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DERIVATION OF EQUIVALENT LINEAR PROPERTIES OF BOUC-WEN HYSTERETIC SYSTEMS FOR SEISMIC RESPONSE SPECTRUM ANALYSIS VIA STATISTICAL LINEARIZATION

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Abstract.*Anewly proposed statistical linearization based formulation is used to derive effective linear properties (ELPs), namely damping ratio and natural frequency, for stochastically excited hysteretic oscillatorsinvolving the Bouc-Wen force-deformation phenomenological model. This is achieved by first using a frequency domain statistical linearization step to substitute a Bouc-Wen oscillator by a third order linear system. Next, this third order linear system is reduced to a second order linear oscillator characterized by a set of ELPs by enforcing equality of certain response statistics of the two linear systems. The proposed formulation is utilized in conjunction with quasi-stationary stochastic processes compatible with elastic response spectra commonly used to represent the input seismic action in earthquake resistant design of structures. Then, the derived ELPs are used to estimate the peak response of Bouc-Wen hysteretic oscillatorswithout numerical integration of the nonlinear equation of motion; this is donein the context of linear response spectrum-based dynamic analysis. Numerical results pertaining to the elastic response spectrum of the current European aseismic code provisions (EC8) are presented to demonstrate the usefulnessof the proposed approach. These results are supported by pertinent Monte Carlo simulations involving an ensemble of non-stationary EC8 spectrum compatible accelerograms. The proposed approach can hopefully be an effective tool in the preliminary aseismic design stages of yielding structures and structural members commonly represented by the Bouc-Wen hysteretic model within either a force-based or a displacement-based context.*

1 INTRODUCTION

The technique of statistical linearization has been used for over six decades to determine response statistics of stochastically excited non-linear dynamical systems. Arguably, it has become the most widely used alternative to the computationally demanding Monte Carlo simulations[1]. It relies on the consideration of surrogate (equivalent) linear systems whose properties arederived based on various probabilistic criteria. In this context, the authors[2,3] have proposed a frameworkcombiningearly statistical linearization schemes[1,4] in conjunction with response spectrum compatible power spectra to estimate the peak seismic response of various nonlinear oscillators without performing non-linear dynamic response history analysis. The aforementioned framework offers an alternative method to the use of deterministic linearization techniques assuming harmonic excitation of the non-linear systems. Such techniques are widely used by theearthquake engineering community for various purposes includingfor the design of yielding earthquake resistant structures[5] and for deriving of inelastic response spectra[6] and R-μ-T_n (strength reduction factor-ductility-natural period) relationships[7].

Recently, Giaralis et al.[8] introduced a “dimension reduction” step in conjunction with higher-order statistical linearization schemes[1] to derive linear time invariant single-degree-of-freedom systems characterized by an effective stiffness and dampingto approximate the displacement and velocity variance of the response of stochastically excited non-linear oscillators governed by integro-differential equations[9]. Further,a similar dimension reduction step has been considered by Kougioumtzoglou and Spanos[10] to approximate the response statistics of nonlinear MDOF systems exposed to non-stationary stochastic excitations by linear time-varying SDOF systems. This has rendered tractable the determination of the time-evolving response probability density function of nonlinear MDOF systems by means of statistical linearization in combination with stochastic averaging. Moreover, Spanos and Giaralis[11] has demonstrated that the incorporation of the aforementioned reduction step within the framework of Giaralis and Spanos[3]provide dependable peak response estimates of seismically excited bilinear hysteretic systems exhibiting strongly non-linear behavior.

Herein, the aforementioned dimension reduction step is utilized to derive effective linear properties (ELPs), namely damping ratio and natural frequency, for seismically excited hysteretic systems involving the Bouc-Wen force-deformation model[12,13]. This hysteretic modelhas been extensively used to model yielding structures and structural members under earthquake excitation[10,14,15].Following the framework of Giaralis and Spanos[2,3], an input power spectrum compatible with a given response spectrum is assumed in the derivation of the ELPs.the latter spectrum is utilized together with the ELPs to estimate the peak non-linear response of theBouc-Wen oscillators without resorting to numerical integration. Numerical data pertaining to the response spectrum of the European aseismic code provisions[16] are presented to demonstrate the applicability of the proposed approach.

2 EFFECTIVE LINEAR PROPERTIES OF STOCHASTICALLY EXCITED BOUC-WEN OSCILLATORS

2.1 Frequency domain statistical linearization solution forBouc-Wen oscillators

Consider a quiescent nonlinear hysteretic SDOF system with ratio of critical viscous damping ζ, base-excited by a stationary zero-mean Gaussian acceleration process $g(t)$. Denote by $\tilde{x}$ the response displacement process of this system relative to the ground motion and assume that its nonlinear behavior traces the Bouc-Wen model. The motion of the considered system is governed by the following system of differential equations[12]

$$\begin{aligned} \ddot{x}(t)+2\zeta\omega_n\dot{x}(t)+a\omega_n^2 x(t)+(1-a)\,\omega_n^2 z(t) &= -g(t)/x_y \\ \dot{z}(t) &= -\gamma\left|\dot{x}(t)\right| z(t)\left|z(t)\right|^{n-1}-\beta\dot{x}(t)\left|z(t)\right|^{n}+A\dot{x}(t) \end{aligned}, \qquad (1)$$

where x_y is a nominal yielding displacement, x is the relative non-dimensional response displacement process normalized by the nominal yielding displacement (i.e. $x=\tilde{x}/x_y$), z is an additional state associated with the Bouc-Wen hysteretic model, γ, β, n, A are constant parameters which control the shape of the hysteretic loops, α is a generalized post-yield to pre-yield stiffness ratio, and $\omega_n=\sqrt{f_y/x_y}$ is the pre-yielding natural frequency with f_y being a nominal yielding strength. In the above equation and hereafter the dot over a symbol denotes differentiation with respect to time t.

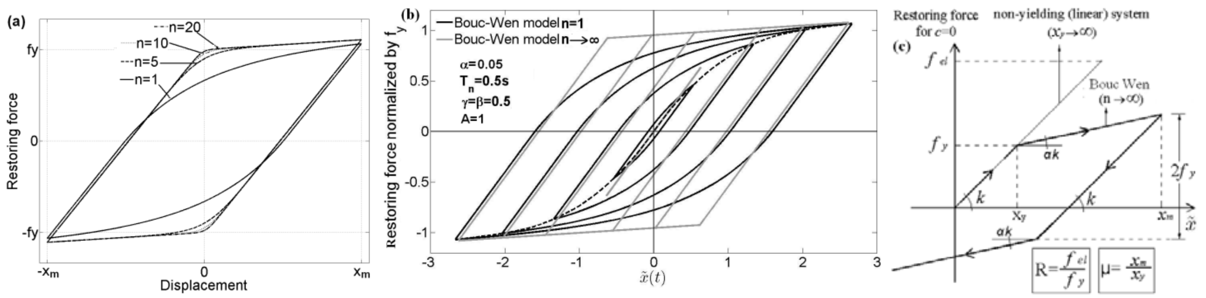

Figure 1.Bouc-Wen model for variousn parameters and definition of ductility μand strength reduction factor R.

In Eq.(1),a can be construed as a parameter governing the severity of nonlinear response ranging from linear system (α=1) to the perfectly “elasto-plastic” system (α=0)[9].Further, Wen[12] has showed that by varying theγ, β,and A parameters, the state z forms varioushysteretic loop shapes with x. For A=1 and for $\beta=\gamma$=0.5 the non-linear restoring force of the oscillator governed by Eq.(1)is of the “softening type” and constitutes a reasonable model to capture the response of yielding structures and structural members exposed to earthquake induced strong ground motions[13,15]. Figure 1includes plots of the restoring force of the aforementioned softening type of hysteretic Bouc-Wen oscillators for various values of the exponent n, and for various amplitudes of harmonic excitation. It can be seen that the exponent n governs the “smoothness” of the hysteretic loops with n=1 being the smoothest possible. For n>20 the Bouc-Wen oscillator traces closely a “bilinear” hysteretic law[15].

Application of the statistical linearization procedure proposed by Wen[13] yields a 3rd-order equivalent linear system (ELS) governed by the system of linear differential equations

$$\begin{aligned} \ddot{x}(t)+2\zeta\omega_n\dot{x}(t)+a\omega_n^2 x(t)+(1-a)\,\omega_n^2 z(t) &= -g(t)/x_y \\ \dot{z}(t)+c_{eq}\dot{x}(t)+k_{eq}z(t) &= 0 \end{aligned}, \qquad (2)$$

where the linearization parameters c_{eq} and k_{eq} are determined by requiring minimization of the mean square error in replacing Eq.(1)by Eq.(2). By approximating the processes $\dot{x}(t)$, and $z(t)$ as jointly Gaussian, the following expressions for determining c_{eq} and k_{eq} are reached[17]

$$c_{eq} = \gamma F_1 + \beta F_2 - A \quad and \quad k_{eq} = \gamma F_3 + \beta F_4 , \tag{3}$$

where

$$\begin{gathered} F_1 = \frac{\sigma_z^n}{\pi}\Gamma\left(\frac{n+2}{2}\right)2^{n/2}P \quad ; \quad F_2 = \frac{\sigma_z^n}{\sqrt{\pi}}\Gamma\left(\frac{n+1}{2}\right)2^{n/2} \\ F_3 = \frac{n\sigma_{\dot{x}}\sigma_z^{n-1}}{\pi}\Gamma\left(\frac{n+2}{2}\right)2^{n/2}\left(2\left(1-\rho^2\right)^{\frac{n+1}{2}} + \rho P\right) \quad ; \quad F_4 = \frac{n\rho\sigma_{\dot{x}}\sigma_z^{n-1}}{\sqrt{\pi}}\Gamma\left(\frac{n+1}{2}\right)2^{n/2} \end{gathered} , \tag{4}$$

in which

$$P = 2\int_L^{\pi/2} \sin^n\theta \, d\theta \quad and \quad L = \tan^{-1}\left(\frac{\sqrt{1-\rho^2}}{\rho}\right). \tag{5}$$

In the preceding equations, $\Gamma(\cdot)$ is the standard gamma function, while $\sigma_{\dot{x}}^2, \sigma_z^2$, and ρ denote,respectively,the variance of the processes $\dot{x}(t)$ and $z(t)$, and their correlation.That is,

$$\sigma_{\dot{x}}^2 = E\left\{\dot{x}(t)^2\right\} \quad ; \quad \sigma_z^2 = E\left\{z(t)^2\right\} \quad ; \quad \rho = \frac{E\left\{\dot{x}(t)z(t)\right\}}{\sigma_z\sigma_{\dot{x}}}. \tag{6}$$

In the last equation and henceforth $E\{\cdot\}$ denotes the mathematical expectation operator.

From the above expressions it is seen that the parameters c_{eq} and k_{eq} depend on the variance of the response processes $\dot{x}(t)$ and $z(t)$ andon their correlation. To this end, a frequency domain formulation relying on the spectral input/output relations for linear systems is devised to calculate these response moments as[11,14]

$$\sigma_{\dot{x}}^2 = \int_0^\infty \omega^2 \left|\frac{i\omega + k_{eq}}{\sum_{k=0}^{3}(i\omega)^k A_k}\right|^2 \frac{G(\omega)}{x_y^2} d\omega \quad and \quad \sigma_z^2 = \int_0^\infty \left|\frac{-i\omega c_{eq}}{\sum_{k=0}^{3}(i\omega)^k A_k}\right|^2 \frac{G(\omega)}{x_y^2} d\omega , \tag{7}$$

in which $i = \sqrt{-1}$, $G(\omega)$ is the one-sided power spectrum representing in the domain of frequencies ω the input process $g(t)$

$$A_0 = a\omega_n^2 k_{eq} \quad ; \quad A_1 = a\omega_n^2 + 2\zeta\omega_n k_{eq} - (1-a)\omega_n^2 c_{eq} \quad ; \quad A_2 = k_{eq} + 2\zeta\omega_n \quad ; \quad A_3 = 1 . \tag{8}$$

while the cross-variance term can be computed by the equation

$$E\left\{\dot{x}(t)z(t)\right\} = -\frac{k_{eq}}{c_{eq}}\sigma_z^2 . \tag{9}$$

Equations (3), (7), and (9) form a system of non-linear equations with five unknowns: c_{eq}, k_{eq}, $\sigma_{\dot{x}}^2$, σ_z^2 ,and $E\{\dot{x}z\}$.This five-by-five system of equations can be readily written as a standard minimization problem and solved numerically by any qualified optimization routine. In of the all ensuing numerical examples aMATLAB®built-in optimization algorithmemploying a trust region dog-leg search method is used for the purpose[18].

Note that the solution of the aforementioned system of equations establishesthe 3rdorder ELS of Eq.(2) governed by the linearization parametersc_{eq}, k_{eq}. Various researchers[1,13,19] have demonstrated theoretically and through numerical experimentation that this particular higher-than-a-second-order linear system captures the response statistics of hysteretic systems better than the early 2ndorder statistical linearization schemes[4,20]. However, this 3rd order ELS does not correspond to any particular mechanical dynamical system and, thus, the linearization parameters c_{eq}, k_{eq}bear limitedphysical significance.To this end, in the next section an approach to reduce the system order of the 3rdELS to a 2nd order linear SDOF oscillator defined by physically meaningful parameters (i.e. natural frequency and critical damping ratio)is considered.

2.2 System order reduction relying on a response statistics criterion

Consider an effective quiescent linear single-degree-of-freedom (SDOF) oscillator of critical viscous damping ζ_{eff} and natural frequency ω_{eff}base-excited by the stationary zero-mean Gaussian acceleration process $g(t)$. The governing equation of motion in terms of the deflection$y$ of this auxiliary system normalized by x_y reads as

$$\ddot{y}(t)+2\zeta_{eff}\,\omega_{eff}\,\dot{y}(t)+\omega_{eff}^2\,y(t)=-g(t)/x_y\,. \tag{10}$$

Following Giaralis et al.[8] and Spanos and Giaralis[11], the above 2nd order system is related to the 3rd order ELS of Eq. (2) by equating the variances of the processes $x(t)$ and $y(t)$. That is,

$$\sigma_x^2=\int_0^\infty \frac{G(\omega)/x_y^2}{\left(\omega^2-\omega_{eff}^2\right)^2+\left(2\zeta_{eff}\,\omega_{eff}\,\omega\right)^2}d\omega\,. \tag{11}$$

andthe variances of the first derivative of the above processes (relative velocities $\dot{x}(t)$ and $\dot{y}(t)$), that is,

$$\sigma_{\dot{x}}^2=\int_0^\infty \frac{\omega^2 G(\omega)/x_y^2}{\left(\omega^2-\omega_{eff}^2\right)^2+\left(2\zeta_{eff}\,\omega_{eff}\,\omega\right)^2}d\omega\,. \tag{12}$$

The variance appearing in the lhs of Eq.(11)can be calculated by the expression

$$\sigma_x^2=\int_0^\infty \left|\frac{i\omega+k_{eq}}{\sum_{k=0}^{3}(i\omega)^k A_k}\right|^2 \frac{G(\omega)}{x_y^2}d\omega\,, \tag{13}$$

while the variance appearing in the lhs of Eq.(12)is determined upon solving the five-by-five system of equations considered in the statistical linearization solution of the previous section. In this regard, Eqs.(11)and(12) define a two-by-two system of nonlinear equations which can be solved for the unknown effective linear properties ζ_{eff} and ω_{eff}(ELPs) of the 2ndorder linear system corresponding to a linear SDOF oscillator. To this aim, the same optimization algorithm used to obtain the statistical linearization solution is employed to solve the above two-by-two system of non-linear equations in obtaining the numerical resultsdiscussedin section 4.1.

3 RESPONSE SPECTRUM COMPATIBLE QUASI-STATIONARY POWER SPECTRA

The novel formulation detailed in section 2 to derive effective linear properties corresponding to a Bouc-Wen oscillator can be used in conjunction with any stationary process $g(t)$ represented in the frequency domain by a power spectrum $G(\omega)$. For the purposes of this work, the above formulation is used within the statistical linearization-based framework of Giaralis and Spanos[3]to determine the peak inelastic response of seismically excited nonlinear systems by considering response spectra of linear SDOF oscillators. In the adopted framework, the seismic input action is defined in terms of a pseudo-acceleration response spectrum $S_\alpha(T,\zeta)$ with $T=2\pi/\omega_j$ being the natural period of a linear SDOF oscillator with ratio of critical viscous damping ζ. Further, $g(t)$ is a "quasi-stationary" process of finite duration $T_s$related to the response spectrum $S_\alpha$via the concept of a "peak factor" η_j by the equation[21,22]

$$S_a\left(\frac{2\pi}{\omega_j},\zeta\right)=\eta_j\omega_j^2\sqrt{\int_0^\infty \frac{G(\omega)}{\left(\omega^2-\omega_j^2\right)^2+\left(2\zeta\omega\omega_j\right)^2}d\omega}\,. \tag{14}$$

In determining the peak factor η_j appearing in Eq.(14)the following approximate semi-empirical expression can be adopted[21]

$$\eta_j=\sqrt{2\ln\left\{2v_j\left[1-\exp\left(-q_j^{1.2}\sqrt{\pi\ln\left(2v_j\right)}\right)\right]\right\}}\,. \tag{15}$$

with

$$v_j=-\frac{T_s^*}{2\pi\ln(0.5)}\omega_j \quad ; \quad T_s^*=T_s\exp\left(-2\left(\frac{1-\exp\left(-2\zeta\omega_k T_s\right)}{1-\exp\left(-\zeta\omega_k T_s\right)}-1\right)\right). \tag{16}$$

and

$$q_j = \sqrt{1-\frac{1}{1-\zeta_j^2}\left(1-\frac{2}{\pi}\tan^{-1}\frac{\zeta_j}{\sqrt{1-\zeta_j^2}}\right)^2} \quad ; \quad \zeta_j = \frac{\zeta}{1-\exp\left(-2\zeta\omega_j T_s\right)}. \tag{17}$$

Equations(15) and (17) allow for calculating reliably the median peak factor of a linear oscillator with properties ω_j and ζ subject to clipped white noise input of duration T_s. In this regard, Eq.(14)establishes the criterion: considering an ensemble of realizations of the process $g(t)$, half of the population of their response spectra will lie below S_a (i.e. S_a is the median response spectrum)[3,14,21,22].

Given a response spectrum S_a, an estimate of the power spectrum $G(\omega)$ conforming with the aforementioned criterion can be recursively evaluated at a specific set of M equally spaced natural frequencies $\omega_k= \omega_0+ (j-0.5)\Delta\omega$; $j= 1,2,\ldots,M$using the equation[23]

$$G\left[\omega_k\right] = \begin{cases} \dfrac{4\zeta_k}{\omega_k\pi - 4\zeta_k\omega_{k-1}}\left(\dfrac{S_\alpha^2\left(2\pi/\omega_k,\zeta\right)}{\eta_k^2} - \Delta\omega\sum_{i=1}^{k-1}G\left[\omega_i\right]\right), \omega_k > \omega_0 \\ 0\,,\, 0 \le \omega_k \le \omega_0 \end{cases}. \tag{18}$$

In the last equation, ω_0 is the lowest frequency for which Eq.(15) is defined. An approximation of the pseudo-acceleration response spectrum $A[2\pi/\omega_k,\zeta]$ corresponding to the power spectrumobtained by Eq.(18)can be determined by using Eq.(14)[3,11].In general, $A[2\pi/\omega_k,\zeta]$ may not lie as close as desired to the target spectrum S_α for all the considered ω_k natural frequencies. In this respect, $G[\omega_k]$ can be further modified iteratively to improve the point-wise matching of the response spectrum $A[2\pi/\omega_k,\zeta]$ with the target spectrum by means of the following equation written at the N-th iteration[3,24]

$$G^{(N+1)}\left[\omega_k\right] = G^{(N)}\left[\omega_k\right]\left(\frac{S_a\left[2\pi/\omega_k,\zeta\right]}{A^{(N)}\left[2\pi/\omega_k,\zeta\right]}\right)^2. \tag{19}$$

In the ensuing numerical workthe thus obtained response spectrum compatible power spectra are used to determine ELPs associated with hystereticBouc-Wen oscillators excited by a given response spectrum.Conveniently, there exist spectral moment formulae to numerically evaluate the integrals in Eqs.(7) and(11) to (13)for "discrete" power spectra known at equally spaced natural frequencies $\omega_k$in a computationally efficient manner[3,25].

4 NUMERICAL APPLICATION TO THE EC8 RESPONSE SPECTRUM

4.1 Derivation of EC8-compatible effective linear properties

The elastic response spectrum of the current aseismic code provisions effective in Europe (EC8)[16] is herein considered as a paradigm to assess the usefulness and applicability of the proposed approach to derive effective linear properties (ELPs) corresponding to various Bouc-Wen hysteretic oscillators. Specifically, the EC8 (target) pseudo-acceleration response spectrum for peak ground acceleration 0.36g (g=981cm/sec^2), ground type "B" and damping ratio ζ= 5% (gray thick line in Figure 2(a)), is considered to represent the induced seismic action.Figure 2(b) plots a discrete power spectrum compatible with the EC8 target spectrum computed by means of Eq.(19) after three iterations assuming T_s = 20s and $\Delta\omega$= 0.1rad/s. Further, the median spectral ordinates of an ensemble of 1000 20s long stationary signals compatible with the power spectrum of Figure 2 (b) are also included in Figure 2(a). These signals have been generated using a random field simulation technique based on an auto-regressive-moving-average filter[26]. The latter Monte Carlo-based analysis ensures numerically that the criterion prescribed by Eqs.(14)~(17) is satisfied by the EC8 compatible power spectrum considered.

Next, the EC8 compatible power spectrum of Figure 2(b) is used to obtain ELPs$T_{eff}= 2\pi/\omega_{eff}$ and ζ_{eff}via the statistical linearization-based formulation detailed in sections 2 for various Bouc-Wen hysteretic oscillators. In particular, the five-by-five system of nonlinear Eqs.(3), (7), and (9)is solved in series with the two-by-two system of nonlinear Eqs. (11)and(12) using the '`fsolve`' built-in MATLAB routine for viscously damped Bouc-Wen oscillators with ζ=5%,shape parameters β=γ=0.5 and A=1, pre-yield natural period T_n=0.5s, 1s, and 2s, rigidity ratios α ranging from 0.4 to 0.05 and for several values of yielding deformation x_y. The latter parameter is varied to achieve different levels of nonlinear behavior. In all cases considered, convergence has been achieved within a few seconds assuming initial conditions k_{eq}=0.1 and c_{eq}=-1. Further details on the numerical/algorithmic aspects for solving the above five-by-five system of non-linear equations can be found in the literature[27].

The thus obtained ELPs are plotted inFigure 3 versus the "ductility" max|y| defined in Eq. (10)to quantify the severity of the nonlinear response. The considered EC8 spectrum for different damping ratios has been used together with the ELPs to compute this ductility parameter as detailed in the following section. Data included in Figures 3(a) and 3(b) are in agreement with engineering intuition: the departure from the linear response yields “softer” effective linear systems characterized by longer natural periods. Furthermore, the effective damping ratio increases monotonically with max|y| to account for the increased energy dissipation through more severe plastic/ hysteretic behaviour of the corresponding nonlinear oscillators. As expected, the above effects become more prominent as the rigidity ratio decreases. Figures 3(c) and 3(d) gauge the influence of the Bouc-Wen exponent n to the ELPs. Notably, for relatively small values of the exponent n the equivalent linear system properties for max|y|=1 deviate significantly from the values expected for “small oscillations” (no yielding condition). This is due to the fact that for small values of the exponent (quite “smooth” loops) the Bouc-Wen oscillator forms hysteretic loops even for very low excitation amplitudes compared to the “nominal” yielding displacement $x_y$as shown in Figure 1. The latter can be unambiguously defined only for large values of n (i.e. n>20) where the Bouc-Wen model approximates well the bilinear hysteretic law (Figure 1)[15]. Thus, the value max|y|=1 should not be interpreted as “linear” condition for low values of n. In other words, for a fixed level of max|y|, the “severity” of hysteretic response is not uniform for all values of n. A similar observation holds for Bouc-Wen oscillators characterized by relatively long periods T_n for small oscillations (“flexible” systems), as shown in Figures 3(e) and 3(f). Further, in the latter plots it is observed that the increase of the effective damping ratios tend to saturate and even to slightly decrease as the level of yielding increases and more flexible oscillators are considered. Such trends have been identified in the literature in the context of conventional statistical linearization techniques applied to bilinear hysteretic[21]. Overall, the above discussion confirms the robustness and validity of the herein proposed approach to yield ELPs consistent with the engineering intuition whichcharacterize well the actual hysteretic response ofBouc-Wen oscillators.

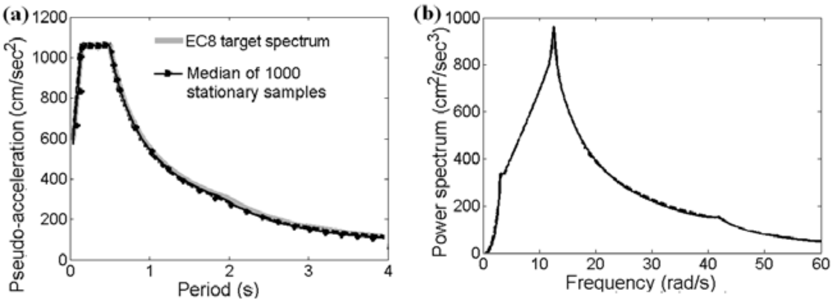

Figure 2.Considered input EC8 compatible power spectrum.

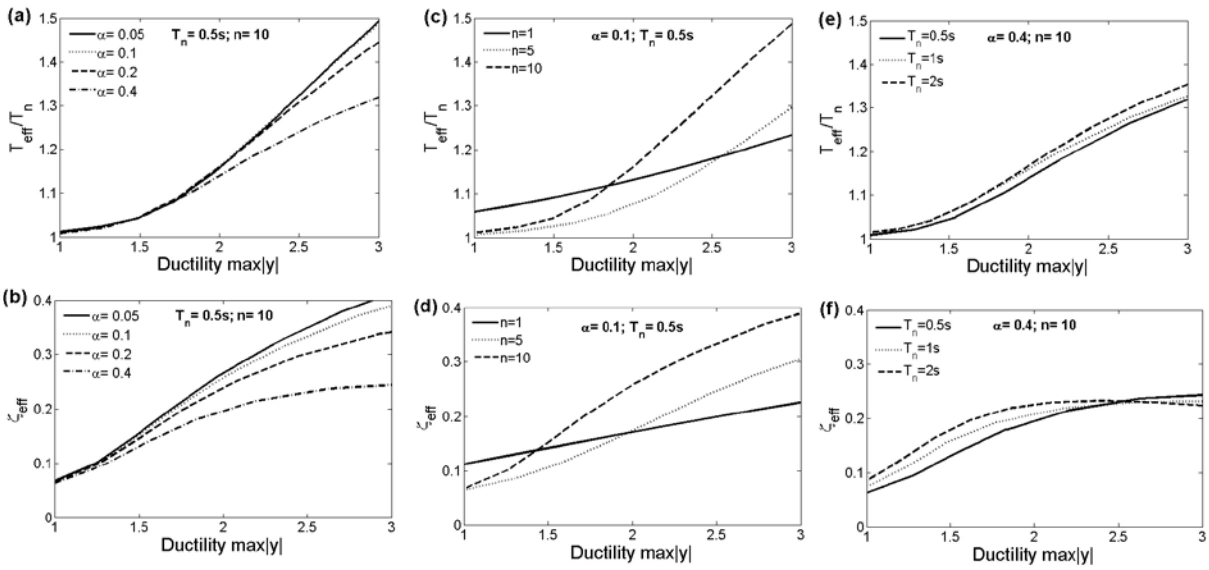

Figure 3. Effective natural period and damping ratio properties for various Bouc-Wen oscillators exposed to the EC8 compatible power spectrum of Figure 2.

4.2 Peak non-linear response determination and R-μ-Tn relationships

Figure 4exemplifies the manner in which the response spectrum compatible ELPs derived from the proposed statistical linearization procedure can be used to approximate the peak non-linear response of the associated Bouc-Wenoscillators by using the EC8 design spectrum for different levels of viscous damping. In particular, consider a specific viscously damped Bouc-Wenoscillator with damping ratio ζ= 5% and pre-yield natural period

T_n exposed to theEC8 elastic design spectrum (vertical broken lines). One can move, following the horizontal arrows, to a vertical solid line corresponding to an effective linear system characterized by T_{eff} and ζ_{eff} obtained by the statistical linearization based methodology herein adopted and “read” the related spectral value ordinate. In this manner, an estimate of the peak response of the considered structural system is achieved without the need to have available suites of spectrum compatible accelerograms and to numerically integrate the governing nonlinear equation of motion[3,11].

Finally, in Figure 5(b), the ductility μ of a Bouc-Wenoscillator (dots of various shapes) computed from ensemble averaging of the systems’ nonlinear responses is plotted versus the strength reduction factor R (R-μ-Tn relationship) in a Monte Carlo analysis context. On the same Figure, the thus obtained R-μ-Tn relationships are compared vis-à-vis the peak response normalized by the yielding deformation x_y of effective linear oscillators (curves of various types) whose properties (ELPs) have been derived as detailed in the previous sub-section from the considered nonlinear oscillator. An ensemble of 40 artificial non-stationary accelerograms compatible with the EC8 spectrum ofFigure 2 are used to compute the ductility values of Figure 5(b) using standard numerical integration routines. These signals have been generated by the wavelet-based stochastic approach proposed by Giaralis and Spanos[28].Pertinent statistics of the spectral ordinates of the considered accelerograms in terms of pseudo-acceleration are shownin Figure 5(a), and compared with the target EC8 spectrum.Evidently, the average response spectrum of the 40 considered accelerograms practically coincides with the EC8 spectrum and thus these signals are consistent with the compatibility criterion utilized in deriving the input power spectrum of Figure 2 considered in defining the considered linear systems. Overall, the quality of the achieved approximation of the peak nonlinear responses by the peak responses of the corresponding heavily damped linear oscillators deteriorates as the level of nonlinear behavior increases in terms ofthe strength reduction factor R. This is in alignment with the well-studied approximations involved in the application of statistical linearization[1].

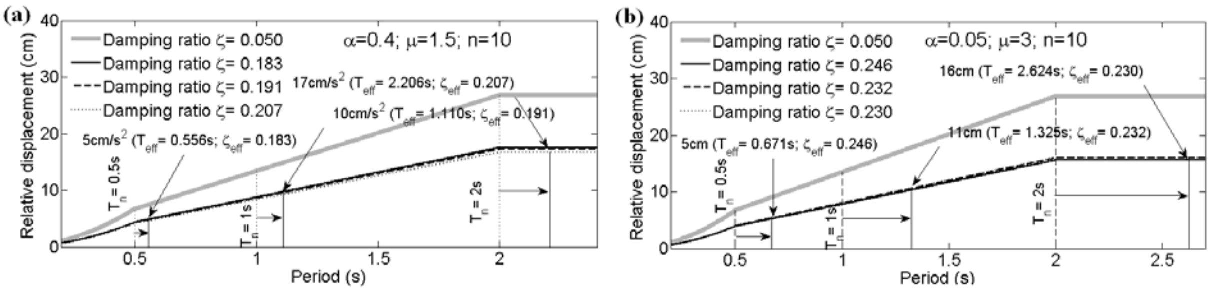

Figure 4.Peak response estimation of Bouc-Wen hysteretic oscillators using the derived ELPs and the EC8 elastic response spectrum.

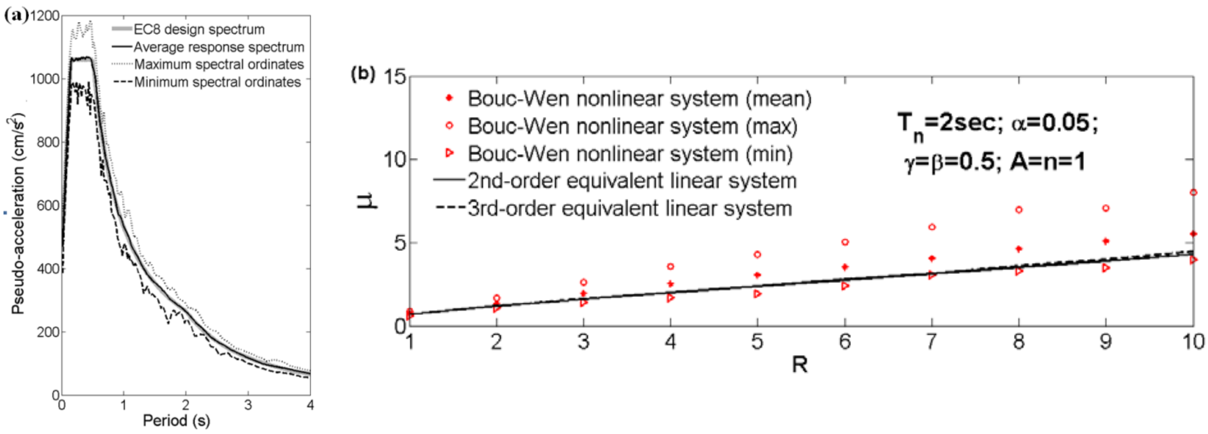

Figure 5.Peak responses of Bouc-Wen hysteretic oscillators and of their corresponding effective linear systems subject to 40 EC8 compatible accelerograms.

5 CONCLUDING REMARKS

A statistical linearization based approach has been proposed to derive effective linear properties (ELPs), namely damping ratio and natural frequency, for hysteretic oscillators following the versatile Bouc-Wen force-deformation law[12,13]. The oscillators are subjected to seismic excitations specified by an elastic response/design spectrum. An efficient numerical scheme is adopted to derive a power spectrum, satisfying a certain statistical criterion, which is compatible with the considered response spectrum. The thus derived power spectrum is used in conjunction with a frequency domain higher-order statistical linearization formulation to substitute a Bouc-

Wen oscillator by a 3rd order linear system. Then, this linear system is reduced to a 2nd order linear oscillator characterized by a set of ELPs by enforcing equality of certain response statistics of the two linear systems. Finally, the ELPs are utilized to estimate the peak response of the considered hysteretic oscillator in the context of linear response spectrum-based dynamic analysis. In this manner, the need for numerical integration of the nonlinear equation of motion is circumvented. Numerical results pertaining to the elastic response spectrum of the current European aseismic code provisions (EC8) are presented to demonstrate the usefulness and reliability of the proposed approach. These results are supported by Monte Carlo simulations involving an ensemble of 250 non-stationary artificial EC8 spectrum compatible accelerograms. The proposed approach can hopefully be an effective tool in the preliminary aseismic design stages of yielding structures following either a force-based or a displacement-based methodology.

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