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Omitted budget constraint bias in discrete-choice demand models

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ABSTRACT

A large body of discrete-choice demand studies estimate a demand model in which the consumer's budget constraint is not taken into account. We illustrate how incorrectly specifying the consideration set, when in fact the budget constraint binds for some products, may bias the demand estimates. We illustrate and quantify the nature of the bias in three ways: (i) in analytical examples; (ii) in field data commonly used in the literature and (iii) in a Monte Carlo study. We find that the price sensitivity can be substantially lower when correctly imposing the budget constraint, and own-price elasticities are typically overestimated although the direction of the own-price elasticity bias is in general ambiguous and depends on the income distribution.

1. Introduction

Discrete choice models of product differentiation have gained considerable importance in empirical work. The seminal work of Berry et al. (1995, 1999) (hereafter BLP) has made the random coefficient logit model a popular tool. Many empirical studies following this approach share two features: (i) all products are assumed affordable for all consumer, and (ii) the price sensitivity in the choice utility is modelled as a function of income. Yet, with available income data, and when low-income consumers find some products unaffordable or outside their budget, then this restriction on the choice set can be taken into account in the estimation.

This paper explores how affordability restrictions affect demand elasticity estimates. Affordability considerations can matter especially for expensive products. For products such as cars or houses a subset of consumers may in fact be restricted in their choices. Budget constraints may also play a role for low priced items. For example, Miravete and Seim (2015) find that the volume of alcohol purchases is higher immediately after pay days. This suggests that some consumers may have little money available for alcoholic beverages towards the end of the pay cycle.

Standard consumer theory emphasizes the role of the budget constraint when deriving Marshallian demand. For the discrete choice demand framework, McFadden (1981) explores the utility maximization subject to a budget constraint and shows how to aggregate the model across consumers when product affordability is taken into account. Indeed,

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Xiao et al. (2017) follow this approach when studying the vehicle quota system in China. They explicitly take the product affordability into account. We complement the prior work by illustrating how misspecification of the consideration set may bias the estimates.

Many empirical studies of house or automobile purchase decisions do not account for product affordability, including (Berry et al., 1995; 1999), (Goldberg, 1995; Bayer et al., 2007; Copeland et al., 2011; Beresteanu and Li, 2011; Schiraldi, 2011; Huse and Lucinda, 2013) and (Grigolon and Verboven, 2014). Yet, some of the empirical findings may in part be the misspecification of the consumers' affordable set. For example, Goldberg (1995) finds that low-income consumers exhibit a stronger distaste for automobile price than high-income consumers. The stronger effect could in fact arise because of the omission of consumers' budget constraints. Reducing low-income consumers' choice sets by excluding cars not affordable at their income level may in fact lower their price sensitivity estimates.

The affordability of housing and automobiles in relation to household income is well documented in the urban economics literature. Moos and Mendez (2015) describe suburban living in terms of the trade-off between homeownership and automobile use. Litman (2013) documents the vehicles affordable to households are strongly linked to income quintiles with households spending less than 10% of their budget on mobility needs on average. In high housing cost areas the amount is reduced to 5%. This evidence suggests that using the annual disposable income is a conservative measure of potential biases.[1]

We view the omission of the affordability constraint as a potentially misspecified demand model. Our goal is to explore how the omission of this constraint may result in biased demand elasticity estimates. We also illustrate that with available income data the affordability restriction on the choice set can be readily taken into account in the estimation. We quantify and illustrate this bias in three ways: (i) in analytical examples; (ii) in field data commonly used in the literature and (iii) in a Monte Carlo study. We find that the price sensitivity can be substantially lower when correctly imposing the budget constraint, and own-price elasticities are typically overestimated although the direction of the own-price elasticity bias is in general ambiguous and depends on the income distribution.

Estimating demand elasticities correctly matters for applied work. It is important for assessing mark-ups, for calculation merger welfare effects, for assessing conduct in markets, and for quantifying welfare changes due to new product introduction.

There is a closely related literature which explores how the set of available products affects demand estimates including the literature on consideration sets, inattention and limited information. See Sovinsky Goeree (2008), Conlon and Holland Mortimer (2013), and Dinerstein et al. (2018). Sovinsky Goeree (2008) explores how advertising may influence the available choice set. Conlon and Holland Mortimer (2013) study how stock-outs affect product choice. Dinerstein et al. (2018) study how platform design affects product choice in online markets. These studies are related but focus on choice set selection processes other than income. The inattention literature relaxes the full information assumption and explores alternatives to capture the notion that the choice set may not include all available alternatives. Budget constraints differ from inattention. Income is known to consumers. It is typically observed by the econometrician and in fact frequently used as an explanatory variable to measures price sensitivity of consumers. The effect of income on the choice set can be readily taken into account in many settings.

Section 2 provides two analytic examples which illustrate the bias on aggregate demand stemming from appropriately specifying the consumers' affordable choice sets. The examples show that there are two opposing forces in play when rationalizing purchase data without using a budget constraint. First, the price sensitivity parameter is artificially increased to rationalize that fewer consumers buy when price increases. The price sensitivity parameter compensates for the omitted budget set. Holding all else equal, the increased price sensitivity parameter makes the demand curve steeper. Second, the correctly specified model takes into account that an increase in price will reduce the mass of consumers that can afford the product. The misspecified model ignores this effect. If this mass is large, then the misspecified model may in fact underestimate the demand elasticity as it makes the demand curve flatter than it should. Which of these two opposing effects dominates at a particular point depends on the shape of the income distribution. We shall provide analytical examples in which the omitted budget constraint bias increases, respectively decreases, the price elasticity estimate.

Section 3 describes the standard random-coefficients, discrete-choice aggregate-demand model and discusses the role of the budget constraint. We shall comment on McFadden's original formulation and discuss how the recent literature has deviated.

Section 4 illustrates the misspecification bias when the budget constraint is not taken into account for data and demand models studied in the literature, see Goldberg and Verboven (2001). We shall illustrate the own-price elasticity bias for automobile data for three selected OECD countries. We find that the price sensitivity parameter is overestimated with the average overestimate ranging from 11% to 78% across countries depending on the specification considered. We find that price elasticities for the top quality automobiles are overestimated by 28% to 60% across countries depending on the specification considered.

Section 5 illustrate the misspecification bias when the budget constraint is not taken into account in a Monte Carlo study which varies the fraction of budget constraint consumers, the number of products available and the number of markets. Our

[1] If the affordability condition differs from the observed income by a constant share, then this can be readily taken into account.

Monte Carlo results illustrate the direction of the omitted budget constraint bias as the number of products and fraction of income constraint consumers is varied.

2. Examples

This section describes two examples to illustrate the nature of the bias that emerges from omitting the budget constraint. The examples show that the direction of the bias will depend on the shape of the income distribution and is in general not determined. The first example considers a discrete income distribution in which income takes on two values, while the second example considers a uniform income distribution. Both examples share the standard modelling assumptions for single-product discrete choice with uniformly distributed random utility component. In example one, the bias results in an overestimated aggregate demand elasticity, while example two results in an underestimated aggregate demand elasticity.

Suppose that consumers' indirect utility of buying the product equals $\varepsilon - \frac{\alpha}{y} \cdot p$ where ε is a random utility element uniformly distributed on the interval [0,10], p is the price of the product and y is income. The parameter α measures the price sensitivity or distaste for price, with $0 < \alpha \leq 10$. The price sensitivity parameter is normalized by income allowing for differential price sensitivity based on income. The specification $\frac{\alpha}{y}$ is commonly imposed in applied work. Not buying the product yields zero utility.

A consumer with income y will buy the product if the product is in the budget $p \leq y$ and the purchase utility exceeds the outside alternative $\varepsilon - \frac{\alpha}{y} \cdot p \geq 0$. The expected demand by consumers with income y is given by $Pr(\varepsilon - \frac{\alpha}{y} \cdot p \geq 0) = \max(\int_{\alpha p/y}^{10} \frac{1}{10} d\varepsilon, 0) = \max(1 - \frac{\alpha \cdot p}{10y}, 0)$.

Individual choices can be aggregated across the population by using the income distribution. We shall consider two income distributions: (i) income y follows a discrete distribution taking on two values $\{4, 10\}$ with equal probability and (ii) income y is uniformly distributed on [1,10]. We shall illustrate the demand function in both cases.

Figure 1 plots the aggregate demand functions for the discrete income distribution in which half the population has income of 10 and the other half has income of 4. Price is depicted on the horizontal axis and quantity on the vertical axis. The solid function represents the true demand function, which incorporates consumers' budget constraints correctly, and is

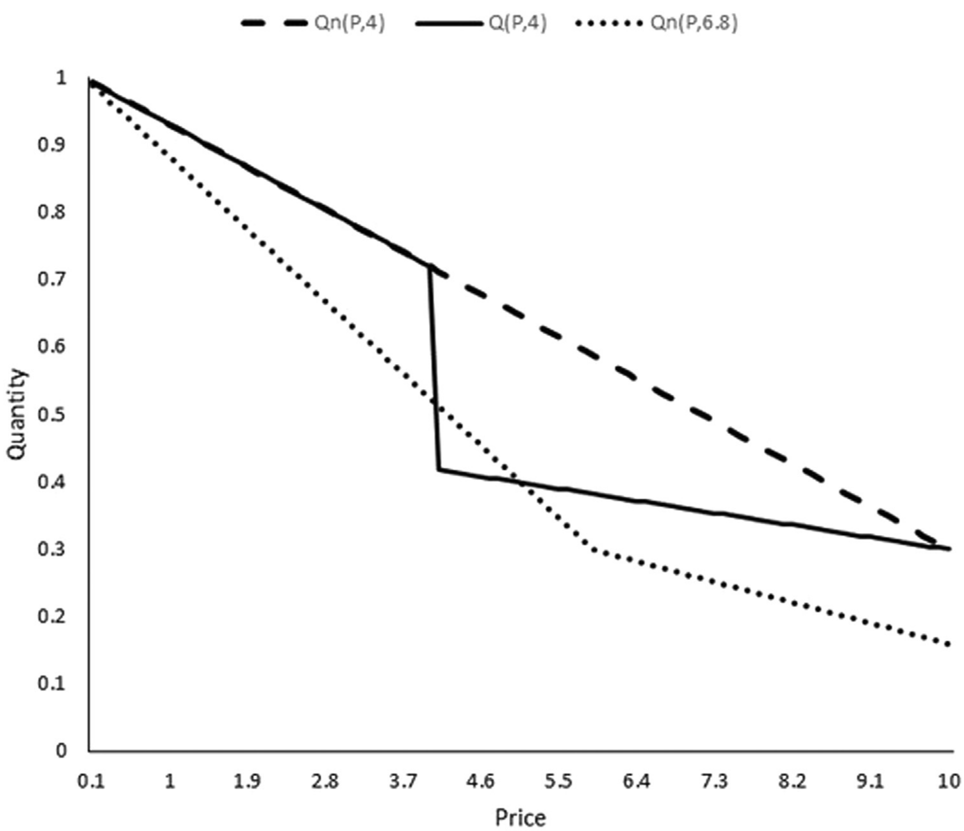

Fig. 1. Aggregate demand functions: income in $\{4, 10\}$ with equal probability.

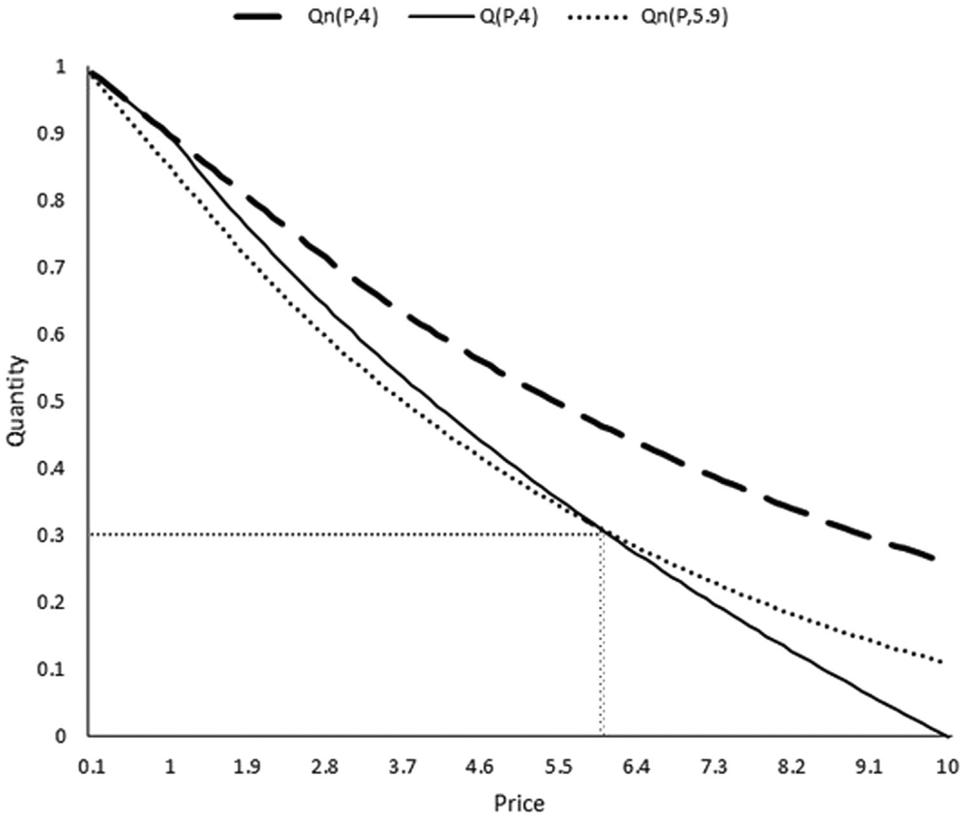

Fig. 2. Aggregate demand functions: income uniformly distributed on [1,10].

given by

$$Q(P;\alpha) = \begin{cases} \frac{1}{2}\max(1-\frac{\alpha P}{100},0)+\frac{1}{2}\max(1-\frac{\alpha P}{40},0) & \text{for } P \le 4; \\ \frac{1}{2}\max(1-\frac{\alpha P}{100},0) & \text{for } P \in (4,10]; \\ 0 & \text{for } P > 10. \end{cases}$$

The figure assumes parameter $\alpha = 4$. Notice that the true demand function has a jump at the price $P = 4$ when half the population becomes income constraint.

The dashed and the dotted function in Fig. 1 represent two misspecified demand functions, obtained by ignoring consumers' budget constraint. The misspecified demand function $Q^n(.;\alpha)$ for any parameter α is given by

$$Q^n(P;\alpha) = \frac{1}{2}\max(1-\frac{\alpha P}{100},0)+\frac{1}{2}\max(1-\frac{\alpha P}{40},0).$$

The dashed and the dotted function assume parameter values of $\alpha = 4$ and $\alpha = 6.8$, respectively. They do not exhibit a jump as the budget constraint is ignored. The dotted demand function $Q^n(.;6.8)$ has a kink at the point where low income consumers no longer buy the product.

Suppose the researcher observes that 40 percent of consumers buy at a price $P = 5$. In Fig. 1 the demand functions $Q(.;4)$ and $Q^n(.;6.8)$ go through that point. A researcher that considers the family of demand functions without budget constraints $Q^n(.;\alpha)$ will erroneously obtain the best fit when the price sensitivity coefficient $\widehat{\alpha} = 6.8$ (instead of 4). The misspecified demand functions Q^n has a demand elasticity of -1.5 at $P = 5$. In contrast, the correctly specified demand function $Q(.;4)$ has a demand elasticity of -0.25 at a price $P = 5$. Thus, erroneously interpreting the data with the misspecified family of demand functions Q^n, obtained by ignoring the consumers' budget constraints, results in an over-estimated demand elasticity.

Figure 2 plots the aggregate demand functions with the uniformly distributed income on [1,10]. The solid curve represents the true demand function, obtained by incorporating the budget constraint, and given by $\int_{\max(P,1)}^{10}\left(1-\frac{\alpha P}{10y}\right)\frac{dy}{9}$. Solving the integral yields

$$Q(P;\alpha) = \frac{10-\max(P,1)}{9} - \frac{\alpha P}{90}[\ln(10)-\ln(\max(P,1))].$$

The solid curve assumes parameter $\alpha = 4$.

The dashed and the dotted function in Fig. 2 represent the misspecified demand function, which is obtained by ignoring consumers' budget constraint. The misspecified demand function $Q^n(.;\alpha)$ is given by $\int_{\max(\frac{\alpha P}{10},1)}^{10}\left(1-\frac{\alpha P}{10y}\right)\frac{dy}{9}$. Solving the integral yields

$$Q^n(P;\alpha)=\frac{10-\max(\frac{\alpha P}{10},1)}{9}-\frac{\alpha P}{90}\Big[\ln(10)-\ln(\max(\frac{\alpha P}{10},1))\Big].$$

The dashed and the dotted function in Fig. 2 assume parameters of $\alpha=4$ and $\alpha=5.9$, respectively. Observe that the misspecified demand function $Q^n(.;4)$ implies that more consumers buy at a given price than with correctly specified demand function $Q(.;4)$, resulting in a flattening of the demand curve.

Suppose the researcher observes that 30 percent of consumers buy at a price of 6. This is the point in Fig. 2 where the correctly specified demand function $Q(.;4)$ intersects the misspecified dotted demand function $Q^n(.;5.9)$ from above. A researcher that fits the family of demand models Q^n will erroneously obtain the best fit when the price sensitivity coefficient equals $\widehat{\alpha}=5.9$ (instead of 4). The misspecified demand function increases the price sensitivity parameter α, from 4 to 5.9, but also increases the mass of consumers buying the car. The first effect makes the demand curve steeper, while the second effect makes it flatter. With the uniform income distribution the second effect dominates. Overall the demand curve $Q^n(.;5.9)$ is flatter than the demand curve $Q(.;4)$ at the price of 6. Thus, the demand elasticity will be under-estimated. The demand elasticity of the misspecified demand function $Q^n(.;5.9)$ is -1.33 at a price of 6. The price elasticity of the correctly specified demand function $Q(.;4)$ at the price of 6 is -1.74.

Figures 1 and 2 share the feature that omitting the budget constraint will artificially increase the price sensitivity parameter α and thus bias the demand elasticity estimate. The direction of the demand elasticity bias depends on the shape of the income distribution. For cases similar to Fig. 1 where the mass of consumers with income close to the price being considered is small, we would expect demand elasticities to be overestimated. For cases similar to Fig. 2 where the mass of consumers with income close to the price being considered is large, the demand elasticity may in fact be underestimated.

In practice rich discrete choice models are estimated which include a number of products with distinct prices and product characteristics. We would expect that the omitted budget constraint bias effects not only the elasticity of demand but also the product characteristics' coefficient estimates. Incorrectly omitting the budget constraint may have a strong effect on the indirect utility of high priced products and less of an effect for low priced products. We would expect a differential effect for high and low priced products respectively. Shaked and Sutton (1982) show that price levels reflect product quality. We may thus conjecture that omitting the budget constraint may artificially reduce the preference for quality. On the other hand, for product markets where price and income are positively correlated, the bias concerns may be less severe.

The next section describes the role of the budget constraint in the standard random-coefficient discrete-choice aggregate demand model. We contrast two models, with and without a budget constraint.

3. The role of the budget constraint

This section illustrates the role of the budget constraint in the standard random-coefficients, discrete-choice aggregate-demand model. Our illustration is based on McFadden (1981) and Berry et al. (1995, 1999). We follow McFadden in deriving the aggregate demand function obtained from consumer level discrete choice conditional on the choice being contained in the budget set.[2]

There is a composite good c and J differentiated products $\mathbf{J}=\{0,1,2,\ldots,J\}$. Each product $j\in\mathbf{J}$ is described by a vector of publicly observable characteristics x_j and a scalar of unobservable characteristics ξ_j, which are observed by the consumers but not by the researcher. Purchasing no product is denoted by $j=0$, the outside good.

As is shown in Berry et al. (1995) the random coefficient specification can be derived as the indirect utility obtained from a Cobb-Douglas utility function involving a composite good c and the log-linear utility of the product purchased with public characteristics (x_j,ξ_j) and private taste shock ν_i. Suppose that consumer i solves the following standard utility maximization problem with a budget constraint:

$$\max_{c\geq 0,j} c^{\alpha_i}\cdot e^{x_j\beta+\sum_k\sigma_k x_{jk}\nu_{ik}+\xi_j+\varepsilon_{ij}} \qquad \text{subject to } c+p_j\leq y_i$$

where y_i is the income of consumer i, β is common across consumers, σ_k is the characteristic component k specific standard deviation across consumers and ε_{ij} is a separable utility shocks unobservable to the researcher. At the optimum choice of the consumption good c and the discrete good j, the budget constraint holds with equality and the composite good c can be replaced with y_i-p_j in the utility function to obtain an indirect utility expression. Following BLP by taking the logarithm of the indirect utility of buying product j and the composite good c yields an expression equal to

$$\alpha_i\log(y_i-p_j)+x_j\beta+\sum_k\sigma_k x_{jk}\nu_{ik}+\xi_j+\varepsilon_{ij} \tag{1}$$

[2] See equation (5.5.) on page 207 in McFadden (1981).

and the outside good has (log) indirect utility $\alpha_i \log(y_i) + \xi_0 + \sigma_0 \nu_{i0} + \varepsilon_{i0}$. Berry et al. (1999) take a first-order Taylor series approximation of the logarithmic function $\log(y_i - p_j)$ around the point $p_j = 0$.[3] Berry et al. (1999, page 407) show that this approximation yields $\alpha_i \log(y_i - p_j) = \alpha_i \log(y_i) + \alpha_i \frac{1}{y_i}(-p_j)$+remainder. Based on this Taylor approximation, and using the notation $\alpha_i(y_i) = \alpha_i \frac{1}{y_i}$, Eq. (1) can be equivalently written as:

$$-\alpha_i(y_i)p_j + x_j\beta + \sum_k \sigma_k x_{jk}\nu_{ik} + \xi_j + \varepsilon_{ij}, \tag{2}$$

which is the specification commonly adopted in the empirical literature. The Eq. (2) consists of a sum of three terms: (i) a mean utility term $x_j\beta + \xi_j$ common to all consumers; (ii) an individual-specific utility term $\sum_k \sigma_k x_{jk}\nu_{ik} - \alpha_i(y_i)p_j$; and (iii) an individual error term ε_{ij} specific to each product j and individual i which is assumed to be EV distributed.

Berry et al. (1995, 1999) omit budget constraint considerations in their analysis.[4] Yet, strictly speaking the indirect utility derived in Eq. (1) and (2) is only defined for products that are in the budget set, that is for $p_j \le y_i$. For products outside the budget set, the substitution $c = y_i - p_j$, would yield a negative consumption level of the composite good, which is not possible.

The mathematical expression of the conditional indirect utility in Eq. (2), conditional on product j being in the budget, can be evaluated for any income level y_i even when the product is not in the budget. It has become widespread practice in the literature to use this expression as a measure of the indirect utility for products even when they are not in the budget set. However, this measure is only valid conditional on the product being in the budget set. It is not valid for products outside the budget set. In fact, correctly imposing a budget constraint excludes expensive good for which $p_j > y_i$ from the available choice set, or equivalently sets the indirect utility to $-\infty$.[5]

McFadden (1981) observes that the budget constraint limits the consumer's choice set. Consumer i chooses the high utility alternative from all alternatives contained in the budget set. The predicted market share of product j in each market is the probability that product j yields the high utility among all alternatives contained in the budget set. Let $\mathbf{J}(\mathbf{y}) = \{j \in \mathbf{J} : p_j \le y\}$ denote the set of products in the budget set for a consumer with income y. Let $P(y, \nu)$ denote the joint distribution function of income and taste in the population. Let $\theta = (\alpha, \beta, \sigma)$ denote the parameter vector of interest. The predicted market share is given by the logit choice probability expression, integrated over the income and tastes of consumers:

$$s_j(\mathbf{x}, \mathbf{p}, \xi; \theta) = \int \frac{\exp(x_j\beta + \xi_j + \sum_k \sigma_k x_{jk}\nu_k - \alpha_i(y)p_j) \cdot 1\big(y - p_j\big)}{1 + \sum_{l\in\mathbf{J}(\mathbf{y})} \exp(x_l\beta + \xi_l + \sum_k \sigma_k x_{lk}\nu_k - \alpha_i(y)p_l)} dP, \tag{3}$$

where $1(x)$ denotes the indicator function, $1(x) = 1$, if $x \ge 0$; and $1(x) = 0$, otherwise. Note, that the budget constraint enters in two places: (i) the summation in the denominator is over the constrained set $\mathbf{J}(\mathbf{y})$ which consists of all products in the budget set of consumers with income $\mathbf{y}$ and (ii) the numerator only takes into account products which are in the budget for consumer i.

In contrast, the unconstrained optimization problem in Berry et al. (1999), that is when the budget constraint is omitted, leads to a different optimization problem. In the unconstrained case consumers are assumed to choose the highest conditional utility among all alternatives in the unconstrained choice set $\mathbf{J}$ which leads to the following unconstrained market share equation:

$$\widetilde{s}_j(\mathbf{x}, \mathbf{p}, \xi; \theta) = \int \frac{\exp(x_j\beta + \xi_j + \sum_k \sigma_k x_{jk}\nu_k - \alpha_i(y)p_j)}{1 + \sum_{l\in\mathbf{J}} \exp(x_l\beta + \xi_l + \sum_k \sigma_k x_{lk}\nu_k - \alpha_i(y)p_l)} dP. \tag{4}$$

Note, that the summation in the denominator in the unconstrained case is over all products $\mathbf{J}$.

While the original problem is consistently formalized as the solution to the constrained optimization problem (3), most of the recent empirical literature has carried out the estimation by studying the solution to the unconstrained optimization problem (4). The budget constraint plays no role in shaping the affordable choice set consumers face. Every household is allowed to choose among all available products. Clearly such an estimation procedure may lead to potentially severe bias in the price parameter as well in all other parameters when the relevant goods are expensive, as is the case with houses or automobiles. Some households can only afford a subset of all available products.

Berry et al. (1999) specify the consumer's distaste for price $\alpha_i(y_i)$ as inversely proportional to individual income level, $\alpha_i(y_i``````) = \frac{\alpha}{y_i}$ where α is a parameter identical across consumers. Recent applications with this specification include Beresteanu and Li (2011), Copeland et al. (2011), Schiraldi (2011) and Grigolon and Verboven (2014). Alternative functional form specifications for the consumer's distaste for price $\alpha_i(y_i)$ have been adopted in Goldberg (1995), Nevo (2001),

[3] Berry et al. (1999) additionally assume identical price coefficients for all consumers, $\alpha_i = \alpha$, which is not essential as the argument applies at the consumer level.

[4] The omission of the budget constraint appears common in automobile market demand studies. Manski (1983) and Bresnahan (1987), who estimates discrete choice automobile demand, do not include a budget constraint.

[5] Berry et al. (1995) consider the logarithmic specification described in equation (1). Equations (2.6) and (2.7a) in Berry et al. (1995, page 848 and 849) are well-defined for $p_j \le y_i$ only. For $p_j > y_i$, these equations are ill-defined. Computationally, the budget constraint can be incorporated by setting the log of the utility in equation (2.7a) to a large negative number for $p_j > y_i$, but this is not discussed in BLP(1995). The vast majority of subsequent empirical papers adopt the specification in Berry et al. (1999) which ignores the conditioning argument and specifies the indirect utility of product j based on Eq. (2) even if the product is not in the budget set of the consumer.

Table 1
Prices and Income in thousand 1990 US dollars.

	Belgium	France	Germany
Mean Car Price	14.8	15.4	15.8
Median Car Price	13	13.8	14.8
Top 5 Most Expensive Cars	30–38	30–42	27–33
Mean Income	33.7	40.5	31.6
5th Income Percentile	13.8	12.6	11.1
25th Income Percentile	22.5	22.2	16
50th Income Percentile	31.3	32.2	24.9
75th Income Percentile	42.6	46.2	37.6
95th Income Percentile	63.7	102.3	68.9

Goolsbee and Petrin (2004), Mortimer (2007), Nakamura and Zerom (2010), Crawford and Yurukoglu (2012), Houde (2012), Lee (2013), Goldberg and Hellerstein (2008), Huse and Lucinda (2013), and Conlon and Gortmaker (2020) among others, where usually the consumer's distaste for price is assumed to take a linear functional form $\alpha_i(y_i) = \alpha + \pi_p y_i$ with parameters α and π_p identical across consumers. Nevo (2001) assumes a normal distribution for $\alpha_i(y_i)$ with household specific covariates affecting the mean of the normal. Bayer et al. (2007) allow the price coefficient to depend on income and additional consumers' characteristics. Goldberg (1995) allows for distinct distaste for price α_i for low and high income consumers.[6] All these applications have in common that they consider the unconstrained optimization problem (4) in which the budget constraint is omitted. A notable exception is a recent paper by Xiao et al. (2017) which in fact estimates the constrained optimization problem (3).

The effect of incorrectly ignoring the budget constraint is illustrated in the following section by comparing the estimates of the demand with and without budget constraints in field data commonly used in the literature.

4. The automobile market

This section illustrates the omitted budget constraint bias in field data commonly used in applied work. We use data on automobile market shares and product characteristics for some selected OECD countries. The next subsection explains in detail the dataset. Section 4.2 describes the estimation procedures, and Section 4.2 discusses the estimates from the correctly and misspecified aggregate demand model.

4.1. Data

The demand estimates for new automobiles in three countries are examined: Belgium, France, and Germany. The data cover the years 1990–1999 and have previously been used in Goldberg and Verboven (2001).

The variables in the data set include quantity, price, fuel consumption (measured in number of liters necessary to cover 100 kilometers), size (measured as length times width), manufacturer origin, horse-power, weight and year. The data set includes this information on the majority of models marketed during the period. Since models both appear and exit over this period, this gives us an unbalanced panel.

Following Berry et al. (1995) and most of the literature we use annual disposable household income as the measure for y_i in Eq. (3). For capital constrained households, which according to Jappelli (1990) are about 20% of households, an annual income measure appears well suited. For consumers with access to capital markets the permanent discretionary income may be an alternative measure. Attanasio et al. (2008) and Einav et al. (2012) find strong evidence that low income consumers tend to be liquidity constraint for car loans. Since the income constraint is binding at the lower end of the income distribution, using the annual disposable income appears reasonable. Litman (2013) argues that lower income households can only afford to spend a fraction of their annual budget on automobile costs, which suggests that using the annual disposable income is a conservative measure of potential biases.

The income variable is obtained from the EIU Market Indicators & Forecasts for the years 1990–1999. The (disposable) income data is organized in seven bands; the number (as well as the percentage) of households earning the income in each band is provided. For the estimation purposes, the income within each band has been approximated by a uniform distribution so the whole income distribution has been approximated by a piecewise linear distribution function.

Table 1 illustrates percentiles of the income distribution in relation to car prices in the year 1990. The table reports the mean and median car price as well as the five most expensive cars. The bottom 5th percentile of the income distribution can afford about half the cars in the data, the 25th percentile in the income distribution can afford most of the cars except the very expensive ones, while the 75th percentile in the income distribution can afford all the cars.

[6] Goldberg (1995) does not include unobserved characteristics ξ as the discrete choice demand model is estimated using data on individual consumers' decisions.

4.2. Estimation

We estimate the parameters of the models (3) and (4) using the data described in Section 4.1 by following the algorithm proposed by BLP. However, we do not include the supply function.

The estimation algorithm uses a population moment condition to form a nonlinear GMM estimator. Let Z be a set of instruments such that $E[Z'\xi(\theta^*))] = 0$, where the vector unobserved product characteristics ξ is a function of the true parameters θ^*. The GMM estimator is defined as:

$$\theta = \arg\max_{\theta} \xi(\theta)'ZA^{-1}Z'\xi(\theta), \tag{5}$$

where A^{-1} equals $Z'Z$. The unobserved characteristics ξ, as function of data and parameters, is obtained by finding the solution to the system of market share equations:

$$\boldsymbol{s_t}(\mathbf{x}, \boldsymbol{p_t}, \xi, \theta) = \boldsymbol{S_t} \tag{6}$$

where the vector of (theoretical) shares $\boldsymbol{s_t}$ in market t is defined in Eq. (3) for the (misspecified) model without budget constraint and in equation (4) for the model with budget constraints. $\boldsymbol{S_t}$ is the is the vector of observed market shares in market t. A market t is defined as a year. We estimate the parameters separately for each country. We take a large number of 1000 Haton draws for income and private taste shocks for each market and solve numerically the integrals in Eqs. (3) and (4).

To the extent that unobserved product characteristics ξ are anticipated by firms when making the price choice this may result in endogeneity concerns as $Cov(p_{jt}, \xi_{jt}) \neq 0$. To obtain more precise estimates, we use two set of instrumental variables: (i) BLP type instruments and (ii) cost shifters. As BLP type instruments we include the sum of the fuel consumption of other cars for the same manufacturer, the sum of weight/hp for the other cars for the same manufacturer, the sum of the size of the other cars for the same manufacturer, the sum of cylinders of the other cars for the same manufacturer, the number of other car from the same manufacturer sold in a given market, and the total number of brands minus the brands owned by the same manufacturer in a given market. As cost shifters we include the price of aluminium times the size of the car and the lagged price of aluminium times the size of the car.

Alternative sets of instruments have been proposed in the literature. Reynaert and Verboven (2014) and Conlon and Gortmaker (2020) suggest the use of optimal instruments following Chamberlain (1987), while Davis and Schiraldi (2014) and Gandhi and Houde (2019) propose the use of distance differentiation instruments. We attempted the second approach but did not observe any relevant change in the significance of the parameters possibly due to the small number of markets.

4.3. Results

Table 2 reports the parameter estimates across countries obtained from models (3) and (4), that is with and without budget constraint. We report estimates varying the size of the outside option. As is discussed in Dube et al. (2009) the size of the outside option ranges between 50% and 90% in empirical studies. Table 2 assumes that the share of the outside good is around 80% in each year. Table 3 assumes that the share of the outside option is smaller and around 60%.

Table 2
Demand parameter estimates (i) - Outside option 80%.

		Belgium		France		Germany	
		1990–1999		1990–1999		1990–1999	
		With Budget	No Budget	With Budget	No Budget	With Budget	No Budget
α	Price Coefficient	4.040	4.930	4.635	5.146	4.741	5.283
		(1.253)	(1.076)	(1.187)	(1.071)	(1.591)	(1.426)
β	Size	0.061	0.066	0.040	0.042	0.042	0.044
(Mean		(0.011)	(0.010)	(0.011)	(0.010)	(0.010)	(0.009)
of)	Hp/Weight	-0.046	-0.052	0.040	0.038	0.029	0.025
		(0.030)	(0.029)	(0.034)	(0.037)	(0.019)	(0.018)
	Fuel Consumption	-0.943	-0.937	-0.149	-0.134	-0.115	-0.110
		(0.401)	(0.407)	(0.270)	(0.232)	(0.051)	(0.049)
	Trend	0.966	0.986	-0.027	-0.047	-0.611	-0.585
		(0.222)	(0.229)	(0.145)	(0.145)	(0.976)	(0.962)
	Constant	-4.430	-4.406	-8.588	-8.627	-7.504	-7.465
		(1.206)	(1.249)	(1.263)	(1.172)	(0.497)	(0.496)
σ	Size	0.003	0.004	0.003	0.002	0.001	0.001
(Std		(0.012)	(0.011)	(0.007)	(0.006)	(0.004)	(0.004)
of)	Hp/Weight	0.031	0.030	0.004	0.003	0.001	0.001
		(0.078)	(0.076)	(0.172)	(0.212)	(0.461)	(0.446)
	Fuel Consumption	0.561	0.565	0.080	0.066	0.001	0.000
		(0.265)	(0.269)	(0.357)	(0.365)	(0.331)	(0.328)

Table 3
Demand parameter estimates (ii) - Outside option 60%.

		Belgium		France		Germany	
		1990–1999		1990–1999		1990–1999	
		With Budget	No Budget	With Budget	No Budget	With Budget	No Budget
α	Price Coefficient	3.805	4.705	4.041	4.676	3.372	4.193
		(1.439)	(1.209)	(1.148)	(0.994)	(1.374)	(1.167)
β	Size	0.053	0.059	0.038	0.042	0.037	0.041
(Mean		(0.010)	(0.009)	(0.010)	(0.009)	(0.011)	(0.010)
of)	Hp/Weight	-0.035	-0.042	0.042	0.039	0.035	0.030
		(0.056)	(0.063)	(0.038)	(0.038)	(0.019)	(0.019)
	Fuel Consumption	-0.249	-0.231	-0.125	-0.119	-0.126	-0.119
		(0.185)	(0.153)	(0.154)	(0.152)	(0.094)	(0.096)
	Trend	0.962	0.987	-0.025	-0.051	-0.660	-0.621
		(0.221)	(0.230)	(0.147)	(0.146)	(1.005)	(0.984)
	Constant	-5.748	-5.770	-7.718	-7.707	-6.583	-6.488
		(1.044)	(1.111)	(0.896)	(0.894)	(0.543)	(0.551)
σ	Size	0.001	0.001	0.003	0.003	0.001	0.001
(Std		(0.009)	(0.008)	(0.010)	(0.010)	(0.007)	(0.007)
of)	Hp/Weight	0.005	0.008	0.003	0.003	0.001	0.001
		(0.336)	(0.274)	(0.296)	(0.301)	(0.268)	(0.268)
	Fuel Consumption	0.050	0.038	0.041	0.039	0.009	0.009
		(0.484)	(0.540)	(0.522)	(0.549)	(0.681)	(0.686)

The first panel of Tables 2 and 3 reports the price sensitivity coefficient estimate $\widehat{\alpha}$, the second shows the mean of the distribution of the marginal utilities $\widehat{\beta}$, and the third panel gives the standard deviation estimates $\widehat{\sigma}$. The standard deviations are not sharply estimated with only the standard deviation of Fuel Consumption being statistically significant. The mean of Size is positive and statistically significant for all countries in Tables 2 and 3. In both specifications, Hp/Weight is negative (and insignificant) for Belgium and positive (and insignificant) for the other countries. Fuel Consumption has a negative and significant effect in all countries in both specifications.[7]

The key finding in Tables 2 and 3 is that omitting the budget constraint artificially increases the magnitude of the price coefficient estimate $\widehat{\alpha}$. This effect arises in all countries and in both specifications. The omitted budget constraint bias for the point estimate $\widehat{\alpha}$ equals 15% on average across countries in the specification (i) when the market share of the outside good is around 80%. The bias appears negatively related to the market share of the outside option. The bias increases to 21% in specification (ii) when the market share of the outside good is 60%. The bias is strongest in Belgium with 24% in the second specification and weakest in France with 11% in the first specification. The bias on other coefficient estimates is of small magnitude and without a clear pattern.

Tables 4 and 5 report selected own price elasticity estimates[8] for 15 automobile models. We select the five highest, five lowest and five around the median automobile prices in the year 1990. The bias artificially increases the elasticity estimates (in absolute values) for the vast majority of cases. The effect arises in all countries. For the most expensive automobiles the bias amounts to 24% on average across countries in specification (i) and 29% in specification (ii). The bias is highest for Belgium in specification (ii) with 51% and lowest for France in specification (i) with 12%. For the median and bottom automobile price the bias amounts to 13% on average in specification (i) and 18% in specification (ii).

We examined the null hypothesis that the own price elasticity with and without budget constraint is equal for individual products and countries. Following Nevo (2001), we compute the standard errors of the elasticities, which are required for the test, by using a parametric bootstrap based on the asymptotic normality of the estimator.[9] Therefore we test if the own-price elasticities estimated for the constrained and unconstrained problem are drawn from a distribution with the same mean using a simple two-sample t-test assuming that the two samples are independent. The independence assumption may be violated because the constrained and unconstrained own-price elasticities are estimated from the same data and they are clearly positively correlated. However, the independent two-sample t-test will reject less often the null hypothesis than the correct test. Hence, the percentage calculated underestimates the true one. We find that in all three countries in all specifications the null of equality is rejected for all products.

Table 6 presents a sample of own and cross price elasticities. We select the three most expensive cars (Honda Legend, Opel Senator, and Mercedes 200–300) and the three cheapest cars (Fiat Panda, Citroen 2 CV 6, and Fiat 126) in Germany in the year 1990. We assume the outside option has 80% and report the demand elasticity of the column car when the price

[7] Even though the specifications are not the same, Grigolon and Verboven (2014) and Noton (2016) obtain similar estimates regarding common covariates, namely size and fuel consumption.

[8] See Nevo (2000) for the own and cross-price elasticities formulas.

[9] See Ketz (2019) for a discussion of inference in random coefficient models

Table 4
Own price elasticities (i) - Outside option 80%.

		Belgium		France		Germany	
		1990–1999		1990–1999		1990–1999	
		With Budget	No Budget	With Budget	No Budget	With Budget	No Budget
Top	1st	-3.013	-4.378	-2.853	-3.329	-2.718	-3.180
	2nd	-2.847	-4.289	-2.514	-2.846	-2.687	-3.125
	3rd	-2.438	-3.613	-2.514	-2.845	-2.490	-2.853
	4th	-2.610	-3.572	-2.488	-2.796	-2.481	-2.829
	5th	-2.736	-3.750	-2.436	-2.725	-2.515	-2.863
	m+2	-1.753	-2.144	-1.575	-1.699	-1.870	-2.048
	m+1	-1.886	-2.360	-1.604	-1.725	-1.864	-2.043
Median	m	-1.771	-2.162	-1.552	-1.673	-1.865	-2.043
	m-1	-1.878	-2.296	-1.564	-1.683	-1.845	-2.021
	m-2	-1.824	-2.231	-1.567	-1.687	-1.844	-2.019
	J-4	-1.184	-1.385	-1.000	-1.074	-1.340	-1.460
	J-3	-1.039	-1.231	-0.955	-1.027	-1.151	-1.254
	J-2	-1.033	-1.229	-0.947	-1.017	-1.084	-1.181
	J-1	-0.962	-1.142	-0.891	-0.961	-1.057	-1.151
Botton	J	-0.876	-1.052	-0.883	-0.946	-1.004	-1.093

Table 5
Own price elasticities (ii) - Outside option 60%.

		Belgium		France		Germany	
		1990–1999		1990–1999		1990–1999	
		With Budget	No Budget	With Budget	No Budget	With Budget	No Budget
Top	1st	-2.749	-3.725	-2.515	-3.156	-2.107	-2.933
	2nd	-2.704	-3.657	-2.262	-2.726	-2.091	-2.884
	3rd	-2.505	-3.295	-2.253	-2.713	-1.920	-2.611
	4th	-2.425	-3.135	-2.217	-2.648	-1.938	-2.604
	5th	-2.451	-3.149	-2.202	-2.619	-1.989	-2.655
	m+2	-1.471	-1.764	-1.441	-1.636	-1.550	-1.909
	m+1	-1.480	-1.773	-1.479	-1.678	-1.543	-1.901
Median	m	-1.457	-1.747	-1.422	-1.614	-1.546	-1.904
	m-1	-1.447	-1.734	-1.441	-1.635	-1.530	-1.881
	m-2	-1.433	-1.717	-1.450	-1.645	-1.529	-1.880
	J-4	-0.882	-1.062	-0.932	-1.052	-1.130	-1.357
	J-3	-0.831	-1.002	-0.890	-1.006	-0.977	-1.170
	J-2	-0.820	-0.991	-0.880	-0.995	-0.919	-1.099
	J-1	-0.726	-0.880	-0.833	-0.943	-0.898	-1.075
Botton	J	-0.708	-0.860	-0.815	-0.922	-0.854	-1.020

of the row car is increased by one percent. Interestingly, cross price elasticities are overestimated in the model without a budget constraint.

The budget constraint bias also impacts the welfare analysis. We illustrate this impact by computing the compensating variation caused by a 10% increase in all car prices. Table 7 reports the results assuming that the outside option is 80%. The model without budget constraint underestimates the compensating variation for all countries in all years. The bias is highest in Germany where the bias amounts to $142.38 on average across the years. France has the smallest bias with $88.42 on average. For Belgium the average bias equals $94.4. We considered the null hypothesis that the average compensating variation is equal for the model estimates obtained using the with and without budget constraint. We tested the null separately for individual year-country observations using a procedure similar to the one used to test the equality of the own price elasticities. We can reject the null hypothesis of equality for all years in all countries.

This section has shown that ignoring the budget constraint in the empirical estimation leads to serious bias in the estimated demand elasticity for automobile data commonly used in applied work. Across countries the magnitude of the own-price elasticity bias for ranges from 11% to 24%. As a consequence, welfare analysis generates misleading estimations of the compensating variation. Next, we explore the nature of the bias and the mechanism that underlies it more closely in a Monte Carlo.

Table 6
A Sample of estimated own and cross-price elasticities for Germany 1990 - Outside option 80%.

	With Budget Constraint						No Budget Constraint					
	Honda Legend	Opel Senator	Mercedes 200–300	Fiat Panda	Citroen 2 CV 6	Fiat 126	Honda Legend	Opel Senator	Mercedes 200–300	Fiat Panda	Citroen 2 CV 6	Fiat 126
Legend	-2.7177	0.0047	0.0881	0.0000	0.0002	0.0018	-3.1795	0.0048	0.0939	0.0001	0.0003	0.0021
Senator	0.0007	-2.6871	0.0869	0.0000	0.0002	0.0018	0.0008	-3.1252	0.0929	0.0001	0.0003	0.0022
200–300	0.0007	0.0042	-2.4897	0.0000	0.0003	0.0020	0.0007	0.0045	-2.8531	0.0001	0.0003	0.0023
Panda	0.0002	0.0016	0.0364	-1.0837	0.0004	0.0028	0.0003	0.0019	0.0422	-1.1807	0.0004	0.0031
2 CV 6	0.0003	0.0017	0.0376	0.0001	-1.0566	0.0028	0.0003	0.0020	0.0437	0.0001	-1.1514	0.0031
126	0.0003	0.0017	0.0382	0.0001	0.0004	-1.0036	0.0003	0.0020	0.0443	0.0001	0.0004	-1.0933

Table 7
Average Compensating Variation (thousands of dollars) - Outside option 80%.

	Belgium		France		Germany	
	With Budget	No Budget	With Budget	No Budget	With Budget	No Budget
1990	1.18	1.10	1.16	1.13	1.47	1.30
1991	1.15	1.09	1.21	1.16	1.46	1.37
1992	1.32	1.24	1.25	1.20	1.53	1.44
1993	1.39	1.27	1.36	1.26	1.56	1.45
1994	1.48	1.36	1.40	1.30	1.61	1.48
1995	1.63	1.54	1.61	1.49	1.88	1.70
1996	1.72	1.57	1.56	1.45	1.85	1.67
1997	1.28	1.22	1.41	1.30	1.67	1.52
1998	1.32	1.25	1.39	1.31	1.68	1.53
1999	1.52	1.40	1.52	1.37	1.76	1.60

5. Monte-Carlo study

This section reports the results of a Monte-Carlo study. We illustrate the magnitude and nature of the omitted budget constraint bias as the number of available products, the income distribution and the size of the data set is varied. Our Monte Carlo considers a varying but small set of products. We consider a random utility model based on Eq. (2). The aggregate market share estimation method commonly used in the literature is considered. We compare the estimates obtained from the correctly specified model to estimates obtained by the misspecified model which omits the budget constraint. We first describe the Monte Carlo design in more detail. Then, we describe our estimators, and finally we comment on the findings.

5.1. Monte Carlo design

Our Monte Carlo study assumes an indirect utility based on equation (2). The utility of consumer i in market t with income y_i buying product $j \in J(y_i)$ is given by

$$(j - \frac{M}{2}) \cdot \beta - \frac{\alpha}{y_i} \cdot p_{jt} + \xi_{jt} + \varepsilon_{ijt} \tag{7}$$

where the parameters of interest (α, β) consist of a scalar price sensitivity α and a scalar characteristic's parameter β. M denotes the number of products. The income variable y_i is drawn independently from a lognormal distribution with parameters $(\mu, 1)$ denoting the mean and variance of the underlying normal. The market and product-specific demand shock ξ_{jt} is uniformly distributed on the interval $[-0.5, 0.5]$. The random utility component ε_{ijt} is extreme value distributed. We assume that products with higher characteristics fetch a higher price, $p_{jt} = j$. The utility of the outside alternative u_{i0} is set equal to ε_{i0t}.

We fix the price sensitivity parameter $\alpha = 4$ and the characteristic's parameter $\beta = 3$ when simulating the data. We vary three key elements: (i) the size of the product set $M = 2, 4, 6, 8, 10$; (ii) the income distribution by changing the parameter μ, and (iii) the number of simulated markets $T = 5, 10, 15$. We construct a discrete approximation to the income distribution by taking $N = 250$ sample draws from the standard-normal. We add the mean μ and apply the exponential function. The parameter μ is varied but the underlying sample normal sample draws are held fixed in the Monte Carlo. ρ denotes the fraction of consumers which are budget constrained for at least one product.

When simulating the market share data we use the fact that the probability that consumer i in market t with income y_i buys good j takes the usual logit expression with the modification that only products in the budget set are taken into account,

$$\Pr(z_{ijt} = 1|y_i) = \begin{cases} \dfrac{\exp\left((j-\frac{M}{2})\beta - \frac{\alpha}{y_i} p_{jt} + \xi_{jt}\right)}{1+\sum_{l\in \mathbf{J}(\mathbf{y}_i)} \exp\left((l-\frac{M}{2})\beta - \frac{\alpha}{y_i} p_{lt} + \xi_{lt}\right)} & if\, p_{jt} \le y_i; \\ 0 & otherwise, \end{cases} \tag{8}$$

where the variable $z_{it} = (z_{it0}, \ldots, z_{itM})$ denotes the purchase decision for consumer i in market t and ξ_{jt} is an iid draw from the uniform distribution on the interval $[-0.5, 0.5]$. We use the simulated purchase probabilities to calculate market shares in market t, $s_{jt} = \frac{1}{N}\sum_{i=1}^{N} \Pr(z_{ijt} = 1|y_i)$ for all $j \in J$. Thus, variation in market shares across markets arise due to the market and product-specific demand shock only. Next, we briefly summarize our estimation methods.

5.2. Estimation method

The estimation procedure is the same described in Section 4.2. It is based on a GMM estimator for the unobserved market and product-specific characteristic ξ. Let $g(\xi(\theta))$ denote the vector of moment conditions for $\theta = (\alpha, \beta)$. The moments include the mean of ξ, the variance of ξ, and the mean of $P'\xi$.

The correctly specified GMM estimator is defined as the solution

$$\widehat{\theta} = \arg\min[g(\xi(\theta))' I_{3\times 3} g(\xi(\theta))]$$

where $\xi(\theta)$ is found by solving the system of market share equations $\boldsymbol{s_t}(\xi;\theta) = \boldsymbol{s_t}$ with

$$s_{jt}(\xi;\theta) = \int \frac{\exp\left((j-\frac{M}{2})\beta - \frac{\alpha}{y_i}p_{jt} + \xi_{jt}\right)\cdot 1\left(y - p_{jt}\right)}{1 + \sum_{l\in\mathbf{J}(\mathbf{y}_i)} \exp\left((j-\frac{M}{2})\beta - \frac{\alpha}{y_i}p_{lt} + \xi_{lt}\right)} g(y)dy, \tag{9}$$

The misspecified GMM estimator is defined as the solution

$$\widetilde{\theta} = \arg\min[g(\widetilde{\xi}(\theta))' I_{3\times 3} g(\widetilde{\xi}(\theta))]$$

where $\widetilde{\xi}(\theta$ is found by solving the system of equations $\tilde{\boldsymbol{s}}_{\boldsymbol{t}}(\xi;\theta) = \boldsymbol{s_t}$ with

$$\widetilde{s}_{jt}(\xi;\theta) = \int \frac{\exp\left((j-\frac{M}{2})\beta - \frac{\alpha}{y_i}p_{jt} + \xi_{jt}\right)}{1 + \sum_{l\in\mathbf{J}} \exp\left((j-\frac{M}{2})\beta - \frac{\alpha}{y_i}p_{lt} + \xi_{lt}\right)} g(y)dy, \tag{10}$$

We use the contraction mapping described in Berry et al. (1995) to numerically calculate ξ from the market share equation.

The next section reports our results.

5.3. Monte Carlo results

Tables 8 reports the results of the Monte Carlo simulations. The parameter values are chosen as follows. The number of markets equals $T = 10$. The income distribution is log-normal with mean μ chosen such that the fraction of consumers ρ who are budget constraint for at least one product ranges between 0.2 and 0.5. The variance of the log-normal is fixed at 1. The fraction ρ is reported in column 1 and the corresponding mean μ is reported in column 2. We use 100 random draws from the log-normal to calculate the expectations in Eq. (9) and (10) which are held fixed throughout. The third column reports the number of products M which takes on values 2, 4 and 8. The fourth and fifth column report the estimator $\widehat{\theta} = (\widehat{\alpha}, \widehat{\beta})$ of the correctly specified model obtained by imposing the budget constraint in the market share equations. The eigth and ninth column report the estimator $\widetilde{\theta} = (\widetilde{\alpha}, \widetilde{\beta})$ of the misspecified model obtained without the budget constraint. Columns six and ten report the mean squared error (MSE) of the estimates. The seventh and 11th column report the average own-price elasticity for the estimates with budgets constraint, respectively without a budget constraint. To illustrate the elasticity bias, we report in column 12 the percentage point difference in the average own-price elasticity estimate stemming from the misspecified model and the correctly specified model. We use 100 replications to calculate standard errors which are reported in parenthesis.

Table 8
Monte Carlo Results, log-normal.

ρ	μ	M	$\hat{\alpha}$	$\hat{\beta}$	MSE	Elast. on Avg	$\tilde{\alpha}$	$\tilde{\beta}$	MSE	Elast. on Avg	Elast. Bias in %
0.2	1.584	2	4.019	3.032	0.101	-0.698	4.189	3.026	0.132	-0.756	8.3
			(0.26)	(0.18)	(0.13)	(0.04)	(0.25)	(0.18)	(0.17)	(0.03)	
0.3	1.271	2	3.998	3.000	0.087	-0.808	4.183	2.979	0.117	-0.887	9.7
			(0.25)	(0.15)	(0.09)	(0.04)	(0.24)	(0.16)	(0.13)	(0.03)	
0.4	0.932	2	4.021	3.013	0.074	-0.938	4.204	2.99	0.112	-1.046	11.5
			(0.22	(0.16)	(0.10)	(0.04)	(0.21)	(0.16)	(0.15)	(0.03)	
0.5	0.691	2	4.018	3.015	0.08	-1.004	4.265	2.968	0.150	-1.169	16.4
			(0.22	(0.18)	(0.10)	(0.04)	(0.21)	(0.18)	(0.17)	(0.03)	
0.2	2.277	4	3.994	3.001	0.012	-1.183	5.013	2.79	1.082	-1.363	15.2
			(0.10)	(0.05)	(0.02)	(0.03)	(0.09)	(0.04)	(0.18)	(0.03)	
0.3	1.965	4	4.005	3.001	0.009	-1.336	4.892	2.766	0.859	-1.516	13.4
			(0.08)	(0.05)	(0.01)	(0.03)	(0.08)	(0.04)	(0.13)	(0.03)	
0.4	1.625	4	3.984	2.985	0.011	-1.431	4.783	2.746	0.688	-1.670	16.7
			(0.09)	(0.05)	(0.02)	(0.03)	(0.09)	(0.05)	(0.12)	(0.03)	
0.5	1.384	4	4.011	3.007	0.011	-1.444	4.855	2.708	0.825	-1.786	23.6
			(0.10)	(0.05)	(0.01)	(0.04)	(0.09)	(0.04)	(0.13)	(0.03)	
0.2	2.970	8	3.996	2.999	0.002	-1.497	6.625	2.363	7.298	-2.451	63.7
			(0.05)	(0.02)	(0.00)	(0.02)	(0.05)	(0.02)	(0.26)	(0.02)	
0.3	2.658	8	3.999	3.000	0.002	-1.571	6.409	2.362	6.213	-2.623	66.9
			(0.05)	(0.02)	(0.00)	(0.02)	(0.04)	(0.01)	(0.19)	(0.02)	
0.4	2.318	8	4.000	3.002	0.002	-1.656	6.421	2.371	6.257	-2.873	73.4
			(0.04)	(0.02)	(0.00)	(0.02)	(0.04)	(0.01)	(0.17)	(0.02)	
0.5	2.077	8	4.005	3.002	0.003	-1.659	6.31	2.365	5.741	-2.951	77.8
			(0.05)	(0.02)	(0.00)	(0.02)	(0.05)	(0.01)	(0.21)	(0.02)	

Table 9
Monte Carlo Results, mixture of log-normals.

ρ	μ_1	M	$\hat{\alpha}$	$\hat{\beta}$	MSE	Elast. on Avg	$\tilde{\alpha}$	$\tilde{\beta}$	MSE	Elast. on Avg	Elast. Bias in %
0.08	1.584	2	4.024 (0.34)	3.032 (0.18)	0.151 (0.19)	-0.521 (0.04)	4.19 (0.33)	3.038 (0.18)	0.178 (0.24)	-0.555 (0.03)	6.5
0.10	1.271	2	3.996 (0.34)	3.000 (0.16)	0.142 (0.15)	-0.575 (0.04)	4.172 (0.32)	3.008 (0.16)	0.157 (0.18)	-0.619 (0.04)	7.6
0.11	0.932	2	4.026 (0.29)	3.014 (0.17)	0.111 (0.16)	-0.656 (0.04)	4.12 (0.27)	3.022 (0.17)	0.117 (0.18)	-0.688 (0.04)	4.8
0.12	0.691	2	4.023 (0.28)	3.016 (0.19)	0.113 (0.15)	-0.704 (0.04)	4.116 (0.27)	3.024 (0.19)	0.121 (0.17)	-0.740 (0.04)	5.1
0.14	2.277	4	3.994 (0.1)	3.001 (0.05)	0.012 (0.02)	-1.12 (0.03)	4.895 (0.10)	2.858 (0.04)	0.834 (0.18)	-1.211 (0.03)	8.0
0.16	1.965	4	4.006 (0.09)	3.001 (0.05)	0.010 (0.01)	-1.209 (0.03)	4.885 (0.09)	2.864 (0.04)	0.810 (0.14)	-1.337 (0.03)	10.6
0.19	1.625	4	3.982 (0.10)	2.984 (0.05)	0.013 (0.02)	-1.260 (0.03)	4.767 (0.10)	2.862 (0.05)	0.619 (0.13)	-1.436 (0.03)	13.9
0.26	1.384	4	4.012 (0.10)	3.007 (0.05)	0.013 (0.02)	-1.302 (0.04)	4.807 (0.10)	2.82 (0.04)	0.695 (0.15)	-1.52 (0.03)	16.7
0.21	2.970	8	3.996 (0.05)	2.999 (0.02)	0.002 (0.00)	-1.539 (0.02)	6.419 (0.05)	2.427 (0.02)	6.182 (0.23)	-2.45 (0.02)	59.2
0.24	2.658	8	3.999 (0.05)	3.000 (0.02)	0.002 (0.00)	-1.622 (0.02)	6.540 (0.04)	2.405 (0.01)	6.808 (0.20)	-2.700 (0.02)	66.4
0.35	2.318	8	4.000 (0.04)	3.002 (0.02)	0.002 (0.00)	-1.643 (0.02)	6.242 (0.04)	2.473 (0.01)	5.306 (0.15)	-2.811 (0.02)	71.1
0.45	2.077	8	4.005 (0.05)	3.002 (0.02)	0.003 (0.00)	-1.581 (0.02)	6.172 (0.05)	2.43 (0.01)	5.046 (0.21)	-2.855 (0.02)	80.6

Reassuringly, the estimator $(\hat{\alpha}, \hat{\beta})$ in Tables 8, obtained from the correctly specified model, performs well. For most specifications the coefficient estimates are close to the true value and the mean squared error is small. There is no evidence of a bias.

The estimator $(\tilde{\alpha}, \tilde{\beta})$, obtained from the misspecified model which omits the budget constraint, over-estimates the price sensitivity parameter estimates α and under-estimates the product differentiation parameter β in all specifications but one. The MSE increases as the number of products increases but is not monotone in income. The non-monotonicity appears in accordance with the theoretical findings illustrated in Figs. 1 and 2.

The misspecified model over-estimates the own-price elasticity by 33% on average. The bias equals 8% with 2 products when 20% of the population budget constraint. The bias increases to 78% with 8 products when 50% of the population is budget constraint.

Table 9 considers an alternative income distribution consisting of a bi-modal distribution. It is generated using a mixture of two log-normals. The income variable is drawn with probability 1/2 from the log-normal with mean μ_1 as reported in Table 8 and with probability 1/2 is drawn from a log-normal with mean 1.8. The fraction of constraint consumer ρ increases in μ_1 but less strong than in Table 8 because the mean of the second mixture distribution is held fixed.

Interestingly, now there is a region of income expansion in which the elasticity bias is no longer monotone in ρ. The elasticity bias when ρ equals 0.11 or 0.12 is smaller than when ρ equals 0.08 or 0.10. We interpret this non-monotonicity finding as consistent with the earlier theoretical illustration in Figs. 1 and 2.

Summarizing, we considered a Monte Carlo study with price sensitivity parameter $\alpha = 4$, which is close the field data estimates in Section 4. Omitting the budget constraint results in overestimated price coefficient parameters α and underestimated preference for quality parameter β. Varying the number of products M and the income distribution, we found that the bias on the price sensitivity parameter increases with the number of products but exhibits no clear pattern induced by shifts in the income distribution. The bias on the preference for quality parameter β increases with the number of products.

6. Conclusions

Discrete choice models including the random coefficient multinomial logit model are widely used to estimate demand of various goods by scholars and practitioners. These models are typically derived and estimated from an unconstrained utility maximization problem in which the consumers' budget constraints are omitted. While for inexpensive good such feature may be of less concern, it becomes relevant for expensive products, such as cars or houses. For such products, a substantial subset of consumers may in fact be restricted in their choices.

The magnitude of the bias is illustrated by estimating the demand for automobile in four different countries. We find that the price sensitivity parameter is overestimated with the average overestimate ranging from 12% to 51% across countries depending on the specification considered. We find that own-price elasticities for automobiles are overestimated by 11% to 24% across countries depending on the specification considered.

Additionally, the mechanism of the bias was illustrated in an analytical example and by using a Monte Carlo study. The analytical examples showed that the direction of the demand elasticity bias is in general not determined and depends on the income distribution. Our Monte Carlo study uses a price sensitivity parameter $\alpha = 4$, which is close to automobile field data estimates, and suggests that omitting the budget constraint leads to over-estimated price coefficient parameters α and underestimates the preference for quality parameters β in all our specifications. The magnitude of the price coefficient bias increased with the number of products yet was unaffected by changing the fraction of budget constraint consumers. The magnitude of the product differentiation coefficient bias increased with both, the number of products and the fraction of budget constraint consumers.

Data availability

Data is available online as described in the paper.

CRediT authorship contribution statement

Martin Pesendorfer: Conceptualization, Investigation, Writing – original draft, Writing – review & editing. **Pasquale Schiraldi:** Conceptualization, Investigation, Writing – review & editing. **Daniel Silva-Junior:** Data curation, Investigation, Writing – original draft, Writing – review & editing.

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