Excess Verdicts Insurance

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Abstract

This paper thoroughly studies the impact of excess verdicts on the insurance industry and further explores the optimal insurance strategy from the policyholder's perspective, especially when the court awards compensation beyond the insurance coverage, i.e., excess verdicts. This is particularly prominent in wrongful death cases, where both financial and legal risks are considerable. We introduce a mathematical model that simplifies these complex interactions into manageable components, such as loss amount and legal stage, to understand better when insurers need to cover excesses over policy limits. Our approach uses Value-at-Risk and Conditional Value-at-Risk under the premium principle to transform this infinite dimensional challenge into a tractable optimization problem. Our study shows that optimal insurance contracts employ multiple layers of indemnity to accommodate different risk environments, resulting in optimal risk allocation between insurers and policyholders, minimizing legal costs, and ensuring fair coverage.

Keywords: Excess verdicts; Legal implications; Risk management; Optimal insurance; Multiple indemnity environments; Value-at-Risk; Conditional Value-at-Risk.

1 Introduction

1.1 Excess Verdicts and Insurance Impact

In the insurance field, which is closely related to litigation, each verdict has the potential to recalibrate risk assessments and premium decisions. A critical concern is the increase in excess verdicts, often referred to as "nuclear verdicts." These verdicts are becoming increasingly common, creating uncertainty, protracted legal battles, and unforeseen liabilities for insurers and policyholders. In particular, they may directly impact insurance policies and coverage standards. This trend might imply a shift in societal perspectives on litigation, an evolution of legal strategies, and a change in society's view of compensation and justice. This paper provides a deeper exploration of the complexity of excess verdicts and their impact on stakeholders. Further, it proposes new models and policies in insurance contracts to address these challenges effectively.

Within the framework of the law, excess verdicts occur when a court judgment requires the defendant to compensate the plaintiff more than the defendant's insurance limits. This trend is particularly evident in personal injury and wrongful death cases, as it is increasingly recognized that socio-emotional damages outweigh quantifiable economic losses. Therefore, liability for emotional, social, and psychological damages also tends to exceed the measurable standards established by traditional insurance policies. In such cases, the apportionment of liability between the insurer and the policyholder is bound to be more complex, and it is accordingly often difficult to cover large claims under the terms of a traditional policy.

The development of financial markets has led to a paradigm change in the insurance industry towards layered coverage. Richmond (2000) articulated the fundamental role of primary insurance in providing baseline coverage, with excess insurance gaining relevance only after primary policy limits are exhausted. Their study emphasized the significant challenges posed to the excess insurance industry by large-scale judgment cases, particularly nuclear verdicts. Richmond (2000) further criticized that the delineation of liability coverage in excess insurance lacked the overall standardization evident in primary liability insurance. This discrepancy, coupled with the outdated nature of agreements such as the *1974 Guiding Principles for Primary and Excess Insurers*, fails to meet the requirements of the modern insurance industry, thus exacerbating problems for policyholders and insurers.

In nuclear verdict cases, policyholders implicated in the litigation may find themselves responsible for contributing to an eventual settlement. This involvement may cause them significant personal financial stress and, at worst, even lead to bankruptcy. For insurers, a major nuclear verdict case could not only emphasize their potentially catastrophic financial distress, which in turn threatens their solvency, but it could also damage their reputation, especially if the court interprets their actions as bad faith, which would result both in irreparable damages and a shaking of the fundamental integrity of the industry. Despite these considerable risks, the insurance market often ignores nuclear verdicts as financial anomalies without thoroughly investigating their causes, effects, and possible prevention strategies. This negligence underestimates the far-reaching impact these verdicts can have on policyholders and the insurance industry as a whole.

To fully understand the complex legal and insurance effects of nuclear verdicts, the evolution of insurance contracts and the changing legal landscape must be explored. Historically, the settlement of claims involving insurers and policyholders has been closely linked to risk events and resulting losses. However, in today's evolving risk environment, policyholders tend to seek compensation for losses that exceed policy limits. This trend increasingly combines the utilization of excess insurance with innovative litigation strategies.

O'Connor (2003) provided an illustration of this emerging phenomenon, emphasizing the challenges of introducing excess insurer liability when negotiating settlements with primary insurers, especially when those settlements are below policy limits. This situation might inadvertently transfer financial liability from the policyholder and primary insurer to the excess insurer not involved in the settlement. O'Connor (2003) further noted that the judicial system, instead, prioritizes holding policyholders liable for the coverage gap created by such settlements. This legal position not only preserves the basic principle of excess insurance but also prevents excess insurance from being transformed into primary or secondary insurance.

Epps and Chappell (1958) examined the excess liability of insurers, particularly where claims exceeded policy limits, revealing the evolution of tort theories and their corresponding role in expanding litigation. This exploration primarily involved the principles of "bad faith" and "negligence." "Bad faith" requires evidence of fraudulent or bad faith behavior from the insurer, while "negligence" involves a lack of due diligence in settling a claim. Gallogly (2006) further expanded on these concepts by articulating the standards that insurers should comply with in good faith when dealing with third-party claims. They emphasized that bad faith is not just a lack of good faith, but also necessitates that the insurer's actions have an objectively unreasonable basis. This includes cases where the insurer fails to notify the policyholder of the settlement. Nonetheless, the survey clarified that a simple rejection of a settlement offer or an unfavorable outcome does not constitute liability for bad faith, which rather stems from an unreasonably dismissive attitude toward the settlement offer. Furthermore, Asmat and Tennyson (2014) used U.S. automobile insurance claims data from 1972 to 1997 to empirically demonstrate that insurer liability for bad faith torts leads to higher settlements and reduces the likelihood of inadequate claims. Their study stressed that the legal threat of bad faith could significantly influence insurers' settlement behavior.

The excess verdict case takes on additional dimensions in litigation due to the contrasting strategies utilized by the defense and plaintiffs' attorneys. Murray et al. (2020) observed that defense attorneys representing insurers often use tactics that increase financial and procedural barriers to deter plaintiffs. This approach is rooted in the dual objectives of minimizing costs and efficiently preparing for litigation. Conversely, plaintiffs' attorneys take a "high-risk, high-reward" attitude, investing significant financial resources in the case while considering the significant financial risk of an unsatisfactory outcome. The report further illuminated the differences in strategic messaging and communication between these legal factions. Defense attorneys generally tend to support business-centric entrepreneurship, prioritizing confidentiality and competitive advantage. In contrast, plaintiffs' attorneys prefer to build collaborative networks and share strategies and opinions more openly. This tendency is partially motivated by the likelihood that plaintiffs' attorneys would receive a considerable portion of their honorariums from large settlements, which incentivizes them to seek higher levels of compensation, this may conversely exacerbate the litigation process and potentially lengthen the duration of the litigation.

The impact of nuclear verdicts extends beyond litigation and sheds light on the broader phenomenon of "social inflation." This concept encapsulates the forces that make insurance claims cost more than the rate of general economic inflation, as measured by indices such as the Consumer Price Index (CPI), Retail Price Index (RPI), and wage inflation. Nuclear verdicts do not just affect legal proceedings and insurance claims, but also broader economic and social tendencies. The legal impact of such variations in social sentiment is particularly evident in specific areas of insurance, including private and commercial personal accident insurance, medical professional liability, product liability insurance, and other general liability areas. Given the growing importance of nuclear verdicts in claim settlement, insurers have become increasingly aware of the risks associated with these societal evolutions. This awareness has led to major changes in claims practices and premium-setting methods. While responding to the direct impact of nuclear verdicts, these adjustments have also had a notable impact on social inflation indicators, shaping the insurance industry's response to these transformations[*].

Variations in jurisdictional legal frameworks significantly amplify the challenges inherent in insurance litigation, especially regarding nuclear verdicts. In the United States, the handling of excess verdict liability diverges across states: some states place excess liability burdens on policyholders, while others allow such damages to be collected directly from insurers. Despite these differences, a unifying principle across jurisdictions is the insurer's obligation to endeavor to settle within policy limits. Pennsylvania exemplifies jurisdictions that have strict requirements for insurers. Gallogly (2006) highlighted the evolution of the law in this state, which initially lacked a common law right to sue insurers for bad faith in settlements. The introduction of the bad faith statute in 1990 changed the landscape of the insurance industry significantly, allowing insurers to take legal action against bad faith behavior. Additionally, Deng and Zanjani (2018) identified litigation activity such as insurance loss and tort cases as the primary motivation for states to adopt tort reforms between 1971 and 2005. This finding has emphasized that the diverse legal frameworks governing insurance cross-jurisdictional litigation, especially in the case of nuclear indemnity verdicts, vary not only in terms of excess verdict management but also in the propensity of states to engage in legal reforms driven by litigation pressures.

Moreover, insurers facing an increasing number of bad-faith behavior suits may lead to a proliferation of excess verdicts in the states. Examples of such conduct include *delayed settlement of claims, denial of claim defenses, exceeding policy limits during settlement, refusal to pay claims, and conducting bad faith investigations.* These practices, are elaborated upon in a spectrum of legal cases (see examples in Appendix Table 6 for details), highlighting the multifaceted difficulties that insurers face in different jurisdictions, with each state's legal framework shaping its litigation and settlement strategies.

[*]For more details on social inflation, please see here

1.2 A Case Analysis for Nuclear Verdicts

An extensive analysis by the *American Trucking Transportation Institute* reveals patterns of excess verdicts and key factors in the evolution of the trucking industry landscape. A pivotal moment occurred in 1994 when the jury awarded $2.7 million in the famous "*hot coffee*[†]" case (Murray et al., 2020). This decision marked a watershed in the trend of excess verdicts, initiating an era where lawsuit damages frequently surpassed the million-dollar mark. The upward trend in verdicts was further highlighted by a 2011 case in which a victim of a fatal truck accident was awarded $40 million. These instances underscored an emerging pattern linking the magnitude of damages, the elapsed duration from accident occurrence to verdict, and the consequent escalation in insurance premiums (Murray et al., 2020).

Despite the decline in the incidence of serious trucking accidents, the surge in verdicts and settlements has further exacerbated the financial pressures faced by the trucking industry (Sharma, 2023). The landmark 1977 case of *Bates v. State Bar of Arizona*[‡] marked the beginning of an era when law firms stepped up their marketing efforts, resulting in increased public awareness of litigation rights and a more litigation-friendly environment. Sharma (2023) emphasized that the contemporary legal environment in the trucking industry is affected not only by the proliferation of large-scale settlements and verdicts, but also by challenges such as fabricated accidents, direct contact between attorneys and accident victims, and the intentional conflation of quantifiable economic damages with more subjective non-economic damages, such as mental anguish in wrongful death cases.

These changing legal practices not only motivate plaintiffs' attorneys but also influence the trajectory of judicial decisions, leading to increased insurance costs in industries such as trucking. Globally, the emphasis on non-economic damages, such as the assertion of wrongful death claims in the 1999 report of the *British Law Commission,* reflects a change in the legal framework toward consistency with current societal values. Chang et al. (2015) underlined the international differences in this regard, noting in particular the cultural differences between European and East Asian countries in valuing pain and suffering. Their empirical investigation revealed the impact of family relationships on traffic accident damages. For example, spouses often receive greater compensation for pain and suffering compared to their adult children due to deep emotional ties. Such discrepancies highlight the limitations of traditional economic valuation methods in fully recognizing the value of individuals and underscore the need for a balanced approach to recognizing grief and allocating damages. In this context, Heaton and Lucas (2000) argued that while certain legal amendments may cut the cost of automobile insurance and affect coverage, the impact is incomplete. Thus tort reforms and their broader impacts need to be carefully assessed, including potential spillover effects in a variety of personal injury cases. The inclusion of non-economic dam-

[†]For more information on this case, please see here

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ages (e.g., pain and suffering) in wrongful death cases is therefore critical and represents a major transformation in legal attitudes towards the value of life and loss.

However, Sharma (2023) also revealed a paradox in the analysis of the trucking industry: despite surging premiums, the insurers serving the trucking industry have experienced a large decline in profitability. This phenomenon underscores the impact of claim severity and frequency in shaping the industry's financial landscape. As a result, excess coverage has become unpredictable and expensive, forcing numerous insurers to either withdraw it altogether or drastically scale back the comprehensive insurance coverage they offer to the trucking industry. This has forced trucking companies to assume greater risk and, in many cases, struggle to obtain comprehensive coverage.

In addition, the prevalence of large verdicts significantly influences the settlement trends in this industry. Sharma (2023) noted that the increase in nuclear verdicts may force insurers to agree to higher settlement amounts, often exceeding typical jury awards for similar claims. This phenomenon, known as "settlement creep," arises from insurers' concern about facing disproportionate verdicts, leading insurers to settle for large amounts even when the trucking company's liability is uncertain or disputed. Silverman and Appel (2023) further emphasized that plaintiffs' attorneys are increasingly using the "reptile theory," a strategy that inspires the innate human motivation to be safe and complicate the litigation landscape by emphasizing that more serious injuries may occur in the future if trucking companies are not severely penalized. This approach seeks to amplify the fear and anger of juries and judicial officers by stressing the overall behavior of the industry rather than specific incidents, which encourages more punitive verdicts. Correspondingly, an empirical study conducted by Chang et al. (2015) on pain and suffering damages in wrongful death cases showed that a plaintiff's initial request for damages dramatically shapes a court's decision. This phenomenon illustrates an "anchoring effect" in which the size of the plaintiff's claim largely determines the court's decision, independent of the objective specifics of the case.

The impact of large verdicts and settlements extends far beyond the trucking industry, ultimately affecting the consumer economy. In the face of the rising impact of these excess verdicts and settlements, the trucking industry inevitably faces a dramatic increase in operating expenses. Rising costs put intense pressure on the industry and have a broader economic impact by passing cost increases on to consumers through higher prices for goods and services. Sharma (2023) highlighted this critical issue by showing how inflated litigation settlements increase insurance expenditures for trucking companies, providing examples of the broader impact of these legal developments on the economic landscape.

1.3 Our Innovations and Contributions on Excess Verdicts

Based on the discussion of excess verdicts and their multifaceted impact on the insurance industry, our study aims to fill a major gap in the current literature by exploring the relationship between these verdicts and the multiple indemnity environment framework. This study aims to strengthen the existing academic foundation while providing new perspectives on the evolving nature of the insurance industry. Our analysis explores how various potential risk factors play a role in excess verdicts, particularly when the scope of damages now includes emotional and social aspects. The broadening of the definition of damages, which effectively obscures the once-explicit economic connection to a particular event, necessitates a closer review of these broader standards of damages.

In this paper, we aim to propose innovative insurance policy frameworks and models addressing contract structures that are subject to external triggers or exogenous risk factors. These triggers are evident in products such as multi-risk and index-linked insurance, although they may not always reflect direct intrinsic losses. Building on the research of Asimit et al. (2021), our study explores the mechanisms of insurance contracts, with a focus on the impact of exogenous triggers. These external factors, mainly characterized by unforeseen risks, play a key role in determining indemnity coverage, especially in multi-risk insurance models that underwrite unforeseen events and are not related to any actions of the insured party. By analyzing these events, insurers can effectively rebalance their risk portfolios and minimize the capital reserves required to cover potential losses, thereby improving financial stability. Moreover, our model extends its application to optimal insurance settings. While Asimit et al. (2021) focus on achieving Pareto optimality in multiple indemnity environments, our research prioritizes the development of optimal risk-sharing models in these settings from the policyholder's perspective.

To explain it comprehensively, our optimal insurance model is based on Wang's premium principle described in Wang (1996), which emphasizes mainly on the study of potential loss functions and is complemented by analyzing the conditional probability distributions of these losses in different risk environments. This allows us to construct a model in which the primary risk holder and the insurance seller work together to develop a fair indemnity contract. In assessing the risk profile of the buyer, we use Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) as indicators. Subsequently, we introduce the optimal insurance model and its corresponding optimal indemnity profile derived on this basis. It is important to emphasize that exogenous risks, manifested through the realization of different triggers, operate in an environment that is not influenced by any strategic decision of the buyer or the seller and are mutually exclusive.

In our exploration of the multiple indemnity environment paradigm, we place emphasis on the special case of excess verdicts. In such scenarios, the insurer's behavior, whether in good faith or not, becomes a key exogenous risk factor in the compensation process. Inspired by the concept of "parametric insurance", we propose an innovative model that strategically allocates pre-determined liability within a multiple indemnity framework. The model utilizes a "court judgment index" as an indicator to assess whether an insurer acts in good faith in the event of an excess verdict. Unlike traditional approaches, our methods incorporate a "sequential trigger" mechanism into the parametric insurance model. The process is divided into two distinct phases: first, the existence of an excess verdict is evaluated. After the initial trigger is activated, the second stage begins with the determination of total compensation and its distribution between the policyholder and the insurer. This sequential approach is specialized in identifying excess verdicts, thereby facilitating proceedings and improving their efficiency, reasonableness, and predictability.

The process of resolving legal disputes arising from excess verdicts is simplified when the policyholder and the insurer mutually agree to the terms outlined in the policy. Such agreements greatly reduce the time and expense of protracted legal proceedings. At the heart of this efficiency is a predetermined allocation of liability if the insurer is deemed to have acted in bad faith, rather than leaving the outcome to unpredictable and lengthy court proceedings. This structured approach improves clarity and minimizes the unforeseeability often associated with legal proceedings. One must recognize that our proposed model may have potentially significant implications for litigation law firms, as by reducing the incentive for prolonged legal battles, this approach may lead to a reduction in related litigation activity. Furthermore, it may help to mitigate the effects of social inflation, as the potential gains from extensive legal battles become less attractive.

Our study makes three major contributions to the field of insurance studies. First, it provides an in-depth exploration of the legal and economic implications of excess verdicts, particularly focusing on wrongful death cases. This includes an analysis of the broader implications for insurance policymaking and the marketplace, supplemented by a case study focused on the trucking industry. Second, we introduce a theoretical model designed to address the issue of excess verdicts, a relatively unexplored but critical area for financial risk management in the insurance industry. To the best of our current knowledge, this is the first theoretical model to provide insurers with a comprehensive framework for effectively addressing the complex issues associated with the huge and unpredictable legal judgment process.

Third, our findings suggest that optimal insurance policies maintain the same deductible across different risk environments. This simplicity benefits insurers by making it easier for them to formulate policies, assess risks, and set premiums, increasing customer satisfaction and trust in the insurer. Additionally, the stability of deductibles across risk scenarios mitigates the problems associated with moral hazard and adverse selection, as policyholders are less likely to change their behavior to benefit from different deductibles, resulting in a more robust and predictable risk pool. For customers, this means that insurance terms are easier to understand, which benefits them in comparing different insurance options. This clarity can lead to more competition among insurers to provide better provisions or services, thereby increasing the fairness and efficiency of the insurance market.

This paper is organized as follows: Section 1 lays the groundwork by exploring in detail the concept of excess verdicts. Here, we emphasize the new contributions of our study and discuss its implications in the insurance industry. Section 2 provides a comprehensive literature review focusing on optimal risk-sharing mechanisms and the background risk environment relevant to our study. Section 3 defines issues relevant in multiple indemnity environments. This section describes a customized insurance model designed specifically for these settings and analyzes it from the buyer's perspective. The model combines various risk preferences, in particular, Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), resulting in a layered solution within an optimal indemnity insurance structure. Special emphasis is given to excess verdict modeling in Section 4 within a multiple indemnity framework. Section 5 presents numerical simulations of the multiple indemnity model and analyzes the results. Finally, Section 6 summarizes our main findings and outlines possible future research directions in this area. More detailed information is provided in Appendix A, and full proof of the main result is provided in Appendix B.

2 Literature Review

This paper investigates the optimal design of insurance and reinsurance contracts in a multiple indemnity environment. A common feature of insurance markets is the existence of indemnity contracts that are activated based on specific, mutually exclusive events or "triggers." These triggers are external and not necessarily directly related to the potential loss. Such contract structures are widely found in markets dealing with multiple risks and index-linked insurance, as well as in catastrophe bonds and other risk-related securities. For example, Miranda and Vedenov (2001) examined risk securitization as a new approach to managing agricultural risk in developing countries, focusing on weather-related risks. Their work emphasized the use of index-based insurance derivatives and capital market integration to address the uncertainties inherent in weather-dependent agriculture.

Following Borch (1960)'s foundational principles on reinsurance calculations using the expected value principle and Arrow (1963)'s validation of stop-loss contracts for optimizing wealth in risk-averse insurers, significant developments have occurred in optimal insurance contract design. Raviv (1979) explored the process involved in developing an optimal insurance policy with an emphasis on deductibles, coinsurance, and the overall cost of insurance, addressing multiple loss scenarios and optimal coverage limits. Young (1999) then constructed optimal insurance contracts by focusing on maximizing the expected utility of risk-averse policyholders based on Wang's premium principle, highlighting its effectiveness against distortion in insurance pricing. Most recently, Boonen and Ghossoub (2020) focused on contracts that navigate different utility preferences and risk limits within a framework that deals with heterogeneous beliefs and constraints by analyzing optimal risk-sharing contracts in more complex scenarios and providing a mathematical approach to formulate these contracts.

As risk quantification gained focus, Cai and Tan (2007) introduced a reinsurance model using Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) for minimizing insurers' risk exposures. This led to further studies by Cai et al. (2008), Guerra and Centeno (2012), Asimit et al. (2013a, 2013b), Cheung et al. (2015), exploring premium principles and optimal reinsurance under various constraints. Chi and Tan (2011, 2013) provided a thorough study of the premium principle that satisfied general assumptions and showed how layered-type contracts remain optimal under both VaR and CVaR. Boonen and Ghossoub (2019, 2021) developed a model of a representative reinsurer integrating the risk preferences and premium principles of individual reinsurers with simplified optimal reinsurance contract formation, addressing the complexity of heterogeneous beliefs and distorting risk measures through layered insurance indemnities, simplifying negotiations in a diversified market.

Multi-risk insurance contracts combine a variety of coverages arising from different, mutually exclusive risks, focusing on losses triggered by multiple unrelated events. These triggers range from physical threats like fire, theft, and storms to quantifiable indices like the catastrophic loss index. Each of the risks described defines the specifics of the potential loss, prompting the design of policies that comprehensively cover these different risks. In these contracts, coverage is activated upon predetermined events that require independent verification. The insured pays a premium based on the probability of these triggering events, and the insurer covers the loss up to specified limits. Goodwin (1993) explored this framework in the U.S. Federal Crop Insurance Program to identify the determinants that influence farmers' preferences for multiple-risk crop insurance (MPCI), where varying degrees of farmers' risk of loss affect their insurance needs. In more complex situations, indemnification may involve the sequential occurrence of multiple specified events, thus increasing insurance coverage in a multi-risk contract. This is an important factor in the excess verdicts discussed in this paper and is usually associated with situations where multiple triggers lead successively to an overall loss outcome.

The field of behavioral economics is increasingly concerned with how background risk affects individuals' risk-taking and insurance decisions. The key work of Gollier and Pratt (1996) introduced the concept of risk vulnerability in economic decision-making, showing how background risk enhances an individual's aversion to independent risk. The analysis challenged the conventional perspective of risk substitutability, exposing that risk aversion is exacerbated in the presence of zero-mean background risk. Through mathematical modeling, Gollier and Pratt (1996) identified sufficiently necessary conditions for this effect and revealed how background risk affects market behavior, including the pricing of risky assets and equity premiums. Eeckhoudt et al. (1996) further investigated the impact of changes in background risk on individual risk-taking behavior. Their study shows that changes in background wealth significantly affect individuals' risk aversion. They identified conditions under which various types of changes in background risk, categorized as first- and second-order stochastic dominance, lead to increased risk aversion.

Heaton and Lucas (2000) explored the impact of background risks, including labor income and proprietary business income, on portfolio allocation decisions. Their study combined theoretical models with cross-sectional data to emphasize how these risks lead to heterogeneity in investors' portfolio holdings across different segments. More recently, Strobl (2022) shifted their focus to developing countries, in particular Kenya, to examine the impact of healthcare costs, a major background risk, on investment decisions. Their findings suggested that in this context, risk-averse individuals tend to choose less risky investments when faced with healthcare-related risks. In addition, the study also found that the provision of insurance does not consistently promote riskier investments, suggesting that factors such as limited insurance comprehension and disincentives to pay premiums play a crucial role in influencing investment behavior under background risk.

Looking more deeply into the insurance perspective, some attempts have been made to examine the issue of optimal indemnity insurance given the presence of background risk. Lu et al. (2018) explored optimal insurance contracts, emphasizing the importance of the relationship between background risk and insurable risk, while preserving the stop-loss order. This study suggested that deductible insurance is optimal, especially when the background risk grows stochastically relative to the insurable risk. Chi and Wei (2018) established the optimality of stop-loss insurance under various positive correlation structures between insurable and background risks, and comprehensively analyzed how changes in initial wealth or transfers of background risks affect the optimal insurance retention. Chi and Wei (2020) extended this to cover different correlation structures, including positive, moderately negative, and significantly negative correlations. Their findings indicated that the nature of dependence between background and insurable individuals mainly influences optimal insurance strategies, emphasizing stochastic dependence's important role in influencing the insured's risk transfer decisions.

Furthermore, Chi and Tan (2021) emphasized the impact of background risk on the enforcement of incentive-compatible conditions in policy design, especially when negatively correlated with insurable risk. Their study played a crucial role in reducing ex-post moral hazards, such as the temptation of the insured to exaggerate losses. Hinck and Steinorth (2023) further stressed the increased demand for insurance due to risk vulnerability and loss-dependent background risk, especially in cases where potentially large losses exceed premiums. This complements the explorations of Hofmann et al. (2019), who explored how limited policyholder liability and background risk can lead to deviations from traditional insurance models, particularly affecting the demand for excess insurance under negatively correlated risks. Altogether, these studies indicate that exogenous events may be important for optimal insurance contract design.

Recognizing the challenges posed by various environments and insurable risks, recent studies have proposed new approaches to designing insurance contracts. The framework proposed by Asimit et al. (2021) highlighted the role of external events as potential catalysts for different indemnity contracts. If these events are used effectively, insurers can manage their risk exposure and optimize the capital reserved for potential losses. In this model, the primary risk taker (the buyer of protection) and the insurer (the seller of protection) cooperate to develop a satisfactory indemnity contract profile. When taking the buyer's perspective, some studies hypothesize that the buyer's risk preferences can be summarized in terms of VaR and CVaR to reveal the optimal reinsurance model and its corresponding indemnity profile. Crucially, these models emphasize exogenous risk, characterized by the realization of triggers in the external environment that are independent and unaffected by buyer or seller decisions. Maintaining transparency and fairness for the insured remains a central concern of these advances.

3 Optimal Insurance with Multiple Indemnity Environments

The concept of excess verdict insurance can be represented with four distinct and mutually exclusive environments, each of which depends on the progress of the legal proceedings and the conduct of the insurer. The first situation is where there is no loss. The second scenario corresponds to the case in which the damages awarded remain within the prescribed insurance limit. In the third scenario, the damages awarded exceed the limit specified in the insurance policy. However, the subsequent litigation does not reveal any bad faith or misconduct on the part of the insurer. The last scenario describes a situation where, after the compensation awarded exceeds the insurance limit, a further lawsuit also reveals bad faith from the insurer. This categorization, pertinent to excess verdict insurance, motivates the study of the broader conceptual framework of "multiple indemnity environments," a notion rigorously examined through the lens of Pareto-optimal risk-sharing in the work by Asimit et al. (2021). In the following subsections, we state and solve an optimal insurance problem where the indemnity function depends on the prevailing environments. This discussion encompasses the problem's definition, optimization using VaR and CVaR, and further analysis through the Proportional Hazard Transform.

3.1 Problem Definition

Let $(\Omega, \mathscr{F}, \mathbb{P})$ be a probability space on which all random variables are defined. We consider a one-period economy where a primary risk holder is endowed with a non-negative loss X which is payable at a fixed future time $T > 0$. It is assumed that $0 < \mathbb{E}[X] < \infty$, where $\mathbb{E}$ is the expectation under $\mathbb{P}$.

The primary risk holder, or (insurance) buyer, intends to share the loss at time T with another party, or (insurance) seller and accepts to pay a premium at time 0. Both parties agree to achieve optimality in terms of their risk positions by choosing appropriate amounts of indemnity and premium. However, unlike classical risk-sharing problems, this paper considers a setting such that the indemnity level depends upon an external factor, which cannot be influenced by either party, yet can be precisely observed at time T.

To this end, let Y be the trigger characterizing the exogenous environment so that the sample space Ω is partitioned into $m+1$ disjoint subsets given by events $\{\omega \in \Omega : Y(\omega) = k\}$, for $k = 0, 1, \dots, m$. Moreover, if $Y = 0$ then $X = 0$, implying that under the environment $Y = 0$ there is no loss. For each remaining environment $k = 1, \dots, m$, the loss is risky, in the sense that $\mathbb{P}(X > 0 | Y = k) > 0$. Thus, we explicitly assume that the random variables X and Y are *not* independent.

If the realized environment is non-risky, i.e. $Y = 0$, no indemnity transfer is required. Moreover, if the prevailing environment is $Y = k$, for some $k = 1, \ldots, m$, the buyer will transfer the amount $I_k(X)$ to the seller at time T and retain the amount $R_k(X) = X - I_k(X)$, where $I_k : [0, \infty) \to \mathbb{R}$ is called an indemnity function and $R_k : [0, \infty) \to \mathbb{R}$ is called a retention function. Note that both parties have to agree at time 0 on a profile of indemnity functions $(I_1, \ldots, I_m)$ since the exogenous environment is not revealed until time T.

A profile of indemnity functions is admissible if it belongs to the set

$$\mathcal{I} = \{(I_1, \ldots, I_m) : 0 \le I_k \le \mathrm{Id},\ R_k = \mathrm{Id} - I_k,\ I_k \text{ and } R_k \text{ are non-decreasing for all } k = 1, \ldots, m\},$$

where Id denotes the identity function. Hence, under each environment, the indemnity is at most the loss, and misrepresentation of the loss is disincentivized precluding ex-post moral hazard from both parties, as suggested by Huberman et al. (1983). We refer to a tuple $(I_1, \ldots, I_m) \in \mathcal{I}$ as a *contract*.

For each contract $(I_1, \ldots, I_m) \in \mathcal{I}$, the realized risk position of the buyer is given by

$$\mathbf{B}(I_1, \ldots, I_m) = \sum_{k=1}^{m} R_k(X) \mathbb{1}_{\{Y=k\}} + (1+\rho) P_g \left(\sum_{k=1}^{m} I_k(X) \mathbb{1}_{\{Y=k\}} \right), \tag{3.1}$$

where $\mathbb{1}_A$ is the indicator function of an event $A \subset \Omega$. On the right-hand side of (3.1), the first component of the loss is retained by the buyer, which depends on the prevailing environment. The second term is the seller's premium, calculated with Wang's Premium Principle P_g and inflated by the explicit safety load $\rho \ge 0$. For any loss Z, $P_g(Z)$ is defined as

$$P_g(Z) = \int_0^\infty g(S_Z(z))\, dx, \tag{3.2}$$

where $g : [0,1] \to [0,1]$ is a non-decreasing concave function with $g(0) = 0$, $g(1) = 1$, and S_Z is the survival function of Z.

Let φ denote the buyer's risk measure, designed to rank their risk preferences at time $t = 0$. Formally, φ is a real function defined on a linear space of losses containing the constants. We assume φ to be translation invariant, therefore ensuring consistency in the evaluation of risk positions with respect to capital injections. With this in mind, the buyer's risk position at $t = 0$ corresponding to (3.1) can be expressed as

$$\begin{aligned} F_\varphi(I_1, \ldots, I_m) &= \varphi(\mathbf{B}(I_1, \cdots, I_m)) \\ &= \varphi\left(\textstyle\sum_{k=1}^m R_k(X) \mathbb{1}_{\{Y=k\}}\right) + (1+\rho) P_g\left(\textstyle\sum_{k=1}^m I_k(X) \mathbb{1}_{\{Y=k\}}\right). \end{aligned} \tag{3.3}$$

3.2 Optimality with VaR and CVaR Preferences

In this section, we assume that the risk preferences φ of the buyer are represented by either Value-at-Risk or Conditional Value-at-Risk. Then $\varphi = \mathrm{VaR}$ or $\varphi = \mathrm{CVaR}$.

Recall that for a loss Z, the Value-at-Risk at level $\alpha \in (0,1)$ is

$$\mathrm{VaR}_\alpha(Z) = \inf\{z \in \mathbb{R} : \mathbb{P}(Z > z) \leq 1-\alpha\}.$$

The ruin probability α is associated with the buyer's risk tolerance level.

The Conditional Value-at-Risk at level $\alpha \in (0,1)$ is

$$\mathrm{CVaR}_\alpha(Z) = \frac{1}{1-\alpha}\int_\alpha^1 \mathrm{VaR}_s(Z)\, ds.$$

The CVaR is alternatively called Expected Shortfall and has gained practitioners' interest since the introduction of Basel III regulations, see McNeil et al. (2015) for further discussion.

The buyer seeks to minimize his/her risk position at time $t = 0$, given by (3.3), over all admissible indemnity profiles. The buyer's minimization problem is given by

$$\min_{(I_1,\ldots,I_m)\in\mathcal{I}} F_\varphi\left(I_1,\ldots,I_m\right). \tag{3.4}$$

Consider now the following subset of admissible indemnity profiles

$$\begin{aligned}\mathcal{I}^* = \Big\{\big(I_1,\ldots,I_m\big) \in \mathcal{I}\colon \quad & \text{for each } k = 1,\ldots,m, \text{ there exist } m_k \in [0, \operatorname{ess\,sup}(X)], \\ & \text{and } n_k \in [m_k, \operatorname{ess\,sup}(X)], \text{ such that } I_k(x) = \big(x-m_k\big)_+ - \big(x-n_k\big)_+\Big\}.\end{aligned}$$

where $\operatorname{ess\,sup}(X)$ is the essential supremum of X and $(x)_+ = \max\{x, 0\}$. Each indemnity profile in $\mathcal{I}^*$ features layer-type transfers where, in any exogenous environment, the indemnity is full insurance up to a deductible m_k and beyond an upper limit n_k. The notation makes clear that the deductible and upper limit may depend on the environment.

The buyer may want to find an optimal contract in the sub-class $\mathcal{I}^*$, therefore restricting the choice to layer-type transfers. From a mathematical point of view, the latter problem involves a finite number of decision variables, unlike (3.4) which is infinite-dimensional. In general, this will result in a sub-optimal contract. However, the next theorem shows that, when risk preferences are VaR or CVaR, any solution of (3.4) is indeed a layer-type indemnity profile.

Theorem 3.2.1. *Let $\varphi = VaR_\alpha$ or $\varphi = CVaR_\alpha$. For any $\rho \geq 0$ and $(I_1,\ldots,I_m) \in \mathcal{I}$, there exists $(\tilde{I}_1,\ldots,\tilde{I}_m) \in \mathcal{I}^*$ such that $F_\varphi(\tilde{I}_1,\ldots,\tilde{I}_m) \leq F_\varphi\left(I_1,\ldots,I_m\right)$.*

The proof of this theorem is in Appendix B.1 for $\varphi = \mathrm{VaR}_\alpha$ and in Appendix B.2 for $\varphi = \mathrm{CVaR}_\alpha$.

Remark 3.2.1. *Theorem 3.2.1 states that any admissible indemnity profile* $(I_1, \ldots, I_m)$ *is dominated by a layer-type risk transfer* $(\tilde{I}_1, \ldots, \tilde{I}_m) \in \mathcal{I}^*$*. Inspection of the proof of Theorem 3.2.1 further shows that* $(\tilde{I}_1, \ldots, \tilde{I}_m)$ *can be chosen so that the deductibles in each environment coincide,* $m_1 = \cdots = m_k$*. Therefore, the optimal indemnity profile requires covering the entire loss up to a deductible that is not affected by the prevailing exogenous environments.*

3.3 Optimality with the Proportional Hazard Transform

Consider a modification of the premium principle employed in (3.1), that allows us to weigh losses differently in each environment. Specifically, we utilize the *Proportional Hazard (PH) transform* (see Appendix A.3), which features a unique concave distortion function for different risk environments, i.e., $g_k(z) = z^{\beta_k}$, where $0 < \beta_k \leq 1$, see Wang (1995). The proposed principle of risk-adjusted premiums is advantageous as it factors in the buyer's risk aversion and, more importantly, it allows for a proper weighing of events from riskier environments. Thus, for any given indemnity profile $(I_1, \ldots, I_m) \in \mathcal{I}$, the realized risk position of the buyer can be expressed as

$$\mathbf{B}(I_1, \cdots, I_m) = \sum_{k=1}^{m} R_k(X) \mathbb{1}_{\{Y=k\}} + \sum_{k=1}^{m} P_{g_k}\left(I_k(X) \mathbb{1}_{\{Y=k\}}\right), \tag{3.5}$$

Analogously, with respect to equation (3.5), the buyer's risk position at $t = 0$ is articulated as

$$\varphi\left(\mathbf{B}(I_1, \cdots, I_m)\right) = \varphi\left(\sum_{k=1}^{m} R_k(X) \mathbb{1}_{\{Y=k\}}\right) + \sum_{k=1}^{m} P_{g_k}\left(I_k(X) \mathbb{1}_{\{Y=k\}}\right) \tag{3.6}$$

The main theorem in Subsection 3.2 still works if the objective function in (3.4) is replaced by (3.6). The proof is omitted as it is similar to that in the main theorem.

4 Excess Verdicts

Insurance contracts are essential to mitigate the financial impact of unforeseen events. However, in some jurisdictions, there may be discrepancies between what these contracts cover and what the courts decide in the case of litigation. Our research seeks to develop a model that simplifies the understanding of these differences, focusing on the joint decision-making process of (insurance) buyers and (insurance) sellers, in determining the allocation of liability in challenging scenarios.

Consider a situation where a loss, denoted as X, is caused by an external factor such as bodily injury or property damage. Let L be the loss threshold resulting in the attainment of the policy limit. Losses up to this limit of L are shared between the buyer and the seller according to the provisions of the policy. However, when the loss surpasses L, the excess is usually the responsibility of the buyer. It is important to recognize that deviations from this rule are at times observed in some well-developed jurisdictions, such as some U.S. states, Canadian provinces, and the United

Kingdom, reflecting the intricate interplay of socio-political and legal factors. When a plaintiff brings a claim against the buyer (defendant), the latter may be more inclined to take legal action against the seller, especially if they are facing a serious case such as a *wrongful death*. When this happens, a court of law may force the seller to bear the cost of exceeding policy limits in certain circumstances.

In the legal sphere, compensatory damages and non-economic damages, including punitive damages, emotional distress, and other related liabilities, are subject to strict judicial scrutiny. The actions and behavior of the seller play a critical role in these legal assessments. Apparent misconduct or bad faith practices by the seller can significantly increase the financial damages awarded by the court. As mentioned before, the seller may be liable for damages above the policy limits. The exact allocation of the damages between the buyer and the seller, including the substantial legal fees, is left to the discretion of the court. Thus, the primary purpose of this paper is to evaluate the design of insurance contracts that pre-allocate the financial liability between buyers and sellers based on prospective judicial decisions. The inclusion of such predictive clauses has the potential to reduce legal costs, expedite protracted legal proceedings, and improve the seller's position in legal disputes, ultimately reducing financial burdens.

Consistently with Section 3, let Y be a variable that concisely classifies the stages and intricacies of the legal process, especially the excess verdicts related to the seller's behavior. As exemplified by the flowchart in Figure 3, the case $Y = 0$ denotes a situation where there is no loss; $Y = 1$ indicates that, even if legal action may be initiated because of the plaintiff's claim, the damages awarded are within the insurance limit, which corresponds to $X \leq L$, and there is no excess verdict; $Y = 2$ occurs when $X > L$, implying that the damages awarded exceed the insurance limit, but the subsequent litigation does not reveal any misconduct or bad faith on the part of the seller; finally, the case $Y = 3$ takes place when, after the verdict has exceeded the insurance limit, a further lawsuit determines that there is bad faith on the seller's part. Clearly, in the latter case, $X > L$.

It is essential to note that our analysis deliberately excludes considerations of the buyer's and seller's financial solvency, such as a potential bankruptcy. It is important to note the two-stage nature of the judicial decision-making process. The initial stage involves determining whether there is an excess verdict. The second stage distinguishes between $Y = 2$ and $Y = 3$, depending on the good faith or bad faith of the seller. In the upcoming subsections, we explore scenarios with and without environment-contingent policies in contracts and conduct a comparative analysis.

4.1 Contract Without Environment Contingent Provisions

In this section, let $\hat{I}$ represent the indemnity function and $\hat{R}(X) = X - \hat{I}(X)$ be the retention function in a contract in which specific provisions for pre-determining the liability allocation in the event of an excess verdict are absent. The sharing rule established in the contract applies to scenarios $Y = 1$ and $Y = 2$. In particular, when $Y = 2$, and so $X > L$, the payment obligation of the seller is $\hat{I}(X) = \hat{I}(L)$, where $\hat{I}(L)$ is the policy limit. In this case, the buyer's liability for the excess over L is $\hat{R}(X) = \hat{R}(L) + (X - L)$.

In the third scenario, $Y = 3$, where the total loss X surpasses L, the apportionment of the liability between the buyer and seller is subject to judicial determination following extensive and prolonged legal proceedings, resulting in an allocation different than the one agreed in the contract. Denote by $\hat{I}^c(X)$ the obligation of the seller, and by $\hat{R}^c(X) = X - \hat{I}^c(X)$ that of the buyer, as decided by the adjudicating entity, whether a jury or judge. The duration of the legal proceedings is directly correlated with the cumulative liability incurred, amplifying the plaintiff's emotional distress as the litigation extends. Therefore, conditional on $Y = 3$, the distribution of the loss X will be greater than in the states $Y = 1$ or $Y = 2$. The seller's obligation $\hat{I}^c(X)$ will sensibly exceed the agreed indemnity $\hat{I}(X) = \hat{I}(L)$.

4.2 Contract With Environment Contingent Provisions

Let us assume now that the insurance contract has a provision according to which the buyer and seller share damages when the seller's bad faith has been established through a post-excess verdict determination. As argued before, the rationale for designing the contract including such provision is to stabilize the excess risk loss, primarily by abbreviating the duration of intricate legal proceedings.

When $Y = 1$ or $Y = 2$, the structure of the contract will be similar to that described in Section 4.1, with an allocation of liabilities based on the indemnity functions I_1, I_2 and the corresponding retention functions R_1, R_2. In particular, when $Y = 2$, the loss X extends over the limit L, i.e. $X > L$, and the excess is suffered by the buyer so that $R_2(X) = R_2(L) + (X - L)$. When there is an excess verdict, $Y = 3$ and $X > L$, the splitting of the liability will be based on the predetermined indemnity I_3 and retention R_3. It is clear that the distribution of the loss in the scenario $Y = 3$ will be modified by the presence of such environment contingent provisions. These clauses will also impact the shape of the contract in scenarios $Y = 1$ and $Y = 2$. In particular, the limit L may be different from the one considered in Section 4.1.

Following the theoretical results in Section 3.2, the shape of the optimal contract in each environment can therefore be inferred. It is suggested that the equilibrium rule includes a shared deductible m and an environment-specific upper limit n_i for $i = 1, 2, 3$. It is plausible that, under normal circumstances, i.e. $Y = 1$ or $Y = 2$, the indemnity will be specified through a single expression as the range of the loss is different in each case. The threshold L can be identified with the limit $n_1 = n_2$.

In the case of the excess verdict, $Y = 3$, the losses exceeding L are now split as follows. The seller agrees to cover all the losses up to a limit $n_3 = \tilde{L}$, which, because of the nature of the problem, is such that $\tilde{L} > L$. The indemnity paid by the seller is therefore $I_3(X) = X - R_3\left(\tilde{L}\right)$ if $X \leq \tilde{L}$ and $I_3(X) = \tilde{L} - R_3\left(\tilde{L}\right)$ if $X > \tilde{L}$. The corresponding retention is $R_3(X) = R_3\left(\tilde{L}\right)$ if $X \leq \tilde{L}$ and $R_3(X) = R_3\left(\tilde{L}\right) + \left(X - \tilde{L}\right)$ if $X > \tilde{L}$.

Table 1 summarizes the payments of buyer and seller under the presence or not of environment contingent provisions.

Environment	Party	Without Provisions	With Provisions
$Y = 1$	Buyer	$\hat{R}(X)$	$R_1(X)$
	Seller	$\hat{I}(X)$	$I_1(X)$
$Y = 2$	Buyer	$\hat{R}(X) = \hat{R}(L) + (X - L)$	$R_2(X) = R_2(L) + (X - L)$
	Seller	$\hat{I}(X) = \hat{I}(L)$	$I_2(X) = I_2(L)$
$Y = 3$	Buyer	$\hat{R}^c(X)$	$R_3(\tilde{L}) + (X - \tilde{L})_+$
	Seller	$\hat{I}^c(X)$	$I_3(X) = X - R_3(\tilde{L}) - (X - \tilde{L})_+$

Table 1: Payments of the buyer and seller in the contract with/without environment contingent provisions across different environments.

5 Numerical Optimization Analysis

In this section, we introduce the foundational model setup for numerical optimization and examine the outcomes of the simulations.

5.1 Basic Model Setting

We consider three risk environments, in each of which the loss (in thousands of monetary units) is modeled with a Type II Pareto distribution, as detailed in Table 2, together with the corresponding scenario probabilities.

Risk Environment	$\mathbb{P}(Y = k)$	λ	α	$\mathbb{E}[X \vert Y = k]$	$SD[X \vert Y = k]$
$Y = 1$	60%	40	5	10	12.91
$Y = 2$	30%	200	3	100	173.21
$Y = 3$	10%	1,500	2.5	1,000	2236.07

Table 2: Risk environment parameters and their statistical properties.

We use $\text{CVaR}_{95\%}$ as risk measure and the proportional hazard transformation for adjusting risk premiums in heavy-tailed scenarios introduced in Section 3.3. Figure 1 shows the inverse relationship between environmental-specific premia (for full insurance coverage) and the parameter

β defining the distortion function g. Smaller values of β result in more concave distortions and, in turn, in higher premium loadings.

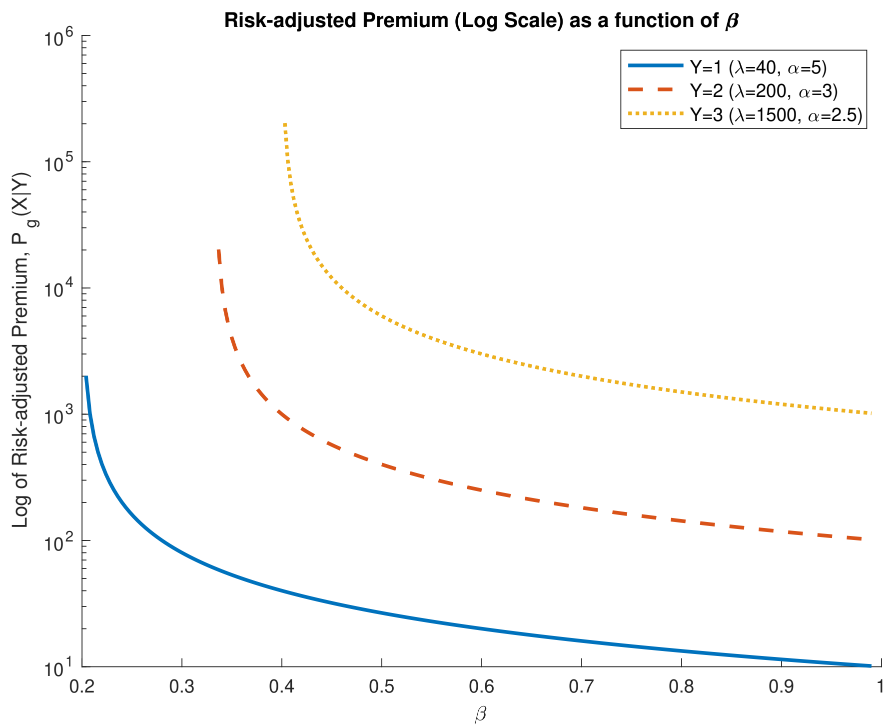

Figure 1: Log-scale for the risk-adjusted premium vs. β values

5.2 Model Results Analysis

In this section, we show some results on the optimal contracts defined in Section 3 and, in particular, in Section 3.3. According to Theorem 3.2.1, these contracts feature a common deductible $m = m_1 = m_2 = m_3$ and environment-specific limits n_1, n_2, n_3. We explore the finite-dimensional nature of the problem to find the optimal contract by minimizing, using a routine numerical algorithm, the objective function over the parameters m, n_1, n_2, n_3. We analyze the interplay between the distortion parameters $\beta_1, \beta_2, \beta_3$ and the policyholder coverage preferences across different risk scenarios.

We use as baseline values for the distortion coefficients $\beta_1 = 0.65$, $\beta_2 = 0.55$, $\beta_3 = 0.45$, so that the loading increases with the riskiness of the scenario. Tables 3, 4, and 5 show the quantiles, conditional on each environment, corresponding to the deductible m and the limits n_1, n_2, n_3, as each of the distortion coefficients is separately stressed.

An increase in β_k results in a decrease in risk aversion, leading to cheaper insurance, in relative terms, and eventually to more extensive coverage, i.e., lower deductible and higher upper limit. However, the upper limits in environments other than k^{th} are affected to a very limited extent.

	Risk environment Y_1		Risk environment Y_2		Risk environment Y_3	
β_1	$F_X(m_1)$	$S_X(n_1)$	$F_X(m_2)$	$S_X(n_2)$	$F_X(m_3)$	$S_X(n_3)$
0.45	93.13%	0.72%	32.80%	0.43%	4.57%	4.31%
0.55	89.44%	0.21%	27.58%	0.43%	3.69%	4.31%
0.65	85.40%	0.03%	23.60%	0.43%	3.06%	4.31%
0.75	81.19%	0.00%	20.48%	0.43%	2.60%	4.31%
0.85	76.93%	0.00%	17.95%	0.43%	2.24%	4.31%
0.95	72.71%	0.00%	15.87%	0.43%	1.95%	4.31%

Table 3: CDF at the deductible and survival function at the upper limit, conditional on each scenario, for different values of β_1.

	Risk environment Y_1		Risk environment Y_2		Risk environment Y_3	
β_2	$F_X(m_1)$	$S_X(n_1)$	$F_X(m_2)$	$S_X(n_2)$	$F_X(m_3)$	$S_X(n_3)$
0.45	90.69%	0.032%	29.12%	1.44%	3.94%	4.31%
0.55	85.40%	0.032%	23.60%	0.43%	3.06%	4.31%
0.65	79.93%	0.032%	19.67%	0.06%	2.48%	4.31%
0.75	74.51%	0.032%	16.72%	0.00%	2.07%	4.31%
0.85	69.28%	0.00%	14.41%	0.00%	1.75%	4.31%
0.95	64.32%	0.00%	12.57%	0.00%	1.51%	4.31%

Table 4: CDF at the deductible and survival function at the upper limit, conditional on each scenario, for different values of β_2.

On the other hand, in the k^{th} scenario, the upper limit increases with β_k quickly attaining full insurance above the deductible, a result reminiscing of classical optimal insurance paradigms. This is true also for the high-risk environment $Y = 3$, although the pricing rule must embody a much limited distortion before full insurance is attained.

6 Conclusions and Future Research

In this paper, we have studied the optimal insurance problem from the buyer's perspective across multiple indemnity environments, with a particular focus on excess verdicts and their legal and

	Risk environment Y_1		Risk environment Y_2		Risk environment Y_3	
β_3	$F_X(m_1)$	$S_X(n_1)$	$F_X(m_2)$	$S_X(n_2)$	$F_X(m_3)$	$S_X(n_3)$
0.45	85.40%	0.032%	23.60%	0.43%	3.06%	4.31%
0.55	78.72%	0.032%	18.95%	0.43%	2.38%	1.28%
0.65	72.53%	0.00%	15.80%	0.43%	1.94%	0.19%
0.75	67.09%	0.00%	13.56%	0.43%	1.64%	0.006%
0.85	62.45%	0.00%	11.94%	0.43%	1.43%	0.00002%
0.95	58.57%	0.00%	10.73%	0.43%	1.27%	0.00%

Table 5: CDF at the deductible and survival function at the upper limit, conditional on each scenario, for different values of β_3.

financial impacts. Our model analyzes the risk-sharing between policyholders and insurers, especially when legal rulings may impose damages beyond the policy limits. The phenomenon of excess verdicts — court-mandated payments exceeding insured amounts — demonstrates the practical relevance of our framework. Our research shows that optimal insurance contracts feature layered indemnities in each risk environment. We employ risk measures like VaR and CVaR to simplify complex optimization problems into tractable forms, facilitating numerical optimization and the decision-making process.

While VaR and CVaR are established risk measures in the insurance industry, considering alternative risk measures may enhance our grasp of risk sharing across various indemnity contexts. Further, assessing the enforceability of anticipatory clauses across jurisdictions can improve the excess verdict model's legal soundness. Additionally, empirical studies utilizing actual insurance data, particularly in cases involving excess verdicts, are crucial for validating our theoretical constructs. Such investigations promise to enrich the practical relevance of our framework, and we reserve these areas for future exploration.

A Ancillary Results

A.1 Left and right continuous inverses

Given the role of left and right continuous inverse functions in the proof of the main result of this paper, Theorem 3.2.1. We provide in this section their definitions and state some of their properties.

Definition A.1. *Let $f:\mathbb{R}\to\mathbb{R}$ be a real function. The right-continuous inverse of f is given by*

$$f^{-1+}(y)=\inf\{x\in\mathbb{R}: f(x)>y\},\ \ y\in\mathbb{R}.$$

The left-continuous inverse of $f:\mathbb{R}\to\mathbb{R}$ is given by

$$f^{-1}(y)=\inf\{x\in\mathbb{R}: f(x)\geq y\},\ \ y\in\mathbb{R}.$$

In this definition, we use the convention that $\inf\varnothing=+\infty$.

Note that the $\mathrm{VaR}_u(Z)$ of a random variable Z with cumulative distribution function F, defined in Section 3.2, is the left-continuous inverse of F.

The next result states some properties of the right-continuous inverse.

Proposition A.1. *Let $f:\mathbb{R}\to\mathbb{R}$ be a real function. Then, for a given $y\in\mathbb{R}$ we have that*

a) *$f^{-1+}(y)=-\infty$ if and only if $f(x)>y$ for all $x\in\mathbb{R}$; further, $f^{-1+}(y)=+\infty$ if and only if $f(x)\leq y$ for all $x\in\mathbb{R}$;*

b) *Assume f is right-continuous and $f^{-1+}(y)<\infty$, then $f\left(f^{-1+}(y)\right)\geq y$; further, if f is continuous, then $f\left(f^{-1+}(y)\right)=y$;*

c) *$x>f^{-1+}(y)$ implies that $f(x)>y$, and the reverse implication holds if f is left-continuous; further, $f(x)\leq y$ implies that $x\leq f^{-1+}(y)$, and the reverse implication holds if f is letf-continuous.*

Proof. For any $y\in\mathbb{R}$, define $A_y=\{z\in\mathbb{R}: f(z)>y\}$.

For Part a), the result is immediate from Definition A.1.

For the first claim in Part b). If $f^{-1+}(y)<\infty$, then $A_y\neq\varnothing$. By definition, $f^{-1+}(y)=\inf A_y$. For any sequence $(x_n)_{n\in\mathbb{N}}$ with $x_n\in A_y$ and $x_n\geq f^{-1+}(y)$ for all n, it is observed that $x_n\downarrow f^{-1+}(y)$ as $n\to\infty$ ($x_n\downarrow f^{-1+}(y)$ indicates x_n is monotonically decreasing and converges to $f^{-1+}(y)$.). From the right-continuity of f at $f^{-1+}(y)$, $\lim_{n\to\infty}f(x_n)=f(f^{-1+}(y))$ follows. Furthermore, since $f(x_n)>y$ for all n, we obtain $f(f^{-1+}(y))\geq y$.

For the second claim in Part b), given the right-continuity of f, $f(f^{-1+}(y))\geq y$ has been established. To derive the equality $f(f^{-1+}(y))=y$, it is necessary to show $f(f^{-1+}(y))\leq y$. Consider a sequence $(y_n)_{n\in\mathbb{N}}$ for all n such that $y_n<f^{-1+}(y)$ and $y_n\uparrow f^{-1+}(y)$ as $n\to\infty$ ($y_n\uparrow f^{-1+}(y)$ means the sequence y_n monotonically increases and converges to $f^{-1+}(y)$.). With the left-continuity property of f at $f^{-1+}(y)$, $\lim_{n\to\infty}f(y_n)=f(f^{-1+}(y))$ is inferred. Given that $f(y_n)\leq y$ for all n, $\lim_{n\to\infty}f(y_n)\leq y$ follows. Therefore, $f(f^{-1+}(y))=y$ is established, which completes the proof.

For the first statement of Part c), assume $x>f^{-1+}(y)$. Given $f^{-1+}(y)=\inf A_y$, it follows that $x\in A_y$ and $f(x)>y$. If $f(x)>y$, then $x\geq f^{-1+}(y)$. For the case $x=f^{-1+}(y)$, consider a sequence

$(y_n)_n$ for all n where $y_n < f^{-1+}(y)$ and $y_n \uparrow f^{-1+}(y)$ as $n \to \infty$. Using the result from the proof of Part b) because of the left-continuity of f, $f(f^{-1+}(y)) \le y$ is derived, implying $f(x) \le y$. This is a contradiction, so $x > f^{-1+}(y)$.

For the second statement of Part c). Suppose $f(x) \le y$, it follows that $x \notin A_y$, establishing x as a lower bound for A_y. As $f^{-1+}(y) = \inf A_y$, we have $x \le f^{-1+}(y)$. Conversely, suppose $x \le f^{-1+}(y) = \inf A_y$, if $x < \inf A_y$, which implies $x \notin A_y$, so $f(x) \le y$; if $x = f^{-1+}(y)$, we can utilize the result from the proof of Part b), which applies due to the left-continuity of f. From this, we infer $f(f^{-1+}(y)) \le y$, and consequently, $f(x) \le y$ follows. Thus, in either case, $f(x) \le y$ holds, which completes the proof. □

The next result states a property of left-continuous inverse. A more in-depth discussion of the left-continuous inverse functions can be found in Embrechts and Hofert (2013).

Proposition A.2. *Let f be a non-decreasing and right-continuous function. Then $f^{-1}(y) \le x$ and $f(x) \ge y$ are equivalent for any $(x, y) \in \mathbb{R}^2$.*

Proof. For any $y \in \mathbb{R}$, define $B_y = \{z \in \mathbb{R} : f(z) \ge y\}$.

First, assume $f^{-1}(y) \le x$. By definition, $f^{-1}(y) = \inf B_y$. Given $x \ge \inf B_y$, a sequence $(x_n)_{n \in \mathbb{N}}$ exists such that for all n, $x_n \in B_y$ and $x_n \downarrow f^{-1}(y)$ as $n \to \infty$. With the right-continuity of f and $f(x_n) \ge y$ for all n, it follows that $\lim_{n\to\infty} f(x_n) = f(f^{-1}(y)) \ge y$. The non-decreasing nature of f then implies that if $x \ge f^{-1}(y)$, $f(x) \ge f(f^{-1}(y))$. Hence, $f(x) \ge y$.

Second, if $f(x) \ge y$, then by the definition of the left-continuous inverse, $x \in B_y$ and $x \ge \inf B_y$. Thus, $f^{-1}(y) = \inf B_y$ yields $f^{-1}(y) \le x$. □

A.2 Stochastic Ordering

We recall the formal definition of stop-loss order and collect some related results, see Rolski et al. (1999) and Shaked and Shanthikumar (2007).

Definition A.2. *Let X and Y be two real-valued random variables.*
We say that X is smaller than Y in stop-loss order, and write $X \le_{sl} Y$, if for all non-decreasing convex function $g : \mathbb{R} \to \mathbb{R}$:

$$\mathbb{E}[g(X)] \le \mathbb{E}[g(Y)],$$

provided the expectations $\mathbb{E}[g(X)]$ and $\mathbb{E}[g(Y)]$ are finite.

Proposition A.3. *Let X and Y be two real-valued random variables.*

a) *If $X \le Y$ almost surely, then $X \le_{sl} Y$.*

b) *Assume X and Y have finite expectations and $\mathbb{E}[X] \le \mathbb{E}[Y]$. If, for some $t_0 \in \mathbb{R}$, $F_X(t) \le F_Y(t)$ for $t < t_0$ and $F_X(t) \ge F_Y(t)$ for $t \ge t_0$, then $X \le_{sl} Y$.*

c) *The following statements are equivalent:*

   i) $X \leq_{sl} Y$.

   ii) *For all* $d \in \mathbb{R}$, $\mathbb{E}[(X-d)_+] \leq \mathbb{E}[(Y-d)_+]$.

d) *The premium principle in* (3.2) *preserves the stop-loss order:*

$$X \leq_{sl} Y \Rightarrow P_g(X) \leq P_g(Y).$$

The proof of part d) can be found in Wang (1996).

A.3 Proportional Hazards Transform

Here, we introduce the proportional hazard transformation and the associated risk-adjusted premiums used in Section 3.3. Full details can be found in Wang (1995).

Definition A.3. *Given any random variable* X *with survival function* S_X *and* $0 < \beta \leq 1$, *the equation*

$$S_Y(t) = S_X(t)^{\beta}, \quad t \in \mathbb{R},$$

defines another random variable Y *with survival function* S_Y. *The mapping:* $\Pi_\beta : X \mapsto Y$ *is called the proportional hazards (PH) transform.*

Definition A.4. *For a risk* X *with survival function* S_X *and* $0 < \beta \leq 1$, *the risk-adjusted premium is defined as*

$$\pi_\beta(X) = \mathbb{E}\left[\Pi_\beta(X)\right] = \int_0^{\infty} S_X(t)^{\beta} dt$$

where $\frac{1}{\beta}$ *is called the (risk-averse) index. When* $\beta = 1$, $\pi_1(X) = \int_0^\infty S_X(t)dt = \mathbb{E}[X]$, *which is the net expected loss.*

B Proofs of the Main Results

We now present the proof of our main results. For clarification, we will focus on using the Wang's premium principle. Although our provided numerical optimization is based on the risk-adjusted premium of the *PH transform*, the proof approaches are similar. Therefore, we do not provide a separate proof for the *PH transform* method.

B.1 Proof of Theorem 3.2.1 with VaR Preferences

Fix $(I_1, \ldots, I_m) \in \mathcal{I}$ and let $R_k = Id - I_k$. Define

$$b := \mathrm{VaR}_\alpha\left(\sum_{k=1}^{m} R_k(X)\, \mathbb{1}_{\{Y=k\}}\right). \tag{B.1}$$

Fix now $k = 1, \ldots, m$ and let R_k^{-1+} denote the right-continuous inverse of R_k and note that $R_k(R_k^{-1+}(b)) = b$ provided $R_k^{-1+}(b) < +\infty$, see Proposition A.1b). In this case, it holds $b \le R_k^{-1+}(b)$ since $R_k + I_k = Id$. The same inequality holds if $R_k^{-1+}(b) = +\infty$. Consequently, define $m_k = b$, $n_k = R_k^{-1+}(b)$ and $\tilde{I}_k$ by

$$\tilde{I}_k(x) = (x-b)_+ - \left(x - R_k^{-1+}(b)\right)_+ = \begin{cases} 0 & \text{if } 0 \le x < b, \\ x - b & \text{if } b \le x \le R_k^{-1+}(b), \\ R_k^{-1+}(b) - b & \text{if } R_k^{-1+}(b) < x. \end{cases} \tag{B.2}$$

It follows that

$$\tilde{R}_k(x) = Id(x) - \tilde{I}_k(x) = \begin{cases} x & \text{if } 0 \le x < b, \\ b & \text{if } b \le x \le R_k^{-1+}(b), \\ x - R_k^{-1+}(b) + b & \text{if } R_k^{-1+}(b) < x. \end{cases} \tag{B.3}$$

It is understood that, in (B.2) and (B.3), only the first two cases apply when $R_k^{-1+}(b) = +\infty$. So the first step is to demonstrate

$$\{R_k(X) > b\} = \left\{\tilde{R}_k(X) > b\right\} \quad \text{for any} \quad k = 1, \ldots, m. \tag{B.4}$$

According to Proposition A.1a), if $R_k^{-1+}(b) = +\infty$ then $R_k(x) \le b$ for all x. But it is seen from (B.3) that the latter is equivalent to $\tilde{R}_k(x) \le b$ for all x. Therefore, we only need to consider the case when $R_k^{-1+}(b) < +\infty$. Suppose $R_k(x) > b$. According to Proposition A.1c), we deduce that $x > R_k^{-1+}(b)$ and from (B.3), we obtain $\tilde{R}_k(x) > b$. Conversely, suppose $\tilde{R}_k(x) > b$. From (B.3) it follows that $x > R_k^{-1+}(b)$ and Proposition A.1c) implies that $R_k(x) > b$. Therefore, (B.4) holds and the first part of the proof is complete.

In the second step, we aim to prove that $\tilde{I}_k(x) \le I_k(x)$ for all x, from which

$$\tilde{I}_k(X) \le I_k(X) \quad \text{for any} \quad k = 1, \ldots, m. \tag{B.5}$$

We proceed by cases on the value of x.

For $0 \le x < b$, based on (B.2), we have $\tilde{I}_k(x) = 0 \le I_k(x)$. For $b \le x \le R_k^{-1+}(b)$, utilizing (B.2), we find $\tilde{I}_k(x) = x - b$. Therefore, $\tilde{I}_k(x) \le I_k(x)$ if and only if $R_k(x) \le b$. If $R_k^{-1+}(b) = +\infty$, this follows from Proposition A.1a). If instead $R_k^{-1+}(b) < +\infty$, then $R_k(x) \leq R_k\left(R_k^{-1+}(b)\right) = b$ by Proposition A.1b). Finally, assume $R_k^{-1+}(b) < x$, so that $R_k^{-1+}(b) < +\infty$. From (B.2), we have $\tilde{I}_k(x) = R_k^{-1+}(b) - b$. Therefore, $\tilde{I}_k(x) \leq I_k(x)$ if and only if $R_k(x) \leq x - R_k^{-1+}(b) + b$. By the 1-Lipschitz-continuity of R_k, we have $0 \leq R_k(x) - R_k\left(R_k^{-1+}(b)\right) \leq x - R_k^{-1+}(b)$, from which the conclusion follows since $R_k\left(R_k^{-1+}(b)\right) = b$ by Proposition A.1b). Thus, (B.5) is obtained and the second part of the proof is complete.

The third step is to demonstrate

$$\mathrm{VaR}_\alpha\left(\sum_{k=1}^m \tilde{R}_k(X)\mathbb{1}_{\{Y=k\}}\right) \leq \mathrm{VaR}_\alpha\left(\sum_{k=1}^m R_k(X)\mathbb{1}_{\{Y=k\}}\right). \tag{B.6}$$

Let $R_Y(X) = \sum_{k=1}^m R_k(X)\mathbb{1}_{\{Y=k\}}$ and $\tilde{R}_Y(X) = \sum_{k=1}^m \tilde{R}_k(X)\mathbb{1}_{\{Y=k\}}$. From (B.4) we get that $\mathbb{P}(R_k(X) > b) = \mathbb{P}\left(\tilde{R}_k(X) > b\right)$ for $k = 1, \ldots, m$. Consequently, we deduce that $\mathbb{P}(R_Y(X) > b) = \mathbb{P}\left(\tilde{R}_Y(X) > b\right)$. Recall that, by Proposition A.2, for any random variable Z we have $\mathrm{VaR}_\alpha(Z) \leq x$ if and only if $\mathbb{P}(Z > x) \leq 1-\alpha$, for any $x \in \mathbb{R}$. Since, by definition, $b = \mathrm{VaR}_\alpha(R_Y(X))$, it follows that $\mathbb{P}\left(\tilde{R}_Y(X) > b\right) = \mathbb{P}(R_Y(X) > b) \leq 1-\alpha$, from which $\mathrm{VaR}_\alpha\left(\tilde{R}_Y(X)\right) \leq b = \mathrm{VaR}_\alpha(R_Y(X))$. Therefore, (B.6) holds and the third part is done.

The last step establishes the inequality

$$P_g\left(\sum_{k=1}^m \tilde{I}_k(X)\mathbb{1}_{\{Y=k\}}\right) \leq P_g\left(\sum_{k=1}^m I_k(X)\mathbb{1}_{\{Y=k\}}\right). \tag{B.7}$$

Let $I_Y(X) = \sum_{k=1}^m I_k(X)\mathbb{1}_{\{Y=k\}}$ and $\tilde{I}_Y(X) = \sum_{k=1}^m \tilde{I}_k(X)\mathbb{1}_{\{Y=k\}}$. Recall that $\tilde{I}_k(X) \leq I_k(X)$ for all $k = 1, \ldots, m$, which implies that $\tilde{I}_Y(X) \leq I_Y(X)$. Furthermore, recall from (3.2) that

$$P_g(I_Y(X)) = \int_0^\infty g(\mathbb{P}(I_Y(X) > z))\,dz,$$

and analogously,

$$P_g\left(\tilde{I}_Y(X)\right) = \int_0^\infty g\left(\mathbb{P}\left(\tilde{I}_Y(X) > z\right)\right)dz.$$

since $\mathbb{P}\left(\tilde{I}_Y(X) > x\right) \leq \mathbb{P}(I_Y(X) > x)$ for all $x \in \mathbb{R}$ and g is a non-decreasing concave function, we can conclude that

$$g\left(\mathbb{P}\left(\tilde{I}_Y(X) > x\right)\right) \leq g(\mathbb{P}(I_Y(X) > x)).$$

From which $P_g\left(\tilde{I}_Y(X)\right) \leq P_g(I_Y(X))$ follows and (B.7) holds, which completes the last step.

Finally, (B.6), together with (B.7) show that, for $(\tilde{I}_1, \ldots, \tilde{I}_m) \in \mathcal{I}_1$ and any $\rho > 0$,

$$
\begin{aligned}
F\left(\tilde{I}_1, \ldots, \tilde{I}_m\right) &= \mathrm{VaR}_\alpha\left(\sum_{k=1}^m \tilde{R}_k(X)\, \mathbb{1}_{\{Y=k\}}\right) + (1+\rho) P_g\left(\sum_{k=1}^m \tilde{I}_k(X)\, \mathbb{1}_{\{Y=k\}}\right) \\
&\leq \mathrm{VaR}_\alpha\left(\sum_{k=1}^m R_k(X)\, \mathbb{1}_{\{Y=k\}}\right) + (1+\rho) P_g\left(\sum_{k=1}^m I_k(X)\, \mathbb{1}_{\{Y=k\}}\right) \\
&= F(I_1, \ldots, I_m).
\end{aligned}
$$

B.2 Proof of Theorem 3.2.1 with CVaR Preferences

Fix $(I_1, \ldots, I_m) \in \mathcal{I}$ and define b as in the Appendix B.1. For $k = 1, \ldots, m$, define $m_k = b$ and n_k should be a value which satisfies $n_k \geq R_k^{-1+}(b)$. Further, define $\tilde{I}_k$ by

$$
\tilde{I}_k(x) = (x-b)_+ - (x-n_k)_+ = \begin{cases} 0 & \text{if } 0 \leq x < b, \\ x - b & \text{if } b \leq x \leq n_k, \\ n_k - b & \text{if } n_k < x, \end{cases} \tag{B.8}
$$

and

$$
\tilde{R}_k(x) = Id(x) - \tilde{I}_k(x) = \begin{cases} x & \text{if } 0 \leq x < b, \\ b & \text{if } b \leq x \leq n_k, \\ x - n_k + b & \text{if } n_k < x. \end{cases} \tag{B.9}
$$

In the initial step, we confirm that there exists an $n_k \geq R_k^{-1+}(b)$ for which

$$
\mathbb{E}\left[\left(\tilde{R}_k(X) - b\right)+\right] = \mathbb{E}\left[\left(R_k(X) - b\right)_+\right] \quad \text{holds for every } \; k = 1, \ldots, m. \tag{B.10}
$$

According to Proposition A.1a), if $n_k = R_k^{-1+}(b) = +\infty$, then $R_k(x) \leq b$ for all x, and also $\tilde{R}_k(x) \leq b$ for all x from (B.9). Consequently, both sides of (B.10) equate to zero. Given this, we restrict our attention only to the case $R_k^{-1+}(b) < +\infty$.

For $x \leq R_k^{-1+}(b)$, it follows from Proposition A.1c) that $R_k(x) \leq b$ and from (B.9), it leads to $\tilde{R}_k(x) \leq b$, which further means that

$$
\mathbb{E}\left[\left(\tilde{R}_k(X) - b\right)_+ \mathbb{1}_{\{X \leq R_k^{-1+}(b)\}}\right] = \mathbb{E}\left[\left(R_k(X) - b\right)_+ \mathbb{1}_{\{X \leq R_k^{-1+}(b)\}}\right] = 0. \tag{B.11}
$$

For $x > R_k^{-1+}(b)$, using the 1-Lipschitz-continuity of R_k, it follows that $0 \leq R_k(x) - R_k\left(R_k^{-1+}(b)\right) \leq x - R_k^{-1+}(b)$, and then by Proposition A.1b), we have $R_k(x) \leq x - R_k^{-1+}(b) + b$ for all $x > R_k^{-1+}(b)$. If we consider the case $R_k(x) = x - R_k^{-1+}(b) + b$ for all $x > R_k^{-1+}(b)$, which can be visualized in the left-hand plot of Figure 2. Then we can choose $n_k = R_k^{-1+}(b)$ exactly, so $\tilde{R}_k(x) = x - R_k^{-1+}(b) + b$ holds for $x > n_k = R_k^{-1+}(b)$ from (B.9), which implies that $R_k(X)\mathbb{1}_{\{X > R_k^{-1+}(b)\}} = \tilde{R}_k(X)\mathbb{1}_{\{X > R_k^{-1+}(b)\}}$.

This can further lead to $\mathbb{E}\left[\left(\tilde{R}_k(X)-b\right)_+ \mathbb{1}_{\{X>R_k^{-1+}(b)\}}\right] = \mathbb{E}\left[\left(R_k(X)-b\right)_+ \mathbb{1}_{\{X>R_k^{-1+}(b)\}}\right]$, which can be combined with (B.11) to obtain (B.10) holds. If we consider the case $R_k(c_k) < c_k - R_k^{-1+}(b) + b$ for some $c_k > R_k^{-1+}(b)$, which can be visualized in the right-hand plot of Figure 2. Then we can have $R_k^{-1+}(b) < c_k - R_k(c_k) + b$. Furthermore, we can suppose there exists a $n_k > R_k^{-1+}(b)$ such that $R_k(c_k) = c_k - n_k + b$ holds, which implies that $n_k = c_k - R_k(c_k) + b < c_k$ since $c_k > R_k^{-1+}(b)$ implies $R_k(c_k) > b$ from Proposition A.1c). Besides, we also have $\tilde{R}_k(c_k) = c_k - n_k + b$ from (B.9) since $c_k > n_k$, so $R_k(c_k) = \tilde{R}_k(c_k)$ means that for $x > R_k^{-1+}(b)$, $R_k(x)$ and $\tilde{R}_k(x)$ can always intersect at a point with $x = c_k > R_k^{-1+}(b)$, then there exist a $n_k > R_k^{-1+}(b)$ satisfying $n_k = c_k - R_k(c_k) + b$ such that, for $x \in \left(R_k^{-1+}(b), c_k\right]$, $\tilde{R}_k(x) \le R_k(x) < x - R_k^{-1+}(b) + b$ is established, which further yields that

$$\mathbb{E}\left[R_k(X)\mathbb{1}_{\{R_k^{-1+}(b)<X\le c_k\}}\right] \ge \mathbb{E}\left[\tilde{R}_k(X)\mathbb{1}_{\{R_k^{-1+}(b)<X\le c_k\}}\right]. \tag{B.12}$$

And for $x > c_k$, the inequalities $R_k(x) \le \tilde{R}_k(x) < x - R_k^{-1+}(b) + b$ is held, which further yields that

$$\mathbb{E}\left[R_k(X)\mathbb{1}_{\{X>c_k\}}\right] \le \mathbb{E}\left[\tilde{R}_k(X)\mathbb{1}_{\{X>c_k\}}\right]. \tag{B.13}$$

Then we can define a function as $f(c_k) = \mathbb{E}\left[\tilde{R}_k(X)\mathbb{1}_{\{X>R_k^{-1+}(b)\}}\right] - \mathbb{E}\left[R_k(X)\mathbb{1}_{\{X>R_k^{-1+}(b)\}}\right]$ for $c_k > R_k^{-1+}(b)$. If $c_k \uparrow +\infty$, then $n_{k_{max}} \uparrow c_k - R_k(c_k) + b$, we can further obtain that $0 \le \mathbb{E}\left[\tilde{R}_k(X)\mathbb{1}_{\{X>c_k\}}\right] - \mathbb{E}\left[R_k(X)\mathbb{1}_{\{X>c_k\}}\right] \to 0$, then combined with the inequality from (B.12), it further means that

$$f(c_k) = \mathbb{E}\left[\tilde{R}_k(X)\mathbb{1}_{\{R_k^{-1+}(b)<X\le c_k\}}\right] - \mathbb{E}\left[R_k(X)\mathbb{1}_{\{R_k^{-1+}(b)<X\le c_k\}}\right] \le 0 \tag{B.14}$$

If $c_k \downarrow R_k^{-1+}(b)$ then $n_k \downarrow R_k^{-1+}(b)$, we can further deduce that $0 \le \mathbb{E}\left[R_k(X)\mathbb{1}_{\{R_k^{-1+}(b)<X\le c_k\}}\right] - \mathbb{E}\left[\tilde{R}_k(X)\mathbb{1}_{\{R_k^{-1+}(b)<X\le c_k\}}\right] \to 0$, then combined with the inequality from (B.13), which further implies that

$$f(c_k) = \mathbb{E}\left[\tilde{R}_k(X)\mathbb{1}_{\{X>c_k\}}\right] - \mathbb{E}\left[R_k(X)\mathbb{1}_{\{X>c_k\}}\right] \ge 0. \tag{B.15}$$

Furthermore, we know that $f(c_k)$ is non-decreasing and continuous for $c_k > R_k^{-1+}(b)$, so from (B.14) and (B.15), it is evident to find a suitable $c_{k_0} > R_k^{-1+}(b)$ to satisfy that $R_k(c_{k_0}) < c_{k_0} - R_k^{-1+}(b) + b$, then for $n_k > R_k^{-1+}(b)$, we can have that

$$f(c_{k_0}) = \mathbb{E}\left[\tilde{R}_k(X)\mathbb{1}_{\{X>R_k^{-1+}(b)\}}\right] - \mathbb{E}\left[R_k(X)\mathbb{1}_{\{X>R_k^{-1+}(b)\}}\right] = 0. \tag{B.16}$$

This further indicate that $\mathbb{E}\left[\left(\tilde{R}_k(X)-b\right)_+ \mathbb{1}_{\{X>R_k^{-1+}(b)\}}\right] = \mathbb{E}\left[\left(R_k(X)-b\right)_+ \mathbb{1}_{\{X>R_k^{-1+}(b)\}}\right]$ since for $x > R_k^{-1+}(b)$, $R_k(x) > b$ from Proposition A.1c) and $\tilde{R}_k(x) \ge b$ from (B.9), which can be combined with (B.11) to obtain (B.10) holds.

In our second step, we aim to demonstrate that

$$\tilde{I}_k(X) \le_{sl} I_k(X) \quad \text{for any} \quad k = 1, \dots, m. \tag{B.17}$$

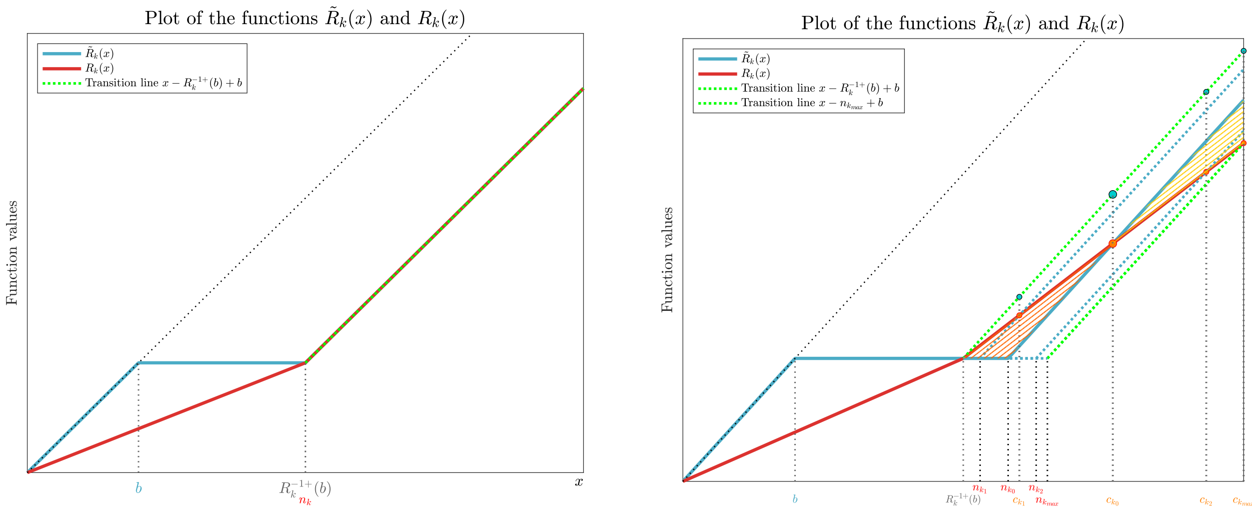

Figure 2: Constructions of $\tilde{R}_k$ and R_k under the conditions for CVaR risk preference: *(Left)* $R_k(x) = x - R_k^{-1+}(b) + b$ for $x > R_k^{-1+}(b)$; *(Right)* $R_k(x) < x - R_k^{-1+}(b) + b$ for $x > R_k^{-1+}(b)$, it is obvious that as c_k increasing, the orange area is also increasing but the yellow is rather decreasing, which is exactly as what we described in our proof. Besides, linear R_k is chosen for graphical convenience.

which means that $\tilde{I}_k(X)$ is smaller than $I_k(X)$ in stop-loss order based on (B.10). We proceed by cases on the value of x.

For $0 \leq x \leq R_k^{-1+}(b) \leq n_k$, by cross-referencing with our earlier derivations in Appendix B.1, we can confirm that $\tilde{I}_k(x) \leq I_k(x)$ for all $x \leq R_k^{-1+}(b)$. Taking this a step further, define $P(X) = \tilde{I}_k(X)\mathbb{1}_{\{X \leq R_k^{-1+}(b)\}}$ and $Q(X) = I_k(X)\mathbb{1}_{\{X \leq R_k^{-1+}(b)\}}$. So for any realization of X, we have $P(X) \leq Q(X)$, which further leads to $P(X) \leq_{sl} Q(X)$ from Proposition A.3a).

For $x > R_k^{-1+}(b)$, define $M(X) = \tilde{I}_k(X)\mathbb{1}_{\{X > R_k^{-1+}(b)\}}$ and $N(X) = I_k(X)\mathbb{1}_{\{X > R_k^{-1+}(b)\}}$. If for the case $R_k(x) = x - R_k^{-1+}(b) + b$, we have $\tilde{R}_k(x) = R_k(x)$ for $x > R_k^{-1+}(b)$ from the first step, which mean that $\tilde{I}_k(x) = I_k(x)$ for $x > R_k^{-1+}(b)$ by using the identity $I_k = Id - R_k$, this further lead to $M(X) = N(X)$, then we can get that

$$
\begin{aligned}
\tilde{I}_k(X) &= \tilde{I}_k(X)\mathbb{1}_{\{X \leq R_k^{-1+}(b)\}} + \tilde{I}_k(X)\mathbb{1}_{\{X > R_k^{-1+}(b)\}} \\
&= P(X) + M(X) \\
&= P(X) + N(X) \\
&\leq Q(X) + N(X) \\
&= I_k(X)\mathbb{1}_{\{X \leq R_k^{-1+}(b)\}} + I_k(X)\mathbb{1}_{\{X > R_k^{-1+}(b)\}} \\
&= I_k(X)
\end{aligned}
$$

This implies (B.17) holds by applying Proposition A.3a). If for the case $R_k(x) < x - R_k^{-1+}(b) + b$, then focusing on $x \in \left(R_k^{-1+}(b), c_k\right]$, we have $\tilde{R}_k(x) \leq R_k(x)$ from the first step, which guarantees that $\tilde{I}_k(x) \geq I_k(x)$. This further implies that $\tilde{I}_k(X)\mathbb{1}_{\{R_k^{-1+}(b)<X\leq c_k\}} \geq I_k(X)\mathbb{1}_{\{R_k^{-1+}(b)<X\leq c_k\}}$, which can be written as $\tilde{I}_k(X)\mathbb{1}_{\{X>R_k^{-1+}(b)\}}\mathbb{1}_{\{X\leq c_k\}} \geq I_k(X)\mathbb{1}_{\{X>R_k^{-1+}(b)\}}\mathbb{1}_{\{X\leq c_k\}}$. So for any realization of X, we have $M(X)\cdot\mathbb{1}_{\{X\leq c_k\}} \geq N(X)\cdot\mathbb{1}_{\{X\leq c_k\}}$. Then it means that

$$\mathbb{P}\left(M(X) \leq t, X \leq c_k\right) \leq \mathbb{P}\left(N(X) \leq t, X \leq c_k\right) \tag{B.18}$$

Then for $x > c_k$, $R_k(x) \leq \tilde{R}_k(x)$ in the initial step can lead to $I_k(x) \geq \tilde{I}_k(x)$. This further implies that $I_k(X)\mathbb{1}_{\{X>c_k\}} \geq \tilde{I}_k(X)\mathbb{1}_{\{X>c_k\}}$, which can be written as $I_k(X)\mathbb{1}_{\{X>R_k^{-1+}(b)\}}\mathbb{1}_{\{X>c_k\}} \geq \tilde{I}_k(X)\mathbb{1}_{\{X>R_k^{-1+}(b)\}}\mathbb{1}_{\{X>c_k\}}$. So for any realization of X, $N(X)\cdot\mathbb{1}_{\{X>c_k\}} \geq M(X)\cdot\mathbb{1}_{\{X>c_k\}}$ holds. Then it leads to

$$\mathbb{P}\left(N(X) \leq t, X > c_k\right) \leq \mathbb{P}\left(M(X) \leq t, X > c_k\right). \tag{B.19}$$

Besides, from the definition of M and N, it is clear that $M(x) = N(x) = 0$ for $x \leq R_k^{-1+}(b)$. For $x \in \left(R_k^{-1+}(b), c_k\right]$, we know that $\tilde{I}_k(x) \in \left(R_k^{-1+}(b) - b, n_k - b\right]$ from (B.8), which implies that $M(x) \in \left(R_k^{-1+}(b) - b, n_k - b\right]$. Furthermore, we have $I_k\left(R_k^{-1+}(b)\right) = R_k^{-1+}(b) - R_k\left(R_k^{-1+}(b)\right) = R_k^{-1+}(b) - b$ by using the Proposition A.1b), and $I_k(c_k) = \tilde{I}_k(c_k) = n_k - b$ since $R_k(c_k) = \tilde{R}_k(c_k)$ from the first step and the identity $I_k = Id - R_k$, which implies that $I_k(x) \in \left(R_k^{-1+}(b) - b, n_k - b\right]$ for $x \in \left(R_k^{-1+}(b), c_k\right]$. So we obtain that $N(x) \in \left(R_k^{-1+}(b) - b, n_k - b\right]$ for $x \in \left(R_k^{-1+}(b), c_k\right]$. For $x > c_k$, we have $\tilde{I}_k(x) = n_k - b$ from (B.8), which means that $M(x) = n_k - b$. Besides, we also have that $I_k(x) \geq \tilde{I}_k(x)$ for $x > c_k$, which means that $N(x) \geq n_k - b$ for $x > c_k$. Therefore, we can summarize M as

$$M(x) = \begin{cases} 0 & \text{if } x \leq R_k^{-1+}(b), \\ \in \left(R_k^{-1+}(b) - b, n_k - b\right] & \text{if } x \in \left(R_k^{-1+}(b), c_k\right], \\ n_k - b & \text{if } x > c_k, \end{cases} \tag{B.20}$$

and N as

$$N(x) = \begin{cases} 0 & \text{if } x \leq R_k^{-1+}(b), \\ \in \left(R_k^{-1+}(b) - b, n_k - b\right] & \text{if } x \in \left(R_k^{-1+}(b), c_k\right], \\ \geq n_k - b & \text{if } x > c_k. \end{cases} \tag{B.21}$$

Now, for $t < 0$, we have $F_{M(X)}(t) = F_{N(X)}(t) = 0$ from (B.20) and (B.21). For $t \in [0, n_k - b)$, from (B.20) and (B.21), we know that for $x > c_k$, $N(x) \geq n_k - b = M(x)$, it further leads to

$\mathbb{P}(M(X) \le t, X > c_k) = \mathbb{P}(N(X) \le t, X > c_k) = 0$. Therefore, we have

$$
\begin{aligned}
F_{M(X)}(t) &= \mathbb{P}(M(X) \le t, X \le c_k) + \mathbb{P}(M(X) \le t, X > c_k) \\
&= \mathbb{P}(M(X) \le t, X \le c_k) \\
&\le \mathbb{P}(N(X) \le t, X \le c_k) \\
&= \mathbb{P}(N(X) \le t, X \le c_k) + \mathbb{P}(N(X) \le t, X > c_k) \\
&= F_{N(X)}(t).
\end{aligned}
$$

The third inequality comes from (B.18). For $t \ge n_k - b$, we have $F_{M(X)}(t) = 1$ since $M(x) \le n_k - b$ from (B.20), and $F_{N(X)}(t) \le 1$ because from (B.21), we have $N \ge n_k - b$ when $x > c_k$. Consequently, $F_N(t) \le F_M(t)$ for $t \ge n_k - b$. So denote $t_0 = n_k - b$, when $t < t_0$, we have the condition $F_{M(X)}(t) \le F_{N(X)}(t)$, when $t \ge t_0$, $F_{N(X)}(t) \le F_{M(X)}(t)$. Besides, (B.16) and the identity $I_k = Id - R_k$ give that $\mathbb{E}\left[I_k(X)\mathbb{1}_{\{X > R_k^{-1+}(b)\}}\right] = \mathbb{E}\left[\tilde{I}_k(X)\mathbb{1}_{\{X > R_k^{-1+}(b)\}}\right]$, which means that $\mathbb{E}[M(X)] = \mathbb{E}[N(X)]$ and applying Proposition A.3b), we can assert that $M(X) \le_{sl} N(X)$. Now, for any $d \ge 0$, we have

$$
\begin{aligned}
\mathbb{E}\left[\left(\tilde{I}_k(X) - d\right)_+\right] &= \mathbb{E}\left[\left(\tilde{I}_k(X) - d\right)_+ \mathbb{1}_{\{X \le R_k^{-1+}(b)\}}\right] + \mathbb{E}\left[\left(\tilde{I}_k(X) - d\right)_+ \mathbb{1}_{\{X > R_k^{-1+}(b)\}}\right] \\
&= \mathbb{E}\left[\left(\tilde{I}_k(X)\mathbb{1}_{\{X \le R_k^{-1+}(b)\}} - d\right)_+\right] + \mathbb{E}\left[\left(\tilde{I}_k(X)\mathbb{1}_{\{X > R_k^{-1+}(b)\}} - d\right)_+\right] \\
&= \mathbb{E}\left[(P(X) - d)_+\right] + \mathbb{E}\left[(M(X) - d)_+\right] \\
&\le \mathbb{E}\left[(Q(X) - d)_+\right] + \mathbb{E}\left[(N(X) - d)_+\right] \\
&= \mathbb{E}\left[\left(I_k(X)\mathbb{1}_{\{X \le R_k^{-1+}(b)\}} - d\right)_+\right] + \mathbb{E}\left[\left(I_k(X)\mathbb{1}_{\{X > R_k^{-1+}(b)\}} - d\right)_+\right] \\
&= \mathbb{E}\left[(I_k(X) - d)_+ \mathbb{1}_{\{X \le R_k^{-1+}(b)\}}\right] + \mathbb{E}\left[(I_k(X) - d)_+ \mathbb{1}_{\{X > R_k^{-1+}(b)\}}\right] \\
&= \mathbb{E}\left[(I_k(X) - d)_+\right].
\end{aligned}
$$

The justification for the fourth inequality is grounded in the established stop-loss orders $P(X) \le_{sl} Q(X)$ and $M(X) \le_{sl} N(X)$, coupled with the application of Proposition A.3c). Besides, $\tilde{I}_k(x) \le I_k(x)$ for all $x \le R_k^{-1+}(b)$ means that $\mathbb{E}\left[\tilde{I}_k(X)\mathbb{1}_{\{X \le R_k^{-1+}(b)\}}\right] \le \mathbb{E}\left[I_k(X)\mathbb{1}_{\{X \le R_k^{-1+}(b)\}}\right]$, combined with the fact that $\mathbb{E}\left[\tilde{I}_k(X)\mathbb{1}_{\{X>R_k^{-1+}(b)\}}\right] = \mathbb{E}\left[I_k(X)\mathbb{1}_{\{X>R_k^{-1+}(b)\}}\right]$, then we have for any $d<0$ that

$$\begin{aligned}
\mathbb{E}\left[\left(\tilde{I}_k(X)-d\right)_+\right] &= \mathbb{E}\left[\tilde{I}_k(X)-d\right] \\
&= \mathbb{E}\left[\tilde{I}_k(X)\right]-d \\
&= \left(\mathbb{E}\left[\tilde{I}_k(X)\mathbb{1}_{\{X\leq R_k^{-1+}(b)\}}\right]+\mathbb{E}\left[\tilde{I}_k(X)\mathbb{1}_{\{X>R_k^{-1+}(b)\}}\right]\right)-d \\
&\leq \left(\mathbb{E}\left[I_k(X)\mathbb{1}_{\{X\leq R_k^{-1+}(b)\}}\right]+\mathbb{E}\left[I_k(X)\mathbb{1}_{\{X>R_k^{-1+}(b)\}}\right]\right)-d \\
&= \mathbb{E}\left[I_k(X)\right]-d \\
&= \mathbb{E}\left[I_k(X)-d\right] \\
&= \mathbb{E}\left[\left(I_k(X)-d\right)_+\right]
\end{aligned}$$

Therefore, all $d\in\mathbb{R}$, we have $\mathbb{E}\left[\left(\tilde{I}_k(X)-d\right)_+\right] \leq \mathbb{E}\left[\left(I_k(X)-d\right)_+\right]$, then by applying Proposition A.3c), we can get that (B.17) holds and this completes the second part of the proof.

The third step demonstrates that

$$\mathrm{CVaR}_\alpha\left(\sum_{k=1}^m \tilde{R}_k(X)\mathbb{1}_{\{Y=k\}}\right) \leq \mathrm{CVaR}_\alpha\left(\sum_{k=1}^m R_k(X)\mathbb{1}_{\{Y=k\}}\right). \tag{B.22}$$

Let $R_Y(X)=\sum_{k=1}^m R_k(X)\mathbb{1}_{\{Y=k\}}$ and $\tilde{R}_Y(X)=\sum_{k=1}^m \tilde{R}_k(X)\mathbb{1}_{\{Y=k\}}$. Recall from (B.10), we can deduce $\mathbb{E}\left[\left(\tilde{R}_Y(X)-b\right)_+\right]=\mathbb{E}\left[\left(R_Y(X)-b\right)_+\right]$. Utilizing the dual representation of CVaR, we have

$$\mathrm{CVaR}_\alpha\left(R_Y(X)\right)=\inf_{t\in\mathbb{R}}\left\{t+\frac{1}{1-\alpha}\mathbb{E}\left[\left(R_Y(X)-t\right)_+\right]\right\}.$$

where the infimum is achieved at $t^*=b$, yielding $\mathrm{CVaR}_\alpha\left(R_Y(X)\right)=b+\frac{1}{1-\alpha}\mathbb{E}\left[\left(R_Y(X)-b\right)_+\right]$. Therefore, we have

$$\begin{aligned}
\mathrm{CVaR}_\alpha\left(\tilde{R}_Y(X)\right) &= \inf_{t\in\mathbb{R}}\left\{t+\frac{1}{1-\alpha}\mathbb{E}\left[\left(\tilde{R}_Y(X)-t\right)_+\right]\right\} \\
&\leq b+\frac{1}{1-\alpha}\mathbb{E}\left[\left(\tilde{R}_Y(X)-b\right)_+\right] \\
&= b+\frac{1}{1-\alpha}\mathbb{E}\left[\left(R_Y(X)-b\right)_+\right] \\
&= \mathrm{CVaR}_\alpha\left(R_Y(X)\right).
\end{aligned}$$

Thus, we establish (B.22) and the third part of the proof is done.

In the final step, we aim to show that

$$P_g\left(\sum_{k=1}^m \tilde{I}_k(X)\mathbb{1}_{\{Y=k\}}\right) \leq P_g\left(\sum_{k=1}^m I_k(X)\mathbb{1}_{\{Y=k\}}\right), \tag{B.23}$$

Let $I_Y(X) = \sum_{k=1}^m I_k(X)\mathbb{1}_{\{Y=k\}}$ and $\tilde{I}_Y(X) = \sum_{k=1}^m \tilde{I}_k(X)\mathbb{1}_{\{Y=k\}}$. Recall from (B.17), which further implies $\tilde{I}_Y(X) \le_{sl} I_Y(X)$ for any $k = 1,\ldots,m$. Applying Proposition A.3d), we obtain $P_g(\tilde{I}_Y(X)) \le P_g(I_Y(X))$. Therefore, (B.23) holds.

Thus, combining (B.22) and (B.23) demonstrates that, for $(\tilde{I}_1,\ldots,\tilde{I}_m) \in \mathcal{I}_2$ and $\rho > 0$,

$$\begin{aligned} F(\tilde{I}_1,\ldots,\tilde{I}_m) &= \mathrm{CVaR}_\alpha\left(\sum_{k=1}^m \tilde{R}_k(X)\mathbb{1}_{\{Y=k\}}\right) + (1+\rho)P_g\left(\sum_{k=1}^m \tilde{I}_k(X)\mathbb{1}_{\{Y=k\}}\right) \\ &\le \mathrm{CVaR}_\alpha\left(\sum_{k=1}^m R_k(X)\mathbb{1}_{\{Y=k\}}\right) + (1+\rho)P_g\left(\sum_{k=1}^m I_k(X)\mathbb{1}_{\{Y=k\}}\right) \\ &= F(I_1,\ldots,I_m). \end{aligned}$$

C Flow charts: Court Process

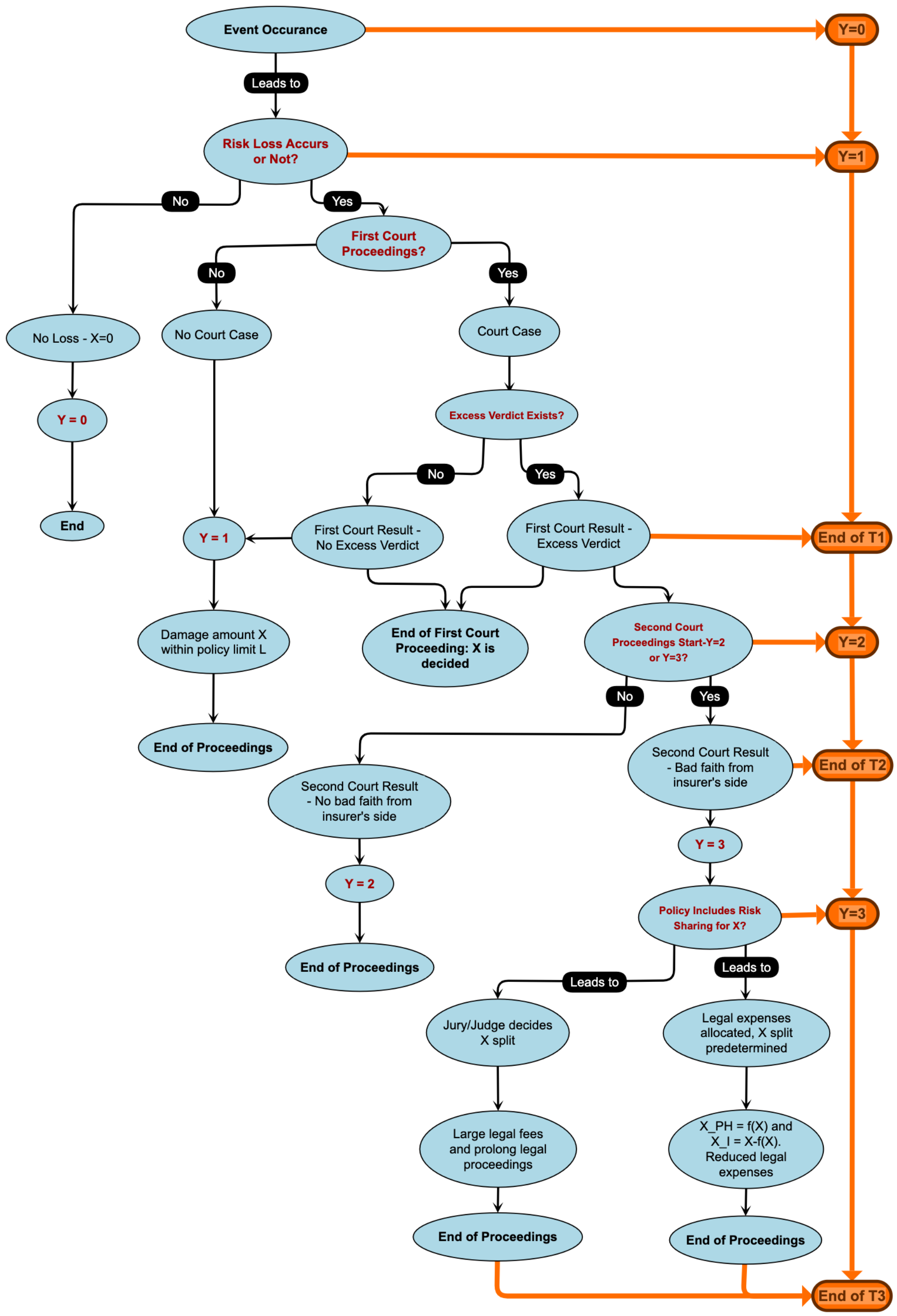

Figure 3: Flowchart illustrating the stages of legal proceedings concerning insurance claims and the subsequent apportionment of liabilities based on the loss threshold and seller conduct.

D Excess Verdict in Auto & Liability Insurance

Case	Event Time	Court Start Time	Verdict Time	Compensatory Damage Cost	Punitive Damage Cost	Total Cost	Excess Verdict Trigger	Type of Policy
Dock vs McLendon et al.	Jan. 26, 2019	July 27, 2021	July 30, 2021	$66.5 million	N/A	$66.5 million	Motor Vehicle	Auto Insurance
Cargal vs Forehand & FedEx	Sep. 8, 2018	Oct. 15, 2021	Oct. 24, 2021	$30,000,000	N/A	$30,000,000	Motor Vehicle	Auto Insurance
Godwin vs Carroll & Eaton Asphalt Paving Co., Inc.	Jan. 9, 2019	July 12, 2021	N/A	$24,000,000	$50,000,000	$74,000,000	Motor Vehicle	Auto Insurance
Leslie vs Rodriguez	May 1, 2017	March 6, 2020	N/A	1.82 million & $2.8 million	N/A	$4.62 million	Motor Vehicle	Auto Insurance
Pedro Pasillas-Sanchez vs Consolidated Materials, Inc. & Lee	March 26, 2018	Nov. 13, 2020	N/A	$9,000,000	N/A	$9,000,000	Motor Vehicle	Auto Insurance
Ware vs Home Opportunity, LLC, Ewing & Marchman	Oct. 2, 2016	Jan. 22, 2020	N/A	$9,689,948.18	N/A	$9,689,948.18	Premises Liability	Liability Insurance
Church & Austin vs Case New Holland Industrial of America, LLC	March 2, 2016	Nov. 12, 2020	N/A	$3,000,000	$10,000,000	$13,000,000	Products Liability	Liability Insurance
Madere & Thomas vs Greenwich Insurance Company et al.	July 18, 2016	Aug. 23, 2019	Aug. 28, 2019	$180,065,000	$100,000,000	$280,065,000	Negligence	Auto Insurance
Mayfield & Phillips vs Kennison	April 10, 2006	Feb. 26, 2019	March 2, 2019	$33,413,000	N/A	$32,412,610 (After Comparative Negligence Adjustment)	Motor Vehicle	Auto Insurance
Garmon vs Jenkins and Atlas Excavating/Atlas Trucking	Sept. 7, 2012	Oct. 3, 2019	Oct. 10, 2019	$22,144,971.88	$10,000,000	$32,144,971.88	Negligent Hiring	Auto Insurance
Plascencia & Trujillo vs Newcomb etc.	April 19, 2014	March 25, 2019	N/A	$30,000,000	N/A	$12,000,000 (After Apportionment)	U-Turn	Auto Insurance
Willoughby vs Ellison & 21st Century Centennial Insurance Company	Nov. 2, 2012	March 15, 2019	March 22, 2019	$30,101,599	N/A	$34,668,619	Passenger	Auto Insurance
Thornton vs Ralston GA LLC d/b/a The Ralston etc.	July 6, 2017	July 1, 2019	July 7, 2019	$35,000,000	$50,000,000	$125,000,000	Negligent Repair	Liability Insurance
Enriquez, Jr., Martinez & Irene Gonzalez vs Lasko Products, Inc.	Jan. 3, 2016	Nov. 21, 2019	Nov. 28, 2019	$36,240,000	N/A	$36,240,000	Manufacturing Defect	Liability Insurance
Johnson vs Lee & Corrugated Replacements, Inc.	July 1, 2011	Sep. 14, 2018	Sep. 21, 2018	$128,813,522	N/A	$128,813,522	Motor Vehicle	Auto Insurance
Herrera & Sweeting vs Extended Stay America, Inc., etc.	Nov. 12, 2014	Nov. 12, 2018	Nov. 20, 2018	$46,000,000	N/A	$41,400,000 (After Apportionment)	Negligence	Liability Insurance
Barron vs B & G Crane Service etc.	May 11, 2016	Sep. 13, 2018	N/A	$44,370,000	N/A	$20,791,235.34	Negligence	Liability Insurance
The Estate of Kari Dunn vs OM Lodging LLC etc.	Dec. 1, 2013	June 22, 2018	June 26, 2018	$41,550,000	N/A	$2,400,000	Negligence	Liability Insurance
Anaya vs Superior Industries Inc. et al.	Oct. 7, 2013	March 19, 2018	March 26, 2018	$30,000,000	N/A	$30,000,000	Negligence	Auto Insurance
Sitton et al. v. Ceeda Enterprises, Inc.	March 28, 2016	July 17, 2018	July 19, 2018	$27,091,054	N/A	$27,091,054	Negligence	Auto Insurance
Dougherty & Forester vs WCA of Florida, LLC	Oct. 28, 2016	Oct. 5, 2018	Oct. 10, 2018	$25,000,000	N/A	$20,000,000 (After 20% Comparative Negligence Reduction)	Right Turn Motor Vehicle	Auto Insurance
Braswell vs The Brickman Group Ltd, LLC. et al.	May 16, 2014	May 3, 2017	May 9, 2017	$39,960,000	N/A	$27,172,800 (After the Reduction for Comparative Fault)	Motor Vehicle	Liability Insurance
Jester vs Utilimap Corporation & Duke Energy Ohio, Inc.	Feb. 27, 2014	Jun. 7, 2017	Jun. 28, 2017	$27,871,944	N/A	$27,871,944	Negligent Training	Liability Insurance
Cruz et al. vs Methenge et al.	Aug. 29, 2012	Jul. 21, 2017	Aug. 10, 2017	$24,931,109	N/A	$24,931,109	Design Defect	Auto Insurance
Angulo & Lopez vs J. Calero et al.	May 28, 2015	Oct. 26, 2017	N/A	$20,000,000	$25,005,000	$45,005,000	Negligence	Liability Insurance
Debra Morris et al. vs AirCon Corporation, et al.	April 26, 2014	Nov. 1, 2017	Nov. 10, 2017	$18,460,279	N/A	$923,014	Negligence	Liability Insurance
Stolowski et al. vs 234 East 178th Street LLC & City N.Y	Jan. 23, 2005	Feb. 22, 2016	June, 2016	$140,100,000	N/A	$183,261,737	Negligence	Liability Insurance
Garcia vs Manhattan Vaughn JVP et al.	Dec. 4, 2013	Feb. 10, 2016	April 29, 2016	$53,852,558	N/A	$55,834,971.47 (Final Judgment)	Worker/Workplace Negligence	Liability Insurance
Garcia, et al. vs O'Reilly Auto Enterprises, LLC & Shoots	Feb. 28, 2015	Jul. 19, 2016	Jul. 25, 2016	$37,945,000	N/A	$9,000,000 (Reduced due to High/Low Agreement)	Motor Vehicle	Auto Insurance
Swenson et a. vs Troy et al.	May 22, 2012	April 18, 2016	May 1, 2016	$35,029,371	$100,000	$35,129,371	Motor Vehicle	Auto Insurance
Dubuque vs Cumberland Farms, Inc. & V.S.H. Realty, Inc.	Nov. 28, 2008	Feb. 23, 2016	March 8, 2016	$32,369,024.30	$10	$32,369,034.30	Negligence	Auto insurance
Gonzalez et al. vs Atlas Construction Supply Inc. et al.	Aug. 2, 2011	July 27, 2016	Aug. 8, 2016	$26,920,170	N/A	$16,345,170 (No jointly liability)	Negligence	Liability Insurance
Jacobs Engineering Group Inc. vS ConAgra Foods Inc	2013	March 25, 2016	April 22, 2016	$108,913,520.89	N/A	$108,913,520.89	Exposition	Liability Insurance
Hinson et al. vs Dorel Juvenile Group, Inc et al.	May 15, 2013	June 17, 2016	June 21, 2016	$24,438,000	$10,000,000	$34,438,000	Failure to Warn	Liability Insurance

Table 6: Wrongful Deaths Cases: Auto & Liability Insurance

* *Source:* Case details from Report 1, Report 2, Report 3, Report 4, Report 5, and Report 6

References

Arrow, K. J. (1963). Uncertainty and the welfare economics of medical care. *American Economic Review 53*(5), 941–973.

Asimit, A. V., A. M. Badescu, and T. Verdonck (2013). Optimal risk transfer under quantile-based risk measurers. *Insurance: Mathematics and Economics 53*(1), 252–265.

Asimit, V., A. M. Badescu, and A. Tsanakas (2013). Optimal risk transfers in insurance groups. *European Actuarial Journal 3*(1), 159–190.

Asimit, V., T. J. Boonen, Y. Chi, and W. F. Chong (2021). Risk sharing with multiple indemnity environments. *European Journal of Operational Research 295*(1), 587–603.

Asmat, D. P. and S. Tennyson (2014). Does the threat of insurer liability for "bad faith" affect insurance settlements? *The Journal of Risk and Insurance 81*(1), 1–26.

Boonen, T. J. and M. Ghossoub (2019). On the existence of a representative reinsurer under heterogeneous beliefs. *Insurance: Mathematics and Economics 88*(1), 209–225.

Boonen, T. J. and M. Ghossoub (2020). Bilateral risk sharing with heterogeneous beliefs and exposure constraints. *ASTIN Bulletin: The Journal of the IAA 50*(1), 293–323.

Boonen, T. J. and M. Ghossoub (2021). Optimal reinsurance with multiple reinsurers: distortion risk measures, distortion premium principles, and heterogeneous beliefs. *Insurance: Mathematics and Economics 101*, 23–37.

Borch, K. H. (1960). An attempt to determine the optimum amount of stop loss reinsurance. *In: Transactions of the 16th International Congress of Actuaries, 2*, 597–610.

Cai, J. and K. S. Tan (2007). Optimal retention for a stop-loss reinsurance under the var and cte risk measures. *ASTIN Bulletin: The Journal of the IAA 37*(1), 93–112.

Cai, J., K. S. Tan, C. Weng, and Y. Zhang (2008). Optimal reinsurance under var and cte risk measures. *Insurance: Mathematics and Economics 43*(1), 185–196.

Chang, Y.-c., T. Eisenberg, H.-w. Ho, and M. T. Wells (2015). Pain and suffering damages in wrongful death cases: An empirical study. *Journal of Empirical Legal Studies 12*, 128–160.

Cheung, K., W. Chong, and S. Yam (2015). Convex ordering for insurance preferences. *Insurance: Mathematics and Economics 64*, 409–416.

Chi, Y. and K. S. Tan (2011). Optimal reinsurance under var and cvar risk measures: a simplified approach. *ASTIN Bulletin: The Journal of the IAA 41*(2), 487–509.

Chi, Y. and K. S. Tan (2013). Optimal reinsurance with general premium principles. *Insurance: Mathematics and Economics 52*(2), 180–189.

Chi, Y. and K. S. Tan (2021). Optimal incentive-compatible insurance with background risk. *ASTIN Bulletin: The Journal of the IAA 51*(2), 661–688.

Chi, Y. and W. Wei (2018). Optimum insurance contracts with background risk and higher-order risk attitudes. *ASTIN Bulletin: The Journal of the IAA 48*(3), 1025–1047.

Chi, Y. and W. Wei (2020). Optimal insurance with background risk: An analysis of general dependence structures. *Finance Stochastics 24*, 903–937.

Deng, Y. and G. Zanjani (2018). What drives tort reform legislation? an analysis of state decisions to restrict liability torts. *The Journal of Risk and Insurance 85*(4), 959–991.

Eeckhoudt, L., C. Gollier, and H. Schlesinger (1996). Changes in background risk and risk taking behavior. *Econometrica 64*(3), 683–689.

Embrechts, P. and M. Hofert (2013). A note on generalized inverses. *Mathematical Methods of Operations Research 77*, 423–432.

Epps, A. C. and R. H. Chappell (1958). Insurer's liability in excess of policy limits: Some aspects of the problem. *Virginia Law Review 44*(2), 267–277.

Gallogly, A. J. (2006). Survey of pennsylvania "bad faith" law. Available at: https://tinyurl.com/3jnt55e2.

Gollier, C. and J. W. Pratt (1996). Risk vulnerability and the tempering effect of background risk. *Econometrica 64*(5), 1109–1123.

Goodwin, B. K. (1993). An empirical analysis of the demand for multiple peril crop insurance. *American Journal of Agricultural Economics 75*(2), 425–434.

Guerra, M. and M. Centeno (2012). Are quantile risk measures suitable for risk-transfer decisions? *Insurance: Mathematics and Economics 50*(3), 446–461.

Heaton, J. and D. Lucas (2000). Portfolio choice in the presence of background risk. *The Economic Journal 110*(460), 1–26.

Hinck, S. and P. Steinorth (2023). Insurance demand in the presence of loss-dependent background risk. *Journal of Risk and Insurance 90*(4), 991–1026.

Hofmann, A., O. V. Häfen, and M. Nell (2019). Optimal insurance policy indemnity schedules with policyholders' limited liability and background risk. *The Journal of Risk and Insurance 86*(4), 973–988.

Huberman, G., D. Mayers, S, and C. W. S. Jr. (1983). Optimal insurance policy indemnity schedules. *Bell Journal of Economics 14*(2), 415–426.

Lu, Z., S. Meng, L. Liu, and Z. Han (2018). Optimal insurance design under background risk with dependence. *Insurance: Mathematics and Economics 80*, 15–28.

McNeil, A. J., R. Frey, and P. Embrechts (2015). *Quantitative Risk Management: Concepts, Techniques and Tools Revised edition*. Number 10496 in Economics Books. Princeton University Press.

Miranda, M. and D. V. Vedenov (2001). Innovations in agricultural and natural disaster insurance. *American Journal of Agricultural Economics 83*(3), 650–655.

Murray, D., N. Williams, and E. Speltz (2020). Understanding the impact of nuclear verdicts on the trucking industry. Available at: https://tinyurl.com/3td78mnd.

O'Connor, J. F. (2003). Insurance coverage settlements and the rights of excess insurers. *Maryland Law Review 62*(1), 30.

Raviv, A. (1979). The design of an optimal insurance policy. *The American Economic Review 69*(1), 84–96.

Richmond, D. R. (2000). Rights and responsibilities of excess insurers. *Denver Law Review 78*(1), 29–123.

Rolski, T., H. Schmidli, V. Schmidt, and J. Teugels (1999). *Stochastic Processes for Insurance and Finance*. John Wiley & Sons.

Shaked, M. and J. G. Shanthikumar (2007). *Stochastic Orders*. Springer.

Sharma, P. (2023, July). Roadblock: The trucking litigation problem and how to fix it. Available at: https://tinyurl.com/4ah987kd.

Silverman, C. and C. E. Appel (2023). Nuclear verdicts: Trends, causes, and solutions. Available at: https://tinyurl.com/467a6mz6.

Strobl, R. (2022). Background risk, insurance and investment behaviour: Experimental evidence from Kenya. *Journal of Economic Behavior & Organization 202*(C), 34–68.

Wang, S. (1996). Premium calculation by transforming the layer premium density. *ASTIN Bulletin: The Journal of the IAA 26*(1), 71–92.

Wang, S. S. (1995). Insurance pricing and increased limits ratemaking by proportional hazards transforms. *Insurance: Mathematics and Economics 17*, 43–54.

Young, V. R. (1999). Optimal insurance under wang's premium principle. *Insurance: Mathematics and Economics 25*(2), 109–122.