# Sound generation by entropy perturbations passing through short circular holes

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**Abstract**

It is well known that unsteady combustion can generate temperature fluctuations (also known as entropy waves or “hot/cold spots”). These entropy waves remain silent when advected by a non-accelerated mean flow, but generate additional sound (or "noise/acoustics") when accelerated. The mean flow can be accelerated, for instance, by a nozzle or a sudden cross-sectional area change, i.e. a hole. Several analytical models that predict entropy noise in nozzles exist for a wide range of parameters. However, these models, which are often based on inviscid and one-dimensional assumptions, are believed to be inadequate for holes. Several mechanisms may need to be considered when entropy waves interact with holes, e.g. flow separation, vortex shedding, multi-dimensional effects or non-isentropicity. In the present work, we study the effect of the entropy profile (mainly affected by flow separation) on the generation of entropy noise. To this end, an analytical model based on a strict acoustic analogy formulation of the problem is developed. Subsequently, an analytical solution is obtained by using the Green’s function method.

Keywords: Entropy noise, Hole acoustics, Flow contraction, Flow expansion

## 1 INTRODUCTION

Temperature fluctuations are also known as entropy waves or “hot/cold spots”. Physically, they can be generated by any unsteady heat release, unsteady heat transfer or viscous effects. Entropy waves remain silent when advected by a non-accelerated mean flow but generate additional sound when accelerated [1]. In many laboratory-scale experiments which study this phenomenon, the mean flow acceleration is achieved by using sudden cross-sectional area changes, such as a hole. Several analytical models that predict entropy noise in nozzles exist in the literature, such as [10, 5, 2]. However, these models are found to be inadequate in predicting entropy noise in holes [11]. Several mechanisms may be responsible for the mismatch between theory and experiments: flow separation, vortex shedding [5, 8, 9], shear dispersion [7] or non-isentropic effects [6] among others. A detailed quantification of these mechanisms is still needed in order to physically clarify their importance.

It is well known that in geometries with sharp edges (such as sudden contractions/expansions) the flow separates, leading to low-speed recirculation regions (see Fig. 4). Entropy is advected by the local mean flow velocity and, therefore, the flow separation strongly affects its distribution in the domain. Since entropy is the sound source, the acoustic response of the hole is expected to be modified by the separation. In this paper, we study the effect of the mean flow separation on the propagation of entropy and, in turn, its effect on the generation of sound.

A solution to this problem requires careful resolutions of both the acoustic field (with boundaries following the wall) and the entropy source field (mainly following the bulk mean flow profile). To this end, a novel way to write the acoustic governing equation based on a strict acoustic analogy formulation of the problem (with other acoustic sources/sinks neglected and mean flow effects simplified) is firstly derived in this paper. This equation is then solved using the Green’s function method to obtain the generated sound.

## 2 THEORY

Neglecting any mass sources, the mass conservation equation can be written as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0, \tag{1}$$

where $\rho$ denotes the density, $t$ the time and $\boldsymbol{u}$ the velocity. By neglecting the viscosity and any volume force, the momentum equation can be written in the form of stagnation enthalpy, vorticity and entropy [4]

$$\frac{\partial \boldsymbol{u}}{\partial t} + \nabla B = \boldsymbol{u} \times \boldsymbol{\omega} + T \nabla s, \tag{2}$$

where $B = h + |\boldsymbol{u}|^2/2$ ($h$ is the enthalpy, $\mathrm{d}h = \mathrm{d}p/\rho + T\mathrm{d}s$, and thus $\mathrm{d}B = \mathrm{d}p/\rho + T\mathrm{d}s + \mathrm{d}(|\boldsymbol{u}|^2/2)$), $\boldsymbol{\omega} = \nabla \times \boldsymbol{u}$, $T$ the temperature and $s$ the entropy. If we neglect any heat loss/addition and viscosity (this is equal to an isentropic assumption), the conservation of energy can be written as

$$\rho T \frac{\mathrm{D}s}{\mathrm{D}t} = 0, \tag{3}$$

where $\mathrm{D}[\ ]/\mathrm{D}t = (\partial/\partial t + \boldsymbol{u} \cdot \nabla)[\ ]$ denotes the material derivation.

By combining Eqs.(1)(2)(3), and using the thermodynamic relation $\mathrm{d}\rho = \mathrm{d}p/c^2 + \mathrm{d}s \cdot \rho/C_p$ ($c$ is the speed of sound and $C_p$ the heat capacity at constant pressure), we get

$$\left( \rho \frac{\mathrm{D}}{\mathrm{D}t} \left( \frac{1}{c^2} \frac{\mathrm{D}}{\mathrm{D}t} \right) - \nabla \cdot (\rho \nabla) \right) B = -\nabla \cdot (\rho \boldsymbol{u} \times \boldsymbol{\omega} + \rho T \nabla s). \tag{4}$$

Note that Eq. (4) is strict and general except for the assumptions previously outlined. In the present paper, we focus on studying the effect of the bulk mean flow profile on affecting the entropy sound generation. To study this, three additional assumptions are subsequently introduced. Firstly, the effect of vortex shedding is neglected, i.e. the vorticity term on the right-hand side of Eq. (4) vanishes. Secondly, the acoustics are kept 2-D ($\hat{x}$, $\hat{r}$), but we assume the mean flow to be one-dimensional (the entropy wave propagates only in the axial direction). Shear dispersion [7] and non-isentropic effects [6] are neglected and will be addressed in following studies. Finally, the mean flow Mach number is kept small ($\bar{M} \ll 1$), its effect on the acoustic propagation is neglected. It is crucial to note that neglecting mean flow effect should only be done after the linearisation of the equation and carefully separating the acoustic source terms (the mean flow effect needs to be kept for these terms) from the terms affecting the acoustic propagation [3, 4]. This very low Mach number assumption is sufficient to demonstrate the effect of the bulk mean flow profile on the entropy sound generation. Higher Mach number mean flows could be considered and are left for future work.

By writing all the terms in Eq. (4) as the sum of a mean value and a small perturbation ($[\ ] = \bar{[\ ]} + [\ ]'$) and subtracting the mean parts from both sides, we obtain

$$\frac{1}{\bar{c}^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \frac{\partial}{\partial x} \left( \frac{\mathrm{d}\bar{p}}{\mathrm{d}x} \frac{s'}{C_p} \right). \tag{5}$$

This equation clearly shows how the entropy perturbation acts as an acoustic source when it passes through a non-homogeneous mean flow field. The mean pressure profile is obtained by giving a 1-D bulk mean flow profile shape (provided by any given function or extracted from CFD) and solving the governing equations for 1-D isentropic nozzle flows. Both ends of the system are assumed to be anechoic. The entropy profile is obtained by solving the linearised energy equation

$$\frac{\partial s'}{\partial t} + \bar{u}_x \frac{\partial s'}{\partial x} = 0, \tag{6}$$

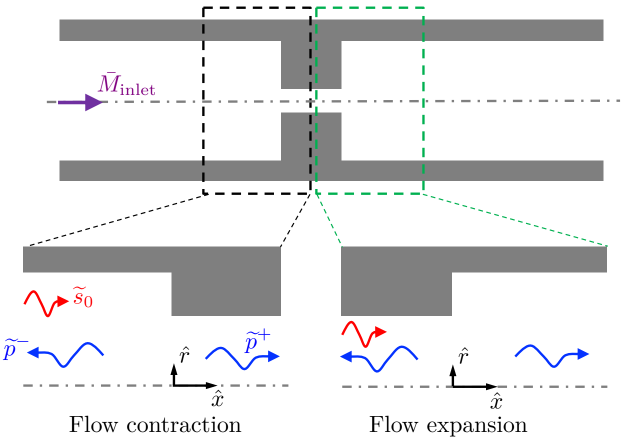



Figure 1. A short circular hole with mean flow considered as the connection of two parts: a flow contraction and a flow expansion. $\hat{x}$ and $\hat{r}$ are normalised by the hole radius $R_h$. For each part, $\widetilde{s}_0$ is the incoming entropy wave at the inlet, $\widetilde{p}^-$ and $\widetilde{p}^+$ the reflected and transmitted acoustic waves. All waves are plane.

subject to $s_0'$ at the inlet.

We now solve Eq. (5) in the frequency domain (with $\widetilde{[\ ]}$ denoting the Fourier amplitude of $[\ ]'$). To this end, we introduce a Green's function $\widetilde{G}(\boldsymbol{x},\boldsymbol{y},\omega)$ defined as

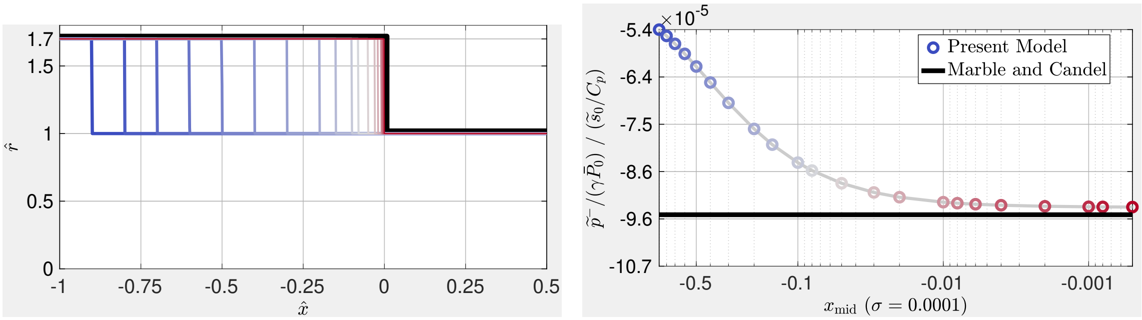
$$\left(\frac{\omega^2}{\bar{c}^2}+\nabla^2\right)\widetilde{G}=\delta(\boldsymbol{x}-\boldsymbol{y}). \tag{7}$$

As shown in Fig.1, instead of considering a hole directly, we split it into a flow contraction and a flow expansion and treat them separately (a combination of these two to study the hole acoustics will be given in following studies). A detailed explanation of the solution procedure for both configurations is given by [8, 9]. For the sake of completeness, we briefly summarise here the procedural steps: (i) obtaining Green's functions for both the small and large cylindrical ducts by using the Fourier-Bessel expansion, (ii) expanding the unknown velocity oscillation at the duct interface ($\hat{x}=0$) as the sum of a series of Bessel functions, (iii) combining Eqs.(5,7) to write the pressure perturbations just before ($\hat{x}=0^-$) and after ($\hat{x}=0^+$) the duct interface as a function of the given entropy perturbation and the unknown velocity oscillation at the duct interface, (iv) applying pressure continuity across the interface to resolve the velocity oscillation, and (v) obtaining both the reflected and transmitted plane acoustic waves by applying the Green's function method again.

## 3 THE EFFECT OF THE DETAILED ENTROPY PROFILE ON ENTROPY NOISE

In this section, we illustrate the sensitivity of entropy sound generation to the detailed entropy distribution. To this end, we study a sudden flow contraction and a sudden flow expansion using artificially enforced mean flow profiles. These profiles are obtained by assuming that the geometry is a quasi-one-dimensional isentropic nozzle with an arbitrary cross-sectional area distribution (hereafter called mean flow envelop shape). The governing equations for such configuration are solved and the entropy field is then obtained using this velocity field together with Eq. (6). Note that the acoustics are solved in the actual geometry.

As shown in Fig.2a, a cylinder with radius $\hat{r}=1.7$ is connected at $\hat{x}=0$ to a smaller cylinder with $\hat{r}=1$. A series of mean flow envelop shapes contracting from $\hat{r}=1.7$ to 1 are considered. These shapes are given by a Gauss error function with the form $\hat{r}=1.7-0.35\times\left(1+\mathrm{erf}(\frac{\hat{x}-x_{\text{mid}}}{\sqrt{2}\sigma})\right)$ where $\sigma=0.0001$ is fixed to ensure a steep contraction and $x_{\text{mid}}$ varies from $-0.9$ to $-0.0005$. $x_{\text{mid}}\to 0$ means the mean flow envelop shape tends




(a) (b)

Figure 2. A flow contraction with enforced bulk mean flow envelop shapes and plane entropy waves. (a) The bulk mean flow envelop shapes, $M_{\mathrm{inlet}} = 0.01$, (b) entropy-generated outgoing plane acoustic wave at the upstream duct, $S_t = \omega R_h / U_h = 0.0035$, $U_h$ is the mean velocity at the smaller duct. The Marble and Candel result is obtained using the compact nozzle model [10].

to attach to the wall during the contraction. The entropy generated acoustic waves at the upstream side ($\tilde{p}^-$) are shown in Fig.2b. It can be clearly seen that when the mean flow envelop shape tends to attach to the wall, the reflected acoustic wave tends to match the compact nozzle theory [10]. However, a slight shift of the mean flow envelop to the upstream side significantly changes the reflected acoustics, e.g. shifting the mean flow envelop 0.1 ahead of the duct interface ($x_{\mathrm{mid}} = -0.1$) changes the acoustic reflection from about $-9.5 \times 10^{-5}$ to $-8.3 \times 10^{-5}$.

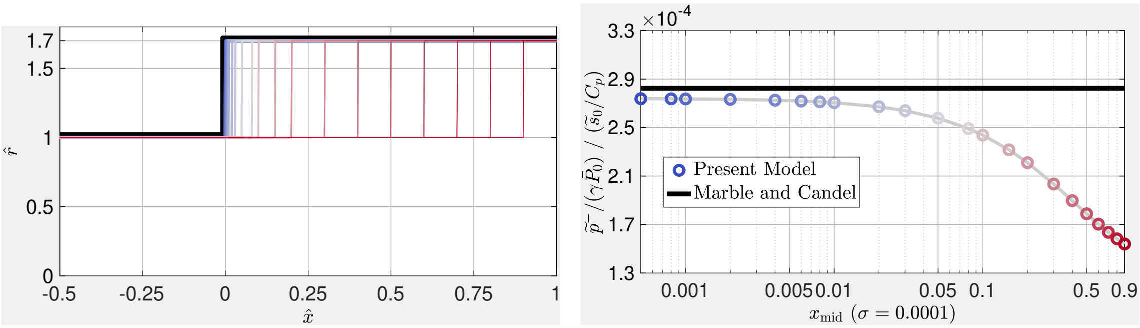
For flow expansions, a cylinder with radius $\hat{r} = 1$ connecting at $\hat{x} = 0$ to a larger cylinder with $\hat{r} = 1.7$ is considered in Fig. 3a. Similarly, a series of mean flow envelop shapes with $\hat{r} = 1 + 0.35 \times \left(1 + \mathrm{erf}(\frac{\hat{x} - x_{\mathrm{mid}}}{\sqrt{2}\sigma})\right)$ where $\sigma = 0.0001$ and $x_{\mathrm{mid}}$ varying from 0.0005 to 0.9 are studied. $x_{\mathrm{mid}} \to 0$ means the mean flow envelop shape tends to attach to the wall during expansion. Similar conclusions compared to the contraction are obtained (see Fig.3b) : (i) the predicted acoustic reflection tends to the compact nozzle model when the mean flow envelop shape tends to attach to the wall, and (ii) a slight shift of this mean flow envelop shape away from the wall leads to significant sound generation differences.

# 4 APPLICATION TO SUDDEN CROSS-SECTIONAL AREA CHANGES WITH REALISTIC ENTROPY PROFILE ENVELOPES

In this section, the geometries presented in the previous section are studied using realistic mean flow envelop shapes. To correctly account for the flow separation, the mean flow is obtained as the solution of the incompressible RANS equations. A steady solution of the governing equations is obtained using the finite volume solver OpenFOAM. The adopted turbulence model is the $k - \omega$ SST and the Reynolds number is $Re = 10^5$.

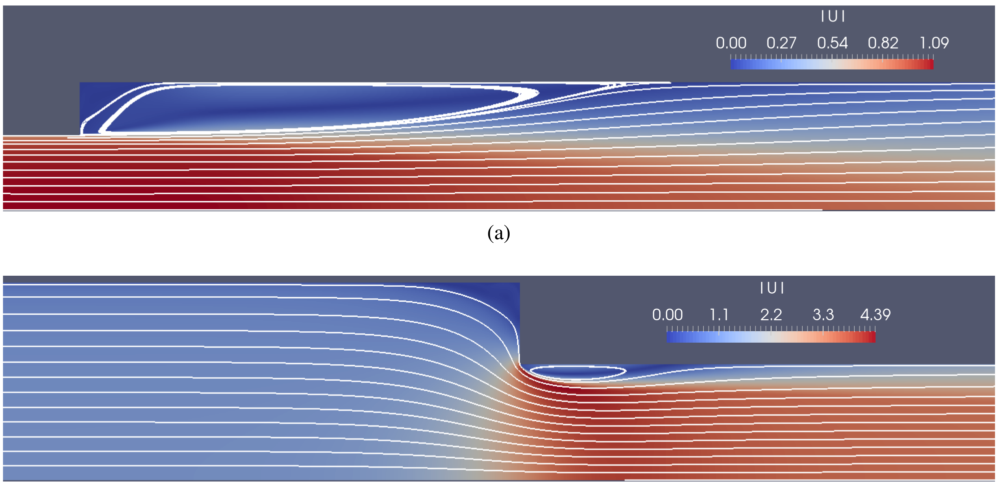
Fig. 4 depicts the velocity fields obtained for both the sudden area expansion and contraction. The flow is separated around sharp edges, leading to low-speed recirculation regions. It is clear that, in absence of diffusive terms, entropy waves cannot be transported inside these recirculating zones (Eq. (6)). Consequently, the distribution of entropy is greatly affected by the separation. To quantify this effect, we assume that the boundary of the recirculation bubble is defined by the streamline passing through the separation point at $(\hat{x} = 0, \hat{r} = 1)$. These streamlines are extracted from the CFD results and used as the mean flow envelop shapes.

The shapes for the flow contraction and expansion are shown in Figs.5a and 6a respectively. Apparently, these flow envelopes do not always attach to the walls, so when entropy waves are inserted from the upstream inlets, even if we consider the wave to be plane (as shown in Figs.5b and 6b) and the frequency to be very low, the




(a) (b)

Figure 3. A flow expansion with enforced bulk mean flow envelop shapes and plane entropy waves. (a) The bulk mean flow envelop shapes, $M_{\text{inlet}} = 0.0289$, (b) entropy-generated outgoing plane acoustic wave at the upstream duct, $S_t = \omega R_h / U_h = 0.0035$. The Marble and Candel result is obtained using the compact nozzle model [10]




(a)

(b)

Figure 4. Mean flow velocity magnitude $\bar{\boldsymbol{u}}$ for (a) a sudden area expansion and (b) a sudden area contraction. White lines denote streamlines. The domains are axisymmetric with the bottom line being the axis of symmetry.

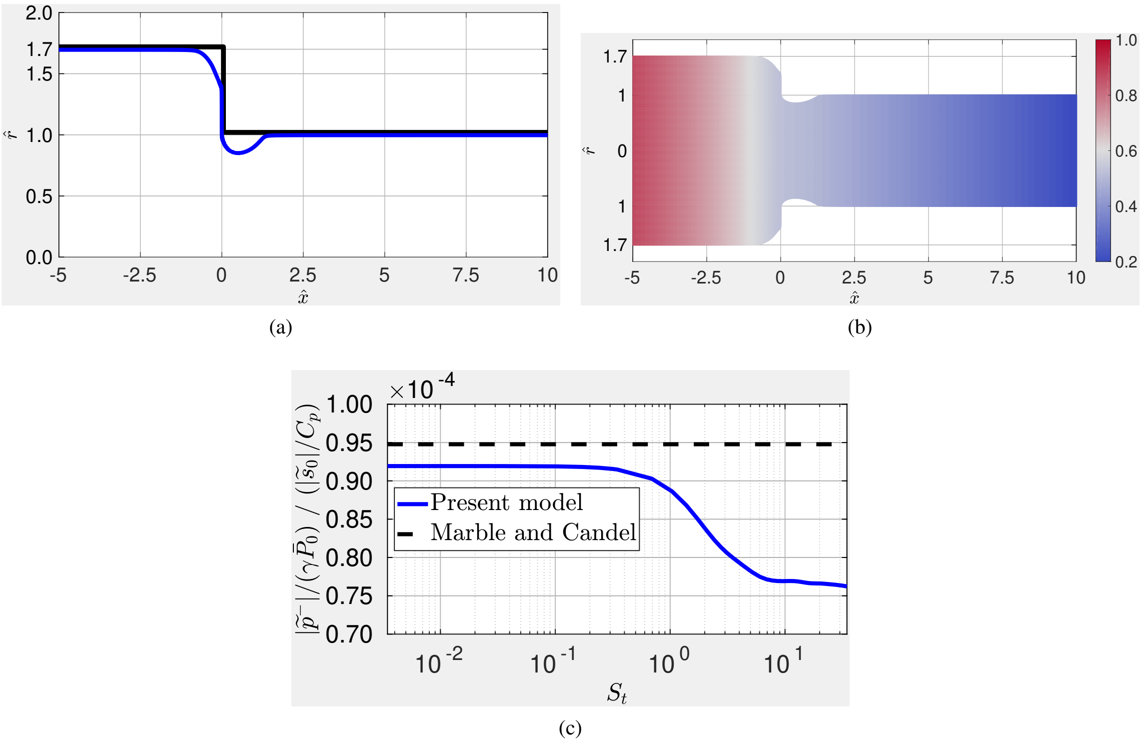
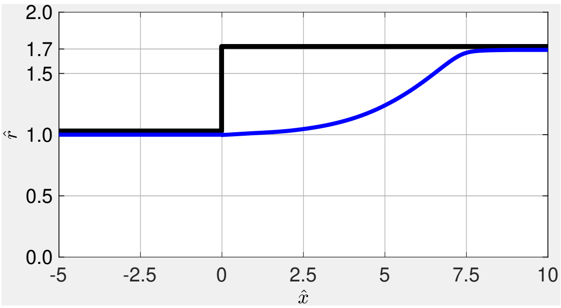
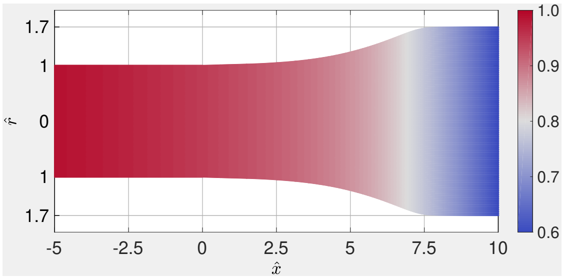


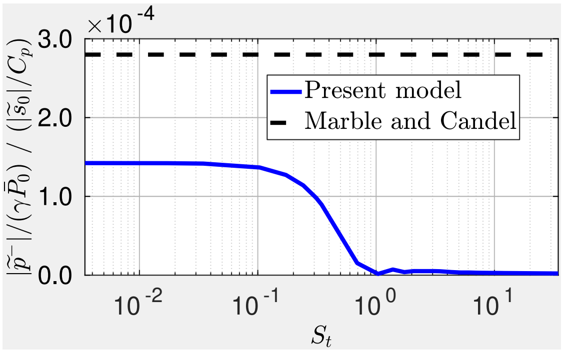

Figure 5. A flow contraction with CFD mean flow envelop shapes and plane entropy waves. (a) The bulk mean flow envelop shape comes from CFD, $M_{\text{inlet}} = 0.01$, (b) a plane entropy wave with $S_t = \omega R_h / U_h = 0.035$, (c) entropy generated outgoing plane acoustic wave at the upstream duct. The Marble and Candel result is obtained using the compact nozzle model [10].




(a)




(b)




(c)

Figure 6. A flow expansion with CFD mean flow envelop shapes and plane entropy wave. (a) The bulk mean flow envelop shape comes from CFD, $M_{\text{inlet}} = 0.0289$, (b) a plane entropy wave with $S_t = \omega R_h/U_h = 0.035$, (c) entropy generated outgoing plane acoustic wave at the upstream duct. The Marble and Candel result comes from the compact nozzle model in [10].

reflected acoustic pressures are different from those predicted by the compact nozzle theory. This is clearly seen in Figs.5c and 6c where results at the low frequency limit ($S_t < 10^{-2}$) predicted by the present model slightly deviates from the compact nozzle model for the contraction but are significantly lower for the expansion case. This is expected as the contraction case does not see significant mismatch between the bulk mean flow envelop shape and the geometry boundary but the expansion case sees a big difference due to the flow separation and thus the formation of a large recirculation region.

## 5 CONCLUSIONS

When a mean flow passes through a circular hole connecting two large coaxial cylinders, the difference between the bulk mean flow profile and the geometry boundary can be significant due to the flow separation/recirculation regions near the hole. Entropy perturbations are carried by the mean flow passing through it, and their sound generation can be significantly affected by the detailed mismatch between the bulk mean flow profile and the geometry boundary. This is studied for the first time in the present work by using an analytical model based on a strict acoustic analogy formulation of the problem and a solution using the Green's function method.

The main conclusion of the present study is that the difference between the bulk mean flow profile and the geometry boundary for the hole case (or any other cases with flow separations) may be an important factor for the mismatch between the experiment measurements and the predictions of existing entropy sound models [11]. Further detailed analysis and numerical validations are under active study by the authors. The authors are also working to relax some of the assumptions of the model in an attempt to study the coupling of additional physical mechanisms.

## ACKNOWLEDGEMENTS

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