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# ***Effect of the Covid-19 frailty heterogeneity on the future evolution of mortality by stratified weighting***

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## **Abstract.**

The starting point of our research is the inadequacy of assuming, in the construction of a model of mortality, that frailty is constant for the individuals comprising a demographic population. This assumption is implicitly made by standard life table techniques. The substantial differences in the individual susceptibility to specific causes of death lead to heterogeneity in frailty, and this can have a material effect on mortality models and projections – specifically a bias due to the underestimation of longevity improvements. Given these considerations, in order to overcome the misrepresentation of the future mortality evolution, we develop a stochastic model based on a stratification weighting mechanism, which takes into account heterogeneity in frailty. Furthermore, the stratified stochastic model has been adapted also to capture Covid-19 frailty heterogeneity, that is a frailty worsening due to the Covid-19 virus. Based on different frailty levels characterising a population, which affect mortality differentials, the analysis allows for forecasting the temporary excess of deaths by the stratification schemes in a stochastic environment.

## **1. Introduction**

Mortality improvement trends have been mainly studied by means of stochastic mortality models (Lee and Carter 1992, Cairns *et al.* 2006) and affine models (Schrager 2006, Luciano and Vigna 2008, Blackburn and Sherris 2013). Nevertheless, the analysis of the changes in the mortality trend as the underlying risk factors vary remains relatively unexplored, as pointed out in Xu *et al.* (2019): the main risk factors affecting older adults that have been codified in the literature include high blood pressure, cancer, and heart problems.

As noted by Fried *et al.* (2004), the terms comorbidities (or multiple chronic conditions), frailty and disability are often used interchangeably in the identification of the vulnerable elderly. For example, poorer physical health has been defined in terms of frailty (Fried *et al.* 2001, Jones *et al.* 2004). There is now a growing consensus that comorbidities, frailty and disability are distinct entities in clinical terms but there is as yet no consensus on definitions in geriatric medicine. Focusing on the concept

of frailty, we find in the medical literature, relevant approaches include the phenotype model and the cumulative deficit model (Chen *et al.* (2014), Clegg *et al.* 2013, Rodríguez-Mañas *et al.* 2013). Further, Chen *et al.* (2014) and Clegg *et al.* 2013 argue that frailty is a measure of physical health and is associated with a considerably increased risk of adverse health outcomes such as falls, hospitalization, long-term care, institutionalization and mortality (Ahrenfeldt 2019).

In demography and actuarial science, a different approach to frailty has been pursued. Thus, Vaupel *et al.* (1979) have introduced frailty models based on a risk factor representing an individual's susceptibility to death. Some authors, such as Fong *et al.* (2017), describe the frailty as an individual's relative susceptibility to death compared to a standard and connect it to the different health status that emerges from disability surveys. As an individual's disability level increases, so a higher level of frailty develops. Traditional frailty models generally assume that frailty is fixed throughout a person's lifetime (Haberman and Butt 2004, Su and Sherris 2012). Conversely, stochastic ageing models represent the process of deterioration in terms of the human body's physiological capacity and the status of an individual's physiological capacity is called 'physiological age' (Fong *et al.* 2017) allowing for randomly changing frailty. Then, susceptibility to death depends on the physiological changes and environmental influences (Yashin *et al.* 1994, Lin and Liu 2007). As pointed out by Vaupel *et al.* (1979), Xu *et al.* (2019) and others, frailty is an unobserved risk factor in these demographic and actuarial models of frailty. According to this line of research, frailty represents an unobserved covariate that impacts mortality heterogeneity.

Systematic mortality improvement trends vary with the risk characteristics of individuals in a population (including by age and gender) and this variation determines the degree of mortality heterogeneity within a population (Vaupel *et al.* 1979, Meyricke *et al.* 2013). Further, some authors have shown that population heterogeneity can impact on population dynamics and on aggregate mortality trends (Kakai *et al.* 2019) Our paper relies on the assumption that mortality improvement trends differ due to different health status across individuals with the same age and gender, based on evidence that health status (especially chronic illness) is significantly correlated with disability (Sherris 2021) and mortality (Brown *et al.* 2013, Koijen *et al.* 2016, Yogo, 2016).

In this paper, we investigate the systematic effects on mortality rates due to the Covid-19 pandemic. Based on the intrinsic connection among frailty and mortality which impacts on the future evolution of mortality rates, we propose a more accurate estimate of the mortality differentials on the basis of the different frailty levels of a population. Indeed, the main underlying assumption of our study consists in detecting the material effect of frailty heterogeneity on mortality projections. It does not seem plausible to assume that frailty is constant in a demographic population. As is emphasised by Vaupel *et al.* (1979), if heterogeneity in frailty is substantial in a population, then the analysis of

population mortality needs to take it into account in terms of its impact on age specific mortality measures including life tables.

We also assume that the mortality rates in a population may experience sudden jumps, due to a critical change in living conditions, such as recently it occurred with the Covid-19 pandemic, where the Sars-Cov2 virus has amplified the excess of deaths in presence of the pre-existing comorbidities, by leading to a worsening of the health status of individuals (Carannante et al. 2022, a), b)). The *acceleration* of mortality represents the underlying insight whereby deaths are ‘accelerated’ ahead of schedule (or brought forward in time) due to Covid-19 (Cairns et al. 2020).

Another main assumption underlying our study relies on the effects of the Covid-19 vaccination measures. These medical treatments produce temporary effects, not providing a structural immunity against the virus, so that we consider a normal virus spread by neglecting the vaccination effects.

Since the structure of a population is composite, characterised by different segments with different levels of comorbidities, we propose a stratified sampling stochastic model, in order to take into account the different frailty levels in projecting the future mortality rates. In our approach, we use 2 different versions of the concept of the frailty.

As discussed earlier, the classical actuarial literature has defined a frailty measure that includes all unobservable factors affecting individual mortality (Pitacco et al. 2009). According to Carannante et al. 2022 a), the concept of implied frailty represents an adaptation of the standard actuarial concept of frailty which takes into account all indistinct unobservable risk factors determining the mortality deviations between the expected baseline mortality and the aggregate observed mortality including Covid-19 mortality. In this context, we provide a stratified approach to frailty, with the mortality heterogeneity being affected by the comorbidities instead of indistinct unobservable risk factors. Our main empirical findings show that the implied frailty concept tends to overestimate the actual mortality for all classes of age, except for young ages, in the case of the countries that we consider, i.e. England and Wales, Northern Ireland and Scotland. This is probably due to the all unobservable risk factors embedded in this measure. The stratified frailty concept that we develop for modelling mortality trends accurately captures the frailty heterogeneity in the population, allowing us to obtain more consistent forecasts.

The layout of the paper is as follows. Section 2 explains the rationale of the research. Section 3 introduces the concept of frailty in the actuarial literature. We illustrate the implied frailty approach in section 4. Section 5 introduces the stratified weighting frailty for the two different schemes. The approach to modelling and forecasting mortality is described in section 6. The main empirical outcomes are shown in section 7. Section 8 concludes.

## 2. Motivation

In general terms, “a stochastic mortality model should allow for the several types of possible deviations in the frequency of death in respect of the forecasted mortality rate” (Pitacco et al. 2009). In particular, the systematic deviations from expected values may come from “a misspecification of the relevant mortality model, namely model risk (e.g. because the time-pattern of actual mortality differs from that implied by the adopted mortality table) or a biased assessment of the relevant parameters (e.g. due to a lack of data)”, *i.e.* the parameter risk (Pitacco et al. 2009). The uncertainty risk refers to model and parameter risk jointly, meaning uncertainty in the description of the evolution of future mortality.

The deviations due to the shocks caused by period effects (*i.e.* catastrophe risk), involve the risk of a sudden and short-term rise in the frequency of deaths. Mortality rates in a population may experience sudden jumps, due to adverse living conditions, such as an influenza epidemic, or other severe environmental conditions, such as natural disasters.

Embedding the sources of randomness in the mortality model is a critical task, in order to accurately represent the mortality phenomenon. In our study, we address the uncertainty risk, in its systematic long-run component by providing a stratification scheme based on the introduction of different levels of population frailty (*i.e.* the frailty heterogeneity). Moreover, we address the catastrophe risk, that has recently appeared through the short term excess of deaths caused by the Covid-19 pandemic, by providing a stratification scheme based on frailty related to Covid-19.

Broadly speaking, different levels of comorbidities that vary by age give rise to frailty heterogeneity. Some studies (Shepard et al., 1975, 1977; Tolley et al., 1978; Manton et al., 1979) have analysed the differences in individual susceptibility to specific causes of death and their effects on mortality heterogeneity in the trends (Meyricke et al. 2013, Xu et al. 2019). Thus, they implicitly explore the impact of frailty heterogeneity on mortality heterogeneity. Vaupel (1979) show that by ignoring the heterogeneity in frailty according to the standard life table methods leads to an underestimation of the future progress in reducing mortality rates, with an impact on differentials in mortality. For a general approach to heterogeneity models in the actuarial field, we refer readers to the monograph of Cummins et al (1983).

In order to manage the heterogeneity by capturing any prior information on the frailty of a population, we develop a stratified weighting which allows us to obtain higher efficiency by reducing the forecasting errors for mortality projections. The forecasts of future mortality evolution would then be based on homogeneous groups such as those that are identified by a specified level of frailty, with the assumption that frailty within a group is homogeneous. One of the main advantage due to the stratification comes from the sample size of the strata which is under the analyst's control.

We propose two different stratified weighting schemes for the partition of the population into distinct subpopulations, so that each subpopulation is more homogeneous than in the original population. One scheme is based on the age structure of the population characterized by different levels of the comorbidities that correspond to general frailty of the individuals in the sample (general frailty scheme). The other scheme is based on the Covid-19 infection rate by age, so that frailty is specifically related to the virus which leads to a deterioration in the comorbidity conditions (specific frailty scheme due to the Covid-19 impairment of a frail population). In order to embed the systematic and accidental sources of randomness in the mortality projections, we define two multiplicative models, the former being the General Frailty Stochastic Model (GFSM) based on the general frailty scheme and the latter the Specific Frailty Stochastic Model (SFSM) based on the specific frailty scheme due to the Covid-19.

### 3. Frailty

The literature has considered observable and unobservable factors explaining the heterogeneity of a population in respect of its mortality experience. The concept of “frailty” denotes an unobservable factor. In the actuarial literature, the concept of ‘frailty’ represents a tool to describe heterogeneity in populations, due to unobservable risk factors, based on a non-negative real valued variable, i.e the frailty, whose role is to include all unobservable factors affecting an individual’s mortality rate (Pitacco et al. 2009). According to the observable risk factors, the population is instead assumed to be homogeneous. The idea that individuals with higher frailty die on average earlier than others dates back to Beard (1959, 1971) and it has been extensively explored by Vaupel *et al.*(1979).

The literature assumes the invariance of the specific value of the frailty of an individual in respect of time. We denote  $Z_x$  to be the continuous random frailty at age  $x$ , with a continuous probability density function,  $g_x(z)$ . Let  $\mu_x(z)$  denote the conditional force of mortality for an individual in a population group at age  $x$ , and with a frailty level of  $z$ :

$$\mu_x(z) = \lim_{t \rightarrow 0} \frac{P(T_x \leq t | Z_x = z)}{t} \quad (1)$$

where  $T_x$  being the remaining lifetime and  $Z_x = z$  for simplicity  $z$ .

According to Vaupel et al. (1979), the force of mortality depends on a multiplicative frailty factor which refers to the force of mortality at frailty level  $z = 1$  for of an individual at age  $x$ , as in the following formula:

$$\mu_x(z) = z\mu_x \quad (2)$$

Let us consider a person at age 0. The survival function up to age  $x$  for a person with frailty  $z$  is

$$S(x|z) = e^{-\int_0^x \mu_t(z)dt} = e^{-zH(x)} \quad (3)$$

with  $H(x)$  the cumulative standard force of mortality in the interval  $(0, x)$ .

#### 4. Implied Frailty

According to the traditional strand of literature on frailty (Beard 1959, 1971; Vaupel *et al.* 1979; Pitacco *et al.* 2009), referring to a generic individual in a given (heterogeneous) cohort, we would assume that his/her frailty remains constant throughout the whole life span.

A more complex model would recognize that frailty probably changes as time passes and depends on a large number of factors, such as the comorbidities of an individual at age  $x$ .

In the following discussion in sections 4 and 5, we introduce other definitions of frailty.

According to Carannante *et al.* (2022b), implied frailty is a tool to measure the sensitivity of a population to an exogenous shift of mortality. The idea of implied frailty (in the context of Covid-19) is that the pandemic did not affect the entire population without distinction, but the mortality shocks depend on the presence of previous conditions that the infection has aggravated. We begin with formula 1 of Cairns *et al.* (2020):

$$A(x, t) = q(x, t) \times \pi^S(x, t) \quad (4)$$

where  $A(x, t)$  are the accelerated deaths due to Covid-19 infection at age  $x$  and time  $t$ ,  $q(x, t)$  represents the deaths for age  $x$  at time  $t$  for all-cause mortality and  $\pi^S(x, t)$  the stochastic acceleration factor. Following Cairns *et al.* (2020), we regard  $\pi^S(x, t)$  as the product of two elements: the first is  $\lambda(x, t)$ , which represents the infection rate, and the second is the Implied Relative Frailty, or  $IRf(x, t)$ , which represents the effects of comorbidity at Covid-19 diagnosis for age  $x$  at time  $t$ .

We know that  $IRf(x, t) \geq 0$  the greater the  $IRf$  value, the greater the impact of Covid-19 on mortality and, consequently, the greater the accelerated deaths. However, it is an unknown term that includes a number of accidental factors affecting changes in mortality. In this sense, it is possible to give a frailty score starting from the known terms of the formula (4). Letting  $t = 2020$ , the implied frailty at age  $x$  is estimated as follows:

$$IRf(x, 2020) = \frac{A(x, 2020)}{q(x, 2020) \times \lambda(x, 2020)} \quad (5)$$

In this case, as can be seen from the formulas (4-5), the frailty is closely linked to deaths caused by the Covid-19 infection. Implied frailty is an immediate approach for detecting the effects of an accidental component on mortality, even if it only includes unspecified factors that affect Covid-19 mortality.

## 5. General and specific frailties based on stratified weighting schemes

In this section, we propose other frailty measures based on a stratified weighting scheme, in order to capture the a priori information on the frailty heterogeneity of a population, allowing to obtain higher efficiency by reducing the sampling errors for mortality projections. The forecasts of future mortality evolution are based on homogeneous groups, such as the strata that represent in some measure frailty cohorts, it being assumed that the frailty intra-group is homogeneous.

Starting from a generic population frailty index, in particular an indicator of disability prevalence, we design a weighted indicator by using two different approaches. Disability prevalence is an important indicator related to the concept of healthy life expectancy, which is often estimated by the Sullivan method (Jagger et al. 2006). The basic idea is that the health status affects the mortality behaviour in a certain population, and it is necessary to define an indicator that reflects the current state of health of a real population, adjusted for mortality levels and independent of age structure. The method consists of computing (indirectly) the number of person years lived in the healthy state from that age at the particular time, without using data from individual longitudinal studies. Following this approach, we use disability prevalence as an indicator that allows measuring a time-invariant health-based frailty status by age. The disability prevalence is the unweighted measure of frailty used to build the two stratified frailty measures defined below.

The first approach concerns the definition of a general frailty, the second a specific frailty due to Covid-19 infection. The underlying idea is the same as that of post-stratification, which is widely used in survey analysis, to avoid some bias that could be affecting the data (Holt and Smith 1979; Little 1993). In a similar way, we consider an estimator of the frailty of a country's population on the basis of some variables that could correct the bias from the use of the original indicator. In this sense, we define stratified frailty  $Sf_i(x)$  as a function of a measure of frailty  $f_i(x)$  and two different weighting schemes  $w_{i,k}(x)$ , for including in the general frailty or the specified frailty approaches. A general formula for stratified frailty for an age  $x$  is defined by (6)

$$Sf_i(x) = \sum_{k=1}^K w_{i,k}(x)f(x, k) \quad (6)$$

where:

$w_{i,k}(x)$  are the weights assigned according to one of the two weighting schemes;

$f(x, k)$  is a health-based measure of frailty, that is the disability prevalence;

$i$  is an index defining the general or the specified frailty measurement approach.

The general frailty approach is defined by a weighting scheme that depends on the age structure of the population. Let  $x$  be the age and  $k$  the UK region of residence of a certain individual. The weight based on the general frailty  $w_{\text{GFSM},k}(x)$  is defined as follows:

$$w_{\text{GFSM},k}(x) = \frac{N_k(x)}{N(x)} \quad (7)$$

where:  $N_k(x)$  is the number of individuals at age  $x$  in region  $k$  and

$N(x)$  is the number of total individuals at age  $x$ .

The Covid-19 specific frailty approach is defined by a weighting scheme that depends on the infection numbers by age in the population. The weight based on the Covid-19 specific frailty  $w_{\text{SFMSM},k}(x)$  is defined as follows:

$$w_{\text{SFMSM},k}(x) = \frac{I_k(x)}{I(x)} \quad (8)$$

where:  $I_k(x)$  is the number of Covid-19 infected at age  $x$  in region  $k$  and

$I(x)$  is the number of total Covid-19 infected at age  $x$ .

The comparison between implied frailty and stratified frailty, allows us to consider the phenomenon from 2 different points of view. Thus, we can compare the frailty at Covid-19 diagnosis (implied), with the frailty of the overall population ignoring the pandemic effects (stratified with  $w_{\text{GFSM},k}(x)$ ) and the frailty of all those positive with Covid-19 for all infection conditions, without symptoms, with non severe symptoms or hospitalized (stratified with scheme  $w_{\text{SFMSM},k}(x)$ ).

## 6. Mortality projections allowing for stratified weighting of frailty

Equations (6), (7) and (8) allow us to define mortality projection models including the frailty-effect in a multiplicative way. Let consider  $m_{x,t}$  denote the force of mortality for age  $x$  and time  $t$ . We estimate a log-bilinear model assuming a Poisson distribution of deaths with a log-link function (in a generalized linear model setting). In particular, considering the maximum likelihood estimates (MLE) of the central mortality rates. The general formula of a standard Lee-Carter model (Lee and Carter 1992) defines the force of mortality as follows:

$$m_{x,t} = \exp(\alpha_x + \beta_x \kappa_t + \varepsilon_{x,t}) \quad (9)$$

To estimate parameters, we follow the approach based on heteroscedastic Poisson error structures, that is the Poisson log-bilinear version of the LC model (Renshaw and Haberman 2003). In this approach, the number of deaths  $d_{x,t}$  follows a Poisson distribution:

$$d_{x,t} \approx \text{Poisson}(\varepsilon_{x,t}, m_{x,t}) \text{ with } m_{x,t} = \exp(\alpha_x + \beta_x \kappa_t) \quad (10)$$

$m_{x,t}$  has a log-bilinear form:

$$\log(m_{x,t}) = \alpha_x + \beta_x \kappa_t \quad (11)$$

where the parameters  $\alpha_x$ ,  $\beta_x$  and  $\kappa_t$  have the same meaning as in the standard Lee-Carter model, but estimated using the MLE method assuming a Poisson random distribution of the random part of the model.

Starting from  $\dot{m}_{x,t+s}$  projections, we define two multiplicative models, on the basis of the multiplicative effects of the frailty on the force of mortality, as defined by equation (2). The first is the General Frailty Stochastic Model (GFSM) obtained by the product of the projected aggregate central mortality rate and the general frailty indicator (obtained from (6) and (7)), obtaining projections  $\dot{m}_{x,t+s}^{GFSM}$ :

$$\dot{m}_{x,t+s}^{GFSM} = Sf_{GFSM}(x) \cdot \dot{m}_{x,t+s} \quad (12)$$

The second model is the Specific Frailty Stochastic Model (SFSM) obtained by the product of the projected central mortality rate and the specific frailty indicator (obtained from (6) and (8)), obtaining projections  $\dot{m}_{x,t+s}^{SFSM}$ :

$$\dot{m}_{x,t+s}^{\text{SFMSM}} = Sf_{\text{SFMSM}}(x) \cdot \dot{m}_{x,t+s} \quad (13)$$

## 7. Numerical Applications: UK Mortality

In this section, we apply the above methodology to data for the countries of the UK. For Covid-19 and all-causes of deaths by country and age, we consider the following sources of weekly data: Office for National Statistics for England and Wales (ONS 2020), Northern Ireland Statistics and Research Agency for Northern Ireland (NISRA 2020) and National Records of Scotland for Scotland (NRS 2020). The daily infection rates by country and age are collected by Public Health of England (PHE 2020), Public Health of Wales (PHW 2020), Public Health of Scotland (PHS 2020) and Department of Health of Northern Ireland (DOH 2020). For disability prevalence rates and the population by age, the source is the Office for National Statistics for all of the countries (ONS 2017).

The various sources provide very inhomogeneous data, so that a phase of data pre-processing is required to be able to calculate the indexes. In particular, for the implied frailty calculation, the steps listed below have been followed:

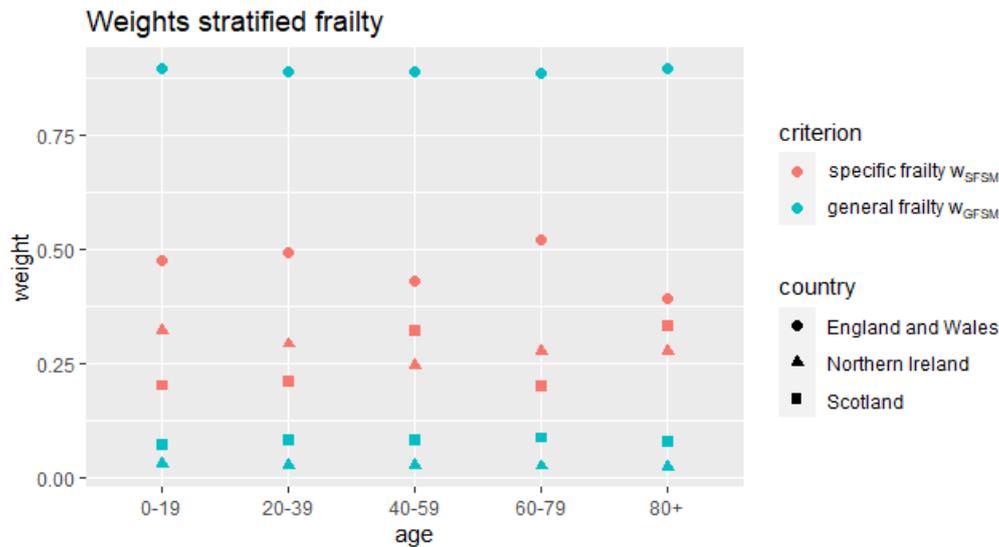
1. Harmonization of age groups for England and Wales;
2. Aggregation of mortality data from weekly to annual levels of granularity;
3. Combining rows of the country datasets;
4. Computing of annual infection rate as the mean of log-difference of daily infections;
5. Combining of mortality and infections datasets using country and age as a key variable.

And for the stratified sampling method, the steps listed below are followed:

1. Harmonization of age rates for England and Wales;
2. Aggregation of infection data from daily to annual;
3. Weights construction, as defined by formulas (4-5);
4. Computing of the stratified frailty index;
5. Combining of implied frailty and stratified frailty index datasets using age as a key variable.

Figure 1 shows the weights according to the different two schemes by country and age:

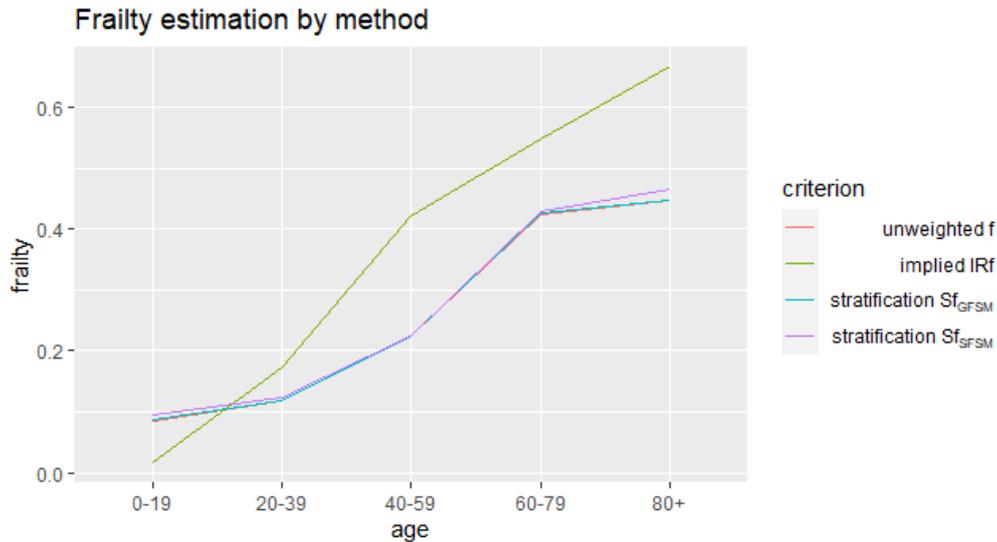
*Figure 1. Weights of stratified frailty by country and age*



As shown by Figure 1, the weighting schemes differ in a marked way. In particular, we can observe that for the age structure of the population, England and Wales represent the most important area (about the 0.90) of the UK population for all the age classes. In contrast, the weights of the infection rate are much more balanced and show an infection rate that also changes depending on the area and age. In particular, for England and Wales, the rate is higher than 0.5 of the total for the 60-79 age group, while for Northern Ireland the infection rate is always higher than that for Scotland, with the exception of the age group, 40-59. It can also be observed that for the 80+ age group the infection rates tend to be more similar for all countries, while there is more variability for the other age groups. If we consider that the different rates of contagion strongly depend on the mobility of people and on compliance with the rules for social mixing and interpersonal distancing, differences by age are expected. It is also important to take into account that the number of infections and, consequently, the number of persons who are Covid positive per country is influenced by the number of tests performed.

Figure 2 shows the different estimates of frailty: the frailty without stratification (pink), the frailty stratified by age structure of population (purple), the frailty stratified by infections (blue) and the implied frailty (green):

Figure 2. Frailty estimation by age



As shown in Figure 2, implied frailty and stratified frailty differ both in functional form and in terms of age. While implied frailty has much higher values except for the 0-19 class, the various indicators based on disability prevalence have rather similar values. Furthermore, the functional form remains the same and is similar to a logistic curve, while the implied frailty appears as a function with downward concavity with respect to age. While the differences with the method used to assess frailty seem marked, there are not huge differences with the weighting scheme used, even if the population age structure scheme assumes slightly higher values for the older population. The basic idea that justifies the considerable differences between the two methods of estimation lies in the concept of frailty that they intend to measure. Implied frailty measures the excess of mortality by Covid-19 without any consideration about the possible underlying phenomena causing the excess deaths. In this sense, implied frailty respects the definition of equation (4), where frailty is a series of accidental unknown causes that create discrepancies with mortality projections. In contrast, stratified frailty focuses our attention on the effects of Covid-19 on the deterioration in disability status, that could adversely affect mortality experience.

Comparing the different approaches of stratified frailty measures, we can observe that the differences with the disability prevalence index are in the extreme ages only, with the general frailty index showing a lower difference with respect to the specific Covid-19 index. In this sense, a health-based co-morbidity index allows us to correct the excess of mortality for younger and older ages, both considering general condition and a specific event as a pandemic.

## 7.1 Covid-19 mortality rate projections

Mortality projections rates by age  $x$  and time  $t$  are obtained on the basis of stochastic modelling of general mortality of the population, using the Renshaw-Haberman model (Renshaw and Haberman, 2003). The mortality data for the UK countries populations are downloaded from the Human Mortality Database ranging from 1950 to 2018, aggregated by gender for all the ages from 0 to 100. Figures 3-5 shows the projections of the accelerated mortality by age and country, considering the three estimation methods of frailty:

Figure 3. Projection of Covid-19 mortality rates by age and country. Implied frailty method

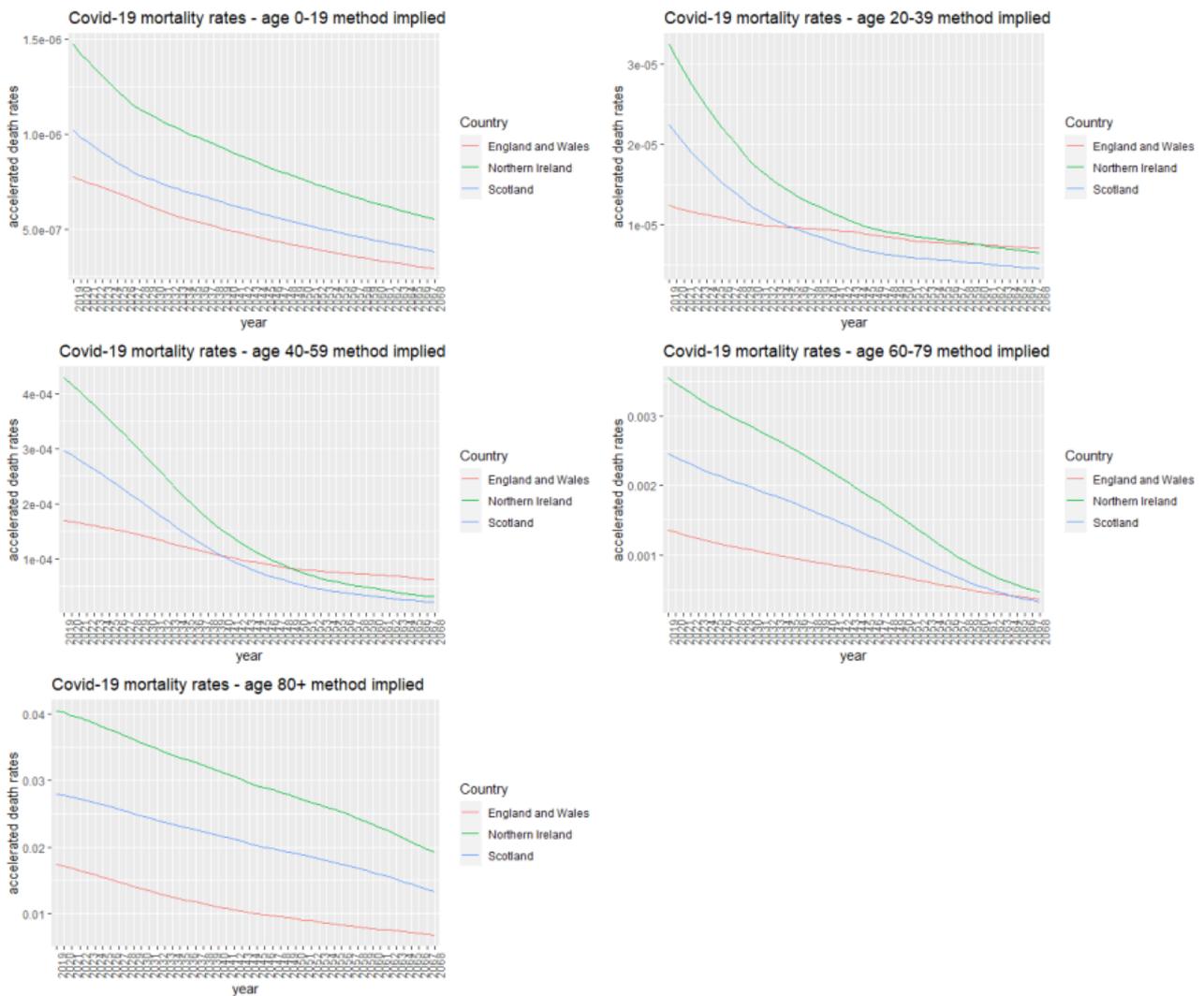


Figure 4. Projection of Covid-19 mortality rates by age and country. Stratified infection frailty method

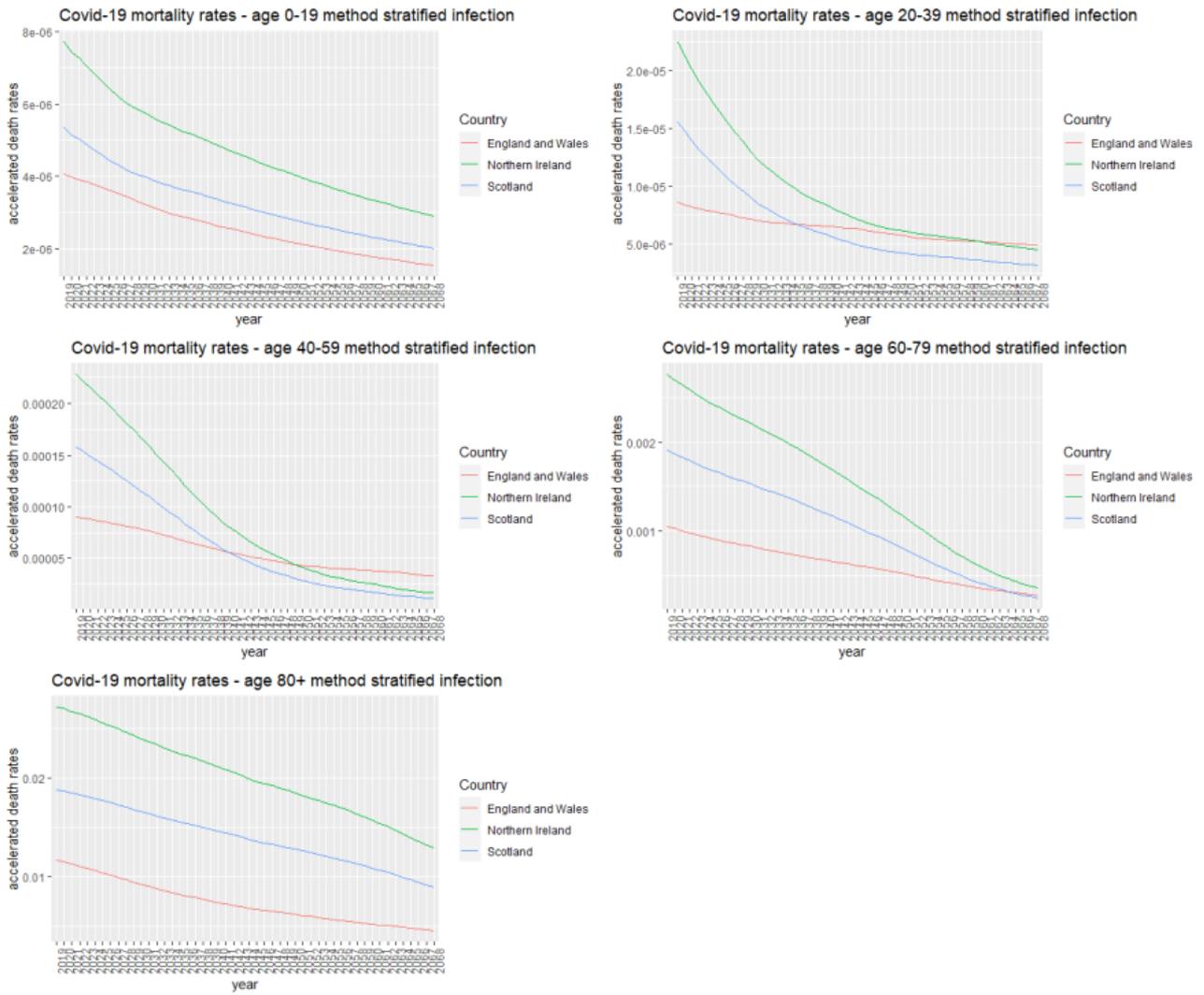
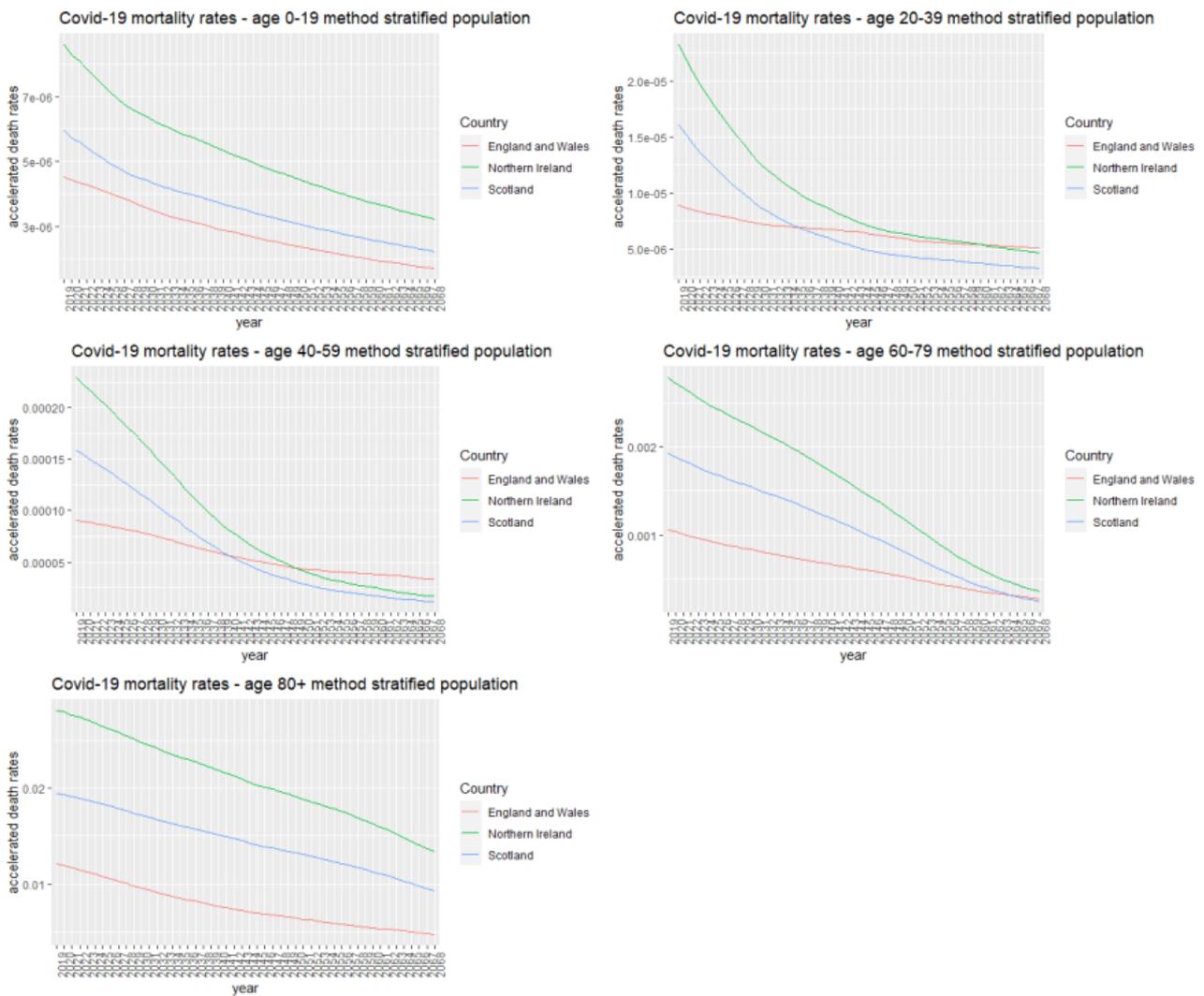


Figure 5. Projection of Covid-19 mortality rates by age and country. Stratified population frailty method



Overall, Figures 3-5 show that the Covid-19 projected mortality rates are the lowest for England and Wales for all age groups, while it is the highest for Northern Ireland in the short term. However, for the 20-39 and 40-59 age groups there is a trend reversal whereby mortality rates seem to be higher in England and Wales in the long term, reaching the values of Northern Ireland in the first case and exceeding them in the second one. We note that the mortality rates obtained with implied frailty (Figure 3) are higher than those obtained with the 2 approaches based on stratified frailty (Figures 4 and 5), which are similar to each other.

Figures 6-8 show mortality rate projections by frailty estimation method, for each country and age group: Figure 6 considers England and Wales, Figure 7 considers Northern Ireland and Figure 8 considers Scotland.

Figure 6. Projection of Covid-19 mortality rates by frailty estimation method and age group. England and Wales

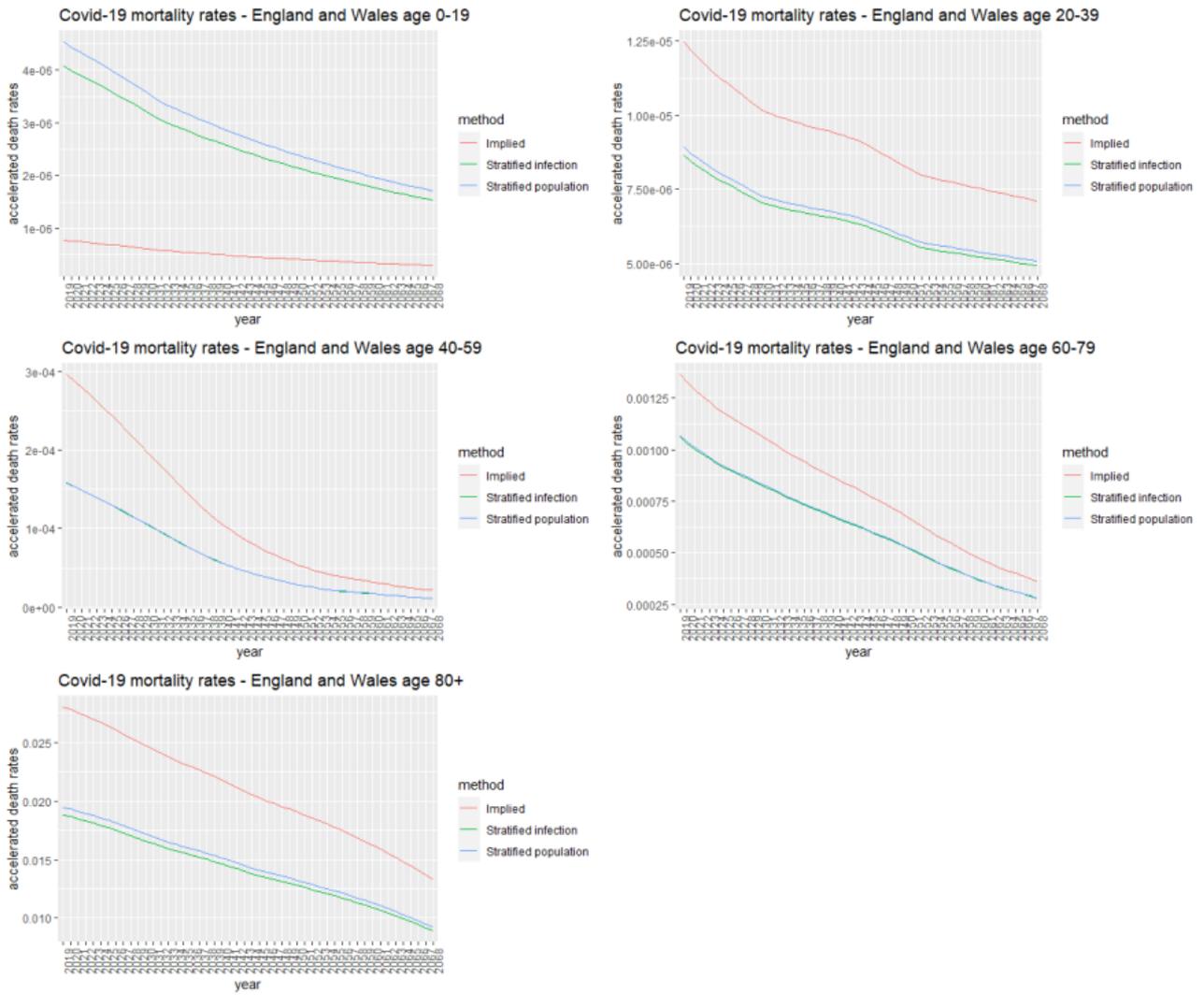


Figure 7. Projection of mortality rates by frailty estimation method and age group. Northern Ireland

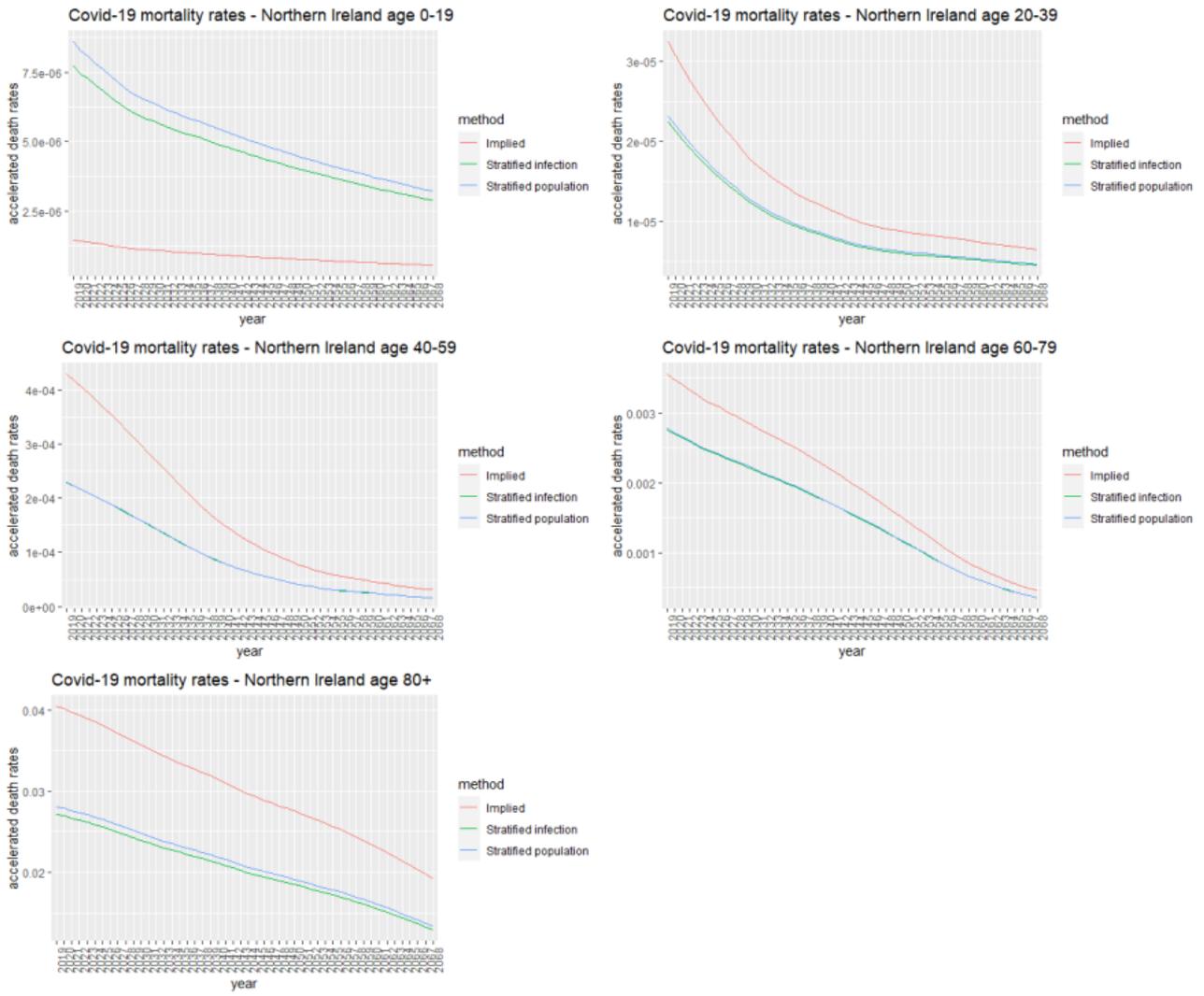
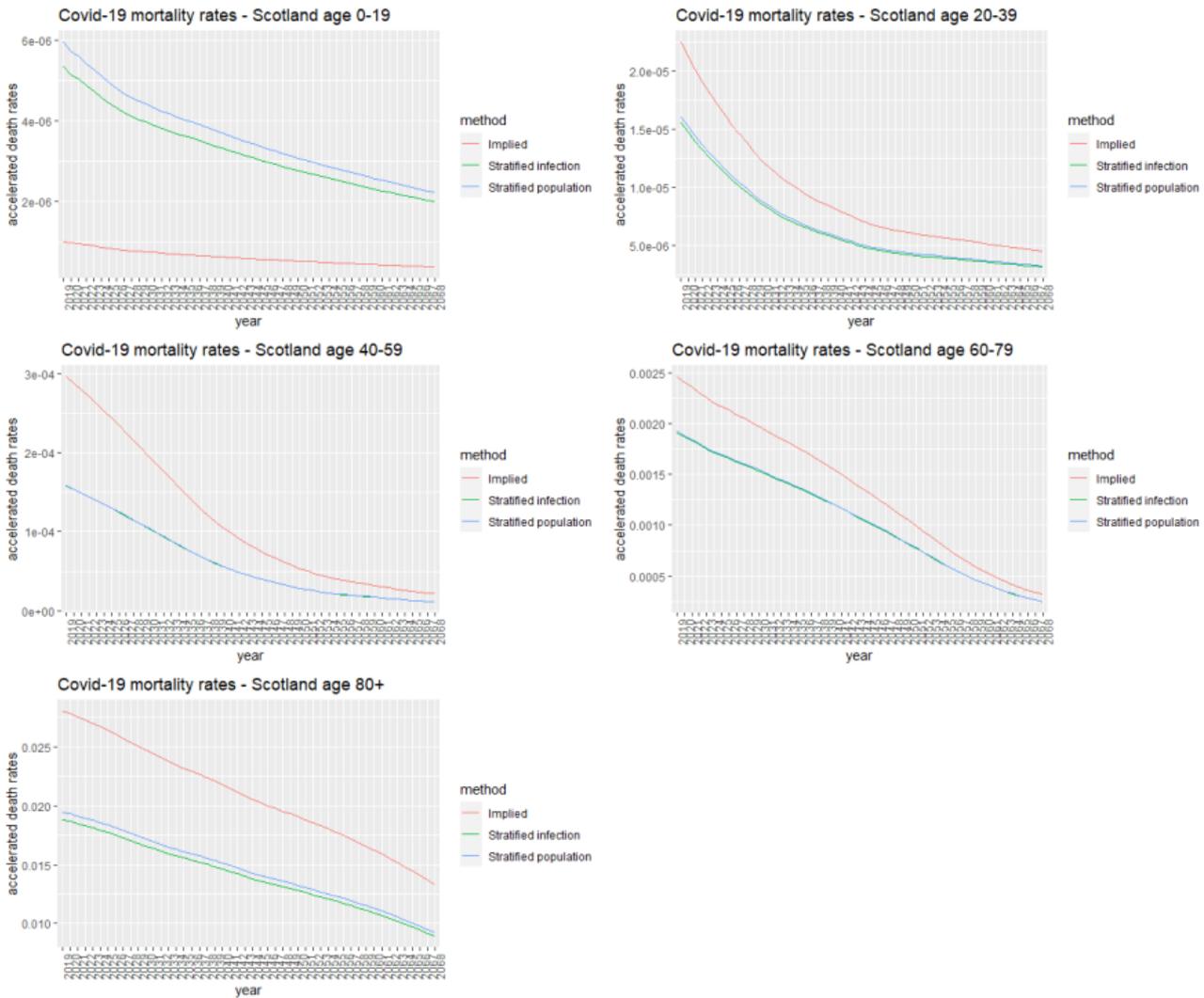


Figure 8. Projection of mortality rates by frailty estimation method and age group. Scotland



As observed from Figures 6-8, the projected mortality rates for each country change with age in ways that depend on the frailty estimation method used. In particular, for age group 0-19, the implied frailty method estimates a much lower mortality rate than the stratified frailty methods. In addition, the decreasing trend over time of the mortality rate (for age group 0-19) obtained with implied frailty is much less marked than the others. For the adult age groups, on the other hand, the opposite trend is observed, that is the implied frailty method tends to lead to a higher estimate of the mortality rate compared to the stratified frailty methods, although with very similar trends over time. Regarding the two stratification methods, they are quite similar for the central age groups, while for ages up to 19, the estimate is higher if stratification per population is used and for over 80 higher if stratification per infection is used. For England and Wales, there are also higher projected Covid-19 mortality rates using the stratification per population method for the 20-39 age group. These results reflect the different age profiles of the frailty estimates (shown in Figure 2) and the different patterns of weights used in the two stratification schemes.

## 6. Concluding Remarks

The paper focus on the heterogeneity in frailty of a demographic population that determines differentials in mortality. In particular, the literature shows that neglecting this feature leads to a bias in projecting the longevity phenomenon. Accordingly, to avoid a misrepresentation of the longevity, we develop a stochastic model based on a stratification weighting mechanism, which allows taking into account the different levels of the population frailty.

Basically, in our paper we introduce the concepts of the general and specific frailty. The former corresponds to the general frailty of the individuals in the sample, the latter being caused by the Covid-19 infection rate by age, so that frailty is specifically related to the virus which leads to a deterioration in the comorbidity conditions. The idea underlying the research is that the general frailty cause structural, long-run deviations in the baseline of the mortality, where the specific frailty determines period effects shocks, i.e. the risk of a sudden and short-term rise in the frequency of deaths.

In our study, we address these systematic long-run and short-run components of mortality by providing the stratification schemes based on the introduction of different levels of population frailty (i.e. the frailty heterogeneity).

In terms of limitations, this study takes into account only the snapshot of frailty observed cross-sectionally in a period, and a possible improvement in research would involve allowing for dynamic changes over time in frailty in order to obtain more stable projections in the long run.

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