The Impact of Immigration on Public Debt: A Dynamic Macroeconomic Analysis

Azar Sultanov
City, University of London

December, 2020

A thesis submitted to

the Academic Faculty

by

Azar Sultanov

In partial fulfillment of the requirements for the degree of

Doctor of Philosophy

Department of Economics

City, University of London

London, United Kingdom

December, 2020

Declaration

I certify that the thesis I have presented for examination for the PhD degree of the City, University of London is solely my own work. The copyright of this thesis rests with the author. Quotation from it is permitted, provided that full acknowledgement is made. This thesis may not be reproduced without my prior written consent. I warrant that this authorisation does not, to the best of my belief, infringe the rights of any third party. I declare that my thesis consists of approximately 45,500 words.

London, December 2020
Azar Sultanov

Acknowledgements

This thesis is dedicated to my dear family, my mother Mariya, father Arifsah, sister Gular, brother-in-law Abidin, brother Ariz, sister-in-law Rana, my beautiful nices Maryam and Asel and nephews Sardar and Alp, who supported and encouraged me every and each day of my life. I owe them all my achievements.

I am extremely grateful to my supervisors, Prof. Michael Ben Gad and Prof. Joseph Pearlman, for their endless support and invaluable guidance. I would have never been able to complete my PhD without them. I learned from them far more what I could expect. The words of appreciation for their contributions to my academic and personal development can never go far enough.

I thank all members of staff within the Department of Economics for their kind support and encouragement. I also thank all PhD students, both past and present, for our conversations, mutual support, and friendship, especially Ekaterina Kuznetsova and Anna Kaliciak. They all encouraged me throughout and shared great memories. I owe a lot to Zamid Aligishiev, who has become my true and best friend.

Finally, I would like to thank all my friends. I have been fortunate to meet such amazing people helping and supporting me throughout my life.

Abstract

In this thesis we extend both DSGE and VECM models to study the economic impact of immigration and other demographic changes and how these relate to dynamic fiscal policy.

The first chapter extends the Dynamic Stochastic General Equilibrium (DSGE) model to also incorporate overlapping infinite-lived dynasties and considers what this richer demographic structure implies about both immigration policy and how immigration may influence policy makers willingness to rely on deficit finance. We extend previous work incorporating overlapping infinite-lived dynasties into a neoclassical growth model facilitating more robust welfare analysis of immigration policy to include endogenous labour supply, by assuming agents have GHH preferences—an approach that makes these types of models aggregable without generating a negative labour supply. This makes for a much richer general equilibrium analysis of factor taxation, public debt, government consumption, transfer payments and changes to immigration with a particular focus on the welfare of the incumbent population.

The second chapter focuses on Bayesian estimation and evaluation of the DSGE model with overlapping dynasties extended to incorporate a rich description of fiscal policy and immigration, to examine the debt dynamics using US data. By estimating a DSGE model that incorporates a detailed description of fiscal policy and immigration, we can estimate how government debt has been financed historically. Moreover, we can examine how adjustments in each fiscal instrument and immigration have affected the observed equilibrium. To accurately predict the impact of fiscal policy and immigration, it is essential to understand the magnitude and speed of their response to debt. The model is rich enough to provide a satisfactory empirical account of the impact of immigration in the US, and exogenous

shocks reflect unanticipated changes in fiscal policy and immigration to assess the role of these shocks in explaining the variance of the model's endogenous variables. Results for the policy parameter estimates indicate that several distortionary fiscal instruments have played an essential role in financing debt innovations. Although the response of capital taxes to debt innovations are highest, the response of other fiscal instruments and immigration are also important. Effect of an increase in immigration in the short-run dilute capital and hence lower per capita output, consumption, and tax receipts. However, we can observe a modest short-term rise in hours worked, government spending, and transfers. Immigration appears to have only a little short-run impact on real wages and output, while there is a positive reaction of interest rates and debt to immigration shocks. Government spending, transfers, investment and hours worked together with investment-specific and technology shocks have been a significant driver of immigration. However, response of immigration to consumption, labour and capital income tax shocks are negative. Results indicate that innovations to the flow of immigration have relatively little impact on the US economy. This is largely due to the long-run adjustment the economy undergoes when it absorbs immigrants and to the fact that we only measure the impact of changes to the flow of immigrants, not the impact of the flow itself or the stock of immigrants that accumulates over time.

The final chapter investigates the relationship between immigration and fiscal policy by estimating with US data a structural vector error correction model (SVECM) that provides a new practical approach with many advantages over conventional VAR analysis. As there are only a few papers that investigate the macroeconomic effects of immigration using time series techniques this study is unique in evaluating the long-run as well as the contemporaneous impact of immigration on debt formation and fiscal sustainability. We incorporate the government budget constraint within the empirical model to capture the long-run relationship and dynamic interactions between government spending, tax revenues, debt, interest rate and immigration. By isolating and assessing the impact of immigration on debt and fiscal financing, we can examine debt stabilizing changes in fiscal policy that can be attributed to immigration in the short and long horizon. This macroeconometric model generates results largely comport with the prediction generated by the more theoretical approaches in Chapters 1 and 2. The main findings are as follows. Immigration shocks are of minor importance for the US economy. The number of immigrants (flow) in any given year is not significant, which explains the magnitude of the impact. While the arrival of immigrants is associated with an initial dilution of public debt, in the long-run, this relationship is reversed. Immigration, although contributing to the increase of debt, on the other hand, can help to lessen the burden of public debt for the descendants of the natives in the US. Immigration may also help to alleviate the demographic problems through a positive long-term contribution to the revenues and thus to the welfare of the native population. Overall, the empirical model captures well the dynamics following immigration shocks predicted by the theoretical model developed in Chapter 1.

Contents

Introduction

This thesis collects three papers studying the effect of immigration on economic activity in the context of modern macroeconomic and macroeconometric modelling frameworks. In this thesis we extend both DSGE and VECM models to study the economic impact of immigration and other demographic changes and how these relate to dynamic fiscal policy. It is widely accepted that low population growth and ageing demographics make immigration a significant factor in the US economy. The main objective is to evaluate the effects of immigration on key macroeconomic variables, and study the dynamic interaction between fiscal and immigration policy in the US, which contributes to the current academic literature and provides guidance for government policies.

The first chapter extends the Dynamic Stochastic General Equilibrium (DSGE) model to also incorporate overlapping infinite-lived dynasties and considers what this richer demographic structure implies about both immigration policy and how immigration may influence policy makers willingness to rely on deficit finance. As Ben-Gad (2004, 2008) demonstrates, incorporating overlapping infinite-lived dynasties into a neoclassical growth model allows for a clear distinction between natural population growth and immigration and facilitates more robust welfare analysis of immigration policy. Ben-Gad (2018) uses this type of model to demonstrate how the prospect of future immigration may induce deficit bias in the receiving countries. We extend this work to include endogenous labour supply, by assuming agents have GHH preferences—an approach Ascari and Rankin (2007) demonstrate makes these types of models aggregable without generating a negative labour supply. This makes for a much richer general equilibrium analysis of factor taxation, public debt, government consumption, transfer payments and changes to immigration with a particular focus on the welfare of the incumbent population.

The second chapter focuses on Bayesian estimation and evaluation of the DSGE model with overlapping dynasties extended to incorporate a rich description of fiscal policy and immigration, to examine the debt dynamics using US data. The chapter's core contribution lies in its detailed specification of fiscal policy instruments and including the immigration into the analysis of the state of the public debt. Immigration time series are constructed and extended using the decomposition of changes in the US working-age population following Kiguchi and Mountford (2013) and Weiske (2017, 2019). As unlike in Chapter 1, here immigration is endogenously determined yet also reacts to the innovations, we can use the model to study not only its impact on the economy but how it responds to other innovations. Therefore in addition to specifying policy rules for capital, labour and consumption taxes, government expenditure and transfers, and allowing for contemporaneous responses to output and dynamic responses to government debt as in Leeper et al. (2010), there is also an analogous rule for immigration that responds to wages, the level of economic activity and the state of public debt. This motivated by a host of immigration models in which the decision to migrate depends not only on conditions in a sending country but also on the conditions in the destination country.

By estimating a DSGE model that incorporates a detailed description of fiscal policy and immigration, we can estimate how government debt has been financed historically. Moreover, we can examine how adjustments in each fiscal instrument and immigration have affected the observed equilibrium. To accurately predict the impact of fiscal policy and immigration, it is essential to understand the magnitude and speed of their response to debt. The model is rich enough to provide a satisfactory empirical account of the impact of immigration in the post-war US, and exogenous shocks reflect unanticipated changes in fiscal policy and immigration to assess the role of these shocks in explaining the variance of the model's endogenous variables. Results for the policy parameter estimates indicate that several distortionary fiscal instruments have played an essential role in financing debt innovations. Although the response of capital taxes to debt innovations are highest, the response of other fiscal instruments and immigration are also important. Unlike Leeper et al. (2010), the labour taxes have responded strongly to debt in the presence of immigration. The results also show that capital and labour tax rates have had a highly procyclical response to the level of aggregate output, while immigration is less responsive. However, immigration is responsive to wage rate innovations. Finally, we observe that exogenous changes to capital and labour tax rates affect the two rates simultaneously as in Leeper et al. (2010), suggesting that typical tax legislation tends to change both tax rates. In line with Leeper et al. (2010), exogenous changes to consumption tax rates do not affect the capital or labour tax rates. Effect of an increase in immigration in the short-run is dilutive for per capita output, consumption, capital and tax receipts. However, we can observe a short-term modest rise in hours worked, government spending, and transfers. Immigration appears to have only a little short-run impact on real wages and output, while there is a positive reaction of interest rates and debt to immigration shocks as predicted by Ben-Gad (2018). Government spending, transfers, investment and hours worked together with investment-specific and technology shocks have been a significant driver of immigration. However, response of immigration to consumption, labour and capital income tax shocks are negative. Results indicate that innovations to the flow of immigration have relatively little impact on the US economy. This is largely due to the long-run adjustment the economy undergoes when it absorbs immigrants and to the fact that we only measure the impact of changes to the flow of immigrants, not the impact of the flow itself or the stock of immigrants that accumulates over time. Since debt-financed fiscal changes trigger very-long lived dynamics, even in entirely conventional models, short-run impacts can differ sharply from long-run effects, even being of different signs (Leeper et al., 2010). The estimated model can be used to evaluate the effect of counterfactual fiscal policies and immigration. My third chapter offers alternative approach—an estimated structural VECM model that can be used to answer specific fiscal and immigration policy questions.

The final chapter investigates the relationship between immigration and fiscal policy by estimating with US data a structural vector error correction model (SVECM) that provides a new practical approach with many advantages over conventional VAR analysis. As there are only a few papers that investigate the macroeconomic effects of immigration using time series techniques this study is unique in evaluating the long-run as well as the contemporaneous impact of immigration on debt formation and fiscal sustainability. We incorporate the government budget constraint within the empirical model to capture the long-run relationship and dynamic interactions between government spending, tax revenues, debt, interest rate and immigration. By isolating and assessing the impact of immigration on debt and fiscal financing, we can examine debt stabilizing changes in fiscal policy that can be attributed to immigration in the short and long horizon. This macroeconometric model generates results largely comport with the prediction generated by the more theoretical approaches in Chapters 1 and 2. The main findings are as follows. Immigration shocks are of minor importance for the US economy. The number of immigrants (flow) in any given year is not significant, which explains the magnitude of the impact. While the arrival of immigrants is associated with an initial dilution of public debt, in the long-run, this relationship is reversed. Immigration, although contributing to the increase of debt, on the other hand, can help to lessen the burden of public debt for the descendants of the natives in the US. Immigration may also help to alleviate the demographic problems through a positive long-term contribution to the revenues and thus to the welfare of the native population. Overall, the empirical model captures well the dynamics following immigration shocks predicted by the theoretical model developed in Chapter 1 as an extension of Ben-Gad (2018).

Chapter 1

Immigration and Fiscal Policy in a Model of Optimal Growth with Endogenous Labour Supply

1.1. Introduction

Migration is a dynamic process and begins with the very history of humanity. As Gumilev (1990) defines: *"the species Homo sapiens has repeatedly and constantly during its existence, modified its distribution over the earth's surface"*. Beginning with the movement of out of Africa across Eurasia about 1.75 million years ago, early humans migrated due to many factors such as changing climate, landscape, and inadequate food supply. Following movements of the population had a significant impact on a global scale for centuries to come in many ways; in setting the societies, nations, economies and states.

The literature generally distinguishes between three major types of migration: labour migration, refugee migration, and urbanization. Industrialization encouraged migration wherever it appeared. Moreover, the increasingly global economy globalized the labour market. Transnational labour migration after reaching its peak of three million migrants per year in the early twentieth century fell to a lower level from the 1930s to the 1960s and then rebounded. In reaction different countries have enacted immigration restrictions. Arguments around these restrictions and the broader impact of immigration on destination countries have informed debates for generations. Immigration interacts with nearly every policy area, from employment and the economy to education, healthcare, the national budgets, and factors into a nearly endless list of social and economic subjects (Blau and Mackie, 2017). The problem is that the immigration debate, as Borjas (1999) puts it, is like most debates over social policy, frames the issues in black and white, and as with most things in life, there is a range of policy options in varying shades of grey. From the time of its founding, the United States has been a nation of immigrants. It remains a primary destination for wouldbe migrants from nearly every corner of the globe. Hence our focus will be on examining these questions within a US context.

My primary motivation in writing this chapter is to evaluate some of the policy options related to immigration, by focusing on the interaction between immigration flows and fiscal policy within the context of a dynamic growth model and then focus on how these impact on the welfare of the incumbent population. At the same time as rates of fertility have dropped throughout the developed world, the native population has aged rapidly necessitating more spending on old age pensions and health care. In the United States much of this increased spending has coincided with an increase in debt. At 78% of gross domestic product (GDP), federal debt held by the public in 2018 was at its highest level since shortly after World War II (see Figure 1.1).

In the past decade, the debt burden was exacerbated by large increases in government spending and shortfalls in revenues related to the 2007-2009 recession. Spending in every other area, including defence, all other appropriations, and other federal entitlements, though significant, make up a smaller share of the economy than has historically been the case. Nevertheless, the big three – Social Security, Medicare and Medicaid.[1] - remain at the centre of the problem. We also have to consider a fourth, relatively new Federal assistance program, the Affordable Care Act of 2010. The US Congressional Budget Office (CBO) projects that before today's 25-year-olds are ready to receive Medicare, these four programs alone: Social Security, Medicare, Medicaid and the Affordable Care Act, are expected to cost

[1]Social Security is the US government pension program for people over 65; Medicare is a government health insurance program for people over 65, and Medicaid is a government health assistance program for people who cannot afford to buy insurance.

more than all the money the government collects in taxes.

Since 1975, each year the CBO produces two different estimates representing the agency's best assessment of future spending, revenues, deficits, and debt for the decades to come. The first one is the Extended Baseline Forecast which is based on the assumption that current legislation will be implemented unchanged, some mandatory programs extended after their authorizations lapse, and that spending for Medicare and Social Security continues as scheduled even if their trust funds are exhausted. According to CBO projections from June 2018, the federal budget deficit, relative to the economy's size, was forecasted to reach 152% by 2048—the highest in US history so far. Moreover, if lawmakers follow their previous practice of renewing temporary measures that for example reduce individual income till 2026, the debt will rise faster still.

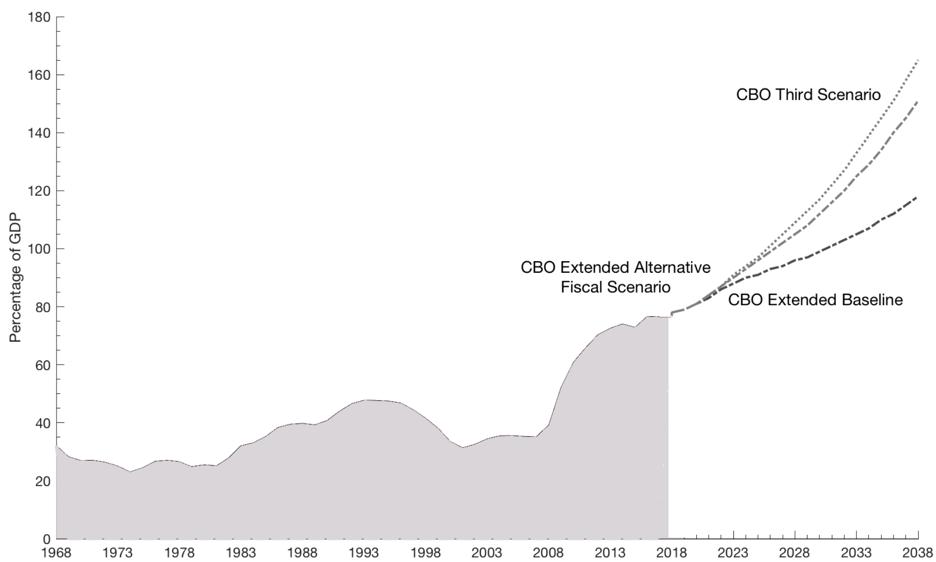


Figure 1.1: *Historical and Projected by CBO Federal Debt held by the public, June 2018*

In particular, over the next 30 years, spending as a share of GDP will increase for Social Security, the major health care programs (primarily Medicare), and interest on the government's debt. In CBO's projections, most of the spending growth for Social Security and Medicare results from the population's ageing. As members of the baby-boom generation (people born between 1946 and 1964) age and as life expectancy continues to rise, the percentage of the population age 65 or older will grow sharply, boosting the number of beneficiaries of those programs. In 1940, when the first Social Security payment was made, the average life expectancy of an individual in the US was 64 years. Today it is almost 80. The potential support ratio – the number of people aged 20-64 divided by the number of people aged 65 or over – for the US today is close to 4.6, and it is projected to decline to 1.9 by the year of 2100 (Fischetti, 2014).

Rising health care costs per person also drive growth in spending on Medicare and the other major health care programs. To give but one example, Medicare Part B, the part of Medicare that covers physicians' services, was initially projected to cost $500 million a year. In 2012 it cost $164 billion.

Furthermore, the CBO projects that the federal government's net interest costs will eventually climb sharply as a percentage of GDP as interest rates rise from their currently low levels as the debt continues to accumulate. Revenues are initially lower as a share of GDP. However, they are ultimately higher because individual income taxes are now projected to grow more quickly due to provisions of the 2017 Tax Cuts and Jobs Act).

Along with the Extended Baseline Forecast, the CBO also publishes *The Long-Term Budget Outlook Under Alternative Scenarios for Fiscal Policy.* This expands on CBO's extended baseline projections by showing how the federal budget and the nation's economy would evolve under three alternative scenarios. Those scenarios anticipate that some legislation will be altered to maintain certain policies beyond their current expiration dates.

In the first scenario, the current law is changed to maintain certain major policies that are now in place. Including the individual income tax provisions of Public Law 115-97 (originally called the Tax Cuts and Jobs Act, in short, the 2017 tax act), which scheduled to expire in 2026 under current law. Most other parts of the tax system's structure are left unchanged, including those that cause revenues to rise as a percentage of GDP. Also, discretionary spending equals a larger percentage of GDP than under the extended baseline, and that percentage remains roughly flat after 2028. In that scenario, the CBO projected in August 2018 that deficits will be even larger than under the extended baseline. Federal debt will equal 148% of GDP in 2038 and continue to rise after that.

In the second scenario after 2028 tax policy is assumed to change so that revenues remain flat as a percentage of GDP, rather than growing over time. In that scenario, debt would equal 151% of GDP in 2038.

The third scenario is similar to the second, except that tax policy is assumed to change so that revenues remain flat equal to 165% of GDP in 2038 and keeps rising. Under all three scenarios, economic output in 2038 would be smaller than under CBO's extended baseline. In any of the three scenarios, debt would exceed 200% of GDP by 2048 - but those models probably understate the increase in debt (see Figure 1.1)

The ageing of the population plays an essential role in the growth of the debt burden. CBO (2018) projects average annual US population growth from 2018 to 2092 to be 0.5%, however, for the same period the growth rate for people aged 65 and over is expected to be 1.1% per annum. This means that by 2092 the share of people aged 65 and over will increase to 25%. During this period, it is expected that the rate of legal and other immigration on average will be 3 per thousand per year (see Figure 1.2) and as fertility rates drop below replacement levels for US natives, immigrants will be the only source of population growth.

Looking forward, how might the US government eventually stabilise its debt? It could increase taxes, cut spending or perhaps increase the age at which Americans receive social security to reflect the longer lifespan. Could immigration provide a different answer to these challenges? The CBO also produced a report named *How Changes in Immigration Policy Might Affect the Federal Budget*, released on January 15, 2015, examining proposals by Congress to modify the immigration system and how these changes would affect the federal budget. CBO (2015) predicted that changes to immigration policy could significantly affect the size and composition of the non-citizen population and, as a result, alter rates of participation in federal programs and the payment of taxes. For that reason, when estimating the

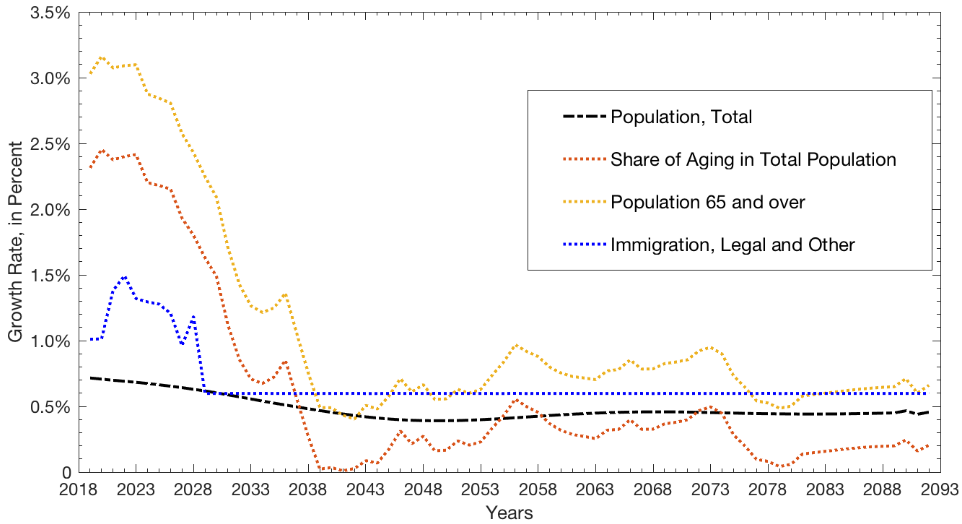


Figure 1.2: *CBO's Projected Growth Rates for the US: Total Population, Share of Aging Population, Population aged 65 and over, Legal and Other Immigration. Source: Congressional Budget Office, 2018.*

budgetary effects of proposals, the CBO considers the demographic and labour force characteristics of foreign-born people, their eligibility for and participation in federal programs, their tax liability, changes in the economy, and several other factors.

According to the CBO (2015) of the 41 million foreign-born people living in the US in 2012, about 22 million were non-citizens. Non-citizens differ from foreign-born and native-born citizens across several demographic dimensions, especially in terms of their skills and employment status. In particular, non-citizens are much more likely to be working age (between 25 and 64 years old) and much less likely to be aged 65 years or older. They also reflect a broad spectrum of education and skills (CBO, 2015).

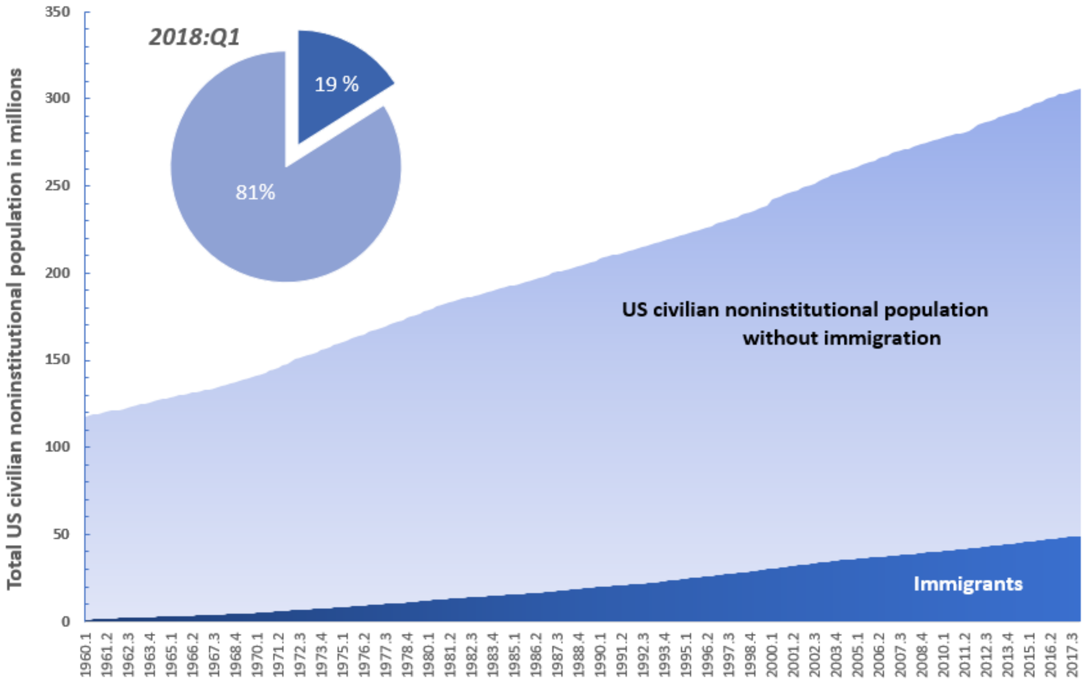


Figure 1.3: *Immigration and US civilian noninstitutional population, 1960:Q1-2018:Q1.*
Source: U.S. Bureau of Labor Statistics, CNP16OV-Civilian noninstitutional population is defined as persons 16 years of age and older. Retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CNP16OV, December 29, 2020.
Immigration data series constructed using CNP16OV series. See Chapter 2.5.2.

Size and share of immigrants in the US civilian noninstitutional population is presented in Figure 1.3. Similar estimates published by many organizations and think tanks. Migration Policy Institute estimated the immigrants' share of the US civilian labour force to be 17.2% in 2018.[2] Pew Research Center projects the US foreign-born population to reach 78 million by 2065 from 45 million in 2015.[3] Figure 1.4 displays the immigration to US noninstitutional population and

[2] "Immigrant Share of the U.S. Population and Civilian Labor Force, 1980 - Present" From: https://www.migrationpolicy.org/programs/data-hub/charts/immigrant-share-us-population-and-civilian-labor-force?width=1000&height=850&iframe=true.

[3] "Key findings about US Immigrants", by A.Budiman, August 20, 2020. From: https://www.pewresearch.org/fact-tank/2020/08/20/key-findings-about-u-s-immigrants/

growth rate for US noninstitutional civilian population aged 16 and over. The latter constitute the working age population in the US.

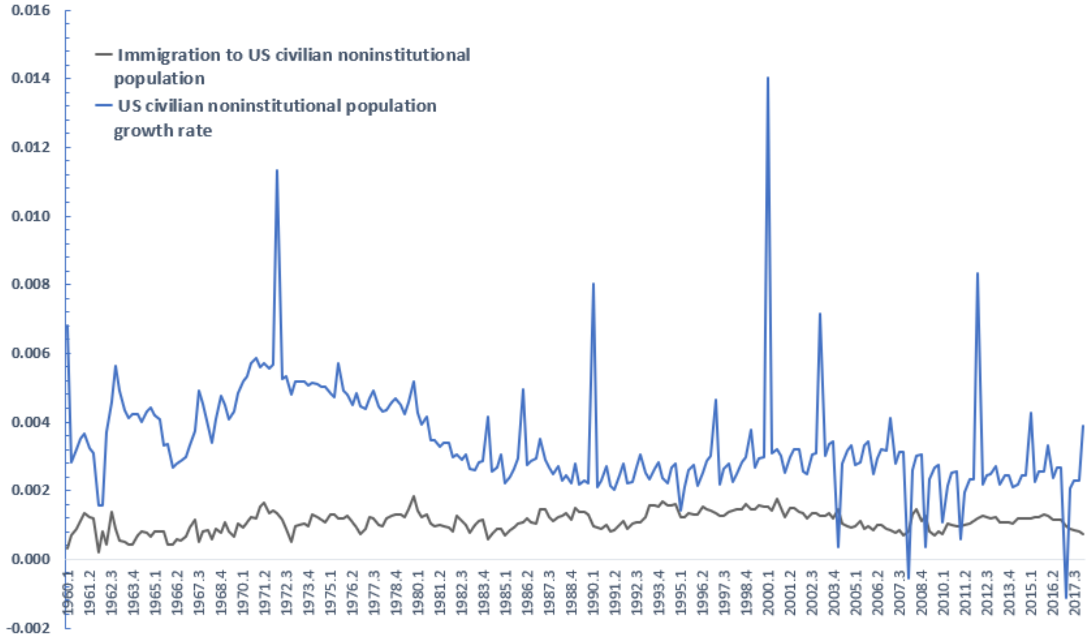


Figure 1.4: *Immigration and US civilian noninstitutional population dynamics, 1960:Q1-2018:Q1.*
Source: Own analysis based on the data series from U.S. Bureau of Labor Statistics.

Because most non-citizens who live and work in the US are subject to taxation, changes to federal immigration policy would affect the amount of revenue the government collects, also dilute the existing debt. A policy that led to a significant increase in the working-age population would expand the labour force and lead to a significant amount of additional revenues from income and payroll taxes.

Besides, the impact on taxes paid directly by immigrants is only part of the picture. Descendants of immigrants had, on average, the more favourable net fiscal impact for all government levels. By their slightly higher educational achievements and their higher wages and salaries, they contribute more in taxes than did their parents and dilute the existing debt (Blau and Mackie, 2017). According to Pew Research Center projections, the US-born children of immigrants (second-generation Americans) in 2020 already make up 12% of the US population and by 2050, immigrants and their descendants could account for 19% and 18% of the population, respectively.[4]

In this chapter, we evaluate the macro-economic and welfare impacts of immigration on a destination country and how that interacts with fiscal policy. Models that analyze immigration in the context of the neoclassical growth model are few. There is Canova and Ravn (2000) and Ben-Gad (2004, 2008, 2018) who developed a neoclassical growth model based on Weil (1989) overlapping infinite-lived dynasties designed to both accommodate and draw a distinction between natural population growth and immigration. Ben-Gad (2004, 2008) explores the welfare consequences of changes in immigration flows and Ben-Gad (2018) demonstrates how these immigration flows may generate a political bias in favour of fiscally imbalanced policies by receiving country governments.

In order to study the effects of fiscal shocks on equilibrium outcomes, we employ a non-stochastic optimal growth model with overlapping dynasties in which decision-makers have perfect foresight about future government decisions as in Ben-Gad (2018). We extend Ben-Gad (2018) by introducing a type of preferences Ascari and Rankin (2007) demonstrate can allow the model to accommodate endogenous labour supply with the type of infinite-lived overlapping dynasties first introduced into the literature by Weil (1989). Aside from endogenous labour supply, our model includes savings and capital accumulation, factor taxes, exogenous government spending and transfer payments. We study the impact of immigrants and changes to fiscal policy on the welfare of the native-born population, via changes in factor prices, the wages and return to capital. Unlike Ben-Gad (2004, 2008, 2018), the timing in this model is discrete rather than continuous and hence forms the basis for an extended version in Chapter 2 that can be estimated using Bayesian techniques.

To solve the model, we need to compute an equilibrium allocation when there are distorting taxes. For this, we have to solve a system of non-linear difference equations consisting of the first-order conditions for decision-makers presented in the following section.

[4] "Facts on U.S. immigrants, 2018. Statistical portrait of the foreign-born population in the United States" by A.Budiman, C.Tamir, L.Mora and L.Noe-Bustamante, August 20, 2020. From: https://www.pewresearch.org/hispanic/2020/08/20/facts-on-u-s-immigrants/

Since we are interested in welfare effects rather than merely the impulse responses of the key variables and want to analyze large policy changes that shift the economy far from steady-state, the usual methodologies of approximating the dynamic behaviour of models with saddle path equilibria using first or second-order approximations are inappropriate. They can lead to not only quantitative, but even in terms of welfare, qualitatively wrong answers. Therefore, we solve the model numerically by relying on an iterative shooting methodology. First, we solve the impulse responses, the consumption and labour behaviour that satisfy the consumer's transversality condition. Secondly, we choose associated fiscal consolidation that satisfies the government's budget constraint, employing an iterative algorithm that alters a particular distorting tax until we can ensure a balanced budget.[5]

1.2. The Basic Model

1.2.1 The Household

We assume an economy that is closed in every way, but open to immigration. Immigrants arrive from abroad at an annual rate of m_t to join the economy as workers, consumers and savers. Each of these immigrants is a founding member of the new infinite lived dynasty, indexed by s, which grows at a rate of n. At time t a member of a dynasty maximizes the infinite stream of discounted utility starting in period s when it joins the economy:

$$\sum_{t=s}^{\infty} \beta^{t-s} (1+n)^{t-s} \ln \left(c_{s,t} - d \left(l_{s,t} \right) \right) \tag{1.1}$$

where $c_{s,t}$ and $l_{s,t}$, denote consumption and hours worked for the members of dynasty s at time t. Subject to a time t budget constraint ($\forall\ s, t$):

$$(1+n)^{t-s+1} a_{s,t+1} = (1+n)^{t-s} \left[(1-\tau_t^w) w_t l_{s,t} + (1 + (1-\tau_t^r) r_t) a_{s,t} - (1+\tau_t^c) c_{s,t} + z_{s,t} \right] \tag{1.2}$$

We denote $a_{s,t}$ total financial assets as a sum of the holdings of government bonds $b_{s,t}$ and physical capital $k_{s,t}$. Following Ben-Gad (2018) we do not differentiate

[5] Matlab codes available upon request.

between the interest rate applied to the bond and capital holdings and the returns on these assets are taxed at the same rate of τ_t^r. The wage rate w_t and rate of return on capital r_t are common across dynasties as are the subjective discount rate β and the rate n at which each dynasty itself is growing. The earnings from labour supply taxed at the rate τ_t^w. Income received from government transfer payments is denoted by $z_{s,t}$.

The transversality condition is obtained when taking derivatives of the Lagrangian with respect to the total assets a_{t+1}:

$$\lim_{t\to\infty}(1+n)^{t+1}\prod_{s=1}^{t}\frac{1}{1+(1-\tau_s^r)\,r_s}a_{t+1}=0 \tag{1.3}$$

1.2.2 Labour-Leisure Choice

Homothetic preferences are aggregable. However, the income effect will eventually cause dynasties that own capital to hit a zero limit on their labour supply. Ascari and Rankin (2007) demonstrate that the preferences introduced in Greenwood et al. (1988) are non-homothetic but satisfy the conditions of the Gorman polar form meaning they are aggregable.[6] Named after Jeremy Greenwood, Zvi Hercowitz, and Gregory Huffman, the Greenwood-Hercowitz-Huffman (henceforth "GHH") utility function is a particular functional form of aggregable preferences where consumption and labour are not additively separable. In a model with these preferences there is no wealth effect on the labour supply and so incumbent dynasties do not reduce their labour supply to zero as a result of the higher rates of return on capital they enjoy due to continuous inflows of new immigrants. In a model with GHH preferences, the income effect is only expressed through consumption, whereas the labour supply is determined by the substitution effect only.

We follow Ascari and Rankin (2007) and adopt the specific functional form in

[6] In the Gorman polar form an indirect utility takes the general form: $v(p,I)=\frac{I-f(p)}{g(p)}$ where I is income and f and g are homogenous of degree one functions of the price vector p.

Greenwood et al. (1988):

$$d\left(l_{s,t}\right)=\left(\frac{\eta}{\varepsilon}\right)l_{s,t}^{\varepsilon}\qquad\varepsilon\geq1 \tag{1.4}$$

where $d\left(l_{s,t}\right)$ is a function giving disutility of labour supply, with $d',d''\geq0$ and η represents the productivity of workers (in our model we assume it is equal to one). Lastly, $\varepsilon=\theta+1$, where θ is the inverse of the Frisch elasticity of labour supply, which captures the elasticity of hours worked to the wage rate.[7]

From the first-order conditions,

$$\frac{U_{l}(c_{s,t},l_{s,t})}{U_{c}\left(c_{s,t},l_{s,t}\right)}=w_{t}\,\frac{\left(1-\tau_{t}^{w}\right)}{\left(1+\tau_{t}^{c}\right)}$$

This is a period-by-period relationship between optimal consumption and labour supply, showing how the optimal labour supply relates to the wage, the level of consumption and taxes.

Differentiating with respect to c and l of the utility function (see Appendix 1.9.) we obtain labour supply equation, which is not a function of accumulated wealth and common across all households participating in the economy at time t:

$$l_{s,t}=\left[\frac{\left(1-\tau_{t}^{w}\right)}{\left(1+\tau_{t}^{c}\right)}\,\frac{w_{t}}{\eta}\right]^{\frac{1}{\epsilon-1}} \tag{1.5}$$

1.2.3 Firms

The single firm chooses $\{k_t,l_t\}$, labour and physical capital, as factors to produce a single good in the economy and to maximize profits using standard neoclassical specification and Cobb-Douglas production function.

$$F\left(k_{t},l_{t}\right)=k_{t}^{\alpha}l_{t}^{1-\alpha}$$

Both input factors receive their marginal products,

[7]The Frisch elasticity measures the substitution effect of a change in the wage rate on labour supply (Burkhard and Maussner, 2005).

$$r_t = \alpha k_t^{\alpha-1} l_t^{1-\alpha} - \delta$$

$$w_t = (1-\alpha)\, k_t^{\,\alpha}\, l_t^{-\alpha}$$

1.2.4 The Government and Fiscal Policies

The government purchases goods in the market and also pay transfer payments to households. They are financed by the proceeds from the sale of government debt B_t, net of the payment of interest and principal and the revenue from the three different flat rate distorting taxes. We assume government behaviour is exogenous.

Unlike other economic models, we do not assume the government can impose lump-sum taxes. To ensure that the government balances its budget, we need to calculate each solution using two loops which assure that the model is initially solved and also that taxes and expenditures are such that the government budget constraint is satisfied for all $t \geq 0$. By employing such an iterative algorithm that alters a particular distorting tax until the government budget constraint satisfied as in Braun (1994) and McGrattan (1994), we can ensure that the government's intertemporal budget constraint is ultimately balanced.

1.2.5 Laws of Motions

The solution to the optimization problem yields the evolution of consumption for each individual dynasty s over time:

$$c_{s,t} - \frac{\eta}{\epsilon}\, l_{s,t}^{\epsilon} = \frac{(1+\tau_s^c)}{(1+\tau_t^c)} \left(c_{s,s} - \frac{\eta}{\epsilon}\, l_{s,s}^{\epsilon} \right) \left\{ \prod_{s}^{t} \beta \left(1 + (1-\tau_t^r)\, r_t\right) \right\} \tag{1.6}$$

The economy-wide feasibility and the governments budget constraints are:

$$K_{t+1} = F(K_t, L_t) - G_t - C_t + (1-\delta)\, K_t + M_{t+1} k_{t+1,t+1} \tag{1.7}$$

$$B_{t+1} = G_t - \tau_t^w w_t L_t - \tau_t^r r_t K_t - \tau_t^c C_t + (1-\tau_t^r)\, r_t B_t + B_t + Z_t + M_{t+1} b_{t+1,t+1} \tag{1.8}$$

Aggregate capital, aggregate consumption, publicly held government debt and aggregate transfer payments across different dynasties represented as,

$$K_t = \sum_{s=0}^{t} (1+n)^{t-s} M_s k_{s,t}$$

$$C_t = \sum_{s=0}^{t} (1+n)^{t-s} M_s c_{s,t}$$

$$B_t = \sum_{s=0}^{t} (1+n)^{t-s} M_s b_{s,t}$$

$$Z_t = \sum_{s=0}^{t} (1+n)^{t-s} M_s z_{s,t}$$

and the size of population is

$$P_t = \sum_{s=0}^{t} (1+n)^{t-s} M_s$$

where n is the growth rate of the dynasties, M_s.

The terms $b_{t+1,t+1}$ and $k_{t+1,t+1}$ represent any asset, in the form of either bonds or capital, that new immigrants arriving at time t may import with them.

Integrating the first-order conditions of the individual maximization problem and the time t budget over time, we obtain the consumption rule for the dynasty s at time t:

$$c_{s,t} = \frac{(1-\beta(1+n))}{(1+\tau_t^c)} \left[\omega_{s,t} + (1 + (1-\tau_t^r) r_t) a_{s,t}\right] + d(l_{s,t}) \tag{1.9}$$

where

$$\begin{aligned} \omega_{s,t} &= \sum_{h=0}^{\infty} (1+n)^h \prod_{i=1}^{h} \frac{1}{\left(1 + \left(1-\tau_{t+i}^r\right) r_{t+j}\right)} \\ &\times \left[\left(1-\tau_{t+h}^w\right) w_{t+h}\, l_{s,t+h} - \left(1+\tau_{t+h}^c\right) d\left(l_{s,t+h}\right)\right] \end{aligned} \tag{1.10}$$

is the present discounted value of all future labour income from time t forward, for dynasty s.

Aggregating (1.8) over all dynasties that arrived at time t, substituting (1.6) and (1.7), and re-writing in terms of stationary *per capita* variables yields (see Appendix 1.9):

$$\begin{aligned} & c_{t+1} - d(l_{t+1}) \\ = \; & \left(1 + \left(1 - \tau^{r}_{t+1}\right) r_{t+1}\right) \\ \times \; & \left[\beta \frac{\left(1 + \tau^{c}_{t}\right)}{\left(1 + \tau^{c}_{t+1}\right)} \left(c_t - d(l_t)\right) - \frac{\left(1 - \beta \left(1 + n\right)\right)}{\left(1 + \tau^{c}_{t+1}\right)} m_{t+1} \left(b_{t+1}\phi_{t+1} + k_{t+1}\kappa_{t+1}\right)\right] \end{aligned} \tag{1.11}$$

where $\kappa_{t+1} = \frac{k_{t+1} - k_{t+1,t+1}}{k_{t+1}}$ is the fractional difference between per-capita physical capital and the physical capital owned by new immigrants at the moment their arrival and $\phi_{t+1} = \frac{b_{t+1} - b_{t+1,t+1}}{b_{t+1}}$ is the analogous term for government debt. Going forward, we assume that immigrants exhaust all their capital and bonds travelling to their new country, so that κ_{t+1} and ϕ_{t+1} are set equal to 1 .

The economy-wide feasibility or law of motion for capital and the government budget constraint in *per capita* terms are:

$$k_{t+1} = \frac{f(k_t, l_t) - g_t - c_t + (1 - \delta)k_t}{(1 + n)\left(1 + m_{t+1}\right)} \tag{1.12}$$

$$b_{t+1} = \frac{g_t + q_t - \tau^{w}_{t} w_t l_t - \tau^{r}_{t} r_t \left(k_t + b_t\right) - \tau^{c}_{t} c_t + r_t b_t + b_t}{(1 + n)\left(1 + m_{t+1}\right)} \tag{1.13}$$

1.2.6 Competitive Equilibria with Distorting Taxes

Each vintage s household chooses sequences $\{c_{s,t}, l_{s,t}, k_{s,t}\}$ to maximize the utility function (1.1), subject to (1.2) the budget constraint. Firms choose $\{k_t, l_t\}$ to maximize profits. A budget-feasible government policy is an expenditure plan $\{g_t, q_t\}$ and tax plan and borrowing plan that satisfy (1.4). Once the households are aggregated, a feasible allocation is a vector of time series $\{c_t, l_t, k_t\}$ that satisfies (1.7).

As defined by Sargent and Ljingqvist (2012), competitive equilibrium with distorting taxes is a budget-feasible government policy, a feasible allocation and price system such that, given price system and government policy, the allocation solves the household's problem and firm's problem.

1.3. Welfare Analysis

To measure and analyze the welfare implications of different fiscal policies and the economic impact of immigration, we need to measure the welfare effect of the policy change.[8] Therefore, the similar consumption path should be given to the incumbent population along the old consumption path, making its members indifferent between the old policy and the new one. In other words, we need to compare the discounted welfare generated by the evolution the *per capita* consumption, $c_{0,t}$ of the native population already resident in the country at time $t = 0$, against the discounted welfare generated by the analogous counterfactual consumption path, $\bar{c}_{0,t}$ as in Ben-Gad (2004, 2008, 2018):

$$\sum_{t=s}^{\infty} \beta^{t-s} (1+n)^{t-s} \left[\ln \left(c_{0,t} - \frac{\eta}{\epsilon} \, l_{0,t}^{\epsilon} \right) \right] = \sum_{t=s}^{\infty} \beta^{t-s} (1+n)^{t-s} \left[\ln \left[\left(1 + \frac{\Delta_T}{100} \right) \bar{c}_{0,t} - \frac{\eta}{\epsilon} \, \bar{l}_{0,t}^{\epsilon} \right] \right] . \tag{1.14}$$

Inserting (11) yields:

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^{t} (1+n)^{t} \left\{ \ln \left[\left(c_{0,0} - \frac{\eta}{\epsilon} \, l_{0,0}^{\epsilon} \right) \beta^{t} \frac{(1+\tau_0^c)}{(1+\tau_t^c)} \prod_{0}^{t} \left(1 + (1-\tau_t^r) \, r_t \right) \right] \right\} \\ = \; & \sum_{t=0}^{\infty} \beta^{t} (1+n)^{t} \left\{ \ln \left[\left(1 + \frac{\Delta_T}{100} \right) \bar{c}_{0,t} - \frac{\eta}{\epsilon} \, \bar{l}_{0,t}^{\epsilon} \right] \right\} \end{aligned} \tag{1.15}$$

The welfare effect is measured as a compensating differential—a permanent percentage Δ_T of consumption sufficient to compensate native households for not deviating from the baseline fiscal policy. A negative value would mean that there is a welfare loss associated with the fiscal reform. Solving for Δ_T. (see Appendix

[8] Borjas (1995) developed the concept of *Immigration Surplus* to measure how economic theory can be used to analyze the economic impact of immigration and to quantify, abstracting from fiscal effects in terms of the welfare of native-born population (see also Blau and Mackie (2017)).

1.9.5)

$$\Delta_T = 100\times\left[-1 + \frac{\eta}{\epsilon}\frac{\bar{l}^{\epsilon}_{0,0}}{\bar{c}_{0,0}} + \frac{1}{\bar{c}_{0,0}}\left(c_{0,0} - \frac{\eta}{\epsilon}\, l^{\epsilon}_{0,0}\right)^{(1-\beta(1+n))} e^{\left\{(1-\beta(1+n))\sum_{t=0}^{\infty}\beta^t(1+n)^t\left[\ln\left(\prod_1^t(1+(1-\tau_t^r)r_t)\right)\right]\right\}}\right] \tag{1.16}$$

1.4. Calibrating the Model

In the baseline calibration, we take the period to correspond to a year and set model parameters to fit the US economy. The values are determined by using long-run averages and ratios from 1981 to 2014 for the United States. Similar values can be seen in Ben-Gad (2018) and Trabandt and Uhlig (2011). An overview of the calibration is provided in Table 2.2.[9]

Parameters	Value	Description
α	0.3700	Capital share
n	0.0064	Population growth rate
κ	1	Fractional difference, Capital Holdings
ϕ	1	Fractional difference, Bond Holdings
FE	$\{0.27; 0.40\}$	Frisch elasticity of labour supply
τ^r	0.387	Tax on Asset Income
τ^w	0.256	Tax on Wage Income
τ^c	0.059	Tax on Consumption
B/Y	0.426	Debt to Output ratio
K/Y	2.918	Capital to Output ratio
C/Y	0.586	Consumption to Output ratio
G/Y	0.197	Government Consumption to Output ratio

Table 1.1: *Long-run averages and ratios for the US, 1981-2014.*

From the Euler's equation assuming that immigration is equal to zero at steady

[9]For steady-state calculation and also formulae used to calibrate the model of parameters see Appendix 1.9.

state we pin down value for the β:

$$\beta = \frac{1}{\left(1 + (1-\tau^r_{ss})\left(\frac{\alpha - 1 + \frac{C}{Y} + \frac{G}{Y}}{\left(\frac{K}{Y}\right)} + n\right)\right)}, \tag{1.17}$$

and from Law of Motion for Capital we solve for depreciation rate δ,

$$\delta = \frac{1 - \frac{C}{Y} - \frac{G}{Y} - n\frac{K}{Y}}{\frac{K}{Y}}. \tag{1.18}$$

The value of δ and of capital share in production α, together with capital tax rate τ^r, labour tax rate τ^w and consumption tax rate τ^c largely agree with those in Trabandt and Uhlig (2011) which in turn followed the methodology Mendoza et al. (1994) to calculate average effective tax rates from national product and income accounts for the US from 1995 to 2007.

1.4.1 The Role of the Frisch Elasticity

One particular parameter, the Frisch elasticity of labour supply, is of central importance for determining the economic effect of different fiscal policies. In deciding on the supply of the amount to work, consume and save, household respond to incentives, which in part are also determined by the different tax rates, both present and future. The responsiveness of the supply of labour to changes in fiscal policies is determined by the Frisch elasticity, which is the sum of substitution elasticity and measure of people's willingness to trade work for consumption over time (Reichling and Whalen, 2012, 2017). As Peterman (2013) notes, the optimal capital and labour tax rates are highly sensitive to the Frisch elasticity.

Estimates of the Frisch elasticity can be generated by using micro or macro data. In contrast to large (from 2 to 4) calibration values for macroeconomic models, the seminal microeconomic estimates of the Frisch elasticity are much smaller—falling in the range of zero to 0.54 (MaCurdy, 1981; Altonji, 1986). Keane and Rogerson (2012) conclude that estimates of small labour supply elasticities based on micro data are entirely consistent with large aggregate labour supply elasticities. As labour supply elasticities are a function of preference parameters and all other aspects of the economic environment, we need structural modelling of the complete environment to model labour supply. The estimation of individual preferences alone will not be adequate, mainly when predicting the effects of changes in wages and taxes. Keane and Rogerson (2012) show that even in simple models, changes in after-tax wages can have effects on labour supply that differ significantly and simply being able to reconcile aggregate labour supply responses with observations from micro data are not in itself sufficient.

Peterman (2015) concluded that one explanation for this gap is that they capture fundamentally different notions of the Frisch labour elasticity. According to Peterman (2015), the seminal microeconometric estimates include two restrictions that are relaxed in the macroeconomic calibration values. First, while the micro Frisch elasticity restricts the sample to include a subset of the population, typically focusing on prime-aged married males, the macro Frisch elasticity represents the entire population. In addition, unlike the macroeconomic Frisch elasticity that includes labour fluctuations on both the intensive and extensive margin, the seminal microeconometric estimates incorporate fluctuations on the intensive margin only Peterman (2015).

Reichling and Whalen (2017) from their review of the literature concluded that most relevant for fiscal policy analysis estimate of the Frisch elasticity range from 0.27 to 0.53, with the central estimate of 0.4. They concluded that the same range is also used by the non-partisan CBO, which bases all of its macroeconomic analysis on parameter estimates that encompass a wide array of economists views about economic relationship.[10] The literature, including but not limited to Trabandt and Uhlig (2011), Peterman (2013) and Reichling and Whalen (2012, 2017) show that the choice of Frisch elasticity can have a significant influence on analyses of the economic effects of fiscal policy changes. Our model mainly uses the central estimate for Frisch elasticities; 0.27 and 0.4.

1.5. The Impact of Immigration

Consider the effect of different rates of immigration with different Frisch elasticities of labour supply. Consider first how the economy would behave if the rate of immigration increases by different increments from an initial value of zero to either

[10]For a discussion of how the CBO arrived that range see (Reichling and Whalen, 2012, 2017)

2, 4, 6, 8 or 10 per thousand for 40 years, while all taxes and government spending remain stable. After that all immigration ceases. To solve the model, and assuming that fiscal consolidation occurs through the tax on wages, we need to calculate the new tax on wage earnings necessary to stabilize government finances and satisfy the government budget constraint from year 40 onward. Here we alternate between three values for the Frisch elasticity of labour supply; zero, the inelastic labour supply, then 0.27 and 0.40, the most relevant for fiscal policy analysis estimates of the Frisch elasticity.

Further results are presented in Table 1.2. Immigration dilutes pre-existing public debt in the short-run (40 years) and in the longer perspective, after government stabilization, with the immigration levels increasing from 2 per thousand to 10, debt as a per cent of output decreases as much as from 1.5% to 6%. As a result of decreasing debt levels, the resulting new tax rate on wage income that would stabilize the debt at a particular level is also decreasing from 0.256 to 0.255.

Immigrants arrive in this economy without any capital, thereby driving down per-capita income, but potentially raising the income enjoyed by natives. However, this immigration surplus enjoyed by natives may disappear if the extra burden of funding of government spending falls disproportionately on natives.

Inflows of immigrants steadily increase the rate of return on capital from its steady-state level of 5.9% to 6.0% when immigration is 2 per thousand, up to 6.44% when it is 10 per thousand and depressing the wage rates (Figure 1.5). Expecting an increase in the rate of return with the arrival of immigrants, the native population, which owns the country's capital, immediately decreases consumption by 0.75% when immigrants allowed to the country 2 per thousand and up to by 3.72% for 10 per thousand immigrants (Figure 1.8).

How sensitive are these results in introducing elastic labour supply and period immigrants allowed in the country? Table 1.2 presents results for different values of the Frisch elasticities for 40 years. We can also see the comparison between immigration surges that last 40, 55 and 70 years with the respective Frisch Elasticity equals to 0.27 (Table 1.3) and 0.40 (Table 1.4). Changes in the rate of return and the wage rate for Frisch Elasticity equal to zero can be seen in Figure 1.5.

Figure 1.6 we can see impulse responses for the rate of return for different annual rates of immigration for 40 and 70 years when the Frisch Elasticity equals 0.40.

Perhaps one unexpected finding is that, when Frisch elasticity sufficiently increases from zero, the surge of immigration causes the amount of public debt to increase by a small magnitude, both by year 40, 55 and 70, as well as in the very long-run. Since the economy has reached a new steady-state, we observe that public debt decreases if Frisch elasticity is zero, is flat if Frisch elasticity is 0.27 and increases when 0.4. Consequently, the tax rate that will stabilize this increase is also increasing as we increase the time span of the policy experiment from 40 to 55 and 70 years and immigration in increments from zero to 10 per thousand (see Tables 1.3 and 1.4).

Although the rate of return increases as much as in inelastic labour supply case (Figure 1.7) and initial change in native consumption has the similar magnitude, consumption ends up in a new steady-state value which is below initial steady-state level as seen in Figure 1.9 when the Frisch elasticity equals 0.40. In this case, the welfare as a per cent of consumption drops from -0.03% for 2 per thousand immigrants to -0.07% for 10 per thousand immigrant.

Results for elastic labour supply shows that higher the Frisch elasticity the less welfare benefit natives enjoy. This is because of the immigration surplus, that is also a function of the labour supply. As a simple theoretical model of the labour market predicts, immigrants' inflow initially drives down wages. However, native incomes still rise in aggregate due to immigration surplus, due to the rise in the return to capital (Blau and Mackie, 2017). However, the wage effect is smaller than in the case where the native labour supply is fixed. When the labour supply is elastic, some natives could supply less labour in response to immigration. Therefore, in the case elastic supply, the size of immigration surplus shrinks.

Immigrants enjoy the same government spending and transfer as natives. They also pay taxes on their wages, but they do not initially own capital though as optimising dynasties they may acquire it over time once they arrive. Given that in the US and most of the country's capital gains are taxed at a higher rate than labour earnings, in our case it is 0.387 and 0.256 respectively, the burden of government expenditure falls more heavily on natives. That is more than enough to eliminate the immigration surplus, as immigration surplus declines in any event as the elasticity of labour supply increases.

Table 1.2: *The Impact of Increasing the Immigration rate for T=40*

The Frisch Elasticity of Labour Supply equals [0.00, 0.27, 0.40]

New immigrants per thousand m	Δ Debt as percent of output at T $100 \times \Delta\frac{b(T)}{y(T)}$	Δ Debt as percent of output long-run $100 \times \Delta\frac{b(\infty)}{y(\infty)}$	Tax rate after T $\tau^w(t > T)$	Initial change in native consumption $100 \times \left[\frac{c_{0,0}}{\bar{c}_{0,0}} - 1\right]$	Welfare as percent of permanent consumption Δ_T
Frisch Elasticity = 0.00					
0	0	0	0.2560	0	0
2	-1.5	-0.7	0.2557	-0.754	0.002
4	-2.9	-1.2	0.2554	-1.502	0.009
6	-4.1	-1.5	0.2553	-2.246	0.019
8	-5.1	-1.8	0.2552	-2.985	0.034
10	-6.0	-1.8	0.2551	-3.719	0.053
Frisch Elasticity = 0.27					
0	0	0	0.256	0	0
2	-0.3	1.1	0.257	-0.733	-0.028
4	-0.5	2.4	0.257	-1.461	-0.050
6	-0.5	3.8	0.258	-2.183	-0.066
8	-0.4	5.3	0.259	-2.898	-0.077
10	-0.2	7.0	0.260	-3.608	-0.081
Frisch Elasticity = 0.40					
0	0	0	0.256	0	0
2	0.3	2.0	0.257	-0.726	-0.047
4	0.7	4.2	0.259	-1.445	-0.087
6	1.3	6.5	0.260	-2.159	-0.121
8	1.9	8.9	0.261	-2.866	-0.149
10	2.7	11.4	0.263	-3.566	-0.169

Table 1.3: *The Impact of Increasing Immigration and Stabilizing Tax on Wage Earnings*

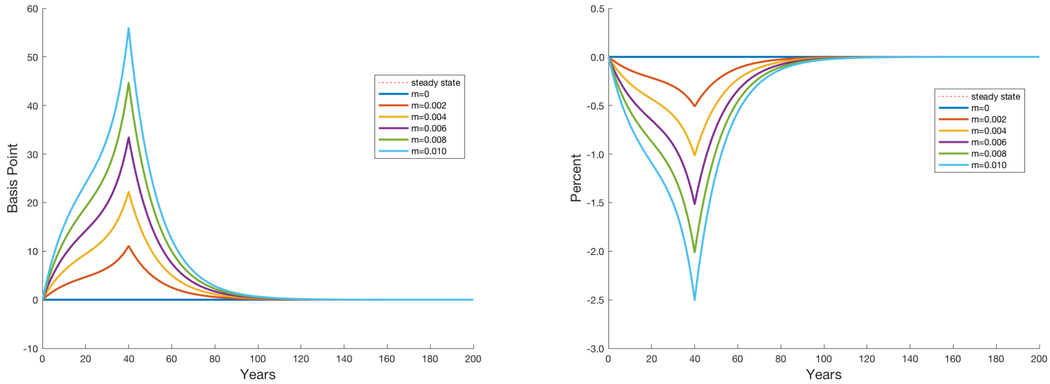
The Frisch Elasticity of Labour Supply equals 0.27

New immigrants per thousand m	Δ Debt as percent of output at T $100 \times \Delta \frac{b(T)}{y(T)}$	Δ Debt as percent of output long-run $100 \times \Delta \frac{b(\infty)}{y(\infty)}$	Tax rate after T $\tau^w(t > T)$	Initial change in native consumption $100 \times \left[\frac{c_{0,0}}{\bar{c}_{0,0}} - 1\right]$	Welfare as percent of permanent consumption Δ_T
$T = 40$					
0	0.0	0.0	0.256	0.000	0.000
2	-0.3	1.1	0.257	-0.733	-0.028
4	-0.5	2.4	0.257	-1.461	-0.050
6	-0.5	3.8	0.258	-2.183	-0.066
8	-0.4	5.3	0.259	-2.898	-0.077
10	-0.2	7.0	0.260	-3.608	-0.081
$T = 55$					
0	0.0	0.0	0.256	0.000	0.000
2	-0.5	1.0	0.257	-0.739	-0.020
4	-0.7	2.3	0.257	-1.473	-0.033
6	-0.6	3.8	0.258	-2.201	-0.040
8	-0.3	5.6	0.259	-2.923	-0.039
10	0.1	7.6	0.260	-3.639	-0.031
$T = 70$					
0	0.0	0.0	0.256	0.000	0.000
2	-0.7	0.7	0.256	-0.740	-0.015
4	-1.0	2.0	0.257	-1.475	-0.023
6	-0.8	3.7	0.258	-2.204	-0.024
8	-0.2	5.8	0.259	-2.927	-0.016
10	0.7	8.2	0.261	-3.644	0.002

Table 1.4: *The Impact of Increasing Immigration and Stabilizing Tax on Wage Earnings*

The Frisch Elasticity of Labour Supply equals 0.40

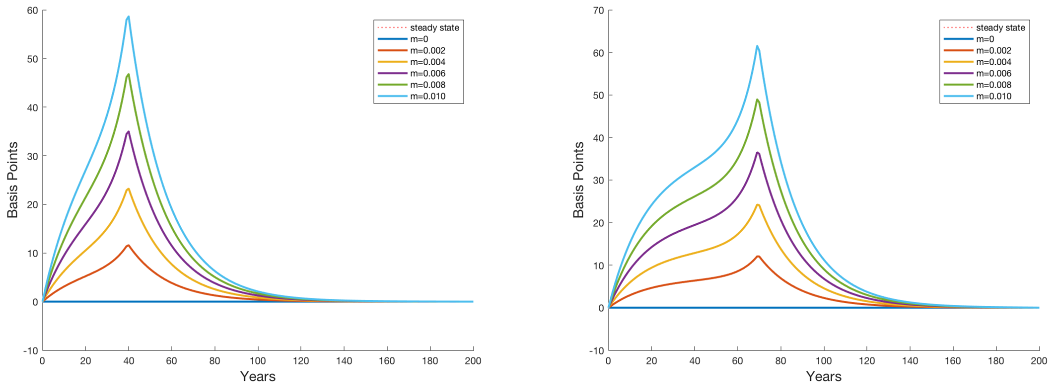
New immigrants per thousand m	Δ Debt as percent of output at T $100 \times \Delta \frac{b(T)}{y(T)}$	Δ Debt as percent of output long-run $100 \times \Delta \frac{b(\infty)}{y(\infty)}$	Tax rate after T $\tau^w(t > T)$	Initial change in native consumption $100 \times \left[\frac{c_{0,0}}{\bar{c}_{0,0}} - 1\right]$	Welfare as percent of permanent consumption Δ_T
$T = 40$					
0	0.0	0.0	0.256	0.000	0.000
2	0.3	2.0	0.257	-0.726	-0.047
4	0.7	4.2	0.259	-1.445	-0.087
6	1.3	6.5	0.260	-2.159	-0.121
8	1.9	8.9	0.261	-2.866	-0.149
10	2.7	11.4	0.263	-3.566	-0.169
$T = 55$					
0	0.0	0.0	0.256	0.000	0.000
2	0.6	2.4	0.257	-0.732	-0.038
4	1.5	5.0	0.259	-1.458	-0.069
6	2.6	7.9	0.261	-2.178	-0.092
8	3.8	11.0	0.263	-2.892	-0.106
10	5.3	14.3	0.265	-3.599	-0.111
$T = 70$					
0	0.0	0.0	0.256	0.000	0.000
2	1.1	2.9	0.258	-0.733	-0.033
4	2.7	6.3	0.260	-1.461	-0.057
6	4.6	9.9	0.262	-2.182	-0.071
8	6.7	14.0	0.264	-2.897	-0.076
10	9.1	18.2	0.267	-3.606	-0.070



(a) Rate of Return

(b) Wage Rate

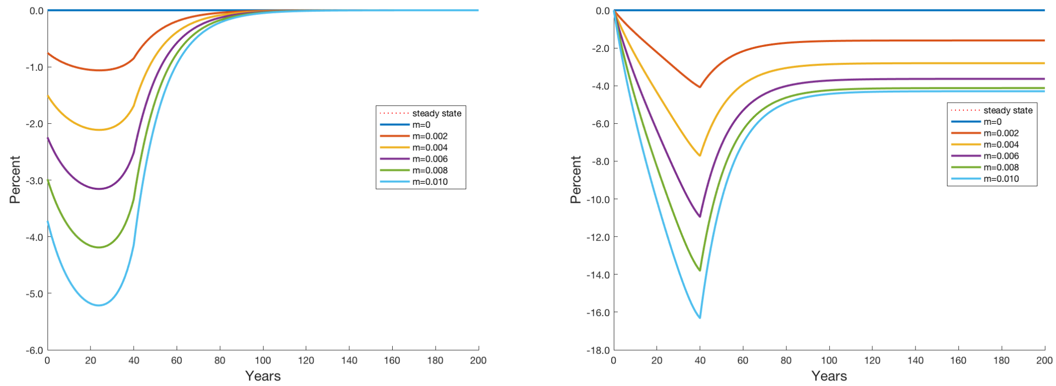
Figure 1.5: *Impulse Responses for the Rate of Return, Change in Basis Points and Wage Rate, Change in Percents, for Different Annual Rates of Immigration, Raising the Tax Rate on Wage Earnings in T=40 Years when the Frisch Elasticity equals zero*



(a) 40 Years

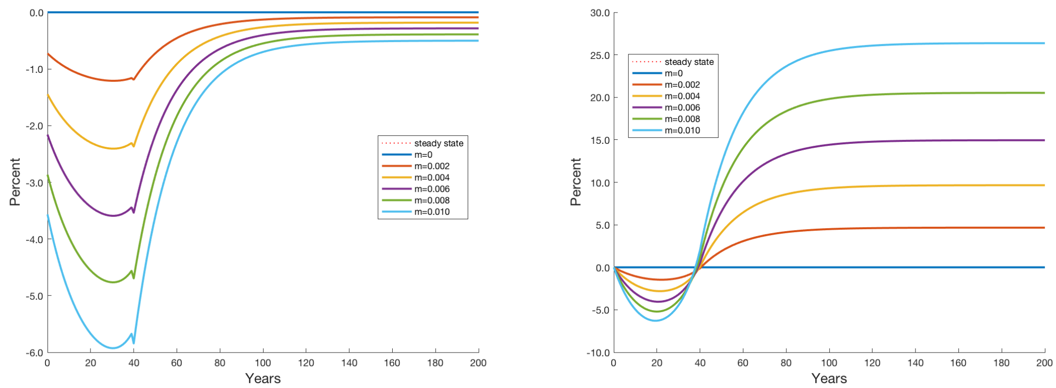
(b) 70 Years

Figure 1.6: *Impulse Responses for the Rate of Return, Change in Basis Points, for Different Annual Rates of Immigration, Raising the Tax Rate on Wage Earnings in T=40 or 70 Years when the Frisch Elasticity equals 0.40*



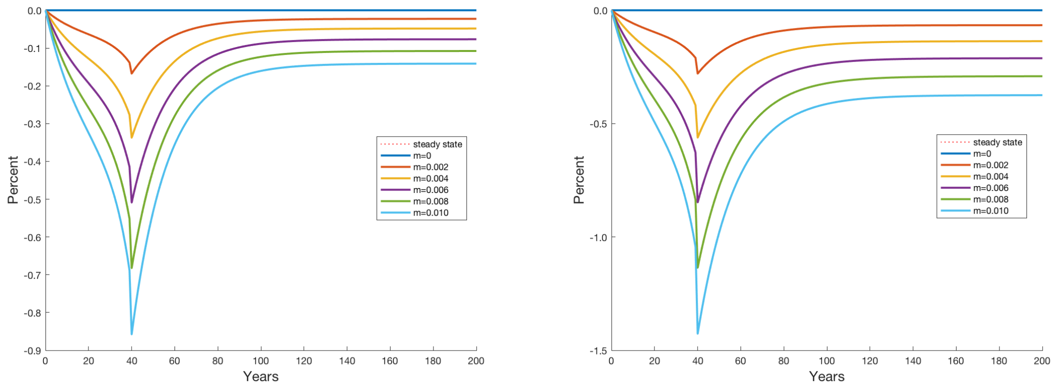
(a) Consumption (b) Debt

Figure 1.7: *Impulse Responses for the Consumption and Debt, Change in Percents, for Different Annual Rates of Immigration, Raising the Tax Rate on Wage Earnings in T=40 when the Frisch Elasticity equals zero*



(a) Consumption (b) Debt

Figure 1.8: *Impulse Responses for the Consumption and Debt, Change in Percents, for Different Annual Rates of Immigration, Raising the Tax Rate on Wage Earnings in T=40 Years when the Frisch Elasticity equals 0.40*



(a) Frisch Elasticity 0.27 (b) Frisch Elasticity 0.40

Figure 1.9: *Impulse Responses for the Labour Supply, Change in Percents, for Different Annual Rates of Immigration, Raising the Tax Rate on Wage Earnings in T=40 when the Frisch Elasticity equals 0.27 and 0.40*

1.6. Intertemporal Shifts in the Tax on Asset Income

In the previous section, we considered the impact of a surge of immigration into the US economy. Here we consider the effect of immigration when fiscal policy is also changing.

Consider the effect of permanently lowering the tax rate on asset income by 0.05 from 0.387 to 0.337 while allowing immigration to once again increase temporarily from to rates of 2, 4, 6, 8 and 10 per thousand for 40, 55 and 70 years.

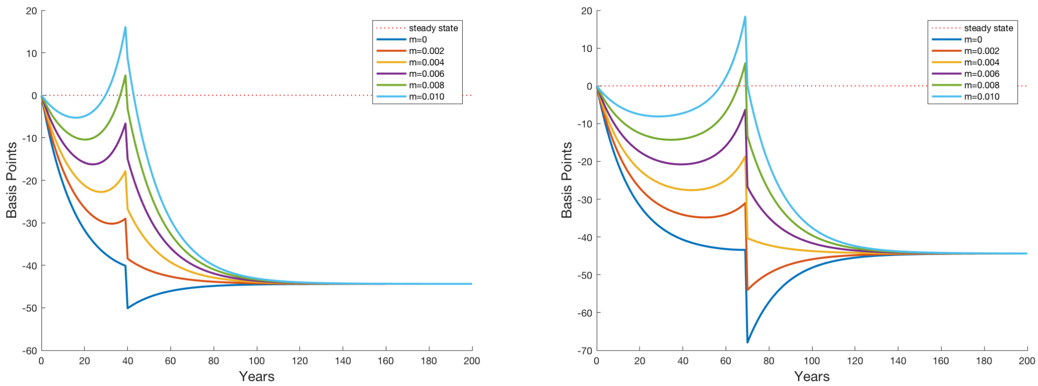
Once again, immigration raises the return to capital, making it more productive and increasing income to owners of capital. As the rate of return on capital is directly affected by the law of motion for consumption (9) and indirectly through the government budget constraint (11) in the long-run, high debt translates into permanently higher rates of return, even accounting for the higher taxes paid on capital gains. Table 1.5 and 1.6 show the impact of this policy experiment.

Figure 1.10 and 1.11 present impulse responses for the rate on capital and the wage rate for different annual rates of immigration when the Frisch Elasticity equals 0.27.

With the consequent change in tax on wage earnings, natives enjoy the benefit of shifting permanently from tax on asset income to tax on wage income, which now is shared with the immigrants who are already in the economy and with their descendants.

The rise in immigration occurs over the course of 40 periods, in the long-run, higher debt translates into comparatively higher rates of return. With the influx of immigrants, natives immediately lower their consumption to take advantage of higher rates of return.

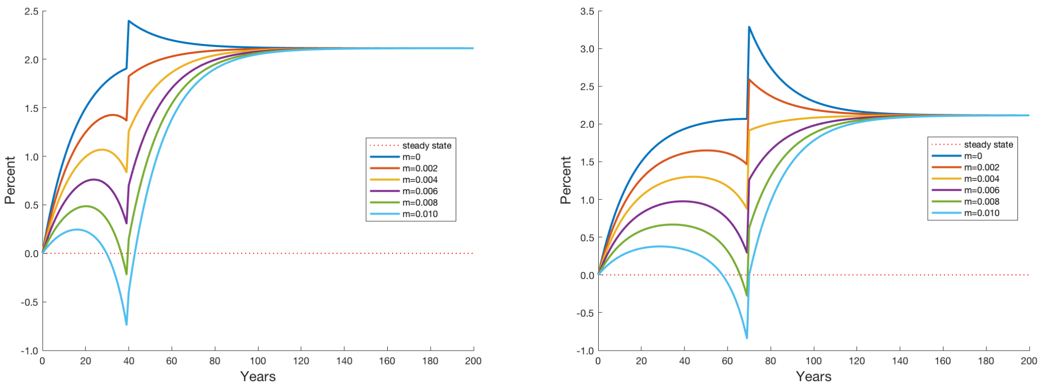
As we can see from the Table 1.5 and 1.6 the more elastic is the labour supply, here when Frisch elasticity is equal to 0.40, the less natives will enjoy any benefit from the surge in immigration. This is because the rise in the return to native-owned capital is somewhat more suppressed. In Figure 1.11, we can observe how the wage rate is affected by immigration and permanent changes in taxes on assets.



(a) 40 Years

(b) 70 Years

Figure 1.10: *Impulse Responses for the Rate of Return, Change in Basis Points, for Different Annual Rates of Immigration, after Permanently Lowering the Tax Rate on Asset Income from 0.387 to 0.337 and then Raising the Tax Rate on Wage Earnings in T=40 or 70 Years when the Frisch Elasticity equals 0.27*



(a) 40 Years

(b) 70 Years

Figure 1.11: *Impulse Responses for the Wage Rate, Change in Percents, for Different Annual Rates of Immigration, after Permanently Lowering the Tax Rate on Asset Income from 0.387 to 0.337 and then Raising the Tax Rate on Wage Earnings in T=40 or 70 Years when the Frisch Elasticity equals 0.27*

Table 1.5: *The Impact of Permanently Lowering the Tax Rate on Asset Income from 0.387 to 0.337*

The Frisch Elasticity of Labour Supply equals 0.27

New immigrants per thousand m	Δ Debt as percent of output at T $100 \times \Delta \frac{b(T)}{y(T)}$	Δ Debt as percent of output long-run $100 \times \Delta \frac{b(\infty)}{y(\infty)}$	Tax rate after T $\tau^w(t > T)$	Initial change in native consumption $100 \times \left[\frac{c_{0,0}}{\bar{c}_{0,0}} - 1\right]$	Welfare as percent of permanent consumption Δ_T
$T = 40$					
0	53.1	52.2	0.294	-1.891	0.385
2	51.2	52.0	0.294	-2.620	0.409
4	49.4	52.0	0.294	-3.344	0.437
6	47.9	52.2	0.294	-4.061	0.471
8	46.6	52.6	0.294	-4.772	0.511
10	45.4	53.2	0.294	-5.478	0.556
$T = 55$					
0	91.7	89.5	0.315	-1.892	0.379
2	87.0	86.9	0.313	-2.627	0.436
4	82.9	84.8	0.312	-3.356	0.497
6	79.3	83.1	0.311	-4.080	0.563
8	76.1	81.9	0.310	-4.797	0.635
10	73.3	81.0	0.310	-5.509	0.714
$T = 70$					
0	151.8	147.5	0.347	-1.892	0.369
2	141.8	140.0	0.343	-2.628	0.458
4	133.0	133.7	0.339	-3.358	0.550
6	125.3	128.4	0.336	-4.083	0.645
8	118.6	123.9	0.334	-4.802	0.745
10	112.7	120.3	0.332	-5.515	0.850

Table 1.6: *The Impact of Permanently Lowering the Tax Rate on Asset Income from 0.387 to 0.337*

The Frisch Elasticity of Labour Supply equals 0.40

New immigrants per thousand m	Δ Debt as percent of output at T $100 \times \Delta \frac{b(T)}{y(T)}$	Δ Debt as percent of output long-run $100 \times \Delta \frac{b(\infty)}{y(\infty)}$	Tax rate after T $\tau^w(t > T)$	Initial change in native consumption $100 \times \left[\frac{c_{0,0}}{\bar{c}_{0,0}} - 1\right]$	Welfare as percent of permanent consumption Δ_T
$T = 40$					
0	51.6	49.9	0.294	-1.814	0.362
2	50.4	50.8	0.295	-2.534	0.367
4	49.3	51.8	0.296	-3.248	0.377
6	48.5	53.0	0.296	-3.956	0.394
8	47.8	54.4	0.297	-4.657	0.416
10	47.3	55.9	0.298	-5.352	0.446
$T = 55$					
0	88.5	84.7	0.316	-1.814	0.352
2	85.2	83.8	0.315	-2.541	0.391
4	82.5	83.4	0.315	-3.262	0.435
6	80.1	83.6	0.315	-3.976	0.485
8	78.2	83.9	0.315	-4.684	0.542
10	76.6	84.6	0.316	-5.386	0.607
$T = 70$					
0	146.1	138.7	0.350	-1.814	0.336
2	138.7	134.3	0.347	-2.542	0.407
4	132.2	130.8	0.345	-3.264	0.483
6	126.8	128.2	0.343	-3.980	0.564
8	122.2	126.4	0.342	-4.689	0.652
10	118.4	125.2	0.341	-5.392	0.748

1.7. Intertemporal Shifts in the Tax on Wage Income

In the simple theoretical model of the labour market with inelastic labour supply, the influx of new immigrants initially drives down wages. However, native incomes still rise in the aggregate due to a rise in capital income yielding an immigration surplus. However, when the labour supply is sufficiently elastic, and the tax rate on capital income is higher than the tax on wage earnings, that same immigration surplus shrinks.

Tax revenues collected on labour earnings enter the model directly through the government budget constraint (11). The tax rate on labour earnings affects consumption, investment, and the rate of return on capital through the labour supply equation (13) and only when the economy accepts new immigrants.

Consider the effect of lowering the tax rate on wage income by 0.05 from 0.256 to 0.206 for 40, 55 and 70 years, with subsequent fiscal consolidation when taxes change to satisfy the government budget constraint (11). This period lasts 40, 55 and 70 years and only during this period, immigrants are allowed to enter the country.

Both the native population and immigrants initially benefit from the lowering of the tax rate on labour income. However, immigrants enjoy it more, as they arrive in the country without asset holdings, so their income is wholly derived from wage earnings. As fiscal consolidation is four, five or seven decades away, the more debt accumulates and the larger the future corresponding tax increase is necessary to satisfy the transversality condition and the government intertemporal budget constraint. Therefore, the new tax rate on wages is adjusted to continue funding the higher transfer payments and government spending and any finance the additional public debt that has accumulated in the interim.

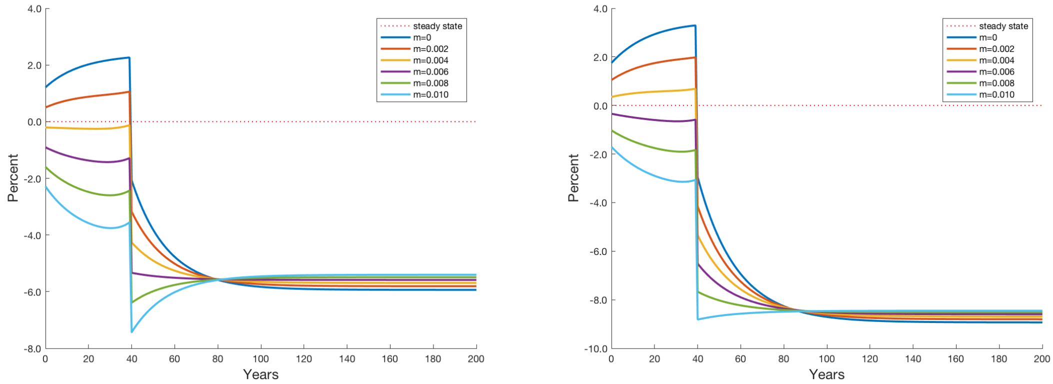
For each immigration rate m second and third columns, Table 1.7 and 1.8 present the changes to the debt burden by the end of the 40, 55 and 70 year periods. When the Frisch elasticity is equal to 0.40 after 40 years, the additional accumulated debt is equivalent to 202.9% of output if the rate of immigration is zero and 178.3% if the surge in immigration is 10 per thousand. Corresponding income tax rates are listed in column 4, as we can see the tax rates necessary to serve the debt accumulated in term and after the date of fiscal consolidation range between 37.4% to 36.8% for the debt as mentioned earlier.

The magnitude of the response of the rate of return on assets directly relates to the immigration rate. The term $k_{t+1}\kappa_{t+1}$ in (9) multiplied by the rate of immigration shows how the supply of labour provided by immigrant workers complements the stock of native-owned capital and raises its rate of return. This also increases the welfare of the native population in the last column of Table 1.7 and 1.8 from negative to a significant positive number.

Arriving at the destination country without assets, new immigrants pay higher taxes on their earnings to service debt accumulated before they arrived and help fund transfer payments. If the rate of immigration is 4 per thousand the benefit the native population derives is equivalent to a permanent increase in consumption by 0.071% when the Frisch elasticity is 0.27 and continues to grow as we increase the rate of immigration.

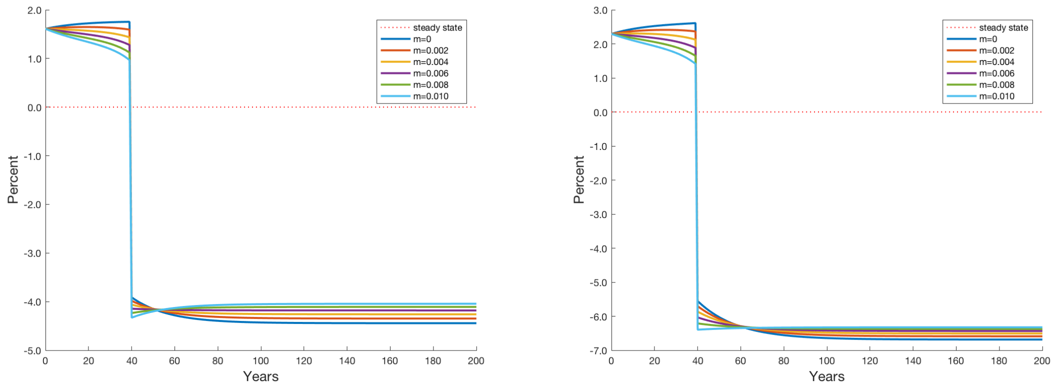
What we observe from both tables Table 1.7 and 1.8, is that in the absence of immigration lowering the tax rate on wage earnings, and then subsequently increasing it to a higher new level lowers welfare by introducing an intertemporal distortion. We can observe impulse responses for this policy experiment for consumption on Figure 1.12 and labour supply on Figure 1.13 when the Frisch elasticity equals 0.27 and 0.40.

Allowing immigrants to enter the country and join the workforce can change the welfare impact of this policy. This policy of postponing wage taxation into the future shifts the burden further away from natives and towards immigrants as they arrive in the destination country with little or no capital. We can observe the impact of temporarily lowering the tax rate on wage income by 0.05 from 0.256 to 0.206 for the Frisch Elasticity equals 0.27 at last column of Table 1.7 and 0.40 at Table 1.8.



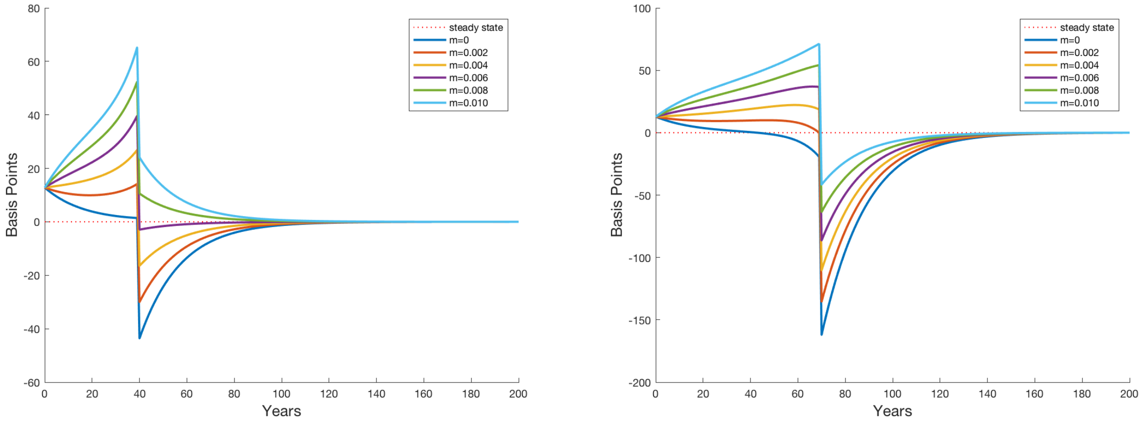
(a) Frisch Elasticity 0.27 (b) Frisch Elasticity 0.40

Figure 1.12: *Impulse Responses for the Consumption, Change in Percents, for Different Annual Rates of Immigration, after Temporarily Lowering the Tax Rate on Wage Earnings from 0.256 to 0.206 and then Rising the Tax Rate on Wage Earnings in T=40 Years when the Frisch Elasticity equals 0.27 and 0.40*



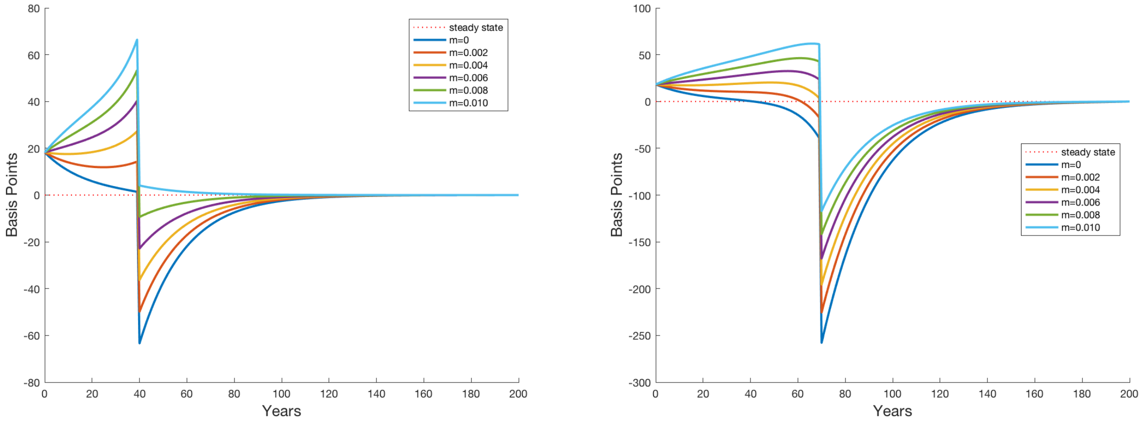
(a) Frisch Elasticity 0.27 (b) Frisch Elasticity 0.40

Figure 1.13: *Impulse Responses for the Labour Supply, change in Percents, for Different Annual Rates of Immigration, after Temporarily Lowering the Tax Rate on Wage Earnings from 0.256 to 0.206 and then Rising the Tax Rate on Wage Earnings in T=40 Years when the Frisch Elasticity equals 0.27 and 0.40*



(a) 40 Years (b) 70 Years

Figure 1.14: *Impulse Responses for the Rate of Return, Change in Basis Points, for Different Annual Rates of Immigration, after Temporarily Lowering the Tax Rate on Wage Earnings from 0.256 to 0.206 and then Rising the Tax Rate on Wage Earnings in T=40 or 70 Years when the Frisch Elasticity equals to 0.27*



(a) 40 Years (b) 70 Years

Figure 1.15: *Impulse Responses for the Rate of Return, Change in Basis Points, for Different Annual Rates of Immigration, after Temporarily Lowering the Tax Rate on Wage Earnings from 0.256 to 0.206 and then Rising the Tax Rate on Wage Earnings in T=40 or 70 Years when the Frisch Elasticity equals to 0.40*

Table 1.7: *The Impact of Temporarily Lowering the Tax Rate on Wage Earnings from 0.256 to 0.206*

The Frisch Elasticity of Labour Supply equals 0.27

New immigrants per thousand m	Δ Debt as percent of output at T $100 \times \Delta\frac{b(T)}{y(T)}$	Δ Debt as percent of output long-run $100 \times \Delta\frac{b(\infty)}{y(\infty)}$	Tax rate after T $\tau^w(t > T)$	Initial change in native consumption $100 \times \left[\frac{c_{0,0}}{\bar{c}_{0,0}} - 1\right]$	Welfare as percent of permanent consumption Δ_T
$T = 40$					
0	212.8	204.3	0.371	1.210	-0.179
2	206.0	200.2	0.369	0.499	-0.054
4	199.7	196.5	0.367	-0.206	0.071
6	193.7	193.1	0.365	-0.905	0.198
8	188.1	190.1	0.363	-1.599	0.327
10	182.8	187.3	0.361	-2.287	0.459
$T = 55$					
0	388.7	368.1	0.469	1.208	-0.358
2	371.6	355.7	0.461	0.494	-0.114
4	355.6	344.3	0.454	-0.216	0.124
6	340.7	333.7	0.448	-0.920	0.359
8	326.9	323.9	0.442	-1.618	0.591
10	314.1	314.9	0.436	-2.311	0.821
$T = 70$					
0	678.1	616.7	0.633	1.208	-0.799
2	639.9	590.8	0.614	0.493	-0.383
4	604.6	566.3	0.597	-0.217	0.009
6	572.0	543.2	0.581	-0.922	0.382
8	541.9	521.5	0.566	-1.621	0.740
10	514.2	501.3	0.553	-2.314	1.086

Table 1.8: *The Impact of Temporarily Lowering the Tax Rate on Wage Earnings from 0.256 to 0.206*

The Frisch Elasticity of Labour Supply equals 0.40

New immigrants per thousand m	Δ Debt as percent of output at T $100 \times \Delta \frac{b(T)}{y(T)}$	Δ Debt as percent of output long-run $100 \times \Delta \frac{b(\infty)}{y(\infty)}$	Tax rate after T $\tau^w(t > T)$	Initial change in native consumption $100 \times \left[\frac{c_{0,0}}{\bar{c}_{0,0}} - 1\right]$	Welfare as percent of permanent consumption Δ_T
$T = 40$					
0	202.9	188.5	0.374	1.749	-0.277
2	197.3	186.0	0.373	1.046	-0.177
4	192.0	183.9	0.371	0.349	-0.075
6	187.1	182.1	0.370	-0.342	0.030
8	182.6	180.5	0.369	-1.028	0.137
10	178.3	179.2	0.368	-1.707	0.248
$T = 55$					
0	373.2	337.6	0.478	1.749	-0.576
2	358.4	328.7	0.471	1.040	-0.352
4	344.6	320.6	0.465	0.338	-0.133
6	331.9	313.2	0.460	-0.358	0.083
8	320.2	306.4	0.455	-1.048	0.298
10	309.4	300.4	0.451	-1.732	0.512
$T = 70$					
0	662.4	542.7	0.670	1.748	-1.478
2	627.6	527.9	0.649	1.040	-1.016
4	595.8	513.0	0.631	0.337	-0.592
6	566.6	498.4	0.615	-0.360	-0.197
8	539.9	484.3	0.601	-1.051	0.176
10	515.4	471.0	0.588	-1.736	0.534

1.8. Conclusion

Borjas (1995) introduced the concept of an immigration surplus to measure immigration's welfare implications in a static general equilibrium setting. Ben-Gad (2004, 2008) demonstrates that incorporating overlapping infinite-lived dynasties into a neoclassical growth model allows for a clear distinction between natural population growth and immigration and facilitates a more robust welfare analysis of immigration policy. Ben-Gad (2018) uses this type of model to demonstrate how the prospect of future immigration may induce deficit bias in the receiving countries. We extend this work to include endogenous labour supply, by assuming agents have GHH preferences—an approach Ascari and Rankin (2007) demonstrate makes these types of models aggregable without generating a negative labour supply. This makes for a much richer general equilibrium analysis of factor taxation, public debt, government consumption, transfer payments and changes to immigration, focusing on the welfare of the incumbent population.

To illustrate the various channels through which immigration affects labour and asset markets and how the economy's adjustments mitigate those effects over time, we developed a simple model where policymakers can simultaneously adjust fiscal and immigration policy. We find that the quantitative and even qualitative impact of immigration depends on the size of the inflow and characteristics of the destination country economy, such as the speed at which capital can accumulate and the initial level of public debt. The initial level of the destination country's public debt is important, as natives initially own the entire stock of debt and benefit when interest rates increase (we assume that the government refinances its debt each period). The primary motivation of the households to hold government debt in a steady state is the rate of return. In our deterministic model without the risk premium and arbitrage-free pricing, we did not differentiate between the interest rate applied to the bond and capital holdings following Ben-Gad (2018). As government debt pays the same rate of return as capital, in the steady-state households are indifferent between holding debt and capital. The influx of immigrants raises returns on both assets in the same manner. Natives who own more capital and initial debt will receive more income from the immigration surplus than natives who own less capital and debt, which can be adversely affected. Moreover, we observe that the elasticity of labour supply has a significant impact on results.

Appendix

1.9. Calibration and Steady State Calculations

$$Y = F\left(k_{ss}, l_{ss}\right) = k_{ss}^{\alpha} l_{ss}^{1-\alpha} = K^{\alpha} L^{1-\alpha}$$

$$\frac{K}{Y} = \frac{K}{K^{\alpha} L^{1-\alpha}} = \frac{K^{1-\alpha}}{L^{1-\alpha}} = \left(\frac{K}{L}\right)^{1-\alpha}$$

$$\frac{K}{L} = \left(\frac{K}{Y}\right)^{\frac{1}{1-\alpha}}$$

$$K = \left(\frac{K}{Y}\right)^{\frac{1}{1-\alpha}} L$$

$$\frac{L}{Y} = \frac{L}{K^{\alpha} L^{1-\alpha}} = \frac{1}{K^{\alpha} L^{-\alpha}} = \left(\frac{L}{K}\right)^{\alpha} = \left(\frac{L}{\left(\frac{K}{Y}\right)^{\frac{1}{1-\alpha}} L}\right)^{\alpha} = \left(\frac{K}{Y}\right)^{-\frac{\alpha}{1-\alpha}}$$

$$F_K\left(k_{ss}, l_{ss}\right) = r_{ss} + \delta$$

$$\alpha k_{ss}^{\alpha-1} l_{ss}^{1-\alpha} = r_{ss} + \delta$$

$$r_{ss} = \alpha \left(\frac{K}{L}\right)^{\alpha-1} - \delta$$

$$r_{ss} = \alpha k_{ss}^{-1}\left(k_{ss}^{\alpha} l_{ss}^{1-\alpha}\right) - \delta$$

$$r_{ss} = \alpha \left(\frac{Y}{K}\right) - \delta = \frac{\alpha}{\left(\frac{K}{Y}\right)} - \delta$$

$$w_{ss} = (1-\alpha)\, k_{ss}^{\alpha} l_{ss}^{-\alpha} = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}$$

$$\frac{w_{ss} L}{Y} = \frac{(1-\alpha)\, K^{\alpha} L^{1-\alpha}}{K^{\alpha} L^{1-\alpha}} = (1-\alpha)$$

1.9.1 From Euler's equation

$$c_{ss} - d(l_{ss}) = \left(1 + \left(1 - \tau_{ss}^{r}\right) r_{ss}\right)\left[\beta \frac{\left(1+\tau_{ss}^{c}\right)}{\left(1+\tau_{ss}^{c}\right)}\left(c_{ss} - d(l_{ss})\right) - \frac{\left(1 - \beta\left(1+n\right)\right)}{\left(1+\tau_{ss}^{c}\right)} m_{ss}\left(b_{ss}\phi_{ss} + k_{ss}\kappa_{ss}\right)\right]$$

We assume steady-state immigration is equal to zero, $m_{ss} = 0$

$$1 = \left(1 + \left(1 - \tau^r_{ss}\right) r_{ss}\right) \beta$$

$$\left(1 - \tau^r_{ss}\right) r_{ss} = \frac{1}{\beta} - 1$$

$$r_{ss} = \frac{\frac{1}{\beta} - 1}{\left(1 - \tau^r_{ss}\right)}$$

$$\beta = \frac{1}{\left(1 + \left(1 - \tau^r_{ss}\right) r_{ss}\right)}$$

$$\beta = \frac{1}{\left(1 + \left(1 - \tau^r_{ss}\right)\left(\frac{\alpha}{\left(\frac{K}{Y}\right)} - \delta\right)\right)}$$

$$\beta = \frac{1}{\left(1 + \left(1 - \tau^r_{ss}\right)\left(\frac{\alpha - 1 + \frac{C}{Y} + \frac{G}{Y}}{\left(\frac{K}{Y}\right)} + n\right)\right)}$$

1.9.2 From Law of Motion for Capital

$$\left(1 + n\right)\left(1 + m_{ss}\right) k_{ss} = F(k_{ss}, l_{ss}) - g_{ss} - c_{ss} + (1 - \delta) k_{ss}$$

$$\left(1 + n\right) k_{ss} = F(k_{ss}, l_{ss}) - g_{ss} - c_{ss} + (1 - \delta) k_{ss}$$

$$\left(n + \delta\right) k_{ss} = F(k_{ss}, l_{ss}) - g_{ss} - c_{ss}$$

$$F(k_{ss}, l_{ss}) = c_{ss} + g_{ss} - \left(n + \delta\right) k_{ss}$$

$$Y = C + G + \left(n + \delta\right) K$$

$$1 = \frac{C}{Y} + \frac{G}{Y} + n\frac{K}{Y} + \delta\frac{K}{Y}$$

$$\delta = \frac{1 - \frac{C}{Y} - \frac{G}{Y} - n\frac{K}{Y}}{\frac{K}{Y}}$$

$$\delta = \frac{1 - \frac{C}{Y} - \frac{G}{Y}}{\frac{K}{Y}} - n$$

1.9.3 Consumption-Leisure choice

$$\eta\, l_t^{\epsilon-1} = \frac{(1-\tau_t^w)}{(1+\tau_t^c)}\, w_t$$

$$l_t^{\epsilon-1} = \frac{(1-\tau_t^w)}{(1+\tau_t^c)}\, \frac{(1-\alpha)\, k_{ss}^{\alpha} l_{ss}^{-\alpha}}{\eta}$$

$$L^{\epsilon-1+\alpha} = \frac{(1-\tau_t^w)}{(1+\tau_t^c)}\, \frac{(1-\alpha)\, k_{ss}^{\alpha}}{\eta}$$

$$L^{\epsilon-1+\alpha} = \frac{(1-\tau_t^w)}{(1+\tau_t^c)}\, \frac{(1-\alpha)}{\eta} \left[\left(\frac{K}{Y}\right)^{\frac{1}{1-\alpha}} L\right]^{\alpha}$$

$$L^{\epsilon-1} = \frac{(1-\tau_t^w)}{(1+\tau_t^c)}\, \frac{(1-\alpha)}{\eta} \left[\left(\frac{K}{Y}\right)\right]^{\frac{\alpha}{1-\alpha}}$$

$$L = \left(\frac{(1-\tau_t^w)}{(1+\tau_t^c)} \frac{(1-\alpha)}{\eta} \left[\left(\frac{K}{Y}\right)\right]^{\frac{\alpha}{1-\alpha}}\right)^{\frac{1}{\epsilon-1}}$$

1.9.4 From The Law of Motion for Public Debt:

$$(1+n)\,(1+m_{ss})\, b_{ss} = g_{ss} + q_{ss} - \tau_{ss}^w w_{ss} l_{ss} - \tau_{ss}^r r_{ss}\,(k_{ss}+b_{ss}) - \tau_{ss}^c c_{ss} + r_{ss} b_{ss} + b_{ss}$$

$$b_{ss}\,(n - r_{ss}\,(1-\tau_{ss}^r)) = g_{ss} + q_{ss} - (\tau_{ss}^w w_{ss} l_{ss} + \tau_{ss}^r r_{ss} k_{ss} + \tau_{ss}^c c_{ss})$$

$$\frac{B}{Y}\,(n - r_{ss}\,(1-\tau_{ss}^r)) = \frac{G}{Y} + \frac{Q}{Y} - \left(\tau_{ss}^w w_{ss} \frac{L}{Y} + \tau_{ss}^r r_{ss} \frac{K}{Y} + \tau_{ss}^c \frac{C}{Y}\right)$$

$$\frac{Q}{Y} = \frac{B}{Y}\,(n - r_{ss}\,(1-\tau_{ss}^r)) - \frac{G}{Y} + \left(\tau_{ss}^w w_{ss} \frac{L}{Y} + \tau_{ss}^r r_{ss} \frac{K}{Y} + \tau_{ss}^c \frac{C}{Y}\right)$$

Replace, $w_{ss}\frac{L}{Y} = (1-\alpha)$

$$\frac{Q}{Y} = \frac{B}{Y}\,(n - r_{ss}\,(1-\tau_{ss}^r)) - \frac{G}{Y} + \left(\tau_{ss}^w\,(1-\alpha) + \tau_{ss}^r r_{ss} \frac{K}{Y} + \tau_{ss}^c \frac{C}{Y}\right)$$

From $r_{ss} = \alpha\left(\frac{Y}{K}\right) - \delta$ and $\delta\frac{K}{Y} = 1 - \frac{C}{Y} - \frac{G}{Y} - n\frac{K}{Y}$

$$r_{ss}\frac{K}{Y} = \left(\alpha\left(\frac{Y}{K}\right) - \delta\right)\frac{K}{Y} = \alpha - \delta\frac{K}{Y} \qquad or \qquad r_{ss}\frac{K}{Y} = \alpha - 1 + \frac{C}{Y} + \frac{G}{Y} + n\frac{K}{Y}$$

Replace $(1-\tau_{ss}^r)\, r_{ss} = \frac{1}{\beta} - 1$

$$\frac{Q}{Y} = \frac{B}{Y}\left(n - \frac{1}{\beta} + 1\right) - \frac{G}{Y} + \left(\tau^w_{ss}\left(1-\alpha\right) + \tau^r_{ss}\left(\alpha - \delta\frac{K}{Y}\right) + \tau^c_{ss}\frac{C}{Y}\right)$$

1.9.5 Derivation of Compensating Differential

$$U\left(c_{s,t}, l_{s,t}\right) = \sum_{t=s}^{\infty} \beta^{t-s}\left(1+n\right)^{t-s} \ln\left(c_{s,t} - \frac{\eta}{\epsilon}\, l^{\epsilon}_{s,t}\right)$$

$$\sum_{t=s}^{\infty} \beta^{t-s}\left(1+n\right)^{t-s}\left[\ln\left(c_{0,t} - \frac{\eta}{\epsilon}\, l^{\epsilon}_{0,t}\right)\right] = \sum_{t=s}^{\infty} \beta^{t-s}\left(1+n\right)^{t-s}\left[\ln\left[\left(1+\frac{\Delta_T}{100}\right)\bar{c}_{0,t} - \frac{\eta}{\epsilon}\, \bar{l}^{\epsilon}_{0,t}\right]\right]$$

Evolution of consumption for each individual dynasty s over time is,

$$c_{s,t} - d\left(l_{s,t}\right) = \frac{\left(1+\tau^c_s\right)}{\left(1+\tau^c_t\right)}\left[c_{s,s} - d\left(l_{s,s}\right)\right]\left\{\beta^t \prod_{s}^{t}\left(1 + \left(1-\tau^r_t\right) r_t\right)\right\}$$

When $s = 0$

$$c_{0,t} - \frac{\eta}{\epsilon}\, l^{\epsilon}_{0,t} = \frac{\left(1+\tau^c_0\right)}{\left(1+\tau^c_t\right)}\left[c_{0,0} - d\left(l_{0,0}\right)\right]\left\{\beta^t \prod_{0}^{t}\left(1 + \left(1-\tau^r_t\right) r_t\right)\right\}$$

Replace

$$\begin{aligned} &\sum_{t=0}^{\infty} \beta^t\left(1+n\right)^t\left\{\ln\left[\left(c_{0,0} - \frac{\eta}{\epsilon}\, l^{\epsilon}_{0,0}\right)\beta^t\frac{\left(1+\tau^c_0\right)}{\left(1+\tau^c_t\right)}\prod_{0}^{t}\left(1+\left(1-\tau^r_t\right) r_t\right)\right]\right\} \\ = \;&\sum_{t=0}^{\infty} \beta^t\left(1+n\right)^t\left\{\ln\left[\left(1+\frac{\Delta_T}{100}\right)\bar{c}_{0,t} - \frac{\eta}{\epsilon}\, \bar{l}^{\epsilon}_{0,t}\right]\right\} \end{aligned}$$

Rearrange

$$\begin{aligned} &\ln\left[\left(c_{0,0} - \frac{\eta}{\epsilon}\, l^{\epsilon}_{0,0}\right)\right] + \sum_{t=1}^{\infty} \beta^t\left(1+n\right)^t\left\{\ln\left[\left(c_{0,0} - \frac{\eta}{\epsilon}\, l^{\epsilon}_{0,0}\right)\beta^t\frac{\left(1+\tau^c_0\right)}{\left(1+\tau^c_t\right)}\prod_{1}^{t}\left(1+\left(1-\tau^r_t\right) r_t\right)\right]\right\} \\ = \;&\sum_{t=0}^{\infty} \beta^t\left(1+n\right)^t\left\{\ln\left[\left(1+\frac{\Delta_T}{100}\right)\bar{c}_{0,t} - \frac{\eta}{\epsilon}\, \bar{l}^{\epsilon}_{0,t}\right]\right\} \end{aligned}$$

$$\ln\left[\left(c_{0,0} - \frac{\eta}{\epsilon}\, l^{\epsilon}_{0,0}\right)\right] + \sum_{t=1}^{\infty} \beta^t (1+n)^t \left\{\ln\left[\left(c_{0,0} - \frac{\eta}{\epsilon}\, l^{\epsilon}_{0,0}\right) \beta^t \frac{(1+\tau^c_0)}{(1+\tau^c_t)} \prod_1^t \left(1 + (1-\tau^r_t)\, r_t\right)\right]\right\}$$
$$= \left\{\ln\left[\left(1+\frac{\Delta_T}{100}\right) \bar{c}_{0,t} - \frac{\eta}{\epsilon}\, \bar{l}^{\epsilon}_{0,t}\right]\right\} \sum_{t=0}^{\infty} \beta^t (1+n)^t$$

Replace $\sum_{t=0}^{\infty} \beta^t (1+n)^t = \frac{1}{1-\beta(1+n)}$

$$\ln\left[\left(c_{0,0} - \frac{\eta}{\epsilon}\, l^{\epsilon}_{0,0}\right)\right] + \sum_{t=1}^{\infty} \beta^t (1+n)^t \left\{\ln\left[\left(c_{0,0} - \frac{\eta}{\epsilon}\, l^{\epsilon}_{0,0}\right) \beta^t \frac{(1+\tau^c_0)}{(1+\tau^c_t)} \prod_1^t \left(1 + (1-\tau^r_t)\, r_t\right)\right]\right\}$$
$$= \left\{\ln\left[\left(1+\frac{\Delta_T}{100}\right) \bar{c}_{0,t} - \frac{\eta}{\epsilon}\, \bar{l}^{\epsilon}_{0,t}\right]\right\} \frac{1}{1-\beta(1+n)}$$

$$(1-\beta(1+n)) \ln\left[\left(c_{0,0} - \frac{\eta}{\epsilon}\, l^{\epsilon}_{0,0}\right)\right]$$
$$+ (1-\beta(1+n)) \sum_{t=1}^{\infty} \beta^t (1+n)^t \left\{\ln\left[\left(c_{0,0} - \frac{\eta}{\epsilon}\, l^{\epsilon}_{0,0}\right) \beta^t \frac{(1+\tau^c_0)}{(1+\tau^c_t)} \prod_1^t \left(1 + (1-\tau^r_t)\, r_t\right)\right]\right\}$$
$$= \ln\left[\left(1+\frac{\Delta_T}{100}\right) \bar{c}_{0,t} - \frac{\eta}{\epsilon}\, \bar{l}^{\epsilon}_{0,t}\right]$$

$$e^{\left\{(1-\beta(1+n)) \ln\left[\left(c_{0,0} - \frac{\eta}{\epsilon} l^{\epsilon}_{0,0}\right)\right] + (1-\beta(1+n)) \sum_{t=1}^{\infty} \beta^t (1+n)^t \left\{\ln\left[\left(c_{0,0} - \frac{\eta}{\epsilon} l^{\epsilon}_{0,0}\right) \beta^t \frac{(1+\tau^c_0)}{(1+\tau^c_t)} \prod_1^t (1+(1-\tau^r_t) r_t)\right]\right\}\right\}}$$
$$= \left(1+\frac{\Delta_T}{100}\right) \bar{c}_{0,0} - \frac{\eta}{\epsilon}\, \bar{l}^{\epsilon}_{0,0}$$

$$\left(c_{0,0} - \frac{\eta}{\epsilon}\, l^{\epsilon}_{0,0}\right)^{(1-\beta(1+n))} e^{\left\{(1-\beta(1+n)) \sum_{t=1}^{\infty} \beta^t (1+n)^t \ln\left[\left(c_{0,0} - \frac{\eta}{\epsilon} l^{\epsilon}_{0,0}\right) \beta^t \frac{(1+\tau^c_0)}{(1+\tau^c_t)} \prod_1^t (1+(1-\tau^r_t) r_t)\right]\right\}}$$
$$= \left(1+\frac{\Delta_T}{100}\right) \bar{c}_{0,0} - \frac{\eta}{\epsilon}\, \bar{l}^{\epsilon}_{0,0}$$

Suppose we assume that $\tau^c_t = \tau^c_0$

$$\left(c_{0,0} - \frac{\eta}{\epsilon}\, l^{\epsilon}_{0,0}\right)^{(1-\beta(1+n))} e^{\left\{(1-\beta(1+n)) \sum_{t=1}^{\infty} \beta^t (1+n)^t \ln\left[\left(c_{0,0} - \frac{\eta}{\epsilon} l^{\epsilon}_{0,0}\right) \beta^t \frac{(1+\tau^c_0)}{(1+\tau^c_t)} \prod_1^t (1+(1-\tau^r_t) r_t)\right]\right\}}$$
$$= \left(1+\frac{\Delta_T}{100}\right) \bar{c}_{0,0} - \frac{\eta}{\epsilon}\, \bar{l}^{\epsilon}_{0,0}$$

$$\frac{\Delta_T}{100} = \left\{ \frac{\eta}{\epsilon}\, \bar{l}^{\epsilon}_{0,0} + \left(c_{0,0} - \frac{\eta}{\epsilon}\, l^{\epsilon}_{0,0} \right)^{(1-\beta(1+n))} e^{\left\{ (1-\beta(1+n)) \sum_{t=0}^{\infty} \beta^t (1+n)^t \left[\ln\left(\prod_1^t (1+(1-\tau_t^r) r_t) \right) \right] \right\}} \right\} / \bar{c}_{0,0} - 1$$

$$\Delta_T = 100 \times \left[-1 + \frac{\eta}{\epsilon} \frac{\bar{l}^{\epsilon}_{0,0}}{\bar{c}_{0,0}} + \frac{1}{\bar{c}_{0,0}} \left(c_{0,0} - \frac{\eta}{\epsilon}\, l^{\epsilon}_{0,0} \right)^{(1-\beta(1+n))} e^{\left\{ (1-\beta(1+n)) \sum_{t=0}^{\infty} \beta^t (1+n)^t \left[\ln\left(\prod_1^t (1+(1-\tau_t^r) r_t) \right) \right] \right\}} \right]$$

Chapter 2

Bayesian Estimation and Evaluation of a Model of Optimal Growth with Endogenous Labour Supply and Immigration

2.1. Introduction

Most developed countries have seen the share of foreign-born populations rise over the last few decades. The rise in the share the foreign born population United States, from 4.7% in 1970 to 13.7% in 2018 means that it is now in the middle of the range for OECD countries in terms of the percentage of its population that is foreign-born (NAS, 2017).[1]

The debate in the US about immigration and its impact has persisted for more than two centuries—concerns about the effects of immigration on the economic prospects of the native-born and fiscal balances at all levels of government are not new. Immigrants comprise 13 per cent of the population overall, but 16 per cent of the civilian workforce ages 16-64 (Singer, 2012) and it is evident, in significant part reflecting current demographics, immigrants and their children will account for the vast majority of current and future net workforce growth. These and

[1] Abby Budiman, Pew Research Center, https://www.pewresearch.org/fact-tank/2020/08/20/key-findings-about-u-s-immigrants/.

recent developments further highlight the importance and urgency to understand the resultant economic and fiscal impacts of immigration. One set of headline questions concerns the economy, specifically jobs, output and wages.

– What is the impact of immigration on jobs and wages?

– Does immigration lower the wage? If so, what is the magnitude of its impact?

Other questions arise about taxes and public spending:

– What are the fiscal impacts of immigration?

– In particular, what is the impact on debt and tax policy?

– To what extent is the sustainability of government expenditure affected by immigration and immigration policy?

Given the complexity of mechanisms through which immigration can impact the economy, it is not surprising that the empirical literature has produced a range of wage, employment and fiscal impact estimates. The main goal of this chapter is to explore the economic and fiscal consequences of immigration in the context of a DSGE model. Herbst and Schorfheide (2016) notes that the first small-scale New Keynesian DSGE model (including Smets and Wouters (2003)) were explicitly designed for the study of monetary policy and so assume fiscal policy to be passive and the taxes that finance government spending to be non-distortionary. As a result of the 2007-2009 recession, more attention to tax changes and government spending in DSGE model has become the norm, resulting in a more accurate representation of the fiscal policy. This study is unique in a way that it extends Leeper et al. (2010)—which examines the source and timing of fiscal financing estimated in the post-war US—and providing a new practical approach to evaluate the impact and interactions between immigration and fiscal policy in the United States over the course of the last sixty years.

The overlapping dynasties model with endogenous labour supply, factor taxes, public debt, government spending, transfer payments and immigration flows presented in this chapter allows not only capital, labour and consumption tax rates, and the migration, to react to the state of the economy, in particular, the level of output, consumption, investment, debt and factor prices. These are subject to exogenous shocks reflecting unanticipated changes in fiscal and immigration policy. The impact of these extensions to the model explained in detail in the following sections. There are numerous reasons why immigrants are drawn to the US. However, the search for economic opportunity for themselves and their children has usually been the driving force and that is the assumption in this model.

The main contributions of this chapter are:

- By including immigration within the context of a DSGE model we can study its impact on the economy.
- By endogenising immigration flows we can estimate its responsiveness to changes in the economy.
- Estimating the model and performing a shock decomposition to assess the role of immigration shocks in explaining the variance of the model's endogenous variables. The estimated model is used to evaluate the effect of counterfactual fiscal policies and immigration policies.
- After estimating the model parameters that include debt and output response parameters, exogenous tax co-movement parameters, and endogenous propagation parameters we simulate the model to understand the effect of different shocks.
- We isolate and assess the effect of migration on the factor prices, such as wages and interest rates, together with assessing the effect of immigration on tax policy in the US
- We use the extended data series for the US covering the period from 1960:Q1 to 2018:Q1 on nine series to estimate the model: the demeaned (untrended) deviations of log real per capita output, hours worked, government spending, transfers, capital tax revenues, labour tax revenues, consumption tax revenues and immigration flows. The time series for immigration flows as it impacts the labour force is derived using census data.

The chapter is structured as follows. Section 2 contains the theoretical framework, a DSGE model that incorporates endogenous labour supply, factor taxes, public debt, government spending, transfer payments and through the addition of overlapping dynasties, immigrants that also act as optimising agents in the economy. This section also incorporates our assumptions regarding fiscal policies, and the technique used to solve the model. Fiscal policy rules introduced in this section describe the reaction of the fiscal authority to the level of output and debt in the economy. There is also an immigration rule which responds to change in wages, output and debt. In section 3, we present the construction of our time series, with particular focus on our measure of historical immigration flows as they effect the size of the labour force. Section 4 outlines the techniques we use to estimate the model and our assumptions regarding prior distributions and calibrated values. In Section 5 we presents our findings, including the estimation results and their implications, while section 6 concludes.

2.2. The Model

In this section, we introduce a DSGE that incorporates endogenous labour supply, factor taxes, public debt, government spending, transfer payments, extended to include immigration. Rather than representative agents, in this economy we follow Weil (1989) and Ben-Gad (2004, 2008, 2018) and assume agents are member of overlapping dynasties, so that immigrants are also acting as optimising agents in the economy, choosing how much labour to supply and how much to consume and save as they arrive in the economy. In addition there is a representative firm and a government that funds its operations through taxes on labour, capital income, consumption and the sale of debt.

The model is estimated using nine different time series which necessitates the introduction of nine stochastic shocks, denoted by u_t's. These include a preference shock specific to labour supply, neutral technology shock, shocks to fiscal instruments and an immigration shock.

2.2.1 Households

We assume an economy that is closed in every way, except that it does absorb new immigrant dynasties, who arrive from outside the country at an annual rate of m_t to join to the economy as workers, consumers and savers. Each immigrant is a founding member of the new infinite-lived dynasty, indexed by s. The utility of the members of each dynasty is a positive function of consumption, c_t and a negative function of hours worked, l_t. Following Ascari and Rankin (2007) the disutility from work is specified by:

$$d\left(l_{s,t}\right)=\left(\frac{\eta}{\varepsilon}\right)\ l_{s,t}^{\varepsilon}. \tag{2.1}$$

We also introduce a preference shock specific to labour supply, u_t^l. Specifically, at time t the members of a dynasty maximise an infinite stream of discounted utility starting in period s, which is the moment when it first joins the economy:

$$\sum_{i=s}^{\infty}\beta^{i-s}\left(1+n\right)^{i-s}\ \ln\left(c_{s,t}-u_t^l\ \frac{l_{s,t}^{\varepsilon}}{\varepsilon}\right), \tag{2.2}$$

subject to a time t budget constraint:

$$\begin{aligned}&\underbrace{\left(1+\tau_t^c\right)c_{s,t}}_{consumption}+\underbrace{\left(1+n\right)b_{s,t+1}}_{\substack{purchase\ of\\ risk-free\ bonds}}+\underbrace{\left(1+n\right)k_{s,t+1}-\left(1-\delta\right)k_{s,t}}_{investment\ in\ physical\ capital} \\ =\ &\underbrace{\left(1-\tau_t^w\right)w_t\,l_{s,t}}_{\substack{after\ tax\\ wage\ income}}+\underbrace{\left(1-\tau_t^r\right)r_tk_{s,t}}_{\substack{after\ tax\\ interest\ income}}+\underbrace{\left(1-\tau_t^r\right)r_tb_{s,t}+b_{s,t}}_{bond\ income}+\underbrace{z_{s,t}}_{transfers}\end{aligned} \tag{2.3}$$

where $\beta\in(0,1)$ is the subjective discount factor, n is the population growth rate, $\varepsilon=1+1/FE$, where FE is the Frisch elasticity of labour supply. The stock of financial assets owned by dynasty s at time t is the sum of that dynasty's holdings of government bonds $b_{s,t}$ and physical capital $k_{s,t}$.[2]

The shock u_t^l follows the AR(1) process given by:

$$\ln(u_t^l)=\rho^l\ln(u_{t-1}^l)+\sigma_l\epsilon_t^l,\quad \epsilon_t^l\sim N(0,1). \tag{2.4}$$

The wage rate w_t and rate of return on capital r_t are common across dynasties as is the subjective discount factor β and the rate n at which natural population growth causes each dynasty to increase in size over time. Income received from

[2]Following Ben-Gad (2018) we did not differentiate between the interest rate applied to the bond and capital holdings and the returns on these assets are taxed at the rate of τ_t^r.

government transfer payments is denoted by $z_{s,t}$.

2.2.2 Firms

The representative firm chooses $\{k_t, l_t\}$, labour and physical capital, as factors to produce a single good in the economy and to maximize profits (π) using a standard neoclassical specification and the Cobb-Douglas production function.

$$\pi = \underbrace{u_t^a\; k_t^\alpha\; l_t^{1-\alpha}}_{technology} - \underbrace{w_t l_t}_{labour\ cost} - \underbrace{r_t k_t}_{capital\ cost}, \tag{2.5}$$

where k_t is beinning of period capital stock. $\alpha \in [0.1]$ and u_t^a denotes a neutral technology shock that assumed to follow the $AR(1)$ process.

$$\ln(u_t^a) = \rho^a \ln(u_{t-1}^a) + \sigma_a \epsilon_t^a, \quad \epsilon_t^a \sim N(0,1). \tag{2.6}$$

Total output at period t is given by:

$$y_t = u_t^a\; k_t^\alpha\; l_t^{1-\alpha}. \tag{2.7}$$

Both input factors receive their marginal products:

$$r_t = \alpha\, \frac{y_t}{k_t}, \qquad w_t = (1-\alpha)\, \frac{y_t}{l_t}. \tag{2.8}$$

The model also includes a shock to the efficiency of the investment process, denoted u_t^i, which enters the aggregate capital accumulation equation as follows:

$$(1+n)\,(1+m_{t+1})\,k_{t+1} = (1-\delta)\,k_t + u_t^i\; i_t. \tag{2.9}$$

What this means is that an increase in u_t^i implies a higher amount of capital in the next period for a given amount of investment. The shock u_t^i assumed to follow the AR(1) process.

$$\ln(u_t^i) = \rho^i \ln(u_{t-1}^i) + \sigma_i \epsilon_t^i, \quad \epsilon_t^i \sim N(0,1). \tag{2.10}$$

2.2.3 The Government

The government spends resources on its own consumption and also on distributing transfer payments to all agents in the model. It finances its expenditure and the interest on its accumulated debt by taxing labour earnings, capital income, and consumption and by issuing new debt which it sells to the public. The government's budget constraint is

$$\underbrace{B_{t+1}}_{\text{issued debt}} + \underbrace{T_t^w + T_t^k + T_t^c}_{\text{tax revenues}} = \underbrace{G_t}_{\text{gov. spending}} + \underbrace{Z_t}_{\text{transfers}} + \underbrace{(1-\tau_t^r)\, r_t B_t}_{\text{interest payments}} + \underbrace{B_t}_{\text{previous debt}} \tag{2.11}$$

where $r_t^b = (1-\tau_t^r) r_t$ is the after-tax interest rate for riskless government bonds, G_t is government expenditure and X_t represents the aggregate level of any variable x. As mentioned above, there are three sources of time-varying distortionary taxes in the model, levied on consumption, capital and labour income. The tax revenues derived from each are defined as the respective tax rate multiplied by the tax base:

$$T_t^w = \tau_t^w w_t L_t = \tau_t^w (1-\alpha)\, Y_t, \tag{2.12}$$

$$T_t^k = \tau_t^r r_t K_t = \tau_t^r \alpha\, Y_t, \tag{2.13}$$

$$T_t^c = \tau_t^c C_t, \tag{2.14}$$

When restated in per capita terms, the government budget constraint also includes the impact of the two sources of population change in the economy, natural population growth and immigration:

$$(1+n)(1+m_{t+1})\, b_{t+1} + \tau_t^w (1-\alpha)\, y_t + \tau_t^r \alpha\, y_t + \tau_t^c c_t = g_t + z_t + (1-\tau_t^r)\, r_t b_t + b_t \tag{2.15}$$

2.3. Fiscal Policy Rules

The tax rates in the model are functions of the state of the economy and some exogenous shocks. These functions are called fiscal policy rules, because they in part describe the reaction of the fiscal authority to the level of output and debt in the economy (Herbst and Schorfheide, 2016).

The model applies fiscal policy rules as defined in Leeper et al. (2010) which embed three features:

1. They incorporate some "automatic stabilised" component to movements in fiscal variables. These are modelled as a contemporaneous response to deviations of output from steady state, which are represented as $\varphi_x \, \hat{y}_t$;

2. All instruments except consumption taxes are permitted to respond to the state of government debt, through $\gamma_x \, \hat{b}_t$;

3. Technically exogenous movement in one tax category can affect the other tax rates. Another possibility is that fiscal policymakers often consider changes in tax rates jointly. For this reason Leeper et al. (2010) allows shocks affecting one tax rate to also affect other tax rates contemporaneously and the degree of co-movement is controlled by the parameters $\phi_{xz} \, \hat{u}_t$. These parameters also control how much unpredicted movement in one tax rate is due to an exogenous shock to another tax rate (Leeper et al., 2010).

In terms of log deviations from steady state, the policy rules are:

$$\hat{g}_t = \underbrace{-\varphi_g \hat{y}_t}_{\text{"automatic stabilizer"}} - \underbrace{\gamma_g \hat{b}_t}_{\text{debt response}} + \underbrace{\hat{u}_t^g ,}_{\text{exogenous policy}} \qquad \hat{u}_t^g = \rho_g \hat{u}_{t-1}^g + \sigma_g \epsilon_t^g, \tag{2.16}$$

$$\hat{z}_t = -\varphi_z \hat{y}_t - \gamma_z \hat{b}_t + \hat{u}_t^z, \qquad \hat{u}_t^z = \rho_z \hat{u}_{t-1}^z + \sigma_z \epsilon_t^z, \tag{2.17}$$

$$\hat{\tau}_t^r = \varphi_{\tau r} \hat{y}_t + \gamma_{\tau r} \hat{b}_t + \underbrace{\phi_{rw} \hat{u}_t^{\tau w} + \phi_{rc} \hat{u}_t^{\tau c}}_{\text{"tax comovements"}} + \hat{u}_t^{\tau r}, \qquad \hat{u}_t^{\tau r} = \rho_{\tau r} \hat{u}_{t-1}^{\tau r} + \sigma_{\tau r} \epsilon_t^{\tau r}, \tag{2.18}$$

$$\hat{\tau}_t^w = \varphi_{\tau w} \hat{y}_t + \gamma_{\tau w} \hat{b}_t + \phi_{rw} \hat{u}_t^{\tau r} + \phi_{wc} \hat{u}_t^{\tau c} + \hat{u}_t^{\tau w}, \quad \hat{u}_t^{\tau w} = \rho_{\tau w} \hat{u}_{t-1}^{\tau w} + \sigma_{\tau w} \epsilon_t^{\tau w}, \tag{2.19}$$

$$\hat{\tau}_t^c = \underbrace{\phi_{rc} \hat{u}_t^{\tau r} + \phi_{wc} \hat{u}_t^{\tau w} + \hat{u}_t^{\tau c},}_{\text{"completely exogenous"}} \qquad \hat{u}_t^{\tau c} = \rho_{\tau c} \hat{u}_{t-1}^{\tau c} + \sigma_{\tau c} \epsilon_t^{\tau c}, \tag{2.20}$$

where hats denote log-deviations of variables and each of the ϵ's are distributed *i.i.d.* $N(0, 1)$.

The fiscal rule for government spending and lump-sum transfers is a function of current output and the previous period's debt, controlled by the parameters φ_z and γ_z, respectively. Transfers and spending are also affected by an exogenous

$AR(1)$ process, u_t^x. The level of debt adjusts to ensure that the government budget constraint is satisfied. To capture unexpected changes in fiscal policy each rule x is augmented by an $i.i.d$ error term, ϵ_t^x.

2.4. Immigration Policy Rule

In addition to the fiscal policy rules there is also a policy rule that governs the rate of immigration in terms of its log deviations from steady state:

$$\hat{m}_t = \underbrace{\varphi_m \hat{y}_t}_{\text{"automatic stabilizer"}} + \underbrace{\gamma_{1,m} \hat{b}_t}_{\text{debt response}} + \underbrace{\gamma_{2,m} \hat{w}_t}_{\substack{\text{wage rate} \\ \text{response}}} + \hat{u}_t^m, \qquad \hat{u}_t^m = \rho_m \hat{u}_{t-1}^m + \sigma_m \epsilon_t^m \tag{2.21}$$

Unlike other instruments immigration, in addition to debt, is permitted to respond to the wage rate, through $\gamma_{2,w}\ \hat{w}_t$. Immigration is also allowed to respond to deviations of output from steady state, which is represented above as $\varphi_x\, \hat{y}_t$. As we demonstrate below both responses, to output and wages, are positive as we would expect—higher output and wages in the United States attracts more immigrants.

2.5. Data Construction

2.5.1 Standard Time Series

The model is estimated using US quarterly data from 1960:Q1 to 2018:Q1. All data are in real terms and we restrict all the fiscal variables, including government consumption and investment, transfers and tax revenues to those associated with the US federal government. We abstract from the fiscal policy associated with states and localities for two reasons. First, immigration policy is strictly within the purview of the Federal government and so the interaction between fiscal and immigration policy is best analysed at this level. Second, Leeper et al. (2010) argues that since most states and localities have little scope to rely on deficit finance to fund their current expenditures, public debt is mostly federal.

In all there are nine time series: real output, real investments, real hours worked, real government consumption and investment, net of government consumption of fixed assets, real government transfers, real consumption tax revenues, real labour tax revenues, real capital tax revenues and immigration. We detrended the natural logarithm of each variable using a linear trend. Therefore, each empirical per capita variable enters the model as the log deviation from the long-term trend with zero mean. Hence, specifying the observation equation is redundant as we directly 'observe' $\hat{x}_t$ and $x_t^{obs} = \hat{x}_t$ (Pfeifer, 2018). Details regarding the construction of the standard time series can be found in Appendix 2.10.

2.5.2 Constructing the Historical Rate of Immigration

Official data for immigration in the US is not useful for modeling purposes because it fails to account not only for both the inflows and outflows of those who arrive illegally, but even account for those who arrive legally until the moment when they are granted official status as immigrants. Therefore to construct a useful immigration time series, we follow Kiguchi and Mountford (2013) and Weiske (2017, 2019) and use a decomposition of changes in the US working-age population that accounts for the likely number of surviving natives born sixteen years earlier. Quarterly changes in the US working age population are as follows:

$$\Delta CNP16OV_t = \underbrace{(b_{t-16y,t} \times Births_{t-16y} - Deaths_t)}_{\Delta N_{1,t}} - \Delta Military_t + Revisions_t + \Delta N_{2,t}, \tag{2.22}$$

where $CNP16OV_t$ is the civilian noninstitutional population 16 years and older, $b_{t-16y,t}$ is the survival probability of a newborn to age 16, $Births_{t-16y}$ is the number of live births 16 years ago, $Deaths_t$ is the number of deaths 16 years and older, $\Delta N_{1,t}$ is the natural population change, $\Delta Military_t$ is the change in worldwide US military personnel, $Revisions_t$ are Current Population Survey (CPS) data revisions unrelated to migration, and $\Delta N_{2,t}$ *is the residual time series that represents the estimated net flow of migrants to the US civilian population and accounts for the change in the civilian noninstitutional population that is not due to past changes in fertility, current deaths or net flows to the US military.* In the analysis and estimation, we use the $\Delta N_{2,t}$ as a proxy for immigration, both legal and illegal.[3]

[3]See also Appendix 2.10 Data Construction

The civilian noninstitutional population series is collected and calculated on a 'best level' basis, meaning that the data at any point in time reflects the best estimate for the level of this series, given all information that was available up to that moment in time. Any additional information becoming available at a later date is only used on a 'forward basis', to estimate later data points, but not for updating previous time points. This is the reasoning used Edge et al. (2016), and also cited by Weiske (2019) and Pfeifer (2018).

Because of the irregular updating of the time series, one observes artificial dynamics in the measured data that is not present in the underlying object. In other words, unrelated sizable peaks in the time series and provides a misleading picture as noted by Edge et al. (2016). The reason for these spikes is not due to any regular population dynamics but reflect the revisions due to new information that has not been adequately accounted for in the past.

Table 2.10, Appendix 2.10 contains details on the CPS data revisions and an overview of the sources of population data provided in Table 2.9. Some of them directly attributable to immigration, but some not. Most of the revisions also reflect other population dynamics, and without further information, it is not possible to properly extract revisions due to immigration only. For example, the revision made in January 1962 is attributed to the 1960 census, whereas the revision made in January 1972 is attributed to the 1970 census. These revised estimates include total estimates which in turn include net international immigration, updated vital statistics, methodological changes and other population-related information. There are also revisions related to immigrants specifically; some of them attributed to the same period, for example, 76,000 Vietnamese refugees arrived after the fall of Saigon in July 1975, and some reflect updated information on immigrants such as 400,000 undocumented immigrants and emigrants (legal) since 1980 reported in January 1986.

As mentioned above, the CPS data revisions do not update previous time points, thus creating the artificial dynamics in the measured data. Therefore, to have a real picture without unrelated sizable peaks, we edited some time points by attributing the revisions related to immigration to their respective periods. As the CPS makes data revisions periodically and they are usually attributed to the last census, we updated previous time points by attributing revisions related to immigration. Figure 2.1 compares the official statistics by U.S. Department of Homeland Security with our proxy for immigration time series.

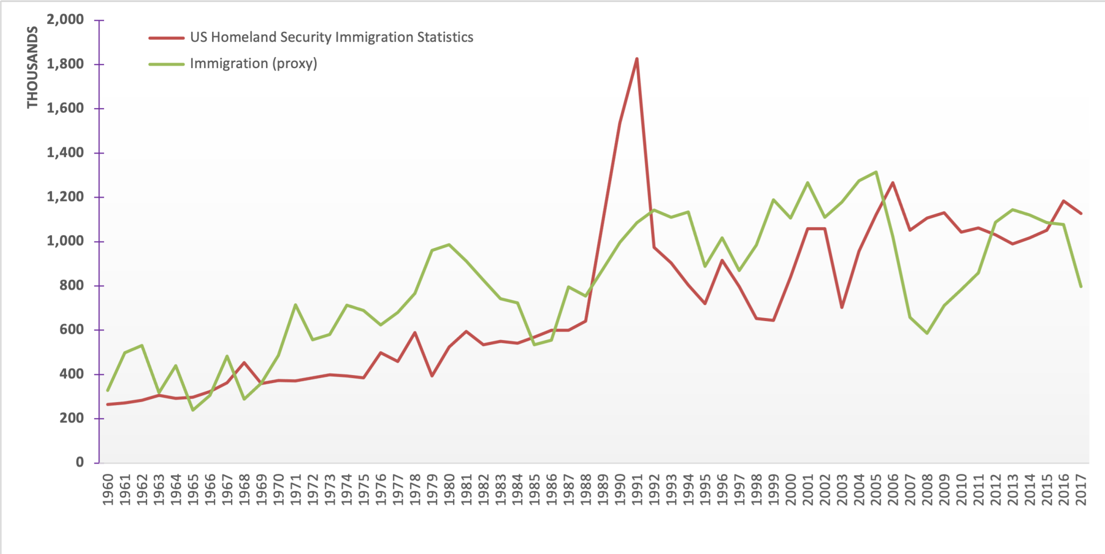


Figure 2.1: *Comparison: U.S. Department of Homeland Security Data vs. Proxy for immigration.*
Source: U.S. Department of Homeland Security. Immigration Data and Statistics. 2019 Yearbook of Immigration Statistics, Persons Obtaining Lawful Permanent Resident Status: Fiscal Years 1820 to 2019 (Table 1). Retrieved from: https://www.dhs.gov/immigration-statistics/yearbook/2019/table1.
Immigration data series constructed using CNP16OV series. See Chapter 2.5.2.

2.6. Estimation

We estimate the model using Bayesian methods, using a Monte-Carlo based optimization routine to maximize the posterior log function combining the priors and the likelihood function.[4] For samples from the posterior distribution, we use the

[4]We used Dynare (version 4.5.6) and set mode compute = 4. In some situations, the posterior mode (that will be used to initialize the Metropolis-Hastings (MH) and to define the jumping distribution) is hard to obtain with standard (Newton-Raphson) optimisation routines. The user, by specifying *mode compute = 6*, can trigger a Monte-Carlo based optimization routine. See https://archives.dynare.org/DynareWiki/MonteCarloOptimization. Retrieved on 22.04.2019.

random walk Metropolis-Hastings algorithm with a total number of 250,000 MH draws to estimate the parameter means and standard errors and the first 125,000 draws used as a burn-in period.[5] After running the MH algorithm, we perform diagnostics to ensure convergence of the MCMC chain.

The figures related to the estimated parameters (Figure 2.11) and shocks (Figure 2.12) and also results mode checks on priors (Figures 2.13 and 2.14) can be found in Appendix 2.12. Resulting figures for prior and posterior distributions are presented in Figures 2.15 and 2.16 in Appendix 2.12.

2.6.1 Choice of Priors and Calibrated Values

To form a posterior distribution using Bayesian inference, the researcher needs to combine the likelihood function with a prior distribution, that describes our state of knowledge about the parameters before observing the sample data. Their principal role is to allow us to integrate that information not contained in the estimation sample into the empirical analysis.

As noted by Del Negro and Schorfheide (2008), it is very convenient to group the elements of parameter vector into different categories;

(i) parameters that affect the steady-state of the model, θ_{ss},

(ii) parameters that characterizes the law of motion of the exogenous shock processes, θ_{exo},

(iii) parameters that control the endogenous propagation mechanisms without affecting the steady-state of the model, θ_{endo}.

Upon having grouped the parameters into one of the three categories, we can consider a priori plausible ranges for these parameters. While we can form priors by pure introspection, in reality, most priors, should be based on some empirical observations and probability distributions, with the domain set to accord with economic theory or a priori beliefs. Usually, prior distributions for parameters

[5] When setting the Frisch elasticity to equal 0.40, the current acceptance ratio is 54.55% and log data density [Laplace approximation] is -5239.78, for the Frisch elasticity to equal 2, the current acceptance ratio per chain is 53.98% and log data density is -5239.83.

on the real line are assumed Normal, while priors for non-negative parameters can be log Normal, Gamma, or Inverse Gamma, and priors for parameters on a bounded interval could be set to truncated Normal or a Beta distribution. Table 2.1 summarizes the prior distribution we use for the estimation of the model.

Prior Shape	**Distribution**	**Range**
Normal PDF	$\mathcal{N}(\mu, \sigma)$	$\mathbb{R}$
Gamma PDF	$\mathcal{G}(\mu, \sigma, p_3)$	$[p_3, +\infty)$
Beta PDF	$\mathcal{B}(\mu, \sigma, p_3, p_4)$	$[p_3, p_4]$
Inv.Gamma PDF	$\mathcal{IG}(\mu, \sigma)$	$\mathbb{R}^+$
Uniform PDF	$\mathcal{U}(p_3, p_4)$	$[p_3, p_4]$

Table 2.1: *Prior Distributions*

As is the usual practice, we based the priors for steady-state, θ_{ss} on pre-sample averages. An overview of priors of the parameters that affect the steady-state of the model, θ_{ss}, is provided in Table 2.2. These parameters can be seen as a very strict priors and were kept fixed from the start of the exercise. Given that the data set is already demeaned, we cannot pin them down in the estimation procedure (see Smets and Wouters (2002)). For the calibrated parameters, we

Parameter	**Value**	**Description**
α	0.35	Capital share
β	0.99	Time preference
δ	0.025	Depreciation rate of capital
n	0.0064	Population growth rate, annually
m	0.0034	Immigration rate, annually
τ^r	0.195	Capital tax rate, mean value from data
τ^w	0.227	Labour tax rate, mean value from data
τ^c	0.027	Consumption tax rate, mean value from data
B/Y	0.340	Ratio of Federal Government Debt to Quarterly Output
G/Y	0.092	Federal Government Consumption to Output ratio

Table 2.2: *Calibrated Parameters for Estimated Model*

used those commonly found in the literature (see Leeper et al. (2010), Smets and Wouters (2002), Herbst and Schorfheide (2016), Sungbae and Schorfheide (2007), Del Negro and Schorfheide (2008)). We set the capital share α to equal to 0.30, which roughly implies a steady-state labour share of 70%. The discount factor, β, is calibrated to be 0.99, implying an annual steady real interest rate of 4%. The quarterly depreciation rate of capital, δ, is set equal to 0.025, implying an annual depreciation on capital equal to 10%. For the ratios of government spending and debt to the output, we used the mean values of the data set. Similarly, we calculated the annual civilian population growth rate and the steady state rate of immigration and present them in Table 2.2, but when estimating the model they are first converted into quarterly terms.

We follow the common practice in the literature and base priors for parameters that control the endogenous propagation mechanisms, θ_{endo} on microeconometric evidence. As exogenous processes are generally unobservable, priors for θ_{exo} are the most difficult to specify. Being conditional on θ_{ss} and θ_{endo}, the exogenous shock parameters determine the volatility and persistence of y_t one suggestion is to elicit priors for θ_{exo} indirectly (Del Negro and Schorfheide, 2008). Finally, the distributions for the innovation standard deviations of the exogenous shock processes are chosen to obtain realistic magnitudes for the volatilities of the variables. The low degree of freedom of the inverse Gamma distributions creates somewhat dispersed priors for the σ-s. Prior distributions for all model parameters can be seen in Table 2.3 and 2.4. Similar priors have been used elsewhere in the empirical DSGE model literature (see Leeper et al. (2010) and Herbst and Schorfheide (2016)).

	Prior Distribution			Mean of Posterior Distribution		
Parameter	Density	Mean	St. Dev.	Posterior mean	90% HPD interval	
Output Response Parameters						
Gov.spend. output coeff. φ_g	$\mathcal{G}$	0.07	0.05	0.0654	0.0034	0.1290
Immigration output coeff. φ_m	$\mathcal{G}$	0.07	0.05	0.0702	0.0037	0.1411
Labour tax output coeff. $\varphi_{\tau w}$	$\mathcal{G}$	0.50	0.25	0.9654	0.4143	1.4886
Capital tax output coeff. $\varphi_{\tau r}$	$\mathcal{G}$	1.00	0.30	2.7116	1.9926	3.4140
Transfers output coeff. φ_z	$\mathcal{G}$	0.20	0.10	0.1258	0.0276	0.2116
Debt Response Parameters						
Gov. spend debt coeff. γ_g	$\mathcal{G}$	0.4	0.2	0.1639	0.0520	0.2726
Immigration debt coeff. $\gamma_{1,m}$	$\mathcal{G}$	0.4	0.2	0.2498	0.0668	0.4306
Immigration wage coeff. $\gamma_{2,m}$	$\mathcal{G}$	0.4	0.2	0.3837	0.0978	0.6540
Labour tax debt coeff. $\gamma_{\tau w}$	$\mathcal{G}$	0.4	0.2	0.2085	0.0536	0.3592
Capital tax debt coeff. $\gamma_{\tau r}$	$\mathcal{G}$	0.4	0.2	0.7415	0.4075	1.0357
Transfers debt coeff. γ_z	$\mathcal{G}$	0.4	0.2	0.1455	0.0391	0.2469
Exogenous Tax Co-movement Parameters						
Capital/Cons. co-term ϕ_{rc}	$\mathcal{N}$	0.05	0.1	0.0093	-0.0019	0.0207
Capital/Labour co-term ϕ_{rw}	$\mathcal{N}$	0.25	0.1	0.3511	0.3047	0.3952
Labour/Cons. co-term ϕ_{wc}	$\mathcal{N}$	0.05	0.1	0.0030	-0.0083	0.0147
Exogenous Process Parameters						
Technology AR coeff. ρ_a	$\mathcal{B}$	0.7	0.2	0.9316	0.9081	0.9561
Labour AR coeff. ρ_l	$\mathcal{B}$	0.7	0.2	0.9563	0.9334	0.9803
Investment AR coeff. ρ_i	$\mathcal{B}$	0.7	0.2	0.9620	0.9352	0.9894
Gov.spend AR coeff. ρ_g	$\mathcal{B}$	0.7	0.2	0.9656	0.9443	0.9894
Labour tax AR coeff. $\rho_{\tau w}$	$\mathcal{B}$	0.7	0.2	0.8896	0.8371	0.9425
Capital tax AR coeff. $\rho_{\tau r}$	$\mathcal{B}$	0.7	0.2	0.9299	0.8934	0.9648
Consumption tax AR coeff. $\rho_{\tau c}$	$\mathcal{B}$	0.7	0.2	0.9835	0.9698	0.9978
Immigration AR coeff. ρ_m	$\mathcal{B}$	0.7	0.2	0.9835	0.9698	0.9978
Transfers AR coeff. ρ_z	$\mathcal{B}$	0.7	0.2	0.9227	0.8836	0.9633
Technology std. σ_a	$\mathcal{IG}$	1	4	2.8667	2.6436	3.0816
Labour std. σ_l	$\mathcal{IG}$	1	4	2.6169	2.4200	2.8187
Investment std. σ_i	$\mathcal{IG}$	1	4	4.0279	3.6817	4.3623
Gov.spend std. σ_g	$\mathcal{IG}$	1	4	3.0835	2.8459	3.3335
Transfers std. σ_z	$\mathcal{IG}$	1	4	4.0264	3.7278	4.3322
Immigration std. σ_m	$\mathcal{IG}$	1	4	16.1716	14.9060	17.3782
Labour tax std. $\sigma_{\tau w}$	$\mathcal{IG}$	1	4	6.3489	5.8199	6.8360
Capital tax std. $\sigma_{\tau r}$	$\mathcal{IG}$	1	4	6.7976	6.2482	7.3441
Consumption tax std. $\sigma_{\tau c}$	$\mathcal{IG}$	1	4	0.6812	0.6286	0.7333

Table 2.3: *Prior and Posterior Distributions for the Model Estimation, when Frisch elasticity equals 0.40*

	Prior Distribution			Mean of Posterior Distribution		
Parameter	Density	Mean	St. Dev.	Posterior mean	90% HPD interval	
Output Response Parameters						
Gov.spend. output coeff. φ_g	$\mathcal{G}$	0.07	0.05	0.0624	0.0031	0.1225
Immigration output coeff. φ_m	$\mathcal{G}$	0.07	0.05	0.0694	0.0041	0.1343
Labour tax output coeff. $\varphi_{\tau w}$	$\mathcal{G}$	0.50	0.25	3.2362	2.6834	3.8969
Capital tax output coeff. $\varphi_{\tau r}$	$\mathcal{G}$	1.00	0.30	3.4355	2.8408	4.1542
Transfers output coeff. φ_z	$\mathcal{G}$	0.20	0.10	0.1309	0.0321	0.2315
Debt Response Parameters						
Gov. spend debt coeff. γ_g	$\mathcal{G}$	0.4	0.2	0.2755	0.0825	0.4558
Immigration debt coeff. $\gamma_{1,m}$	$\mathcal{G}$	0.4	0.2	0.3061	0.0696	0.5155
Immigration wage coeff. $\gamma_{2,m}$	$\mathcal{G}$	0.4	0.2	0.3938	0.1021	0.6852
Labour tax debt coeff. $\gamma_{\tau w}$	$\mathcal{G}$	0.4	0.2	0.2718	0.0670	0.4733
Capital tax debt coeff. $\gamma_{\tau r}$	$\mathcal{G}$	0.4	0.2	0.7426	0.3221	1.1618
Transfers debt coeff. γ_z	$\mathcal{G}$	0.4	0.2	0.2086	0.0490	0.3496
Exogenous Tax Co-movement Parameters						
Capital/Cons. co-term ϕ_{rc}	$\mathcal{N}$	0.05	0.1	0.0098	-0.0016	0.0212
Capital/Labour co-term ϕ_{rw}	$\mathcal{N}$	0.25	0.1	0.3418	0.2905	0.3906
Labour/Cons. co-term ϕ_{wc}	$\mathcal{N}$	0.05	0.1	0.0118	0.0016	0.0220
Exogenous Process Parameters						
Technology AR coeff. ρ_a	$\mathcal{B}$	0.7	0.2	0.8840	0.8514	0.9147
Labour AR coeff. ρ_l	$\mathcal{B}$	0.7	0.2	0.9161	0.8830	0.9484
Investment AR coeff. ρ_i	$\mathcal{B}$	0.7	0.2	0.9318	0.8985	0.9670
Gov.spend AR coeff. ρ_g	$\mathcal{B}$	0.7	0.2	0.9648	0.9416	0.9881
Labour tax AR coeff. $\rho_{\tau w}$	$\mathcal{B}$	0.7	0.2	0.8953	0.8487	0.9428
Capital tax AR coeff. $\rho_{\tau r}$	$\mathcal{B}$	0.7	0.2	0.9275	0.8907	0.9630
Consumption tax AR coeff. $\rho_{\tau c}$	$\mathcal{B}$	0.7	0.2	0.9833	0.9699	0.9982
Immigration AR coeff. ρ_m	$\mathcal{B}$	0.7	0.2	0.7366	0.6607	0.8074
Transfers AR coeff. ρ_z	$\mathcal{B}$	0.7	0.2	0.9244	0.8850	0.9624
Technology std. σ_a	$\mathcal{IG}$	1	4	3.0871	2.8421	3.3223
Labour std. σ_l	$\mathcal{IG}$	1	4	1.7379	1.6039	1.8745
Investment std. σ_i	$\mathcal{IG}$	1	4	5.2919	4.7791	5.7864
Gov.spend std. σ_g	$\mathcal{IG}$	1	4	3.0724	2.8476	3.3013
Transfers std. σ_z	$\mathcal{IG}$	1	4	4.0222	3.7118	4.3261
Immigration std. σ_m	$\mathcal{IG}$	1	4	16.1985	14.9808	17.3856
Labour tax std. $\sigma_{\tau w}$	$\mathcal{IG}$	1	4	6.8748	6.3023	7.4632
Capital tax std. $\sigma_{\tau r}$	$\mathcal{IG}$	1	4	6.7405	6.2071	7.2555
Consumption tax std. $\sigma_{\tau c}$	$\mathcal{IG}$	1	4	0.6778	0.6230	0.7305

Table 2.4: *Prior and Posterior Distributions for the Model Estimation, when Frisch elasticity equals 2.00*

2.7. Estimation Results

2.7.1 Policy parameters estimates

Table 2.3 and Table 2.4 reports the means and HDP of the posterior distribution for the parameters of the model estimated when Frisch elasticity is 0.40 and 2.00 respectively. Except for the terms associated with the co-movement between consumption, labour and capital taxes, all parameter estimates are statistically significant. The non-policy parameter estimates are similar to previous estimates found in the literature. All the persistent shocks are estimated to have an autoregressive parameter which is higher than the mean of 0.7 assumed in the prior distribution and by Leeper et al.(2010).

$$\hat{\tau}_t^r = 2.71\ \hat{y}_t + 0.74\ \hat{b}_t + 0.35\ \hat{u}_t^{\tau w} + 0.01\ \hat{u}_t^{\tau c} + \hat{u}_t^{\tau r}$$

$$\hat{\tau}_t^w = 0.97\ \hat{y}_t + 0.20\ \hat{b}_t + 0.35\ \hat{u}_t^{\tau r} + 0.003\ \hat{u}_t^{\tau c} + \hat{u}_t^{\tau w}$$

$$\hat{\tau}_t^c = 0.01\ \hat{u}_t^{\tau r} + 0.003\ \hat{u}_t^{\tau w} + \hat{u}_t^{\tau c}$$

$$\hat{m}_t = 0.07\ \hat{y}_t + 0.25\ \hat{b}_t + 0.38\ \hat{w}_t + \hat{u}_t^m$$

$$\hat{z}_t = -0.13\ \hat{y}_t - 0.15\ \hat{b}_t + \hat{u}_t^z$$

$$\hat{g}_t = -0.07\ \hat{y}_t - 0.16\ \hat{b}_t + \hat{u}_t^g$$

Table 2.5: *Policy Parameter Estimates when Frisch Elasticity equals 0.40*

Results of the exercise for the estimated rules are presented in Table 2.5 and Table 2.6. Examining the estimates of the policy parameter, we can see that capital and labour tax rates both respond to debt innovations, the former at 0.74 more than the latter at 0.20. By contrast, Leeper et al. (2010), calculates the analogous labour tax debt coefficient to be only 0.049, and the capital tax debt coefficient is 0.39. Immigration too, which is absent from Leeper et al. (2010), responds also in a significant manner to innovations in the debt burden.

$\hat{\tau}^r_t = 3.44\ \hat{y}_t + 0.74\ \hat{b}_t + 0.34\ \hat{u}^{\tau w}_t + 0.01\ \hat{u}^{\tau c}_t + \hat{u}^{\tau r}_t$
$\hat{\tau}^w_t = 3.24\ \hat{y}_t + 0.27\ \hat{b}_t + 0.34\ \hat{u}^{\tau r}_t + 0.012\ \hat{u}^{\tau c}_t + \hat{u}^{\tau w}_t$
$\hat{\tau}^c_t = 0.01\ \hat{u}^{\tau r}_t + 0.012\ \hat{u}^{\tau w}_t + \hat{u}^{\tau c}_t$
$\hat{m}_t = 0.07\ \hat{y}_t + 0.31\ \hat{b}_t + 0.39\ \hat{w}_t + \hat{u}^m_t$
$\hat{z}_t = -0.13\ \hat{y}_t - 0.21\ \hat{b}_t + \hat{u}^z_t$
$\hat{g}_t = -0.06\ \hat{y}_t - 0.28\ \hat{b}_t + \hat{u}^g_t$

Table 2.6: *Policy Parameter Estimates when Frisch Elasticity equals 2.00*

The results also show that capital and labour tax rates generate a highly procyclical response to the level of aggregate output, while immigration is less responsive. However, immigration is strongly responsive to wage rate innovations. This result supports quite a few empirical studies that find the wage rate is among the main drivers of immigration. As mentioned above, immigration also appears to respond to public debt, particularly if we set the labour elasticity equal to 2. One plausible explanation is that in the last four decades in the US, growing government expenditures have not necessarily been followed by an increase in taxes but rather by increases in public debt, implying a possible intertemporal shift in taxation. Low natural population growth and the subsequent aging of the population means that the underlying immigration flows gradually become a significant factor in changing the size and composition of the labour force. Taking into account the declining birth rates in the US, perhaps the native population is relying on immigration to dilute the debt burden for them and their descendants. Furthermore, the debt stock which is mostly domestically owned generates income for the incumbent population and this income effect drives down their labour supply, while still allowing them more leisure and consumption. This could explain why increases in debt might yield more permissive immigration policies.

When the labour supply is elastic, the response of both capital and labour taxes to output is strong. What is interesting the labour and consumption tax co-movement increases with the elasticity of labour supply. The response of immigration to wage rate is strong under both cases. Finally, we observe that exogenous changes to capital and labour tax rates affect the two rates simultaneously as in Leeper et al. (2010), suggesting that typical tax legislation tends to change both tax rates. In line with Leeper et al. (2010), exogenous changes to consumption tax rates have little effect on the capital or labour tax rates.

2.7.2 Variance decomposition

Table 2.7 and Table 2.8 summarise the results for the variance decomposition. First, labour and capital tax shocks do play some role in explaining the variation in output, consumption and debt, but these effects are smaller than the shocks to investment, labour and technology. As most consumption taxes are not collected at the federal level, it is not surprising that the impact of the shocks to the consumption tax is very small.

Second, shocks to the rate of immigration account for little of the variance of the other endogenous variables in the model. This is in line with Weiske (2017) estimates of the macroeconomic effects of immigration in the US using a VAR analysis. Similar to Weiske (2017), neutral technology shocks have little influence on immigration at all. Third, shocks to the rate of immigration account for only 0.01% of the variation of wages, which is even less than the value in Weiske (2017) (4% in 5 years and 1% later 20 years according to his forecast error variance decomposition). Immigration accounts for only 0.01% of the variation in interest rates, and output, less than result in Weiske (2017) and minimal variation in consumption and debt. The plausible explanation is that quarterly changes in the number of immigrants to the United States are usually not sufficient to compete with the other shocks that typically buffet the economy.

Shock decompositions of output and debt can be seen in Figures 2.17 and 2.18 of Appendix 2.13. Figures 2.19 and 2.19 present the shock decompositions of hours worked and the wage rate. In each of the figures the black line depicts the deviation of the smoothed value of the corresponding endogenous variable from its steady state at the specified parameter set. The colored bars correspond to the contribution of the respective smoothed shocks to the deviation of the smoothed endogenous variable from its steady state. 'Initial values' in the graphs refers to the part of the deviations from steady state not explained by the smoothed shocks, but rather by the initial value of the state variables (see Pfeifer (2018)).

Shocks / **Variables**	*Investment* ε_t^i	*Hours* ε_t^l	*Technology* ε_t^a	*Gov.spen.* ε_t^g	*Transfers* ε_t^z	*Immigration* ε_t^m	*Capital tax* $\varepsilon_t^{\tau r}$	*Labour tax* $\varepsilon_t^{\tau w}$	*Cons.tax* $\varepsilon_t^{\tau c}$
Output, $\hat{y}_t$	69.33	1.22	25.13	0.05	0.06	0.01	3.92	0.29	0.00
Consumption, $\hat{c}_t$	76.26	0.92	18.94	0.53	0.04	0.00	3.06	0.24	0.00
Debt, $\hat{b}_t$	67.20	1.13	22.85	1.81	2.81	0.00	0.43	3.68	0.09
Capital, $\hat{k}_t$	95.06	0.01	0.11	0.06	0.07	0.01	4.63	0.06	0.00
Gov.spending, $\hat{g}_t$	1.44	0.03	0.53	97.53	0.14	0.00	0.08	0.26	0.01
Transfers, $\hat{z}_t$	0.20	0.01	0.20	0.08	98.94	0.00	0.24	0.33	0.01
Labour taxes, $\hat{T}_t^w$	61.74	1.10	22.87	0.01	0.01	0.01	0.57	13.54	0.15
Capital taxes, $\hat{T}_t^r$	69.59	1.26	26.48	0.01	0.02	0.01	2.52	0.12	0.00
Consumption taxes, $\hat{T}_t^c$	74.41	0.90	18.48	0.51	0.03	0.00	2.77	0.22	2.67
Investments, $\hat{i}_t$	96.86	0.02	0.35	0.01	0.02	0.01	2.59	0.15	0.00
Interest rate (bond), $\hat{r}_t^b$	98.07	0.02	0.68	0.00	0.01	0.01	1.13	0.09	0.00
Interest rate, $\hat{r}_t$	85.02	0.59	10.43	0.05	0.06	0.00	3.83	0.01	0.01
Hours worked, $\hat{l}_t$	29.75	50.18	10.65	0.04	0.05	0.00	4.33	4.93	0.06
Wage rate, $\hat{w}_t$	70.57	0.49	25.65	0.04	0.05	0.01	3.18	0.01	0.01
Immigration, $\hat{m}_t$	0.17	0.06	0.17	0.04	0.06	99.18	0.18	0.13	0.00

Table 2.7: *Variance Decomposition (in percent) when the Frisch Elasticity equals 0.40.*

Variable / **Shocks**	*Investment* ε_t^i	*Hours* ε_t^l	*Technology* ε_t^a	*Gov.spen.* ε_t^g	*Transfers* ε_t^z	*Immigration* ε_t^m	*Capital tax* $\varepsilon_t^{\tau r}$	*Labour tax* $\varepsilon_t^{\tau w}$	*Cons.tax* $\varepsilon_t^{\tau c}$
Output, $\hat{y}_t$	68.63	1.96	22.72	0.02	0.02	0.00	5.03	1.62	0.01
Consumption, $\hat{c}_t$	76.80	1.42	16.36	0.60	0.01	0.00	3.62	1.19	0.00
Debt, $\hat{b}_t$	71.40	1.95	22.43	1.00	1.60	0.00	0.95	0.65	0.02
Capital, $\hat{k}_t$	94.87	0.04	0.17	0.02	0.02	0.01	4.80	0.07	0.00
Gov.spending, $\hat{g}_t$	3.12	0.09	1.06	95.48	0.12	0.00	0.03	0.10	0.00
Transfers, $\hat{z}_t$	0.48	0.02	0.33	0.04	98.89	0.00	0.07	0.16	0.00
Labour taxes, $\hat{T}_t^w$	70.31	2.03	23.64	0.01	0.01	0.01	1.79	2.18	0.02
Capital taxes, $\hat{T}_t^r$	70.87	2.10	24.61	0.00	0.00	0.01	2.02	0.38	0.01
Consumption taxes, $\hat{T}_t^c$	74.43	1.37	15.85	0.58	0.01	0.00	3.21	0.96	3.58
Investments, $\hat{i}_t$	97.86	0.03	0.26	0.00	0.00	0.01	1.78	0.05	0.00
Interest rate (bond), $\hat{r}_t^b$	96.91	0.12	1.82	0.00	0.00	0.01	1.11	0.03	0.00
Interest rate, $\hat{r}_t$	86.44	0.97	8.80	0.02	0.01	0.00	3.62	0.12	0.01
Hours worked, $\hat{l}_t$	3.54	59.91	1.08	0.02	0.02	0.00	5.97	29.10	0.37
Wage rate, $\hat{w}_t$	71.99	0.80	24.05	0.02	0.01	0.01	3.02	0.10	0.01
Immigration, $\hat{m}_t$	0.18	0.08	0.20	0.02	0.03	99.42	0.06	0.02	0.00

Table 2.8: *Variance Decomposition (in percent) when the Frisch Elasticity equals 2.00.*

2.7.3 Impulse response analysis

Having estimated the parameters in the model in the previous section we can now gauge both the qualitative and quantitative behaviour of economy to each of the exogenous shocks. In Figures 1 to 9, we present the impulse responses of the most salient variables in the model such as output, consumption, debt, investment, hours, government spending, transfers, factor prices, tax rates and revenues following a temporary one standard deviation in each of the nine different exogenous shocks. Throughout we use the mean estimates of the posterior distribution to generate the solid black line which captures the behaviour of the impulse responses themselves, while the grey area that surround them represent the 5th and 90th percentiles based on the posterior distributions.

In general, Figures 1 through 8 demonstrate that the qualitative and quantitative effects of fiscal shocks are mostly in line with Leeper et al. (2010). The increase in investment (Figure 2.3) increases output and consumption above its steady-state level. Initially, government spending crowds out investment and a negative wealth effect causes consumption to decline. However, these effects vary depending on the elasticity of labour supply. The reduction in hours worked is less when the Frisch elasticity equals 2, and therefore the fall in the output below its initial steady-state value is less than when the Frisch elasticity is set to 0.40. In Leeper et al. (2010) if only capital and labour taxes are expected to increase in the future to finance the expansion in public debt, households cut back on investment and hours worked because the return on future capital and labour is expected to decline. This causes output to fall below its initial steady-state level and remains so for over ten years as in Leeper et al. (2010), and Forni et al. (2009) who only allow distortionary taxes to respond to debt innovations in their New Keynesian model.

The impact of a shock to transfer spending can be seen in Figure 2.5. As in Leeper et al. (2010), lump-sum transfers are non-distortionary in our model, so the responses of output, consumption, and investment are driven entirely by agents' expectations of how the resulting increase in debt will be financed. We observe government spending to be below its steady-state level for almost ten years. As wage rate and hours worked fall below the equilibrium values taxes on labour income also fall.

The impact of a 1 per cent increase in the rate of taxation on capital income appears in Figure 2.7. We observe that the behaviour of each of the variables is in line with the standard theory that immediate investment, labour, and output decrease when the return on capital is taxed more heavily. As expected, households' consumption initially rises as they respond to lower after tax returns on their savings by increasing consumption. By contrast in Leeper et al. (2010), consumption decreases on impact following a capital tax increase. Leeper et al. (2010) explains this reduction in consumption as stemming from the correlation between capital and labour taxes; when capital taxes increase, labour taxes increase as well, which causes households to work and consume less. In our model this correlation is stronger still, but transfers also increase and this effect and the disincentive to save dominates, but only for one period.

Figure 2.8 demonstrates the effects of a 1 per cent increase in labour tax rates. Regardless of whether the Frisch elasticity equals 0.4 and 2, the responses of output and consumption are conventional, although slightly differ in magnitude. As a response to higher labour taxes, households reduce their labour, also reducing income and consumption. The decline in the labour supply also induces a fall in the return to capital which in turn causes a decrease in investment. However, the impulse responses show that when distortionary fiscal instruments adjust to the debt, the investment increases on impact. We observe a decrease in capital taxes and debt that further increases the return to investment.

The impact of a shock to consumption taxes are shown in Figure 2.9. It initially tends to lower output, consumption and debt. When all fiscal instruments stabilize debt, as in our model, output, consumption rise within a few of years, returning to the steady-state level when the Frisch elasticity equals to 0.4 within a year.

The effects of a positive shock to the rate of immigration is presented in Figure 2.10. The influx of new immigrants initially reduces per capita output, consumption and tax revenue as it dilutes the capital stock. At the same time there is a slight short-term rise ($\times 10^{-3}$) in government spending, and transfers since immigrants receive transfer payments and different kinds of aid from the government. An initial rise in government expenditure related to immigration is soon followed by a fall of equal magnitude so it returns to its steady state value in two years. Immigration appears to have only a small short-run impact on real wages and output, while there is a positive reaction of interest rates and debt to immigration shocks. Though natives suffer from the fall in wages this is offset by an increase in the return to capital and induces further investment. The fall in per-capita consumption is less when the Frisch elasticity of labour supply is equal to 2, than when it equals 0.4. The Impact of immigration on the debt is positive and in line with the Ben-Gad (2018). At the same time these results indicate that overall immigration has had relatively little impact on the US economy in stark contrast to micro-based models such as Borjas (1995). Why the difference? First, we model immigration as a flow rather than an abrupt change in the stock of immigrants—the change in the size of the US working-age population that can be attributed to immigration is not significant from one quarter to the next. Second, we are not measuring the impact of immigration itself. Immigrants continue to arrive in this economy even when they are no shocks. Instead, here we are isolating the impact of a one time shock to the underlying flow of immigrants that arrive. Third, in this model capital adjusts in response to any surge in immigration, which means that the impact of temporary shocks to immigration are mostly transitory.

In contrast to chapter 1, here we assume immigration is endogenously determined by variables in the model whose signs and magnitudes can only be determined by estimation, since there is no appropriate theory or microeconometric study that would inform a calibration of the parameters that directly govern its behaviour. More broadly, when we consider the behaviour of the model itself, government spending, transfers, investment and hours (worked) together with technology and investment-specific shocks impact immigration in a positive manner. Shocks to consumption, labour and capital income taxes impact the immigration negatively.

Figure 2.2: *Bayesian IRF. Orthogonalized shock to Technology,* ε_t^a

The Frisch Elasticity equals 0.40. *The Frisch Elasticity equals 2.00*

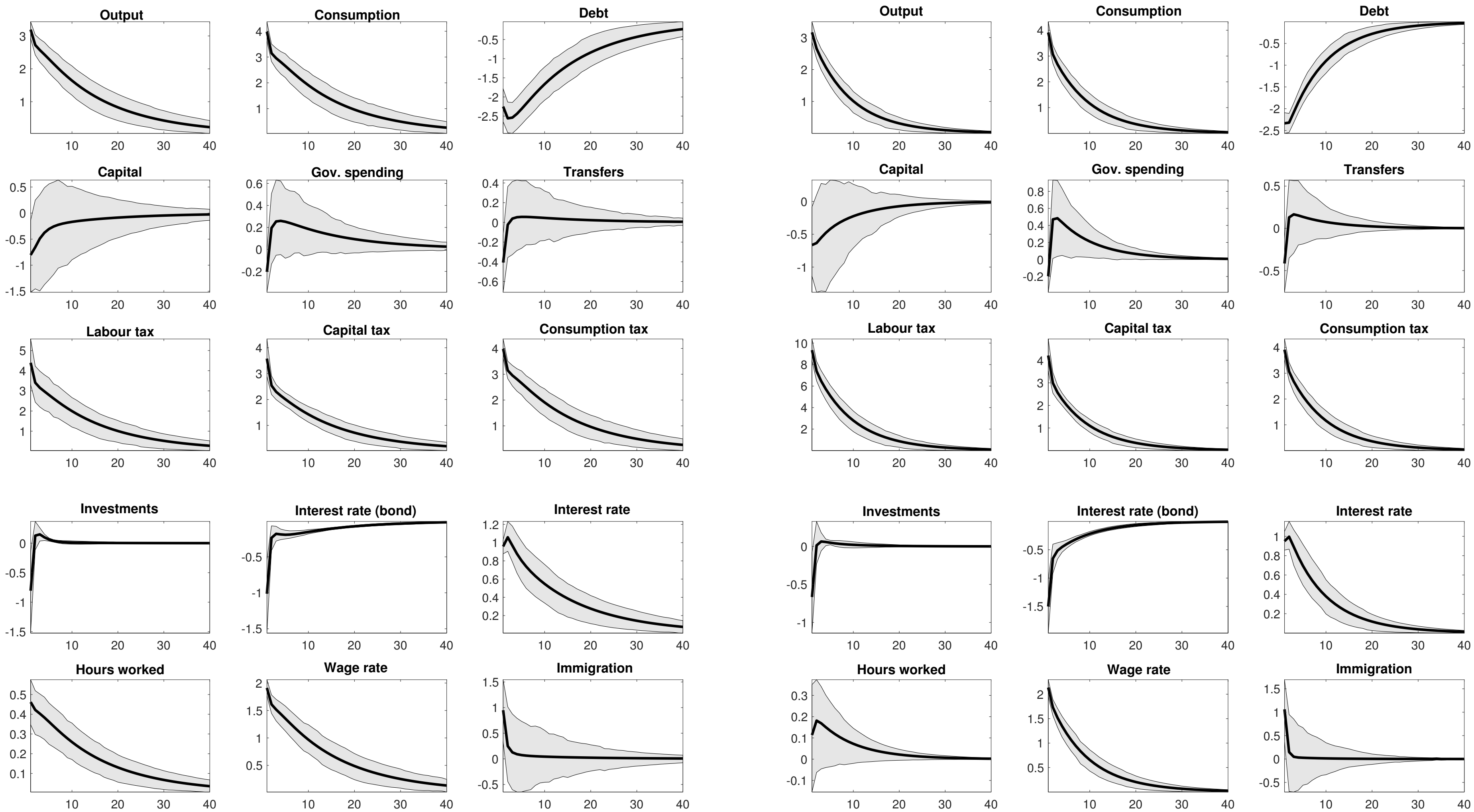
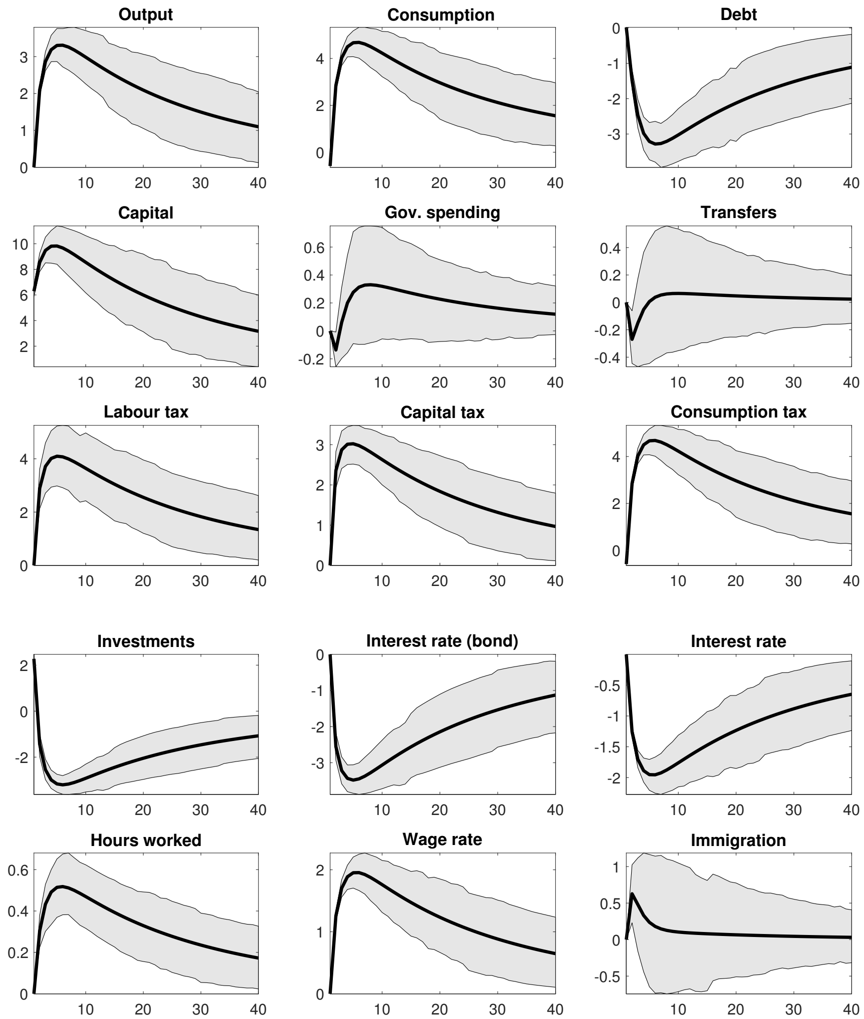


Figure 2.3: *Bayesian IRFs. Orthogonalized shock to Investments,* ε_t^i

The Frisch Elasticity equals 0.40.



The Frisch Elasticity equals 2.00

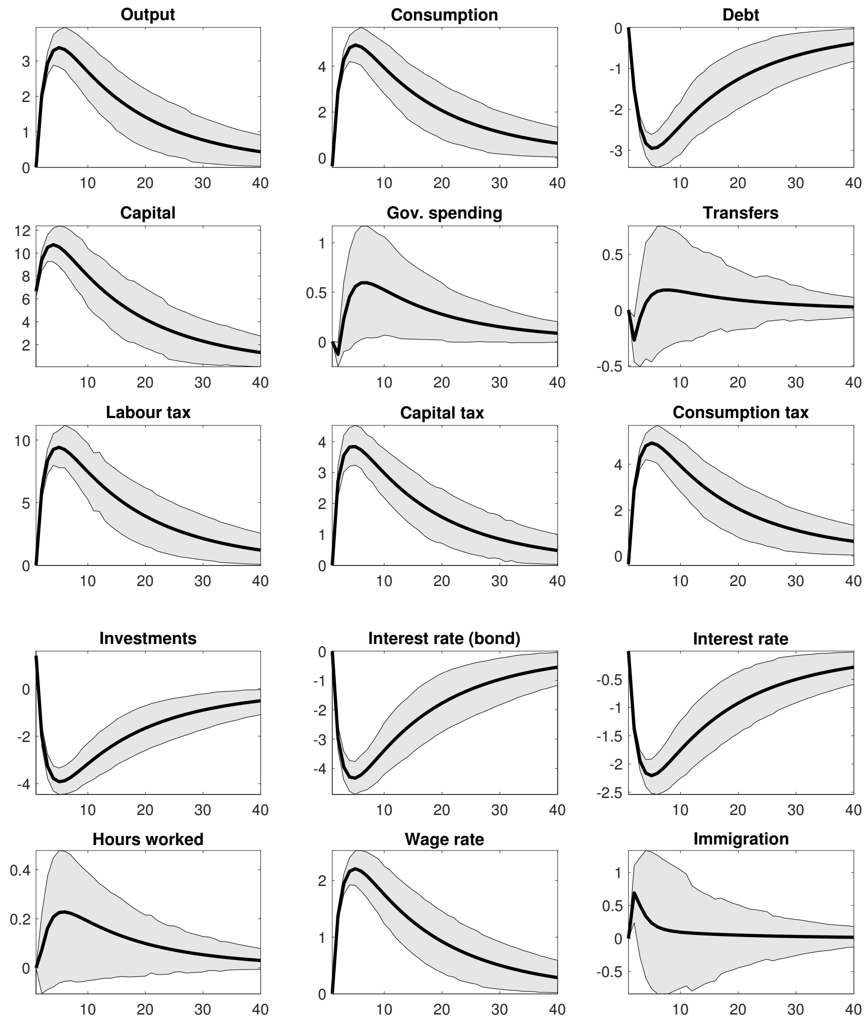


Figure 2.4: *Bayesian IRFs. Orthogonalized shock to Government spending,* ε_t^g

The Frisch Elasticity equals 0.40.

The Frisch Elasticity equals 2.00

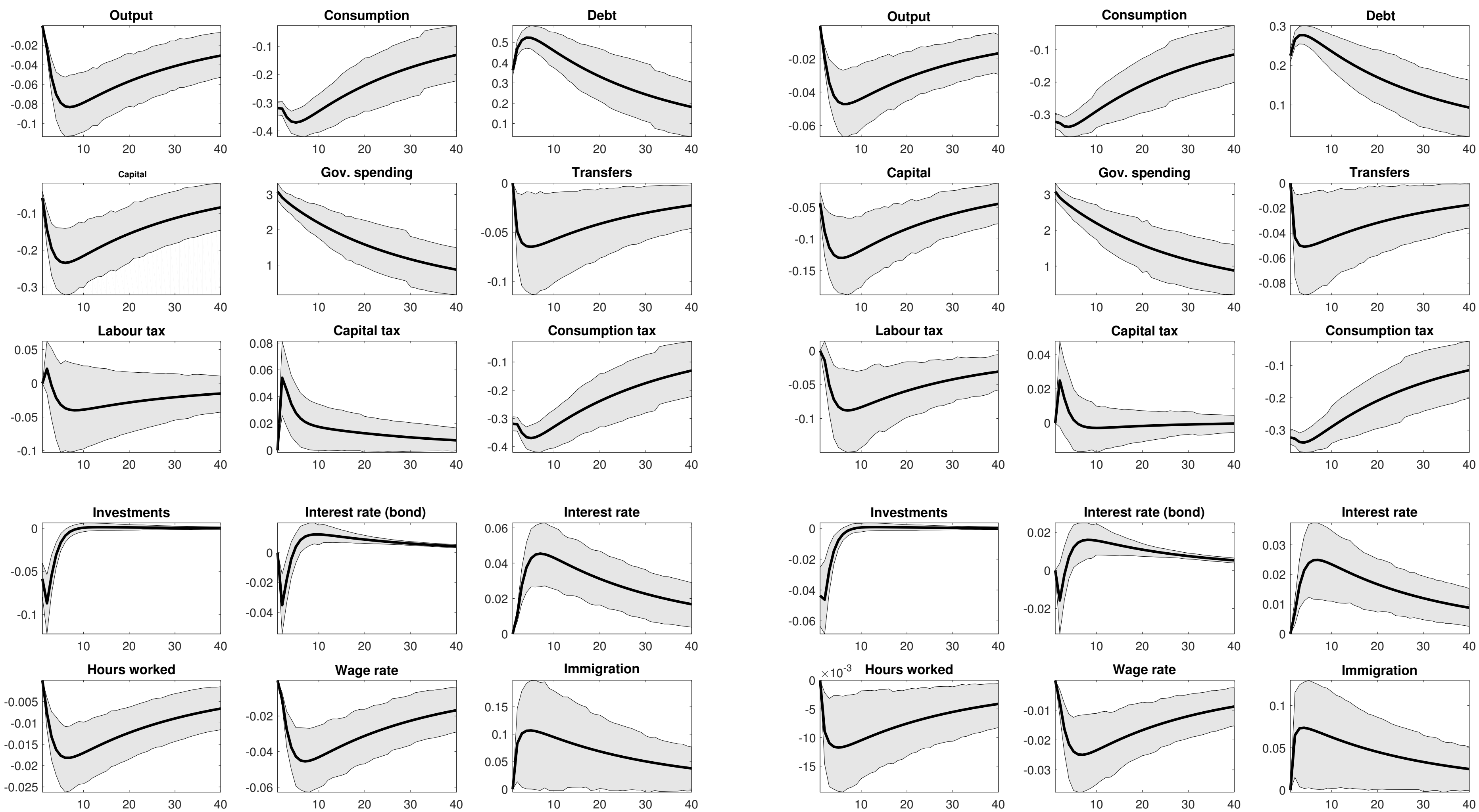


Figure 2.5: *Bayesian IRFs. Orthogonalized shock to Transfers,* ε_t^z

The Frisch Elasticity equals 0.40. *The Frisch Elasticity equals 2.00*

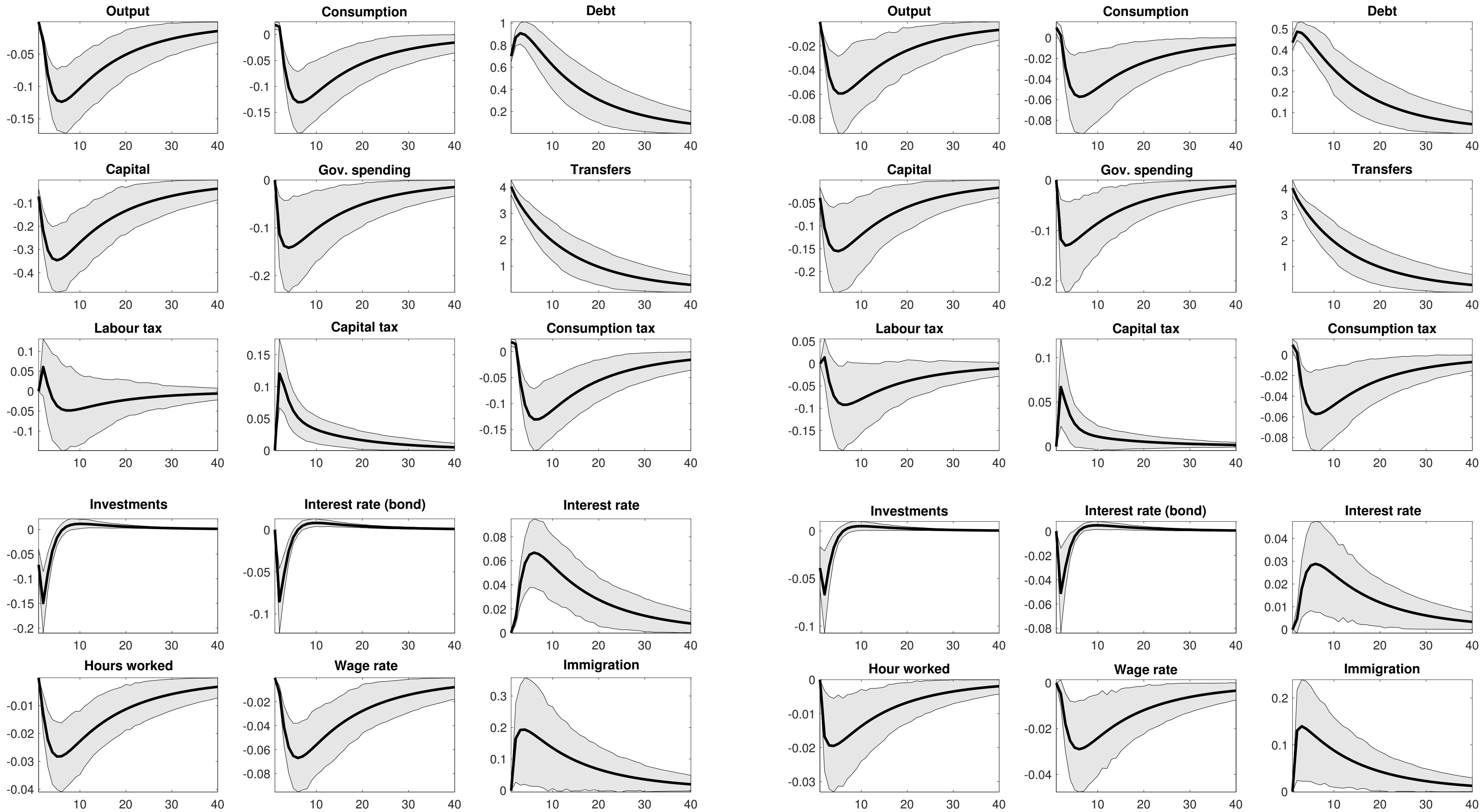


Figure 2.6: *Bayesian IRFs. Orthogonalized shock to Hours worked* ε_t^l

The Frisch elasticity equals 0.40. *The Frisch elasticity equals 2.00*

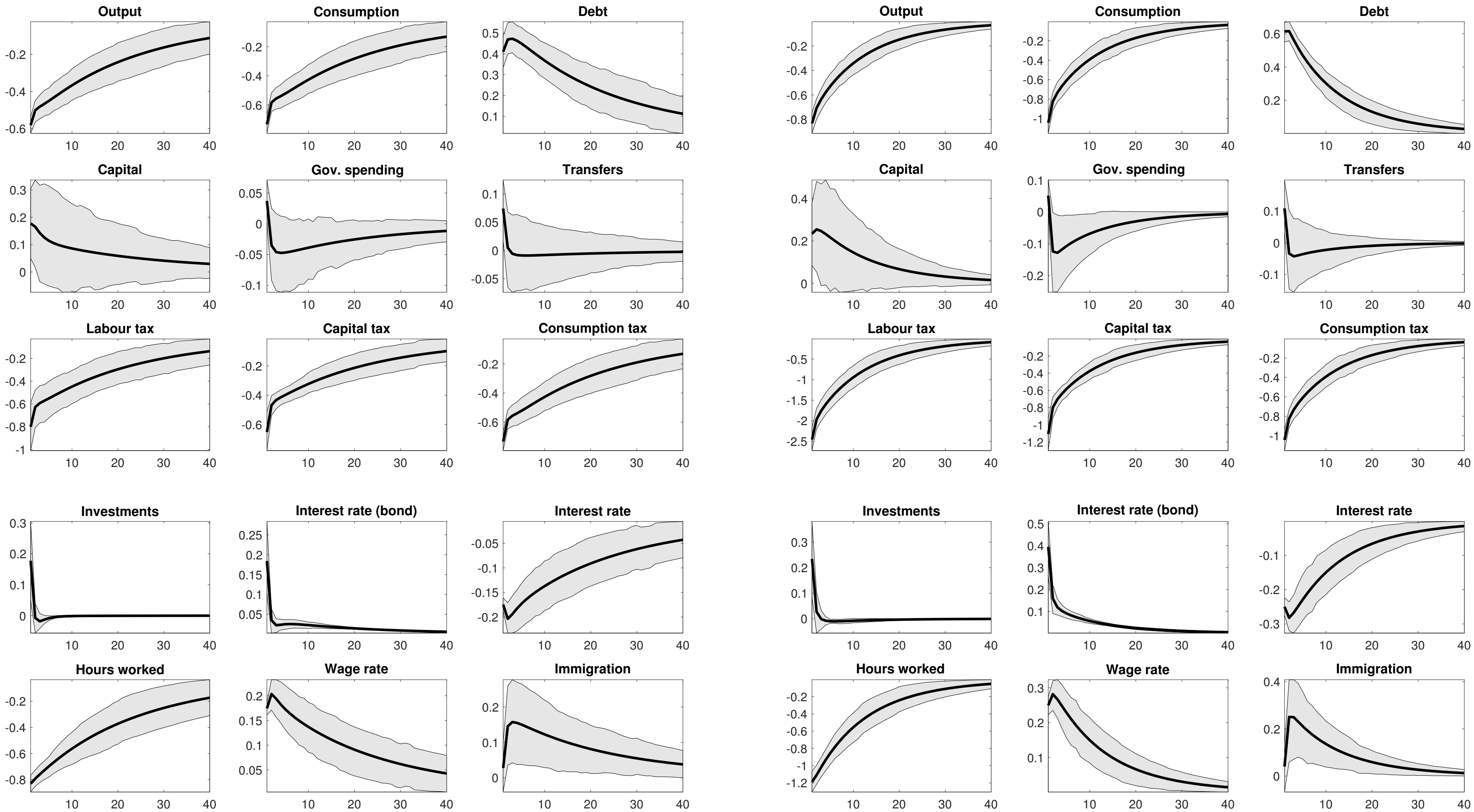


Figure 2.7: *Bayesian IRFs. Orthogonalized shock to Capital tax,* $\varepsilon_t^{\tau r}$

The Frisch elasticity equals 0.40. *The Frisch elasticity equals 2.00*

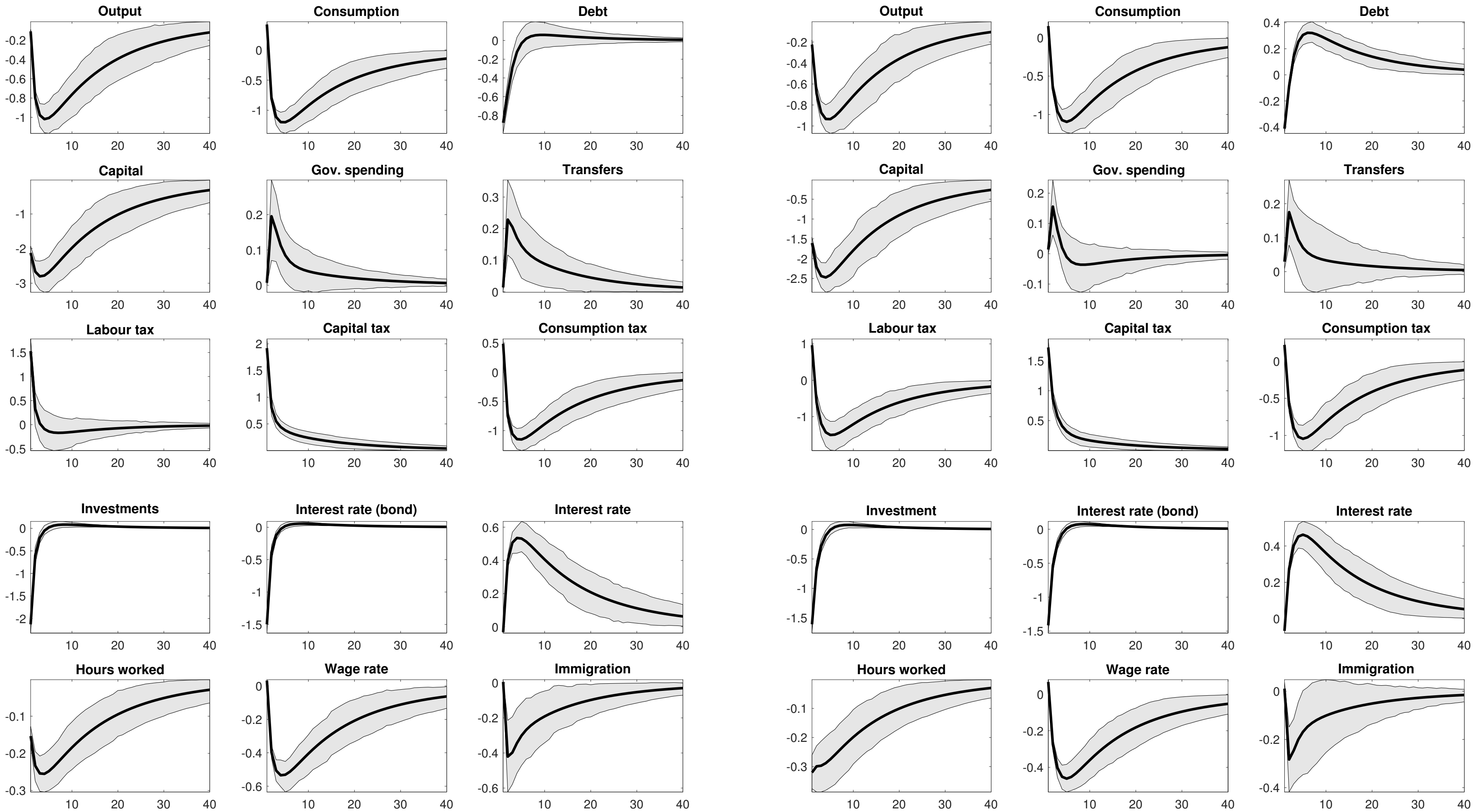
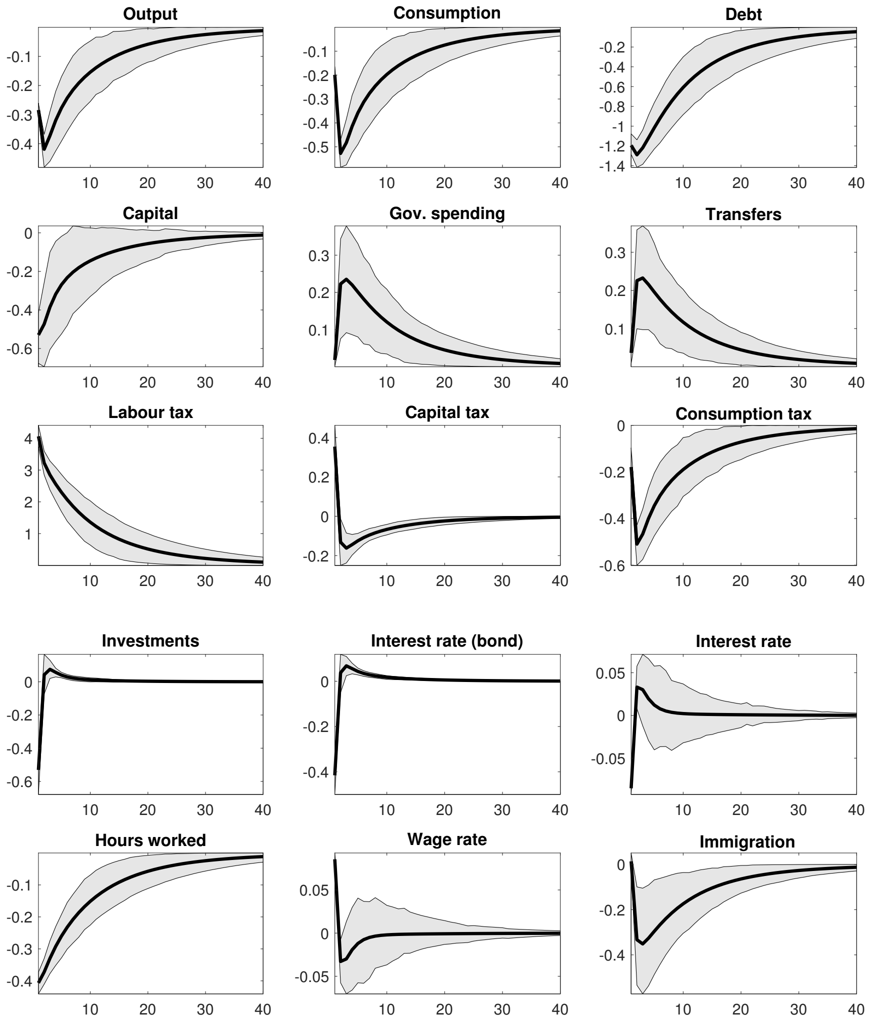


Figure 2.8: *Bayesian IRFs. Orthogonalized shock to Labour tax,* $\varepsilon_t^{\tau w}$

The Frisch Elasticity equals 0.40.



The Frisch Elasticity equals 2.00

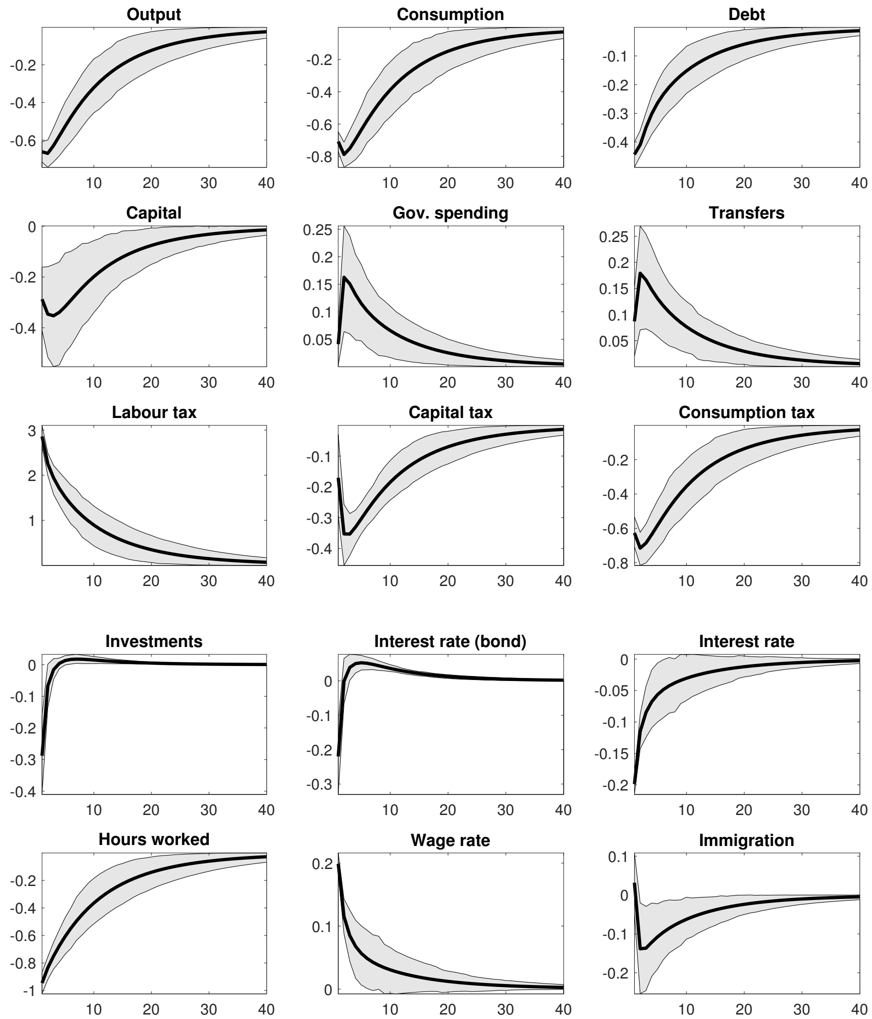


Figure 2.9: *Bayesian IRFs. Orthogonalized shock to Consumption tax,* $\varepsilon_t^{\tau c}$

The Frisch Elasticity equals 0.40.

The Frisch Elasticity equals 2.00

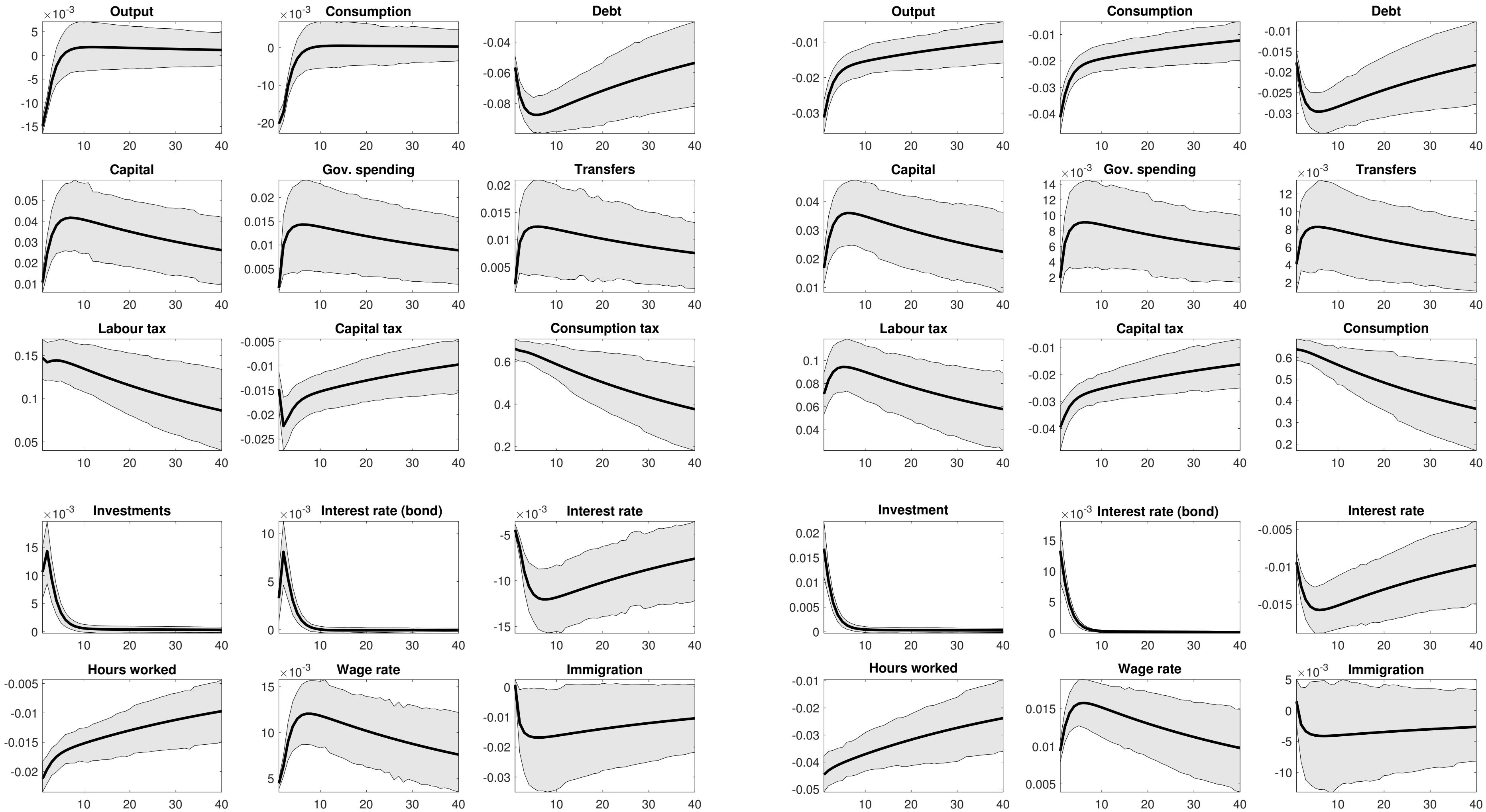
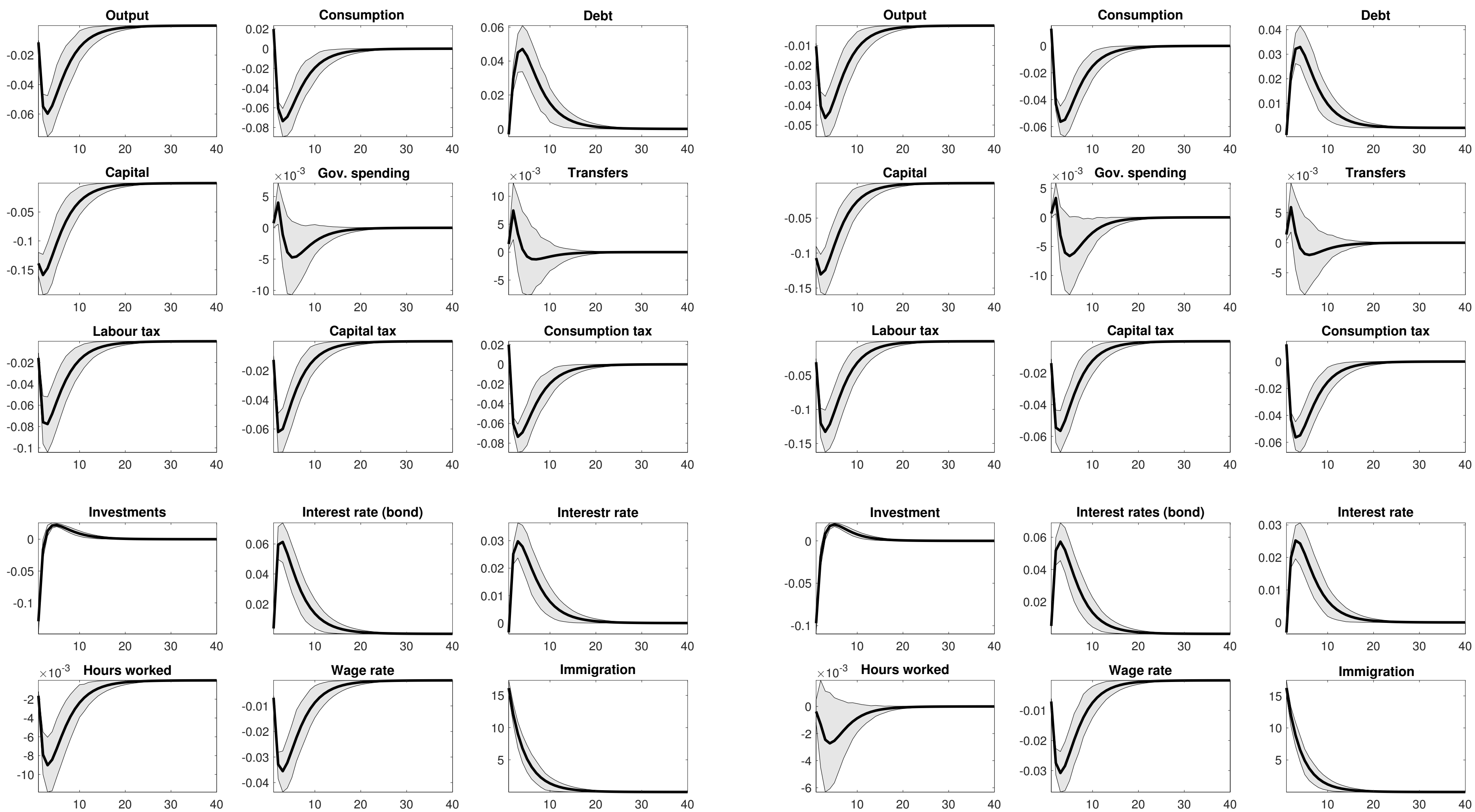


Figure 2.10: *Bayesian IRFs. Orthogonalized shock to Immigration,* ε_t^m

The Frisch Elasticity equals 0.40. *The Frisch Elasticity equals 2.00*



2.8. Conclusion

In this chapter, we adapt a DSGE model to incorporate an overlapping dynasties structure that can then be used to estimate the impact of endogenously determined immigration and fiscal policy and how the two might interact. Following Kiguchi and Mountford (2013) and Weiske (2017, 2019), we contruct an extended time series representing the quarterly net flow of migrants that join the US working-age population employing using data from successive reports from the U.S. Bureau of Labor's Current Population Survey and then employ Bayesian techniques to estimate the model's parameters. The chapter's core contribution lies in its detailed specification of fiscal policy instruments, deficit finance and immigration.

As unlike in Chapter 1, here immigration is endogenously determined yet also reacts to the innovations, we can use the model to study not only its impact on the economy but how it responds to other innovations. Therefore in addition to specifying policy rules for capital, labour and consumption taxes, government expenditure and transfers, and allowing for contemporaneous responses to output and dynamic responses to government debt as in Leeper et al. (2010), there is also an analogous rule for immigration that responds to wages, the level of economic activity and the state of public debt. This motivated by a host of immigration models in which the decision to migrate depends not only on conditions in a sending country but also on the conditions in the destination country.

To accurately predict the impact of fiscal policy and immigration, it is essential to understand the magnitude and speed of their response to debt and changes in output. The model is rich enough to provide a satisfactory empirical account of the impact of immigration in the post-war US, and exogenous shocks reflect unanticipated changes in fiscal policy and immigration to assess the role of these shocks in explaining the variance of the model's endogenous variables.

Results for the policy parameter estimates allow us to quantify the role the the different distortionary fiscal instruments play in financing debt innovations. Although the response of capital taxes to debt innovations are highest, the response of other fiscal instruments are also important. Unlike Leeper et al. (2010), the labour taxes respond strongly to debt. The results also show that capital and labour tax rates have had a highly procyclical response to the level of aggregate output. On the other hand immigration which is absent from Leeper et al. (2010) is less responsive but to output but is responsive to wage rate innovations. Finally, we observe that exogenous changes to capital and labour tax rates affect the two rates simultaneously as in Leeper et al. (2010), suggesting that typical tax legislation tends to change both tax rates.

In line with Leeper et al. (2010), exogenous changes to consumption tax rates do not affect the capital or labour tax rates. Effect of an increase in immigration in the short-run dilute capital and hence lower per capita output, consumption, and tax receipts. However, we can observe a modest short-term rise in hours worked, government spending, and transfers. Immigration appears to have only a little short-run impact on real wages and output, while there is a positive reaction of interest rates and debt to immigration shocks as predicted by Ben-Gad (2018). Government spending, transfers, investment and hours worked together with investment-specific and technology shocks have been a significant driver of immigration. However, response of immigration to consumption, labour and capital income tax shocks are negative. Results indicate that innovations to the flow of immigration have relatively little impact on the US economy. This is largely due to the long-run adjustment the economy undergoes when it absorbs immigrants and to the fact that we only measure the impact of changes to the flow of immigrants, not the impact of the flow itself or the stock of immigrants that accumulates over time.

At large, academic research usually has focused on the flows of immigrants. Yet, it is commonly accepted that for a complete understanding of the migration process and its impact on the economy, isolating and measuring both stocks and flows is desirable. It is apparent that changes in each can be due to diverse factors and lead to different effects. Unlike in Ben-Gad (2008) and Smith and Thoenissen (2019), the labour is undifferentiated concerning skills in our model. In the context of our model, changes in the flow or stock of immigrants do not have much impact on the steady state since the economy will converge to the previous steady state over time when immigration is ceased. In the models with the differentiated skills as in Ben-Gad (2008), where the long-run impact of immigration on wages is not a function of the rate at which immigration occurs but its composition, immigration surges affect wages in the long term only if they alter the long-run ratio of skilled to unskilled workers. Furthermore, in these models change in the composition of the labour force, not in immigrants' numbers, impacts the positive effect on the rate of return from capital dilution (Ben-Gad, 2008). Smith and Thoenissen (2019) also observed that the impact of migration shocks on macroeconomic aggregates is larger when migrants' human capital is greater than those of locals in New Zealand.

Since debt-financed fiscal changes trigger very-long lived dynamics, even in entirely conventional models, short-run impacts can differ sharply from long-run effects, even being of different signs (Leeper et al., 2010). The estimated model can be used to evaluate the effect of counterfactual fiscal policies and immigration. My third chapter offers alternative approach—an estimated structural VECM model that can be used to answer specific fiscal and immigration policy questions.

Appendix

2.9. Derivations of Model Equations

2.9.1 Equilibrium Conditions

From the FOC we derive the *Euler's Consumption Equation*:

$$\begin{aligned} & \left(1+\tau_{t+1}^{c}\right)\left(c_{t+1}-u_{t+1}^{l}\frac{\eta\, l_{t+1}^{\varepsilon}}{\varepsilon}\right) \\ = & \left[r_{t+1}^{b}u_{t+1}^{i}+1\right]\left(\frac{u_{t}^{i}}{u_{t+1}^{i}}\right)\times \\ & \left[\beta\left(1+\tau_{t}^{c}\right)\left(c_{t}-u_{t}^{l}\frac{\eta\, l_{t}^{\varepsilon}}{\varepsilon}\right)-\left(1-\beta\left(1+n\right)\right)m_{t+1}\left(b_{t+1}+k_{t+1}\right)\right] \end{aligned} \tag{2.23}$$

where *Disutility-of-Work* is,

$$d\left(l_{t}\right)=u_{t}^{l}\,\frac{\eta\, l_{t}^{\varepsilon}}{\varepsilon} \tag{2.24}$$

Investments follow,

$$i_{t}=\left(1+n\right)\left(1+m_{t+1}\right)\frac{k_{t+1}}{u_{t}^{i}}-\left(1-\delta\right)\frac{k_{t}}{u_{t}^{i}} \tag{2.25}$$

The per capita *Law of Motion for Capital*,

$$\left(1+n\right)\left(1+m_{t+1}\right)k_{t+1}=u_{t}^{i}\;i_{t}+\left(1-\delta\right)k_{t} \tag{2.26}$$

The *Global Constraint of Resources* in the economy is,

$$y_{t}=c_{t}+i_{t}+g_{t} \tag{2.27}$$

The per capita *Law of Motion for Public Debt*,

$$\left(1+n\right)\left(1+m_{t+1}\right)b_{t+1}+T_{t}^{w}+T_{t}^{k}+T_{t}^{c}=g_{t}+z_{t}+b_{t}+\left(1-\tau_{t}^{r}\right)r_{t}b_{t} \tag{2.28}$$

where per capita tax revenues:

$$T_{t}^{w}=\tau_{t}^{w}w_{t}l_{t}=\tau_{t}^{w}\left(1-\alpha\right)y_{t} \tag{2.29}$$

$$T_t^k = \tau_t^r r_t k_t = \tau_t^r \alpha \, y_t \tag{2.30}$$

$$T_t^c = \tau_t^c c_t \tag{2.31}$$

Consumption-Leisure choice

$$u_t^l \, l_t^{\varepsilon} \, (1 + \tau_t^c) = (1 - \tau_t^w) \, (1 - \alpha) \, y_t \tag{2.32}$$

Production function is Cobb-Douglas,

$$y_t = u_t^a \, k_t^{\alpha} \, l_t^{1-\alpha} \frac{1}{1 + m_t} \tag{2.33}$$

And both input factors receive their marginal products,

$$r_t = \alpha \, \frac{y_t}{k_t} \tag{2.34}$$

$$w_t = (1 - \alpha) \, \frac{y_t}{l_t} \tag{2.35}$$

$$r_t^b = (1 - \tau_t^r) \, r_t \tag{2.36}$$

2.9.2 Steady State

In steady state all shocks are equal to one.

$$s_i = \frac{I}{Y} \, \in (0, 1) \tag{2.37}$$

$$s_g = \frac{G}{Y} \, \in (0, 1) \tag{2.38}$$

$$s_b = \frac{B}{Y} \, \in (0, 1) \tag{2.39}$$

$$\frac{K}{Y} = \frac{\frac{I}{Y}}{((1 + n)(1 + m) - (1 - \delta))} \tag{2.40}$$

$$r = \alpha \left(\frac{Y}{K} \right) = \alpha \left(\frac{K}{Y} \right)^{-1} \tag{2.41}$$

$$\frac{K}{L} = \left(\frac{K}{Y} \right)^{\frac{1}{1-\alpha}} \tag{2.42}$$

$$w = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha} \tag{2.43}$$

$$\frac{L}{Y} = \frac{(1-\alpha)}{w} \tag{2.44}$$

$$\frac{Y}{L} = \frac{w}{(1-\alpha)} \tag{2.45}$$

$$L = \left[\frac{(1-\tau^w)(1-\alpha)\frac{Y}{L}}{(1+\tau^c)}\right]^{\frac{1}{\varepsilon-1}} \tag{2.46}$$

$$K = \left(\frac{K}{Y}\right)^{\frac{1}{1-\alpha}} L \tag{2.47}$$

$$Y = K^{\alpha} L^{1-\alpha} \tag{2.48}$$

$$C = Y(1 - s_g - s_i) \tag{2.49}$$

$$Z = Y\left[\left((1+n)(1+m) - (1+r^b)\right)s_b - s_g\right] + \tau^w(1-\alpha)Y + \tau^r \alpha Y + \tau^c C \tag{2.50}$$

$$G = s_g Y \tag{2.51}$$

$$B = s_b Y \tag{2.52}$$

$$I = s_i Y \tag{2.53}$$

2.9.3 Log-Linearised System

Steady state values of the variables presented in without time subscript, i.e. $x_{ss} = x$. Hats denote log-deviations of variables.
Consumption Euler Equation:

$$\begin{aligned} &\frac{c}{c-d}(\hat{c}_{t+1} - \hat{c}_t) - \frac{d}{c-d}(\hat{d}_{t+1} - \hat{d}_t) + \frac{\tau^c}{1+\tau^c}\left(\hat{\tau}^c_{t+1} - \beta\hat{\tau}^c_t\right) + \hat{u}^i_{t+1} \\ = \;& \hat{r}^b_{t+1} + \hat{u}^i_t - (1-\beta(1+n))\left[\hat{m}_{t+1} - \frac{\left(b\hat{b}_{t+1} + k\hat{k}_{t+1}\right)}{b+k}\right] \end{aligned} \tag{2.54}$$

Accounting Identity:

$$y\hat{y}_t = c\hat{c}_t + i\hat{i}_t + g\hat{g}_t \tag{2.55}$$

Capital Accumulation Equation:

$$(1+n)(1+m)\left[\hat{k}_{t+1}+\frac{m}{1+m}\hat{m}_{t+1}\right]=(1-\delta)\,\hat{k}_t+\frac{i}{k}\hat{i}_t+\frac{i}{k}\hat{u}_t^i \tag{2.56}$$

Government Budget Constraint:

$$\begin{aligned}&(1+n)\,\hat{b}_{t+1}+\left[(1-\alpha)\,\tau^w y\left(\hat{\tau}_t^w+\hat{y}_t\right)+\alpha\tau^r y\left(\hat{\tau}_t^r+\hat{y}_t\right)+\tau^c c\left(\hat{\tau}_t^c+\hat{c}_t\right)\right]\\ =\;& g\hat{g}_t+z\hat{z}_t+b\hat{b}_t+r^b b\left(\hat{r}^b+\hat{b}_t\right)-(1+n)\,\frac{m}{1+m}\hat{m}_{t+1}\end{aligned} \tag{2.57}$$

Consumption-leisure choice:

$$\hat{u}_t^l+\epsilon\,\hat{l}_t+\frac{\tau^c}{1+\tau^c}\,\hat{\tau}_t^c=(1-\alpha)\left[\hat{y}_t-\frac{\tau^w}{1-\tau^w}\hat{\tau}_t^w\right] \tag{2.58}$$

Disutility of Labor:

$$\hat{d}_t=\frac{\eta}{\varepsilon}\left(\hat{u}_t^l+\varepsilon\,\hat{l}_t\right) \tag{2.59}$$

Output:

$$\hat{y}_t=\hat{u}_t^a+\alpha\hat{k}_t+(1-\alpha)\,\hat{l}_t-\frac{m}{1+m}\hat{m}_t \tag{2.60}$$

Price of the capital:

$$\hat{r}_t=\alpha\left(\hat{y}_t-\hat{k}_t\right) \tag{2.61}$$

After tax interest rate:

$$\hat{r}_t^b=\hat{r}_t-\frac{\tau^r}{1-\tau^r}\,\hat{\tau}_t^r \tag{2.62}$$

Economy wide wage rate:

$$\hat{w}_t=(1-\alpha)\left(\hat{y}_t-\hat{l}_t\right) \tag{2.63}$$

Tax revenues:

$$\hat{T}_t^r=\alpha\left(\hat{\tau}_t^r+\hat{y}_t\right) \tag{2.64}$$

$$\hat{T}_t^w=(1-\alpha)\left(\hat{\tau}_t^w+\hat{y}_t\right) \tag{2.65}$$

$$\hat{T}_t^c=\hat{\tau}_t^c+\hat{c}_t \tag{2.66}$$

Fiscal policy rules and immigration rule:

$$\hat{g}_t = -\varphi_g \hat{y}_t - \gamma_g \hat{b}_t + \hat{u}_t^g \tag{2.67}$$

$$\hat{z}_t = -\varphi_z \hat{y}_t - \gamma_z \hat{b}_t + \hat{u}_t^z \tag{2.68}$$

$$\hat{\tau}_t^r = \varphi_{\tau r} \hat{y}_t + \gamma_{\tau r} \hat{b}_t + \phi_{rw} \hat{u}_t^{\tau w} + \phi_{rc} \hat{u}_t^{\tau c} + \hat{u}_t^{\tau r} \tag{2.69}$$

$$\hat{\tau}_t^w = \varphi_{\tau w} \hat{y}_t + \gamma_{\tau w} \hat{b}_t + \phi_{rw} \hat{u}_t^{\tau r} + \phi_{wc} \hat{u}_t^{\tau c} + \hat{u}_t^{\tau w} \tag{2.70}$$

$$\hat{\tau}_t^c = \phi_{rc} \hat{u}_t^{\tau r} + \phi_{wc} u_t^{\tau w} + \hat{u}_t^{\tau c} \tag{2.71}$$

$$\hat{m}_t = \varphi_m \hat{y}_t + \gamma_{1,m} \hat{b}_t + \gamma_{2,m} \hat{w}_t + \hat{u}_t^m \tag{2.72}$$

AR(1) processes in terms of log deviations from steady state for labour, productivity and investments are:

$$\hat{u}_t^l = \rho_l \hat{u}_{t-1}^l + \sigma_l \epsilon_t^l \tag{2.73}$$

$$\hat{u}_t^a = \rho_a \hat{u}_{t-1}^a + \sigma_a \epsilon_t^a \tag{2.74}$$

$$\hat{u}_t^i = \rho_i \hat{u}_{t-1}^i + \sigma_i \epsilon_t^i \tag{2.75}$$

AR(1) processes in terms of log deviations from steady state for fiscal policy and immigration rules:

$$\hat{u}_t^g = \rho_g \hat{u}_{t-1}^g + \sigma_g \epsilon_t^g \tag{2.76}$$

$$\hat{u}_t^z = \rho_z \hat{u}_{t-1}^z + \sigma_z \epsilon_t^z \tag{2.77}$$

$$\hat{u}_t^{\tau r} = \rho_{\tau r} \hat{u}_{t-1}^{\tau r} + \sigma_{\tau r} \epsilon_t^{\tau r} \tag{2.78}$$

$$\hat{u}_t^{\tau w} = \rho_{\tau w} \hat{u}_{t-1}^{\tau w} + \sigma_{\tau w} \epsilon_t^{\tau w} \tag{2.79}$$

$$\hat{u}_t^{\tau c} = \rho_{\tau c} \hat{u}_{t-1}^{\tau c} + \sigma_{\tau c} \epsilon_t^{\tau c} \tag{2.80}$$

$$\hat{u}_t^m = \rho_m \hat{u}_{t-1}^m + \sigma_m \epsilon_t^m \tag{2.81}$$

where each of ϵ's is distributed $i.i.d.$ $N(0,1)$.

2.10. Data Construction

The estimation of the model based on nine time series from 1960:Q1 to 2018:Q1. All data are quarterly and in real terms. Unless otherwise noted, all data are from the Bureau of Economic Analysis' NIPA (National Income and Product Account). Nominal data converted to real values by dividing the GDP deflator, P_t. Data are converted to per-capita by dividing the population index, POP_t. Each series has a (separate) linear trend removed prior to estimation[6].

P_t: The GDP deflator for personal consumption expenditures (line 2 in Table 1.1.4 - Price Indexes for Gross Domestic Product).

POP_t : Index of population, the number of civilian noninstitutional population, ages 16 years and over(FRED mnemonic CNP16OV[7]). Originally CNP16OV is not seasonally adjusted. We constructed quarterly data from monthly data and used X-13 ARIMA-SEATS for quarterly seasonal adjustment.[8] Data normalized so that its 2012Q3 value is equal to 1.

1. Real Output. Take the Gross Domestic product (line 1 of Table 1.1.5. Gross Domestic Product) call it GDP_t and deflate it by the GDP deflator for personal consumption, P_t. Take the number of employed civilians, POP_t. Then

$$Real\,Output = 100 \times \ln\left(\frac{GDP_t/P_t}{POP_t}\right) \tag{2.82}$$

2. Real Consumption. We take nominal personal consumption (line 2 of Table 1.1.5), call it PCE_t and deflate it by the GDP deflator for personal consumption, P_t. Take the number of employed civilians, POP_t. Then

$$Real\,Consumption = 100 \times \ln\left(\frac{PCE_t/P_t}{POP_t}\right) \tag{2.83}$$

[6]For this we use Matlab function *detrend*().

[7]Federal Reserve Economic Data, Federal Reserve Bank of St. Louis, https://fred.stlouisfed.org

[8]http://www.seasonal.website

3. Real Consumption Tax Revenues. We take federal government current tax receipts from production and imports (line 4 of Table 3.2. Federal Government Current Receipts and Expenditures) , call it $CTAX_t$. Then

$$Real\,Consumption\,Tax\,Revenues = 100 \times \ln\left(\frac{CTAX_t/P_t}{POP_t}\right) \tag{2.84}$$

4. Real Labour Tax Revenues. Following the Jones (2002) we calculate average personal income tax rate by the formula

$$\tau_t^p = \frac{IT_t}{W_t + PRI_t/2 + CI_t} \tag{2.85}$$

where IT_t is personal current tax revenues (line 3 of Table 3.2 Federal Government Current Receipts and Expenditures), W_t is wage and salary accruals (line 3 of Table 1.12 National Income by Type of Income) and PRI_t is a proprietors' income (line 9 Table 1.12). CI_t is the capital income is defined as rental income, $RENT_t$(Table 1.12 line 12), corporate profits, $PROF_t$ (Table 1.12 line 13), interest income, INT_t (line 18 of Table 1.12) and $PRI_t/2$:

$$CI_t = RENT_t + PROF_t + INT_t + PRI_t/2 \tag{2.86}$$

Take contributions for government social insurance, CSI_t (Table 3.2 line 10) and EC_t, compensation for employees (Table 1.12 line 2). The average labour income tax rate is calculated as,

$$\tau_t^w = \frac{\tau_t^p\,(W_t + PRI_t) + CSI_t}{EC_t + PRI_t/2} \tag{2.87}$$

Take the tax base,

$$BASE_t = PCE_t + PCED_t \tag{2.88}$$

where PCE_t is nominal personal consumption (line 2 of Table 1.1.5), and $PCED_t$ is nominal personal consumption on durable goods (line 4 of Table 1.1.5). Then,

$$Real\,Labour\,Tax\,Revenues = 100 \times \ln\left(\frac{\tau_t^w\,BASE_t/P_t}{POP_t}\right) \tag{2.89}$$

5. Real Capital Tax Revenues. We take federal government current tax receipts from corporate income (line 8 of Table 3.2. Federal Government Current Receipts and Expenditures), call it CT_t, and take property taxes, PT_T (Table 3.3 line 9). Define the average capital income tax rate as,

$$\tau_t^r = \frac{\tau_t^p \, CI_t + CT_t}{CI_t + PT_t} \tag{2.90}$$

Then, multiplying it to tax base gives,

$$Real\, Capital\, Tax\, Revenues = 100 \times \ln\left(\frac{\tau_t^r \, BASE_t / P_t}{POP_t}\right) \tag{2.91}$$

6. Real Hours Worked. We construct hours worked using the index of average weekly nonfarm business hours (Nonfarm Business, All persons, average weekly hours duration: index, 2012Q3=100 from U.S. Department of Labour) called H_t and number of employed civilians 16 years and over (FRED mnemonic "CE16OV" Civilian Employment Level over 16), normalized so that its 2012Q3 values is 100, EMP_t. Hours worked are then defined as

$$Real\, Hours\, Worked = 100 \times \ln\left(\frac{(H_t \times EMP_t)/100}{POP_t}\right) \tag{2.92}$$

7. Real Government Expenditure. We take government consumption expenditure (Table 3.2 line 25), call it GC_t. Take governmet gross investment (Table 3.2 line 45), call it GI_t. Take government net purchases of non-produced assets (Table 3.2 line 47), call it GP_t. Finally, take government consumption of fixed capital (Table 3.2 line 48), call it GCK_t and define government expenditure G_t as,

$$G_t = GC_t + GI_t + GP_t - GCK_t \tag{2.93}$$

Then Real Government Expenditure is

$$Real\, Government\, Expenditure = 100 \times \ln\left(\frac{G_t/P_t}{POP_t}\right) \tag{2.94}$$

8. Real Government Transfers. We take current transfer payments (Table 3.2 line 26) call it $TRANSPAY_t$ and take current transfer receipts (Table 3.2 line

19), call it $TRANSREC_t$. Define net current transfers as,

$$CURRTRANS_t = TRANSPAY_t - TRANSREC_t \tag{2.95}$$

To calculate net capital transfers $CAPTRANS_t$ we take capital transfer payments (Table 3.2 line 46), name it $CAPTRANSPAY_t$ and subtruct from it capital transfer receipts $CAPTRANSREC_t$ (Table 3.2 line 42).

$$CAPTRANS_t = CAPTRANSPAY_t - CAPTRANSREC_t \tag{2.96}$$

Now, in order to calculate tax residuals, $TAXRESID_t$ we take current tax receipts, $TAXREC_t$ from Table 3.2 line 2, take income receipts on assets (from line 13), call it $INCREC_t$ and take the current surplus of government enterprises (line 23), call it $GOVSRP_t$. Define the total tax revenue, T_t, as the sum on consumption, labour and capital tax revenues (we multiply tax rate to tax base). To calculate tax residuals we just need the contributions for government social insurance, CSI_t (Table 3.2 line 10).

$$TAXRESID_t = TAXREC_t + INCREC_t + GOVSRP_t + CSI_t - T_t \tag{2.97}$$

Transfers Z_t defined as net current transfers, net capital transfers and subsidies, minus the tax residual.

$$Z_t = CURRTRANS_t + CAPTRANS_t - TAXRESID_t \tag{2.98}$$

Then,

$$Real\ Government\ Transfers = 100 \times \ln\left(\frac{Z_t/P_t}{POP_t}\right) \tag{2.99}$$

9. Real Investment. Take $PCED_t$, nominal personal consumption on durable goods (Table 1.1.5 line 3) and gross private domestic investment (Table 1.1.5 line 6) call it $GPDI_t$, add them up, call and deflate it by the GDP deflator for personal consumption P_t, then divide it by POP_t

$$Real\ Investment = 100 \times \ln\left(\frac{(PCED_t + GPDI_t)/P_t}{POP_t}\right) \tag{2.100}$$

10. Immigration. Take $\Delta N_{2,t}$ - the residual time series that represents the estimated net flow of migrants to the US civilian population and accounts for the change in the civilian noninstitutional population that is not due to past changes in fertility, current deaths or net flows to the US military. In order to calculate immigration rate we normalize above series (so that its 2012(3) value is equal to 1), call it M_t and divide by POP_t. By multiplying the series by 100 as other variables and applying the same detrending procedure above, makes the resulting series being interpreted as percentage deviation of immigration rate from a time-varying steady state (trend), $\hat{m}_t$.[9]

$$m_t^{obs} = m_t^{data} - m_{trend,t}^{data} = \hat{m}_t \tag{2.101}$$

We thank Sebastian Weiske for kindly sharing his calculations and immigration time series. Using the method by Kiguchi and Mountford (2013) and Weiske (2017) we re-constructed and extended the time series based on the latest data. For our analysis we transformed the series. As CPS makes data revisions periodically and they are usually attributed to the last census, we updated previous time points by attributing revisions related to immigration.

[9]See Pfeifer (2018)

2.11. Tables

Variable	Description	Frequency	Source
$CNP16OV_t$	*Civilian Noninst. population 16 years and older*	Monthly	FRED/CPS
$b_{t-16y,t}$	*Survival probability of a newborn to age 16*	Decennial	NCHS
$Births_{t-16y}$	*Live births 16 years ago*	Monthly	NCHS
$Deaths_t$	*Deaths 16 years and older*	Annual	NCHS
$Revisions_t$	*CPS data revisions unrelated to migration*		BLS/CPS
$Military_t$	*US military personnel, worldwide*	Quarterly	S.Cocuiba[2]

$CNP16OV_t$ and $Births_{t-16y}$ are seasonally adjusted using X-13 ARIMA-SEATS quarterly seasonal adjustment method. The numbers for $b_{t-16y,t}$ and $Deaths_t$ are interpolated to quarterly frequency. The series $Military_t$ ends in 2017Q4.
(2). Retrieved on May 16, 2019: https://sites.google.com/site/simonacociuba/research

Table 2.9: *Immigration data sources*

Date	Number	Explanation
Nov. 1956-1957	38,000	Immigrants from Hungary after a failed uprising against the Soviets*.
1960-1962	18,000	Refugee children from Cuba
January 1960	500,000	incl. Alaska and Hawaii
January 1962	-50,000	1960 census
January 1972	800,000	1970 census
July 1975	76,000	Vietnamese refugees. Fall of Saigon on April 30, 1975
Apr.-Oct. 1980	125,000	Mariel boatlift
January 1986	400,000	Undocumented immigrants and emigrants (legal) since 1980
January 1994	1,100,000	1990 census (adjustment effective in January 1990)
January 1997	470,000	Updated information on immigrants
January 1999	310,000	Updated information on immigrants
January 2000	2,600,000	2000 census
January 2003	941,000	2000 census
January 2004	-560,000	Revised estimates of net international migration for 2000 - 2003
January 2005	-8,000	4
January 2006	-67,000	4
January 2007	321,000	4
January 2008	-745,000	4
January 2009	-483,000	4
January 2010	-258,000	4
January 2011	-347,000	4
January 2012	1,510,000	2010 census
January 2013	138,000	4
January 2014	2,000	4
January 2015	528,000	4
January 2016	265,000	4
January 2017	-831,000	4
January 2018	488,000	4

Sources: Bureau of Labor Statistics (BLS),"Adjustments to Household Survey Population Estimates in January 20XX". (Online: https://www.bls.gov/cps/documentation.html).
*They were among the first Cold War refugees. The US would admit over 3 million refugees during the Cold War.
4–Revised estimates of net international migration adding up updated vital statistics, methodological changes and other information as in Weiske (2017)

Table 2.10: *CPS Data Revisions, 1957-2018.*

2.12. Figures

2.12.1 Variables and Shocks

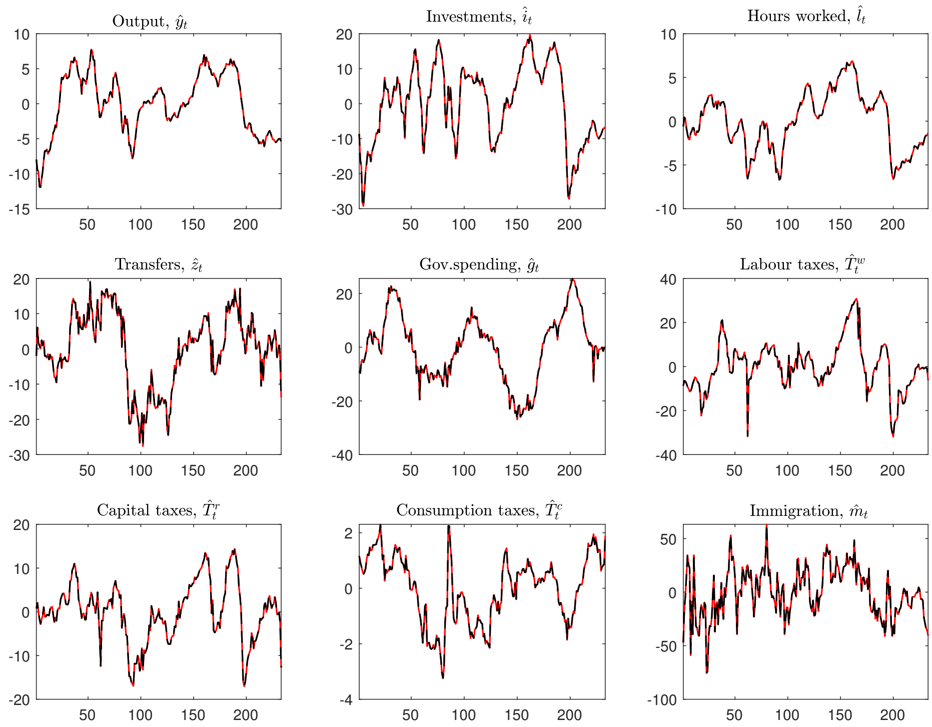


Figure 2.11: *Historical and Smoothed variables*

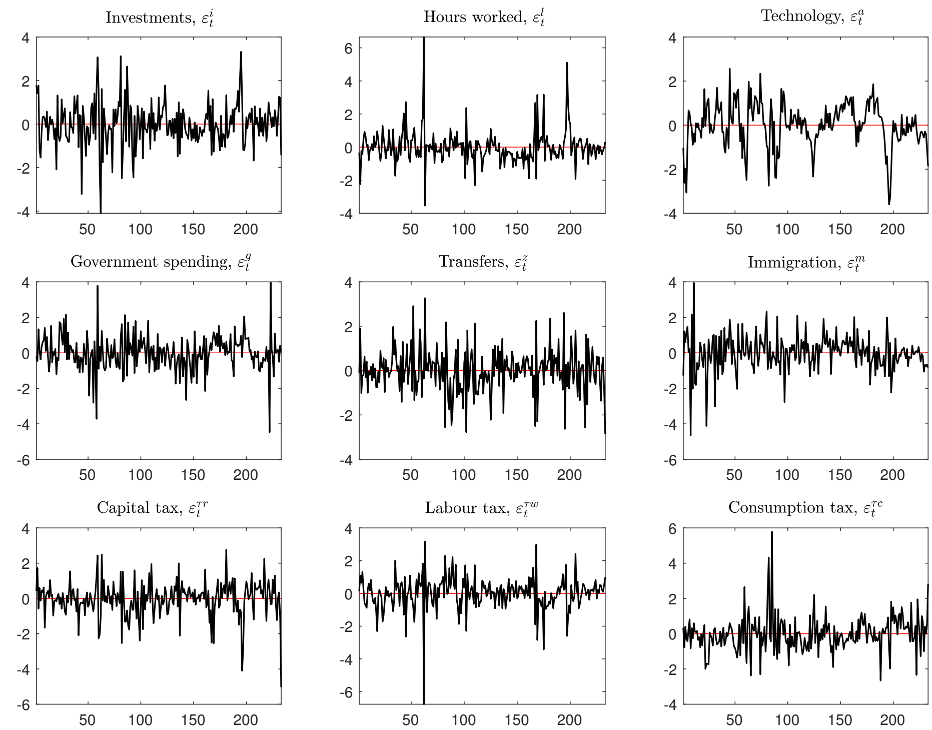


Figure 2.12: *Smoothed Shocks*

2.12.2 Mode Check Plots

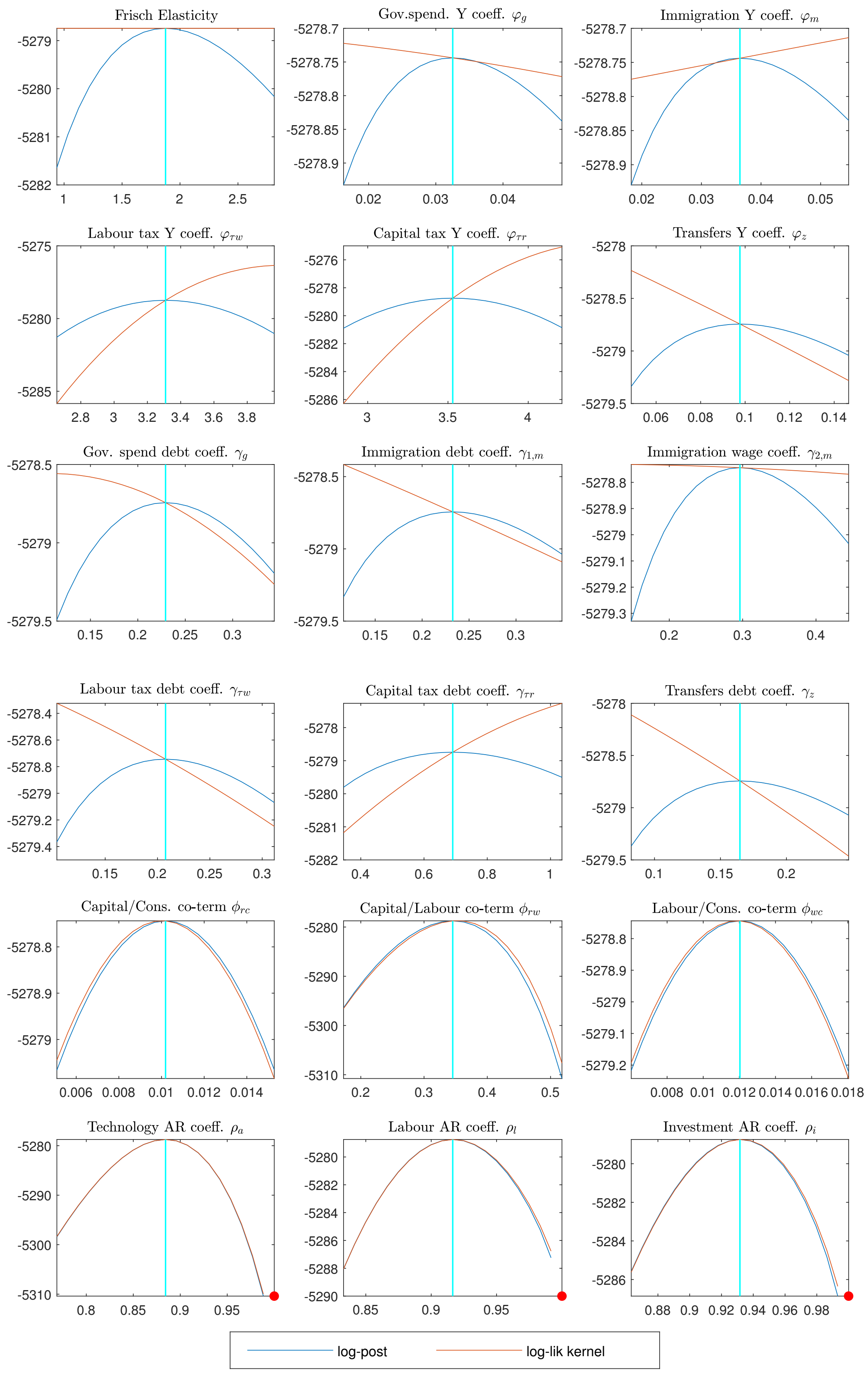


Figure 2.13: *Mode Check Plots for Priors (The Frisch elasticity equals 2.)*

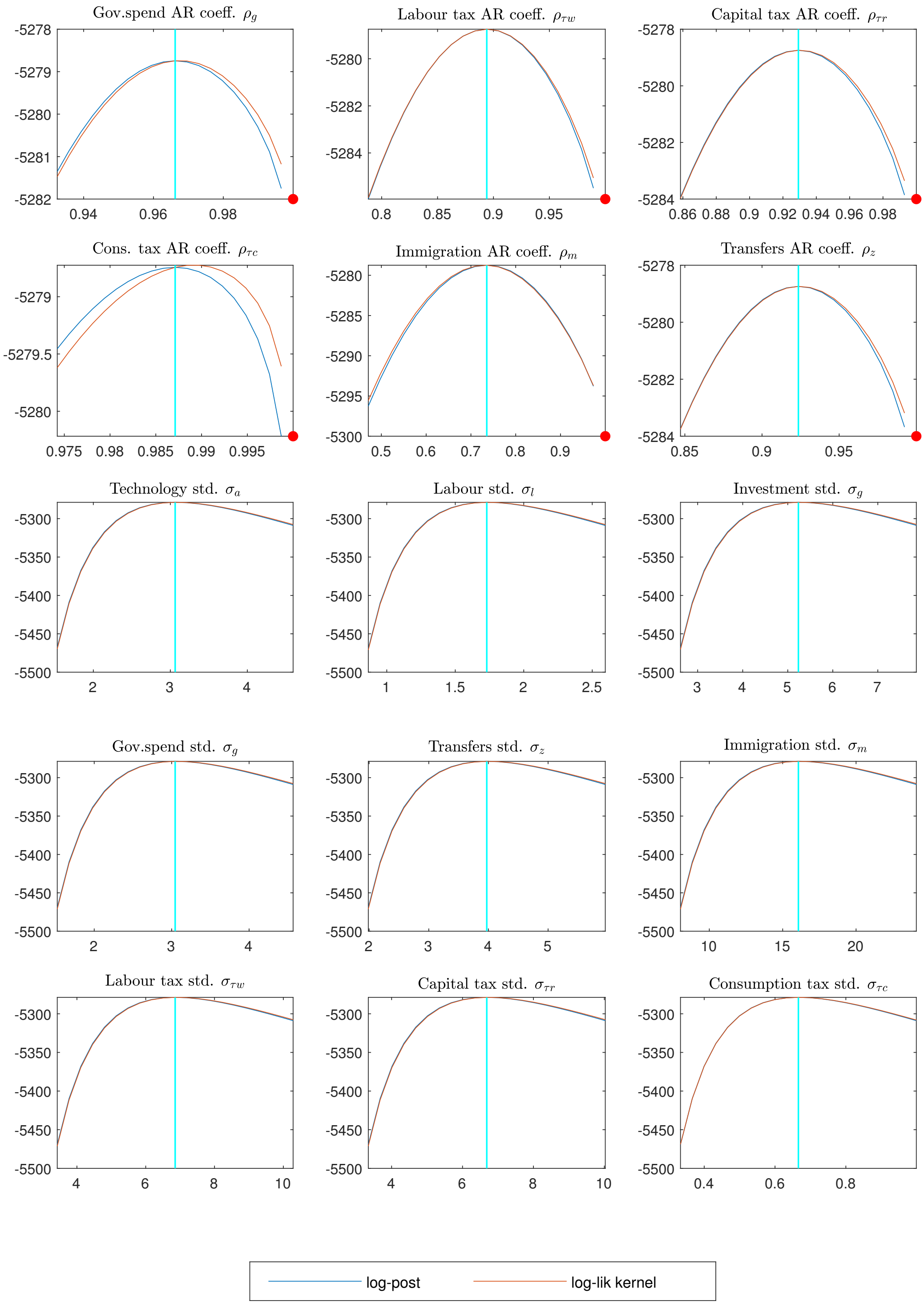


Figure 2.14: *Mode Check Plots for Priors (The Frisch elasticity equals 2.)*

2.12.3 Priors and Posteriors

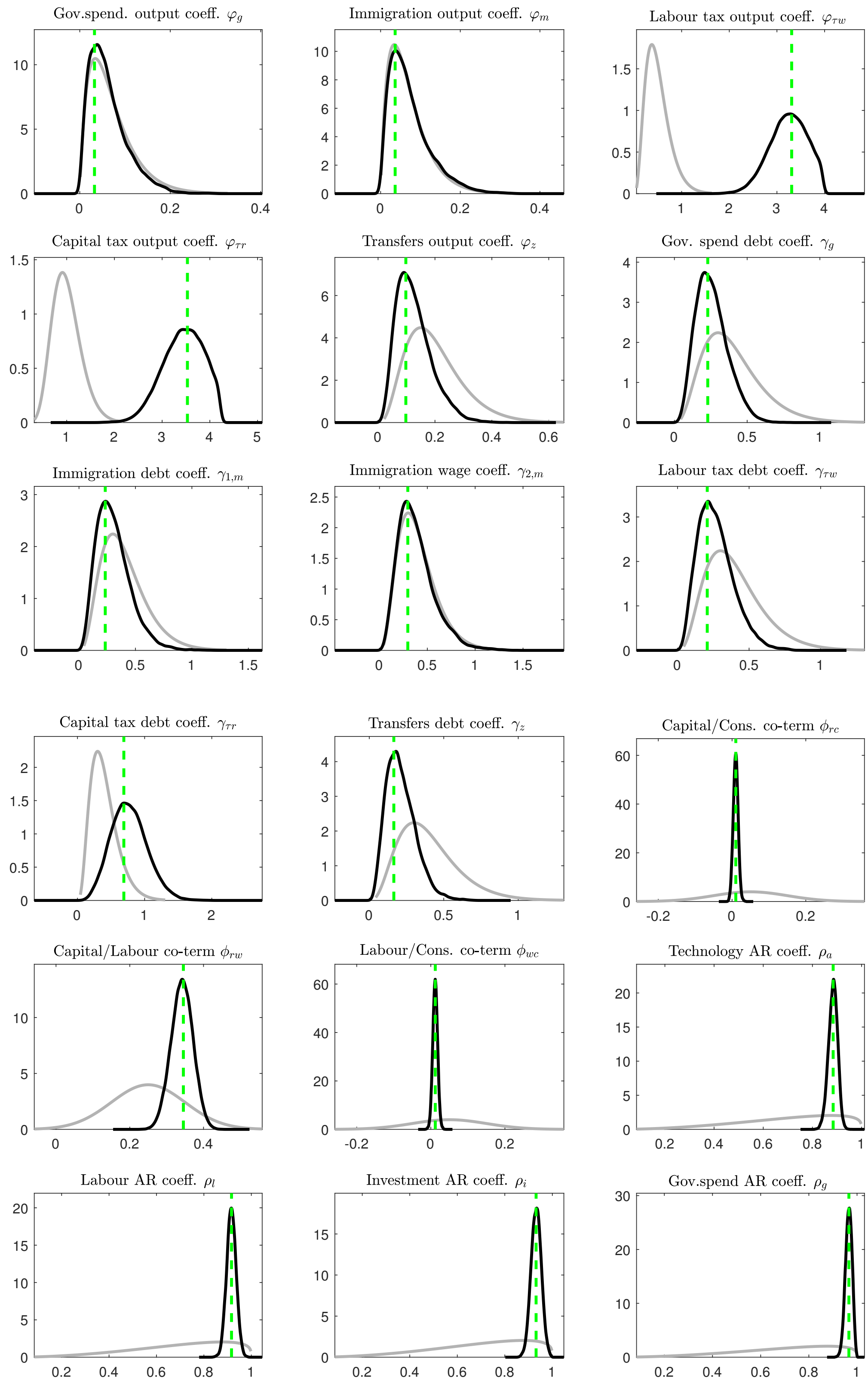


Figure 2.15: *Priors and Posteriors (The Frisch elasticity equals 2.)*

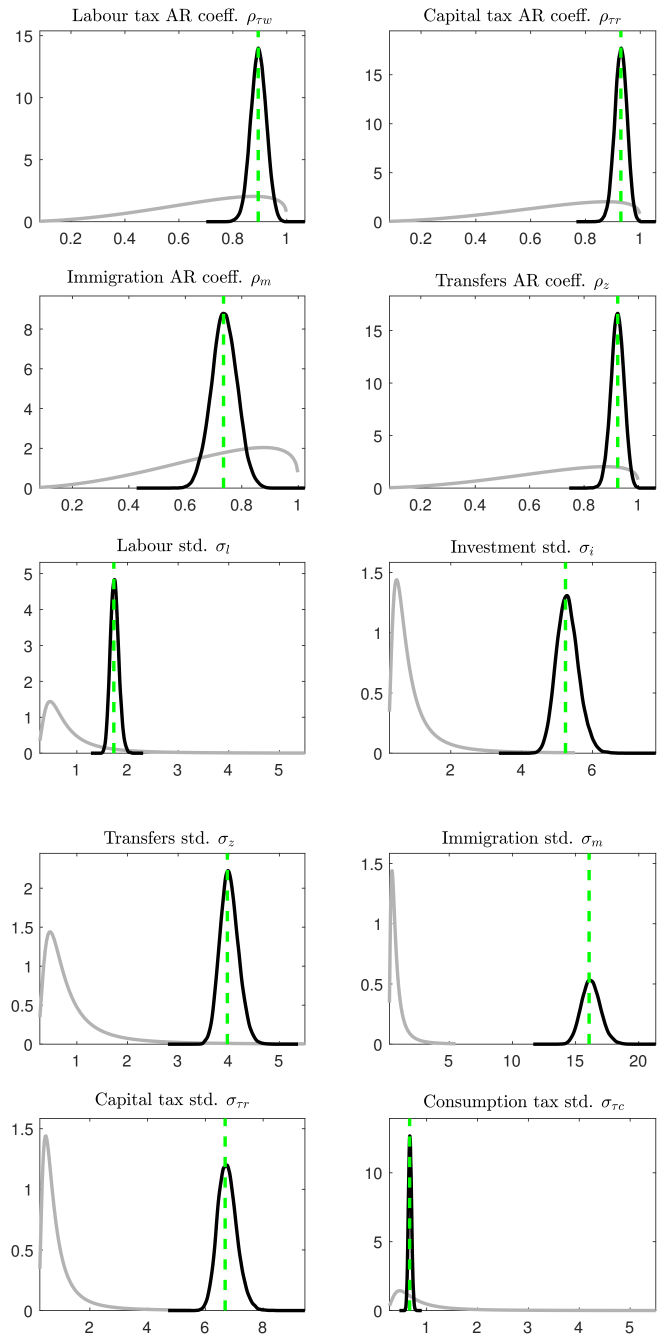


Figure 2.16: *Priors and Posteriors (The Frisch elasticity equals 2.)*

2.13. Shock decompositions

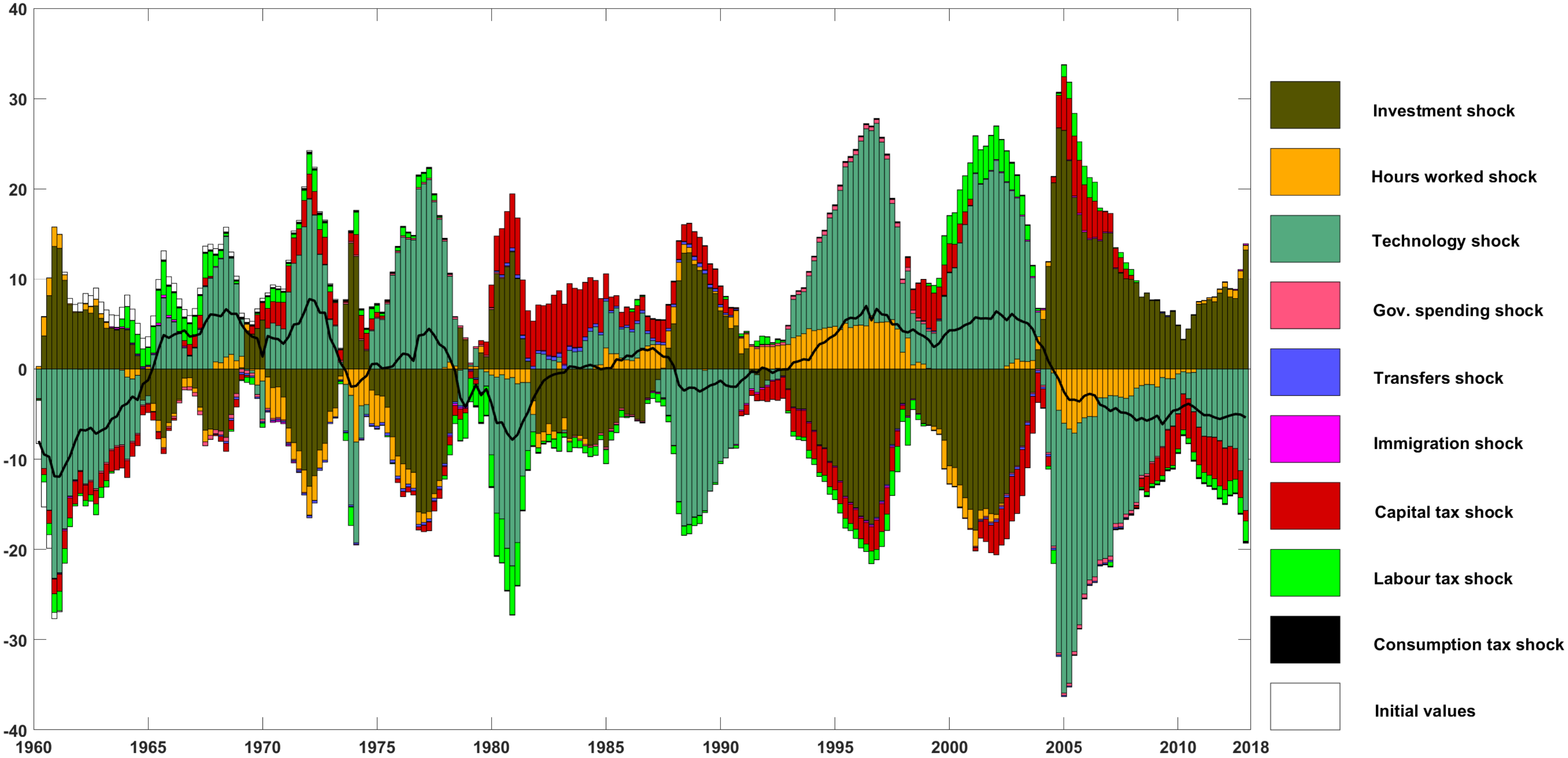


Figure 2.17: *Shock Decomposition of Output,* $\hat{y}_t$.

** The black line depicts the deviation of the smoothed value of the corresponding endogenous variable from its steady state at the specified parameter set. The colored bars correspond to the contribution of the the respective smoothed shocks to the deviation of the smoothed endogenous variable from its steady state. 'Initial values' in the graphs refers to the part of the deviations from steady state not explained by the smoothed shocks, but rather by the initial value of the state variables.

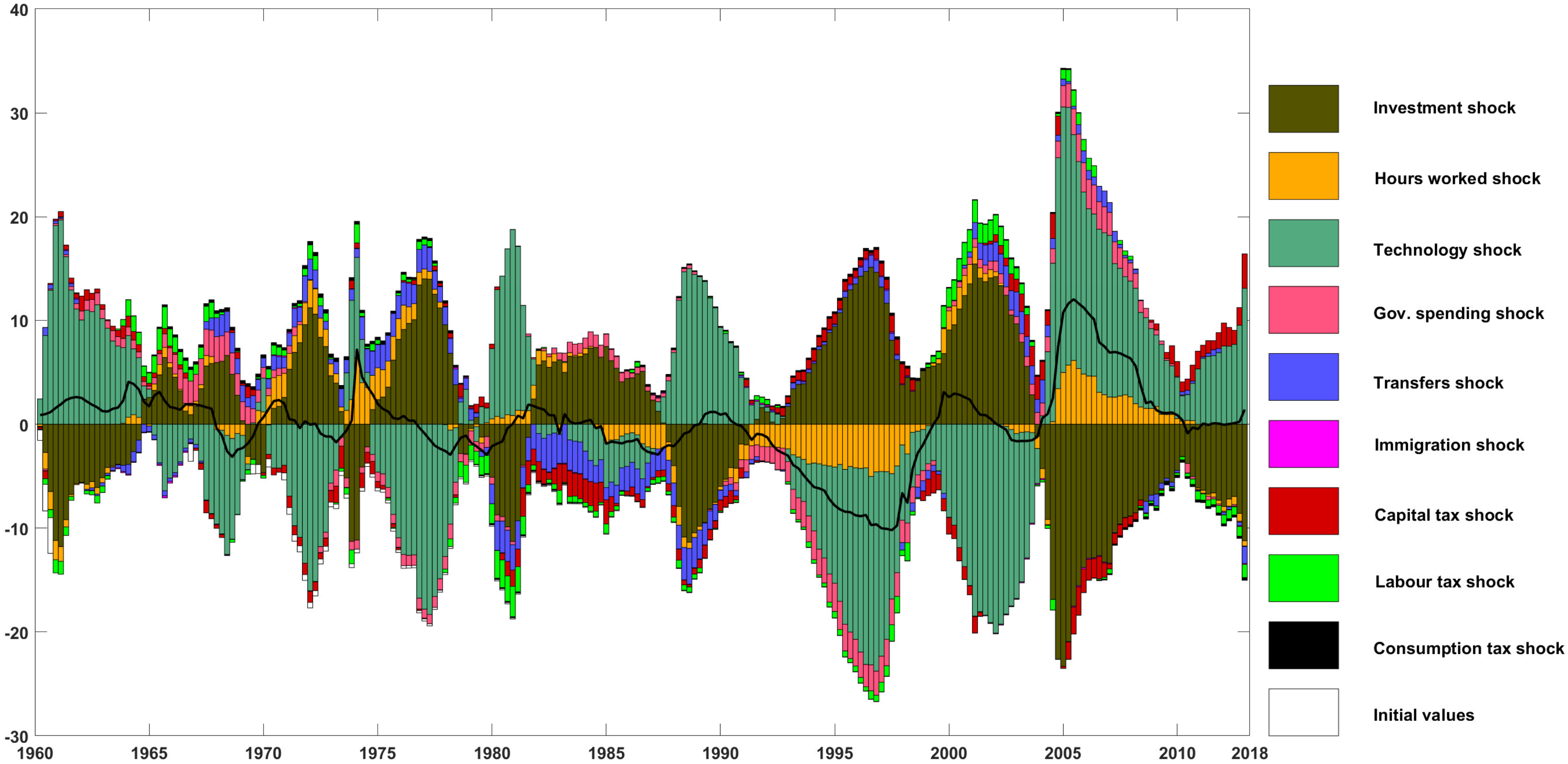


Figure 2.18: *Shock Decomposition of Debt,* $\hat{b}_t$.

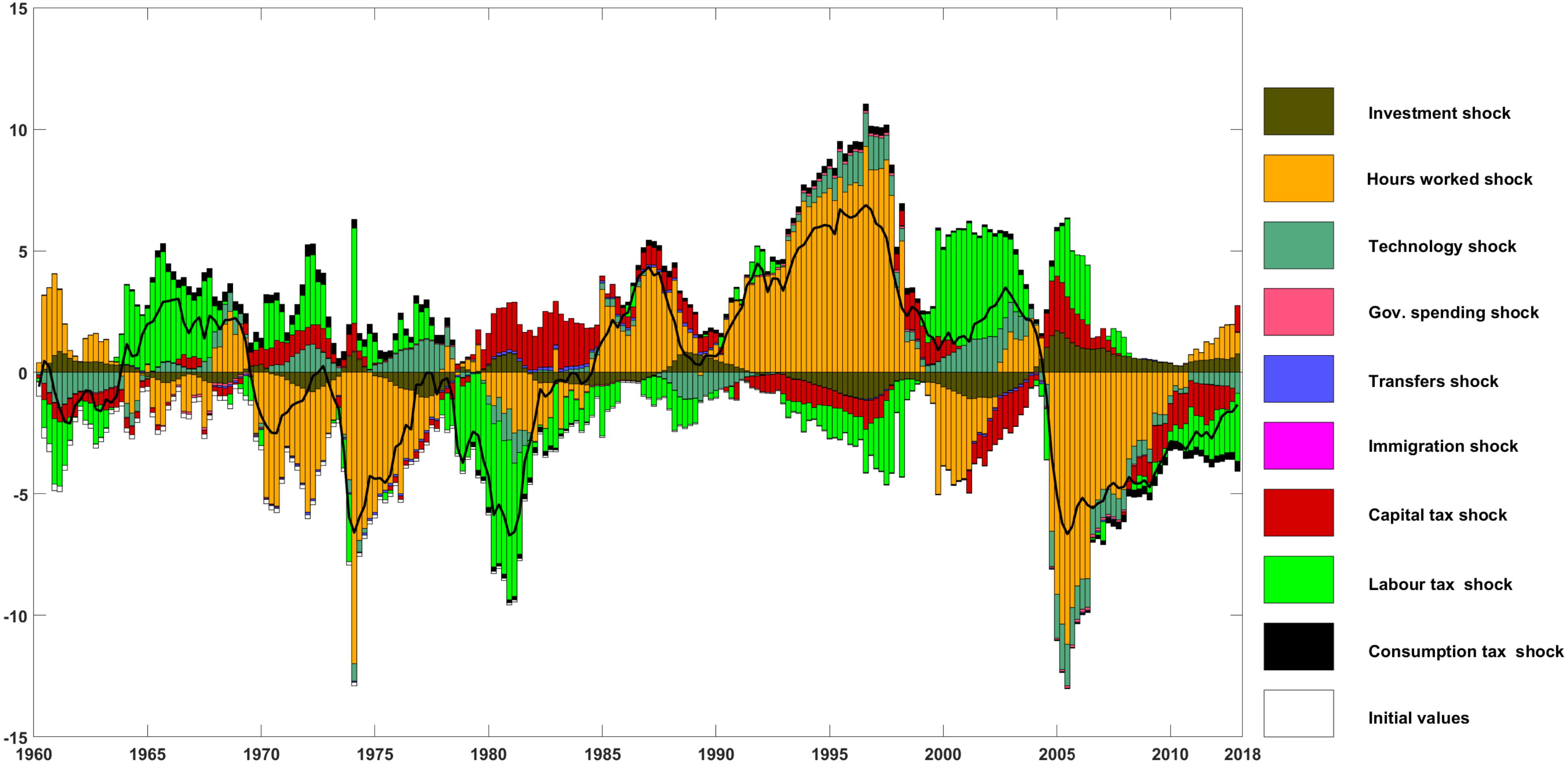


Figure 2.19: *Shock Decomposition of Hours worked* $\hat{l}_t$.

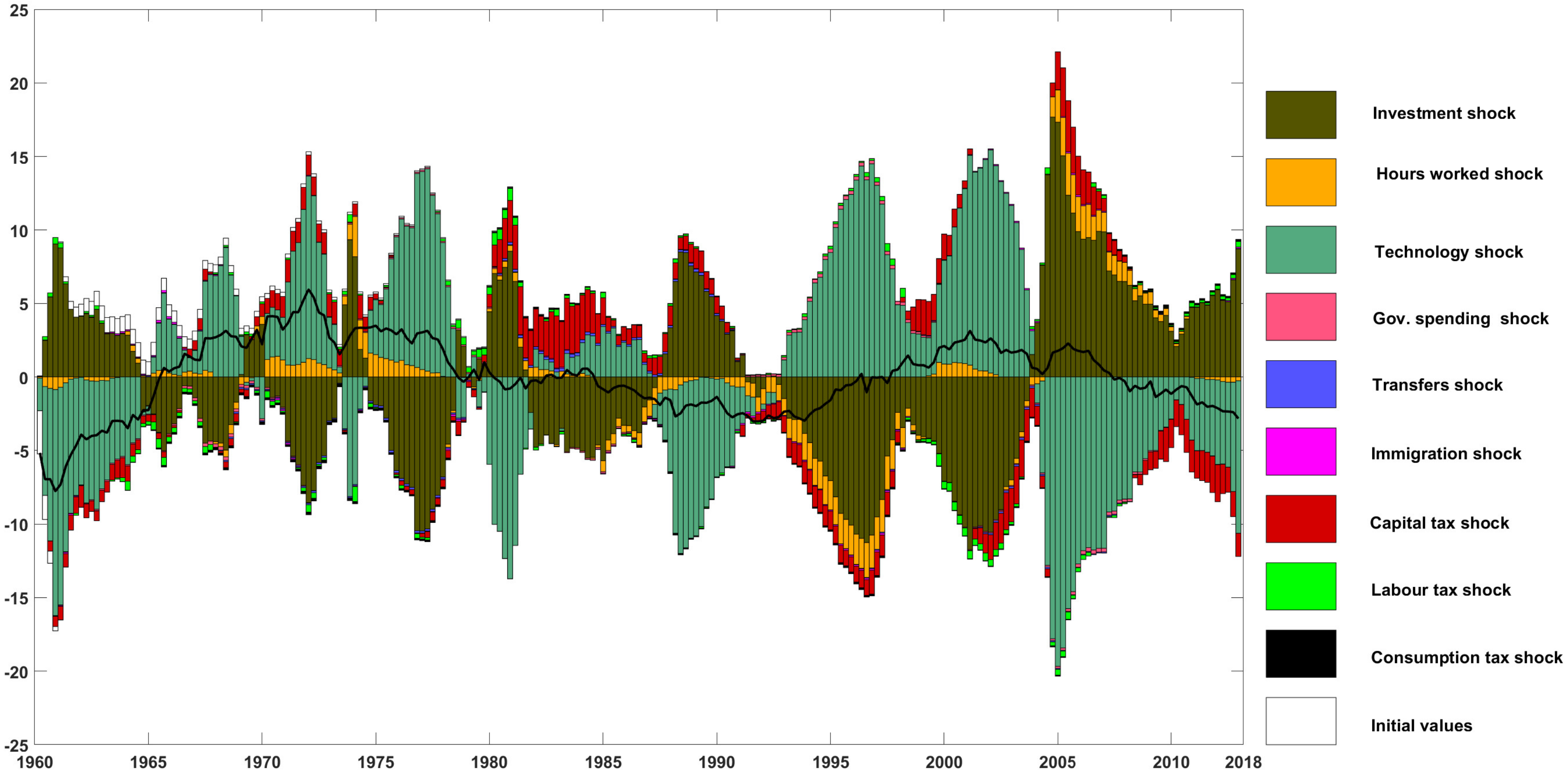


Figure 2.20: *Shock Decomposition of Wages, $\hat{w}_t$.*

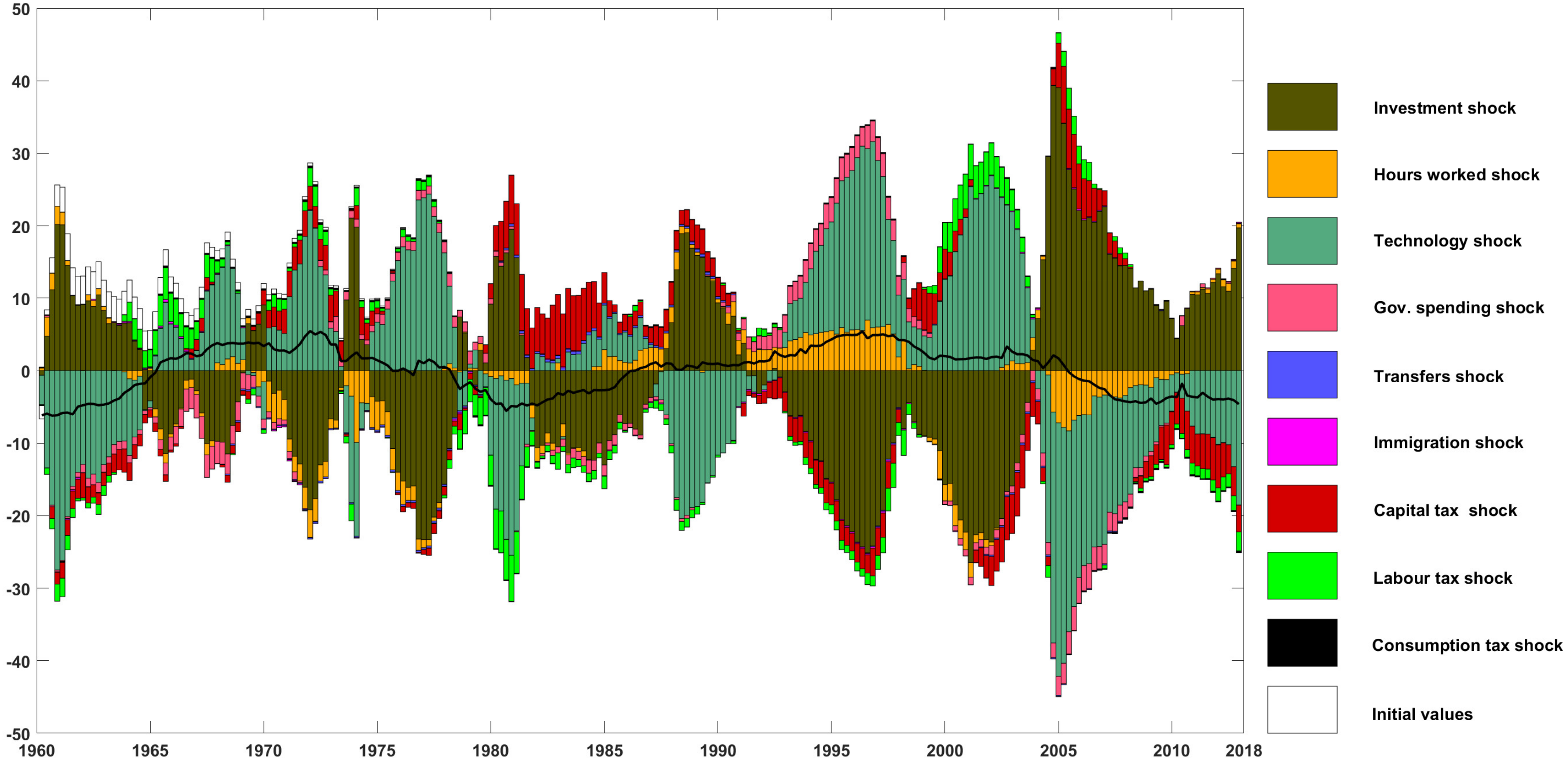


Figure 2.21: *Shock Decomposition of Consumption,* $\hat{c}_t$.

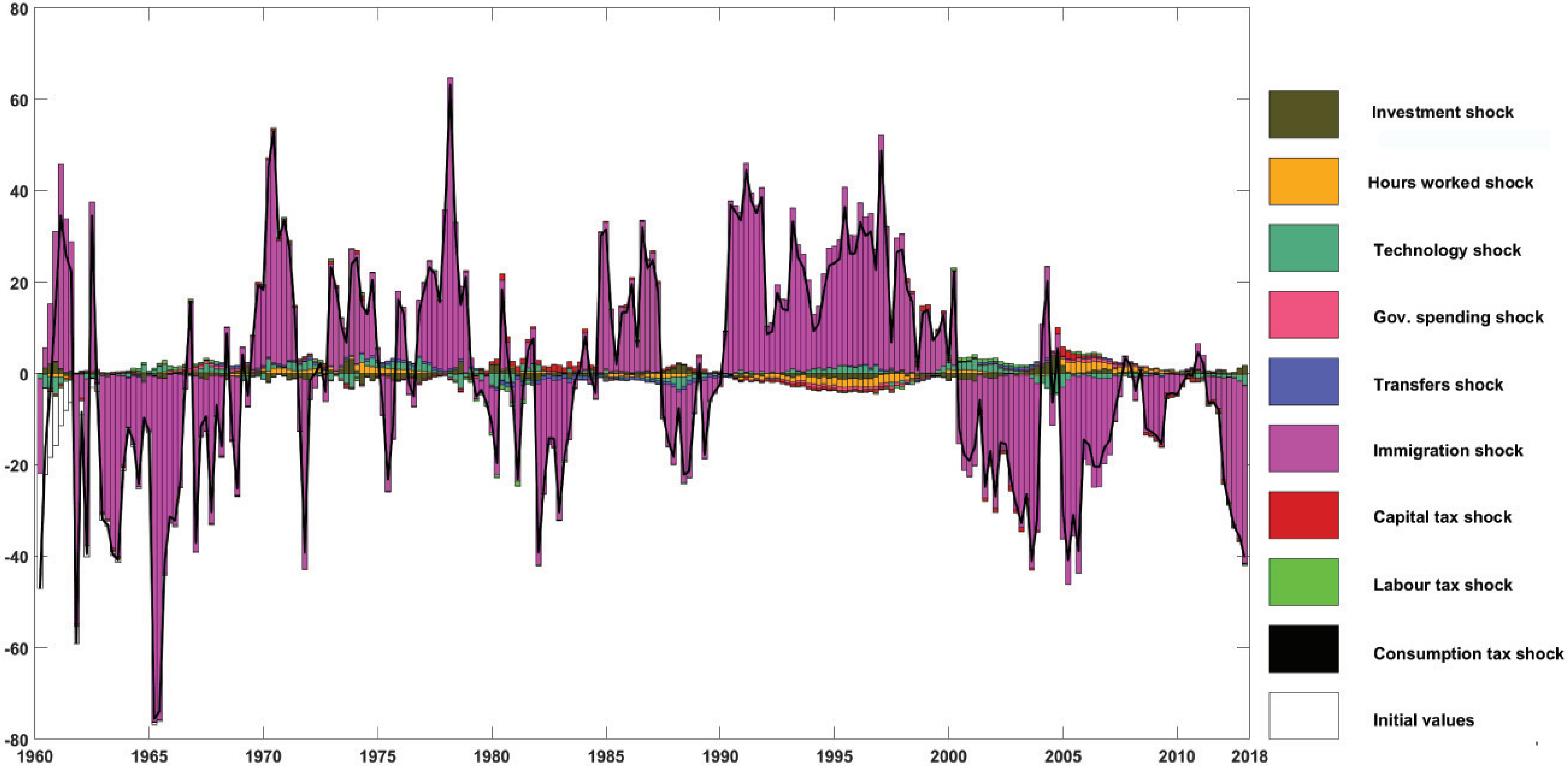


Figure 2.22: *Shock Decomposition of Immigration,* $\hat{m}_t$.

Chapter 3

Immigration and Fiscal Sustainability: A Structural VECM Analysis

3.1. Introduction

Every year the Congressional Budget Office (CBO) provides budget and economic information to the United States Congress, through its Long-Term Budget Outlook. The Outlook incorporates various scenarios for spending, revenue, deficits, and debt based on the estimates of multiple factors, such as productivity growth and interest rates on the federal debt. The 2019 Outlook that mainly covers the 30 years through 2049 projects large budget deficits over this period that will result in an unprecedented level of growth in the federal debt, to 144% of GDP by 2049 from 78% in 2009 (CBO, 2019). Together with the reality of the ageing demographics and rising spending, with increasing part of the healthcare costs in the US, this projections leads to concerns about the long-term sustainability of the federal government's fiscal policies.

There is considerable interest, especially from the academic and policy side, in the fiscal sustainability question. This interest is fuelled in part by the fact that following the 2007-2008 financial crisis, many governments found themselves faced with unexpected revenue shortfalls and increased expenditures leading to higher levels of debt. This naturally raises some critical questions in the academic literature about the quality of the theoretical models and empirical tests on fiscal sustainability. Bohn (1995) found existing empirical tests on US fiscal policy sustainability were based on 'too simple and inappropriate' theoretical models. Bohn (1992, 1995) stressed the importance of searching for more credible empirical tests for future research.

There have been many empirical studies that address this question, starting with the work of Barro (1974). Bohn (1995) re-examined the theoretical foundation of the sustainability problem by studying government policies in an explicitly stochastic general equilibrium model. He based his study on an empirical observation that historically, interest rates on 'safe' US government bonds have been significantly below the average rate of economic growth. Another study by Bohn (1998) examined the sustainability of the US's fiscal policy from 1916 to 1995. Canzoneri et al. (2001) used a two-variable VAR analysis to test the existence of Ricardian regime for the US. Markov-switching regression method employed by Favero and Monacelli (2005) to examine fiscal policy rules for the US between 1960 and 2002. Leeper (1991) and Davig and Leeper (2007) analyzed the fiscal policy over the post-war period in the US. Mountford and Uhlig (2005) introduced a new method that employed sign restrictions to identify fiscal policy surprises in vector autoregressions. Gale and Orszag (1992) used VAR based analysis of budget deficits, national savings and interest rates for the US.

At the same time, according to the (CBO, 2019), low population growth and ageing demographics make immigration a significant factor in the US economy. During the next ten years projected net inflows of immigrants are expected to account for approximately half of population growth and eventually almost four-fifths (CBO, 2019), changing both the size and composition of the US labour force. All these naturally raise some critical questions about whether policy makers should consider immigration's role when determining their fiscal policy and whether existing empirical studies should include immigration when performing fiscal sustainability tests. Although there are some studies on the consequences of demographics on public finance such as Elmendorf and Sheiner (2017), there is almost no study on the relationship between fiscal sustainability and immigration.

In light of recent changes, the impact of immigration on fiscal sustainability in the short and long-horizon naturally raises academicians' interest. Ben-Gad (2018) developed an overlapping generation model to study the relationship between the level of motivation of the native population to favour greater reliance on deficit finance of government expenditures through an intertemporal shift in factor taxation in the existence of rising immigration coupled with the declining birth rates in the US. The model demonstrates that the growth in public debt and unfunded liabilities in the US since the early 1980s, as well as large increases in debt projected by the CBO over the next few decades, may be explained by the rate at which the US economy absorbs new immigrant dynasties (Ben-Gad, 2018).

This chapter focuses on time series econometrics complements the more theoretical analyses that study and estimate the interaction between fiscal policy and immigration in Chapters 1 and 2. We aim to assess immigration's role in the long-term in the fiscal policy sustainability framework by applying a structural vector error correction model (SVECM) for the US from 1961:Q1 to 2018:Q1. After theoretically deriving and estimating the cointegrating relationship, we tried to answer this particularly interesting question from a macro-econometric perspective, which has important policy implications for many developed countries and contributes to the current academic literature.

This study is unique because it provides a new practical approach to evaluate the long-run impact of immigration on fiscal sustainability. This chapter's main contribution is to empirically evaluate immigration within the context of a structural error correction model. This allows studying its long-run and short-run impact on the debt formation in the destination economy, as it is crucial to study the impact of immigration on public finances when formulating immigration policy.

We incorporate the government budget constraint directly in the empirical model and extend the analysis to capture the long-run relationship between spending, tax revenues, debt, interest rate and immigration. To assess the importance of immigration within a structural vector error correction model (SVECM) by identifying important shocks, the respective impulse response functions and forecast error variance decompositions are quite useful in explaining the model's endogenous variables' variance. We also employed a structural vector error correction model (SVECM) to analyze these variables' dynamic interactions. Since the economic interpretation of fiscal policies that lead to maintaining sustainability over time has an important policy implication for governments, this study will contribute to the existing literature on fiscal sustainability and to what degree it might be influenced by immigration.

The estimated model can be used to evaluate the effect of counterfactual fiscal policies and immigration policies. Isolating and assessing the effect of immigration on debt and assessing the effect of migration on fiscal financing in the US and studying debt stabilizing (error correction) changes in fiscal policy related to immigration in the short, and long-horizon has many advantages over conventional VAR analysis.

The chapter is structured as follows. Section 2 contains the theoretical framework and shows how a cointegrating relationship can be derived from the log-linearized budget constraint. Section 3 defines and estimates the vector error correction model (VECM) and shows how structural vector error correction (SVECM) can be estimated in the presence of parameter restrictions. This section uses the theoretical model and economic theory to motivate the identifying assumptions for the structural analysis. In section 4, we present an impulse response analysis. Section 6 compares results from VECM and SVECM, and section 7 concludes.

3.2. The Theoretical Framework

This section introduces the theoretical steady-state or equilibrium relationship, which we expect to see in the data. We will review the intertemporal budget constraint and its implication for the stochastic processes of debt, tax receipts, government spending, discount rate and most importantly, immigration. The main intention in this part is to build the theoretical framework that integrates the data's common trends and analyses it using the most plausible scenarios. This will help evaluate the role of immigration on fiscal sustainability, provided government policy is subject to an intertemporal budget constraint.

3.2.1 The Government Budget Constraint

Almost all analytical discussions of fiscal sustainability as the starting point take a model where the government satisfies an intertemporal budget constraint and a static budget constraint for each period. Hence, this section starts with a theoretical framework that considers a nominal intertemporal government budget constraint (IGBC):

$$G_t + (1 + R_t) B_{t-1} = T_t + B_t \tag{3.1}$$

where G_t is the aggregate government expenditure including transfers to households, R_t is the interest rate on bonds issued at the end of period $t-1$, B_t is the nominal value of government bonds issued at the end of period t and T_t is the total taxes. For per capita budget constraints we divide it by the population defined as P_t

$$\frac{B_t}{P_t} - (1 + R_t) \frac{B_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} = \frac{G_t}{P_t} - \frac{T_t}{P_t} \tag{3.2}$$

We define:

$$\frac{P_{t-1}}{P_t} = \frac{1}{(1 + n_t)(1 + m_t)} \tag{3.3}$$

where the change in population decomposed to natural population growth rate n and inflow of immigrants, m_t (See Appendix 3.13). Therefore, the government budget constraint in per capita terms is,

$$b_t - \frac{(1 + R_t)}{(1 + n_t)(1 + m_t)} b_{t-1} = g_t - \tau_t \tag{3.4}$$

As noted by Bohn (1991) in essence, all the analysis based on the government budget constraint can be stated in terms of real variables, nominal variables, or relative to a scale variable like GDP, provided discount rate is interpreted appropriately. For the empirical analysis and in this theoretical section, it is convenient to use output shares. For this reason, we divide per capita variables by per capita output.

$$\frac{b_t}{y_t} - \frac{(1+R_t)}{(1+n_t)(1+m_t)(1+\gamma_t)}\frac{b_{t-1}}{y_{t-1}} = \frac{g_t}{y_t} - \frac{\tau_t}{y_t} \tag{3.5}$$

where γ_t is the output growth rate.[1]

The total government deficit as a proportion of GDP is defined as,

$$\frac{D_t}{y_t} = \frac{g_t}{y_t} - \frac{\tau_t}{y_t} + \frac{R_t}{(1+n_t)(1+m_t)(1+\gamma_t)}\frac{b_{t-1}}{y_{t-1}} \tag{3.6}$$

where the right-hand side shows the net borrowing required to fund the deficit expressed as a proportion of output. We also define nominal primary deficit or the total deficit exclusive interest payments as d_t

$$\frac{d_t}{y_t} = \frac{D_t}{y_t} - \frac{R_t}{(1+n_t)(1+m_t)(1+\gamma_t)}\frac{b_{t-1}}{y_{t-1}} = \frac{b_t}{y_t} - \frac{1+R_t}{(1+n_t)(1+m_t)(1+\gamma_t)}\frac{b_{t-1}}{y_{t-1}} \tag{3.7}$$

This is non-linear difference equation in b_t/y_t. As noted in Bohn (1991), interest payments are taken out of spending because the primary deficit is relevant for the intertemporal budget constraint.

We need to incorporate immigration as a variable into the intertemporal budget constraint. For this, we first define an interest rate adjusted for economic growth and natural population growth,

$$1+\rho_t = \frac{1+R_t}{(1+\gamma_t)(1+n_t)}, \tag{3.8}$$

(note $\rho_t \simeq R_t - \gamma_t - n_t$). Dividing both sides by $(1+m_t)$,

$$\frac{1+R_t}{(1+\gamma_t)(1+n_t)(1+m_t)} = \frac{1+\rho_t}{1+m_t}, \tag{3.9}$$

[1]Previous period debt b_{t-1} by present day output, y_t gives $\frac{b_{t-1}}{y_{t-1}}\frac{y_{t-1}}{y_t}$ where the second term is equal to $\frac{1}{(1+\gamma_t)}$.

we can rewrite the budget constraint to include the immigration rate

$$\frac{b_t}{y_t} = \left(\frac{1+\rho_t}{1+m_t}\right)\frac{b_{t-1}}{y_{t-1}} + \frac{d_t}{y_t}. \tag{3.10}$$

This is the key equation for determining the sustainability of fiscal policy that also includes the immigration rate, which dilutes the previous period debt.[2]

Solving the government budget constraint forwards over and n-period horizon gives present value budget constraint:

$$\frac{b_t}{y_t} = \mathbb{E}_t\left[\left(\prod_{s=0}^{n}\frac{1+m_{t+1}}{1+\rho_{t+1}}\right)\frac{b_{t+n}}{y_{t+n}}\right] - \mathbb{E}_t\left[\sum_{s=1}^{n}\left(\prod_{i=1}^{s}\frac{1+m_{t+1}}{1+\rho_{t+1}}\right)\frac{d_{t+s}}{y_{t+s}}\right] \tag{3.11}$$

where $\left(\prod_{s=0}^{n}\frac{1+m_{t+1}}{1+\rho_{t+1}}\right)$ can be interpreted as the discount factor applying between periods t and $t+n$.

This equation can be interpreted as follows; to cover the difference between the initial debt stock and the present value of the terminal debt stock, we need the future value of future primary surpluses to exceed the present value of primary deficits by a sufficient amount. This is the requirement of sustainability (or solvency) of debt.

The next step is to apply a no-Ponzi game restriction to the equation, which in the literature is typically regarded as synonymous with sustainability (Chalk and Hemming, 2000). O'Connell and Zeldes (1988) studied a 'rational Ponzi game' where all principal repayments and interest are forever rolled over. Which naturally raises a question: If the government debt is only financed by issuing new debt, is it feasible for a government to incur debt and never pay back any principal or interest?

O'Connell and Zeldes (1988) have demonstrated that the feasibility of a rational Ponzi game depends on some key characteristics of the economy whose agents are

[2]The resulting per capita intertemporal budget constraint is not different from the ones in the literature for determining the sustainability of fiscal policy: $\frac{b_t}{y_t} = (1+\rho_t)\frac{b_{t-1}}{y_{t-1}} + \frac{d_t}{y_t}$. See Polito and Wickens (2012).

going to hold the debt. They demonstrate that it is not feasible for the government to run such a Ponzi game in a model where agents care only about lifetime consumption (or in a model with a finite number of agents). When a rational agent faces holding debt and having a lower consumption, thus welfare, at least one period, is strictly dominated by holding no debt. As a result, a government attempting to run a Ponzi game will find no rational individual is willing to hold its liabilities.

Therefore, fiscal sustainability depends on the transversality condition being satisfied, since private agents cannot end up being indebted to the government for at least one period to forgone some portion of their consumption:

$$\lim_{n\to\infty} \mathbb{E}_t \left[\left(\prod_{s=0}^{n} \frac{1+m_{t+1}}{1+\rho_{t+1}} \right) \frac{b_{t+n}}{y_{t+n}} \right] = 0 \tag{3.12}$$

In other words, fiscal policy sustainability requires that the expected value of the discounted debt-GDP ratio must tend to zero in the limit. If it is satisfied, then the resulting equation for the present value of the debt-output ratio is as following:

$$\frac{b_t}{y_t} = \mathbb{E}_t \left[\sum_{s=1}^{n} \left(\prod_{i=1}^{s} \frac{1+m_{t+1}}{1+\rho_{t+1}} \right) \left(\frac{-d_{t+s}}{y_{t+s}} \right) \right] \tag{3.13}$$

This equation shows that sustainability requires that the present value of current and future primary surpluses must be sufficient to offset current debt liabilities, all expressed as a portion of GDP. Immigration dilutes the per capita public debt as the population grows; in other words, immigrants inherit the portion of the outstanding debt. Equation (3.13) shows that the higher the immigration, the lower the primary surplus needed to stabilize the debt. Initially, immigrants are likely to absorb public resources, however as they join the economy as workers, consumers, and taxpayers, their contribution to primary balance becomes positive in the long term. The upward economic mobility and taxpaying lifetime of immigrants and their descendants over the long term more than offset the initial fiscal burden due to the provision of public services to immigrants and their families, making them net positive contributors to the federal budget (NRC, 1997). We assume the natural population growth rate remains constant and is the same for everyone.

3.2.2 Log-linearising the budget constraint with immigration

In order to have an interest rate and immigration as an additional variable in the budget constraint, we employ a log-linear transformation of the budget constraint. Log-linearising also will able us to analyze fiscal policy when the immigration and discount rate are time-varying.

A log-linear approximation to the GBC about the steady-state is derived as

$$\ln\left(\frac{b_t}{y_t}\right) \simeq c + \frac{g}{b}\ln\left(\frac{g_t}{y_t}\right) - \frac{\tau}{b}\ln\left(\frac{\tau_t}{y_t}\right) + \left(\frac{1+\rho}{1+m}\right)\left[\ln(1+\rho_t) - \ln(1+m_t) + \ln\left(\frac{b_{t-1}}{y_{t-1}}\right)\right], \tag{3.14}$$

where constant c is defined only by steady-state values, which is very useful to define the equilibrium (steady) state relations between our variables:

$$c = -\left(\frac{\rho - m}{1+m}\right)\ln\left(\frac{b}{y}\right) - \frac{g}{b}\ln\left(\frac{g}{y}\right) + \frac{\tau}{b}\ln\left(\frac{\tau}{y}\right) + \left(\frac{1+\rho}{1+m}\right)\left[\ln(1+m) - \ln(1+\rho)\right] \tag{3.15}$$

Substituting $\ln(1+\rho_t) \simeq \rho_t$ and $\ln(1+m_t) \simeq m_t$

$$c = -\left(\frac{\rho - m}{1+m}\right)\ln\left(\frac{b_t}{y_t}\right) - \frac{g}{b}\ln\left(\frac{g_t}{y_t}\right) + \frac{\tau}{b}\ln\left(\frac{\tau_t}{y_t}\right) + \left(\frac{1+\rho}{1+m}\right)\left[m_t - \rho_t\right] \tag{3.16}$$

and

$$\ln\left(\frac{b_t}{y_t}\right) \simeq c + \frac{g}{b}\ln\left(\frac{g_t}{y_t}\right) - \frac{\tau}{b}\ln\left(\frac{\tau_t}{y_t}\right) + \left(\frac{1+\rho}{1+m}\right)\left[\rho_t - m_t + \ln\left(\frac{b_{t-1}}{y_{t-1}}\right)\right] \tag{3.17}$$

Now, fiscal sustainability may be analysed with a linear model, even though the interest (discount) rate and immigration are time-varying. The stability of the log-linearised government budget constraint depends on the sign of ρ and m. Assuming that both $\rho > 0$ and $m > 0$, we solve equation forwards to obtain:

$$\ln\left(\frac{b_t}{y_t}\right) = \left(\frac{1+m}{1+\rho}\right)^n \mathbb{E}_t\left(\ln\left(\frac{b_{t+n}}{y_{t+n}}\right)\right) - \sum_{s=1}^{n}\left(\frac{1+m}{1+\rho}\right)^n \mathbb{E}_t\left(k_{t+s}\right), \tag{3.18}$$

where k_t is logarithmic equivalent of the primary deficit, d_t.

$$k_t = c + \frac{g}{b}\ln\left(\frac{g_t}{y_t}\right) - \frac{\tau}{b}\ln\left(\frac{\tau_t}{y_t}\right) + \left(\frac{1+\rho}{1+m}\right)[\rho_t - m_t] \tag{3.19}$$

Fiscal sustainability depends on transversality condition being satisfied,

$$\lim_{n\to\infty}\left(\frac{1+m}{1+\rho}\right)^n \mathbb{E}_t\left(\ln\frac{b_{t+n}}{y_{t+n}}\right) = 0 \tag{3.20}$$

If it is satisfied then it implies that,

$$\ln\left(\frac{b_t}{y_t}\right) = -\sum_{s=1}^{\infty}\left(\frac{1+m}{1+\rho}\right)^n \mathbb{E}_t\left(k_{t+s}\right). \tag{3.21}$$

If k_t is stationary then $\ln\left(\frac{b_t}{y_t}\right)$ and hence $\left(\frac{b_t}{y_t}\right)$, remains stationary and finite. This may occur due to the individual terms of k_t being stationary, or due to some terms being $I(1)$, but being *cointegrated* with the appropriate cointegrating vector. As noted by Polito and Wickens (2012), if k_t and its components are $I(1)$ then, if they are also cointegrated with the cointegrating vector given by the coefficients in the definition of c, then fiscal sustainability is still satisfied.

Nevertheless, before moving to the derivation of the cointegrating vector in the next section, we continue with the log-linearised budget constraint:

$$\Delta\ln\left(\frac{b_t}{y_t}\right) \simeq c + \frac{g}{b}\ln\left(\frac{g_t}{y_t}\right) - \frac{\tau}{b}\ln\left(\frac{\tau_t}{y_t}\right) + \left(\frac{\rho-m}{1+m}\right)\ln\left(\frac{b_{t-1}}{y_{t-1}}\right) + (1+\rho)[\rho_t - m_t] \tag{3.22}$$

Left-hand side of the equation, $\Delta\ln\left(\frac{b_t}{y_t}\right)$, is stationary and in order equation to be consistent the right-hand side also has to be stationary.

3.3. The Cointegration Vector

Cointegration implies that certain linear combinations of the variables of the vector process are integrated of a lower order than the processes themselves, and such a relation can often be interpreted as long-run economic steady-state or equilibrium relation. Therefore, intuition guides look for a cointegrating vector in c (Eq. 3.16) and solve for debt to derive the following cointegration vector.

$$\ln\left(\frac{b_t}{y_t}\right) \simeq \left(\frac{1+m}{\rho - m}\right)\left[-c - \frac{g}{b}\ln\left(\frac{g_t}{y_t}\right) + \frac{\tau}{b}\ln\left(\frac{\tau_t}{y_t}\right)\right] + \left(\frac{1+\rho}{1+m}\right)[m_t - \rho_t] \tag{3.23}$$

We define $\phi_1 = \left(\frac{1+m}{m-\rho}\right)$ and $\phi_2 = \left(\frac{1+\rho}{m-\rho}\right)$, then cointegrating equation, i.e. relation between debt and other non-stationary and stationary variables becomes

$$\ln\left(\frac{b_t}{y_t}\right) \simeq \phi_1\frac{g}{b}\ln\left(\frac{g_t}{y_t}\right) - \phi_1\frac{\tau}{b}\ln\left(\frac{\tau_t}{y_t}\right) - \phi_2 m_t + \phi_2\rho_t + \phi_1 c \tag{3.24}$$

This relationship embodies the view that the public debt-output ratio must be $I(0)$, even when some of its components are not. It can also be presented in the levels of variables as following

$$\ln b_t = \phi_1\frac{g}{b}\ln g_t - \phi_1\frac{\tau}{b}\ln\tau_t - \left(1 + \phi_1\frac{g}{b} + \phi_1\frac{\tau}{b}\right)\ln y_t + \phi_2\rho_t - \phi_2 m_t + \phi_1 c$$

3.4. Empirical Model

The error correction models (ECMs) represent a theoretically-driven approach useful for estimating a time series' short-term and long-term effects on each other. The term error-correction relates to the fact that the last period's deviation from a long-run equilibrium, the error, influences its short-run dynamics. Thus ECMs directly estimate the speed at which a dependent variable returns to equilibrium after a change in other variables. Because of the stochastic nature of the trend, it is not possible to break up integrated series into a deterministic (predictable) trend and a stationary series containing deviations from the trend. Even in deterministically detrended random walks, spurious correlations will eventually emerge. Thus detrending does not solve the estimation problem.

When the VECM is correctly specified and estimated, we can represent the estimates as a VAR model in levels to construct forecasts and impulse responses. Besides, the primary motivation is to use VECMs because they facilitate the imposition of restrictions on the long-run effects of structural shocks in the VAR model, which extends the range of identifying assumptions used for structural impulse response analysis (Kilian and Lutkepohl, 2017). In some cases, these restrictions are implied by economic theory. We can specify a VECM if that model can be economically motivated, and the data do not object to this specification. In other cases, they may be suggested by statistical tests (Kilian and Lutkepohl, 2017).

Before starting our further analysis, we need to highlight several general comments on model dynamics, specification, estimation and structure. The roots of the model's companion matrix determine its dynamic properties. Therefore, impulse response analysis is an alternative way of presenting this information and data properties as well as model specification affect impulse response function (Ericsson et al., 1998).

We employ the data-based modelling approach used by Johansen and Juselius (1994), which in brief can be defined as *"to look at the data as structured by the statistical model from the point of view of a variety of economic theories or hypothesis"*. Therefore, we need to examine if it is possible to extract economically meaningful results from the empirical analysis of the given information set. By starting with a well-defined statistical model, employing this approach will allow us to reveal the basic features of the data generating process (DGP) and not restrict ourselves to several hypotheses relevant for one economic model (Johansen and Juselius, 1992).

In this section, we report the results from estimates of the model with immigration. The empirical problem in this section can be seen as a direct continuation of the analysis in fiscal sustainability in the literature developed in Trehan and Walsh (2007), Bohn (1991, 1992, 1995, 1998) and Polito and Wickens (2007, 2012). The aim of the empirical analysis is twofold:

- To investigate whether it is possible to find a relation between debt formation, fiscal sustainability and immigration using the multivariate cointegration analysis, and
- whether we could empirically verify the theoretical steady-state relation (equilibrium relation) and thus the cointegrating vector developed in previous sections.

3.5. The Basic Statistical Model

In this section we will follow the basic steps in Johansen's methodology described in Johansen and Juselius (1992), Juselius (2006), Lutkepohl and Kratzig (2004) and Kilian and Lutkepohl (2017). First, we will specify and estimate a VAR(p) model for data series. Then we will construct likelihood ratio tests for the rank of Π to determine the number of cointegrating vectors. If necessary, we will impose normalization and identify restrictions on the cointegrating vector(s). Finally, given the normalized cointegrating vectors, estimate the resulting cointegrated VECM by maximum likelihood (ML).

The basic model is an unrestricted VAR model which is estimated on the following assumptions:

$$x_t = A_1 x_{t-1} + ... + A_k x_{t-k} + \Phi D_t + \epsilon_t, \quad t = 1, ..., T \tag{3.25}$$

where ϵ_t are *iid* $N_p(0, \Omega)$ and D_t is a vector of deterministic variables such as a constant, linear trend and seasonal or intervention dummies. The estimates reported are calculated by running an OLS regression equation bu equation. As long as no restrictions have been imposed on the VAR model, these estimates are maximum likelihood estimates.

Unit root test: The first step is to test the individual time series in order to confirm that they are non-stationary in the first place. After checking the time series for stationarity and integration, we will be able to categorize variables as integrated steady-state trends, enabling us to solve and analyze for long-term and short term effects in the model. In the case of cointegration between $I(1)$ variables, we can use error correction formulation, which gives the flexibility to capture the system's dynamics while integrating the equilibrium suggested by economic theory.

From the visual inspection of time series in levels and their first differences (Figure 3.9 (b)-(f), Appendix 3.17), we can say that except the series for immigration and interest rate, time series are non-stationary and difference stationary. First differences of the time series all seem $I(0)$, which also is checked for the unit root. There appears a break in the trend growth of debt-output, spending-output, tax receipts-output ratios, and interest rate, especially around 2000 and 2008. Such behaviour is compatible with the series being integrated of some order. We do not observe obvious seasonality in the data.

The results for the Augmented Dickey-fuller (ADF) tests are presented in Table 3.6 in Appendix 3.15. Results show that the series for the logs of debt-output, spending-output and tax revenues-output ratios have unit root and difference stationary $I(1)$ series. On the contrary, both the adjusted interest rate ρ_t and immigration m_t, are stationary series. We also check for the unit root in the first differences of the time series to see if we have $I(2)$ series. The results showed that none of the time series is $I(2)$. The presence of the $I(2)$ variables usually complicates the econometric implementation and the corresponding economic interpretation (Ericsson et al., 1998).

Lag length analysis: The test procedure for determining lag length includes the Akaike, the Schwartz and Hannan-Quinn information criteria. The idea is to calculate the test criterion for different values of k and then choose the value of k that corresponds to the smallest value (Lutkepohl and Kratzig, 2004; Juselius, 2006). The test completed for lag determination implemented for the initial VAR in differences with four and eight lags and unrestricted constant with sample 1960:Q1 - 2018:Q1 for estimation. Lag structure analysis is made by checking both the lag length information criteria and the LM test of serial correlation of the residuals. Test results presented at Table 3.7 in Appendix 3.15. We can conclude that up to two lags is an appropriate order for the analysis, as we know that this number is one less than the VAR order. As Juselius (2006) notes, it is seldom the case that a well-specified model needs more than two lags. The lag structure analysis tells us that it is useful to start with two lags VECM model, search for structural shifts, and, if necessary, re-specify the model.

Unrestricted VAR estimate. To start the analysis, and to have the unrestricted VAR estimates, we formulate series as, $x_t = \{\ln(\frac{b_t}{y_t}), \ln(\frac{g_t}{y_t}), \ln(\frac{\tau_t}{y_t}), \rho_t, m_t\}$ for the sample period for estimation from 1961:Q1 to 2018:Q1, keeping restricted constant for the mean specification. With only exception of immigration and discount rate all other variables are $I(1)$, i.e. difference stationary.

We have restricted constant in the model. We do not include time trend into the model as our tests show it is insignificant. In practice, few empirical macroeconomics researchers include a deterministic time trend in the VAR model (Juselius, 2006). Macroeconomic time series are highly persistent, and a deterministic time trend in conjunction with a (near) unit root in the autoregressive lag order polynomial implies a (near) quadratic trend in variables (Kilian and Lutkepohl, 2017).

Cointegration rank. Johansen's cointegration test result with lag interval 2, allowing for a restricted deterministic with no trend in cointegrating equation (CE), with 0.05 MHM critical values can be seen at Table 3.8 of Appendix 3.15. The test results imply that we cannot reject unit root; thus $\lambda_3 = \lambda_4 = \lambda_5 = 0$ with the given p-value. Hence the rank test statistics suggest that the system has three common trends and two cointegrating relations. Based on this finding, the VEC model for series should be applied.

In the literature, the cointegration relationships are usually seen as linear combinations of $I(1)$ variables. However, there is also a case that some of the elements of a vector of variables are $I(0)$ in levels. In this case, there is an additional cointegrating relationship for each stationary component of x_t as these cointegrating vectors are linearly independent columns of coefficients of cointegration vector, β, the cointegrating rank must be at least as large as the number of $I(0)$ variables in the system (Lutkepohl and Kratzig, 2004; Juselius, 2006; Kilian and Lutkepohl, 2017). As we have two stationary variables in our model, we expect to see at least two cointegration relationships.

3.5.1 Testing the Adequacy of the Model

The choice of the cointegrating rank r should be based on a well-specified model, so it is recommended to use the residuals from the unrestricted model to decide whether our model is acceptable or not. This can be done by resetting the rank to $r = p$, in which case the residuals are the OLS-estimates from the model. Adequacy of a given model can be assessed in several ways: by visual inspection of the key properties of the variables, misspecification tests of the estimated residuals, and parameter constancy. Given the current model specification, many software packages provide a residual analysis consisting of various descriptive statistics and misspecification tests based on the estimated residuals.

Roots of the companion matrix. Figure 3.9(a) in Appendix 3.17 displays the plot of roots of the companion matrix showing graphically the eigenvalues of the companion matrix for the current rank r. The eigenvalues should be inside the unit disc or equal to 1 under the cointegrated VAR model's assumptions. Eigenvalues outside the unit disc correspond to explosive processes. If the process is $I(1)$, the number of unit roots is equal to $(p - r)$, which is also the number of common stochastic trends in the model. In our case, for the unrestricted model ($r = 5$), all the roots are inside the unit disc with three roots close to unity. We are setting $r = 2$, suggesting that the rank of Π is two.

Visual inspection. Figure 3.9, (b)-(f) of Appendix 3.17 demonstrate the default residual graphics for the estimated model, plotting the estimated residuals of the unrestricted VAR model of $r = p$ rank with no restrictions imposed. We note some very large positive residuals or increases in the first middle panel with debt levels, labelled [$r : Dln\_by$], which can be explained by the rise in public debt from $3.34 trillion in September 2001 to $6.37 trillion by the end of 2008. In the aftermath of the global recession of 2007-2009, due to the related significant revenue declines and spending increases, the public debt rose to $11.9 trillion by the end of July 2013, under the presidency of Barack Obama. Revenue declines and spending increases can also be observed in government spending, and tax receipts share in GDP graphs.

Residual analysis. When modelling economic behaviour, we assume that the economic agents learn from their experience and adjust their behaviour accordingly. In other words, our assumption of agents' rationality leads to the belief that the residual, the unexpected component, behaves like a normal innovation process (Johansen, 1996; Juselius, 2006). Significant political or institutional interventions and reforms usually violate normality assumptions, as they frequently account for the extraordinarily large and non-normal shocks in VAR analysis (Juselius, 2006). Therefore, we need to include several dummy variables to deal with the lack of normality in the residuals caused by the shocks from significant interventions, reforms, crises.

Analysis of outliers in data. The 2009 spending level was the highest relative to GDP in 40 years, while the tax receipts were the lowest relative to GDP in the same period. The next highest spending year was 1985, 22.8%(CBO, 2010). Congressional Budget Office attributes 72% to legislated tax rate cuts and spending increases and 27% to economic and technical factors. Of the latter, 56% occurred from 2009 to 2011 (CBO, 2012). The difference between the projected and actual debt, according to the Congressional Budget Office, could be mainly attributed to economic and political interventions, such as lower than expected tax revenues and higher safety net spending due to recession, Bush tax cuts, wars in Afghanistan and Iraq and President Obama's stimulus and tax cuts (ARRA and Tax Act of 2010). In order to account for the above political interventions, we defined specific dummies.

Quantitative Easing. We mentioned before about impact of significant political or institutional interventions in VAR analysis. Quantitative Easing (QE) is not considered to be one of these interventions. When the targeted federal funds rate fell to "zero lower bound" interval, the Fed initiated a massive purchases of Treasury securities and mortgage-backed securities in hope to influence longer-term interest rates. However, reviewing the effects of the Fed's asset holdings on long-term interest rates over 2009 to 2019, Hamilton (in Cochrane and Taylor (2020), Chapter 4, p.174) concluded that the impact on these rates was ultimately not very large. Hamilton (2018) and Greenlaw et al. (2018) review recent U.S. monetary policy experience with large scale asset purchases and come to a similar conclusion. Greenlaw et al. (2018) argue that the consensus from previous studies that these purchases reduced the yields on 10-year Treasuries is overstated and the effects of Fed actions did not persist. Likewise, inspection of the time series for rate on a 10-year U.S. Treasury security reveals a long downward trend and there is not much evidence of more than a temporary change for Quantitative Easing. In Figure 3.1, the long-term government bond yields do not exhibit evidence of a structural break similar to what we observe in the federal funds rate.

Deterministic components in the model. Consistent with the above discussions, we re-estimated the model with five dummies to capture shocks to the model variables: public debt, interest rate, immigration, government spending and tax revenues ($Ddebt$, Dek, $Dimm$, $Dspen$, $Dtax$). The residuals' properties from the estimated VAR model after including dummies have now considerably improved compared to the unrestricted model, and the model does noticeably better at describing the data. Graphics and test results indicate that the dummies have taken care of the largest residuals, the hypothesis of normality can be accepted. Vector LM-test on the system indicates some autocorrelated errors. While uncorrelated errors would be desirable, they are not a precondition for the validity of the cointegration tests (Juselius, 2006).

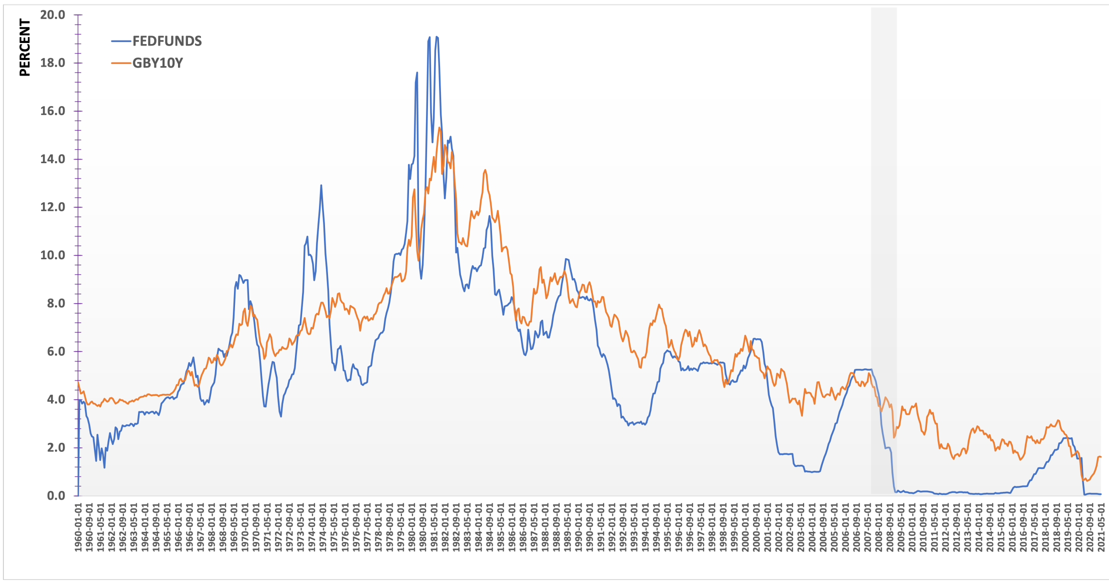


Figure 3.1: *Effective Federal Funds Rate vs Long-Term Government Bond Yields: 10-year.*
Source: FRED Federal Reserve Bank of St. Louis. Monthly, Percent, Not Seasonally Adjusted. Retrieved from: Retrieved from: https://fred.stlouisfed.org/.

Recursive analysis. The recursive analysis presented in Figure 3.8 in Appendix 3.17 shows the log-likelihood constancy and recursive evaluation of the Bartlett-corrected trace test statistics, together with the time path of the sum of the transformed eigenvalues. Hence, results support our choice of $r = 2$.

3.6. Estimating the VECM

The basic VAR model in (3.25) is generally sufficient to accommodate variables with a stochastic trend. However, we need to look for a more convenient model setup for cointegrating analysis. That is the most suitable to display the cointegration relations explicitly (Lutkepohl and Kratzig, 2004). By substructing x_{t-1} from both sides of levels VAR form (3.25) and rearranging the terms, we can obtain the vector error-correction form.

$$\Delta x_t = \Pi x_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \Phi D_t + \epsilon_t, \quad t = 1, ..., T \tag{3.26}$$

where $\Pi = \sum_{i=1}^{k} A_i - I_p$ and $\Gamma_i = -\sum_{i=i+1}^{k} A_i$.

If the VAR process has unit roots, the matrix, Π, can be written as a product of matrices α and β as follows: $\Pi = \alpha\beta'$ (Johansen, 1996; Lutkepohl and Kratzig, 2004; Juselius, 2006; Kilian and Lutkepohl, 2017). β is a cointegration matrix, and α contains the weights attached to the cointegrating relations in the model's individual equations, called the loading matrix. Therefore, the term $\Pi = \alpha\beta'$ contains the cointegration relations and sometimes called the long-run part. Γ_is are often referred to as the short-run parameters.

Both rank-test statistics, trace and eigenvalue tests, indicate the existence of about two cointegration relations in the data. Accordingly, we estimate VECM's cointegration vectors. Software packages allow computing β coefficients and their respective signs of the vectors. With the data sample from 1961:Q1 to 2018:Q1 and a restricted constant in the cointegration vector, we have two cointegration vectors presented in Table 3.1.

The first cointegration vector in Table 3.1 may be interpreted as an equation in which public debt is related to the primary deficit, interest rate and immigration. As we can see, the signs of beta coefficients of the cointegration vector are in accordance with the derived theoretical cointegration equation, except for the constant.

The second cointegration vector shows how the interest rate relates to the immigration, public debt and primary balance elements; government expenditure and tax revenues. In line with theoretical arguments as in Ben-Gad (2004, 2008, 2018) and Kiguchi and Mountford (2013) immigration has a positive impact on interest rates, which is also confirmed by the data, as the coefficient for immigration is positive and significant.

It is worth mentioning the literature's approach regarding the impact of the primary deficit on interest rates. Gale and Orszag (1992) noted that the evidence from the empirical literature on the impact of fiscal policy on interest rates as a whole is mixed. This is partly due to the statistical issues, such as different definitions of deficits and debt, whether deficits or debt should be the variable of interest, the difficulty of distinguishing expected and unexpected changes, and

	$\ln\left(\frac{b_t}{y_t}\right)$	$\ln\left(\frac{g_t}{y_t}\right)$	$\ln\left(\frac{\tau_t}{y_t}\right)$	ρ_t	m_t	c
$CVec(1)$:	1	-11.8	12.8	-126	164	6.33
$CVec(2)$:	0.008	-0.043	0.104	1	-15.3	0.143
$\hat{\alpha}_1'$:	-0.006 (-8.5)	-0.001 (-0.9)	-0.002 (-1.5)	0.002 (0.5)	0.000 (-0.3)	–
$\hat{\alpha}_2'$:	-0.174 (-2.1)	0.231 (2.0)	-0.406 (-2.1)	-0.203 (-3.9)	0.0034 (2.4)	–
$\Pi = \alpha\beta'$, the long-run coefficients:						
$\Delta \ln\left(\frac{b_t}{y_t}\right)$	-0.00685 (-7.5)	0.0726 (8.6)	-0.0883 (-7.4)	0.518 (4.5)	1.77 (1.4)	-0.0597 (-4.8)
$\Delta \ln\left(\frac{g_t}{y_t}\right)$	0.001 (0.8)	-0.000566 (-0.0)	0.0139 (0.8)	0.332 (2.0)	-3.67 (-2.0)	0.028 (1.6)
$\Delta \ln\left(\frac{\tau_t}{y_t}\right)$	-0.00554 (-2.5)	0.0458 (2.2)	-0.0727 (-2.5)	-0.106 (-0.4)	5.82 (1.9)	-0.0732 (-2.4)
$\Delta\rho_t$	-0.00139 (-2.4)	0.00652 (1.2)	-0.0187 (-2.5)	-0.227 (-3.1)	3.14 (4.0)	-0.0279 (-3.6)
Δm_t	2.34e-05 (1.5)	-0.000111 (-0.8)	0.000315 (1.5)	0.00379 (1.9)	-0.0525 (-2.4)	0.000467 (2.2)
Note: t-statistics in parentheses.						

Table 3.1: *Cointegration vectors and loading parameters for VECM with two lagged differences and cointegrating rank r=2.*

the potential endogeneity of many of the key explanatory variables. An excellent literature analysis on this topic is completed by Barth et al. (1991), which was later updated by Gale and Orszag (1992).[3]

3.6.1 Estimation of the Subset VECM

Usually, vector error-correction and structural VEC models are estimated by Maximum Likelihood (ML), and many software packages are available to solve the

[3]The updated survey of Barth et al. (1991) by Gale and Orszag (1992) is presented in Table 3.9 in Appendix 3.16. Barth et al. (1991) surveyed 47 studies through 1989, of which 17 found a 'predominately significant, positive' effect of deficits on interest rates (that is, larger deficits raised interest rates); 6 found mixed effects, and 19 found 'predominately insignificant or negative' effects.

computationally demanding problem. However, because the likelihood function is concentrated with respect to α and Γ_i maximum likelihood (ML) estimation of VECMs and structural VECMs does not allow for additional restrictions on the short-run dynamics, which in turn affect the precision of estimation as reflected by wide confidence bands of the impulse response functions. Accordingly, it is desirable to apply restrictions on short-run parameters based on statistical procedures to have better and precise estimates. We employed the two-stage procedure to estimate restricted subset VECM, where restrictions can also be imposed on the loading coefficients, the short-term dynamics and the deterministic terms as in Lutkepohl and Kratzig (2005). In the first stage, we specified the estimation of cointegration relation, and in the second stage, we can choose between OLS, GLS and 3SLS, or let the software package choose the estimation method.

Some variables can be considered weakly exogenous for the long-run parameters of β. In other words, a weakly exogenous variable has the property that has no permanent effect on any of the other variables in the system. The usefulness of estimated models for an economic policy depends on many factors, including valid exogeneity assumptions, that permit simpler and less computational modelling strategies and help isolate invariants of the economic mechanism (Ericsson et al., 1998). The two-stage procedure developed by Lutkepohl and Kratzig (2004) allows specifying subset model where we can impose zero restrictions on the parameters of a model based on the t-ratios or restrictions individual parameters or groups of parameters may be based on model selection criteria. Many software packages offer suitable model selection procedures. Sequential Elimination of Regressors (SER) strategy sequentially eliminates the regressors with the smallest absolute values of t-ratios until all t-ratios (in absolute value) are greater than some threshold value.

We obtained from the two stage procedure estimate two cointegration relations with normalization in log of debt-output, $\ln\left(\frac{b_t}{y_t}\right)$ and in ρ_t as seen in Table 3.2.

	$\ln\left(\frac{b_t}{y_t}\right)$	ρ_t	$\ln\left(\frac{g_t}{y_t}\right)$	$\ln\left(\frac{\tau_t}{y_t}\right)$	m_t	c
$CVec(1):$	1	0	-9.148 (-4.953)	7.799 (3.822)	-521.03 (-1.793)	2.926 (0.639)
$CVec(2):$	0	1	0.013 (0.758)	-0.007 (-0.356)	-7.991 (-2.860)	0.018 (0.413)
$\hat{\alpha}'_1:$	-0.009 (-8.877)	-0.002 (-2.983)	–	-0.003 (-1.511)	–	
$\hat{\alpha}'_2:$	0.545 (5.746)	-0.242 (-3.759)	–	–	0.003 (1.748)	
Note: t-statistics in parentheses.						

Table 3.2: *Cointegration vectors and loading parameters for VECM with two lagged differences and cointegrating rank r=2.*

The diagnostic tests for the VECM do not indicate signs of misspecification. Since general test results do not rise a concern about the instability of the model, we can regard the VECM as an adequate description of the data.

We have to mention that our choice of variables for normalization and weak exogeneity (implying zero restriction to the loading coefficient α) also supported by tests of restriction on coefficients of the cointegration vectors. An interesting result can be seen when looking at the results of the unit vector in alpha and long-run weak exogeneity tests reported at Table 3.3.

Test of Restrictions			
Test of weak exogeneity			
Test of $\ln\left(\frac{b_{t-1}}{y_{t-1}}\right)$:	$\chi^2(2)$	=	72.569 [0.0000]**
Test of $\ln\left(\frac{g_{t-1}}{y_{t-1}}\right)$:	$\chi^2(2)$	=	5.2055 [0.0741]
Test of $\ln\left(\frac{\tau_{t-1}}{y_{t-1}}\right)$:	$\chi^2(2)$	=	5.5529 [0.0623]
Test of ρ_{t-1}:	$\chi^2(2)$	=	10.723 [0.0047]**
Test of m_{t-1}:	$\chi^2(2)$	=	3.1682 [0.2051]
Test of unit vector in alpha			
Test of $\ln\left(\frac{b_{t-1}}{y_{t-1}}\right)$:	$\chi^2(3)$	=	5.0593 [0.1675]
Test of $\ln\left(\frac{g_{t-1}}{y_{t-1}}\right)$:	$\chi^2(3)$	=	12.035 [0.0073]**
Test of $\ln\left(\frac{\tau_{t-1}}{y_{t-1}}\right)$:	$\chi^2(3)$	=	30.348 [0.0000]**
Test of ρ_{t-1}:	$\chi^2(3)$	=	11.188 [0.0108]*
Test of m_{t-1}:	$\chi^2(3)$	=	25.579 [0.0000]**

Table 3.3: *Test results for alpha restrictions.*

Results show that a unit vector in alpha for US public debt can be accepted with a p-value of 0.17. We conclude that US public debt has been purely adjusting. This implies that the shocks to the public debt have not had any permanent effects on any variables in the system inclusive itself.[4] The batch test long-run weak exogeneity checks whether any of the variables can be regarded as weakly exogenous when β is unrestricted. By looking at the results, we note that government spending, taxes and immigration can be considered weakly exogenous. These test results support two-stage estimations imposition of zero on loading coefficients α.

The first cointegration vector at Table 3.2 may be interpreted as the equation in which debt is related to primary deficit and immigration. It implies the significant positive relationship between the debt-output ratio and government spending to output and the negative relationship between taxes and output ratio. Intuitively, high government expenditure, if not offset by revenues, contributes to the pile-up

[4] A variable having a unit vector in α implies that the variable in question is purely adjusting, i.e. its disturbances have no permanent effect on the system, hence do not enter the common stochastic trends. Such a variable can be considered endogenous in the VAR system (Doornik and Juselius, 2018).

of public debt stock. Also, due to normalization, the constant and immigration could pick up the effect of interest rate on debt. As we can see, immigration enters the relation with the positive sign. This result is in line with Ben-Gad (2018) and the results of Chapters 1 and 2.

The second cointegration vector shows how interest rate related to government expenditure, tax revenues and immigration. In line with theoretical arguments and empirical studies such as Ben-Gad (2004, 2008, 2018) immigration has a positive impact on the interest rate, which also confirmed by the data. The coefficient for immigration is positive and significant. Although less significant, the coefficients for government spending and taxes to output ratios also display expected signs.

By applying the SER strategy to our model, we have reduced the number of parameters substantially. The estimates of coefficients for cointegration vectors, lags and deterministic of the model is presented in the next page.

The Estimates of Subset VEC model

$$
\begin{bmatrix} \Delta\ln(\frac{b_t}{y_t}) \\ \Delta\rho_t \\ \Delta\ln(\frac{g_t}{y_t}) \\ \Delta\ln(\frac{\tau_t}{y_t}) \\ \Delta m_t \end{bmatrix} =
$$

$$
\overbrace{\begin{bmatrix} \underset{(-8.877)}{-0.009} & \underset{(5.746)}{0.545} \\ \underset{(-2.983)}{-0.002} & \underset{(-3.759)}{-0.242} \\ -- & -- \\ \underset{(-1.511)}{-0.003} & -- \\ -- & \underset{(-1.748)}{-0.003} \end{bmatrix}}^{\alpha}
\overbrace{\begin{bmatrix} 1 & -- & \underset{(-4.953)}{-9.148} & \underset{3.822}{7.799} & \underset{-1.793}{-521.029} \\ -- & 1 & \underset{(0.758)}{0.013} & \underset{(-0.356)}{-0.007} & \underset{(-2.860)}{7.991} \end{bmatrix}
\begin{bmatrix} \ln(\frac{b_{t-1}}{y_{t-1}}) \\ \rho_{t-1} \\ \ln(\frac{g_{t-1}}{y_{t-1}}) \\ \ln(\frac{\tau_{t-1}}{y_{t-1}}) \\ m_{t-1} \end{bmatrix} + \begin{bmatrix} \underset{(0.639)}{2.926} \\ \underset{(0.413)}{0.018} \end{bmatrix} \begin{bmatrix} con. \end{bmatrix}}^{\beta}
$$

$$
+ \begin{bmatrix} \underset{(1.697)}{0.107} & \underset{(-5.297)}{-0.434} & \underset{(-2.606)}{-0.110} & \underset{(-4.900)}{-0.118} & \underset{(1.595)}{4.457} \\ \underset{(-2.423)}{-0.100} & \underset{(-3.866)}{-0.238} & \underset{(-3.800)}{-0.106} & \underset{(-6.634)}{-0.111} & -- \\ -- & -- & -- & \underset{(-5.283)}{-0.171} & -- \\ \underset{(-1.622)}{-0.205} & \underset{(1.756)}{0.334} & \underset{(-3.080)}{-0.267} & \underset{(-2.082)}{-0.101} & -- \\ -- & -- & \underset{(-2.119)}{-0.002} & -- & \underset{(-2.240)}{-0.135} \end{bmatrix}
\begin{bmatrix} \Delta\ln(\frac{b_{t-1}}{y_{t-1}}) \\ \Delta\rho_{t-1} \\ \Delta\ln(\frac{g_{t-1}}{y_{t-1}}) \\ \Delta\ln(\frac{\tau_{t-1}}{y_{t-1}}) \\ \Delta m_{t-1} \end{bmatrix}
$$

$$
+ \begin{bmatrix} -- & -- & -- & \underset{(-2.896)}{-0.074} & \underset{(1.628)}{4.639} \\ -- & -- & \underset{(-1.490)}{-0.040} & \underset{(-3.266)}{-0.058} & -- \\ \underset{(-1.688)}{-0.082} & \underset{(2.041)}{0.190} & \underset{(3.380)}{0.187} & \underset{(-3.033)}{-0.098} & \underset{(1.534)}{6.087} \\ \underset{(1.487)}{0.190} & -- & -- & -- & -- \\ \underset{(-2.167)}{-0.002} & -- & \underset{(2.518)}{0.002} & -- & \underset{(-2.645)}{-0.161} \end{bmatrix}
\begin{bmatrix} \Delta\ln(\frac{b_{t-2}}{y_{t-2}}) \\ \Delta\rho_{t-2} \\ \Delta\ln(\frac{g_{t-2}}{y_{t-2}}) \\ \Delta\ln(\frac{\tau_{t-2}}{y_{t-2}}) \\ \Delta m_{t-2} \end{bmatrix}
$$

$$
+ \begin{bmatrix} \underset{(6.771)}{0.030} & \underset{(3.718)}{0.019} & -- & -- & -- \\ -- & \underset{(1.954)}{0.007} & -- & -- & -- \\ -- & \underset{(1.964)}{0.014} & \underset{(-2.577)}{-0.019} & \underset{(8.056)}{0.059} & -- \\ \underset{(-3.480)}{-0.036} & \underset{(2.909)}{0.029} & -- & -- & \underset{(13.648)}{0.084} \\ -- & -- & \underset{(4.579)}{0.001} & -- & -- \end{bmatrix}
\begin{bmatrix} Ddebt \\ Dek \\ Dimm \\ Dspen \\ Dtax \end{bmatrix} + \begin{bmatrix} u^b \\ u^\rho \\ u^g \\ u^\tau \\ u^m \end{bmatrix}
$$

Note: t-statistics in parentheses.

3.7. Structural VECM Analysis

The structural VAR technique can also be applied to the vector error correction models and called structural VECMs. Structural VAR has many advantages over the standard VAR. It serves as an important tool in macroeconomics that can be used to identify the shocks to be traced in an impulse response analysis. Structural VAR models are an adequate framework for the link between forecast errors of the system and fundamental shocks formed by employing the economic theory (Kilian and Lutkepohl, 2017). SVEC models' popularity can also be explained by the fact that economic theory may have more to say about the long run than the short run. Moreover, SVECM allows us to impose both long-run and contemporaneous restrictions.

As we mentioned in previous sections, impulse response functions describe the dynamics of the model. Making policy inferences from an empirical impulse response analysis requires a congruent model invariant to an extension of the information set used. If obtained from the unrestricted VAR with cointegrated variables, estimated impulse responses at long horizons might be inconsistent as noted by Ericsson et al. (1998), and Phillips (1998). Furthermore, ignoring cointegration by differencing all the variables also not a solution because it confounds short-term and long-term properties that are key to impulse response analysis (Ericsson et al., 1998; Phillips, 1998).

Since our empirical analysis aims to implement theoretical results from the previous section, we developed two alternative models from the subset five-variable VECM with two cointegrating vectors. The impulse responses are constructed from those models and the original VECM.

3.7.1 The Moving Average representation

A moving average (MA) representation is critical to understanding the impact of the innovations on the model variables. We know that a stationary VAR model could be directly inverted into the moving average form; when the process is $I(0)$, the effects of shocks in the variables of a given system easily seen in its Wold moving average (MA) representation (Johansen, 1996). In this instance, the orthogonal innovations that use Cholesky decomposition of the covariance matrix (Σ_u) are preferred in an impulse response analysis.[5]

That is not the case for the models containing unit roots as the autoregressive lag polynomial becomes non-invertible. However, impulse response matrices can be computed based on VARs with integrated variables or the levels version of a VECM (Lutkepohl and Kratzig, 2004). From Johansen's version of Granger's Representation Theorem we know that if series x_t is generated by a reduced form VECM, it has the following MA representation known as a multivariate version of the Beveridge-Nelson decomposition:[6]

$$x_t = C\sum_{i=1}^{t} u_i + C^*(L)\,u_t + x_0^* \tag{3.27}$$

where:

$$C = \beta_\perp\left[\alpha'_\perp\left(I_k - \sum_{i=1}^{p-1}\Gamma_i\right)\beta_\perp\right]^{-1}\alpha'_\perp \qquad \textit{and} \qquad C^*(L)\,u_t = \sum_{j=0}^{\infty} C_j^* u_{t-j} \tag{3.28}$$

$\beta_\perp$ and $\alpha_\perp$ are orthogonal complements of β and α. The term $\sum_{i=1}^{t} u_i$ is a K-dimensional random walk. Rank of the C is equal to the number of common trends, $(K - r)$. This representation is also known as the common-trends representation (Kilian and Lutkepohl, 2017).

Matrix C represents the long-run effects of forecast error impulse responses, whereas the C^*s contain transitory effects, therefore, decomposing the process x_t into $I(1)$ and $I(0)$ components (Johansen, 1996; Lutkepohl and Kratzig, 2004; Juselius, 2006; Doornik and Juselius, 2018).

According to Lutkepohl and Kratzig (2004) and also in Doornik and Juselius (2018) and Juselius (2006) since the structural shocks are obtained from the reduced-form errors by a linear transformation, $w_t = B_0 u_t$, we can replace u_t by $B_0^{-1} w_t$ in the Granger

[5] If B is a lower triangular such that $\Sigma_u = BB'$, the orthogonalized shocks given by $\varepsilon = B^{-1}u_t$. Wold MA representation then takes a form of $y_t = \Psi_0\varepsilon_t + \Psi_1\varepsilon_{t-1} + ...$ where $\Psi_i = \Phi_i B$. Here $\Psi_0 = B$ is lower triangular. Thus an ε shock in the first variable may have an instantaneous effect on all the variables. In contrast, a shock on the second variable cannot have an instantaneous effect on the first variable, but only on the other variables and so on. See Chapter 4 in Lutkepohl and Kratzig (2004), Chapters 5 and 15 in Juselius (2006), and Chapter 4 in Johansen (1996)

[6] Theorem 4.2. See Johansen (1996) p. 49

representation to obtain

$$x_t = \Upsilon \sum_{i=1}^{t} w_i + C^* (L) B_0^{-1} w_t + x_0^* \tag{3.29}$$

where $\Upsilon = CB_0^{-1}$ is long-run impact matrix.

Also known as the matrix of long-run multipliers, Υ, directly shows the long-run or permanent effects of the structural shocks on the level of the variables x_t, because the coefficient matrices $C_j^* B_0^{-1}$ in the stationary term $C^* (L) B_0^{-1} w_t$ approaches to zero as j goes to infinity. Therefore, restrictions on the long-run effects of the shocks can be imposed directly on Υ. Since it is obtained from C by the non-singular transformation, it has the same rank, $K - r$. If a shock does not have any long-run effects at all, the corresponding column in Υ is restricted to zero (Lutkepohl and Kratzig, 2004; Juselius, 2006; Doornik and Juselius, 2018).

We need to identify permanent and transitory shocks to the model to compute impulse responses. For the shocks that have the long-run or permanent effect, we have to impose restrictions on the matrix of long-run effects of shocks, $\Upsilon = CB_0^{-1}$. For the transitory shocks that have an instantaneous impact, we impose restrictions on the matrix of contemporaneous effects of the shocks, B_0^{-1} restrictions matrix.[7]

As in every structural analysis, the results may depend to some extent on the specification of the reduced-form model and the choice of identifying assumptions (Lutkepohl and Kratzig, 2004). The robustness of the results can be checked with respect to alternative identifying assumptions and model specifications.

3.7.2 Identification in SVECMs

We start our analysis based on the estimated VECM with two lagged differences and a cointegrating rank of two ($r = 2$). In this five-variable system, ($K = 5$) an additional two cointegrating relationship arises representing the budget equation with immigration and interest rate equation where debt, government spending and tax revenues as shares of output are $I(1)$ variables. Although the nominal interest rate is generally assumed to

[7]JMulti has additional econometric procedures in time series analysis that are unavailable in other packages for VAR and VEC modelling. In particular, JMulti is specially designed to allow for the imposition and testing of restrictions on contemporaneous and long-run impact matrices.

be $I(1)$, the rate adjusted to the output growth and natural population growth is $I(0)$ according to the unit root tests results.

Two stationary time series, accordingly, two cointegrating relationships translate to three common trends and three permanent shocks in the model. Altogether we need $K(K-1)/2$, in our case, ten linearly independent restrictions to just identify the structural VECM. With cointegrating rank two ($r = 2$), at most two shocks have transitory effects, whereas $k^* = K - r$) or three shocks has long-run effects in the VECM. To exactly identify permanent shocks, we need additional three restrictions, $k^*(k^* - 1)/2$, whereas $r(r-1)/2$, or two restrictions are enough to identify transitory shock.

The cointegration vectors show that debt and interest rate relations are stationary.[8] Besides, from tests of the unit vector in alpha, we conclude that the shocks to debt have not had any permanent effects on any variables in the system inclusive itself. Accordingly, debt and interest rate shocks have no long-run impact on the variables. This corresponds to two zero columns of identified long-run impact matrix Υ. Each zero column represents three restrictions. Therefore, with two columns of zero, we have imposed six restrictions on the long-run impact matrix. To just identify the system, we can additionally impose three zero restrictions to the long-run impact matrix and one contemporaneous restriction on B_0^{-1} matrix. The alternative identification includes imposing two zero restrictions on the long-run impact matrix and two restrictions on the transitory matrix.

3.8. The Impulse Response Analysis

This section derives impulse responses for five variable VECM model. We calculate the impulse responses of $\Delta \ln(\frac{b_t}{y_t})$, $\Delta \ln(\frac{g_t}{y_t})$, $\Delta \ln(\frac{\tau_t}{y_t})$, $\Delta \rho_t$ and Δm_t to orthogonilized shocks given by a vector of innovations $\varepsilon_t = \left\{\varepsilon_t^{debt}, \varepsilon_t^{int.rate}, \varepsilon_t^{gov.spd.}, \varepsilon_t^{tax}, \varepsilon_t^{immig.}\right\}$.

A basic definition of the shock is the change in the existing policy. Our identifying assumption includes restrictions imposed on the cointegration vector coefficients, β and loading factor α together with zero restrictions. We identify immigration shock as an unprecedented change in immigration. Our data series includes legal and illegal immigrants who joined the workforce in the US economy; therefore, any change in immigration can

[8] Also the tests of variable stationarity show that ρ_t is stationary with $\chi^2(3) = 5.5543[0.1354]$.

result from a government policy or seen as a flow of legal or illegal immigrants to a given economy by any other factor.

A fiscal policy encompasses a wide variety of policies where changes could occur. Fiscal policy shocks can be viewed as existing in a two-dimensional space spanned by basic vectors, a public debt shock, tax revenue shock and government spending shock (Mountford and Uhlig, 2005). Different fiscal policy shocks can be described as different linear combinations of these fundamental shocks in this setup.

The impulse responses of the fundamental shocks can be seen in Figures 3.10, 3.11 and 3.12 in Appendix 3.18. We plotted the impulse responses of all our five variables to the shocks and provided 68% bootstrap confidence bands computed by the percentile method proposed by Hall (1992) and recommended by Lutkepohl and Kratzig (2004), and Kilian and Lutkepohl (2017).

3.9. Estimation of impact matrices: Set 1

As we mentioned before, long-run identifying assumptions are typically derived from economic theory; therefore, these are the ones most economists prefer. When checking the long-run impact matrix, we can see that the two first columns are zero columns. Our theoretical model assumes that immigration shocks have both contemporaneous and long-run impacts on model variables. Thus, we need to impose additional restrictions on the long-run impact matrix.

The first set of identifying assumptions contains two zero restrictions on the long-run impact matrix. We assume that in the long-run one-off government expenditure shock does not have any permanent effect on tax revenues and vice versa. An increase in government expenditure does not necessarily be financed by taxes. In the last four decades in the US, growing government expenditures have not necessarily been followed by increased taxes but rather by increases in public debt, implying a possible intertemporal shift in taxation. The second identifying assumption concerns tax revenues; an increase in tax receipts might be utilized to finance the government expenditures or service the debt. The direct result will depend on the fiscal policy followed by the government. By applying these restrictions, we want to see the responses of variables to shocks in the presence of immigration. With the above two permanent restrictions, we need to impose one additional restriction on the long-run impact matrix and one restriction on the short-run impact matrix to identify the system exactly. Another possibility is to impose two restrictions but to the short-run impact matrix.

In addition to the above restrictions, we impose one additional zero restriction on the long-run impact matrix. We assume that a one-off tax innovation in the first quarter will not affect the decision to immigrate in the long run. A contemporaneous zero restriction is imposed on the immigration equation. As immigrants came with little or no capital, we assume that they will not be affected by the shock in interest rates in the short run.

$$B_0^{-1} = \begin{bmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & 0 & * & * & * \end{bmatrix} \quad \text{and} \quad \Upsilon = \begin{bmatrix} 0 & 0 & * & * & * \\ 0 & 0 & * & * & * \\ 0 & 0 & * & 0 & * \\ 0 & 0 & 0 & * & * \\ 0 & 0 & * & 0 & * \end{bmatrix}$$

Above identifying assumptions correspond to the following structure on the contemporaneous impact matrix B_0^{-1} and the identified long-run impact matrix $\Upsilon = C^* B_0^{-1}$. Long-run restrictions provide nine independent restrictions and contemporaneous restriction provides one additional restriction.[9]

$$\tilde{B}_0^{-1} = \begin{bmatrix} \begin{matrix}0.0009\\(1.2695)\end{matrix} & \begin{matrix}0.0079\\(10.3941)\end{matrix} & \begin{matrix}0.0025\\(3.1455)\end{matrix} & \begin{matrix}-0.0012\\(-0.9847)\end{matrix} & \begin{matrix}0.0021\\(2.3918)\end{matrix} \\ \begin{matrix}0.0047\\(1.0965)\end{matrix} & \begin{matrix}0.0017\\(3.2540)\end{matrix} & \begin{matrix}0.0006\\(0.6575)\end{matrix} & \begin{matrix}-0.0014\\(-1.5532)\end{matrix} & \begin{matrix}0.0030\\(3.9785)\end{matrix} \\ \begin{matrix}0.0000\\(-0.1445)\end{matrix} & \begin{matrix}0.0000\\(0.6397)\end{matrix} & \begin{matrix}0.0070\\(3.7714)\end{matrix} & \begin{matrix}-0.0020\\(-1.0018)\end{matrix} & \begin{matrix}0.0094\\(4.3523)\end{matrix} \\ \begin{matrix}0.0031\\(0.9230)\end{matrix} & \begin{matrix}0.0032\\(1.6304)\end{matrix} & \begin{matrix}0.0007\\(0.2742)\end{matrix} & \begin{matrix}0.0163\\(8.3887)\end{matrix} & \begin{matrix}0.0044\\(1.3693)\end{matrix} \\ \begin{matrix}0.0000\\(-0.9732)\end{matrix} & -- & \begin{matrix}-0.0001\\(-4.7338)\end{matrix} & \begin{matrix}0.0000\\(-0.9658)\end{matrix} & \begin{matrix}0.0001\\(3.5450)\end{matrix} \end{bmatrix}$$

[9]ML Estimation, Scoring Algorithm (see Amisano & Giannini (1992)). Log-Likelihood: 5704.1778. Structural VAR is just identified.

$$\tilde{\Upsilon} = \begin{bmatrix} -- & -- & \underset{(0.6023)}{0.0078} & \underset{(-4.2499)}{-0.0428} & \underset{(1.7595)}{0.0229} \\ -- & -- & \underset{(-5.9915)}{-0.0009} & \underset{(4.2499)}{0.0000} & \underset{(2.3998)}{0.0003} \\ -- & -- & \underset{(4.7928)}{0.0068} & -- & \underset{(4.1100)}{0.0070} \\ -- & -- & -- & \underset{(4.2499)}{0.0055} & \underset{(5.5141)}{0.0083} \\ -- & -- & \underset{(-5.7055)}{-0.0001} & -- & \underset{(2.8912)}{0.0000} \end{bmatrix}$$

In parenthesis, we have provided bootstrapped t-values obtained using 2,000 bootstrap replications.[10] Impulse responses to all five shocks can be seen at Figure 3.11 in Appendix 3.18.

3.9.1 The Immigration Shock

Figure 3.2 illustrates the impulse responses of variables following a temporary one standard deviation innovation in immigration. We also provide bootstrap confidence bands computed by the percentile method proposed by Hall (1992) and recommended by Lutkepohl and Kratzig (2004), and Kilian and Lutkepohl (2017). In general, graphs demonstrate that the responses of variables to immigration shocks are in line with the expectations. Our primary interest is the response of debt-output ratio the innovation to the immigration rate.

The relative importance of the identified immigration shocks can be assessed by listing the different horizons of the forecast error variance decomposition (FEVD) of debt as in Table 3.4. As we can see in a short horizon, a small variation in the debt-output ratio can be attributed to the immigration shock. However, in the longer horizon, immigration shock plays an important role. Another interesting result from this table is the effect of tax revenues on debt. Although tax revenue innovation is small in the shorter horizons, they become more important as the horizon increases.

What is a little surprising is the response of government expenditure and tax revenues to the immigration shock. One plausible, although not the only, possible explanation

[10]In applied work, bootstrap methods frequently used to constructs confidence intervals for impulse responses. By employing the bootstrap methods, more reliable small sample inference is occasionally possible than by using asymptotic theory and the precise expressions of the variances of the impulse responses are not needed (Lutkepohl and Kratzig, 2004).

horizon	ε_t^{debt}	$\varepsilon_t^{int.rate}$	$\varepsilon_t^{gov.spnd.}$	ε_t^{taxes}	$\varepsilon_t^{immig.}$
1	0.01	0.83	0.08	0.02	0.06
4	0.03	0.49	0.04	0.36	0.09
8	0.05	0.21	0.03	0.59	0.12
12	0.04	0.11	0.03	0.68	0.15
24	0.02	0.03	0.03	0.74	0.18
48	0.01	0.01	0.03	0.76	0.20
80	0.00	0.01	0.03	0.76	0.21

Table 3.4: *Forecast error variance decomposition (FEVD) of debt.*

is that immigration also yields higher costs of financing the debt because of the higher interest rate it generates. Tax revenues consist of taxes on capital income, wage income and consumption. Immigration affects almost all of these taxes at different levels, directly or indirectly, by stimulating the consumption and income of natives. Although immigrants own almost no or minimal capital when they arrive in the destination country, through their contribution to the appreciation of the value of capital holdings of natives, they contribute to the increase in capital tax revenues. When they arrived in a destination country, immigrants immediately start to consume and join the workforce, accordingly paying taxes on consumption and wage income. In the long run, immigrants contribute positively to the welfare of natives by simulating their consumption; accordingly, contributing to the increase in consumption tax revenues.

3.9.2 Interest Rate Shock

As we can observe in Figure 3.2, responses of variables are consistent with the conventional view that a surprise rise in interest rates leads to an initial rise in the debt-output and tax revenues-output ratios. We observe that shock to interest rates results in the decline of government expenditures initially; however, it returns to its initial steady-state value in the long run.

3.9.3 Fiscal Policy Shocks

The initial response of government expenditures to an increase in public debt is a fall. However, tax revenues immediately rise. In the long run, both tax revenues and government spending return to their previous steady-state values. Figure 3.3 demonstrates

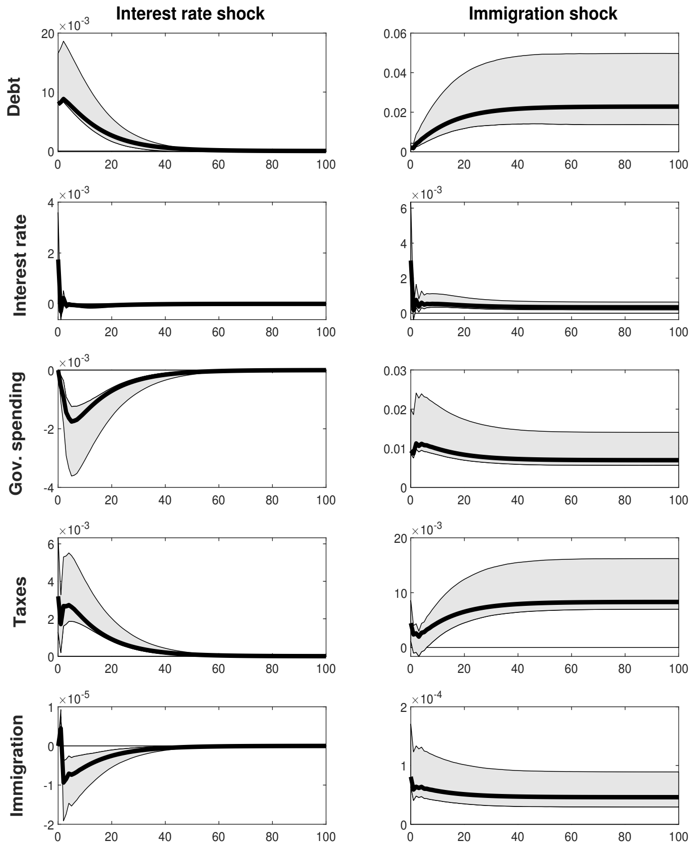


Figure 3.2: *Structural VECM Forecast Error Impulse Responses to interest rate and immigration shocks. With 68% (shaded area) Hall bootstrap confidence intervals based on 2000 bootstrap replications. (Set-1 identification assumptions.)*

forecast error impulse responses of all five variables to debt, government spending and tax shocks.

The response of interest rate to the increase in debt and deficits is quite interesting. An increase in debt leads to an immediate increase followed by a fall in the interest rate. As we mentioned in the previous section, the response of interest rate to deficit and debt is mixed in the literature. Because of the simulation of different policies in the literature, the result also varies widely, as noted by Gale and Orszag (1992). Some major macroeconometric models imply an economically significant connection between changes in budget deficits and long-term interest rates. The precise effect depends on various factors, including whether the change in the deficit is caused by a change in taxes or spendings. Some of the most cited papers using vector autoregressions, including Evans (1987); Plosser (1987) find no effect of deficits on interest rates. Discussion and critique of the VAR based studies can be found in Bernheim (1987) and Elmendorf and Mankiw (1999). They imply that VAR-based projections are more likely to suffer from measurement error and thus are biased toward showing no effects of deficits on interest rates.

The response of immigration to fiscal policy shocks is also interesting. We can notice that debt and tax shocks initially result in a fall in immigration from its steady-state level. However, the change is minimal. Still, although small, the rise in government spending results in an increase in immigration, which is plausible.

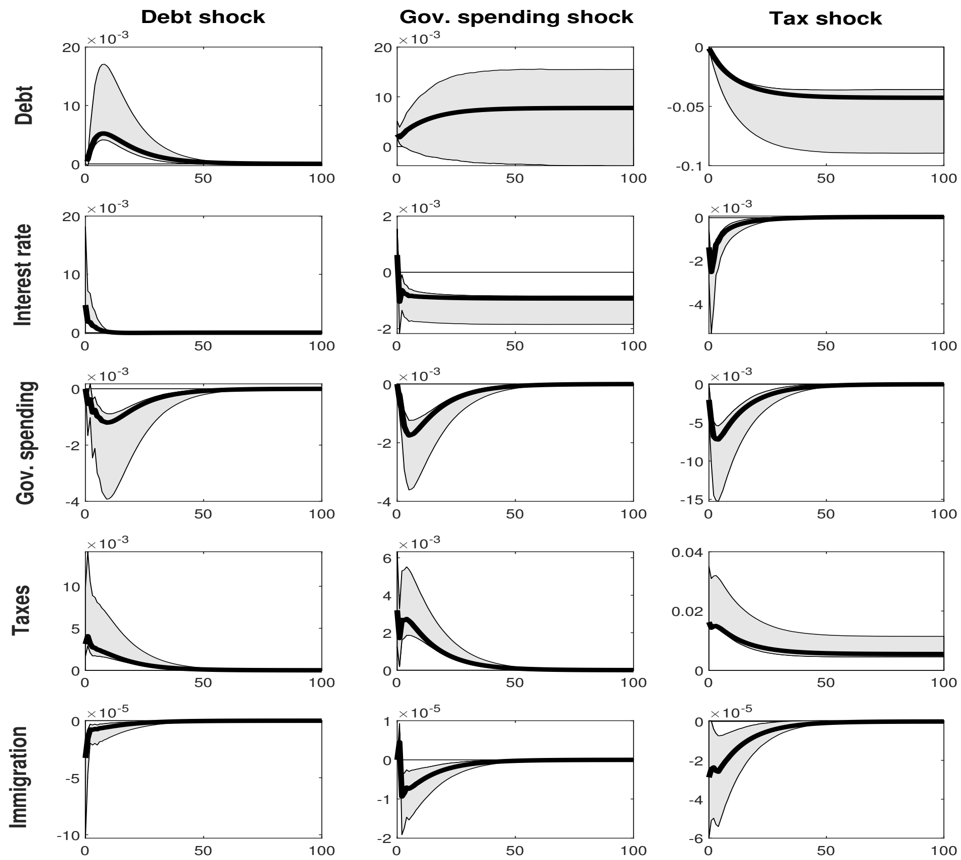


Figure 3.3: *Structural VECM Forecast Error Impulse Responses to debt, government spending and tax shocks. With 68% (shaded area) Hall bootstrap confidence intervals based on 2000 bootstrap replications.(Set-1 identification assumptions.)*

3.10. Estimation of impact matrices: Set 2

In this identification scheme, in addition to our previous two zero restrictions on the long-run estimation matrix, we impose an additional restriction assuming that innovation to government spending does not affect interest rates in a longer horizon. Moreover, we also impose a zero restriction on the short-run impact matrix, assuming that debt innovation does not affect the decision to immigrate in the short run. Finally, another zero restriction on the short-run impact matrix is imposed as zero contemporaneous effect of government spending shock to tax equation.

$$B_0^{-1} = \begin{bmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & 0 & * & * \\ 0 & * & * & * & * \end{bmatrix} \quad \text{and} \quad \Upsilon = \begin{bmatrix} 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * \\ 0 & 0 & * & 0 & * \\ 0 & 0 & * & * & * \\ 0 & 0 & * & * & * \end{bmatrix}$$

Above identifying assumptions correspond to the following structure on the contemporaneous impact matrix B_0^{-1} and the identified long-run impact matrix $\Upsilon = C^* B_0^{-1}$. Long run restrictions provide eight independent restrictions, and contemporaneous restriction provides two additional restrictions.

$$\tilde{B}_0^{-1} = \begin{bmatrix} \underset{(12.9717)}{0.0079} & \underset{(0.9109)}{0.0009} & \underset{(4.8576)}{0.0030} & \underset{(-1.0455)}{-0.0011} & \underset{(-1.2106)}{-0.0012} \\ \underset{(3.3455)}{0.0017} & \underset{(10.4731)}{0.0047} & \underset{(7.5491)}{0.0033} & \underset{(-0.2854)}{-0.0003} & \underset{(0.7076)}{0.0006} \\ \underset{(0.1056)}{0.0000} & \underset{(0.4407)}{0.0000} & \underset{(12.1880)}{0.0113} & \underset{(-0.7441)}{-0.0009} & \underset{(-1.9826)}{-0.0036} \\ \underset{(1.6489)}{0.0032} & \underset{(1.6333)}{0.0031} & -- & \underset{(4.4248)}{0.0151} & \underset{(-1.3565)}{-0.0075} \\ -- & \underset{(-1.8165)}{0.0000} & \underset{(1.6715)}{0.0000} & \underset{(1.0765)}{0.0001} & \underset{(5.2170)}{0.0001} \end{bmatrix}$$

$$\tilde{\Upsilon} = \begin{bmatrix} -- & -- & \underset{(3.7073)}{0.0345} & \underset{(-2.2816)}{-0.0305} & \underset{(1.0682)}{0.0171} \\ -- & -- & -- & \underset{(1.8090)}{0.0005} & \underset{(4.7698)}{0.0008} \\ -- & -- & \underset{(5.9741)}{0.0085} & -- & \underset{(-3.1014)}{-0.0048} \\ -- & -- & \underset{(3.5447)}{0.0061} & \underset{(4.2840)}{0.0077} & \underset{(-0.4119)}{-0.0012} \\ -- & -- & \underset{(5.8727)}{0.0000} & \underset{(1.6694)}{0.0001} & \underset{(4.8997)}{0.0001} \end{bmatrix}$$

Results show that the structural VAR is just identified with Log-Likelihood equals 5704.1778. Impulse responses to all shocks can be seen in Figure 3.12 in Appendix 3.18.

3.10.1 Immigration Shock

Figure 3.4 illustrates the impulse responses of variables following a temporary one standard deviation innovation to immigration. As expected, immigration shock positively and significantly affects debt-output and interest rate. The initial response of government spending and tax revenues is negative, as expected. Following the initial response for about a year, government spending slightly increases, in part probably due to the spendings and transfer payments allocated to new immigrants, but later stabilizes at a new level. The initial response of taxes on immigration is a fall, followed by a gradual return to the previous steady-state level. One possible explanation of this is the rise in the tax contributions by immigrants joining the economy.

3.10.2 Interest Rate Shock

Interest rate shock results in almost ten periods rise in debt-output ratio before returning to its pre-shock steady-state level (Figure 3.4). Thus, although the interest rate shock initially increases tax revenues, as a substantial part of taxes constitutes the tax on capital gains, they return to the pre-shock level in the long run.

3.10.3 Fiscal Policy Shocks

As we can see in Figure 3.5 this set of restrictions produce slightly different impulse responses. First of all, we can observe that the confidence band for government spending becomes more precise than in the previous set of imposed restrictions. The most significant change is observed in the response of interest rate to spending and revenue shocks. As we anticipate, an increase in the primary deficit immediately pushed the interest rate up. Shock to tax revenues initially decreases interest rate; however, they return to a new positive level in the long run. We also observe that immigration negatively responds to government spending shock. This and the response of immigration to the tax shock is not quite in line with what we observed in Chapters 1 and 2.

SVEC forecast error variance decomposition of debt reflects what was expected from the

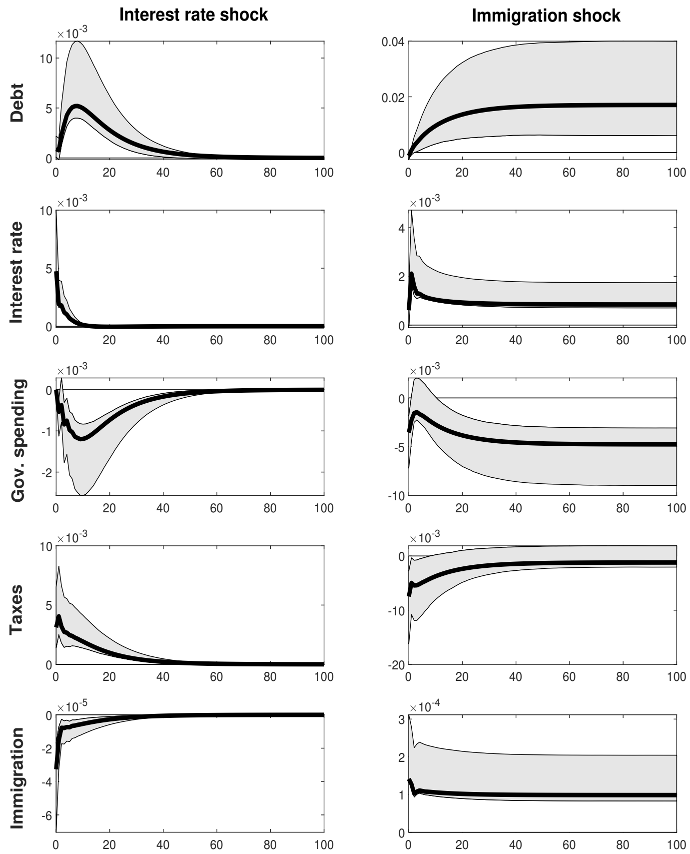


Figure 3.4: *Structural VECM Forecast Error Impulse Responses to interest rate and immigration shocks. With 68% (shaded area) Hall bootstrap confidence intervals based on 2000 bootstrap replications. (Set-2 identification assumptions.)*

theoretical and empirical literature concerning the long-run effect of the primary deficit on debt levels.

horizon	ε_t^{debt}	$\varepsilon_t^{int.rate}$	$\varepsilon_t^{gov.spnd.}$	ε_t^{taxes}	$\varepsilon_t^{immig.}$
1	0.83	0.01	0.12	0.02	0.02
4	0.49	0.03	0.23	0.22	0.04
8	0.21	0.05	0.33	0.34	0.07
12	0.11	0.04	0.38	0.38	0.09
24	0.03	0.02	0.44	0.40	0.11
48	0.01	0.01	0.47	0.39	0.12
80	0.01	0.00	0.48	0.39	0.12

Table 3.5: *Forecast error variance decomposition (FEVD) of debt.*

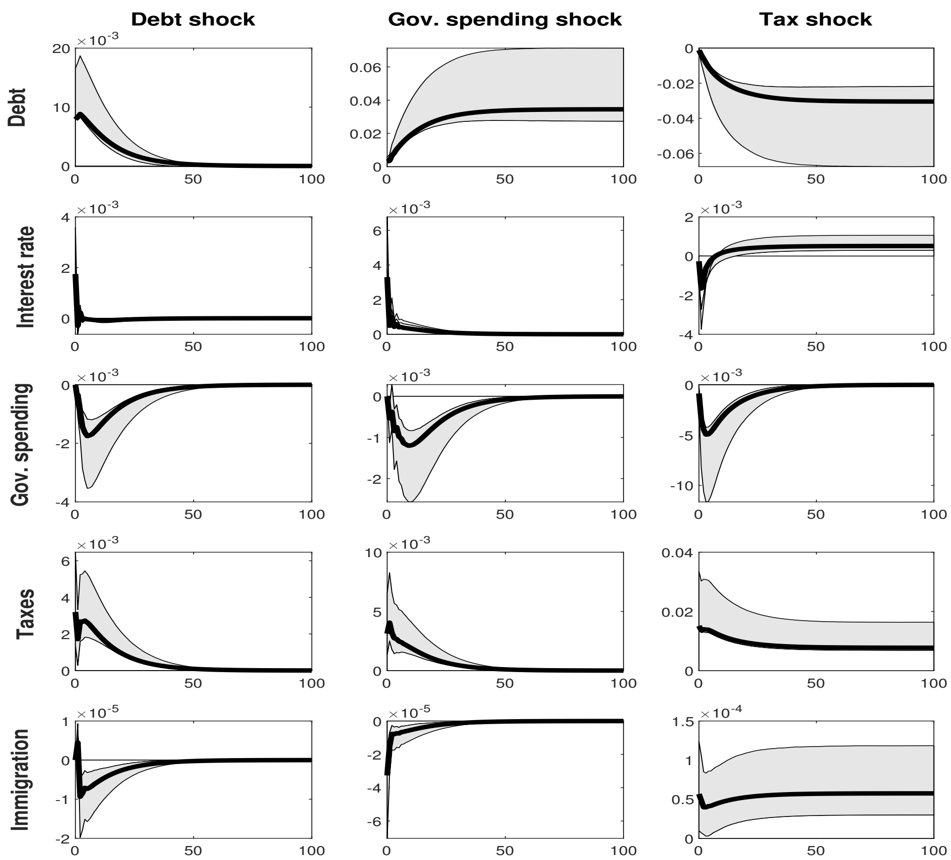


Figure 3.5: *Structural VECM Forecast Error Impulse Responses to debt, government spending and tax shocks. With 68% (shaded area) Hall bootstrap confidence intervals based on 2000 bootstrap replications.(Set-2 identification assumptions.)*

3.11. Comparison of Results

We have developed two alternative schemes from the subset of five-variable VECM with two cointegrating vectors. Respective impulse responses are constructed from those identification sets and the original subset VECM. In this section, first, the estimated results from the subset VEC model will be compared to the results of the alternative structural VEC models, then impulse responses from two structural VECMS are going to be compared with each other to find out:

- whether the structural identification scheme improves our understanding,
- whether it significantly changes our previous conclusions, and
- whether the two structural identification schemes tell a similar story.

Figures 3.10, 3.11 and 3.12 presented in Appendix 3.19 show forecast error impulse responses for subset VECM and both structural VECMs' identification schemes respectively. We can say that both sets of structural identifications are a significant improvement compared to the subset VEC model. Structural VECMs significantly improved our understanding of fiscal policy and immigration shocks' impact on the model variables. We observe more precise and significant coefficients after imposing the identifying restrictions to the subset VECM.

Figure 3.6 displays the impulse responses from two sets of identifying restrictions. Two sets of imposed restrictions produce the same log-likelihood value, and in both cases, the structural VAR is just identified.

In the first set of restrictions, the government expenditure is assumed to have zero long-run impact on tax revenues. Respectively, a shock to tax revenues is considered not to influence government spending. The third long-run identifying assumption imposes a zero impact of the tax revenues on immigration. Moreover, as immigrants usually come to a destination country with little or no capital, we assume that immigration will not be affected by the shock to the interest rate in the short run. The second identification set constitutes two long-run restrictions and two restrictions on the short-run impact matrix. In the latter, we assume that debt innovation has no impact on the decision to immigrate in the short run, furthermore, government spending does not have a contemporaneous effect on tax receipts.

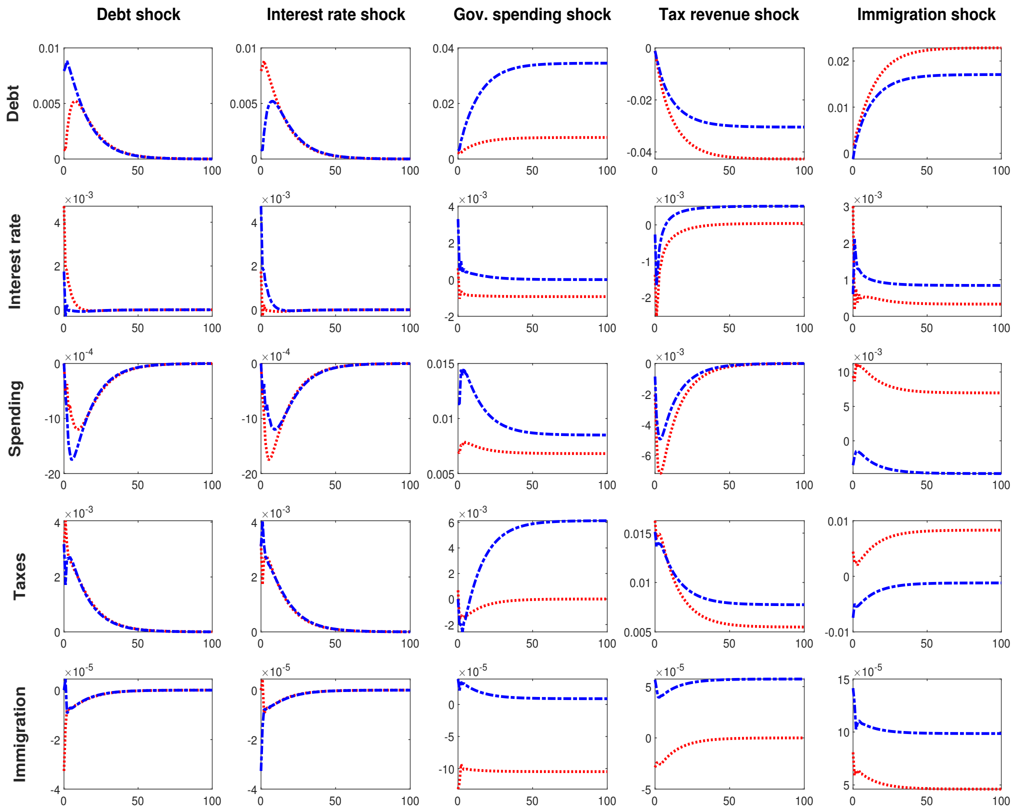


Figure 3.6: *Structural VECM Forecast Error Impulse Responses of two identification assumptions (red dotted line presents Set-1, blue dotted line Set -2).*

Figure 3.6 shows that our contemporaneous restrictions in both sub-systems only have affected the magnitude of the impact in the short horizon. Impulse responses when long-run restrictions are imposed are more interesting. The first set of restrictions produced the result in line with our previous findings. In Chapter 2, we observed that shocks to consumption tax, labour, and capital income taxes negatively impact immigration. Although both sets of restrictions assume no long-run effect of taxes on spending, the second set produced slightly different impulse responses. It is useful to mention that impulse response functions describe the dynamics of a model as a whole; accordingly, the difference in responses needs to be interpreted considering the whole set of restrictions (Ericsson et al., 1998).

An interesting result can be observed when examining the impulse responses of variables to debt innovation. Results hint at fiscal sustainability in the US. Bohn (1998) provided strong evidence of corrective action for the US, showing that the US primary surplus is an increasing function of the debt-output ratio. Historically, the US government responded to a rise in the debt-output ratio by reducing the primary deficit. Bohn (1998) also notes that the positive response of primary surplus to debt is clouded by war-time spending and by cyclical variations, thus it becomes highly significant when corrected for fluctuations in government spending and output. He concludes that the positive response of the primary surplus to changes in debt verifies that US fiscal policy satisfies an intertemporal budget constraint for the sample period 1916-1995 (Bohn, 1998).

The difference between immigrants' tax contributions and government spending on public benefits and services they receive constitutes the first-order net impact of immigrants (NRC, 1998). It is complicated to account the full fiscal impact of immigration, as incoming immigrants also affect the fiscal equation for many natives and their descendants. Although it is expected that initially, immigrants are likely to absorb public resources, thus exerting a net negative impact on public finances, however as they join the workforce and pay taxes, they typically become net contributors to public finances (NRC, 1997). By employing a partial equilibrium analysis Lee and Miller (2000) show that the initial fiscal impact of immigrants and their descendants is negative, but after about 16 years, it turns and remains positive. However, the positive impact is mostly at the federal level; at state and local levels, it is typically negative (Lee and Miller, 2000; NRC, 1997).

In conclusion, we can say that the structural identification scheme improved our understanding and significantly changed the magnitude of the impact compared to the subset VEC model. Two identification schemes, in general, tell a similar story, which is in line with the previous research.

3.12. Conclusion

In this chapter, we estimated the effects of immigration in the structural VECM model using the extended time series of the quarterly net flow of migrants to the US from the Current Population Survey. This chapter's econometric approach is influenced by the general-to-specific principle in econometric modelling prevailing in the literature on cointegration analysis. The data analysis is based on the approach implying to start with a general well-defined statistical model and then testing downwards as advised by Juselius (2006).

In the long-run structure, we found two relations in line with the theoretical model developed in Section 3.2. Altogether, the VECM model follows the theoretical cointegrating equation that we derived following Polito and Wickens (2007, 2012). Our contribution was including immigration into the government budget constraint in the theoretical model. We also derived the cointegrating vector with immigration required for our empirical analysis using the log-linearised budget constraint. Next, we empirically tested the theoretical model.

When analysing the impulse responses, we observed the following; the long-run impact of immigration on the debt-output ratio is positive. There is also a positive reaction of interest rates to immigration innovations. Results indicate that overall immigration has had relatively little impact on the US public debt and public finances, which can be explained by the number of immigrants (flow) in any given quarter is not significant. This is the plausible explanation for the magnitude of the responses to impact. The long-run inflow of immigration increases the total per capita debt, which is in line with Ben-Gad (2004, 2008, 2018). The conclusion is that immigration may alleviate some demographic problems through a positive long-term contribution to the factor prices and revenues. While contributing to the rise of public debt, thus increasing the welfare of the natives who are the primary debt owners, immigration can help lessen the burden of public debt for the descendants of the natives in most advanced economies, being the net contributor to public finances. Overall, the empirical model captures well the dynamics following immigration shocks predicted by the theoretical model developed in Chapter 1 as an extension of Ben-Gad (2018).

This chapter illustrates how an estimated structural VECM model can answer specific fiscal and immigration policy questions. Since in formulating immigration policy, the projections of the impact of immigration on public finances are essential.

Appendix

3.13. Immigration in OLG Model

Total population is the sum of overlapping generations M_s. Each generation grows by a constant rate n.

$$P_t = \sum_{s=0}^{t} (1+n)^{t-s} M_s \tag{3.30}$$

Immigrants enter in the economy and found new generations.Here M_{t+1} is the new generation that formed by incoming immigrants.

$$P_{t+1} = \sum_{s=0}^{t+1} (1+n)^{t-s+1} M_s = \sum_{s=0}^{t} (1+n)^{t-s+1} M_s + M_{t+1} \tag{3.31}$$

$$\frac{P_{t+1}}{P_t} = \frac{\sum_{s=0}^{t} (1+n)^{t-s+1} M_s + M_{t+1}}{\sum_{s=0}^{t} (1+n)^{t-s} M_s} = (1+n) + \frac{M_{t+1}}{P_t} \tag{3.32}$$

Define $\frac{(1+n)P_t}{P_{t+1}-M_{t+1}} = 1$ and $\frac{M_{t+1}}{P_{t+1}-M_{t+1}} = m_{t+1}$. And from these we derive,

$$P_{t+1} - M_{t+1} = \sum_{s=0}^{t} (1+n)^{t-s+1} M_s + M_{t+1} - M_{t+1} = \sum_{s=0}^{t} (1+n)^{t-s+1} M_s \tag{3.33}$$

$$P_{t+1} - M_{t+1} = (1+n) \sum_{s=0}^{t} (1+n)^{t-s} M_s = (1+n) P_t \tag{3.34}$$

$$\frac{M_{t+1}}{P_{t+1} - M_{t+1}} = \frac{M_{t+1}}{(1+n) P_t} \tag{3.35}$$

$$\frac{M_{t+1}}{P_t} = (1+n) m_{t+1} \tag{3.36}$$

Then, we can define:

$$\frac{P_{t+1}}{P_t} = (1+n) + (1+n) m_{t+1} = (1+n)(1+m_{t+1}) \tag{3.37}$$

We define:

$$\frac{P_{t-1}}{P_t} = \frac{1}{(1+n)(1+m_t)} \tag{3.38}$$

3.14. Variable Selection and Data

The estimation of the model based on five time series from 1960:Q1 to 2018:Q1. All data are quarterly. Unless otherwise noted, all data are from the Bureau of Economic Analysis' NIPA (National Income and Product Account). Data for output, spending, taxes and interest rates are nominal. Data are converted to per-capita deflating by the number of civilian noninstitutional population, ages 16 years and over (FRED[11] mnemonic $CNP16OV$). Originally $CNP16OV$ is not seasonally adjusted and monthly. We interpolated $CNP16OV$ time-series to the quarterly frequency and seasonally adjusted using X-13 ARIMA-SEATS quarterly seasonal adjustment method[12]. Each series has a (separate) linear trend. Except interest rate and immigration all data series are taken (natural) logarithm.

1. Output. We take the seasonally adjusted Gross Domestic Product (line 1 of Table 1.1.5. Gross Domestic Product) call it GDP_t. For per capita output we deflated it by the number of civilian noninstitutional population, ages 16 years and over.

2. Government Expenditure (exclusive interest payments). We take Government consumption expenditures (Table 3.1 line 21), add to it Government social benefits to persons (Table 3.1 line 24), Gross government investment (Table 3.1 line 36) and Capital transfer payments (Table 3.1 line 40). For government expenditure to output ratio we deflated the time series by GDP.

3. Current Tax Receipts. We take government current tax receipts (line 2 of Table 3.1)and deflate by GDP.

4. Public Debt. We take federal government debt, Federal Debt: Total Public Debt ($GFDEBTN$), quarterly, not seasonally adjusted. We seasonally adjusted it using X-13 ARIMA-SEATS quarterly seasonal adjustment method and call B_t. As FRED only have quarterly debt starting from 1966:Q1, for a period of 1960:Q1 to 1965:Q4, we used data kindly submitted by M. Ben-Gad, for which I cordially thank him. For public debt to output ratio, We deflated time series by GDP.

[11] Federal Reserve Economic Data, Federal Reserve Bank of St. Louis, https://fred.stlouisfed.org

[12] http://www.seasonal.website

4. Interest Rate. Take nominal rate for 10 year bonds, annualized and quarterly frequency, named it R_t. Then adjusted it for output growth rate, γ and population growth rate, n_t as in the model;

$$\rho_t \simeq R_t - \gamma_t - n_t$$

Population growth rate. n_t is calculated using growth rate of the seasonally adjusted time series for the number of civilian noninstitutional population, ages 16 years and over ($CNP16OV$).

Output growth rate. γ is calculated as growth rate of the GDP of the US for the given period.

Nominal rate for bonds. R_t is the Long-Term Government Bond Yields, 10-year for the United States, in percent, quarterly, not seasonally adjusted data ($IRLTLT01USQ156N$). After calculating respective quarterly returns we seasonally adjusted the time series.

6. Immigration. We used $\Delta N_{2,t}$ - the residual time series that represents the estimated net flow of migrants to the US civilian population and accounts for the change in the civilian noninstitutional population that is not due to past changes in fertility, current deaths or net flows to the US military (See Chapter 2.5.2 and Appendix 2.10). We deflated the series by ($CNP16OV$). For the analysis we used transformed series, as CPS makes data revisions periodically and they are usually attributed to the last census we updated previous time points by attributing revisions related to immigration.

3.15. Test Results

Null Hypothesis: **LN_BY** has a unit root			
Exogenous: Constant			
Lag Length: 2 (Automatic - based on SIC, maxlag=14)			
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		0.194238	0.9718
Test critical values:	1% level	−3.458719	
	5% level	−2.873918	
	10% level	−2.573443	
Null Hypothesis: **LN_GY** has a unit root			
Exogenous: Constant			
Lag Length: 2 (Automatic - based on SIC, maxlag=14)			
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		−2.620631	0.0902
Test critical values:	1% level	−3.458719	
	5% level	−2.873918	
	10% level	−2.573443	
Null Hypothesis: **LN_TY** has a unit root			
Exogenous: Constant			
Lag Length: 0 (Automatic - based on SIC, maxlag=14)			
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		−3.030006	0.0336
Test critical values:	1% level	−3.458470	
	5% level	−2.873809	
	10% level	−2.573384	
Null Hypothesis: **M_T** has a unit root			
Exogenous: Constant			
Lag Length: 0 (Automatic - based on SIC, maxlag=14)			
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		−14.22896	0.0000
Test critical values:	1% level	−3.458470	
	5% level	−2.873809	
	10% level	−2.573384	
Null Hypothesis: **RHO** has a unit root			
Exogenous: Constant			
Lag Length: 1 (Automatic - based on SIC, maxlag=14)			
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		−5.547393	0.0000
Test critical values:	1% level	−3.458594	
	5% level	−2.873863	
	10% level	−2.573413	
**MacKinnon (1996) one-sided p-values.*			

Table 3.6: *ADF Unit Root Test Results*

VAR Lag Order Selection Criteria						
Endogenous variables: LN_BY LN_GY LN_TY RHO M_T						
Exogenous variables: C						
Sample: 1960:Q1-2018:Q1						
Included observations: 225						
Lag	LogL	LR	FPE	AIC	SC	HQ
0	2584.691	NA	7.57e-17	-22.93058	-22.85467	-22.89994
1	3945.960	2649.938	5.25e-22	-34.80853	-34.35305*	-34.62470*
2	3987.621	79.24786	4.53e-22	-34.95663	-34.12158	-34.61960
3	4012.907	46.97585	4.52e-22*	-34.95917*	-33.74456	-34.46895
4	4031.392	33.51977	4.80e-22	-34.90126	-33.30708	-34.25784
5	4060.591	51.65032*	4.64e-22	-34.93859	-32.96484	-34.14198
6	4066.296	9.837373	5.52e-22	-34.76707	-32.41376	-33.81727
7	4084.165	30.01978	5.92e-22	-34.70369	-31.97081	-33.60068
8	4106.387	36.34613	6.10e-22	-34.67900	-31.56655	-33.42280

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Table 3.7: *Lag-length Test Results.*

Johansen Cointegration Test

Sample (adjusted): 1961:Q1 2018:Q1
Included observations: 229 after adjustments
Trend assumption: Linear deterministic trend
Series: LN_BY LN_GY LN_TY RHO M_T
Lags interval (in first differences): 1 to 3

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.201312	118.5299	69.81889	0.0000
At most 1 *	0.157308	67.05418	47.85613	0.0003
At most 2	0.073352	27.86008	29.79707	0.0823
At most 3	0.044093	10.41457	15.49471	0.2502
At most 4	0.000383	0.087791	3.841466	0.7670

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.201312	51.47569	33.87687	0.0002
At most 1 *	0.157308	39.19410	27.58434	0.0011
At most 2	0.073352	17.44551	21.13162	0.1520
At most 3	0.044093	10.32677	14.26460	0.1914
At most 4	0.000383	0.087791	3.841466	0.7670

Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Table 3.8: *Johnsen's Cointegration Test.*

3.16. Tables

Empirical studies of deficits and interest rates		
Predominately positive significant effect	**Mixed effect**	**Predominately insignificant effect**
	Expected or unanticipated deficit	
1. Makin and Tanzi (1984)	1. Sinai and Rathjens	1. Bradley (1986)
2. Feldstein (1986)	2. Kim and Lombra (1989)	
3. Wachtel and Young (1987)	3. Cohen and Garnier (1991)	
4. Bovenberg (1988)	4. Quigley and Porter-Hudak (1994)	
5. Thomas and Abderrezak (1988a)	5. Engen and Hubbard (2004)	
6. Thomas and Abderrezak (1988b)		
7. Barth and Bradley (1989)		
8. Thorbecke (1993)		
9. Elmendorf (1993)		
10. Elmendorf (1996)		
11. Kitchen (1996)		
12. Canzoneri, Cumby, and Diba (2002)		
13. Laubach (2003)		
	VAR-based dynamics	
1. Miller and Russek (1991)	1. Mountford and Uhlig (2000)	1. Plosser (1982)
2. Tavares and Valkanov (2001)	2. Perotti (2002)	2. Evans (1985)
3. Dai and Phillipon (2004)	3. Engen and Hubbard (2004)	3. Evans (1987a)
		4. Evans (1987b)
		5. Plosser (1987)
		6. Evans (1989)
	Current deficit or debt	
1. Feldstein and Eckstein (1970)	1. Echols and Elliott (1976)	1. Feldstein and Chamberlain (1973)
2. Kudlow (1981)	2. Dewald (1983)	2. Canto and Rapp (1982)
3. Carlson (1983)	3. Tanzi (1985)	3. Frankel (1983)
4. Hutchison and Pyle (1984)	4. Zahid (1988)	4. Hoelscher (1983)
5. Muller and Price (1984)	5. Coorey (1992)	5. Makin (1983)
6. Barth, Iden, and Russek (1985)		6. Mascaro and Meltzer (1983)
7. de Leew and Hollaway (1985)		7. Motley (1983)
8. Hoelscher (1986)		8. Tatom (1984)
9. Cebula (1987)		9. U.S. Treasury (1984)
10. Cebula (1988)		10. Giannaros and Kolluri (1985)
11. Cebula and Koch (1989)		11. Kolluri and Giannaros (1987)
12. Cebula and Koch (1994)		12. Swamy et al (1988)
13. Miller and Russek (1996)		13. Calomiris, Engen,Hassett, and Hubbard (2004)
14. Kitchen (2002)		
15. Kiley (2003)		
16. Cebula (2000)		

Table 3.9: *Source: Table 3 from Gale and Orszag (1992).*

3.17. Graphs

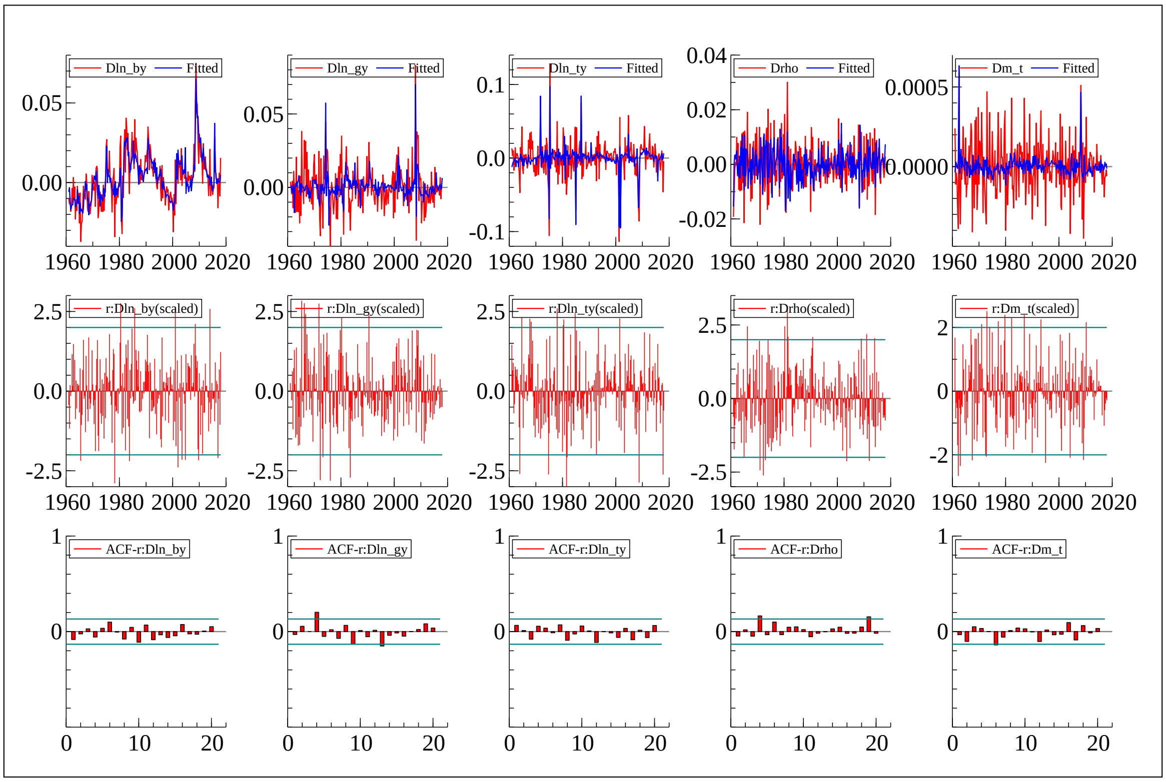


Figure 3.7: *CATS output with the default residual graphics for the estimated model.*

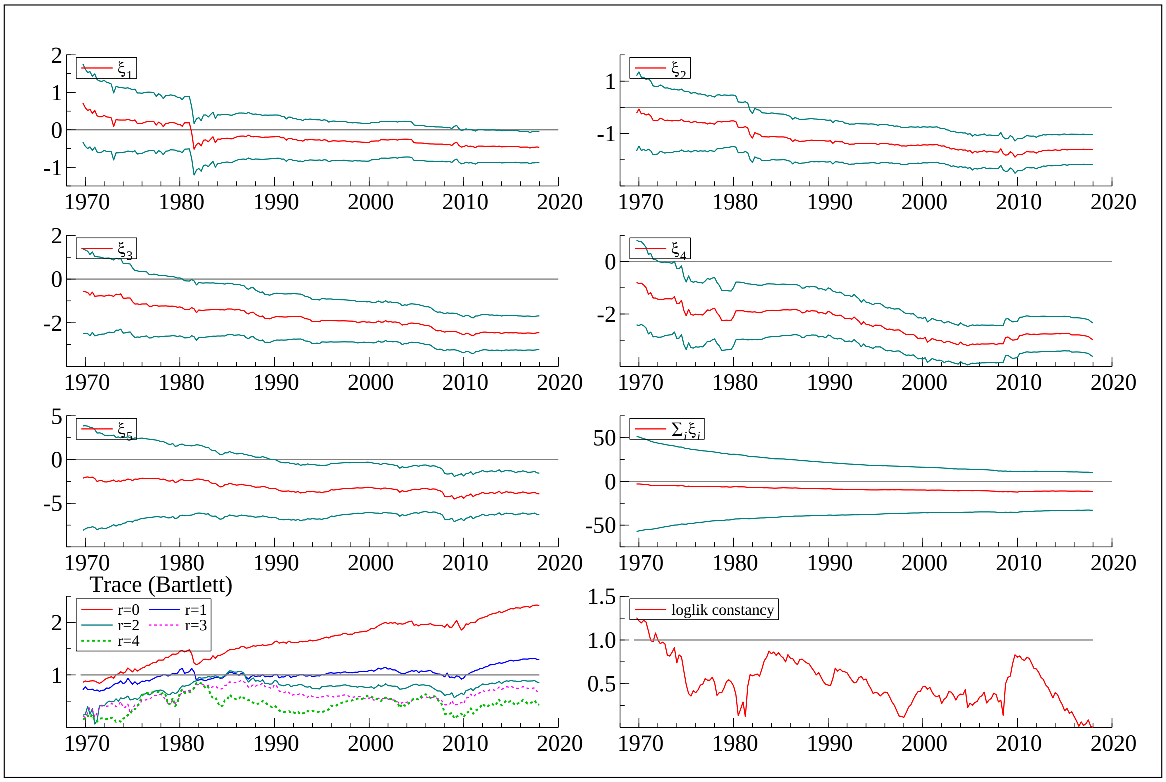
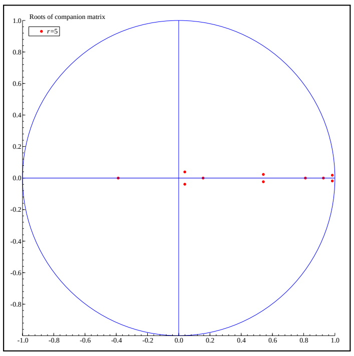
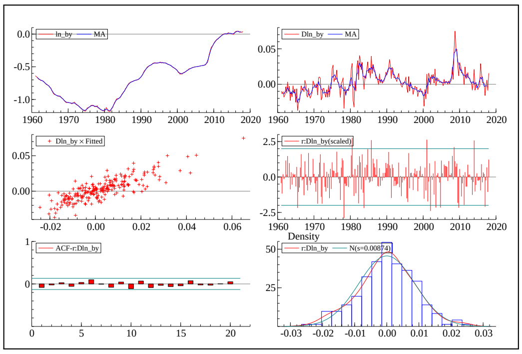


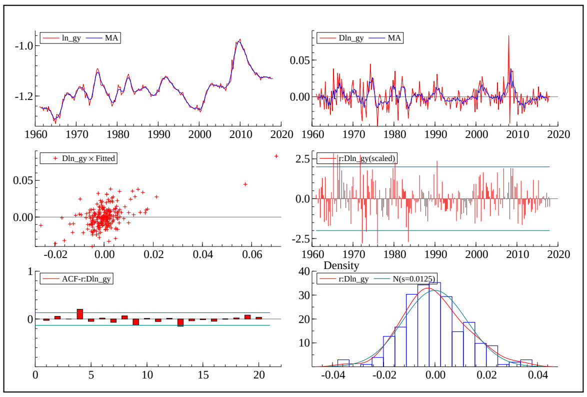
Figure 3.8: *Specification tests for the unrestricted VAR(2) model with dummies.*



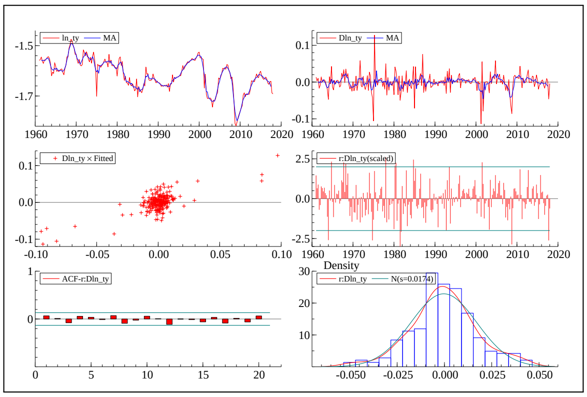
(a) The roots of the companion matrix plotted for $r = 2$.



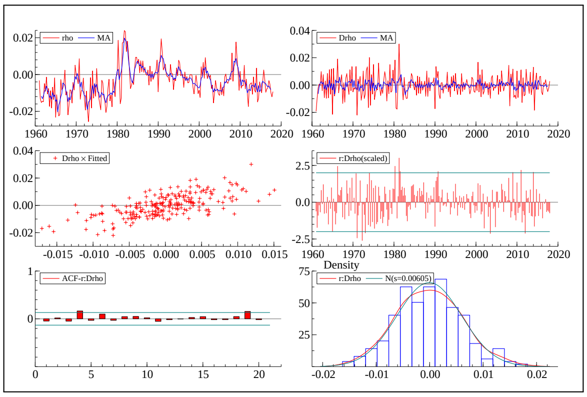
(b) Debt to output.



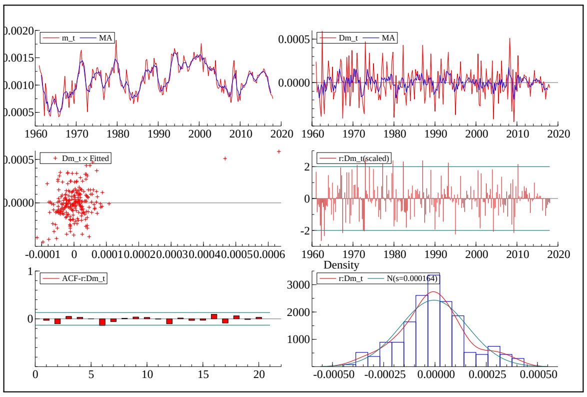
(c) Government spending to output



(d) Tax receipts to output



(e) Discount rate, ρ_t



(f) Immigration, m_t.

Figure 3.9: *Model variables*

3.18. Impulse Responses

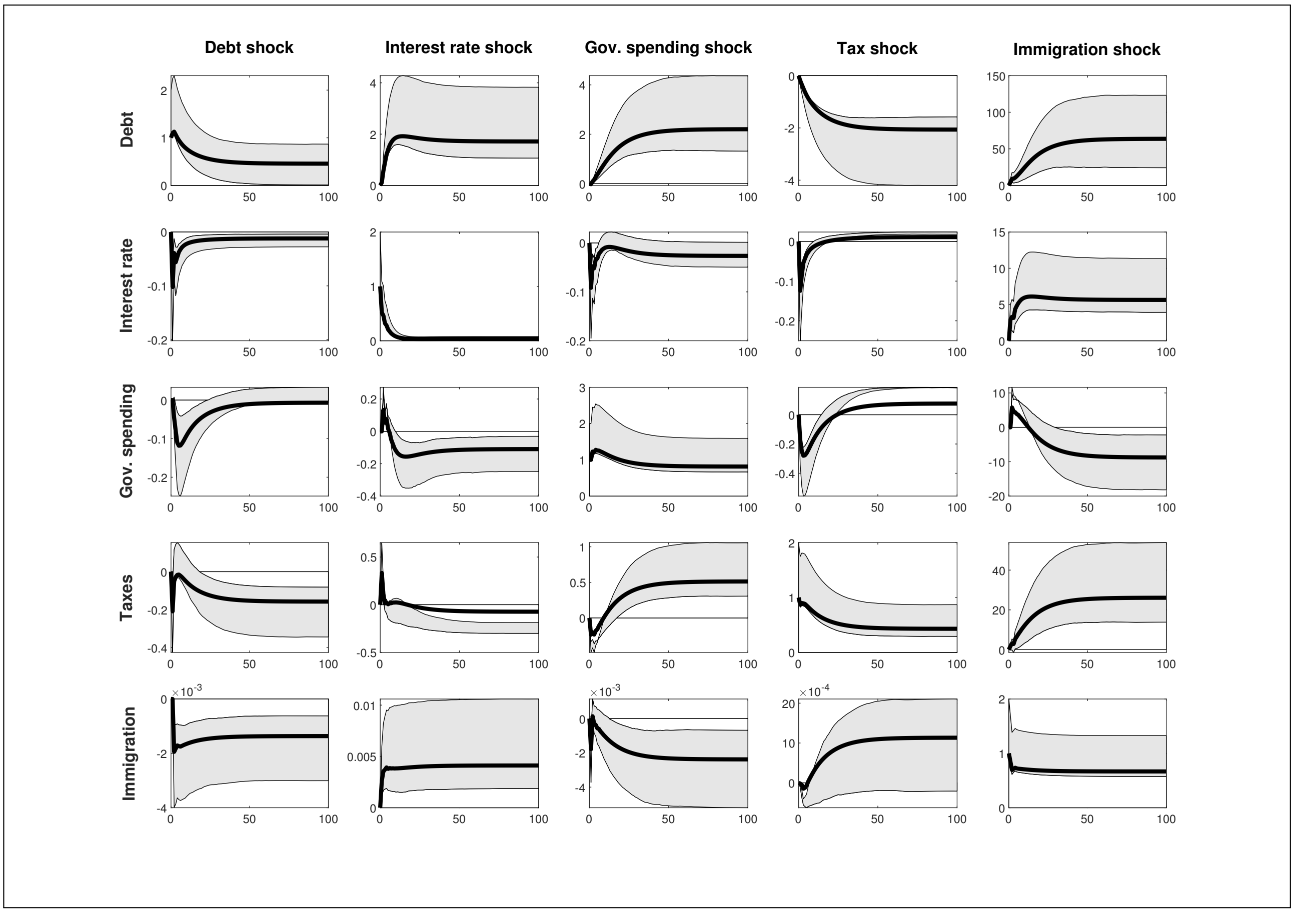


Figure 3.10: *Subset VECM Forecast Error Impulse Responses with 68% (dotted line) Hall bootstrap confidence intervals based on 2000 bootstrap replications.*

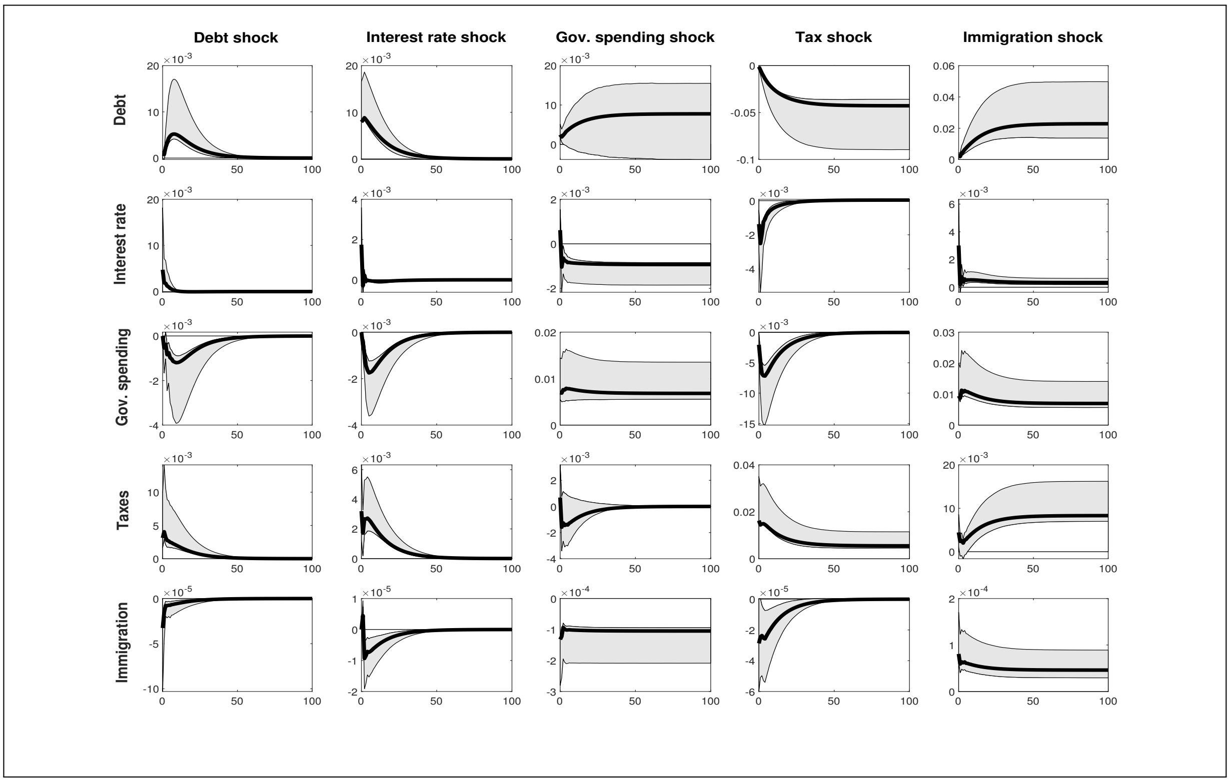


Figure 3.11: *Structural VECM Forecast Error Impulse Responses with 68% (dotted line) Hall bootstrap confidence intervals based on 2000 bootstrap replications.(Set-1 identification assumptions.)*

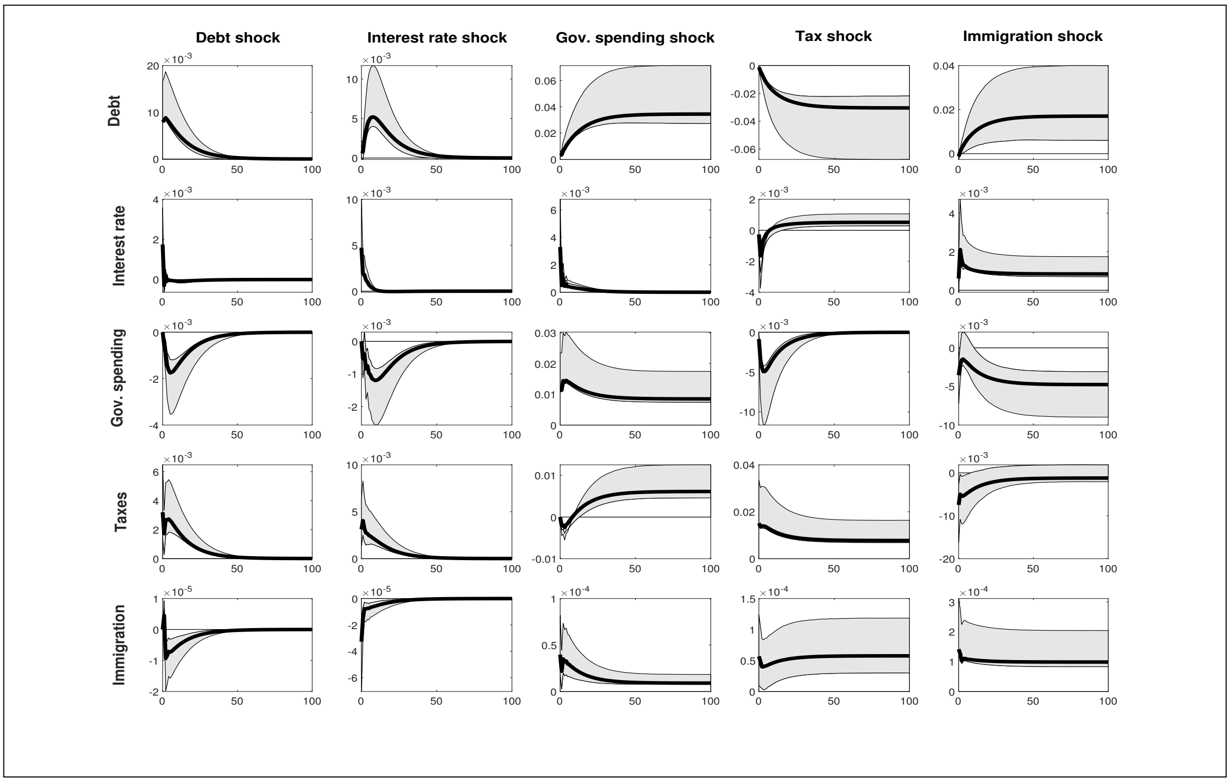


Figure 3.12: *Structural VECM Forecast Error Impulse Responses with 68% (dotted line) Hall bootstrap confidence intervals based on 2000 bootstrap replications.(Set-2 identification assumptions.)*

Chapter 4

Bibliography

Adjemian, R.S., Bastani, H., Karamé, F., Juillard, M., Maih, J., Mihoubi, F., Perendia, G., Pfeifer, J., Ratto, R. and Villemot, S. (2011). *"Dynare: Reference Manual, version 4"*. Dynare Working Papers 1. CEPREMAP.

Altonji, J.G (1986). "Intertemporal Substitution in Labour Supply: Evidence from Micro Data". *Journal of Political Economy*, 94(3), S176-215.

Ascari, G. and Rankin, N. (2007). "Perpetual Youth and Endogeneous Labour Supply: A Problem and a Possible Solution". *Journal of Macroeconomics,* 29, 708-723.

Barth, J.R., Iden, G., Russek, F.S. and Wohar M. (2010). "The Effects of Federal Budget Deficits on Interest Rates and the Composition of Domestic Output". In *The Great Fiscal Experiment*, edited by Rudolph G. Penner. Washington: Urban Institute Press.

Bartlett, B. (2012). "The Fiscal Legacy of George W. Bush". *The New York Times. June 12, 2012.*

Bernheim, B.D. (1987). "Ricardian Equivalence: An Evaluation of Theory and Evidence". *NBER Working Paper 2330.*

Ben-Gad, M. (2004). "The Economic Effects of Immigration-A Dynamic Analysis". *Journal of Economic Dynamics and Control,* 28, 1825-1845.

Ben-Gad, M. (2008). "Capital-skill Complementarity and the Immigration Surplus". *Review of Economic Dynamics,* 11, 335-365.

Ben-Gad, M. (2017). "The Optimal Taxation of Asset Income when Government Consumption is Endogenous: Theory, Estimation and Welfare". *Economic Enquiry,* 55(4), 1689-1711.

Ben-Gad, M. (2018). "On the Political Economy of Deficit Bias and Immigration". *The Economic Journal,* 128(614), 2191-2221.

Blau, D. and Mackie, C. (2018). *The Economic and Fiscal Consequences of Immigration.* The National Academies Press, Washington, DC.

Bohn, H. (1991). "Budget Balance through Revenue or Spending Adjustments? Some historical evidence for the United States". *Journal of Monetary Economics,* 27, 333-359.

Bohn, H. (1992). "Budget Deficit and Government Accounting". *Carnegie-Rochester Conference Series on Public Policy,* 37, 1-84.

Bohn, H. (1995). "The Sustainability of Budget Deficits in a Stochastic Economy". *Journal of Money, Credit and Banking,* 27, 257-271.

Bohn, H. (1998). "The Behavior of US Public Debt and Deficits". *Quarterly Journal of Economics,* 113, 949-963.

Bohn, H. (2005). "The Sustainability of Fiscal Policy in the United States". *CESifo Working Paper No. 1446.*

Borjas, G. (1995). "The Economic Benefits from Immigration". *Journal of Economic Perspectives,* 9(2), 3-22.

Borjas, G. (1999). *Heavens Door: Immigration Policy and the American Economy.* The Princeton University Press, New Jersey.

Braun, R.A. (1994). "Tax Disturbances and Real Economic Activity in the Postwar United States". *Journal of Monetary Economics,* 33(3), 441-462.

Burkhard, H. and Maussner, A. (2005). *Dynamic General Equilibrium Modelling.* Springer. p. 192. ISBN 978-3-540-22095-4.

Carey, D. and Rabesona, J. (2003). "Tax Ratios on Labour and Capital Income and on Consumption". *OECD Economic Studies,* 2002(2), 129-174.

Canova, F. and Ravn, M. (2000). "The Macroeconomic Effects of German Unification: Real Adjustments and the Welfare State". *Review of Economic Dynamics,* 3(3), 423-460.

Canzoneri, M.B., Cumby, R.E. and Behzad, T.D. (2001). "Is the Price Level Determined by the Needs of Fiscal Solvency?" *American Economic Review,* 91(5), 1221-1238.

Canova, F. and Ravn, M. (2000). "The Macroeconomic Effects of German Unification: Real Adjustments and the Welfare State". *Review of Economic Dynamics,* 3(3), 423-460.

Cochrane, J.H. and Taylor, J.B. (2020). *Strategies for Monetary Policy.* Hoover Institution Press.

Congressional Budget Office. (2010). *The Budget and Economic Outlook: Fiscal Years 2010 to 2020.* Congressional Budget Office. January 26, 2010. https://www.cbo.gov/ftpdocs/108xx/doc10871/HistoricalTables.pdf

Congressional Budget Office. (2012). *Changes in CBO's Baseline Projections Since 2001.* Congressional Budget Office. June 7, 2012. https://www.cbo.gov/publication/41463

Congressional Budget Office. (2015). *How Changes in Immigration Policy Might Affect the Federal Budget.* Washington, DC: United States Congress.

Congressional Budget Office. (2018). *The 2018 Long-Term Budget Outlook.* Washington, DC: United States Congress.

Congressional Budget Office. (2019). *The 2019 Long-Term Budget Outlook.* Congressional Budget Office. June 25, 2019. https://www.cbo.gov/publication/55331.

Chalk, N. and Hemming, R. (2000). "Assessing Fiscal Sustainability in Theory and Practice". *IMF Working Paper*, WP/00/88.

Davig, T. and Leeper, E.M. (2007). "Fluctuating Macro Policies and the Fiscal Theory". *In NBER Macroeconomics Annual 2006,* Volume 21, MIT Press, Cambridge, 247-298.

Del Negro, M. and Schorfheide, F. (2008). "Forming Priors for DSGE Models (and How it Affects the Assessment of Nominal Rigidities"). *Journal of Monetary Economics,* 55(7), 1191-1208.

Doornik, J.A. and Hendry D.F. (2013). Modelling Dynamic Systems: PcGive 14 Volume II . *Published by Timberlake Consultants Ltd, www.timberlake.co.uk.*

Doornik, J.A. and Juselius K. (2018). Cointegration Analysis of Time Series using CATS 3 for OxMetrics. *Published by Timberlake Consultants Ltd, www.timberlake.co.uk.*

Edge, R.M., Gurkaynak, R.S. and Kisacikoglu, B. (2016). "Judging the DSGE model by its forecast". *Unpublished manuscript.*

Elmendorf, D.W. and Mankiw, G.N. (1999). Government Debt. In *Handbook of Macroeconomics.* Volume 1C, edited by John B. Taylor and Michael Woodford, 1615-1669. Amsterdam: Elsevier Science B.V.

Elmendorf, D.W. and Sheiner, L.M. (2017). "Federal Budget Policy with an aging population and persistantly low interest rates". *Journal of Economic Perspectives,* 31(3), 94-175.

Ericsson, N.R., Hendry, D.F. and Mizon, G.E. (1998). "Exogeneity, Cointegration, and Economic Policy Analysis". *Journal of Business & Economic statistics,* 16(4), 370-387.

Evans, B.D. (1987). "Interest Rates and Expected Future Budget Deficits in the United States". *Journal of Political Economy,* 95(11), 34-58.

Favero, C. and Monacelli, T. (2005). *"Fiscal Policy Rules and Regime (In)Stability: Evidence from the US".* IGIER Working Paper Series 282.

Forni, L., Monteforte, L. and Sessa, L. (2009). "The General Equilibrium Effects of Fiscal Policy: Estimates for the Euro Area". *Journal of Public Economics,* 93(3-4), 559-585.

Gale, W.G. and Orszag, P.R. (2004). "Budget Deficits, National Saving, and Interest Rates". *Brookings Institution and Tax Policy Center.* Paper was prepared for the Brookings Panel on Economic Activity, September 9-10, 2004.

Greenlaw, D., Hamilton, J.D., Harris, E. and West, K.D. (2018). "A Skeptical View of the Impact of the Fed's Balance Sheet". *NBER Working Paper* No. 24687.

Greenwood, J., Hercowitz, Z. and Huffman, G. (1988). "Investment, Capacity Utilization and the Real Business Cycle". *American Economic Review,* 78, 402-417.

Gumilev, L.N. (1990). *Ethnogenesis and the Biosphere of Earth* Progress Publishers, Moscow.

Hall, P. (1992). *The Bootstrap and Edgeworth Expansion.* New York, Springer.

Hamilton, J.D. (2018). "The Efficiacy of Large-Scale Asset Purchases when the Short-term Interest Rate is at its Effective Lower Bound". *Brookings Papers on Economic Activity*, Fall 2018.

Herbst, E.P. and Schorfheide, F. *Bayesian Estimation of DSGE Models.* Princeton University Press, Princeton and Oxford, 2016.

Johansen, S. and Juselius K. (1992). "Identification of the Long-run and the Short-run Structure: An Application to the ISLM model". *Institute of Mathematical Statistics, University of Copenhagen,* Preprint May 3, 1992.

Johansen, S. and Juselius K. (1994). "Identification of the Long-run and the Short-run Structure: An Application to the ISLM model". *Journal of Econometrics,* 63(1), 7-36.

Johansen, S. (1996). Likelihood-Based Inference in Cointegrated Vector Autoregressive Models. *Oxford University Press.*

Juselius, K. (2006). The Cointegrated VAR Model: Methodology and Applications. *New York, Oxford University Press.*

Keane, M. and Rogerson, R. (2012). "Micro and Macro Labor Supply Elasticities: A Reassessment of Conventional Wisdom". *Journal of Economic Literature,* 50(2), 464-76.

Kiguchi, T. and Mountford, A. (2013). "The macroeconomics of immigration". *MPRA Paper from University Library of Munich, Germany.*

Lee, R. and Miller, T. (2000). "Immigration, Social Security, and Broader Fiscal Impacts". *American Economic Review: Papers and, Proceedings,* 90(2), 350-354.

Kilian, L. and Lutkepohl,H. (2017). Structural Vector Autoregressive Analysis. *Cambridge University Press.*

Leeper, E.M. (1991). “Equilibria under ’active’ and ’passive’ monetary and fiscal policies”. *Journal of Monetary Economics,* 27(2001), 129-147.

Leeper, E.M., Plante, M. and Traum, N. (2010). “Dynamics of Fiscal Financing in the United States”. *Journal of Econometrics,* 156(2010) 304-321.

Ljungqvist, L. and Sargent, T.J. (2000). Recursive Macroeconomic Theory. *The MIT Press.*

Lutkepohl H. and Kratzig, M. (2005). VECM Analysis in JMulti. *http://www.jmulti.de/.*

Lutkepohl H. and Kratzig M. (2004). Applied Time Series Econometrics. *Cambridge University Press.*

MaCurdy, T.E. (1981). “An empirical Model of Labour Supply in a Life-Cycle Setting”. *Journal of Political Economy,* 89(6), 1059-85.

Fischetti, M. (2014). “Ratio of Workers to Retirees will plummet worldwide”. *Scientific American*

McGrattan, E.R. (1994). “The Macroeconomic Effects of Distortionary Taxation”. *Journal of Monetary Economics,* 33, 573-601.

Mendoza, E., Razin, A. and Tesar, L. (1994). “Effective Tax Rates in Macroeconomics: Cross-country Estimates of Tax Rates on Factor Incomes and Consumption”. *Journal of Monetary Economics,* 34, 297-323.

Mountford, A. and Uhlig, H. (2005). “What are the effects of fiscal policy shocks?” *CERP discussion paper,* No. 3338.

Muysken, J. and Ziesemer, T.H.W. (2013). “A Permanent Effect of Temporary Immigration on Economic Growth”. *Applied Economics,* 45(28), 4050-4059.

Muysken, J. and Ziesemer, T.H.W. (2014). “The Effect of Immigration on Economic Growth in an Ageing Economy”. *Bulletin of Applied Economics,* 1(1), 35-63.

National Academies of Sciences, Engineering, and Medicine. (2017). *The Economic and Fiscal Consequences of Immigration.* Washington, DC: The National Academies Press. https://doi.org/10.17226/23550.

National Research Council. (1998). *The Immigration Debate: Studies on the Economic, Demographic, and Fiscal Effects of Immigration.* Washington, DC: The National Academies Press. https://doi.org/10.17226/5985.

National Research Council. (1996). *Statistics on US Immigration: An Assessment of Data Needs for Future Research.* Washington, DC: The National Academies Press. https://doi.org/10.17226/4942.

National Research Council. (1997). *The New Americans: Economic, Demographic, and Fiscal Effects of Immigration.* Washington, DC: The National Academies Press. https://doi.org/10.17226/5779.

O'Connell, S.A and Zeldes, S.P. (1988). "Rational Ponzi Games". *International Economic Review,* 29(3), 431-450.

Peterman, W. (2013). "Determining the Motives for a Positive Optimal Tax on Capital". *Journal of Economic Dynamics and Control,* 37(1), 265-95.

Peterman, W. (2015). "Reconciling Micro and Macro Estimates of the Frish Labor Supply Elasticity". *Economic Inquiry,* 54(1), 100-120.

Pfeifer, J. (2017). *An Introduction to Graphs in Dynare.* University of Cologne. June 29, 2017.

Pfeifer, J. (2018). *A Guide to Specifying Observation Equations for the Estimation of DSGE Models.* University of Cologne. July 8, 2018.

Phillips, P.C.B. (1998). "Impulse Response and Forecast Error Variance Asymptotics in Nonstationary VARs". *Journal of Econometrics,* Elsevier, 83(1-2), 21-56.

Plosser, C.I. (1987). "Fiscal Policy and the Term Structure". *Journal of Monetary Economics,* 20(6), 343-367.

Polito, V. and Wickens, M. (2012). "A model-based Indicator of the Fiscal Stance". *European Economic Review,* 56, 526-551.

Polito, V. and Wickens M. (2007). "Measuring the Fiscal Stance". *Discussion Papers in Economics, The University of York,* No. 2007/14.

Reichling, F. and Whalen, C. (2012). “Review of Estimates of the Frisch Elasticity of Labour Supply”. CBO Working Paper 2012-13.

Reichling, F. and Whalen, C. (2017). “Estimates of the Frisch Elasticity of Labour Supply: A Review” *Eastern Economic Journal,* 43(1), 37-42.

Sargent, T.J. and Ljingqvist, L. (2012). *Recursive Macroeconomic Theory.* The MIT Press, Cambridge, Massachusetts.

Singer, A. (2012). *Immigrant Workers in the US Labor Force,* Washington, DC: Brookings Institution Press.

Smets, F. and Wouters, R. (2002). “An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area”. *European Central Bank*, Working Paper No.171, August 2002.

Smith, C. and Thoenissen, C. (2019). “Skilled migration and business cycle dynamics”. *Journal of Economic Dynamics and Control*, 109(2019), 103781.

Sungbae, A. and Schorfheide, F. (2007). “Bayesian Analysis of DSGE models”. *Econometric Reviews*, 26 (2-4), 113-172.

Trabandt, M. and Uhlig, H. (2011). “The Laffer Curve Revisited”. *Journal of Monetary Economics*, 58, 305-327.

Trehan, B. and Walsh C. (1988). “Common Trends, The Government’s Budget Constraint, and Revenue Smoothing”. *Journal of Economic Dynamics and Control*, 12, 425-444.

Weil, P. (1989). “Overlapping families of infinitely-lived agents”. *Journal of Public Economics*, 38(2), 183-198.

Weiske, S. (2017). “On the macroeconomic effects of immigration: A VAR analysis for the US”. *Goethe University, Frankfurt.* June 9, 2017.

Weiske, S. (2019). “On the macroeconomic effects of immigration: A VAR analysis for the US”. *German Council of Economic Experts.* Working Paper 02/2019.