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A bi-level model for optimal capacity investment and subsidy design under risk aversion and uncertainty

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ABSTRACT

Meeting ambitious sustainability targets motivated by climate change concerns requires the structural transformation of many industries and the careful alignment of firm- and Government-level policymaking. While private firms rely on Government support to achieve timely the necessary green investment intensity, Governments rely on private firms to tackle financial constraints and technology transfer. This interaction is analysed in the real options literature only under risk neutrality, and, consequently, the implications of risk aversion due to the idiosyncratic risk that green technologies entail are overlooked. To analyse how this interaction impacts a firm's investment policy and a Government's subsidy design under uncertainty and risk aversion, we develop a real options framework, whereby: (i) we solve the firm's investment problem assuming an exogenous subsidy; (ii) conditional on the firm's optimal investment policy, we address the Government's optimisation objective and derive the optimal subsidy level; (iii) we insert the optimal subsidy level in (i) to derive the firm's endogenous investment policy. Contrary to existing literature, results indicate that greater risk aversion lowers the amount of installed capacity yet postpones investment. Also, although greater uncertainty raises the optimal subsidy under risk neutrality, the impact of uncertainty is reversed under high levels of risk aversion.

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1. Introduction

The need for sustainable green growth has motivated the implementation of a wide range of environmental regulations to promote the structural transformation for many industries. However, this transformation is a capital intensive process that requires the collaboration between Governments and private firms in order to overcome budgetary and technology constraints. Indeed, Governments rely on private firms in order to tackle financial constraints and access technological innovations. Similarly, private firms rely on Government support, since they are expected to engage at unprecedented levels in renewable energy (RE) investment as well as research and development (R&D). In this paper, we develop an analytical framework in order to explore this interaction and derive a solution that is optimal from both a firm's and Government's perspective. This is a notoriously challenging task because we must take into account various trade-offs that impact not only a Government's but also a private firm's optimisation objectives.

i. From the perspective of a private firm, capital intensive projects are particularly risky, since they require accurate investment timing and capacity sizing decisions. However, a large capacity raises the firm's exposure to downside risk, while a small capacity implies that revenues could be forgone if market conditions suddenly become favourable.
ii. The Government's objective is twofold:
iii. First, the level of subsidy should stimulate investment in a way that environmental targets are met both in terms of timing and investment intensity. However, a high (low) subsidy may result in early (later) investment, but in a smaller (bigger) project.
iv. Second, if the subsidy takes the form of a cash grant (Lukas & Thiergart, 2019), then, upon the firm's investment, the Government receives a tax in the form of a concession fee from the operating project's cash flows, reflecting a fair return for the Government investment. Hence, the Government must balance the level of support so as to maximize its net tax income at the time of investment.

If the interaction among the aforementioned optimisation objectives is not properly understood, then subsidies and incentives will not be properly

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set, resulting in a dynamic inefficiency with possible cycles of under or over investment, and, in consequence, increased regulatory risk when corrective policy actions are required.

From the perspective of Government policymaking and subsidy design, many countries have implemented a wide range of support schemes, including various forms of feed-in tariffs and tax incentives to promote green capacity investment (European Commission, 2005). The diversity of support schemes has motivated academic research on which scheme is more effective in fostering the diffusion of RE technologies. However, the comparison has focused mainly on price support systems, subsidy schemes and fiscal incentives (Boomsma et al., 2012; Boomsma & Linnerud, 2015), thus ignoring the contribution of schemes for overcoming budgetary constraints, such as the public-private partnerships (PPPs)(Krüger, 2012; Power et al., 2016). The latter, are legal partnerships between public and private actors for joint equity investments in infrastructure projects. One of the challenges associated with PPPs is that of identifying *ex-ante* the level of Government support that will facilitate the timely development and adoption of technological innovations.

Further complicating the interaction between firm- and Government-level policy-making is that investment within the R&D-based sector of the economy often entails idiosyncratic risk that cannot be diversified. Therefore, decision-making at the firm level should account for risk aversion. Indeed, the underlying commodities of such projects are not likely to be freely traded, which prevents the construction of a replicating portfolio as the assumption of hedging via spanning assets breaks down. Therefore, in this paper, we develop a utility-based, bi-level framework between a private firm and a Government in order to analyse how the optimal capacity investment and subsidy design are affected by not only economic uncertainty but also attitudes towards risk. Our analysis is based on real options theory, as it accounts for decision making under uncertainty, while reflecting the flexibility from embedded managerial discretion. Within the context of PPPs, real options models analysing the interaction between firm- and Government-level policy-making have been developed under the assumption of risk neutrality (Lukas & Thiergart, 2019).

Within this context, we assume that the subsidy takes the form of a cash grant, which, alongside tax credits and infrastructure assistance, is one of the common types of Governmental support. For example, in 2010 the EU granted roughly €10.9 billion to promote corporate R&D investments, while, since 2004, the Texas Enterprise Fund has awarded nearly $600 million to companies such as Apple Inc., eBay, Lockheed Martin, Samsung and T-Mobile (Lukas & Thiergart, 2019). In addition, within the context of green investment, cash grants and PPP agreements have been increasingly utilised to support investment in RE infrastructure (Giles & Clark, 2012; REN21, 2018), have been recognised for their contribution in the diffusion of wind power (Dinica, 2008; Martins et al., 2011) and have been proposed as a valid alternative to traditional policies for promoting climate change (Buso & Stenger, 2018; Nagy et al., 2019; Rossi et al., 2019). Consequently, the contribution of our work is threefold: **i.** First, we develop a bi-level framework to analyse how the objective of a Government and a private firm can be coupled to meet ambitious sustainability targets under uncertainty and risk aversion; **ii.** Second, we derive the optimal investment threshold and optimal capacity of a private firm, as well as the Government's optimal subsidy policy (analytically where possible); and **iii.** Third, we provide policy and managerial insights based on analytical and numerical results.

We proceed by discussing some related work in Section 2 and present the analytical framework in Section 3. First, we introduce assumptions and notation in Section 3.1 and then we derive the optimal investment policy of the private firm in Section 3.2 assuming that the subsidy is defined exogenously. Subsequently, we derive the equilibrium characterised by the capacity investment decision of a private firm and the Government's optimal subsidy policy in Section 3.3. Section 4 presents numerical examples as well as policy implications based on numerical and analytical results, while Section 5 concludes the paper offering directions for further research.

2. Related works

Following the seminal work of McDonald and Siegel (1985) and Dixit and Pindyck (1994), the real options literature has grown considerably to include various models that analyse the implications of uncertainty for capacity investment decisions within a wide range of contexts. Early examples include Pindyck (1988), who considers a firm that expands its capital stock incrementally with operational flexibility. Also, discrete capacity sizing is addressed in Dixit (1993), who develops a model for choosing among mutually exclusive projects under uncertainty. An extension of this line of work that allows for a continuously scalable capacity is presented in Dangl (1999), who finds that demand uncertainty raises the optimal capacity and makes waiting the optimal strategy even when demand is high. Extending the framework of Dixit (1993), Décamps et al. (2006) identifies a second waiting region around the indifference point between the net present values (NPVs) of two projects. Within this region, a firm will select the smaller (larger) project if the price drops (increases) sufficiently. Recent extensions of this line of work include Hagspiel et al. (2016a) and Wen et al. (2017b), who allow for volume flexibility and other types of demand functions.

Applications of real options models for capacity investment under uncertainty to the energy sector are presented in Fleten et al. (2007) and Bøckman et al. (2008), while policy-oriented applications are presented in Siddiqui and Fleten (2010), Boomsma et al. (2012), Boomsma and Linnerud (2015), Chronopoulos et al. (2016) and Wen et al. (2017a). More recently, Bigerna et al. (2019) develop a real options model for optimal capacity investment in RE under market demand uncertainty within the context of the Italian strategy for RE deployment under the EU policy. Their results confirm that a given environmental target cannot be reached by simply scaling the amount of support, and show that there exists an optimal subsidy level. The contribution of this line of work is that it provides policy insights in terms of the likelihood of meeting environmental targets set at a Government level by exploring how capacity investment decisions of private firms are affected by market and policy risk.

However, a limitation of the aforementioned literature is that the optimal investment policy and subsidy design are typically determined *ex-post* and not via the strategic interactions and the resulting equilibrium between a firm's and a Government's optimisation objective. Such strategic interactions are considered in the real options literature within the context of PPPs for investment in infrastructure projects. For example, Pennings (2000) uses a real options model to study how a Government can manipulate a firm's investment decision via combinations of tax reduction and investment subsidy. Results indicate that if tax collections and investment subsidy are combined in a way that they offset each other, then the entry trigger is a decreasing function of the tax rate. This implies that an investment subsidy is more efficient than a tax cut, which is also confirmed in Yu et al. (2007). A similar model is presented in Sarkar (2012), who examines the net benefit to a Government from using tax cut and/or investment subsidy as investment incentives. Unlike Pennings (2000) and Yu et al. (2007), Sarkar (2012) allows the Government's and the firm's discount rate to differ. Under this assumption, results indicate that it can be optimal to use a combination of investment subsidy and tax cut in order to encourage immediate investment.

The implications of PPP agreements for capacity expansion in road infrastructure are analysed in Krüger (2012), who develops a real options model in order to evaluate the capacity expansion option, assess the implications of ownership for exercising the option and identify when it is socially optimal to transfer ownership. Results indicate that the value of the expansion option is considerable and may entail a long waiting period prior to investment, yet the timing of capacity expansion depends on the level of traffic demand. Also, early expansion may take place if congestion costs are sufficiently high, but this decision depends also on the ownership of the project. Specifically, if a private company owns the right to expand, then this will result to a delay in the decision to expand beyond the point of time that is socially optimal.

In the same line of work, Power et al. (2016) develop an analytical framework to value strategic options in a specific type of PPP agreement, the Comprehensive Development Agreements (CDAs), emphasising on new transportation projects. The strategic options considered are buyout and conditional buyout options, revenue-sharing options for the public agency and minimum revenue guarantee options for the private sector. Results indicate that baseline buyout, revenue-sharing and minimum revenue guarantee options have significant value relative to the value of the concession, because there is a high probability that the options will be exercised. However, the baseline conditional buyout option has a small value, since it is likely to be out-of-the-money throughout the life of the concession. Additionally, the results suggest that strategic options are useful to reduce revenue risk, yet scaling down their payoffs or designing them as initially out-of-the-money may be required to keep their cost low.

Apart from supporting infrastructure projects, PPPs have become a valuable instrument for financing projects within the area of green energy and the R&D-based sector of the economy (Alloisio & Carraro, 2015; Carbonara & Pellegrino, 2020). Indeed, RE technologies are highly capital intensive, and, therefore, private sector capital, technology and innovation are often procured via PPPs to supplement limited public sector funding. Cedrick and Long (2017) find that positive externalities and PPP mechanisms present considerable incentives in the increase of RE projects in some countries. Also, Buso and Stenger (2018) compare policy responses to climate change and find that PPPs are more effective as a climate change policy than public subsidies.

A bi-level real options model within the context of PPPs is presented in Lukas and Thiergart (2019). Specifically, they develop a game-theoretic real options model between a risk-neutral firm and a

Government in order to analyse the effect of uncertainty and investment stimulus, in the form of a cash grant, on optimal investment timing, financing and investment scaling. As part of their analysis, they derive closed-form expressions for the firm's optimal investment threshold and optimal capacity, as well as for the optimal subsidy. However, this work overlooks the implications of attitudes towards risk that become particularly relevant with the increasing amount of investment in emerging markets, since the assumption of hedging via spanning assets breaks down, thus preventing risk-neutral valuation.

Examples of early work in the area of investment under risk aversion and uncertainty include Henderson and Hobson (2002) and Henderson (2007). The former introduce market incompleteness via the inclusion of a risky asset on which no trading is allowed, while the latter assumes that the uncertainty associated with the investment payoff can be hedged only partly via a risky asset that is correlated with the investment payoff. Results indicate that higher risk aversion due to greater idiosyncratic risk raises the incentive to reduce uncertainty by investing at a lower threshold in order to lock-in a value for the investment payoff. By contrast, Hugonnier and Morellec (2013) show that risk aversion erodes the value of a project and raises the required investment threshold. However, allowing for flexibility over project scale Chronopoulos et al. (2013) show how greater risk aversion accelerates investment by reducing the amount of installed capacity. Extensions of this line of work that allow for Markov regime-switching but ignore capacity sizing include Matomäki (2013) and Chronopoulos and Lumbreras (2017). Despite their obvious policy-making implications at a firm and Government level, such insights are not considered in bi-level models for investment under uncertainty that are typically developed under the assumption of risk neutrality.

In this paper, we extend the real options literature that analyses capacity investment decisions within the context of PPP agreements to allow for risk aversion with the objective to extend the application potential of options theory within bi-level optimisation frameworks for optimal subsidy design. Thus, this paper builds upon and expands the literature that tackles the problem of *ex-post* optimal subsidy design and investment under uncertainty and risk-neutrality (Lukas & Thiergart, 2019; Bigerna et al., 2019), so that risk preferences are taken into account and the optimal subsidy is determined *ex-ante*. We assume that a firm has a perpetual option to invest in a project facing economic uncertainty, and, therefore, its objective is to determine the investment threshold and project scale that maximise the time-zero expected discounted utility of the project's cash flows. Economic uncertainty is reflected in the output price that follows a geometric Brownian motion (GBM), while the firm's risk preferences are modelled via hyperbolic absolute risk aversion (HARA) utility function. The Government has a similar objective, since, upon the firm's investment, it receives taxes from the operating project's cash flows. Hence, the objective of the bi-level framework is to establish the appropriate subsidy that the Government should offer *ex-ante* so that the firm exercises its investment option timely.

Results indicate that the impact of risk aversion within a strategic bi-level framework is different than that presented in the existing literature. Specifically, we find that although greater risk aversion lowers the amount of installed capacity, it does not accelerate investment, as demonstrated in Chronopoulos et al. (2013). Instead, greater risk aversion raises the incentive to postpone investment. Intuitively, the decrease in project scale due to greater risk aversion lowers the optimal subsidy, thereby resulting in an increase of the required investment threshold. Also, although greater uncertainty raises the optimal subsidy under risk neutrality, the impact of uncertainty is reversed under high levels of risk aversion.

3. Model

3.1. Assumptions and notation

We consider a firm with a perpetual option to invest in a project of infinite lifetime facing uncertainty over future revenue steams. The firm has discretion over both the time of investment and the size of the project. The output price process $\{X_t, t \geq 0\}$, where t denotes time, is exogenous and follows a GBM that is described in (1), where μ denotes the annual growth rate, σ denotes the annual volatility and $\mathbf{d}Z_t$ denotes the increment of the standard Brownian motion. Also, ρ denotes the subjective discount and r is the risk-free rate.

$$\mathbf{d}X_t = \mu X_t \mathbf{d}t + \sigma X_t \mathbf{d}Z_t, \quad X_0 \equiv X>0, \qquad (1)$$

The assumption that the output price follows a GBM is often made in order to enable mathematical tractability and to facilitate closed-form solutions, however, it is also supported via empirical evidence. Indeed, although Pindyck (1999) finds that energy prices are mean reverting after analyzing 127 years of commodity prices (including coal, natural gas, and oil), he also finds that the rate of mean reversion is low, so that using a GBM to model electricity prices is unlikely to lead to large errors for the purposes of investment analysis.[1]

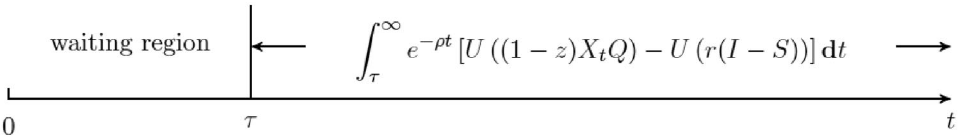

Figure 1. Irreversible Investment.

The firm's preferences are described by a HARA utility function (Sendstad & Chronopoulos, 2021), as indicated in (2), where $\gamma > 0$ is the risk aversion parameter. However, our framework can accommodate a wide range of utility functions, such as constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) utility function (Conejo et al., 2016).

$$U(X_t) = \frac{X_t^{\gamma}}{\gamma} \tag{2}$$

The investment cost $I(\cdot)$ (\$) is described in (3) and implies that a higher investment level leads to a higher cash flow with decreasing returns to scale.[2] Evidence that support such a cost function within the context of green energy are presented in Bøckman et al. (2008), Dismukes and Upton (2015) and Mignacca and Locatelli (2020). Also, in line with Huisman and Kort (2015), we assume that the firm always produces at full capacity, Q. This is often referred to as the *clearance* assumption and arises when it is costly to ramp up and down capacity or when commitments to workers and suppliers hinders temporary adjustments (Hagspiel et al., 2016b). For ease of exposition we set $I(Q) \equiv I$.

$$I(Q) = \frac{Q}{1-Q} \tag{3}$$

We assume that the firm receives Government support in the form of a cash grant, which is derived as the outcome of a non-cooperative game between participants $i = \{\mathrm{f}, \mathrm{g}\}$, denoting the firm and the government. The optimal time of investment is denoted by τ, while the optimal investment threshold and the optimal capacity are denoted by X_τ and Q, respectively, when the subsidy S is defined exogenously. If the subsidy is defined endogenously, then the equilibrium investment threshold, the equilibrium capacity and the equilibrium subsidy are denoted by $\hat{X}_\tau, \hat{Q}$ and $\hat{S}$, respectively. Also, $F_i(\cdot)$ is the maximised expected value of the option to invest and $\Phi_i(\cdot)$ is the maximised expected utility of the active project.

3.2. Firm-level investment

We begin by analysing the investment decision at the firm level. Notice that the utility function is not separable, and, therefore, following the same approach as Hugonnier and Morellec (2013) and Conejo et al. (2016), the key insight is to decompose the cash flows of the project into disjoint time intervals, as illustrated in Figure 1. Therefore, we assume that the amount of capital required for investment is exchanged at investment for the risky cash flows of the project. Specifically, up to time τ, the firm earns a risk-free rate, r, by placing the capital required for investment in an interest bearing account. At time τ, the firm swaps the risk-free cash flow for the risky cash flow that the project generates, fixes the capacity of the project and incurs the investment cost reduced by the subsidy.

The firm's objective is to determine the investment policy that maximises the time-zero expected discounted utility of all the cash flows of the project. This is described in (4), where τ is the random first-passage time of the state variable X_t through the investment threshold from below, i.e., $\tau = \inf\{t > 0 : X_t \geq X_\tau\}$, and $\mathcal{S}$ is the set of stopping times generated by the filtration of the process $\{X_t, t \geq 0\}$. Also, $\mathbb{E}_X[\cdot]$ is the expectation operator conditional on the initial value X of the price process.

$$F_{\mathrm{f}}(X) = \sup_{\tau \in \mathcal{S}} \mathbb{E}_X\left[\int_\tau^\infty e^{-\rho t}[U((1-z)X_tQ) - U(r(I-S))]\mathbf{d}t\right] \tag{4}$$

Next, we rewrite the right-hand side in (4) as in (5) using the law of iterated expectations and the strong Markov property of the GBM. The latter states that the values of the price process after time τ are independent of the values before τ and depend only on the value of the process at the time of investment, τ.

$$F_{\mathrm{f}}(X) = \sup_{\tau \in \mathcal{S}} \mathbb{E}_X\left[e^{-\rho\tau}\mathbb{E}_{X_\tau}\left[\int_0^\infty e^{-\rho t}[U((1-z)X_tQ) - U(r(I-S))\mathbf{d}t]\right]\right] \tag{5}$$

Note that the inner conditional expectation's independence from X_τ means that the two expectations in (5) may be separated as indicated in (6)

$$F_{\mathrm{f}}(X) = \sup_{\tau \in \mathcal{S}} \mathbb{E}_X[e^{-\rho\tau}]\Phi_{\mathrm{f}}(X_\tau) \tag{6}$$

as the product of the stochastic discount factor (first term) and the expected utility of the project's cash flows maximised with respect to capacity (second term), i.e.:

$$\Phi_{\mathrm{f}}(X_\tau) = \mathbb{E}_{X_\tau}\left[\int_0^\infty e^{-\rho t}[U((1-z)X_tQ) - U(r(I-S))\mathbf{d}t]\right] \tag{7}$$

As indicated in (8), it is possible to derive the analytical expression for the expected utility of a perpetual stream of cash flows when X_t follows a GBM

$$\mathbb{E}_X\left[\int_0^\infty e^{-\rho t}U((1-z)X_tQ)\mathbf{d}t\right] = \mathcal{A}U((1-z)XQ) \quad (8)$$

where $\mathcal{A} = \frac{\beta_1\beta_2}{\rho(\gamma-\beta_1)(\gamma-\beta_2)}$ and β_j denotes the positive (β_1) or negative (β_2) root of the quadratic $\frac{1}{2}\sigma^2\beta(\beta-1) + \mu\beta - \rho = 0$, i.e.:

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left[\frac{\mu}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2\rho}{\sigma^2}} > 1 \quad (9)$$

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left[\frac{\mu}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2\rho}{\sigma^2}} < 0 \quad (10)$$

The first step in solving the firm's capacity investment problem is to assume that it does not have the option to delay investment and must exercise the investment opportunity immediately. If the firm were to exercise a *now-or-never* investment opportunity, then the current output price is known and the firm would only need to determine the optimal size of the project that corresponds to the current level of the price process. Hence, the objective of the firm when exercising a *now-or-never* investment opportunity is $\max_Q \Phi_f(X)$, where:

$$\Phi_f(X) = \mathcal{A}U((1-z)XQ) - \frac{1}{\rho}U(r(I-S)) \quad (11)$$

By applying the first-order necessary condition (FONC) to the unconstrained optimisation problem (11), we obtain the condition for the optimal capacity, which is described in (12).

$$\mathcal{A}[(1-z)X]^\gamma Q^{\gamma-1} - \frac{1}{\rho}\left[r\left(\frac{Q}{1-Q} - S\right)\right]^{\gamma-1}\frac{r}{(1-Q)^2} = 0 \quad (12)$$

Next, we assume that X_t is too low to justify immediate investment, and, therefore, the firm needs to wait until X_t hits a sufficiently high threshold. Note that we can write the stochastic discount factor as $\mathbb{E}_X[e^{-\rho\tau}] = \left(\frac{X}{X_\tau}\right)^{\beta_j}$ (Dixit & Pindyck, 1994). Therefore, $F_f(X)$ can be expressed as in (13) for $X < X_\tau$.

$$F_f(X) = \max_{X_\tau > X}\left(\frac{X}{X_\tau}\right)^{\beta_1}\Phi_f(X_\tau) \quad (13)$$

By applying the FONC to the unconstrained optimisation problem (13), we obtain the expression for the optimal investment threshold that is indicated in (14). Subsequently, by inserting (14) into (12) we derive the analytical expression for optimal capacity, which is indicated in (15).

Proposition 1. *The optimal capacity Q(S) and the corresponding optimal investment threshold $X_\tau(S)$ are given by*

$$Q(S) = \frac{2\beta_1 S + \gamma + \sqrt{\Delta}}{2\beta_1(1+S)} \quad (14)$$

$$X_\tau(S) = \frac{r}{1-z}\left(\frac{\beta_2-\gamma}{\beta_2}\right)^{\frac{1}{\gamma}}\left[\frac{1}{1-Q(S)} - \frac{S}{Q(S)}\right] \quad (15)$$

provided that $\Delta = \gamma^2 - 4\beta_1 S(\beta_1 - \gamma) > 0$.

Under risk neutrality ($\gamma = 1$), (14) and (15) simplify to the analytical solution for the optimal investment threshold and optimal capacity described in Lukas and Thiergart (2019). Note that the second-order sufficiency condition (SOSC) requires the objective function to be concave at X_τ, which is shown in Proposition 2.

Proposition 2. *The objective function $F_f(\cdot)$ is strictly concave at X_τ for all $\gamma < 1$.*

The impact of greater risk aversion on the optimal investment threshold is described in Proposition 3. Intuitively, greater risk aversion lowers the investment propensity by decreasing the expected utility of the project and raising the required investment threshold.

Proposition 3. *For a given subsidy level, greater risk aversion lowers both the optimal capacity and the required investment threshold.*

In Proposition 4, we show that greater volatility increases the optimal capacity. Intuitively, the firm mitigates economic uncertainty by postponing the investment decision in order to learn about future market conditions, and, in case of favourable conditions, the firm will invest later in a greater capacity.

Proposition 4. *Greater economic uncertainty raises the optimal capacity and increases the required investment threshold.*

Alongside the implications of σ and γ on the optimal investment policy, of similar interest and relevance is their impact on the likelihood of investment. Therefore, in order to analyse the uncertainty-investment relationship, we also need to account for the probability of investment within a specific time horizon, denoted by T. This is described in (16), where $\mathrm{N}(\cdot)$ denotes the cumulative distribution function of the standard normal distribution, and it depends on the level of the subsidy through the critical threshold $X_\tau(S)$.

$$\mathbb{P}(X \geq X_\tau(S)|X_0) = \mathrm{N}\left(\frac{\ln\left[\frac{X}{X_\tau(S)}\right] + (\mu - 0.5\sigma^2)T}{\sigma\sqrt{T}}\right) + \left(\frac{X_\tau(S)}{X}\right)^{\frac{2\mu}{\sigma^2}-1}\mathrm{N}\left(\frac{\ln\left[\frac{X}{X_\tau(S)}\right] - (\mu - 0.5\sigma^2)T}{\sigma\sqrt{T}}\right) \quad (16)$$

Note that the analysis of the firm's investment strategy so far assumes that the subsidy is determined exogenously. As such, it does not allow for any interaction with decisions made at a Government level; at least not in a way that would impact the firm's investment policy *ex-ante*. In Section 3.3, we will extend this framework by analysing how the firm's and the Government's optimisation objective can be brought together within a single bi-level framework.

3.3. Government-level investment

Here, we analyse the Government's subsidisation policy. Note that upon the firm's investment, the Government receives taxes from the operating project's cash flows and incurs the cost reflected in the provision of the cash grant. The Government's net income received upon investment is described in (17). Following the same argument as Arrow and Lind (1970), we assume that the Government is risk neutral because it can hold a well-diversified portfolio and distribute the risk over a large number of shareholders.

$$\Phi_g(X) = z\frac{X_\tau(S)Q(S)}{r-\mu} - S \tag{17}$$

The trade-off implied by (17) gives rise to the question of what is the appropriate level of the subsidy. We assume that the Government will choose the level of subsidy S so as to maximise its net tax income at the time of investment. Following the same approach as in the firm-level investment, the value of the Government's investment opportunity is described in (18).

$$F_g(X) = \max_{S>0}\left(\frac{X}{X_\tau(S)}\right)^{\beta_1}\left[z\frac{X_\tau(S)Q(S)}{r-\mu} - S\right] \tag{18}$$

The right-hand side of (18) represents the present value of all future tax revenues the Government will receive from the moment the firm invests. However, this present value is received in exchange for paying a subsidy S at the moment of investment. In order to assess the profitability associated with S, this net income has to be discounted because the firm controls the timing and is likely to delay the investment decision. Unlike Lukas and Thiergart (2019), here the optimal subsidy is obtained numerically, since the FONC for the unconstrained optimisation problem (18) is highly non-linear. Nevertheless, as indicated in Section 4, the numerical results confirm those of Lukas and Thiergart (2019), while also extending them to allow for risk aversion.

Proposition 5. *The optimal subsidy $\hat{S}$ is obtained implicitly as the solution to the following equation*

$$\left[\frac{z}{\rho-\mu}\left(X_\tau(S)\frac{\partial Q(I(S))}{\partial I} + (1-\beta_1)Q(I(S))\frac{\partial X_\tau(S)}{\partial I}\right) + \beta_1\frac{S}{X_\tau(S)}\frac{\partial X_\tau(S)}{\partial I}\right]\frac{dI(S)}{dS} = 1 \tag{19}$$

Since the firm's investment policy is subject to the chosen subsidy, the solution for the equilibrium investment threshold $\hat{X}_\tau \equiv X_\tau(\hat{S})$ and investment level $\hat{Q} \equiv Q(\hat{S})$ can be obtained by setting the solution of (19) into (14) and (15).

4 Numerical examples

To demonstrate the application potential of our model and position it better within the energy sector, we consider the case of RE investment and utilise data from the Italian power sector (IEA, 2016) to estimate the values of relevant parameter, such as μ and σ. The estimated parameter values will serve as the baseline values that will be extended to facilitate sensitivity and robustness analysis within a range that satisfies the condition indicated in Proposition 1, i.e., $\Delta = \gamma^2 - 4\beta_1 S(\beta_1 - \gamma) > 0$. Note that estimation of the risk aversion parameter (γ) requires an empirical analysis of data on investors' risk preferences, which is outside the scope of this paper.

The estimate for r is taken from Monte dei Paschi di Siena (2011), a report on the status of the RE sector in Italy. The reported discount rate relates to a large plant (1 MW) in 2010 and is estimated at 6%. Also, the estimate for μ is derived assuming a positive correlation between X_t and consumers' willingness to pay. The latter can be estimated using data on the income growth and the expected reduction of the unit cost from 2010 to 2015. During this period, the average growth rate, and, in turn, the estimated sample mean of the log change of X_t is 1.83%. Finally, we assume that σ can be estimated as the historic annual standard deviation of price changes (Fleten et al., 2007) and we use the estimated value of the sample variance of the log change of X_t, which is $\sigma^2 = 0.004726$ or $\sigma = 0.07$.

The left panel in Figure 2 illustrates the firm's option and project value for different volatility levels. Notice that greater economic uncertainty raises the opportunity cost of investing, and, in turn, the value of waiting, thereby increasing the required investment threshold. Similarly, as the right panel illustrates, both the optimal investment threshold and the corresponding optimal capacity increase with greater z. This happens because greater z lowers the expected utility of the project's cash flows and raises the firm's incentive to compensate the loss in value by raising the required investment threshold.

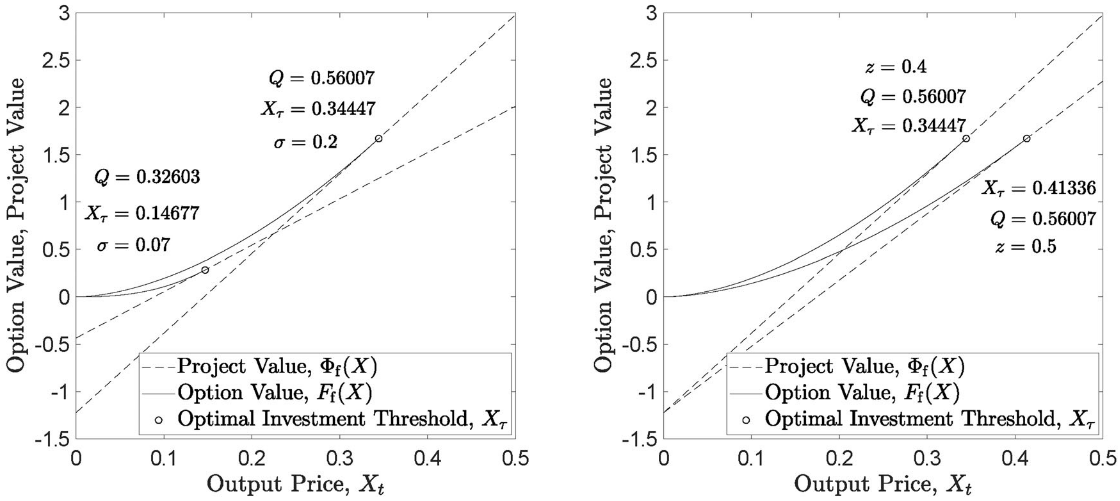

Figure 2. Option and project value for $\sigma = 0.07, 0.2$ (left panel) and for $z = 0.4, 0.5$ (right panel) for $S = 0.05$. Greater economic uncertainty raises the value of waiting, and, in turn, the required investment threshold and optimal capacity, while greater z lowers the expected utility of the revenues and increases the incentive to postpone investment.

The impact of economic uncertainty and risk aversion on the optimal investment threshold and the optimal capacity is illustrated in Figure 3. If the subsidy is defined exogenously (top panels), then an increase in risk aversion lowers both the optimal investment threshold (left panel) and the optimal capacity (right panel). This seemingly counter-intuitive result happens because the investment decision is irreversible, and, therefore, the firm is reluctant to postpone investment with greater risk aversion if this entails the installation of a bigger project. However, in direct contrast to the existing literature, we find that if the subsidy is defined endogenously, then the equilibrium investment threshold increases with greater risk aversion, whereas the equilibrium capacity decreases. This implies that greater risk aversion not only delays investment but also raises the incentive to install a smaller project. Intuitively, this happens because of the interaction between two opposing forces. On the one hand, greater risk aversion increases the incentive to install a smaller project, thus lowering the investment cost and the required investment threshold (Chronopoulos et al., 2013; Sendstad et al., 2021). On the other hand, a decrease in the optimal capacity lowers the optimal level of the subsidy, thereby raising the cost of investment and the required investment threshold. Interestingly, although the balance between these two effects is not readily obvious, Figure 3 indicates that the latter effect dominates in equilibrium. In turn, this demonstrates an implication of risk aversion that is not pronounced when the interaction between firm- and Government-level decision making is ignored and the subsidy is defined exogenously (Boomsma & Linnerud, 2015; Bigerna et al., 2019).

As illustrated in the left panel of Figure 4, the increase in the optimal investment threshold with greater risk aversion and uncertainty implies that the probability of hitting that threshold within a specific time interval decreases. However, a lower tax rate raises the probability of investment by increasing the expected utility of the project's cash flows, which lowers the required investment threshold. Also, as the right panel indicates, economic uncertainty impacts the level of subsidy that the Government grants to the firm, but the effect is ambiguous and depends crucially on the level of risk aversion. Indeed, for low levels of risk aversion, the right panel indicates that an increase in uncertainty leads to an increase in the subsidy level, whereas, for high levels of risk aversion, an increase in economic uncertainty may lower the level of subsidy.

Intuitively, subsidies affect investment timing, and, in turn, the discounting of the Government's payoff. Provision of a high subsidy mitigates the effect of discounting, yet has a negative influence on the Government's payoff. The strength of these effects is heavily determined by the level of uncertainty. Under risk neutrality, an increase in the level of subsidy leads to benefits that outweigh the costs. Indeed, an increase in the level of uncertainty raises the investment intensity and the firm's incentive to delay investment. Anticipating this reaction, the Government responds by increasing the subsidy in order to limit the negative effect of discounting, thereby maximizing its payoff. Interestingly, however, this result does not survive and is in fact reversed under high levels of risk aversion, thereby demonstrating the adverse impact of risk aversion on Government policymaking.

5. Conclusion

Although the implications of uncertainty and risk aversion for optimal investment have been explored in the real options literature within a wide range of

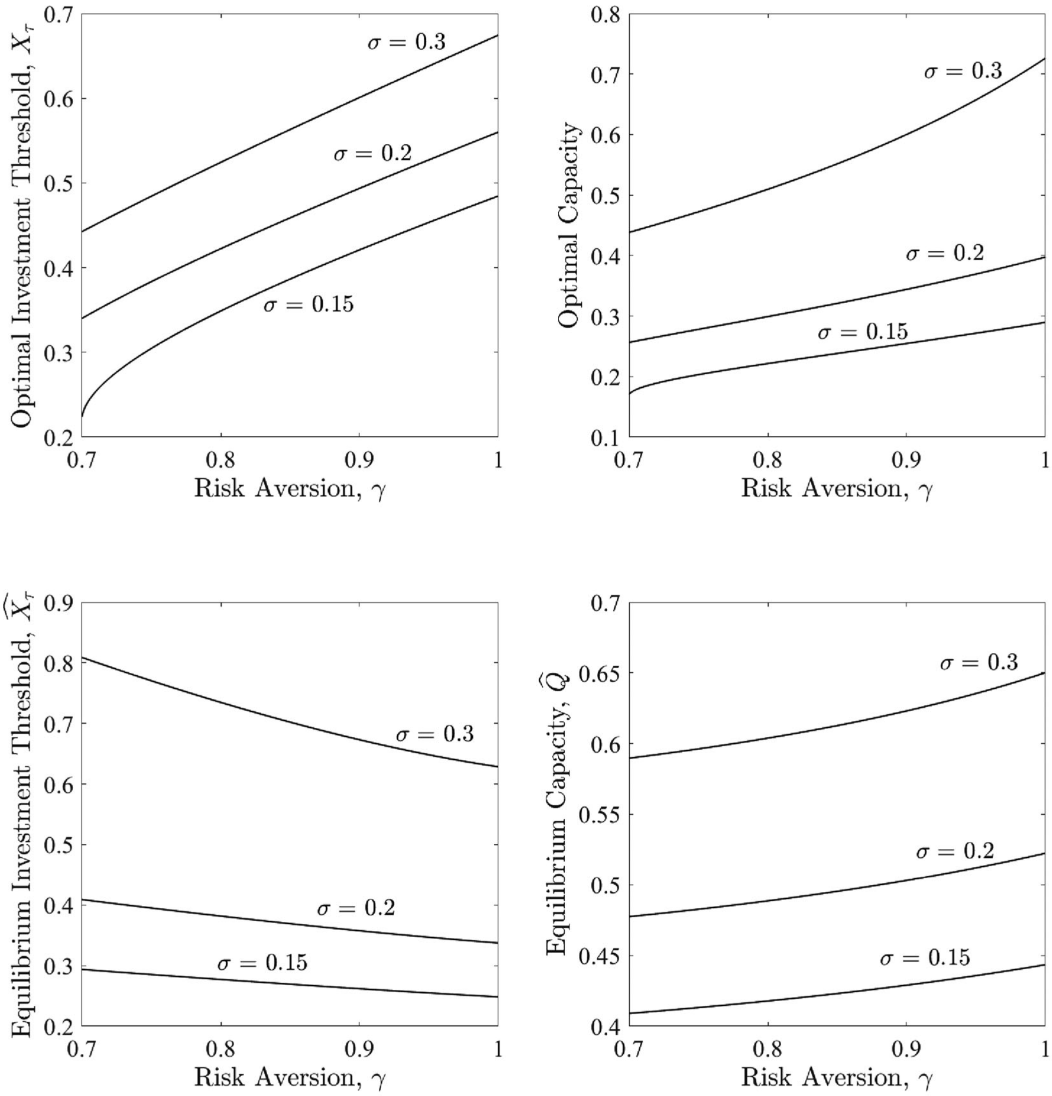

Figure 3. Optimal investment threshold (left panels) and optimal capacity (right panels) versus γ for $\sigma = 0.15, 0.2, 0.3$. Greater risk aversion accelerates (postpones) investment and lowers (lowers) the amount of installed capacity when the subsidy is defined exogenously (endogenously).

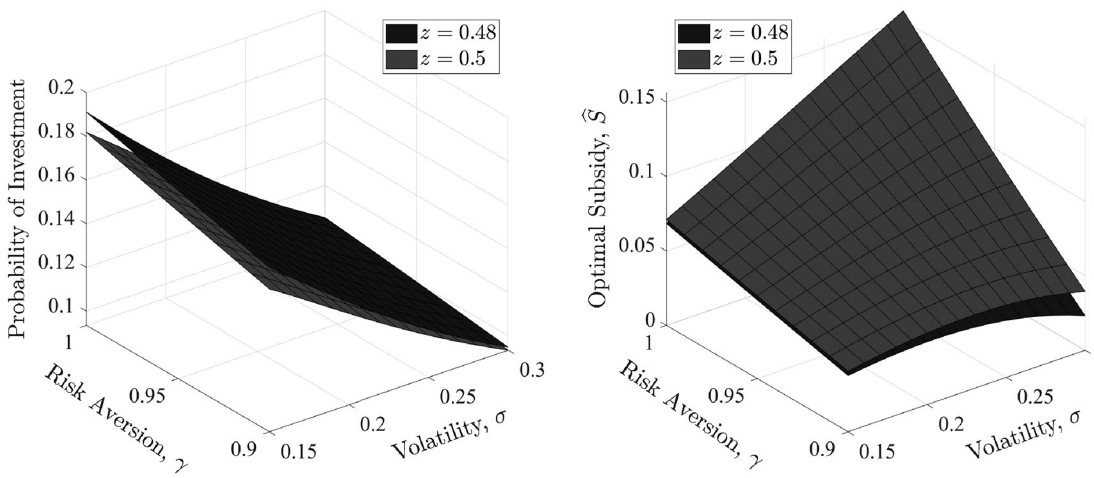

Figure 4. Probability of investment (left panel) and optimal subsidy level (right panel) versus γ and σ. Greater risk aversion and economic uncertainty lower the probability of investment but the impact of economic uncertainty on the optimal subsidy is ambiguous.

contexts, policy-oriented applications with the context of PPPs remain somewhat limited. Therefore, we develop a utility-based, bi-level real options framework, where decisions over capacity investment and subsidy design are determined endogenously taking into account the interaction between firm- and Government-level policymaking. Results indicate that the impact of risk aversion on capacity investment decisions is different than what existing literature indicates. Indeed, we find that greater risk aversion lowers the optimal capacity of a project but raises the required investment threshold. This is contrary to Chronopoulos et al. (2013), who find that greater risk aversion lowers both the optimal capacity and the required investment threshold. In addition, we confirm that greater economic uncertainty raises the optimal subsidy under risk neutrality, as shown in Lukas and Thiergart (2019), but we also find that the impact of economic uncertainty is reversed under high levels of risk aversion.

These results emphasise the implications of risk aversion for policymaking at a firm and a Government level when support takes the form of a cash grant. Within this context, greater risk aversion reduces not only the investment intensity but also the likelihood of investment. Despite the benefits that PPPs may present over public subsidies in terms of promoting RE investment (Buso & Stenger, 2018), it seems that PPP agreements must be designed carefully to take into account how risk aversion may hinder Government planning that aims to achieve specific targets, in terms of both investment timing and intensity. In addition, results suggest that optimal subsidy design at a Government level must consider the interaction between risk aversion and economic uncertainty, since the nature of this interaction under risk neutrality cannot be transposed naturally within a risk averse context.

A limitation of our model is that it ignores strategic interactions, as it considers a single firm, and, therefore, a natural extension of the existing framework would be to allow for oligopolistic competition. Hence, a directions for further research may include the implementation of strategic interactions, whereby two firms compete in the procurement of a PPP agreement. Following the same approach as Takashima et al. (2008), it would be interesting to explore how asymmetric competition and managerial flexibility may impact the strategic advantage of a firm and the outcome of competition in the procurement of a PPP agreement. In the same direction, another extension may include not only the analysis of duopolistic competition, but also the implications for social welfare, as in Huisman and Kort (2015). Also, to explore the application potential of our and other similar utility-based frameworks, a direction for further research could involve the development of an empirical framework for estimating the level of risk aversion within the context of RE investment (Kim & Lee, 2012; Eisenhauer & Ventura, 2003). In turn, this will support the development of a realistic case study.

Furthermore, the implications of utilising a different discount rate at a firm and a Government level, as in Pennings (2000), could provide further insights to the line of work that tackles the problem of optimal combination of investment subsidy and tax cut under risk aversion. In addition, by assuming an inverse demand function, as in Sendstad and Chronopoulos (2021), we may relax the assumption of a price-taking firm, which deprives our analysis of insights related to the feedback effect of capacity expansion on the price process. Finally, our framework may be extended to allow for different investment cost functions (Dangl, 1999), operational flexibility in the form of suspension and resumption options (Chronopoulos et al., 2011) and capacity switching options (Siddiqui & Takashima, 2012).

Notes

1. Although the GBM facilitates mathematical tractability, a limitation of the GBM for modelling energy prices is that it is not valid for short-term operational analysis, as it cannot capture strong mean reversion and spikes (Deng, 2005).
2. It is possible to apply a wide range of investment cost functions within the same framework, e.g., $I(Q) = aQ^b$ where a and b are positive constants, in order to analyse the implications of economies of scale, however, the specific choice is motivated by Lukas and Thiergart (2019), as it facilitates analytical tractability.

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Appendix

Proof of Proposition 1 We can derive the optimal investment threshold and the optimal capacity by decomposing the investment decision in two stages. First, the firm exercises a *now-or-never* investment opportunity, which implies that the output price at investment is known and the firm needs to determine only the optimal capacity. The optimisation objective when a firm exercises a *now-or-never* investment decision is described in (A-1).

$$\max_{Q}\left\{\mathcal{A}U((1-z)XQ)-\frac{1}{\rho}U(r(I-S))\right\} \tag{A-1}$$

By applying the first-order necessary condition (FONC) to the unconstrained optimisation problem (A-1), we obtain (A-2).

$$\begin{aligned}&\frac{\partial}{\partial Q}\left\{\mathcal{A}U((1-z)XQ)-\frac{1}{\rho}U(r(I-S))\right\}=0\\ &\Rightarrow \mathcal{A}[(1-z)X]^{\gamma}Q^{\gamma-1}-\frac{1}{\rho}\left[r\left(\frac{Q}{1-Q}-S\right)\right]^{\gamma-1}\frac{r}{(1-Q)^2}=0\end{aligned} \tag{A-2}$$

Next, we assume that the firm has the option to delay investment and that the required investment threshold exceeds the current output price. The optimisation objective is described in (A-3).

$$F_f(X)=\max_{X_\tau>X}\left(\frac{X}{X_\tau}\right)^{\beta_1}\left[\mathcal{A}U((1-z)X_\tau Q)-\frac{1}{\rho}U(r(I-S))\right] \tag{A-3}$$

By applying the FONC to the unconstrained optimisation problem (A-3) we obtain

$$\mathcal{A}((1-z)X_\tau Q)^{\gamma}\frac{\beta_1-\gamma}{\beta_1}-\frac{1}{\rho}\left[r\left(\frac{Q}{1-Q}-S\right)\right]^{\gamma}=0 \tag{A-4}$$

and solving with respect to X_τ we have:

$$X_\tau=\frac{r}{1-z}\left(\frac{\beta_2-\gamma}{\beta_2}\right)^{\frac{1}{\gamma}}\left[\frac{1}{1-Q}-\frac{S}{Q}\right] \tag{A-5}$$

Inserting the expression for X_τ from (A-5) into (A-2) we obtain (A-6).

$$\beta_1(1+S)Q^2-(2\beta_1S+\gamma)Q+\beta_1S=0 \tag{A-6}$$

The analytical expression for the optimal capacity is obtain by solving (A-6) and is described in (A-7), where $\Delta=\gamma^2-4\beta_1S(\beta_1-\gamma)$

$$Q=\frac{2\beta_1S+\gamma+\sqrt{\Delta}}{2\beta_1(1+S)} \tag{A-7}$$

and, thus, the expression for $I(Q)$ is:

$$I(Q) = \frac{Q}{1-Q} = \frac{\gamma + \sqrt{\Delta}}{2(\beta_1 - \gamma)} \tag{A-8}$$

□

Proof of Proposition 2 The second-order sufficiency condition requires that the objective function is concave at the critical threshold X_τ. Hence, we first need to calculate the second derivative of $F_f(\cdot)$ and evaluate it at X_τ.

$$\frac{\partial^2 F_f(X)}{\partial X_\tau^2} = \beta_1(\beta_1+1)\left[\mathcal{A}\frac{[(1-z)X_\tau Q]^\gamma}{\gamma} - \frac{1}{\rho}\frac{[r(bQ-S)]^\gamma}{\gamma}\right] -2\beta_1\mathcal{A}[(1-z)X_\tau Q]^\gamma + \mathcal{A}(\gamma-1)[(1-z)X_\tau Q]^\gamma \tag{A-9}$$

The SOSC requires that $\frac{\partial F_f(X)}{\partial X_\tau}<0$. Substituting for X_τ we have:

$$\frac{1}{\beta_1-\gamma}(\beta_1(\beta_1+1) - 2\beta_1\gamma + \gamma(\gamma-1)) - \beta_1 - 1 < 0 \tag{A-10}$$

After simplifying the above expression we have $\frac{\partial F_f(X)}{\partial X_\tau}<0 \iff \gamma(\gamma-\beta_1)<0$, which holds for all $\gamma<1$, i.e., the range of γ corresponding to risk aversion. □

Proof of Proposition 3 The partial derivative $\frac{\partial Q}{\partial \gamma}>0$ is described in (A-11). Note that greater risk aversion is reflected in lower values of γ. Hence, $\frac{\partial Q}{\partial \gamma}>0$ implies that Q increases with lower risk aversion, i.e., as γ increases, or, equivalently, that Q decreases with greater risk aversion, i.e., as γ decreases.< >

$$\frac{\partial Q}{\partial \gamma} = \frac{1 + \frac{1}{2}\left(\gamma^2 - 4\beta_1 S(\beta_1-\gamma)\right)^{-\frac{1}{2}}(2\gamma + 4\beta_1 S)}{2\beta_1(1+S)} = \frac{\sqrt{\Delta} + \gamma + 2\beta_1 S}{2\beta_1(1+S)\sqrt{\Delta}} > 0 \tag{A-11}$$

Note also that the optimal investment threshold X_τ is a monotonic function of the optimal capacity, and, therefore, a greater γ should also raise the optimal investment threshold. Intuitively, a greater output price is required to install a bigger project, as shown in (A-12).

$$\frac{\partial X_\tau}{\partial Q} = \frac{1}{1-z}\left(\frac{\beta_2-\gamma}{\beta_2}\right)^{\frac{1}{\gamma}} \underbrace{\left[\frac{1}{(1-Q)^2} + \frac{S}{Q^2}\right]\frac{\partial Q}{\partial \gamma}}_{>0} \tag{A-12}$$

□

Proof of Proposition 4 The expression for $\frac{\partial Q}{\partial \sigma^2}$ is:

$$\frac{\partial Q}{\partial \sigma^2} = \frac{\frac{\partial \beta_1}{\partial \sigma^2}\left[2S + \frac{1}{2\sqrt{\Delta}}(4\gamma S - 8S\beta_1)2\beta_1(1+S) - 2(1+S)\left(2\beta_1 S + \gamma + \sqrt{\Delta}\right)\right]}{4\beta_1^2(1+S)^2} = \frac{\frac{\partial \beta_1}{\partial \sigma^2}\left[4S\sqrt{\Delta} + 8(\gamma S - 2S\beta_1)\beta_1(1+S) - 4\sqrt{\Delta}(1+S)\left(2\beta_1 S + \gamma + \sqrt{\Delta}\right)\right]}{4\beta_1^2(1+S)^2} \tag{A-13}$$

It suffices to show that the numerator in (A-13) is positive. Since $\frac{\partial \beta_1}{\partial \sigma}<0$, we must show that the term within the brackets is negative, i.e.:

$$S\sqrt{\Delta} + 2(\gamma S - 2S\beta_1)\beta_1(1+S) - \sqrt{\Delta}(1+S)\left(2\beta_1 S + \gamma + \sqrt{\Delta}\right) < 0 \tag{A-14}$$

The last expression can be rewritten as:

$$S\sqrt{\Delta}[1-(1+S)2\beta_1] + 2S(\gamma - 2\beta_1)\beta_1(1+S) - (1+S)\left(\gamma\sqrt{\Delta} + \Delta\right) < 0 \tag{A-15}$$

Notice that all the terms in (A-15) are negative, and, thus, the last inequality holds. □

Proof of Proposition 5 The Government's optimisation objective is described in (A-16).

$$F_g(S) = \max_S \left(\frac{X}{X_\tau(S)}\right)^{\beta_1}\left[z\frac{X_\tau(S)Q(S)}{r-\mu} - S\right] \tag{A-16}$$

If we express $Q(S)$ in terms of $I(S)$, then we obtain the equivalent expression

$$F_g(S) = \max_S \left(\frac{X}{X_\tau(S)}\right)^{\beta_1}\left[z\frac{X_\tau(S)Q(I(S))}{r-\mu} - S\right] \tag{A-17}$$

where $Q(I(S)) = \frac{I(S)}{1+I(S)}$. The objective is to solve the equation $\frac{\mathbf{d}F_g(S)}{\mathbf{d}S} = 0$. Note that $\frac{\mathbf{d}F_g(S)}{\mathbf{d}S}$ can be expressed as in (A-18).

$$\frac{\mathbf{d}F_g(S)}{\mathbf{d}S} = \left[\frac{\partial F_g(S)}{\partial I(S)} + \frac{\partial F_g(S)}{\partial X_\tau(S)}\frac{\partial X_\tau(S)}{\partial I(S)}\right]\frac{\mathbf{d}I(S)}{\mathbf{d}S} + \frac{\partial F_g(S)}{\partial S} = \left(\frac{X}{X_\tau(S)}\right)^{\beta_1}\left[\left(\frac{z}{\rho-\mu}\left(X_\tau(S)\frac{\partial Q(I(S))}{\partial I} + (1-\beta_1)Q(I(S))\frac{\partial X_\tau(S)}{\partial I}\right)\beta_1\frac{S}{X_\tau(S)}\frac{\partial X_\tau(S)}{\partial I}\right)\frac{\mathbf{d}I(S)}{\mathbf{d}S} - 1\right] \tag{A-18}$$

Since (A-18) is highly non-linear, it is not possible to obtain an analytical solution for the optimal subsidy, and, therefore, we solve for S numerically. □