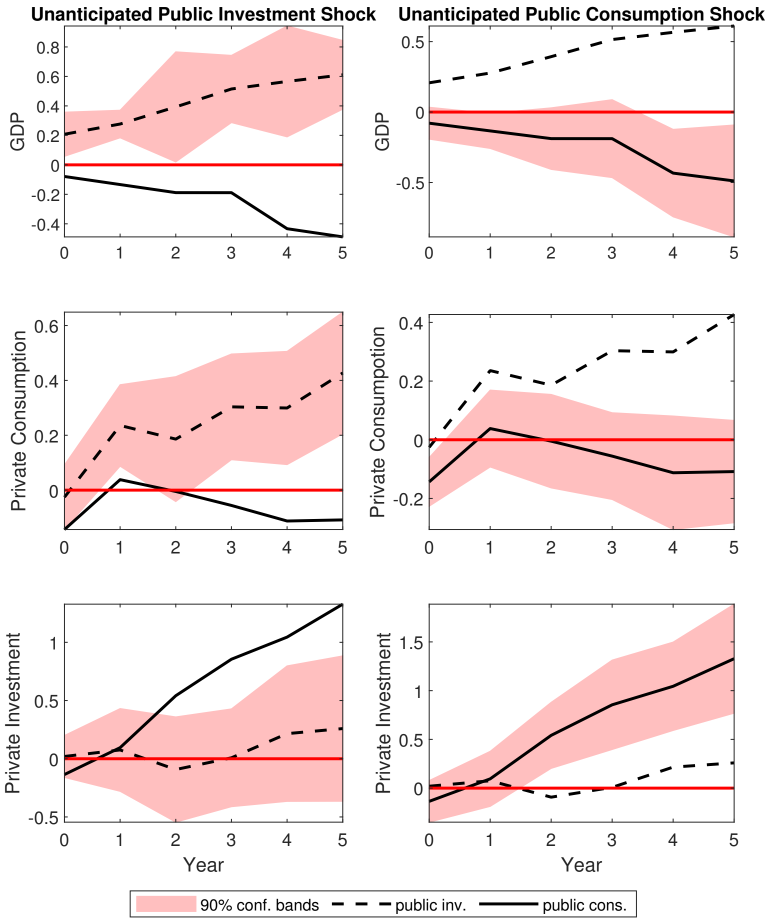
This section presents fiscal multipliers calculated using the baseline model specification for the joint group of emerging and developing economies.[6] Figure 4.4.1 reports government consumption and investment multiplier estimates; a multiplier is defined as the cumulative change of the response variable to a unit fiscal shock, both measured in terms of the national currency. Focusing on the two types of government expenditure, namely investment and consumption, it is obvious that these two types of stimuli not only have different overall impact on GDP but also on its components . An additional unit of local currency spent on investing in public capital stock tends to significantly increase output by 0.21 units in the same year. This effect remains significant and positive for the following five years, delivering a cumulative increase in GDP of 0.61 units, five years after the shock ensued. On the other hand, government consumption expenditure tends to deliver a negative, yet statistically insignificant response for the first three years, on output. The cumulative government consumption multiplier becomes significant four years after the shock takes place, reaching a value of -0.49 five years later. This result is also supported by the earlier evidence in Ilzetski et al. (2013). Government consumption expenditure in developing economies is counterproductive and tends to reduce output further.

An interesting pattern can be observed once we analyse the impact of fiscal expenditure on GDP component—each type of government expenditure is effective in stimulating the opposite private counterpart. Government investment stimulates private consumption expenditure while failing to have a significant effect on private investment. Government consumption has a significant negative short-term implication for private consumption, but it is successful in stimulating private investment activity in the mid- and long-term. Government investment shocks do not have a significant effect on pri-

[6]Details on the sample of countries can be found in Appendix 4.7.

vate consumption, yet, private consumption increases by 0.24 in the following year and builds up to a 0.43-unit cumulative response by the end of the fifth consecutive year. The effect on private investment fluctuates around zero for the entire six-year period and does not become significant. Private consumption has an immediate negative effect on private consumption spending of -0.14 units of the national currency; mid- and long-term effects remain insignificant. Private investment, as compared to private consumption, seems to be more responsive to government consumption stimuli. The cumulative response of government investment becomes significant two years after the original shock and continues to increase from 0.54 to 1.33 units by the fifth year.

During the five years after the original injection of fiscal stimulus, additional units of local currency spent on government investment increase output by 0.61 units and private consumption by 0.43 units. During a similar period, the government consumption shock decreases output by 0.49 units and increases private investment by 1.33 units. It also reduces private consumption significantly by 0.14 units during the impact year.



Note: 90% confidence bands in red.

Figure 4.4.1: Cumulative response of GDP, private consumption and investment in local currency units to an unanticipated government investment (right) and government consumption (left) shocks of 1 unit in local currency.

Based on the considerations expressed in the previous section, we demonstrate how the inclusion of both shocks affects the multiplier estimates. Table 4.4.1 presents the cumulative output responses at different horizons to the two types of government expenditure shocks.

		$h=0$	$h=1$	$h=2$	$h=3$	$h=4$	$h=5$
FS^I	Baseline	0.21** (0.09)	0.28*** (0.06)	0.39* (0.23)	0.51*** (0.14)	0.57** (0.23)	0.61*** (0.14)
	Isolated TZ	0.20** (0.09)	0.26*** (0.05)	0.36* (0.21)	0.48*** (0.12)	0.48** (0.22)	0.50*** (0.12)
	Isolated	0.20** (0.09)	0.30*** (0.06)	0.45** (0.22)	0.56*** (0.15)	0.55** (0.22)	0.54*** (0.12)
FS^C	Baseline	-0.08 (0.07)	-0.13* (0.08)	-0.19 (0.13)	-0.19 (0.17)	-0.43** (0.19)	-0.49** (0.24)
	Isolated TZ	-0.06 (0.07)	-0.11 (0.08)	-0.16 (0.12)	-0.15 (0.17)	-0.37** (0.18)	-0.42 (0.25)
	Isolated	-0.06 (0.07)	-0.13* (0.07)	-0.17 (0.13)	-0.16 (0.16)	-0.36* (0.20)	-0.43* (0.25)
N		1.674	1,540	1,411	1,294	1,179	1,072
Standard errors in parentheses $^*p < 0.1, ^{**}p < 0.05, ^{***}p < 0.01$							

Note: FS^I– government investment shock, FS^C– government consumption shock. Dependent variable: $ln(GDP_{i,t+h}) - ln(GDP_{i,t-1})$

Table 4.4.1: Comparison of the baseline multiplier estimation results with approaches that do not account for the hitherto unnoticed bias and bias resulting from omitting relevant shocks.

We compare estimates resulting from our analysis (Baseline) to the case where we would calculate the effects of the two types of expenditure in isolation. We also consider the case when shocks are estimated in isolation but incorporate Tseulings and Zubanov's (2014) approach (Isolated TZ). Adding future shock realisations to the isolated estimations shows that simple LP application tends to overestimate the government investment multiplier. The difference in government consumption multipliers is negligible. Estimating the model with both shocks and future realisations shows that the simple isolated LP estimation tends to underestimate government investment and overestimate government consumption multipliers; bias increases with the forecast horizon. Our approach signifies that an additional currency unit of government investment stimulates output by an extra 0.07 units of national currency than the simple LP estimation implies. Similarly, government consumption spending would reduce output by an additional 0.06 units of national currency.[7]

4.4.2 Economick slack

The interest in dependence of fiscal policy's impact on the business cycle stage has been reanimated considering the recent economic developments, even more so for economic policy making. The Keynesian view dictates that in times of economic slack, injection of an additional fiscal stimulus can boost income and stimulate private expenditure through a multiplicative effect. This section compares the effects of fiscal expenditure between periods of economic slack and boom. In the absence of a narrative recession measure, such as the US National Bureau of Economic Research (NBER) recession dates, for the entire sample of countries under analysis, this approach focuses on two economic states that are similar in nature: positive and zero/negative growth. As discussed in Section 4.3, these states are identified via the introduction of dummy variables.[8] The results on estimating cumulative responses of output, private consumption, and private investment to two types of government expenditure shocks, that is, government consumption and government investment are presented in Figures 4.4.2 and 4.4.3 below.

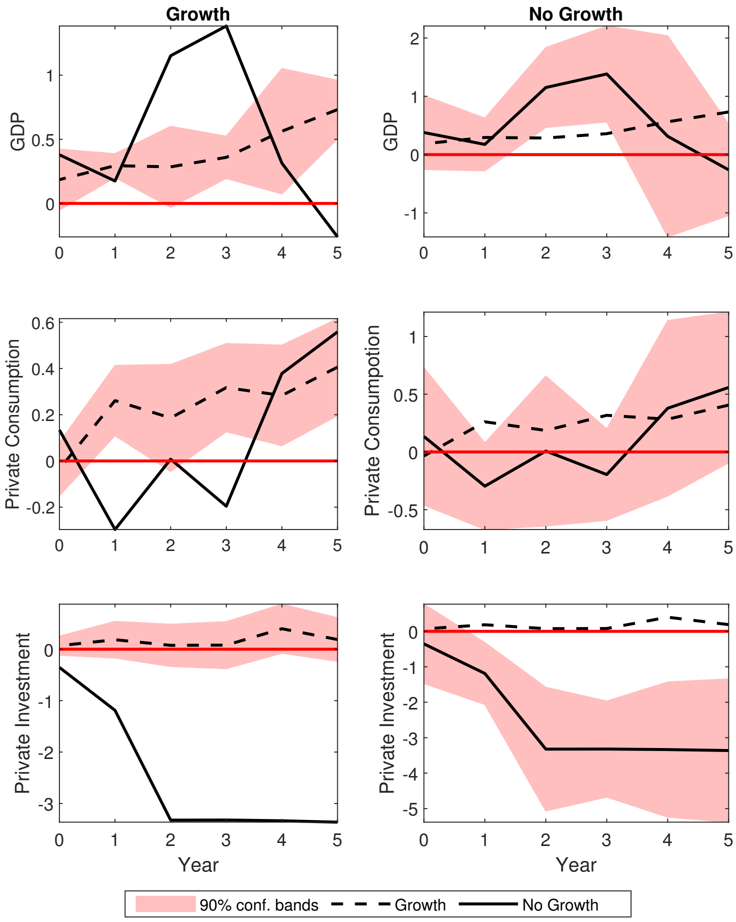
During periods of economic slack, the model estimates higher output responses to both unanticipated government investment and consumption injections. At times when the GDP falls, the cumulative response of output to a government investment shock peaks at the three-year horizon with 1.38

[7] Estimations were carried on a panel of countries with variables denominated in respective national currencies. Since we estimate multipliers rather than elasticities, IRFs should be interpreted as responses measured in national currency to a unit shock denominated in the respective national currency.

[8] This approach delivers results that are not caused by spurious correlations, as may be the case in some applications of alternative approaches used in the literature. For example, Auerbach and Gorodnichenko (2013a) use a continuous indicator of the state that enters the shock calculation. Although in many applications it is a completely feasible strategy, the authors use the Hodrick and Prescott filter to obtain the underlying variable in the indicator calculation. Following Hamilton's (2018) line of thought, such an approach may create correlations between macroeconomic variables, both contemporaneous and lagged, that are not present in the true data-generating process.

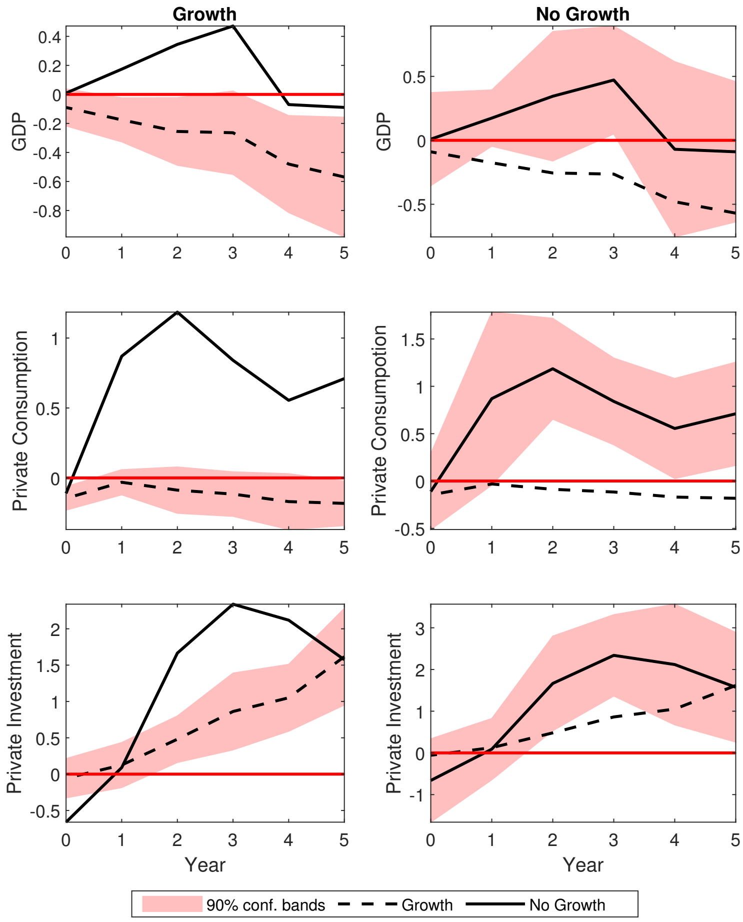
significant increase in output . At times of positive economic growth, the response is small, with a 0.36 unit increase. The impact of government injections in the form of investment fade faster during periods of economic slack. The cumulative output response slips into negative space and becomes insignificant four years after the shock. During non-slack periods, on the other hand, GDP continues to grow at a slow pace even five years after the impact. Government consumption spending tends to have a negative cumulative effect on output during periods of positive economic growth, yet the significance of the result only increases between four- and five-year horizons. During periods of economic slack, government consumption has a modest positive short- and mid-term effect. In the long-term, analogous to government investment, the cumulative GDP response plunges into negative space.

The crucial difference between the two types of government expenditure is the fact that government consumption seems to have the ability to stimulate private expenditure during periods of economic slack, while government investment does not. Furthermore, at times of negative or zero growth, a government investment shock tends to significantly crowd out private investment. The cumulative response of private investment falls to -3.32 units two years after the original fiscal injection and remains at the same level up to the fifth-year horizon. An unanticipated government consumption shock that hits the economy during periods of slack has a significant positive effect on private consumption; cumulative response peaks at 1.19, two years after the shock. During periods of positive growth, the impact on private consumption is negative, small, and insignificant. The effect on private investment is similar to the non-state-dependent baseline case (Figure 4.4.1); however, during periods of economic slack, there seems to be an additional boost in mid-term efficacy. Private investment response peaks at 2.34, three years after the original injection.



Note: Cumulative response of GDP, private consumption, and private investment in local currency units to unit shock. Positive GDP growth (left) versus negative/zero growth (right). 90% confidence bands in red.

Figure 4.4.2: IRFs and economic slack: unanticipated government investment shock.



Note: Cumulative response of GDP, private consumption, and private investment in local currency units to a unit shock. Positive GDP growth (left) versus negative/zero growth (right). 90% confidence bands in red.

Figure 4.4.3: IRFs and economic slack: unanticipated government consumption shock.

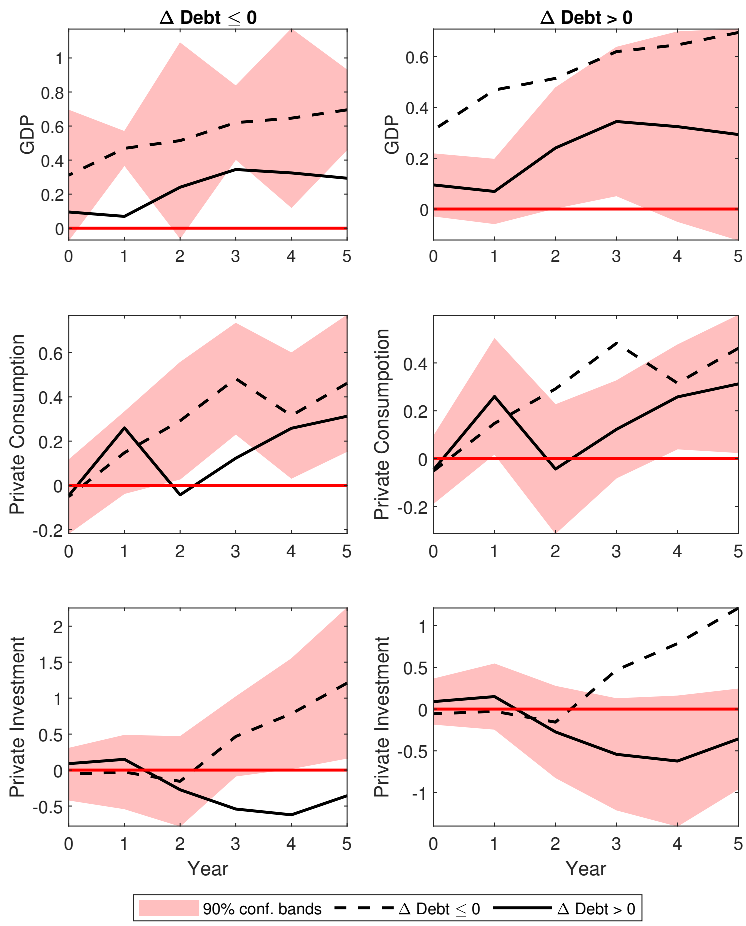
4.4.3 Debt burden

Another interesting dimension of the analysis considers how multipliers change with levels of outstanding government debt. Ilzetzki et al. (2013) identifies a 60% debt-to-GDP threshold, concluding that higher levels of debt negatively affect the potency of government consumption stimuli. Focusing on debt burden levels, when a fiscal shock hits the economy, the study methodology delivers counter-intuitive results. The basic idea underlying such an analysis is that an already high debt level increases the cost of additional borrowing, as potential lenders may not deem the country's fiscal position sustainable, thereby raising the risk premium and the market interest rates, in turn.

The rationale behind such a decision-making process is yet more complicated than simply focusing on the face value of government debt, or its ratio to GDP. Factors such as currency risks, maturity structure, level of implicit interest rates on outstanding debt or projections of a country's future growth are usually considered when making such decisions. Thus, focusing on debt level may not necessarily deliver robust results in some cases, as a variety of other factors should also be considered.

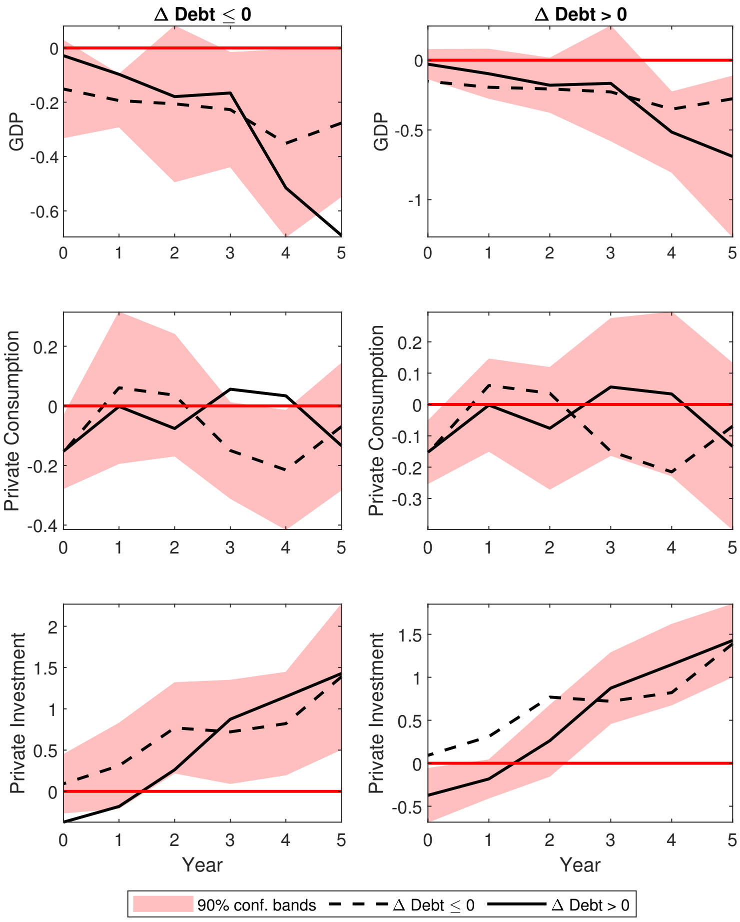
This section will focus on understanding how raising additional debt can alter the potency of fiscal stimuli. Thus, we define two states: when outstanding government debt increased during a calendar year, and when it remained the same or decreased. Therefore, periods when countries expanded their borrowing levels, would be associated with increasing interest rates and vice versa. A neoclassical view dictates that an increase in debt crowds out investment in productive physical capital. One way to consider this is that injection of additional government bonds will attract part of the household's income that would otherwise be invested in private debt. Decline in investment leads to lower capital, increasing the marginal product of capital. The higher marginal product of capital will increase the interest rates, as firms struggle to attract investments.

When estimating the impact of fiscal stimuli for the two states of the economy, we observe that the effect of a government investment shock seems to depend on public debt. If the shock took place in the year when the debt-to-GDP ratio decreased or remained consistent, the cumulative response of output was higher for all five impulse response horizons, as compared to the periods when the ratio increased. On impact, a unit shock to government investment increased output by 0.31 units when the debt was not growing and by 0.09 otherwise; yet, both values were not statistically significant. The only horizon, at which responses for both states are statistically significant, is three years after the original shock, with the cumulative output response of 0.62- and 0.34-units during periods of negative/zero and positive debt-to-GDP growth, respectively. Thus, increasing debt levels tend to decrease output response by almost a factor of 2, on average.



Note: Cumulative response of GDP, private consumption, and private investment in local currency units to a unit shock. Zero/negative debt-to-GDP ratio growth (left) versus positive debt-to-GDP ratio growth (right). 90% confidence bands in red.

Figure 4.4.4: IRFs and debt dynamics: unanticipated government investment shock.



Note: Cumulative response of GDP, private consumption, and private investment in local currency units to a unit shock. Zero/negative debt-to-GDP ratio growth (left) versus positive debt-to-GDP ratio growth (right). 90% confidence bands in red.

Figure 4.4.5: IRFs and debt dynamics: unanticipated government consumption shock.

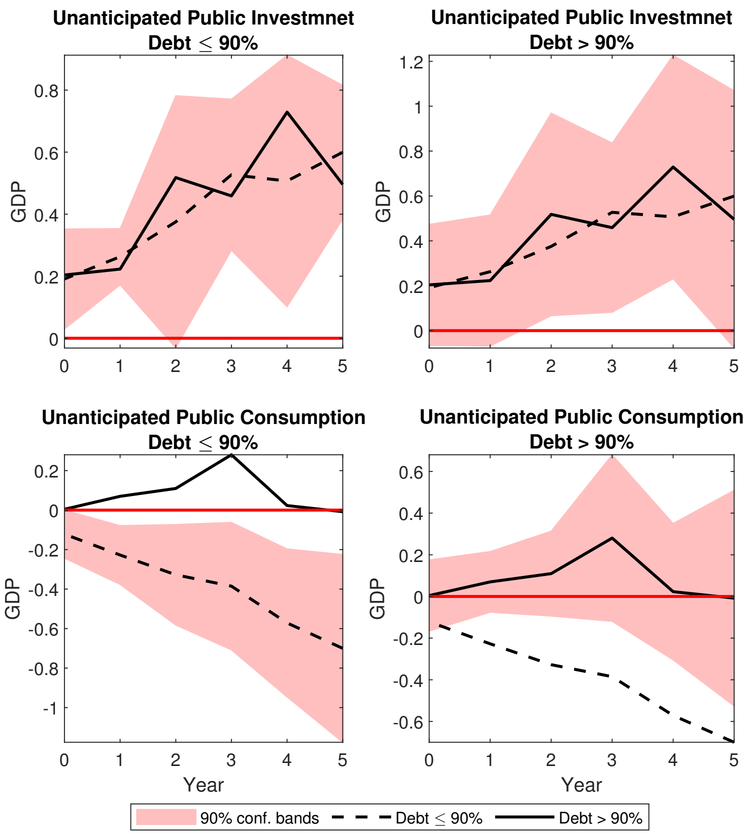
The effect on private consumption when debt is decreasing or constant is higher and significant at mid-term, whereas the response is mostly insignificant when the debt levels are rising. The response of private investment in the long run is positive when the debt level remains constant or decreases, but declines in the negative space when the debt levels are rising; private investment responses are insignificant for both the states. As can be seen in

Figure 4.4.5, GDP, private consumption, and private investment responses to a government consumption shock do not demonstrate large differences between the two states. One interesting fact is that the impact response of private investment to a government consumption shock is negative and significant when the debt level is rising, whereas it is positive and insignificant when it remains constant or decreases.

Another interesting aspect to focus on is whether there is any significant difference in the fiscal policy's potency when the debt levels are substantially higher. However, it is important to keep in mind that the issues raised earlier in this section apply to this analysis. Rather than making an inference on the difference in transmission of fiscal policy shocks between periods of high and low debt-to-GDP ratios, we simply seek to determine the difference in the effect on output in a sample-split fashion. The threshold for analysis is chosen to be 90% debt-to-GDP ratio, identified by Reinhart and Rogoff in their analysis of growth during times of debt.[9] As can be seen in Figure 4.4.6, the impact of government investment on GDP does not vary substantially between times when the debt-to-GDP ratio is below and above the proposed threshold. In both states, the response of GDP on impact is approximately equal to 0.2 units; the response is insignificant in the case of high debt levels. Overall, the cumulative responses of output to a government investment shock do not demonstrate significant differences. Government consumption tends to significantly decrease output when the debt level is below the threshold, culminating at -0.7-units cumulative response of output five years after fiscal injection had ensued. When the debt-to-GDP ratio is above 90%, the output response is positive at mid-term, yet insignificant at every horizon.[10]

[9]It worth reminding that this study acknowledges the fact that there was an error in the original calculation. This part seeks to test if the mentioned debt threshold has any significance in the case of fiscal policy's efficacy.

[10]Appendix 4.12 presents the results of the robustness checks. The pattern observed in Figure 6 holds for 70\% and 80\% debt-to-GDP threshold definitions. A 100% debt-to-GDP threshold delivers government consumption multipliers that are positive, both above and below the threshold.



Note: Cumulative response of GDP, private consumption, and private investment in local currency units to a unit shock. Debt-to-GDP ratio less or equal to 90% of GDP (left) versus higher than 90% (right). 90% confidence bands in red.

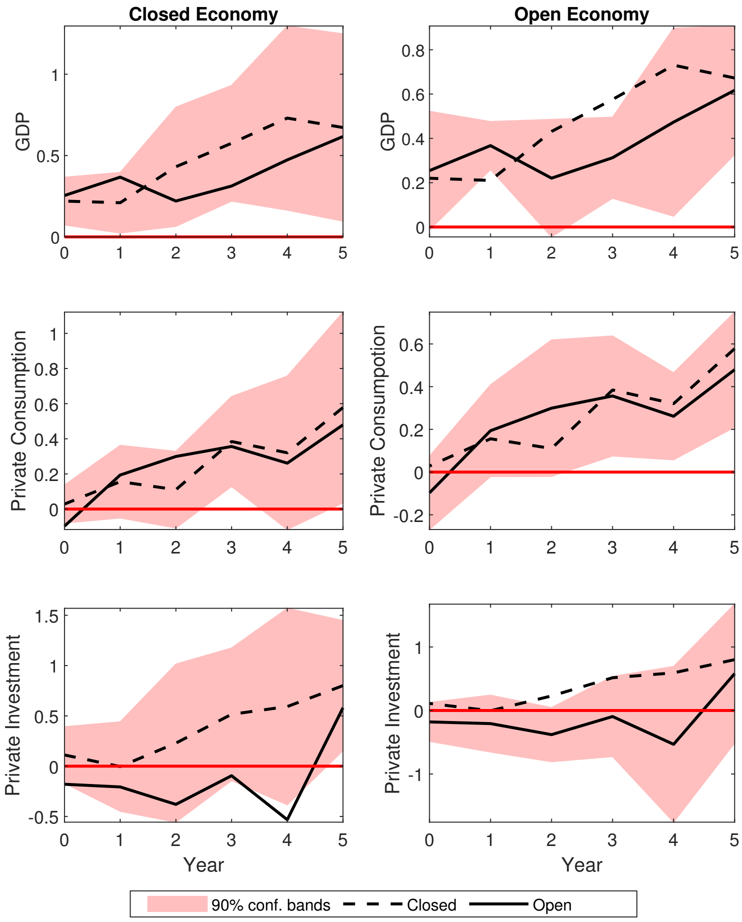
Figure 4.4.6: IRFs and the debt level: unanticipated government investment and consumption shocks.

4.4.4 Openness to trade

We now consider openness to trade. The textbook Mundell-Fleming model predicts that any fiscal injection in an open economy will have limited power in stimulating economic activity, as the positive effect on output will be partly offset by a reduction in net exports through exchange rate appreciation. Ilzetski et al. (2009) identified 60% exports+imports-to-GDP threshold, and estimated higher multipliers for the group of countries that have trade volumes higher than the specified level. This study does not split the sample into open and closed economies. Alternatively, we identify openness to trade through the median level of trade-to-GDP, calculated using the whole sample. Then, using model (4.3.4), we estimate the differences in fiscal policy's potency based on whether a specific observation in the sample was above or below the threshold. In other words, this approach does not restrict the ability of any given country in the sample to move from the group of relatively open to relatively closed economies or vice versa.

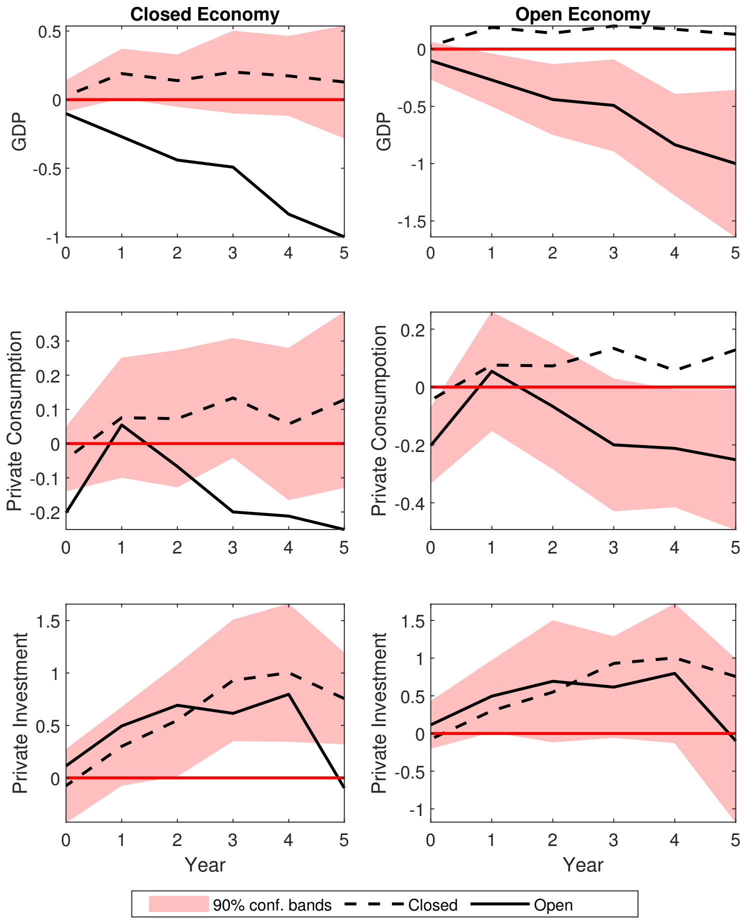
Ilzetski et al. (2009) estimated statistically insignificant negative government consumption multipliers in open economies and statistically significant positive multipliers for closed economies. The LP method delivers similar results, as unanticipated government consumption shock tends to deliver a positive, yet statistically insignificant, cumulative output response for relatively closed economies. The cumulative output response in the case of an open economy is negative and becomes statistically significant one year after the impact; output falls by 1 unit of national currency five years after the original fiscal injection. Government investment does not seem to lose much potency in the case of an open economy. For both states of the economy , the cumulative response of output is positive and mostly significant. However, government investment expenditure seems to stimulate GDP more rapidly, as the mid-term effect is larger in the case of a relatively closed economy.

The degree of openness to trade also seems to affect the potency of the two types of government expenditure in stimulating their private counterparts. In the closed economy case, government consumption shock delivers a positive but statistically insignificant cumulative private consumption response. In the case of an open economy, the cumulative private consumption response on impact and in long-term is negative and statistically significant. In a closed economy, the cumulative private investment response remains positive and becomes significant only five years after the investment shock takes place. In an open economy, it is insignificant and remains negative for most of the impulse response horizon. The cumulative private investment response to a government consumption shock is significant and positive at mid- and long-term, with a 0.76 unit increase by the end of the fifth year after the shock takes place.



Note: Cumulative response of GDP, private consumption, and private investment in local currency units to a unit shock. Closed economy case (left) versus open economy case (right). 90% confidence bands in red.

Figure 4.4.7: IRFs and involvment in trade: unanticipated government investment shock.



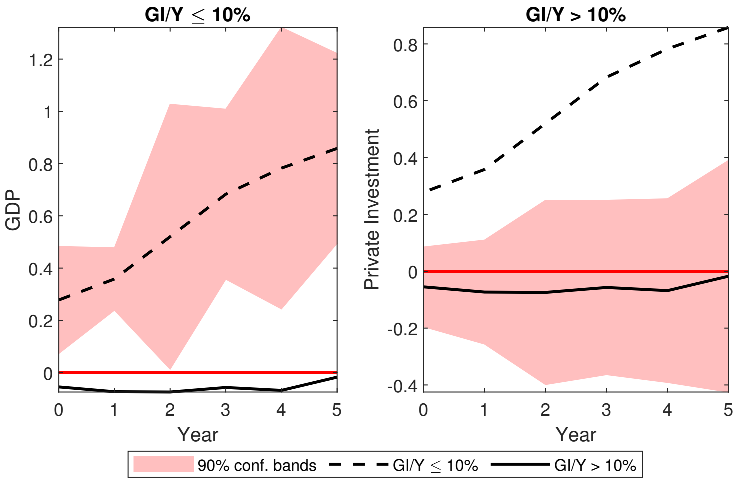
Note: Cumulative response of GDP, private consumption, and private investment in local currency units to a unit shock. Closed economy case (left) versus open economy case (right). 90% confidence bands in red.

Figure 4.4.8: IRFs and involvment in trade: unanticipated government consumption shock.

4.4.5 Size of the public sector

Finally, this section seeks to determine if the size of the public sector affects the potency of fiscal expenditure. We consider two criteria. First, we examine how the stock of public capital affects government investment multipliers. The macroeconomic theory dictates that the higher the stock of public capital, the smaller will be the effect of a marginal increase in government investment on output. Second, we investigate how the size of the public labour force affects both government expenditure multipliers. There are two possible ways to examine the size of the public labour force as a factor capable of affecting fiscal expenditure potency. On the one hand, the larger the share of public employee compensation in total employee compensation, the stronger will be the impact of changing government consumption on the real economy. A substantial share of government consumption consists of public employee compensation. Thus, an increase in government consumption can boost private expenditure directly by paying public employees more, or alternatively, hiring more from the labour force. On the other hand, a large public labour force indicates an oversized public sector, which can sometimes make it difficult for the fiscal stimulus to find its way into the real economy due to bureaucratic and efficiency issues. In order to identify the implications of the size of the public sector, we chose a 10% threshold for both government investment-to-GDP and public employee compensation-to-GDP ratios.

Given that a measure of public capital stock is not easily obtainable, we use the government investment-to-GDP ratio to approximate it. The cumulative investment multipliers, shown in Figure 4.4.9, suggest that countries with a government-investment-to-GDP ratio smaller than 10% have higher government investment multipliers. When the level of government investment is small, an unanticipated government investment shock delivers a statistically significant positive output response on impact; the simultaneous response of output equals 0.43 units of national currency. In the mid- to long-term, government investment delivers a positive and growing cumulative response of output, which remains statistically significant and builds up to 1.1 units in the fifth year after the original injection. When the level of investment is high, the simultaneous response of output equals -0.01 and is statistically insignificant. It remains around zero in the mid-term and declines into negative space in the fourth year following the shock, where it remains statistically insignificant.

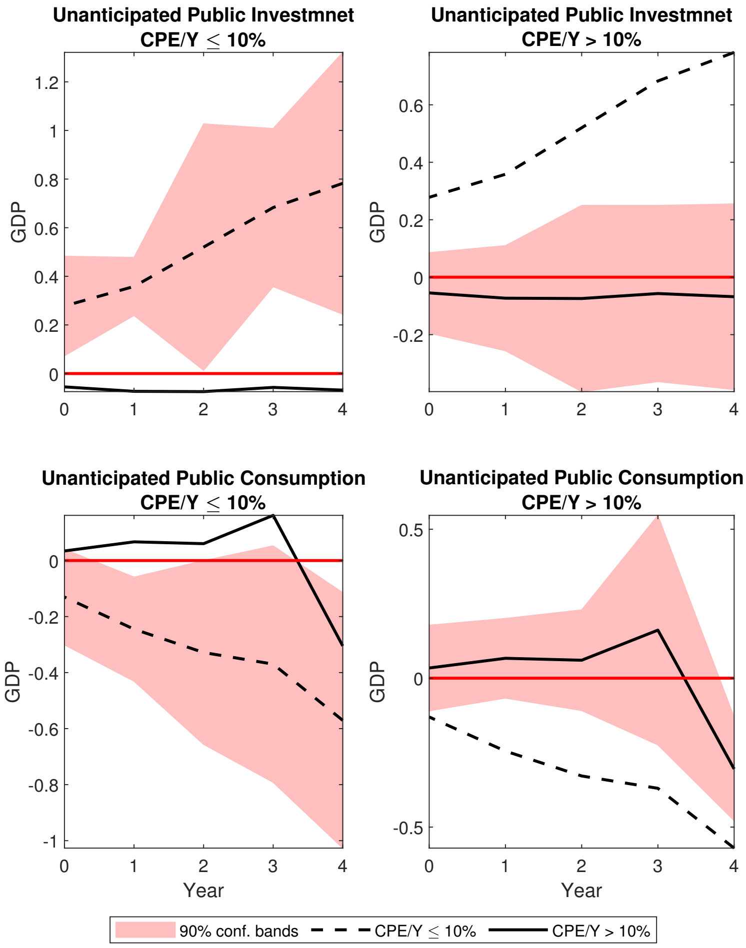


Note: Cumulative response of GDP in local currency units to a unit shock. Government investment less or equal to 10% of GDP (left) versus more than 10% of GDP (right). 90% confidence bands in red.

Figure 4.4.9: Output repsonse and public investment level: unanticipated government investment shock.

The size of the public labour force has similar implications for government investment, although it delivers opposite results for government consumption. Government investment expenditure delivers a positive and significant output response both in the short- and long-term when the size of public labour force compensation is less than 10% of GDP. The output response on impact equals 0.28 units of national currency and increases to 0.86 units at the five-year horizon. When the ratio is above 10%, the cumulative output response to government investment shock remains negative and statistically insignificant at each horizon. On the other hand, government consumption delivers a negative but occasionally significant cumulative response of output when the public labour compensation is less than 10% of GDP. The output response is positive and insignificant in the short- and mid-term if the labour compensation constitutes a larger share of the GDP. In both cases, long-term responses of output are negative and significant, with cumulative responses of -0.57 and -0.45 units in case of low and high compensation ratios, respectively.

The government investment expenditure multiplier is higher when the size of the public sector and the level of government investment are small, yet the size of the government consumption multiplier is larger when the share of the public labour force compensation is more than 10%.



Note: Cumulative response of GDP, private consumption, and private investment in local currency units to a unit fiscal shock. Total compensation of public employees less or equal to 10% of GDP (left) versus more than 10% of GDP (right). 90% confidence bands.

Figure 4.4.10: Output response and size of public labour force: unanticipated government investment and consumption.

4.5 Conclusion

As the debate about the appropriate way to compute fiscal multipliers evolves constantly, we deliver an empirical analysis of the potency of government consumption and government investment expenditure in stimulating GDP and its components via the LP approach. We incorporate the recent developments from the fiscal multiplier and LP literature, and raise attention regarding possible biases that could arise if relevant shocks are excluded from the estimation. Assessing the overall and state-dependent multipliers on a panel of 107 emerging and developing economies, we summarise our analysis in the following six points.

First, failure to control for Tseulings and Zubanov's (2014) hitherto unnoticed bias, combined with the estimation of public investment and public consumption effects in isolation, results in the underestimation of the positive effect of public investment and overestimation of the negative effect of public consumption on economic growth.

Second, unanticipated government investment and consumption stimuli produce conflicting responses of output. The government investment stimulus delivers a positive cumulative output response that is significant for all IRF horizons. It gradually increases from 0.21 units of the national currency on impact to 0.61 units, five years after the shock occurred. Government investment significantly increases future private consumption, although it does not have a significant impact on private investment. Unanticipated government consumption stimulus delivers a significant negative response of the output in the long-term. In the fifth year after the original shock, the output decreased by -0.49. Surprisingly, government consumption significantly stimulates private investment in the mid- to long-term, raising private capital stock by 1.33 units of national currency after five years.

Third, periods of negative or zero growth are associated with higher long-term multipliers for both unanticipated government consumption and investment. During periods of economic slack, government investment tends to crowd out private investment, and additional government consumption tends to fuel private consumption.

Fourth, government investment shocks that occur during years of rising government debt tend to deliver lower multiplier values. The effect seems to be propagated through the crowding out of private investment due to rising market rates, even though the relationship is insignificant.

Fifth, government consumption leads to significant crowding out of output in relatively open economies. In relatively closed economies the effect is positive, yet statistically insignificant.

Finally, government investment shocks significantly boost output when the government investment-to-GDP ratio is below 10%, leading to a cumulative output response higher than one unit of national currency. When the level of government investment is already high, the marginal increase in capital stock does not cause the output to change significantly. Additionally, high levels of public employee compensation are associated with lower investment and higher consumption multipliers.

Appendix

4.6 Data

Variable	Transformation	Source
Forecast of Government Consumption	% of GDP	WEO
Forecast of Government Investment	% of GDP	WEO
Government Consumption	% of GDP	WEO
Government Investment	% of GDP	WEO
Public Employee Compensation	% of GDP	WEO
Real GDP Growth	% of GDP	SWEO
Total Outstanding Public Debt	% of GDP	FAD
Trade Openness	% of GDP	PWT 9.0

Table 4.6.1: List of emerging economies

4.7 List of Emerging and Developing Economies in the Sample

Country	Sample	County	Sample
Argentina	1999–2016	Mexico	1996–2016
Brazil	1998–2016	Peru	1996–2016
Bulgaria	1999–2016	Philippines	1996–2016
Chile	1996–2016	Poland	2013–2016
Colombia	1996–2016	Russia	2003–2016
China	2004–2016	South Africa	1996–2016
India	1996–2016	Thailand	1996–2016
Indonesia	2004–2016	Turkey	1996–2016
Malaysia	1996–2016	Venezuela	1996–2014

Table 4.7.1: List of emerging economies

Country	Sample	Country	Sample
Afghanistan	2010–2016	Eritrea	2001–2016
Albania	2007–2016	Ethiopia	1997–2016
Algeria	2000–2016	Gabon	1996–2016
Angola	2002–2016	Gambia	2008–2016
Antigua and Barbuda	2003–2016	Georgia	2004–2015
Armenia	2000–2016	Grenada	1996–2016
Azerbaijan	2003–2016	Guatemala	2002–2016
Bahrain	1998–2016	Guinea	1996–2016
Bangladesh	2000–2016	Guinea-Bissau	2013–2016
Barbados	2000–2016	Haiti	2003–2016
Belize	2000–2016	Honduras	1996–2016
Benin	1996–2016	Jordan	1996–2016
Bhutan	2004–2015	Kazakhstan	2006–2016
Bolivia	1996–2016	Kenya	1996–2016
Bosnia and Herzegovina	2004–2016	Kuwait	1996–2016
Botswana	1996–2016	Lebanon	1999–2016
Burkina Faso	2005–2016	Madagascar	1993–2016
Burundi	1997–2016	Malawi	2008–2016
Côte d'Ivoire	1996–2016	Mali	1996–2016
Cabo Verde	2003–2016	Mauritius	1996–2016
Cambodia	2009–2016	Moldova	2000–2016
Cameroon	1996–2016	Mongolia	2011–2016
Central African Republic	1996–2016	Morocco	1998–2016
Chad	2007–2016	Mozambique	2000–2016
Comoros	1996–2016	Myanmar	2004–2016
Costa Rica	2002–2016	Namibia	1997–2016
Croatia	1996–2016	Nepal	1998–2016
Djibouti	1997–2015	Oman	1997–2016
Dominican Republic	2003–2016	Pakistan	2007–2016
Ecuador	2002–2016	Paraguay	1996–2016
Egypt	1998–2016	Rep. of Montenegro	2009–2014
El Salvador	1996–2016	Republic of Congo	1997–2016
Equatorial Guinea	1999–2016	Romania	2002–2016

Table 4.7.2: List of developing economies

Country	Sample	Country	Sample
Rwanda	1997–2016	Tanzania	1998–2016
Sâo Tome and Principe	2011–2016	Togo	1999–2016
Saudi Arabia	1996–2016	Tunisia	2002–2016
Senegal	1997–2016	Turkmenistan	2005–2016
Serbia	2008–2016	Uganda	1998–2016
Seychelles	2004–2016	Ukraine	2000–2016
Sierra Leone	2006–2016	United Arab Emirates	2006—2016
St. Kitts and Nevis	1996–2016	Uruguay	1996–2016
St. Vincent and the Grenadines	1998–2016	Uzbekistan	2001–2016
Sudan	2001–2016	Vietnam	2000–2016
Syria	1999–2010	Yemen	2001–2014
Tajikistan	2006–2016		

Table 4.7.3: List of developing economies (Cont.)

4.8 Local Projection Method

This section explains the choice of model specification. The LP method develops from a specific peculiarity of a Vector Autoregression model, pointed out by Jorda (2005). It delivers misspecification robust IRFs if the true data generating process is a VAR. Let us consider a workhorse VAR model for fiscal policy analysis, namely the Blanchard and Perotti (2002) model. For simplicity, assume Cholesky identification in cases where the fiscal expenditure variable is ordered first and net taxes last.

$$y_t = c + B_1 y_{t-1} + B_2 y_{t-2} + ... + B_k y_{t-k} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \Omega)$$

Or, alternatively, with structural shocks obtained through Cholesky:

$$y_t = c + B_1 y_{t-1} + B_2 y_{t-2} + ... + B_k y_{t-k} + A^{-1}\Sigma u_t, \qquad u_t \sim N(0, I_n)$$

As shown in Hamilton (1994) and argued in Jorda (2005), the following relationship holds for the above model:

$$
\begin{aligned}
(y_{t+s} - \mu) = B_1^{s+1}(y_{t-1} - \mu) + B_2^{s+1}(y_{t-2} - \mu) + ... + B_k^{s+1}(y_{t-k} - \mu) \\
+ \Phi_s \varepsilon_t + \Phi_{s-1}\varepsilon_{t+1} + ... + \Phi_2 \varepsilon_{t+s-2} + \Phi_1 \varepsilon_{t+s-1} + \varepsilon_{t+s}
\end{aligned}
$$

,where μ is a $n \times 1$ vector containing means of the endogenous variables and Φ_S are the coefficients of a MA(∞) representation of a VAR, that is, the inverted VAR lag polynomial $B(L)$. Introducing structural shocks instead of reduced-form residuals, the above equation can be rearranged into:

$$
\begin{aligned}
y_{t+s} = \left(I_n - B_1^{s+1} - B_2^{s+1} - ... - B_k^{s+1}\right)\mu + B_1^{s+1} y_{t-1} + B_2^{s+1} y_{t-2} + ... \\
+ B_k^{s+1} y_{t-k} + \Phi_s A^{-1}\Sigma u_t + \Phi_{s-1} A^{-1}\Sigma u_{t+1} + ... + \Phi_2 A^{-1}\Sigma u_{t+s-2} \\
+ \Phi_1 A^{-1}\Sigma u_{t+s-1} + A^{-1}\Sigma u_{t+s}
\end{aligned}
$$

From the conventional Structural VAR analysis, coefficients before the structural shocks deliver the IRFs. Thus, responses for all endogenous variables at horizon s are given by $\Phi_s A^{-1}\Sigma$. Jorda (2005) suggests that these responses can be estimated directly without first estimating the underlying multivariate system. The author proposes estimating the following system for each horizon s of interest:

$$
\begin{aligned}
y_{t+s} &= \alpha_s + \beta_{1,s} y_{t-1} + \beta_{2,s} y_{t-2} + ... + \beta_{k,s} y_{t-k} + v_{t+s,s} v_{t+s,s} \\
&= \phi_s u_t + \phi_{s-1} u_{t+1} + ... + \phi_2 u_{t+s-2} + \phi_1 u_{t+s-1} + \phi_0 u_{t+s}
\end{aligned}
$$

,where ϕ_0 is a lower unitriangular matrix. Although this is still a multivariate vector system, it can be estimated equation by equation; moreover, one can focus only on equations of the response variables of interest. Thus, once again, let us return to the Blanchard and Perotti (2002) model, the LP equivalent of estimating the response of output to government expenditure shock would be (assuming 1 lag for simplicity):

$$\begin{aligned}GDP_{t+s} = \alpha_s + \left(\beta^{11}_{1,s}G_{t-1} + \beta^{12}_{1,s}GDP_{t-1} + \beta^{13}_{1,s}NT_{t-1}\right) + \\ + \left(\phi^{21}_{s}u^{G}_{t} + \phi^{22}_{s}u^{GDP}_{t} + \phi^{23}_{s}u^{NT}_{t}\right) + ...+ \\ + \left(\phi^{21}_{1}u^{G}_{t+s-1} + \phi^{22}_{1}u^{GDP}_{t+s-1} + \phi^{23}_{1}u^{NT}_{t+s-1}\right) \\ + \left(\phi^{21}_{0}u^{G}_{t+s} + \phi^{22}_{0}u^{GDP}_{t+s} + \phi^{23}_{0}u^{NT}_{t+s}\right)\end{aligned} \tag{4.8.1}$$

A conventional SVAR approach would require $\phi^{23}_{s} = ... = \phi^{23}_{0} = 0$, as it is a necessary condition to identify structural shocks. The LP methodology does not require such an assumption, as structural shocks can be identified prior to estimation. Furthermore, if identified shocks are truly orthogonal, lags of endogenous variables can be omitted without causing bias in estimating IRFs using the LP method. As argued by the method developer, they may still increase efficiency. Abandoning the lags of endogenous variables, equation (4.8.1) dictates the set of linear regression models necessary to estimate the IRF up to \$h\$ periods ahead are:

$$\begin{aligned}&s = 0: \quad GDP_{t+0} = \alpha_0 + \phi^{21}_{0}u^{G}_{t} + \phi^{23}_{0}u^{NT}_{t} + v_t \\ &s = 1: \quad GDP_{t+1} = \alpha_1 + \phi^{21}_{1}u^{G}_{t} + \phi^{23}_{1}u^{NT}_{t} + \phi^{21}_{0}u^{G}_{t+1} + \phi^{23}_{0}u^{NT}_{t+1} + v_{t+1} \\ &\vdots \\ &s = h: \quad GDP_{t+h} = \alpha_h + \phi^{21}_{h}u^{G}_{t} + \phi^{23}_{h}u^{NT}_{t} + ... + \phi^{21}_{0}u^{G}_{t+h} + \phi^{23}_{0}u^{NT}_{t+h} + v_{t+h}\end{aligned}$$

,where v_t can be considered as a linear combination for all extra relevant determinants, standardized to have a mean of zero, which were not included in the model, that is, the forecast error. As can be seen from above, all the shocks should remain in the equation, if the econometrician believes that they are not independent. Future realizations of the shocks should also be present.

4.9 Bias Resulting from the Exclusion of Relevant Shocks

For simplicity, assuming only one control, the fixed effects estimator minimizes the sum of squared residuals[11]. The results of such a minimization problem can be obtained as a solution to the following system:

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\tilde{FS}_{i,t}\left(\tilde{\Delta Y}_{i,t+h}-\hat{\beta}\tilde{FS}_{i,t}-\hat{\delta}\tilde{X}_{i,t}\right)=0 \tag{4.9.1}$$
$$\sum_{i=1}^{N}\sum_{t=1}^{T}\tilde{X}_{i,t}\left(\tilde{\Delta Y}_{i,t+h}-\hat{\beta}\tilde{FS}_{i,t}-\hat{\delta}\tilde{X}_{i,t}\right)=0$$

, where tilde denotes 'within' transformation,[12] for example:

$$\tilde{FS}_{i,t}=FS_{i,t}-\frac{1}{T}\sum_{t=1}^{T}FS_{i,t}$$

Given the value of $\hat{\delta}$, the multiplier can be obtained from (4.9.1). As the fixed-effects model assumes the overall intercept to be equal to zero, applying a simple OLS estimator on the pooled dataset of all the countries and years, once the within transformation, (6) can be represented as:[13]

[11] This section provides an explicit derivation of bias arising from omitting the relevant shocks. The derivation follows a conventional method of depicting the omitted variable biases. In order to sustain consistency in the experiment at hand, the derivation is applied to a panel estimator case.

[12] Within transformation is the conventional way to transform the variables used to compute the fixed-effects estimator or the within estimator, as it is often called.

[13] The fixed-effect estimation is analogous to a pooled ordinary least squares (OLS) estimation with a zero constant term and explanatory variables entering a regression equation as deviations from the country-specific time average.

$$
\begin{aligned}
0 &= \sum_{i=1}^{N}\sum_{t=1}^{T} \tilde{FS}_{i,t}\left(\tilde{\Delta Y}_{i,t+h} - \left[\bar{\tilde{\Delta Y}}_{i,t+h} - \hat{\beta}\bar{\tilde{FS}}_{i,t} - \hat{\delta}\bar{\tilde{X}}_{i,t}\right] - \hat{\beta}\tilde{FS}_{i,t} - \hat{\delta}\tilde{X}_{i,t}\right) \\
&= \sum_{i=1}^{N}\sum_{t=1}^{T} \tilde{FS}_{i,t}\left(\tilde{\Delta Y}_{i,t+h} - \bar{\tilde{\Delta Y}}_{i,t+h}\right) - \hat{\beta}\sum_{i=1}^{N}\sum_{t=1}^{T} \tilde{FS}_{i,t}\left(\tilde{FS}_{i,t} - \bar{\tilde{FS}}_{i,t}\right) \\
&- \hat{\delta}\sum_{i=1}^{N}\sum_{t=1}^{T} \tilde{FS}_{i,t}\left(\tilde{X}_{i,t} - \bar{\tilde{X}}_{i,t}\right) \\
&= \sum_{i=1}^{N}\sum_{t=1}^{T} \left(\tilde{FS}_{i,t} - \bar{\tilde{FS}}_{i,t}\right)\left(\tilde{\Delta Y}_{i,t+h} - \bar{\tilde{\Delta Y}}_{i,t+h}\right) - \hat{\beta}\sum_{i=1}^{N}\sum_{t=1}^{T} \left(\tilde{FS}_{i,t} - \bar{\tilde{FS}}_{i,t}\right)^2 \\
&- \hat{\delta}\sum_{i=1}^{N}\sum_{t=1}^{T} \left(\tilde{FS}_{i,t} - \bar{\tilde{FS}}_{i,t}\right)\left(\tilde{X}_{i,t} - \bar{\tilde{X}}_{i,t}\right)
\end{aligned}
$$

and, thus, delivers the estimator formula:

$$
\begin{aligned}
\hat{\beta} = &\frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\tilde{FS}_{i,t} - \bar{\tilde{FS}}_{i,t}\right)\left(\tilde{\Delta Y}_{i,t+h} - \bar{\tilde{\Delta Y}}_{i,t+h}\right)}{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\tilde{FS}_{i,t} - \bar{\tilde{FS}}_{i,t}\right)^2} \\
&- \hat{\delta}\frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\tilde{FS}_{i,t} - \bar{\tilde{FS}}_{i,t}\right)\left(\tilde{X}_{i,t} - \bar{\tilde{X}}_{i,t}\right)}{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\tilde{FS}_{i,t} - \bar{\tilde{FS}}_{i,t}\right)^2}
\end{aligned}
$$

As $\bar{\tilde{FS}}_{i,t} = 0$ by definition, (4.3.3) is obtained:

$$
\begin{aligned}
\hat{\beta} = &\frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(FS_{i,t} - FS_i\right)\left(\Delta Y_{i,t+h} - \Delta Y_i\right)}{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(FS_{i,t} - FS_i\right)^2} \\
&- \delta\frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(FS_{i,t} - FS_i\right)\left(X_{i,t} - X_i\right)}{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(FS_{i,t} - FS_i\right)^2}
\end{aligned} \tag{4.9.2}
$$

, where FS_i is a shortcut for $\frac{1}{T}\sum_{t=1}^{T} FS_{i,t}$.

Additionally, (4.9.2) can be shown to depict the fiscal multiplier. For simplicity, consider only the former part of the right-hand side of (4.9.2) for now:

$$\beta^I_{FE} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(FS^I_{i,t} - FS^I_i\right)\left(\Delta Y_{i,t+h} - \Delta Y_i\right)}{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(FS^I_{i,t} - FS^I_i\right)^2}$$
$$= \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(FS^I_{i,t} - FS^I_i\right)^2 \frac{\Delta Y_{i,t+h} - \Delta Y_i}{FS^I_{i,t} - FS^I_i}}{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(FS^I_{i,t} - FS^I_i\right)^2}$$
$$= \sum_{i=1}^{N}\sum_{t=1}^{T} W_{i,t}\frac{\Delta Y_{i,t+h} - \Delta Y_i}{FS^I_{i,t} - FS^I_i} \tag{4.9.3}$$

, where $W_{i,t}$ is a country/year-specific weight on the county-wise year-to-year multipliers. The formula in (4.9.3) would indeed deliver the multiplier β^I_{FE} if there were no relevant additional controls in the out model. In our case, the multiplier formula would be:

$$\hat{\beta}^I_{FE} = \sum_{i=1}^{N}\sum_{t=1}^{T} W_{i,t}\frac{\Delta Y_{i,t+h} - \Delta Y_i}{FS^I_{i,t} - FS^I_i} - \delta\sum_{i=1}^{N}\sum_{t=1}^{T} W_{i,t}\frac{X_{i,t} - X_i}{FS^I_{i,t} - FS^I_i} \tag{4.9.4}$$

, where the same derivation was applied to the right-hand side. Now, it is appropriate to show that all the fiscal shocks should be present in the model simultaneously. Meanwhile, consider that the extra control $X_{i,t}$ is the unanticipated government consumption shock – $FS^C_{i,t}$. Then, (4.9.4) can be rearranged into:

$$\hat{\beta}^I_{FE} = \sum_{i=1}^{N}\sum_{t=1}^{T} W_{i,t}\frac{\Delta Y_{i,t+h} - \Delta Y_i - \delta\left(FS^C_{i,t} - FS^C_i\right)}{FS^I_{i,t} - FS^I_i}$$

, where δ now represents the government consumption multiplier. In the above case, the investment country/year specific multiplier is derived after first eliminating the share of output change caused by the government consumption expenditure that ensued in the same country/year. In other words, if the second fiscal shock variable was omitted from the model, the results would be biased. This bias would not be an issue only if:

- The extra fiscal shock has no impact on the output, that is, government consumption multiplier δ equals 0;

- There is no variation in country-specific government consumption in the entire sample under analysis, that is, $FS^C_{i,t} - FS^C_i = 0$;

- Unanticipated government investment and government consumption shocks are uncorrelated, that is, $corr(FS^I_{i,t}, FS^C_{i,t}) = 0$.

We can reject the first condition, as Section 4.4 shows that the government consumption multiplier is non-zero and significant. The second is rejected because of the properties of the data at hand. The third condition can be rejected by observing the correlations between the shocks and evaluating the significance of the fixed effects regression coefficient of one shock on the other. Figure 4.9.1 presents such statistics for the simultaneous relationship between shocks; although the relationship is not large, it is very significant. A similar reasoning can be used to show that additional bias arises in the application of Tseulings and Zubanov's (2014) extension of the LP method. If the LP model equation includes future realisations of the shocks, they may be correlated with the current or past realisations of another shock. Thus, Figure 4.9.2 shows that such a relationship between the first lead of unanticipated government investment shock and the current realisation of unanticipated government consumption shock exists and is significant, although extremely small. Supported by the above reasoning, we include both unanticipated government consumption and investment shocks in a joint estimation. The model would further benefit with the inclusion of tax and monetary policy measures; however, we do not have such a series for the entire sample at our disposal.[14]

[14] It is worth mentioning, that some of the robustness checks employed by the relevant literature would not result in the bias discussed in this section. For instance, using residuals from running a regression with one shock being the dependent variable, and the remainder shocks at hand as explanatory. Nonetheless, such an approach would fail to account for the fact that these residuals are estimates; thus, simple significance evaluations may produce misleading results.

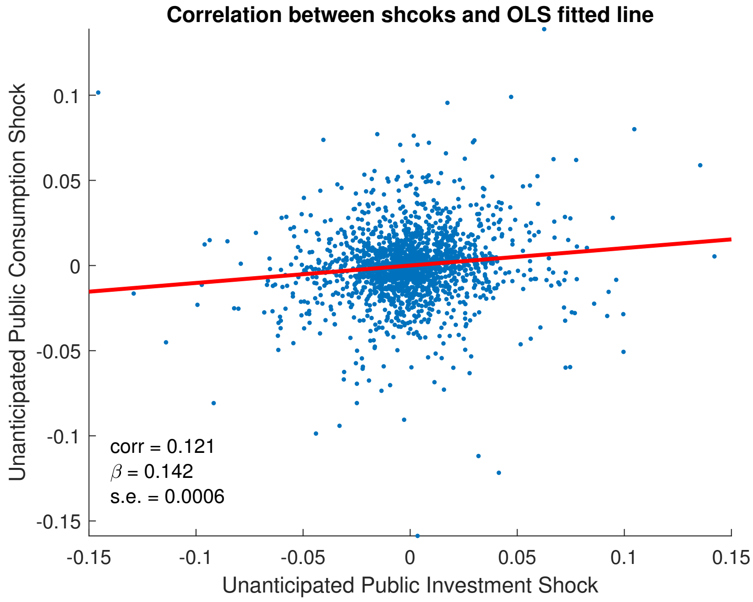


Figure 4.9.1: Correlation and fixed effects fitted line of unanticipated government consumption shock on unanticipated government investment shock.

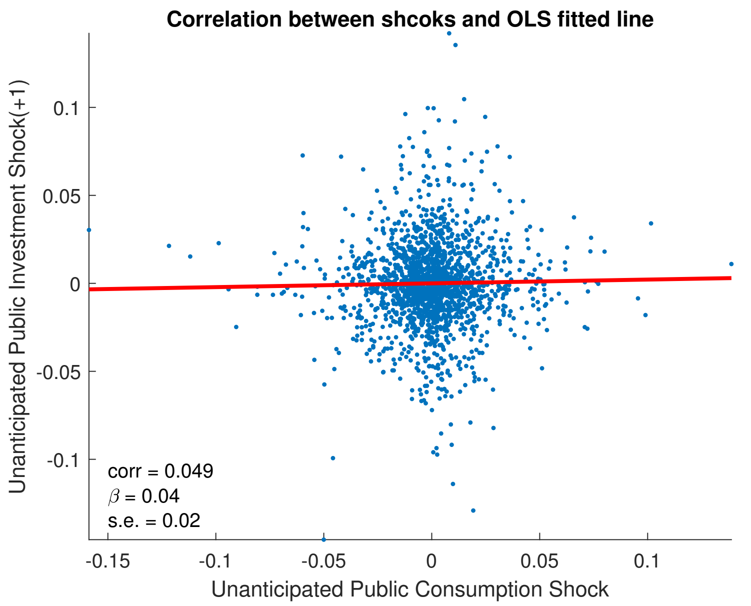


Figure 4.9.2: Correlation and fixed effects fitted line of unanticipated government investment shock on current unanticipated government consumption shock.

4.10 Robustness check. Sample Trimming and Multiplier Weights

An important issue regarding the common approach in measuring fiscal multipliers, using the LP method, is the observation weights. Consider, once again, a model with only one shock present. Following Hall (2009), the multiplier formula is given by:

$$\beta_{FE}^{I} = \sum_{i=1}^{N} \sum_{t=1}^{T} W_{i,t} \frac{\Delta Y_{i,t+h} - \Delta Y_i}{FS_{i,t}^{I} - FS_i^{I}}$$

, where the weights on country-and-year specific multipliers ($W_{i,t}$) are given by:

$$W_{i,t} = \frac{\left(FS_{i,t}^{I} - FS_i^{I}\right)^2}{\sum_{i=1}^{N} \sum_{t=1}^{T} \left(FS_{i,t}^{I} - FS_i^{I}\right)^2}$$

In this way, higher weights will be assigned to observations with a higher deviation of the discretionary spending shock from its country-specific year average. Thus, the resulting multiplier will shift towards multiplier values observed during such periods and countries, primarily to compensate for the lack of data on large fiscal injections. This can clearly be seen in Figures 4.10.1 below.

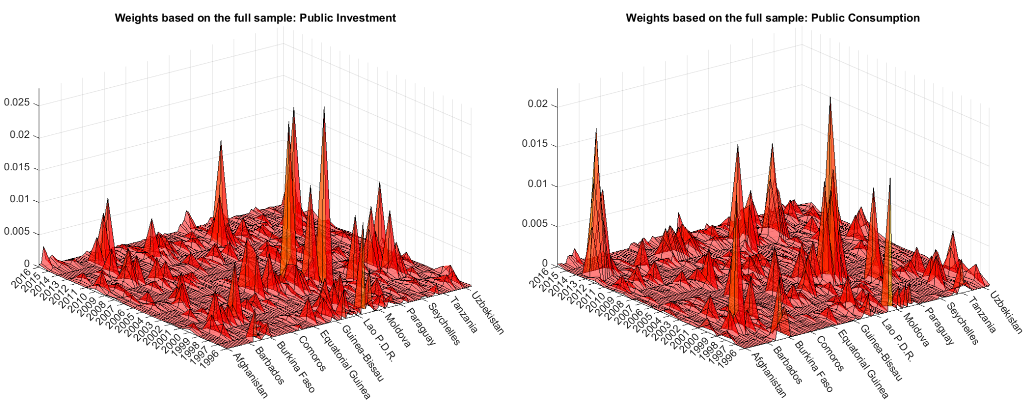


Figure 4.10.1: Observation-wise weights in the multiplier calculation. Case of Government Investment.

The above figure presents weights that are used in the multiplier calculation, provided the LP model is estimated on the full sample of observations. Considering the nature of the multiplier calculation, we can conclude that our multiplier estimates can only be immune to sample trimmings if the impact of the fiscal shock does not depend on its size, that is, if the effect of the additional US$1 of government consumption on the economy is equal to 10^{-6} times the effect of additional US$1,000,000 worth of stimulus spending.

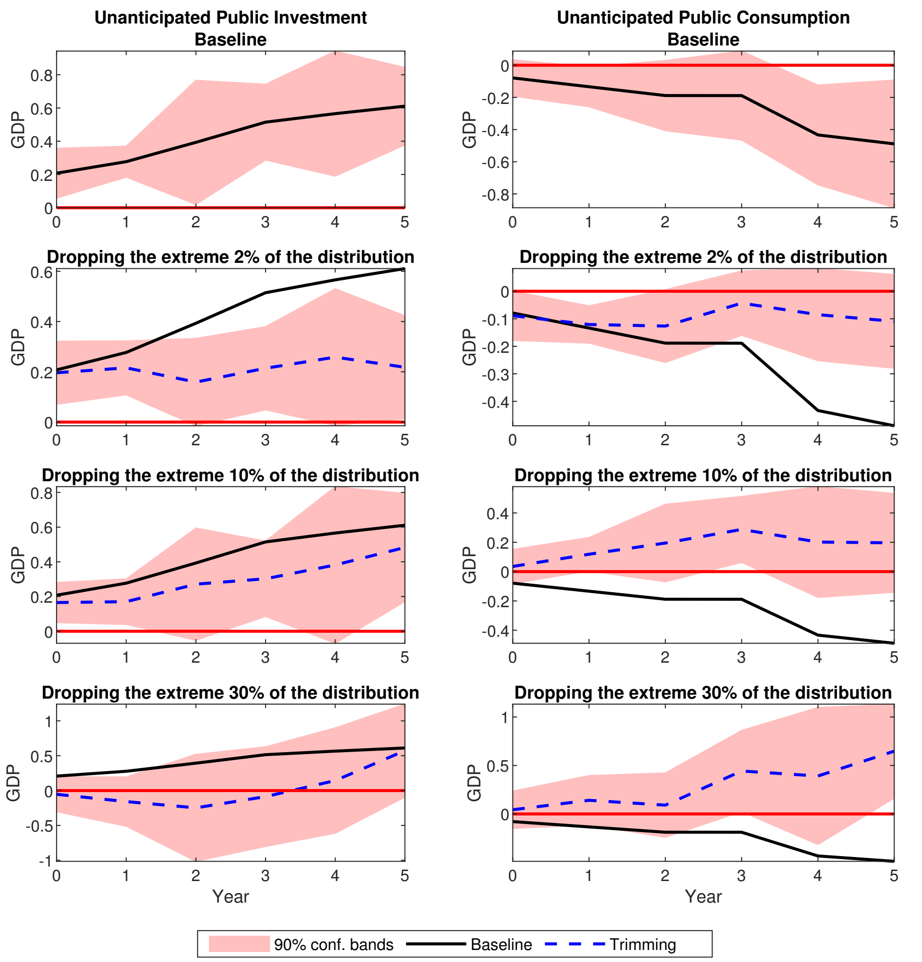


Figure 4.10.2: Comparison of baseline estimates (top) and estimates obtained via alternative shock trimming strategies. Estimates based on omitting the 1st and 99th percentiles (second row), 5th and 95th percentiles (third row), and 15th and 85th percentiles (last row).

Trimming the shock distribution of high values can potentially deliver evidence to assess the correctness of the claim. This experiment should be carried out with a certain degree of caution. First, shock distributions with large omitted tail segments may no longer represent Gaussian distributions. Performing simple significance inference on such results can be misleading. Second, results obtained through excessive trimming may not represent explicit evidence of the differences between the impact of large and small fiscal injections. Such an experiment still constitutes interest, as it is a rather simple way to see whether decreasing shocks under analysis deliver different results. Figure 4.10.2 presents the baseline result, along with the estimations based on trimming the 1st/99th, 5th/95th, and 15th/85th percentiles of the distributions; Figures 4.10.3 and 4.10.4 plot the respective distributions. Figure 4.10.2 delivers a basis to suspect that larger government investment shocks are associated with larger medium-term responses of output, yet estimation on smaller government consumption shocks tends to deliver positive GDP response.

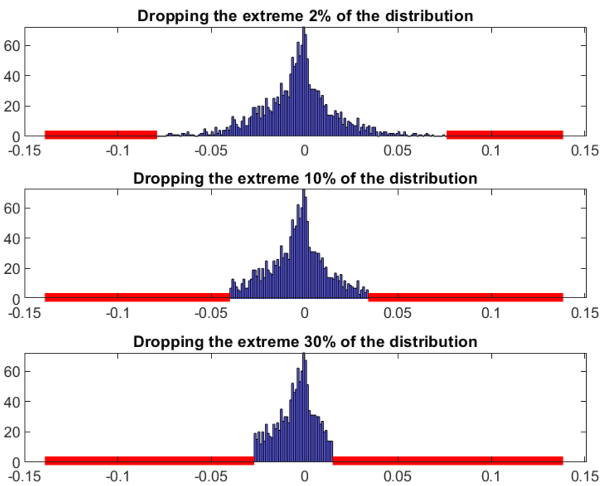


Figure 4.10.3: Trimmed distributions used to obtain results for government investment injections from Figure 4.10.2. The red areas represent parts of distributions that were omitted from estimation.

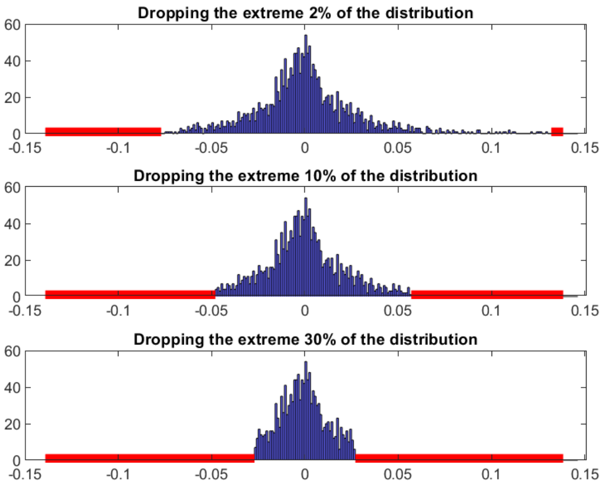


Figure 4.10.4: Trimmed distributions used to obtain results for government consumption injections from Figure 4.10.2. The red areas represent parts of distributions that were omitted from the estimation.

4.11 Robustness check. Adding the forecast error of GDP in the baseline specification

As discussed in Section 4.3, fiscal shocks are identified in the following way. First, we obtain the forecasts for government consumption-to-GDP and government investment-to-GDP ratios. Next, we calculate the difference between the forecast and the observed ratios for a given year. In order to obtain the fiscal shocks expressed in the national currency, we re-scale the forecast error of the ratio using the observed GDP. In this way, uncertainty regarding the end-of-year GDP makes its way into the shock definition. Nonetheless, if the forecast error of the GDP drives our results, including it separately as an additional control would alter the estimated multipliers. As can be seen in Figure 4.11.1, this is not the case.

Figure 4.11.1 can also help prove that our results are unaffected by the endogeneity concerns. Since we define shocks as the difference in the forecast of the end-of-the-year fiscal expenditure made in October of the same year, any output innovations that took place before October, and could potentially cause the level of government spending to change, would be considered by the forecaster and incorporated into the forecast. Thus, the forecast error would not include such an effect. Nevertheless, there remains a possibility that output innovations that took place in the last quarter could potentially drive the value of fiscal expenditure in the same quarter, that is, simultaneously. If that was an issue, the inclusion of the forecast error of GDP would alter our results.

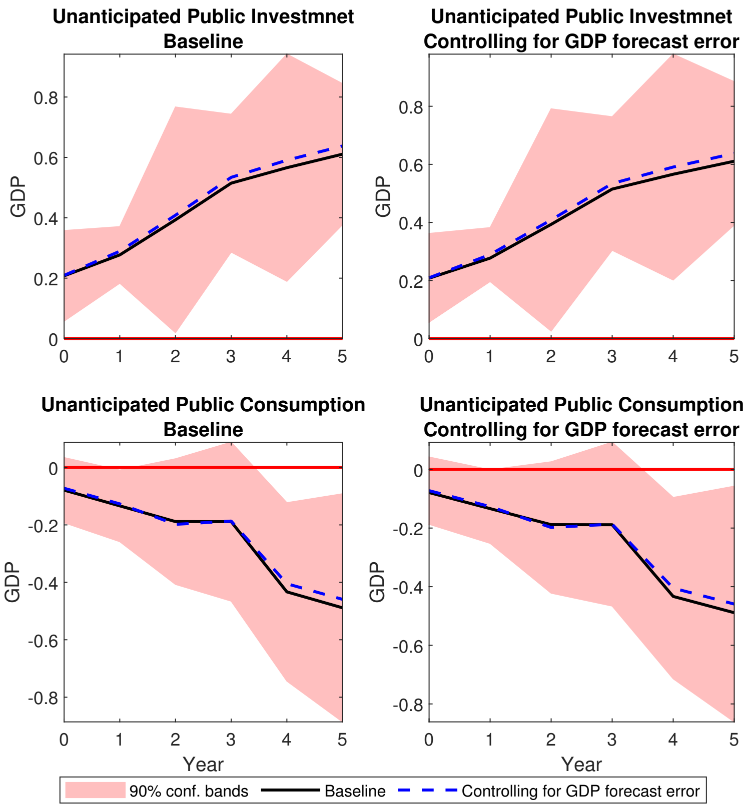


Figure 4.11.1: Comparison of baseline estimates (left) and estimates obtained by controlling for current and future realizations of the GDP forecast error.

4.12 Robustness check. Government expenditure multipliers and debt-to-GDP level

This section delivers a robustness check of the results depicted in Figure 4.4.6 in Section 4.4.3. The results obtained using the 90% debt-to-GDP threshold are compared to alternative threshold choices: 70%, 80%, and 100%. As can be seen in Figure 4.12.1, cumulative GDP responses to a government investment shock do not show consistent difference between the low and high debt cases.

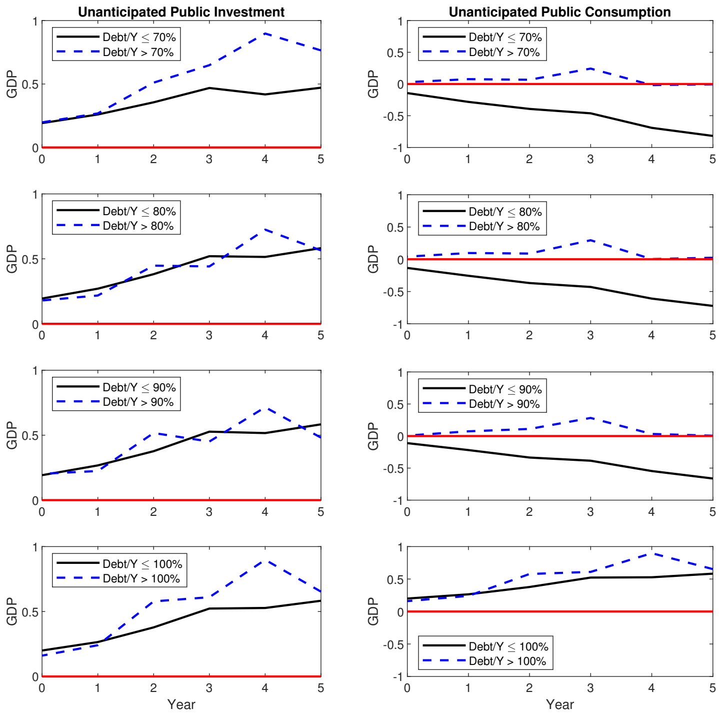


Figure 4.12.1: Comparison of estimated cumulative GDP responses at different debt-to-GDP thresholds. Government investment (left) and government consumption (right) shocks.

As for the government consumption shock, the main result obtained using the 90% threshold does not change if we split the sample using the 70% or the 80% thresholds; government consumption delivers a higher multiplier during times of higher debt. Using the 100% debt-to-GDP value to split the sample, does not follow a similar fashion. Both responses become positive, potentially indicating that observations falling between 90% and 100% debt-to-GDP ratio in the sample tend to deliver higher government consumption multipliers.

Chapter 5

Thesis conclusions

This thesis is a collection of three papers in macroeconomics studying the effects of government expenditure using recent developments in empirical and applied macroeconomic modelling.

The first chapter of the thesis contributes to the debate on the fiscal policy at times of economic slack. Using a Time-Varying Parameter version of the Balnchard and Perotti (2002) model, we show that the relationship between the government expenditure multiplier and the phase of the business cycle is more complicated than widely considered in the relevant research. Based on our results, we can divide the estimated multiplier series into two distinct periods—the period between 1949 and late 1980s, wherein the fiscal expenditure multiplier is counter-cyclical and the subsequent pro-cyclical period.

We extend the model to respond to a wide range of bias and misspecification concerns, often outlined in the relevant literature. The results are robust to the inclusion of unanticipated fiscal policy shocks and are not subject to the re-scaling bias outlined in Ramey (2011). Focusing on the components of government expenditure, we show that the fluctuations of the multiplier values observed throughout the business cycle originate in non-defence public consumption expenditure. Public investment multipliers do not exhibit pronounced state-dependent behaviour. Finally, we argue that change in the dynamics of the fiscal multiplier observed after the mid-1980s may originate in the shift in the monetary policy objective that took place under Paul Volcker's Federal Reserve.

The second chapter of this thesis performs optimised simple policy rules analysis on a foundation of Drautzburg and Uhlig (2015) model. The model includes a rich set of features that can alter the results of optimal policy analysis. This paper has four notable results. First, the historical formulation of the simple monetary policy rule in the Post-WWII U.S. did not constitute a welfare-maximising rule. The U.S. population welfare could be improved through a higher reaction of the nominal interest rate to inflation. Second, an optimised implementable public investment rule is characterised by a modest response to past debt and zero response to output fluctuations. Third, welfare maximising implementable public investment rule does not deliver substantially higher welfare levels than a public investment policy aiming at stimulating production. Fourth, the welfare gain associated with switching to an optimised simple monetary policy rule is considerably larger than that associated with optimising the fiscal rule.

We also show that the choice of the aggregator function matters for optimal policy analysis using the second-order approximation of DSGE models. The second part of the chapter shows that a simple monetary policy rule that maximises the public consumption multiplier is characterised by the smallest response to inflation while satisfying the Taylor principle. Additionally, a range of positive responses to the growth gap, that is the difference in the growth rates between the sticky- and flexible-price output, can substantially increase the government consumption multiplier in the long-run, conditional on weak inflation targeting.

The final chapter of the thesis evaluates the impact of government consumption and investment stimuli in a large sample of emerging and developing economies. The literature on the effects of fiscal policy in developing economies is scarce due to data limitations. We construct a series of unanticipated government expenditure shocks making use of the vintages from the World Economic Outlook database of the International Monetary Fund. Our results show that the potency of the fiscal stimulus in emerging and developing economies indeed is in line with the Local Projection literature on advanced economies. Public consumption delivers a negative statistically significant cumulative long-term response of output, while public investment is associated with a positive statistically significant cumulative output multiplier. Furthermore, unanticipated investment in public capital tends to stimulate private consumption expenditure.

By evaluating the state-dependent nature of the fiscal stimulus, we find several notable results. First, an unanticipated public investment shock stimulates output more during periods of zero or negative output growth, while negative output implications of the public consumption injections are minimised under such circumstances. Second, we find that public investment stimulus taking place during periods of public debt build-up tends to deliver a lower impact on output. Third, public consumption multipliers are lower in relatively open economies. Finally, we find that public investment multipliers are lower if the level of investment in public capital stock is already high and that government consumption stimulates output better if public sector employs a larger share of the workforce.

To conclude, this thesis contributes to policy debate along several dimensions. First, it delivers empirical evidence that government expenditure multipliers can fall during some recessions. This finding raises the importance of further studying the interaction of monetary and fiscal policies during crisis periods. Second, we show that if the U.S. Federal Reserve's objective is to stabilise business cycles and maximise population welfare, it should incorporate a higher response to inflation in its policy rule. Third, the U.S. government should refrain from using simple debt-targeting public investment rules if the ultimate objective is the maximisation of household welfare. Finally, we show that international organisations, such as the International Monetary Fund, should consider a wide range of factors in evaluating the projected impact of a proposed government expenditure interventions in emerging and developing economies.

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