Elsevier Editorial System(tm) for Journal of Sound and Vibration
Manuscript Draft

Manuscript Number: JSV-D-19-01808R1

Title: Free Vibration of functionally graded beams and frameworks using the dynamic stiffness method

Article Type: Invited Commentary

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Authors' Reply

We thank Professor Popov [1] for his interest in our work [2] and we are pleased for his detailed assessment of our paper and for his alternative viewpoint. In particular, he has shown that our published results in [2] computed by applying the dynamic stiffness method (DSM) can be obtained by using the transfer matrix method (TMM) through the computation of matrix exponentials. It is admitted that Professor Popov is correct in his assertion that both DSM and TMM give accurate results. However, there are some issues regarding the advantages and disadvantages of the two methods, and after careful considerations, we are responding to his comments as follows.

The authors would like to point out that there are significant differences between the DSM and TMM and in this respect, the following points are pertinent.

1) It is well recognised that the TMM relies on a sequence of matrix operations which are often very extensive, requiring considerable computational efforts, even though such operations are generally carried out on small matrices. Because of the applications of successive matrix operations, the TMM can lead to inaccuracies and numerical ill-conditioning, particularly when analysing complex structures for which the values of various stiffness components of constituent structural elements can vary significantly, maybe by orders of magnitude. The DSM, particularly when applied in conjunction with explicit stiffness expressions [2], overcomes this difficulty and thus becomes computationally more efficient and less error prone than the TMM. Although computer power is continuing to grow at an ever-increasingly rapid rate, we believe that computational efficiency is still very important, particularly when solving large scale problems. Computer power used without responsibility can be misguided and uninstructive in dealing with structural, fluid or other engineering problems.

2) In our experience, the use of explicit dynamic stiffness expressions as opposed to using successive matrix operations reduces the computational time by a huge margin. For the type of problems investigated in [2] there can be at least a fifteen-fold advantage in computational time when using explicit dynamic stiffness expressions as opposed to using matrix operations which, of course, form the fundamental basis of the TMM. The first author of [2] made a direct comparison of the CPU time by using the two independent approaches and he provided this estimate of computational time saving in his earlier publication, see Table 2 of [3].

3) It is significant to note that the explicit stiffness expressions are particularly useful when some but not all of the stiffness coefficients are needed.

4) The DSM investigation carried out in [2] will have the added advantage in carrying out further optimisation studies [4] where repetitive computation of sensitivities of the objective function with respect to different stiffness parameters is required. The TMM is expected to perform poorly in this respect and is envisaged to be computationally expensive. Furthermore, Professor Popov's approach requires the use of matrix exponentials which are not straightforward to calculate although there are available tools such as MATLAB.

Based on the above, the authors would like to point out that in terms of computational time as well as accuracy of results, the TMM or any matrix manipulation-based method would perform inefficiently when compared with the DSM coupled with the application of explicit analytical expressions. The first author of [2] used the TMM in an earlier investigation [5] and he experienced this first-hand and became aware of the shortcomings of TMM and concluded that the DSM was superior to TMM.

Professor Papov's TMM formulation [1] is primarily based on his Eq. (5) which considers the effect of rotatory inertia of the beam, i.e. it is for a Rayleigh beam. He stated [1] that his results matched perfectly with those of Lee and Lee [6] (Ref. [30] of [2]). It should be noted that Lee and Lee [6] made it clear in the title of their paper that their free vibration analysis for functionally graded beams was carried out by using Bernoulli-Euler theory through the application of the TMM and there was no suggestion from the title that they were embarking on the Rayleigh beam theory. It was therefore, not expected that Professor Papov would get a perfect match. This prompted an additional inquiry into the matter, and upon further inspection of [6], it appears that Lee and Lee [6] have eventually used Rayleigh's theory as opposed to Bernoulli-Euler theory even though the title of their paper suggests otherwise and thus, a perfect match with Professor Popov's results [1] becomes possible.

However, Professor Popov has made a perfectly valid and legitimate point that at the beginning of section 2.1 of our paper [2] it is clearly stated that we have used Bernoulli-Euler theory, but our governing differential equation (Eq. (3) of [2]) contains Rayleigh's rotatory inertia terms. Surely, there is an anomaly here and Professor Popov's misunderstanding or rather misgiving is totally understandable. We apologise for the confusion we have caused. We have essentially used Bernoulli-Euler theory by cutting out the rotatory inertia terms in Eq. (3) of [2] when developing our computer programs and subsequently obtaining our numerical results. This was well suspected by Professor Popov and he is right. With hindsight we should have made it clearer in our paper that all rotatory inertia terms were dropped when obtaining the results. The comparative values of the first five natural frequencies shown in Table 1 of [2] show that our results are slightly higher than those of Lee and Lee [6] (maximum discrepancy being around 4%) which is qualitatively anticipated because the effect of rotatory inertia is expected to diminish the natural frequencies.

Professor Popov is certainly right that the accuracy of our plotted mode shapes in Fig.4 of [2] is not so great, particularly when satisfying the zero-slope boundary condition for the clamped end. We understand Professor Popov's concern. However, the illustrative mode shapes are not merely sketches, but they are intended to demonstrate the bending and axial deformation dominated modes of the functionally graded beams for different boundary conditions. The authors were conscious of consuming not too excessive journal space when writing the paper and particularly, when presenting the mode shapes. They showed four sets of mode shapes (Fig.4 of [2]) as concisely as possible, i.e. within the confines of one single page. This would not have been possible if the mode shapes were presented too accurately. Nevertheless, they take Professor Popov's point on board and perhaps on second thought, they should have presented the mode shapes more accurately.

About the second illustrative example given in Table 2 of [2] for which the data as well as comparative results were taken from [7] (Ref. [41] of [2]), Professor Popov is right in his comment that we did not provide enough data in [2] for the reconstruction of our results. This was simply an inadvertent risk and we anticipated that readers of our paper [2] would find without much difficulty the required data in [7] which was referred to in our paper. In retrospect, it was probably advisable to duplicate the data of [7] in [2] in order to make our paper completely self-contained. The results of Table 2 in [2] were computed using the

Bernoulli-Euler theory as was the case with Table 1 in [2]. We thank Professor Popov for his care.

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