MMSE Based Beamforming Techniques for Relay Broadcast Channels

Kanapathippillai Cumanan, *Member, IEEE,* Yogachandran Rahulamathavan, *Member, IEEE,* Sangarapillai Lambotharan *Senior Member, IEEE,* and Zhiguo Ding, *Member, IEEE,*

***Abstract*—We propose minimum mean square error (MMSE) based beamforming techniques for a multiantenna relay network, where a base station (BS) equipped with multiple antennas communicates with a number of single antenna users through a multiantenna relay. We specifically solve three optimization problems: a) sum-power minimization problem b) mean square error (MSE) balancing problem and c) mixed quality of services (QoS) problem. Unfortunately, these problems are not jointly convex in terms of beamforming vectors at the BS and the relay amplification matrix. To circumvent this non-convexity issue, the original problems are divided into two subproblems where the beamforming vectors and the relay amplification matrix are alternately optimized while other one is fixed. Three iterative algorithms have been developed based on convex optimization techniques and general MSE duality. Simulation results have been provided to validate the convergence of the proposed algorithms.**

***Index Terms*—Relay networks, convex optimization, amplify and forward relays, multiple-input multiple-output (MIMO) systems, quality of service (QoS) requirements.**

I. Introduction

Amplify and forward based relays have been attractive due to low computational complexity, low processing time and viable practical implementation as compared to decode-and-forward relays. For the amplify and forward relays, signals received at the relay is amplified and possibly phase-rotated before transmission towards receiver. However, in decode and forward relays, the received signals should be decoded and re-encoded before transmission which increases relative complexity [1]–[8]. In [1], optimal relay matrix design has been proposed for a single user multiple-input multiple-output (MIMO) amplify and forward relay network. A sum-rate duality has been established between the broadcast channel and the multiple access channel for an amplify and forward based multihop relay network in [2]. In [3], signal to interference plus noise ratio (SINR) based uplink-downlink duality has been derived for a multihop amplify and forward based MIMO relay network. A novel low complexity based linear and non-linear transceiver designs have been proposed in [9]. Relay matrix design and power allocation techniques based on quality of service (QoS) requirements have been investigated for a two-hop MIMO relay network in [9]. In [5], beamforming vectors and relay amplification matrix have been designed for a multiantenna relay broadcast channel to satisfy SINR target for each user. In this work, the beamforming vectors and the relay amplification matrix were alternately optimized while other is fixed. The design of beamfroming vectors is formulated into a convex optimization framework whereas the relay amplification matrix design was approximated into a convex problem. This approximated optimization approach cannot be directly applied to solve either SINR balancing or mixed QoS requirement problems that provide an attractive formulation to have feasible solutions all the time and to satisfy different QoSs to various users respectively. In this paper, we show that by considering minimum mean square error (MMSE) based beamforming techniques, the relay amplification matrix design can be formulated into a convex optimization framework, and we solve transceiver design based on three different MMSE criteria.

A. Motivations and Contributions

In our work, a base station (BS) equipped with multiple antennas communicates with a number of single antenna users through a multiantenna amplify and forward relay. We consider three MMSE based optimization criteria. Unfortunately, the optimization framework is not jointly convex in terms of the beamforming vectors at the BS and the relay amplification matrix. The relay design using the SINR criterion cannot be expressed in a convex form whereas the design based on MMSE can be formulated in convex form through some algebraic manipulations. Hence, MMSE is opted in our optimization problems.

A1. Sum-power Minimization: We first consider an optimization problem where each user should be satisfied with a predefined QoS, measured in terms of mean square error (MSE). This scenario could arise in a network consisting of users with delay-intolerant real-time services (real-time users) [10]. These users should achieve their required QoS all the time regardless of channel conditions.

A2. MSE Balancing: Due to insufficient transmission power at either or both the BS and the relay or due to bad channel conditions, it is not always possible to achieve MSE thresholds for every users, and hence the sum-power minimization problem might turnout to be infeasible. In

K. Cumanan and Z. Ding are with Newcastle University, UK. (Emails: Kanapathippillai.Cumanan@ncl.ac.uk; Zhiguo.Ding@ncl.ac.uk). Y. Rahulamathavan is with City University London, UK. (Email: Yogachandran.Rahulamathavan.1@city.ac.uk). S. Lambotharan is with Loughborough University, UK.(Email: S.Lambotharan@lboro.ac.uk).

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this case, the MSE thresholds should be increased and the optimization should be performed repeatedly until the problem becomes feasible. This requires considerable complexity as the minimum MSE value is unknown a priori. This motivates the MSE balancing criterion, where MSEs of all users are balanced and minimized while satisfying the transmission power constraint. This practical scenario could arise in a network consisting of users with delay-tolerant packet data services (non-real time users) [11], [12] where packet size could be varied according to the achievable MSE value. In contrast to the criterion in (*A1*), the optimization based on MSE balancing is always feasible.

A3. Mixed QoS Requirement Problem: A network might consist of both the real-time and the non real-time users requiring a mixed QoS requirement. The real-time users should be satisfied with their required QoS all the time and a fairness should be maintained in providing QoS for the non real-time users with available transmission power. This has motivated design based on the mixed QoS requirement.

II. System Model

The BS and the relay are equipped with N_T and N_R antennas, respectively. There are K users, each with single antenna. In the first time-slot, the transmitted signal from the BS can be written as $\mathbf{x} = \tilde{\mathbf{U}}\mathbf{s}$, where $\mathbf{s} = [s_1 \cdots s_K]^T \in \mathbb{C}^{K\times 1}$, s_k is the symbol intended for the k^{th} user, $\mathbb{E}(\mathbf{s}\mathbf{s}^H) = \mathbf{I}$ and $\tilde{\mathbf{U}} = [\tilde{\mathbf{u}}_1 \cdots \tilde{\mathbf{u}}_K] = \mathbf{U}\mathbf{P} \in \mathbb{C}^{N_T\times K}$. $\mathbf{U} = [\mathbf{u}_1 \cdots \mathbf{u}_K] \in \mathbb{C}^{N_T\times K}$ and $\|\mathbf{u}_k\|_2 = 1$. The diagonal matrix $\mathbf{P} = diag[\sqrt{p_1} \cdots \sqrt{p_K}] \in \mathbb{R}^{K\times K}$ consists of power allocation for the users. The BS power can be expressed as $P_t = \text{Tr}\{\tilde{\mathbf{U}}\tilde{\mathbf{U}}^H\}$. The received signal at the relay can be written as $\mathbf{y}_r = \mathbf{H}_0\mathbf{x} + \mathbf{n}_r$, where $\mathbf{H}_0 \in \mathbb{C}^{N_R\times N_T}$ is the corresponding MIMO channel and $\mathbf{n}_r$ is the noise vector at the relay with zero-mean and covariance matrix $\sigma^2\mathbf{I}$. In the second time-slot, the relay forwards the received signal to the users using amplify and forward technique. The transmitted signal can be written as $\mathbf{x}_r = \mathbf{F}\mathbf{y}_r$, where $\mathbf{F} \in \mathbb{C}^{N_R\times N_R}$ is the relay amplification matrix. The transmission power consumed at the relay can be expressed as

$$P_r = \mathbb{E}\{\mathbf{x}_r\mathbf{x}_r^H\} = \text{Tr}\{\mathbf{F}(\mathbf{H}_0\tilde{\mathbf{U}}\tilde{\mathbf{U}}^H\mathbf{H}_0^H + \sigma^2\mathbf{I})\mathbf{F}^H\}. \quad (1)$$

The received signal at the k^{th} user can be written as $y_k = \mathbf{h}_k^H\mathbf{x}_r + n_k$, where $\mathbf{h}_k \in \mathbb{C}^{N_R\times 1}$ is the channel vector from the relay station to the k^{th} user and $\mathbf{H}_1 = [\mathbf{h}_1 \cdots \mathbf{h}_K]$. We assume that n_k is a zero-mean circularly symmetric additive white Gaussian noise component with variance σ_k^2. The estimated signal at the k^{th} user can be stated as $\hat{s}_k = a_k y_k$, where a_k is the receiver coefficient at the k^{th} user. Let $\mathbf{A} = diag([a_1\ a_2 \cdots a_K]^T)$. The MSE value of the k^{th} user is $\varepsilon_k = 1 - 2\Re(a_k\mathbf{h}_k^H\mathbf{F}\mathbf{H}_0\tilde{\mathbf{u}}_k) + a_k\mathbf{h}_k^H\mathbf{F}(\mathbf{H}_0\tilde{\mathbf{U}}\tilde{\mathbf{U}}^H\mathbf{H}_0^H + \sigma^2\mathbf{I})\mathbf{F}^H\mathbf{h}_k a_k^* + a_k a_k^*\sigma_k^2$.

III. Sum-Power Minimization

The aim is to design $\tilde{\mathbf{U}}, \mathbf{F}$ and $\mathbf{A}$ to minimize the total transmission power at the BS and the relay while ensuring MSE of each user does not exceed a threshold. We assume that the perfect channel state information (CSI) $\mathbf{H}_0$ and $\mathbf{H}_1$ is available at the relay where the optimization is performed. Since both channels $\mathbf{H}_0$ and $\mathbf{H}_1$ are required for the design, it is more convenient to perform optimization at the relay rather than at the BS. The relay can send the required beamformers to the BS through a dedicated feedback channel. The sum-power minimization problem can be stated as

$$\min_{\tilde{\mathbf{U}},\mathbf{F},\mathbf{A}} \quad P_t + \alpha_0 P_r, \text{ s.t. } \varepsilon_k \le \gamma_k, \quad k = 1, \cdots, K, \quad (2)$$

where γ_k is the MSE threshold of the k^{th} user and α_0 is a positive weight which determines the proportion of the total power that is spent for the relay transmission. It can be observed that the sum-power minimization in (2) is not convex jointly in terms of $\tilde{\mathbf{U}}, \mathbf{F}, \mathbf{A}$. Therefore, the original problem in (2) is divided into two subproblems where the beamforming vectors and the relay amplification matrix are successively optimized and an iterative algorithm is proposed.

A. Beamformer Design at the Base Station

The beamformer design at the BS is formulated into a second order cone programming (SOCP) (convex problem) for a fixed relay amplification matrix. The received signal at the k^{th} user can be written

$$y_k = \mathbf{h}_k^H\mathbf{F}\mathbf{H}_0\tilde{\mathbf{U}}\mathbf{s} + \mathbf{h}_k^H\mathbf{F}\mathbf{n}_r + n_k = \tilde{\mathbf{h}}_k^H\tilde{\mathbf{U}}\mathbf{s} + z_k, \quad (3)$$

where $\tilde{\mathbf{h}}_k^H = \mathbf{h}_k^H\mathbf{F}\mathbf{H}_0$ and $z_k = \mathbf{h}_k^H\mathbf{F}\mathbf{n}_r + n_k$. The MSE of the k^{th} user can be formulated as $\varepsilon_k = 1 - 2\Re(a_k\tilde{\mathbf{h}}_k^H\tilde{\mathbf{u}}_k) + |a_k|^2\tilde{\mathbf{h}}_k^H\tilde{\mathbf{U}}\tilde{\mathbf{U}}^H\tilde{\mathbf{h}}_k + |a_k|^2\eta_k^2$, where $\eta_k^2 = \sigma^2\mathbf{h}_k^H\mathbf{F}\mathbf{F}^H\mathbf{h}_k + \sigma_k^2$. For a given set of beamformers and relay amplification matrix, the optimum receiver filter coefficient for the k^{th} user can be obtained as

$$\bar{a}_k = \frac{\tilde{\mathbf{u}}_k^H\tilde{\mathbf{h}}_k}{\tilde{\mathbf{h}}_k^H\tilde{\mathbf{U}}\tilde{\mathbf{U}}^H\tilde{\mathbf{h}}_k + \eta_k^2}. \quad (4)$$

The MSE of the k^{th} user for the $\bar{a}_k$ in (4) can be written as $\varepsilon_k = \frac{\sum_{i\neq k}\tilde{\mathbf{h}}_k^H\tilde{\mathbf{u}}_i\tilde{\mathbf{u}}_i^H\tilde{\mathbf{h}}_k + \eta_k^2}{\sum_{i=1}^K\tilde{\mathbf{h}}_k^H\tilde{\mathbf{u}}_i\tilde{\mathbf{u}}_i^H\tilde{\mathbf{h}}_k + \eta_k^2}$. The beamformer design at the BS for a fixed relay amplification matrix can be formulated into a SOCP (convex problem) by introducing a slack variable τ as follows:

$$\begin{aligned}
\min_{\tilde{\mathbf{U}},\tau} \quad & \tau, \\
\text{s.t.} \quad & \begin{bmatrix} \tau \\ \left[\|\bar{\mathbf{A}}\tilde{\mathbf{u}}_1\|_2 \|\bar{\mathbf{A}}\tilde{\mathbf{u}}_2\|_2 \cdots \|\bar{\mathbf{A}}\tilde{\mathbf{u}}_K\|_2\right]^T \end{bmatrix} \succeq_K 0, \\
& \begin{bmatrix} \frac{1}{\sqrt{1-\gamma_k}}\tilde{\mathbf{h}}_k^H\tilde{\mathbf{u}}_k \\ \left[\tilde{\mathbf{h}}_k^H\tilde{\mathbf{U}}\right]^T \\ \eta_k \end{bmatrix} \succeq_K 0, k = 1, .., K,
\end{aligned} \quad (5)$$

where $\bar{\mathbf{A}} = \mathbf{H}_0^H\mathbf{F}^H\mathbf{F}\mathbf{H}_0 + \mathbf{I}$. Once, the beamformers are obtained, the optimal receiver coefficients can be determined from (4).

B. Relay Amplification Matrix Design

As shown in Appendix I, the relay amplification matrix design is formulated into a quadratically constrained quadratic program (QCQP) (convex problem) for a given set of beamformers at the BS and the receiver coefficients as

$$\begin{aligned}\min_{\mathbf{f},\xi} \quad & \xi, \\ \text{s.t.} \quad & \mathbf{f}^H\mathbf{B}\mathbf{f} \le \xi, \\ & 1 - 2\Re(\mathbf{g}_k^T\mathbf{f}) + \mathbf{f}^H\mathbf{D}_k\mathbf{f} + |\bar{a}_k|^2\sigma_k^2 \le \gamma_k, \\ & \qquad k = 1, \cdots, K, \end{aligned} \quad (6)$$

where

$$\begin{aligned}\mathbf{f} &= \text{Vec}(\mathbf{F}),\ \mathbf{B} = \left[\mathbf{R}_r^{1/2} \otimes \mathbf{I}\right]^T \left[\mathbf{R}_r^{T/2} \otimes \mathbf{I}\right] \succeq \mathbf{0}, \\ \mathbf{R}_r &= \mathbf{H}_0\tilde{\mathbf{U}}\tilde{\mathbf{U}}^H\mathbf{H}_0^H + \sigma^2\mathbf{I}, \\ \mathbf{D}_k &= \left[\mathbf{R}_r^{1/2} \otimes a_k^*\mathbf{h}_k^T\right]^T \left[\mathbf{R}_r^{T/2} \otimes \bar{a}_k\mathbf{h}_k^H\right] \succeq \mathbf{0}, \\ \mathbf{g}_k &= \text{Vec}(\bar{a}_k\mathbf{h}_k^*\tilde{\mathbf{u}}_k^T\mathbf{H}_0^T). \end{aligned} \quad (7)$$

The proposed sum-power minimization is summarized in Table I. Since each subproblem is convex, the total transmission power monotonically decreases with iteration as observed in the simulation results. This confirms the convergence of the algorithm.

Table I: Sum-power Minimization Algorithm.

1) Initialize: $\mathbf{F} = \mathbf{F}_0$.
2) **Repeat**
   a) Solve the problem in (5) for a fixed relay amplification matrix $\mathbf{F}$. Obtain optimal beamformers $\tilde{\mathbf{U}}$ and receiver coefficients $\bar{a}_k \ \forall\ k$ using (4).
   b) Solve the problem in (6) for a fixed set of beamformers $\tilde{\mathbf{U}}$. Obtain the optimal relay amplification matrix $\mathbf{F}$ using (6).
3) **Until** the required accuracy.

IV. MSE Balancing

Motivated as in (*A2*), we consider a formulation known as MSE balancing in this section, where the MSE of the worst-case user is minimized while satisfying the transmission power constraint as follows:

$$\begin{aligned}\min_{\mathbf{U},\mathbf{p},\mathbf{A},\mathbf{F}} \max_{1\le k\le K} \quad & \frac{\varepsilon_k(\mathbf{U},\mathbf{p},\mathbf{A},\mathbf{F})}{\gamma_k}, \\ \text{s.t.} \quad & \mathbf{1}^T\mathbf{p} = P_t \le P_1^{(b)},\ P_r \le P_2^{(r)}, \end{aligned} \quad (8)$$

where $P_1^{(b)}$ and $P_2^{(r)}$ are the maximum available transmission power at the BS and the relay, respectively, and $\mathbf{p} = [p_1 \cdots p_K]^T$ consists of the power allocation for the users. Unfortunately, this MSE balancing problem is also not jointly convex in terms of $\mathbf{U}, \mathbf{p}, \mathbf{A}$ and $\mathbf{F}$. Therefore, we consider two subproblems as in the following subsections: a) beamformer design and power allocation problem at the BS and b) relay amplification matrix design.

A. Beamformer Design and Power Allocation at the Base Station

For a given relay amplification matrix, $\mathbf{U}$ and $\mathbf{p}$ at the BS are determined to ensure that the MSEs of all users are balanced while satisfying the power constraints at the BS and relay. Since, the transmission power at the relay also depends on the the beamformers at the BS, the power constraint at the relay should be incorporated in the beamformer design at the BS. In this case, we can ensure that the balanced MSE will decrease monotonically with each iteration. For a given $\mathbf{F}$, the MSE balancing problem can be formulated as

$$\begin{aligned}\min_{\mathbf{U},\mathbf{p},\mathbf{A}} \max_{1\le k\le K} \quad & \frac{\varepsilon_k(\mathbf{U},\mathbf{p},\mathbf{A})}{\gamma_k}, \\ \text{s.t.} \quad & \mathbf{1}^T\mathbf{p} = P_t \le P_1^{(b)},\ P_r \le P_2^{(r)}. \end{aligned} \quad (9)$$

It is not straight forward to solve (9) in the downlink due to the coupled structure of the beamformers and transmission powers. However, (9) can be represented as follows using a virtual uplink framework and introducing auxiliary variables as follows [12]:

$$\begin{aligned}\min_{\mathbf{Q},\mathbf{A}} \max_{1\le k\le K} \quad & \frac{\varepsilon_k(\mathbf{Q},\mathbf{A})}{\gamma_k}, \\ \text{s.t.} \quad & \text{Tr}\{\mathbf{Q}\mathbf{C}_1\} \le P_{\max},\ \mathbf{Q} \succeq \mathbf{0}, \end{aligned} \quad (10)$$

where $\mathbf{Q} = \tilde{\mathbf{U}}\tilde{\mathbf{U}}^H$, $\mathbf{C}_0 = \mathbf{H}_0^H\mathbf{F}^H\mathbf{F}\mathbf{H}_0$, $\mathbf{C}_1 = \lambda_1\mathbf{I} + \lambda_2\mathbf{C}_0$, $P_{\max} = \lambda_1 P_1^{(b)} + \lambda_2 P_3^{(r)}$ and $\lambda_1 > 0, \lambda_2 > 0$. The solution of the problem in (10) will be an upper-bound of that in (9) using the same argument in [12]. Note that the optimal solution of the original problem in (9) can be obtained by solving this problem with appropriate values of auxiliary variables which will be obtained using subgradient adaptation. The solution of (10) is determined by solving an equivalent uplink problem using the general MSE duality.

General MSE duality: The same MSE values can be obtained in both the downlink and the uplink systems with the linear constraints $\text{Tr}\{\mathbf{Q}\mathbf{C}_1\} \le P_{\max}$ and $\sum_{i=1}^K \eta_i^2 q_i \le P_{\max}$, respectively. The transmit beamformers and receiver filter coefficients in the downlink can be determined from the uplink receiver beamformers and the uplink transmit powers to achieve the same MSE values as in the uplink system by determining the positive constants $\alpha_k \ \forall\ k$ for all users and vice-versa.

Proof: Please refer to Appendix II. ■

The equivalent uplink problem can be defined based on general MSE duality as follows:

$$\begin{aligned}\min_{\mathbf{V},\mathbf{q}} \max_{1\le k\le K} \quad & \frac{\varepsilon_k^{(u)}(\mathbf{v}_k,\mathbf{q})}{\gamma_k}, \\ \text{s.t.} \quad & \textstyle\sum_{i=1}^K \eta_i^2 q_i \le P_{\max}, \end{aligned} \quad (11)$$

where $\mathbf{V} = [\mathbf{v}_1 \cdots \mathbf{v}_K]$ contains the uplink receiver beamformers and $\mathbf{q} = [q_1 \cdots q_K]^T$ contains the uplink power allocation for all users. The uplink MSE of the k^{th} user is represented by $\varepsilon_k^{(u)}$.

For a given set of uplink power allocation $\mathbf{q}$, the uplink receiver beamformers of all users $\mathbf{V} = \tilde{\mathbf{V}}\boldsymbol{\Theta}$ can be obtained by minimizing the sum-MSE of all users as follows:

$$\tilde{\mathbf{V}}\boldsymbol{\Theta}\tilde{\mathbf{Q}}^{-1/2} = (\mathbf{C}_1 + \tilde{\mathbf{H}}\tilde{\mathbf{Q}}\tilde{\mathbf{H}}^H)^{-1}\tilde{\mathbf{H}}\tilde{\mathbf{Q}}^{1/2}, \quad (12)$$

where $\tilde{\mathbf{H}} = [\tilde{\mathbf{h}}_1 \cdots \tilde{\mathbf{h}}_K]$, $\tilde{\mathbf{Q}} = diag\{q_1 \cdots q_K\}$ and $\tilde{\mathbf{V}} = [\tilde{\mathbf{v}}_1 \cdots \tilde{\mathbf{v}}_K]$ consists of normalized beamformers and $\boldsymbol{\Theta} = diag\{\theta_1 \cdots \theta_K\}$ is a diagonal matrix. The power allocation problem in the equivalent uplink to balance the MSEs of all users can be formulated into the following geometric programming (GP) (convex problem) [13]:

$$\begin{aligned} \min_{t,\mathbf{q}} \quad & t, \\ \text{s.t.} \quad & q_k^{-1}\left[(\boldsymbol{\Omega} + \boldsymbol{\Theta}^2\boldsymbol{\Psi})\mathbf{q} + \boldsymbol{\Theta}^2\boldsymbol{\rho}\right]_k \le t\gamma_k, \forall\, k, \\ & \textstyle\sum_{i=1}^K \eta_i^2 q_i \le P_{\max},\ q_k \ge 0,\ t \ge 0, \end{aligned} \quad (13)$$

where

$$\begin{aligned} \boldsymbol{\Omega}_{ij} &= \begin{cases} 1-2\theta_i\Re(\tilde{\mathbf{v}}_i^H\tilde{\mathbf{h}}_i)+\theta_i^2\tilde{\mathbf{v}}_i^H\tilde{\mathbf{h}}_i\tilde{\mathbf{h}}_i^H\tilde{\mathbf{v}}_i, & i=j; \\ 0, & i\neq j, \end{cases}, \\ \boldsymbol{\Psi} &= \begin{cases} \tilde{\mathbf{v}}_i^H\tilde{\mathbf{h}}_j\tilde{\mathbf{h}}_j^H\tilde{\mathbf{v}}_i & i\neq j; \\ 0, & i=j \end{cases}, \\ \text{and } \boldsymbol{\rho} &= [\tilde{\mathbf{v}}_1^H\mathbf{C}_1\tilde{\mathbf{v}}_1 \cdots \tilde{\mathbf{v}}_K^H\mathbf{C}_1\tilde{\mathbf{v}}_K]^T. \end{aligned} \quad (14)$$

From these solutions, the corresponding downlink beamformers and transmission power allocation can be determined through the general MSE duality. The auxiliary variables λ_1 and λ_2 are updated based on a subgradient method as follows:

$$\begin{aligned} \lambda_1^{(n+1)} &= \lambda_1^{(n)} + \mu(\text{Tr}\{\tilde{\mathbf{Q}}^{(n)}\} - P_1^{(b)}), \\ \lambda_2^{(n+1)} &= \lambda_2^{(n)} + \mu(\text{Tr}\{\tilde{\mathbf{Q}}^{(n)}\mathbf{C}_0\} - P_3^{(r)}), \end{aligned} \quad (15)$$

where μ is the step-size of the subgradient method. The proposed MSE balancing algorithm for a given $\mathbf{F}$ is summarized in Table II.

Table II: Beamformer Design and Power Allocation.

1) Initialize: λ_1, λ_2, $q_k = P_{max}/K \ \ \forall k$.
2) **Repeat**
3) **Repeat**
   a) Obtain uplink receiver beamformers from (12).
   b) Obtain uplink power allocation by solving (13).
4) **Until** the required accuracy.
5) Obtain $\mathbf{Q}$ and $\mathbf{A}$ using general MSE duality.
6) Update λ_1 and λ_2, using the subgradient method in (15).
7) **Until** the required accuracy.

B. Relay Amplification Matrix Design

We determine $\mathbf{F}$ to balance the MSEs of all users for a given $\mathbf{U}$ and $\mathbf{p}$ at the BS and $\mathbf{A}$ at the receivers. This problem can be formulated into an QCQP (convex problem) by introducing new variable t as follows:

$$\begin{aligned} \min_{\mathbf{f},t} \quad & t, \\ \text{s.t.} \quad & \mathbf{f}^H\mathbf{B}\mathbf{f} \le P_2^{(r)}, \\ & 1 - 2\Re(\mathbf{g}_k^T\mathbf{f}) + \mathbf{f}^H\mathbf{D}_k\mathbf{f} + |\bar{a}_k|^2\sigma_k^2 \le t\gamma_k, \\ & k = 1, \cdots, K, \end{aligned} \quad (16)$$

where $\mathbf{f}, \mathbf{B}, \mathbf{g}_k$ and $\mathbf{D}_k$ are defined in (7). The proposed MSE balancing algorithm is summarized in Table III.

Table III: MSE Balancing Algorithm.

1) Initialize: $\mathbf{F} = \mathbf{F}_0$.
2) **Repeat**
   a) Obtain the beamformers and power allocation for a given relay amplification matrix $\mathbf{F}$ from the algorithm in Table II.
   b) Solve the problem in (16) for a given set of beamformers $\tilde{\mathbf{U}}$. Obtain the optimal relay amplification matrix $\mathbf{F}$ using (16).
3) **Until** the required accuracy.

V. Mixed Quality of Service Requirement

We solve a mixed QoS requirement problem where a set of users should be satisfied with specific MSE thresholds and the remaining users' MSEs should be balanced and minimized while satisfying the power constraints. Motivated as in (*A3*), we consider a network where the first K_1 users (real-time users) employ delay intolerant real-time services whose MSEs should not exceed certain thresholds all the time, the remaining users (non-real-time users) employ delay tolerant packet data services. In order to maintain user fairness, the MSEs of these non-real-time users should be balanced and minimized while satisfying the overall transmission power constraints. This mixed QoS requirement problem can be formulated as,

$$\begin{aligned} \min_{\mathbf{U},\mathbf{p},\mathbf{A},\mathbf{F}} \max_{K_1+1\le k\le K} \quad & \frac{\varepsilon_k(\mathbf{U},\mathbf{p},\mathbf{A},\mathbf{F})}{\delta_k},\ k = K_1+1,\cdots,K, \\ \text{s.t.} \quad & \varepsilon_k \le \gamma_k, \quad k = 1,\cdots,K_1, \\ & \mathbf{1}^T\mathbf{p} = P_t \le P_1^{(b)},\ P_r \le P_2^{(r)}, \end{aligned} \quad (17)$$

where δ_k is the preferred MSE threshold of k^{th} non-real time user, γ_k is the MSE threshold for the k^{th} real-time user. The MSE thresholds of the real-time users (i.e., $1 \le k \le K_1$) should be satisfied. Please note that this problem might turnout to be infeasible due to insufficient transmission power to satisfy the required QoSs of the real-time users. This mixed QoS problem is not jointly convex in terms of $\mathbf{U}, \mathbf{p}, \mathbf{A}$ and $\mathbf{F}$. Hence, we consider two subproblems and propose an iterative algorithm.

A. Beamformer Design and Power Allocation at the Base Station

We obtain $\mathbf{U}$ and $\mathbf{p}$ for a given relay matrix $\mathbf{F}$. These beamformers and power allocation ensure that the real-time users achieve their MSE thresholds and the MSEs of the non-real-time users are balanced while satisfying the power constraints at the BS and the relay. The mixed QoS problem can be formulated as

$$\begin{aligned} \min_{\mathbf{U},\mathbf{p},\mathbf{A}} \max_{K_1+1\le k\le K} \quad & \frac{\varepsilon_k(\mathbf{U},\mathbf{p},\mathbf{A})}{\delta_k},\ k = K_1+1,\cdots,K, \\ \text{s.t.} \quad & \varepsilon_k \le \gamma_k, \quad k = 1,\cdots,K_1, \end{aligned}$$

$$\mathbf{1}^T\mathbf{p} = P_t \le P_1^{(b)},\ P_r \le P_2^{(r)}. \quad (18)$$

The problem in (18) can be efficiently solved by considering the equivalent uplink problem and introducing auxiliary variables similar to (10) as follows:

$$\begin{aligned} \min_{\mathbf{Q},\mathbf{A}} \max_{K_1+1\le k\le K} \quad & \frac{\varepsilon_k(\mathbf{Q},\mathbf{A})}{\delta_k},\ k = K_1+1,\cdots,K, \\ \text{s.t.} \quad & \varepsilon_k(\mathbf{Q},\mathbf{A}) \le \gamma_k,\ k = 1,\cdots,K_1, \\ & \text{Tr}\{\mathbf{Q}\mathbf{C}_1\} \le P_{\max},\quad \mathbf{Q} \succeq \mathbf{0}. \end{aligned} \quad (19)$$

Note that the solution of the problem in (19) will yield an upper-bound of the problem in (18) [12]. However, the optimal solution of the original problem in (18) can be obtained by solving (19) with appropriate values of auxiliary variables. In addition, this problem is difficult to solve in the downlink. Hence, we consider an equivalent uplink problem using general MSE duality. By introducing new variables $\delta_k, k = 1\cdots K_1$, the above problem can be formulated into a MSE balancing problem in the uplink as follows:

$$\begin{aligned} \min_{\mathbf{V},\mathbf{q}} \max_{1\le k\le K} \quad & \frac{\varepsilon_k^{(u)}(\mathbf{v}_k,\mathbf{q})}{\delta_k},\quad k = 1,\cdots,K, \\ \text{s.t.} \quad & \textstyle\sum_{i=1}^K \eta_i^2 q_i \le P_{\max}, \end{aligned} \quad (20)$$

where $\delta_k,\quad k = 1\cdots K_1$ can be updated such that the real-time users achieve their MSE thresholds. The beamformers can be obtained by minimizing the sum-MSE as in (12). The uplink power allocation problem can be formulated into a GP (convex problem) as follows:

$$\begin{aligned} \min_{t,\mathbf{q}} \quad & t, \\ \text{s.t.} \quad & q_k^{-1}\left[(\mathbf{\Omega}+\mathbf{\Theta}^2\mathbf{\Psi})\mathbf{q}+\mathbf{\Theta}^2\boldsymbol{\rho}\right]_k \le t\delta_k, \\ & \qquad\qquad k = K_1+1,\cdots,K, \\ & q_k^{-1}\left[(\mathbf{\Omega}+\mathbf{\Theta}^2\mathbf{\Psi})\mathbf{q}+\mathbf{\Theta}^2\boldsymbol{\rho}\right]_k \le \gamma_k, \\ & \qquad\qquad k = 1,\cdots,K_1, \\ & \textstyle\sum_{i=1}^K \eta_i^2 q_i \le P_{\max},\ q_k \ge 0,\ t \ge 0, \end{aligned} \quad (21)$$

where $\mathbf{\Omega}, \mathbf{\Theta}, \mathbf{\Psi}$ and $\boldsymbol{\rho}$ are defined in (14) and (12), respectively. The auxiliary variables λ_1 and λ_2 are updated based on (15).

B. Relay Amplification Matrix Design

For a given $\mathbf{U}$ and $\mathbf{p}$, the relay matrix $\mathbf{F}$ is designed to satisfy the mixed QoS requirement using GP (convex problem) as follows:

$$\begin{aligned} \min_{\mathbf{f},t} \quad & t, \\ \text{s.t.} \quad & \mathbf{f}^H\mathbf{B}\mathbf{f} \le P_2^{(r)}, \\ & 1 - 2\Re(\mathbf{g}_k^T\mathbf{f}) + \mathbf{f}^H\mathbf{D}_k\mathbf{f} + |\bar{a}_k|^2\sigma_k^2 \le t\delta_k, \\ & \qquad\qquad k = K_1+1,\cdots,K, \\ & 1 - 2\Re(\mathbf{g}_k^T\mathbf{f}) + \mathbf{f}^H\mathbf{D}_k\mathbf{f} + |\bar{a}_k|^2\sigma_k^2 \le \gamma_k, \\ & \qquad\qquad k = 1,\cdots,K_1, \end{aligned} \quad (22)$$

where $\mathbf{f}, \mathbf{B}, \mathbf{g}_k$ and $\mathbf{D}_k$ are defined in (7). The algorithm for mixed QoS requirement problem is summarized in Table IV.

Table IV: Mixed QoS Algorithm.

1) **Repeat**
   a) Initialize: λ_1, λ_2, $q_k = P_{max}/K \quad \forall k$.
   b) **Repeat**
   c) **Repeat**
      i) Obtain uplink receiver beamformers using (12).
      ii) Obtain uplink power allocation by using (21).
   d) **Until** the required accuracy.
   e) Obtain $\mathbf{Q}$ and $\mathbf{A}$ from general MSE duality.
   f) Update λ_1 and λ_2, using subgradient method as in (15).
   g) **Until** the required accuracy.
2) Obtain $\mathbf{F}$ by solving (22).
3) **Until** the required accuracy.

VI. Simulation Results

In order to validate the convergence of the proposed algorithms, we consider three single antenna users. The base station and the relay consist of four and three antennas, respectively. All the channel coefficients have been generated using zero-mean circularly symmetric *iid* complex Gaussian random variables. It is assumed that all the channel coefficients are available at the relay. Please note that the imperfect CSI, for example due to quantization of CSI, may degrade the overall performance, however it is not expected to change convergence behaviour of the proposed algorithms. i.e., the iterative algorithm will still converge with monotonically decreasing MSE values, as both the subproblems are individually convex problems. The noise power at the user terminals and noise covariance matrix at the relay have been assumed 0.05 and 0.05 $\mathbf{I}$, respectively.

In order to evaluate the convergence of the sum-power

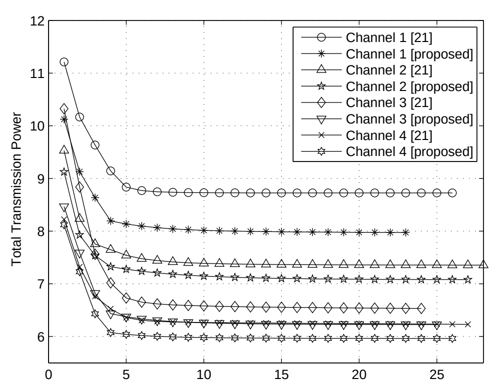

Fig. 1: The performance comparison of the proposed sum-power minimization algorithm with that of the algorithm presented in [5] for different channels.

minimization algorithm, the MSE threshold at each user and α_0 in (2) have been set to 0.1 and 1, respectively. Here, the relay amplification matrices are initialized with zero-forcing based solution. The initialization with zero-forcing relay matrix is the better strategy, because random matrix initialization will change the overall end to end channel matrices, for which the optimization problem might turnout to be infeasible even though the original problem may be feasible with zero-forcing initialization. In addition, we compare the performance of the proposed sum-power minimization algorithm with that of the algorithm presented in [5]. The equivalent target SINR has been set for the algorithm presented in [5]. As seen in Figure 1, the proposed algorithm converges and outperforms the algorithm presented in [5] in terms of total transmission power. Since the optimal solution is obtained from each subproblem, the total transmission power monotonically decreases as observed in Figure 1. This confirms the convergence of the proposed algorithms.

To demonstrate the convergence of the MSE balancing algorithm, the maximum available transmit power at the BS and the relay has been individually set to 2. Figure 2 represents the convergence of the balanced MSEs with zero-forcing and random initialization of relay amplification matrices for different channels. The results confirm the convergence of the MSE balancing algorithm. Lastly, we evaluate the convergence of the mixed QoS algorithm. We consider the same network as in the previous set of simulations, however, with one real-time user and two non-real-time users. The MSE threshold of the real-time user has been set to 0.1. The MSEs of the non-real-time users as in Figure 3 are balanced with zero-forcing and random initialization of relay amplification matrices for different channels, while satisfying the MSE threshold of the real-time user. This result confirms the convergence of the mixed QoS algorithm.

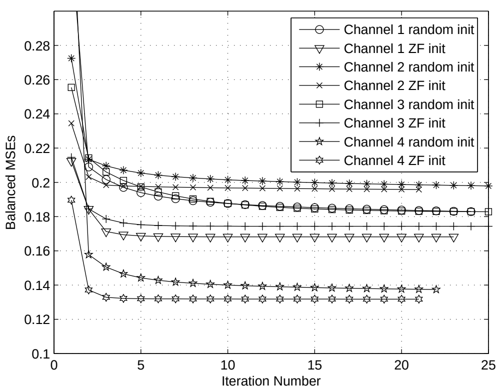

Fig. 2: The performance comparison of MSE balancing algorithm with zero-forcing and random initialization of relay matrices for different channels.

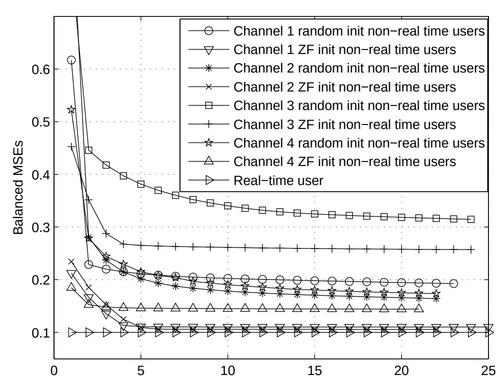

Fig. 3: The performance comparison of mixed QoS algorithm with zero-forcing and random initialization of relay amplification matrices for different channels.

VII. Conclusions

We proposed three MMSE based criteria for an amplify and forward based multiantenna relay network to solve a) sum-power minimization, b) MSE balancing and c) mixed QoS requirement problems. These algorithms were developed based on convex optimization techniques and general MSE duality. Simulation results have been provided to support the convergence of the proposed algorithms.

Appendix I

We provide the proof for the formulation of QCQP in (6). The following matrix identities are used:

$$\begin{aligned} \mathrm{Vec}(\mathbf{AXB}) &= (\mathbf{B}^T \otimes \mathbf{A})\mathrm{Vec}(\mathbf{X}), \\ \mathrm{Tr}(\mathbf{A}^T\mathbf{B}) &= \mathrm{Vec}(\mathbf{A})^T\mathrm{Vec}(\mathbf{B}). \end{aligned} \tag{23}$$

$$\begin{aligned} \mathrm{Tr}\left\{\mathbf{F}\mathbf{R}_r\mathbf{F}^H\right\} &= \mathrm{Tr}\left\{\mathbf{R}_r^{1/2}\mathbf{F}\mathbf{F}^H\mathbf{R}_r^{1/2}\right\} \\ &= \left[\mathrm{Vec}(\mathbf{R}_r^{1/2}\mathbf{F}^H)^T\right]^T \mathrm{Vec}(\mathbf{F}\mathbf{R}_r^{1/2}) \\ &= \left[(\mathbf{R}_r^{1/2}\otimes\mathbf{I})\mathrm{Vec}(\mathbf{F}^*)\right]^T(\mathbf{R}_r^{T/2}\otimes\mathbf{I})\mathrm{Vec}(\mathbf{F}) \\ &= \mathbf{f}^H\Big(\Big[\mathbf{R}_r^{1/2}\otimes\mathbf{I}\Big]^T\Big[\mathbf{R}_r^{T/2}\otimes\mathbf{I}\Big]\Big)\mathbf{f} \\ &= \mathbf{f}^H\mathbf{B}\mathbf{f}. \end{aligned} \tag{24}$$

$$\begin{aligned} a_k\mathbf{h}_k^H\mathbf{F}\mathbf{R}_r\mathbf{F}^H\mathbf{h}_k a_k^* &= \mathrm{Tr}\left\{(\mathbf{R}_r^{1/2}\mathbf{F}\mathbf{h}_k a_k^*)(a_k\mathbf{h}_k^H\mathbf{F}^H\mathbf{R}_r^{1/2})\right\} \\ &= \mathbf{f}^H\left(\left[\mathbf{R}_r^{1/2}\otimes a_k^*\mathbf{h}_k^T\right]^T\left[\mathbf{R}_r^{T/2}\otimes a_k\mathbf{h}_k^H\right]\right)\mathbf{f} \\ &= \mathbf{f}^H\mathbf{D}_k\mathbf{f}, \end{aligned} \tag{25}$$

$$\begin{aligned} \bar{a}_k\mathbf{h}_k^H\mathbf{F}\mathbf{H}_0\tilde{\mathbf{u}}_k &= \mathrm{Tr}(\bar{a}_k\mathbf{H}_0\tilde{\mathbf{u}}_k\mathbf{h}_k^H\mathbf{F}) \\ &= [\mathrm{Vec}(\bar{a}_k\mathbf{H}_0\tilde{\mathbf{u}}_k\mathbf{h}_k^H)]^T\mathrm{Vec}(\mathbf{F}) \\ &= \mathbf{g}_k^T\mathbf{f}. \end{aligned} \tag{26}$$

Appendix II

We consider a downlink network with a single linear constraint, i.e., $\mathrm{Tr}\{\mathbf{Q}\mathbf{C_1}\} \leq P_{\max}$. The MSE of the k^{th} user can be written as follows:

$$\varepsilon_k = 1-2\Re(a_k\tilde{\mathbf{h}}_k^H\tilde{\mathbf{u}}_k)+|a_k|^2\tilde{\mathbf{h}}_k^H\tilde{\mathbf{U}}\tilde{\mathbf{U}}^H\tilde{\mathbf{h}}_k+|a_k|^2\eta_k^2. \tag{27}$$

Next, we derive the MSE of the k^{th} user in the equivalent uplink system where the channel of the k^{th} user is considered as $\mathbf{h}_k$. The noise covariance is $\mathbf{C_1}$ which defines the linear constraint in the downlink system. The power constraint in the uplink system is modified as $\sum_{i=1}^{K}\eta_i^2 q_i \le P_{\max}$. In the uplink, the received signal can be written as $\mathbf{y} = \tilde{\mathbf{H}}\tilde{\mathbf{Q}}\mathbf{s} + \tilde{\mathbf{n}}$, where $\tilde{\mathbf{Q}} = diag\{\tilde{q}_1 \cdots \tilde{q}_K\}$ is a diagonal matrix. The estimated signal of the k^{th} user can be written as $\hat{s}_k = \mathbf{v}_k^H\tilde{\mathbf{H}}\tilde{\mathbf{Q}}\mathbf{s} + \mathbf{v}_k^H\tilde{\mathbf{n}}$, where $\mathbf{v}_k$ is the receiver beamformer of the k^{th} user. From this receiver filter, the MSE of the k^{th} user is

$$\varepsilon_k^{(u)} = 1 - 2\Re\{\tilde{q}_k\tilde{\mathbf{h}}_k^H\mathbf{v}_k\} + \mathbf{v}_k^H\tilde{\mathbf{H}}\tilde{\mathbf{Q}}\tilde{\mathbf{Q}}^H\tilde{\mathbf{H}}^H\mathbf{v}_k + \mathbf{v}_k^H\mathbf{C}_1\mathbf{v}_k. \tag{28}$$

In order to show that, the same MSEs can be achieved in the downlink, the linear relationship is considered as $\mathbf{v}_k = \alpha_k\tilde{\mathbf{u}}_k$ and $\tilde{q}_k = \frac{a_k}{\alpha_k}$ [14]. From this relationship, the MSE of the k^{th} user in the uplink can be written as

$$\begin{aligned}\varepsilon_k^{(u)} \;=\; & 1 - 2\Re\{a_k\tilde{\mathbf{h}}_k^H\tilde{\mathbf{u}}_k\} + \alpha_k^2\tilde{\mathbf{u}}_k^H(\textstyle\sum_{i=1}^{K}\frac{a_i a_i^*}{\alpha_i^2}\tilde{\mathbf{h}}_i\tilde{\mathbf{h}}_i^H)\tilde{\mathbf{u}}_k \\ & + \alpha_k^2\tilde{\mathbf{u}}_k^H\mathbf{C}_1\tilde{\mathbf{u}}_k. \end{aligned} \tag{29}$$

By equating the MSEs of both the uplink and the downlink of each respective user and summing up all K equations, we obtain the following:

$$\begin{aligned}\textstyle\sum_{i=1}^{K}\frac{a_i a_i^*}{\alpha_i^2}\eta_i^2 &= \textstyle\sum_{i=1}^{K}\tilde{\mathbf{u}}_k^H\mathbf{C}_1\tilde{\mathbf{u}}_k, \\ \textstyle\sum_{i=1}^{K} q_k\eta_i^2 &= \mathrm{Tr}\{\mathbf{C}_1\mathbf{Q}\}. \end{aligned} \tag{30}$$

This shows that $\alpha_i, i = 1, \cdots, K$ can be found to achieve the same uplink MSEs of all users as in the downlink with $\sum_{i=1}^{K} q_k\eta_i^2 = \mathrm{Tr}\{\mathbf{C}_1\mathbf{Q}\}$. Similarly, it can be proven that the same uplink MSEs of all users can be achieved in the downlink with $\sum_{i=1}^{K} q_k\eta_i^2 = \mathrm{Tr}\{\mathbf{C}_1\mathbf{Q}\}$. This concludes the proof of the general MSE duality. ■

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