THE APPLICATION OF OPTICAL FEEDBACK IN LASER DIODES TO SENSOR SYSTEMS

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Submitted for the degree of Doctor of Philosophy

to

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London.

September 1997.

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LIST OF ILLUSTRATIONS

ACKNOWLEDGEMENTS

I would like to thank the following people for their help and support during the work leading to this thesis: Ken Grattan, Andy Palmer, Roger Valsler, Kenny Weir, and all the other staff and students in the measurement and instrumentation groups at City University.

I am grateful for financial support to The Engineering and Physical Sciences Research Council for a studentship, and to The Worshipful Company of Scientific Instrument Makers, The Institute of Physics and The Instrument Science and Technology Group of the Institute of Physics for bursaries enabling me to attend international conferences.

DECLARATION

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ABSTRACT

This thesis is concerned with the application of external optical feedback in a laser diode to interferometric optical sensing using the effects of 'self-mixing interference' and examines some of the practical issues for the development of such sensors.

A theory is described which accounts for the behaviour of a laser diode with weak external optical feedback and shows how a phase-dependent interferometric signal can be obtained from such a system.

Experimental techniques are described which have been developed to overcome some of the inherent difficulties in carrying out practical work with optical feedback in laser diodes and enable accurate and systematic interpretation of experimental results. These include the estimation and accurate measurement of feedback strength, the accurate alignment of components and the measurement of wavelength changes due to feedback.

The principles of operation of a simple self-mixing interference-based wide-range displacement sensor have been demonstrated. The choice of components for such a system has been considered in detail and interferometric fringes have been produced for target distances of over one metre.

The problem for interferometric sensing systems of the wavelength instability with respect to temperature variations of laser diodes has been addressed with particular reference to the effects of optical feedback. A method of using weak optical feedback to reduce the wavelength tuning coefficient of a single longitudinal mode has been proposed and demonstrated experimentally.

Experimental results are presented in which the frequency of the intensity modulation due to feedback is twice the value expected for the length of the external cavity. This is shown to be due to misalignment of the external reflector causing light to make a double pass of the external cavity. It is shown that with the external reflector misaligned the system is mechanically more stable and the resulting modulation frequency doubling offers a potential doubling in the resolution of a self-mixing interference based sensing system.

KEY TO ABBREVIATIONS AND SYMBOLS

E	Instantaneous electric field magnitude
E_0	Amplitude of electric field oscillation
ω	Optical angular frequency
t	Time
k	Optical wave constant
z	Direction of propagation of the electric field
x	Direction perpendicular to the direction of propagation
λ	Optical wavelength
I	Optical intensity
V	Interference pattern visibility
ν'	Normalised optical frequency
ϕ	Phase
d	Distance
r	Amplitude reflection coefficient
R	Intensity reflection coefficient
T	Intensity transmission coefficient
$\mathcal{F}$	Fabry Perot finesse
g	Laser gain coefficient
λ	Laser loss coefficient
g_{th}	Threshold gain of a laser
L	Laser cavity length
c	Speed of light in a vacuum
n	Refractive index
ν	Optical frequency
τ	Laser cavity round-trip time
Φ	Round trip phase change due to feedback

ξ	Feedback coupling coefficient
α	Linewidth enhancement factor
C	Feedback strength coefficient
N	Laser diode charge carrier density
J	Laser diode injection current
e	Charge of an electron
v_g	Optical group velocity
τ_c	Charge carrier lifetime
S	Laser diode cavity photon density
κ	Variation of gain with charge carrier density
m	Intensity modulation coefficient
Δv_{xcm}	Longitudinal mode spacing
Δv_{sxm}	Range of single longitudinal mode operation
Δv_{hys}	Hysteresis frequency interval
LD	Laser diode
PD	Photodiode
M	Mirror
ND	Neutral density filter
L	Lens
BS	Beamsplitter
w	Gaussian beam waist radius
θ	Gaussian beam divergence angle
W	Width of a diffracting slit
f	Focal length of a lens
f_c	Electrical cutoff frequency
Θ	Thermal conductivity
P	Thermal power
T	Temperature
V	Voltage

D Optical density of a filter

Chapter 1: Introduction and Background

1.1 Introduction

The development of optical sensors, using semiconductor optical devices and fibre-optics, to replace electro-mechanical devices is an area of considerable ongoing interest. Optical devices have a number of potential advantages including accuracy, reliability, low mass and low cost. A detailed discussion of these familiar aspects is given in a number of texts, e.g. Grattan & Meggitt *(1995)*.

This work is a study of the application of external optical feedback in a laser diode to interferometric sensing techniques. In recent years a number of systems based on the interferometric effects of weak optical feedback in laser diodes have been proposed for sensing quantities such as displacement, range and velocity.

The effects of optical feedback on laser diodes have been extensively studied partly as a problem to be overcome and partly as phenomena with potentially useful applications in areas such as telecommunications and sensing. Effects on the output optical frequency of the laser diode and the linewidth can adversely affect performance in, for example, fibre optic communications systems, where reflections from fibre ends can be very effectively guided back into the laser cavity. These effects can, however, for certain levels of feedback give rise to an modulation in the output power of the laser diode which is similar to the output of a conventional interferometer; this type of effect has been called 'self-mixing interference'*(Shinohara et al., 1986)*.

The research reported here is concerned with the use of self-mixing interference in interferometric sensing; the aims of this thesis and its structure are set out in the following section. In the remainder of this chapter, a historical background is presented, giving an overview of the important aspects of past work on optical feedback in laser diodes as it relates to optical sensing, following which some background physics necessary for the study of optical feedback in laser diodes is discussed.

1.2 Aims and structure of this thesis

The main aims of the work described in this thesis are:

(i) to describe a theory of optical feedback in laser diodes which explains how a laser diode with optical feedback can be used to produce a phase-dependent output which can be used as the basis for an interferometric device and which assists in determining the physical parameters of such a device. This theory is presented in Chapter 2.

(ii) to develop the techniques necessary for accurate and systematic experimental work with optical feedback in laser diodes. These include control of the alignment of components, accurate estimation and measurement of the strength of feedback, and measurement of frequency variations due to feedback. This work is described in Chapter 3.

(iii) to demonstrate the use of optical feedback for optical sensing, to identify the optimum components and to quantify the limits to the performance of such a system. The results of experiments undertaken to demonstrate and configure such a system are presented in Chapter 4.

(iv) to investigate the effects of the sensitivity to temperature of the wavelength of light emitted by a laser diode to the performance of a self-mixing interference based sensing system and to examine the possible use of optical feedback to reduce that temperature sensitivity. Chapter 5 gives an overview of the spectral behaviour of laser diodes, presents a theoretical proposal for the use of feedback to improve frequency stability and describes experimental results obtained to support this theory.

(v) to explain experimental observations which are not consistent with the theory initially outlined and to modify the theory accordingly. These observations are presented and their implications discussed in Chapter 6.

A summary of the results of this work is given in Chapter 7 along with suggestions for future work in this area.

1.3 Historical Background

The first detailed experimental and theoretical study of the effects of external optical feedback in a laser diode was by Lang and Kobayashi in 1980. This described the observed effects of feedback on the light-current characteristics of a laser diode coupled to an external reflector at a distance of approximately 10mm from the laser; undulations in the light current curve and the variation of the amplitude of the undulations with the external cavity length as well as hysteresis in the light-current curve were reported. A qualitative explanation is given for the variation of the light-current undulation amplitude with external cavity length based on the interaction between the longitudinal modes of the laser diode cavity and modes associated with the external cavity formed between the front facet of the laser diode and the external reflector. An analytical theory relating variations in the laser diode cavity refractive index, the cavity temperature, the laser gain and the optical frequency of the light emitted by the laser is presented based on steady-state solutions of a rate equation for the time-dependent electric field containing a time delayed field term representing the feedback. This theory makes an approximation by linearising the variation of cavity refractive index about the equilibrium value which is valid for small variations. The theory successfully predicts the form of the variation of optical frequency and output intensity of a laser diode with feedback and predicts multistability and hysteresis in the frequency variation although the condition for the occurrence of hysteresis is given in terms of an undefined feedback coupling coefficient.

Following this a large amount of work has been carried out describing various effects of feedback in laser diodes and developing a theoretical explanation for these observations. Most work has concentrated on the spectral effects of feedback, notably the effect on linewidth which can be either increased *(Miles et al., 1980)* or decreased *(Kikuchi & Okoshi, 1982; Patzak et al., 1983; Agrawal, 1984)* depending on conditions such as the intensity and phase of the feedback *(Petermann, 1995)*. Other work has described the effects of feedback on the noise content of the laser output (both phase and intensity noise) *(Saito et al., 1982; Biesterbos et al., 1983; Tamburrini et al., 1983)*, and the appearance of longitudinal modes associated with the external cavity *(Osmundsen & Gade, 1983)*.

An important application of these effects has been in controlling and stabilising the spectral characteristics of laser diodes. Most of the work in this area has involved the use of strong, frequency selective feedback *(Agrawal, 1986)* obtained by placing a filtering element such as an atomic absorption cell *(Valenzuela et al., 1988; Choi et al., 1993)* or a Fabry-Perot cavity *(Favre & le Guen, 1980)* in the external cavity or using a diffraction grating as the external reflector *(Sato et al., 1982)*. These techniques work by reducing the cavity loss at the filter wavelength so that the laser will tend to operate at that wavelength.

The effects of feedback strength are described by Tkach and Chraplyvy *(1986)* who define five regimes of feedback strength in each of which a laser diode exhibits a distinct type of behaviour. This work has demonstrated the importance of the distinction between weak and strong feedback which has allowed weak feedback to be treated separately and for simplified approximate theories based on solving the gain and phase conditions for a laser diode with the complex amplitude reflection coefficient of the front facet of the laser cavity modified by the presence of the field reflected from the external reflector to be developed *(de Groot et al.,* *1988; Petermann, 1991).* These theories account for the phase dependent variations in laser diode frequency and threshold gain caused by weak feedback and successfully predict the asymmetric form of the intensity variations and the occurrence of hysteresis in the path taken by the intensity as the feedback phase is varied. Petermann *(1991)* describes explicitly the effect of a parameter called the linewidth enhancement coefficient which relates the laser gain to the cavity refractive index and affects the relative phase of the gain and frequency variations; it is also used to define a dimensionless feedback strength coefficient, C, which is used to categorise the behaviour of a laser diode in terms of feedback strength. The interferometric effects used for sensing applications are observed in the weak feedback regime and so the following chapters will concentrate on the theory and application of weak feedback.

The possibility of obtaining a phase-dependent intensity modulation from a laser diode with weak optical feedback has lead to interest in applying feedback effects to interferometric optical sensing devices. Such devices are attractive because of their simplicity in terms of the number of components compared to conventional interferometric devices or devices using for example helium-neon lasers, which also implies advantages in terms of bulk and cost.

As an illustration of this, displacement sensors have been demonstrated *(Wang et al., 1994), Donati et al., 1995)* which consist basically of just a laser diode, collimating optics and a target reflector and which produce a phase-dependent periodic intensity modulation as the target moves with respect to the laser diode. This type of configuration offers a reduction in complexity, bulk, and cost compared to a conventional system. The output of such systems is therefore similar to that of a conventional interferometer; however in the laser diode feedback systems asymmetric fringes can be obtained which allow the direction of the movement of the target to be determined in contrast to conventional interferometric sensors; for relatively low levels of feedback the fringes have a saw-tooth form and for stronger feedback mode-hopping between longitudinal modes of the external cavity occurs with the direction of the mode-hop depending on the direction of movement of the target. The system demonstrated by Donati et al. *(1995)* shows a maximum dynamic range of 1.2m with a target velocity limited to approximately 20mms^{-1} by the frequency response of the electronics used to process the signal obtained from the laser; the effects of noise in the laser diode output are not discussed but would be expected to limit the maximum target velocity further. It is also acknowledged that the accuracy is limited by the sensitivity of the laser diode wavelength to environmental temperature variations; for single fringe accuracy the maximum target distance is only about 0.5m. It was shown that fringes could be obtained with light fed-back from a scattering surface although the maximum dynamic range for such a system is only about 0.2m.

Range finding devices have been demonstrated *(Beheim & Fritsch, 1986; de Groot et al., 1988; Shinohara et al., 1992)* in which the phase modulation is obtained by modulating the laser diode drive current which in a laser diode leads to a modulation of the optical frequency. This technique is analogous to conventional pseudo-heterodyne interferometric methods, or frequency-modulated continuous wave interferometry *(Kikuta et al., 1986; Chebbour et al., 1994)*. The accuracy of these devices depends on the ability to interpret frequency changes from the obtained intensity modulation pattern and on the number of periods of the feedback-induced intensity variation contained within each period of the applied current modulation, and is typically of the order of millimetres.

1.4 Principles of interferometry

1.4.1 Introduction

In order to be able to describe a theory of optical feedback in laser diodes as it relates to interferometric effects, it is necessary to understand the basic physics of optical interference and the action of conventional interferometers. These topics are discussed briefly in the following sections; detailed coverage of theses issues can be found in many texts, including Born and Wolf *(1980)* and Hecht *(1987)*.

1.4.2 Interference between electromagnetic waves

The propagation of a plane, monochromatic electromagnetic wave can be described by the following expression for the magnitude of the electric field vector:

$$E = E_0 \exp[i(\omega t - kz)] \tag{1.1}$$

where E_0 is the amplitude of the electric field wave, ω is the angular frequency, t is time, k is the wave number, k=2π/λ, and z is the direction of propagation. The intensity observed at a given time and position is given by

$$\begin{aligned} I &= |E|^2 \\ &= EE^* . \end{aligned} \tag{1.2}$$

where E* is the complex conjugate.

In an interferometer, the wave is split into two components which take differing paths and are then recombined. The resulting field at the interferometer output is then given by the linear superposition of the fields in each component of the recombined beam:

$$E = E_1 \exp[-i(\omega t + kz_0)] + E_2 \exp[-i(\omega t + k(z_0 + \delta z))] \tag{1.3}$$

where E_1 and E_2 are the amplitudes of the two components, z_0 is the path length taken by the reference wave and δz is the path length taken by the other component. The intensity at the detector is given by:

$$
\begin{aligned}
I &= EE^* \\
&= E_1^2 + E_2^2 + 2E_1E_2\cos(k\delta z).
\end{aligned}
\tag{1.4}
$$

and in the case where the original field is split into equal parts to that $E_1=E_2$, then

$$I = I_0\left(1+\cos(k\delta z)\right). \tag{1.5}$$

This gives a sinusoidal intensity variation with respect to variations in the optical path difference between the two beams. The amplitude of this variation when the original beam is split into components of equal amplitude is double the intensity of the original beam. If the original beam is split unevenly then the maximum in the intensity distribution is lowered and the minimum is raised and the visibility of the pattern, defined as

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \tag{1.6}$$

is reduced. The visibility function is in effect the magnitude of the envelope of the fringe pattern obtained from the interferometer, and its width is a measure of the temporal coherence of the source.

For a non-monochromatic source the cosine term in equation (1.4) is replaced by an integral over the range of frequency, which, when expressed as $\nu' = \lambda^{-1}$, gives

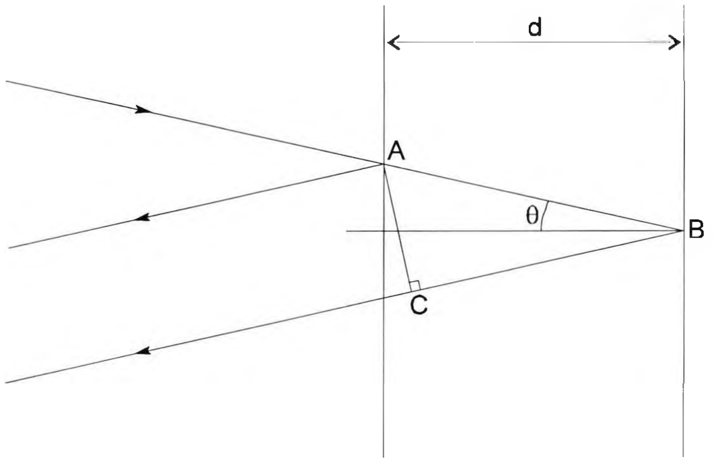
$$I = \int_0^\infty I(\nu')d\nu' + \int_0^\infty I(\nu')\cos(2\pi\nu'\delta z)d\nu' \tag{1.7}$$

where the first term is equal to the intensity of the input beam and the second term is equal to the cosine Fourier transform of the spectral distribution of the source. Applying the inverse transform therefore allows the form of the source spectrum to be inferred from the output of the interferometer. There is, therefore, an inverse relationship between the width of the visibility pattern with respect to the interferometric path length difference and the width of the spectrum of the light source.

Interferograms obtained from lasers with multiple longitudinal modes can be interpreted using the convolution theorem, which states that the Fourier transform of the product of two functions is equal to the convolution of the Fourier transforms of the individual functions. The spectrum of a multimode laser can be represented by the convolving the longitudinal mode line shape function with a comb function of period equal to the longitudinal mode spacing multiplied by the gain spectrum function which determines the relative intensity of the modes. The interferogram obtained from such a source is then equal to the convolution of the transform of the gain spectrum and a comb function with a period inversely proportional to the longitudinal mode spacing multiplied by the transform of the individual mode line shape function. The mean wavelength and the longitudinal mode spacing can therefore easily be determined from such an interferogram.

1.4.3 The Michelson interferometer

One of the simplest and most commonly used types of interferometer is the Michelson interferometer which is described here in order to illustrate the general features of interferometry. In its classical configuration this consists of an extended light source, usually a ground glass plate placed in front of a lamp, from which light is collected by a collimating lens and passes to a beamsplitter. This divides the light into two arms each of which ends in a mirror placed perpendicularly to the incident beam. The reflected beams are recombined at the beamsplitter to form the output beam which can be viewed directly with the naked eye or imaged with a suitable lens. An interference pattern is formed as a result of any difference in optical path length in the two arms of the interferometer. The optical path length difference for a given mirror setting is a function of the angle and is the length



$$AB = \frac{d}{\cos\theta}$$

$$BC = AB\cos 2\theta = \frac{d\cos 2\theta}{\cos\theta} = 2d\cos\theta - \frac{d}{\cos\theta}$$

$$ABC = AB + BC = 2d\cos\theta$$

Fig. 1.1. Optical geometry of the Michelson interferometer.

ABC in Fig. 1.1, which shows the two arms of the interferometer superimposed, and is equal to $2d\cos\theta$. The phase difference between the two components arriving at the image plane is then:

$$\delta\phi = \frac{4\pi}{\lambda} d\cos\theta \qquad (1.8)$$

As the phase difference is a function only of the angle with the optical axis the resulting pattern has circular symmetry and a pattern of rings is observed which expand or contract as the separation of the mirrors is changed.

If a laser source is used and the light passing through the interferometer is well collimated, then a real pattern of fringes is formed, without the use of an imaging lens, by the overlapping wavefronts of each component of the output beam. If both mirrors are perpendicular to the optical axis then the intensity in the output beam varies according to the phase difference in the two arms of the interferometer. Fringes distributed in the plane perpendicular to the optical axis can be observed when either mirror is tilted at an angle to the optic axis or when there is a difference in the curvature of the wavefronts in each component of the output beam. This type of configuration of the Michelson interferometer, with collimated beams, is known as the Twyman-Green interferometer*(Hecht, 1987)*.

If a time variation of the source frequency is introduced then the time delay between the recombining beams in the interferometer causes a difference in frequency between the two beams and hence a difference in phase which can be detected in the output of the interferometer. The phase difference is dependent on the difference in optical path length between the two beams and the instantaneous difference in frequency so if the source frequency is continuously and linearly varied, then an ac signal is obtained from the interferometer, the frequency of which depends on the interferometer path difference. This technique is known as pseudo-heterodyne interferometry or frequency-modulated continuous wave interferometry and can be used to measure absolute distances of stationary targets; laser diodes are particularly suitable as sources for pseudo-heterodyne interferometry because the emitted frequency can be modulated simply by modulating the drive current.

Techniques analogous to those discussed in this section using laser diodes with optical feedback will be discussed in later chapters.

1.4.4 The Fabry-Perot interferometer

There are numerous other configurations of interferometer which all rely on the same basic principle; one type that is of interest here because of its relevance to the action of lasers is the Fabry-Perot interferometer. This consists of two partially reflective plates forming a cavity around which a fraction of the light is repeatedly reflected, introducing a path difference with the fraction that is transmitted from the output plate. The path difference between successive rays at the output as shown in Fig. 1.2 is the same as that calculated for the Michelson interferometer, ie. $\delta z=2d\cos\theta$. The difference in the output between the Fabry-Perot and Michelson interferometers is due to the fact that in the Fabry-Perot interferometer there are many more than two beams interfering at the output.

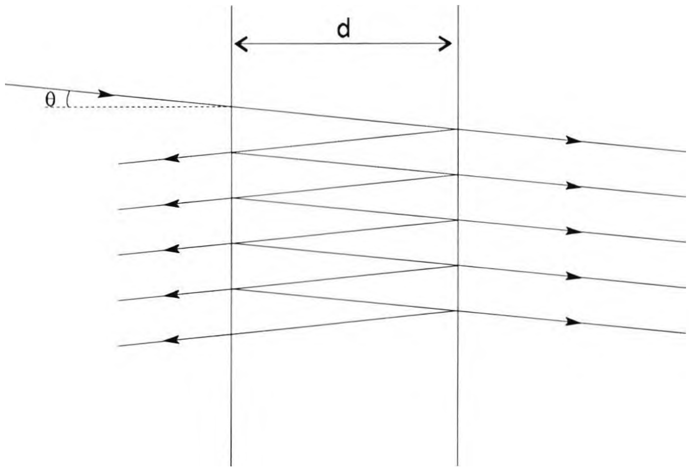


Fig. 1.2. Geometry of the Fabry-Perot interferometer.

If the plates have equal amplitude reflectivity, r, and there is no loss due to absorption so that the amplitude transmission is given by $t^2=1-r^2$, then the magnitude and phase of the nth beam emerging from the cavity is given by:

$$E_n = E_0 t^2 r^{2n} \exp\{i[\phi_0 + (n-1)k\delta z]\} \tag{1.9}$$

where E_0 is the magnitude of the incident field and ϕ_0 is the phase of the first output beam at the plane of detection. The total field at the detector is given by the sum

$$\begin{aligned} E &= E_0 t^2 \sum_{a=1}^{n} r^{2(a-1)} \exp\{i[\phi_0 + (a-1)k\delta z]\} \\ &= E_0 t^2 e^{i\phi_0} \sum_{a=0}^{n-1} \left(r^2 e^{ik\delta z}\right)^a \end{aligned} \tag{1.10}$$

where the result of the second summation is well known and gives

$$E = E_0 t^2 e^{i\phi_0} \left\{ \frac{1 - r^{2n} e^{nik\delta z}}{1 - r^2 e^{ik\delta z}} \right\}. \tag{1.11}$$

Assuming a large number of beams interfering then $r^{2n} \approx 0$ and the observed intensity is then given by

$$I = I_0 \frac{T^2}{1 + R^2 - 2R\cos(k\delta z)} \qquad (1.12)$$

where T and R are the intensity transmission and reflection coefficients. This function gives a periodic series of narrow peaks as a function of path difference rather than the sinusoidal function obtained for the two-beam Michelson interferometer. Equation (1.12) can be rewritten as:

$$\frac{I}{I_0} = \frac{1}{1 + F\sin^2 \frac{k\delta z}{2}} \qquad (1.13)$$

where

$$F = \frac{4R}{(1-R)^2}. \qquad (1.14)$$

The quantity F is a measure the narrowness of the fringes observed; the half-width at half-maximum of the intensity distribution in terms of the phase, $k\delta z$, is approximately $2/\sqrt{F}$ for large F. The finesse of a Fabry-Perot interferometer, $\mathcal{F}$, is defined as the ratio of the separation of the fringes to their half-width; the fringe separation in terms of phase is 2π so the finesse is:

$$\mathcal{F} = \frac{\pi\sqrt{F}}{2}. \qquad (1.15)$$

High finesse can be achieved with highly reflecting cavity mirrors which makes the Fabry-Perot interferometer suitable for high-resolution spectroscopic applications.

The free spectral range of a Fabry-Perot interferometer is the difference between two consecutive frequencies, ν_a and ν_b, which give fringes at the same angular position. The phase differences for the two frequencies are related by $k_a \delta z = k_b \delta z + 2\pi$ from which the frequency difference is $\Delta\nu_{fsr}$=c/2dcosθ. The free spectral range of a Fabry-Perot interferometer is equivalent to the longitudinal mode spacing in a laser.

As with the Michelson interferometer, with an extended source and a lens used to produce and image at a detector, the pattern obtained has radial symmetry and a series of circular fringes is observed. If the input beam is collimated then the intensity in the plane of the detector varies with the separation of the mirrors. In the case a laser diode chip the transmitted intensity is fixed but the spectral properties of the laser output are determined by the properties of the Fabry-Perot cavity formed by the reflective end-faces of the laser chip. These properties are discussed in further detail in the next section.

1.5 The Physics of Laser Diodes

1.5.1 Introduction

The physics of the normal operation of lasers in general and of semiconductor lasers is well known *(Svelto, 1989; Thompson, 1980)* and it is not necessary to describe it here in great detail. The following section gives a brief, mainly qualitative overview of the properties of semiconductors and the p-n junction diode and the general principles of lasers, and of the most important aspects of these relating to semiconductor lasers and their use with optical feedback.

1.5.2 Semiconductors, p-n junctions and diodes

Conductors, insulators and semiconductors can be defined in terms of the band theory, according to which, as a result of the periodic nature of a crystal lattice, the electrons of the outermost shell of the atoms in a crystal may occupy certain fixed bands of energy states in between which there are gaps covering energy values which are not allowed to the electrons. In a conductor there is a partially filled band and so there are higher energy states available for electrons to move into when an electric field is applied and the electrons are accelerated to produce a current. In an insulator there is a completely filled valence band with a large band gap between it and the next band, the conduction band; when a field is applied there are no available energy states in the valence band and the electrons do not have enough energy to be excited into the conduction band and so a current is unable to flow. In a semiconductor there is a filled valence band but the band gap between it and the conduction band is small enough that some electrons can be raised into the conduction band by thermal excitation and can then occupy the available higher energy states in that band when a field is applied so that a current can flow. In a conductor, the conductivity decreases with increasing temperature due to increased scattering of the conduction electrons by lattice phonons; in a semiconductor an increase in temperature excites more electrons into the conduction band and so increases the conductivity.

The properties of tetravalent intrinsic semiconductors such as silicon and germanium can be modified by the addition of small amounts of trivalent or pentavalent impurities. When a pentavalent atom is placed in a crystal of tetravalent atoms it has one electron in the outer shell which is not required for bonding in the lattice and is easily ionised. This means that the energy level of this fifth electron is effectively raised out of the valence band to a level just below the conduction band, and the small amount of energy required to excite it into the conduction band increases the probability of thermal excitation and hence the conductivity of the material. The addition of this type of 'donor' impurity to an intrinsic semiconductor is called n-type doping.

P-type doping occurs when a trivalent atom is added to the tetravalent crystal and there is one electron less than is required for bonding; this electron can be taken from adjacent atoms in the lattice for an energy similar to the ionisation energy of the pentavalent atom in the n-doped material. Thus there is effectively an electron energy state available at a level just above the top of the valence band into which electrons are can easily be excited leaving vacant sites in the valence band which may be used for electrical conduction. The vacant site in the valence band is called a hole and may be treated as a charge carrier identical to an electron but with opposite charge.

When a p-type crystal and an n-type crystal are placed in contact, the electrons in the conduction band in the n-type material can drift across the junction to occupy the empty states in the conduction band of the p-type material. This drift occurs until it is balanced by the electric field created by the charge transfer. There is also an analogous drift of holes from the p-type material to the n-type. If an external electric field is applied in the direction opposite to that created by the charge drift, ie. a positive potential is applied to the p-type side of the junction, then the electrons drifting into the p-type material from the n-type flow are able to flow away from the junction region, electrons will continue to cross the junction and a current flows through the device. If an external electric field is applied in the other direction then it opposes the flow of electrons from the n-type material and as there are very few electrons in the conduction band of the p-type material the amount of current able to flow is small. This is the basis of the operation of the p-n junction diode.

With electrons flowing from the n- to the p-side of the diode and holes flowing in the opposite direction it is possible, in a direct band-gap material where the electron states on either side of the band-gap are matched for momentum, for conduction electrons on either side of the junction to fall into the vacant states in the valence band, the holes, with a release of energy in the form of light equal to the band gap of the material. This is spontaneous emission and is what occurs in a light emitting diode. In order for a light emitting diode to become a laser diode stimulated emission must occur which requires population inversion; this is discussed in the following section.

1.5.3 Basics of semiconductor lasers

'Laser' is an acronym for 'light amplification by stimulated emission of radiation'. Stimulated emission occurs when an atom or molecule in an exited energy state interacts with a photon and emits another photon, with the same frequency and phase as the stimulating photon. If this process creates more photons than are lost through absorption and emission from the device then the light field is amplified and gain occurs. The loss of light from the active medium is minimised by containing the medium in a cavity with reflective end faces.

Most lasers are based on transitions between atomic electron energy levels (helium-neon, neodymium-YAG) or molecular vibrational energy levels (CO_2, dye lasers). In order to obtain gain, it is necessary that the occupation of the higher energy levels is greater than that of the lower ones (population inversion); this can be achieved by exciting the atoms or molecules of the laser medium by, for example, an electrical discharge in a gas laser or by using another light source. In semiconductor lasers the energy level differences are those between the conduction and valence bands at the junction of a p-n diode. Population inversion is achieved when the occupation of electron states in the conduction band is greater than the occupation of electron states in the valence band which requires a high current density through the active region at the junction. A high current density is achieved by confining the current to a narrow stripe at the p-n junction by the construction of the diode chip and by the use of a so-called double-heterostructure in which a narrow layer of relatively narrow band-gap material is placed between the p- and n-type layers; the narrower band gap causes electron-hole recombination to take place preferentially within in this layer. For practical reasons mainly related to the fabrication of complicated multiple-layer devices most semiconductor optical devices use one of two materials systems both based on compounds of trivalent and pentavalent compounds (III-V semiconductors). Devices made of gallium arsenide (GaAs) and gallium arsenide-aluminium arsenide alloy (GaAlAs) operate at wavelengths of around 800nm with the exact wavelength depending on the composition of the GaAlAs mixture. Longer wavelength devices, important for telecommunications applications, are made with a quaternary material system containing gallium arsenide, gallium phosphide, indium arsenide and indium phosphide. Metals such as tin, zinc and cadmium are used as p-type dopants and non-metallic elements such as sulphur, selenium and tellurium are used for n-type doping.

The laser cavity is formed by the cleaved ends of the diode chip, which, uncoated, have typical reflectivities of about 32%.

Such lasers are cheaply manufactured, extremely compact, are available with a range of operating characteristics, are easily intensity- and frequency-modulated and are thus attractive for many applications including sources for interferometric sensing devices.

1.5.4 Lasing conditions

The gain, g, and loss, l, in the active medium of a laser are defined by

$$I' = I_0 \exp[(g - l)z] \qquad (1.16)$$

where I' is the is the optical intensity after a field with initial intensity, I_0, has propagated a distance, z. The ranges of wavelengths for which gain may be experienced, the gain spectrum, depend on the energy gap between electron states, excited molecular states, or the bandgap in a semiconductor and various broadening mechanisms such as the spectral width associated with decay lifetime, Doppler broadening and pressure broadening in gas lasers. In semiconductor lasers, line broadening arises from the intensity noise associated with the stochastic nature of spontaneous emission. The power spectrum is related to the noise spectrum by the Wiener-Kintchine relationship which states that the power spectrum is equal to the Fourier transform of the auto-correlation of the time-dependent field amplitude noise. These broadening mechanisms may be described as either homogeneous or inhomogeneous; with a homogeneous broadening mechanism an excited atom or molecule may interact with any photon within the wavelength range of the gain spectrum whereas in the case of inhomogeneous line broadening each atom or molecule is able to interact with light at a single frequency within the gain spectrum. Lifetime broadening is a homogeneous broadening process; Doppler broadening is inhomogeneous. The line broadening processes in semiconductor lasers are predominantly homogeneous.

The threshold gain, g_{th}, is the gain for which the optical field is unchanged after making one round-trip of the laser cavity. If the cavity end faces have amplitude reflectivities, r_1 and r_2, this condition can be expressed as:

$$r_1 r_2 \exp\left[\left(-ik + \tfrac{1}{2}\left(g_{th} - l\right)\right)2L\right] = 1 \qquad (1.17)$$

where the L is the laser cavity length and k is the propagation constant, $k=2\pi n/\lambda$, where n is the cavity refractive index. The real part of this condition gives the definition of threshold gain as:

$$g_{th} = l + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right). \qquad (1.18)$$

The imaginary part of equation (1.17) gives the condition:

$$2kL = m\pi\,, \qquad (1.19)$$

where m is an integer. This condition describes the longitudinal modes of the laser; the possible frequencies at which the laser can emit light. The frequency separation of adjacent longitudinal modes is given by:

$$\nu_m - \nu_{m+1} = \frac{c}{2nL}. \qquad (1.20)$$

The actual form of the power spectrum of a laser diode depends on a number of factors. Whether a laser operates in a single longitudinal mode or in a number of modes simultaneously depends on the longitudinal mode spacing, the shape and width of the gain spectrum, the linewidth broadening mechanism, and in a semiconductor laser, the optical wave guiding mechanism (gain-guiding/index-guiding). When a laser is pumped at higher levels than are required to achieve threshold the gain remains pinned at threshold as the emission rate is increased and the laser reaches a stationary state at a higher power output. For a laser in which homogeneous line broadening is dominant, the first longitudinal mode that achieves threshold is able to extract energy from all the atoms or molecules in the gain medium and so no other mode is then able to achieve threshold. Where inhomogeneous line broadening dominates, each longitudinal mode takes its energy from a different set of particles in the gain medium and so multiple longitudinal modes may operate simultaneously.

1.5.5 Optical properties of semiconductor lasers

The shape and width of the spectral distribution of the individual longitudinal modes in a semiconductor laser are determined mainly by the frequency noise characteristics of the device. The line shape is generally considered to be Lorentzian and the linewidth, amongst other factors, varies inversely with optical intensity, cavity length, and the cavity mirror reflectivities. The Fabry-Perot finesse of a laser diode cavity is usually low due to relatively low facet reflectivities; an uncoated GaAlAs diode has facet reflectivities of about 32% which give a finesse of about 2.6 and so the line shape is not directly determined by the properties of the Fabry-Perot cavity.

The intensity profile of the light emitted from a laser in the plane perpendicular to the axis of the cavity depends on the geometry of the cavity and the shape of the reflecting end facets. The profile must take a form for which the field is self-replicating upon diffraction at the end facets and so is unchanged by a round trip of the cavity; the intensity profiles which satisfy these conditions are the transverse modes of the laser. For a circularly symmetrical laser with confocal concave facets the transverse mode fields have the form of Hermite-Gauss polynomials; the lowest order mode has a Gaussian intensity profile and higher order modes have increasing numbers of lobes. The appearance of higher order longitudinal modes can be prevented by restricting the diameter of the cavity or the apertures at the facets.

In a laser diode the form of the transverse modes is determined by the characteristics of the waveguide which forms the cavity rather than the shape and dimensions of the end facets. Light can be contained within a narrow active region either by index-guiding or gain-guiding. An index-guided structure consists of a narrow layer of higher refractive index than the surrounding layers in which light is guided by total internal reflection. A gain-guided device uses a layer of material of positive optical gain surrounded by regions of negative gain (loss) to define the active region. Common heterostructure laser diodes are index-guided with the stepped refractive index profile of a slab waveguide. The transverse field distribution for such a waveguide has a cosine form within the central layer with exponentially decaying tails in the outer layers. The number of transverse modes which may exist depends on the width of the central layer with fewer modes allowed by narrower layers. The zero order mode profile, with a single cosinusoidal peak with exponentially decaying wings is very similar to a Gaussian profile and Gaussian beam optics can be used with reasonable accuracy in this case to predict the behaviour of the emitted beam. The modes of a slab waveguide are also defined according to their polarization; transverse electric (TE) modes have the electric field vector parallel to the plane of the waveguide and transverse magnetic (TM) modes have the orthogonal polarization. In a slab waveguide with a single transverse mode, the TE mode experiences a lower threshold gain than the TM mode and so, with homogeneous line broadening the TM mode will not be excited. The output from laser diodes is therefore usually predominantly polarized parallel to the plane of the junction.

1.6 Summary

This introductory chapter has set out the objectives for this thesis and given a brief review of the most important work that has been published on the effects and applications of optical feedback in laser diodes.

A brief overview has been made of the necessary background physics required to understand the properties of laser diodes and the action of interferometers both of which are important in the study of the effects of optical feedback on laser diodes. The theory of optical feedback in laser diodes is discussed in greater detail in the following chapter.

Chapter 2: Theory of External Optical Feedback in Laser Diodes.

2.1 Introduction

The following chapter presents the theoretical background which is necessary to understand the use of optical feedback in a laser diode for interferometric sensing applications and some of the related issues such as frequency stability which are the subject of investigation in later chapters.

A large amount of work has been published on the theory of the effects of external optical feedback on laser diodes. A general theory based on the adding of a feedback term to the rate equation for the time-dependent electric field in a laser diode was originally described by Lang and Kobayashi and this has extended to describe phenomena such as spectral line broadening *(Miles et al., 1980)* and narrowing *(Kikuchi & Okoshi, 1982; Patzak et al., 1983; Agarawal, 1984)*, the effect of feedback on noise characteristics *(Saito et al., 1982; Biesterbos et al., 1983; Tamburrini et al., 1983)* and the effect of 'coherence collapse' *(Lenstra et al., 1985; Cohen et al., 1990; Hamel et al., 1992)*.

The effects of feedback strength have been classified by Tkach and Chraplyvy *(1986)* by defining five regimes of feedback in each of which a laser diode exhibits a distinct type of behaviour. The first two of these regimes account for what can be described as 'weak' feedback and describe the type of phase-dependent intensity variations which may be of use for interferometric sensing and so are of interest here.

Simplified theories for the weak feedback regime based on solving the standard laser gain and phase conditions for a three-mirror compound cavity have been developed by de Groot et al. *(1988)* and Petermann *(1991)*. This chapter presents an overview of this type of theory showing how it accounts for frequency and intensity variations in the output of the laser diode on which interferometric applications of feedback depend. Sections 2.5 and 2.6 present original work on the effect of feedback on the linewidth of a laser diode and its implications for interferometric applications, and on the implications for spectral stability of the presence of external longitudinal modes. The effects of feedback in the other three of the five regimes defined by Tkach and Chraplyvy *(1986)* are also considered briefly in Section 2.7 in order to demonstrate the importance of controlling the feedback strength in practical applications.

The effects of light of light reentering a laser diode cavity and adding coherently to the field inside the cavity are caused primarily by the resulting change in the phase and amplitude of the light inside the laser cavity. These changes lead to a range of interrelated changes in various other parameters, the relationships between which are shown schematically in the block diagram of Fig. 2.1. The change in the round trip phase forces a change in the output frequency and the

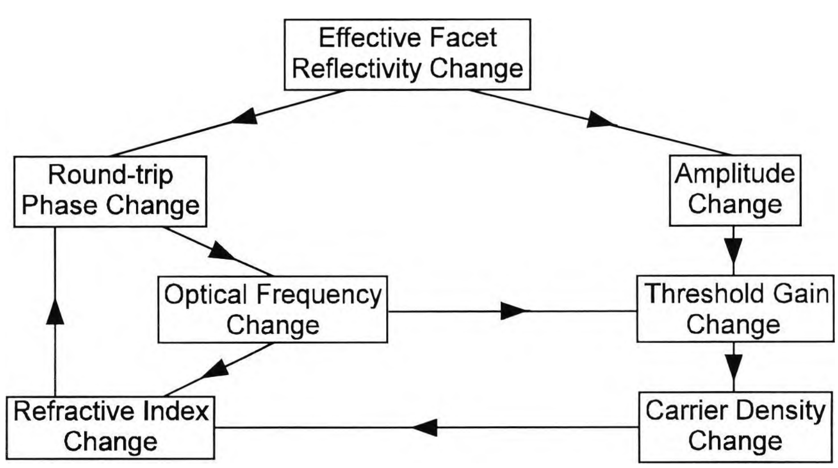


Fig. 2.1 Block diagram showing the relationships between the physical parameters affected by external optical feedback.

change in the round trip amplitude causes a corresponding change in the threshold gain which in turn is frequency dependent. The associated variation in the gain is accompanied by a change in the diode charge carrier density which alters the cavity refractive index which is also frequency dependent and which affects the round trip phase. In the following sections these relationships are described quantitatively and expressions are developed showing the relationships between external cavity phase and the output frequency and power of the laser diode in the presence of feedback.

The theory described here includes approximations which apply to and define a regime of weak feedback within which the observed effects of the feedback on the laser diode are dependent on the round-trip phase for light travelling between the laser and the external reflector. For higher levels of feedback, where the approximations are not valid, the change in threshold gain dominates the phase change and the feedback effects become independent of the feedback phase. The weak feedback regimes are primarily of interest here because the resulting phase-dependence of the laser diode power output enables a laser diode with weak feedback to be used as the basis of an interferometric sensing device.

2.2 Lasing conditions with weak feedback

A simple model of a laser diode with feedback from an external reflector may be obtained by considering the laser diode output facet and the external reflector as a single compound mirror by coherently adding the optical field reflected from the laser facet and the field reentering the laser cavity after reflection from the external reflector. Fig 2.2 shows schematically a laser diode with cavity length, L_1, facet amplitude reflection coefficients, r_0 and r_1 at the rear and output facets respectively, and cavity refractive index, n_1, with an external cavity of length L_2, with refractive index, n_2, formed by an external reflector with amplitude reflectivity, r_2; the complex electric fields at various stages in the round trip of the

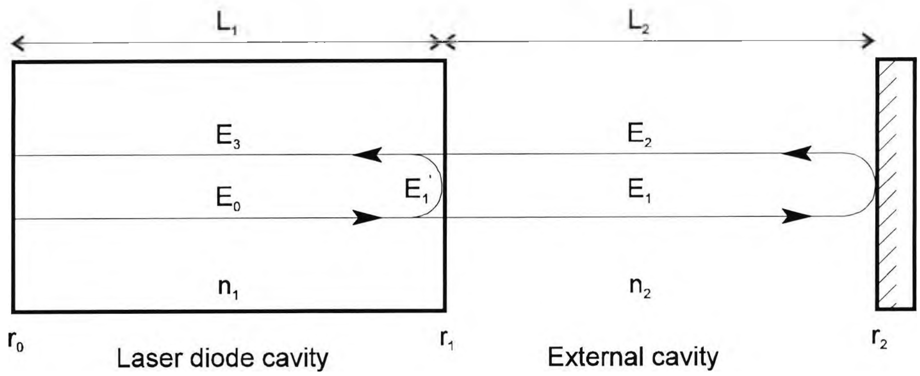


Fig 2.2. Schematic representation of a laser diode with an external reflector showing amplitude reflection coefficients, r_i, effective refractive indices, n_i, electric field amplitudes, E_i, and cavity lengths, L_i.

compound cavity are denoted by E_i. Intensity reflection coefficients will be denoted by R_i.

The amplitude transmission coefficient for the laser front facet is $t = \sqrt{1-R_1}$ and the relative phase of the light having made the round-trip of the external cavity is $\phi_2=2\pi\nu\tau_2$, where ν is the optical frequency, $\tau_2=n_2L_2/c$ is the round trip time for light in the external cavity and c is the speed of light in a vacuum. The field reentering the laser from the external cavity is thus:

$$E_2' = (1 - R_1)r_2E_0\exp(i2\pi\nu\tau_2) \tag{2.1}$$

where E_0 is the initial field magnitude in the laser diode cavity.

Adding this to the field reflected directly from the laser cavity front facet, E_0r_1, and including terms for reflection from the laser diode rear facet, r_0, for the phase change during the round trip of the laser diode cavity, $\phi_1=2\pi\nu\tau_1$, and for the gain and loss in the laser cavity gives the field after one round trip of the compound cavity as:

$$E_3 = E_0 \exp\{(g-l)L_1\}[(1-R_1)r_2 r_0 \exp\{i2\pi\nu\tau_2\} + r_1 r_0]$$

$$= |E_3| \exp\{i\Phi\}$$

(2.2)

where g and l are the cavity gain and loss respectively, and Φ is the round trip phase change for the compound cavity.

This result can be applied to the same threshold gain and round-trip phase conditions that were given for the solitary laser in section 1.4.3. The threshold gain condition is that the intensity after one round trip of the compound cavity is equal to the initial intensity, ie.

$$|E_3|^2 = |E_0|^2 , \tag{2.3}$$

or

$$\left|\exp((g_{th}-l)L_1)((1-R_1)r_2 r_0 \exp(i2\pi\nu\tau_2) + r_1 r_0)\right|^2 = 1 , \tag{2.4}$$

where g_{th} is the threshold gain and is given by:

$$g_{th} = l - \frac{1}{2L_1}\log\left(R_0 R_1 + R_0 R_2 (1-R_1)^2 + R_0 r_1 r_2 (1-R_1)\cos\phi_2\right). \tag{2.5}$$

Putting r_2=0 in equation (2.5) gives the usual expression for the threshold gain of a laser with no external feedback:

$$g_{th}^0 = l - \frac{1}{2L_1}\log(R_0 R_1).$$

(2.6)

The difference in the threshold gain due to feedback is thus:

$$\Delta g_{th} = g_{th} - g_{th}^{0}$$
$$= -\frac{1}{2L_1}\log\left(1+\frac{R_2}{R_1}(1-R_1)^2 + 2\frac{r_2}{r_1}(1-R_1)\cos\phi_2\right) \tag{2.7}$$

Introducing a feedback coupling coefficient,ξ, which is defined as

$$\xi = (1-R_1)\frac{r_2}{r_1}, \tag{2.8}$$

gives the threshold gain change due to feedback as:

$$\Delta g_{th} = -\frac{1}{2L_1}\log\left(1+\xi^2+2\xi\cos\phi_2\right). \tag{2.9}$$

For weak feedback, where $\xi<<1$, equation (2.9) can take the following approximation:

$$\Delta g_{th} = -\frac{\xi}{L_1}\cos\phi_2 . \tag{2.10}$$

Thus with weak feedback, the threshold gain is sinusoidally modulated about the value of the threshold gain in the absence of feedback with an amplitude of modulation proportional to the amplitude reflectivity of the external reflector.

The round trip phase for a laser without feedback, $\phi_0=2\pi\nu\tau_1$, may be used to determine the frequencies of the longitudinal modes of the laser by applying the condition, $\phi_1=2m\pi$, where m is an integer. For a laser diode with feedback the round trip phase for the compound cavity, Φ, can be found by rearranging equation (2.2) as:

$$E_3 = E_0 r_0 \exp\{(g-l)L_1\}[r_1 \exp\{i\phi_1\} + r_2(1-R_1)\exp\{\phi_1+\phi_2\}]$$
$$= |E_3|\exp\{i\Phi\} \tag{2.11}$$

which gives:

$$\tan\Phi = \frac{\sin\phi_1 + \xi\sin(\phi_1+\phi_2)}{\cos\phi_1 + \xi\cos(\phi_1+\phi_2)}. \tag{2.12}$$

The change in round-trip phase relative to the solitary laser diode is found by putting $\Phi=\phi_1+\delta\phi$. In the case of weak feedback, changes in the frequency and in the cavity refractive index are small and so the change in the round-trip phase for the laser diode cavity is also small, ie.$\phi_1\approx 2m\pi$, in which case:

$$\tan\delta\phi \approx \frac{\xi\sin\phi_2}{1+\xi\cos\phi_2} \tag{2.13}$$

and for weak feedback, where $\xi<<1$, this can be further approximated to:

$$\delta\phi \approx \xi\sin\phi_2\,. \tag{2.14}$$

The phase condition for a laser diode with feedback is thus:

$$2m\pi = \phi_1 + \delta\phi$$
$$= \frac{4\pi n_0 \nu L_1}{c} + \xi\sin(2\pi\nu\tau_2) \tag{2.15}$$

In order to maintain this condition as the phase of the fed-back light is varied, the round trip phase for the laser diode cavity must vary. This is achieved by variations in the frequency of the laser light, and a variation in frequency in turn implies a variation in refractive index and threshold gain. In order to determine the lasing frequency, ν, from this condition it is necessary to consider the relationships between frequency, cavity refractive index, threshold gain and the external cavity phase. Differentiating equation (2.15) gives:

$$\Delta\Phi = \frac{4\pi L_1}{c}\left(n_0 \Delta\nu + \nu_0 \Delta n\right) + \xi \sin\left(2\pi\nu\tau_2\right) \qquad (2.16)$$

where n_0 and ν_0 are the cavity refractive index and the lasing frequency in the absence of feedback and Δ indicates the change in a quantity due to feedback. The cavity refractive index is dependent on the charge carrier concentration which in turn affects the laser gain; it is also a function of frequency as in any dispersive medium. Therefore the change in refractive index can be expressed as a function of the changes in both frequency and gain:

$$\Delta n = \left.\frac{\partial n}{\partial g}\right|_{\xi=0} \Delta g + \left.\frac{\partial n}{\partial \nu}\right|_{\xi=0} \Delta\nu \qquad (2.17)$$

$$\Rightarrow \quad \Delta\Phi = \frac{4\pi L_1}{c}\left(\left(n_0 + \nu_0 \left.\frac{\partial n}{\partial \nu}\right|_{\xi=0}\right)\Delta\nu + \nu_0 \frac{\partial n}{\partial g}\Delta g\right) + \xi \sin\left(2\pi\nu\tau_2\right) \qquad (2.18)$$

The term in front of Δν is equal to the cavity group velocity *(Petermann, 1991)*. The relationship between the cavity refractive index and the gain of the cavity medium is expressed in the linewidth enhancement factor, α *(Henry, 1982)*, which is defined as the derivative of the real part of the refractive index to the imaginary part, which is proportional to the gain:

$$\alpha = \frac{\partial n}{\partial n^*}; \quad n^* = \frac{-gc}{4\pi\nu},$$

$$\therefore \quad \left.\frac{\partial n}{\partial g}\right|_{\xi=0} = \frac{-\alpha c}{4\pi\nu_0} \tag{2.19}$$

Using the value for the change in the threshold gain given by equation (2.10):

$$\begin{aligned} \Delta\Phi &= \frac{4\pi n_g L_1}{c}\Delta\nu + \xi(\sin\phi_2 + \alpha\cos\phi_2) \\ &= 2\pi\tau_1(\nu - \nu_0) + \xi\sqrt{1+\alpha^2}\sin(2\pi\nu\tau_2 + \arctan\alpha) \end{aligned} \tag{2.20}$$

where τ_1 is the round-trip time for the laser diode cavity; this is a function of the cavity group refractive index, n_g, because the longitudinal mode is the result of interference effects. To satisfy the phase condition for the compound cavity there must be no change in the overall phase, Φ, ie. $\Delta\Phi=0$. It is convenient to define a feedback coefficient, C, as:

$$C = \frac{L_2}{L_1}\xi\sqrt{1+\alpha^2}\,, \tag{2.21}$$

which gives:

$$\begin{aligned} \Delta\Phi &= 2\pi(\nu - \nu_0)\tau_2 + C\sin(2\pi\nu\tau_2 + \arctan\alpha) \\ &= 0 \end{aligned} \tag{2.22}$$

Note that if the fed-back light were in phase with the light in the laser diode cavity at the solitary laser wavelength then the increase in intensity due to interference would lead to a change in the threshold gain and would also be accompanied by a change in wavelength. The fed-back light would not then be in perfect phase with the light in the laser cavity and the intensity would not be maximised. This is what gives rise to the phase difference, of magnitude arctan(α), between the condition for which the threshold gain reduction is maximised and the condition for which there is no shift in the laser diode wavelength.

2.3 Frequency variation

Fig. 2.3 shows the variation of the compound cavity round-trip phase change, $\Delta\Phi$, with frequency change, $\Delta\nu=\nu-\nu_0$, according to equation (2.22). The allowed frequencies are given by the intersection of the plotted function with the $\Delta\nu$ axis. As the external cavity phase is varied, the intersect will move relative to the origin giving rise to a periodic variation in the frequency change due to feedback. When C is greater than one the variation of $\Delta\Phi$ with $\Delta\nu$ is not monotonic because the derivative, $\partial\Delta\Phi/\partial\Delta\nu$, can take negative values, and so multiple solutions are possible depending on the exact value of C and the external cavity phase. There will be an odd number of solutions but the gradient of the

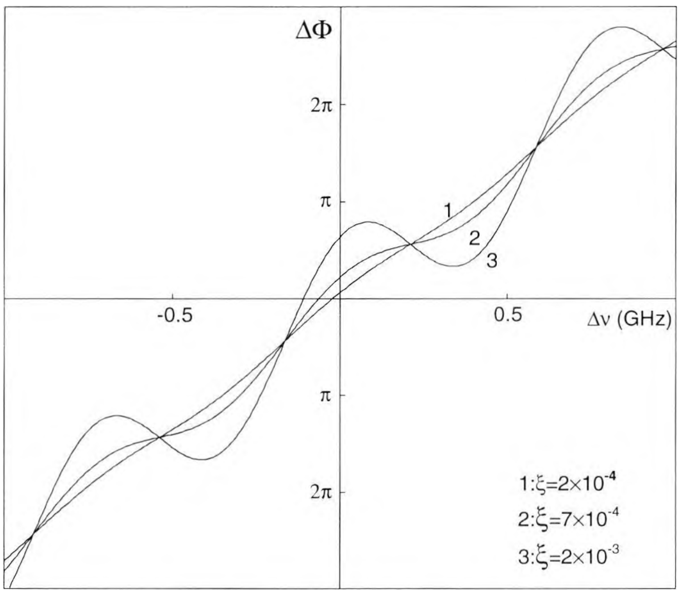


Fig. 2.3. Variation of compound cavity phase change, $\Delta\Phi$ with frequency change, $\Delta\nu$. n_1L_1=1mm, L_2=20mm, α=5.

curve, $\partial\Delta\Phi/\partial\Delta\nu$ is proportional to an effective compound cavity round trip time (by analogy with $\partial\phi_1 / \partial\nu = 2\pi\tau_1$), and so the solutions for which the gradient is negative imply a negative compound cavity round trip time and are not physically realisable.

Equation (2.22) can easily be rearranged to give the following expression relating the frequency emitted by the laser diode with feedback to the frequency emitted by the solitary laser diode:

$$2\pi\nu_0\tau_2 = 2\pi\nu\tau_2 + C\sin(2\pi\nu\tau_2 + \arctan\alpha). \qquad (2.23)$$

The expected form of the frequency variation from this is shown in Fig. 2.4(a) for the case in which the external cavity length is changing and in Fig. 2.4(b) for the case in which the laser diode drive current is varied which adds an additional linear term in frequency.

The real solutions to equation (2.22) represent longitudinal modes of the external cavity; their mean frequency spacing is given by the period of the sinusoidal term in equation (2.22) which, in an analogous way to the spacing of the longitudinal modes of a conventional laser cavity, is

$$\Delta\nu_{xcm} = \frac{c}{\tau_2}. \qquad (2.24)$$

When the laser is operating in a steady state in a region in which there are multiple external cavity longitudinal modes, the laser will operate in only one mode at a time. The mode in which the laser will operate and the stability of that mode are determined by a combination of the phase stability, which is related to the linewidth, and the differences in threshold gain between the individual modes. The laser tends to operate in the mode with the greatest phase stability, or the narrowest linewidth, but as the feedback increases the differences in threshold gain

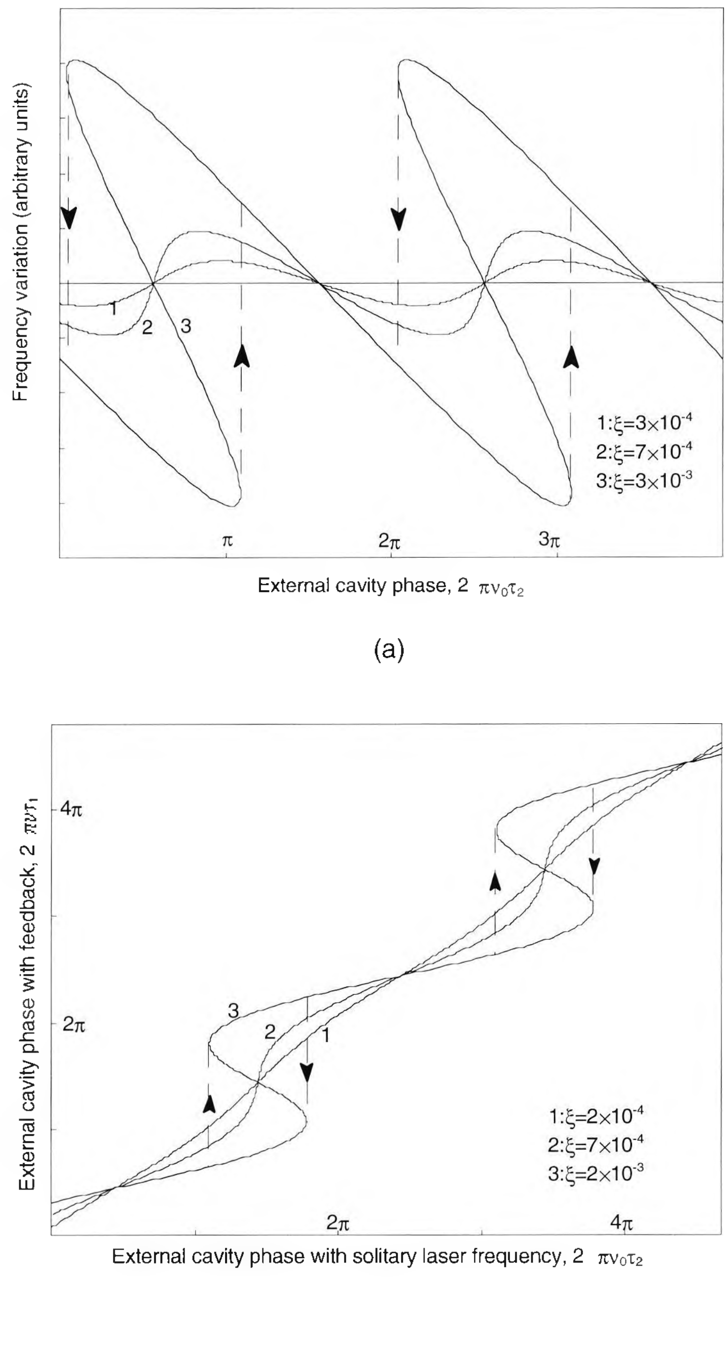


Fig. 2.4. Variation of frequency for a laser diode with feedback with external cavity length (a) and drive current (b). n_1L_1=1mm, L_2=20mm, α=5.

increase and so that factor will tend to dominate. In general, mode-hopping will occur, creating a high level of frequency and intensity noise. This mode hopping will be present at a low enough frequency that in a transient state the output frequency will follow the path shown in fig. 2.4(a) by the arrows and hysteresis will be seen in the frequency path.

2.4 Intensity variation

In most practical situations in which a laser diode is used with feedback the observed parameter is the output intensity of the laser diode. The intensity variation of a laser diode with feedback is related to the variation of the threshold gain which itself is dependent on the variation in frequency.

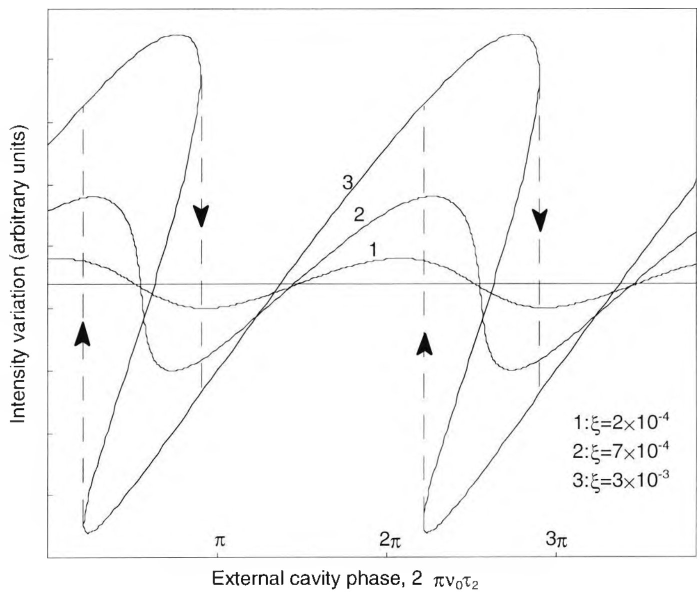
For the weak feedback situation, the behaviour of the output intensity of the laser diode may be modelled using the following rate equation for the charge carrier density, N, in the active region:

$$\frac{\partial N}{\partial t} = \frac{J}{e} - \frac{N}{\tau_c} - v_g g S \qquad (2.25)$$

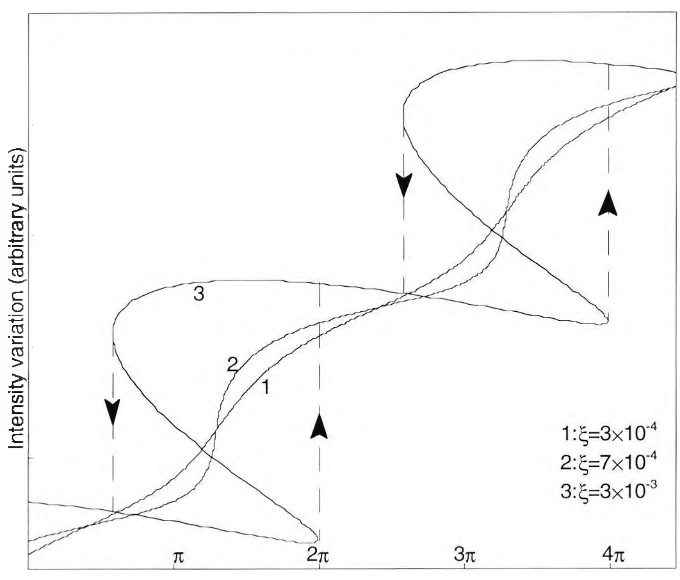
where J is the injection current, e is the electronic charge, τ_c is the mean charge carrier lifetime with respect to spontaneous recombination, v_g is the cavity group velocity, g is the linear gain, and S is the cavity photon density; the last term on the right hand side therefore represents the gain as a function of time and is the rate of stimulated recombination.

Equating the steady state conditions, $\partial N/\partial t=0$ for the laser diode with feedback and the solitary laser diode gives:

$$\frac{N}{\tau_c} + v_g g S = \frac{N_0}{\tau_c} + v_g g_0 S_0 \qquad (2.26)$$



(a)



(b)

Fig. 2.5. Variation of intensity for a laser diode with feedback with external cavity length (a) and drive current (b). n_1L_1=1mm, L_2=20mm, α=5.

where the subscript, 0, indicates the solitary laser diode. Making the substitution $g=g_0+\Delta g$, and introducing a coefficient, $\kappa=\Delta g/\Delta N$, relating the change in gain to the change in the charge carrier density, equation (2.26) can be rearranged to give:

$$S = \frac{S_0 g_0 v_g - \frac{1}{\tau_c}(N - N_0)}{v_g(g_0 + \Delta g)} \tag{2.27}$$

Assuming that $\Delta g \ll g_0$ then the following approximation can be made:

$$S \approx S_0\left(1 - \frac{\Delta g}{g_0}\left(\frac{1}{\kappa v_g \tau_c S_0} + 1\right)\right) \tag{2.28}$$

and using the expression for the gain change due to feedback obtained in Section 2.2 gives:

$$S = S_0(1 + m\cos\phi_2) \tag{2.29}$$

where m is a modulation coefficient defined by

$$m = \frac{\xi}{g_0 L_1}\left(\frac{1}{\kappa v_g \tau_c S_0} + 1\right). \tag{2.30}$$

For low levels of feedback, the variation in intensity is almost sinusoidal, but as the feedback strength increases and the variation of the frequency becomes greater the intensity variations become asymmetric and eventually, for C>1, become multi-valued. The behaviour of the intensity output of the laser diode follows that of the frequency with discontinuous jumps in intensity occurring at the same time as the hops between the external cavity longitudinal modes. The expected form of the variation in output intensity is shown in Fig. 2.5(a) for external cavity phase variation due to movement of the external reflector and due to the variation in frequency accompanying a variation of the laser drive current in Fig 2.5(b). The intensity variation is predicted to have the same periodicity as the frequency variation with external cavity phase. For the case in which the phase change is due to movement of the external reflector there is one period of intensity variation for every half-wavelength of movement of the external reflector which is the same as for a conventional Michelson interferometer. There is an obvious difference between the fringes obtained from a Michelson interferometer and the waveform shown in Fig. 2.5(a) for the signal obtained from a laser diode with feedback which is the asymmetry of the fringes obtained with feedback. As the feedback strength is increased, the fringes progress from a near sinusoidal shape through a saw-tooth form until, for C>1, discontinuous jumps occur between the external cavity modes, shown by the dashed lines in the figure. The direction of the inclination of the saw-tooth fringes or the direction of the intensity change at the mode-hops depends on the direction of the movement of the external reflector. Thus the intensity variation of a laser diode with weak external optical feedback should be suitable to be used in an interferometric sensor capable of measuring both magnitude and direction of the displacement of a target reflector.

In the case in which the phase variation is due to a variation in the frequency, which in turn is caused by a variation in the laser drive current, the frequency of the intensity modulation superimposed on the intensity variation that will also accompany the variation in drive current is proportional to the distance to the external reflector, so by modulating the laser drive current a sensor can be constructed to measure the absolute distance to a target.

2.5 Effect of feedback on linewidth

It is well-known that feedback is capable of having a significant effect on the linewidth of a laser diode *(Petermann, 1995)*. Unwanted feedback can lead to large increases in the linewidth which can degrade the performance of communications systems. Alternatively it has been shown that feedback can be used to narrow the laser diode linewidth from typical values of around 10-100MHz to spectral widths of as little as 10KHz *(Wyatt & Devlin, 1982).* These effects are observed in regions of stronger feedback than is being considered here and although it has been noted that there is a phase-dependent effect on the linewidth in the weak feedback regime, the effects of this in self-mixing interferometric applications have not been stated explicitly.

The spectral width of the emitted light is related to the gradient, $\partial\Delta\Phi/\partial\Delta\nu$ derived from equation (2.22), at $\Delta\Phi=0$. It can be seen qualitatively that when the gradient is small there will be a greater range of frequencies for which equation (2.22) is close to being satisfied than when the gradient is high. For a solitary laser the spectral linewidth is inversely proportional to the inverse square of the cavity round trip time. This result can be extended to the case of a laser diode with external feedback using the effective compound cavity round trip time which was

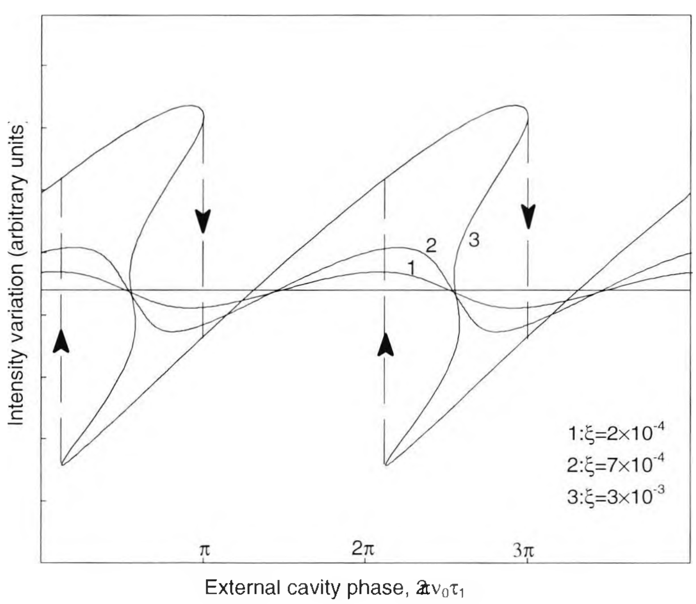


Fig. 2.6. Intensity variation with external cavity phase when linewidth and coherence length are phase dependent.

shown in section 2.3 to be proportional to $\partial\Delta\Phi/\partial\Delta\nu$. This gives the following expression for the spectral linewidth:

$$\delta\nu = \frac{\delta\nu_0}{\left(\frac{\partial\Delta\Phi}{\partial\nu}\right)^2} = \frac{\delta\nu_0}{\left(1 + C\cos(\phi_2 + \arctan\alpha)\right)^2} \tag{2.31}$$

where $\delta\nu$ is the laser diode spectral width with feedback and $\delta\nu_0$ is the spectral width for the solitary laser diode.

By using a simple model in which a reduction in the coherence length of the laser light causes an effective reduction in the reflectivity of the external reflector and hence a reduction in the feedback coupling coefficient, the theoretical variation of intensity with external cavity length takes the form shown in Fig. 2.6. Because the variation of spectral linewidth is in phase with the intensity and frequency variations the effect of including linewidth variation in the theoretical model is only slight; the large variations in linewidth occur when $\cos(\phi_2+\arctan\alpha)\approx 0$ at which point the frequency and intensity variations are close to zero and so the overall effect is small. Thus the curves in Fig. 2.6 do not differ greatly from those shown in Fig. 2.5(a), calculated without taking the effect of feedback on linewidth into account, and the conclusions drawn in section 2.5 about the interferometric sensing applications of feedback in laser diodes are valid.

2.6 Stability considerations

In many practical applications it is necessary to ensure that the laser diode with feedback remains in a single external cavity longitudinal mode and free of mode hopping which results in excessive noise. The stability of the laser against changes in temperature or injection current can be quantified in terms of the range

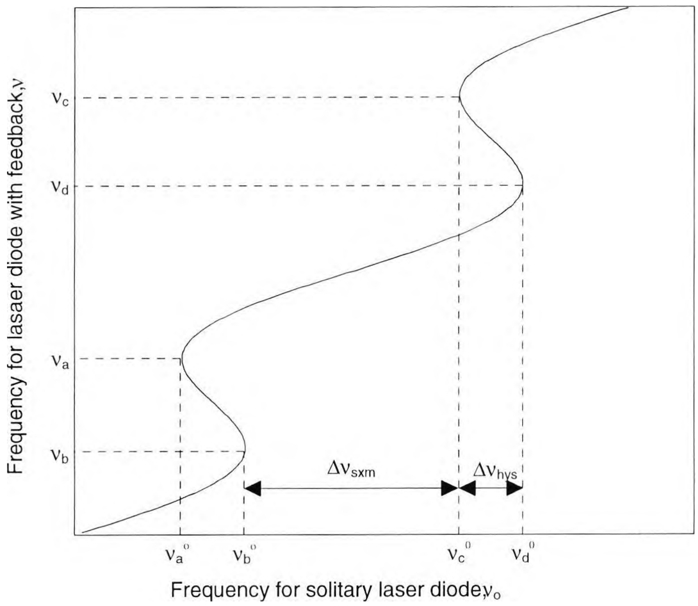


Fig. 2.7. Variation of compound cavity phase change with frequency difference showing single longitudinal mode frequency region, $\Delta\nu_{sxm}$, and hysteresis width, $\Delta\nu_{hys}$.

in the solitary laser frequency domain for which there is a single possible external cavity longitudinal mode for the laser diode with feedback. This is the region between, shown in ν_b^0 and ν_c^0 in Fig. 2.7. The width of this region can be calculated by differentiating equation (2.23):

$$\frac{\partial \nu_0}{\partial \nu} = 1 + C\cos(2\pi\nu\tau_2 + \arctan(\alpha)) \tag{2.32}$$

The stationary points corresponding to $\nu_{a\text{-}d}$ in Fig. 2.7 are given by

$$\cos(\phi_a) = -\frac{1}{C} \tag{2.33}$$

where $\phi_a = 2\pi\nu_a\tau_2 + \arctan(\alpha)$, and similarly for ν_{b-d}. Substituting these values back into equation (2.33) and using the relationship $\sin(\arccos(x)) = \pm\sqrt{1-x^2}$ gives:

$$\nu_a^0 = \nu_a - \frac{\sqrt{C^2-1}}{2\pi\tau_2} \text{ and } \nu_b^0 = \nu_b + \frac{\sqrt{C^2-1}}{2\pi\tau_2}. \tag{2.34}$$

The frequency range in which there is a single external cavity longitudinal mode is, from Fig 2.7, given by $\Delta\nu_{sxm}^0 = \nu_c^0 - \nu_b^0 = (\nu_c^0 - \nu_a^0) - (\nu_b^0 - \nu_a^0)$. The first term, $(\nu_c^0 - \nu_a^0)$, is just the period in the frequency domain of the sine term in equation (2.22), and is equal to $1/\tau_2$. The second term, from the above, is:

$$\nu_b^0 - \nu_a^0 = \nu_b - \nu_a + \frac{2\sqrt{C^2-1}}{2\pi\tau_2} \quad . \tag{2.35}$$

Expressing the frequencies ν_a and ν_b in terms of ϕ_a and ϕ_b as defined above and using equation (2.22) and the relationship $\cos(\phi) = \pm\sqrt{(\tan^2(\phi))^{-1}+1}$ gives

$$\nu_b - \nu_a = -\frac{1}{2\pi\tau_2} 2\arctan\left(\sqrt{C^2-1}\right) \tag{2.36}$$

and so

$$\Delta\nu_{sxm}^0 = \frac{1}{\tau_2}\left\{1 - \frac{1}{\pi}\left[\sqrt{C^2-1} - \arctan\left(\sqrt{C^2-1}\right)\right]\right\} \tag{2.37}$$

Thus the width of the single external cavity mode region is inversely proportional to the length of the external cavity and decreases, almost linearly, with increasing feedback strength as is shown in Fig. 2.8. Solving equation (2.37) numerically shows that for $\Delta\nu_{sxm}=0$, $C\approx4.61$; above this level of feedback there are no frequencies for which there is a single external cavity longitudinal mode.

The magnitude of the hysteresis difference between the upward and downward frequency tuning paths is given by the frequency separation:

$$\Delta\nu_{hys} = \nu_b^0 - \nu_a^0 = \frac{1}{\pi\tau_2}\left(\sqrt{C^2-1} - \arctan\sqrt{C^2-1}\right) \qquad (2.38)$$

This quantity can be measured experimentally in order to determine the feedback

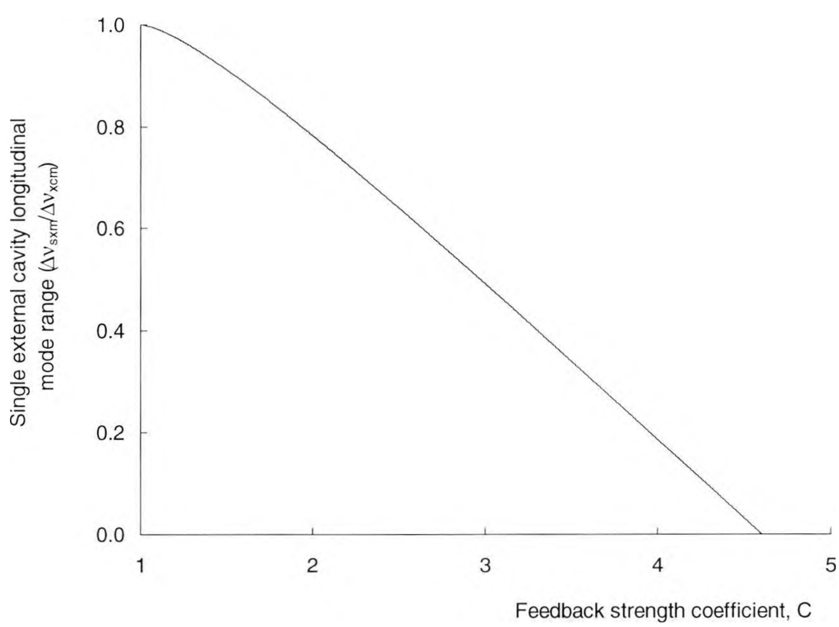


Fig. 2.8. Variation of single external cavity longitudinal mode range, expressed as a fraction of the external cavity mode spacing, with feedback strength.

strength in a system for values of C between 1 and 4.6.

2.7 Higher levels of feedback

The effects described in the previous sections occur for very weak feedback, typically with a coupling coefficient of the order of 10^{-6}. Within this regime of feedback the laser diode operates in either one or two external cavity longitudinal modes with an output which varies periodically with the external cavity phase and so is suited to interferometric applications.

For levels of feedback within the weak feedback regime as defined above but with $C>>1$ there are multiple external cavity longitudinal modes available to the laser diode and it will exhibit instability, hopping between these modes. This effect places a theoretical upper limit on the coupling strength and target distance in a self-mixing interferometric system, although in practice the target distance is usually limited by the degradation of the beam profile and collimation over large distances (the beam emitted by the laser diode is astigmatic and so perfect collimation is not possible with an ordinary microscope objective).

At higher levels of feedback for which the weak feedback condition no longer holds, the behaviour of the laser diode can be classified into a number of distinct regimes *(Tkach & Chraplyvy, 1986)*. The first two regimes of feedback are defined as those with weak feedback as defined above with single or multiple external cavity longitudinal mode operation respectively. In the third regime, as the amplitude of the gain difference modulation due to feedback increases, the gain difference between adjacent external cavity longitudinal modes is sufficient that the laser will operate stably in one mode, which will be the mode with the narrowest linewidth.

In the fourth regime the effect known as coherence collapse occurs, where the linewidth of the emitted light increases dramatically and there can also be large increases in phase and intensity noise. These effects are caused by the presence of relaxation oscillations which are initiated by spontaneous emission noise events.

In the fifth regime the reflectivity of the laser diode front facet is so low relative to the reflectivity of the external reflector that the system operates as a single cavity laser with the cavity defined by the laser rear facet and the external reflector. The linewidth is reduced relative to the solitary laser diode and the output is again independent of the external cavity phase. This regime of operation is that which is used for 'external cavity' semiconductor lasers *(Fujita et al., 1984; Sun et al., 1992)*.

The range of values of feedback strength for which the output is phase dependent and for which the laser diode can be use for interferometric applications is thus a small proportion of the possible values and so care must be taken when using feedback to control the coupling strength accurately.

2.8 Summary

This chapter has presented a theory which explains how weak external optical feedback in a laser diode can cause phase dependent frequency and intensity variations which allow the laser diode with feedback to be used for interferometric applications. The theory shows that the form of these variations, which are associated with longitudinal modes of the external cavity, depends critically on the strength of the feedback coupling and the distance from the laser to the external reflector. The conditions for which stable periodic intensity variations suitable for an interferometric signal may be obtained have been discussed. The effect of linewidth variations due to feedback on the observed intensity variations is considered and shown not to be significant.

In the following chapters the theory described here is tested experimentally; the operation of an interferometric displacement sensor using optical feedback in a laser diode is demonstrated, and the spectral stability with respect to environmental temperature variations and effects related to mechanical alignment are examined.

The next chapter describes the experimental techniques that have been developed to investigate the phenomena described here theoretically and to apply them to an interferometric sensing system.

Chapter 3: Experimental Techniques for Investigating the Effects of Optical Feedback in Laser Diodes.

3.1 Introduction

This chapter considers some of the practical problems which are associated with any experimental work with optical feedback in a laser diode and describes solutions to these. A typical experimental system, which includes the essential elements necessary for experimental investigation of feedback or its practical application, is evaluated in order to make a good estimate of the feedback coupling strength. The effects of the alignment of the components of a system is described theoretically and a method of achieving good alignment is described. Simple techniques for measuring the frequency variations in the laser diode output and the strength of the feedback coupling strength are presented.

A typical experimental set-up used to investigate the effects of external optical feedback in a laser diode is shown in Fig. 3.1. Light is returned to the laser diode, LD1, from the external reflector, M1, which may be a mirror, a diffraction grating or a corner cube prism; the issues determining the choice of reflector in a practical self-mixing interference-based sensing system are discussed in Chapter 4. The feedback strength is controlled by placing a neutral density filter, ND1 between the laser and the external reflector. In order to modulate the external cavity phase by modulating the external cavity length, the mirror M1 was mounted on a loudspeaker cone which was mounted on a micrometer controlled translating stage which provided larger scale longitudinal movement of the reflector. The external cavity phase could also be modulated by modulating the laser diode

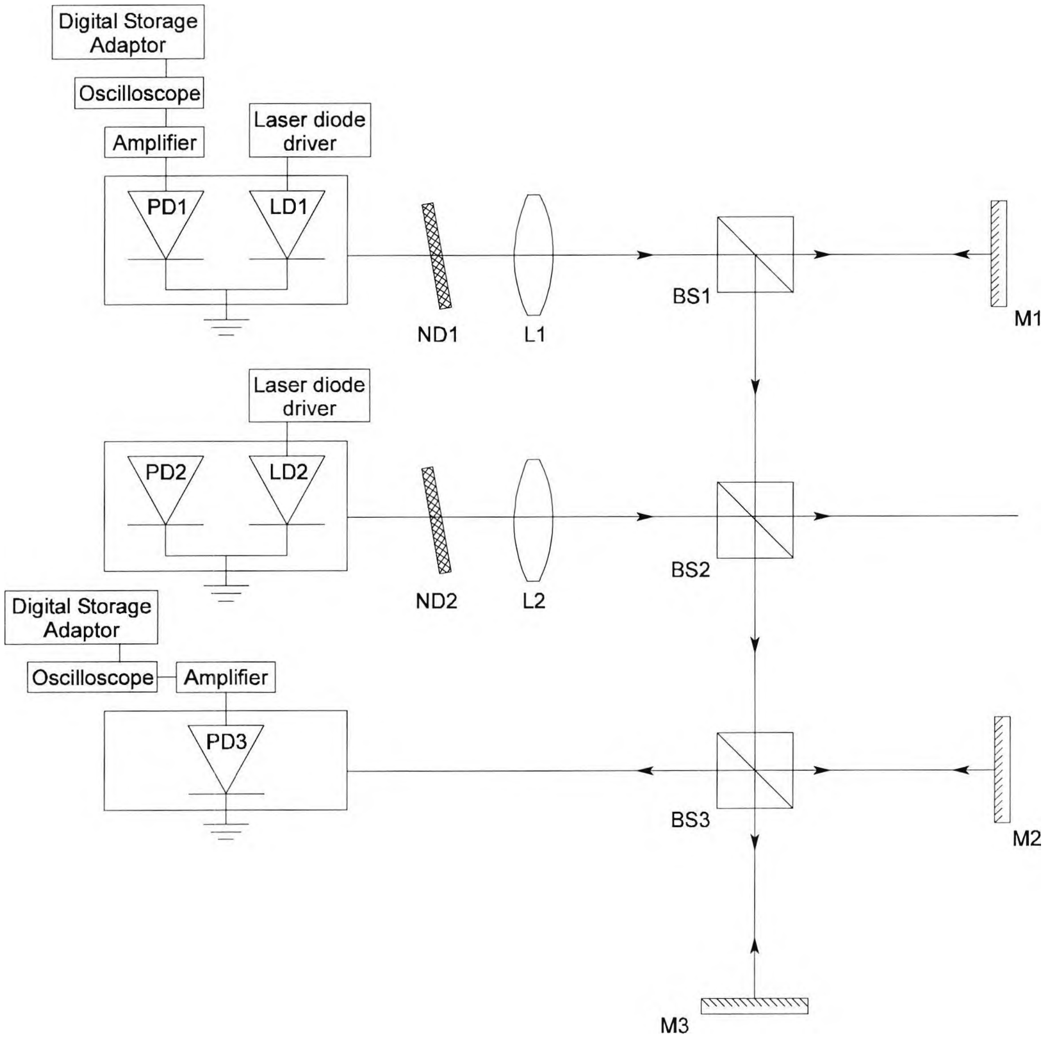


Fig. 3.1. Typical experimental set-up; LD_i: laser diodes; PD_i: monitor photodiodes; L_i: collimating lenses; ND_i: neutral density filters; BS_i: beamsplitters; M_i: mirrors.

frequency which is achieved by adding an AC component to the laser diode drive current; a triangular waveform was used for this modulation to give linear frequency and phase variation with respect to time.

The effects of feedback on the intensity output of the laser are observed via a monitor photodiode incorporated in the laser diode package at the rear facet of the laser; the photodiode current is passed to a transimpedence amplifier and the signal is viewed on an oscilloscope.

Laser diode drivers were used which kept the drive current stable to within 0.01mA and each laser was mounted in a heat-sink with a Peltier temperature controller which stabilised the temperature to within approximately 0.01K. It should be noted that this figure for temperature stability actually applies to the heat-sink in which the thermistor used to control the Peltier device is mounted and so temperature variations at the laser diode due to variations in the laser drive current, which cause variations in the electrical power dissipated in the laser, will not be affected by the temperature stabiliser. The stability of the temperature and current are important because each affects the optical frequency of the laser diode. The overall wavelength stability achieved with this set-up is demonstrated in section 3.5 and the issue of temperature stability is discussed in greater detail in Chapter 5.

The effects of the feedback on the wavelength of the light emitted by the laser diode are observed by comparing the wavelength of emission to that of a reference laser diode, LD2, using the Michelson interferometer made up of the beamsplitter, BS3, and the mirrors, M2 and M3, one of which is fixed and the other mounted on a micrometer-controlled translating stage. The Michelson interferometer was chosen for these measurements because of its simplicity of construction, using readily available optical-bench components, combined with its potential for high spectral resolution; this is described fully in Section 3.5.

Oscilloscope traces of the signals from the monitor photodiode and the Michelson interferometer photodiode were digitised and stored for analysis using a digital storage adaptor.

3.2 Estimation of feedback strength

In order both to be able to set up the experimental system correctly and to be able to evaluate and interpret experimental observations it is necessary to be able to make a good estimate of the feedback strength achievable in the experimental system. For the basic system consisting of a laser diode, a collimating lens and a plane mirror, this requires estimates to be made of the coupling from the laser diode to the lens, losses due to reflection from the lens surface, the reflectivity of the mirror at the laser diode wavelength coupling of the return beam back into the lens and coupling of the image of the laser aperture back into the aperture.

The coupling efficiency for light from the laser into the collimating lens depends on the divergence of the beam from the laser and on the numerical aperture of the lens. The manufacturer's specified beam angles (full angles at half maximum) for the Sharp LT015MD laser diodes used in these experiments are typically 9.5° parallel to the diode junction plane and 27° perpendicular to that plane. With a ×10 magnification microscope objective for the collimating lens the numerical aperture is 0.25 which corresponds to a full angle subtended by the aperture at the focus of 29°; therefore the coupling of the component diverging parallel to the junction is almost 100% and for the other component, where the aperture closely matches the full width at half-maximum, the coupling is approximately 75%, for the reasonable assumption of a Gaussian intensity distribution.

Assuming that the beam is well collimated, the coupling of the return beam into the lens should be close to 100%. The coupling of the imaged spot back into the laser diode aperture is determined by the diffraction pattern of the returned image. As the component of the emitted beam parallel to the diode junction is passed almost entirely by the collimating lens it is reasonable to assume that all the spatial frequencies in that component are passed and a good image is formed by the beam fed back in the direction parallel to the junction. For the other direction, part of the outgoing beam is blocked by the aperture of the collimating lens and so the image formed by the return beam will be spread out due to diffraction. The dimensions of the output aperture can be estimated from the beam divergence angles using the following expression for the beam waist radius at e^{-2} of the maximum intensity, w, in a Gaussian beam with wavelength, λ, and divergence angle, θ:

$$w = \frac{2\lambda}{\pi 6} \qquad (3.1)$$

which, for the beam divergence angles given above, gives dimensions of 3.8μm× 1.3μm. The intensity distribution perpendicular to the junction plane will be pproximately that of a single slit diffraction pattern and for this case the fraction of the power falling within the 0.6μm aperture at the laser diode facet will be approximately 90%.

For the case in which a front surface aluminium mirror is used for the external reflector, the reflectivity at around 830nm is approximately 85% and the reflectivity of each surface of the collimating lens and the window in the laser diode package is assumed to be about 4%.

Taking all these factors into account gives a maximum feedback coupling of approximately 50%. This coupling strength is much greater than that needed for weak feedback, as defined in Chapter 2, and so, with good alignment of the external reflector, the coupling can be controlled by placing neutral density filters in the cavity between the laser diode and the reflector to give any level of feedback coupling within the weak feedback regime.

Using the beam divergence angles given above, the feedback from the window of the laser diode package can be calculated by calculating the area of the reflected beam at the diode facet. The feedback coupling coefficient, ξ, from each surface of the window is appoximately 10^{-3} which is comparable with the coupling coefficients from the external target reflector in the following experiments but does not have a signicant effect on the output of the laser because of the short distance to the window, which is approximately equal to the length of the laser cavity and so gives a very small value of the feedback coefficient, C, also about 10^{-3} in this case. The feedback from the diode window, and also the collimating lens, does not, therefore, affect the observed performance of the laser diode.

The above calculations are made with the assumption that the external reflector is perfectly perpendicular to the optical axis of the laser diode so that the image of the aperture formed by the collimating lens is centred on the aperture itself. In fact the coupling will obviously depend on the alignment of the external reflector which may be difficult in practice to set up and maintain perfectly perpendicular to the axis of the laser diode cavity. In the following section the effect of misalignment of the external reflector is examined in order to quantify this effect.

3.3 Alignment of components and beam collimation

The behaviour of a laser diode with optical feedback is critically dependent on the amount of light reentering the laser cavity. This in turn depends on the alignment and positioning of the components of the system, in particular the collimating lens and the external reflector.

With a 0.25NA microscope objective used as the collimating lens for a Sharp LT015MD laser diode the divergence of the beam parallel to the diode junction is less than the numerical aperture of the lens and so Gaussian beam optics apply and in the unit magnification system used, the image profile in that direction should match the transverse mode profile at the output aperture. In the orthogonal direction, the numerical aperture of the lens is approximately equal to the width of the beam at half-maximum and in this case the image is dominated by the aperture of the lens and has the profile of a single slit diffraction pattern, which is given by

$$I(\theta) = I_0 \frac{W^2}{\pi} \operatorname{sinc}^2(\beta) \quad , \beta = \frac{\pi W}{\lambda} \sin\theta \tag{3.2}$$

where I_0 is the total intensity, W is the width of the diffracting slit and θ is the angle of diffraction. The FWHM of this intensity distribution is given by $\Delta\beta \approx 3.8$ which for the aperture and wavelength in this case corresponds to $\Delta(\sin\theta) \approx 1.25 \times 10^{-4}$. The actual width of the central maximum of the diffraction pattern at the laser output facet is given by $\Delta x = f\Delta(\sin\theta)$, where f is the focal length of the collimating lens, which for the 0.25NA microscope objective is 16mm, giving a width, Δx, of 2.0μm. The FWHM for the variation of the feedback intensity with the displacement of the diffracted image will be approximately equal to the width of the function given by the convolution of the diffraction intensity profile and the transverse mode profile at the laser output aperture which will be approximately the width of the wider function, which in this case is the diffraction profile.

The variation of feedback intensity with the angular position of the external reflector thus has a sinc-squared profile for tilts about an axis parallel to the diode junction plane with an angular full width at half maximum of about 10^{-4}rad. For tilts about the axis perpendicular to the junction plane the variation of the feedback intensity should be approximately Gaussian and the angular width is given by the angle subtended by the width of the transverse mode at the laser front facet at the collimating lens which is also about 10^{-4}rad.

The longitudinal position of the collimating lens also affects the distribution of light at the plane of the laser output facet. Again the two orthogonal components must be considered separately. For the component for which the divergence is less than the numerical aperture of the collimating lens, and which therefore maintains its Gaussian profile, the longitudinal variation of intensity on the optical axis near the focus is approximately inversely proportional to the beam waist width and is given by

$$I = I_0\left(1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2\right)^{-\frac{1}{2}}, \qquad (3.3)$$

where w_0 is the beam waist radius and z is the distance along the optic axis from the beam waist *(Young, 1984)*. The half width of this intensity distribution is therefore

$$\Delta z = \sqrt{3}\,\frac{\pi w_0^2}{\lambda}\,. \qquad (3.4)$$

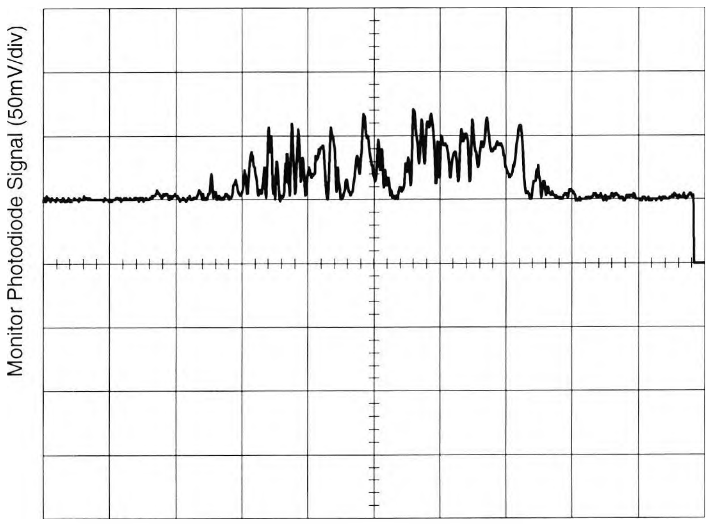
In the case of the laser diode discussed here the beam waist radius, w_0, is 1.9μm which gives a longitudinal FWHM of about 23μm.

For the direction in which the transverse distribution is close to that of a single slit, the longitudinal light distribution near the focus can be assumed to be similar to the distribution for a circular aperture given by Born and Wolf *(1980)* with the diameter of the aperture replaced by the slit width. The FWHM of the central on-axis maximum in this case is approximately 23μm.

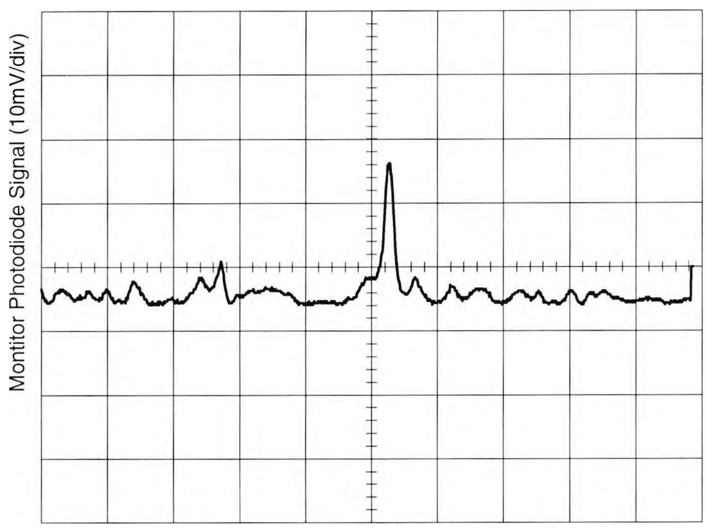
3.4 Alignment of the external reflector

Due to the small size of the laser diode output aperture, approximately 1.6μm× 0.6μm, the performance of a laser diode feedback system is very sensitive to the alignment of the external reflector. In order to make valid estimates of the feedback strength in an experimental situation it is necessary to be able to determine when the external reflector is aligned perpendicularly to the optical axis of the system.

When the alignment is exactly perpendicular the central maximum of the diffracted image of the laser aperture is aligned centrally with the actual aperture and the feedback is maximised. Therefore a simple method of detecting changes in the feedback strength with respect to changes in the angular position of the mirror is required. With no attenuating filters in the external cavity the feedback, when the reflector is well aligned, is strong enough to cause a reduction in the threshold gain independent of the external cavity phase. This reduction in threshold gain causes a



(a)



(b)

Fig. 3.2. Oscilloscope traces showing laser diode output as the external reflector is scanned through the normal position, (a) above threshold, and (b) below threshold.

reduction in the threshold current for the laser diode and at a given current, an increase in the output power of the laser. At drive currents above threshold, as the mirror is tilted and the diffraction pattern of the imaged aperture sweeps across the actual aperture, the output from the laser diodes monitor photodiode shows a complex series of peaks, as shown in Fig. 3.2(a), from which it is very difficult to determine the central maximum of the pattern. If the drive current is reduced below threshold, only those peaks in the pattern with enough power to reduce the threshold current below that level will cause a peak in the monitor photodiode output in this way. By gradually reducing the drive current and scanning the mirror until only one peak is found in the output, the central maximum of the diffraction pattern can be located, as shown in Fig. 3.2(b), and it is then known that the mirror is well aligned.

3.5 Wavelength measurement

Measurements of variations in the wavelength of the light emitted by a laser diode were made using a dual-wavelength Michelson interferometer which measures the difference between the wavelength of interest and a fixed reference wavelength. The experimental arrangement is shown in Fig. 3.1: a portion of the light from the laser under test, LD1, is sampled by the beamsplitter, BS1, and mixed with light from the reference laser, LD2, at the beamsplitter, BS2. The mixed light then enters the Michelson interferometer made up of the beamsplitter, BS3, the mirrors M1 and M2, and the photodiode, PD1. One of the mirrors is mounted on a loudspeaker cone to provide phase modulation and that is mounted on a micrometer-controlled translating stage to provide large scale variations in the interferometer path difference.

If the two component beams have equal intensity at the detector then the output intensity of the Michelson interferometer with two discrete wavelengths, λ_a

and λ_b, as a function of the distance of the adjustable mirror from balance, z, is given by

$$I(z) = 4I_0\left\{1 + \cos\left((k_a + k_b)z\right)\cos\left((k_a - k_b)z\right)\right\} \tag{3.5}$$

where $k_i=2\pi/\lambda_i$. This pattern is simply the sum of the interference patterns due to the individual wavelengths and consists of fringes whose period is the mean of the individual fringe period modulated by a sinusoidal envelope with a period, Δz, inversely proportional to the difference between the two wavelengths given by:

$$\Delta z = \frac{k_a k_b}{2(k_a - k_b)} \quad . \tag{3.6}$$

If the intensities in the two input beams are well-balanced, for example by using neutral density filters in front of the interferometer beamsplitter, then the minima are very well defined and the adjustable mirror can be positioned very accurately at the minima. An example of an observed oscilloscope trace is shown in

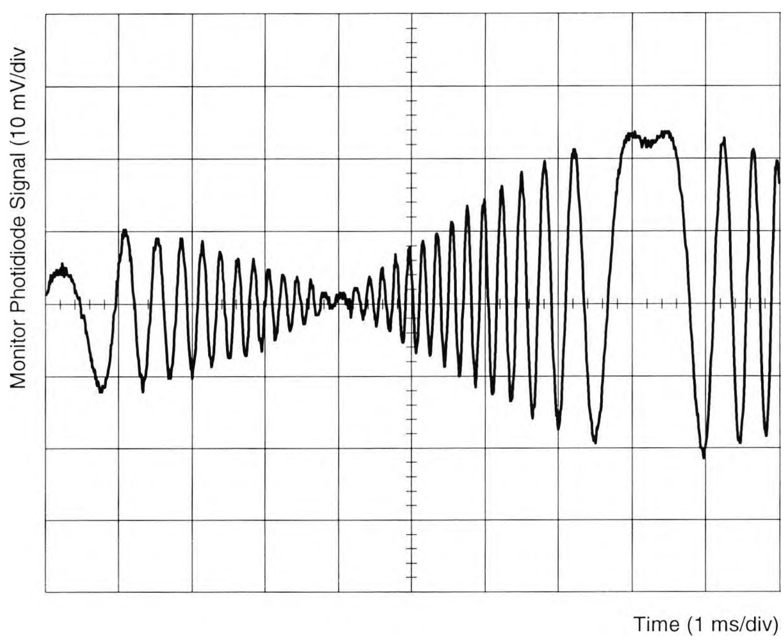


Fig. 3.3 Example of an oscilloscope trace obtained with the dual-wavelength Michelson interferometer.

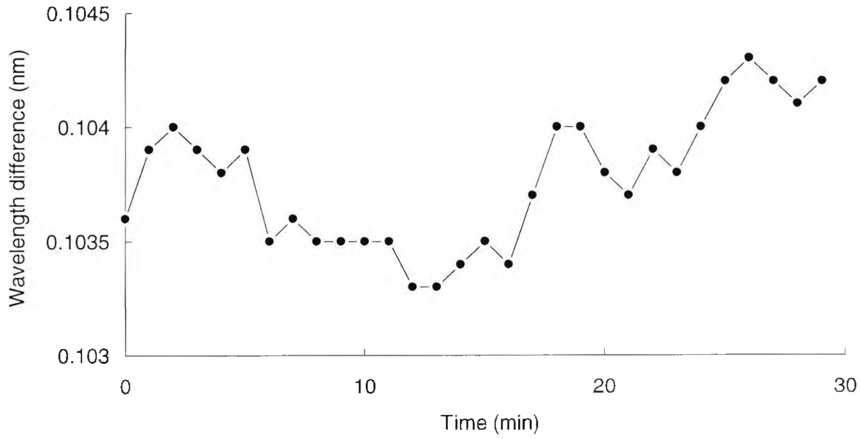


Fig 3.4. Variation over time of measured wavelength difference.

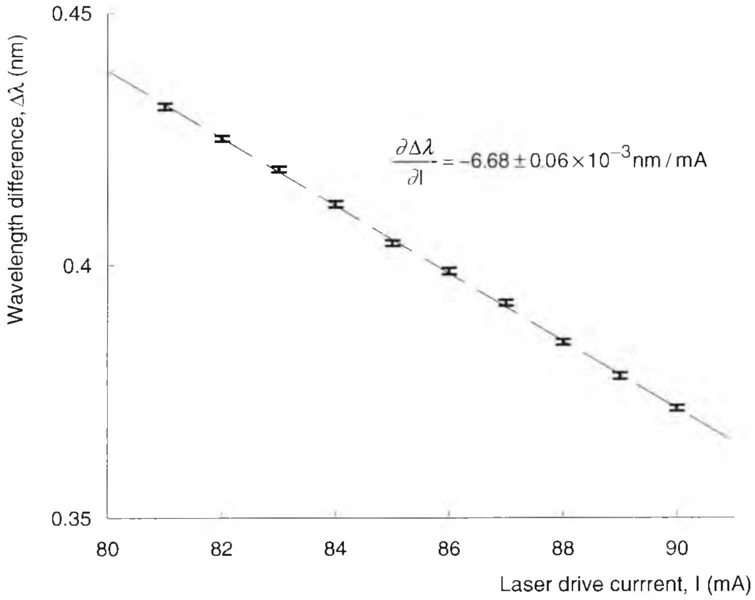
Fig. 3.3 and it can be seen that the minimum can be located with an accuracy of approximately one fringe width, or about 0.4μm of mirror movement.

If the wavelength of one laser diode is varied by, for example varying the drive current or temperature, then the change in the position of one of the visibility pattern minima can be tracked and the wavelength variation thus measured. The limiting factor for the accuracy of this method is the stability of the wavelength of the reference laser diode with respect to temperature fluctuations. Using a LDX LDT5412 temperature controller the laser diode temperature could be stabilised to within 10mK which resulted in an accuracy for the relative wavelength measurements of approximately $\pm 5\times 10^{-4}$nm. This corresponds to a spectral resolution of approximately 25Mhz which compares well with resolutions obtained using scanning Fabry-Perot interferometers for this type of observation, for example a resolution of 30MHz for the Fabry Perot interferometer used by Goldberg et al. *(1982)*. This figure has been confirmed experimentally by making a series of measurements of wavelength difference over a period of 30 minutes, which is typical of the timescale over which measurements are made. The results of this are shown in Fig. 3.4; the standard deviation in these measurements is

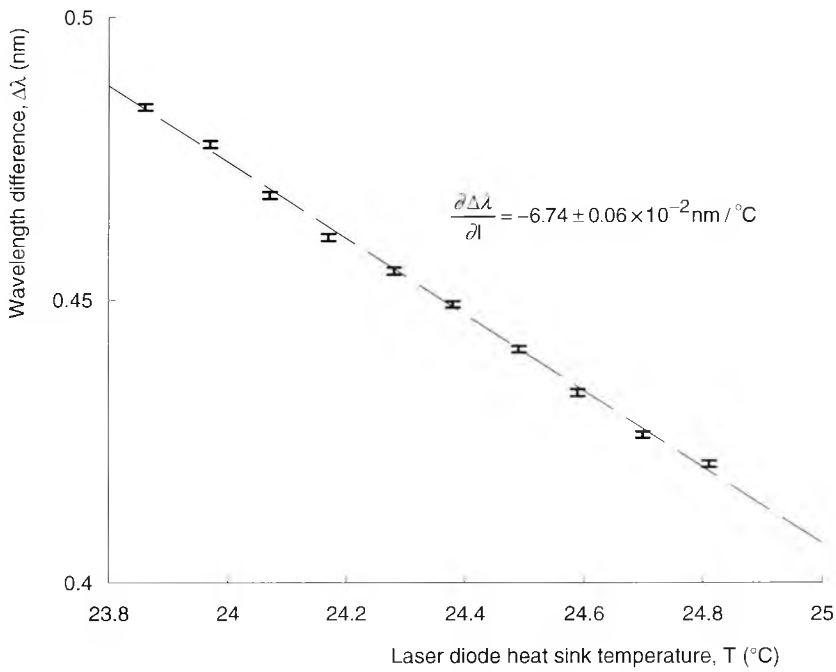
$\sigma(\Delta\lambda)=2.9\times10^{-4}$nm. In cases where the distance measured between minima in the visibility pattern is small, less than about 1mm, the accuracy of the wavelength measurement is limited not by the temperature and current stability but by the accuracy with which that distance can be measured. This can be avoided either by adjusting the temperature and drive current of the reference laser diode so that the wavelength difference is small, thus increasing the period of the visibility pattern, or measuring the movement of the mirror over several periods of the visibility pattern.

Examples of measurements made using this method are shown in Fig. 3.5, where the response of the wavelength to drive current (Fig 3.5(a)) and to the temperature measured by the thermistor at the heat sink in which the laser diode is mounted (Fig. 3.5(b)) for a Sharp LT015MD laser diode. The magnitudes of the wavelength tuning coefficients, found by making a linear least squares fit to the data, are $6.68\pm0.06\times10^{-3}$ nm/mA for the current and $6.74\pm0.06\times10^{-2}$ nm/K for temperature; or 2.88 ± 0.03 GHz/mA and 29.0 ± 0.3 GHz/K respectively in terms of frequency.

By measuring the wavelength tuning coefficient with respect to the laser diode drive current and also the variation of the monitor photodiode output with the drive current, variations in wavelength can be inferred from corresponding variations in the laser power output recorded on an oscilloscope as the drive current is modulated. The variation of intensity with feedback phase has a complex form for which the theoretical prediction was shown in Fig. 2.5. The relationship between intensity and wavelength is not linear but has the form of periodic variations around the linear varition that is seen in the absence of feedback. Therefore if measurements are made of intensity differences between identical points in different periods of the observed waveform then the corresponding frequency difference will be equal to the frequency difference calculated for the same points on the waveform obtained in the absence of feedback for which there is a linear relationship between intensity and wavelength. Values obtained for the



(a)



(b)

Fig. 3.5 Measurements of laser diode wavelength variation with drive current (a), and with heat sink temperature (b) made using the dual-wavelength Michelson interferometer.

external cavity longitudinal mode spacing using this method are shown in Table 3.1 along with the values expected for the length of the external cavity used. It can be seen that in a number of cases the measured external cavity longitudinal mode spacing is lower than the expected value by a factor of 2; this phenomenon is discussed and explained in Chapter 6. Apart from this effect, the measured and predicted values show good agreement with a mean ratio of 1.02 and a standard deviation of 0.08. The major source of error in these results was the measurement of the external cavity length; it can be seen that the first four results for which were recorded with the same external cavity show a systematic error which gives measured mode spacings lower than the expected value or half the expected value.

External cavity length, l_{ext} (mm)	Measured mode spacing, $\Delta\nu_{xcm}$(MHz)	Predicted mode spacing, $c/2l_{ext}$ (MHz)	$\frac{\Delta\nu_{xcm}}{c/2l_{ext}}$
102	728	1470	0.495
102	1461	1470	0.994
102	696	1470	0.473
102	718	1470	0.488
107	655	1402	0.467
107	684	1402	0.488
166	773	904	0.855
181	907	829	1.09
213	796	704	1.13

Table 3.1 Comparison of measured and predicted external cavity longitudinal mode spacings.

This method gives a simple and effective technique for studying the spectral effects of weak feedback on a laser diode and enables the feedback strength to be measured by the method described in the following section.

3.6 Measurement of feedback strength

The methods described above for estimating the strength of feedback in a system involves a number of simplifications and approximations and the alignment of the system is unstable and so it is desirable to be able to measure directly the feedback strength. The measurement and control of the strength of feedback, although of critical importance to the behaviour of the laser diode, are seldom referred to explicitly in published work involving feedback. One simple method that has been used to measure feedback strength *(Kato et al., 1995)* is to take a sample of the reflected beam using a beamsplitter and measure its intensity; this, however, does not take into account the coupling losses at the laser diode and so cannot be considered to be reliable. Another method *(Acket et al., 1984)* involves fitting experimental measurements of the feedback-induced frequency variations made using a Fabry-Perot interferometer to theoretical curves, which is somewhat complicated and time-consuming. The remainder of this section describes a quick and simple method of measuring the feedback strength in a system without disturbing the system, based on measurement of the effects of the feedback on the laser diode frequency.

In section 2.5 it was shown that when C>1 and mode-hopping occurs between external cavity longitudinal modes then upward and downward mode-hops show hysteresis with the magnitude of the hysteresis being a function of the feedback strength:

$$\Delta \nu_{hys} = \frac{1}{\pi \tau_2}\left(\sqrt{C^2 - 1} - \arctan\sqrt{C^2 - 1}\right) \tag{3.7}$$

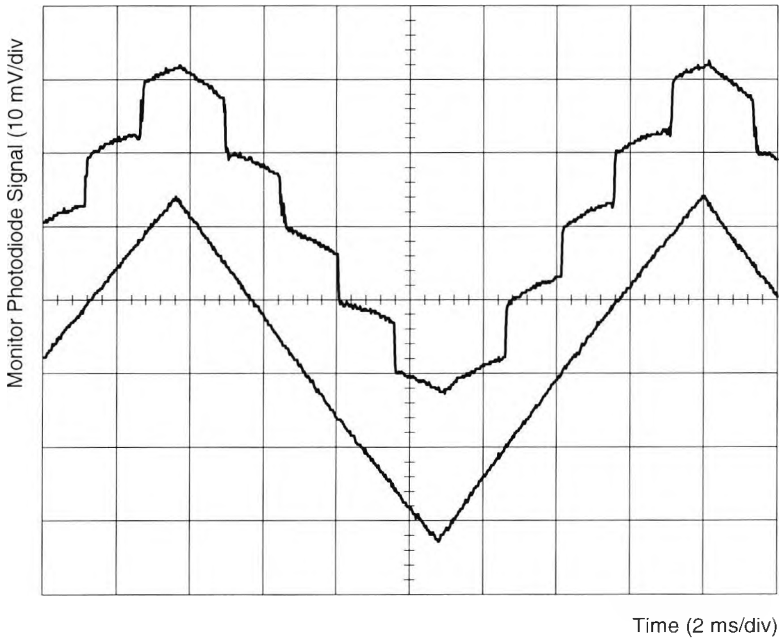


Fig. 3.6 Oscilloscope trace showing monitor photodiode output for a laser diode with feedback (upper trace) and without (lower trace) with triangular wave drive current modulation.

The feedback strength can thus be measured by measuring the frequency separation of the corresponding upward and downward mode-hops due to this hysteresis.

This was done by modulating the laser diode drive current with a triangular wave and digitally recording the resulting oscilloscope trace of the monitor photodiode output; a typical trace is shown in Fig. 3.6 along with the trace obtained with the same current modulation applied to the solitary laser diode. A computer program was written to find the turning points in the triangular wave output of the solitary laser diode, identify the positions of the mode hops and, by converting the monitor photodiode signal to frequency as described in the previous section, calculate the mean hysteresis frequency separation. Fig 3.7 shows the trace after reflection around the turning point in the modulation signal and the hysteresis frequency separation can clearly be seen.

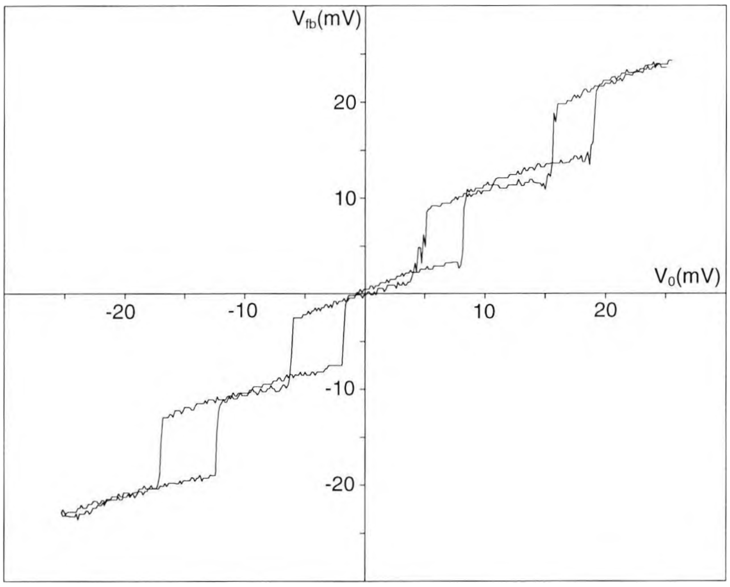


Fig. 3.7. Plot of monitor photodiode output with feedback, V_{fb}, against output without feedback, V_0, with triangular wave drive current modulation.

It has been reported previously (*Saito et al., 1982*) that under dynamic conditions the mode-hops between the external cavity longitudinal modes occur at the turning points of the wavelength variation curve of Fig. 2.4, whereas under static conditions mode hopping occurs at random within the region over which there are multiple external cavity longitudinal modes. Fig. 3.8 shows an oscilloscope trace showing the intensity changes from a laser diode undergoing mode-hopping between external cavity longitudinal modes under stationary conditions with a measured feedback coefficient of C=1.54. The typical time-scale for the mode-hopping here is of the order of 10ms, which, with a drive current modulation frequency of about 50Hz, is large compared to the time in which the multiple external cavity modes exist which in the example shown in Fig. 3.8 is approximately 2ms. With these time constants, the position of the mode hop would, in the majority of cases, not be affected by random changes and the error in the measured hysteresis due to random mode-hopping would be small.

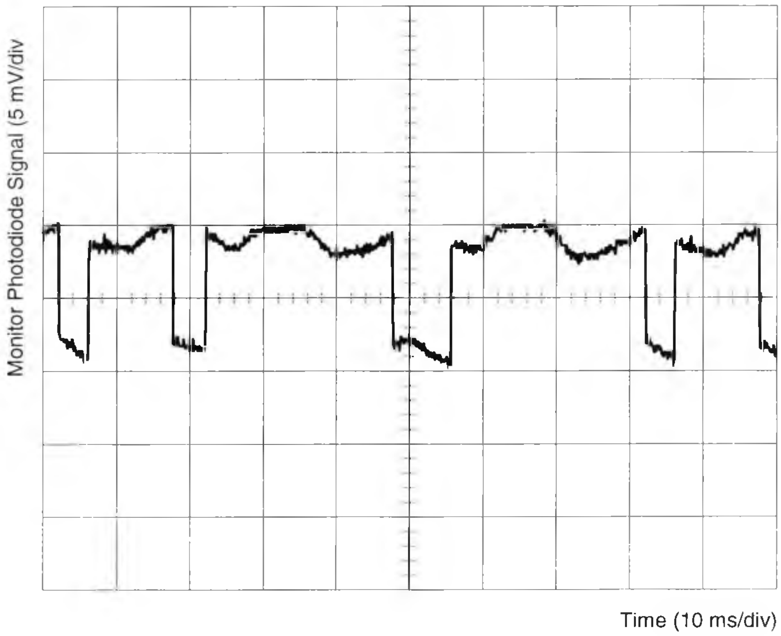


Fig. 3.8 Oscilloscope trace showing laser diode optical power output as mode-hopping occurs between external cavity longitudinal modes under stationary conditions.

It should be noted that this method is applicable to cases where the feedback strength is in the range $1<C<4.6$; for weaker feedback there is no hysteresis and for stronger feedback there may be more than two external cavity longitudinal modes available to the laser and the mode-hopping becomes unpredictable. This does not present a serious problem as this range covers the useful range of feedback strength for self-mixing interference applications, and in cases where the feedback strength is outside this range, by placing filters of known density in the external cavity, the feedback strength can be brought into the measurable range and, by taking into account the filter density, the original feedback strength can be calculated.

This method has the advantage that, relying only on modulation of the laser diode drive current, there is no physical disturbance to the system and so there is no danger of changing the measured quantity by making the measurement.

3.7 Summary

This chapter has dealt with a number of issues which are of importance for developing an effective experimental configuration using a laser diode with external optical feedback. These include the accurate estimation and measurement of feedback coupling strength, the effects of the alignment of the components and methods for ensuring good alignment, and the measurement of wavelength changes in the laser diode due to feedback.

The techniques described here are used in the following chapters to investigate the application of optical feedback in laser diodes to interferometric optical sensing. Chapter 4 describes the demonstration of the principles of operation of a self-mixing interference-based displacement sensing system and the following chapters examine in greater detail issues relating to the thermal and mechanical stability of such a system.

Chapter 4: Optical Sensing Using Self-Mixing Interference

4.1 Introduction

The theory described in Chapter 2 has shown that feedback in a laser diode, under the correct conditions, can be use to generate a phase-dependent variation in the output intensity similar to that obtained from a conventional interferometer such as the Michelson interferometer. A number of sensing techniques have been proposed based on this. The simplest and most common application of feedback in laser diodes is for a range-finding sensor *(Beheim & Fritsch, 1986; Shinohara et al., 1992)* in which the feedback phase, $2\pi\nu\tau_2$, is varied by modulating the laser current and hence the frequency with a triangular waveform, ν; the round-trip time for light between the laser and the target reflector, τ_2, can be determined by measuring the period of the feedback-induced variations in intensity. This technique is analogous to frequency-modulated continuous-wave interferometry performed with a conventional interferometer *(Kikuta et al., 1986; Collins et al.,1993)*.

If the target is moving longitudinally the variation of phase and hence the modulation frequency is increased during one half-period of the current modulation and decreased during the other and so this technique can easily be extended to measure the velocity of the target *(Shinohara et al., 1986)*. The asymmetry of the laser diode intensity modulation waveform, as shown theoretically in Fig. 2.5(a) enables the direction of motion of the target to be determined by spectral analysis of the signal obtained from the laser diode*(de Groot et al., 1988)*.

Relatively little work has been done on the application of feedback in laser diodes to displacement sensing although the principle of counting the discontinuities due to mode-hopping between external cavity longitudinal modes, the direction of which provides information about the direction of the displacement, is not complicated. Donati et al. *(1995)* describe such a system with a half-wavelength resolution over a dynamic range of 1.2m.

This chapter describes a series of experiments carried out to demonstrate the basic principles of operation of a self-mixing interference-based sensing system, to determine the optimum components and configuration of such a system, to quantify the possible range of operation, and to identify some of the main problems and limitations in its use.

The system envisaged here is one for the measurement of large-scale displacement based on fringe counting using the intensity modulation produced by self-mixing interference as a target reflecting light back into a laser diode cavity moves along the optic axis of the laser diode. This type of system would be applicable, for example, to the automated control of machine tools. The basic principles discussed here may also be applied to sensing systems for, for example, absolute distance or vibration.

The simplest possible set-up for a self-mixing interference-based system consists of a laser diode and an external reflector. Laser diodes are available with wide variations in properties such as wavelength, output power and spectral width and the effects of these characteristics is considered. The reflector may be a plain mirror, a prism (right-angled or corner cube), or else feedback may be from a diffusely scattering surface, for example plain white paper, or a retro-reflecting microprism-embedded paper; each of these options is considered.

In addition to these minimum components it is usual to use a collimating lens to gather the light emitted by the laser and refocus the fed-back light. Also it is possible to use optical fibre coupling to enable the output end of the system to be detached from the laser diode. The suitability of different lenses and fibres for these purposes will be considered.

4.2 Choice of laser diode

It has been suggested *(Wang et al., 1993)* that a multimode laser diode is adequate for sensing applications using self-mixing interference because the spacing of external cavity modes, satisfies the condition $m\lambda_1=(m+1)\lambda_2$, where λ_1 and λ_2 are the wavelengths of adjacent external cavity longitudinal modes and m is the order of the first mode, and so the self-mixing interference fringes for each wavelength are in phase and the maximum signal is observed for all target distances. This effect is used to explain the observation that, with a multimode laser diode, the visibility pattern envelope for self-mixing interference is much wider than the corresponding envelope obtained with a Michelson interferometer. However the above condition does not apply universally to the longitudinal modes of the laser diode cavity and so the visibility pattern for self-mixing interference still has minima of zero amplitude with a spacing equal to the length of the laser

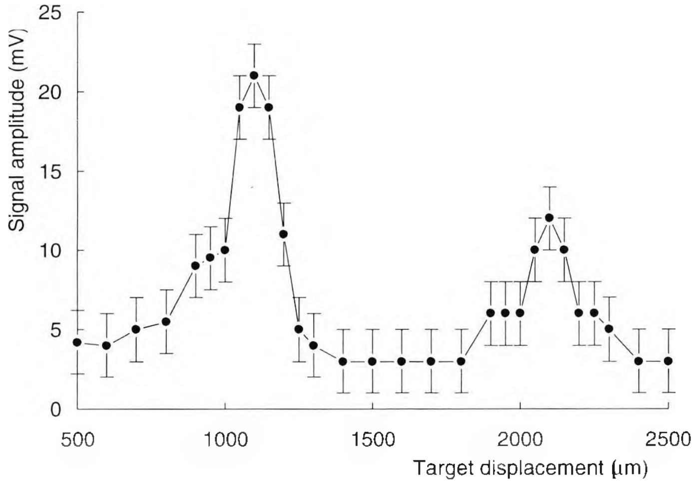


Fig. 4.1. Self-mixing interference fringe amplitude measured against longitudinal target displacement with a multimode laser diode.

diode cavity which occur when the condition $m\lambda_a=(m+\frac{1}{2})\lambda_b$ is satisfied, where λ_a and λ_b are the wavelengths of adjacent laser diode longitudinal modes and m is an integer, as is the case for the Michelson visibility pattern. An example of the measured self-mixing interference visibility pattern for a Sharp LTO23MD multimode laser diode is shown in Fig. 4.1. This measurement was obtained with light coupled from the laser into a fibre and a mirror placed close to the end of the fibre; there is no collimation of light emerging from the fibre which accounts for the decrease in the peak amplitude as the mirror is displaced further from the fibre end. The peak spacing of approximately 1mm is equal to the optical length of the laser diode cavity.

With a single longitudinal mode laser diode, e.g. the Sharp LT015MD, strong feedback can cause the laser to become multi-mode, as is shown by the self-mixing interference visibility pattern of Fig. 4.2 which has similar features to that of the multimode laser visibility pattern of Fig. 4.1. The visibility pattern recorded with the same laser diode with a neutral density filter placed in the external cavity

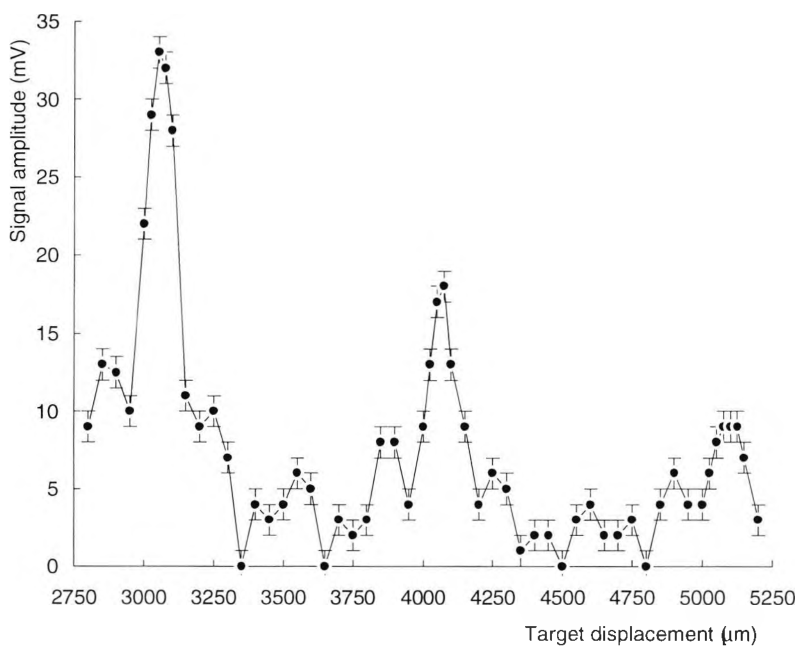


Fig. 4.2. Self-mixing interference visibility pattern for a multimode laser diode with strong feedback.

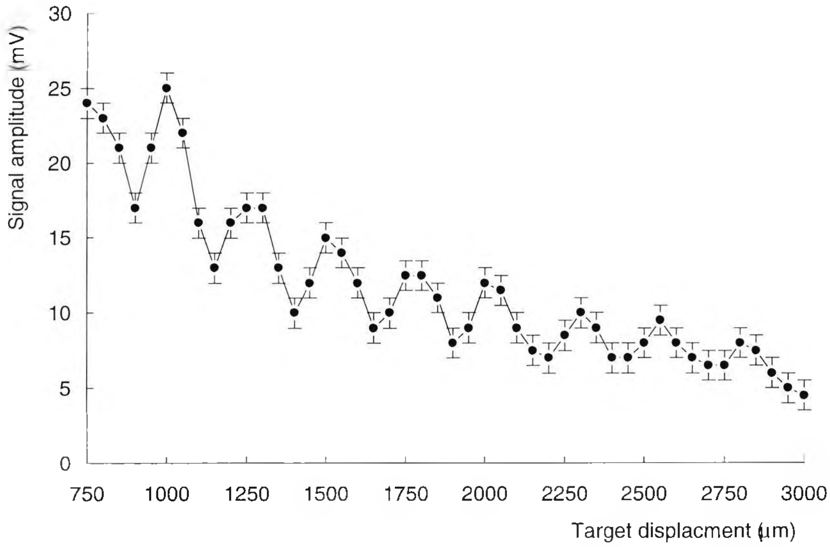


Fig. 4.3. Self-mixing interference visibility pattern for a single mode laser diode with feedback controlled by a neutral density filter.

to reduce the level of feedback is shown in Fig. 4.3. In this case the behaviour is as would be expected from a single longitudinal mode laser diode and there are no localised regions where the fringe visibility approaches zero; the small variation with a period of approximately 0.25mm is probably caused by an oscillation of the mirror related to the movement of the micrometer stage on which it was mounted.

It is obvious therefore that for a displacement sensing system to work continuously over an extended range, a single longitudinal mode laser diode is required.

4.3 Choice of external reflector

The choice of reflector from which light is returned to the laser diode is subject to considerations such as stability, the level of feedback obtainable, and the possibility of speckle in the light fed-back into the laser.

Possible choices for the target material include a plane mirror, a retro-reflecting prism, a scattering surface such as white paper or retro-reflecting microprism embedded paper, each of which has particular advantages and disadvantages, as discussed below, in practical situations.

With a white paper target the amount of light returned to the laser is lower than with a mirror and is also strongly dependent on the distance of the target from the laser. These problems can be alleviated by the use of retro-reflecting paper but this has another disadvantage, which is the creation of laser speckle. Speckle is a random far-field interference pattern formed when coherent light is reflected from a rough surface by the superposition of wavefronts reflected in all directions from all illuminated parts of the surface *(Hecht, 1987)*. The characteristic speckle dimensions for a speckle pattern imaged by a lens is equal to the point spread size for the lens given by the Rayleigh condition:

$$\Delta x = \frac{1.22\lambda f}{D} \qquad (4.1)$$

where λ is the wavelength of the light and D and f are the diameter and focal length of the lens respectively. With a laser diode wavelength of around 800nm and a microscope objective lens with a numerical aperture of 0.25, the speckle size is approximately 1.7μm. The laser diode aperture has dimensions of several microns and so the speckle size is quite significant in relation to this.

The exact distribution of the speckle pattern varies with the distance to the scatterer and so the level of feedback can change significantly with the movement of the target, resulting in the signal obtained suffering severely from intensity noise. An example of a self-mixing interference visibility pattern obtained with a retro-reflecting paper target is shown in Fig. 4.4., in which the noise due to speckle has swamped the signal.

With a plane mirror used to reflect light back into the laser, the problem of speckle is avoided, the level of feedback is high and filters can be used to control it accurately. However the small size of the laser diode aperture means that the

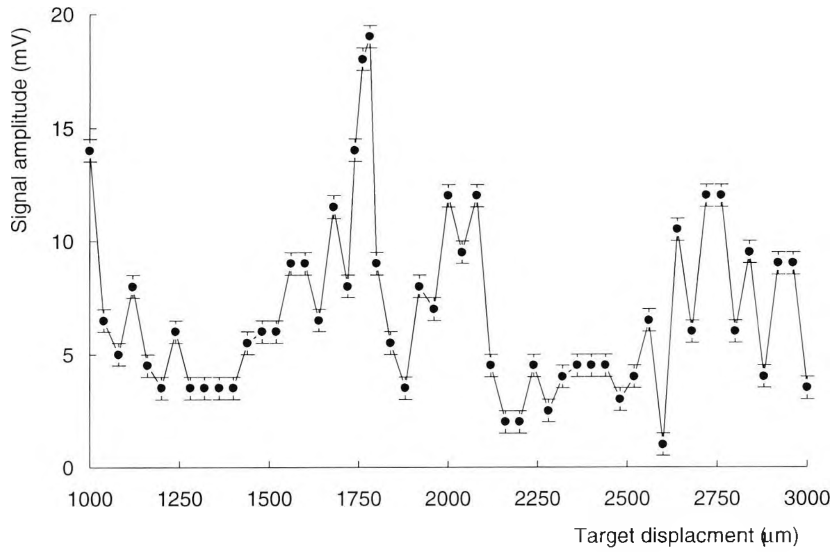


Fig. 4.4. Self-mixing interference visibility pattern recorded using a micro-prism embedded retro-reflecting paper target.

system is very sensitive to the alignment of the mirror. The question of mirror alignment was considered in some detail in Chapter 3; the sensitivity to alignment for a typical system was quantified and a method of achieving good alignment was described. One simple way to alleviate the problem is the use of a retro-reflecting corner cube prism, although there may be some advantage to being able deliberately to deflect the reflected beam away from the optical axis, as will be shown in Chapter 6. The sensitivity to mirror alignment may also be reduced by focusing the collimating beam onto the mirror with a second lens, although this arrangement is somewhat bulky at the probe end for a practical sensing system.

4.4 Other components

The most suitable type of collimating lens for use for a system using a laser diode with feedback was found to be a ×10 magnification (0.25 numerical aperture) microscope objective. The numerical aperture of this type of lens is large enough to be reasonably well matched to the divergence of the output of most laser diodes and not so large that the small depth of focus or short focal length make positioning the lens difficult. In situations where space is important, a graded index rod lens can be used.

For a system with optical fibre, as opposed to free-space coupling, it was found that although a 50μm core multimode fibre was easier to align and could transmit more light, the mode speckle pattern produced at the fibre output made the level of feedback very sensitive to small changes in alignment. A single mode fibre is therefore preferable for this type of system in order to eliminate such effects.

4.5 Demonstration of a sensing system using self-mixing interference

Using the components discussed above, the arrangement shown in Fig. 4.5 was used to demonstrate the principle of operation of a displacement sensing system and the possible range of operation achievable. The feedback is from the mirror, M1 which is mounted on a loudspeaker cone to which a sinusoidal signal is applied in order to provide phase modulation. In this case the feedback is coupled back into the laser via a single mode optical fibre.

Fig 4.6 (a)-(c) shows traces plotted from a digital oscilloscope with self-mixing interference fringes obtained for target distances ranging from 185±5mm to 1130±5mm. The fringes clearly show the expected direction-dependent asymmetry

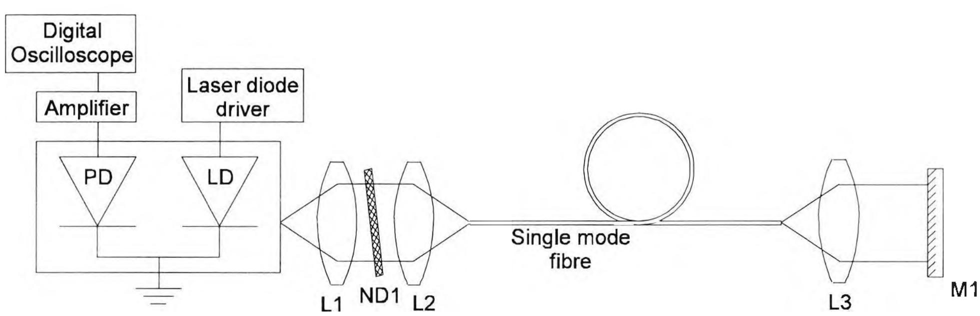
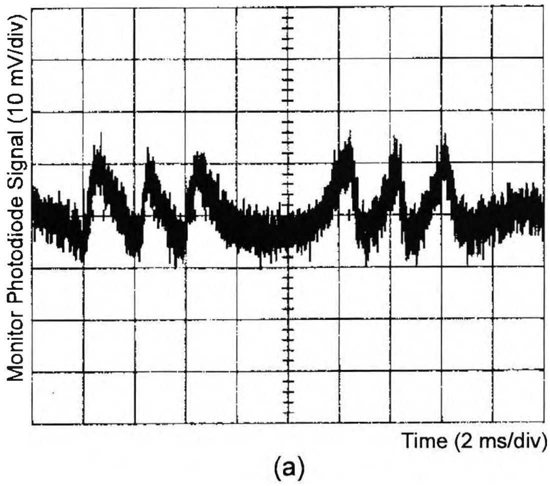
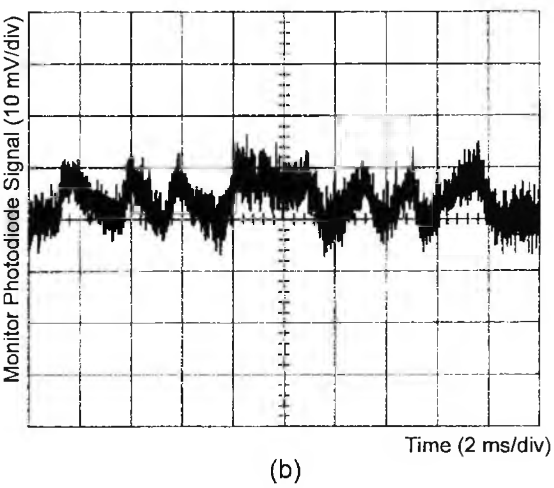


Fig. 4.5. Experimental arrangement used to demonstrate a self-mixing interference-based sensing system.





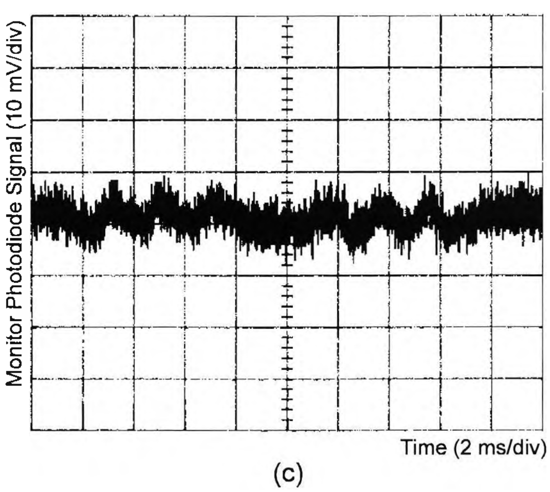


Fig. 4.6. Oscilloscope traces showing self-mixing interference fringes for target distances: (a) 185±5mm, (b) 775±5mm, (c) 1130±5mm. Horizontal scale: 2ms/div. Vertical scale: 10mV/div.

and are still visible for the largest target distance although their amplitude is reduced, due mainly to the difficulty of maintaining the quality of the beam profile over that large a distance.

The length of the fibre coupling used in this experiment was approximately 270mm; this length does not have a strong effect on the results as long as the total optical path length from the laser to the target reflector is within the coherence length of the laser diode, which in this case was of the order of 10m. However in a range finding sensor in which the laser diode drive current is modulated in order to provide the phase modulation, the increased total length of the external cavity due to fibre coupling increases the frequency of the signal obtained offering a potential increase in the resolution of the system. The use of an optical fibre delivery system gives advantages to a system in terms of versatility with the sensing head able to be positioned independently of the laser diode which can be placed in a controlled environment. The fibre performs a spatial filtering operation on the beam which eliminates the astigmatism present in the output of a laser diode which along with the lower beam divergence of the output from a fibre enables the collimation of the output beam to be improved and the range of target displacements for which the feedback coupling strength can be kept within the necessary limits in order to obtain a self mixing interference signal can be increased. However care has to be taken to avoid possible effects of feedback from the ends of the fibre. In the system shown in Fig. 4.5 the feedback from the fibre ends is significantly less than that from the target reflector due simply to the fact that the target reflector has much greater reflectivity and no effects due to feedback from the fibre ends were observed. Ideally the fibre ends should be angle-polished in order to eliminate this potential problem. Another possible disadvantage of using a fibre coupling is the sensitivity of the optical length of the fibre to temperature variations which introduces an extra source of noise to the system.

These results show that it is possible to obtain self-mixing interference fringes for target distances over one metre, although at that distance for these results the signal to noise ratio is rather low. The noise originates mainly in the output of the laser diode itself rather than in the monitor photodiode or the subsequent electronics. Laser diodes are naturally noisy with fluctuations in intensity coused by spontaneous emission events giving rise to fluctuations in frequency due to the coupling between gain and refractive index. These frequency fluctuations are an additional source of noise in an interferometric system due to the associated fluctuations in phase. Other sources of noise include vibration, which can affect the phase and the strength of the feedback, and fluctuations in the laser drive current and temperature. The amplifier attached to the photodiode in this experiment was used in conjunction with a simple low-pass RC filter with a cut-off frequency of approximately f_c=2.5kHz; the fringe frequency was approximately 500Hz which corresponds to a mirror velocity of approximately 1.6 $\times 10^{-4}$ ms^{-1}. Higher mirror velocities would obviously lead to higher fringe frequencies and so a low-pass filter such as the one used here would not be appropriate. In a practical displacement sensing system close attention needs to be paid to the separation of the signal from the noise using appropriate filters for the range of mirror speed and fringe frequency expected.

4.6 Summary

The experiments described here have shown that optical feedback in a laser diode can be used, with a suitable choice of components, to make a simple interferometer suitable for detecting displacement for target distances of up to one metre.

However, with a target distance of over one metre the interferometer output becomes very sensitive to variations in wavelength which are caused in a laser diode by fluctuations in the temperature of the laser and in the laser drive current. The effects of the wavelength instability of a laser diode in an interferometric system and possible methods of reducing that instability using optical feedback are considered in Chapter 5.

A system based on feedback in a laser diode is also sensitive to mechanical factors, in particular the alignment of the external reflector. A possible method for reducing the sensitivity of the system to reflector alignment is presented in Chapter 6.

Chapter 5: Optical Feedback and the Frequency Stability of Laser Diodes

5.1 Introduction

The sensitivity of the output frequency of a laser diode to changes in environmental temperature, and to a lesser extent to variations in the laser drive current, is a well-known problem for laser-diode based interferometric systems *(Bobroff, 1993)*; this chapter explains the physical basis of these effects and shows how they affect the performance of an interferometric sensing system.

The effects of optical feedback on the spectral output of laser diodes have also been studied extensively. The relatively small frequency variations due to weak feedback, within and between longitudinal modes of the external cavity, have been discussed in Chapter 2. In addition to this, at higher levels of feedback the laser undergoes coherence collapse *(Lensta et al., 1985)* and for very strong feedback the reflectivity of the laser front facet becomes insignificant relative to that of the external reflector and the laser behaves as a single cavity laser with a longer cavity *(Tkach & Chraplyvy, 1986)*. This chapter examines methods by which strong feedback has been used to control the spectral behaviour of laser diodes and examines the possibility of using weak feedback to reduce the sensitivity of the laser diode frequency to temperature or current variations in order to increase the potential accuracy of laser-diode based interferometric sensing systems.

5.2 Theoretical Background

The behaviour of a laser diode with respect to temperature is governed by two temperature-dependent quantities, the gain spectrum peak and the laser cavity optical length.

The population inversion condition for gain requires the Fermi energy levels for electrons in the n-type region and holes in the p-type region to be inside the conduction and valence bands respectively. Gain is then experienced for transitions between energy levels above the Fermi level in the valence band and below the Fermi level in the conduction band. The gain spectrum, the range of frequencies for which gain may be experienced, is limited by the band gap and by the difference between the two Fermi levels. Increasing temperature lowers the Fermi level for electrons in the n-type region and raises it for holes in the p-type due to intrinsic electron-hole pair creation by thermal excitation. This reduces the upper limit of the gain spectrum and also lowers the spectrum peak frequency. The magnitude of this effect is typically about 130GHz/K or 0.3nm/K.

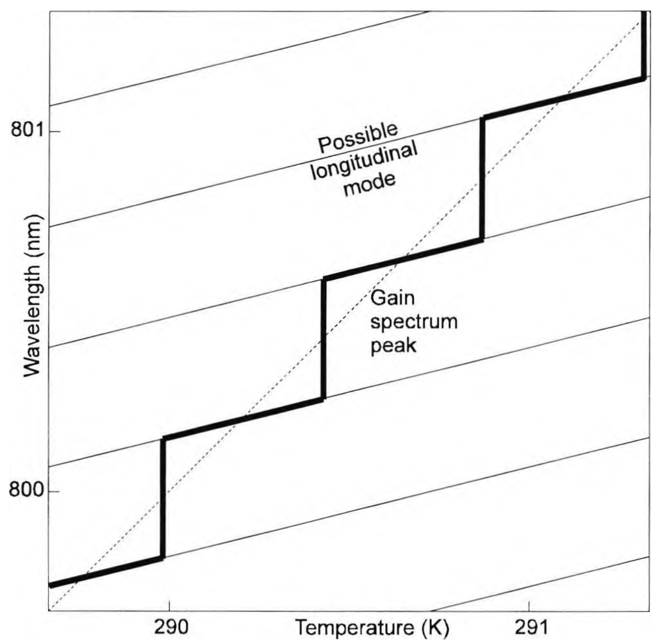


Fig. 5.1 Typical variation of laser diode wavelength with temperature.

The dependence of the cavity optical length on temperature is due to the combined effects of refractive index variation and thermal expansion of the laser material. The temperature coefficient of refractive index ($\partial n/\partial T$, where n is refractive index and T is temperature) for gallium arsenide is approximately 3.2×10^{-4} for wavelengths near 1μm and the thermal expansivity ($1/L.\partial L/\partial T$, where L is length) is about $5.7\times10^{-6}K^{-1}$ at room temperature *(Donaldson & Edwards, 1983)*. For a laser diode operating at 830nm the resulting wavelength tuning coefficient is expected to be about $0.08nmK^{-1}$.

A single longitudinal mode laser diode will tend to operate in the longitudinal mode closest to the gain spectrum peak and so the overall variation of frequency with temperature has the form shown in Fig. 5.1 where, as temperature increases about a typical operating temperature of 290K, the frequency determined by the longitudinal mode diverges from the gain spectrum peak and an adjacent longitudinal mode converges on the gain spectrum peak frequency. Then at some point the laser will hop from one longitudinal mode to the neighbouring one, resulting in the stepped frequency variation shown in Fig. 5.1.

Variations in drive current affect the laser diode optical frequency mainly by their effect on the temperature of the laser. The thermal conductivity for gallium arsenide is $0.48W(cmK)^{-1}$. The thermal conductivity, Θ, is defined for a piece of material by

$$\Theta = \frac{P(x)}{A\,^{\partial T}\!/_{\partial x}}\,, \qquad (5.1)$$

where P is the power transmitted, A is the cross sectional area of the piece and $\partial T/\partial x$ is the temperature gradient through the material. For the laser diode, which consists of a single object subject to resistive heating with a heat sink at one end, the power, P, is a function of x and is equal to the power generated to the nonheat sink side of the position, x. If x is measured from the heat sink then, for a uniform current distribution throughout the laser chip,

$$P(x) = \left(1 - \frac{x}{L}\right) P_0, \qquad (5.2)$$

where P_0 is the total power generated in the laser diode and L is the length of the diode. The temperature as a function of position is then given by

$$\begin{aligned} T &= T_0 + \frac{1}{\Theta A}\int_0^x P(x)dx \\ &= T_0 + \frac{P_0}{\Theta A} x\left(1 - \frac{x}{2L}\right) \end{aligned} \qquad (5.3)$$

were T_0 is the temperature at the heat sink. The total power, P_0, is related to the laser diode drive current, J, and the voltage across the diode, V, by $P_0=VJ$, and so the variation of temperature with the drive current as a function of position is given by:

$$\frac{\partial T}{\partial J} = \frac{V}{\Theta A} x\left(1 - \frac{x}{2L}\right). \qquad (5.4)$$

A typical Sharp laser diode such as has been discussed in previous chapters has a cross section with respect to the current flow of 250μm×300μm and height of 125μm, with the active layer approximately in the centre of the chip and the operating voltage is typically 1.75V. These figures give an estimated temperature variation at the active region of $\partial T/\partial J$=0.02K/mA. However, this calculation has not taken into account that the current is not uniform throughout the diode but is guided through a narrow strip at the active region; the power distribution and the effective cross-section area are therefore changed. The effect of channelling the current through a narrow region is to increase the power generated in that region; the effect of this on the integral of P(x) in equation (5.3) is a small increase with respect to the uniform current distribution but the reduction in the effective cross-section area through which the power is transmitted to the heat sink should give an increase of about a factor of two. An estimate of about 0.05K/mA can therefore be made for the dependence of the temperature of the laser diode on its drive current which lead to values for the wavelength dependence on current of 0.015 nm/mA for the gain spectrum peak and 4×10^{-3} nm/mA for the longitudinal mode.

Measurements made with the laser diodes used in this work using the dual wavelength Michelson interferometer described in Chapter 3, give mean tuning coefficients of -3.1±0.4 GHz/mA and -29.8±0.7 GHz/K for current and temperature tuning respectively. The increase in the measured value for the current tuning coefficient over the theoretically estimated value may be due to the increase in the cavity refractive index due to the increased charge carrier density when the current is increased.

For the figures given, the mean width of each longitudinal mode in terms of temperature is approximately 1.5K and so it is not difficult to stabilise the laser diode sufficiently with respect to temperature to ensure single longitudinal mode operation, although for applications in which a large continuous tuning range or a specific wavelength is required, the limited tuning range and the gaps in the spectrum may cause problems. However, in developing an interferometric sensing device using a laser diode, accuracy will be limited by the variation of the wavelength within the longitudinal mode with changes in environmental temperature and to a certain extent with fluctuations in the drive current. The following sections quantify this effect and examine the possibility of using the variations in output frequency caused by weak optical feedback in a laser diode to reduce the sensitivity of the output frequency of a laser diode to such variations.

5.3 The effects of temperature sensitivity in interferometry

As mentioned in the previous section, a laser diode operating in a single longitudinal mode responds to temperature variations with a frequency shift of about -30GHz/K. The most common method of limiting this effect is to control the temperature of the laser. This is usually achieved by mounting the laser on a heat sink connected to a Peltier heat exchanger *(Okoshi & Kikuchi, 1980)*. The temperature of the laser diode may monitored using a thermistor embedded in the heat sink near the laser; the resistance of the thermistor is compared to a fixed reference to determine the current applied to the Peltier element. Such systems are limited mainly by their time response characteristics; a balance has to be achieved between a reasonable response time and overshooting and oscillation which occurs with too fast a response. Typically the temperature of a laser diode can be stabilised to within ±10mK using such a system *(Saunders & Kane, 1992)*.

In a self-mixing interference-based displacement measuring system using fringe counting, the order of the fringe, m, is given by $m\lambda=2L_2$, were L_2 is the external cavity length and λ is the wavelength of the laser light, and so a change in the output frequency of the laser diode, as well as a change in the length of the external cavity, will lead to a change in the fringe order and thus give rise to a false reading from the sensor.

The rate of change of the fringe order with respect to optical frequency, and hence the magnitude of the error due to fluctuations in temperature and drive current, is given by:

$$\frac{\partial m}{\partial \nu} = \frac{2L_2}{c} \qquad (5.5)$$

With a sensitivity to temperature of -30GHz/K and a temperature stability of ±10mK, the resulting uncertainty in frequency is ±300MHz. This instability places limits on either the accuracy of the device or the maximum distance of the target reflector from the laser diode. For example, with the above conditions, the maximum target distance is given by:

$$L_2 = \frac{c\Delta m}{2\Delta \nu}, \qquad (5.6)$$

where, for single finge accuracy, Δm is equal to one and Δv is equal to 300MHz, which gives a maximum distance of 0.5m. It has been demonstrated in Chapter 4 that self-mixing interference systems are capable of operating over greater distances than this and so the sensitivity of the laser diode wavelength to temperature is a major limiting factor in the performance of such systems. The following sections examine the possibility of using the effects on the frequency of optical feedback to stabilise the frequency of a laser diode with respect to temperature.

5.4 The use of optical feedback to extend the tuning range

Two different methods using optical feedback have been used to extend the range of single longitudinal mode operation in laser diodes. Each technique involves reducing the sensitivity to external factors of the gain spectrum peak.

The first method, used by Sharp in their LT080 laser diode *(Laser Diode Users Manual)*, uses an external cavity which is shorter than the actual laser diode cavity. The gain spectrum is modified by the feedback according to equation (2.22), and if the thermal expansion of the external cavity can be controlled then the peak of the modified gain spectrum can be controlled and its variation with respect to temperature reduced.

The second method *(Agrawal, 1986)* uses strong frequency selective feedback from a filter or a diffraction grating. If the feedback is strong enough then there is a reduction in threshold gain for all values of external cavity phase for a narrow range of frequencies at the filter pass band. The laser will then

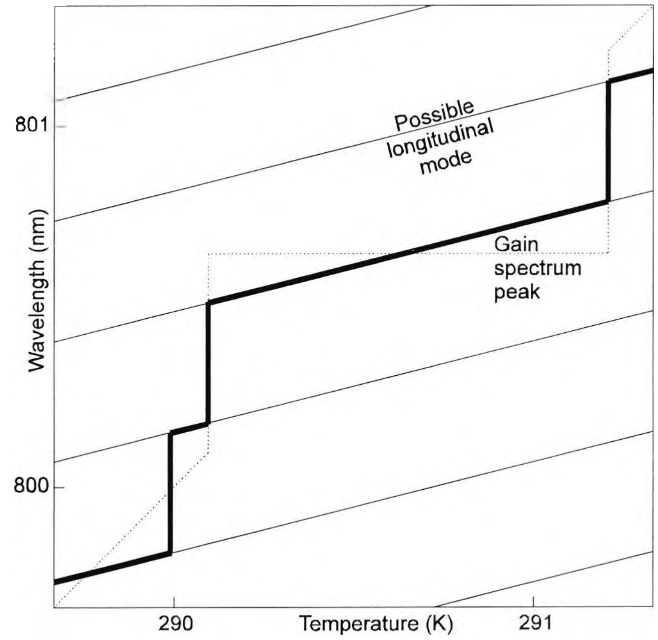


Fig. 5.2 Typical variation of wavelength with temperature for a laser diode with longitudinal mode stabilisation.

operate at around the filter frequeny over a range of temperatures or drive currents for which the threshold gain reduction due to feedback is greater than the difference between the gain in the solitary laser at that temperature or current and the peak gain.

In both cases the result is a flat region in the variation of the gain spectrum peak with respect to temperature, as shown in Fig 5.2, and there is a greater temperature range over which one longitudinal mode is closest to the gain spectrum peak, and over which the laser will operate in that mode.

5.5 The use of frequency selective feedback to modify the frequency characteristics of a laser diode

Several methods of selecting the frequency of the light fed back to a laser diode are available including filters, atomic absorption cells *(Valenzuela et al., 1988; Ludvigsen & Holmund, 1992)*, Fabry-Perot cavities *(Favre & Le Guen,* *1980)* and diffraction gratings *(Sato et al., 1982, Maki et al., 1993)*. The usual use of these devices is with strong, phase independent feedback which reduces the threshold gain across a narrow band around the wavelength to which the device is tuned *(Agrawal, 1986)*. If the gain spectrum peak coincides exactly with the centre of the feedback spectrum then the gain is pinned at the reduced threshold at this point and, because laser diodes are dominated by homogeneous line broadening, the entire gain spectrum curve is shifted down by the same amount. As the gain spectrum shifts due to a temperature change then, assuming that the feedback spectrum is narrow compared to the gain spectrum, the reduction in threshold gain at the centre of the feedback spectrum may be greater than the difference between the gain at the peak of the gain spectrum and the gain close to the centre of the feedback spectrum, and so the laser will continue to operate at a wavelength close to the centre of the feedback spectrum. This effectively results in a region in which the variation of the gain spectrum peak with temperature is reduced and so the frequency of mode-hopping, which results from the difference between the temperature variations of the gain peak spectrum and the permitted longitudinal mode wavelengths, can be reduced.

In the case of weak feedback, in order to modify the wavelength characteristics of the laser, it would be necessary for the width of the feedback spectrum to be less than the external cavity longitudinal mode spacing. Taking the case of a diffraction grating as an example, for a grating with 1800 lines/mm and an incident wavelength, λ, of 830nm, the angular dispersion in the first order is given by:

$$\frac{\partial \lambda}{\partial \theta} = \frac{2}{N} \cos \theta . \qquad (5.7)$$

where θ is the angle of diffraction and N is the grating density in lines/mm. The angular width of the feedback distribution, Δθ, has been calculated in Chapter 3 to be approximately 0.1 mrad , and so the feedback bandwidth, Δλ, is given by:

$$\Delta\lambda = \frac{2\Delta\theta}{N}\cos\theta \tag{5.8}$$

which gives $\Delta\lambda \approx 0.1$nm.

This is greater than the external cavity longitudinal mode spacing for any external cavity length greater than about 3mm, and so for any practically realistic external cavity length the external cavity longitudinal mode spacing will be less than the feedback bandwidth. In this case the feedback would not be able to affect the wavelength behaviour on the scale of the single external cavity longitudinal mode as adjacent external cavity longitudinal modes would experience similar levels of feedback.

The bandwidth calculated above is, however, less than the typical intrinsic diode cavity mode spacing, which is normally about 0.3nm, which is why feedback can be used in this situation to alter the behaviour of the laser diode with respect to these modes.

5.6 The use of optical feedback to reduce temperature sensitivity

In Chapter 2, the following relationship between the output frequency, ν, of a laser diode with weak external optical feedback and the frequency of the solitary laser diode, ν_0, was obtained:

$$2\pi\nu\tau_1 + C\sin(2\pi\nu\tau_1 + \arctan(\alpha)) = 2\pi\nu_0\tau_1, \tag{5.9}$$

where C is the feedback strength coefficient, τ_1, is the round trip time for light in the external cavity and α is the linewidth enhancement factor. The frequency for the solitary laser diode varies approximately linearly with temperature or drive current and so the term on the right hand side of equation (5.9) can be replaced by a term proportional to either of these quantities. The variation of frequency for a laser diode with weak optical feedback with temperature therefore has the form shown in fig. 5.3. The straight diagonal line on this figure shows the variation of frequency with temperature for the solitary laser diode and typically has a gradient of about 3GHz/K. With the feedback present the frequency variation becomes non-linear, oscillating about the solitary laser diode values, and so there are regions in which the gradient of the curve is reduced with respect to that of the solitary laser diode.

Differentiating equation (5.9) with respect to the solitary laser frequency, ν_0, gives the reduction in frequency sensitivity due to feedback to be:

$$\frac{\partial \nu}{\partial \nu_0} = \left(1 + C\cos(2\pi\nu\tau_1 + \arctan(\alpha))\right)^{-1} \qquad (5.10)$$

The minimum gradient occurs at the point where the argument of the sine term in equation (5.9) is equal to $2m\pi$ and so the value of the minimum gradient is:

$$\left(\frac{\partial \nu}{\partial \nu_0}\right)_{min} = \frac{1}{C+1}. \qquad (5.11)$$

The reduction in sensitivity of the laser diode frequency to temperature variation thus increases with increasing feedback strength. However, for $C \geq 1$, it has been shown that there are regions in which there are multiple possible frequencies, the external cavity longitudinal modes, in which the laser can operate and that in these regions the laser diode will show instability, hopping randomly between the modes. The width of the region of stable, single external cavity longitudinal mode operation was given in Chapter 2 and is defined by

$$\Delta\nu_{sxm} = \frac{1}{2\pi\tau_2}\left(1 - 2\arctan\left(\sqrt{C^2-1}\right) + 2\sqrt{C^2-1}\right) \qquad (5.12)$$

where τ_2 is the external cavity round-trip time.

This function is valid for C>1; for C<1 there are no unstable, multiple external cavity longitudinal mode regions. The width of the single mode region decreases monotonically from $\Delta\nu_{sxm}=1/2\pi\tau_2$ at C=1 and so there is a trade-off to be made between the reduction in sensitivity to temperature and the external longitudinal mode stability. Solving $\Delta\nu_{sxm}=0$ numerically gives $C\approx4.61$; for feedback stronger than this there are multiple external cavity longitudinal modes for all values of ν_0, and so this places a limit on the possible reduction in the laser temperature response.

For a given value of C, the width of the single longitudinal mode region is inversely proportional to the length of the external cavity, so ideally in a practical setup for temperature sensitivity reduction, the external cavity should be as short as possible with the feedback intensity strong enough to give the desired value of C but not so strong that the system is outside the weak feedback regime.

5.7 Feedback from two external reflectors

In the preceding section the equations used were for a system consisting of a laser diode and a single external reflector. However, for a system using feedback in order to stabilise the laser in this way as well as to provide the sensing information, it is necessary to consider an arrangement with two external reflectors. The first reflector, closer to the laser diode provides the stabilising effect and is at a fixed position and is partially reflecting. The second reflector is the sensor target as in the normal system.

The theory described in Chapter 2 can easily be extended for the case of a laser diode with feedback from two external reflectors shown in Fig 5.3. If the round trip phase between the laser front facet and the first reflector, ϕ_2, and the feedback coefficient for the first reflector, ξ_1, are defined as in Chapter 2 then the round trip phase for the cavity formed by the first and second reflectors, ϕ_3, and the feedback coefficient for the second reflector, ξ_2, can be defined as:

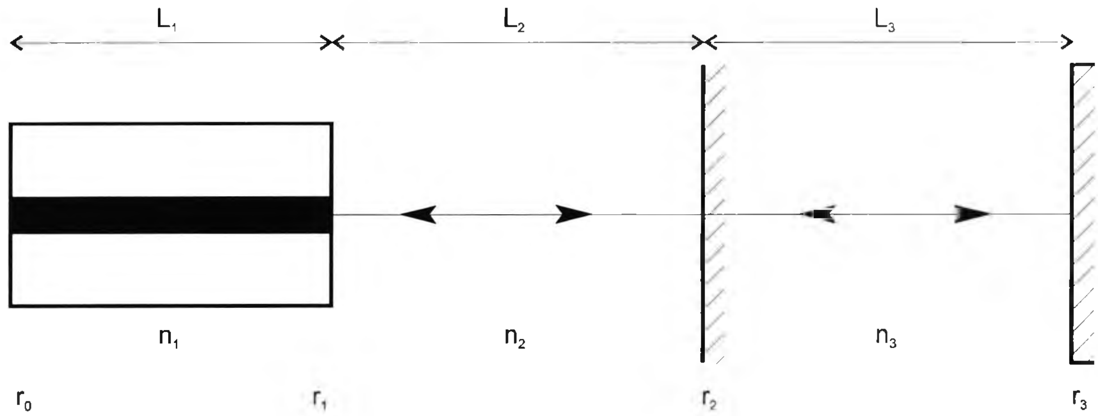


Fig. 5.3. Schematic representation of a laser diode with feedback from two external cavities.

$$\phi_3 = \frac{2\pi n_3 L_3}{c} \qquad (5.13)$$

$$\xi_2 = (1 - R_1)(1 - R_2)\frac{r_3}{r_1} \quad . \qquad (5.14)$$

where the terms are as defined in Chapter 2, with the subscript '3' referring to the second external cavity.

The threshold gain change, Δg, and the change in round trip phase, $\delta\phi$, are then:

$$\Delta g = -\frac{1}{L_1}\left(\xi_1 \cos\phi_2 + \xi_2 \cos(\phi_2 + \phi_3)\right) \qquad (5.15)$$

$$\delta\phi = \xi_1 \sin\phi_2 + \xi_2 \sin(\phi_2 + \phi_3), \qquad (5.16)$$

and the output frequency, ν, is given by:

$$2\pi(\nu-\nu_0)\tau_3 + C_1\sin(2\pi\nu\tau_3 + \arctan\alpha) + \frac{\tau_2}{\tau_3}C_2\sin(2\pi\nu(\tau_2+\tau_3) + \arctan\alpha) = 0$$

(5.17)

where C_1 is the feedback strength coefficient for the single external cavity case and C_2 is a similar coefficient for the second external reflector defined by

$$C_2 = \frac{L_2}{L_1}\xi_2\sqrt{1+\alpha^2} \qquad (5.18)$$

The first two terms here are the same as those in equation 2.22 for the laser diode with feedback from a single reflector. The term in C_2, therefore, has the effect of adding a perturbation to the form of the variation of frequency shown in Fig. 5.4 for the single external cavity. This is shown in Fig. 5.4 for the following conditions: L_1=1mm, L_2=25mm, L_3=300mm, R_1=0.075, $R_2=2\times10^{-5}$, and $R_3=10^{-6}$.

It is not possible to express equation (5.17) parametrically in terms of the external cavity phase as was the case with the single reflector due to the presence of the two phase terms, $\phi_2=2\pi\nu\tau_2$, and $\phi_3=2\pi\nu\tau_3$. However, from Fig. 5.3, it can be seen that the behaviour of the system will depend strongly on the round trip phase in the first external cavity. For $1<C_1<4.6$ there is a region in which there is a single external mode associated with the first cavity, the width of which is given by equation (2.37). The single mode region is centred around the frequency for which the argument of the first sine term in equation (5.17) is equal to an even integral multiple of π. The reduced response to temperature variation is provided by a reduction in the gradient of the ν versus ν_0 curve in the single mode region with respect to the unity gradient for the case of zero feedback, obtained by putting C=0 into equation (2.23). The gradient at the centre of the single mode region is given by;

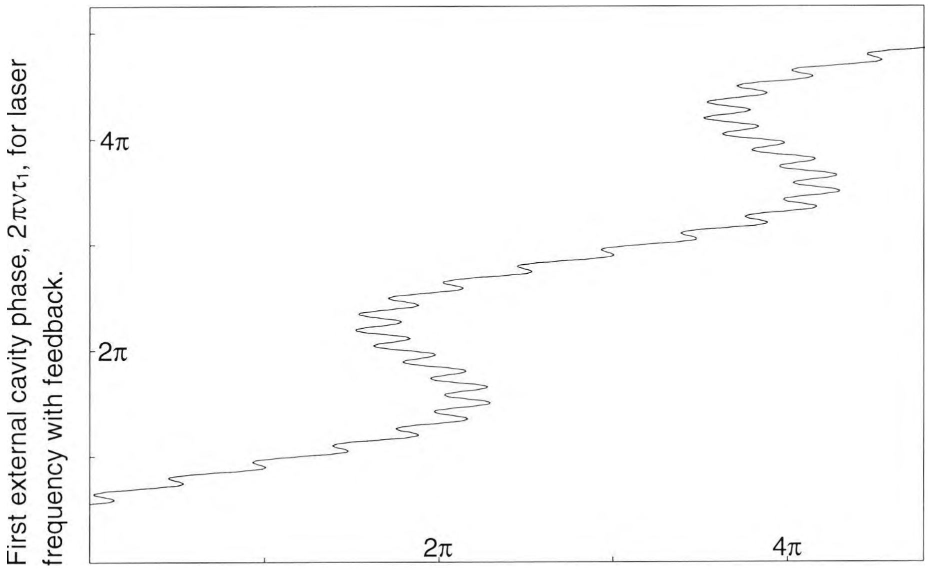


Fig. 5.4. Variation of laser frequency with feedback from two external reflectors with solitary laser frequency, expressed in terms of round trip phase in the first external cavity, for L_1=25mm, L_2=300mm, $R_2=2.5\times10^{-5}$, $R_3=10^{-6}$.

$$\frac{\partial \nu}{\partial \nu_0} = \frac{1}{C+1} \quad . \tag{5.19}$$

Using a similar derivation for intensity as was used in Section 2.4 for the single external cavity case, the following expression is obtained:

$$S = S_0\left(1+\frac{1}{L_0 g_0}\left(\xi_1 \cos\phi_1 + \xi_2 \cos\phi_1\right)\left(\frac{1}{\kappa v_g \tau_e S_0}+1\right)\right) \tag{5.20}$$

where $\phi_1 = 2\pi\nu\tau_1$ and $\phi_2 = 2\pi\nu\tau_2$ and the frequency, ν, is given by the solution to equation (5.17). For the single cavity it is possible to obtain a pair of parametric equations relating the intensity and the external cavity length or the solitary laser diode frequency to the actual emitted frequency using the phase, ϕ, as the parameter. For the double cavity case, the presence of ϕ_1 and ϕ_2 means it is not possible to define a single parameter relating the intensity to either the cavity lengths or the solitary laser frequency, even for the simplified case where the first external cavity phase is fixed.

However it has been shown for the single external cavity case that the intensity variation is broadly similar in form to the frequency variation in magnitude, and it is reasonable to expect the same to be true for the double cavity case. In particular discontinuities in the intensity variation will be seen when mode hopping occurs between the external cavity modes associated with either reflector.

5.8 Experimental Verification

In the following experiments, variations of the wavelength of laser diodes have been measured with respect to their drive currents because, in practice, the drive current can be controlled more accurately than can the temperature. However, as the wavelength variation due to changes in the drive current is mainly due to the associated temperature change, the form of the temperature variation can be inferred from the measured wavelength variation.

The variation with drive current of the wavelength of light emitted by a laser diode with feedback has been measured directly using the dual wavelength Michelson interferometer arrangement shown in Chapter 3 and the result is shown in Fig. 5.5. The measured wavelength variation shows a step pattern, with regions of reduced gradient, but the wavelength difference between the external cavity modes is only slightly larger than the uncertainty in the measured values of wavlength and so it is impossible to measure accurately the reduction in wavelength tuning coefficient.

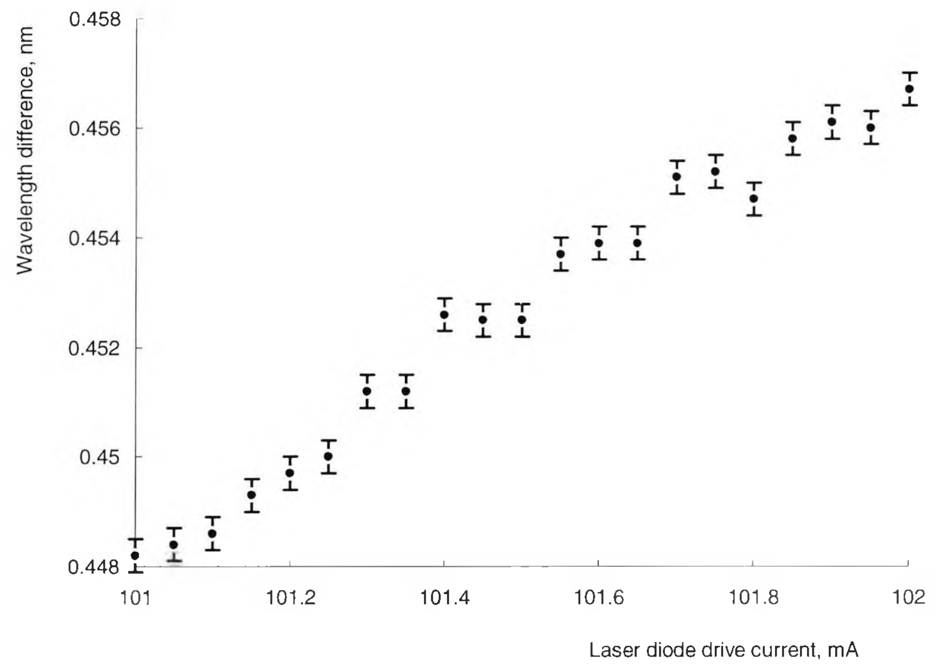
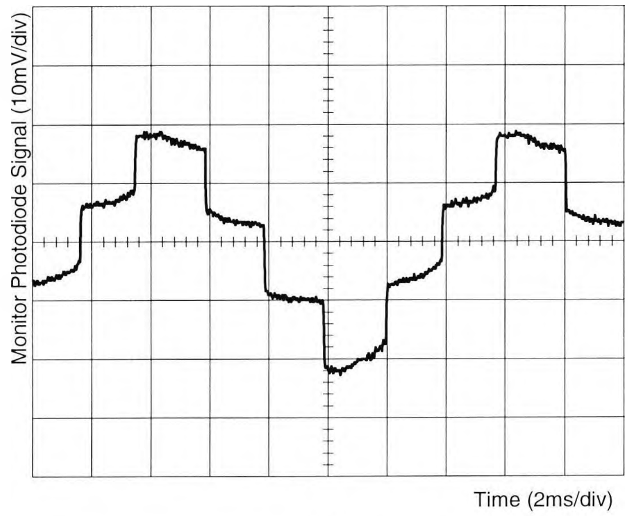
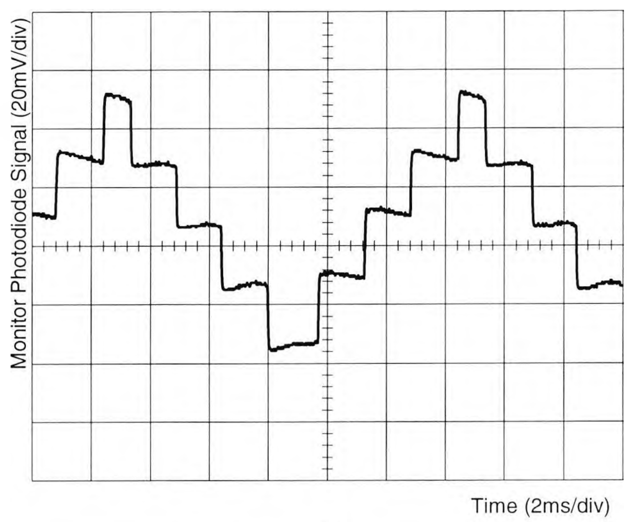


Fig. 5.5 Variation of wavelength for a laser diode with feedback measured using the dual-wavelength Michelson interferometer.

A similar step pattern can be seen clearly in the intensity variation from the laser diode as the drive current is modulated with a triangle wave AC component. The shape of the steps in the intensity variation varies in a rather complex way with the modulation coefficient of equation (2.32) which is a function of feedback strength, solitary laser cavity photon density, laser diode cavity optical length and cavity group velocity; the solitary laser photon density depends on the injection current and the laser cavity optical length and group velocity depend on injection current and temperature. As a result, the exact shape of the steps varies somewhat unpredictably depending on the precise conditions under which the laser is operating, as can be seen from the two examples shown in Fig. 5.6 which were made with separate Sharp LT015MD laser diodes operating at different drive currents and temperatures. Therefore it is not possible to make a direct measurement of the reduction of the wavelength tuning coefficient within the region of the steps. However the feedback strength can be calculated from a



(a)



(b)

Fig. 5.6. Oscilloscope traces of monitor photodiode output obtained with a laser diode with feedback and the drive current modulated with a triangle wave signal.

measurement of the hysteresis, as described in Section 3.6, and the reduction in wavelength tuning coefficient then obtained from equation (5.19).

To demonstrate the behaviour of a laser diode with feedback from two external reflectors, a diffraction grating was placed close to the collimating lens with the first order diffracted beam directed back into the laser and the zero order

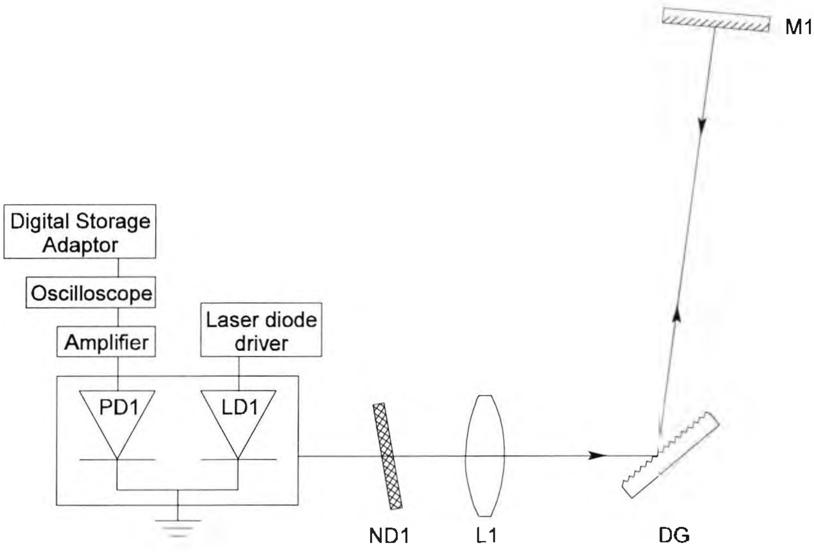


Fig. 5.7. Experimental arrangement with feedback from two external reflectors. LD1: laser diode; PD1: monitor photodiode; ND1: neutral density filter; DG: L1: collimating lens; diffraction grating; M1: mirrror.

beam incident on the second reflector, a plane mirror; this arrangement is shown in Fig. 5.7.

Fig. 5.8 shows an oscilloscope trace recorded with this arrangement with cavity optical lengths, L_1=78mm and L_2=342mm with triangular wave modulation of the laser drive current. The densities of the filters place in the first and second external cavities were, respectively, D_1=0.6 and D_2=1.4. A trace was also recorded with feedback from only the first reflector in order to estimate the strength of the feedback from the mode-hop hysteresis, which for the trace shown was found to be C=3.4.

The modulation in the intensity due to the second reflector is clearly shown, superimposed on the large scale modulation due to the feedback from the grating. This demonstrates the potential for the use of a double external cavity self mixing interference system for reducing the wavelength sensitivity with respect to the single external cavity configuration while still providing a useful interferometric

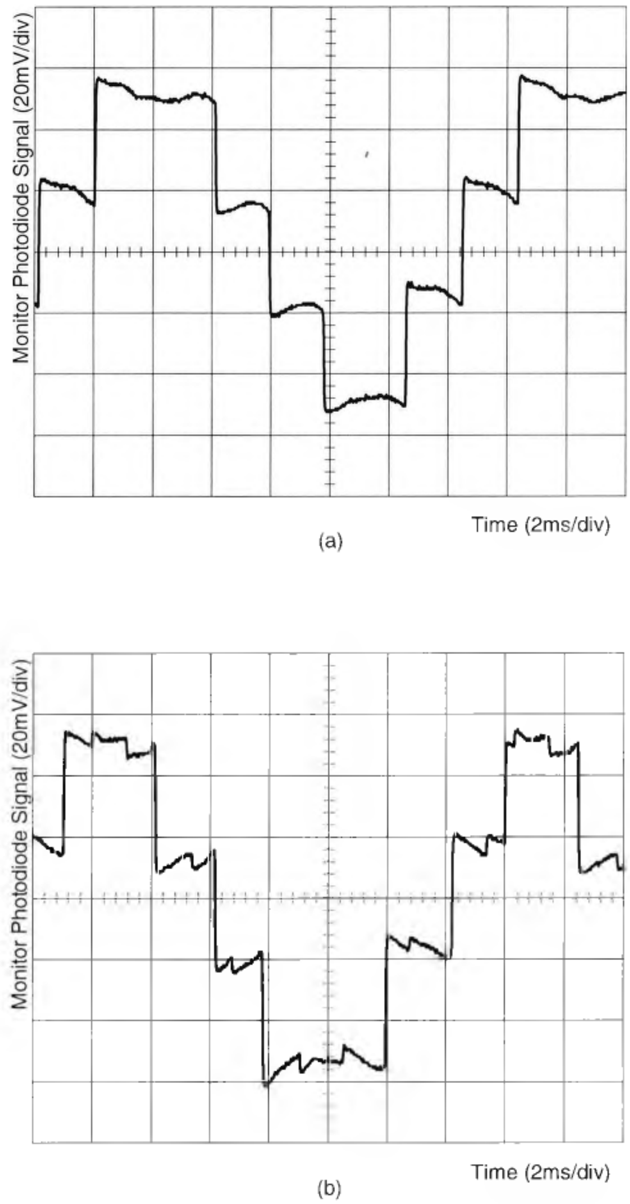


Fig. 5.8. Oscilloscope trace recorded with a laser diode with triangular wave modulation of the drive current with feedback from one external reflector (a) and two external reflectors (b).

output. The measured feedback strength coefficient for feedback from the grating in this case of C=3.4 implies a reduction in wavelength sensitivity by a factor of 4.4, using equation (5.19). This is relatively high and the distance from the laser to the grating was relatively large, which results in a narrow single frequency range and only approximately one fringe due to the target reflector within each modulation period due to the grating. The use of a graded index rod lens for collimating can reduce the distance from the laser to the first external reflector to as little as 10mm and this along with correct control of the level of feedback would extend the useful single external cavity longitudinal mode range.

For example, a reflector at 100mm from the laser with a feedback coefficient of C=2 the width of the single external cavity mode region is, from equation (5.12), $\Delta\nu_{sxm}$=815MHz, which for a diode cavity longitudinal mode temperature tuning coefficient of 30GHz/K implies a stable temperature range of 0.27K which can be achieved reasonably easily with a Peltier temperature controller. The feedback coefficient of C=2 would give rise to a reduction of the temperature tuning coefficient within the external cavity longitudinal mode by a factor of three. For an interferometric displacement sensor with the parameters given in Section 5.3 the uncertainty in frequency would thus be reduced from ±300MHz to ±100MHz and the range over which single fringe accuracy is achievable would be increased from 0.5m to 1.5m.

5.9 Summary

This chapter has examined the question of the stability of the wavelength of the light emitted by a laser diode with respect to variations in environmental temperature or laser drive current. Laser diodes are relatively sensitive to temperature changes in particular and this can limit the performance of interferometric devices using laser diodes because interferometry uses the wavelength of the light in the interferometer as the unit of measurement.

It has been seen that the use of feedback, either with a short external cavity, or with wavelength selectivity, can be used to reduce the temperature sensitivity of the gain spectrum peak which can be used to extend the temperature range over which the laser diode operates in a single longtiudinal mode. However, this does not reduce the variation of frequency to temperature within that longitudinal mode, and so the use of the phase-dependent frequency variations produced by weak feedback to reduce the frequency variation has been considered and demonstrated experimentally. It has been shown that with a two cavity arrangement, the first reflector fixed close to the laser provides a reduction in the wavelength tuning coefficient, while the second reflector provides the feedback for the interferometry. There is a tradeoff to be made between the width of the region for which a single stable external cavity longitudinal mode exists and the reduction achieved in the wavelength sensitivity to temperature. However for a first external cavity length of 20mm a feedback strength coefficient of C=2, giving a reduction in tuning coefficient by a factor of three, can be achieved with a single external cavity longitudinal mode region extending over a frequency range of about 6GHz. This corresponds to a temperature range of about 0.2K can easily be maintained for a laser diode using a Peltier-based temperature controller.

Chapter 6: Modulation Frequency Doubling

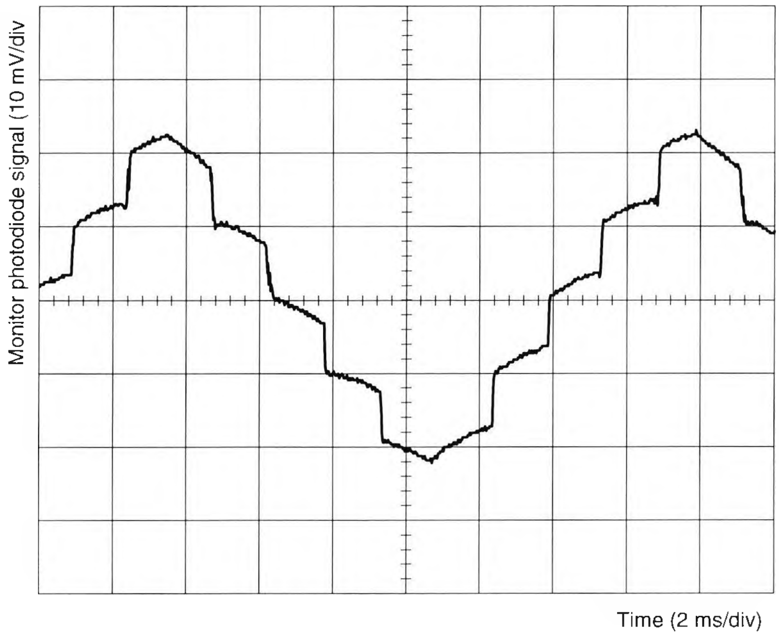
6.1 Introduction

This chapter presents experimental results which show the frequency of the intensity modulation of the laser diode output due to feedback to be twice that expected for the length of the external cavity. An explanation based on a similar phenomenon reported by other authors is shown to be inapplicable to these observations and an alternative explanation of these observations is made. The consequences of this effect for self-mixing interference based systems is discussed.

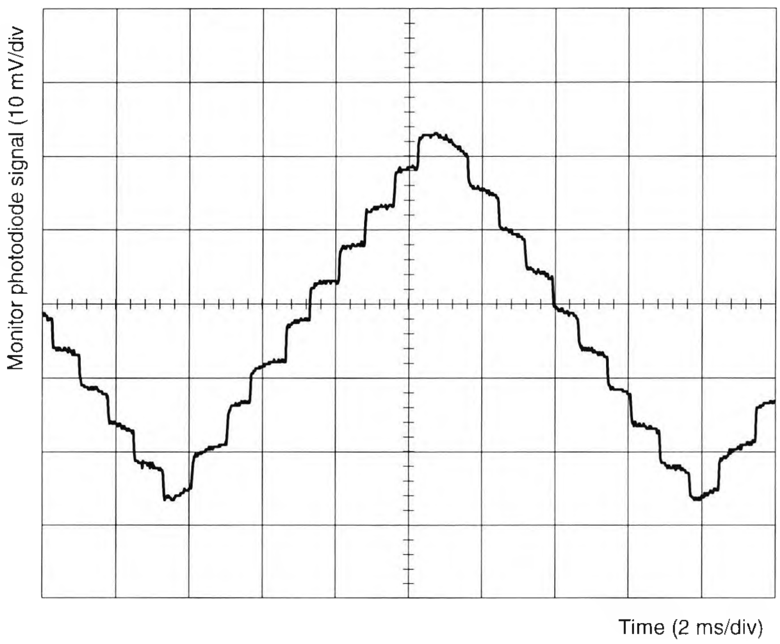
6.2 Experimental observations

In experiments such as those described in Chapter 4 to demonstrate the operation of a self-mixing interference based sensing system, in which the feedback was obtained from a mirror and the phase was modulated either by modulating the laser drive current or by mounting the mirror on a loudspeaker cone, the intensity modulation of the laser diode has frequently been observed to have a frequency which is twice that expected for the length of the external cavity used. The measured external cavity longitudinal mode spacing was thus half the value expected. The presence of these higher frequency fringes or steps was found to depend on the value of the neutral density filter in the external cavity and the precise angular position of the mirror.

Examples of this are shown in Fig. 6.1 and Fig 6.2 for signals obtained using drive current modulation and modulation of the mirror position respectively. For

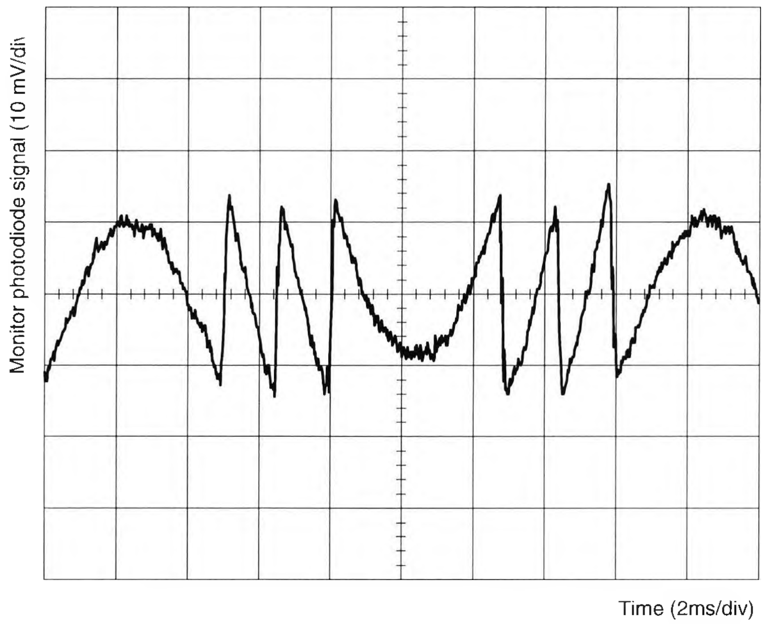


(a)

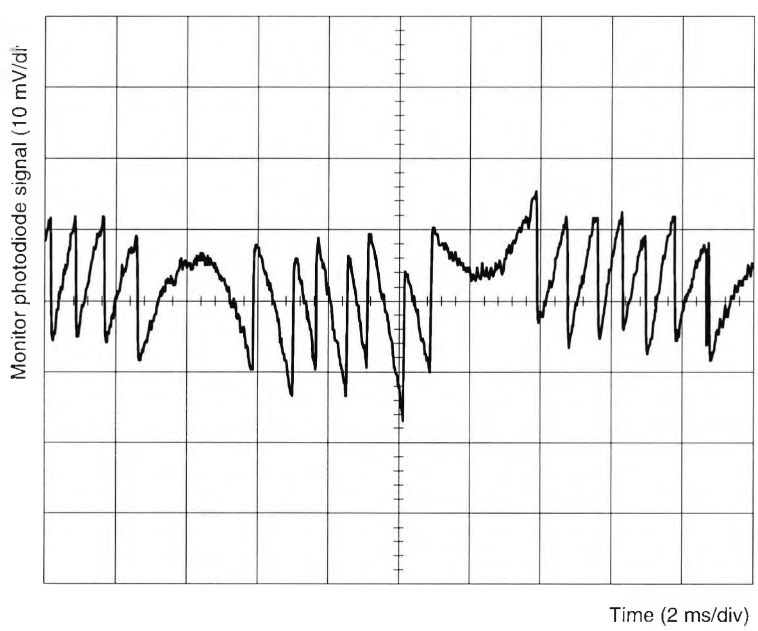


(b)

Fig. 6.1 Two oscilloscope traces obtained with the same external cavity length with the laser diode drive current modulated.



(a)



(b)

Fig. 6.2 Two oscilloscope traces obtained with the same external cavity length with the external reflector position modulated.

all four traces the external cavity optical length was 252±2mm; for the trace in Fig. 6.1,(a) and Fig. 6.2(a) the density of the neutral density filter in the external cavity was D=2.5 and for the remaining traces the density was D=0.7. Measurement of the feedback coefficient from the two current modulation traces gave C=2.8 for Fig. 6.1(a) and C=2.3 for Fig. 6.1(b). The measured external cavity longitudinal mode spacings were 677MHz and 337MHz for Fig. 6.1(a) and Fig. 6.1(b) respectively. The presence of the higher frequency intensity modulation was not dependent on the length of the external cavity over a distance of about 1mm.

6.3 Theoretical considerations

Similar effects to those described in the previous section involving a higher frequency component in the intensity output of a laser diode with self-mixing interference have been reported previously (*Lang & Kobayashi, 1980; Kato et al., 1995*) and have been explained as being connected with mode hopping between the intrinsic diode cavity longitudinal modes of the laser diode. This work is examined below and the observations described above are compared to the behaviour predicted by these theories.

According to these accounts, when the ratio of the lengths of the external cavity to the laser diode cavity is a half-integer, then if the longitudinal mode of the laser diode cavity closest to the gain spectrum peak coincides with a longitudinal mode of the external cavity, the adjacent laser diode modes will each be exactly mid-way between two external cavity longitudinal modes. As the external cavity phase is changed, the external cavity longitudinal modes move relative to the laser diode cavity modes and eventually a situation is reached where the laser cavity longitudinal mode closest to the gain spectrum peak is midway between two external cavity modes and the adjacent laser diode modes are each coincident with an external cavity longitudinal mode.

It is claimed that when a laser diode cavity longitudinal mode and an external cavity longitudinal mode are aligned, ie. $\nu=\nu_{th}$, then the power output of the laser diode is maximised. This does not take into account the phase difference, arctan(α), between the maximum gain condition and the zero frequency shift condition. However this phase difference is constant and so does not affect the implementation of this theory in practice.

Thus if the laser is initially operating in the diode cavity mode closest to the gain spectrum peak and with the external cavity phase such that the threshold gain is minimised and the external cavity phase is then shifted by π, then the threshold gain will be minimised for the external cavity longitudinal modes closest to one of the laser cavity modes adjacent to that nearest to the peak of the gain spectrum. If the reduction in threshold gain due to feedback is greater than the threshold gain difference between the adjacent laser diode modes at the gain spectrum peak, then the laser cavity mode with the lowest threshold gain will be the mode adjacent to the mode at the gain spectrum peak and the laser may switch to that mode. The change in laser cavity longitudinal mode will be accompanied by a sudden change in the output intensity of the laser diode due to the corresponding sudden change in the threshold gain.

As the external cavity phase continues to change, then at some point in the following half-period the system will return to a state where the threshold gain is lowest for the mode closest to the gain spectrum peak and so the laser will move back into that mode. Therefore there will be two visible mode hops for a change in the external cavity phase of one period instead of the one that would be otherwise expected, due to the type of frequency and intensity variations shown in Fig. 2.4 and Fig. 2.5.

In order to give rise to a component in the intensity modulation with exactly double the expected frequency, the external cavity length must satisfy the condition $L_2=(m+\frac{1}{2})L_1$ where m is an integer. If L_2 is an integral multiple of L_1 then the relative positions of the diode cavity and longitudinal cavity modes are the same at each diode cavity mode and so there is no cause for mode hopping between the diode cavity modes unless the modes are moved relative to the gain spectrum by a change in the drive current. For external cavity lengths in between these two cases, it is possible for mode hopping to occur between a larger number of diode cavity modes, which is determined by the maximum gain difference due to the feedback.

It was noted earlier that the higher frequency fringes shown in Fig. 6.1(b) and Fig. 6.2(b) were not affected as the external reflector was moved over a distance equal to the length of the diode cavity and so the explanation described above does not seem to be applicable to these observations. In addition Fig. 6.3 shows an oscilloscope trace with the output of the dual wavelength Michelson interferometer and a simultaneous trace showing the monitor photodiode output with laser drive current modulation in which the intensity modulation due to feedback is double the expected frequency. If the laser diode was switching between diode cavity modes then the relatively large changes in wavelength would cause large changes in the fringe amplitude of the Michelson output, as shown by the simulated visibility pattern shown in Fig. 6.4 which has been calculated for the same conditions under which the trace shown in Fig. 6.3 was recorded. The

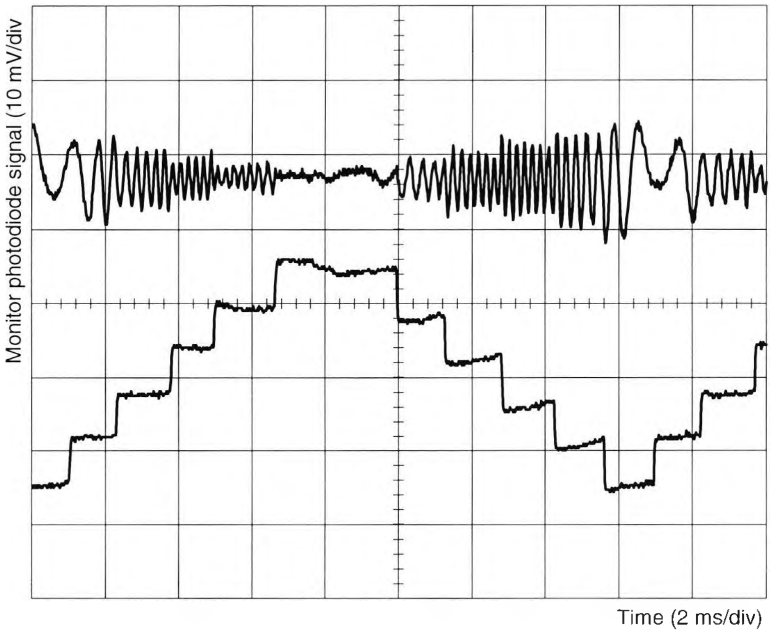


Fig. 6.3 Oscilloscope traces showing simultaneous output from laser diode with double frequency intensity modulation and output from a dual wavelength interferometer.

discontinuities in the observed Michelson fringe pattern are in fact very slight, indicating that there is no mode hopping between the longitudinal modes of the laser diode cavity.

An explanation more consistent with the observations made here is provided by a mechanism which has previously been used to explain certain effects observed with very strong feedback in laser diodes *(Seo et al., 1989a &1989b)* and is shown in Fig. 6.5. If the external reflector is tilted with respect to the optical axis, then an image of the laser diode aperture will be formed in the plane of the aperture but displaced transversely from the aperture. The end facet of the laser diode chip is a smooth reflective surface and so the light forming this image will be reflected back into the external cavity and make a second pass of that cavity. As a result of the fact that the reflection occurs in the focal plane of the collimating lens, the beam

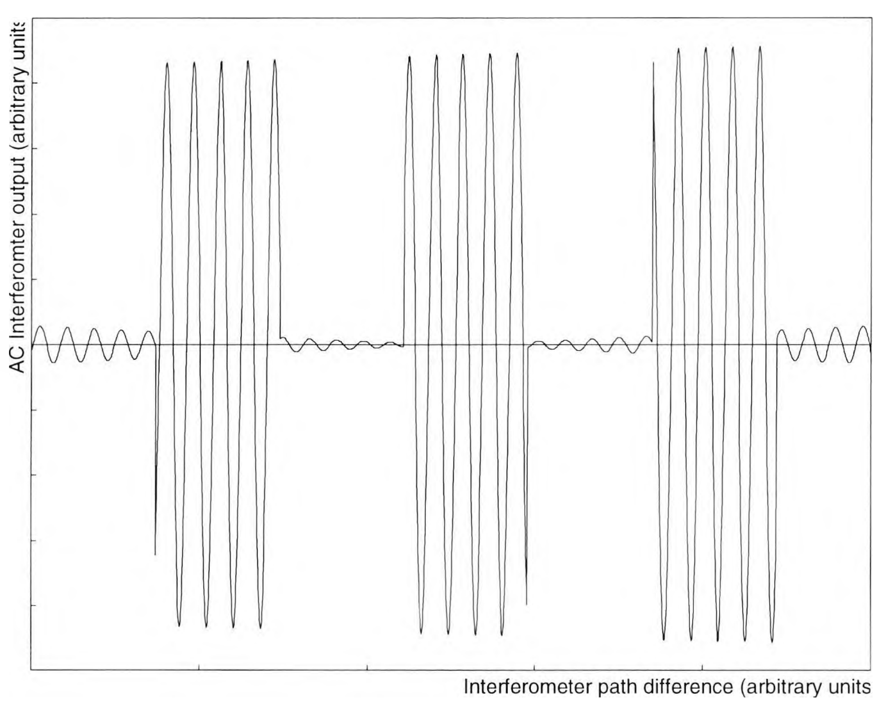


Fig. 6.4. Calculated visibility pattern for a dual wavelength interferometer when one laser diode undergoes hopping between laser diode cavity longitudinal modes.

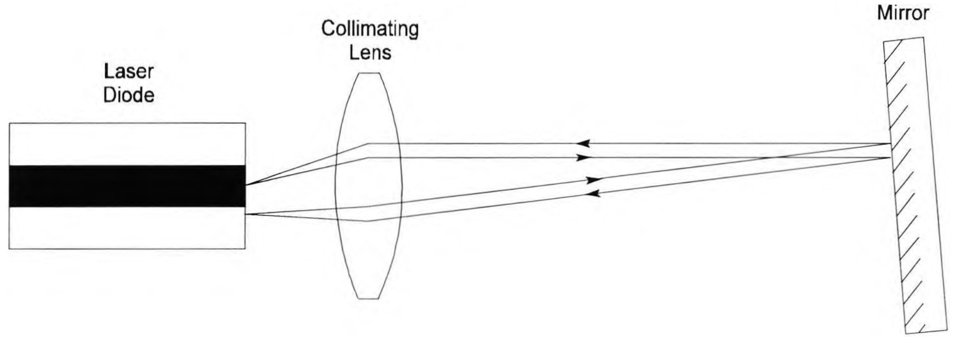


Fig. 6.5. Path taken by a single ray in an external cavity with a tilted reflector.

making the second pass is still collimated and because the laser aperture and its first image are conjugate points in a system effectively made up of two lenses, the image formed after the second pass of the external cavity will, assuming paraxial optics, be located precisely at the actual aperture.

When fringes are formed as a result of the light that has made a double pass of the external cavity, the external cavity length has effectively been doubled and so the fringe frequency will also be doubled.

The effects of multiple reflections in feedback systems with laser diodes are often ignored because of the combination of the low reflectivity of the laser facet (compared to a helium-neon laser, for example) and the losses in the external cavity (which are also greater than for a system using a helium-neon laser due to the larger beam divergence as well as the possible use of attenuating filters) which can result in successive orders of feedback having intensities reduced by several orders of magnitude. The laser diode used to obtain the results shown here had a quarter-wavelength aluminium oxide anti-reflection coating, the intensity reflectivity of which is estimated to be approximately 0.07. Combined with the other losses in the external cavity, excluding any filters used, which were detailed in Section 3.2, the attenuation resulting from a second pass of the external cavity is approximately 0.05. The density of the filter required to maintain the coupling coefficient is then reduced according to

$$D_2 = \frac{1}{2}\left(D_1 + \frac{1}{2}\log(0.05)\right)$$

where D_2 is the filter density for the double pass case and D_1 is the density for the single pass. For D_1=2.5 this gives D_2=0.9 which is in reasonable agreement with the observations described in section 6.2 where the fringes for the single pass case were obtained with a filter density of 2.5 and for the double pass case with a filter density of 0.7.

6.4 Discussion of the observations

If the external cavity contains filters of sufficient density to reduce the amount of feedback to the weak feedback level required for self-mixing interference with the first order reflection then after the second pass of the external cavity the intensity of the light fed back is likely to be too low to have a significant effect on the laser. It is thus possible to prevent the occurrence of double-frequency intensity modulation by careful control of the feedback intensity by the use of filters in the external cavity. However the effect of multiple reflections in the external cavity should not be ignored because when a collimating lens is used in front of the laser, the second order reflection is automatically aligned and so if the external reflector is not in perfect alignment, feedback from the second order reflection may be greater than that from the first order. If the filter density is determined experimentally, by finding a value for which fringes are obtained, then it is likely that those fringes will be from the second order reflection and it is possible that any results obtained may be misinterpreted. This demonstrates the importance of being able both to align the external reflector accurately and to estimate and measure the level of feedback when experimenting with external optical feedback in a laser diode, using the methods discussed in Chapter 3.

However if the system is set up with a lower density filter in the external cavity and with the external reflector tilted slightly with respect to the optical axis so that the double frequency fringes are obtained, then because of the automatic alignment of the second order image with the laser diode aperture, the system should then be more stable against any additional tilt of the external reflector. In section 3.2 it was shown that an angle of the order of 0.1mrad between the reflector normal and the optical axis is enough to ensure that the central maximum of the first order image does not coincide with the aperture in which case the second order reflection dominates and the sensitivity of the system to the positioning of the external reflector is reduced. In addition, if the frequency of the fringes or intensity steps is doubled, then the potential resolution of a self-mixing interference based sensing system will be doubled, with one fringe produced for every quarter-wavelength of target displacement instead of one fringe per half-wavelength in the single pass case. As an illustration of this effect, for a displacement sensor based on fringe-counting, the fringe frequency will be one per quarter-wavelength displacement, giving a resolution of 0.2μm for a laser diode wavelength of 800nm.

6.5 Summary

This chapter has described experimental observations in which the observed intensity modulation from a laser diode due to feedback had a frequency which was double the expected value for the length of the external cavity used. A possible explanation based on the interaction between the longitudinal modes of the laser diode cavity and those of the external cavity has been considered but further experimental observations have been made which are not consistent with this theory. The observed effects can, instead, be explained by light making a double pass of the external cavity and so effectively doubling the external cavity length.

It is important for experimental accuracy that the possibility of this modulation frequency doubling effect is considered when working with optical feedback to a laser diode from a plane reflector. The effect can be prevented by careful choice of filter density and precise alignment of the reflector, but because the coupling of the feedback after making a double pass of the external cavity is independent of the angular position of the external reflector so this effect can be used to increase the mechanical stability in a feedback-based system.

Chapter 7: Conclusion

7.1 Overall summary

Numerous authors have proposed interferometric sensing devices using the effects of weak external optical feedback in laser diodes which produces what is known as 'self-mixing interference'. The work reported in this thesis has examined some of the practical issues related to such systems with the aim of improving their performance.

In Chapter 2 a simple theory was presented to show how weak optical feedback modifies the lasing conditions of the laser diode causing changes in the frequency and intensity of the emitted light. The magnitude of these changes was seen to be dependent on the phase of the light reentering the laser cavity and so the observed intensity of the laser diode output contains interferometric information about the position of the external reflector. A laser diode with optical feedback is thus analogous to a Michelson interferometer, with the laser diode output facet being the reference mirror of the interferometer, and can be used for interferometric sensing. The physical basis of self-mixing interference is more complex than that of conventional interference and this work has shown that there are some important differences in the observed effects. The main differences are related to the variation of the form of the observed intensity variations with the strength of the feedback; for very low levels of feedback a nearly sinusoidal modulation similar to that produced by a conventional interferometer is observed but as the feedback strength is increased the interferometric fringes become markedly asymmetric, taking a saw-tooth waveform. This asymmetry allows information about the direction of movement of the external reflector to be obtained unlike in a conventional interferometer. The theory presented here shows that as the frequency variations increase with increasing feedback, separate longitudinal modes of the external cavity appear and hysteresis occurs in the path followed by frequency and intensity as the external cavity phase is varied. Stability criteria related to the external cavity longitudinal modes have been calculated in this thesis in terms of the feedback strength.

Because of the sensitivity of the effects of feedback to the feedback strength and because of the small size of the laser diode output aperture into which light has to be redirected, great care must be taken when working with feedback in laser diodes. Chapter 3 describes techniques that have been developed to enable systematic and accurate experimental work to be carried out on laser diodes with feedback. A typical experimental arrangement has been developed and analysed to provide an accurate estimate of the feedback strength attainable. Methods are described for achieving optimal collimation of the laser beam and for accurately aligning the external reflector. In order to interpret experimental results it is necessary to be able to measure the feedback strength and a simple method of doing this has been developed and presented based on measuring the degree of hysteresis in the frequency path followed when the laser drive current is modulated. The techniques described here are shown to be relevant to any experimental work or practical application using feedback in a laser diode.

Chapter 4 described the experimental demonstration of a simple self-mixing interference-based displacement sensing system. The implications for the system of the choice of component for each element of the system, laser diode, collimating lens, target reflector, and fibre optic link, were examined and considered in detail. Self-mixing interference fringes were obtained for target distances of over one metre, showing the direction dependent saw-tooth form predicted by the theory.

A major limiting factor for the accuracy and range of any laser diode-based interferometric device is the sensitivity of the laser diode wavelength to environmental temperature fluctuations. This problem was discussed in detail in Chapter 5 with reference to the effects of external optical feedback. Optical feedback has previously been used to modify the gain characteristics of a laser diode and extend the region over which the laser operates in a single longitudinal mode but without affecting the sensitivity to temperature of the wavelength within that longitudinal mode. In this work it has been shown that the wavelength variations caused by weak feedback can be used to reduce the temperature tuning coefficient of wavelength thus offering a potential increase in range and accuracy of a self-mixing interference based displacement sensor.

Chapter 6 presented experimental results from which the calculated external cavity longitudinal mode spacing was calculated to be double the value expected for the length of the external cavity used. This effect was shown to be different from similar higher order effects observed in the signal from a laser diode with feedback by other authors and was shown to be due to light being reflected from the laser front facet and making a double pass of the external cavity before reentering the laser cavity when the external reflector is misaligned. This demonstrates the importance of being able to align the components of a feedback-based system accurately as was discussed in Chapter 3 as misalignment could cause an error by a factor of two in interferometric results. However it may be advantageous to deliberately misalign the reflector because the image formed by light making a double pass of the external reflector is automatically aligned with the laser diode aperture and the doubled intensity modulation obtained can double the resolution of an interferometric sensor.

7.2 Suggestions for future work

The use of external optical feedback in laser diodes for interferometric sensing applications is still a relatively new field of research and there is the potential for much further work in this area. Important areas of work carrying on from the work presented in this thesis could include:

(i) development of a signal processing scheme for a self-mixing interference based displacement sensor that overcome the problems associated with the relatively small modulation depth of the fringes obtained and the relatively high noise of the laser diode output at displacement velocities appropriate to practical applications.

(ii) consideration of the problem of wavelength drift over the lifetime of a laser diode *(Wieman & Holberg, 1991)* and its implications in self-mixing interferometry.

(iii) more work to develop a practical configuration incorporating the two reflector system which has been demonstrated in principle in Chapter 5 for reducing the temperature dependence of the laser diode wavelength, in particular the question of the accurate positioning of the first reflector needs to be addressed.

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Effects of External Reflector Alignment in Sensing Applications of Optical Feedback in Laser Diodes

Richard C. Addy, Andrew W. Palmer, and Kenneth Thomas Victor Grattan

***Abstract*—The alignment of the components in sensor systems based on external optical feedback is a critical factor due to the small size of laser aperture into which the returned light must be coupled. This paper presents experimental results and a theoretical explanation which show that the fringe frequency of a self-mixing interferometric sensor system can be doubled due to misalignment of the external reflector. The double frequency fringes are less sensitive to the alignment of the reflector than the normal fringes and offer a potential increase in the resolution of a sensing system of a factor of two.**

I. Introduction

SENSING systems based on self-mixing interference in laser diodes have been of interest due to the simple, compact, and inexpensive nature of such systems of which the major components are a laser diode, basic collimating optics, and one of a range of possible reflectors (e.g., mirror, corner cube, diffraction grating) [1], [2]. Such systems rely on changes in the laser output intensity caused by changes of phase produced between the laser diode and input from an external reflector; range-finding systems have been proposed in which the external cavity phase is changed by modulating the laser diode drive current and thus changing the lasing frequency [3], [4], and displacement sensors have been proposed in which the phase change is due to the displacement of the external reflector [5], [6].

The effects of misalignment of the external reflector have previously been described for external cavity semiconductor lasers which operate in a high feedback regime [7]. However, the results of these studies are not applicable to the weak feedback region in which self-mixing interference systems operate. This paper presents the results of experiments with weak feedback systems, demonstrating the effects of external reflector misalignment, gives a simple theoretical explanation for the effects observed and shows how these effects may offer advantages in designing self-mixing interference based sensor systems.

II. Background Theory

The theory of weak optical feedback in laser diodes has been described by various authors [8]–[10] and is briefly summarized below.

Fig. 1 shows a schematic self-mixing interference system using such a laser diode source. This laser diode has an optical length, L_0, and rear and front facets with respective amplitude reflection coefficients r_1 and r_2. The laser output is collimated by the lens L1 and reflected back into the laser cavity by the mirror M1 which form an external cavity with optical length L_{ext}. The amplitude reflection coefficient for the external reflector r_3 combines the mirror reflectivity with attenuation losses and losses due to imperfect coupling between the laser diode and the collimating lens and between the diffraction limited image of the laser aperture and the aperture itself. The front facet amplitude reflectivity r_2 can be replaced by an effective amplitude reflectivity r_2', which combines r_2 and r_3 and the round-trip phase in the external cavity and is given by

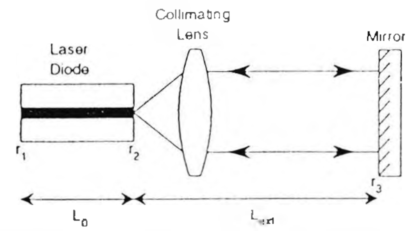

Fig. 1. Schematic representation of a laser diode self-mixing interference system.

$$r_2' = r_2(1 + \xi \exp[-2\pi i \nu \tau_2]) \quad (1)$$

where ν is the laser output frequency, τ_2 is the round-trip time for the external cavity and ξ is a feedback coupling coefficient defined by

$$\xi = (1 - |r_2|^2)\frac{r_3}{r_2} \quad (2)$$

Multiple reflections in the external cavity can be ignored at this stage because of the high attenuation in the external cavity in most practical situations. The effective amplitude reflecivity can be used in the phase and gain conditions for laser action to calculate changes in the threshold gain and the round-trip phase. The optical frequency, the threshold gain, the laser diode carrier density, the cavity refractive index, and the compound cavity round trip phase are all closely interrelated using the linewidth enhancement factor α to relate the gain and the cavity refractive index leads to the following relationship between the solitary laser threshold frequency ν_{th} and the

Manuscript received May 6, 1996; revised August 6, 1996.
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Publisher Item Identifier S 0733-8724(96)08921-9.

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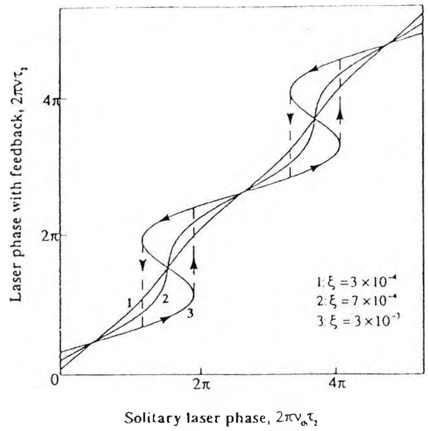


Fig. 2. Theoretical variation of frequency for a laser diode with feedback ν with solitary laser frequency ν_{th} for three values of feedback coupling coefficient ξ with external cavity length L_{ext} = 200 mm. Frequencies expressed in terms of external cavity phase.

lasing frequency with feedback ν

$$2\pi(\nu - \nu_{th})\tau_2 + C\sin[2\pi\nu\tau_2 + \arctan(\alpha)] = 0 \quad (3)$$

where C is the feedback coefficient defined by

$$C = \frac{L_{ext}}{L_0}\xi\sqrt{1+\alpha^2} \quad (4)$$

Solutions to (3) for lasing frequency ν have the form shown in Fig. 2 for varying levels of feedback. For $C > 1$ there are regions in which there exist multiple solutions which represent discrete external cavity longitudinal modes whose mean frequency spacing is

$$\Delta\nu_{xcm} = \frac{C}{2L_{ext}} \quad (5)$$

The mode hops between the external cavity modes occur at opposite turning points in curve 3 of Fig. 2 depending on whether the frequency is increasing or decreasing [11], giving rise to hysteresis as shown by the dashed lines and arrows in the figure. The frequency separation of upward and downward mode-hops, $\Delta\nu_{hys}$, is a function of the feedback strength and is given by

$$\pi\tau_2\Delta\nu_{hys} = \sqrt{C^2-1} - \arctan\sqrt{C^2-1} \quad (6)$$

The change in output intensity due to feedback is proportional to the change in threshold gain which in turn is related to the change in frequency. The important feature of the variation in the intensity output is that discrete jumps will occur at the same time as the frequency hops between external cavity modes, and the same hysteresis effect will be seen. The resulting variation in output intensity is shown in Fig. 3(a) for the case in which the external cavity phase is changing due to a change in the external cavity length, and in Fig. 3(b) for the case in which the laser diode drive current is

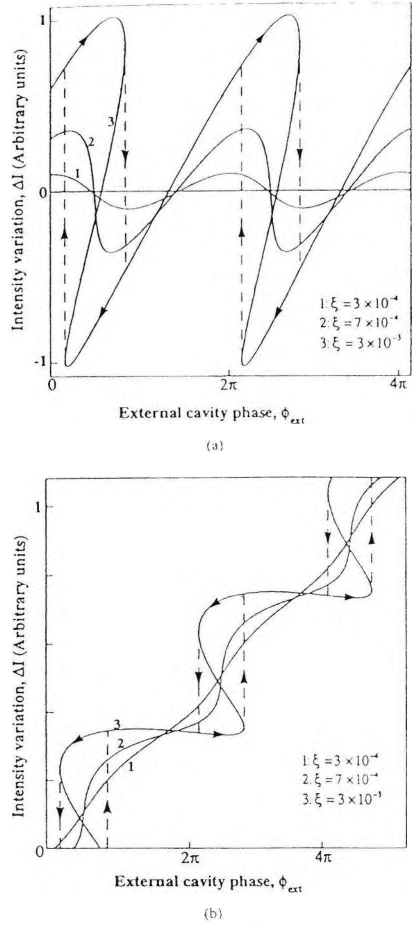


Fig. 3. Theoretical variation of output intensity ΔI of a laser diode with feedback with external cavity phase $\phi_{ext} = 2\pi\nu\tau_2$ due to (a) variation in external cavity length and (b) variation in drive current for three values of feedback coupling coefficient ξ.

ramped and the external cavity phase changes as a result of the change in the intrinsic output frequency, ν_{th}. These periodic variations form the basis for self-mixing interference-based sensing systems; the fringes due to changes in the external cavity length can be used to measure displacement in the same way as a conventional interferometer and the spacing of the steps in the pattern due to modulation of the drive current can be used to measure absolute distance between the laser and the external reflector.

Solving (6) for $\Delta\nu_{hys} = 0$ gives $C \approx 4.61$. For values of C greater than this there exist regions in which there are multiple solutions to (3) where the laser will not operate stably [6]. For a constant level of feedback this gives a theoretical limit to the operating range of a self-mixing interference-based sensing system.

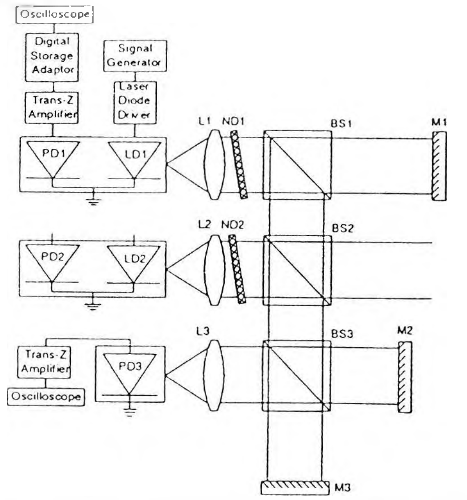


Fig 4. Experimental arrangement used to study feedback; LD1–2: laser diodes, PD1–3: photodiodes; L1–3: lenses; BS1–3 cube beamsplitters; M1–3: plane mirrors; ND1–2: neutral density filters.

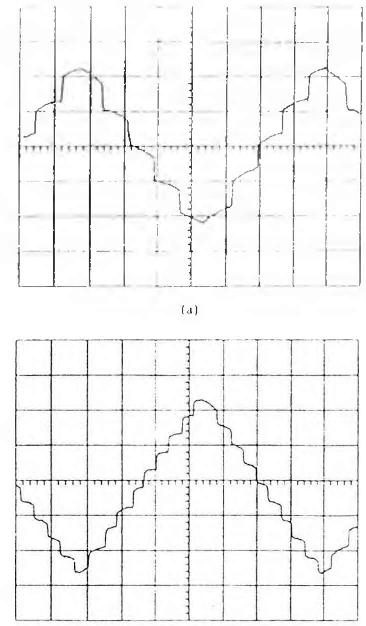
III. Experimental Arrangement

The experimental arrangement used to study the effects of feedback on the frequency and power output of a laser diode is shown in Fig. 4. The feedback system comprised the laser diode LD1 (Sharp LT015MD) the collimating microscope objective L1 (0.25 NA) the mirror M1 which was mounted on a loudspeaker cone in order to provide modulation of the external cavity phase, and the neutral density filter ND1 which was used to control the feedback coupling strength. The output of this laser was sampled by the beamsplitter BS1 and changes in wavelength relative to that of a reference laser diode LD2 were measured using the Michelson interferometer comprising the beamsplitter BS3, the mirrors M2 and M3, and the photodiode PD3.

The visibility pattern produced by the Michelson interferometer with two wavelengths has a sinusoidal component with sharply defined minima whose positions can be measured to an accuracy of ± 2 μm in the mirror position which enables relative measurements of wavelength, accurate to within $\pm 5 \times 10^{-4}$ nm.

The feedback into LD1 from M1 was maximized by finding the mirror position which gave the maximum reduction in threshold current. The intensity output of LD1 was observed on an oscilloscope via the photodiode at the rear facet of the laser and oscilloscope traces were digitized and stored using a digital storage adaptor. By measuring the wavelength tuning with respect to drive current using the Michelson Interferometer and measuring the variation of photodiode output with drive current, any changes in wavelength could then be inferred from associated changes in the photodiode output.

Values of the feedback coefficient C were obtained by measuring the hysteresis separation of the mode-hops in the upward and downward halves of the modulation period. This was done using the digitally stored oscilloscope traces, identifying the positions of the mode-hops and then finding their distance from the turning point in the modulation signal. The measured hysteresis frequency separation was then used to solve (6) numerically for the value of C.



(a)

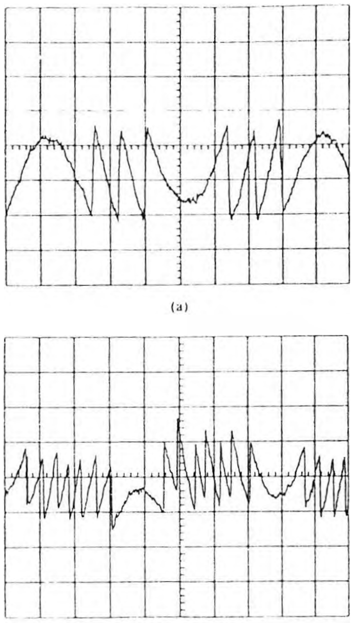
(b)

Fig. 5. Oscilloscope traces showing output intensity of a laser diode with feedback with the drive current modulated with a triangular waveform. (a) Normal intensity modulation frequency. (b) Double intensity modulation frequency. (Horizontal scale: 2 ms/div; vertical scale: 10 mV/div.)

IV. Results

Fig. 5 shows two oscilloscope traces each of which represents the variation of intensity output of the laser diode with feedback as the drive current was modulated with a triangular wave with a peak to peak amplitude of 2 mA. Fig. 6 shows the variation of intensity as the length of the external cavity was modulated by applying a signal to the loudspeaker cone on which it was mounted, each trace having been recorded under the same conditions as those in Fig. 5, respectively. The optical length of the external cavity was 252 ± 1 mm; the differences introduced between (a) and (b) were a change in the density of the neutral density filter from $D = 2.5$ to $D = 0.5$ and a slight adjustment to the angular position of the mirror.

The steps in the intensity output traces in Fig. 5 correspond to the external cavity longitudinal modes of the laser diode. The mean frequency separation of these modes was 677 MHz for (a) and 337 MHz for (b) and the respective values of C obtained were 2.78 and 3.30. The expected longitudinal



(a)

(b)

Fig. 6. Oscilloscope traces showing output intensity of a laser diode with feedback with the external cavity length modulated using a loudspeaker mounted mirror. (a) Normal intensity modulation frequency. (b) Double intensity modulation frequency. (Horizontal scale: 2 ms/div; vertical scale 2 mV/div.)

mode spacing for an external cavity of length 252 ± 1 mm is 595 ± 2 MHz.

These results show that the frequency of fringes or intensity steps produced in a self-mixing interference system may be double the value expected for certain levels of feedback intensity and certain alignments of the external reflector. This effect has been observed to be independent of the external cavity length on a scale of about 1 mm.

Fig. 7 shows two oscilloscope traces recorded simultaneously; one showing the intensity output of the laser diode for a case where the step frequency is double the expected value, the other the output from the Michelson interferometer with the adjustable mirror placed close to a minimum in the dual wavelength visibility pattern. The interferogram shows small steps in the visibility pattern coincident with the steps in the intensity output indicating that there is no change in the intrinsic laser diode longitudinal mode throughout the current modulation period.

V. Discussion

Higher order effects have previously been reported in a self-mixing interference displacement sensing system [12]. These effects were shown to be due to mode hopping between the intrinsic laser diode modes as the external and intrinsic longitudinal mode patterns moved relative to each other and were observed only for cases where the external cavity length was equal to a half-integer multiple of the laser diode cavity optical length. The effects reported in this paper have been shown to be independent of the external cavity length on the scale of the laser cavity length and to take place without changes in frequency of the order of the intrinsic laser diode longitudinal mode spacing.

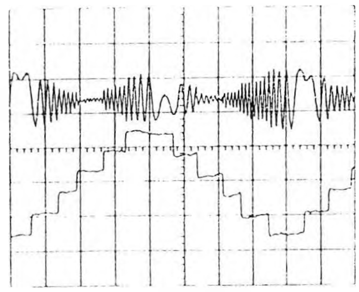


Fig. 7. Oscilloscope traces showing intensity output of a laser diode with feedback with drive current modulation (lower trace) and simultaneous dual wavelength Michelson interferometer output (upper trace). (Horizontal scale 2 ms/div; vertical trace 10 mv/div.)

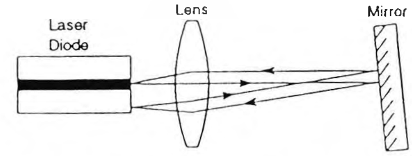


Fig. 8. Path taken by a single ray in an external cavity with a tilted reflector.

Fig. 8 shows the mechanism previously used to describe the effects of reflector misalignment for external cavity semiconductor lasers [7] and which is consistent with the results reported here. A single ray emitted on the optic axis is shown. When the reflector is misaligned, the image of the laser aperture formed by the first round trip of the external cavity will be displaced from the actual aperture. The light will not then be fed back into the cavity, however it may be reflected from the laser facet and make a second pass of the external cavity in which case, because the point of reflection is in the focal plane of the collimating lens, the aperture and the first image are effectively conjugate points in a two-lens system and the second image of the laser aperture will always coincide with the aperture. The effective length of the external cavity is then doubled and so the spacing of external cavity longitudinal modes is halved.

The intensity of the beam will be further attenuated during the second pass of the external cavity and so in order to observe similar feedback effects with a double pass and a single pass, the attenuation in the external cavity needs to be reduced for the double pass case. The laser diode used has a quarter-wavelength aluminum oxide coating on the front face with an estimated intensity reflection coefficient of $R_2 = 0.07$. This loss, in addition to the extra loss caused by light passing through the external cavity twice, means that a neutral density filter with density $D = 2.5$ for the single-pass case should be replaced by a filter of density $D = 0.5$ for the double pass case in order to maintain a similar coupling efficiency. These predictions, based on the condition of a double pass of the external cavity, are confirmed by the experimental results given in the previous section.

The double frequency steps or fringes may thus be avoided by ensuring a high level of attenuation in the external cavity. However when the system is set up to obtain feedback from a single pass of the external cavity the coupling efficiency is extremely sensitive to the alignment of the external reflector. With a collimating microscope objective lens of NA = 0.25, an angular displacement of approximately 200 μrad in the direction perpendicular to the junction plane will shift the diffraction pattern of the imaged laser aperture so that the first minimum of that pattern is at the center of the laser aperture instead of the central maximum. However, with the system set up so that feedback is due to a double pass of the cavity, the feedback strength is almost independent of the alignment of the reflector; it is only necessary to ensure that the alignment is far enough away from the perpendicular that light from the side modes of the single pass image does not affect the system.

In addition, if the frequency of fringes or intensity steps is doubled then the potential resolution of a self-mixing interference based sensing system will be doubled, with one finge produced for every quarter-wavelength of target displacement instead of one fringe per half-wavelength in the single pass case. For a displacement sensor based on fringe counting, the fringe frequency will be one per quarter-wavelength instead of one per half-wavelength, giving a resolution of 0.2 μm for a laser diode wavelenth of 800 nm.

Thus, for reasons of ease of alignment, stability and system performance, it is advantageous to set up self-mixing interference-based sensing systems with the external reflector tilted with respect to the beam normal by an angle of the order of 1 mrad.

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Applied Optics and Optoelectronics 1996

Proceedings of the Applied Optics Divisional Conference of The Institute of Physics, held at Reading, 16–19 September 1996

Edited by K T V Grattan

Institute of Physics Publishing
Bristol and Philadelphia

Paper presented at Appl Opt Div Conf, Reading, 16 19 September 1996

Modulation Frequency Doubling in Self-Mixing Interference with Laser Diodes.

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Abstract. Self-mixing interference in laser diodes has been proposed as the basis of a simple and inexpensive alternative to conventional interferometric sensing systems. Normally, the performance of a self-mixing interference based system depends critically on the alignment of the external reflector. This paper shows that the sensitivity to alignment may be reduced by introducing a degree of tilt to the external reflector and that at the same time the intensity modulation frequency and thus the potential resolution of any self-mixing interference system may be doubled.

1. Introduction

Self-mixing interference in laser diodes occurs when light is reflected from an external target and reenters the laser cavity. This fed-back light introduces a change in the effective round-trip phase for the laser which results in a change in the lasing frequency and, in turn, a change in the output intensity which varies periodically with the round-trip phase of light in the external cavity in a similar way to the output of a conventional Michelson interferometer. Thus a simple, inexpensive interferometric sensing system can be constructed, based on just a laser diode, a collimating lens and a mirror [1]. Displacement sensors have been discussed in which the external cavity phase is modulated by the movement of the external reflector [2], and range-finding sensors have been proposed in which the phase is varied by changing the laser drive current and thereby modulating the laser frequency [3].

The output of a laser diode with feedback is critically dependent on the feedback coupling strength which is dependent on the alignment of the external reflector. This paper shows how the sensitivity of a self-mixing interference based system to external reflector alignment can be reduced by deliberately tilting the mirror with respect to the beam and that this also results in a doubling of the frequency of the laser intensity modulation with a consequent doubling in the potential resolution of a sensing system.

2. Background Theory

The theory of the behaviour of laser diodes with weak optical feedback has been described by various authors [4] and is summarized here. For a system consisting of a laser diode with a cavity of optical length, L_0, and a front facet amplitude reflectivity of r_2, and an external reflector at a distance L_{ext} from the laser with and amplitude reflectivity of r_3, the feedback coupling coefficient, ξ, is defined by

$$\xi = (1-|r_2|^2)\frac{r_3}{r_2}, \tag{1}$$

with a feedback coefficient, C, defined by

$$C = \frac{L_{ext}}{L_0}\xi\sqrt{1+\alpha^2}, \tag{2}$$

where α is the linewidth enhancement factor.

By considering the interrelated effects of feedback on the threshold gain, the lasing frequency, the compound cavity round-trip phase, and the cavity refractive index, the following relationship between the solitary laser threshold frequency, ν_{th}, and the actual lasing frequency, ν, is obtained:

$$2\pi(\nu-\nu_{th})\tau_2 + C\sin[2\pi\nu\tau_2 + \arctan(\alpha)] = 0, \tag{3}$$

where τ_2 is the round trip time for light in the external cavity and α is the linewidth enhancement factor.

Solutions to equation (3) for the lasing frequency, ν, have the form shown in Fig. 1. For C>1 there are regions in which there are multiple solutions to equation (3), each of which represents a discrete external cavity longitudinal mode. The mean frequency separation of these modes, $\Delta\nu_{xcm}$, is

$$\Delta\nu_{xcm} = \frac{c}{2L_{ext}}. \tag{4}$$

The mode-hops between the external cavity modes occur at opposite turning points in curve 3 of Fig. 1 depending on whether the frequency is increasing or decreasing [5], giving rise to hysteresis, as shown by the dashed lines in the figure. The feedback coefficient, C, can be determined from a measurement of the frequency separation of the upward and downward mode-hops, $\Delta\nu_{hys}$, using the following relationship:

$$\pi\tau_2\Delta\nu_{hys} = \sqrt{C^2-1} - \arctan\sqrt{C^2-1} \tag{5}$$

The variation of the output intensity due to feedback is proportional to the change in threshold gain, Δg, which is given by

$$\Delta g = \frac{\xi}{L_0}\cos(2\pi\nu\tau_2), \tag{6}$$

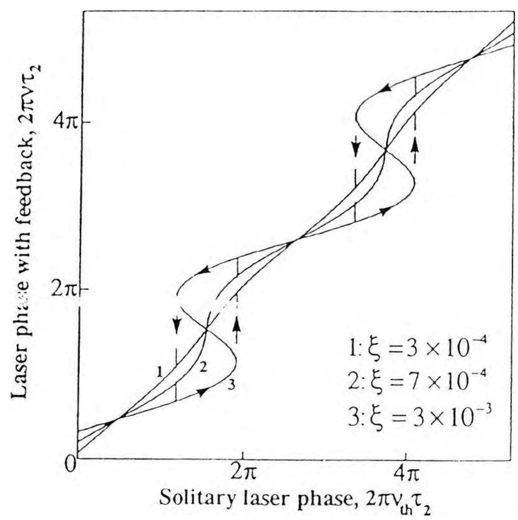


Fig. 1. Theoretical variation of frequency for a laser diode with feedback, ν, with solitary laser frequency, ν_{th}, for three values of feedback coupling coefficient, ξ, with external cavity length, L_{ext}=200mm. Frequencies expressed in terms of external cavity phase.

The important features of the intensity variation are that the periodicity is the same as for the frequency variation and that discrete jumps in intensity occur at the same time as the mode-hops in the frequency variation.

3. Experiment

The experimental arrangement used to study the effects of feedback on the frequency and power output of a laser diode is shown in Fig. 2. The feedback system comprised the laser diode, LD1 (Sharp LT015MD), the collimating microscope objective, L1 (0.4 NA), the mirror, M1, which was mounted on a loudspeaker cone in order to provide a modulation of the external cavity phase, and the neutral density filter, ND1, which was used to control the feedback coupling strength. The output of this laser was sampled by the beamsplitter, BS1, and changes in wavelength relative to that of a reference laser diode, LD2, were measured using the Michelson interferometer comprising the beamsplitter, BS3, the mirrors, M2 and M3, and the photodiode, PD2.

The visibility pattern produced by the Michelson interferometer with two wavelengths has a sinusoidal component with sharply defined minima whose positions can be measured to an accuracy of ±2μm in the mirror position which enables relative measurements of wavelength, accurate to within $\pm5\times10^{-4}$nm.

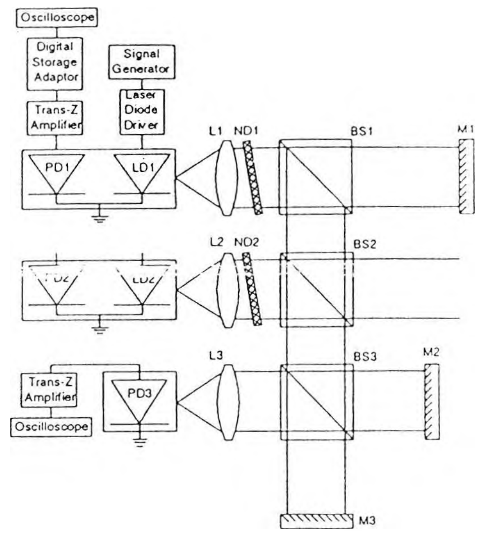


Fig. 2. Experimental arrangement used to study feedback. LD1-2: laser diodes.
PD1-3: photodiodes: L1-3: lenses; BS1-3: cube beamsplitters,
M1-3: plane mirrors; ND1-2: neutral density filters.

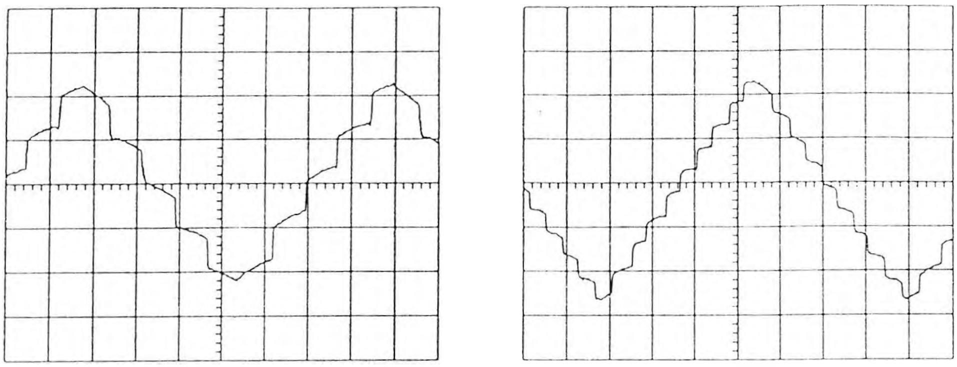
The feedback into LD1 from M1 was maximized by finding the mirror position which gave the maximum reduction in threshold current. The intensity output of LD1 was observed on an oscilloscope via the photodiode at the rear facet of the laser and oscilloscope traces were digitized and stored using a digital storage adaptor. By measuring the wavelength tuning with respect to drive current using the Michelson interferometer and measuring the variation of photodiode output with drive current, any changes in wavelength could then be inferred from associated changes in the photodiode output.

Values of the feedback coefficient, C, were obtained by measuring the hysteresis separation of the mode-hops in the upward and downward halves of the modulation period. This was done using the digitally stored oscilloscope traces, identifying the positions of the mode-hops and then finding their distance from the turning point in the modulation signal. The measured hysteresis frequency separation was then used to solve equation (5) numerically for the value of C.

4. Results

Fig. 3 shows two oscilloscope traces representing the output of a laser diode with feedback and with the drive current modulated by a triangle wave of approximately 2mA amplitude. Each trace shows the discontinuities in intensity associated with the external cavity mode-hops. The external cavity length was 252±1mm for each trace; the only difference in the experimental set up was a change in the value of the neutral density filter from D=2.5 to D=0.7 and a small adjustment to the angular position of the mirror.

The mean frequency separation of the mode-hops was 677MHz for (a) and 337MHz for (b) and the measured values of feedback coefficient, C, were, respectively, 2.78 and



(a) (b)

Fig. 3. Oscilloscope traces showing output intensity of a laser diode with feedback with the drive current modulated with a triangular waveform. (Horizontal scale: 2ms/div; Vertical scale: 10mV/div.)

3.30. The calculated external cavity longitudinal mode spacing for an external cavity length of 252mm is 595MHz. When the external cavity length was modulated using a loudspeaker mounted mirror, the frequency of the fringes was able to be varied by a factor of two under the same conditions.

The appearance of an intensity modulation with approximately twice the expected frequency was found to be dependent only on the value of the attenuating filter in the external cavity and the angular position of the mirror and not on the longitudinal position of the mirror.

Observations with the dual wavelength Michelson interferometer showed that there was no significant variation in lasing frequency occurring while the double-frequency intensity modulation was taking place.

5. Discussion

Higher order effects in the intensity modulation pattern of a laser diode with feedback have previously reported [6] and were shown to be due to mode-hopping between the longitudinal modes of the laser diode cavity and occurred only when the external cavity length was equal to a half-integer multiple of the diode cavity length. The effects reported in the previous section are clearly different.

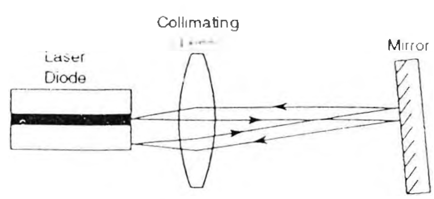


Fig. 4. Path taken by a single ray in an external cavity with a tilted reflector.

Fig. 4 shows the mechanism previously used to explain the effects of reflector misalignment in external cavity semiconductor lasers operating in the strong-feedback regime [7]. When the reflector is tilted with respect to the beam, the image of the laser aperture formed by the first round trip of the external cavity is displaced from the actual aperture. This image may then be reflected from the laser facet and make a second pass of the external cavity. Because the aperture and its first image are conjugate points in the optical system, the second image will coincide with the actual aperture irrespective of the angle of external reflector. The effect of light making a double pass of the external cavity is effectively to double the length of the external cavity.

The attenuation on reflection from the laser facet and at other stages in the second pass of the external cavity explain the reduction in the value of the attenuating filter required in order to observe the double frequency modulation.

Thus it may be advantageous to set up a self-mixing interference based system with the reflector tilted so that there is no feedback from the first pass of the external cavity; using the feedback from the second pass reduces the sensitivity to the reflector alignment and the doubled frequency of the intensity modulation thus obtained offers a doubling in the potential resolution of the system.

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Aspects of the use of self-mixing interference in laser diodes for displacement sensing

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ABSTRACT

Self-mixing interference in laser diodes occurs when light is reflected from an external surface back into the laser cavity. A displacement sensor based on this effect is attractive for its simplicity and low cost; the only necessary optical components are the laser diode, collimating optics, and the reflector. This paper examines the limitations placed on such a system by the effects of thermal and mechanical instabilities. The accuracy and dynamic range of any laser diode-based interferometric sensor is limited by the sensitivity of the operating wavelength to temperature fluctuation. It is shown here that optical feedback can be used to reduce the wavelength sensitivity, and hence increase the potential accuracy, by a factor of two. In the case of misalignment of the reflector, it is shown that, although the output of a laser diode with optical feedback is normally very sensitive to the alignment, if a small alignment is introduced deliberately, feedback can be obtained from light making a double-pass of the external cavity. This leads to a reduction of the sensitivity to alignment and, at the same time, doubles the potential resolution of a sensing system.

Keywords: laser diode, optical feedback, displacement sensing.

1. INTRODUCTION

Coherent optical feedback from an external reflector into a laser diode changes the phase and threshold gain conditions for the laser which leads to changes in the frequency and the intensity of the emitted light. With an appropriate level of feedback, these changes vary periodically with the round-trip phase of the light in the external cavity and so the output of the laser can be modulated in a similar way to the output of a conventional two-beam interferometer; this effect is known as self-mixing interference[1]. Self-mixing interference-based sensing systems are attractive in terms of cost, simplicity and bulk compared to systems based on conventional interferometers. Displacement sensors have been described in which the external cavity phase variation is caused by the displacement of a target reflector[2,3], and range-finding sensors have been proposed in which the phase variation is provided by modulating the laser diode drive current and thus varying the intrinsic laser output frequency[4,5].

All laser diode based interferometric sensors are limited by the strong dependence of the laser output frequency on temperature. The sensitivity of frequency to temperature within a single longitudinal mode is typically about 30 GHz/K. For a self-mixing interference-based displacement sensor with a dynamic range of 1m, the required temperature stability in order to achieve single fringe accuracy is $\Delta T < 4\text{mK}$[6]. A typical temperature stabilizer using a thermistor and a Peltier element can achieve a temperature stability of around 10mK[7]. Optical feedback has previously been used to stablize the frequency of laser diodes by extending the range of single longitudinal mode operation, but without affecting the frequency tuning properties within a longitunal mode. This paper shows theoretically how optical feedback may be used to reduce the sensitivity of the frequency of a laser diode within a single longitudinal mode.

The behaviour of a laser diode with optical feedback depends critically on the strength of the feedback. Due to the small size of the output aperture of a typical laser diode aperture (around 3μm × 5μm) the level of feedback is very sensitive to the alignment of the external reflector; in a system using a 0.4NA collimating lens, a tilt from the normal of about 0.2 mrad will result in the first minimum in the diffraction limited image of the aperture being positioned at the centre of the aperture. This paper will show that by deliberately introducing a small amount of tilt into the target reflector the sensitivity to alignment of the target reflector can be reduced and at the same time the self-mixing interference modulation frequency, and hence the potential resolution of the sensing system, can be doubled.

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$$\phi_2 = \tan^{-1}\frac{\xi\sin(2\pi\nu\tau_2)}{1+\cos(2\pi\nu\tau_2)} \tag{5}$$

$$\approx \xi\sin(2\pi\nu\tau_2). \tag{6}$$

Taking account of the interrelated effects of the changes in threshold gain, round trip phase, and the implied changes in cavity refractive index and lasing frequency gives the following relationship between the solitary laser threshold frequency, ν_{th}, and the lasing frequency with feedback, ν:

$$2\pi(\nu-\nu_{th})\tau_2 + C\sin[2\pi\nu\tau_2 + \arctan(\alpha)] = 0, \tag{7}$$

where C is the feedback coefficient defined by

$$C = \frac{L_{ext}}{L_0}\xi\sqrt{1+\alpha^2}, \tag{8}$$

and α is the linewidth enhancement factor.

Solutions to equation (7) for the lasing frequency, ν, have the form shown in Fig. 2 for varying levels of feedback.. For C>1 there are regions in which there exist multiple solutions which represent discrete external cavity longitudinal modes whose mean frequency spacing is

$$\Delta\nu_{xcm} = \frac{c}{2L_{ext}}, \tag{9}$$

where c is the speed of light.

The mode hops between the external cavity modes occur at opposite turning points in curve 3 of Fig. 2 depending on whether the frequency is increasing or decreasing[10], giving rise to hysteresis as shown by the dashed lines and arrows in the figure. The frequency separation of the upward and downward mode-hops, $\Delta\nu_{hys}$, is a function of the feedback strength and is given by

$$\pi\tau_2\Delta\nu_{hys} = \sqrt{C^2-1} - \arctan\sqrt{C^2-1}. \tag{10}$$

The change in output intensity due to feedback is proportional to the change in threshold gain which in turn is related to the change in frequency, as shown in equation (4). The important feature of the variation in the intensity output is that discrete jumps will occur at the same time as the frequency hops between external cavity modes, and the same hysteresis strength. The effect will be seen. The resulting variation in output intensity is shown in Fig. 3(a) for the case in which the external cavity phase is changing due to a change in the external cavity length, and in Fig. 3(b) for the case in which the laser diode drive current is ramped and the external cavity phase changes as a result of the change in the intrinsic output frequency, ν_{th}. These periodic variations form the basis for self-mixing interference-based sensing systems; the fringes due to changes in the external cavity length can be used to measure displacement in the same way as a conventional interferometer and the spacing of the steps in the pattern due to modulation of the drive current can be used to measure absolute distance between the laser and the external reflector.

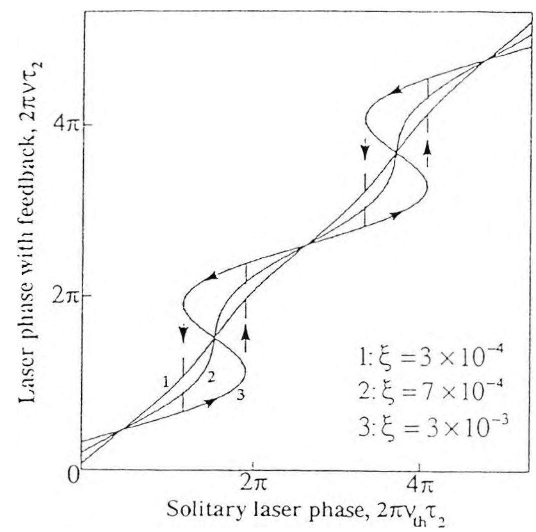


Fig. 2. Theoretical variation of frequency for a laser diode with feedback, ν, with solitary laser frequency, ν_{th}, for three values of feedback coupling coefficient, ξ, with external cavity length, L_{ext}=200mm. Frequencies expressed in terms of external cavity phase.

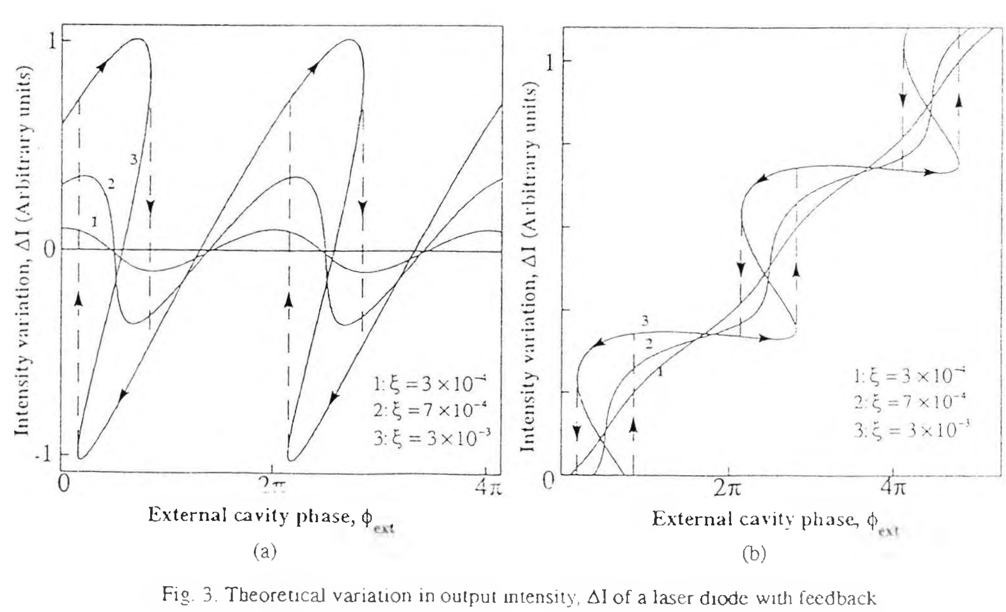


Fig. 3. Theoretical variation in output intensity, ΔI of a laser diode with feedback with extenal cavity phase, $\phi_{ext}=2\pi\nu\tau_2$ due to (a) variation in external cavity length, and (b) variation in drive current, for three values of coupling coefficient, ξ.

3. FREQUENCY CONTROL USING OPTICAL FEEDBACK

The analysis of the previous section can be extended to the case in which feedback occurs from two external surfaces. This situation may arise due to reflection from the front end of a coupling optical fibre or from any other partial reflector placed in the external cavity. In this case the relationship between the solitary laser frequency and the frequency with feedback is given by

$$2\pi(\nu - \nu_{th})(\tau_2 + \tau_3) + \frac{\xi_2}{\xi_1} C_1 \sin(2\pi\nu\tau_1 + \arctan\alpha) + C_2 \sin(2\pi\nu(\tau_2 + \tau_3) + \arctan\alpha) = 0 \quad (11)$$

where ξ_1 and C_1 are the feedback coupling and strength coefficients for the first reflector and ξ_2 and C_2 those for the second reflector and τ_3 is the round trip time between the two reflectors.

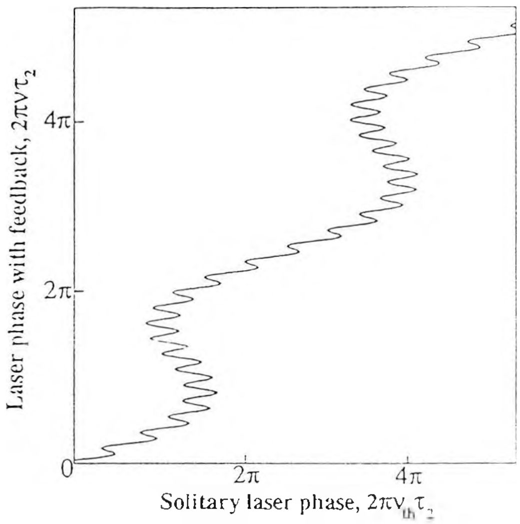


Fig. 4. Theoretical variation of frequency, ν, for a laser diode with two extenal reflectors with variation in solitary laser frequency, ν_{th}. Frequencies expressed in terms of first external cavity phase.

Fig 4. shows an example of a solution to equation (11) for a system with the first reflector 20mm from the laser diode and the second reflector 250mm from the first and respective intensity reflection coefficients of $R_1=3\times10^{-5}$ and $R_2=2\times10^{-6}$. The effect of the first reflector is to give an underlying non-linear in frequency and thus there will be regions in which the sensitivity to of the laser frequency to external factors such as temperature variation will be reduced with respect to the solitary laser diode.

The amount of this reduction and the useful range over which it may be obtained depend on the distance from the laser of the first reflector and the strength of the feedback from it. The gradient at the midpoint of the linear region of the underlying curve in Fig. 3 is $(C+1)^{-1}$, however for C>1 there are regions of multiple solution in which the laser will be unstable, hopping between longitudinal modes created by the first external reflector and for C>4.6 there are multiple solutions for all values of υ_{th}.

Thus with a short, fixed external cavity between the laser and the target reflector of a sensing system, with a feedback strength coefficient of $C_1 = 1$, a reduction in sensitivity of the laser frequency to temperature by a factor of about two can be obtained.

4. EXPERIMENTAL ARRANGEMENT.

The experimental arrangement used to study the effects of feedback on the frequency and power output of a laser diode is shown in Fig. 5. The feedback system comprised the laser diode, LD1 (Sharp LT015MD), the collimating microscope objective, L1 (0.4 NA), the mirror, M1, which was mounted on a loudspeaker cone in order to provide modulation of the external cavity phase, and the neutral density filter, ND1, which was used to control the feedback coupling.

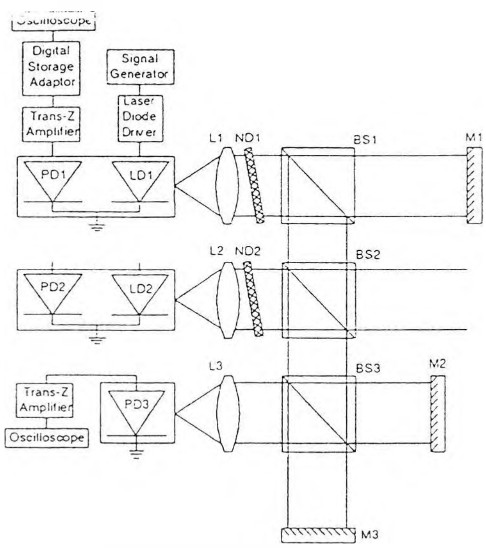


Fig. 5. Experimental arrangement used to study feedback. LD1-2: laser diodes;PD1-3: photodiodes: L1-3: lenses; BS1-3: cube beamsplitters; M1-3: plane mirrors; ND1-2: neutral density filters.

output of this laser was sampled by the beamsplitter, BS1, and changes in wavelength relative to that of a reference laser diode, LD2, were measured using the Michelson interferometer comprising the beamsplitter, BS3, the mirrors, M2 and M3, and the photodiode, PD3.

The visibility pattern produced by the Michelson interferometer with two wavelengths has a sinusoidal component with sharply defined minima whose positions can be measured to an accuracy of $\pm 2\mu m$ in the mirror position which enables relative measurements of wavelength, accurate to within $\pm 5\times 10^{-4}$nm.

The feedback into LD1 from M1 was maximized by finding the mirror position which gave the maximum reduction in threshold current. The intensity output of LD1 was observed on an oscilloscope via the photodiode at the rear facet of the laser and oscilloscope traces were digitized and stored using a digital storage adaptor. By measuring the wavelength tuning with respect to drive current using the Michelson Interferometer and measuring the variation of photodiode output with drive current, any changes in wavelength could then be inferred from associated changes in the photodiode output.

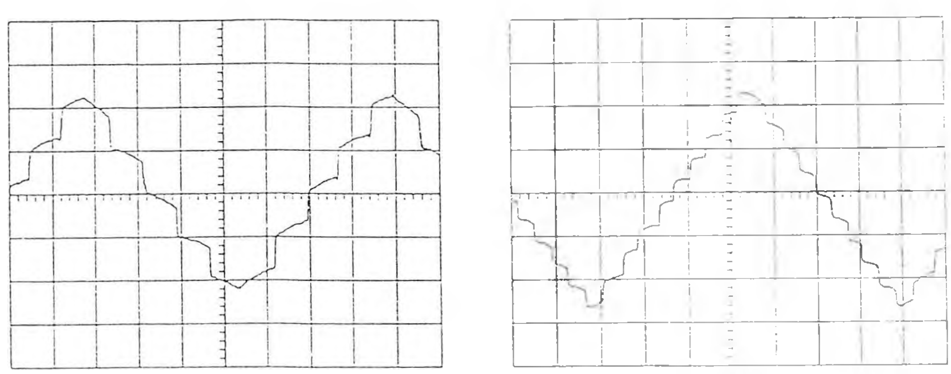
Values of the feedback coefficient, C, were obtained by measuring the hysteresis separation of the mode-hops in the upward and downward halves of the modulation period. This was done using the digitally stored oscilloscope traces, identifying the positions of the mode-hops and then finding their distance from the turning point in the modulation signal. The measured hsyteresis frequency separation was then used to solve equation (10) numerically for the value of C.

5. RESULTS.

Fig. 6 shows two oscilloscope traces, each of which represents the variation of intensity output of the laser diode with feedback as the drive current was modulated with a triangular wave with a peak to peak amplitude of 2mA. Fig. 7 shows the variation of intensity as the length of the external cavity was modulated, each trace having been recorded under the same conditions as those in Fig. 6 respectively. The optical length of the external cavity was 252± 1mm; the differences introduced between (a) and (b) were a change in the density of the neutral density filter from D=2.5 to D=0.5 ($D = -\log_{10}T$, where T is the intensity transmittance of the filter) and a slight adjustment to the angular position of the mirror.

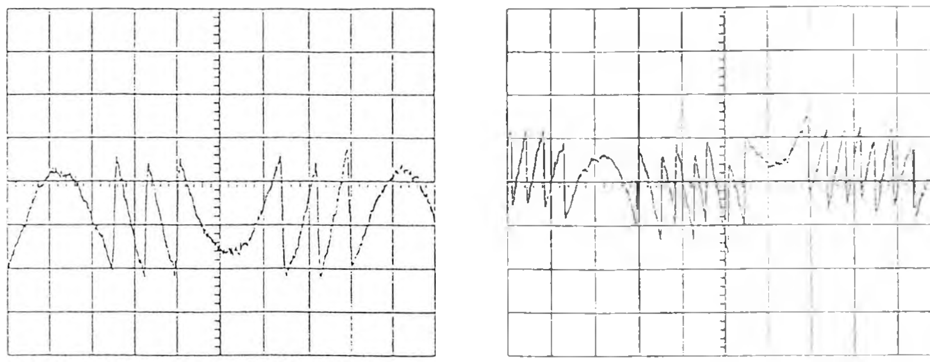
The steps in the intensity output traces in Fig 6 correspond to the external cavity longitudinal modes of the laser diode. The mean frequency separation of these modes was 677MHz for (a) and 337MHz for (b) and the respective values of C obtained were 2.78 and 3.30. The expected longitudinal mode spacing for an external cavity of length 252±1mm is 595±2MHz. Fig. 4 shows traces obtained with the same feedback conditions as for those in Fig. 3 but with the laser drive current held constant and the position of the mirror modulated by applying a signal to the loudspeaker cone on which it is mounted.

These results show that the frequency of fringes or intensity steps produced in a self-mixing interference system may be double the value expected for certain levels of feedback intensity and certain alignments of the external reflector. This effect has been observed to be independent of the external cavity length on a scale of about 1mm.



(a) (b)

Fig. 6. Oscilloscope traces showing output intensity of a laser diode with feedback with the drive current modulated with a triangular waveform. (Horizontal scale:2ms/div; vertical scale: 10mV/div.)



(a) (b)

Fig. 7. Oscilloscope traces showing output intensity of a laser diode with feedback with the external cavity length modulated using a loudspeaker mounted mirror. (Horizontal scale:2ms/div; vertical scale: 10mV/div.)

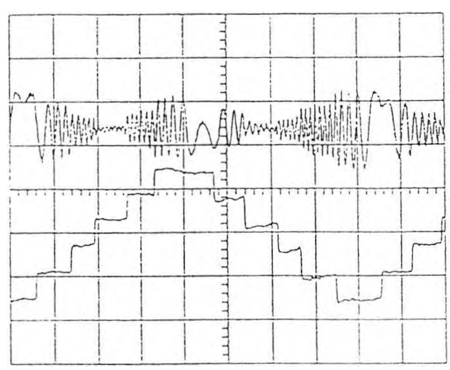


Fig. 8. Oscilloscope traces showing intensity output of a laser diode with feedback with drive current modulation (lower trace) and simultaneous dual-wavelength Michelson interferometer output (upper trace). (Horizontal scale: 2ms/div; Vertical trace: 10mv/div).

Fig. 8 shows two oscilloscope traces recorded simultaneously; one showing the intensity output of the laser diode for a case where the step frequency is double the expected value, the other the output from the Michelson interferometer with the adjustable mirror placed close to a minimum in the dual wavelength visibility pattern. The interferogram shows small steps in the visibility pattern coincident with the steps in the intensity output indicating that there is no change in the intrinsic laser diode longitudinal mode throughout the current modulation period

6. DISCUSSION

The results reported in the previous section show that for certain combinations of feedback strength and reflector alignment the self-mixing interference intensity modulation frequency is double the expected value. A similar effect has been reported before[11] and was shown to be due to hopping between the intrinsic laser diode longitudinal modes and occurred only when the external cavity length was equal to a half-integer multiple of the laser diode cavity

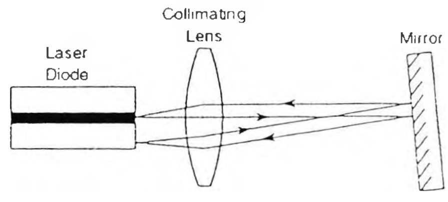


Fig. 9. Path taken by a single ray in an external cavity with a tilted reflector

length. However, the effect reported here has been shown to be independent of the external cavity length on the scale of the length of the laser cavity and to take place without jumps in frequency of the magnitude of the intrinsic laser diode longitudinal mode spacing.

The effects of reflector misalignment have previously been discussed for the case of external cavity semiconductor lasers which operate in a high feedback regime[12]. The effects described for the strong feedback case are not applicable to the weak feedback regime used for sensing applications, but the mechanism used to explain those effects can be used to explain the modulation frequency doubling described here. Fig. 9 shows the path of a single ray emitted on the optic axis and reflected from a mirror tilted at an angle to the beam normal. The first image of the laser aperture is displaced from the actual aperture and the feedback strength will be reduced. however this image may be reflected from the laser facet and, because the aperture and the first image are conjugate points in the system, a second image will be formed coincident with the laser aperture irrespective of the tilt of the reflector. The effective length of the external cavity and consequently the self-mixing interference modulation frequency are thus doubled.

The intensity of the beam wil be reduced by reflection from the laser facet and at the other elements during the second pass of the external cavity. The laser diode used in the experiments has an aluminium oxide quarter-wavelength coating on the output facet, with an estimated intensity reflectivity of R_2=0.07 and so in order to achieve the same value
of feedback strength coefficient, the value of the neutral density filter in the external cavity should be reduced by approximately 2, as was observed.

The modulation frequency doubling effect may thus be avoided by an appropriate choice of attenuating filters in the external cavity. However there are advantages for self-mixing interference systems in setting the attenuation and the angular position of the external reflector so that the system uses the feedback from the second pass of the external cavity: the independence of the postion of the second pass image to the mirror tilt reduces the sensitivity of the system to mechanical instability and the doubling of the modulation frequency offers a doubling in the potential resolution for displacement or range-finding sensors.

7. ACKNOWLEDGMENTS

The authors acknowledge the support of the Engineering and Physical Sciences Research Council for this work. RCA is grateful to the Worshipful Company of Scientific Instrument Makers, London for a bursary enabling his attendance at this conference.

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ASPECTS OF THE USE OF OPTICAL FEEDBACK FOR FREQUENCY STABILIZATION OF LASER DIODES

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INTRODUCTION

Self-mixing interference in laser diodes occurs when light is reflected from an external target back into the laser cavity. The changes in the gain and phase conditions for the resulting compound cavity result in changes in the optical frequency and the intensity of the laser output. The laser diode output may thus be modulated by a change in phase in the external cavity to give a signal similar to the output of a conventional Michelson interferometer. This enables the construction of simple, inexpensive interferometric sensing systems, the necessary components of which are just the laser diode, collimating optics and a target reflector[1].

Such a system for displacement sensing has been demonstrated with a dynamic range for which a useful signal was obtained of over 1m[2]. However for such a large target distance the interferometer output has a high sensitivity to wavelength, which, for a laser diode, is sensitive to temperature. To achieve single fringe accuracy with a target reflector distance of 1m requires a wavelength stability of 3×10^{-4}nm or better; for a typical wavelength sensitivity to temperature of about 30nm/K, this requires a temperature stability of 4mK or better, whereas a typical temperature controller using a thermistor and a Peltier element can provide a stability of about 10mK[3].

Optical feedback has previously been used for wavelength stabilisation in the sense of extending the range of operation in a single longitudinal mode. This is achieved by controlling the variation with temperature of the gain spectrum peak and does not affect the wavelength sensitivity within the longitudinal mode. This paper considers the possibility of using the changes in frequency caused by external feedback to provide wavelength stabilization of laser diodes within a longitudinal mode.

BACKGROUND THEORY

For a system consisting of a laser diode with cavity length, L_0, and front facet amplitude reflectivity, r_1, and an external reflector at a distance from the laser front facet of L_1 with and amplitude reflectivity r_2, the output frequency for the laser with feedback is found by replacing the front facet amplitude reflection coefficient with an effective amplitude coefficient taking into account the amplitude and phase of the light re-entering the laser cavity from the external reflector and then applying the usual laser gain and phase conditions. This leads to the following relationship between the solitary laser frequency, ν_{th}, and the frequency for the laser with feedback, ν:

$$2\pi(\nu - \nu_{th})\tau_1 + C\sin[2\pi\nu\tau_1 + \arctan(\alpha)] = 0, \tag{1}$$

where C is the feedback coefficient defined by:

$$C = \frac{L_1}{L_0}\left(1 - |r_1|^2\right)\frac{r_2}{r_1}\sqrt{1+\alpha^2}, \tag{2}$$

and α is the linewidth enhancement factor and τ_1 is the round trip time for light in the external cavity[4].

Solutions to equation (1) are shown in Fig. 1. for various values of the feedback coefficient, C. For C>1 there exist regions in which there are multiple solutions for ν which correspond to discrete external cavity longitudinal modes with a frequency separation given by:

$$\Delta\nu_{xcm} = \frac{1}{\tau_1}. \tag{3}$$

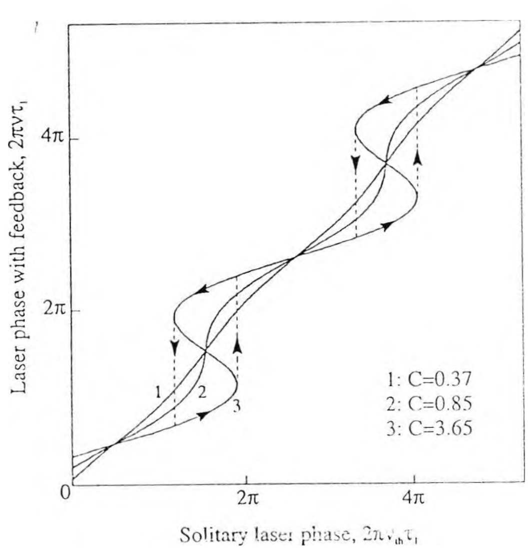

Fig. 1. Variation of laser frequency with feedback from a single external reflector with solitary laser frequency, expressed in terms of external cavity round trip phase, for various feedback strengths.

The hops between these modes occur at opposite turning points on the curve depending on whether the frequency is increasing or decreasing[5]. The strength of this hysteresis is dependent on the feedback strength and is given in terms of the solitary laser frequency by:

$$\Delta\nu_{hys} = \frac{1}{\pi\tau_1}\left(\sqrt{C^2-1} - \arctan\sqrt{C^2-1}\right), \qquad (4)$$

The width of the region of stable operation in which there is a single solution to equation (1) is the difference between the external cavity longitudinal mode spacing and the hysteresis frequency separation given by equation (4). The intensity output of the laser is closely related to the optical frequency and so hopping between external cavity longitudinal modes leads to discontinuities in the intensity.

THEORY OF WAVELENGTH STABILIZATION USING OPTICAL FEEDBACK

For a system consisting of the laser diode with two external reflectors, at distances L_1 and L_2 from the laser front facet and with an amplitude reflectivities r_2 and r_3 respectively, the output frequency for the laser with feedback is given by:

$$2\pi(\nu-\nu_{th})\tau_2 + \frac{L_2}{L_1}C_1\sin(2\pi\nu\tau_1+\arctan\alpha) + C_2\sin(2\pi\nu\tau_2+\arctan\alpha) = 0 \qquad (5)$$

where C_1 and C_2 are feedback coefficients for the first and second external reflectors and τ_1 and τ_2 are the round trip times for light in the first and second external cavities. Solutions to equation (5) are shown in Fig. 2 for a system with L_0=1mm, L_1=25mm and L_2=300mm and with intensity reflection coefficients, $R_1=0.075$, $R_2=2\times10^{-5}$ and $R_3=10^{-6}$. The first reflector provides a large-scale modulation of the frequency characteristics which can be used to used to provide stabilization while the second reflector superimposes the interferometric sensing signal.

The effectiveness of the first reflector in stabilizing the output frequency of the laser diode depends on the distance to the first reflector and the strength of the feedback from it. The reduction in wavelength sensitivity is given be the gradient of the underlying large scale component of the curve in Fig. 2 at the midpoint of the flat region, which is :

$$\frac{\partial\nu_1}{\partial\nu_{th}} = \frac{1}{C_1+1} \qquad (6)$$

and the width of the frequency band in which the laser will operate stably in a single longitudinal mode of the first external cavity is given by equation (4). There is thus a trade-off to be made between obtaining a large reduction in wavelength sensitivity by having a large feedback coefficient and a large region of stable single mode operation, which requires a low value of C as well as a short distance between the laser and the reflector. For $C_1=1$, there are no multiple external cavity modes and the maximum reduction in wavelength sensitivity is of a factor of 2. Any further reduction in wavelength sensitivity is at the expense of the width of the single longitudinal mode band, the minimum width of which will be determined by the temperature stability with which the laser diode is operated; for example, for a laser diode with temperature tuning coefficient in the absence of feedback of

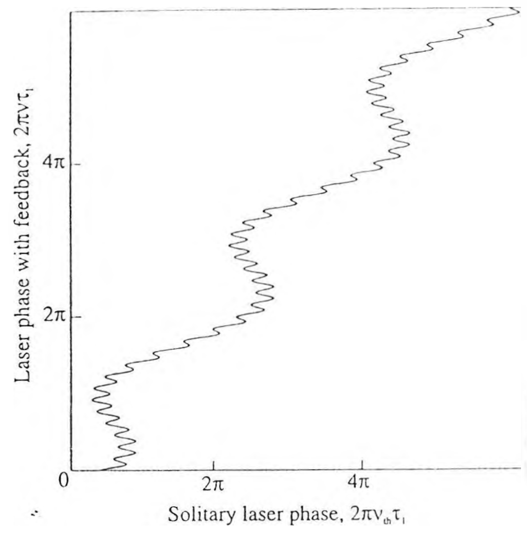

Fig. 2. Variation of laser frequency with feedback from two external reflectors with solitary laser frequency, expressed in terms of round trip phase in the first external cavity, for L_1=25mm, L_2=300mm, $R_2=2\times10^{-5}$, $R_3=10^{-6}$

30GHz/K and a first external cavity of 25mm, if the single mode region is half the width of the external longitudinal mode spacing the maximum temperature variation allowed to keep the laser operating in the single longitudinal mode region is about 0.1K which is easily achieved with most temperature stabilizers. The wavelength sensitivity reduction achieved in the case of this example would be of about a factor of four.

EXPERIMENTAL ARRANGEMENT

Experiments have been performed using Sharp LT015MD single mode laser diodes which have a nominal wavelength of 830nm and have a quarter-wavelength aluminium oxide coating on the front facet which gives an estimated intensity reflectivity coefficient, R_1, of 0.075. The laser diode package has an integral monitor photodiode at the rear facet of the laser.

The frequency tuning coefficient with respect to drive current was measured for a laser diode using a dual wavelength Michelson interferometer in which changes in the output frequency of the laser under test were measured with respect to the output frequency of a similar reference laser diode. This measurement along with a measurement of the variation of the monitor photo diode output with drive current allows variations in frequency to be determined from variations in the photodiode output.

The effects of feedback on a laser diode have been studied by observing the intensity output of the laser as the drive current was modulated with a triangular wave signal. A digital storage adaptor was used to record oscilloscope traces from which measurements of the external cavity longitudinal mode spacing and the hysteresis frequency separation were made in order to make a comparison with the theory.

The effects of feedback with two serial reflectors were observed using a diffraction grating (holographic, 1800 lines/mm) from which the first diffracted order was directed straight back to the laser and the zero order beam was incident on the second reflector. The fraction of light directed into the first diffracted order was approximately 0.1. Neutral density filters were used between the laser and the grating and between the grating and the mirror to control the levels of feedback.

Estimation of Feedback Strength

In any practical implementation of a feedback-based system it is important to be able to make a reasonable estimate of the strength of feedback that can be achieved in order to be able to operate within the narrow range of useful feedback strength.

The coupling from the laser to the collimating lens can be calculated simply from the laser's beam angles and the numerical aperture of the collimating lens. If the laser beam angle in either direction is greater than the numerical aperture of the lens than the return beam then the size of the focussed spot on the laser front facet will be determined by diffraction; the dimensions of the laser active region can be estimated, assuming Gaussian beam optics, from the beam angles, and so the coupling between the reflected beam and the laser cavity can be calculated. Other sources of loss are the reflectivity of the external reflector and reflections from the surfaces of the lens.

Making these calculations for the system used here gives a maximum value for R_2 of approximately 0.5. In practice this value is an upper limit because it does not take into account such factors as the alignment and positioning of components and beam profile matching; it does however provide a useful for ensuring that a system is operating in the correct feedback regime and for optimizing the position and alignment of components.

RESULTS AND DISCUSSION

Fig,. 3 shows a recording of an oscilloscope trace of the monitor photodiode output with the laser drive current modulated with a triangle wave with an amplitude of approximately 2mA. Feedback was from a single mirror at a distance of 340mm with a

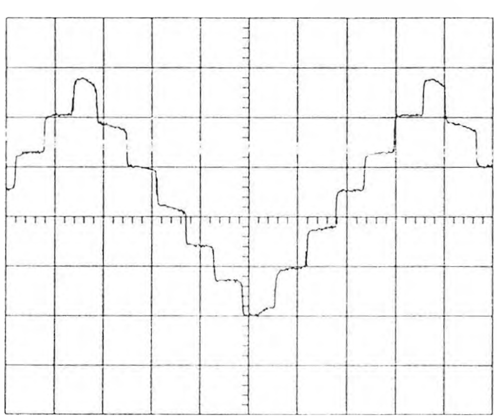

Fig. 3. Oscilloscope trace showing monitor photodiode output from laser with feedback from a single external reflector and with modulated drive current. (Horizontal scale: 2ms/div; vertical scale: 10mV/div.)

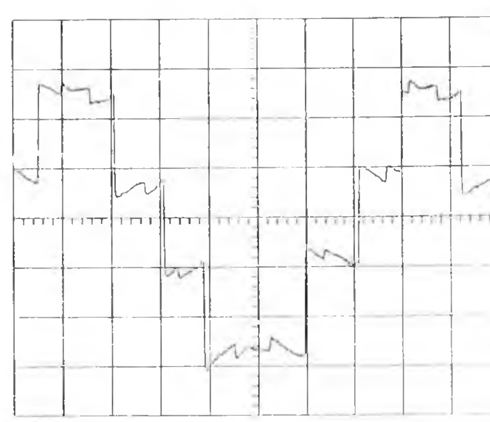

Fig. 4. Oscilloscope trace showing monitor photodiode output from laser with feedback from two external reflectors and with modulated drive current. (Horizontal scale: 2ms/div; vertical scale: 10mV/div.)

neutral density filter of density 2.4 in the cavity. The discontinuities due to mode-hopping between external cavity modes are clearly visible. The external longitudinal mode spacing was found by measuring the mean separation of the flat regions of the trace and the hysteresis frequency separation was found by measuring the positions of the upward and downward mode-hops relative to the turning point in the modulation signal. The measured external longitudinal mode spacing was 412MHz which is in good agreement with the value predicted using the cavity length in equation (4). The hysteresis frequency separation was measured to be 370MHz from which the feedback coefficient, C was found to be 4.1. This value of C implies an effective external cavity reflectivity, R_2, of 0.31 in the absence of the neutral density filters.

Fig. 4 shows a trace obtained with feedback from the first order of the diffraction grating and from the zero order reflected by a mirror. The distance from the laser to the grating was 78mm and from the grating to the mirror, 342mm, and there were neutral density filters of density 1.4 between the collimating lens and the grating and 0.6 between the grating and the mirror. Without the feedback from the mirror the feedback coefficient was found to be C=3.4. The modulation in the intensity due to the second reflector is clearly shown superimposed on the large scale modulation due to the feedback from the grating. This demonstrates the potential for the use of a double external cavity self mixing interference system for reducing the wavelength sensitivity with respect to the single external cavity configuration while still providing a useful interferometric output. In this case the feedback coefficient was measured with feedback from the grating only and was found to be C=3.4. This is relatively high and the distance from the laser to the grating was relatively large which results in a narrow single frequency range and only approximately one fringe due to the target reflector within each modulation period due to the grating. The use of a graded index rod lens for collimating can reduce the distance from the laser to the first external reflector to about 10mm and this along with correct control of the level of feedback would extend the useful single external cavity longitudinal mode range.

ACKNOWLEDGEMENTS

This work is supported by the Engineering and Physical Sciences Research Council. R. Addy is grateful to the Engineering and Physical Sciences Research Council, the Worshipful Company of Scientific Instrument Makers, and the Instrument Science and Technology Group of the Institute of Physics for funding his attendance at this conference.

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Sensors and their Applications VII

Proceedings of the seventh conference on Sensors and their Applications, held in Dublin, Ireland, 10–13 September 1995

Edited by

A T Augousti

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Institute of Physics Publishing
Bristol and Philadelphia

Wavelength Stabilisation of Laser Diodes Using Optical Feedback

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Abstract. Wavelength stability is an important factor in the potential accuracy of a laser diode based interferometric sensor. Two aspects of the use of optical feedback for wavelength stabilisation are discussed; the use of weak feedback for longitudinal mode stabilisation and the use of strong feedback to reduce the temperature dependence of the longitudinal mode wavelength. Experimental results are presented to demonstrate the latter effect, and confirm its value in optical measurement techniques.

1. Introduction

The effects of self-mixing interference in laser diodes [1] have recently been employed for various sensor applications including velocimetry and the measurement of displacement [2,3]. Self-mixing interference occurs when light emitted by a laser diode is reflected from an external surface (which, in practice, may be a mirror, a grating, or a scattering surface) and re-enters the laser cavity. Coupling between the cavity photon density and the injected charge carrier density results in a modulation of, in turn, the cavity refractive index, the threshold gain, and the intensity of the laser output. With the correct level of feedback a self-mixing interference-based system produces a modulated output with one fringe per half-wavelength of movement of the external reflector, similar to the output of a conventional interferometric system. Self-mixing interference- based systems, however, are of particular interest due to their potential advantages in terms of the size of the equipment, simplicity and cost.

2. Brief Description of Feedback

In order to illustrate both the effects and the use of such a feedback system, Fig. 1 shows the compound optical system formed by a laser diode with an external reflector. The laser diode has a cavity length, l, and facets with amplitude reflection coefficients r_1 and r_2. The external reflector has an amplitude reflection coefficient, r_3, and forms an external cavity of length, L.

The amount of light re-entering the cavity is characterised by the coupling coefficient, K, where

$$K = (1 - |r_2|^2)\, r_3 / r_2 \qquad (2.1)$$

The strength of the feedback in terms of its effects depends not only on the coupling strength but also on the length of the external cavity and is quantified by the feedback coefficient, C, where

$$C = K(1 + \alpha^2)^{1/2} L / nl \qquad (2.2)$$

where n is the laser cavity refractive index and α is the linewidth enhancement factor and is a measure of the coupling between the real and imaginary parts of the refractive index [4].

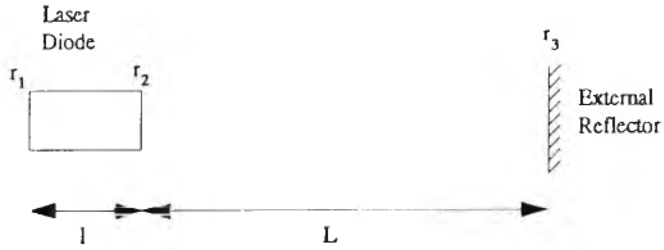

Fig. 1 Schematic representation of a iaser diode with an external reflector.

3. Wavelength Stabilisation of Laser Diodes

There are a number of major considerations in the use of such systems in practical optical sensor devices. One important factor in the accuracy of a laser diode-based interferometric sensor is the stability of the wavelength of the light emitted by the laser diode, which typically varies with the laser diode drive current and with temperature. In practice the sensitivity to temperature is the more significant and that will considered here, the effect of the laser drive current, above threshold, is mainly due to its effect on the temperature of the device and is qualitatively similar to the effect of any other externally caused temperature change.

There are two major temperature-dependent factors which affect the wavelength: the gain peak wavelength shift of typically about 0.3 nm / K and the change of the refractive index and hence the optical length of the cavity, which shifts the longitudinal mode wavelength by about 0.07 nm / K [5]. The combined result of these effects is that there are discrete regions with a lower sensitivity in-between "hops" between longitudinal modes, as shown in Fig. 2.

Two possible methods of improving the wavelength-temperature characteristics by using either weak or strong feedback are discussed below, with the aim of offering a system of satisfactory stability for practical applications.

3.1 Weak Feedback

A technique already in commercial use (e.g. the Sharp LTO80MD laser diode [6]) uses an external cavity which is shorter than the laser cavity, in which case the feedback coefficient is low ($C < 1$) and the feedback can be described as *weak*. The periodic variation of threshold gain with feedback phase modulates the wavelength gain distribution and determines the position of the gain peak and therefore the longitudinal

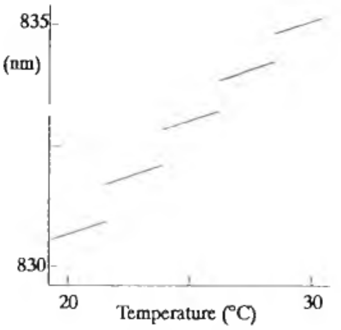

Fig. 2 Typical variation of laser diode wavelength, λ, with temperature.

mode in which the laser diode operates. If the variation of the feedback phase with temperature can be controlled so that the gain peak wavelength has the same temperature dependence as the longitudinal mode wavelength, then the wavelength interval between mode hops can be greatly increased.

Whilst this technique may eliminate potential problems caused by longitudinal mode hopping, it does not affect the temperature sensitivity of the wavelength within the longitudinal mode, and thus is limited in its applicability.

3.2 Strong Feedback

For very high accuracy applications, or for the use of a self-mixing interference system with a long external cavity, the wavelength stability criteria may need to be very strict. For example in a self-mixing interference system with an external cavity of length, L, and a laser diode wavelength, λ, the fringe number, n, is given by

$$n = 2L / \lambda \tag{3.1}$$

and the variation of n with wavelength is thus:

$$\partial n / \partial \lambda = - 2L / \lambda^2 \tag{3.2}$$

For an external cavity length of 1m with a laser diode operating at 830 nm, the wavelength tolerance for single fringe accuracy is $\Delta\lambda < 3 \times 10^{-4}$ nm, and with a temperature sensitivity of 0.07 nm / K, the corresponding temperature tolerance required is $\Delta T < 4$ mK. A typical laser diode temperature controller using a thermistor and a Peltier element can achieve a long term stability of $\Delta T \approx 10$ mK [7], and so it is desirable to be able to reduce the temperature sensitivity within a single longitudinal mode.

Previously published theoretical studies of feedback in laser diodes [8,9] suggest that this is possible when the feedback coefficient is high ($C > 1$), which can be achieved by a combination of high coupling and the use of an external cavity longer than the diode cavity, producing *strong* feedback. Under these conditions, external cavity longitudinal modes exist whose spacing is less than the spacing of the intrinsic laser diode modes and whose temperature sensitivity is also less.

4. Experiment

4.1 Method

The wavelength stability characteristics of a laser diode have been measured with respect to drive current as this offers greater resolution and accuracy, but the qualitative features of the results may equally well be applied to temperature stability.

The experimental arrangement used to measure the wavelength change in a laser diode with feedback is shown in Fig. 3. Light was fed back into the laser diode, LD1, from a 1800 line / mm holographic diffraction grating, DG, which formed an external cavity of length, L. Part of the beam from LD1 was diverted by the beam-splitter, BS1 and mixed with the beam from a second laser diode, LD2, by the beam-splitter, BS2. The combined beam was analysed by the Michelson interferometer consisting of the beam-splitter, BS3, the mirrors, M1 and M2, and the photodiode, PD. The neutral density filters, ND1 and ND2, were used to minimise feedback from the interferometer into the two lasers. The difference in wavelength between the outputs of the two lasers was found by measuring the separation of the minima in the visibility pattern observed with the Michelson interferometer. Both lasers were temperature stabilised to about 10 mK and their drive currents were stable to about 0.02 mA. These were the main factors affecting the uncertainty in the wavelength difference measurements, which was estimated in this work to be $\pm 5 \times 10^{-4}$ nm.

4.2 Results

Fig. 4 shows the measured variation of wavelength difference, δλ, with the drive current of the laser diode with feedback. For this measurement, the optical length of the external cavity was 195 ± 5 mm.

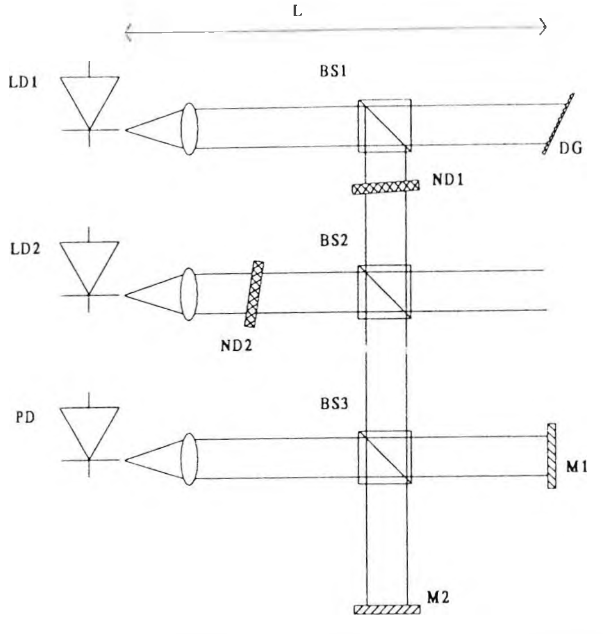

Fig. 3 Experimental arrangement; LD1, LD2: Laser diodes; DG: Diffraction grating; PD: Photodiode; BS1, BS2, BS3: Beam-splitters; ND1, ND2: Neutral density filters; M1, M2: Mirrors.

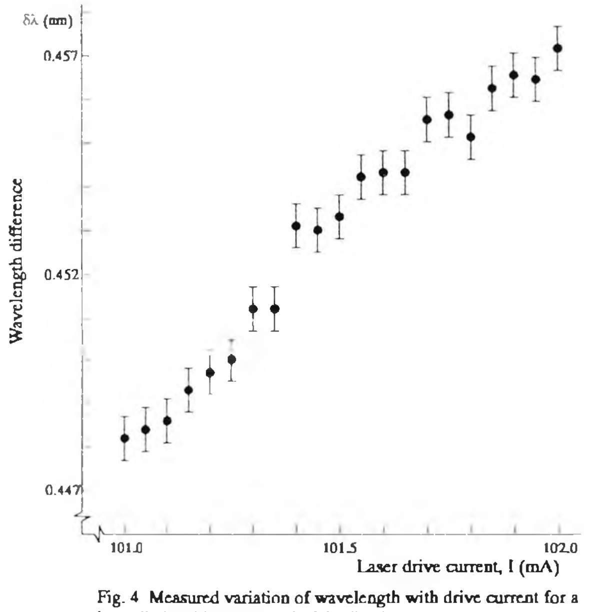

Fig. 4 Measured variation of wavelength with drive current for a laser diode with strong optical feedback.

The average gradient of the graph of wavelength difference versus drive current is 8.9 ± 0.6 nm / mA but it can be seen that there are groups of two or three points in which the gradient is significantly less than the large-scale average and between which there are sudden jumps, corresponding to hops between external cavity longitudinal modes.

The mean wavelength separation of adjacent groups is $1.3 \pm 0.2 \times 10^{-3}$ nm, which is in reasonable agreement with the calculated external cavity mode spacing of $1.75 \pm 0.05 \times 10^{-3}$ nm

5. Conclusion

Two different aspects of the use of optical feedback for wavelength stabilisation have been discussed. The first, using weak feedback to reduce the occurrence of longitudinal mode hops, is already in current commercial use. This work has emphasised an approach which builds upon this to enable the use of a laser diode-based system in optical measurement applications by developing a method of reducing the temperature dependence of the wavelength within a given longitudinal mode.

Further work is required, to achieve a greater experimental resolution, in order to determine the practical limitations of this technique in optical measurement situations. Specific areas for future investigation include the process of transition between the external cavity modes and the possibility of bistability around the region of transition.

6. Acknowledgements

The authors are pleased to acknowledge the support of the Engineering and Physical Sciences Research Council in this work.

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