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**Citation:** Piernikowska, S., Tomas-Rodriguez, M. & Santos Penas, M. (2022). Floating offshore wind turbine stability study under self-induced vibrations. In: Guedes Soares, C & Santos, TA (Eds.), Trends in Maritime Technology and Engineering. . London, UK: Routledge. ISBN 9781003320289 doi: 10.1201/9781003320289

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# Floating offshore wind turbine stability study under self-induced vibrations

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**ABSTRACT:** Floating Offshore Wind Turbines (FOWT), due to their geographical location are subjected to strong dynamic loadings originated from external sources such as wind and waves. Consequently, these systems are highly sensitive to a specific range of operational frequencies. If a dynamical system is under resonance conditions, a significant amplification of the dynamic response amplitude is prone to occur. The aim of this work is to study the response of a particular FOWT model with an externally applied periodic force so that the system may reach resonance conditions. The authors make use of a dynamic model described by a set of second order differential equations. Time and frequency domain analysis are performed with a major focus on the response of the system.

## 1 INTRODUCTION

Renewable energies and in particular, Wind turbine technology became crucial in the last decade due to the increasing possibilities of fully exploiting natural energy resources. Wind as a source of energy has many advantages: it is cost-effective and truly clean and in addition to this, it does not contribute to CO<sub>2</sub> footprint increase in the generation process. Wind energy has enough potential to replace classically used energy sources (e.g., coal or fuel) and it is predicted to produce the most significant amount of the electric power from all industrially available renewable sources. Wind onshore technology is highly reliable and environmentally friendly. However, it naturally carries some well-known disadvantages (Leithead, 2006). On the other hand, offshore wind technology offers possible solutions which overcome the drawbacks of onshore technology. Offshore wind turbines are mounted off land, usually in seawater or in freshwater, at different depths. Offshore location allows these systems to take full advantage of stronger and more constant winds than those fixed inland, hence, to extract more energy from the wind. Geographical location of wind farms is also an important factor when determining the environmental impact. Ideally, visual and noise pollutions should be kept to a minimum. From very early stages of wind technology, EU countries have been leading the development of this source of energy. Denmark, started to harvest electrical energy from the wind in early 1970s. Nowadays, at least 40% of their energy production comes from the wind. Other

EU countries which also have devoted resources and research to implement this type of clean energy are Germany, Belgium, Netherlands, France, or Italy. Outside of Europe, China is the current leading country in wind technology developments- as an example, the very large wind farm at the Gobi Desert (Jha, 2010). Currently, United Kingdom is becoming a leader in offshore power harvesting. UK owns over 34% of the total global installations with the largest offshore wind farm build in England (Broom, 2020).

Vibrations in mechanical systems are often a response to disturbances from their equilibrium point. One of the factors that could induce oscillations in FOWT systems is the presence of loadings originated by the wind itself or waves/currents. Also, the rotating nature of the rotor blades do transfer vibrations along the system's tower into the barge/platform.

Self-induced vibrations are a natural phenomenon often experienced by mechanical systems of rotating nature (Dinh & Basu, 2015) and in the case of wind turbines, this is a challenge faced by floating turbines due to the lack of fixed foundation. In the case of FOWT, above certain windspeed, maximum energy extraction is achieved by means of blade-pitch control routines that can induce increasing amplitude barge pitch motions, this is what appears in the literature as "self-induced unstable vibrations" (Jonkman, 2007). Consequently, a significant reduction in power generation could happen.

Moreover, in FOWT, structural costs are primarily associated to the floating foundation stabilization (Roddi, 2010). Hence, the main control challenge is focused on reduction of the unwanted oscillation to extend the operation life of the structure.

## 2 MODEL DESCRIPTION

FOWTs are highly complex mechanical structures composed of three main subsystems, platform (barge), tower and nacelle (Fig.1).

As mentioned in the introduction, these structures are exposed to challenging environmental conditions due to the geographical location of the wind farms. By means of mathematical simplifications it is possible to represent a FOWT system as a simplified rigid-body model composed by three Degrees Of Freedom (DOF). The approach followed in this work uses the benchmark model used in (Jonkman, 2008) which is a widely used approach in several studies i.e. (Jonkman 2009, Stewart et al. 2011, He et al. 2017, Tomás-Rodríguez & Santos 2018) just to cite a few.

The used model is the 5MW wind turbine developed by National Renewable Energy Laboratory (NREL) and studied by (Jonkman, 2009). The model is a three bladed upwind system with a 90 m hub height and a 126 m rotor diameter. In (Jonkman, 2008), this model was described by a set of limited degrees-of-freedom differential equations.

Figure 2 illustrates the simplified system as a diagram where each component of the structure is represented as an individual block with springs and dampers characterizing various flexibilities and torsion properties. For the simplicity, the subscripts ‘p’, ‘t’ and ‘T’ indicate the platform, tower, and nacelle. The  $k$  elements, appear on Figure 2 as  $k_p$ ,  $k_t$ ,  $k_T$ , represent the rotational and linear stiffnesses. Similarly,  $d$  terms ( $d_p$ ,  $d_t$ ,  $d_T$ ) describe the rotational and linear damping constants. The center of masses of each of the subsystem’s elements are represented by  $m_p g$ ,  $m_t g$ ,  $m_T g$  (Stewart & Lackner, 2011).

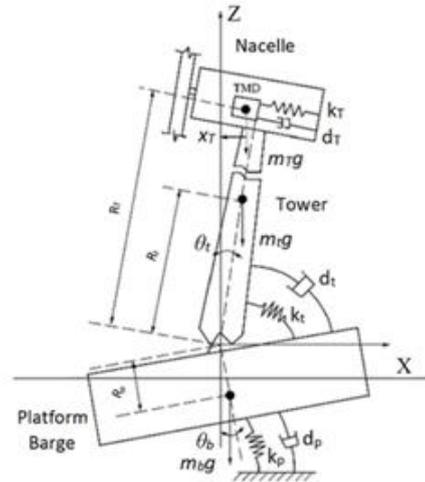


Figure 1. 5 MW benchmark model (He et al. 2017).

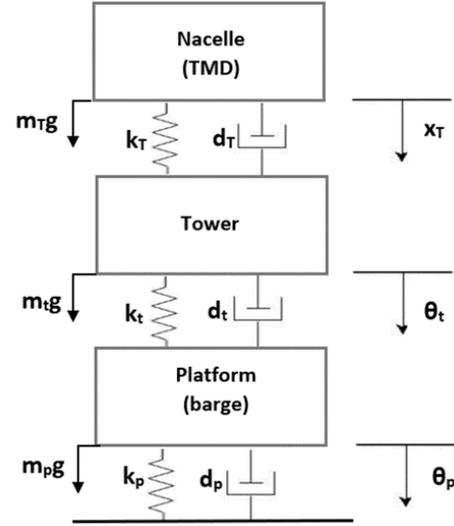


Figure 2. Rigid-body platform-pitch model of FOWT.

Another important factor is the relative motion (degree of freedom) between components. Hence, the existing rotation between the tower and the barge ( $\theta_t$ ) and barge and the surface of the water ( $\theta_p$ ) are also considered in this model. The existing motion between the tower and the TMD mass in the nacelle can be assumed as translational ( $x_T$ ) due to the restricted displacement of the TMD inside the nacelle. Based on the simplified model, a mechanical problem can now be fully described by a set of 3 single-degree-of-freedom second order differential equations presented in Equation 1, 2 and 3.

$$I_p \ddot{\theta}_p = -d_p \dot{\theta}_p - k_p \theta_p - m_p g R_p \theta_p + k_t (\theta_t - \theta_p) + d_t (\dot{\theta}_t - \dot{\theta}_p) \quad (1)$$

$$I_t \ddot{\theta}_t = -m_t R_t \theta_t - k_t (\theta_t - \theta_p) - d_t (\dot{\theta}_t - \dot{\theta}_p) - k_T R_T (R_T \theta_t - x_T) - d_T R_T (R_T \dot{\theta}_t - \dot{x}_T) - m_T g (R_T \theta_t - x_T) \quad (2)$$

$$m_T \ddot{x}_T = k_T (R_T \theta_t - x_T) + d_T (R_T \dot{\theta}_t - \dot{x}_T) + m_T g \theta_t \quad (3)$$

where  $I$  = rotational inertia;  $R$  = various distances of the structural elements from its centers of the mass. For further details on this model and how to obtain Equations 1-3, the reader is referred to (Hen et al. 2017).

The dynamic equations of motion (Eq. 1-3) are derived based on the simplified model (Fig. 2) and full details on how to obtain them can be found in (He et al. 2017) In our study, environmental factors such as wind and wave loading can initially be omitted as structural dynamics of the system are independent of those type of forcing. The control of the blade-pitch for above-rated wind speeds is critical for the overall system’s stability. It has been demonstrated how blade pitching for maximum power extraction can lead to an overall negative damping of

the tower and floating platform (Jonkman 2008, Jose et al. 2018, Nielsen et al. 2008, Larsen et al. 2007).

Hence, the overall damping of the barge-pitch mode needs to be kept positive. This barge-pitch damping problem can be further analyzed by considering the rigid-body platform-pitch subsystem as a second order single DOF system as in (Jonkman, 2008). Following this approach, the platform equation of motion (Eq. 1) can be re-written as a second order differential equation as follows:

$$(I_{Mass} + A_{Radiation})\ddot{\xi} + (B_{Radiation} + B_{Viscous})\dot{\xi} + (C_{Hydrostatic} + C_{Lines})\xi = L_{HH}T \quad (4)$$

where  $\xi$  = platform-pitch angle (rotational displacement);  $I_{mass}$  = pitch inertia associated with wind turbine and barge mass;  $A_{Radiation}$  = added inertia (added mass) associated with hydrodynamic radiation in pitch;  $B_{Radiation}$  = damping associated with hydrodynamic radiation in pitch;  $B_{Viscous}$  = linearized damping associated with hydrodynamic viscous drag in pitch;  $C_{Hydrostatic}$  = hydrostatic restoring in pitch;  $C_{Lines}$  = linearized restoring in pitch from all mooring lines (can be compared to the stiffness induced by a spring);  $T$  = aerodynamic rotor thrust;  $L_{HH}$  = hub height (i.e., rotor-thrust moment arm).

According to (Jonkman, 2008), in FOWT, the rotor thrust has an impact into the platform-pitch damping. This depends on the relative wind speed at the hub due to the motion of the hub, rotor speed, and blade-pitch angle. The translational displacement and platform-pitch angle can be linearly related to each other as  $x = L_{HH} \times \xi$  by the assumption that the pitch angle is small enough. If the hub translation varies slowly, the response of the wake of the rotor is identical for changes in hub and wind speed. If only variation of the rotor thrust with the speed of the hub is considered, a first-order Taylor series can be applied to express the aerodynamic rotor thrust as shown in Equation 5.

$$T = T_0 - \frac{\partial T}{\partial V} \dot{x} = T_0 - \frac{\partial T}{\partial V} L_{HH} \dot{\xi} \quad (5)$$

where  $T_0$  = rotor thrust at a linearization point;  $V$  = rotor-disk-averaged wind speed.

The equation of motion of the platform-pitch mode presented in Equation 4 can now be expressed in terms of the translational motion of the hub (Eq. 5) and this is shown in Equation 6:

$$\underbrace{\left( \frac{I_{Mass} + A_{Radiation}}{L_{HH}^2} \right)}_{M_x} \ddot{x} + \underbrace{\left( \frac{B_{Radiation} + B_{Viscous}}{L_{HH}^2} + \frac{\partial T}{\partial V} \right)}_{C_x} \dot{x} + \underbrace{\left( \frac{C_{Hydrostatic} + C_{Lines}}{L_{HH}^2} \right)}_{K_x} x = T_0 \quad (6)$$

Therefore, the isolated rigid-body platform-pitch model will respond as a second-order system with certain natural frequency and damping ratio as presented in Equations 7 and 8. The overall damping coefficient can be extracted from Equation 6 and is shown in Equation 9 as  $C_x$ .

$$\omega_{xn} = \sqrt{\frac{K_x}{M_x}} \quad (7)$$

$$\zeta_x = \frac{C_x}{2\sqrt{K_x M_x}} \quad (8)$$

$$\underbrace{\left( \frac{B_{Radiation} + B_{Viscous}}{L_{HH}^2} + \frac{\partial T}{\partial V} \right)}_{C_x} \quad (9)$$

where  $\omega_{xn}$  = natural frequency;  $\zeta_x$  = damping ratio;  $C_x$  = overall damping coefficient.

Just above rated wind speeds, the damping coefficient could become negative. In consequence, the platform may experience increasing amplitude of oscillations in its pitch mode. The high sensitivity of the platform-pitch to loads originated on the nacelle/rotor can be seen. For future control development, it is important to analyze the stability of the system in this disturbed state.

In this paper, the stability of the FOWT is examined by application of BIBO stability criterion and by analysis of the time and frequency domain performances.

Equations 1-3 were used to derive a set of transfer functions describing the model. The transfer function indicates the relationship between the input and the output of the system in the Laplace domain.

$$TF_1 = \frac{\theta_p}{x_T} = \frac{s^2(I_p m_T g + k_T R_T) + s(m_T g d_t + m_T g d_p + k_T R_T d_t) + (m_T g k_t + m_T g k_p + m_p g^2 R_p m_T)}{s^3(m_T d_t) + s^2(d_T d_t + m_T k_t) + s(d_t k_T + d_T k_t) + (k_t k_T)} \quad (10)$$

$$TF_2 = \frac{\theta_t}{\theta_p} = \frac{s d_t + k_t}{s^2 I_p + s(d_t + d_p) + (k_t + k_p + m_p g R_p)} \quad (11)$$

$$TF_3 = \frac{\theta_t}{x_T} = \frac{(m_t g + k_t R_t)}{s^2 m_T + s d_T + k_T} \quad (12)$$

where  $TF1$  = transfer function 1 (motion of the platform with respect to the mass  $M$  translation located in the nacelle);  $TF2$  = transfer function 2 (rotation of the tower with respect to the platform's rotation);  $TF3$  = transfer function 3 (rotation of the tower with respect to the mass  $M$  translation located in the nacelle).

### 3 SIMULATIONS

The fundamental analysis of the behavior of any system involves the study of the denominator of its transfer functions. The operation provides the information about the BIBO stability of the computed model. MATLAB was used to apply classic control theory. Figure 3 and Figure 4 show the pole-zero mapping and root locus of the platform-pitch, respectively.

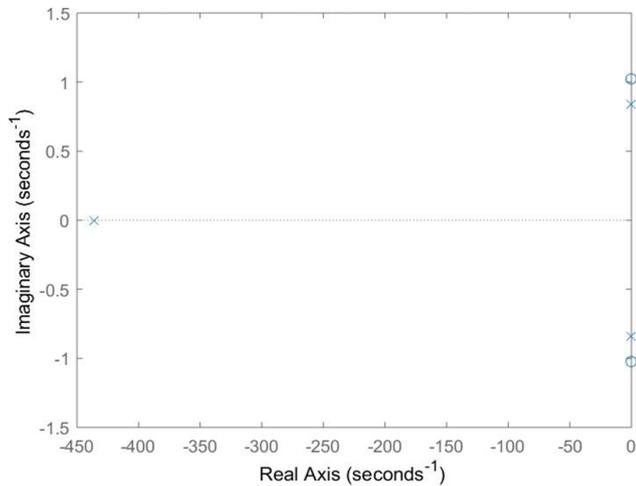


Figure 3. Pole-zero map platform-pitch mode (TF1).

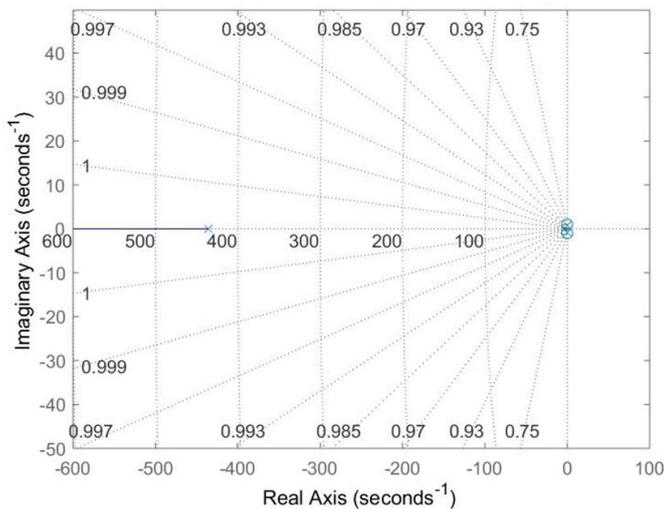


Figure 4. Root Locus of platform-pitch mode (TF1).

Figure 3 is the representation of the pole-zero map computed from the  $TF1$  relating the rotation of the platform with respect to the translation of the TMD

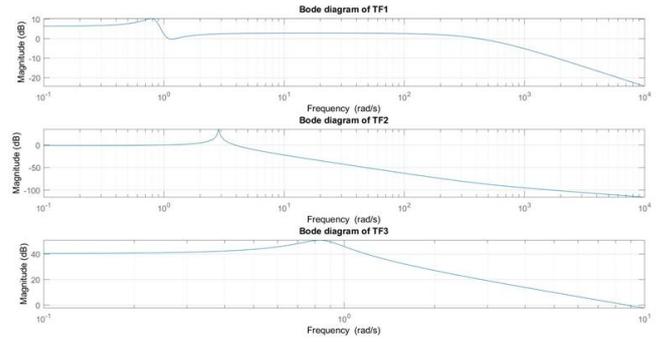


Figure 5. Magnitude-Bode plots of the system.

mass  $M$  located in the nacelle. There is a pair of pure complex conjugate modes indicating the oscillatory nature of this subsystem and one stable single pole. Figure 4 represents the roots location of the closed-loop poles obtained as a solution of the characteristic polynomial. The plot shows one real pole position on the real axis looking for its zero in infinity. Also, there is a complex conjugate pair with zero real parts. The presence of the purely imaginary complex conjugate pair indicates the oscillation with constant amplitude. However, due to the location of the third pole, the system behaves an underdamped system with a decaying amplitude of oscillation.

The frequency domain studies the behavior of the system within certain range of operational frequencies and describes how the system energy is dissipated. Any possible peak in its magnitude would indicate near resonance conditions.

Figure 5 shows the resultant Bode plots obtained for this model's transfer functions. The barge-pitch mode is the one with the lowest resonance frequency. It can be deduced that generally; floating wind turbines experience low structural frequencies. Therefore, there is a risk that the system could reach its resonance condition and as result to it, the oscillations amplitude would significantly increase possibly causing damages to the system.

### 4 RESULTS

The magnitude-Bode plot for the barge-pitch mode was used to extract the value of the resonance condition of the model. The resonance frequency was established as 0.829 Hz and further applied into the time domain analysis as a reference value for the range of the operational frequencies.

The mathematical model has been also validated by the theoretical calculations of the resonance frequency. The characteristic polynomial describing the dynamic behavior of the system can be expressed in term of natural angular frequency and the damping ratio as shown in Equation 13. The theoretical resonance frequency was calculated as 0.8486 Hz which

falls within the numerical percentage error of  $\pm 2.31\%$ .

$$m\ddot{x} + b\dot{x} + kx = 0 \rightarrow \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = 0 \quad (13)$$

Figure 6 shows the oscillation of the barge for different cases of TMD mass oscillation inside the nacelle. The model was tested with a sinusoidal input at different frequencies  $w = \{0.6, 0.829, 1\}$  Hz. This periodic input was introduced to simulate a periodic blade-pitch change as it is usually done to carry out blade control in above-rated wind conditions. It can be seen how at the resonance condition; the system displays the highest amplitude of oscillation. When the system is subjected to lower/higher frequencies than resonance, it oscillates with similar amplitude.

Figure 7 shows the behavior of the barge, tower, and TMD mass when no external force is applied. The Figure 8 shows the comparison of dynamic responses of the model both in the presence of a sustained external excitation and in the case of an initial natural disturbance from the equilibrium position. The displacement plot represents the transient response of the system subjected to the disturbance from its equilibrium position.

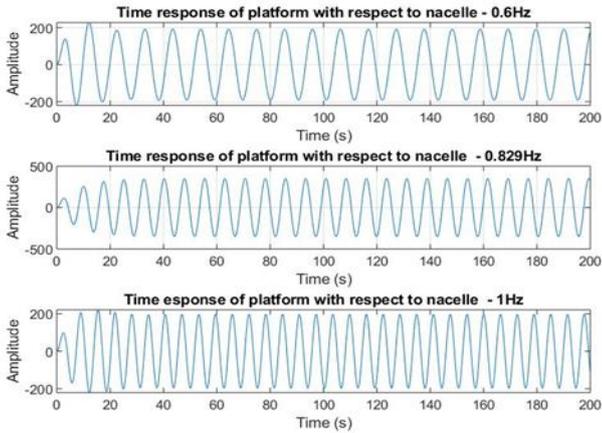


Figure 6. Time domain analysis.

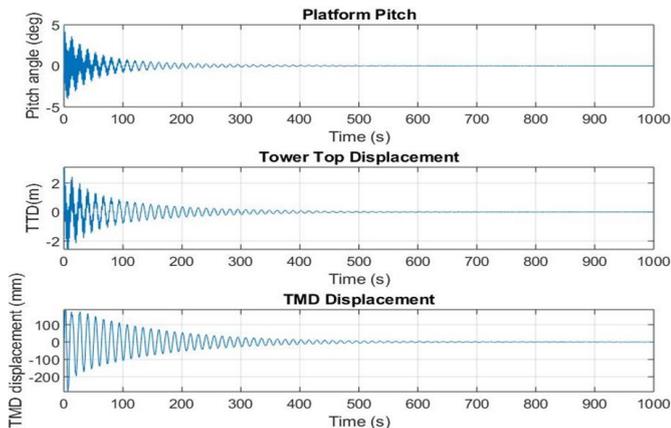


Figure 7. Time response of the model (no external force).

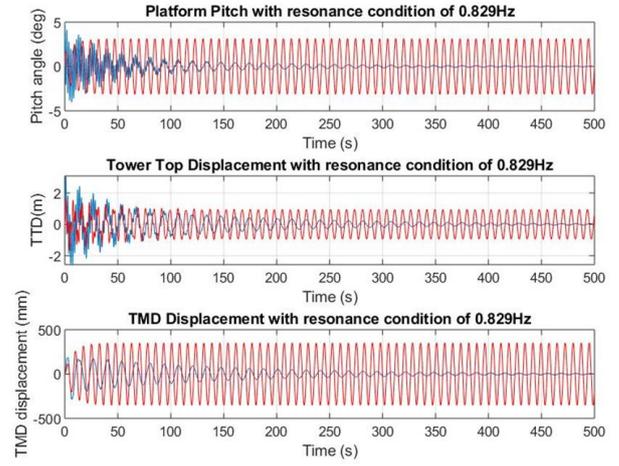


Figure 8. Comparison of system's response with (red line) and without (blue line) periodic external force.

The model will vibrate after excitation until it reaches its initial state and then it goes to rest. The sinusoidal time response describes reaction of the model exposed to the external force. Again, the system will oscillate with the frequencies introduced by input.

Figure 8 shows the system's response when conditions simulate periodic blade-pitch changes that could induce periodic rotor thrust changes, therefore, this could lead to potential instabilities of the system if the overall coefficient  $C_x$  is negative. In here, unstable conditions have not been reached, but it is clear to see that in the absence of external forcing the system's oscillations tend to zero after certain time, whilst in the case of an external periodic force with frequency near the resonant frequency of the system, the amplitude grows and remains as such for the entire simulation time interval.

## 5 CONCLUSIONS

In this article, the authors have studied potential instabilities as a result of self-induced vibrations. The mathematical modelling and computer simulations allow the analysis of FOWs. The operating model is a simplification of the complex turbine structure into a rigid-body platform-pitch mode by isolation from the environmental factors. It has been shown that the system can be fully described by a set of second-order differential equations upon such simplification.

The main aim of this work was to study the oscillatory behavior of FOWTs when this might have been naturally induced by the rotor during the blade-pitch control routine at above-rated wind speeds. In order to do so, a simplified simulation model has been used. The platform oscillatory motion has been studied for various frequencies of the input (applied to the nacelle). It has been shown by analysis of the pole-zero map and root locus, that the model is be-

having as an underdamped system when no external force is applied.

By means of a simplified model and with application of the Bode plot, the response of the system to the externally applied force with the frequency around its resonance condition has been studied. It has been proven that the amplitude of oscillation of the model significantly increased and such behavior remain through all simulation time.

It can be concluded that blade-pitch control introduces instabilities to FOWTs which results in appearance of unwanted oscillation on the structure.

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