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Speculative trade and the value of public information

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Abstract

In environments with expected utility, it has long been established that speculative trade cannot occur and that the value of public information is negative in economies with risk-sharing and no aggregate uncertainty. We show that these results are still true even if we relax expected utility, so that either Dynamic Consistency (DC) or Consequentialism is violated. We characterize no speculative trade in terms of a weakening of DC and find that Consequentialism is not required. Moreover, we show that a weakening of both DC and Consequentialism is sufficient for the value of public information to be negative. We therefore generalize these important results for convex preferences which contain several classes of ambiguity averse preferences.

1 | INTRODUCTION

In markets with subjective expected utility (SEU), it has long been established by Milgrom and Stokey (1982) that speculative trade cannot occur. This means that if the market is at an ex ante Pareto optimal allocation, differential information among traders in the interim stage can never imply that it is common knowledge that there is another allocation which Pareto dominates it. Moreover, in standard neoclassical economies with complete markets and symmetric information, it is shown with an example by Hirshleifer (1971) and more generally by Schlee (2001) that public information makes everyone weakly worse off, as it destroys opportunities for mutual insurance.

By adopting the SEU model, both these results implicitly assume the following two properties. The first is *Dynamic Consistency* (DC), which requires that an action plan is optimal when evaluated with the updated preferences of a later period, *if and only if* it is optimal when evaluated with the preferences of an earlier period. DC ensures that an ex ante optimal action plan will remain optimal at every period and irrespective of how information is updated. In the SEU model, DC is expressed through the Bayesian updating of the prior. The second property, *Consequentialism*, requires that conditional preferences do not depend on past actions, foregone payoffs or unrealized events. However, for preferences that are not SEU, at least one of the two properties must be relaxed (Siniscalchi, 2009).

The purpose of this paper is to examine whether no speculative trade and the negative value of public information for mutual insurance are still valid in general models with convex preferences, where either DC or Consequentialism is violated. We find that Consequentialism can be weakened significantly, whereas DC less so, confirming Machina (1989), who argued that DC is more natural to retain for non-SEU preferences.

We first show that Consequentialism can be dropped completely and that the “if” part of weak DC, as formulated and motivated in Galanis (2020), is the minimum requirement which ensures that there is no speculative trade. This means that, if every trader's preferences satisfy this axiom, there can be no speculative trade, whereas if at least one trader's preferences violate it, there are examples with speculative trade. Galanis (2020) showed that the “if” part of weak DC characterizes the Bayesian updating of subjective beliefs, which are identified by Rigotti, Shannon, and Strzalecki (2008) in the context of convex preferences and can be interpreted as the prices for Arrow-Debreu securities which characterize Pareto efficient allocations. Hence, an intuitive axiom for single-agent environments has natural implications for a multiagent environment.

Second, we find that, if Consequentialism is weakened to Status Quo Bias, the “only if” part of DC implies that the value of public information is negative for mutual insurance, so traders would prefer not to receive free information. We distinguish between two definitions of receiving more information and find that the stronger is satisfied for preferences that are smooth, so that subjective beliefs are unique (e.g., the smooth ambiguity model of Klibanoff, Marinacci, & Mukerji, 2005).

Status Quo Bias has been proposed in axiomatic work by Masatlioglu and Ok (2005), Sagi (2006) and Ortoleva (2010), and identified experimentally by Samuelson and Zeckhauser (1988). Moreover, the “only if” part of weak DC implies that the value of public information is weakly negative, so that the traders would prefer to mix more with less information, instead of getting less information for sure. These results mirror the findings of Galanis (2020) for single-agent environments: the “only if” part of DC characterizes an agent who always prefers receiving more to less information, whereas the “only if” part of weak DC characterizes an agent who prefers to mix more with less information, instead of getting less information for sure. In other words, if information is (weakly) valuable for each agent, then public information is (weakly) not valuable for all agents.

1.1 | Related literature

Rigotti et al. (RSS; 2008) identify the subjective beliefs generated by a large number of models of ambiguity aversion, based on an idea of Yaari (1969), making our approach very general. We are not aware of any papers that examine the value of public information in settings that go beyond SEU preferences. Instead, our model allows a wide variety of ambiguity averse preferences.

Examples are the convex Choquet model of Schmeidler (1989), the multiple priors model of Gilboa and Schmeidler (1989), the variational preferences model of Maccheroni, Marinacci, and Rustichini (2006), the multiplier model of Hansen and Sargent (2001), the smooth second-order prior models of Klibanoff et al. (2005) and Nau (2006), the confidence preferences model of Chateauneuf and Faro (2009) and the second-order expected utility model of Ergin and Gul (2009).[1]

Ma (2001) proves that there is no speculative trade, using DC and piecewise monotonicity, whereas Galanis (2018) examines speculative trade under unawareness, where DC may be violated, but Consequentialism is not. Halevy (2004) shows that Consequentialism can be weakened, using Resolute Conditional preferences and Conditional Decomposition, but retains the full force of DC. In this paper, we prove the same result by weakening both DC and Consequentialism. Dominiak, Duersch, and Lefort (2012) show experimentally that subjects violate DC.

Following Kajii and Ui (2009), Martins-da-Rocha (2010) also shows that Bayesian updating of subjective beliefs precludes speculative trade, therefore his result applies to the same preferences as the current model. Martins-da-Rocha (2010) does not discuss Consequentialism, whereas we show that it is not needed. Moreover, we identify a particular axiom on preferences that is necessary and sufficient for no speculative trade, Weak Forward Consistency, instead of focusing on the Bayesian updating of subjective beliefs, which is a condition on utility functions.

The paper proceeds as follows. Section 2 presents the model. In Section 3, we show that the "if" part of weak DC is the minimum requirement that precludes speculative trade. In Section 4, we show that if each agent individually considers information to be (weakly) valuable, then public information is not (weakly) valuable in competitive risk-sharing environments with no aggregate uncertainty. All proofs are contained in the appendix.

2 | MODEL

2.1 | Preliminaries

Fix a finite set of payoff relevant states S, with typical element s. The set of consequences is $\mathbb{R}_+$, interpreted as monetary payoffs. Let $\mathcal{F} = \mathbb{R}_+^S$ be the set of acts, with the natural topology. An act $f \in \mathcal{F}$ maps each state s to a monetary payoff. Given $x \in \mathbb{R}_+$, let $x \in \mathcal{F}$ be the constant act with payoff x at each state s. Let X be the set of constant acts. An act f is strictly positive if $f(s) > 0$ for all $s \in S$. Let $\mathcal{F}_+$ be the set of strictly positive acts.

For any two acts $f, g \in \mathcal{F}$ and event $E \subseteq S$, we denote by fEg the act h such that $h(s) = f(s)$ if $s \in E$ and $h(s) = g(s)$ if $s \notin E$. Define $f \geq_E g$ if $f(s) \geq g(s)$ for all $s \in E$, with strict inequality for some $s \in E$. Equality $f =_E g$ and strict inequality are similarly defined. Let E^c be the complement of E with respect to S.

Given events $E, F \subseteq S$ and probability measure $p \in \Delta E$, where $F \subseteq E$ and $p(F) > 0$, denote by $p_F \in \Delta F$ the measure obtained through Bayesian conditioning of p on F. Formally, for any event $G \subseteq S$, $p_F(G) = \frac{p(G \cap F)}{p(F)}$. We write $\mathbb{E}_p f := \sum_{s \in E} p(s) f(s)$ for the expectation of f given p.

[1]Note that RSS adopts a domain of preferences over monetary acts, whereas these models allow for more general domains. Galanis (2020) discusses their differences.

Let $\mathcal{E}$ be a collection of nonempty events $E \subseteq S$ which contains S. A partition Π of S is a collection of mutually disjoint events, whose union is S. It is finer than another partition Π' if, for each $E' \in \Pi'$, there exists $E \in \Pi$ with $E \subseteq E'$. We then say that Π' is coarser than Π.

2.2 | Updating

We consider two periods. In period 0, no information is revealed and the agent's ex ante preference relation is denoted by $\succsim$. In period 1, the agent learns that an event $E \in \mathcal{E}$ has occurred and updates his preferences. Updating is not standard because it depends not only on the event E, but also on the act f that was chosen in period 0.

Formally, the decision maker is endowed with a collection of conditional preference relations, $\{\succsim_{E,h}\}_{E\in\mathcal{E},h\in\mathcal{F}}$, one for each event $E \in \mathcal{E}$ and each act $h \in \mathcal{F}$. The interpretation is that in period 0 the agent had chosen act h and in period 1 he learns that event E has occurred. His updated preference relation is then $\succsim_{E,h}$. The ex ante preference relation $\succsim_{S,h}$ does not depend on the act h and is denoted by $\succsim$.

To provide an example, let E be the event that a major economic shock occurs (e.g., because of COVID-19), whereas the complement E^c describes business as usual. The agent has a choice of three acts f, g, and h, where f denotes betting on Delta Airlines shares, g denotes betting on Apple shares and h denotes betting on Amazon shares.

Suppose first that the agent has SEU preferences, represented by $U(f) = \sum_{s\in S} \pi(s)u(f(s))$, where π is his prior. In period 0, the agent chooses f, which is the act of betting on Delta Airlines. In period 1, he is informed that event E has occurred, so he knows there is a COVID-19 pandemic. His utility from choosing act g is $\sum_{s\in E} \frac{\pi(s)}{\pi(E)}u(g(s))$, so it is independent of the act that was chosen in the previous period. That is, irrespective of whether he had chosen f, g, or h in period 0, his valuation of act g (betting on Apple shares) would be exactly the same in period 1, given event E. These are the standard *Consequentialist* preferences, that do not depend on what was chosen in the previous period.

To model non-consequentialist preferences, we allow the updating of preferences to depend not only on the event, but also on the act that was chosen in the previous period. For instance, we could specify that in period 1, the agent's utility from act f is $\sum_{s\in E} \frac{\pi(s)}{\pi(E)}u_{E,f}(f(s))$ if in period 0 he chose f, whereas it is $\sum_{s\in E} \frac{\pi(s)}{\pi(E)}u_{E,h}(f(s))$, if in period 0 he chose h, allowing for $u_{E,f} \neq u_{E,h}$. If the agent chose act f (Delta Airlines) in period 0 and E occurs, he may feel disappointed in period 1, so he may become more risk averse. This could be modeled by $u_{E,f}$ being a concave function of $u_{E,h}$. In Section 2.5, we provide formal definitions for Consequentialism and the weaker property, Status Quo Bias.

2.3 | Revealed preference

Given preference relation $\succsim_{E,h}$, we say that act f is *revealed preferred* to act g, written $f \succsim^*_{E,h} g$, if $f \succsim_{E,h} \alpha g + (1-\alpha)f$ for all $\alpha \in [0,1]$, so that f is weakly preferred to all convex combinations of f and g. Preference relation $\succsim^*_{E,h}$ is neither transitive nor complete.[2]

The interpretation of $f \succsim^*_{E,h} g$ is that f is weakly preferred to g under $\succsim_{E,h}$ and g is inside a "budget set," which is constructed given f as the agent's endowment and some prices for the

[2]The revealed preference relation is discussed extensively in Galanis (2020).

Arrow-Debreu securities, one for each state. If these prices were to prevail and the agent chose f, it would be revealed that the agent prefers f over g.

2.4 | Convex preferences

We consider the following axioms on preferences $\{\succsim_{E,h}\}_{E\in\mathcal{E},h\in\mathcal{F}}$, for all events $E\in\mathcal{E}$ and acts $h\in\mathcal{F}$.

Axiom 1 (Preference). *$\succsim_{E,h}$ is complete and transitive.*

Axiom 2 (Continuity). *For all $f\in\mathcal{F}$, the sets $\{g\in\mathcal{F}: g\succsim_{E,h} f\}$ and $\{g\in\mathcal{F}: f\succsim_{E,h} g\}$ are closed.*

Axiom 3 (Strong Monotonicity). *For all $f\neq_E g$, if $f\geq_E g$, then $f\succ_{E,h} g$.*

Axiom 4 (Convexity). *For all $f\in\mathcal{F}$, the set $\{g\in\mathcal{F}|g\succsim_{E,h} f\}$ is convex.*

These axioms are standard and imply that each $\succsim_{E,h}$ is represented by a continuous, increasing and quasiconcave function $U_{E,h}:\mathcal{F}\to\mathbb{R}$. We say that preferences $\{\succsim_E\}_{E\in\mathcal{E},h\in\mathcal{F}}$ are convex if they satisfy Axioms 1 through 4 and strictly convex if they additionally satisfy Axiom 5.

Axiom 5 (Strict Convexity). *For all $f\neq_E g$ and $\alpha\in(0,1)$, if $f\succsim_{E,h} g$, then $\alpha f+(1-\alpha)g\succ_{E,h} g$.*

Axioms 2 and 3 imply the following property, specifying that if two acts are equal on E, then, conditional on E, they are indifferent.[3]

Property 1 (Conditional Preference). For all $f,g\in\mathcal{F}$, if $f=_E g$ then $f\sim_{E,h} g$.

2.5 | Consequentialism

Consequentialism requires that the agent's preferences depend only on the received information and not on the act that was chosen in the previous period.[4]

Axiom 6 (Consequentialism). *For all $f,g\in\mathcal{F}$ and events $E\in\mathcal{E}$, $\succsim_{E,f}=\succsim_{E,g}$.*

A weakening of Axiom 6 has been proposed in axiomatic work by Masatlioglu and Ok (2005), Sagi (2006), and Ortoleva (2010), where preference relation $\succsim_{E,h}$ depends on a "status quo" act (or frame) h. It specifies that if the agent ever prefers f over g (given some status quo h),

[3] I thank a referee for pointing out the connection. To see why it is true, suppose that $f=_E g$ but $f\succ_{E,h} g$. Let g' such that $g'(s)=g(s)+\epsilon$, $\epsilon>0$, if $s\in E$ and $g'(s)=g(s)$ otherwise. From Axiom 3 we have $g'\succ_{E,h} f$. Taking ϵ to 0 and using Axiom 2, we have $g\succsim_{E,h} f$, a contradiction.

[4] Some papers refer to Consequentialism as the conjunction of Property 1 and Axiom 6.

then he would also prefer it if the status quo was f. In other words, the status quo exerts attraction towards itself.

Axiom 7 (Status Quo Bias). *For all $f, g, h \in \mathcal{F}$ and events $E \in \mathcal{E}$, if $f \succsim_{E,h} g$ then $f \succsim_{E,f} g$.*

Consider again the example of Section 2.2, with the same three acts: f is betting on Delta Airlines, g is betting on Apple and h is betting on Amazon. Event E describes that there is a COVID-19 pandemic, whereas E^c describes business as usual.

Suppose that in period 0 the agent chose h, betting on Amazon. In period 1, event E occurs and the agent prefers betting on Delta Airlines over Apple ($f \succsim_{E,h} g$). Status Quo Bias implies that $f \succsim_{E,f} g$, so that if he had chosen f in period 0, betting on Delta Airlines, then he would still prefer f over g in period 1. The intuition is that the agent has a bias towards the status quo act that he chose in the previous period. However, Status Quo Bias allows $g \succ_{E,g} f$, so that if the agent had chosen Apple in period 0, he would still prefer it over Delta Airlines in period 1. This reversal of preferences ($f \succsim_{E,f} g$ and $g \succ_{E,g} f$) violates Consequentialism, because it requires that preferences conditional on any event E should be the same.

Note that SEU preferences satisfy Status Quo Bias because they also satisfy Consequentialism, which is a stronger property. Status Quo Bias requires that if f is preferred to g given some status quo act h, then f is also preferred to g given the status quo act f. Consequentialism further requires that f is preferred to g given any status quo act k, because $\succsim_{E,k} = \succsim_{E,h}$ for all acts k, h.

As pointed by Masatlioglu and Ok (2005), Status Quo Bias is documented not only by experimental studies but also by empirical work in actual markets. For instance, Madrian and Shea (2001) examined how the default choice influenced participation in 401(k) saving plans, whereas Samuelson and Zeckhauser (1988) identified Status Quo Bias experimentally, in a study concerning portfolio choices.

2.6 | Dynamic Consistency

DC provides restrictions on how two acts, which are identical outside of the conditioning event E, should be compared before and after E is known to have occurred. We break DC into two Axioms, Forward and Backward Consistency.[5]

Axiom 8 (Forward Consistency). *For all acts $f, g \in \mathcal{F}$ and events $E \in \mathcal{E}$, if $f \succsim g$ and $f =_{E^c} g$ then $f \succsim_{E,f} g$.*

Axiom 9 (Backward Consistency). *For all acts $f, g \in \mathcal{F}$ and events $E \in \mathcal{E}$, if $f \succsim_{E,f} g$ and $f =_{E^c} g$ then $f \succsim g$.*

Suppose that f and g specify the same payoff at each state not belonging to event E and that f is weakly preferred to g ex ante. Forward Consistency says that if the agent has chosen f ex

[5] I thank the Associate Editor for suggesting these names. Ghirardato (2002) refers to these Axioms as Consistency of Implementation and Information is Valuable, respectively.

ante and he is informed that event E has occurred (so that his preferences are $\succsim_{E,f}$), then in the interim stage f is still weakly preferred to g. Backward Consistency specifies the converse.

In a single-agent setting, Galanis (2020) provides an extensive discussion of DC and motivates the following version of weak DC, using the revealed preference relation, $\succsim^*$.

Axiom 10 (Weak Forward Consistency). *For all acts $f \in \mathcal{F}_+$, $g \in \mathcal{F}$ and events $E \in \mathcal{E}$, if $f \succsim^* g$ and $f =_{E^c} g$ then $f \succsim_{E,f} g$.*

Axiom 11 (Weak Backward Consistency). *For all acts $f \in \mathcal{F}$, $g \in \mathcal{F}_+$ and events $E \in \mathcal{E}$, if $f \succsim_{E,f} g$, $f =_{E^c} g$ and $g \succ f$ then $g \not\succsim^* f$.*

Weak Forward Consistency requires that if f is revealed preferred to g ex ante, then f is weakly preferred to g, conditional on E.[6] Weak Backward Consistency specifies that if f is weakly preferred to g conditional on E and f but ex ante strictly preferred to f, then g is not revealed preferred to f ex ante.

2.7 | Subjective beliefs

RSS define the *subjective beliefs at an act f* and preference relation $\succsim_{E,h}$ to be the set of all normals (normalized to be probabilities) of the supporting hyperplanes of f,

$$\pi_{E,h}(f) = \{p \in \Delta S : \mathbb{E}_p g \geq \mathbb{E}_p f \quad \text{for all } g \succsim_{E,h} f\}.$$

RSS provide two alternative definitions for subjective beliefs and show that all three coincide for strictly positive acts. First, suppose that the agent's endowment is act f and we interpret a probability measure as a set of prices, one for each Arrow-Debreu security which pays 1 in a particular state and 0 otherwise. Given preference relation $\succsim_{E,h}$, the subjective beliefs revealed by unwillingness to trade at f contain the measures (prices) for which the agent would be unwilling to trade his endowment,

$$\pi^u_{E,h}(f) = \{p \in \Delta S : f \succsim_{E,h} g \quad \text{for all } g \quad \text{such that } \mathbb{E}_p g = \mathbb{E}_p f\}.$$

Second, let P be a set of measures (prices) such that whenever another act k is unaffordable for every $p \in P$, then there exists a mixture of k with endowment f that the agent would strictly prefer to his endowment. The smallest such P of measures contains the *subjective beliefs revealed by willingness to trade at f*. Formally, let $\mathcal{P}_{E,h}(f)$ denote the collection of all compact, convex sets $P \subseteq \Delta S$ such that if $\mathbb{E}_p g > \mathbb{E}_p f$ for all $p \in P$, then $\epsilon g + (1-\epsilon)f \succ_{E,h} f$ for sufficiently small $\epsilon > 0$. Then, the subjective beliefs revealed by willingness to trade at f are denoted by $\pi^w_{E,h}(f) = \bigcap \mathcal{P}_{E,h}(f)$. RSS show that for strictly positive acts f, $\pi_{E,h}(f) = \pi^u_{E,h}(f) = \pi^w_{E,h}(f)$.

It is important to emphasize that the subjective beliefs are not the beliefs that are used in the utility representation of preferences, such as SEU or Maxmin Expected Utility (Gilboa & Schmeidler, 1989). First, they vary as the act f on which they are evaluated changes, even if we assume Consequentialism. Second, they are only defined for convex preferences, although

[6] We also require that f is a strictly positive act.

there are some extensions to nonconvex preferences (Ghirardato & Siniscalchi, 2018). Since they are the normals of the supporting hyperplane at an allocation, this allocation is Pareto optimal if and only there is a common subjective belief. Galanis (2020) shows that their Bayesian updating is equivalent to Weak Forward Consistency. We employ this result in the current paper and use subjective beliefs to construct full insurance allocations.

3 | SPECULATIVE TRADE

In this and the next section, we show that weak DC has economic content in multiagent settings, such as financial markets. First, we show that Axiom 10 is the minimum requirement which precludes speculative trade. In a single-agent setting, Galanis (2020) shows that Axiom 10 is equivalent to Bayesian updating of subjective beliefs.

Consider an economy consisting of I agents, with $|I| = m$ and typical element i. Each agent's consumption set is the set of acts $\mathcal{F}$. He is endowed with a collection of convex preferences $\{\succsim^i_{E,h}\}_{E\in\mathcal{E},h\in\mathcal{F}}$.

An economy is a tuple $\langle\{\succsim^i_{E,h}\}_{i\in I,E\in\mathcal{E},h\in\mathcal{F}}, e, \{\Pi^i\}_{i\in I}\rangle$, where $e \in \mathbb{R}^S_{++}$ is the aggregate endowment and $\{\Pi^i\}_{i\in I}$ denotes the information structure, where each $\Pi^i \subseteq \mathcal{E}$ is a partition of S. If in period 0 the resulting allocation is f, then in period 1 and at state s, agent i considers states in $\Pi^i(s)$ to be possible and has conditional preferences $\succsim^i_{\Pi^i(s),f^i}$.

An allocation is a tuple $f = (f^1, \ldots, f^m) \in \mathcal{F}^m$. It is feasible if $\sum_{i=1}^m f^i = e$. It is interior if $f^i(s) > 0$ for all $s \in S$ and for all i. Given an event $E \subseteq S$, let $K^i(E) = \{s \in S : \Pi^i(s) \subseteq E\}$ be the set of states where i knows E. Event E is self evident if $E \subseteq K^i(E)$ for all $i \in I$. That is, an event is self evident if whenever it happens, everyone knows it. An event F is common knowledge at s if and only if there exists a self evident event E such that $s \in E \subseteq F$ (Aumann, 1976).

We say that there is speculative trade if an allocation is ex ante Pareto efficient (according to preferences $\{\succsim^i\}_{i\in I}$) but at some state $s \in S$ it is common knowledge that there exists a Pareto improvement.

Definition 1. There is speculative trade in economy $\langle\{\succsim^i_{E,h}\}_{i\in I,E\in\mathcal{E},h\in\mathcal{F}}, e, \{\Pi^i\}_{i\in I}\rangle$ at an ex ante Pareto efficient allocation f if there is agent $j \in I$, state s' and feasible allocation g such that event $H = \{s \in S : g^i \succsim^i_{\Pi^i(s),f^i} f^i \text{ for all } i \in I \text{ and } g^j \succ^j_{\Pi^j(s),f^j} f^j\}$ is common knowledge at s'.

We now show that Axiom 10 is necessary and sufficient for preventing speculative trade. In particular, if all agents' preferences satisfy Axiom 10 then there is no speculative trade, whereas if at least one fails it, there are economies with speculative trade. Let $\mathbb{P}$ be the collection of convex preferences $\{\succsim_{E,h}\}_{E\in\mathcal{E},h\in\mathcal{F}}$. Note that we do not require Consequentialism or Status Quo Bias.

Proposition 1. *If $\{\succsim^i_{E,h}\}_{i\in I,E\in\mathcal{E},h\in\mathcal{F}} \in \mathbb{P}^I$ satisfy Axiom 10 then there is no speculative trade in any economy $\langle\{\succsim^i_{E,h}\}_{i\in I,E\in\mathcal{E},h\in\mathcal{F}}, e, \{\Pi^i\}_{i\in I}\rangle$ and at any interior allocation f. Conversely, if $\{\succsim^k_{E,h}\}_{E\in\mathcal{E},h\in\mathcal{F}} \in \mathbb{P}$ fails Axiom 10 then there exist preferences $\{\succsim^i_{E,h}\}_{i\in I,E\in\mathcal{E},h\in\mathcal{F}} \in \mathbb{P}^I$ satisfying Axiom 10 and economy $\langle\{\succsim^i_{E,h}\}_{i\in I\cup\{k\},E\in\mathcal{E},h\in\mathcal{F}}, e, \{\Pi^i\}_{i\in I\cup\{k\}}\rangle$ such that there is speculative trade at an allocation f.*

To provide a sketch of the proof for one direction, suppose by contradiction that allocation $\{f^i\}_{i\in I}$ is ex ante Pareto efficient but in the interim it is common knowledge at some state s' that allocation $\{g^i\}_{i\in I}$ is a Pareto improvement. From Aumann (1976), there exists a self evident event F containing s', where $s \in F$ implies $g^i \succ_{\Pi^i(s),f^i} f^i$ for all $i \in I$.[7] Property 1 implies $g^i\Pi^i(s)f^i \succ_{\Pi^i(s),f^i} f^i$. Because each Π^i partitions F, Axiom 10 implies $\mathbb{E}_p(g^iFf^i) > \mathbb{E}_p f^i$ for all $p \in \pi^i(f^i)$. By convexity of preferences, $\epsilon g^iFf^i + (1-\epsilon)f^i \succ^i f^i$ for small enough $\epsilon > 0$, which contradicts that $\{f^i\}_{i\in I}$ is ex ante Pareto efficient. Proposition 1 is also true if we replace Axiom 3 (Strong Monotonicity) with Monotonicity, Property 1 and Axiom 8 (Weak Full Support) in Galanis (2020).

Proposition 1 cannot be proven just by assuming Dynamic Consistency. Ma (2001) and Halevy (2004), whose setting is in terms of abstract preferences and drop Consequentialism, require an "additivity" property, that replaces Consequentialism. For example, Ma (2001) requires that if $f \succsim_{E_i,g} g$ for each $i = 1, 2, 3$, where E_1, E_2, E_3, form a partition of E, then $f \succsim_{E,g} g$. In the current setting, this additivity property is true for some acts and only in terms of their expected value, given a subjective belief. In other words, the property is true only for the revealed preference relation $\succsim^*$. However, this is enough, together with Weak Forward Consistency, to prove the result. Although the additivity property is not implied by Consequentialism, it is jointly implied by Dynamic Consistency and Consequentialism, because these two axioms imply SEU preferences, together with some standard axioms (Ghirardato, 2002).

4 | THE VALUE OF PUBLIC INFORMATION

In a single-agent setting, Galanis (2020) shows that information is (weakly) valuable if and only if Axiom 9 (Axiom 11) is satisfied.[8] In this section, we explore whether public information is valued in multiagent settings where each agent values information individually. In the standard environment with SEU preferences, where each agent values information, Hirshleifer (1971) first argued with an example that if agents trade to mutually insure, then more public information could make everyone worse off. Schlee (2001) generalized this result, showing that the value of public information is negative in an expected utility model with a common prior, risk aversion and no aggregate uncertainty. His comparative statics are with respect to competitive equilibria, whereas Campbell (2004) establishes Hirshleifer's result in the more general solution concept of implementable allocations.[9] We concentrate on competitive equilibria, thus following Schlee (2001).

For strictly convex preferences, we find that as long as Weak Forward Consistency and Status Quo Bias are satisfied, if every agent (weakly) values information, then public information is locally (weakly) not valuable.[10] Additionally, if preferences are smooth, so that subjective beliefs are unique at each act, public information is (weakly) not valuable.

[7]We can assume strict preference for everyone due to Strong Monotonicity.

[8]We only examine information in terms of its nonintrinsic value, as it enables an agent to make better contingent plans. Grant, Kajii, and Polak (1998) study the case where preferences for information are intrinsic. For example, an agent may feel better (or worse) if he knows whether he has a certain incurable disease.

[9]These two papers use the Blackwell (1951) criterion of more information. Moreover, Schlee (2001) proves this result in two other cases, that we do not examine. First, there are some risk neutral agents who fully insure the risk averse ones. Second, all agents are risk averse and the economy has a representative agent.

[10]Similar results are shown by Galanis (2015, 2016), in an environment with unawareness, where DC is violated but Consequentialism is not.

To rule out pure indifference to betting, we assume strictly convex preferences. We also assume the following axiom, which is proposed by RSS.

Axiom 12 (Translation Invariance at Certainty). *For all acts $h \in \mathcal{F}$ and events $E \in \mathcal{E}$, for all $g \in \mathbb{R}^S$ and all constant bundles $x, x' > 0$, if $x + \lambda g \succsim_{E,h} x$ for some $\lambda > 0$, then there exists $\lambda' > 0$ such that $x' + \lambda' g \succsim_{E,h} x'$.*

RSS show that Axiom 12 is satisfied by most classes of ambiguity averse preferences and it implies that subjective beliefs do not change across constant acts: $\pi^i_{E,h}(x) = \pi^i_{E,h}(x')$ for all constant acts $x, x' > 0$. We henceforth write $\pi^i_{E,h}$ instead of $\pi^i_{E,h}(x)$ for all constant acts $x > 0$.

We also impose a slight variation of Axiom 10. First, we weaken it by applying it only to constant acts, x. Second, we strengthen it by applying it not only between the ex ante preference relation $\succsim^*$ and the interim $\succsim_{E,x}$, but also between $\succsim^*_{F,h}$ and $\succsim_{E,x}$, where $E \subseteq F$ and $F \in \mathcal{E}$. In other words, it is as if we consider a multiperiod model where the agent first learns F and then E.

Axiom 13 (Multiperiod Weak Forward Consistency). *For all acts $x, g, h \in \mathcal{F}$, where $x > 0$ is constant, and events $F, E \in \mathcal{E}$ with $E \subseteq F$, if $x \succsim^*_{F,h} g$ and $x =_{E^c} g$ then $x \succsim_{E,x} g$.*

There are two periods, 0 and 1. In period 0, the agents have not received any information but they have a common information structure about period 1, which is represented by partition Π of S. Hence, there is symmetric information among all agents. The initial allocation is $\{e^i\}_{i \in I}$, where $e^i \in \mathcal{F}_+$.The aggregate endowment is $\sum_{i \in I} e^i = e \in \mathbb{R}^S_{++}$. We assume that there is no aggregate uncertainty, so e is constant across all states in S. The economy in period 0 is a tuple $\langle S, \succsim^1, \ldots, \succsim^m, e \rangle$.

In period 1, all agents are informed that some event $E \in \Pi$ has occurred and trade, using their conditional preferences. Hence, information is symmetric. Trading at each $E \in \Pi$ generates an act for each agent, which is evaluated in period 0 using preference relation $\succsim^i$.

Given event $E \in \Pi$, an allocation for economy $\langle E, \succsim^1_{E,h^i}, \ldots, \succsim^m_{E,h^m}, e \rangle$ is a tuple $f_E = (f^1_E, \ldots, f^m_E) \in \mathcal{F}^m$.[11] An allocation is feasible if $\sum_{i=1}^m f^i_E = e$. It is interior if $f^i_E(s) > 0$ for all $s \in E$ and for all i. A feasible allocation f_E is full insurance if each f^i_E is constant across all states in E. It is Pareto optimal if there is no feasible allocation g_E such that $g^i_E \succsim^i_{E,h^i} f^i_E$ for all $i \in I$ and $g^j_E \succ^j_{E,h^j} f^j_E$ for some $j \in I$.

Fix a collection of convex preferences $\{\succsim^i_{E,h}\}_{i \in I, E \in \mathcal{E}, h \in \mathcal{F}}$. Let $\mathcal{M} = \{\Pi, e\}$ be an aggregate decision problem, where $\Pi \subseteq \mathcal{E}$ is a partition of S and $e \in \mathbb{R}^S_{++}$ is the aggregate endowment, assumed constant across states.

Given event $E \in \Pi$ and economy $\langle E, \succsim^1_{E,h^i}, \ldots, \succsim^m_{E,h^m}, e \rangle$, define $f_E = \{f^i_E\}_{i \in I} \in \mathcal{F}^m$ to be an equilibrium allocation if it is feasible and there are prices $p \in \mathbb{R}^S_+$, with $p(s) = 0$ if $s \notin E$, such that, for each $i \in I$, $\mathbb{E}_p f^i_E \leq \mathbb{E}_p e^i$ and $f^i_E \succsim^i_{E,h^i} g$ for all g such that $\mathbb{E}_p g \leq \mathbb{E}_p e^i$.

We say that interior allocation $\{f^i\}_{i \in I}$ is admissible for aggregate decision problem $\mathcal{M} = \{\Pi, e\}$ if, for each agent $i \in I$, for each $E \in \Pi$, $f^i =_E f^i_E$, where $f_E = \{f^i_E\}_{i \in I}$ is an equilibrium allocation of economy $\langle E, \succsim^1_{E,f^1_E}, \ldots, \succsim^m_{E,f^m_E}, e \rangle$. In words, $\{f^i\}_{i \in I}$ is admissible if, under

[11]Note that we define the aggregate endowment of the economy as a map from S (rather than E) to $\mathbb{R}_{++}$. This is without loss of generality because, from Property 1, what the endowment prescribes outside of E is irrelevant. For consistency, we do the same for all subsequent acts.

some equilibrium, agent i receives f_E^i at each $E \in \Pi$. Note that for admissibility we require that the status quo act for preference $\succsim^i_{E,f_E^i}$ is the allocation that is actually chosen, f_E^i.

We compare aggregate decision problems by evaluating the admissible acts they generate.

Definition 2. Aggregate decision problem $\mathcal{M}_2 = \{\Pi_2, e\}$ is locally more valuable than $\mathcal{M}_1 = \{\Pi_1, e\}$ if whenever $\{g^i\}_{i\in I}$ is admissible for $\mathcal{M}_2$, there exists $\{f^i\}_{i\in I}$ which is admissible for $\mathcal{M}_1$ and $g^i \succsim^i f^i$, for all $i \in I$. It is locally weakly more valuable if $ag^i + (1-a)f^i \succsim f^i$ for some $a \in (0, 1]$.

Aggregate decision problem $\mathcal{M}_2$ is locally more valuable than $\mathcal{M}_1$ if for every allocation which is admissible for $\mathcal{M}_2$, there is an admissible allocation $\mathcal{M}_1$, such that every agent ex ante prefers the former over the latter. It is locally weakly more valuable if each agent prefers a convex combination of the two. The "local" indicates that it is possible that there is an admissible allocation for $\mathcal{M}_1$, such that no admissible allocation for $\mathcal{M}_2$ is ex ante weakly preferred by all agents. We eliminate this case with the following, stronger definition.

Definition 3. Aggregate decision problem $\mathcal{M}_2 = \{\Pi_2, e\}$ is more valuable than $\mathcal{M}_1 = \{\Pi_1, e\}$ if whenever $\{f^i\}_{i\in I}$ is admissible for $\mathcal{M}_1$, there exists $\{g^i\}_{i\in I}$ which is admissible for $\mathcal{M}_2$ and $g^i \succsim^i f^i$, for all $i \in I$. It is weakly more valuable if $ag^i + (1-a)f^i \succsim f^i$ for some $a \in (0, 1]$.

If $\mathcal{M}_1$ differs from $\mathcal{M}_2$ only because partition Π_1 is finer than Π_2, we can examine whether more information has any value for the agents, by comparing how they evaluate the admissible allocations ex ante. We say that public information is locally (weakly) not valuable if an aggregate decision problem with a coarser partition is always locally (weakly) more valuable than a decision problem with a finer partition, ceteris paribus.

Definition 4. Public information is locally (weakly) not valuable for $\{\succsim^i_{E,h}\}_{i\in I, E\in\mathcal{E}, h\in\mathcal{F}}$ if, for all endowments e and partitions Π_1, Π_2 of S, Π_1 finer than Π_2, aggregate decision problem $\mathcal{M}_2 = \{\Pi_2, e\}$ is locally (weakly) more valuable as $\mathcal{M}_1 = \{\Pi_1, e\}$.

If we drop the term "locally," we have that public information is (weakly) not valuable if an aggregate decision problem with a coarser partition is always (weakly) more valuable than a decision problem with a finer partition, ceteris paribus.

Definition 5. Public information is (weakly) not valuable for $\{\succsim^i_{E,h}\}_{i\in I, E\in\mathcal{E}, h\in\mathcal{F}}$ if, for all endowments e and partitions Π_1, Π_2 of S, Π_1 finer than Π_2, aggregate decision problem $\mathcal{M}_2 = \{\Pi_2, e\}$ is (weakly) more valuable than $\mathcal{M}_1 = \{\Pi_1, e\}$.

The following Proposition shows that if information is (weakly) valuable for each agent, then public information is locally (weakly) not valuable. To show the stronger result that information is (weakly) not valuable, we need to assume that preferences are smooth at certainty, so that subjective beliefs are unique at constant acts. Formally, for all constant acts x, all agents $i \in I$ and events $E \in \mathcal{E}$, $\pi_E^i(x)$ is a singleton. Examples of smooth preferences at certainty are SEU, smooth ambiguity (Klibanoff, Marinacci, & Mukerji, 2009) and smooth variational preferences (Maccheroni et al., 2006). For example, Hanany and Klibanoff (2009) propose the smooth updating rule, which Galanis (2020) shows that it satisfies Weak Forward Consistency. Moreover, we need to assume Status Quo Bias.

Proposition 2. *Suppose strictly convex preferences* $\{\succsim^i_{E,h}\}_{i\in I,E\in\mathcal{E},h\in\mathcal{F}}$ *satisfy Axioms 7, 12, and 13. Then, Axiom 9 (Axiom 11) implies that public information is locally (weakly) not valuable. If, additionally, preferences are smooth at certainty, then public information is (weakly) not valuable.*

Proposition 2 requires Axiom 3 (Strong Monotonicity), to ensure that the equilibrium allocations are interior, because all states are nonnull. Strong Monotonicity is important in determining whether representing information through partitional structures is without loss of generality. In particular, the no speculative trade theorem of Milgrom and Stokey (1982) and the value of public information theorem of Schlee (2001) use a signal structure, where there is a set S of payoff relevant states and a set Z of signals. The full state space is $\Omega = S \times Z$. Each signal structure with nonnull states generates an equivalent partitional structure, with the same state space Ω, where a partition element is defined as $\Pi(\{s_k, z_k\}) = \{\{s_l, z_l\} \in \Omega : z_l = z_k\}$. However, a signal structure with some null states will generate a partitional structure with some null states, so Strong Monotonicity is violated. This implies that we cannot apply Proposition 2 to a signal structure where some states are null. However, as explained in Section 3, Proposition 1 is true even without Strong Monotonicity.

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DATA AVAILABILITY STATEMENT

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APPENDIX A

Proof of Proposition 1. Fix $j \in I$. From Axioms 2 and 3 we have that $H = G \equiv \{s \in S : g_0^i \succ^i_{\Pi^i(s), f^i} f^i \text{ for all } i \in I\}$ for some feasible allocation g_0, as we can always distribute a small enough portion of j's allocation to everyone else. We subsequently show that G cannot be common knowledge at any s, denoting g_0 by g.

Because f is an ex ante efficient allocation, there does not exist a feasible allocation h such that $h^i \succ^i f^i$ for all $i \in I$. Suppose that there exists feasible allocation g such that G is common knowledge at $s \in S$. Let F be a self evident event such that $s \in F \subseteq G$. Note that each Π^i partitions F. Then, we have that for each $i \in I$, for each $s' \in F$, $g^i \succ^i_{\Pi^i(s'), f^i} f^i$. From Property 1, $g^i \Pi^i(s') f^i \succ^i_{\Pi^i(s'), f^i} f^i$. Using Axiom 10 we have that $f^i \not\succsim^{*i} g^i \Pi^i(s') f^i$. Noting that f^i is strictly positive, so that $\pi^{ui} = \pi^i$, and from the definition of π^{ui}, we have that $\mathbb{E}_p(g^i \Pi^i(s') f^i) > \mathbb{E}_p f^i$ for all $p \in \pi^i(f^i)$. Because this is true for all $s' \in S$ such that $\Pi^i(s') \subseteq F$ and Π^i partitions F, we have that $\mathbb{E}_p(g^i F f^i) > \mathbb{E}_p f^i$, for all $p \in \pi^i(f^i)$. Define $h^i = g^i F f^i$ and $h = \{h^i\}_{i \in I}$.

Allocation f is interior, hence $\pi^{ui}(f^i) = \pi^{wi}(f^i) = \pi^i(f^i)$. Because $\mathbb{E}_p h^i > \mathbb{E}_p f^i$ for all $p \in \pi^{wi}(f^i)$, we have that for small enough ϵ^i, $\epsilon^i h^i + (1 - \epsilon^i) f^i \succ^i f^i$. By taking $\epsilon < \epsilon^i$ for all $i \in I$, we have that $\epsilon h^i + (1 - \epsilon) f^i \succ^i f^i$ for all $i \in I$. Moreover, $\epsilon h + (1 - \epsilon) f$ is feasible because both f and h are feasible. Hence, f is not ex ante efficient, a contradiction.

Conversely, suppose that $\{\succsim^1_{E,h}\}_{E \in \mathcal{E}, h \in \mathcal{F}}$ fails Axiom 10. This means that for some event $E \in \mathcal{E}$ and acts $f \in \mathcal{F}_+$, $g \in \mathcal{F}$, with $f =_{E^c} g$, we have $g \succ^1_{E,f} f$ and $\mathbb{E}_{p_0} g \le \mathbb{E}_{p_0} f$ for some $p_0 \in \pi^{1u}(f)$. Consider an economy with two agents, 1 and 2. Their information structure is identical, so that $\Pi^1 = \Pi^2 = \Pi = \{E, E^c\}$. Let $e = f + g$. Agent 2 has preferences represented by expected utility. In particular, $h \sim^2 h'$ if and only if $\mathbb{E}_{p_0} h = \mathbb{E}_{p_0} h'$. His conditional preferences given E or E^c are given by updating p_0 using Bayes' rule. This is well defined because Axiom 3 implies Axiom 8 (Weak Full Support) in Galanis (2020), hence from Lemma 2 in that paper we have that $p(E), p(E^c) > 0$.

We next show that allocation $h = \{f, g\}$ is ex ante Pareto efficient. Suppose there exists allocation $\{x, y\}$ such that $x \succ^1 f$ and $y \succsim^2 g$. Because $p_0 \in \pi^{1u}(f)$, we have that $\mathbb{E}_{p_0} x > \mathbb{E}_{p_0} f$. Moreover, $\mathbb{E}_{p_0} y \ge \mathbb{E}_{p_0} g$. These inequalities imply that $\mathbb{E}_{p_0}(x + y) > \mathbb{E}_{p_0}(f + g) = \mathbb{E}_{p_0} e$, which implies that $x + y \neq e$, hence $\{x, y\}$ is not feasible. A similar argument applies if $x \succsim^1 f$ and $y \succ^2 g$.

Note that $f \succsim^2 g$ because $\mathbb{E}_{p_0} g \le \mathbb{E}_{p_0} f$. Given E and since $f =_{E^c} g$, we have that $\mathbb{E}_{p_{0E}} g \le \mathbb{E}_{p_{0E}} f$, which implies $f \succsim^2_{E,g} g$. Because $g \succ^1_E f$, at each $s \in E$ it is common knowledge that allocation $h' = \{g, f\}$ Pareto dominates $h = \{f, g\}$, hence there is speculative trade. $\square$

Proof of Proposition 2. First note that because Axiom 5 implies Axiom 7 (No Flat Kinks) in Galanis (2020), Proposition 1 in that paper implies that if information is (weakly) valuable then $\{\succsim^i_{E,h}\}_{i\in I,E\in\mathcal{E},h\in\mathcal{F}}$ satisfy Axiom 9 (Axiom 11), for each $i \in I$. Let e be the endowment and suppose partition Π_1 is finer than partition Π_2. Let $\{g^i\}_{i\in I}$ be admissible for $\mathcal{M}_2 = \{\Pi_2, e\}$, defined as follows. Let $\{g_{E_2}\}_{E_2\in\Pi_2}$ be a tuple where, for each $E_2 \in \Pi_2$, g_{E_2} is an equilibrium allocation for economy $\langle E_2, \succsim^1_{E_2,g^1_{E_2}}, \ldots, \succsim^m_{E_2,g^m_{E_2}}, e\rangle$ with (normalized) prices $p^{E_2} \in \Delta S$, such that $p(s) = 0$ if $s \notin E_2$. For each $E_2 \in \Pi_2$, let $g^i =_{E_2} g^i_{E_2}$. From the first welfare theorem, g_{E_2} is Pareto optimal. From Proposition 9 in RSS, g_{E_2} is a full insurance allocation. Recall that a feasible allocation g_{E_2} is full insurance if each $g^i_{E_2}$ is constant across all states in E_2. From Axiom 3, $p^{E_2}(s) > 0$ for all $s \in E_2$ and $g^i_{E_2}(s) = \mathbb{E}_{p^{E_2}} e^i$, for all $s \in E_2$ and all $i \in I$. Hence, g_{E_2} is also an interior allocation. Proposition 9 in RSS implies that $p^{E_2} \in \bigcap_i \pi^i_{E_2,g^i_{E_2}}$, where $\pi^i_{E_2,g^i_{E_2}}$ is the same for all constant acts, from Axiom 12.

Proposition 1 in RSS shows that $\pi_{E,g^i_{E_2}}(f) = \pi^{ui}_{E,g^i_{E_2}}(f)$ for all strictly positive acts f. We have already shown in the previous paragraph that $p^{E_2}(E_1) > 0$ for all $E_1 \subseteq E_2$, because Axiom 3 implies that $p^{E_2}(s) > 0$ for all $s \in E_2$.[12] It is straightforward that we can use the proof of Proposition 2 in Galanis (2020), but applying Axiom 13 instead of Axiom 10, to show that there is Bayesian updating of subjective beliefs at a constant act, between any events $F, E \in \mathcal{E}$, where $E \subseteq F$. We therefore have $p^{E_2}_{E_1} \in \bigcap_i \pi^i_{E_1,x}$, for each $E_1 \subseteq E_2$, where x is any constant act, $E_1 \in \Pi_1$ and $p^{E_2}_{E_1}$ is the Bayesian update of p^{E_2} on E_1.

Define allocation $\{f^i\}_{i\in I}$ as follows. If $E_1 \subseteq E_2$, where $E_1 \in \Pi_1$ and $E_2 \in \Pi_2$, then $f^i =_{E_1} f^i_{E_1}$, where $f_{E_1} = (f^1_{E_1}, \ldots, f^m_{E_1})$ is such that, for each $i \in I$, $f^i_{E_1}(s) = \mathbb{E}_{p^{E_2}_{E_1}} e^i$ for all $s \in E_1$. Hence, f_{E_1} is a full insurance allocation and we set $x = f^i_{E^1}$ for the Bayesian updating of subjective beliefs, $p^{E_2}_{E_1} \in \bigcap_i \pi^i_{E_1,x}$. Because $p^{E_2}_{E_1} \in \pi^i_{E_1,x}$, $f^i_{E_1}$ is weakly preferred to each act h that is affordable given prices $p^{E_2}_{E_1}$. Because f_{E_1} is feasible, it is an equilibrium allocation of economy $(E_1, \succsim^1_{E_1,f^1_{E_1}}, \ldots, \succsim^m_{E_1,f^m_{E_1}}, e)$. By repeating this for all E_1, E_2, $\{f^i\}_{i\in I}$ is admissible for $\mathcal{M}_1$.

By construction, $\mathbb{E}_{p^{E_2}} f^i = \mathbb{E}_{p^{E_2}} g^i_{E_2}$. Because $p^{E_2} \in \bigcap_i \pi^i_{E_2,g^i_{E_2}}$, we have that $g^i_{E_2} \succsim^i_{E_2,g^i_{E_2}} f^i$, for all $i \in I$ and all $E_2 \in \Pi_2$. Property 1 implies that $g^i \succsim^i_{E_2,g^i_{E_2}} f^i$ and Axiom 7 implies $g^i \succsim^i_{E_2,g^i} f^i$, for each $E_2 \in \Pi_2$ and each $i \in I$.

Enumerate the partition cells of $\Pi_2 = \{E_1, \ldots, E_n\}$. If $n = 1$ then $\Pi_2 = \{S\}$ is the uninformative partition and $g^i \succsim^i f^i$ for each $i \in I$, so we are done. Suppose that $n \geq 2$. For cell $1 \leq k \leq n$ define act h^i_k as follows. Let $h^i_k(s) = g^i(s)$ if $s \in E_j$, where $1 \leq j \leq k$, and $h^i_k(s) = f^i(s)$ otherwise. Note that $h^i_n = g^i$ and let $h^i_0 = f^i$. For each $1 \leq k \leq n$, from Property 1, we have that $g^i \succsim^i_{E_k,g^i} f^i$ implies $h^i_k \succsim^i_{E_k,g^i} h^i_{k-1}$. Axiom 7 implies $h^i_k \succsim^i_{E_k,h^i_k} h^i_{k-1}$. Applying Axiom 9 we have $h^i_k \succsim^i h^i_{k-1}$. By Axiom 1, we have that

[12]Alternatively, Axiom 3 implies Axiom 8 (Weak Full Support) in Galanis (2020), hence Lemma 2 in that paper implies $p^{E_2}(E_1) > 0$.

$g^i \succsim^i f^i$, for each $i \in I$, which implies that $\mathcal{M}_2$ is locally more valuable than $\mathcal{M}_1$. Therefore, Axiom 9 implies that public information is locally not valuable.

We next show that Axiom 11 implies that public information is locally weakly not valuable. For each $E_2 \in \Pi_2$ define $h^i_{E_2} = g^i_{E_2} E_2 f^i$. From Property 1, $g^i_{E_2} \succsim^i_{E_2, g^i} f^i$ implies $h^i_{E_2} \succsim^i_{E_2, g^i} f^i$. Axiom 7 implies $h^i_{E_2} \succsim^i_{E_2, h^i_{E_2}} f^i$. From Axiom 11, either $h^i_{E_2} \succsim^i f^i$ or $f^i \not\succ^i h^i_{E_2}$ and $f^i \not\succsim^{*i} h^i_{E_2}$. If $h^i_{E_2} \succsim^i f^i$, Axiom 5 implies that for all $a \in (0, 1)$, $ah^i_{E_2} + (1 - a)f^i \succ^i f^i$. This means that $\mathbb{E}_p(ah^i_{E_2} + (1 - a)f^i) > \mathbb{E}_p f^i$ for all $p \in \pi^i(f^i)$, or that $\mathbb{E}_p h^i_{E_2} > \mathbb{E}_p f^i$. Hence, in both cases we have that $\mathbb{E}_p h^i_{E_2} > \mathbb{E}_p f^i$ for all $p \in \pi^i(f^i)$. Repeating this argument for all $E_2 \in \Pi_2$, we have that $\mathbb{E}_p g^i > \mathbb{E}_p f^i$ for all $p \in \pi^i(f^i)$. By definition, for small enough $\epsilon^i > 0$, $\epsilon^i g^i + (1 - \epsilon^i)f^i \succ^i f^i$. By taking $\epsilon < \epsilon^i$ for all $i \in I$, we have that $\epsilon g^i + (1 - \epsilon)f^i \succ^i f^i$ for all $i \in I$, hence public information is locally weakly not valuable.

We now prove that if preferences are smooth at certainty, Axiom 9 (Axiom 11) implies that public information is (weakly) not valuable. Let $\{p^i\} = \pi^i(x)$ be the unique subjective belief for the ex ante preference relation $\succsim^i$ and $\{p^i_E\} = \pi^i_{E,x}(x)$ be the Bayesian update for event $E \in \mathcal{E}$ and status quo act x. From Axiom 12 we have $\pi^i(x) = \pi^i(x')$ for all constant acts x, x'. Since subjective beliefs are unique and they are updated using Bayes' rule, we have that $\pi^i_E(x) = \pi^i_E(x')$ for all $E \in \mathcal{E}$ and x, x'.

We need to show that for any allocation $\{f^i\}_{i \in I}$, admissible for $\mathcal{M}_1$, there is an allocation $\{g^i\}_{i \in I}$, admissible for $\mathcal{M}_2$, that dominates it ex ante. Suppose that $\{f^i\}_{i \in I}$ is admissible for $\mathcal{M}_1$, defined in a similar way as $\{g^i\}_{i \in I}$ for $\mathcal{M}_2$, in the beginning of the proof. That is, $\{f_{E_1}\}_{E_1 \in \Pi_1}$ is a tuple where, for each $E_1 \in \Pi_1$, f_{E_1} is an equilibrium allocation for economy $\langle E_1, \succsim^1_{E_1, f^1_{E_1}}, \ldots, \succsim^m_{E_1, f^m_{E_1}}, e \rangle$ with (normalized) prices $p^{E_1} \in \Delta S$, such that $p^{E_1}(s) = 0$ if $s \notin E_1$. For each $E_1 \in \Pi_1$, let $f^i =_{E_1} f^i_{E_1}$. Using the same arguments as in the first paragraph of this proof, f_{E_1} is an interior, Pareto optimal and full insurance allocation. Moreover, $\{p^{E_1}\} = \pi^i_{E_1, f^i_{E_1}} = \{p^i_{E_1}\}$ for all $i \in I$. In other words, the equilibrium price is just the updated subjective belief given E_1, that is therefore common among all agents, denoted p_{E_1}. Since the subjective belief is unique, the equilibrium allocation is unique and $f^i_{E_1} = E_{p_{E_1}} e^i$. Similarly, we can define admissible allocation $\{g^i\}_{i \in I}$ for $\mathcal{M}_2$ and show that it is interior, Pareto optimal and full insurance, where $g^i_{E_2} = \mathbb{E}_{p_{E_2}} e^i$ and p_{E_2} is equilibrium price and the common subjective belief, updated given E_2. Since Π_1 is finer than Π_2 and the update of p_{E_2} given E_1 is p_{E_1}, we have that $\mathbb{E}_{p_{E_2}} f^i = \mathbb{E}_{p_{E_2}} g^i_{E_2}$. The rest of the proof is exactly the same as the last three paragraphs of the proof that public information is locally (weakly) not valuable. □