Disruption Mitigation and Pricing Flexibility

Oben Ceryan
Bayes Business School
City, University of London
oben.ceryan@city.ac.uk

Florian Lücker
Bayes Business School
City, University of London
florian.lucker@city.ac.uk

July 27, 2023

Book Chapter, Forthcoming, Foundations and Trends in Technology, Information and Operations Management

Abstract

We study a firm that is exposed to random supply chain disruptions while producing a single product. During a disruption, the firm may use reserve inventory and/or reserve capacity to serve customer demand. As supply in the form of reserve inventory and reserve capacity is often lower than demand during a disruption, the firm may choose to increase the price of the product during the disruption. An increase in price reduces demand during the disruption, which may help better match supply and demand during the disruption. We find that pricing flexibility (i.e., the ability to increase the price during a disruption) may complement or substitute the operational mitigation levers of holding reserve inventory or reserve capacity. Specifically, when a firm has pricing flexibility, it may be economical to increase the use of reserve inventory or reserve capacity relative to a setting without pricing flexibility.

1. Introduction and Motivation

When supply chain disruptions occur, customer demand often exceeds the available supply. Some firms opt to increase the price when supply is limited. Consider the 2011 earthquake in Japan. The disaster affected production at Honda and Nissan and resulted in a reduction of inventory of cars from both companies. Nissan Rogue's price rose by 3% after the earthquake (while the average industry price increase was 0.3%). Honda Fit's price rose by nearly 6% (while the average increase in the prices of compact cars was 2.3%). Price increases when demand exceeds supply are not limited to the automotive industry. Coffee prices soared 44% from June to September 2010 due to bad weather in South America threatening crops. Further, U.S. egg prices hit record high due to bird flu in 2015.

In this research, we want to better understand the role of pricing flexibility, i.e., the flexibility to increase prices when demand exceeds supply during a disruption, in reducing profit losses during a disruption. Our goal is to understand how pricing affects risk mitigation levers that have been widely studied in the literature such as holding additional inventory or reserve capacity (Tomlin 2006, Chopra et al. 2007, Qi 2013, Qi and Lee 2015).

We consider a firm producing a single product that is exposed to random supply disruptions resulting in a production stop for a random time length. We model a two-stage problem where in the first stage the firm decides on the optimal amount of reserve inventory and/or reserve capacity to carry as a protection from profit losses during a disruption. When a disruption occurs, the firm decides, in the second stage, on the optimal price, given the available reserve inventory and/or reserve capacity amount. Clearly, both decisions (first and second stage) are interrelated. If the firm has significant pricing flexibility to manage demand during a disruption, a price increase typically results in a reduction of demand, which then affects the optimal amount of reserve inventory and/or reserve capacity to carry in the first place.

We find that pricing flexibility may complement or substitute the use of reserve inventory. While without pricing flexibility it may be optimal not to hold any reserve inventory [or little reserve inventory], once there is sufficient pricing flexibility, it may be economical to use reserve inventory [increase the use of reserve inventory]. Similarly, we find that the reserve capacity may complement or substitute the use of pricing flexibility.

Pricing flexibility has been studied in the operations literature. Van Mieghem and Dada (1999) study the value of pricing flexibility when a firm takes capacity and inventory decisions while facing uncertain demand. The authors consider a two-stage problem where in the first stage inventory is decided and in the second stage the price is set when the uncertainty is resolved. Tang and Yin (2007) extend this work to include supply yield uncertainty. In their model, a retailer faces uncertain supply yield. Once this uncertainty is resolved, the retailer sets the optimal price to maximize the profit during the selling season. While the model is very similar to our setup, we focus on the inventory and reserve capacity decisions that have to be committed to before a disruption takes place. Our goal is to understand how the optimal inventory and capacity decisions change with the pricing flexibility. Further literature considers the interplay between pricing flexibility and supplier diversification when some supply is uncertain (Dong et al. n.d., Li et al. 2017, 2013), or the role of risk aversion under pricing flexibility and supply yield uncertainty (Kouvelis et al. 2021).

2. Model

2.1 Model Preliminaries

We consider a firm that produces a single product and that employs *reserve inventory* and/or *reserve capacity* to mitigate the effects of random disruptions. The firm also possesses some *pricing flexibility* to increase its price during a disruption to better align demand with its limited supply. Our main objective in this study is to provide insights on the impact of pricing flexibility during a disruption on a firm's reserve inventory and capacity decisions in anticipation of disruptions.

The reserve inventory is a fixed quantity of I units that the firm decides to hold during the non-disrupted time periods with a holding cost rate of h per unit and per unit time. The firm can then sell from the reserve inventory to partially or fully meet the demand during a disruption. We let u denote the replacement cost per unit for the reserve inventory, i.e., the production (replenishment) cost per unit is u. The reserve capacity, on the other hand, is the level of ancillary production (replenishment) capability that the firm has obtained access to during a disruption and can provide a steady production of a units per unit time during the disruption at a cost of $c_a > u$ per unit. We note that as the replacement cost for reserve inventory is lower than the unit cost of producing through reserve capacity, the firm first utilizes its reserve inventory before resorting to the reserve capacity during a disruption.

To model the firm's pricing decisions, we assume a linear price-demand relationship with a demand rate of $d(p) = b_0 - b_1 p$, where p is the price the firm charges, b_0 is the demand intercept, and b_1 is the price sensitivity coefficient. (We assume $b_0, b_1 > 0$.) For future reference, the profit maximizing price for the non-disrupted period, $p_o := \arg\max_p (p-u)\,(b_0 - b_1 p)$, can be found as $p_o = \frac{b_o}{2b_1} + \frac{u}{2}$, which we will refer to as the *base price*. We also let $r(p_o) := (p_o - u)\, d(p_o)$ denote the base profit rate per unit time.

The timeline of the firm's decisions is as follows: First, the firm decides on the level of reserve inventory, I, and/or reserve capacity, a, to invest in before experiencing the disruption. When a disruption occurs, the firm may adjust its price taking into account its reserve inventory and/or capacity level as well as the length of the disruption, k. For the remainder of this chapter, we assume that once a disruption occurs, the firm is able to foresee the length of the disruption.

We will first describe the firm's optimal pricing problem during a disruption given reserve inventory and reserve capacity levels and then formulate the initial problem of setting these optimal reserve inventory and capacity levels taking into account the optimal pricing decisions that the firm will apply during a subsequent disruption.

2.2 Pricing During Disruption

Suppose the firm enters a disruption period of length k with a reserve inventory of I units and a reserve capacity of a units per unit time.

The firm's optimal price setting problem can be stated as:

$$\tilde{\Pi}_d(I, a, k) = \max_p \quad \Big((p-u) \min\{I, (b_0 - b_1 p)\, k\} + (p - c_a) \min\{a\, k, \max\{(b_0 - b_1 p)\, k - I, 0\}\} \Big) \tag{1}$$

where $\tilde{\Pi}_d(I, a, k)$ denotes the optimal disruption profit. As the formulation given in (1) indicates, the objective function is piecewise in disruption price. Specifically, we have the following three cases:

(i) $(b_0 - b_1 p)\, k < I$. In this case, the reserve inventory exceeds disruption demand at price p. Therefore, a partial use of the reserve inventory without employing any of the reserve capacity is sufficient to meet the disruption period demand. The firm's disruption profit function becomes: $(p - u)\,(b_0 - b_1 p)\, k$.

(ii) $(b_0 - b_1 p)\, k - a\, k < I \leq (b_0 - b_1 p)\, k$. In this case, the reserve inventory is not sufficient to cover all disruption demand by itself, but a simultaneous utilization of the entire reserve inventory along with a partial use of reserve capacity is sufficient to meet disruption period demand at price p. The firm's corresponding disruption profit function is given by $(p - u)\, I + (p - c_a)\,((b_0 - b_1 p)k - I)$, where the first term refers to the profit obtained through selling the entire reserve inventory of I units and the second term refers to the profit obtained from meeting the remaining demand through utilizing the reserve capacity.

(iii) $I \leq (b_0 - b_1 p)\, k - a\, k$. In this case, both the reserve inventory and reserve capacity are used fully. The profit function thus becomes $(p - u)\, I + (p - c_a)\, a\, k$, where the first term is the profit obtained through selling the entire reserve inventory and the second term is the profit obtained through utilizing the entire reserve capacity.

2.3 Reserve Inventory and Capacity Decisions

Next, we describe the initial problem of setting reserve inventory and capacity levels. During the non-disrupted stage, and in anticipation of a future disruption, the firm sets a reserve inventory and/or reserve capacity level that it may utilize during a subsequent disruption. We aim to maximize long-run expected profit per unit time. We assume that after every disruption, the supply chain returns to the undisrupted stage before the next disruption occurs. This allows us to define a renewal cycle as a period of no disruption followed by a period with one disruption. The duration of a renewal cycle is then defined as the expected time duration of not being disrupted plus the expected time duration of one disruption. Specifically, let α and β denote, respectively, the disruption rate and the recovery rate. Hence, the expected time of not being disrupted is $\frac{1}{\alpha}$ while the expected time duration of one disruption is $\frac{1}{\beta}$. Thus, the expected renewal cycle length is given by $\frac{1}{\alpha}+\frac{1}{\beta}$. Based on this definition of the renewal cycle we use the well-known renewal-reward theorem to calculate the long-run expected profit per unit time. The long-run expected profit $\mathbb{E}[\Pi(I,a)]$ is the ratio of the expected profit per cycle and the expected renewal cycle length. In order to determine the expected profit per cycle, we first introduce the expected profit when there is no disruption, $\mathbb{E}[\Pi_0(I,a)]$, and the expected profit during a disruption, $\mathbb{E}[\Pi_d(I,a)]$.

The expected profit when there is no disruption, $\mathbb{E}[\Pi_0(I,a)]$, includes the base profit rate $r(p_o)$ minus the costs associated with holding the reserve inventory and reserve capacity. Specifically, if the firm holds I units of reserve inventory at a cost of h per unit per unit time, and has access to a reserve capacity with a production rate of a units per unit time at a capacity reservation cost of c per reserve capacity production rate, the expected profit when there is no disruption can be stated as:

$$\mathbb{E}[\Pi_0(I,a)] = \frac{r(p_o) - h\,I - c\,a}{\alpha}. \tag{2}$$

The expected profit when there is a disruption, $\mathbb{E}[\Pi_d(I,a)]$, takes into account the optimal pricing decision corresponding to (1). In addition, in order to present a simple framework to incorporate uncertainty around the disruption length when making the initial stage decisions, we assume that the the disruption will have a length of k_s with probability q or a length of $k_l \geq k_s$ with probability $1-q$. (Therefore, we have $1/\beta = q\,k_s + (1-q)\,k_l$). Consequently, the expected profit when there is a disruption is given by

$$\mathbb{E}[\Pi_d(I,a)] = q\,\tilde{\Pi}_d(I,a,k_s) + (1-q)\,\tilde{\Pi}_d(I,a,k_l) \tag{3}$$

The long-run expected profit is thus given by

$$\mathbb{E}[\Pi(I,a)] = \frac{\mathbb{E}[\Pi_0(I,a)] + \mathbb{E}[\Pi_d(I,a)]}{\frac{1}{\alpha}+\frac{1}{\beta}} \tag{4}$$

and the firm's first stage reserve inventory and reserve capacity problem can be stated as $\max_{I,a} \quad \mathbb{E}[\Pi(I,a)]$.

3. Results and Insights

3.1 Optimal Pricing During Disruption

We first provide the characterization of the optimal disruption period price $p^*(I,a,k)$ for a given reserve inventory level I, a reserve capacity of a units per unit time, and a disruption of length k. (We omit the proof for brevity.)

Proposition 1 *For a given reserve inventory level I, reserve capacity level of a, and disruption length of k, the optimal disruption price, $p^*(I,a,k)$ is as follows:*

$$p^*(I,a,k) = \begin{cases} \frac{b_o}{2b_1} + \frac{u}{2}, & \textit{if } k < \frac{2I}{b_0 - ub_1} \\ \\ \frac{b_0 - I/k}{b_1}, & \textit{if } \frac{2I}{b_0 - ub_1} \leq k < \frac{2I}{b_0 - c_a b_1} \\ \\ \frac{b_o}{2b_1} + \frac{c_a}{2}, & \textit{if } \frac{2I}{b_0 - c_a b_1} \leq k < \frac{2I}{b_0 - c_a b_1 - 2a} \\ \\ \frac{b_0 - (I/k + a)}{b_1}, & \textit{if } k \geq \frac{2I}{b_0 - c_a b_1 - 2a} \end{cases} \tag{5}$$

The result summarized in (5) indicates that the optimal disruption price may take four different forms depending on the disruption length.

For short disruptions, i.e., if $k < \frac{2I}{b_0 - ub_1}$, it is optimal for the firm to charge $p^*(I,a,k) = \frac{b_o}{2b_1} + \frac{u}{2}$, i.e., to continue applying the list price p_o. In this case, the disruption price is independent of the disruption length.

If the disruption length k is slightly longer, satisfying $\frac{2I}{b_0 - ub_1} \leq k < \frac{2I}{b_0 - c_a b_1}$, then it is optimal for the firm to charge $p^*(I,a,k) = \frac{b_0 - I/k}{b_1}$, which is the price that will suppress demand such that the reserve inventory will be depleted just at the end of the disruption period. In this case, the disruption price is increasing in the disruption length.

For a longer disruption such that $\frac{2I}{b_0 - c_a b_1} \leq k < \frac{2I}{b_0 - c_a b_1 - 2a}$, the firm increases its price to $p^*(I,a,k) = \frac{b_o}{2b_1} + \frac{c_a}{2}$, which is in a similar form as the base price except for taking into account the higher production cost associated with the reserve capacity. Consequently, it is once again a constant price level independent of the disruption length.

Lastly, for disruptions of further length, the firm sets the price to $p^*(I,a,k) = \frac{b_0 - (I/k + a)}{b_1}$. The price in this region is again decreasing in the disruption length and suppresses demand to a level that will be met by the full extent of reserve inventory and reserve capacity throughout the disruption.

We also note the following sensitivity results regarding the optimal price during a disruption. (We use the terms increasing and decreasing in the weak sense.)

Proposition 2 *The optimal price, $p^*(I,a,k)$, is decreasing in the reserve inventory level I, decreasing in* *reserve capacity production rate a, and increasing in the disruption length k. In addition, the optimal price is increasing in the demand intercept b_o, decreasing in the price sensitivity coefficient b_1, increasing in the reserve inventory usage (replacement) cost u, and increasing in the reserve capacity production cost c_a.*

Next, we study how the level of pricing flexibility a firm has during a disruption impacts its reserve inventory and reserve capacity decisions. To do so, we first introduce an exogenous upper bound on the price that a firm can charge during the disruption denoted by $\bar{p}$ such that $p_o \leq \bar{p} \leq b_0/b_1$. That is, the firms optimal price selection for the disruption period is $\min\{p^*(I, a, k), \bar{p}\}$. We then investigate how an increase in this price upper bound impacts the firm's reserve inventory and reserve capacity decisions. For the remainder of this work, we limit our attention to a firm's reserve inventory and reserve capacity decisions separately.

3.2 Reserve Inventory and Pricing Flexibility

In this section, we study how the level of pricing flexibility a firm has during a disruption impacts its reserve inventory decisions.

Recall from Proposition 1 that when the firm enters a disruption period of length k with a reserve inventory level of I and with no reserve capacity (i.e., $a = 0$), it will continue to apply the base price p_o if $k < \frac{2I}{b_0 - ub_1}$. For longer disruptions, i.e., for $k \geq \frac{2I}{b_0 - ub_1}$, and in the presence of a price bound $\bar{p}$, it will charge $\min\{\frac{b_0 - I/k}{b_1}, \bar{p}\}$. Intuitively, as the firm increases its inventory level, it may shift its pricing policy from charging $\bar{p}$ to $\frac{b_0 - I/k}{b_1}$, and finally to p_o. As the disruption length can be either short or long, a particular reserve inventory position may lead the firm to charge different prices for different disruption lengths. Therefore, depending on the disruption length, these price transitions can occur at different inventory levels. While we omit the details of arising cases for brevity, we find that the firms objective function of expected profit per unit time results in a piecewise unimodal function with respect to the reserve inventory. We provide the main findings below.

We omit the derivation details for brevity and provide the main findings below.

Proposition 3 *(a) If $\bar{p} < \frac{h}{\alpha} + u$, the firm does not invest in reserve inventory, i.e., $I^* = 0$. If $\bar{p} \geq \frac{h}{\alpha} + u$, the firm selects a reserve inventory level of $I^* \geq k_s(b_0 - b_1\,\bar{p})$, i.e., at least to cover demand during a short disruption at the price bound.*
(b) The firm's optimal reserve inventory level may increase or decrease with or be independent of the price bound $\bar{p}$.

There are two main implications of Proposition 3. First, part (a) indicates that the price bound the firm is allowed to charge up to ($\bar{p}$) must be at least at some sufficiently high level for it to justify the firm to hold reserve inventories. In other words, this implies that there may need to be at least some pricing flexibility for the firm to hold reserve inventories. As can be intuitively expected, the required minimum price bound is increasing in the inventory holding cost (h), the length of the non-disrupted stage the firm will be holding this inventory for (i.e., $1/\alpha$), and the production cost of the item (u). We also find that the inventory level the firm selects will at the minimum fully meet the demand at the price bound for a short disruption length.

Second, part (b) of Proposition 3 indicates that the optimal inventory level may be increasing or decreasing with the price bound or be independent of the price bound. The explanation is as follows: In instances where the firm's optimal inventory selection is such that it charges $\frac{b_0 - I/k}{b_1}$ if the disruption is short and $\bar{p}$ if the disruption is long, a marginal increase in $\bar{p}$ prompts the firm to increase the inventory level as the disruption period profit it gains from this additional inventory outweighs the additional holding cost. As a side note, in this case, the firm's disruption profit is linearly increasing in inventory for long disruptions (as each additional inventory can be sold at $\bar{p}$) and concavely increasing in inventory (i.e., with diminishing returns) for short disruptions (as each additional inventory will now result in a slightly lower price to be sold throughout the short disruption duration). In other instances, the firm's optimal inventory selection may be such that it targets to fully meet either the short disruption demand at the price bound or the long disruption demand at the price bound. For these instances, the firms optimal inventory level decreases with the price bound. Lastly, the optimal inventory decisions may be such that the firm applies p_o during a short disruption and either $\frac{b_0 - I/k}{b_1}$ or p_o in a long disruption, for which the firm's reserve inventory decisions are independent of the price bound. Overall, Proposition 3 indicates that pricing flexibility and reserve inventories may be complements or substitutes.

3.3 Reserve Capacity and Pricing Flexibility

Next, we study how the level of pricing flexibility a firm has during a disruption impacts its optimal reserve capacity level. We note that in this section, our focus will now be limited to a firm which invests only in reserve capacity and not in reserve inventory. Similar to the previous section, we investigate how an increase in the upper price bound $\bar{p}$ impacts the firm's reserve capacity decisions.

Recalling again the optimal pricing decisions outlined in Proposition 1, we observe that if the firm enters a disruption with a reserve capacity level of a and with no reserve inventory (i.e., $I = 0$), then, it will charge $\min\{\frac{b_0 - a}{b_1}, \bar{p}\}$. That is, the firm will charge the price bound $\bar{p}$ for low levels of reserve capacity (i.e., $a < b_0 - b_1 \bar{p}$) and charge $\frac{b_0 - a}{b_1}$ for higher reserve capacity levels (i.e., $a \geq b_0 - b_1 \bar{p}$). Note that in this case, the firm's optimal price selection does not depend on the length of the disruption. We again find that the firm's expected profit is piecewise in its reserve capacity level. We provide our main findings below (we again omit the proof for brevity).

Proposition 4 *(a) If $\bar{p} < \frac{c\beta}{\alpha} + c_a$, the firm does not invest in reserve capacity, i.e., $a^* = 0$. If $\bar{p} \geq \frac{c\beta}{\alpha} + c_a$, the firm selects a reserve capacity level $a^* \geq b_0 - b_1 \bar{p}$, i.e., at least to cover demand rate at the price bound.*

(b) Once it is optimal for a firm to invest in reserve capacity, the optimal reserve capacity level may decrease with or be independent of the price bound $\bar{p}$.

Part (a) of Proposition 4 indicates that, similar to our previous finding regarding the reserve inventory decisions, we find that the firm will invest in reserve capacity only if there is sufficient pricing flexibility, i.e., if $\bar{p} \geq \frac{c\beta}{\alpha} + c_a$. Specifically, the firm may not find it economical to invest in reserve capacity if the price bound is relatively low, cost of reserving the reserve capacity, c, is high, the emergency production fee through this capacity, $c_a > u$, is high, the expected non-disrupted stage is long, or the expected disruption is short. Further, if the firm decides to invest in reserve capacity, it does so to be able to at least cover demand during disruption at a demand rate corresponding to the price bound $\bar{p}$.

Proposition 4 part (b) shows that the optimal capacity investment may decrease with the price bound or may be independent of the price bound. Specifically, if the optimal reserve capacity decision exactly targets to cover demand during disruption at a demand rate corresponding to the price bound $\bar{p}$, then the required capacity decreases as $\bar{p}$ increases. If, on the other hand, the firm's optimal capacity selection exceeds this rate, then the optimal capacity selection is found to be independent of the price bound.

In summary, Proposition 4 indicates that the firm may require some pricing flexibility to justify investing in reserve capacity and would then select a (weakly) lower capacity level as it has further pricing flexibility. Therefore, similar to our results regarding the reserve inventory in the preceding section, we find that reserve capacity and disruption period pricing flexibility may again be either complements or substitutes. The main difference between the relationship between the optimal reserve capacity and the price bound as compared to the relationship between the optimal reserve inventory and the price bound is that once a firm decides to invest in reserve capacity, any increase in pricing flexibility may only decrease the optimal reserve capacity level whereas an increase in pricing flexibility may first increase and then decrease the optimal reserve inventory levels.

4. Conclusion and future research

We study a firm that is exposed to random supply chain disruptions while producing a single product. During a disruption, the firm may use reserve inventory and/or reserve capacity to serve customer demand. Pricing flexibility is used to manage demand during a disruption.

We find that pricing flexibility may complement or substitute the use of reserve inventory or capacity. Specifically, when a firm has pricing flexibility, it may be economical to increase the use of reserve inventory/capacity.

In this research, we assume that the disruption length is known when the disruption starts. However, in practice, it is often difficult to estimate the disruption length. It might be interesting to explore how uncertainty in the estimation of the disruption time affects the insights.

Further, we analyzed the following two cases under pricing flexibility independently: 1) holding reserve inventory only (no reserve capacity used) and 2) holding reserve capacity only (no reserve inventory used). It would be interesting to investigate how the joint dynamic plays out.

References

Chopra, S., Reinhardt, G. and Mohan, U. (2007), 'The importance of decoupling recurrent and disruption risks in a supply chain', *Naval Research Logistics* **54 (5)**, 544–555.

Dong, L., Xiao, G. and Yang, N. (n.d.), 'Supply diversification under random yield: The impact of price postponement', *Production and Operations Management* .

Kouvelis, P., Xiao, G. and Yang, N. (2021), 'Role of risk aversion in price postponement under supply random yield', *Management Science* **67**(8), 4826–4844.

Li, T., Sethi, S. P. and Zhang, J. (2013), 'Supply diversification with responsive pricing', *Production and Operations Management* **22**(2), 447–458.

Li, T., Sethi, S. P. and Zhang, J. (2017), 'Mitigating supply uncertainty: The interplay between diversification and pricing', *Production and Operations Management* **26**(3), 369–388.

Qi, L. (2013), 'A continuous-review inventory model with random disruptions at the primary supplier', *European Journal of Operational Research* **225**, 59–74.

Qi, L. and Lee, K. (2015), 'Supply chain risk mitigations with expedited shipping', *Omega* **57**, 98–113.

Tang, C. S. and Yin, R. (2007), 'Responsive pricing under supply uncertainty', *European journal of operational research* **182**(1), 239–255.

Tomlin, B. (2006), 'On the value of mitigation and contingency strategies for managing supply chain disruption risks', *Management science* **52**(5), 639–657.

Van Mieghem, J. A. and Dada, M. (1999), 'Price versus production postponement: Capacity and competition', *Management Science* **45**(12), 1639–1649.