A rejoinder to "Thirty years on: A review of the Lee-Carter method for forecasting mortality"

It is an honour to have the opportunity to respond to this excellent review paper which celebrates the 30th anniversary of the publication of the Lee-Carter model for fitting and forecasting mortality rates.

The review paper provides a critical examination of the original publication and the huge literature that has followed it. In so doing, the authors identify the most substantial extensions and improvements to the Lee-Carter model that have appeared in the literature and have been successfully applied.

The authors begin the review by first describing the Lee-Carter model (as set out in the 1992 paper) and then testing the accuracy of the 30-year forecasts (based on the original model) over the period 1990 to 2019. In section 4, they then review the technical details of the model and the key issues arising. They then cover the developments and modifications that have been subsequently proposed to address the identified limitations. Section 5 looks at the performance of the extensions to the LC model in terms of their ability to forecast accurately future mortality rates. Section 6 steps back to consider the alternative underlying statistical assumptions that have been made in these various model extensions, and then looks at potential future avenues for research.

Structuring the review in this manner is very helpful to the reader. It allows us to identify the key elements and assumptions in the Lee-Carter methodology and the ways in which these have been analysed and then improved. It provides a coherent structure to the paper, as well to the review of the background literature. This will be very useful to students and researchers in the future who are interested in the subject.

In reading this review, I have come across a few areas where I would like to have seen more emphasis or elaboration. Perhaps, this is inevitable when the original paper has attracted over 3700 citations and the derived literature is so large.

One of the key outcomes of the Lee-Carter model, with its use of mortality forecasting based on stochastic models, has been the ability to forecast the central trend in mortality rates but also facilitate the calculation of (model-based) prediction intervals. This has been revolutionary for the insurance and pensions industries, which are faced with systematic increases in life expectancy. This has led to consideration of "longevity risk" – the risk that those receiving regular benefits through their remaining lifetimes via annuities or pensions will live longer than anticipated which would thereby threaten the financial sustainability of the provider (insurance company or pension scheme). Various forms of longevity risk management and risk transfer have been developed in the theoretical literature and then applied practically using a range of stochastic mortality models, the origin of which can be traced back to the Lee-Carter model.

As the authors point out in section 4, the Lee-Carter model can be seen within a broader structure. It is noteworthy that Hunt and Blake (2021) pursue this point by enhancing the Lee-Carter model through the inclusion of a sum of *N* terms of the form $b_x k_t$ and hence creating a generalized family of non-linear stochastic mortality models. In this framework, Hunt and Blake allow the b_x functions to be either parametric or non-parametric functions of age.

In section 4.5 (and section 6), the authors discuss the choice (and implicitly the length) of the fitting period. A determining factor that could be also mentioned is the proposed length of the forecasting period. Time series models are at the heart of the Lee-Carter model and, in many forecasting applications of time series models, a key consideration is the relative lengths of the fitting and forecasting periods. There is a commonly quoted "rule of thumb" that the fitting period should be roughly twice the length of the forecasting period. This creates a tension when the application requires forecasting over long periods – many pensions applications would involve 30 year forecasts, for example – and when the data in the fitting period exhibit structural changes. There is no easy answer to this problem, although some researchers have explored the use of Markov regime-switching mortality models to allow for historic structural breaks in the mortality dynamics (Milidonis et al, 2011).

A development in the last decade in stochastic mortality modelling has been a switch to considering the modelling of mortality improvement rates, rather than mortality rates. One of the motivations has been a practical one. Many standard mortality tables used by actuaries (and required by regulators) for annuity pricing or reserving became based on an assumption about the dynamics of suitably defined mortality improvement rates. Thus, Denuit and Trufin (2016) specifically mention then current actuarial practice in Austria, Belgium, Denmark, Switzerland, UK and US that uses mortality improvement rates as a building block.

A second motivation for the interest of the academic literature has been the recognition that there may be theoretical and practical advantages in modelling mortality improvement rates. A key issue in modelling mortality dynamics is understanding the dominant downwards time trend that has manifested itself over at least the last 70 years. It is well known in time series work that there are advantages if the underlying stochastic process that generates the time trend is time-invariant. One of the common methods in time series analysis of transforming a so-called non-stationary time series into a stationary one is by de-trending the series i.e., taking first differences: this has been implemented Bohk-Ewald and Rau (2017), Haberman and Renshaw (2012, 2013), Hunt and Villegas (2023), Li and Liu (2020), Ludkovski et al (2018) and Mitchell et al (2013) inter alia. This transformation implies that the mortality trend relates to the previous year's mortality rates rather than the trend in a hidden mortality factor, like k_t in the Lee and Carter model.

In the early studies of modelling mortality improvement rates, the researchers encountered difficulties with robust estimation and a high level of parameter uncertainty – these arise from the improvement rate being defined in terms of the time differential (or finite time difference) in the underlying mortality rates. In their comprehensive and rigorous investigation, Hunt and Villegas (2023) argue that it is sounder to model mortality rates and then use the model to derive the implied mortality improvement rates. This has been pursued recently in an application to modelling trends in US mortality rates by cause of death: see Villegas et al (2023).

The discussion in section 4.9 on coherence is comprehensive. The novel approach developed by Shang (2016) for modelling subnational mortality rates and ensuring coherence with the mortality model for the aggregate population looks to be promising. It enables the reconciliation of subnational mortality forecasts so that they aggregate adequately across levels of a group hierarchy and allows for the combination of forecasts to achieve improved forecast accuracy.

It is noteworthy that the title of the Lee-Carter paper refers both to "modelling and forecasting". The authors of the review paper correctly emphasise (particularly in section 6) that goodness of fit and forecasting accuracy are two key dimensions for measuring the performance for potential models. Also, they stress that point and interval forecast accuracy and bias should be investigated.

The authors conclude with identifying some important signposts to future research directions, regarding the Lee-Carter model and mortality forecasting, that would be worthy of exploration:

- addressing the significant limitation of a fixed b_x;
- investigating the potential of modelling mortality data using a cohort perspective (as in, for example, Haberman and Renshaw, 2013);
- looking at the impact of the choice of the benchmark population in coherent forecasting for multiple populations;
- a more rigorous approach to comparing model goodness of fit and forecast performance, so that a consensus around significant model extensions can be reached;
- developing models where exposure counts are endogenous in order to improve model estimation methods;
- use of averaging techniques for the choice of jump-off values and of model averaging for mortality forecasts.

Another avenue that I would add to the list is the potential for further applications of machine-learning techniques, as mentioned in section 4.3.

Overall, this is a masterly and welcome review of developments over the last thirty years in the field of mortality forecasting that has arisen from the seminal contribution of Lee & Carter.

Additional References

Bohk-Ewald, C., Rau, R. (2017). Probabilistic mortality forecasting with varying age-specific survival improvements. *Genus* 73:1.

Denuit, M., Trufin, J. 2016. From regulatory life table to stochastic mortality projections: The exponential decline model. *Insurance: Mathematics & Economics* 71, 295-303.

Haberman, S., Renshaw, A.E., 2012. Parametric mortality rate modelling and projecting. *Insurance: Mathematics & Economics* 50, 309-333.

Haberman, S., Renshaw, A.E., 2013. Modelling and projecting mortality improvement rates using a cohort perspective. *Insurance: Mathematics & Economics* 53, 150-168

Hunt, A., Blake, D., 2021. On the structure and classification of mortality models. *North American Actuarial Journal*, 25(S1), S215-S234.

Hunt, A., Villegas, A.M., 2023. Mortality improvement rates: modelling, parameter uncertainty and robustness. *North American Actuarial Journal*, to appear.

Li, J.S.H., Liu, Y., 2020. The heat wave model for forecasting 2-dimensional mortality improvement rates with measures of uncertainty. *Insurance: Mathematics & Economics* 93, 1-26.

Ludkovski, M., Risk, J., Zail H., 2018. Gaussian process models for mortality rates and improvement factors. *ASTIN Bulletin* 48, 1307-1347.

Milidonis, A., Liu, Y., Cox, S.H., 2011. Mortality regimes and pricing. *North American Actuarial Journal* 15 (2) 266-289.

Mitchell, D., Brockett, P.L., Mendoza-Arriaga, R., Muthuraman, K., 2013. Modelling and forecasting mortality rates. Insurance: Mathematics & Economics 52, 275-285.

Villegas A.M., Bajekal, M., Haberman, S., Zhou, L., 2023. Key drivers of long-term rates of mortality improvements in the U.S.: period, cohort and cause of death analysis, 1959-2016. *North American Actuarial Journal*, to appear.