THE BEHAVIOUR OF VOLATILITY AND OPTIONS PRICING

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Abstract

The overall objective of the thesis is to understand volatility and to derive implications for options pricing with particular reference to the Nikkei 225 index, which has not been widely researched. Our brief survey shows us that the volatility models including the GARCH family can be applied for forecasting the volatility of the Nikkei 225 index daily returns. In addition, forecasting power of volatility is much stronger when using implied volatility rather than using historical volatility and the GARCH estimates. We observe the smile effect and term structure of implied volatility in the Nikkei 225 options market. From the perspective of the international linkage of the world major markets, both historical and implied volatilities spill over from one market to another. We can utilise those characteristics for two applications. One is that we can trade options by forecasting volatilities with volatility models. If our forecasts were higher than the implied volatility in the market, we would go short in some series of options with delta neutral hedging. If volatility has declined over the option period, we could capitalise the forecast with the option positions. Moreover, the information in the FTSE and S&P markets (historical and implied volatilities) is useful to forecast the Nikkei volatility, when we trade the Nikkei options. The other application is to evaluate option positions from the risk management point of view. Middle-office managers are concerned with profits and losses if the market and volatility move in a particular way. By using the information of smiles and term structures of implied volatilities, managers can evaluate their positions more accurately for risk management.

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1

Introduction and Overview

Naoki Kamiyama

1. Introduction and Overview

The overall objective of the thesis is to understand volatility and to derive implications for options pricing with particular reference to the Nikkei 225 index, which has not been widely researched.

As an option trader, one of the most important decisions for trading is to estimate volatility for any type of option pricing formula. Even though the Black-Scholes formula has been widely accepted, we know that historical volatility of the underlying asset return does not seem constant, but changing over time. On the other hand, we also observe that volatility figures implied in the option prices are affected by time to maturity (implied volatility term structure) and exercise price (smile effect).

Not only for trading but also for managing the risk of a financial institution, precise evaluation of financial products has become more important than ever, while derivative products have become more complicated. Many of the financial derivatives which are managed in financial institutions are option-related, and risk measurement and management for them are not so straightforward as for traditional financial instruments. The most difficult part of risk management for option related products is to estimate volatility, as described above. The middle office, measuring and managing risks for a whole institution, must be familiar with models of changing volatility, including term structure and smile effects.

In this thesis, we look carefully at both approaches to estimating volatility (historical and implied). We begin in Chapter 2 (Review of the Concepts and Models of Volatility), with a review of several methods for estimating volatility, including the historical volatility calculations that utilise extreme values such as high/low prices. Then the ARCH models, now the most widely recognised in finance, are applied for the Nikkei index. JP Morgan's RiskMetrics™ is examined in comparison with GARCH, and, in the case of the Nikkei, we find that RiskMetrics can estimate the volatility as effectively as GARCH, even though RiskMetrics is very easy to implement.

In trading of options, which volatility estimate is most powerful? Chapter 3 (Forecasting Power of Volatility: Historical Data vs. the Market) examines whether forecasts of volatility from past data or from implied volatility are more accurate, using the Nikkei 225 index. In comparison of HV22 (22-day historical volatility), HV60 (60-day historical volatility), implied volatility (OTM, ATM, and ITM), and GARCH estimates, we find that GARCH has the smallest RMSE (root mean square error). Implied volatility overpredicts realised volatility, but implied volatility is a better forecast than simple historical volatility for the Nikkei 225 traded options. We also find that a combined forecast may be best: encompassing regressions suggest optimal weights of 80% implied volatility and 20% GARCH.

In Chapter 4 (Implied Volatility Shapes: the Nikkei 225 Case), we investigate the skewness of the smile. We find a left-skewed smile, and a time effect, ie., the shorter the time to maturity, the larger smile. We employ two different regression methods to find these effects, and find similar results by both ways.

Chapter 5 (Stock Return and Volatility Transmission: The Nikkei 225 and other major markets) investigate the linkage of the world markets. It is shown that the return and volatility on the Japanese stock market (Nikkei 225) are affected by the American (S&P 500) and British (FTSE 100) markets. In this chapter, a general model is developed which integrates return spillovers with volatility spillovers. We find that traders can utilise the quantitative results of spillovers in pricing of options, by watching the volatility of the market which has just closed.

If historical volatility spills over, how about implied volatilities and smiles? In Chapter 6 (Implied Volatilities and Skewness across the Index Options Markets: Comparison and Transmission), we examine three subjects, (1) transmission of implied volatility across time zones, (2) transmission of skewness across time zones, and (3) domestic influences on skewness. Across a three-zone world, a change in IV spills over to the next-opening market, but the shape of volatility smile does not: it is a local phenomenon.

We often observe that the implied volatility for short-term options is different from that for long-term options. How are they related to each other? Is the relationship predictable? In Chapter 7 (The Implied Volatility Term Structure: Cointegration of the Short- and Long-term Implied Volatilities), a cointegration analysis is applied to the term structure of implied volatilities for the Nikkei options on futures traded on SIMEX. We find that their relationships are potentially useful in risk management and in trading; it allows us to forecast the implied volatility level in the near future. We find that the short- and long-term volatilities are co-integrated, and that both short- and long-term volatilities tend to correct any disturbance from equilibrium by changing their levels over time.

Although several papers derive the implied risk-neutral distribution for an asset by using the shape of implied volatility smile[1], this approach is not very

[1] For example, implied binomial tree is suggested by Rubinstein (1994).

useful unless the shape is stable over time. In Chapter 8 (Stability of Implied Volatility Functions: A Test on the Nikkei Options), we examine the stability of the shape of implied volatility smile of the Nikkei 225 options, and find that the stability is not sufficient to be useful in forecasting option prices. This casts doubt on so-called "deterministic volatility function" models, such as implied binomial trees.

Finally, in Chapter 9, conclusions are given. Having examined several aspects of volatilities, we find some important characteristics of volatility of asset returns. Our brief survey shows us that the volatility models including the GARCH family can be applied for forecasting the volatility of the Nikkei 225 index daily returns. In addition, forecasting power of volatility is much stronger when using implied volatility rather than using historical volatility and the GARCH estimates. We observe the smile effect and term structure of implied volatility in the Nikkei 225 options market. From the perspective of the international linkage of the world major markets, both historical and implied volatilities spill over from one market to another.

We can utilise those characteristics for two applications. One is that we can trade options by forecasting volatilities with volatility models. If our forecasts were higher than the implied volatility in the market, we would go short in some series of options with delta neutral hedging. If volatility has declined over the option period, we could capitalise the forecast with the option positions. Moreover, the information in the FTSE and S&P markets (historical and implied volatilities) is useful to forecast the Nikkei volatility, when we trade the Nikkei options. The other application is to evaluate option positions from the risk management point of view. Middle-office managers are concerned with profits and losses if the market and volatility move in a particular way. By using the information of smiles and term structures of implied volatilities, managers can evaluate their positions more accurately for risk management.

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2

Review of the Concepts and Models of Volatility

Naoki Kamiyama

2. Review of the Concepts and Models of Volatility

2.1. Introduction

Estimating volatility is becoming more important, not only for academics but also practitioners in the financial world. These days, more people are familiar with the word "volatility" as option contracts are traded more frequently. After Black and Scholes (1973) revealed their option pricing model, which includes volatility as an input, traders and users of options should have estimated volatility for "theoretical" prices to deal with the instruments everyday. It has been a challenge to find a appropriate volatility input into the Black-Scholes formula in order to evaluate options because volatility is unobservable.

We have two types of estimated volatilities, historical and implied. At the beginning stage of utilising the Black and Scholes option pricing model (B-S model), historical volatility (HV) was mainly calculated for input in order to derive theoretical option prices. Because the B-S model assumes a constant volatility, market participants calculated the standard deviation of past asset returns and used it as a forecast to be inputted into the B-S model. On the other hand, because option prices are available in the market, practitioners may also calculate implied volatility (IV) by inputting the observed price into B-S model to derive volatility. For price-following traders in the market, implied volatility is the most important indicator because it indicates the consensus volatility forecast of market participants.

The historical volatility calculation has been modified by Parkinson (1980) and Garman and Klass (1980). They expand the volatility calculation from close-to-close returns to more informative high-to-low, open-to-close, and close-to-next-open returns. Because they assume a constant variance for stationary return series, (which is considered incorrect these days), they calculate volatility as if trading continues in periods when the market is closed. This modification (Modified Garman-Klass and Modified Parkinson) leads to a rather high estimate of volatility compared with the classical close-to-close volatility estimator, as demonstrated in Figure 2.1. In the case of the Nikkei 225 as shown in Table 2.1, the close-to-next-open volatility is much smaller than the close-to-close volatility, therefore it seems unreasonable to expand the assumed measurement period of volatility from trading time to 24 hour trading by using the assumption of normally distributed returns for an asset.

Volatility has been investigated and analysed by many researchers and practitioners, after practitioners noticed that volatilities of assets do not seem constant, that is, the underlying asset returns are not stationary. For the purposes of options trading and return forecasting as well, researchers have started modelling volatility as a time series and forecasting volatility statistically. An early example of comprehensive volatility studies is Taylor (1986), who focused on squared returns on many kinds of asset and commodity prices in both cash and futures markets, and modelled volatility that is changing dynamically. GARCH models are one of the most popular volatility forecasting models in the financial society at this moment. In the seminal paper, Engle (1982) introduced autoregressive conditional heteroskedasticity (ARCH) to examine non-stationary economic data, the U.K. inflation rate being used as an example. The ARCH model has the disturbance term affecting conditional volatility in the next time period. Because there are many financial time series, which are non-stationary due to changing variance, the ARCH methodology is accepted widely by researchers in finance. Bollerslev (1986) generalised ARCH (GARCH) by adding the autoregressive conditional volatility term, which is now the most widely recognised ARCH model in finance. Engle (1993) summarised the recent volatility models including GARCH. Bollerslev, Chou, and Kroner (1992) also introduce GARCH models with detailed analysis for applications.

Stochastic volatility models[1] have been used in several works such as Wiggins (1987) and Chesney and Scott (1989). In their studies, the mean reversion of volatility was assumed. This model is applied to the Nikkei 225 later, but the estimates do not seem plausible in this paper. The daily variance, or daily squared return, may be unstable when we use a simple autoregression model for forecasting.

Recently, risk management has become more important than ever in financial institutions all over the world, and more practitioners are interested in volatility estimates and forecasts for asset prices to evaluate their position risks and values. JP Morgan's RiskMetrics™ (Zangari (1995)) is one of the major methodologies accepted by risk management professionals, in which an exponential weighted moving average is used to estimate and forecast volatility of asset prices. Because their methodology is broadly accepted, it is worthwhile to double-check its ability to estimate and forecast volatility with special interest in this paper on the Nikkei index. The difference in methods for VaR (value at risk) calculation often leads to significantly different risk measurement results, as Beder (1995) explained.

The purpose of this paper is (1) to apply major volatility calculation methods to the Nikkei 225 index, and characterise and compare the methods and results, and (2) to review major volatility forecasting models with the Nikkei 225,

[1] Taylor (1992) calls it ARV (autoregressive random variance model).

including stochastic volatility models, GARCH models, and the exponential smoothing model suggested by JP Morgan. It is interesting to examine whether the efforts to introduce more information such as high / low prices and the 24-hour-trading assumption, made by Garman and Klass and Parkinson, are useful or not for volatility calculation. In addition, for the purpose of forecasting, we will examine a GARCH model with the special interest in comparison to JP Morgan's model.

In the next section, the methods for volatility calculation are explained. The classical method is compared to Garman and Klass's, Parkinson's, modified Garman and Klass's, and modified Parkinson's methods. Section 3 deals with stochastic volatility models for the Nikkei index. In Section 4, the ARCH/GARCH family is introduced. In addition, we compare exponential weighted moving average volatility (suggested by JP Morgan as RiskMetrics™) with GARCH in the case of Nikkei, and conclude that this simple method works rather well.

2.2. Volatility Calculation

2.2.1 Calculation Methods

Five methods are explained in this section. The most popular way of historical volatility calculation is to use close-to-close returns as shown in Equation (1).

$$\hat{\sigma}_C^2 = \frac{1}{n-1}\sum_{t=1}^{n}(\ln R_t - m)^2 \tag{1}$$

where,

$$m = \frac{1}{n}\sum_{t=1}^{n}\ln R_t$$

$$R_t = \frac{C_t}{C_{t-1}}$$

C_t: Closing price of the asset price at time t

n: number of observation

Several other ways of calculation have been introduced. Ohmura and Shimizu (1987), and Ikeda (1989)[2] introduced the Parkinson method (Parkinson (1980)), Garman-Klass method, modified Parkinson method, and modified Garman-Klass method (Garman and Klass (1980)) to the Japanese market. The Parkinson method is expressed as following.

$$\hat{\sigma}_P^2 = \frac{1}{n}\sum\frac{(u-d)^2}{4\ln 2} \tag{2}$$

Where,

n = number of days to calculate volatility (number of samples)

u = ln(High) - ln(Open) = normalised high

d = ln(Low) - ln(Open) = normalised low

The Garman-Klass method is as below.

$$\hat{\sigma}_G^2 = \frac{1}{n}\sum\left[0.511(u-d)^2 - (2\ln 2 - 1)c^2\right] \qquad (3)^3$$

where,

c = ln(close) - ln(open) = normalised close

The modified Parkinson method is as below.

$$\hat{\sigma}_{MP}^2 = \frac{1}{n}\sum\left[\frac{.17(\ln O_1 - \ln C_0)^2}{f} + \frac{.83(u-d)^2}{(1-f)\cdot 4\ln 2}\right] \tag{4}$$

[2] He mentioned that some studies are wrong in calculations and the following equations are out of Ikeda's work. All the coefficients in Equations (3), (4), and (5) are same as the ones in Garman and Klass (1980).

[3] Kagraoka (1990) noted that this is $\left[.511(u-d)^2 + .019\left[c(u+d) - 2ud\right] - (2\ln 2 - 1)c^2\right]$, and the second is considered negligible.

where,

f = a proportional day period when the market closed = .8125 = 19.5 / 24 in Japan[4].

The modified Garman-Klass method is as below.

$$\hat{\sigma}^2_{MG} = \frac{1}{n}\sum\left[\frac{.12(\ln O_1 - \ln C_0)^2}{f} + \frac{.88\hat{\sigma}^2_G}{1-f}\right] \tag{5}$$

Note that the Parkinson method and others based upon high-low information require geometric Brownian motion (particularly normal distributions and a continuous price record), otherwise they are biased.

2.2.2 Data

The daily open, high, low, and close prices of the Nikkei 225 index are used from 30 March 1991 to 29 March 1996 to calculate daily returns. The monthly volatilities are calculated as standard deviation of daily returns over one calendar month. In total, there are 60 observations.

2.2.3 Comparison Results

Equations (4) and (5) are designed to allow for the discontinuous trading hours of the stock market. For example, the Japanese equity market is closed and not traded at night in Japan, even though some exceptions exist (for example, ADRs of Sony and some other major Japanese corporates are traded in U.S. trading hours). These two modifications above are designed to solve this problem to obtain the "correct" volatility observation as if trading continued at night.

However, it is not reasonable to consider that the close-to-close returns are similarly distributed to the close-to-next-open returns. As Ikeda (1989)

[4] Trading hour is 4.5 hours (2 hours in the morning and 2.5 hours in the afternoon) a day in

claimed, Garman and Klass's and Parkinson's modifications are based on the assumption of the same volatility distribution even though the market is closed. Table 2.1 shows the difference between close-to-close and close-to-next-open volatilities. In both squared log returns, i. e. (Log of $close_t$ / $close_{t-1})^2$ and squared differences, i. e. $(close_t - close_{t-1})^2$, close-to-close and close-to-next-open data are significantly different by a t test at the 95% confidence interval. As expected, close-to-next-open squared differences and returns are much smaller than close-to-close ones.

Table 2.2.a shows the descriptive statistics of the estimates of those five methods, and Table 2.2.b examines the differences between the results from the various high/low estimators and the simple close-to-close estimator (Equation (1)). The table shows RMSE (root mean square error) and ME (mean error), which are defined as below.

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(X_t - CC_t)^2} \quad (6)$$

$$ME = \frac{1}{N}\sum_{t=1}^{N}(X_t - CC_t) \quad (7)$$

where X is volatility estimator, (e.g. Garman and Klass's), and CC is the close-to-close estimator. Garman and Klass and Parkinson have negative ME relative to close-to-close volatility estimator. The results are consistent with Ikeda (1989), who also finds a downward bias of volatility estimates in the Japanese equity market by using those methods. Both methods include high-to-low and open-to-close, which are normally smaller returns because no price jump between close-to-next-open is taken into account. On the other hand, modified Garman and Klass and modified Parkinson have rather large RMSE

Japan.

and ME relative to the close-to-close volatility estimator. Both modified methods overestimate volatility because they take night time into consideration for volatility calculation. It is also shown in Figure 2.1. The modified Garman-Klass and modified Parkinson results are always higher than the close-to-close estimator over the sample period. The original Garman-Klass and Parkinson methods are lower than the close-to-close volatility in most of the months in the period.

True volatility is likely to be somewhere between classical and Garman and Klass's and/or Parkinson's methods. True volatility is anyway unobservable, but it is reasonable to conclude that we should use the classical volatility estimator for the purpose of historical volatility[5], or realised (observed) volatility[6] at this stage.

2.3. Stochastic Volatility Models

2.3.1 Stochastic Volatility Models

The most popular stochastic volatility model is a mean reverting model, which is explained in Taylor (1986). Equation (8a) shows a concept that the volatility at t eventually depends on the long-run volatility level ($\overline{\sigma}$) and the dispersion of the volatility at t-1 from the long-run volatility ($\sigma_{t-1} - \overline{\sigma}_t$).

$$\sigma_t = \overline{\sigma} + \alpha(\sigma_{t-1} - \overline{\sigma}) + \varepsilon_t \qquad (8a)$$

[5] As described in Appendix, the 245 days are used for calculation of all the annualised volatility hereafter.

$$\text{Annualised Volatility} = \sqrt{\hat{\sigma}^2} \cdot \sqrt{245}$$

[6] Realised volatility means observed volatility in my terminology. However, Taylor (1994) means real (unobservable) volatility by realised volatility.

Reformulating Equation (8a) into (8b), we can estimate a regression in which the intercept is $(1-\alpha)\overline{\sigma}$ and the slope coefficient is α. Therefore, the long-term mean for volatility, $\overline{\sigma}$, may be estimated.

$$\sigma_t = \alpha\sigma_{t-1} + (1-\alpha)\overline{\sigma} + \varepsilon_t \qquad (8b)$$

Second alternative to the above is to use an autoregressive moving average (ARMA(1,1)). Harvey (1989) suggests that this formulation is useful to find a local trend of a time series. The model adds a moving average item to Equation (8a) and is described as below;

$$\sigma_t - \overline{\sigma} = \phi(\sigma_{t-1} - \overline{\sigma}) + e_t + \theta\, e_{t-1} \qquad (9)$$

An additional consideration, Taylor (1986), is to take the logarithm of volatility, which gives a stationary variable so that $\ln(\sigma)$ may be used as dependent variable in Equation (8).

$$\ln(\sigma_t) = \alpha + \phi[\ln(\sigma_{t-1}) - \alpha] + \theta\eta_t \qquad (10)$$

The logarithm transformation is applied for both Equation (8) and (9).

2.3.2 Data

The closing prices of the Nikkei 225 index from 30 March 1991 to 29 March 1996 are utilised to calculate the daily log returns (R_t). Then the daily volatility at time t is defined as $\sigma_t = \sqrt{(R_t - m)^2}$, where m is the mean of the daily log returns. The number of observation is 1,236.

2.3.3 Results

The results of Equations (8) and (9), and those transformed by Equation (10) are shown in Table 2.3. The ARMA (1, 0) and (1, 1) models have significant autoregressive coefficients, and give plausible volatility estimates around 16% annually. The result can be compared with the estimates of

Parkinson (16.8%) and Garman and Klass (15.9%) methods in Table 2.2.a. The ARMA results are closer to the Parkinson's and Garman and Klass's estimates than the one of the Close-to-Close method. The R squared of ARMA (1, 1) is larger than the one of ARMA (1, 0), being .096 and .025 respectively.

The ARMA model with logarithm of observed volatility has no significant autoregressive coefficient. The estimate of real volatility, 9.9% annually, is not plausible, compared with the average level of observed volatility for the Nikkei 225 index. This result may indicate two possible problems; one is that the log transformation may eliminate the autoregressive nature of the data series, and the other is that the daily volatility specification and/or estimation method may include some problems. Even though we can not specify the potential problems, we may conclude that use of logarithm of observed volatility is not appropriate in an autoregressive volatility model in terms of daily volatility of the Nikkei.

2.4. GARCH Models

2.4.1 GARCH Models

The original GARCH model was introduced by Bollerslev (1986), shown as below;

$$\begin{aligned} y_t &= \varepsilon_t \\ \varepsilon_t &= \sqrt{h_t} \cdot \xi_t \\ h_t &= \omega + \alpha \cdot \varepsilon_{t-1}^2 + \beta \cdot h_{t-1} \end{aligned} \qquad (11)$$

where,

y: conditional return

ε : disturbance

h: conditional volatility

ξ : error with standard normal distribution

α : coefficient of lagged disturbance

β : coefficient of lagged conditional volatility

ω : intercept

The disturbance term of returns can be separated into two terms, conditional volatility and a normally distributed error term. The conditional volatility depends on the lagged disturbance term and lagged conditional volatility itself.

Engle, Lilien, and Robins (1987) generalised GARCH (1, 1) further to the GARCH-in-Mean model which is explained as below;

$$\begin{aligned} y_t &= \phi x_t + \delta\sqrt{h_t} + \varepsilon_t \\ \varepsilon_t &= \sqrt{h_t}\xi_t \\ h_t &= \omega + \alpha \cdot \varepsilon_{t-1}^2 + \beta \cdot h_{t-1} \end{aligned} \tag{12}$$

where,

ϕ : coefficients of autoregression of returns

δ : coefficient of feedback from conditional volatility to return

Other notations are same as the ones for Equation (11).

Conditional volatility now has feedback by adding conditional volatility as an independent variable to explain the conditional mean of returns.

2.4.2 Data

The closing prices of the Nikkei 225 index from 30 March 1991 to 29 March 1996 are used to calculate the daily log return, y_t, for estimating the parameters of GARCH. The number of observation is 1,236.

2.4.3 Results

We employed 6 members of the ARCH family as shown in Table 2.4. Each of the 6 members can be defined as a special case of the GARCH-in-Mean model, Equation (12). As a result, the difference of one from another is subtle. The minimum of log of likelihood for the six is 3525.9 (ARCH(1)), and the maximum is 3589.2 (AR(1)-GARCH(1,1)-Mean)[7].

The coefficients are stable within the six models and Alpha0, Alpha1, and Beta1 are significant and strong. The other coefficients, including the coefficients of the lag returns for AR(1) of returns, are weak and mostly not significant. It therefore appears that GARCH(1, 1) is sufficient for estimating the returns and volatilities of the Nikkei, and it is not worthwhile to use more complicated models, such as GARCH(1, 1)-in-Mean.

2.4.4 GARCH estimates vs. RiskMetrics's EWMA estimates

The estimates from GARCH (1, 1)[8] are compared with exponentially weighted moving average (EWMA) estimates, which is formulated as below.

$$EW_t = \lambda \cdot EW_{t-1} + (1-\lambda) \cdot y_t^2 \qquad (13)$$

where,

EW: exponentially weighted moving average volaitlity estimator

λ : decay factor, 0.94, which is used for daily data in RiskMetrics (Zangari (1995))

y_t: daily log return at time t

For the purpose of comparison with RiskMetrics and GARCH, we set decay factor to be the same as the figure optimised by Zangari, i.e. 0.94, setting a tolerance level to 1% with 74 historical data points. This formulation (Equation (13)) enables us to calculate the volatility forecast in the recursive

[7] Two times the difference has a chi-squared distribution in some circumstances. See Xu and Taylor (1995)). We need further research in order to compare the forcasting performance by both in- and out-of-sample error tests (i.e. RMSE), and/or other tests such as regression between each model estimates and observed (realised) data.

[8] The estimated parameters are as shown in Table 2.4, GARCH(1, 1)

manner, so that we can save much system resources in practice. The initial EW value is set as the variance of the 74-day returns prior to the comparison period.

Figure 2.2 shows the annualised daily volatility (estimated as square root of squared daily return), GARCH estimates, and EWMA estimates. The difference between the GARCH and EWMA estimates is rather small. The exponentially weighted moving average is easier to estimate than GARCH and as good in measuring volatility level, as tested for the Nikkei index.

Table 2.5 shows that EWMA is better than GARCH in both mean error and root mean square error. With 1,163 data points, EWMA (74 days) and GARCH forecast the volatility of the day, and the forecasts are compared with the daily squared returns. The mean error of EWMA is 5.0%, whereas the one of GARCH is 5.5% in an annualised term. In the RMSE analysis, 14.7% for EWMA is less than 15.7% for GARCH.

2.5. Summary

We compare historical volatilities including Garman and Klass (1980) and Parkinson (1980) in the case of the Nikkei 225 index, as shown in Figure 2.1. Table 2.1 shows that the variance of close-to-next-open returns is significantly different from the one of close-to-close returns, therefore, we consider that modified Garman and Klass and modified Parkinson methods tend to overestimate real volatility. Stochastic volatility models, or autoregressive volatility models are employed for the Nikkei index, but the estimates are not meaningful for practitioners. On the other hand, GARCH model can utilise to measure volatility for the Japanese equity market. JP Morgan's exponential weighted moving average (EWMA) used for RiskMetrics™ is compared with GARCH, and we conclude that EWMA is as effective as GARCH to the Nikkei index.

Table 2.1. Difference in Close-to-Close and Close-to-Next-Open Volatilities

Squared Log Returns = $(\ln C_t / C_{t-1})^2$

	Close-to-Close	Close-to-Open
Mean	0.000201053	3.58676e-6
Variance	1.93672e-7	2.03405e-11
#Observation	1236	1236
Degrees of Freedom	1235	
t value	15.791	

t value is for the test of difference of means (Close-to-Close vs. Close-to-open)

Squared Difference = $(C_t - C_{t-1})^2$

	Close-to-Close	Close-to-Open
Mean	71651.698	1358.913
Variance	23214857098	3071537.6
#Observation	1236	1236
Degrees of Freedom	1235	
t value	16.236	

t value is for the test of difference of means (Close-to-Close vs. Close-to-open)

Table 2.2.a Descriptive Statistics

	C-to-C	Parkinson	Garman	Mod.Par	Mod.Gar
Mean	20.48%	16.82%	15.90%	35.41%	34.46%
Standard Error	1.08%	0.82%	0.75%	1.73%	1.62%
Median	18.42%	15.58%	15.19%	32.80%	32.92%
Standard Deviation	8.34%	6.35%	5.78%	13.36%	12.52%
Variance	0.007	0.004	0.003	0.018	0.016
Kurtosis	0.219	0.227	0.389	0.227	0.390
Skewness	0.815	0.773	0.811	0.773	0.811
Range	35.42%	27.43%	25.38%	57.69%	54.97%
Minimum	8.33%	7.03%	6.66%	14.84%	14.46%
Maximum	43.76%	34.46%	32.04%	72.53%	69.43%
#Observation	60	60	60	60	60

N.B.: EWMA (exponentially weighted moving average) is also calculated with the same data (monthly volatility of daily returns) of 60 observations. The mean is 21.04% and the standard deviation is 7.58%.

Table 2.2.b Root Mean Square Error and Mean Error relative to Close-to-Close Estimator

X	RMSE	Mean Error (ME)
Parkinson's	4.558%	-3.665%
Garman and Klass's	5.903%	-4.586%
Modified Parkinson's	15.969%	14.930%
Modified Garman and Klass's	15.116%	13.976%

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(X_t - CC_t)^2} \qquad ME = \frac{1}{N}\sum_{t=1}^{N}(X_t - CC_t)$$

Table 2.3. Autoregressive Moving Average (ARMA)

	Sigma		Log Sigma	
	ARMA (1, 0)	ARMA (1, 1)	ARMA (1, 0)	ARMA (1, 1)
Constant (C)	.869964e-02 (21.824)	.243007e-03 (2.283)	-4.84501 (-32.770)	N/A
Autoregression (ϕ)	.158229 (5.629)***	.976209 (96.891)***	.043018 (1.513)	N/A
Moving Average (θ)	--	.905207 (45.810)***	--	N/A
R squared	.02504	.095716	.18510e-2	--
Sigma bar ($\overline{\sigma}$) [Annualised]	0.0103 [16.16%]	0.0102 [15.99%]	0.0063 [9.99%]	Not converged

t values are in brackets (***: the 1% significance level)

Sigma: $\sigma_t - \overline{\sigma} = \phi(\sigma_{t-1} - \overline{\sigma}) + e_t + \theta\, e_{t-1}$

$\overline{\sigma} = C/(1 - \phi)$

Log Sigma: $\ln(\sigma_t) = \alpha + \phi[\ln(\sigma_{t-1}) - \alpha] + \theta\eta_t$

$\overline{\sigma} = \exp(C/(1 - \phi))$

Table 2.4. Autoregressive Conditional Heteroskedasticity (ARCH) Family

	ARCH (1)	AR(1)-ARCH(1)	GARCH(1,1)	AR(1)-GARCH(1,1)	GARCH(1,1)-Mean	AR(1)-GARCH(1,1)-Mean
c	--	-.16523e-03 (-.420)	--	.889835e-04 (.239)	--	-.17879e-02 (-1.112)
lag(Return) (ϕ)	--	.013549 (.458)	--	.119857e-02 (.039)	--	-.45277e-03 (-.015)
Theta (δ)	--	--	--	--	.015726 (.549)	.154306 (1.230)
Alpha0 (ω)	.163038e-03 (29.219)***	.162855e-03 (29.228)***	.493887e-05 (5.376)***	.500647e-05 (5.381)***	.516911e-05 (5.373)***	.489223e-05 (5.288)***
Alpha1 (α)	.201132 (6.338)***	.2019175 (6.354)***	.073131 (7.265)***	.073674 (7.246)***	.074680 (7.209)***	.071060 (6.964)***
Beta1 (β)	--	--	.903275 (69.573)***	.902347 (68.535)***	.900559 (66.983)***	.905251 (68.384)***
H(0)	--	--	--	.701969e-04 (1.416)	.705824e-04 (1.410)	.771520e-04 (1.561)
L	3525.91	3526.09	3588.30	3588.34	3588.47	3589.18

Number of observation = 1,236 c: constant L: Log likelihood
t values are in brackets (***: the 1% significance level)

$$y_t = \phi x_t + \delta\sqrt{h_t} + \varepsilon_t$$

$$\varepsilon_t = \sqrt{h_t}\xi_t$$

$$h_t = \omega + \alpha \cdot \varepsilon_{t-1}^2 + \beta \cdot h_{t-1}$$

Table 2.5. Root Mean Square Error and Mean Error relative to Daily Squared Returns (SR)

X	RMSE	Mean Error (ME)
EWMA	14.7%	5.0%
GARCH	15.7%	5.4%

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(X_t - SR_t)^2} \qquad ME = \frac{1}{N}\sum_{t=1}^{N}(X_t - SR_t)$$

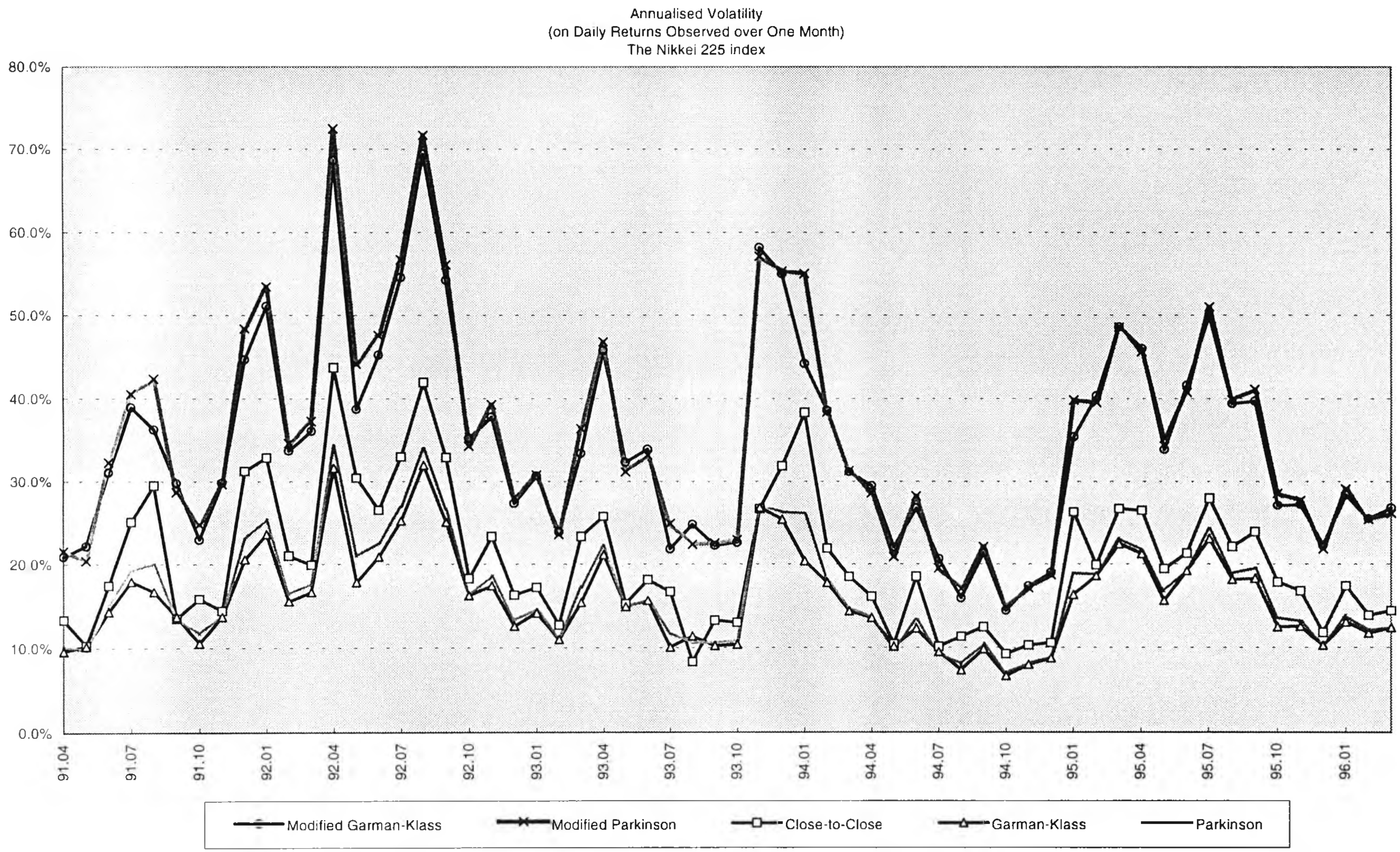

Figure 2.1. Various Volatility Calculations

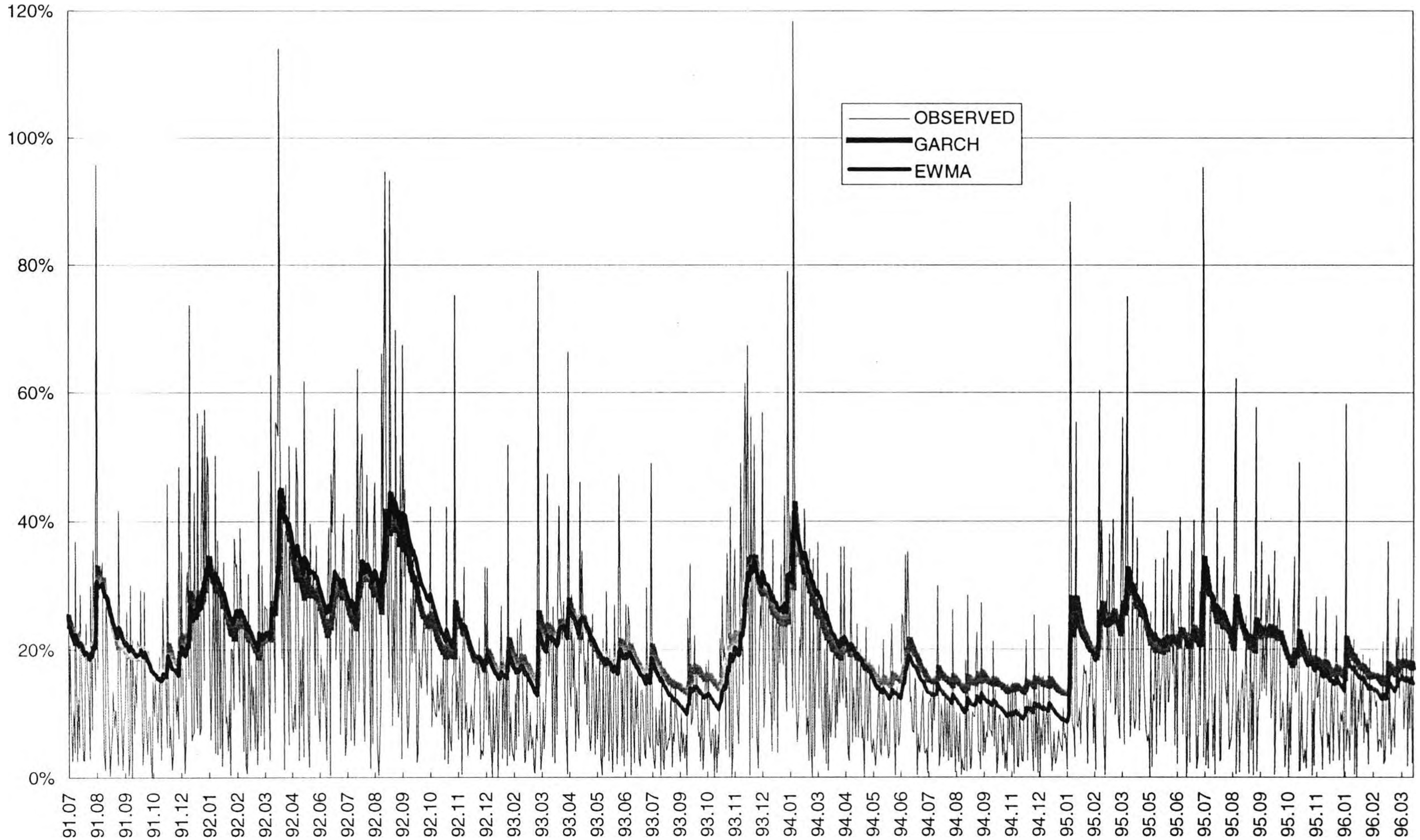

Figure 2.2. Annualised Volatility (Observed) and Estimates (GARCH and EWMA)

Appendix 2.1. Rolling GARCH estimates vs. RiskMetrics's EWMA estimates

The five year rolling (daily) estimates from GARCH (1, 1) are compared with exponentially weighted moving average (EWMA) estimates, also calculated for a rolling five-year period daily. The formulation and optimised decay factor of 0.94 are set in the same manner of Equation (13).

The difference between the rolling GARCH and EWMA estimates is rather small. As Zangari (1995) claims, the exponentially weighted moving average is easier to estimate than GARCH and as good in forecasting volatility level, as tested for the Nikkei index.

Table A1 shows that EWMA is better than GARCH in both mean error and root mean square error. With 1,163 data points, rolling EWMA (74 days) and rolling GARCH (5 year) forecast the volatility of the day, and the forecasts are compared with the daily squared returns. The mean error of EWMA is 5.0%, whereas the one of rolling GARCH is 5.5% in an annualised term. In the RMSE analysis, 14.7% for EWMA is less than 15.8% for rolling GARCH.

Table A1. Root Mean Square Error and Mean Error relative to Daily Squared Returns (SR)

X	RMSE	Mean Error (ME)
EWMA	14.7%	5.0%
Rolling GARCH	15.8%	5.5%

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(X_t - SR_t)^2} \qquad ME = \frac{1}{N}\sum_{t=1}^{N}(X_t - SR_t)$$

Appendix 2.2. Number of Trading Days a Year

It is reasonable to set the number of trading days in Japan 245 days for volatility calculation. We have 104 weekend holidays (= 52 weeks x 2) and 14 to 17 market-closing days. If a national holiday is on Saturday, the holiday is not postponed. If it is on Sunday, the holiday is postponed to Monday.

January	2 national holidays and at most 2 market closing days	= 2 - 4
February	1 national holidays	= 1
March	1 national holidays	= 1
April	1 national holidays	= 1
May	3 national holidays	= 3
June	No national holidays	= 0
July	1 national holidays	= 1
August	No national holidays	= 0
September	2 national holidays	= 2
October	1 national holidays	= 1
November	2 national holidays	= 2
December	1 national holidays and at most 1 market-closing day	= 1 - 2
	Total	= 15 - 18

Therefore, trading days are between 243 to 246 (in a leap year, 248) days. In this paper, all the volatility is annualised with 245 days a year.

1 month means:	20 days	**2 month** means:	41 days
3	61	**4**	82
5	102	**6**	123
7	143	**8**	163
9	184	**10**	204
11	225	**12**	245

In order to annualise volatility, we calculate as following:

$$V^{*} = V \cdot \sqrt{t} \qquad \text{(A1)}$$

where,

V^* : annualised volatility

V : daily volatility

t : number of days a year = 245 in Japan

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3

Forecasting Power of Volatility:

Historical Data vs. the Market

Naoki Kamiyama

3. The Forecasting Power of Volatility: Historical Data vs. the Market

3.1. Introduction

Forecasting volatility is a major issue in options trading. Immediately following the discovery of the Black-Scholes (B-S) formula (Black and Scholes 1973), market participants used the historical volatility (HV) as an input to the equation, based upon some arbitrary sample period. However, they soon also started to pay attention to the implied volatility (IV), because it was positioned to be a rational estimate of future volatility.

Several sophisticated volatility forecasts have been presented by using historical data. An early example is French, Schwert, and Stambaugh (1987), who examined the relation between stock returns and volatility. Engle (1982) introduced the ARCH paradigm and Bollerslev (1986) generalised the method (Generalized ARCH, or GARCH). Bollerslev, Chou, and Kroner (1992) have reviewed the ARCH family of models concisely. Nowadays, GARCH is one of major ways to forecast volatility for option pricing and also to evaluate option positions and portfolios. Many empirical studies for the Japanese stock market have been performed, such as Tokunaga, Iihara, and Kato (1993). In addition to the ARCH family, stochastic volatility (SV) is modeled and examined by Harvey and Shephard (1993). In the stochastic volatility context, volatility is not the variance of the one-step ahead error, but an unobserved variable, the change in which is treated as an independent variable of a stochastic system. Taylor (1994) compared ARCH with SV in the foreign exchange market (DM/$), and concluded that they gave similar estimates of the persistence of volatility shocks (25 - 30 days of half-lives).

The time-series behaviours of implied volatility itself has also been examined. Franks and Schwartz (1991) used an autoregressive model of implied volatility to forecast the one-step ahead weighted implied standard deviation (WISD). A recent study done by Resnick, Sheikh, and Song (1993) showed evidence to improve option pricing models by using expiration-specific WISDs. Harvey and Whaley (1992) also extended Franks and Schwarts (1991) and concluded that change in implied volatility is predictable, but no actual arbitrage is possible when transaction costs are considered. Their model is called implied volatility regression (IVR). Recently, volatility term structures and smile effects have been observed, especially after the crash in 1987. Xu and Taylor (1994) found that a term structure does exist in the foreign exchange option markets. Heynen, Kemna, and Vorst (1994) tested if any of the ARCH family can explain the term structure in Dutch index options.

There are many reports which compare historical volatility forecasts (including GARCH) with implied volatility to forecast realised volatility. An early example is Akgiray (1989), who found that GARCH was superior to HV. Day and Lewis (1992) compared the implied volatility of the S&P 100 options to GARCH and EGARCH, and found that the volatility forecasts by GARCH and EGARCH reflect incremental information relative to implied volatility. Lamoureux and Lastrapes (1993) found that realised volatility (RV) was positively related to implied volatility (IV), negatively related to historical volatility (HV), and not significantly related to GARCH forecasts. Their error analysis showed that IV tends to underestimate RV. Xu and Taylor (1995) found that, when IV is used, neither historical volatility nor a GARCH forecast has incremental forecasting power in forecasting currency volatility. Canina and

Figlewski (1993) used an encompassing regression to examine the forecasting power of implied versus historical volatilities. Their result in the S&P 100 options market is different from the perception normally held by the academic community (for example, Gemmill (1986)), because it shows that implied volatility has weaker power than historical volatility to forecast the future volatility. Figlewski (1994) also found that historical volatility with a long sample period gives a better forecast than one with a short sample period, although his main purpose was to forecast a long-term volatility. He also claimed that GARCH estimates are less accurate than the simple historical volatility. Noh, Engle, and Kane (1994) reported GARCH is better than the IVR modelled in Harvey and Whaley (1992), by simulating straddle trades in the S&P 500 options on futures market.

The purpose of this chapter is to compare the forecasting power of HV, IV, and GARCH, using data on the Nikkei 225 index. Even though this article complements research by Lamoureux and Lastrapes (1993), essential differences are: (1) to use the Nikkei options rather than individual stock options on CBOE; (2) to analyse a range of option (3 moneyness) and forecast horizons (4 periods); and (3) to use two types of historical volatility (22 day and 60 day volatilities) for encompassing regression tests. By analysing 22-day and 60-day HVs, we are able to compare our results with those of Figlewski (1994), reaching a different conclusion.

In the next section, the methods undertaken are shown. Then Section 3 explains the data used in this paper. The test results are analysed in Section 4 and the conclusions are brought together in Section 5.

3.2. The Method

We use the sample period of 12-Jun-89 to 06-May-94. For the initial condition, GARCH (1, 1) is estimated using the data from 06-Sep-85 to 11-Jun-89[1]. In the sample period, daily rolling GARCH (introduced in Bollerslev (1986)) is applied as:

$$y_t = m + \varepsilon_t \quad (1a)$$
$$\varepsilon_t = \sqrt{h_t} \cdot \xi, \quad \xi \sim N(0,1) \quad (1b)$$
$$h_t = \alpha + \beta\varepsilon_{t-1}^2 + \gamma h_{t-1} \quad (1c)$$

y_t is return, m is a constant and ε_t is a disturbance term. ε_t is heteroscedastic as shown in (1b) and (1c). In order to forecast volatility over the option period by GARCH, in addition to the recursive input to h_t, ε_{t-1}^2 is replaced by h_t as well, because $V(\varepsilon_t) = E(\varepsilon_t^2) - (E(\varepsilon_t))^2 = E(\varepsilon_t^2) = h_t$ when $E(\varepsilon_t) = 0$. That is, the expected value of ε_t is equal to h_t. The GARCH parameters are estimated daily in the rolling manner to obtain the GARCH forecasts. Mean error (ME), mean absolute error (MAE), and root mean square error (RMSE) are examined from 12-Jun-89 to 06-May-94. The encompassing regression approach is used. Because of overlapping data of historical volatility, it is too complicated to explicitly show the correlation of the disturbance term. In order to obtain efficient estimates of the parameters, it is appropriate to use GMM, generalised method of moments[2]. One of the early examples of the encompassing regression method is in Hansen and Hodrick (1980). Fair and Shiller (1990) discuss encompassing tests and combination of forecasts. A recent example in stock returns is Canina and Figlewski (1993).

ME, MAE, and RMSE are defined as followed.

[1] The Crash in 1987 period is not left out because the impact was not so large in Japan as in the American market.

[2] The details of GMM implemented here is shown in Appendix B.

$$ME = \frac{1}{N}\sum_{t=1}^{N}(X_t - RV_t) \tag{2a}$$

$$MAE = \frac{1}{N}\sum_{t=1}^{N}|X_t - RV_t| \tag{2b}$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(X_t - RV_t)^2} \tag{2c}$$

where,

X_t ; forecast of volatility from IV, HV22, HV60, or GARCH,

RV; realised volatility during the option period from t

IV; implied volatilities (average of call and put by Near-/In-/Out-of-the-money) at t

HV22; 22-day historical volatility at t

HV60; 60-day historical volatility at t

GARCH; forecasted volatility using (1) at t

The encompassing regression test is based upon:

$$RV_t = a + b \cdot IV_t + c \cdot HV22_t + d \cdot HV60 + e \cdot GARCH_t + \varepsilon_t \qquad (3)[3]$$

The test gives us weights of importance among all the independent variables. The tests are made with four combinations as below.

$$RV_t = a_0 + b_0 \cdot IV_t + c_0 \cdot GARCH_t \tag{4}$$

$$RV_t = a_1 + b_1 \cdot IV_t + c_1 \cdot HV22_t + d_1 \cdot HV60_t \tag{5}$$

$$RV_t = a_2 + b_2 \cdot IV_t + c_2 \cdot HV22_t + d_2 \cdot GARCH_t \tag{6}$$

$$RV_t = a_3 + b_3 \cdot IV_t + c_3 \cdot HV60 + d_3 \cdot GARCH_t \tag{7}$$

$$RV_t = a_4 + b_4 \cdot IV_t + c_4 \cdot HV22_t + d_4 \cdot HV60 + e_4 \cdot GARCH_t \tag{8}$$

3.3. The Data

[3] The disturbance term of the equation may not be normally distributed because all the independent variables are censored (always larger than zero). Therefore, one should note that the t values are biased, when we interpret the regression results. We do not constrain coefficients to sum to one in this paper.

The options were classified according to deepness in the money and time to maturity. The nearest-the-money contract was defined as at-the-money (ATM). In-the-money (ITM) and out-of-the-money (OTM) were chosen to be 500 yen on either side of the ATM options.

There were 1,209 business days during the out-of-sample period, and 157 data-missing days. The remaining 1,052 data are available.

Whole dataset		1,052 data
Subset (Shorter Days to Maturity)	= 5 - 25 days' maturity	725 data
Subset (Longer Days to Maturity)	= 20 - 40 days' maturity	586 data

The implied volatilities of puts and calls are averaged. IV is calculated with the generalised Black-Scholes formula, which is the extended version with the dividend yield input of Black and Scholes (1973). The Nikkei Index options are European-style contracts, so that there is no early-exercise premium and the Black-Scholes formula is appropriate. The daily 3-month CD rates are used as risk free interest rates. The dividend yield rate is assumed constant at 0.7%.

Historical volatility (σ) is calculated as below.

$$\sigma^2 = \frac{1}{n-1}\sum_{t=1}^{n}(\ln R_t - m)^2 \tag{9}$$

where,

$$m = \frac{1}{n}\sum_{t=1}^{n}\ln R_t$$

and $$R_t = \frac{C_t}{C_{t-1}}$$

C_t is Closing price of the Nikkei index. n = 22 (hv22) and n = 60 (hv60) are calculated. The 22 days are representative to one-month of trading days and the 60 to three-months of trading. Realised Volatility (RV) is calculated over each option maturity period in the same way as HV, in which case n means the remaining days of option to expiry.

3.4. Test Results

3.4.1 Forecasting Errors

In Table 3.1.a, descriptive statistics are shown. Table 3.1.b gives the results for simple forecasting errors in terms of ME, MAE, and RMSE. There are three main points to be noted. (1) HV22 has the smallest error in terms of bias (ME) and in terms of MAE as well, and GARCH is the smallest in terms of RMSE. These results hold in shorter maturity subset, but IV (ATM) is best in the case of longer maturity subset. (2) IV gives the smallest RMSE for longer maturity subset. Among IVs, the at-the-money (ATM) forecasts have smaller RMSE than the out-of-the-money (OTM) or in-the-money (ITM) forecasts. (3) HV60 is inferior to HV22 in all respects. This result is different from that of Figlewski (1994), who claimed that the long-term historical volatility is a superior estimator to the short-term volatility.

3.4.2 Encompassing Regressions

In Table 3.2, the results of encompassing regression is shown. Four points are notably observed. (1) Positive weights are observed on IV and GARCH. HV22 has positive weights in Equations (6) and (9), but the positive weights are not significant in Equation (9) with GARCH. HV60 has a negative weight, which is significant in all the subsets. (2) The large coefficients of IV indicates that IV was the most important forecasting component. The coefficients (from 0.6 to 1.0) have significant t values in all the subsets and models. (3) The constants of the regression results are not significant in most of the subsets. It seems no down/upward shift (bias) arises from this analysis. (4) The GARCH coefficients are not significant in 'full models' (Equation (9)) for the longer time to maturity.

In addition, Table 3.3 shows the supplemental results of encompassing regression comparing three of IV (at-, in-, and out-of-the-money)[4]. OTM implied volatilities are better than ATM and ITM.

3.4.3 Interpretations

Four points may help to interpret the results from the error analysis and encompassing regressions. Firstly, implied volatility was significantly upward biased[5]. This means that market participants may require a risk premium in options trading, as Lamoureux and Lastrapes (1993) suggested for stock options in the United States. The observed bias decreases as the days to maturity increase. The market participants may mean that market participants believe that the risk of a change in volatility is larger when the maturity is shorter. Canina and Figlewski (1993), who studied the S&P 100 index options at CBOE, found that IV was not superior to HV, which is opposite to the result shown here. Gemmill (1992) suggested transaction costs to explain this fact in the FTSE index options listed on LIFFE.

Secondly, IV has the largest weight among all the factors. The magnitude is between 0.60 and 0.95. However, this market is not informationally efficient, because GARCH and HV also have significant coefficients. Therefore, market participants do not seem to utilise all the historic information efficiently to set up option prices. In Equation (9), the regression based on all data, all the coefficients (except for HV22) are significant. Thirdly, the GARCH has a positive weight, and HV60 has a negative weight. Lamoureux and Lastrapes

[4] There is col-linearity in this model, but we are able to see which IV most powerful forecast is.

(1993) find that the GARCH forecast has no significant weight, and a large negative weight on HV. The historical information content in the sample variance (HV60) might be totally ignored as Lamoureux and Lastrapes mentioned. The alternative interpretation is an overreaction, which is also suggested by them. However, the HV22 has no significant weight in the 'full model' in the short options case, therefore, the participants do not seem to overreact to the recent information. Finally, the constants of the regression tests are not significant in most of the subsets, but the biases are observed as time to maturity becomes longer. On the other hand, the longer the time to maturity, the smaller the significance of GARCH. The reason seems to be that the GARCH (1,1) model assumes one day autoregressive conditional variance. The recursive one-step-ahead forecasts may cause a large amount of errors as the number of recursive operations increases, as Xu and Taylor (1995) have pointed out.

The OTM implied volatilities are better than ATM in encompassing regression with three different strike prices. The best forecasts by traders may be mostly influenced to OTM because they prefer to trade slightly OTM options to reduce costs (in absolute dollar amount) to control their risk exposures.

3.5. Conclusion

Figure 3.1 shows that the implied volatility tends to follow the realised volatility moderately well. However, it has an upward bias (as shown in Table 3.1). Curiously, the coefficients of the long-term historical volatility such as 60-day volatility in the encompassing regression are significantly negative. Because the options analysed in this paper have 5-40 days to maturity, 60-day volatility is rather longer than the life of the options. This means that the too long cycle is

[5] Note that the option price data were closing prices, therefore no bid-ask bias exists in average.

drawn by the 60-day average compared with the realised volatility movement, as shown in Figure 3.2.

One can improve the forecast of future realised volatility by using both implied volatility and a GARCH historic estimate. However, the GARCH model can only forecast the short-maturity options significantly, because it requires one-step-ahead forecast in the time horizon of one day. Therefore, the GARCH result tends to overreact to a volatility shock when forecasting more than one day ahead. Figure 3.3 shows the overreacted errors. For example, the huge shock which can be observed in the 3rd quarter of 1990 causes a too large GARCH forecast.

It is very interesting to consider which error is most important from the trader's point of view. HV22 has the smallest bias (ME) but the bias is not very important because negative and positive errors are offsetting. A trader would lose money if she made an error in forecasting volatility in either positive or negative manner. Therefore, a comparison band on bias is not good for the trading purpose. Mean absolute error (MAE) may be the best way to choose a single forecast from the candidates. As long as we assume that managers keep a delta neutral position, the profit/loss is generated from the difference between forecasted volatility and realised volatility, multiplied by its position vega[6]. Given a vega, we can estimate the profit/loss amount as MAE multiplied by vega. RMSE exaggerate the forecasting error from the realised volatility if we estimate the profit/loss in the manner mentioned above. In the RMSE calculation, a large error affects more than MAE because it is squared. Therefore, RMSE can not be an exact estimate of the profit/loss impact. However, RMSE may be a good indicator to choose a stable volatility forecast. RMSE pays more attention to a sporadic huge difference between a forecast and a realised volatility than MAE. HV22 is the best in MAE, but GARCH is the best in RMSE. IV is biased upward rather simply, therefore we may be able to adjust the level to the unbiased IV level to forecast the realised volatility. As one can see the encompassing regression, IV has the best forecasting power (the largest weight), if we recognise the bias in practice.

There is no reason to choose a single volatility forecast. The encompassing regression result suggests that a combination of several indicators including historical estimates can enhance volatility forecast. A forecast can be relied on IV in more than 75% in weight, and also on historic estimate such as GARCH in 25%, especially in the case that the number of days to maturity is shorter than 10 days. Although the appropriate weight is stable when the days to maturity is longer than 10, the more the number of days to maturity, the lower the significance. It is rational to forecast volatility by using a composition of several indicators.

Appendix 3.A shows another encompassing regression test to compare with Canina and Figlewski (1993). In the analysis above and the result of Appendix as well, one can find that IV is alway better than HV. The weight for IV is around one compared with that for HV which is around 0.5. We do not reject the hypothesis that the coefficient of IV is greater than zero, whereas Canina and Figlewski (1993) find the hypothesis is not significant.

The implication of all the results above for option practitioners is that IV is the best of all but two points should be added. One is that HV and GARCH analyses can improve the IV forecast. The other is that IV tends to be higher than

[6] Vega is defined as a sensitivity of option's price to a change in volatility. Mathematically, it is defined

the volatility which is realised in the future. Using the encompassing regression coefficients, we can optimise the combination of all kinds of volatility estimates to predict real volatility which is occurring in the future.

as the first derivative of option pricing equation (such as Black-Scholes formula) in volatility.

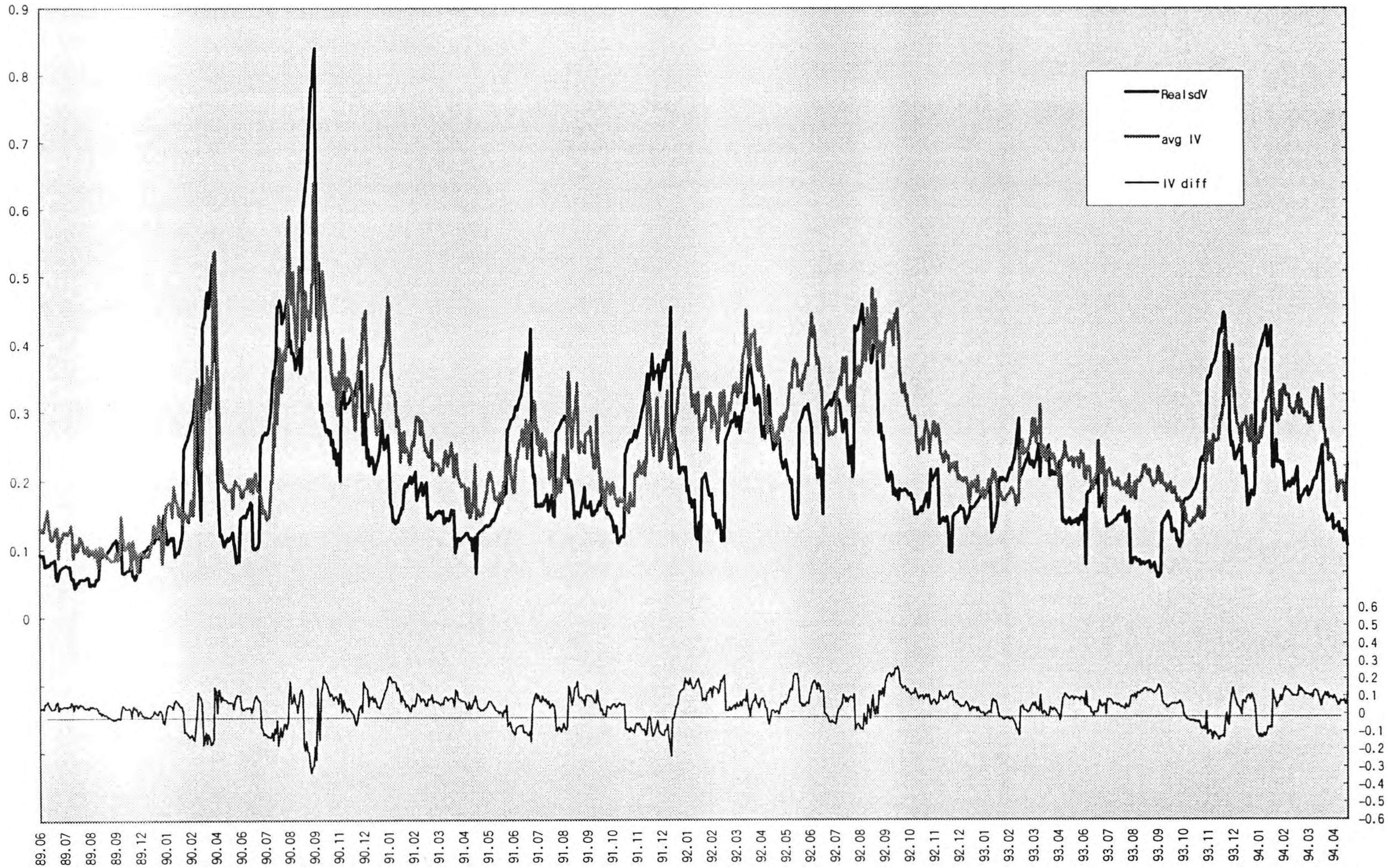

Figure 3.1. Realised Volatility and Implied Volatility

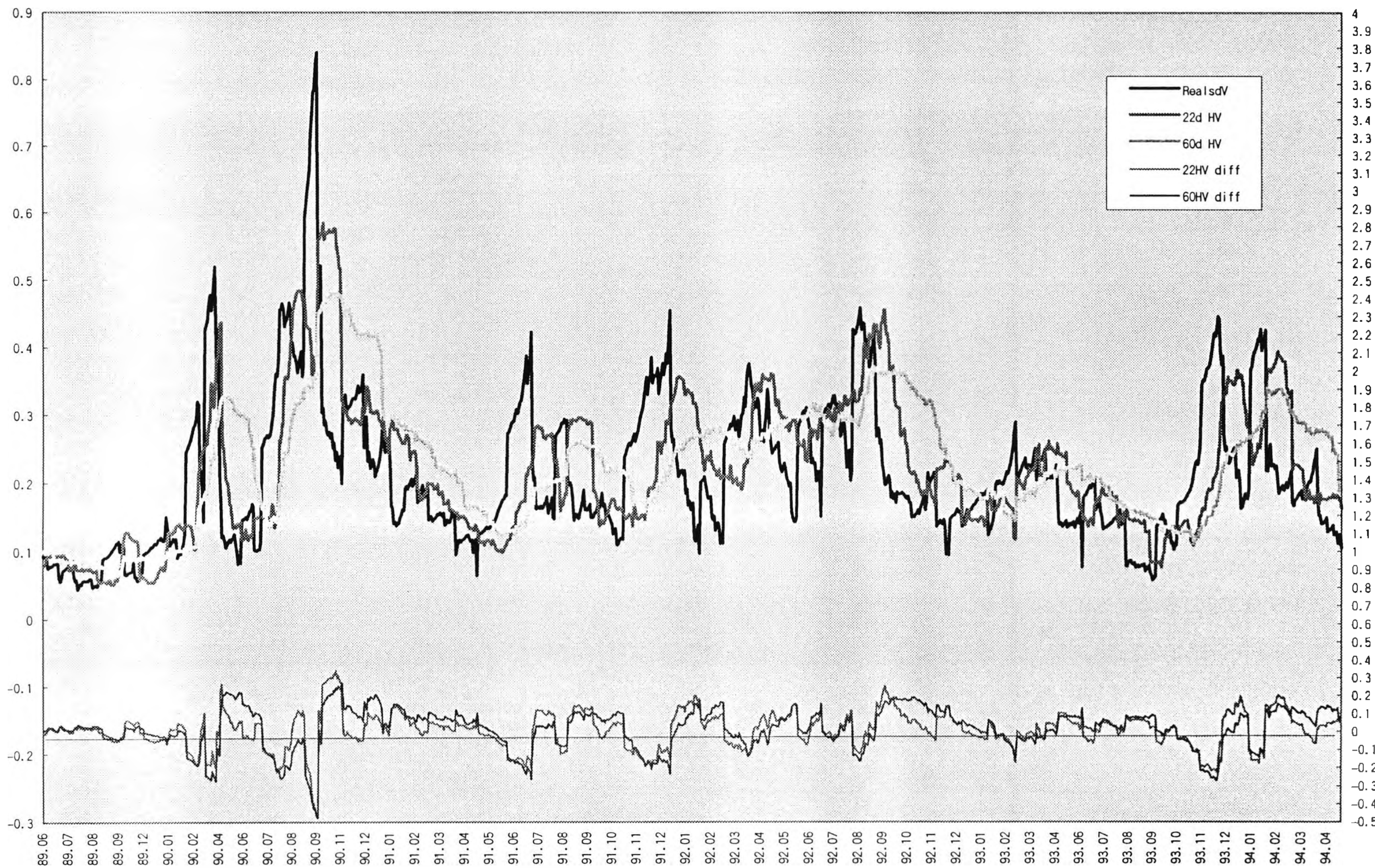

Figure 3.2. Realised Volatility and Historical Volatility

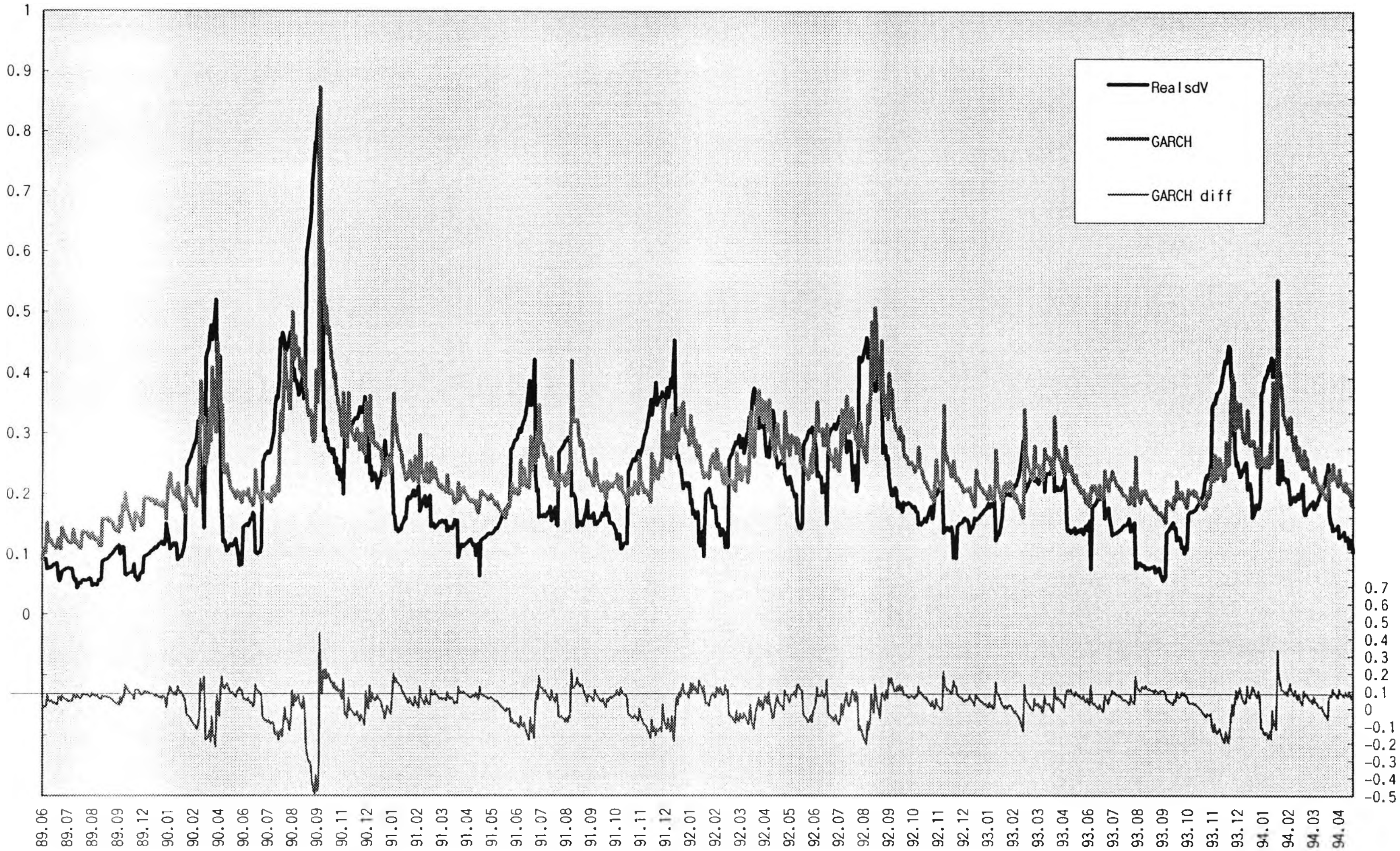

Figure 3.3. Realised Volatility and GARCH Forecast

Table 3.1.a Descriptive Statistics (Annualised Volatilities)

Whole Sample	RV	IV-ATM	IV-ITM	IV-OTM	HV22	HV60	GARCH
Mean	0.21290	0.25410	0.25794	0.25610	0.22213	0.23001	0.24490
Standard Deviation	0.11056	0.09346	0.09268	0.09466	0.10380	0.09047	0.07244
Variance	0.01222	0.00873	0.00859	0.00896	0.01077	0.00819	0.00525
Kurtosis	3.88254	0.27524	0.64964	0.54492	0.78474	-0.09651	7.18373
Skewness	1.45232	0.53654	0.63205	0.58471	0.83565	0.35436	1.61435
#Obs	1052	1052	1052	1052	1052	1052	1052

Shorter (5-25)	RV	IV-ATM	IV-ITM	IV-OTM	HV22	HV60	GARCH
Mean	0.21193	0.26116	0.26623	0.26393	0.22652	0.23236	0.24736
Standard Deviation	0.11736	0.09833	0.09745	0.09954	0.10774	0.09247	0.07842
Variance	0.01377	0.00967	0.00950	0.00991	0.01161	0.00855	0.00615
Kurtosis	4.35875	0.19074	0.59865	0.48509	0.70938	-0.18811	7.07603
Skewness	1.61023	0.55804	0.65158	0.61856	0.88205	0.33628	1.64622
#Obs	696	696	696	696	696	696	696

Longer (20-40)	RV	IV-ATM	IV-ITM	IV-OTM	HV22	HV60	GARCH
Mean	0.21587	0.24540	0.24711	0.24576	0.21725	0.22545	0.24244
Standard Deviation	0.10295	0.08879	0.08792	0.08948	0.10081	0.08950	0.06390
Variance	0.01060	0.00788	0.00773	0.00801	0.01016	0.00801	0.00408
Kurtosis	2.23083	0.08883	0.27851	0.09382	0.83367	0.00124	2.02337
Skewness	1.15772	0.48658	0.54017	0.47227	0.76820	0.38411	0.96206
#Obs	586	586	586	586	586	586	586

Table 3.1.b Analysis of Forecast Errors

- Whole Sample (1,052 data)

	IV-ATM	**IV-ITM**	**IV-OTM**	**HV22**	**HV60**	**GARCH**
ME	0.0412007	0.0450421	0.0432050	0.0092356	0.0171129	0.0320099
(SD)	(0.0854356)	(0.0858755)	(0.0850500)	(0.1011146)	(0.1120616)	(0.0891358)
MAE	0.0793892	0.0811320	0.0799841	0.0771192	0.0889883	0.0772479
RMSE	0.094815	0.096935	0.095359	0.10149	0.11331	0.094669

- Shorter Days to Maturity Subset (5-25 days) (696 data)

	IV-ATM	**IV-ITM**	**IV-OTM**	**HV22**	**HV60**	**GARCH**
ME	0.0492283	0.0542956	0.0520018	0.0145930	0.0204252	0.0354249
(SD)	(0.0894328)	(0.0897844)	(0.0520018)	(0.1073373)	(0.1179410)	(0.0948747)
MAE	0.0859906	0.0883379	0.0859646	0.0814035	0.0948441	0.0814150
RMSE	0.10203	0.10487	0.10176	0.10825	0.11961	0.10121

- Longer Days to Maturity Subset (20-40 days) (586 data)

	IV-ATM	**IV-ITM**	**IV-OTM**	**HV22**	**HV60**	**GARCH**
ME	0.0295292	0.0312434	0.0298957	0.0013798	0.0095839	0.0265746
(SD)	(0.0808312)	(0.0805698)	(0.0811185)	(0.0946217)	(0.1053475)	(0.0838309)
MAE	0.0716120	0.0718385	0.0718992	0.0734302	0.0818161	0.0730510
RMSE	0.085991	0.086351	0.086387	0.094551	0.10569	0.087874

$$ME = \frac{1}{N}\sum_{t=1}^{N}(X_t - RV_t)$$

$$MAE = \frac{1}{N}\sum_{t=1}^{N}|X_t - RV_t|$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(X_t - RV_t)^2}$$

Table 3.2. Encompassing Regression Tests

Whole Sample

A. ATM - Whole data (1,052 data)

Eq	Adjusted R^2	const (t ; GMM)	IV (t ; GMM)	HV22 (t ; GMM)	HV60 (t ; GMM)	GARCH (t ; GMM)
5	0.4453	-0.00750980 (-0.63)	0.620028 (5.80)***	--	--	0.256664 (2.52)***
6	0.4760	0.035129 (2.58)	0.893208 (8.67)***	0.222083 (3.03)***	-0.428359 (-5.83)***	--
7	0.4487	-0.020315 (-1.50)	0.679224 (6.06)***	-0.151777 (-1.78)*	--	0.385196 (3.00)***
8	0.4828	0.00028471 (0.02)	0.836754 (7.09)***	--	-0.374172 (-5.73)***	0.351390 (3.49)***
9	0.4830	0.021681 (0.52)	0.822498 (7.05)***	0.076641 (0.84)	-0.401165 (-5.42)***	0.293320 (2.34)***

B. ITM - Whole data (1,052 data)

Eq	Adjusted R^2	const (t ; GMM)	IV (t ; GMM)	HV22 (t ; GMM)	HV60 (t ; GMM)	GARCH (t ; GMM)
5	0.4423	-0.013420 (-1.09)	0.597680 (5.50)***	--	--	0.294609 (2.86)***
6	0.4649	0.031459 (2.21)	0.835063 (8.08)***	0.258351 (3.55)***	-0.397141 (-5.43)***	--
7	0.4446	-0.024774 (-1.75)	0.644903 (5.74)***	-0.128988 (-1.48)	--	0.408226 (3.08)***
8	0.4737	-0.00819829 (-0.67)	0.777400 (6.62)***	--	-0.337295 (-5.24)***	0.400782 (3.90)***
9	0.4742	0.00019444 (0.01)	0.761762 (6.57)***	0.089674 (0.96)	-0.369560 (-5.06)***	0.331951 (2.57)***

C. OTM - Whole data (1,052 data)

Eq	Adjusted R^2	const (t ; GMM)	IV (t ; GMM)	HV22 (t ; GMM)	HV60 (t ; GMM)	GARCH (t ; GMM)
5	0.4499	-0.00488471 (-0.41)	0.640136 (5.80)***	--	--	0.219844 (2.00)**
6	0.4844	0.034640 (2.59)	0.910236 (8.92)***	0.202497 (2.75)***	-0.434061 (-5.92)***	--
7	0.4532	-0.017399 (-1.27)	0.698232 (6.10)***	-0.151058 (-1.77)*	--	0.347203 (2.53)***
8	0.4886	0.00392512 (0.32)	0.861430 (7.14)***	--	-0.380220 (-5.81)***	0.309554 (2.83)***
9	0.4891	0.011531 (0.80)	0.846647 (7.11)***	0.083478 (0.91)	-0.409983 (-5.50)***	0.246195 (1.83)*

t values in brackets. *: 5%, **: 2.5%, ***: 1% significant

Shorter Days to Maturity Subset

D. ATM - Subset (5-25 days) (696 data)

Eq	Adjusted R^2	const (t ; GMM)	IV (t ; GMM)	HV22 (t ; GMM)	HV60 (t ; GMM)	GARCH (t ; GMM)
5	0.4532	-0.013426 (-1.03)	0.658634 (4.95)***	--	--	0.215675 (1.81)***
6	0.4874	0.027151 (1.63)	0.915199 (7.77)***	0.233131 (2.71)***	-0.460681 (-5.31)***	--
7	0.4571	0.4571 (-1.78)	0.716805 (5.27)***	-0.051566 (-1.75)*	--	0.357375 (2.22)**
8	0.4926	-0.00398333 (-0.30)	0.860422 (6.11)***	--	-0.396080 (-5.24)***	0.336514 (2.76)***
9	0.4931	0.00411238 (0.26)	0.846425 (6.10)***	0.096892 (0.90)	-0.434786 (-4.93)***	0.266190 (1.76)*

E. ITM - Subset (5-25 days) (696 data)

Eq	Adjusted R^2	const (t ; GMM)	IV (t ; GMM)	HV22 (t ; GMM)	HV60 (t ; GMM)	GARCH (t ; GMM)
5	0.4509	-0.020553 (-1.51)	0.636760 (4.74)***	--	--	0.254538 (2.13)**
6	0.4761	0.021446 (1.22)	0.867369 (7.20)***	0.256064 (2.95)***	-0.423644 (-4.84)***	--
7	0.4545	-0.033106 (-2.12)	0.691292 (5.05)***	-0.161292 (-1.63)	--	0.394304 (2.51)***
8	0.4848	-0.013721 (-1.00)	0.805761 (5.75)***	--	-0.362360 (-4.76)***	0.385410 (3.14)***
9	0.4850	-0.00638330 (-0.39)	0.792695 (5.72)***	0.085965 (0.78)	-0.396665 (-4.53)***	0.323308 (2.11)**

F. OTM - Subset (5-25 days) (696 data)

Eq	Adjusted R^2	const (t ; GMM)	IV (t ; GMM)	HV22 (t ; GMM)	HV60 (t ; GMM)	GARCH (t ; GMM)
5	0.4637	-0.011015 (-0.87)	0.702969 (5.10)***	--	--	0.151239 (1.16)
6	0.5021	0.024291 (1.49)	0.946214 (8.18)***	0.196626 (2.27)**	-0.458938 (-5.28)***	--
7	0.4679	-0.023419 (-1.63)	0.761490 (5.46)***	-0.170793 (-1.80)*	--	0.295348 (1.80)*
8	0.5043	-0.00068298 (-0.05)	0.904749 (6.29)***	--	-0.399175 (-5.27)***	0.269134 (2.03)**
9	0.5048	0.00754932 (0.48)	0.891025 (6.29)***	0.099095 (0.92)	-0.439195 (-4.95)***	0.197341 (1.22)

t values in brackets. *: 5%, **: 2.5%, ***: 1% significant

Longer Days to Maturity Subset

G. ATM - Subset (20-40 days) (586 data)

Eq	Adjusted R^2	const (t ; GMM)	IV (t ; GMM)	HV22 (t ; GMM)	HV60 (t ; GMM)	GARCH (t ; GMM)
5	0.4292	0.012397 (0.65)	0.646265 (5.71)***	--	--	0.185112 (1.32)
6	0.4592	0.045622 (3.13)	0.882500 (7.68)***	0.173236 (1.95)*	-0.372372 (-4.11)***	--
7	0.4297	0.001219 (0.06)	0.688187 (6.26)***	-0.100545 (-0.81)	--	0.278882 (1.41)
8	0.4605	0.015901 (0.87)	0.866266 (6.46)***	--	-0.335490 (-3.99)***	0.259955 (1.93)*
9	0.4605	0.025926 (1.12)	0.845058 (6.64)***	0.087954 (0.66)	-0.359072 (-3.92)***	0.183189 (0.93)

H. ITM - Subset (20-40 days) (586 data)

Eq	Adjusted R^2	const (t ; GMM)	IV (t ; GMM)	HV22 (t ; GMM)	HV60 (t ; GMM)	GARCH (t ; GMM)
5	0.4295	0.00970581 (0.52)	0.653592 (5.96)***	--	--	0.184176 (1.37)
6	0.4567	0.042515 (2.94)	0.845808 (7.93)***	0.200426 (2.29)**	-0.351286 (-3.98)***	--
7	0.4291	0.00300025 (0.14)	0.675892 (6.42)***	-0.058688 (-0.47)	--	0.241694 (1.22)
8	0.4559	0.011606 (0.65)	0.840646 (6.69)***	--	-0.302184 (-3.70)***	0.266691 (2.08)**
9	0.4572	0.027015 (1.18)	0.813953 (6.85)***	0.132758 (0.98)	-0.340557 (-3.80)***	0.147059 (0.75)

I. OTM - Subset (20-40 days) (586 data)

Eq	Adjusted R^2	const (t ; GMM)	IV (t ; GMM)	HV22 (t ; GMM)	HV60 (t ; GMM)	GARCH (t ; GMM)
5	0.4281	0.013370 (0.69)	0.638429 (5.65)***	--	--	0.188066 (1.33)
6	0.4622	0.047197 (3.23)	0.900476 (7.71)***	0.171128 (1.93)*	-0.398358 (-4.34)***	--
7	0.4287	0.00181221 (0.08)	0.682484 (6.19)***	-0.104855 (-0.86)	--	0.285039 (1.46)
8	0.4661	0.017990 (0.98)	0.885071 (6.47)***	--	-0.361228 (-4.24)***	0.254904 (1.89)*
9	0.4634	0.028031 (1.22)	0.863830 (6.64)***	0.088394 (0.68)	-0.384511 (-4.12)***	0.177463 (0.07)

t values in brackets. *: 5%, **: 2.5%, ***: 1% significant

Table 3.3. Encompassing Regression with Three Implied Volatilities

Whole Sample (1,052 data)

Eq	Adj R^2	const	IV-ATM	IV-ITM	IV-OTM	HV22	HV60	GARCH
5	0.4505	-0.0072062 (-0.59)	-0.09353 (-0.36)	0.20182 (1.54)	0.54376 (2.04)**	--	--	0.21459 (1.94)*
6	0.4855	0.031337 (2.26)	-0.04187 (-0.17)	0.22234 (1.70)**	0.74749 (3.03)***	0.19558 (2.66)***	-0.43490 (-5.94)***	--
7	0.4542	-0.020735 (-1.48)	-0.04653 (-0.19)	0.21784 (1.67)*	0.54365 (2.09)**	-0.15853 (-1.86)*	--	0.34810 (2.55)***
8	0.4900	0.00096292 (0.08)	0.01838 (0.08)	0.22492 (1.70)*	0.63616 (2.58)***	--	-0.38454 (-5.88)***	0.30532 (2.81)***
9	0.4902	0.00794725 (0.55)	0.00378 (0.02)	0.21891 (1.66)*	0.64258 (2.60)***	0.07525 (0.83)	-0.41104 (-5.52)***	0.24820 (1.86)*

t values by GMM in brackets. *: 5%, **: 2.5%, ***: 1% significant

Shorter Maturity Subset (696 data)

Eq	Adj R^2	const	IV-ATM	IV-ITM	IV-OTM	HV22	HV60	GARCH
5	0.4649	-0.013110 (-0.99)	-0.3446 (-1.15)	0.22079 (1.50)	0.83758 (2.66)***	--	--	0.14232 (1.08)
6	0.5033	0.020637 (1.22)	-0.22852 (-0.80)	0.23947 (1.60)	0.95109 (3.39)***	0.18661 (2.14)**	-0.45652 (-5.24)***	--
7	0.4695	-0.026703 (-1.77)	-0.30012 (-1.03)	0.25169 (1.71)*	0.82591 (2.68)***	-0.17797 (-1.87)*	--	0.29243 (1.76)*
8	0.5058	-0.00391273 (-0.29)	-0.18102 (-0.65)	0.256187 (1.72)*	0.846722 (2.92)***	--	-0.40248 (-5.31)***	0.262596 (1.97)**
9	0.5060	0.00361679 (0.23)	-0.18883 (-0.68)	0.243972 (1.64)	0.853295 (2.94)***	0.088109 (0.82)	-0.43750 (-4.93)***	0.198740 (1.23)

t values by GMM in brackets. *: 5%, **: 2.5%, ***: 1% significant

Longer Maturity Subset (586 data)

Eq	Adj R^2	const	IV-ATM	IV-ITM	IV-OTM	HV22	HV60	GARCH
5	0.4304	0.011991 (0.64)	0.215960 (0.52)	0.339303 (1.67)*	0.115518 (0.26)	--	--	0.159399 (1.14)
6	0.4637	0.043286 (2.98)	-0.02944 (-0.07)	0.342230 (1.78)*	0.601722 (1.37)	0.162903 (1.83)*	-0.39048 (-4.30)***	--
7	0.4308	0.00143233 (0.07)	0.228340 (0.55)	0.323622 (1.59)	0.158209 (0.36)	-0.09614 (-0.77)	--	0.249276 (1.26)
8	0.4640	0.016907 (0.93)	0.006288 (0.02)	0.295515 (1.53)	0.602870 (1.38)	--	-0.35452 (-4.21)***	0.231632 (1.73)*
9	0.4642	0.027625 (1.20)	-0.02069 (-0.05)	0.307823 (1.60)	0.595372 (1.36)	0.094428 (0.71)	-0.37957 (-4.11)***	0.148458 (0.76)

t values by GMM in brackets. *: 5%, **: 2.5%, ***: 1% significant

Appendix 3.A Historical and Implied Volatility

This Appendix shows the result of an encompassing regression which is comparable to Canina and Figlewski. It shows that implied volatility is a better forecast than historical volatility for the Nikkei 225 traded options, which is the opposite to Canina and Figlewski (1993).

A1. The Data

- The 774 data are used from 07-Sep-90 to 08-Nov-93. (The data is different from the data in the main part of this paper.)

- Subsets are defined by the number of days to expiry.

Subset 1 = 5 - 25 days	533 data
Subset 2 = 10 - 30 days	539 data
Subset 3 = 15 - 35 days	504 data
Subset 4 = 20 - 40 days	421 data

A2. The Method

The regression test for rationality of a forecast is expressed as;

$$\sigma = a + b \cdot F(\Phi) + u \qquad \text{(A1)}$$

where, $F(\Phi)$; the forecast of σ based on the information set Φ

u; regression residual

If the forecast is true, a = 0, and b = 1. Any significant deviation form these figures shows the fact that the forecast is biased and inefficient. The encompassing regression is used for analysing the relative information content of two different forecasts.

$$\sigma = a + b \cdot F_1(\Phi_1) + c \cdot F_2(\Phi_2) + u \qquad \text{(A2)}$$

The less informed forecast should have the parameter of 0.

The GMM (Generalised Method of Moments) is used for analysing overlapping time period of data[7]. The time series of forecast errors is serially dependent in this case, therefore the GMM is used to estimate the parameters efficiently in the heteroscedastic systems of a time series. If the system is known as a heteroscedastic, or an unobservable disturbance vector may be serially correlated and nonstationary, the GMM estimates efficient standard errors. The parameters are same as if estimated by the least square method, therefore GMM is good for estimating a heteroscedastic system with an unknown form.

A3. Test Results

$$rv_t(\tau) = a_1 + b_1 \cdot iv_{t,i} + u_{t,i} \quad \text{(A3)}$$

$$rv_t(\tau) = a_2 + b_2 \cdot hv22_{t,i} + u_{t,i} \quad \text{(A4)}$$

$$rv_t(\tau) = a_3 + b_3 \cdot hv60_{t,i} + u_{t,i} \quad \text{(A5)}$$

$$rv_t(\tau) = a_4 + b_4 \cdot iv_{t,i} + c_4 \cdot hv22_{t,i} + u_{t,i} \quad \text{(A6)}$$

$$rv_t(\tau) = a_5 + b_5 \cdot iv_{t,i} + c_5 \cdot hv60_{t,i} + u_{t,i} \quad \text{(A7)}$$

Those above are annualised.

$$yrv_t(\tau) = a_1 + b_1 \cdot yiv_{t,i} + u_{t,i} \quad \text{(A8)}$$

$$yrv_t(\tau) = a_2 + b_2 \cdot yhv22_{t,i} + u_{t,i} \quad \text{(A9)}$$

$$yrv_t(\tau) = a_3 + b_3 \cdot yhv60_{t,i} + u_{t,i} \quad \text{(A10)}$$

$$yrv_t(\tau) = a_4 + b_4 \cdot yiv_{t,i} + c_4 \cdot yhv22_{t,i} + u_{t,i} \quad \text{(A11)}$$

$$yrv_t(\tau) = a_5 + b_5 \cdot yiv_{t,i} + c_5 \cdot yhv60_{t,i} + u_{t,i} \quad \text{(A12)}$$

When we see the result from the annual volatility based analysis, b5 is significantly positive and close to one. This means that implied volatility

[7] See Appendix 3.B.

contents a good deal of information on the future volatility compared with historical volatility, which has negative coefficient of c5.

It is possible to say that the negative coefficient means mean-reverting relationship between historical volatility and realised volatility. In this analysis, the time to maturity of options is shorter than 60 days, therefore, the 60-day historical volatility is likely to be smoother than the realised volatility.

A4. Conclusion

Using GMM to regress overlapping data, we conclude that implied volatility is a better forecast of volatility realised during option period. The 60-day historical volatility is negatively correlated to the realised volatility over the remaining life of options. Despite Canina and Figlewski (1993), the results shown above suggest that implied volatility is a better forecast of future volatility than historical volatility.

A5. Tables

Daily Error Variance Analysis

rv = a1 + b1 * iv; Number of Days to Maturity

	Whole data	5 - 25	10 - 30	15 - 35	20 - 40
a1	-0.00006517	-0.00009729	-0.00007641	-0.00001521	3.22091E-06
SE	0.00003521	0.00004412	0.00004092	0.00002584	0.00002269
t	-1.85	-2.21	-1.87	-0.59	0.14
b1	0.981582	1.049679	1.018698	0.833395	0.804593
SE	0.12947	0.15955	0.14816	0.10493	0.09605
t	7.58	6.58	6.88	7.94	8.38
R^2	0.3754	0.387	0.3712	0.3031	0.3236
N	774	531	539	504	421

rv = a2 + b2 * hv22; Number of Days to Maturity

	Whole data	5 - 25	10 - 30	15 - 35	20 - 40
a2	0.000095358	0.000092605	0.00011266	0.00010771	0.00010408
SE	0.00001469	0.00001914	0.00001444	0.00001395	0.0000151
t	6.49	4.84	7.8	7.72	6.89
b2	0.581318	0.589792	0.554459	0.549292	0.566243
SE	0.06797	0.08932	0.06507	0.06535	0.07219
t	8.55	6.6	8.52	8.41	7.84
R^2	0.1762	0.1567	0.1428	0.1935	0.2343
N	774	533	539	504	421

rv = a3 + b3 * hv60; Number of Days to Maturity

	Whole data	5 - 25	10 - 30	15 - 35	20 - 40
a3	0.00011548	0.00010309	0.00012884	0.00013349	0.00014113
SE	0.00001268	0.00001745	0.00001275	0.00001178	0.00001188
t	9.11	5.91	10.1	11.33	11.88
b3	0.492035	0.541513	0.48433	0.443543	0.417231
SE	0.06269	0.08867	0.06334	0.05361	0.05135
t	7.85	6.11	7.65	8.27	8.13
R^2	0.0792	0.077	0.0626	0.0769	0.0851
N	774	533	539	504	421

Encompassing Regression (1) - Daily RV

rv = a4 + b4 * iv + c4 * hv22; Number of Days to Maturity

	Whole data	5 - 25	10 - 30	15 - 35	20 - 40
a4	-0.00006521	-0.00009626	-0.00007759	-0.00001417	6.51979E-06
SE	0.00003491	0.00004273	0.00003929	0.00002798	0.00002304
t	-1.87	-2.25	-1.97	-0.51	0.28
b4	1.026624	1.112461	1.184867	0.808942	0.717486
SE	0.19593	0.21973	0.23168	0.17057	0.13943
t	5.24	5.06	5.11	4.74	5.15
c4	-0.054056	-0.08114	-0.200774	0.025704	0.089709
SE	0.1324	0.10975	0.12327	0.09734	0.09422
t	-0.52	-0.74	-1.63	0.26	0.95
R^2	0.3761	0.3886	0.3801	0.3033	0.3257
N	774	533	539	504	421

rv = a5 + b5 * iv + c5 * hv60; Number of Days to Maturity

	Whole data	5 - 25	10 - 30	15 - 35	20 - 40
a5	-0.00003191	-0.00005771	-0.00003648	4.50142E-06	0.000021174
SE	0.00002648	0.00003356	0.00002901	0.0000215	0.00001917
t	-1.21	-1.72	-1.26	0.21	1.1
b5	1.264102	1.32002	1.315011	1.051831	1.045199
SE	0.17919	0.20894	0.20151	0.16335	0.16116
t	7.05	6.32	6.53	6.44	6.49
c5	-0.450985	-0.471116	0.50404	-0.331103	-0.339825
SE	0.10428	0.12101	0.12238	0.09823	0.10247
t	-4.32	-3.89	-4.12	-3.37	-3.32
R^2	0.4109	0.4196	0.4109	0.3251	0.3511
N	774	533	539	504	421

Annualised Volatility Analysis

yrv = a1 + b1 * yiv;

	Whole data	5 - 25	10 - 30	15 - 35	20 - 40
		Number of Days to Maturity			
a1	0.01154	-0.00178315	0.0069089	0.025125	0.032861
SE	0.01488	0.01901	0.01777	0.01417	0.01418
t	0.78	-0.09	0.39	1.77	2.32
b1	0.787665	0.81383	0.80712	0.750481	0.743559
SE	0.06064	0.07713	0.07213	0.0589	0.05891
t	12.99	10.55	11.19	12.74	12.62
R^2	0.3684	0.3724	0.3687	0.3518	0.3703
N	774	533	539	504	421

yrv = a2 + b2 * yhv22;

	Whole data	5 - 25	10 - 30	15 - 35	20 - 40
		Number of Days to Maturity			
a2	0.088273	0.085014	0.093487	0.094563	0.093551
SE	0.0077979	0.0098621	0.009014	0.0091657	0.0099266
t	11.32	8.62	10.37	10.32	9.42
b2	0.568144	0.567325	0.561303	0.561114	0.574633
SE	0.03617	0.04665	0.0421	0.04245	0.04591
t	15.71	12.16	13.33	13.22	12.52
R^2	0.2616	0.2387	0.2374	0.2794	0.3111
N	774	533	539	504	421

yrv = a3 + b3 * yhv60;

	Whole data	5 - 25	10 - 30	15 - 35	20 - 40
		Number of Days to Maturity			
a3	0.105752	0.101207	0.110527	0.112276	0.114334
SE	0.008896	0.01175	0.01032	0.0098432	0.01019
t	11.89	8.61	10.71	11.41	11.22
b3	0.474973	0.451178	0.472424	0.469243	0.470579
SE	0.03926	0.05257	0.04578	0.04343	0.04476
t	12.1	9.15	10.32	10.8	10.51
R^2	0.1309	0.1195	0.1197	0.1384	0.1549
N	774	533	539	504	421

Encompassing Regression (2) - Annualised RV

yrv = a4 + b4 * yiv + c4 * yhv22;

	Whole data	5 - 25	10 - 30	15 - 35	20 - 40
		Number of Days to Maturity			
a4	0.012542	-0.00160091	0.00690902	0.028824	0.037059
SE	0.01529	0.0192	0.01858	0.01437	0.01364
t	0.82	-0.08	0.37	2.01	2.72
b4	0.698603	0.766371	0.807113	0.628892	0.575535
SE	0.10513	0.12301	0.13418	0.09556	0.0857
t	6.65	6.23	6.02	6.58	6.72
c4	0.096684	0.053679	7.7159E-06	0.122795	0.169862
SE	0.0639	0.07096	0.08268	0.06527	0.06584
t	1.51	0.76	0	1.88	2.58
R^2	0.3712	0.3733	0.3687	0.356	0.3786
N	774	533	539	504	421

yrv = a5 + b5 * yiv + c5 * yhv60;

	Whole data	5 - 25	10 - 30	15 - 35	20 - 40
		Number of Days to Maturity			
a5	0.025001	0.015517	0.022773	0.034501	0.041226
SE	0.01309	0.0168	0.01523	0.0132	0.01324
t	1.91	0.92	1.5	2.61	3.11
b5	1.003351	1.045495	1.047841	0.928598	0.925407
SE	0.09399	0.11242	0.11245	0.09839	0.10476
t	10.67	9.3	9.32	9.44	8.83
c5	-0.289549	-0.326725	-0.331245	-0.233852	-0.229964
SE	0.06047	0.07427	0.07477	0.06653	0.07201
t	-4.79	-4.4	-4.43	-3.52	-3.19
R^2	0.3894	0.3973	0.3948	0.3664	0.3852
N	774	533	539	504	421

Appendix 3.B GMM, generalised method of moments

This Appendix shows how we implement the GMM for our analysis[8].

$$RV_t = a + b \cdot IV_t + c \cdot HV22_t + d \cdot HV60 + e \cdot GARCH_t + \varepsilon_t \quad \text{(B1)}$$

Consider Equation B1 has the unobservable non-linear disturbance term of ε_t. Note that we do not explicitly assume any autoregressive error term. For simplicity, B1 can be re-written as B2.

$$\begin{aligned} \varepsilon_t &= q(RV_t, x_t, \theta) \\ z_t &= Z(x_t) \end{aligned} \quad \text{(B2)}$$

where x is a vector of explanatory variables, θ is a vector of the parameters, and z is a vector of instruments. The desired condition is $E(\varepsilon_t \otimes z_t) = 0$, that is, the expected crossproducts of the disturbance and functions of the observable variables are set to 0. The first moment of the crossproducts is:

$$\begin{aligned} m_n &= \frac{1}{n}\sum_{t=1}^{n} m(y_t, x_t, \theta) \\ m(y_t, x_t, \theta) &= q(y_t, x_t, \theta) \otimes z_t \end{aligned} \quad \text{(B3)}$$

where y is the dependent variable (RV), and n is number of observations. We estimate the parameters by minimising the objective function of:

$$S(\theta, V) = [nm_n(\theta)]' V^{-1} [nm_n(\theta)] \quad \text{(B4)}$$

where the variance of moment function is defined as:

$V = Cov([nm_n(\theta^0)], [nm_n(\theta^0)]')$, and θ^0 is the true parameter vector. The parameters obtained in minimising the objective function are the GMM estimators. Note that the objective function of the ordinary least square method is $r'r/n$, where r is the vector of residuals.

[8] GMM is calculated with the SAS/ETS® software and its specification.

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4

Implied Volatility Shapes:

The Nikkei 225 Case

Naoki Kamiyama

4. Implied Volatility Shapes: the Nikkei 225 Case

4.1. Introduction

It is becoming known to many option practitioners that implied volatility shape (smile or exercise price effect) is a signal of the market participants' view of the underlying asset return. One practical usage of the shape is to input the skewed volatility figure to evaluate an option into the Black-Scholes (B-S) formula. However, Black and Scholes (1973) assumes a constant volatility of an underlying asset return to formulate their pricing theory, therefore, inputting different volatilities to the B-S formula is not correct from the theoretical point of view.

These days, a tighter control on the derivatives products is desired by the managements of financial institutions world-wide and the financial regulators as well. Many banks and securities firms have introduced a risk management framework such as VaR (value-at-risk) originally implemented by Bankers Trust, and a table of parameters (ex. volatilities and correlations of asset prices) input for evaluation such as RiskMetrics suggested by J.P. Morgan. From the financial professional's point of view, the pitfalls of the B-S formula should be corrected to grasp the exact risks and unrealised profit/loss of the position held.

Many works have been written to explain the smile effect within a context of stochastic volatility. Hull and White (1987) introduced stochastic volatility into option pricing, and showed the smile effect as overpricing of Black-Scholes model. Taylor and Xu (1994a) showed that the smile effect is the logical consequence of stochastic volatility, and that the empirical regression results are robust against the selection of maturities. In their paper, they examine the foreign exchange options listed on the Philadelphia Stock Exchange, set up the model which assumes a symmetric shape of smile depending on moneyness, and find that the magnitude of the smile is decreasing as the time to maturity becomes longer. Bates (1994) developed an American option pricing method which take jump risk and volatility risk into account as non-diversifiable risks. He found that the stochastic volatility-jump-diffusion specification improved the pricing model's ability to fit actual option prices, especially in- and out-of-the-money short-term options which are assumed to have larger smile effects than long-term near-the-money options. Madan and Chang (1995) claim that the Black-Scholes option pricing framework is likely to overstate the value of a short call position, and understate the one of a short put position, compared with their pricing model using a variance-gamma process, which takes the skewness and kurtosis of asset returns into consideration and makes less errors than the Black-Scholes formula. They also conclude that a worst case definition by using the Black-Scholes formula may not be conservative enough.

Shimko (1991) used a polynomial fitting to estimate a shape of implied volatility by exercise price and apply the estimated shape for specifying a return distribution function without a priori parameters such as GARCH. Taylor and Xu (1994b) showed the theoretical background of the shapes of implied volatility when asset prices are correlated with volatility shocks, and made an empirical test on the S&P futures options. They relax the model of 1993, and allow the smile to be asymmetric[1]. Heynen (1994) showed the smile pattern on EOE Dutch index options. Even though Heynen's model is simple and easy to handle, two

[1] Practitioners often claim that one can observe symmetric smiles in the foreign exchange and commodity markets, and asymmetric ones in the equity markets. Taylor (1986) examines a variety of data series including stock prices, indices, foreign exchanges, and commodity prices both in cash and futures markets, and finds that most of the returns have nearly zero skewness and are approximately symmetric. On the other hand, he finds most of all the time series have higher kurtosis than 3.5, which is 3.0 if normally distributed.

consideration should have been added on. Firstly, implied volatilities should be normalised by the overall level of volatility, and secondly, time to maturity effect should be included in the model. Rubinstein (1994) shows an optimisation method to specify a return distribution consistent with a smile for his implied binomial tree.

The purpose of this paper is to examine existence of an exercise effect or 'smile' in the Nikkei 225 traded options market by using the modified Heynen, and Taylor and Xu methods, and observe the shape and characteristics, if any. Neither of these two models is a structural models, but they are mechanically fitted. If we find evidence from the models that the shape of implied volatility is smiling and/or skewed, it is reasonable to take the shape into consideration with the risk management framework and evaluation of positions held in banks. Figure 4.1 shows the average implied volatility relative to at-the-money options, for Nikkei 225 options contracts traded on the Osaka Security Exchange. The graph suggests that there is a rather symmetric smile in the options which are nearest to maturity (less than 10 days), but the smile becomes more skewed as maturity increases. As expected, it is the in-the-money options which have the highest volatilities. However, as maturity decreases, the shape becomes more like smile by increasing the implied volatility of out-of-the-money puts. We will estimate the shape and find the characteristics of the shape to avoid to over/underestimate of the risks and evaluation in the risk management framework built for a financial institution. We do not examine the stability of a "daily" volatility shape, as Dumas, Fleming, and Whaley (1995) have done for S&P 500 options on futures contracts, but try to find the overall characteristics of the shape.

In Section 2, the methods employed are explained. The Heynen's method is modified and named 'Normalised Heynen' method to take the time effect into consideration. The Taylor and Xu method is used as it is shown in Taylor and Xu (1994b). In Section 3, the data used for this analysis are explained, and the results are analysed in Section 4. Section 5 concludes this paper.

4.2. The Methods

4.2.1 Normalised Heynen Method

Heynen (1994) used the formula below to examine the Dutch index options implied volatility shape. Implied volatility with exercise price of X (σ_x) is determined by the function of relative moneyness (y) as;

$$\sigma_X = a_0 + a_1 (y - y_{min}) + a_2 (y - y_{min})^2 + \varepsilon \qquad (1a)$$

where $y = X / Se^{(r-d)t}$, S is underlying index price, and d is dividend yield[2].

However, this formulation is not neutral to an overall level change in level of volatility, or a shift of the smile curve. Because a rather large range (from 4% to 58%) of the implied volatility is observed in the period examined in this chapter, it is better to 'normalise' by using the same approach as the Taylor and Xu method employs as below.

$$\frac{\sigma_X}{\sigma_F} = a_0 + a_1 N + a_2 N^2 + \varepsilon \qquad (1b)$$

where σ_F is implied volatility of ATM forward, F is the forward price, i.e., $F = Se^{(r-d)t}$, and N is Moneyness defined as $\boldsymbol{N = X / F - 1}$. σ_F and σ_X are calculated by the generalised Black-Scholes formula, as explained in Section 3.

[2] Originally Heynen (1994) calculated y by using dividend amount rather than dividend yield, where $y = X / (S-D)e^{rt}$.

Heynen (1994) showed the results by subset of time to maturity. It is possible to consider the time to maturity as a factor of the regression where time has an inverse impact. The inverse interaction also avoids the parallel shift of the curve due to a different time length. This leads to:

$$\frac{\sigma_X}{\sigma_F} = a_0 + a_1 N + a_2 N^2 + a_3 \frac{N}{T} + a_4 \frac{N^2}{T} + \varepsilon \qquad (1c)$$

where T is time to maturity (in year). This formulation does not impose symmetry, nor does it require the minimum point to be at-the-money. It is also formulated to recognise the non-linear decreasing time effect on the shape if a_3 and/or a_4 is significant. That is, the curvature of the shape is expected to decrease as the time to maturity becomes longer. We also include puts in our analysis, whereas Heynen examines only calls.

4.2.2 Taylor and Xu Method

The Taylor and Xu formulation is more complicated. It includes the interaction between moneyness and square root of time, and between moneyness and the volatility level. The curvature of the shape is likely to depend on the square root of time from the view point of probability theory[3]. The regression equation is then:

$$\frac{\sigma_X}{\sigma_F} = a_0 + a_1 \frac{M}{\sqrt{T}} + a_2 \frac{M^2}{\sqrt{T}} + a_3 \frac{M}{T} + a_4 \frac{M^2}{T} + a_5 \frac{M}{\sqrt{T}\sigma_F} + a_6 \frac{M^2}{\sqrt{T}\sigma_F}$$
$$+ a_7 \frac{M}{T\sigma_F} + a_8 \frac{M^2}{T\sigma_F} + \varepsilon \qquad (2)$$

where σ_X is the implied volatility of the option with exercise price of X, σ_F is the implied volatility of the ATM options with a specific expiration, and M is Moneyness defined as **ln(*F*/*X*)**.

3 The equation provided by Taylor and Xu (1994b) is their plausible first guess to estimate some non-linear function of t.

This formulation does not impose symmetry or constrain the minimum volatility to be at-the-money. We include square root of time with inverse interaction to moneyness, so that a larger 'smile' is possible compared with a result of the adjusted 'normalised' Haynen method. From the view point of standard deviation, square root of time should be directly related to the shape of the curve.

We segregate the data into the groups by time to maturity and by type (calls and puts) to characterise the shape of the smile in depth, which is not mentioned by Taylor and Xu (1994b).

4.3. The Data

The raw data are Nikkei 225 option prices from the Osaka Exchange. There were 1,209 business days during the sample period (12-Jun-89 to 26-May-94). When put-call parity ($c + x \cdot e^{-rt} = p + S$) is significantly breached, the data are omitted. The condition of omitting is the ±150 point difference from the parity[4]. The mean of the errors in put-call parity[5] is -4.37 (ATM puts and calls) with SD of 127.09. Data with less than one SD are kept to analyse.

IV is calculated in the generalised Black-Scholes formula, which is the extension version with the dividend yield input of Black and Scholes (1973). IV of puts and calls are averaged. The daily 3-month CD rates are used as risk free interest rates. The dividend yield rate is assumed constant as 0.7%.

Either data series of calls or puts may be biased. Gemmill (1995) reports the 2% put bias, i.e. the mean of put IV is 2% larger than the one of call IV, in the

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[4] The example price level of the call option (24 days to maturity) is 380 point with 20,500 exercise price when the index equals 20,442.6 (nearest-the-money), where the delta is approximately 0.46 and IV is roughly 21% annually. (Closing level as of 19 March 96, April contracts expiring on 12 April 96.) The screening rule of 150 point to omit the parity-breaching data is rather large.

[5] An error in put-call parity (e) is defined as below:

$e = S + P - C - X \exp(-rt)$

case of the FTSE 100 option contracts. In the case of the Nikkei 225 options traded on Osaka. The mean of the call IV is 25.86% in the annual term, and the one of the put IV is 25.35%. The call IV is significantly 0.51% larger than the put IV[6].

There are 392 data-missing days. The remaining 817 days are available. The nearest-the-money contract is defined as at-the-money (ATM). In-the-money (ITM) is defined as ATM - 500 yen in calls and ATM + 500 yen in Puts. OTM is the exact converse. ATM, OTM, and ITM are combined within the option types (Call and Put), so that the number of data for regression analysis is tripled.

The contracts were quasi-American type (exerciseable every Friday) before May 92, and from June 92 contracts, the specification was changed into European. Regressions were initially conducted separately for the pre-June 1992 data (451 days) and the post-May 1992 data (366 days). Because the results were not different, the whole period is reported here (817 days).

Whole data set	2,451 data
Subset 1 = 5 - 10 days	489 data
Subset 2 = 11 - 20 days	663 data
Subset 3 = 21 - 30 days	945 data
Subset 4 = 31 - 40 days	354 data

4.4. Test Results: Goodness of fit

In Table 4.1, the results of the original (but already normalised) (Equation 1b) and the adjusted (time-considered) Heynen method (Equation 1c) are shown. The R squareds (0.02 to 0.10) are lower than the ones in the Taylor and Xu results

[6] H_0: The mean of call IV (x1) is equal to the one of put IV. H_A: The mean of call IV (x2) is not equal to the one of put IV. Reject H_0 if $z < -1.96$ or $z > 1.96$, where $z = (x1 - x2) / $ square root of $(var1 / n1 + var2 / n2)$. Because $z = 2.06$, the call IV is significantly biased to the put IV by 0.51%.

(0.07 to 0.27). The R squareds are also lower than Heynen's result for the Dutch equity index options, which are in the range of 0.30 to 0.70. The adjusted Heynen method improves the R squared compared with the original method, but the improvement is marginal. The estimated coefficients seem very sample-dependent. The instability is likely to be due to the multi-colinearity of those independent variables. The shape of the smile is plotted in Figure 4.2. The curvature of the shape is increasing as the time to maturity is decreasing in both calls and puts. The magnitude of the smile is more in puts than in calls in the sample period.

The result of the Taylor and Xu method is shown in Table 4.2. With the whole-data set, one can see the significant smile and skewness effects, which are shown graphically in Figure 4.3. The magnitude of the smile is more in puts than in calls, which is a similar result to the one of the normalised Haynen method. The R squared is higher, 0.07 to 0.27, than for Heynen's method. The R squared is much lower than the Taylor and Xu's result in the S&P options, which is in the range of 0.7 to 0.9. The subsets results are not stable. In the 11-20 days subset, most of all the t values are less than 2 in absolute term as they are also in the 21-30 days call subset. The reason for this instability is likely to be multi-collinearity of the formulation.

When we impose symmetry by taking out a_1, a_3, a_5, and a_7, R squareds are reduced from 0.1657 and 0.1975 to 0.1157 and 0.1699 of calls and puts, respectively.

4.4.1 *Smile effect*

The Taylor and Xu model shows larger effects in smile than the normalised Heynen model in Figures 4.2 and 4.3[7]. In comparing with actual plotting in Figure 4.1, the Taylor and Xu model seems to have too large curvature to estimate the real curve. It seems because the Taylor and Xu method has some more independent variables to make the curve more convex, so that the possible erroneous data around the deep-in- and -out-of-the-money positions may have a strong impact.

4.4.2 *Time effect*

The time effect is similar in both approaches. It is observed in both models that the shorter the maturity, the more the time effect in smile, as shown in Figures 4.2 and 4.3.

4.5. Conclusion

We find a smiling shape of implied volatility in the case of the Nikkei 225 index options listed on the Osaka Security Exchange, by the fitting models originally introduced by Heynen (1994) and Taylor and Xu (1994b). We confirm the decreasing smile effect with increasing time to maturity. Gemmill and Thomas (1995) suggest that the decreasing smile effect in time is due to a series of mean-reverting asset prices. Their reasoning seems appropriate similarly for the Nikkei options. The magnitude of the smile in puts is larger than the one of calls in the results of both models, but no specific reason is found. It may be due to the erroneous data because of lack of market makers on the Exchange: prices are sometimes left as they were on a few days before, if no trade

[7] Note that the definitions of moneyness, that is, M (Equation (1)) and N (Equation (2)) are conversed in Figures 4.2 and 4.3.

occurs[8]. In conclusion, we characterise the smile, and time effects, all of which are basically the same as the results of Heynen (1994) and Taylor and Xu (1994b).

As many practitioners claim regarding the most of equity markets of the world, the lower the exercise price, the larger the volatility in the case of the Nikkei option contract, as shown in Figure 4.1. We find the existence and characteristics of the shape, and it seems reasonable to use this additional information by fitting a curve to the shape, and then measure risks and evaluate unrealised profit/loss of the derivatives positions. In order to correct the over/underestimate of the value of customised options such as exotic options, we should implement the assessment of the implied volatility shapes. The straight B-S formula application to the position evaluation and risk measurement is rather erroneous.

[8] In an order driven market, we would have no good option price reflected by its current underlying price, because no ask-bid quotation would be available unless there was an order. In an market-making market, a market maker would quote its closing ask-bid quotation for official records.

(June 1989 to May 1994)

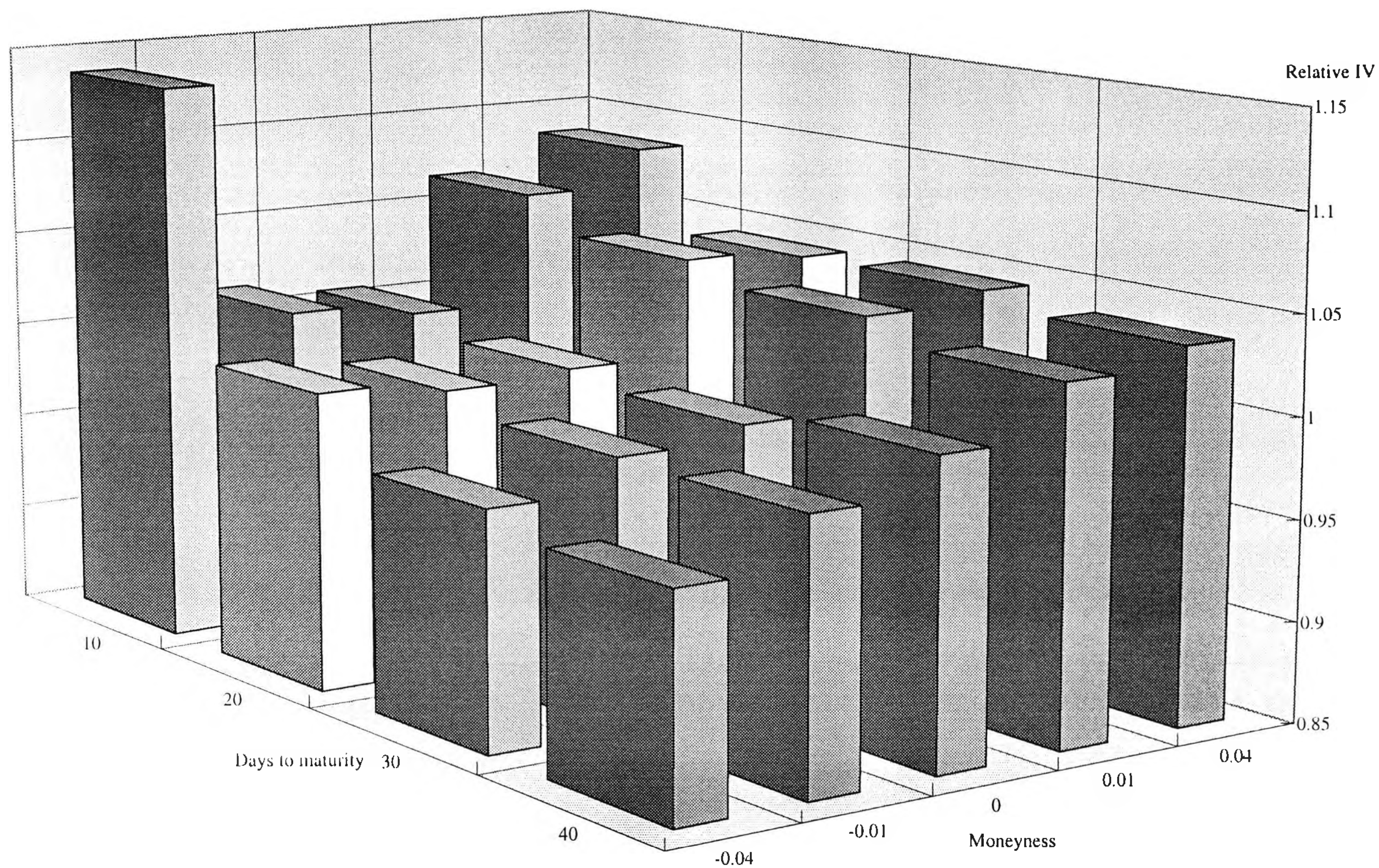
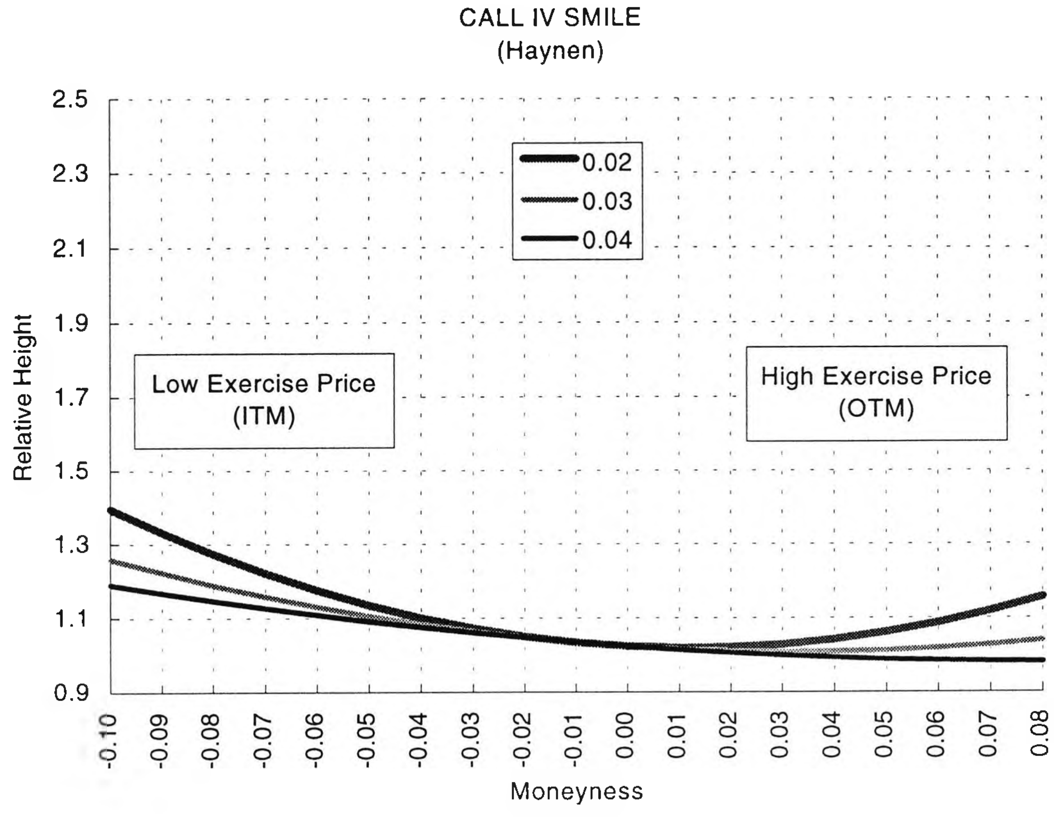


Figure 4.1. Implied Volatility, Maturity, and Moneyness (Nikkei options on Osaka Exchange)

Figure 4.2. Normalised Heynen Methods: Implied Volatility Shape (Whole-data set)

t = 0.02, 0.03, 0.04 (years to maturity of options)



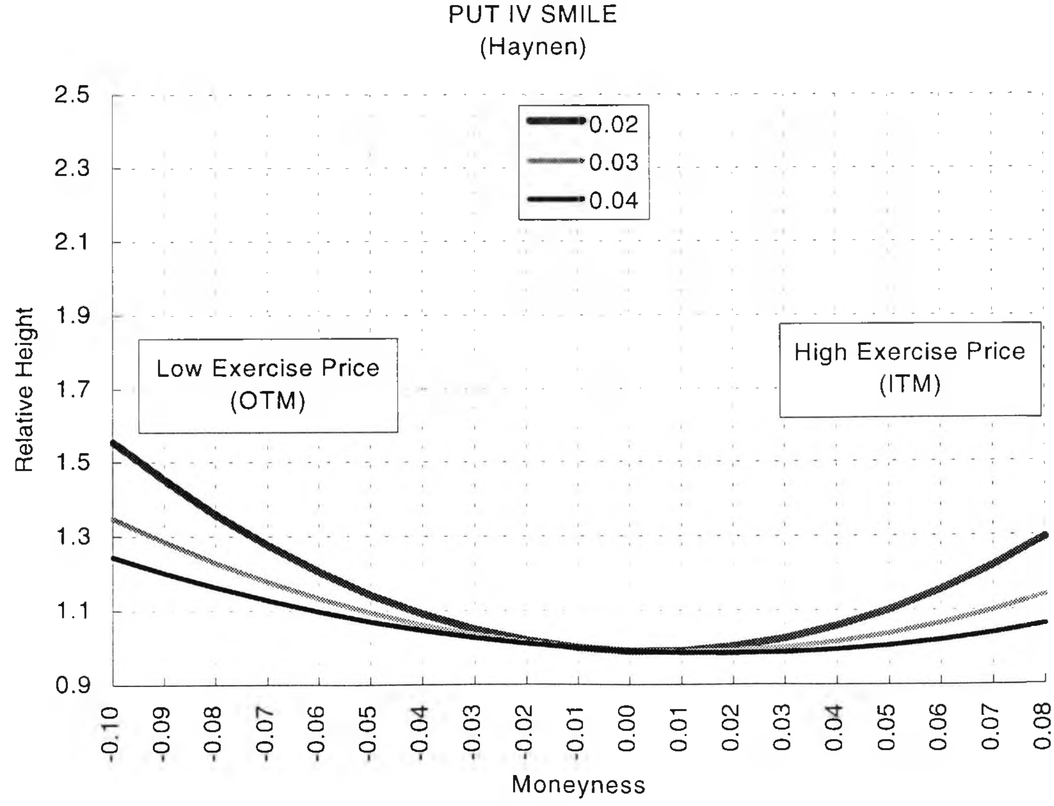
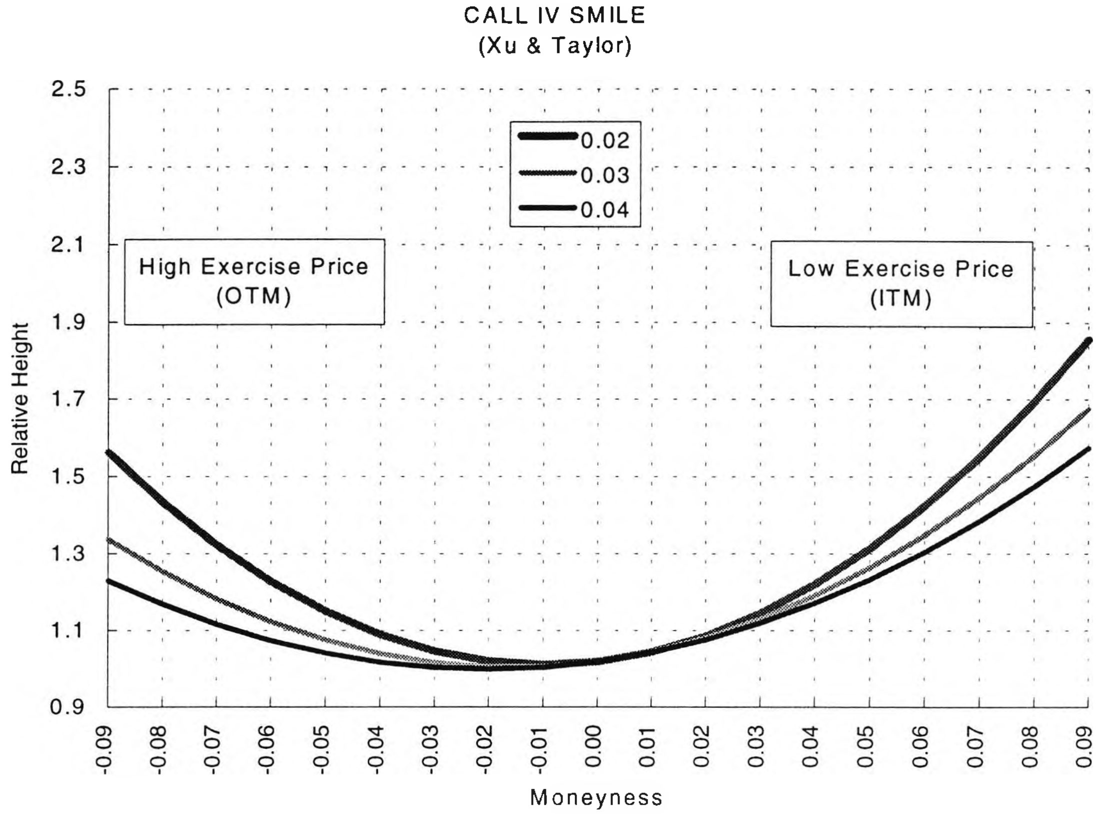


Figure 4.3. Taylor and Xu Methods: Implied Volatility Shape (Whole-data set)

t = 0.02, 0.03, 0.04 (years to maturity of options)



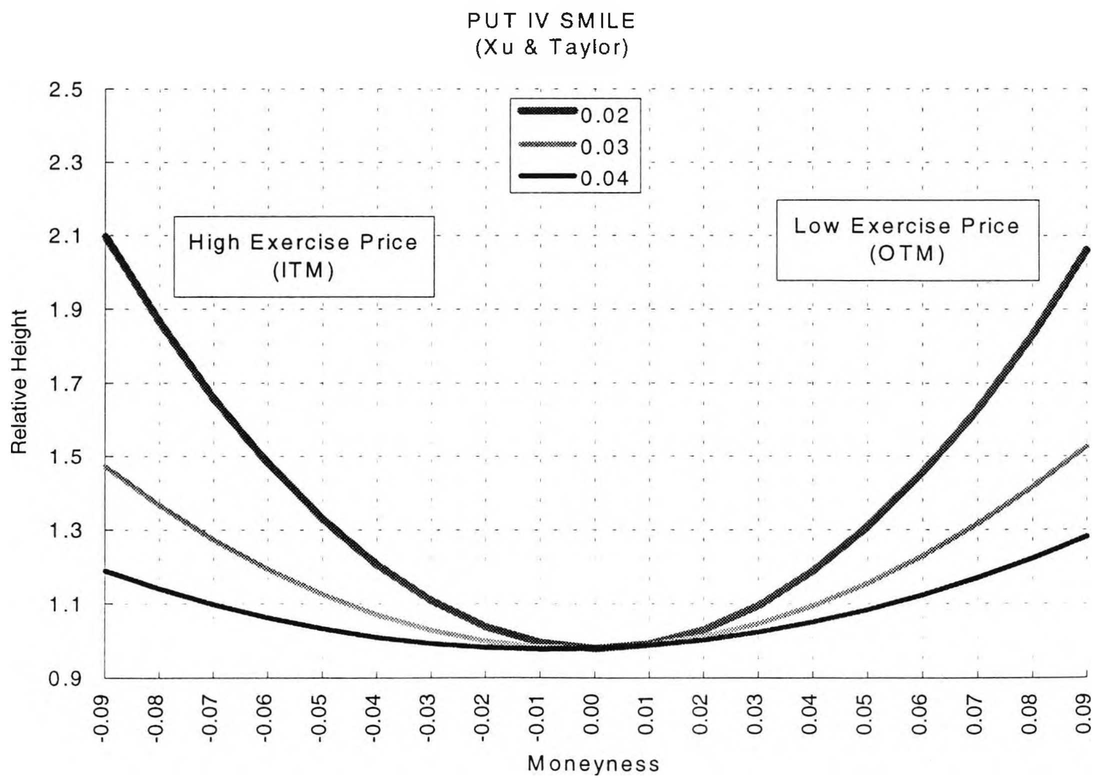


Table 4.1. Heynen Method: Regression Results

CALL

	Whole data	5-10 days	11-20 days	21-30 days	31-40 days
a_0	1.024506 (328.84)	1.022058 (95.24)	1.028059 (176.93)	1.023626 (274.75)	1.020296 (200.10)
a_1	-1.066292 (-10.56)	-0.663033 (-1.81)	-1.103193 (-5.95)	-1.217208 (-10.07)	-1.031258 (-6.51)
a_2	3.146005 (1.02)	29.740290 (2.59)	-0.956498 (-0.16)	-2.517235 (-0.72)	-1.738400 (-0.35)
R^2	0.0441	0.0218	0.0511	0.0975	0.1077

Adjusted

	Whole data	5-10 days	11-20 days	21-30 days	31-40 days
a_0	1.023999 (329.96)	1.022624 (95.20)	1.028249 (176.70)	1.023604 (273.85)	1.020390 (199.59)
a_1	-1.337454 (-6.49)	-0.611878 (-0.26)	-2.366123 (-1.95)	-0.490204 (-0.46)	0.200036 (0.11)
a_2	-17.249152 (-3.25)	-52.137554 (-0.81)	19.747093 (0.65)	13.771763 (0.51)	-2.477477 (-0.06)
a_3	0.012150 (1.51)	-0.00175013 (-0.03)	0.052226 (1.06)	-0.049600 (-0.69)	-0.113266 (-0.66)
a_4	0.943147 (4.74)	1.779647 (1.30)	-0.880722 (-0.71)	-1.102434 (-0.62)	0.065574 (0.02)
R^2	0.0534	0.0254	0.0529	0.0983	0.1088

PUT

	Whole data	5-10 days	11-20 days	21-30 days	31-40 days
a_0	0.990418 (286.73)	1.011423 (76.02)	0.990426 (176.03)	0.979631 (282.69)	0.985603 (172.02)
a_1	-0.663155 (-5.92)	-0.396060 (-0.87)	-0.713994 (-4.05)	-0.752414 (-6.69)	-0.653453 (-3.67)
a_2	14.600694 (4.26)	33.268483 (2.34)	17.085154 (2.98)	11.636497 (3.56)	2.720744 (0.49)
R^2	0.0216	0.0134	0.0361	0.0578	0.0380

Adjusted

	Whole data	5-10 days	11-20 days	21-30 days	31-40 days
a_0	0.989753 (288.44)	1.012256 (76.08)	0.990274 (178.65)	0.979500 (281.88)	0.985894 (174.04)
a_1	-0.871124 (-3.82)	-2.775619 (-0.95)	-0.00327009 (-0.00)	-2.041499 (-2.06)	4.267769 (2.05)
a_2	-14.447583 (-2.46)	-93.674166 (-1.18)	-0.269685 (-0.01)	2.570820 (0.10)	118.373666 (2.40)
a_3	0.00935679 (1.05)	0.051390 (0.81)	-0.029549 (-0.63)	0.087383 (1.31)	-0.452233 (-2.37)
a_4	1.341548 (6.09)	2.748847 (1.62)	0.731938 (0.62)	0.633003 (0.38)	-10.636298 (-2.37)
R^2	0.0365	0.0191	0.0370	0.0595	0.0658

$$\frac{\sigma_X}{\sigma_F} = a_0 + a_1 N + a_2 N^2 + a_3 \frac{N}{T} + a_4 \frac{N^2}{T} + \varepsilon$$

Table 4.2. Taylor and Xu Method: Regression Results

CALL

	Whole data	5-10 days	11-20 days	21-30 days	31-40 days
a_0	1.012961 (332.82)	1.005444 (99.49)	1.018621 (174.32)	1.012816 (271.72)	1.016133 (192.87)
a_1	-0.636587 (-2.83)	-0.163835 (-0.10)	1.821932 (1.46)	3.304304 (1.96)	0.736984 (0.20)
a_2	-14.120431 (-2.31)	-34.505471 (-0.75)	-9.759828 (-0.29)	-88.461911 (-1.77)	73.027677 (0.69)
a_3	0.140646 (3.58)	0.077177 (0.32)	-0.363833 (-1.47)	-0.911359 (-2.09)	-0.186221 (-0.16)
a_4	-0.675849 (-0.65)	2.344110 (0.35)	-1.589984 (-0.24)	18.054597 (1.41)	-25.854473 (-0.81)
a_5	0.265651 (5.15)	0.050742 (0.14)	-0.174970 (-0.61)	-0.922095 (-2.33)	-0.341498 (-0.40)
a_6	2.471363 (1.51)	7.798506 (0.65)	7.372611 (0.85)	20.361386 (1.53)	-21.794801 (-0.81)
a_7	-0.045341 (-5.09)	-0.015464 (-0.29)	0.045397 (0.80)	0.270223 (2.65)	0.118129 (0.45)
a_8	0.604737 (2.22)	-0.155298 (-0.09)	-0.442631 (-0.26)	-3.877498 (-1.14)	7.584678 (0.93)
R^2	0.1657	0.1903	0.1150	0.2057	0.1398

PUT

	Whole data	5-10 days	11-20 days	21-30 days	31-40 days
a_0	0.984261 (300.88)	0.985003 (83.57)	0.984847 (174.98)	0.982157 (267.73)	0.989209 (168.48)
a_1	-0.398272 (-1.65)	-1.838247 (-0.95)	-0.140207 (-0.12)	2.683898 (1.62)	1.548673 (0.37)
a_2	48.465119 (7.39)	-26.290475 (-0.49)	72.435452 (2.27)	-108.309719 (-2.20)	-36.211018 (-0.31)
a_3	0.134433 (3.18)	0.341881 (1.21)	0.066299 (0.28)	-0.663823 (-1.55)	-0.384714 (-0.30)
a_4	-11.238766 (-10.07)	-0.493661 (-0.06)	-16.060243 (-2.53)	29.976169 (2.38)	14.326488 (0.40)
a_5	0.167789 (3.02)	0.589377 (1.36)	0.096759 (0.35)	-0.441518 (-1.13)	-0.945019 (-0.99)
a_6	-14.547056 (-8.28)	3.808385 (0.27)	-20.169724 (-2.40)	31.421651 (2.41)	30.922318 (1.03)
a_7	-0.039491 (-4.13)	-0.099744 (-1.58)	-0.022370 (-0.41)	0.118891 (1.18)	0.279945 (0.96)
a_8	3.573128 (12.21)	0.938234 (0.46)	4.714554 (2.84)	-8.511106 (-2.54)	-10.223053 (-1.12)
R^2	0.1975	0.2773	0.0800	0.0716	0.0878

$$\frac{\sigma_X}{\sigma_F} = a_0 + a_1 \frac{M}{\sqrt{T}} + a_2 \frac{M^2}{\sqrt{T}} + a_3 \frac{M}{T} + a_4 \frac{M^2}{T} + a_5 \frac{M}{\sqrt{T}\sigma_F} + a_6 \frac{M^2}{\sqrt{T}\sigma_F} + a_7 \frac{M}{T\sigma_F} + a_8 \frac{M^2}{T\sigma_F} + \varepsilon$$

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5

Stock Return and Volatility Transmission:

The Nikkei 225 and Other Major Markets

Naoki Kamiyama

5. Stock Return and Volatility Transmission: The Nikkei 225 and other major markets

5.1. Introduction

Volatility forecasting is one of the most important topics for option users, traders, and other market participants in the international derivatives markets. This paper intends to provide an international perspective to forecasting short-run volatility by examining volatility transmission ("volatility spillover") while simultaneously considering return transmission ("contagion[1]" or "return spillover"), using the Nikkei 225, FTSE 100, and S&P 500 indices.

There are several papers examining return spillovers among the stock markets. King and Wadhwani (1990) focus on the return transmission from one market to another, concentrating on the crash of October 1987. They conclude that rational agents use price changes of the most recently traded market as material information, and this causes a contagion effect between the international equity markets. Becker, Finnerty, and Gupta (1990) tested the relation between the Japanese and the U.S. equity markets and found high correlation between the Open-to-Close return of the U.S. market and Japanese market returns, but a relatively small impact of the Japanese market on the U.S. market. Their analysis includes a consideration of local and common currencies in returns. Becker, Finnerty, and Tucker (1993) used the stock index futures prices of Japan, U.K., and U.S. to find that the "US performance has a large impact on the overnight returns in Japan and UK," and concluded that the US market is the dominant market of the world. Although their model only analysed return spillovers, they also performed a daily variance comparison and concluded that the US was also dominant from the volatility perspective. They correctly pointed out that some degree of risk to underestimate the Close-Open returns

[1] "Contagion" is explained in King and Wadhwani (1990) as an effect which occurs between markets "as a result of attempts by rational agents to infer information from price changes in other markets." This provides a channel through which a "mistake" in one market can be transmitted to other markets."

exists in using the opening price data of the cash index, because the opening price of the cash index often includes last traded prices (i.e. last closing) of several constituent shares. The alternative is to use futures price data, but that has its own problem of roll-overs.[2] Cheung and Kwan (1992) provided an interesting comparison of the Canadian market volatility when the US market is closed with the volatility when it is open. They conclude that the trading of the US market affects the Canadian market volatility level through information transmission. The contagion idea is also applicable to the intra- and inter-industry share price linkage: for example, Alli, Thapa, and Yung (1994) examined the US equity market in its way.

Since the ARCH paradigm was developed in the seminal work of Engle (1982) and its generalisation (GARCH) in Bollerslev (1986), much research has been performed with their methods to examine volatility in the financial markets. Bollerslev, Chou, and Kroner (1992) and Engle (1993) provide concise reviews of the ARCH family of models and their application to asset price movements. The GARCH model is a powerful tool to explain the fat-tailedness which prevails for most asset returns[3].

The ARCH paradigm is introduced to examine volatility spillovers among the markets in the other papers. The GARCH concept and the world market spillovers in volatility are combined in Engle, Ito, and Lin (1990), who compare "Heat Wave" and "Meteor Shower" Hypotheses to examine the GARCH effects in the foreign exchange markets. They reject the hypothesis of the independence of the major market places of the world ("Heat Wave") and prefer the world-wide linkage model ("Meteor Shower") as the best representation. The well-presented

[2] Becker, Finnerty, and Tucker (1993) rolled over the futures at every expiry. From the Nikkei 225 experience, early roll-over often occurs in the Osaka Securities Exchange and Singapore International Monetary Exchange (SIMEX) several days prior to the expiry. The roll-over day is normally defined as the day on which the trading volume of the next contract month exceeds the volume of the current contract month. When the contract is rolled over, the trading volume of the nearest contract month decreases, and the opening price of the contract tends to jump from the last closing price. This may cause an overestimate of the price change overnight.

[3] See, for example, Taylor (1986).

econometric background including the log-likelihood function specification is useful and this methodology is followed in our paper. Hamao, Masulis, and Ng (1990) use the ARCH type of model to analyse the volatility spillover between the three major equity markets of the world, with data of April 1985 to Mar 1988. Their specification of the model is slightly different from the one of Engle, Ito, and Lin. In Hamao, Masulis, and Ng, the conditional variance is a linear function of past squared errors of the market, with autoregression of the conditional variance, and the squared residuals of the other market which are derived from an MA(1)-GARCH(1,1)-M applied solely to the return series of the foreign market previously opened. Even though their GARCH formulation is not as comprehensive as the one of Engle, Ito, and Lin (1990) in terms of interdependent determination of volatility, their careful consideration of the decomposition of Close-to-Close return into Close-Open and Open-Close returns is very valuable in delineating the effect of time zone difference for each market. Susmel and Engle (1994) examine the hourly linkage[4] in return and volatility and then show the spillover effects between the UK and US equity markets with consideration of timing of information transmission. In addition to using hourly data, they model asymmetry ('leverage effect') of the conditional variance as affected by the most recent market returns. Their detailed analysis succeeds in showing the precise timing and direction of the spillovers between two markets. A significant spillover is observed at the afternoon session of the London market, which tends to overreact to the opening of the New York market. They also find that both markets are efficient in using past information from the other market in both return and variance. However, they conclude that there is no volatility spillover between these markets. An hourly analysis for Japan is irrelevant because trading hours do not overlap with London or New York. Theodossiou and Lee (1993) expand the analysis of Hamao, Masulis, and Ng by using a multivariate GARCH-M model which enables them to observe any possible

[4] Exactly speaking, they do not use hourly returns but hourly prices to set up several time segments. Therefore the number of observation is 553 from January 1987 to February 1989.

interactions in the mean and volatility of returns among several markets. They examine Canadian and German markets in addition to the major three markets. The results for the mean spillover are not significant from Japan to U.S. and U.K., but the volatility spillover from U.S. to Japan is significant, (the coefficient is 0.0470). Koutmos, Lee, and Theodossiou (1994) investigate ten major equity markets of the industrialised countries and find time-varying betas in relation to the world index and volatility persistence in a GARCH formulation.

Multi-factor ARCH models are suggested in King, Sentana, and Wadhwani (1994) regarding the international major equity markets and Mahien and Schotman (1994) in the foreign exchange markets. Their models contain not only price information (historical returns) but also macroeconomic data and/or other financial market data such as interest rates. Chung and Liu (1994) examine the Pacific rim equity markets to find the common stochastic trends in the long run, using co-integration analysis and factor loading matrices in the analysis of multivariate stock return. However, because of the limited availability of macroecomonic data such as GNP., multi-factor analysis can not be directly applied to daily market linkage and spillovers.

The purpose of this chapter is to examine the linkage of the three major equity markets of the world in order to help market participants forecast the volatility and return of the day from the view point of spillovers, with special interest in the Japanese market. For example, on a Japanese holiday, the S&P index might be declining steeply. How should a Nikkei trader respond? She would also like to know how much bid-ask spread should be increased when there is a volatility increase in the New York market. This chapter will provide a quantitative solution for traders to support their decision making in trading the Nikkei and other major markets.

The plan of this paper is as follows; four alternative types of model are introduced in section 2, including: Return regression, Contagion, GARCH, and General models. The data used for the analysis are specified and the basic autocorrelations are analysed in Section 3. In Section 4, the test results of these four types of model are examined, and the conclusions are drawn together in Section 5.

5.2. The Methods

Four different models are used to relate the stock markets. In the first, the return regression model, returns in one market are related to returns in the most recently closed other market and also related to this market's volatility. In the second, the contagion model of King and Wadhwani (1990), the independent variables are just returns, but at higher lags. In the third, GARCH model, spillovers of the other market volatility are shown. Finally, in the general model, return spillovers are combined with GARCH volatility spillovers.

5.2.1 Return Regression Model

This model explains returns of the markets by 1) autoregression, 2) the other market return, and 3) volatility of the other market. The equation is:

$$R_{j,t} = a_j + b_j \cdot R_{j,t-1} + c_j \cdot R_{j-1,t} + d_j \cdot V_{j-1,t} + e_j \cdot dummy \qquad (1)$$

where,

$R_{j,t}$: Rate of return - defined as:

ln ($Close_t$ / $Close_{t-1}$) in Close-Close Return analysis

ln ($Close_t$ / $Open_t$) in Open-Close Return analysis

$V_{j-1,t}$: Daily volatility of the Other Market - defined as ($R_{j-1,t}$ − **mean of** $R_{j-1,t}$)^2

dummy: 1 if after weekends or missing dates, or 0 if not.

j: the Market (*ft, nk,* or *sp*)

$j-1$: the Other Market (the market most recently closed before the Market)

Equation (1) is more general than the return regression analysis employed by Becker, Finnerty, and Tucker (1993), because it includes the effects of the autoregression of the market return and the daily volatility (defined as a squared excess return of the day) of the other market, as well as the returns of the other market. We also add the dummy variable for non-trading days (weekends). Note that this model examines only return-to-return and volatility-to-return transmission, therefore volatility-to-volatility transmission is omitted in this analysis.

Because of autocorrelation (see below), one can expect slightly positive coefficients for the lagged returns of the market (b_i). We also expect positive coefficients for the other market returns (c_i), and the negative coefficients (due to risk) for the other market volatility (d_i).

5.2.2 Contagion Model

King and Wadhwani (1990) examined the close-close returns with a moving average process as shown below.

$$R_{j,t} = \beta_j \cdot R_{j-1,t} + (1-\theta_j L)\cdot \varepsilon_{j,t} \qquad (2a)$$

where L = lag operator. The contagion coefficients, β_js, measure the effects of the price change on change in the other market. The MA(1) error process acts as the total news items of the market. We make two adjustments to the model, by adding: (i) a weekend dummy, and (ii) a mean return (drift). The full model with both dummy and intercept is:

$$R_{j,t} = \mu_j + \delta_j \cdot dummy + \beta_j \cdot R_{j-1,t} + (1-\theta_j L)\cdot \varepsilon_{j,t} \qquad (2b)$$

In this model, we do not have to add any coefficient for volatility-to-return spillover because the MA coefficients are products of the contagion coefficients[5]. The model focuses on the return-to-return and volatility-to-return spillovers only and no volatility-to-volatility spillover is considered.

Positive signs are expected of the coefficients of returns of the other markets (β_i), as found by King and Wadhwani (1990). The coefficients (θ_j) are expected to be negative because the other market's effect is likely to be reduced by weekends.

[5] See *Appendix 5.1.*

5.2.3 GARCH Model with Volatility Spillovers

The GARCH (1,1) model can be shown as below:

$$
\begin{aligned}
y_{j,t} &= \alpha_j + v_{j,t} + \delta_j \cdot dummy \\
h_{j,t} &= a_j + b_j \cdot h_{j,t-1} + c_j \cdot v^2{}_{j,t-1} + d_j \cdot v^2_{j-1,t} + e_j \cdot dummy
\end{aligned} \tag{3}
$$

To optimise the parameters of Equation (3), the BHHH method[6] is used to maximise the log likelihood function of;

$$
\log L_j = \sum_{t=1}^{N} \left[-0.5 \times (\log h_{j,t} + \frac{v^2_{j,t}}{h_{j,t}}) \right] \tag{4}
$$

Where, j is nk, ft, or sp

and $j+1$ is the other market in the next time zone

The formulation of Equation (3) is the same as the one used by Engle, Ito, and Lin (1990) excluding weekend consideration, but slightly different from the specification of Hamao, Masulis, and Ng (1990). The latter use $h_{j-1,t}$ from the single-market GARCH result rather than $v^2_{j-1,t}$ in Equation (3). The models described as Equation (3) explain volatility-to-volatility spillover only.

In the model shown as Equation (3), one can expect b_i, c_i, and d_i to be positive, because the autoregression of the conditional volatility (of the market) and the squared errors (of the market and of the other market) should all be positively related to the conditional volatility of the market (from a practitioner's viewpoint). The coefficients of the weekend dummies (δ_j) in the return equation are expected to be negative.

5.2.4 General Model (GARCH with Return Spillovers)

The general model is a combination of the return contagion model and the GARCH volatility spillover model. Returns from the most recently closed market affect this market's returns, and the residuals of the recently closed market affect the conditional volatility of this market. Hence, we can combine

[6] The Berndt-Hall-Hall-Hausman method is introduced in Berndt, Hall, Hall, and Hausman (1974).

simultaneously the effects of return and volatility in the most recently closed market (volatility-to-volatility).

$$y_{j,t} = \alpha_j + v_{j,t} + \delta_j \cdot dummy + \beta_j \cdot y_{j-1}$$
$$h_{j,t} = a_j + b_j \cdot h_{j,t-1} + c_j \cdot v^2{}_{j,t-1} + d_j \cdot v^2_{j-1,t} + e_j \cdot dummy \qquad (5)$$

In the model shown as Equation (5), b_i, c_i, and d_i are expected to be positive because of the reason as above in 5.2.3. One can also expect the coefficient of return (β_i) to be positive because of the reason as above in 5.2.1 and 5.2.2.

5.3. The Data

5.3.1 The Data

The Nikkei and the S&P 500 indices data series are from the FutureSource data base. Both are from 2 January 1987 to 17 August 1994. The Nikkei has 1,880 daily data[7], and the S&P has 1,928 data (Open and Close). The FTSE 100 index data series are taken from DataStream, which contains the prices from 2 January 1987 to 19 August 1994. It has 1,992 data (Open and Close). Several Open price data are missing. These three data sets are combined by using the following rule for exclusion:

- If any of the three markets is closed, all the data on that day for all markets, are omitted.
- The rates of return are calculated after omitting the missing days.

The dummy variable is defined as follows; it is 0 if the date is continuous; and it is 1 if there are weekends, and holiday or missing data.

5.3.2 Preliminary Analysis of the Data

Table 5.1 summarises raw returns and autocorrelations up to Lag 10 for both close-to-close and open-to-close data. Several of the correlations are significant at Lags 1, 2, and 4. The S&P is significant at Lag 5, which is

[7] The number of trading days in Japan is smallest among three. The trading days per year are 245 in Japan, 250 in US., and 253 in UK., approximately.

consistent with the weekend effect as Hamao, Masulis, and Ng (1990) explain. The autocorrelations of the Open-Close and Close-Close data are similar except for the FT at Lag 1: it shows positive autocorrelation with Close-Close data, but negative autocorrelation for Open-Close data. The reason may be some degree of high frequency of jumps between Close-to-Open prices, thus the Open-Close return would be negative even though the Close-Close return was positive. Hamao, Masulis, and Ng (1990) report the same result in the autocorrelation table for the pre-crash period.

Table 5.2 shows the ARCH effect of each time series. The significant Q and LM statistics indicate that ARCH effects are present[8].

5.4. Test Results

(Throughout this paper, 'A ← B' means "Market A is affected by Market B")

5.4.1 Return Regression Model

Table 5.3 gives the results from estimating the return regression model (Equation (1)) using Close-to-Close data. We find that: a) As expected, returns are all positively related to the "other" market. The largest coefficient is in the case of SP←FT, 0.47, indicating a 1% change in FTSE is associated with 0.47% change in S&P. b) Returns are negatively influenced by the volatility of the other market (*d* coefficient), and again the largest absolute value of the coefficient is in the case of SP←FT, -2.12. c) Only S&P shows significantly negative autocorrelations (*b* coefficient), although one would expect a positive autocorrelation because the preliminary analysis showed positive autocorrelation at Lag 1 (Close-Close case). d) The weekend dummy is significant only in the NK←lag(SP) case, as would be expected, because Friday's S&P close is reflected in the U.K. on Monday morning.

The analysis of Open-Close Returns *(Table 5.4)* shows similar results with the return spillovers (*c* coefficient) smaller for FT←NK and NK←lag(SP), but

[8] *Appendix 5.2* shows the results of the GARCH model fitted with the individual market returns.

not for SP←FT. The other peculiarity is the much larger impact of volatility (*d* coefficient) in the SP←FT case. In addition, R squareds are lower than for the Close-Close analysis (in the range of 0.02 to 0.16). The FT←NK combination has a particularly low R squared, 0.0192 compared with 0.14 in the Close-Close case result, and this is consistent with the market participants' recognition of only minor linkage between these markets.

5.4.2 Contagion Model of King and Wadhwani

In the model specification of this paper, the non-overlapping markets cases are examined only in Close-Close returns[9] for comparison. Four results are observed in *Table 5.5.* a) All the coefficients of the other market returns are significantly positive and in the range of 0.11 to 0.44. The largest is the NK← lag(SP) case. b) The moving average factor is negative only in the FT←NK case, and positive in others. c) The weekend dummies are significant in the NK← lag(FT) and NK←lag(SP) cases, and d) the mean is not significantly different from zero. The standard errors and AICs[10] have no strong direction in comparing the model with/without the mean. Therefore, the simplest King and Wadhwani model (Equation (2a)) is preferred to the model with the mean on the basis of parsimony.

These results are rather different from those of the return regression model. The most important spillover is from lag(SP) to NK (0.44), whereas it was only 0.27 in the return regression model. This is probably because the contagion model does not explicitly separate the return effect and the volatility effect of the recently closed market. An interesting observation, in comparison with the

[9] King and Wadhwani (1990) specify the model for examining non-overlapping trading hours with the Close-Close returns. They used a different type of model for the overlapping market relationships. Because we focus on the Nikkei, only the non-overlapping model is examined in this paper.
[10] R squared is not utilised because it would be increased by increasing the order. See Harvey (1993). AIC stands for Akaike's Information Criterion. which is defined as -2 ln(maximum likelihood + 2 (number of parameters) to penalise the increasing number of parameters. See an econometrics text book, ex. Judge, Griffiths, Hill, Lütkepohl, and Lee (1984) for detail.

results of King and Wadhwani (1990), is that the effect of Nikkei on FT (FT← NK: 0.24) (for January 87 to August 94) is as large as the one of the crash period (0.26) found by them (for July 87 to February 88). The effect of lag(SP) on Nikkei (0.45) is much larger than the one of the crash period (0.16). The NK← lag(FT) case has a β (return-to-return spillover) of 0.22 that is larger than the one of the just-before-crash of 0.19. From these differences, one may be able to conclude that linkages among these markets became stronger in the testing period of 1987 to 1994, although we need to be aware that the sample period is different in this research from that of King and Wadhwani (1990).

5.4.3 GARCH Model with Volatility Spillovers

In Close-Close returns *(Table 5.6)*, one can observe three points. a) There are significant spillover effects in volatility for the cases of NK←lag(SP), FT← NK, and NK←lag(FT) but not for the SP←NK case. These results are consistent with the ones in Hamao, Masulis, and Ng (1990). Of these, the FT← NK spillover is largest (0.14). The NK←lag(SP) case has only 0.02 spillover coefficient. b) The weekend dummies significantly affect all equations, except the SP←NK return equation. Although the SP←NK case should not have had any weekend effect by definition, the volatility dummy has a small but significant effect because the data are Close-Close returns.

Using Open-Close returns *(Table 5.7)*, the results are familiar. a) The spillover effect in volatility in the cases of the NK←lag(SP), FT←NK, and NK← lag(FT), but the SP←NK case has no significant spillover coefficient. The largest coefficient is 0.104 in the NK←lag(FT). It is expected that a large coefficient in the NK←lag(SP) should exist, but the figure is 0.0367. b) The weekend dummy significantly affects all volatilities, but is not significant for most of the returns (the FT←NK case is only an exception). This suggests that the weekend news in weekend New York may affect the NK volatility after the weekend. However there should be no weekend effect for SP←NK and FT←

NK, because no weekend exists between the markets. The weekend effect even in the continuous market such as the FT←NK in Open-Close prices means that the weekend effect is an own-market effect in the continuous market cases. The weekend dummies of SP←NK (1.619e-5) and FT←NK (1.238e-5) are smaller than NK←lag(SP) (1.693e-5) and NK←lag(FT) (2.309e-5). This seems to imply that the weekend effect is largest for the Nikkei because that is the first market to reopen after the weekend.

This model is not comparable with the two models described so far: it is not formulated to show any effect of the recently traded market's return on the market return. Therefore, it is not comparable with the contagion model which formulates only return spillovers from the most recently traded markets. Also, this model does not include any effect of the recently traded market's volatility on the market return like the return regression model. The GARCH model formulates only the effect of volatility spillovers on the conditional volatility of the market.

5.4.4 General Model (GARCH with Return Spillovers)

There are four results of interest which can be found in *Table 5.8*. a) Compared with the GARCH spillover model in *Table 5.7* (The Open-Close case), the function values (log L) are improved to some extent. b) The return spillover is observed in all the cases to be significant in the Close-Close and Open-Close cases. All the return coefficients are positive from 0.145 (SP←NK) to 0.261 (NK←lag(SP)). c) The volatility spillover is observed significantly except for the SP←NK case. All the volatility coefficients are positive from 0.0162 (NK←lag(SP)) to 0.128 (FT←NK). d) The weekend effect on volatility transmission is observed significantly in all the cases without exception. All the coefficients are positive from 0.0000177 (NK←lag(SP)) to 0.00003228 (FT←NK). The effects on return spillover are not significant except for the FT←NK case (-0.002345).

Comparing the results of the Open-Close and Close-Open cases: i) there are higher coefficients in the Open-Close case than in the Close-Open Case except for the FT←NK case in the return spillovers. This means that the market participants materialise the news at the recently closed market not at the opening, but during trading time of the market. ii) Volatility spillovers in the Close-Open case are not significant except for the NK←lag(FT) case, which means that volatility transmission is also realised during the trading hours.

5.5. Conclusion

We employed four different types of model to analyse return and volatility spillovers between the three major equity markets of the world. The return of the previously closed market affects the return of the next opening market in all the results of the models. The volatility of the previous market affects negatively the next market's returns and positively its conditional volatility.

Using the general model with Close-Close data, which we prefer, there are large and significant return spillovers from the S&P to the Nikkei return, approximately 26%, and from the FTSE to the Nikkei return, which is approximately 17%. The variance spillovers are significant but small from the FTSE to the Nikkei variance, approximately 6.5%, and from the S&P variance to the Nikkei, approximately 1.6%. The volatility spillover from the Nikkei to S&P is not significant, but to FT, it is significant and the coefficient is approximately 13%.

Table 5.9 gives a summary comparison of our result, with those by other researchers. The return spillovers are observed more than ones of King and Wadhwani (K&W). Just after the Crash in 1987, the interrelationship of the markets seemed confused (ex. 0.11 from Lag SP to NK, from Dec 87 to Feb 88 in K&W), but it increases later in all four market relationships (ex. 0.261 from Lag SP to NK, from Jan 87 to Aug 94 in the general model). Only the general model can decompose the return and volatility spillovers from Close to Close into Close-

Open and Open-Close. Decomposition of Close-Close to Open-Close shows us that both return and volatility spillovers are mainly observed during the trading hours (Open-Close). Only the return from Nikkei to FTSE is transmitted at the opening of FTSE. The return and volatility spillovers are simultaneously modeled only in the general model and Theodossiou and Lee (T&L) model, but the T&L model does not find any significant return spillover with their weekly data, although we often experience the daily spillover between the markets. Hamao, Masulis, and Ng (HM&N) find the significant volatility spillover from NK to SP in the period of 1985 to 1988, but the general model does not in the period of 1987 to 1994. It seems because SP was somehow affected by NK before the Crash in 1987, but the effect from NK to SP has become smaller after the event.

The implication for traders of the Nikkei 225 is to take account of the returns and volatilities of the FTSE 100 and S&P 500 indices to forecast the volatility of the Nikkei 225 on the next day. There are strong positive relationships for both return-to-return and volatility-to-volatility. Traders could utilise the quantitative results of spillovers in pricing of options.

Table 5.3 shows the return spillover from FT to SP is the largest in coefficient and R squared. However, the trading time is overlapped (UK afternoon and US morning), and it should be noted that the relationship shown here between FT and SP is not causality from FT to SP but correlation to each other. The coefficient from NK to FT (0.23) includes the effect of the American news to both NK and FT, therefore the coefficient figure does not directly mean any causality from NK to FT. The coefficient from NK to SP is rather small (0.12), that is, NK does not dominant the SP market. Although the U.K. and Japanese markets have a strong interrelationship, this is probably explained by the similar relationship between the U.S. and each market. For example, the US dollar rates are often determined by the relative strength or weakness of the U.S.

economy. Economic news in the U.S. may affect each market independently, and the U.K and Japanese markets tend to move in a similar direction as a result.

Table 5.1. Data Summary: Mean and Correlations Structure of Returns

Means and Standard Deviations

Close-Close Returns

	#obs.	Mean	S.D.
Nikkei	1831	0.000055234	0.0148951
FTSE	1831	0.000350206	0.0103385
S&P	1831	0.000334364	0.0111833

Open-Close Returns

	#obs.	Mean	S.D.
Nikkei	1831	-0.000382878	0.0137571
FTSE	1819	0.000205451	0.0078636
S&P	1832	0.000375642	0.0110009

Autocorrelations

Close-Close returns

	Lag 1	2	3	4	5	6	7	8	9	10
Nikkei										
S.Cor	0.02125	-0.10961**	0.00443	0.04801*	-0.02516	-0.04885*	0.03493	0.05535**	0.00491	-0.00019
(S.E.)	-0.0233	-0.0234	-0.0234	-0.0234	-0.0234	-0.0234	-0.0234	-0.0234	-0.0234	-0.0234
P.Cor	0.02125	-0.11011**	0.00948	0.03606	-0.02605	-0.03914*	0.03164	0.04376*	0.01207	0.01241
FTSE										
S.Cor	0.0608**	0.00389	0.03535	0.0939**	0.02844	-0.0053	-0.0321	0.02002	0.01977	0.02015
(S.E.)	-0.0234	-0.0235	-0.0235	-0.0235	-0.0235	-0.0237	-0.0237	-0.0237	-0.0237	-0.0238
P.Cor	0.0608**	0.0002	0.03523	0.09006**	0.01773	-0.00928	0.02733	0.00696	0.01473	0.01761
S&P										
S.Cor	0.019	-0.07925**	-0.02463	-0.06486**	0.08916**	-0.0193	-0.00854	-0.00125	-0.01749	-0.02764
(S.E.)	-0.0234	-0.0234	-0.0235	-0.0235	-0.0236	-0.0238	-0.0238	-0.0238	-0.0238	-0.0238
P.Cor	0.019	-0.07964**	-0.02164	-0.07077**	0.08901**	-0.03522	0.00478	-0.00723	-0.00631	-0.04036*

Open-Close Returns

	Lag 1	2	3	4	5	6	7	8	9	10
Nikkei										
S.Cor	0.04733*	-0.08123**	0.02543	0.03584	-0.02073	-0.03516	0.01127	0.04865*	0.01916	0.01367
(S.E.)	-0.0234	-0.0234	-0.0236	-0.0236	-0.0236	-0.0236	-0.0237	-0.0237	-0.0237	-0.0237
P.Cor	0.04733*	-0.08366**	0.03387	0.0262	-0.01934	-0.02908	0.00973	0.04305*	0.01926	0.02018
FTSE										
S.Cor	-0.05554**	0.05385*	0.01916	-0.00443	0.02886	-0.02184	-0.01174	0.06681**	-0.01045	0.00547
(S.E.)	-0.0233	-0.0234	-0.0235	-0.0235	-0.0235	-0.0235	-0.0235	-0.0235	-0.0236	-0.0237
P.Cor	-0.05554**	0.05092*	0.02497	-0.00485	0.02618	-0.019	-0.01678	0.06678**	-0.00082	-0.00267
S&P										
S.Cor	0.01939	-0.07538**	-0.02773	-0.06364**	0.09923**	-0.02337	-0.01286	-0.01602	-0.02593	-0.02871
(S.E.)	-0.0234	-0.0234	-0.0235	-0.0235	-0.0236	-0.0238	-0.0239	-0.0239	-0.0239	-0.0239
P.Cor	0.01939	-0.07578**	-0.02484	-0.06877**	0.09879**	-0.03922*	0.00105	-0.02132	-0.014	-0.04596*

Notes:

S.Cor: Sample autocorrelation

P.Cor: Partial autocorrelation

* = Significant at 5% level ($Cor / (1/\sqrt{n}) > 1.645$)

** = Significant at 1% level ($Cor / (1/\sqrt{n}) > 2.326$)

Table 5.2. ARCH Tests

Return on Nikkei

Order	Q	Q>prob	LM	LM>prob
1	83.3466	0.0001	89.2252	0.0001
2	102.496	0.0001	91.7172	0.0001
3	127.456	0.0001	107.452	0.0001
4	154.291	0.0001	118.143	0.0001
5	166.881	0.0001	120.226	0.0001
6	170.682	0.0001	120.268	0.0001
7	176.945	0.0001	121.660	0.0001
8	187.889	0.0001	124.852	0.0001
9	193.309	0.0001	125.285	0.0001
10	194.903	0.0001	125.286	0.0001

Return on FT

Order	Q	Q>prob	LM	LM>prob
1	668.886	0.0001	667.811	0.0001
2	806.306	0.0001	691.740	0.0001
3	843.937	0.0001	696.638	0.0001
4	883.883	0.0001	707.020	0.0001
5	904.668	0.0001	709.513	0.0001
6	910.161	0.0001	709.513	0.0001
7	913.643	0.0001	710.619	0.0001
8	922.301	0.0001	712.179	0.0001
9	929.502	0.0001	712.293	0.0001
10	940.763	0.0001	717.058	0.0001

Return on S&P

Order	Q	Q>prob	LM	LM>prob
1	20.1253	0.0001	20.0944	0.0001
2	60.7558	0.0001	55.2655	0.0001
3	71.0509	0.0001	59.4138	0.0001
4	71.5934	0.0001	59.7955	0.0001
5	106.547	0.0001	87.2228	0.0001
6	107.707	0.0001	87.2237	0.0001
7	107.843	0.0001	88.8118	0.0001
8	111.179	0.0001	90.2948	0.0001
9	112.742	0.0001	91.6560	0.0001
10	112.860	0.0001	92.5552	0.0001

Q test: The probability to see the figure Q is 0.0001 under the null hypothesis of the returns which are serially independent.

LM test: same in the figure LM.

Table 5.3. Return Regression Model (Equation (1))

<u>Close-Close Return</u> (Number of the data used = 1,830)

No Dummy

	FT←NK	SP←FT	SP←NK	NK←lag(FT)	NK←lag(SP)
a	0.00071319 (3.06)***	0.00045084 (1.88)*	-1.63534e-6 (-0.01)	-0.00005982 (-0.17)	0.00005171 (0.16)
b	-0.019849 (-0.88)	-0.160702 (-6.99)***	0.0002630 (0.01)	-0.032231 (-1.31)	-0.027032 (-1.21)
c	0.232918 (15.25)***	0.470286 (18.95)***	0.120752 (6.60)***	0.232543 (6.23)***	0.390968 (11.51)***
d	-1.675228 (-6.17)***	-2.123573 (-4.61)***	1.481087 (4.67)***	0.315713 (0.48)	-1.031933 (-3.50)***
R^2	0.1361	0.2001	0.0357	0.0224	0.1174

With Dummy

	FT←NK	SP←FT	SP←NK	NK←lag(FT)	NK←lag(SP)
a	0.00087322 (3.31)***	0.00037595 (1.37)	0.000055832 (0.18)	0.00081738 (2.04)**	0.00086439 (2.30)**
b	-0.018332 (-0.81)	-0.160849 (-7.00)***	0.00060709 (0.02)	-0.031578 (-1.29)	-0.025248 (-1.14)
c	0.230836 (15.03)***	0.471158 (18.95)***	0.119974 (6.52)***	0.235384 (6.34)***	0.387054 (11.45)***
d	-1.655933 (-6.09)***	-2.128420 (-4.62)***	1.487839 (4.68)***	0.282118 (0.43)	-1.056697 (-3.60)***
e	-0.00069299 (-1.30)	0.00031619 (0.57)	-0.00024838 (-0.41)	-0.00367943 (-4.57)***	-0.0034074 (-4.44)***
R^2	0.1369	0.2002	0.0358	0.0335	0.1268

Keys:

A←B means "Market A is affected by Market B"
a: intercept
b: autoregression coefficient
c: return on the other market coefficient
d: volatility of the other market coefficient
e: weekend dummy coefficient

t-values are in brackets

*: Significant at 5% (t>1.645)
**: Significant at 2.5% (t>1.960)
***: Significant at 1% or better (t>2.326)

Table 5.4. Return Regression Model (Equation (1))

<u>Open-Close Return</u> (Number of the data used = 1,829)

No Dummy

	FT←NK	SP←FT	SP←NK	NK←lag(FT)	NK←lag(SP)
a	0.00036910 (1.96)**	0.00102024 (4.02)***	0.00015937 (0.60)	0.00004633 (0.14)	-0.00029170 (-0.95)
b	-0.081741 (-3.39)***	0.00103829 (0.05)	0.021021 (0.81)	0.026433 (1.15)	0.016270 (0.73)
c	0.052794 (3.88)***	0.477096 (15.77)***	0.080355 (4.14)***	0.307488 (7.60)***	0.276409 (8.63)***
d	-0.795584 (-3.50)***	-12.114670 (-8.25)***	1.176825 (3.49)***	-8.219373 (-4.22)***	-1.593179 (-5.74)***
R^2	0.0192	0.1618	0.0164	0.0460	0.1034

With Dummy

	FT←NK	SP←FT	SP←NK	NK←lag(FT)	NK←lag(SP)
a	0.00037494 (1.76)*	0.00108248 (3.80)***	0.00022228 (0.74)	0.00095809 (2.53)***	0.00055125 (1.57)
b	-0.080675 (-3.39)***	0.00086018 (0.04)	0.021255 (0.82)	0.028805 (1.26)	0.018997 (0.86)
c	0.052704 (3.84)***	0.476879 (15.76)***	0.079391 (4.07)***	0.311632 (7.75)***	0.272249 (8.55)***
d	-0.795376 (-3.49)***	-12.104043 (-8.24)***	1.179316 (3.49)***	-8.354828 (-4.32)***	-1.617938 (-5.86)***
e	-0.00002534 (-0.06)	-0.0026985 (-0.48)	-0.00026859 (-0.45)	-0.00378009 (-5.15)***	-0.00352061 (-4.94)***
R^2	0.0192	0.1619	0.0165	0.0597	0.1153

Keys:

A←B means "Market A is affected by Market B"
a: intercept
b: autoregression coefficient
c: return on the other market coefficient
d: volatility of the other market coefficient
e: weekend dummy coefficient

t-values are in brackets

*: Significant at 5%
**: Significant at 2.5%
***: Significant at 1% or better

Table 5.5. Contagion Model (Close-Close Return)

No Intercept No Dummy (Equation (2a))

Regression	dummy	β	θ	SE / AIC
FT←NK	-	0.23735	-0.05171	0.00970376
		(15.41)***	(-2.19)***	-11776.102
NK←lag(FT)	-	0.21989	0.02493	0.01473912
		(6.24)***	(1.01)	-10239.821
SP←NK	-	0.11875	0.02409	0.01105862
		(6.46)***	(0.97)	-11297.491
NK←lag(SP)	-	0.44582	0.02252	0.01405177
		(15.02)***	(0.95)	-10414.612

No Intercept With Dummy (Equation (2b))

Regression	dummy	β	θ	SE / AIC
FT←NK	-0.0003896	0.23620	-0.05298	0.00970459
	(-0.83)	(15.26)***	(-2.24)***	-11774.792
NK←lag(FT)	-0.0028252	0.22454	0.02414	0.01467837
	(-4.02)***	(6.40)***	(0.97)	-10253.94
SP←NK	0.0002672	0.11964	0.02479	0.01106088
	(0.50)	(6.48)***	(1.00)	-11295.744
NK←lag(SP)	-0.0026502	0.44568	0.02152	0.01399577
	(-3.96)***	(15.07)***	(0.91)	-10428.228

With Intercept No Dummy (Equation (2b))

Regression	mean	dummy	β	θ	SE / AIC
FT←NK	0.0003376	-	0.23730	-0.05074	0.0097011
	(1.42)		(15.41)***	(-2.15)***	-11776.111
NK←lag(FT)	-0.0000235	-	0.21997	0.02495	).01474313
	(-0.07)		(6.23)***	(1.01)	-10237.826
SP←NK	0.0003278	-	0.11888	0.02524	).01105653
	(1.30)		(6.47)***	(1.02)	-11297.185
NK←lag(SP)	-0.0000972	-	0.44611	0.02261	).01405526
	(-0.30)		(15.02)***	(0.96)	-10412.703

With Intercept With Dummy (Equation (2b))

Regression	mean	dummy	β	θ	SE / AIC
FT←NK	0.0005495	-0.0008915	0.23467	-0.05205	0.00969622
	(2.04)***	(-1.68)*	(15.16)***	(-2.20)***	-11776.952
NK←lag(FT)	0.0008613	-0.0037283	0.22446	0.02761	0.01466237
	(2.24)***	(-4.60)***	(6.41)***	(1.12)	-10256.933
SP←NK	0.0003527	-0.0001048	0.11856	0.02514	0.01105947
	(1.21)	(-0.17)	(6.42)***	(1.01)	-11295.214
NK←lag(SP)	0.0007094	-0.0033902	0.44404	0.02398	0.01398544
	(1.93)*	(-4.39)***	(15.02)***	(1.01)	-10429.933

N.B.) $R_{j,t} = \mu_j + \delta_j \cdot dummy + \beta_j \cdot R_{j-1,t} + (1-\theta_j L) \cdot \varepsilon_{j,t}$

β : Return spillover coefficients

θ: Moving average coefficients

SE: Standard error

AIC: AIC (Akaike's Information Criterion) = -2 log L (φ) + 2 n

A←B means "Market A is affected by Market B"

t-values in brackets

*: Significant at 5% **: Significant at 2.5% ***: Significant at 1% or better

Table 5.6. GARCH Models with Volatility Spillover (Equation (3))

CLOSE-CLOSE

Without Dummy

S&P←Nikkei			*Nikkei← lag(SP)*		
Var	Coeff	t value	Var	Coeff	t value
α	5.5493e-4	2.815***	α	9.4971e-4	4.383***
a	4.5063e-6	7.762***	a	1.5796e-6	2.888***
b	0.1397	27.796***	b	0.1419	13.368***
c	0.8225	70.044***	c	0.8487	82.368***
d	4.1811e-4	0.358	d	0.0233	8.400***

logL = 14915.934 #iterat: 103 #obs: 1829

FTSE←Nikkei			*Nikkei←lag(FT)*		
Var	Coeff	t value	Var	Coeff	t value
α	6.6161e-4	3.213***	α	1.0434e-3	4.905***
a	4.0515e-5	11.795***	a	1.1357e-6	1.789*
b	0.1085	3.810***	b	0.1519	12.712***
c	0.1716	3.380***	c	0.8349	73.257***
d	0.1445	21.680***	d	0.0472	13.534***

logL = 14871.880 #iterat: 115 #obs: 1829

With Dummy

S&P←Nikkei			*Nikkei←lag(SP)*		
Var	Coeff	t value	Var	Coeff	t value
α	4.864e-4	2.197**	α	1.243e-3	4.978***
δ	2.787e-4	0.667	δ	-1.576e-3	-2.924***
a	3.242e-6	3.517***	a	-4.989e-6	-6.002***
b	0.817	66.222***	b	0.832	64.984***
c	0.142	26.676***	c	0.149	11.508***
d	5.254e-4	0.431	d	2.170e-2	8.603***
e	6.226e-6	2.532***	e	3.372e-5	8.989***

logL = 14938.427 #iterat: 100 #obs: 1829

FTSE←Nikkei			*Nikkei← lag(FT)*		
Var	Coeff	t value	Var	Coeff	t value
α	9.745e-4	4.355***	α	1.189e-3	4.864***
δ	-1.588e-3	-2.798***	δ	-1.207e-3	-2.317**
a	2.766e-5	7.369***	a	-6.443e-6	-6.999***
b	0.309	5.304***	b	0.810	58.657***
c	0.111	3.666***	c	0.164	12.549***
d	8.594e-2	7.708***	d	5.166e-2	12.549***
e	3.485e-5	5.440***	e	3.887e-5	9.959***

logL = 14911.448 #iterat: 200+ #obs: 1829

N.B.)

$$y_{j,t} = \alpha_j + v_{j,t} + \delta \cdot dummy$$

$$h_{j,t} = a_j + b_j \cdot h_{j,t-1} + c_j \cdot v^2_{j,t-1} + d_j \cdot v^2_{j-1,t} + e_j \cdot dummy$$

A←B means "Market A is affected by Market B

*: Significant at 5% **: Significant at 2.5% ***: Significant at 1% or better

Table 5.7. GARCH Models with Volatility Spillover (Equation (3))

OPEN-CLOSE

Without Dummy

S&P←Nikkei Var	Coeff	t value	*Nikkei←lag(SP)* Var	Coeff	t value
α	1.0990e-3	3.610***	α	5.2201e-4	2.264**
a	1.2614e-5	8.659***	a	2.2944e-6	2.893***
b	0.7355	32.257***	b	0.7206	32.801***
c	0.1724	23.156***	c	0.2385	9.162***
d	7.0925e-3	1.335	d	0.0448	5.822***

logL = 9173.055 #iterat: 98 #obs = 1114

FTSE←Nikkei Var	Coeff	t value	*Nikkei←lag(FT)* Var	Coeff	t value
α	5.6248e-4	2.336***	α	6.7552e-4	2.694***
a	8.9262e-6	7.498***	a	3.8761e-6	4.125***
b	0.7789	24.866***	b	0.6327	25.153***
c	0.0419	2.876***	c	0.3394	9.587***
d	0.0185	4.083***	d	0.1086	10.503***

logL = 8642.398 #iterat: 101 #obs = 1018

With Dummy

S&P←Nikkei Var	Coeff	t value	*Nikkei←lag(SP)* Var	Coeff	t value
α	1.243e-3	3.752***	α	6.565e-4	2.502***
δ	-7.674e-4	-1.060	δ	-7.169e-4	-1.272
a	7.558e-6	2.967***	a	-1.692e-6	-1.310
b	0.758	36.939***	b	0.749	35.831***
c	0.161	22.002***	c	0.213	9.050***
d	5.171e-3	1.035	d	3.673e-2	6.292***
e	1.619e-5	2.278**	e	1.693e-5	3.543***

logL = 9180.694 #iterat: 107 #obs: 1114

FTSE←Nikkei Var	Coeff	t value	*Nikkei←lag(FT)* Var	Coeff	t value
α	1.001e-3	3.512***	α	6.056e-4	2.254**
δ	-1.815e-3	-3.054***	δ	1.030e-4	0.185
a	5.538e-6	3.137***	a	-1.716e-6	-1.258
b	0.786	24.407***	b	0.660	25.382***
c	4.447e-2	3.270***	c	0.306	8.646***
d	1.715e-2	3.667***	d	0.104	10.469***
e	1.238e-5	3.019***	e	2.309e-5	4.484***

logL = 8654.930 #iterat: 130 #obs: 1018

N.B.)

$$y_{j,t} = \alpha_j + v_{j,t} + \delta \cdot dummy$$

$$h_{j,t} = a_j + b_j \cdot h_{j,t-1} + c_j \cdot v^2{}_{j,t-1} + d_j \cdot v^2_{j-1,t} + e_j \cdot dummy$$

A←B means "Market A is affected by Market B"

*: Significant at 5% **: Significant at 2.5% ***: Significant at 1% or better

Table 5.8. General Model ***With Dummy*** **(Equation (5))**

CLOSE-CLOSE (from the Open-Close returns of the other market)

S&P←Nikkei			*Nikkei←lag(SP)*		
Var	Coeff	t value	Var	Coeff	t value
α	1.057e-3	3.118***	α	1.025e-3	3.940***
δ	-6.058e-4	-0.827	δ	-8.808e-4	-1.574
β	0.145	5.114***	β	0.261	8.975***
a	6.400e-6	2.342***	a	-2.008e-6	-1.816*
b	0.748	30.999***	b	0.799	44.852***
c	0.165	19.888***	c	0.187	9.141***
d	7.338e-3	1.231	d	1.617e-2	4.301***
e	2.264e-5	3.188***	e	1.770e-5	3.477***

logL = 9109.508 #iterat: 85 #obs = 1114

FTSE←Nikkei			*Nikkei← lag(FT)*		
Var	Coeff	t value	Var	Coeff	t value
α	1.349e-3	4.292***	α	1.065e-3	3.803***
δ	-2.346e-3	-2.952***	δ	-2.365e-4	-0.407
β	0.234	6.533***	β	0.172	4.679***
a	3.222e-5	6.018***	a	-4.822e-6	-4.241***
b	0.290	3.835***	b	0.756	34.905***
c	0.122	2.699***	c	0.203	8.330***
d	0.128	7.216***	d	6.504e-2	9.718***
e	3.228e-5	3.166***	e	3.000e-5	5.392***

logL = 8423.186 #iterat: 354 #obs = 1019

OPEN-CLOSE (from the Open-Close returns of the other market)

S&P←Nikkei			*Nikkei←lag(SP)*		
Var	Coeff	t value	Var	Coeff	t value
α	1.119e-3	3.408***	α	5.982e-4	2.334***
δ	-5.181e-4	-0.709	δ	-9.833e-4	-1.778*
β	0.124	4.564***	β	0.191	8.303***
a	6.591e-6	2.651***	a	-6.867e-7	-0.559
b	0.770	38.124***	b	0.781	39.384***
c	0.154	21.269***	c	0.191	8.377***
d	1.955e-3	0.372	d	2.234e-2	6.175***
e	1.799e-5	2.641***	e	1.143e-5	2.363***

logL = 9219.532 #iterat: 94 #obs = 1114

FTSE←Nikkei			*Nikkei← lag(FT)*		
Var	Coeff	t value	Var	Coeff	t value
α	9.271e-4	3.220***	α	5.509e-4	2.034**
δ	-1.729e-3	-2.781***	δ	-2.269e-4	-0.413
β	4.530e-2	1.813*	β	0.175	4.628***
a	5.174e-6	3.053***	a	-1.279e-7	-0.091
b	0.788	24.934***	b	0.642	22.464***
c	5.109e-2	3.759***	c	0.338	8.435***
d	1.587e-2	3.519***	d	8.688e-2	8.013***
e	1.250e-5	3.198***	e	1.805e-5	3.379***

logL = 8669.847 #iterat: 96 #obs = 1018

N.B.)

$$y_{j,t} = \alpha_j + v_{j,t} + \delta_j \cdot dummy + \beta_j \cdot y_{j-1}$$

$$h_{j,t} = a_j + b_j \cdot h_{j,t-1} + c_j \cdot v^2{}_{j,t-1} + d_j \cdot v^2_{j-1,t} + e_j \cdot dummy$$

A←B means "Market A is affected by Market B"

*: Significant at 5% **: Significant at 2.5% ***: Significant at 1% or better

Table 5.8. (continued) General Model *With Dummy* (Equation (5))

CLOSE-OPEN (from the Open-Close returns of the other market)

S&P←Nikkei | *Nikkei←lag(SP)*

Var	Coeff	t value	Var	Coeff	t value
α	0.00350	0.938	α	0.00154	0.064
δ	-0.00337	-0.596	δ	-0.0174	-0.022
β	0.00790	0.113	β	0.0767	0.130
a	0.00000572	0.298	a	0.00000185	0.021
b	0.8187	4.603***	b	0.7823	5.212***
c	-0.07633	-0.118	c	0.5462	0.380
d	-0.0290	-1.255	d	-1.604	-0.187
e	0.000169	1.166	e	0.00344	1.639

logL = 7748.768 #iterat: 22 #obs = 1114

FTSE←Nikkei | *Nikkei←lag(FT)*

Var	Coeff	t value	Var	Coeff	t value
α	3.341e-4	1.416	α	6.016e-4	2.689***
δ	-2.270e-3	-3.133***	δ	-2.659e-4	-0.776
β	0.115	10.784***	β	5.133e-3	0.174
a	-1.869e-5	-40.73***	a	2.203e-5	27.539***
b	0.815	53.57***	b	-0.202	-15.201***
c	6.496e-2	7.920***	c	-5.368e-3	-8.848***
d	-6.470e-3	-0.713	d	2.257e-2	21.885***
e	1.501e-4	29.411***	e	7.232e-5	18.168***

logL = 9572.933 #iterat: 44 #obs = 1018

N.B.)

$$y_{j,t} = \alpha_j + v_{j,t} + \delta_j \cdot dummy + \beta_j \cdot y_{j-1}$$

$$h_{j,t} = a_j + b_j \cdot h_{j,t-1} + c_j \cdot v^2{}_{j,t-1} + d_j \cdot v_{j-1,t}^2 + e_j \cdot dummy$$

A←B means "Market A is affected by Market B"

*: Significant at 5% **: Significant at 2.5% ***: Significant at 1% or better

Table 5.9. Comparison Table

Market	Method	Return to Return C-C	O-C	C-O	Volatility to Volatility C-C	O-C	C-O
NK to FT	Return	0.23***	0.05***	0.39***	--	--	--
	Con	0.24***	--	--	--	--	--
	Garch	--	--	--	0.086***	0.017***	--
	Gen	0.23***	0.045*	0.115***	0.125***	0.0159***	0.00647
	K&W	0.16	--	--	--	--	--
	HM&N	--	--	--	--	0.0185***	--
	T&L	-0.0358	--	--	0	--	--
NK to SP	Return	0.12***	0.08***	0.01	--	--	--
	Con	0.12***	--	--	--	--	--
	Garch	--	--	--	0.000525	0.005171	--
	Gen	0.145***	0.124***	0.00790	0.00734	0.001955	-0.02896
	K&W	0.04	--	--	--	--	--
	HM&N	--	--	--	--	0.0159***	--
	T&L	-0.0765	--	--	0	--	--
Lag SP to NK	Return	0.39***	0.27***	0.105**	--	--	--
	Con	0.45***	--	--	--	--	--
	Garch	--	--	--	0.0217***	0.0367***	--
	Gen	0.261***	0.191***	0.0767	0.0162***	0.0223***	-1.6039
	K&W	0.11	--	--	--	--	--
	HM&N	--	--	--	--	0.0519***	--
	T&L	0.0705	--	--	0.0470*	--	--
Lag FT to NK	Return	0.24***	0.31***	-0.035***	--	--	--
	Con	0.22***	--	--	--	--	--
	Garch	--	--	--	0.0517***	0.104***	--
	Gen	0.173***	0.175***	0.0051	0.065***	0.0869***	0.0223***
	K&W	0.06	--	--	--	--	--
	HM&N	--	--	--	--	0.0995***	--
	T&L	-0.018	--	--	0	--	--

Return	Return Regression Model	***: significant at 1%
Con	Contagion Model	**: significant at 2.5%
Garch	GARCH Model	*: significant at 5%
Gen	General Model	02/01/87-31/08/94
K&W	King and Wadhwani results	01/12/87-28/02/88
HM&N	Hamao, Masulis, and Ng results	01/04/85-31/03/88
T&L	Theodossiou and Lee results (Weekly)	11/01/80-27/12/91

With weekend dummy if available

Appendix

Appendix 5.1. King and Wadhwani Model with notations of this chapter

Return of the market is explained as the sum of the cumulative value of news term of the market and the one of the other market. Assuming two markets (j and j-1) exist, we express this condition as follows;

$$R_{j-1,t} = \varepsilon_{j-1,t} + \beta_{j-1,j} \cdot \varepsilon_{j,t-1} \quad \text{(A1)}$$

$$R_{j,t} = \varepsilon_{j,t} + \beta_{j,j-1} \cdot \varepsilon_{j-1,t} \quad \text{(A2)}$$

If we reformulate Equation (A1) as an equation of $\varepsilon_{j-1,t}$, $\varepsilon_{j-1,t}$ can be replaced in Equation (A2), and the new equation is:

$$\begin{aligned} R_{j,t} &= \varepsilon_{j,t} + \beta_{j,j-1}(R_{j-1,t} - \beta_{j,j-1} \cdot \varepsilon_{j,t-1}) \\ &= \beta_{j,j-1} \cdot R_{j-1,t} + \varepsilon_{j,t} - \beta_{j,j-1} \cdot \beta_{j,j-1} \cdot \varepsilon_{j,t-1} \\ &= \beta_{j,j-1} \cdot R_{j-1,t} + (1 - \beta_{j,j-1} \cdot \beta_{j,j-1} \cdot L) \cdot \varepsilon_{j,t} \end{aligned}$$

Then, in the same manner,

$$R_{j-1,t} = \beta_{j-1,j} \cdot R_{j,t-1} + (1 - \beta_{j-1,j} \cdot \beta_{j-1,j} \cdot L) \cdot \varepsilon_{j-1,t} \quad \text{(A1')}$$

$$R_{j,t} = \beta_{j,j-1} \cdot R_{j-1,t} + (1 - \beta_{j,j-1} \cdot \beta_{j,j-1} \cdot L) \cdot \varepsilon_{j,t} \quad \text{(A2')}$$

Therefore, the MA coefficient is the product of the contagion coefficients.

Appendix 5.2. GARCH Model of Single Market

Without Dummy

CLOSE-CLOSE

NK			FTSE			S&P		
Var	Coeff	t value	Var	Coeff	t value	Var	Coeff	t value
α	1.1596e-3	5.359***	α	0.0006357	2.921***	α	5.2704e-4	2.695***
a	5.1929e-6	6.855***	a	0.000009223	7.126***	a	4.5299e-6	7.933***
b	0.7698	77.287***	b	0.7991	35.202***	b	0.8237	70.799***
c	0.2490	37.989***	c	0.1047	7.566***	c	0.1391	27.995***
logL	7135.405		logL	7658.065		logL	7725.408	
#iterat	98		#iterat	76		#iterat	167	
#obs	1830		#obs	1830		#obs	1830	

OPEN-CLOSE

NK			FTSE			S&P		
Var	Coeff	t value	Var	Coeff	t value	Var	Coeff	t value
α	6.3244e-4	2.850***	α	0.0004903	1.951*	α	5.2333e-4	2.793***
a	7.2517e-6	7.388***	a	0.000007858	6.018***	a	3.4577e-6	7.716***
b	0.5938	33.334***	b	0.8056	28.063***	b	0.8456	86.565***
c	0.4685	30.153***	c	0.07746	5.560***	c	0.1266	29.502***
logL	4604.928		logL	4410.088		logL	7770.105	
#iterat	157		#iterat	31		#iterat	166	
#obs	1115		#obs	1019		#obs	1830	

With Dummy

CLOSE-CLOSE

NK			FTSE			S&P		
Var	Coeff	t value	Var	Coeff	t value	Var	Coeff	t value
α	1.248e-3	5.142***	α	8.996e-4	3.716***	α	4.6362e-4	2.098**
δ	-7.559e-4	-1.548	δ	-1.551e-3	-2.946***	δ	2.5673e-4	0.636
a	-3.135e-6	-2.937***	a	6.819e-6	3.033***	a	3.2950e-6	3.721***
b	0.748	64.706***	b	0.714	23.505***	b	0.8185	66.864***
c	0.259	24.704***	c	0.129	8.389***	c	0.1418	26.782***
d	4.356e-5	10.703***	d	3.305e-5	9.766***	d	6.1935e-6	2.552***
logL	7155.854		logL	7672.937		logL	7722.391	
#iterat	150		#iterat	185		#iterat	121	
#obs	1829		#obs	1829		#obs	1829	

OPEN-CLOSE

NK			FTSE			S&P		
Var	Coeff	t value	Var	Coeff	t value	Var	Coeff	t value
α	5.9553e-4	2.380***	α	8.451e-4	2.925***	α	4.648e-4	2.187**
δ	2.1067e-4	0.436	δ	-1.781e-3	-3.024***	δ	2.250e-4	0.548
a	2.2286e-6	1.484	a	4.583e-6	2.847***	a	3.931e-6	5.022***
b	0.6178	32.416***	b	0.803	26.993***	b	0.846	86.716***
c	0.4396	22.521***	c	8.142e-2	5.540***	c	0.127	28.575***
d	1.879e-5	3.864***	d	1.373e-5	3.707***	d	-2.355e-6	-1.022
logL	4605.232		logL	4414.453		logL	7766.206	
#iterat	145		#iterat	40		#iterat	132	
#obs	1114		#obs	1018		#obs	1829	

N.B.)

$$R_{j,t} = \alpha_{j,t} + v_{j,t} + \delta_j \cdot dummy$$

$$h_{j,t} = a_j + b_j \cdot h_{j,t-1} + c_j \cdot v^2{}_{j,t-1} + d_j \cdot dummy$$

*: Significant at 5%   **: Significant at 2.5%   ***: Significant at 1% or better

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6

Implied Volatilities and Skewness across the Index Options Markets:

Comparison and Transmission

Naoki Kamiyama

6. Implied Volatilities and Skewness across the Index Options Markets: Comparison and Transmission

6.1. Introduction

The linkage of the international equity markets has been examined by many academics and practitioners, especially after the crash in 1987. The spillover of returns and volatilities to the next-opening market interests not only finance-related persons but also regulators, including central bankers and governmental officers. Examples of works in this field are Becker, Finnerty, and Gupta (1990), King and Wadhwani (1990), and Hamao, Masulis, and Ng (1990). Kamiyama (1996) developed a general model which integrates both return and volatility spillovers, whereas previous papers concentrated on one or other of them. All of these focus on the returns and/or observed volatilities which affect each other, but few have examined if there is an implied volatility spillover among the major international equity index options markets.

Skewness of the implied volatility is observed in most of the index options markets of the world. For example, Bates (1991) concludes that the crash of 1987 was expected by market participants in U.S. because negative skewness was observed during October 1986 to August 1987 in the S&P 500 futures options market. Gemmill (1995) examines the FTSE options market in pre- and post-crash periods to show whether market participants anticipated the crash and finds no such ability. Taylor and Xu (1994b) find evidence of the skewed shape of the implied volatility vs. exercise price of the S&P 500 options on futures from January 1988 to December 1992. Heynen (1994) analyses the implied volatility pattern of the Dutch index options market, and concludes that the pattern is significantly U-shaped.

Skewness is set by option market participants and so reflects their behaviour. One hypothesis of this behaviour is that option markets participants only passively follow the index return of the day to buy and/or sell options so that implied volatility skewness is formed. If this were the case, skewness would be explained by index returns, but index returns on the next day would not be explained by skewness. The alternative hypothesis of this behaviour is that option-market participants rationally anticipate the next day return of the index and develop the appropriate implied volatility skewed shape. If this were true, the next day's index returns would be influenced by the skewness of today.

Although there are many papers describing how skewness of implied volatility affects the pricing of options (ex. Bates (1994) and Madan and Chang (1995)), there are few papers which focus on the relationship between skewness and index returns. Gemmill (1995) finds that there was a small tendency for FTSE to be left skewed when the market had risen and right skewed when it had fallen. But Gemmill (1995) also finds that FTSE options traders had no premonition of the crash of 1987.

One purpose of this chapter is to examine the implied volatility transmission among the three major index options markets all over the world, including the Nikkei 225 options on futures listed on SIMEX (Singapore International Monetary Exchange), FTSE index options listed on LIFFE (London International Financial Futures Exchange), and S&P 500 options on futures listed on CME (Chicago Mercantile Exchange). Secondly, we compare these markets in terms of (domestic) index returns versus (domestic) implied volatility and skewness, both contemporaneously and with lags. In this way, we examine if the skewness is formed passively by options traders who follow the market direction, or option traders can rationally forecast the next day's return of the index. The results may vary for the three markets.

The third issue in this chapter is the dependence of skewness among the three major index options markets, extending what Gemmill (1995) has done by examining three options markets. Index return spillover is observed by King and Wadhwani (1990) and volatility spillover is also found by Hamao, Masulis, and Ng (1990) in the GARCH methodology. How about spillovers in the skewness of implied volatility? If the skewness of an option market was subject to the day's return of the index, skewness would spillover to the next-opening market because of the index return's spillover.

The chapter is written as follows. In Section 2, the method undertaken is explained. Section 3 describes the data utilised in this chapter, and the test results are shown in Section 4. The conclusions are given in Section 5.

6.2. The Data

For the Japanese market, Nikkei option on future and the underlying future settlement price data have been provided from the Singapore International Monetary Exchange (SIMEX). In order to calculate the implied volatilities, daily three month CD rates are used for all the maturity dates. The nearest contract month is used for Nikkei, because that is the most heavily traded. We roll over the nearest contract when time to maturity becomes less than 5 days. For the British market, FTSE options data have been obtained from Gordon Gemmill of City University London, who collected originally from the Financial Times, the Stock Exchange Daily Official List, and London International Financial Futures Exchange (LIFFE). Three-month interest rates are used for implied volatility calculation, and the rates are obtained from Datastream by

Gemmill. The second-month maturity options are analysed because they are the most liquid. For the U.S. market, S&P 500 options on futures price data have been provided by the Chicago Mercantile Exchange (CME). For the short-term interest rate, the 3-month T-Bill rate is used for any time to maturity. The contract month for the data analysed is the second-nearest contract month, because it is generally most liquid.

The period examined is from 01 April 1992 to 29 December 1995[1] and the total number of daily records of each is 891 for Nikkei, 945 for FTSE, and 928 for S&P. Figure 3 shows the index levels of the three markets during the sample period (all the indices are set at 100 as of 01 April 1992). Figure 6.4 shows the implied volatilities of the three markets.

Implied volatilities for American-type options on futures (Nikkei and S&P) are calculated with the method proposed by Barone-Adesi and Whaley (1987)[2]. The implied volatility calculations for the FTSE options are made with a dividend-adjusted binomial model, because this contract is based on the cash index.

6.3. The Method

6.3.1.a Does implied volatility spillover?

The implied volatility (IV) transmission is examined as below.

$$IV_{nk,t} = c + \beta_1 \cdot IV_{nk,t-1} + \beta_2 \cdot IV_{ft,t-1} + \beta_3 \cdot IV_{sp,t-1} \quad (1a)$$

$$IV_{ft,t} = c + \beta_1 \cdot IV_{ft,t-1} + \beta_2 \cdot IV_{nk,t} + \beta_3 \cdot IV_{sp,t-1} \quad (1b)$$

$$IV_{sp,t} = c + \beta_1 \cdot IV_{sp,t-1} + \beta_2 \cdot IV_{ft,t} + \beta_3 \cdot IV_{nk,t} \quad (1c)$$

where IV is implied volatility, Δ IV is a daily percent change in implied

[1] The SIMEX contract began trading in April 1992, hence the starting data for the data.

volatility, and t is time counted by day. By using Equations (1a), (1b), and (1c), we examine how a ***level*** of IV of a market affects the next-opening market's level of IV[3]. In other words, we examine if a high level of IV in a market spills over and makes the next-opening market's IV higher.

$$\Delta IV_{nk,t} = c + \beta_1 \cdot \Delta IV_{nk,t-1} + \beta_2 \cdot \Delta IV_{ft,t-1} + \beta_3 \cdot \Delta IV_{sp,t-1} \quad (1d)$$

$$\Delta IV_{ft,t} = c + \beta_1 \cdot \Delta IV_{ft,t-1} + \beta_2 \cdot \Delta IV_{nk,t} + \beta_3 \cdot \Delta IV_{sp,t-1} \quad (1e)$$

$$\Delta IV_{sp,t} = c + \beta_1 \cdot \Delta IV_{sp,t-1} + \beta_2 \cdot \Delta IV_{ft,t} + \beta_3 \cdot \Delta IV_{nk,t} \quad (1f)$$

By using Equations (1d), (1e), and (1f), we examine how a ***change*** in IV of a market affects the next-opening market's change in IV[4]. This means that we examine if the large change in IV of a market leads a similar change in IV of the next-opening market.

6.3.1.b Does implied volatility depend on index return of the day?

We analyse the relationship between returns and implied volatilities as shown below.

$$IV_{x,t} = c + \beta_1 \cdot ret_{x,t} + \beta_2 \cdot sqret_{x,t} \quad (1g)$$

$$\Delta IV_{x,t} = c + \beta_3 \cdot ret_{x,t} + \beta_4 \cdot sqret_{x,t} \quad (1h)$$

where,

$$\Delta\ IV = IV_t / IV_{t-1} - 1$$

---

[2] Please see Appendix 6.A.

[3] We employ the SUR (seemingly unrelated regression) process to solve these equations simultaneously because these three may have correlated disturbance terms. The objective function to be minimised is

$S_n = \frac{1}{n}\sum_{t=1}^{n} q(y_t, x_t, \theta)' \Sigma^{-1} q(y_t, x_t, \theta)$, where the model, q, has dependent variables of y_t, independent variables of x_t, and the parameters of θ. The cross-equation covariance matrix, Σ, is estimated by OLS. The SAS/ETS software is used for the process.

[4] The equations are also solved by SUR.

c: Constant

x: Nikkei, FTSE, or S&P

ret: Log return of the cash index

sqret: squared return of the cash index

Δ *IV* is proportional change in skewness, which does not depend on the level of *IV* with respect to the effect from returns. A squared return may also be called a daily observed variance. If squared returns strongly affect implied volatility or change in implied volatility, participants may be considered to buy and/or sell options by watching observed index direction. The results must be compared with the relationship between returns and skewness in Section 3.3.

6.3.2 What is skewness?

Bates (1991) developed a skewness measure based upon interpolated option prices. In this chapter, we will use a measure based upon interpolated volatilities (Gemmill (1995)), which is:

$$skew_t = \frac{\sigma_t(+2\%) + diff - \sigma_t(-2\%)}{\sigma_t(+2\%)} \qquad (2)$$

where,

diff: the difference in implied volatility (IV) of at-the-money puts and at-the-money calls (= ATM Put IV − ATM Call IV),

σ : the implied volatility

(+x%): an exercise price which is x% above at-the-money price

We interpolate implied volatilities by exercise price by using nearest-the-money (NTM) calls and puts, and OTM options (NTM+500 exercise price for calls and NTM-500 for puts), because normally we have no exact at-the-money options in any of three markets. In the same way, we interpolate ATM$\pm$2% implied volatilities. See Figures 6.1.a and 6.1.b for more details in calculating the skewness measure from observed NTM and OTM option implied volatilities. At-the-Money is defined as the day's underlying futures price for Nikkei or S&P. For FTSE options, the forward price of the day for the option's time to maturity is calculated, and at-the-money forward is used for analysis.

The "*diff*" term is included to allow for difference between at-the-money put and call volatilities, as were found by Gemmill for the FTSE index options. In this chapter, we accept the risk free rates from outside of the pricing models, for example, 3-month CD rates in order to discount the Nikkei 225 options. If we would like to omit the *diff* term, we needed to accept the "implied" risk-free rates which would be derived from the option prices which could make the implied volatilities of calls and puts exactly same. On the other hand, we could not find *diff* which may exits, if we accepted the "implied" risk-free rates.

Stationarity of IV and of skewness are examined with the Unit Root (Dickey-Fuller) test regression. The test is designed as shown below.

$$\Delta IV_{x,t} = c + \theta \cdot \Delta IV_{x,t-1} + \beta \cdot IV_{x,t-1} \qquad (3a)$$

$$dskew_{x,t} = c + \theta \cdot dskew_{x,t-1} + \beta \cdot skew_{x,t-1} \qquad (3b)$$

where *dskew* is a lagged difference (not a percent difference here). Over the sample period, it is likely that the IVs and skewness of all three markets are stationary ($\beta < 1$), although IV of FTSE has the relatively high level of lower tail area to make an error.

6.3.3 Is skewness dependent on index return of the day?

If option traders were passive to the market direction of the day to form the smile of implied volatility, we would observe significant coefficients from the regression models as shown below.

$$skew_{x,t} = c + \beta_1 \cdot ret_{x,t} + \beta_2 \cdot sqret_{x,t} \quad (4)$$

$$\Delta skew_{x,t} = c + \beta_3 \cdot ret_{x,t} + \beta_4 \cdot sqret_{x,t} \quad (5)$$

where,

Δ *skew* = $skew_t / skew_{t-1} - 1$

c: Constant

x: Nikkei, FTSE, or S&P

ret: Log return of the cash index

sqret: squared return of the cash index

Δ *skew* is proportional change in skewness, which does not depend on the level of *skew* with respect to the effect from returns. A squared return may also be called a daily observed variance. If squared returns strongly affect skewness or change in skewness, participants may be considered to buy and/or sell options by watching observed volatility rather than index direction.

6.3.4 Can Skewness forecast the next-day index returns?

If option traders rationally forecast the returns of indices on the next day, we would find significant coefficients in the regression model as below.

$$ret_{x,t} = c + \beta_1 \cdot skew_{x,t-1} \quad (6)$$

If β_1 is significant, the skewness level directly affects the next-day return, that is, the traders change the shape of skewness directly affected by their forecast to the market. If no coefficient is significant, we may conclude that skewness does not influence subsequent returns.

6.3.5 How independent is skewness over the world?

Skewness may spill over across markets. Option traders may watch skewness of implied volatilities in the other markets to form their own skewed implied volatilities in their option market. This may be tested as follows:

$$skew_{nk,t} = c + \beta_1 \cdot skew_{nk,t-1} + \beta_2 \cdot skew_{ft,t-1} + \beta_3 \cdot skew_{sp,t-1} \quad (7a)$$

$$skew_{ft,t} = c + \beta_1 \cdot skew_{ft,t-1} + \beta_2 \cdot skew_{nk,t} + \beta_3 \cdot skew_{sp,t-1} \quad (7b)$$

$$skew_{sp,t} = c + \beta_1 \cdot skew_{sp,t-1} + \beta_2 \cdot skew_{ft,t} + \beta_3 \cdot skew_{nk,t} \quad (7c)$$

$$\Delta skew_{nk,t} = c + \beta_1 \cdot \Delta skew_{nk,t-1} + \beta_2 \cdot \Delta skew_{ft,t-1} + \beta_3 \cdot \Delta skew_{sp,t-1} \quad (7d)$$

$$\Delta skew_{ft,t} = c + \beta_1 \cdot \Delta skew_{ft,t-1} + \beta_2 \cdot \Delta skew_{nk,t} + \beta_3 \cdot \Delta skew_{sp,t-1} \quad (7e)$$

$$\Delta skew_{sp,t} = c + \beta_1 \cdot \Delta skew_{sp,t-1} + \beta_2 \cdot \Delta skew_{ft,t} + \beta_3 \cdot \Delta skew_{nk,t} \quad (7f)$$

Similar to the implied volatility transmission tests (Equations (1a) to (1f)), we examine the levels and changes in skewness, i.e. we test if market participants watch and react in the market either to the levels or changes in skewness in their own and other nation's markets[5].

In Figure 6.2.a, we plot the 20-day moving averages of skew to avoid too much noise from the data series. These moving averages may be considered as a measure of optimism / pessimism which is hypothesised to spread across markets.

6.4. Test Results

As a preliminary, Table 6.1 examines the difference between at-the-money put and call IVs. They are significantly different (5% level) for all the three indices. The FTSE options has a "call bias" in the sample period, although a put bias was reported by Gemmill (1995) for FTSE in the earlier sample period of July 1985 to December 1990. The Nikkei and S&P options have a "put bias" on the other hand. A bias in the skewness estimate due to the put/call difference is avoided by subtracting *diff* in Equation (2).

The skewness measures of the three markets are shown in Table 6.2 and

[5] Equations (7a) to (7c) and Equations (7d) to (8f) are solved by SUR.

Figures 6.2.a (20-day moving average) and 6.2.b (5-day average - step 5 days). All three skewness estimates are significantly negative (5% level). The S&P skewness is always negative over the sample period, even though the American equity market was bullish during this period. The S&P skewness is largest among three, although the reason is not clear. The U.S. traders may be more cautious to the down-side risks of the market after the Crash of 1987, than the participants of the other two. The Nikkei skewness is also negative in most of the period, whereas the FTSE occasionally shows positive skewness. FTSE has most volatile skewness movement, and S&P the least. From the figures, it can be expected that the skewness is autoregressive.

Tables 6.3.a and 6.3.b show the results of the augmented Dickey-Fuller test. The Nikkei and S&P IVs are stationary, but it is of some risk to judge that the FTSE IV is stationary. All the skewness are stationary.

6.4.1.a Does implied volatility spillover?

Table 6.4 gives descriptive statistics on the implied volatility data series (average of ATM calls and puts) for the three markets. The mean IV of the Nikkei is the largest of the three (23%), the FTSE is the second (15%), and the S&P lowest (11%). As might be expected, the FTSE and S&P volatilities are highly correlated (0.53), the Nikkei and S&P somewhat (0.22), and the FTSE and Nikkei only slightly (0.11). Table 6.5.a gives the regression results for transmission of the implied volatility levels (Equations (1a) to (1c))[6]. The level of Nikkei implied volatility is independent of those in any other market. However, the S&P IV affects the FTSE IV a little (coefficient is 0.04), and both the Nikkei and FTSE IVs affect the S&P IV significantly, although the effects

[6] The residual covariances are so low to be negligible (from 2.6485e-06 to 0.00001077).

are small (coefficients are 0.006 and 0.032 respectively).

In the long run (as reflected by levels)[7], there appears to be a small impact of one market's IV on another. Durbin's t-tests[8] indicate that the regression results for Nikkei and S&P are suffered from serial correlation even with lagged dependent variables. However, the changes in implied volatility (reflecting short-run effects) (Table 6.5.b) are more interesting (Equations (1d) to (1f))[9]. The change in S&P IV and Nikkei IV affect positively the next day's change in the IV for FTSE (coefficient = 0.1026 and 0.048 respectively). The change in the FTSE IV affects the change in the S&P IV (coefficient = 0.3308), and in the Nikkei IV (coefficient = 0.1954). Table 6.5.b also indicates that IV is mean-reverting because all autoregression coefficients are negative.

From the results above, we conclude that long-term levels of IV are related for FTSE and S&P, but day-to-day changes are more closely related. When the IV of one market changes, the next-opening market's IV is likely to change in the same direction. Options traders tend to react to the change rather than the level of the other market.

These results for spillovers of IV are different from the results for volatility spillovers found in Chapter 5, using equivalent regressions. Nikkei conditional variance (by GARCH) is explained 0.02 by the S&P variance (not significant for IV change), and FTSE variance is explained 0.13 by the Nikkei variance (0.048 for IV change). The spillovers from Nikkei to S&P are not significant in both the GARCH and IV results. Nikkei conditional variance is

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[7] See Appendix 6.B for more detailed analysis by ECM (error correction model).

[8] Durbins's t-test consists of regressing the SUR residuals at t on the explanatory variable and the one at t-1 and testing the significance of the estimate for coefficient of the residual at t-1.

[9] The covariances are also very low (from -8.1232e-06 to 0.0000628).

significantly explained 0.07 by the FTSE variance, and the same direction of IV spillover is significantly observed (0.1954).

6.4.1.b Does implied volatility depend on index return of the day?

Table 6.5.c and 6.5.d show the results from Equations (1g) and (1h), respectively. We find rather low Durbin-Watson statistics for all three markets by using Equation (1g), as shown in Table 6.5.c. First differences (Equation (1h)) removes the autocorrelation of the original data series, and we find significant and strongly negative relationships in Table 6.5.d between changes in implied volatility and returns in all three markets. Also, there is a significant and strongly positive relationship between change in implied volatility and squared return in all three markets, which is no surprise.

This evidence supports the idea that the changes in implied volatilities are domestically determined by returns and volatilities of the underlying asset of the day. As returns fall in the underlying assets, so implied volatility increases in all three markets.

6.4.2 Is skewness dependent on index return of the day?

Table 6.6.a shows the regression results from estimating Equations (4) in which skewness is related to market return. We find low Durbin-Watson statistics for all three markets by using levels (Equation (4)), so those results are ignored.

Taking differences as in Equation (5) (Table 6.6.b), we find little relationship of skewness to returns, except for the S&P case, for which skewness becomes more negative when volatility (squared returns) rises. The result is consistent with the result for implied volatility as in Table 6.5.d. When the S&P index falls, the implied volatility rises and the smile becomes more left-skewed.

6.4.3 Can Skewness forecast the next-day index returns?

Table 6.7 shows the regression results from estimating Equation (6), which tests whether skewness anticipates next day's returns. Only the FTSE index return is significantly dependent on the skewness level of the day before. The coefficient is positive but its size is small.

6.4.4 How independent is skewness over the world?

Tables 6.8.a and 6.8.b show the regression results of Equation (7a) to (7f)[10]. For all three markets, we find a strong and significant autocorrelation of the skewness, but very little else. Furthermore, Durbin's t-tests indicate that all the three regression results are suffered from serial correlation even with lagged dependent variables. On the other hand, the changes in skewness have no significant interrelationship among the three markets. Thus, we conclude that no spillover of skewness exists among the three markets. Market participants do not seem to watch any movement of skewness of another market day by day. We also examine the 5-day average skewness series (Table 6.9), trying to reduce possible noise in the skewness measures. The data are gapped to avoid overlapping. The Durbin-Watson statistics indicate strong autocorrelation and then is no significant relationship among markets.

6.5. Conclusion

Implied volatilities are correlated among the three markets. The relationship between FTSE and S&P is rather strong. In testing both levels and changes, FTSE and S&P affect each other while Nikkei affects FTSE and FTSE affects Nikkei (slightly). The results differ from the GARCH analysis of

[10] See Appendix 6.B for more detailed analysis by ECM (error correction model).

historical volatility spillover in Chapter 5 in terms of size of effect. The level of IV seems to depend mostly on the domestic market matters than what is happening in other markets.

We find strong and significantly negative relationships between changes in implied volatilities and market returns. The fact indicates that the traders' reaction to the downward movement of the domestic market tends to be large increases of implied volatilities, and vice versa. The squared returns (observed volatilities) have positive correlations to the implied volatility shifts. Traders' forecast seems constructed by the recent domestic market direction and volatility, so that implied volatilities are dependent on the underlying asset returns and volatilities strongly.

All markets have left-skewed volatility smiles. However, there is no relationship between skewness across markets (N.B. *Appendix 6.B shows a different result*). Although a change in IV spills over, the subjective distribution of the underlying index of the market of a market is not affected by the one of a trader in any other market. Traders in these three markets do not seem concerned by the implied distribution formed in other options markets. However, we already know that returns spill over to each other. Because returns on the domestic markets affect changes in implied volatilities, the information contained in changes in skewness may be transferred implicitly in the returns from the other market, then traders forecast their own market return distribution seemingly domestically.

Skewness is not related to market returns, but in the U.K. only, market returns are slightly predicted by skewness. By nature, skewness does not mean the next-day return prediction, but means the predicted distribution of the underlying index returns as of options expiry.

Volatility smiles are local phenomena (N.B. *Appendix 6.B shows a different result*). They change frequently but the changes are not caused by factors which have an international character. There are not waves of bullishness or bearishness which sweep across markets. Nevertheless, we still cannot explain the negative skewness of the volatility smiles observed and their frequent changes[11].

[11] For Nikkei, we have also regressed put/call ratio (daily) to skewness, because the ratio is a bullishness measure of traders in options markets. The result is ambiguous yet. The ratio is negatively correlated to skewness, but Durbin-Watson statistic is 1.17, too low. The ratio is not significantly correlated to the first difference of skewness, although the coefficient is negative.

Table 6.1. Difference in ATM implied volatilities between Calls and Puts

	Nikkei	FTSE	S&P
Mean	0.015502614	-0.004875307	0.02279714
Standard Deviation	0.016978888	0.021998138	0.005105232
# of observation	891	947	928
95% confidence interval is between:	0.01661899 and 0.01438624	-0.00347244 and -0.00627817	0.02312603 and 0.02246825
Mean is:	more than 0	less than 0	more than 0

N.B. Difference is defined as Put IV minus Call IV.
FTSE is negatively biased (calls expensive), others positively.

Table 6.2. Skewness in the Markets

	Nikkei	FTSE	S&P
Mean	-0.06780685	-0.07986825	-0.21972618
Standard Deviation	0.081884986	0.04740339	0.057092702
# of observation	891	947	928
95% confidence interval is between:	-0.06219669 and -0.07296468	-0.07666382 and -0.08270983	-0.21604809 and -0.22340427

N.B. $skew_t = \dfrac{\sigma_t(+2\%) + diff - \sigma_t(-2\%)}{\sigma_t(+2\%)}$

Table 6.3.a Unit Root Test (H_0: Implied volatility is stationary)

	Nikkei	FTSE	S&P
θ	-.055878 (-1.556)	-.010887 (-.326)	-.253322 (-7.743)
β	-.032233 (-3.483)	-.019316 (-3.024)	-.074355 (-4.757)
Lower tail area	.04157	.12784	.00099

t values are in bracket.

$\Delta IV_{x,t} = c + \theta \cdot \Delta IV_{x,t-1} + \beta \cdot IV_{x,t-1}$

Table 6.3.b Unit Root Test (H_0: Skewness is stationary)

	Nikkei	FTSE	S&P
θ	-.285256 (-7.657)	-.200678 (-5.921)	-.240527 (-7.225)
β	-.392192 (-10.394)	-.336490 (-11.017)	-.105689 (-6.229)
Lower tail area	.00000	.00000	.00001

t values are in bracket.

$dskew_{x,t} = c + \theta \cdot dskew_{x,t-1} + \beta \cdot skew_{x,t-1}$

Lower tail area: the probability to make an error if H_0:($\beta = \alpha - 1 = 0$) is rejected at the 5% confidence interval with the table provided by Dickey and Fuller (1981)

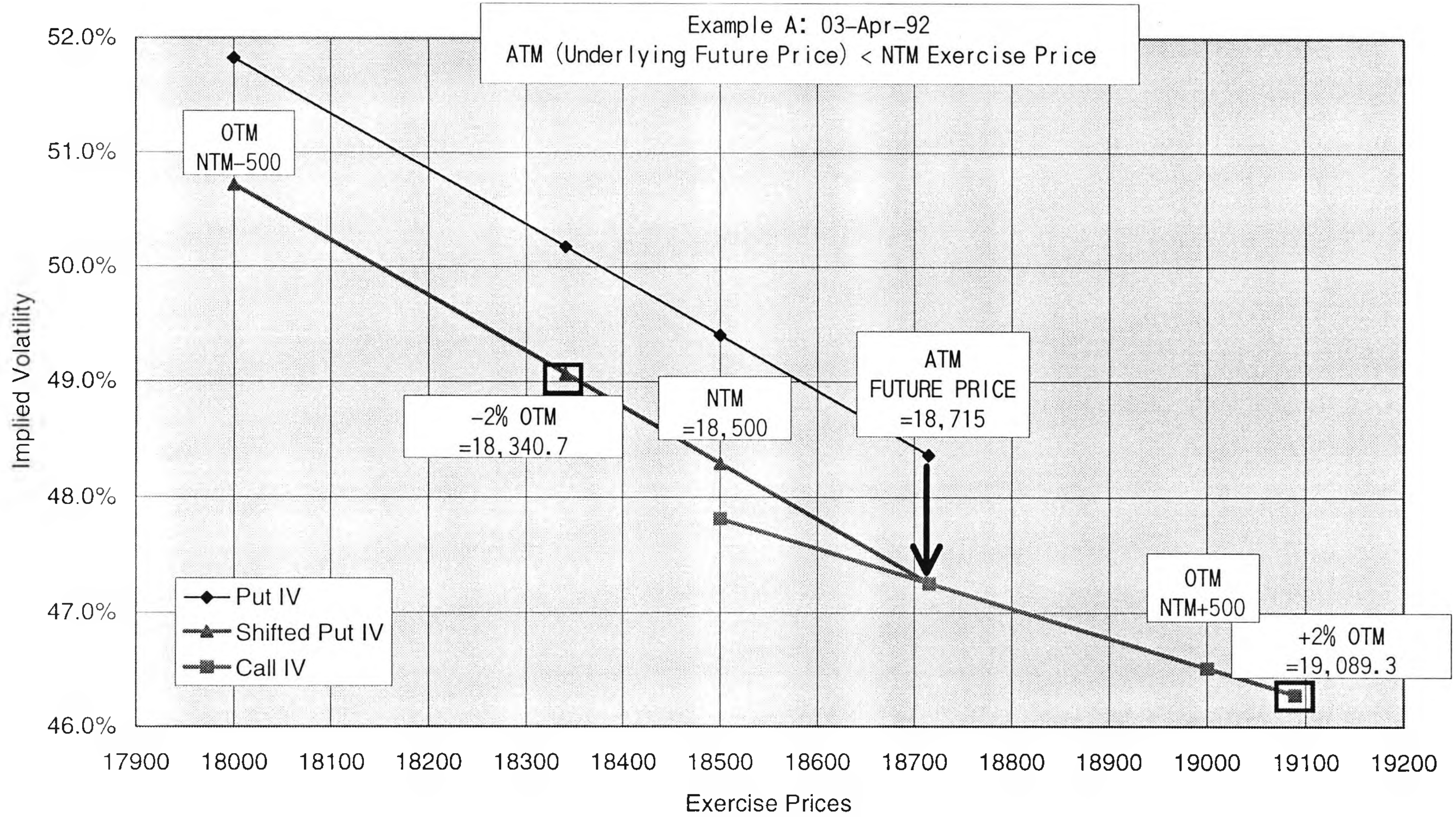

Figure 6.1. a ATM and ±2% Implied Volatility Interpolation: Example A

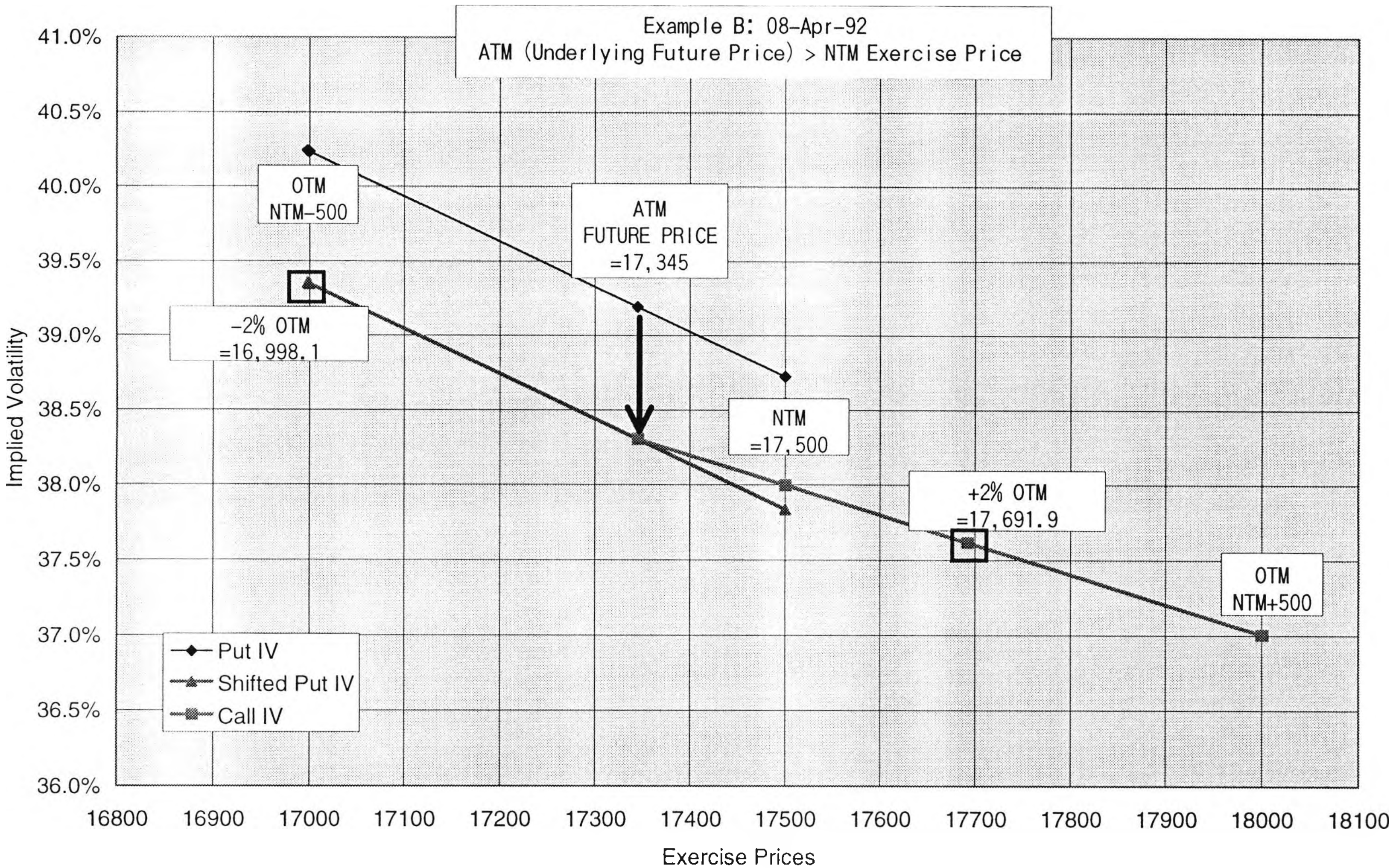

Figure 6.1. b ATM and ±2% Implied Volatility Interpolation: Example B

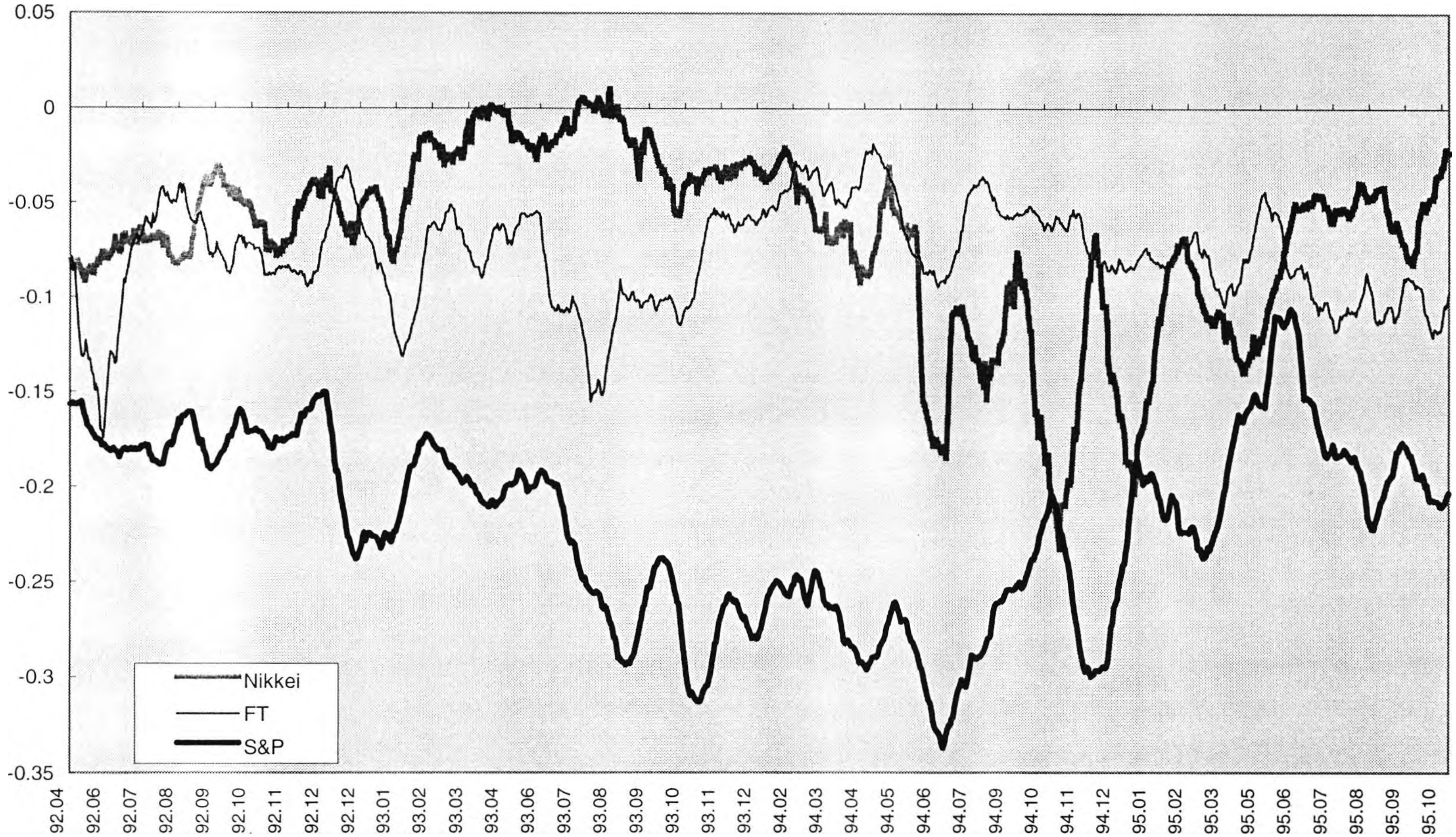

Figure 6.2.a 20-day Moving Average Skewness for Three Markets

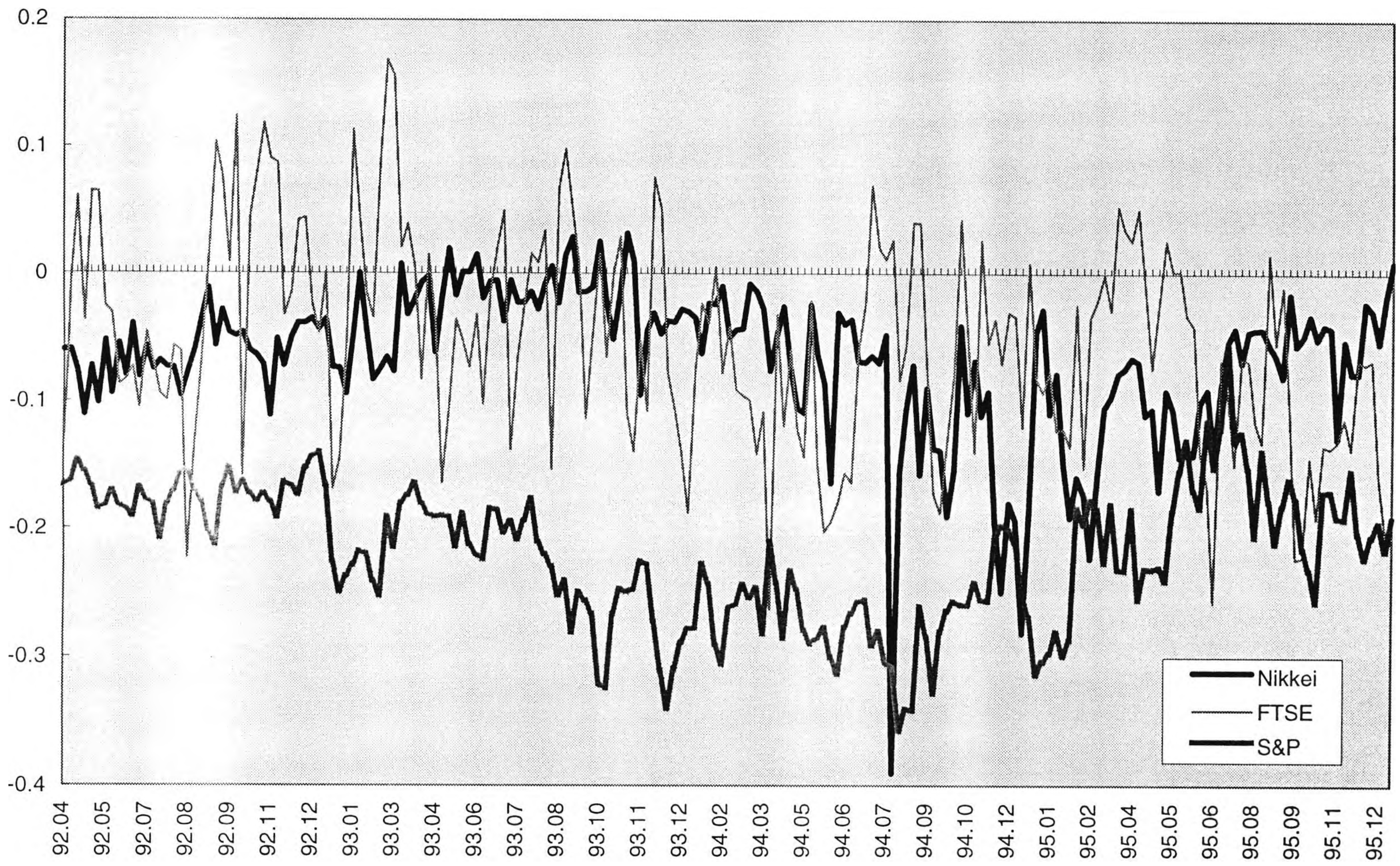

Figure 6.2.b Skewness in the Three Markets

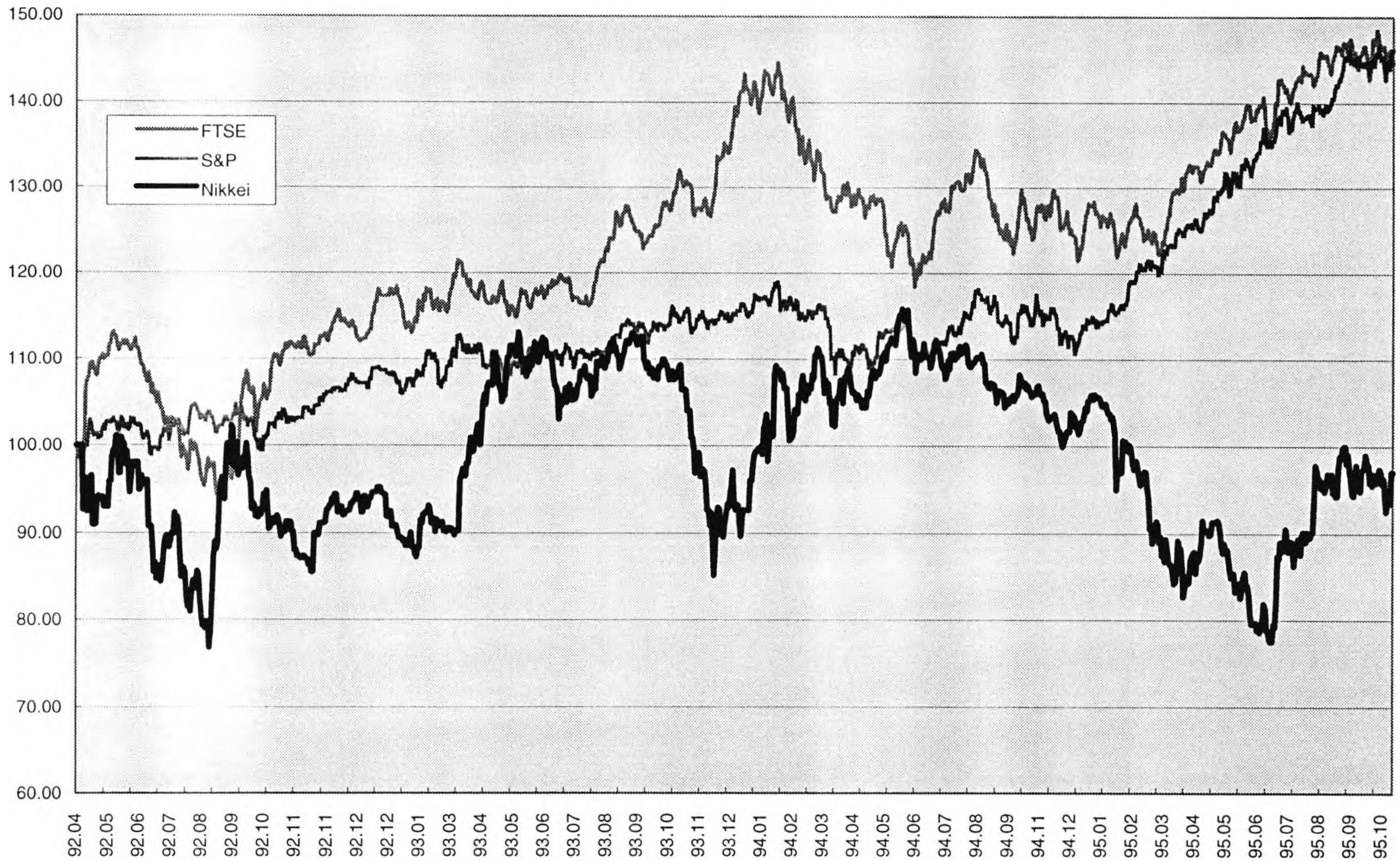

Figure 6.3 Relative Price Movements of the Three Indexes

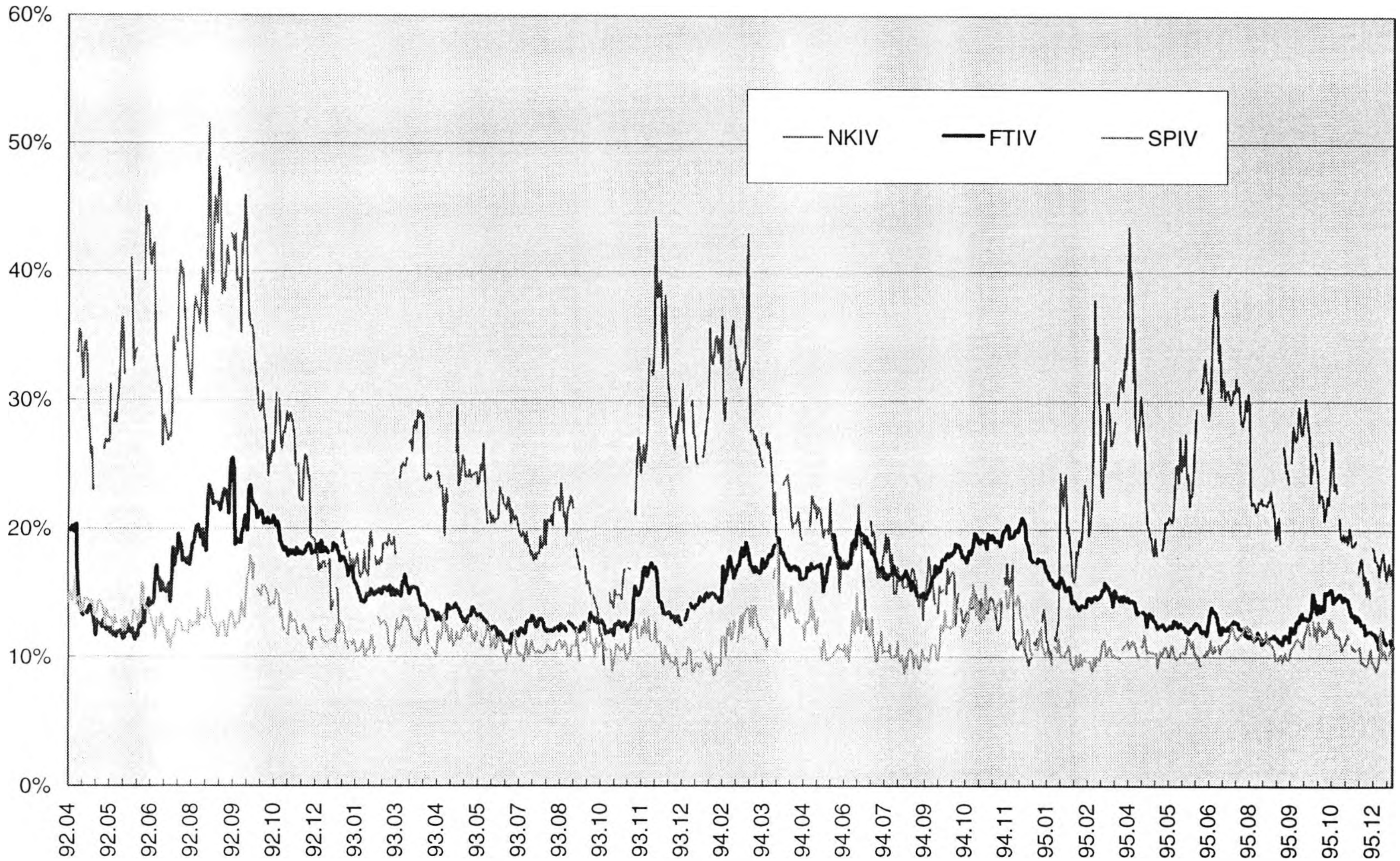

Figure 6.4 Implied Volatilities of the Three Major Markets

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Mean	0.015502614	-0.004875307	0.02279714
Standard Deviation	0.016978888	0.021998138	0.005105232
# of observation	891	947	928
95% confidence interval is between:	0.01661899 and 0.01438624	-0.00347244 and -0.00627817	0.02312603 and 0.02246825
Mean is:	more than 0	less than 0	more than 0

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FTSE is negatively biased (calls expensive), others positively.

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95% confidence interval is between:	-0.06219669 and -0.07296468	-0.07666382 and -0.08270983	-0.21604809 and -0.22340427

N.B. $$skew_t = \frac{\sigma_t(+2\%) + diff - \sigma_t(-2\%)}{\sigma_t(+2\%)}$$

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	Nikkei	FTSE	S&P
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β	-.032233 (-3.483)	-.019316 (-3.024)	-.074355 (-4.757)
Lower tail area	.04157	.12784	.00099

t values are in bracket.

$$\Delta IV_{x,t} = c + \theta \cdot \Delta IV_{x,t-1} + \beta \cdot IV_{x,t-1}$$

Table 6.3.b Unit Root Test (H_0: Skewness is stationary)

	Nikkei	FTSE	S&P
θ	-.285256 (-7.657)	-.200678 (-5.921)	-.240527 (-7.225)
β	-.392192 (-10.394)	-.336490 (-11.017)	-.105689 (-6.229)
Lower tail area	.00000	.00000	.00001

t values are in bracket.

$$dskew_{x,t} = c + \theta \cdot dskew_{x,t-1} + \beta \cdot skew_{x,t-1}$$

Lower tail area: the probability to make an error if H_0:($\beta = \alpha$ -1=0) is rejected at the 5% confidence interval with the table provided by Dickey and Fuller (1981)

Table 6.4. Implied Volatilities (at-the-money) across Markets

	Nikkei (891 data)	FTSE (947 data)	S&P (928 data)
Mean	0.2373991	0.1533680	0.1165869
Standard Deviation	0.0790239	0.0289856	0.0159479
Minimum	0.0931895	0.1072213	0.0847353
Maximum	0.5154700	0.2552574	0.1871148
Correlation to Nikkei	--	--	--
to FTSE	0.11373	--	--
to S&P	0.22851	0.53672	--

Table 6.5.a Transmission of Implied Volatility across Markets - SUR Results

Dependent / Independent	Nikkei	FTSE	S&P
Constant	0.009960 (1.690)	0.0000056 (0.004)	0.009826 (4.939)
Nikkei	0.969702$^{@}$ (99.434)***	-0.001764 (-0.693)	0.006214 (1.897)*
FTSE	0.029705 (0.961)	0.963932$^{@}$ (119.065)***	.032443 (3.079)***
S&P	-0.072490 (-1.226)	0.048930 (3.168)***	0.858836$^{@}$ (42.950)***
R squared	0.9342	0.9658	0.8050
Durbin's t stat	(-1.815)*	(-0.378)	(-5.191)***

N.B. @: autoregressive coefficient (lagged one period)
t values in bracket :*: 5%, **: 2.5%, ***: 1% significant

Table 6.5.b Transmission of Changes in Implied Volatility across Markets - SUR Results

Dependent / Independent	Δ Nikkei	Δ FTSE	Δ S&P
Constant	-0.003716 (-1.193)	-0.000637 (-0.514)	0.000823 (0.365)
Δ Nikkei	-0.073360$^{@}$ (-1.871)*	0.048415 (3.107)***	0.005871 (0.206)
Δ FTSE	0.195431 (2.003)**	-0.084574$^{@}$ (-2.193)**	0.330880 (4.646)***
Δ S&P	0.010104 (0.196)	0.102680 (5.014)***	-0.291126$^{@}$ (-7.751)***
R squared	0.0104	0.0546	0.0979
Durbin's t stat	(0.439)	(-0.206)	(-0.711)

N.B. @: autoregressive coefficient (lagged one period)
t values in bracket :*: 5%, **: 2.5%, ***: 1% significant

Table 6.5.c Factors Determining IV

	Nikkei	FTSE	S&P
Constant	.225263 (79.743)	.150421 (150.334)	.115027 (199.391)
Returns	-.215961 (-1.212)	-.217734 (-1.870)*	-.618633 (-7.037)***
Squared Returns	57.9391 (10.391)***	47.5969 (7.839)***	50.1706 (6.214)***
R squared	.1160	.0627	.0852
F statistics	54.0899	30.8246	40.8688
Durbin-Watson stat	.2533	.1412	.2433

Table 6.5.d Factors Determining Change in IV

	Δ Nikkei	Δ FTSE	Δ S&P
Constant	-.013135 (-5.096)	-.138651e-2 (-1.382)	-.184837e-2 (-0.907)
Returns	-2.45998 (-15.137)***	-2.20653 (-18.911)***	-5.32943 (-17.165)***
Squared Returns	49.2152 (9.673)***	29.2611 (4.809)***	132.395 (4.643)***
R squared	.2687	.2807	.2583
F statistics	151.429	179.747	152.752
Durbin-Watson stat	2.0641	2.2309	2.5342

Table 6.6.a Factors Determining Skewness

	Nikkei	FTSE	S&P
Constant	-.071650 (-22.791)	-.082185 (-51.083)	-.223336 (-102.521)
Returns	.302671 (1.527)	-1.67226 (-8.932)***	.295998 (0.891)
Squared Returns	9.63756 (1.553)	56.0396 (5.740)***	101.027 (3.314)***
R squared	.6174e-2	.0954	.0138
F statistics	2.5597	48.5883	6.1617
Durbin-Watson stat	1.0495	.8515	.2505

Table 6.6.b Factors Determining Changes in Skewness

	Δ Nikkei	Δ FTSE	Δ S&P
Constant	-1.45213 (-0.892)	-0.455168 (-0.971)	.027579 (3.502)
Returns	-12.5634 (-0.122)	59.7632 (1.096)	-1.37605 (1.146)
Squared Returns	1471.48 (0.458)	-313.541 (-0.110)	-316.727 (-2.874)***
R squared	.2645e-2	.1313e-2	.0114
F statistics	.1090	.6056	5.081
Durbin-Watson stat	1.9910	2.0086	2.3093

N.B. t values in bracket :*: 5%, **: 2.5%, ***: 1% significant

Table 6.7 Returns as a Function of Skewness on the Day Before

	Nikkei	FTSE	S&P
Constant	.564033e-03 (0.854)	-.147978e-03 (-0.282)	.424583e-03 (0.549)
Skew (the day before)	.434371e-02 (0.695)	.759690e-02 (-1.335))	-.201861e-03 (-0.059)
R squared	.5865e-03	.1930e-02	.3991e-05
Durbin-Watson stat	1.9238	1.8284	1.9781

t values in bracket

Table 6.8.a Skewness across Markets - SUR Results

	Nikkei	FTSE	S&P
Constant	-0.044909 (-3.665)	-0.039795 (-6.351)	-0.030312 (-6.379)
Nikkei	0.440612@ (12.989)	0.003930 (0.232)	0.013072 (1.009)
FTSE	-0.167529 (-2.703)***	0.559332@ (17.650)	-0.013487 (-0.567)
S&P	0.037128 (0.790)	-0.024058 (-1.007)	0.865390@ (47.186)
R squared	0.2007	0.3047	0.7554
Durbin's t stat	(-2.238)***	(-1.826)*	(-5.544)***

N.B. @ marked for autocorrelation t values in bracket

Table 6.8.b <u>Changes</u> in Skewness across Markets - SUR Results

	Δ Nikkei	Δ FTSE	Δ S&P
Constant	-1.382691 (-0.736)	-0.639483 (-1.050)	0.024358 (2.763)
Nikkei	-0.000513@ (-0.013)	-0.000020493 (-0.002))	0.000056266 (0.305)
FTSE	-0.001094 (-0.009)	0.000448@ (0.011)	0.000344 (0.605)
S&P	-0.516591 (-0.063)	0.730023 (0.273)	-0.182533@ (-4.716)
R squared	0.0000	0.0001	0.0338
Durbin's t stat	(-0.007)	(-0.008)	(-0.152)

N.B. @ marked for autocorrelation t values in bracket

Table 6.9. 5-day Average of Skewness across Markets

	Nikkei	FTSE	S&P
Constant	-.044431 (-2.504)	-.022324 (-0.854)	-.214489 (-34.309)
Nikkei	--	.049348 (0.470)	.044887 (0.678)
FTSE	.045135 (0.900)	--	.045949 (1.005)
S&P	.090993 (1.159)	.123383 (1.082)	--
R squared	.0120	.0077	.0079
Durbin-Watson stat	0.7440	0.9215	0.3261

N.B. t values in bracket *: 5%, **: 2.5%, ***: 1% significant
5-day average is calculated, if any of three markets is open.

Appendix 6.A. Barone-Adesi and Whaley Model

This Appendix shows how American options on futures are priced, which was suggested by Barone-Adesi and Whaley (1987).

Black and Scholes (1973) may be adjusted for pricing options on futures easily as below (it is called Black's model).

$$C = e^{-rt}(F \cdot N(d_1) - K \cdot N(d_2))$$
$$P = e^{-rt}(K \cdot N(-d_2) - F \cdot N(-d_1))$$
$$where \qquad \text{(A1)}$$
$$d_1 = \frac{\ln(F / K) + (\sigma^2 / 2)t}{\sigma\sqrt{t}}$$
$$d_2 = d_1 - \sigma\sqrt{t}$$

C: European-type Call Price $\qquad$ P: European-type Put Price

F: Underlying future price $\qquad$ K: Exercise Price

$N(\cdot)$: Normal distribution density function

σ : Annual volatility

Because American option can be exercised if it is appropriate. If F was more than the price at which it is appropriate to exercise in terms of call options, call price would equal $F - X$. When F^* is defined as the boundary condition to exercise options, American-type options are priced as below:

$$c(F) = \begin{cases} C(F) + A_2(\frac{F}{F^*})^{v2} & \cdots \quad F < F^* \\ F - K & \cdots \quad F \geq F^* \end{cases} \qquad \text{(A2)}$$

$$p(F) = \begin{cases} P(F) + A_1(\frac{F}{F^{**}})^{v1} & \cdots \quad F > F^{**} \\ K - F & \cdots \quad F \leq F^{**} \end{cases} \qquad \text{(A3)}$$

where,

$$v_1 = \left[-(\frac{2r}{\sigma^2} - 1) - \sqrt{(\frac{2r}{\sigma^2} - 1)^2 + \frac{4 \cdot 2r/\sigma^2}{1 - e^{-rt}}} \right] / 2$$

$$v_2 = \left[-(\frac{2r}{\sigma^2} - 1) + \sqrt{(\frac{2r}{\sigma^2} - 1)^2 + \frac{4 \cdot 2r/\sigma^2}{1 - e^{-rt}}} \right] / 2$$

$$A_1 = -\left(\frac{F^{**}}{v_1}\right)\left\{1 - N[-d_1(F^{**})]\right\}$$

$$A_2 = \left(\frac{F^{*}}{v_2}\right)\left\{1 - N[d_1(F^{*})]\right\}$$

$$d_1(F) = \frac{\ln(F/K) + (r + \sigma^2/2)\cdot t}{\sigma\sqrt{t}}$$

The boundary condition is calculated by a numerical method. F^* and F^{**} should have relationships as follow:

$$\text{CALL:}\quad F^* - K = C(F^*) + \{1 - N[d_1(F^*)]\}\frac{F^*}{v_2}$$

$$\text{PUT:}\quad K - F^{**} = P(F^{**}) - \{1 - N[d_1(F^{**})]\}\frac{F^{**}}{v_1}$$

Appendix 6.B. Error Correction Model for the Long-run Relationship

In this Appendix, we examine if the implied volatility levels ***and*** their skewness of major markets spill over to one another. Although the basic results are shown in the chapter, we introduce an Error Correction Model (ECM) in order to examine a long-run relationship among six time series, that is, IV and skewness series of Nikkei, FTSE, and S&P.

Firstly, the cointegration test is examined. If we assume an equilibrium relationship between two time series, we employ the cointegration test to show the existence of the long-term relationship, proposed by Engle and Granger (1987). In the present context (implied volatility), we have:

$$IV_{ft,t} = c + \beta_{ft} \cdot IV_{nk,t} \qquad \text{(B1a)}$$

$$IV_{sp,t} = c + \beta_{sp} \cdot IV_{ft,t} \qquad \text{(B1b)}$$

$$IV_{nk,t} = c + \beta_{nk} \cdot IV_{sp,t-1} \qquad \text{(B1c)}$$

where $IV_{x,t}$ is implied volatility of the x (Nikkei, FTSE, or S&P) market at time t. IV is replaced for *Skew* when we examine the skewness of the three markets. If a long-term equilibrium relationship exists between two markets out of the three, the residual of the regression will be stationary. Stationarity is tested by the Engle-Granger test (EG test), that is:

$$\Delta v_{x,t} = (\alpha - 1) v_{x,t-1} + u_{x,t} \qquad \text{(B2)}$$

where v_t is a residual vector of the volatility regression (B1). The ordinary least square method is employed to estimate if α -1 is equal to zero or not, testing with the Dickey-Fuller critical value table for the t statistic of the coefficient.

Then, the Error Correction Model (ECM) is examined. If two series are cointegrated, an error from the equilibrium is corrected because the error series is stationary. Assuming that the implied volatility of one market is dependent on the implied volatility of the other market which has just closed, as described in Equations (B1a) to (B1c), the equilibrium error is defined as:

$$v_t = IV_{x,t} - c - \beta_x \cdot IV_{x-1,t-1} \quad \text{(B3)}$$

In ECM, the dispersion from the equilibrium is a dependent variable (in the case of Nikkei and S&P as an example) as below.

$$\Delta IV_{nk,t} = a_{nk} + b_{nk} \cdot v_{t-1} + c_{nk} \Delta IV_{sp,t-1} \quad \text{(B4)}$$

$$\Delta IV_{sp,t} = a_{sp} + b_{sp} \cdot v_{t-1} + c_{sp} \Delta IV_{nk,t} \quad \text{(B5)}$$

When the coefficient of the residual in the IV_{nk} equation (b_{nk}) is negative, it means that IV_{nk} corrects the error by changing its own level. When b_{sp} is positive, $IV_{sp,t}$ also corrects the error by changing its own level (all the *IV*s are replaced for *Skew* in the case of examining skewness).

The EG test results are shown in Table B1. There does not seem to be a long-run relationship in IV from Nikkei to FTSE. However, FTSE affects S&P in terms of the IV level. Also, S&P seems to be affected by Nikkei and the Nikkei by S&P in IV. Table B2 shows how the correction process occurs in each combination. FTSE and S&P are corrected to each other to their long-term relationship. On the other hand, only S&P adjusts its IV level to the equilibrium while Nikkei ignores the S&P change in IV.

Table B3 is the EG test results for skewness. In contrast to the results in Section 4.4 in Chapter 5, there seems to be significant relationships among the skewness of the three markets. In all the combinations, the coefficients of residuals are significantly negative. The strongest relationship is from FTSE to S&P, shown as the coefficient of skewness (-.103522). The correction processes to the equilibrium level are shown in Table B4. Between Nikkei and

FTSE, and between FTSE and S&P, skewness is corrected to each other. Both have negative coefficients of change in residuals. Between S&P and Nikkei, we have a confusing result because we have no correction from Nikkei to S&P under the equilibrium relationship from the S&P skewness at t-1 to the Nikkei skewness at t, but a significantly negative (coefficient is -.336211) correction from Nikkei to S&P under the equilibrium relationship from the Nikkei skewness at t to the S&P skewness at t.

In conclusion, we find significant relationships in IVs and skewness among the three markets in the long run (co-integrated), except for the relationship from the Nikkei to FTSE IVs. The correction processes in the short run are varied, but all the processes are significantly corrected to each other.

Table B1. Co-integration (EG) Test Results (Implied Volatility Level)

Nikkei to FTSE

C	NKIV	Residual	
.144136	.039727	-.020098	Lower tail area
(46.393)	(3.202)	(-3.098)	.24179

FTSE to S&P

C	FTIV	Residual	
.072967	.285132	-.143246	Lower tail area
(30.386)	(18.500)	(-7.983)	.00000

S&P to Nikkei

C	$SPIV_{t-1}$	Residual	
.111110	1.09184	-.043854	Lower tail area
(5.629)	(6.496)	(-4.287)	.01192

Nikkei to S&P

C	NKIV	Residual	
.105558	.044884	-.119759	Lower tail area
(63.444)	(6.758)	(-7.138)	.00000

Table B2. Error Correction Model (ECM) Results (Implied Volatility Level)

Nikkei vs. FTSE

	C	Δ Residual	$\Delta\ NKIV_t$	R squared
Δ FTIV	-.146899e-3	-.058192	.029495	.0143
	(-0.741)	(-1.543)	(3.009)***	
	C	Δ Residual	$\Delta\ FTIV_{t-1}$	R squared
Δ NKIV	-.142331e-2	1.47705	-1.08770	.0134
	(-1.907)*	(1.949)*	(-1.448)	

FTSE vs. S&P

	C	Δ Residual	$\Delta\ FTIV_t$	R squared
Δ SPIV	-.154597e-3	-.360159	.249525	.1319
	(-0.638)	(-10.323)***	(5.761)***	
	C	Δ Residual	$\Delta\ SPIV_{t-1}$	R squared
Δ FTIV	-.152818e-3	.149067	.264080e-2	.0358
	(-0.793)	(1.768)*	(0.032)	

S&P vs. Nikkei

	C	Δ Residual	$\Delta\ NKIV_t$	R squared
Δ SPIV	-.150169e-2	-.063853	.021774	.4535e-2
	(-1.908)	(-1.737)*	(0.194)	
	C	Δ Residual	$\Delta\ SPIV_{t-1}$	R squared
Δ NKIV	-.420858e-4	.574168e-3	.016974	.2429e-2
	(-0.151)	(0.045)	(1.242)	

Nikkei vs. S&P

	C	Δ Residual	$\Delta\ SPIV_{t-1}$	R squared
Δ NKIV	-.165614e-2	-.118424	.143501	.2879e-3
	(-2.163)	(-0.308)	(0.389)	
	C	Δ Residual	$\Delta\ NKIV_t$	R squared
Δ SPIV	-.161732e-3	-.311114	.016371	.0971
	(-0.627)	(-8.423)***	(1.286)	

Table B3. Co-integration (EG) Test Results ***(Skewness)***

Nikkei to FTSE

C	NK skew	Residual	
-.082330	-.044648	-.442026	Lower tail area
(-40.426)	(-2.339)	(-14.4887)	.00000

FTSE to S&P

C	FT skew	Residual	
-.227982	-.103522	-.137369	Lower tail area
(-61.681)	(-2.606)	(-7.966)	.00000

S&P to Nikkei

C	SP skew$_{t-1}$	Residual	
-.053861	.062001	-.558026	Lower tail area
(-4.865)	(1.270)	(-16.728)	.00000

Nikkei to S&P

C	NK skew	Residual	
-.217421	.035619	-.134358	Lower tail area
(-85.094)	(1.489)	(-7.441)	.00000

Table B4. Error Correction Model (ECM) Results ***(Skewness)***

Nikkei vs. FTSE

	C	Δ Residual	Δ NK skew$_t$	R squared
Δ FT skew	-.144898e-3	-.506177	-.016783	.1976
	(-0.105)	(-13.323)***	(1.028)	
	C	Δ Residual	Δ FT skew$_{t-1}$	R squared
Δ NK skew	-.109938e-2	-.578051	.324994	.0175
	(-0.350)	(-3.171)***	(2.008)**	

FTSE vs. S&P

	C	Δ Residual	Δ FT skew$_t$	R squared
Δ SP skew	-.468996e-3	-.351191	.740016e-02	.1171
	(-0.475)	(-10.239)***	(0.326)	
	C	Δ Residual	Δ SP skew$_{t-1}$	R squared
Δ FT skew	-.197155e-3	-.870735	.814678	.0310
	(0.133)	(-5.149)***	(4.919)***	

S&P vs. Nikkei

	C	Δ Residual	Δ NK skew$_t$	R squared
Δ SP skew	-.111497e-2	.241765e-2	.016685	.1984e-2
	(-0.936)	(0.118)	(0.994)	
	C	Δ Residual	Δ SP skew$_{t-1}$	R squared
Δ NK skew	-.122690e-2	-.676691	-.914362	.3259
	(-0.450)	(-17.902)***	(-0.098)	

Nikkei vs. S&P

	C	Δ Residual	Δ SP skew$_{t-1}$	R squared
Δ NK skew	.577663e-3	1.69981	-1.72617	.0273
	(0.179)	(4.244)***	(-4.444)***	
	C	Δ Residual	Δ NK skew$_t$	R squared
Δ SP skew	-.990423e-3	-.336211	.013234	.1085
	(-0.909)	(-8.971)***	(1.048)	

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7

The Implied Volatility Term Structure:

Cointegration of the Short- and Long-term Implied Volatilities

Naoki Kamiyama

7. The Implied Volatility Term Structure: Cointegration of the Short- and Long-term Implied Volatilities

7.1. Introduction

Implied volatility is usually derived from the Black-Scholes option pricing model which was originally described in the seminal paper, Black and Scholes (1973). These days, many market participants have noticed that a term structure of implied volatility exists, that is, different levels of implied volatility are observed for short- and long-term maturities. This fact implies that a constant volatility does not normally hold in the real world, and especially for practitioners, volatility of volatility is rather large for the short-term options. As shown in Table 7.1, short-term implied volatility for Nikkei options is in the range from 7% to 52% in the observation period, compared with the range of long-term implied volatility from 11% to 34%. For the risk management and evaluation purposes of trader, it is impossible to ignore the term structure of implied volatilities.

There are a few papers which focus on the implied volatility term structure analysis. Heynen, Kemna, and Vorst (1994) examined the term structure of implied volatilities in the Dutch equity index options market, and by utilising the GARCH frameworks to explain the term structure effect. Xu and Taylor (1994) used the Kalman Filtering to estimate the term structure of implied volatilities in the foreign exchange options-on-futures markets. Takezawa and Shiraishi (1995) tested the Tokyo Currency Option Market by using the similar volatility forecasting models to the model employed by Heynen, Kemna, and Vorst (1994). They concluded that overreaction is observed in implied volatilities compared with all the mean-reverting, GARCH, and EGARCH forecasts.

Cointegration analysis has been utilised for many kinds of econometric research in the recent years, but no application seems exist for implied volatility yet. The classic paper in this area is Engle and Granger (1987), who show the framework of cointegration and error correction. For analysing the term structure of interest rates following Engle and Granger (1987), Bradley and Lumpkin (1992) find the cointegration of seven rates in the Treasury yield curve. A recent example of the methodology is performed by Hiraki and Takezawa (1995) for the term structure of the Japanese Yen-denominated interest rate swap market.

The purpose of this chapter is to prove the existence of the implied volatiity term structure and to examine the cointegrated relationship between the short- and long-term implied volatilities in the Nikkei index options on futures market, and derive any causality to drive each other when the volatility level dynamically changes. As Alexander (1994) points out, it is misleading to use simple correlation analysis to see the relationship of short-term implied volatility to long-term implied volatility, because that assumes a stable (or constant) correlation that is too simplified. A dynamic change in time series of implied volatilities can be captured by cointegration with greater accuracy than by correlation. In addition, the error correction model (ECM) associated with cointegration analysis provides a dynamic correction process and its direction. In employing cointegration and error correction model for the analysis of the implied volatility term structure, we obtain an accurate result in association of volatilities and their dynamic correction process.

The reason for the existence of the term structure is thought to be due to the mean-reversion of an underlying asset price. One possibility is that volatility is mean-reverting. The other is suggested by Gemmill and Thomas (1995) that the mean-reversion of the asset price leads the smile effect of the warrants on investment trusts market. The stronger smile effect, as time to maturity becomes longer, implies that the term structure of implied volatilities is also driven by the mean-reversion of asset prices.

The chapter is written as follows. In Section 2, the method undertaken is explained. Section 3 describes the data utilised in this chapter, and the test results are shown in Section 4. The conclusions are given in Section 5.

7.2. The Method

Firstly, stationarity is examined for two series of data, short- and long-term implied volatilities. The Dickey-Fuller Test is employed and formulated as below.

$$\Delta y_t = \beta_0 + \beta_1 \Delta y_{t-1} + (\alpha - 1) y_{t-1} + \beta_2 \cdot dummy + u_t \quad (1)$$

where the weekend *dummy* is 1 if the data is just after weekend. The theory behind this test is explained in many econometrics books such as Greene (1993). Generally, y_t would be stationary if $|\alpha|$ was less than 1 in the equation of;

$$y_t = \mu + \alpha\, y_{t-1} + u_t \quad (2)$$

where μ is a constant and u is a white noise term. This equation can be transformed with consideration of the auto-regressive first difference of the series. The t statistics which tell us whether α -1=0 can be utilised with the critical value table provided by Dickey and Fuller (1983)[1].

[1] However, we should note that distribution of IV is non-normal, and the result is likely to contain measurement error.

Secondly, the cointegration test is examined. If two time series are nonstationary and we assume an equilibrium relationship between the two, we employ the cointegration test to show the existence of the long-term relationship, proposed by Engle and Granger (1987). In the present context, we have:

$$\text{Liv} = \beta_0 + \beta_1 \text{Siv} + v_t \qquad (3)$$

where,

Liv: Long-term Implied Volatility

Siv: Short-term Implied Volatility

If the long-term equilibrium relationship exists between the long- and short-term implied volatilities, the residual of the regression is stationary. The stationarity is tested by the Engle-Granger test (EG test), that is:

$$\Delta v_t = (\alpha - 1) v_{t-1} + u_t \qquad (4)$$

where v_t is a residual vector of the volatility regression (3). In the similar manner to the unit root test as described above, the ordinary least square method is employed to test if α -1 is equal to zero or not, with the Dickey-Fuller critical value table for the t statistic of the coefficient.

Thirdly, the Error Correction Model (ECM) is examined. If two series are cointegrated, an error from the equilibrium is corrected because the error series is stationary. Assuming that the long-term volatility is dependent on the short-term volatility as described in Equation (3), the equilibrium error is defined as:

$$v_{t-1} = \text{Liv}_{t-1} - \beta_{0,t-1} - \beta_{1,t-1} \text{Siv} \qquad (5)$$

In ECM, the dispersion from the equilibrium is a dependent variable as below.

$$\Delta \, Siv_t = \beta_{S0} + \beta_{S1} \cdot v_{t-1} + \beta_{S2} \cdot \Delta \, Liv_{t-1} \quad (6)$$

$$\Delta \, Liv_t = \beta_{L0} + \beta_{L1} \cdot v_{t-1} + \beta_{L2} \cdot \Delta \, Siv_{t-1} \quad (7)$$

When the coefficient of the residual in the Siv equation (β_{S1}) is negative, it means that Siv corrects the error by changing its own level. When β_{L1} is positive, Liv also corrects the error by changing its own level.

7.3. The Data

The Nikkei option on future and the underlying future settlement prices data are provided from the Singapore International Monetary Exchange (SIMEX). In order to calculate the implied volatilities, the daily three month CD rates are used for all the maturity dates. The period examined is from 01 April 1992 to 28 April 1995 and the total number of the records is 748; (originally 752, but the 4 extraordinary data are omitted).

The options to be examined here are screened as below. The short-term option is defined as the option with the shortest maturity on the day, but the options with less-than-5-day to maturity are omitted beforehand. The long-term option is the option with the second longest time to maturity on the day; (not the longest maturity option, as they are infrequently. After the screening, the short-term options have time to maturity within the range of 0.016 to 0.104 years (0.056 years average). The long-term options have time to maturity within the range of 0.402 to 1.002 years (0.797 years average)[2]. From the short- and long-term options, the near-the-money call and put options are chosen. The near-the-money option is defined as the option with the smallest absolute value of (underlying futures price - exercise price). The data are divided into 5

[2] Note that we should be cautious to the result of this empirical work because the long-term maturity has such a wide range.

subsets with similar numbers of records (approx. 150) in each subset. Detailed information on the data is shown in Table 7.1.

The implied volatility is calculated by the Black's options on futures pricing formula. The formula is as below.

$$
\begin{aligned}
&C = e^{-rt}(F \cdot N(d_1) - K \cdot N(d_2)) \\
&P = e^{-rt}(K \cdot N(-d_2) - F \cdot N(-d_1)) \\
&where \\
&d_1 = \frac{\ln(F / K) + (\sigma^2 / 2)t}{\sigma\sqrt{t}} \\
&d_2 = d_1 - \sigma\sqrt{t}
\end{aligned}
\qquad (8)
$$

C: Call Price	P: Put Price
F: Underlying future price	K: Exercise Price

N(·): Normal distribution density function

σ : Annual volatility

The implied volatilities of the near-the-money call and put of the day are averaged, and the average is defined as the implied volatility of the trading day. This way we set the short- and long-term volatilities of a specific trading day, which are examined if they are cointegrated.

7.4. Test Results

Significant are these differences in the implied volatilities of short- and long-term options. The short-term IV is in the range of 6.79% to 52.37% and the mean is 23.51%. On the other hand, the long-term one is in the range of 11.34% to 33.58% and the mean is 20.86%. We employ the paired-sample t test as below. We set the null hypothesis that the mean of the difference between the short-term implied volatility (x_1) and the long-term implied volatility (x_2) equals zero. The t value is defined as:

$$t = \frac{x}{\sigma / \sqrt{n}} \qquad (9)$$

x: the mean of $x_1 - x_2$

1 denotes short-term implied volatility

2 denotes long-term implied volatility

σ : the standard deviation

n: the number of observation

The hypothesis of equality is rejected for the whole sample and sub-periods except for Subset 4 (February to September 1994). Table 7.1 shows the t values for the test. Then we conclude that the term structure does exist in the Nikkei 225 options market at SIMEX.

As we expected, two series of data, short- and long-term implied volatilities are likely to be nonstationary and cointegrated to each other. There is a long-time equilibrium between the two, and correction is made when a dispersion occurs from the equilibrium. Table 7.2 shows that both the short- and long-term volatility series are likely to be nonstationary, except for the short-term volatility of the All-Data set for which a unit root hypothesis is rejected[3].

Table 7.3 shows the EG test results. We find evidence that the short- and long-term implied volatilities are cointegrated. The regression result between the first difference of the residuals and the lagged residual is negative 11% in the All-Data set with the lower tail area of 0.0%. The lower tail areas in all the subsets are less than 5% except for the Subset 2 (Nov 92 - Jun 93), so that the short- and long-term implied volatilities are cointegrated in most.

The long-term relationship between the short- and the long-term implied

[3] Table 7.2 does not show either series is integrated, because Type II errors are common.

volatilities is also observed in Table 7.3. In the All-Data set (Column 2), the long-term volatility is explained 48% by the short-term one with the R squared of 67%. Period 1 has high volatility (averaging 29.15% for long-term options (Liv)). The result is that there is a weaker long-term volatilities in Subset 1 (see Table 7.3, Column 3). The R squared falls to 8%, but there is still significant cointegration. When the volatility is very low or high, participants may lose the way to make decisions systematically. The long-run volatility relationship between short- and long-term IV may be forgotten in such periods tentatively.

Both the long- and short-term implied volatilities correct an error from the equilibrium, as shown in Table 7.4. In the All-Data subset (Column 2), the short-term volatility has a significant positive coefficient (0.463) for the residual at t-1. However, the result is not consistent in other subsets, with a coefficient significant at the 1% level in only one period. On the other hand, the long-term implied volatility has significant negative coefficients for all the 4 subsets, with a range from -0.545 to -0.339. From Equations (5), (6), and (7), both the positive coefficients of the residual in the short-term volatility equation and the negative coefficients in the long-term residual imply that the short- and long-term volatilities move to correct any disturbance to their long-run equilibrium.

7.5. Conclusions

The relationship between the short- and long-term volatilities are stable in the long-run. The direction and magnitude of the dispersion from the equilibrium give us some information on the short-run movement of the volatility levels and spreads. When the error from the equilibrium expands, both the short- and long-term volatilities tend to revert so that the equilibrium level is re-established.

The reason why the short-term IV is significantly smaller than the long-term IV and those are cointegrated may be due to the mean-reversion effects of the underlying asset prices. Gemmill and Thomas (1995) examine the implied volatility in the warrant markets on investment trusts in the U.K., and conclude that the time effect of the smile (the lower the volatility, the longer time to maturity) is because of the mean-reverting asset prices. On average, one can observe the significantly larger IV in the short-term options than in the long-term options in the case of the Nikkei 225 options on futures listed on SIMEX, as shown in Table 7.1. The mean-reversion of an asset price pulls-in the tails of the return distribution.

For practical purposes, there is an opportunity to trade the implied volatility spread between the short- and long-term volatilities with the information out of the error correction analysis.

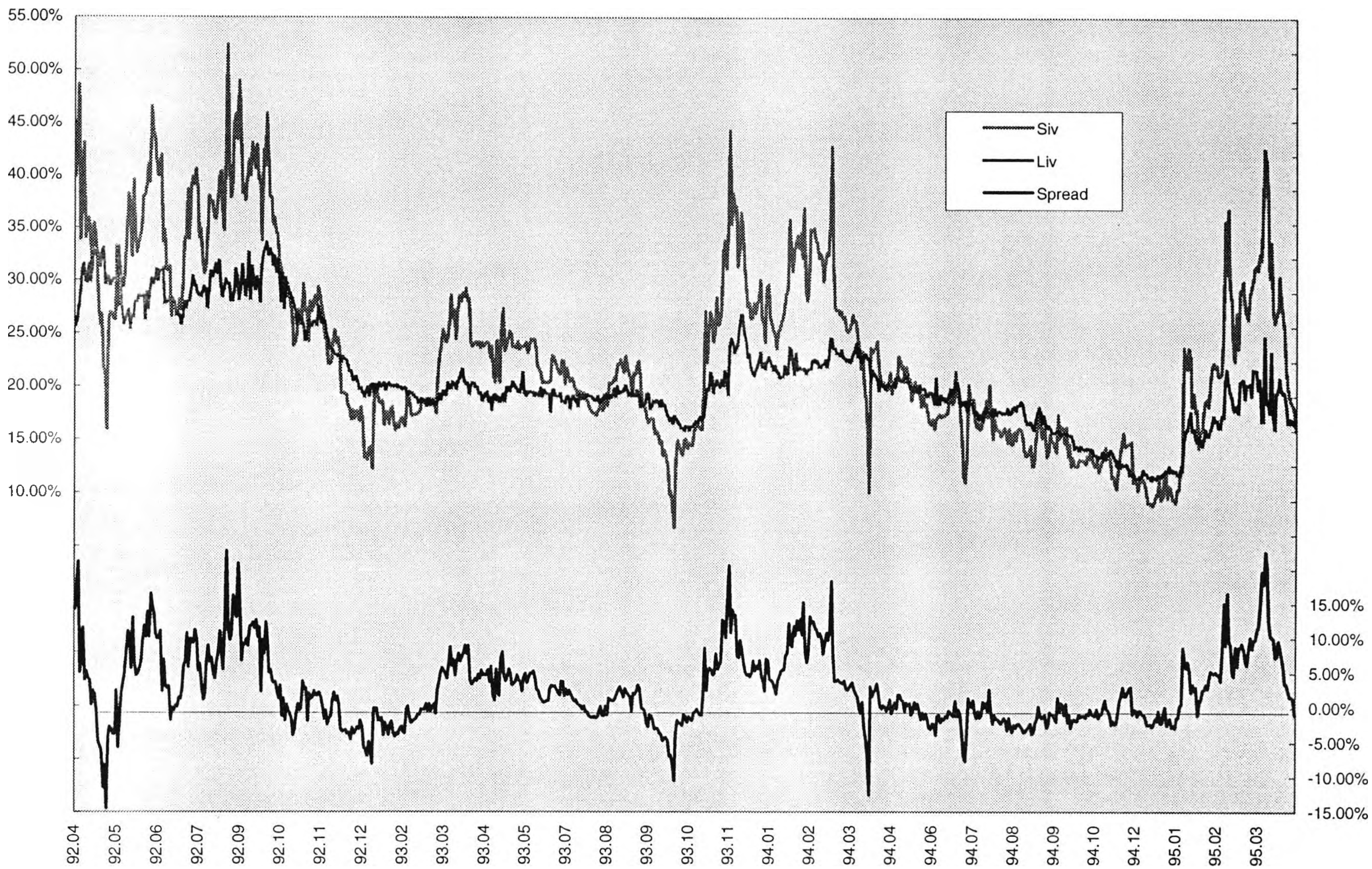

Figure 7.1. Short- and Long-term Volatilities

Table 7.1. Data Summary

H_0: Average of Short-term IV = Average of Long-term IV

All data from 01-Apr-92 to 28-Apr-95 (748 records)

	Short-term Options		Long-term Options	
	Year to Maturity	Implied Volatility	Year to Maturity	Implied Volatility
Minimum	0.016	6.79%	0.402	11.34%
Maximum	0.104	52.37%	1.002	33.58%
Average	0.056	23.51%	0.797	20.86%
Difference in Average		2.64%	t = 13.649	Reject H_0

Subset 1 from 01-Apr-92 to 13-Nov-92 (148 records)

	Short-term Options		Long-term Options	
	Year to Maturity	Implied Volatility	Year to Maturity	Implied Volatility
Minimum	0.016	15.94%	0.402	24.36%
Maximum	0.104	52.37%	0.808	33.58%
Average	0.057	34.58%	0.631	29.15%
Difference in Average		5.43%	t = 10.568	Reject H_0

Subset 2 from 16-Nov-92 to 30-Jun-93 (151 records)

	Short-term Options		Long-term Options	
	Year to Maturity	Implied Volatility	Year to Maturity	Implied Volatility
Minimum	0.016	12.27%	0.498	17.70%
Maximum	0.104	29.41%	0.994	27.52%
Average	0.054	21.51%	0.789	20.14%
Difference in Average		1.37%	t = 4.493	Reject H_0

Subset 3 from 01-Jul-93 to 08-Feb-94 (150 records)

	Short-term Options		Long-term Options	
	Year to Maturity	Implied Volatility	Year to Maturity	Implied Volatility
Minimum	0.016	6.79%	0.736	15.93%
Maximum	0.104	44.33%	0.994	26.79%
Average	0.056	22.81%	0.866	19.91%
Difference in Average		2.91%	t = 6.860	Reject H_0

Subset 4 from 09-Feb-94 to 19-Sep-94 (149 records)

	Short-term Options		Long-term Options	
	Year to Maturity	Implied Volatility	Year to Maturity	Implied Volatility
Minimum	0.016	10.14%	0.693	15.99%
Maximum	0.104	42.81%	0.986	24.59%
Average	0.058	20.28%	0.826	19.78%
Difference in Average		0.60%	t = 1.685	Accept H_0

Subset 5 from 20-Sep-94 to 28-Apr-95 (150 records)

	Short-term Options		Long-term Options	
	Year to Maturity	Implied Volatility	Year to Maturity	Implied Volatility
Minimum	0.016	8.91%	0.712	11.34%
Maximum	0.104	42.56%	1.002	24.69%
Average	0.054	18.38%	0.869	15.45%
Difference in Average		2.93%	t = 6.753	Reject H_0

Table 7.2. Augmented Dickey-Fuller Test

Data Sets	Term	α -1	t-statistics	Lower tail area
All Data	S	-.037104	-3.589	.03073
Apr 92-Apr 95	L	-.014495	-2.103	.61807*
Subset 1	S	-.126357	-2.762	.22598*
Apr 92-Nov 92	L	-.179840	-3.329	.06534*
Subset 2	S	-.073056	-2.405	.41843*
Nov 92-Jun 93	L	-.074780	-3.181	.09193*
Subset 3	S	-.036343	-1.417	.89689*
Jul 93-Feb 94	L	-.045834	-1.709	.80633*
Subset 4	S	-.063272	-2.149	.57785*
Feb 94-Sep 94	L	-.016507	-0.759	.97860*
Subset 5	S	-.038673	-1.578	.85274*
Sep 94-Apr 95	L	-.067593	-1.950	.69348*

S: Short-term implied volatility
L: Long-term implied volatility

$$\Delta y_t = \beta_0 + \beta_1 \Delta y_{t-1} + (\alpha - 1) y_{t-1} + \beta_2 \cdot dummy + u_t$$

dummy: equals 1 if the data is after weekend.
Lower tail area: the probability to make an error if H_0:(α -1=0) is rejected.
*: H_0 is rejected by the 5% confidence interval with the table provided by Dickey and Fuller (1981)

Table 7.3. Cointegration Test Results

	All Data Apr 92-Apr 95	Subset 1 Apr 92-Nov 92	Subset 2 Nov 92-Jun 93	Subset 3 Jul 93-Feb 94	Subset 4 Feb 94-Sep 94	Subset 5 Sep 94-Apr 95
Constant	.095398	.260233	.171404	.133877	.137808	.090851
	(30.863)	(28.894)	(21.829)	(43.676)	(36.635)	(33.736)
Siv	.481748	.090473	.140805	.285055	.293833	.347720
	(38.984)*	(3.534)*	(3.925)*	(22.189)*	(16.670)*	(25.762)*
R squared	.6707	.0788	.0931	.7664	.6509	.8156
Residual	-.111728	-.241810	-.084939	-.249876	-.201500	-.396660
	(-6.667)	(-4.556)	(-3.044)	(-4.665)	(-4.004)	(-6.103)
Lower tail area	.00001	.00712	.27790	.00519	.03244	.00009

Liv = Constant + β Siv

t statistics in bracket.
*: significant at 5% or better
Δ residual$_t$: is equal to β residual$_{t-1}$
Lower tail area: the probability (provided by Dickey-Fuller) to make an error if H_0:(α -1=0) is rejected.

Table 7.4. Error Correction Model

Short-term Implied Volatility

	All Data Apr 92-Apr 95	Subset 1 Apr 92-Nov 92	Subset 2 Nov 92-Jun 93	Subset 3 Jul 93-Feb 94	Subset 4 Feb 94-Sep 94	Subset 5 Sep 94-Apr 95
Constant βs_0	-.41655e-3 (-0.469)	-.1177e-2 (-0.392)	-.4282e-3 (-0.356)	.5209e-3 (0.286)	-.8517e-3 (-0.481)	.7919e-4 (0.040)
Residual βs_1	.463037 (6.030)***	.398766 (0.743)	.174937 (0.372)	.373587 (1.265)	.543801 (1.956)*	.296656 (1.126)
βs_2	-.268263 (-2.594)***	.8601e-2 (0.016)	-.071412 (-0.147)	-.892979 (-3.143)***	.010782 (0.029)	-.289872 (-1.428)
R squared	.0468	.0207	.2173e-2	.0643	.0386	.0139

$$\Delta\, Siv = \beta s_0 + \beta s_1 \cdot v_{t-1} + \beta s_2 \cdot \Delta\, Liv_{t-1}$$

t statistics in bracket
*: significant at 5% (t>1.645)
**: significant at 2.5% (t>1.960)
***: significant at 1% (t>2.326)

Long-term Implied Volatility

	All Data Apr 92-Apr 95	Subset 1 Apr 92-Nov 92	Subset 2 Nov 92-Jun 93	Subset 3 Jul 93-Feb 94	Subset 4 Feb 94-Sep 94	Subset 5 Sep 94-Apr 95
Constant β_{L0}	.1497e-3 (-0.439)	.3433e-4 (0.032)	-.7673e-3 (-1.902)	.2175e-3 (0.376)	-.4286e-3 (-0.949)	-.2429e-4 (-0.024)
Residual β_{L1}	-.339624 (-9.330)***	-.368214 (-4.354)***	-.417763 (-5.599)***	-.390600 (-4.259)***	-.380962 (-4.767)***	-.545333 (-5.564)***
β_{L2}	-.109027 (-5.619)***	-.41369e-2 (-0.139)	-.9225e-2 (-0.311)	.2422e-3 (0.008)	-.070135 (-2.446)***	-.073793 (-1.754)*
R squared	.1069	.1199	.1919	.1415	.1416	.1786

$$\Delta\, Liv = \beta_{L0} + \beta_{L1} \cdot v_{t-1} + \beta s_2 \cdot \Delta\, Siv_{t-1}$$

t statistics in bracket
*: significant at 5%
**: significant at 2.5%
***: significant at 1%

Where,

$$v_{t-1} = Liv_{t-1} - \beta_{0,t-1} - \beta_{1,t-1}\, Siv$$

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8

Stability of Implied Volatility Functions:

A Test on the Nikkei Options

Naoki Kamiyama

8. Stability of Implied Volatility Functions: A Test on the Nikkei Options

8.1. Introduction

The shape of implied volatility smile has become an important issue in evaluating options, because it violates the assumption of constant volatility which indicates the Black-Scholes formula. Especially from the stand point of risk management, the constant volatility assumption is too strong and practitioners start using more sophisticated models including GARCH (Generalised Autoregressive Conditional Heteroscedasticity) proposed by Bollerslev (1986), and EWMA (Exponentially Weighted Moving Average) proposed as a part of RiskMetrics by JP Morgan (Zangari (1995)). Even though the GARCH and EWMA approaches do not have to assume constant volatility, historical returns used in these methods are not directly linked with the implied volatility shape of option markets.

In this chapter, the behaviour of the volatility smile will be examined because we may be able to use the information to improve option valuation and to realise profit. The shape can be estimated as a function which may be linear or quadratic. The shape may also be defined as skewness of the implied volatility, as shown by Gemmill (1995). We need to know not only the shape but also its stability in order to capitalise on the information contained in the shape. If the shape were stable, we could use the information to take option positions to realise profit over the period to maturity. On the other hand, if we found that the shape changed systematically, we would be able to forecast its evolution and take positions to gain returns.

Shimko (1991) suggests curve-fitting to find the volatility function (i.e. shape of implied volatility) and derive the probability distribution implied in the option prices. In Appendix B, we show the implied probability at the end of the Nikkei option contract on futures by deriving the volatility function with Shimko's method. In the example as of 08 February 1993, the underlying futures price was 17,340, whereas the forecasted futures price implied in the options as of the expiry date of 11 March 1993 was 16,867.39. If the shape of implied volatility (therefore the estimated futures price at expiry) was stable over time but any short-term dispersion is observed from the stable shape, we could evaluate each option and buy cheap, sell expensive to capitalise on the effect. As we will see later in this chapter, it does not seem so stable in a week to capitalise on the smile and the forecasted futures price.

Rubinstein (1994) proposes the "implied binomial tree" approach for the evolution of the smile. The approach requires us to input the implied probability distribution to find the path of the underlying asset over the option period. In Appendix C, we see the application of the approach to the Nikkei 225 options on futures. We find that the forecasted futures price implied in the options was 17,881.13, whereas the underlying futures price was 17,435. As described in the prior paragraph, if the shape of implied volatility was stable over a week, the forecast implied in the option prices would enable us to take positions for the period, i.e. to buy if cheaper is the price calculated on the implied probability.

Kuwahara and Marsh (1994) examine Japanese warrants by implied binomial tree. Taylor and Xu (1994a), Heynen (1994), and Kamiyama (1996) examined the shapes of implied volatilities in the Philadelphia Exchange's currency options, the Dutch equity index options on the European Options Exchange, and the Nikkei 225 options, respectively.

Few tests of the stability of the implied volatility shape are found. Dumas, Fleming, and Whaley (1995) show the empirical results to examine the stability of implied volatility functions. They use a sample of S&P 500 index options[1] during the sample period from June 1988 to December 1993. Four different structural models are used in their research, one of which is the Black-Scholes model (constant volatility), and others of which are quadratic curve fitting models. They find that the stability of the estimated volatility function is not stable in a week (the Black-Scholes model is better in error analysis), and conclude that the implied binomial tree approach does not enable us to take positions to capitalise on the smile effect

The purpose of this chapter is to examine the stability of implied volatility functions in the case of the Nikkei 225 options on futures on SIMEX. Although the basic concept of the testing is similar to Dumas, Fleming, and Whaley (1995), we have three important differences (in addition to the market which is examined). One is that we use the shape of the smile of implied volatility as a linear function with respect to exercise price, while they use a curve fitting function. The second point is that we examine the stability of shape of the smile apart from the stability of implied volatility level, so that we can segregate the stability of the shape of the smile from the overall stability of the function. The final point is that we introduce a forecasting model for the shape of the smile, and the model examines if it is worthwhile to forecast shape

[1] S&P 500 index options are traded on the Chicago Board Option Exchange (CBOE). They are European options.

of the smile in order to trade options.

The paper is written as follows. Section 2 describes the data utilised in this chapter. In Section 3, the method undertaken is explained, and the test results are shown in Section 4. The conclusions are given in Section 5. In Appendix 8.A, the forecasting model for shape of the smile is explained in detail. Appendix 8.B shows the implied probability at the end of the Nikkei option contract on futures by deriving the volatility function with Shimko's method. Rubinstein's implied binomial tree approach is used in Appendix 8.C in the case of the Nikkei 225 options on futures.

8.2. The Data

The Nikkei option on future and the underlying future settlement price data are provided by the Singapore International Monetary Exchange (SIMEX). In order to calculate the implied volatilities, the daily three month CD rates are used for all the maturity dates. The contract month is the nearest month for Nikkei, because that contract is the most liquid. We roll over the nearest contract when time to maturity becomes less than 5 days.

The period examined is from 01 April 1992 to 30 December 1995[2], including the in-sample period (total number of records is 891). We examine the out-of-sample period from 04 January 1993 to 30 December 1995, and the total number of daily records is 720.

Implied volatilities for options on futures are calculated with Black's options on futures pricing formula. The formula is as below.

[2] The SIMEX contract began trading in April 1992, hence the starting data for the data.

$$
\begin{aligned}
&C = e^{-rt}(F \cdot N(d_1) - K \cdot N(d_2)) \\
&P = e^{-rt}(K \cdot N(-d_2) - F \cdot N(-d_1)) \\
&where \qquad (1) \\
&d_1 = \frac{\ln(F/K) + (\sigma^2/2)t}{\sigma\sqrt{t}} \\
&d_2 = d_1 - \sigma\sqrt{t}
\end{aligned}
$$

C: Call Price P: Put Price

F: Underlying future price K: Exercise Price

N(・): Normal distribution density function

σ : Annual volatility

8.3. The Method

We examine the stability of the implied volatility functions specified below over one- week periods testing with out-of-sample data, as Dumas, Fleming, and Whaley (1995) have done. We employ four volatility functions, illustrated in Picture 8.1. Model 1 is the Black-Scholes model where the volatility is assumed constant. This takes no account of the volatility smile: the implied volatility at any exercise price at time t is set at the at-the-money value of 5 working days before (t-5).

$$IV_{j,t} = IV_{k,t-5} \qquad (2)$$

for all j (exercise price) and k is near-the-money, where *IV* is implied volatility, and t is time in unit of a day. There is some degree of difference between implied volatility of call and one of put at near-the-money. We would like to be free from the difference in examining the errors, so that we use the implied volatility of the near-the-money call at t-5 as the IV estimates of the near- and out-of the money calls. Therefore, the mean error of the near-the-money call (put) IV means the average change of the near-the-money call (put) IV in 5 business days.

Model 2 assumes that there is a constant shape of smile: implied volatilities at time t are assumed to be the same as at time t-5.

$$IV_{j,t} = IV_{j,t-5} \tag{3}$$

The implied volatilities of OTC call, NTM call, NTM put, and OTC put at t are exactly matched to the ones of OTC call, NTM call, NTM put, and OTC put at t-5 as estimates, respectively. Therefore, the mean error of the call (put) IV means the average change of the call (put) IV in 5 business days.

In Model 3, we keep the shape of smile (skewness) at time t-5, but the level is shifted as of time t. Here, we examine stability of the shape of smile, separately from the implied volatility level[3].

$$IV_{j,t} = c + skew_{t-5} \cdot X_{j,t} \tag{4}$$

where,

$$c = IV_{k,t} - skew_{t-5} \cdot X_{k,t}$$

Model 3 is similar to Model 2, but the intercept is calculated by the implied volatility level at time t (rather than at time t-5).

Model 4 is also similar to Models 2 and 3, but the *skew* term is forecasted by an autoregressive model:

$$IV_{j,t} = c + skew_{forecast,t-5} \cdot X_{j,t} \tag{5}$$

[3]where,

$$skew_t = \frac{IV_{t-5}(+2\%) + diff - IV_{t-5}(-2\%)}{X_{t-5}(+2\%) - X_{t-5}(-2\%)}$$

diff: the difference in implied volatility (IV) of at-the-money puts and at-the-money calls (= ATM Put IV - ATM Call IV),

IV: the implied volatility

(+x%): an exercise price which is x% above at-the-money price

The "*diff*" term is included to allow for difference between at-the-money put and call volatilities, as were found by Gemmill for the FTSE index options. Please also see Chapter 6 of this thesis.

where,

$$c = IV_{k,t} - skew_{forecast,t-5} \cdot X_{k,t}$$

and $skew_{forecast,t-5}$ is forecasted skew at time t-5 (of which the forecasting model is explained in Appendix 8.A).

The implied volatilities derived from the four models described above are inputted into Black's formula, then the theoretical option values are calculated. Forecasting performance is measured by ME (Mean Error), MAE (Mean Absolute Error), and RMSE (Root Mean Squared Error), as defined as followed.

$$ME = \frac{1}{N}\sum_{t=1}^{N}(MP_t - TP_t) \tag{6}$$

$$MAE = \frac{1}{N}\sum_{t=1}^{N}|MP_t - TP|_t \tag{7}$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(MP_t - TP_t)^2} \tag{8}$$

where MP is market price of an option, and TP is the Black's price of an option using forecasted implied volatility derived from the above models. We compare errors across four options types: out-of-the-money calls, nearest-the-money calls, nearest-the-money puts, and out-of-the-money puts.

8.4. Test Results

Table 8.1 shows the results in terms of forecasting errors. Model 1 assumes a constant implied volatility. Bias is observed as ME (Mean Error). The OTM call (C_OTM) has a large negative bias (that is, the market price is undervalued according to the model), and the OTM put (P_OTM) has a large positive bias (that is, the market price is overvalued). In the observed period, the ME is consistent with the fact that implied volatility has a negative sloped smile with respect to exercise price. The average change in implied volatility level is approximately -2 points both in call and put.

Model 2 improves the issue above by taking account of the negatively sloped smile observed at t-5. ME no longer indicates a negative bias (c.f. Model 1). However, MAE (Mean Absolute Error) and RMSE (Root Mean Squared Error) indicate that Model 2 does not improve the forecasts for all four types of option. It means that assuming a stable level and skewness for volatility is still not a very efficient procedure.

Model 3 which allows for a shift in the smile level of volatility shows large improvements over Model 2, in MAE approximately by 25 to 47 points and in RMSE approximately by 47 to 68 points in option price. However, it is not better than Model 2 for ME in the out-of-the-money options. We can conclude that instability of the implied volatility level is the more important matter than instability of skewness.

Model 4 introduces a systematically forecasted shape of volatility as of 5 days before (rolling with the 6 months period of data as of t-5). There is a small improvement in diagnostic errors both for OTM calls and puts. In both MAE and RMSE, our forecasting model can reduce the errors from 2 to 4 points in price.

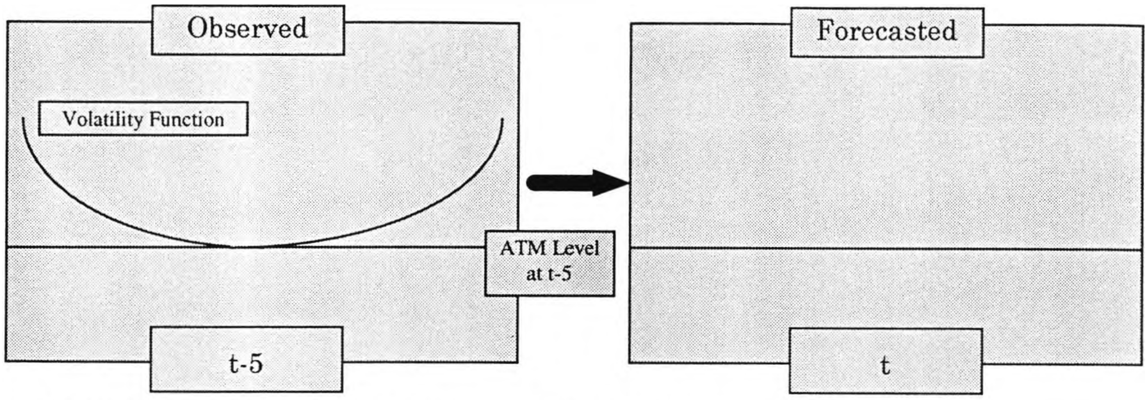
8.5. Conclusion

For the purpose of forecasting the option prices by using the implied volatility function, we conclude that the function is not stable to forecast option prices 5 days ahead, in the case of the Nikkei 225 options on futures. Although it improves the forecast to introduce the shape of volatility, the error may be 39 to 68 points in price, which is very serious if one trades in the market.

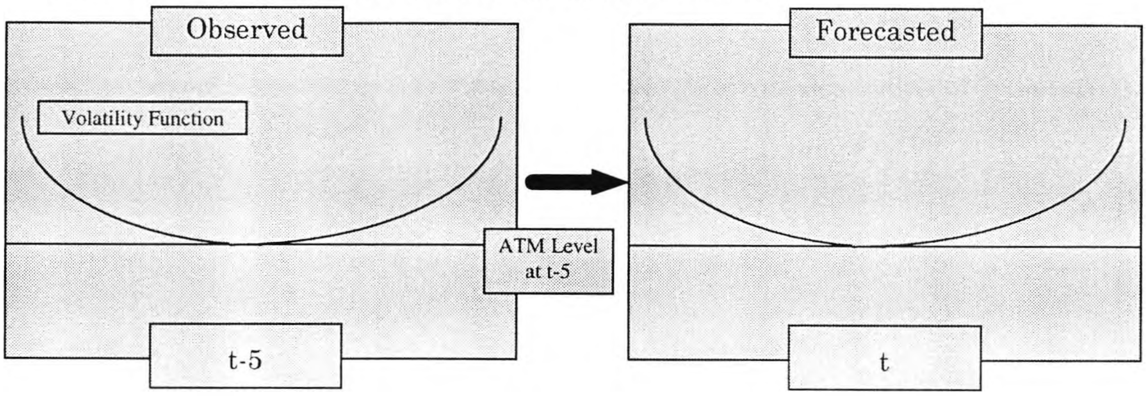
We find that (1) it is good to assume a constant smile in comparing Model 2 with Model 1, (2) it would be even better if volatility level could be forecasted perfectly when we compare Model 3 and Model 2 (we have not used any volatility forecasting model in this chapter, which can be done in the future), and (3) an autoregressive model for the smile is good for OTM calls and puts, but the effect is rather small when Model 4 is compared to Model 3.

Picture 8.1. Illustration of Four Models

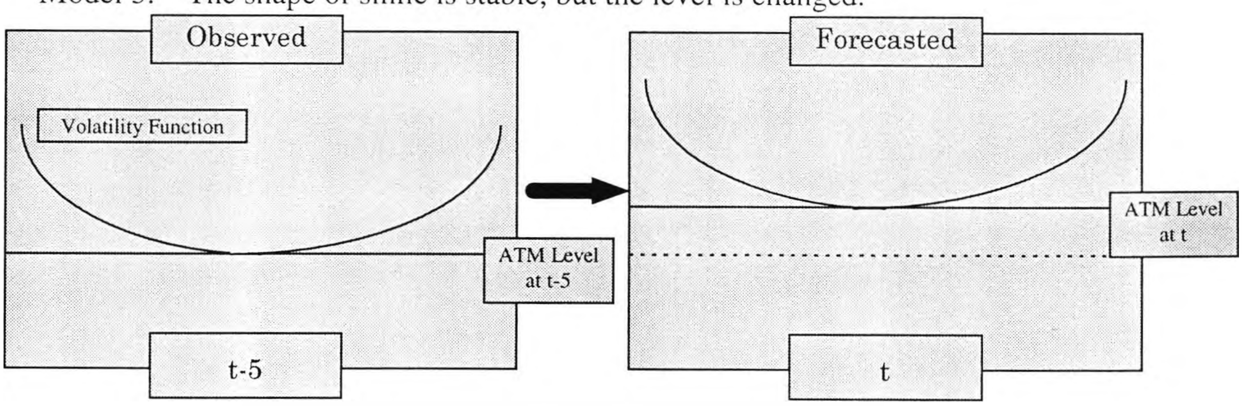
Model 1. A constant volatility is assumed.



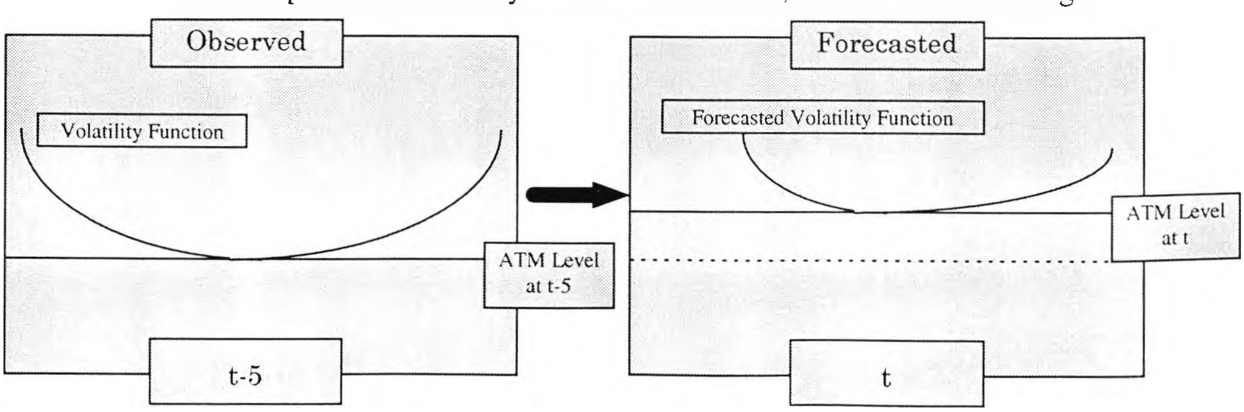
Model 2. Volatility function is assumed stable in a week.



Model 3. The shape of smile is stable, but the level is changed.



Model 4. The shape is forecasted by a time series model, and the level is changed.



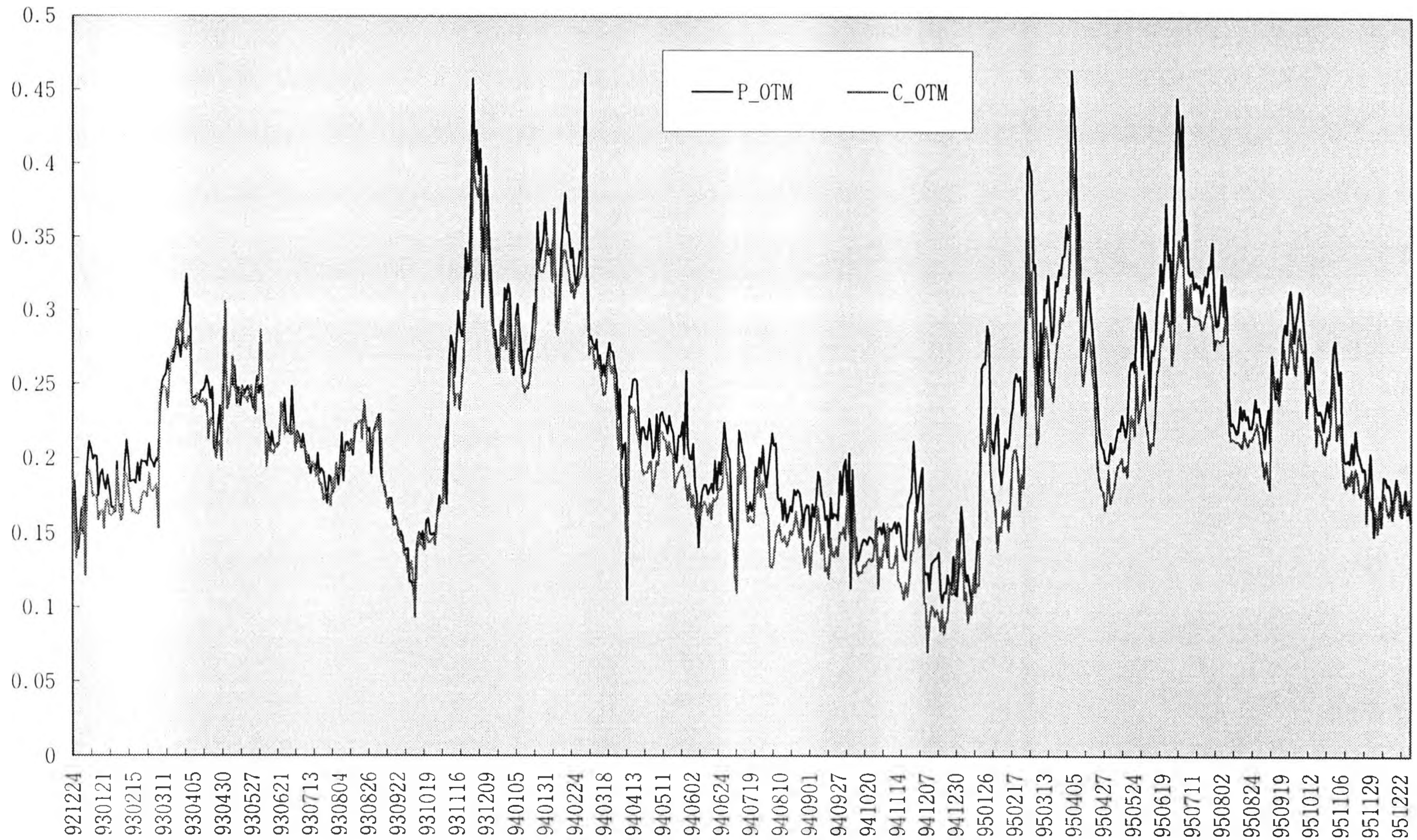


Figure 8.1. Implied Volatility of the Nikkei 225 Options on Futures

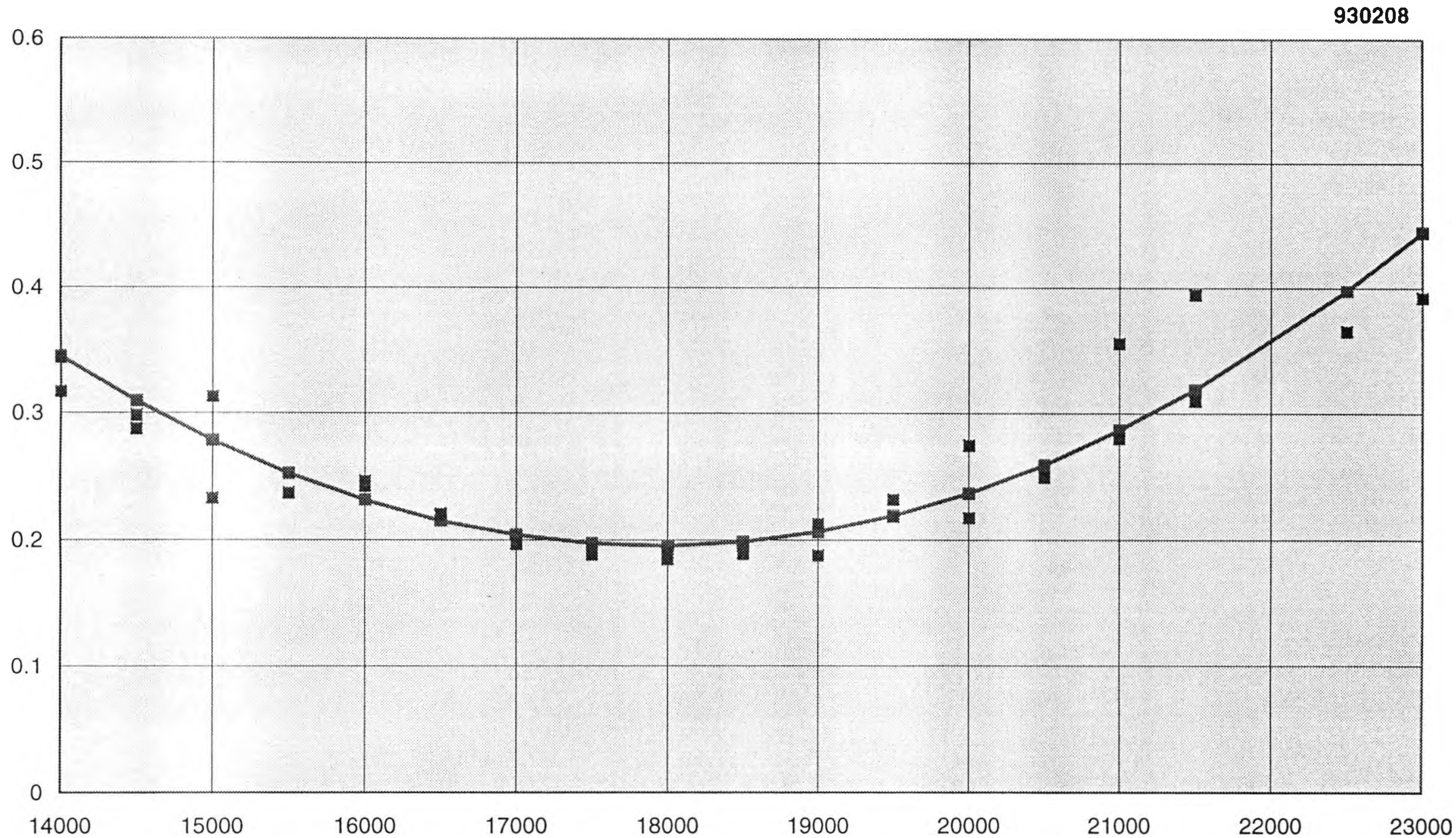


Figure 8.2.a The Curve-fitting as of 08 February 1993 - The Nikkei 225 Options on Futures

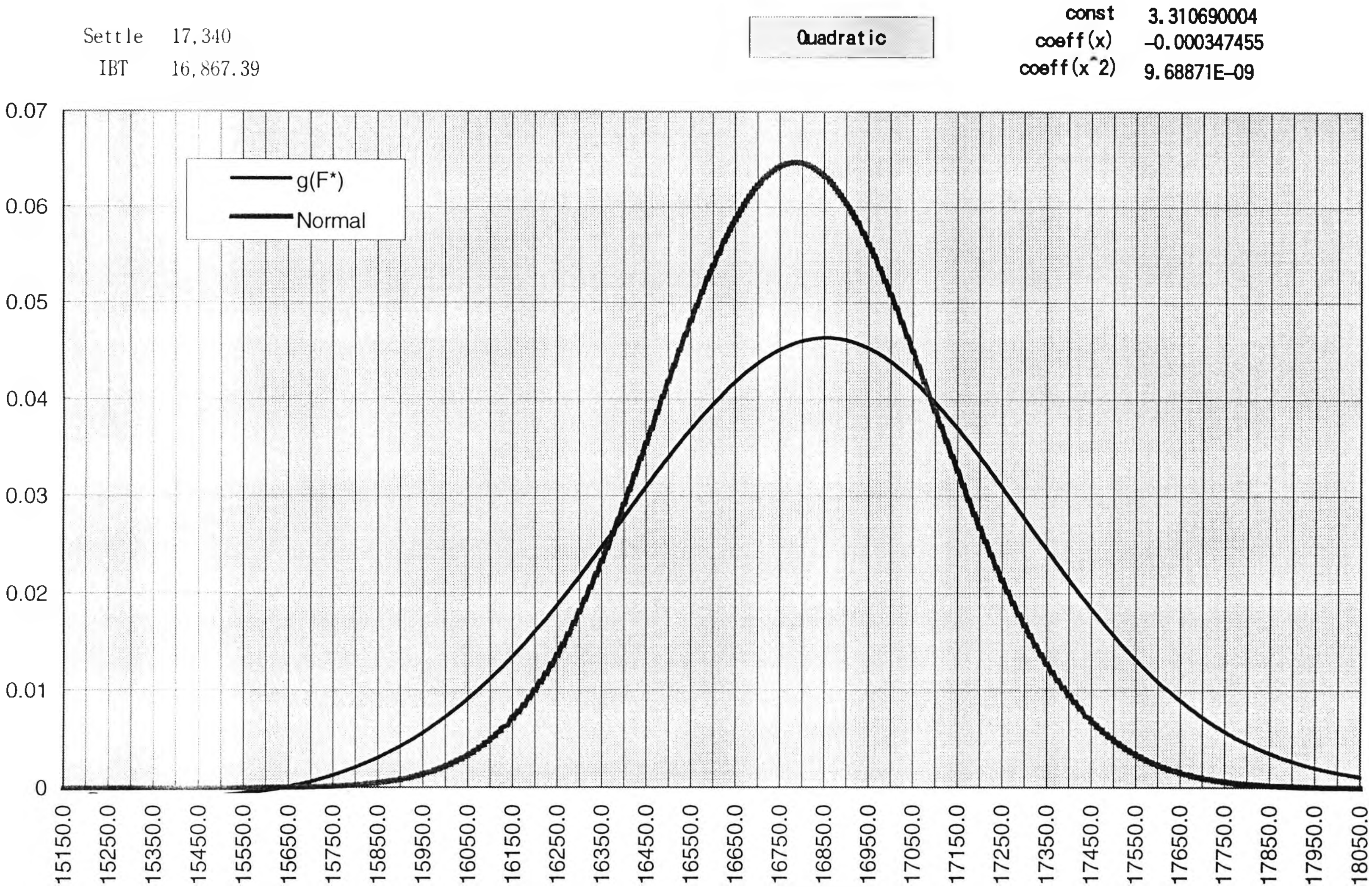


Figure 8.2.b Probability Distribution Implied in the Nikkei 225 Options on Futures as of 08 February 1993

Figure 8.3.a Standard Binomial Tree as of 05 February 1993 - The Nikkei 225 Options on Futures

					Sj'
					17510.12
				17495.07	
			17480.03		17480.03
		17465.01		17465.01	
	17450.00		17450.00		17450.00
17435.00		17435.00		17435.00	
	17420.02		17420.02		17420.02
		17405.04		17405.04	
			17390.08		17390.08
				17375.14	
					17360.20

Figure 8.3.b Implied Binomial Tree - The Nikkei 225 Options on Futures

					Sj
					18118.48
				18466.16	
			18316.19		17485.75
		18144.46		17743.62	
	17989.37		17677.88		16909.86
17881.13		17687.43		17596.09	
	17716.88		17701.39		17333.95
		17752.36		17803.48	
			17810.17		17393.88
				17821.94	
					17360.31

Table 8.1. Error Analysis

MODEL 1

	ME	MAE	RMSE
C_OTM	-7.675	40.641	61.397
C_NTM	-2.014	46.168	67.299
P_NTM	-2.147	47.110	68.539
P_OTM	14.975	43.804	63.233

MODEL 2

	ME	MAE	RMSE
C_OTM	-3.676	39.715	59.698
C_NTM	-2.014	46.168	67.299
P_NTM	-2.147	47.110	68.539
P_OTM	-5.394	43.492	66.695

MODEL 3

	ME	MAE	RMSE
C_OTM	5.819	13.213	18.281
C_NTM	0.000	0.003	0.004
P_NTM	0.000	0.003	0.004
P_OTM	5.649	14.069	19.496

MODEL 4

	ME	MAE	RMSE
C_OTM	6.203	11.346	14.692
C_NTM	0.000	0.003	0.004
P_NTM	0.000	0.003	0.004
P_OTM	5.527	11.929	16.441

Appendix 8.A. The forecasting model for skewness

We assume skewness is mean-reverting and formulate an autoregressive model for forecasting skewness as below:

$$skew_t - skew_{t-5} = \lambda(skew_{t-5} - skew_{mean}) + \varepsilon_t \qquad \text{(A1)}$$

In order to find the mean level of *skew* (i.e. $skew_{mean}$) and the proportional coefficient for reverting (i.e. λ), Equation (A1) may be re-written in the form:

$$skew_t = \alpha + (1+\lambda)\cdot skew_{t-5} + \varepsilon_t \qquad \text{(A2)}$$

where $skew_{mean} = -\ \alpha / \lambda$, and *skew* is defined in footnote 2. The regression result is in Table 8.A.1. The first regression is done for all the data available from 03 April 1992 to 29 December 1995, in total 891 records. Because we approximately take half a year for the regression and roll the half-year period at any time of t, we take roughly 100 records to regress on each day.

As shown in Table 8.A.1, the autocorrelation is significant in taking the 5 day lag. The coefficient (β) is 0.41 with t-value of 13.3. This means that skewness is mean-reverting in 5 days in the proportion (λ) of -0.59 (= 0.41 - 1). It is reasonable to use the result as the systematic forecasting of the shape of implied volatilities.

Table 8.A.1. Regression Results in Appendix A.

	Period	α	β	λ (= β -1)	$skew_{mean}$	R squared
a	Apr 92 - Dec 95	-.11738e-04 (-13.141)	0.410060 (13.326)	-0.58994	0.285e-04	0.167281
b	Jan 93 - Jun 93	-.686917e-05 (-3.524)	0.288144 (3.012)	-0.711856	0.23839e-04	0.089769
c	Jul 93 - Dec 93	-.505555e-05 (-3.714)	0.152906 (1.763)	-0.847094	0.33063e-04	0.022016
d	Jan 94 - Jun 94	-.144134e-04 (-7.936)	-0.030947 (-0.348)	-1.030947	-0.46586e-03	.103422e-02
e	Jul 94 - Dec 94	-.187985e-04 (-6.830)	0.143994 (1.543)	-0.856006	0.13055e-03	0.019148
f	Jan 95 - Jun 95	-.261548e-04 (-7.300)	0.384597 (4.597)	-0.615403	0.68005e-04	0.150785
g	Jul 95 - Dec 95	-.973933e-05 (-5.555)	0.363632 (5.690)	-0.636368	0.26783e-04	0.209724

$skew_t - skew_{t-5} = \lambda(skew_{t-5} - skew_{mean}) + \varepsilon_t$ t-value in bracket, and n = 120 days

Appendix 8.B. The Shimko's volatility function

The Black's formula to derive call option value can be shown as below:

$$
\begin{aligned}
& C = e^{-rt}(F \cdot N(d_1) - X \cdot N(d_2)) \\
& \textit{where} \\
& d_1 = \frac{\ln(F / X)}{v} + \frac{v}{2} \\
& d_2 = d_1 - v \\
& v = \sigma\sqrt{t}
\end{aligned}
\tag{B1}
$$

where C is Call Price, F is the underlying future price, and X is the exercise price. $N(\cdot)$ is normal distribution density function and σ is annualised volatility.

Breeden and Litzenberger (1978) show a generalised option valuation which can be differentiated by exercise price as followed:

$$
\begin{aligned}
& C = B\int_{X}^{\infty}(F^* - X)g(F)dF \\
& C_X = -B[1 - G(F^*)] \\
& C_{XX} = B[g(F^*)]
\end{aligned}
\tag{B2}
$$

where B is a discount factor, g is a distribution function, G is a cumulative distribution function, F^* is a random value of the underlying futures price, and C_X and C_{XX} are the first differential and the second differential of call price with respect to exercise price.

On the other hand, we can differentiate the Black's formula with respect to exercise price as below (see Shimko (1993)):

$$
\begin{aligned}
& C_X = B[X \cdot v' \cdot N'(d_2) - N(d_2)] \\
& C_{XX} = B[X \cdot v'' + (v'(1 - X \cdot d_2 \cdot d_{2X})) - d_{2X}]
\end{aligned}
\tag{B3}
$$

where,

$$
\begin{aligned}
& d_{1X} = -\frac{1}{X \cdot v} + v'(1 - \frac{d_1}{v}) \\
& d_{2X} = d_{1X} - v'
\end{aligned}
$$

From Equations (B2) and (B3), we obtain $G(F^*)$ and $g(F^*)$ as follows:

$$G(F^*) = 1 + X \cdot v' \cdot N'(d_2)$$
$$g(F^*) = [v'' \cdot X + v'(1 - X \cdot d_2 \cdot d_{2X}) - d_{2X}] \cdot N'(d_2) \qquad \text{(B4)}$$

If we assume the volatility function as a quadratic curve, ν can be expressed as below:

$$v = \sigma\sqrt{t} = A_0 + A_1 X + A_2 X^2$$
$$v' = A_1 + 2A_2 X \qquad \text{(B5)}$$
$$v'' = 2A_2$$

We estimate A_0, A_1, and A_2 by regression and then ν' and ν" are calculated and inserted into Equation (B4). In Figure 8.2.a, we show the curve-fitting result as of 08 February 1993, with 35 option prices including both calls and puts. The time to maturity for all is 0.0849 year, with the maturity date of 11 March 1993. The underlying futures price is 17,340.

We now draw a graph of g(F*) as shown in Figure 8.2.b, as a probability distribution function. The probability implied in option prices on the observed date is flatter than normal, and fat-tailed. This fact is consistent to the shape of implied volatility which smiles as shown in Figure 8.2.a.

Appendix 8.C. Implied Binomial Tree

We use the optimisation method suggested by Rubinstein (1994) to find risk neutral probability of each underlying price at the end node of the Nikkei 225 options on futures contract. The "prior guess" or initial input is given by standard binomial tree, that is, the initial probability, P'_j, is determined as below:

$$P'_j = [n!/\, j!(n-j)!] \cdot p'^{\,j} \cdot (1-p')^{n-j} \qquad \text{(C1)}$$

where n is the number of steps (n = 5), and p' is the upside probability in the standard binomial tree. The objective function and constraints for optimisation are shown as below:

$$\begin{aligned} &\min \sum_j (P_j - P'_j)^2 \\ &\text{subject to} \\ &\sum_j P_j = 1 \; and \; P_j \geq 0 \qquad \text{(C2)} \\ &F = \sum_j P_j F_j \Big/ r^n \\ &C_i^b \leq C_i \leq C_i^a \;\; where \;\; C_i = \sum_j P_j \max[0, F_j - K_i] \end{aligned}$$

where r is discount factor ($e^{3.24\% \times 1/365}$) for a step (a constant risk-free rate is assumed), F_j is the underlying futures price at the j-th node of the maturity, and C^b and C^a are bid and ask price of options, respectively. We can obtain P_j and F_j simultaneously by optimising the objective.

Seeking for the implied binomial tree, we go backward from the terminal node, at which we found P_j and F_j as above. Firstly, we find path probability, $P_{terminal}$ at the terminal node. Using two probabilities, P^+ and P^-, we calculate the path probability backward to the second last node, where + denotes one node up and − denotes one down from a specific point of the tree.

$$P_{terminal} = \frac{P_j}{n!/j!(n-j)!} \qquad \text{(C3)}$$
$$P = P^+ + P^-$$

Secondly, we find local probability, p, defined as P^+/P at each point. Finally, we find implied binomial tree of the Nikkei 225 futures prices over the option period. By using the calculation results so far, we calculate R on each point, which is defined as:

$$R = [(1-p)\cdot R^- + p \cdot R^+]/r$$
$$where\ R_j = F_j/F \qquad \text{(C4)}$$

The contract observed here was as of 05 February 1993, and expired on 11 February 1993 (time to maturity is 0.0164 year). The underlying futures price was 17,435. We have 7 options to be examined which did not violate put-call parity, 3 puts and 4 calls. The short-term interest rate of 3.24% (annualised) was used over the period. The range of implied volatilities was from 33% (annualised) of OTM call to 16% of ATM call on that day.

The result is shown in Figures 8.3.a and 8.3.b. Figure 8.3.a is the normal binomial tree as an initial input. Figure 8.3.b is the implied binomial tree. As shown, the probability implied in the option prices at the moment of time was skewed to the bullishness.

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Conclusion

9. Conclusions

We have two major reasons for measuring volatility of asset returns. One is for trading: we can set up option positions to capitalise the forecasted volatility, when the implied volatility is different from the forecasts and the difference can cover the trading costs. The other is for risk management: we can input a more accurate volatility for evaluation, so that we can obtain better quality control of position values in derivatives.

What we have learnt from the empirical work here is 1) that volatility of underlying asset return can be modelled and forecasted, 2) that implied volatility has information which can be utilised, and 3) that there is some degree of international linkage in volatility. Implications for traders are, therefore, 1) that volatility forecasting by a statistical model such as GARCH is worthwhile, 2) that the implied volatility level, smile, and term structure should all be estimated to assess market conditions, and 3) that it is also worthwhile to forecast volatility by using information about the volatility change in other major markets. These conclusions will be reviewed in this chapter.

In Chapter 2, we have a review of estimates of historical volatilities including the high/low methods of Garman and Klass (1980) and Parkinson (1980). We find that modified Garman and Klass and modified Parkinson methods tend to overestimate true volatility. It seems that the assumption of Geometric Brownian motion (i.e. continuous price diffusion even when the market is closed) does not hold. JP Morgan's exponential weighted moving average (EWMA) used for RiskMetrics™ is compared with GARCH, and we conclude surprisingly that EWMA is as effective as GARCH for the Nikkei index.

Chapter 3 examines whether forecasts of volatility from past data or from implied volatility are more accurate, using the Nikkei 225 index. In comparison, GARCH provides the best forecast among major volatility indicators (in terms of RMSE) including historical and implied volatilities. In addition, a combination of several indicators, including historical estimates, can enhance the volatility forecast. In order to forecast volatility, the suggested weights of the combination are 80% implied volatility and 20% historical estimates. The shorter the time to maturity, the more important the historic (GARCH) estimates, because the GARCH model produces recursive one-day ahead estimates. We find that implied volatility tends to overpredict realised volatility, while Lamoureux and Lastrapes (1993) found the opposite for several individual stock options on NYSE. Also, implied volatility is a better forecast than simple historical volatility in our study, which is the opposite of the finding of Canina and Figlewski (1993) for the case of S&P 100 options.

In Chapter 4, Heynen's (1994) and Taylor and Xu's (1994) volatility smile models are fitted to Nikkei 225 options. We confirm that there is a skewed smile in the implied volatility and the steepness of the smile increases as the time to maturity decreases. To evaluate option positions, it is necessary to know that 1) volatility for a specific time to maturity should be varied by exercise price (smile effect) and 2) the effect changes with time (time effect).

In Chapter 5, we find that the return and volatility on the Japanese stock market (Nikkei 225) are affected by the other equity markets, such as the American (S&P 500) and British (FTSE 100). Our general model integrates two approaches, i.e., return spillovers and volatility spillovers in the context of GARCH. Return spillovers are comparatively large; for example the Nikkei return is explained 17% by the FTSE and 26% by the S&P return, independently. Volatility spillovers to Japan are significant but rather small; Nikkei conditional variance is explained 7% by the FTSE and 2% by the S&P variance respectively, so the major part of volatility is explained by purely own-market effects (GARCH). On the other hand, the Japanese market seems to have very little influence on the others, both with respect to return and to volatility. The practical implication is that a Nikkei option trader should take account of movements in both FTSE and S&P in order to forecast the Nikkei return and volatility on the next day.

We examine three subjects in Chapter 6, (1) transmission of implied volatility (IV) across time zones (2) transmission of skewness across time zones, and (3) domestic influences on skewness. We find that IV spills over, but skewness seems to be a domestic phenomenon. A change in IV spills over to the next-opening market across a three-zone world. The IV of the S&P significantly affects Nikkei and FTSE IVs (whether considered in levels or changes in levels). On the other hand, there are three results in terms of skewness of implied volatility. The first is that spillovers of skewness are minor and only significant in relation to UK and US effects on Japan. Secondly, skewness is related to same-day returns in UK and Japan. Thirdly, next-day returns are positively related to skewness in the UK, and negatively in the US.

In Chapter 7, a cointegration analysis is applied to the term structure of implied volatilities for the Nikkei options on futures traded on SIMEX. Cointegration provides an accurate analysis for the association of volatilities (short- and long-term) and their correction process. We find that the short- and long-term volatilities are co-integrated, and that both short- and long-term volatilities tend to correct the disturbance from the equilibrium relationship by changing their levels over time. The implication is that mean-reversion of an asset price makes the volatility expectation vary with maturity and the dispersion is corrected over time as the asset price reverts to the mean.

By using the shape of implied volatility smile, the implied risk-neutral distribution for an asset may be derived. However, this approach is not very useful unless the shape is stable over time. In Chapter 8, we find that the stability is not sufficient to be useful in forecasting option prices in the case of the Nikkei options on futures in SIMEX, confirming the results of Dumas, Fleming, and Whaley (1995) for S&P options.

There are several areas that deserve further research. We have not explicitly tested or compared the forecasting performances among the GARCH family or between GARCH and SV models. For example, EGARCH (exponential GARCH), is not examined in this thesis, but may be better than simple GARCH for the stock return and volatility because the volatility of stock returns seems asymmetric (Nelson (1991)). Taylor (1994) compared GARCH and SV with foreign exchange data. We have not examined the persistence of volatility shocks in the Nikkei daily returns, that may be measured by $\alpha + \beta$ in the GARCH estimates[1]. If the volatility shock is always nearly one, as shown in Table 2.4 in Chapter 2 - Page 28, IGARCH (Integrated GARCH), proposed by Engle and Bollerslev (1986), may be appropriate to forecast the Nikkei volatility.

[1] See Equation (11) in Chapter 2 - Page 21. For example, Watanabe (1997) shows why $\alpha + \beta$ means persistence of volatility shock.

The implied volatility transmission could be examined with more high-frequency data, such as tick-by-tick price data, so that we can avoid overlapping of market opening periods. The overlapping trading hours are approximately half a day between U.K. and U.S.A.. The traders in the afternoon in U.K. may be highly affected by the market direction and volatility of U.S.A.. In such a case, spillovers from/to U.K. would be segregated at noon.

There may be more important factors to explain volatility changes, for example, open interest and trading volumes, which could be driving forces that have not been included in this thesis. Above all, more research is needed on the way in which prices are determined in financial markets, since the distribution of prices which we observe is the result of human behaviour.

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Reference

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