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Citation: Sriram, V., Saincher, S., Yan, S. & Ma, Q. W. (2024). The past, present and future of multi-scale modelling applied to wave–structure interaction in ocean engineering. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 382(2281), 20230316. doi: 10.1098/rsta.2023.0316

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The past, present and future of Multi-Scale Modelling in Ocean Engineering

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Abstract

Concepts and evolution of multi-scale modelling from the perspective of wave-structure interaction have been discussed. In this regard, both domain and functional decomposition approaches have come into being. In domain decomposition, the computational domain is spatially segregated to handle the far-field using potential flow models and the near field using Navier-Stokes equations. In functional decomposition, the velocity field is separated into irrotational and rotational parts to facilitate identification of the free surface. These two approaches have been implemented alongside partitioned or monolithic schemes for modelling the structure. The applicability of multi-scale modelling approaches has been established using both mesh-based and meshless schemes. Owing to said diversity in numerical techniques, massively collaborative research has emerged wherein comparative numerical studies are being carried out to identify shortcomings of developed codes and establish best-practices in numerical modelling. Machine learning is also being applied to handle large-scale ocean engineering problems. This paper reports on the past, present and future research consolidating the contributions made over the past 20 years. Some of these past as well as future research contributions have and shall be actualized through funding from the Newton International Fellowship as the next generation of researchers inherits the present-day expertise in multi-scale modelling.

1 Introduction

In this paper, modelling tools and approximations that are in practice for wave-structure interactions (WSI) are discussed. Emphasis is provided for ocean engineering which encompasses offshore and coastal engineering as well as naval architecture. Thus large time/spatial scale and local time/spatial scales are important. Different modelling aspects based on their level of approximation or theoretical understandings are discussed. This leads to understanding the limitations of the numerical tool, subsequently emphasising how and what to interpret from the results. In recent years, coupling of these standalone tools is being extensively implemented to resolve various levels of the physical process. This is discussed in detail after the brief explanation of the individual tools and how the development took place in each of these modelling efforts.

Now-a-days, these numerical models are available as open-source as well as commercial tools using different numerical methods. Thus, said models have varying degrees of approximations in grid resolution, stability, accuracy and computational efficiency. Hence, one of the recent efforts in the numerical modelling community is the comparative and benchmarking exercises; this shall also be discussed in the present paper. So the readers can test their own development/existing tools using any of these benchmark tests, available theory and open source experimental data.

In this paper, apart from providing an overview of the existing tools, a proper classification of the models, their applicability range, computation and physical processes, a thorough literature review on the history of developments are provided. It should be noted that the

details of each of the presented models is beyond the scope of this paper and the reader may refer the corresponding literature cited.

2 Different Levels of Approximations

A single numerical tool to address all class of problems in ocean engineering is ideal. However such a model is not possible due to the following reasons: (a) a large sea area, having a large range of spatial and time scales, (b) highly nonlinear wave-structure interaction process (here not only fluid, sometimes the structure can also behave nonlinearly such as vegetation or fenders or hydro-elasticity), (c) waves co-exist with nonlinear currents of various levels, sediment transport and others, (d) viscosity, surface tension and turbulence, (e) two phase (air-sea) or multiphase processes (air-sea-oil or air-sea-sediment), (f) violent wave impacts (during cyclonic storm surges, flooding) and aeration on rubble mound structures, green water shipping and slamming. For these above phenomena, one needs to model large spatial/time scale to capture wave propagation phenomenon as well as resolve small spatial/time scale to understand the wave-structure(-soil) interactions processes. Thus, a single mathematical model may not always be a solution for this complex problem. Hence, the researchers have developed various levels of approximations in the mathematical modelling.

The level of approximations in the mathematical modelling is based on two principles: (a) which physical process is governing the problem at hand and (b) strive to reduce the computational effort in the numerical algorithm for the industrial/practical application. In context to the first principle, the requirement of modelling a physical process is mapped with respect to various applications in Table 1. Table 1 lists various applications that require either large domain or local/small domain modelling.

Table 1. Physical processes required to be modelled for various large and small-scale ocean engineering applications.

		APPLICATIONS						
		Large domain modelling			Local/small domain Modelling			
		Current / flow- structure interactions	Wave propagation and interactions	Seakeeping / motion of marine structures	Steep wave and rigid structure	Extreme waves and rigid structure	Wave- structure- soil interaction	Wave- flexible structure- interaction
PHYSICAL PROCESS	Surface tension	Not required	Not required	Not required	Not required	Not required	Not required	May or may not
	Viscosity and / or turbulence	May or may not	May or may not	May or may not	May or may not	May or may not	May or May not	May or may not
	Nonlinearity in fluid	Required	Required	Required	Required	Required	Required	Required
	Nonlinearity in structure	Not required	Not required	May or may not	Not required	Not required	Not required	Required
	Modelling air-phase	Not required	Not required	Not required	Not required	Required	Not required	Required

A typical example for large domain modelling in coastal engineering is wave propagation from offshore to near shore and its interactions with a harbour structure to understand its tranquillity, run-up or inundations. Similar examples from naval architecture and offshore engineering would be ship maneuvering under the action of waves and an offshore wind turbine farm interacting with waves or offshore platform interactions with waves, respectively. For these applications the physical processes such as surface tension and nonlinearities in structural response are not important. Further, complete physics in modelling the air-sea process is also not required; some empirical treatment would be deemed sufficient. Thus, the full continuity and momentum equations can be simplified based on these approximations. Similarly, consider an application of small domain modelling, wherein one is interested in quantifying the forces experienced by structures (such as ships, semi-submersible platforms, seawalls, scour around monopiles and jackets etc.) against operating or extreme sea state conditions. In this scenario, normally researchers would carry out the physical model studies in an experimental wave tank. A similar study can be done using numerical modelling based on so-called numerical wave tanks. A numerical wave tank is a numerical tool that could reproduce the experimental facility as close as possible. Thus, a detailed physical flow process is realized by solving the continuity and momentum equations, only slightly reducing the physical approximations, however, reproducing the dominating forces as close as reality. For instance, in coastal engineering, surface tension, nonlinearity in structure, sediment to sediment interactions or rigid body interactions (say in a rubble-mound breakwater) can be relaxed without greatly compromising the fidelity of the numerical approximation. Thus, for large scale problems, one can employ various levels of approximations based on the wave characteristics and its applications thus leading to savings in the computational cost. This aspect of modelling is further emphasised by means of a bubble plot in Figure 1 wherein the various environmental aspects in spatial and time scale for mathematical modelling of wave-structure interaction as well as wave-propagation are illustrated.

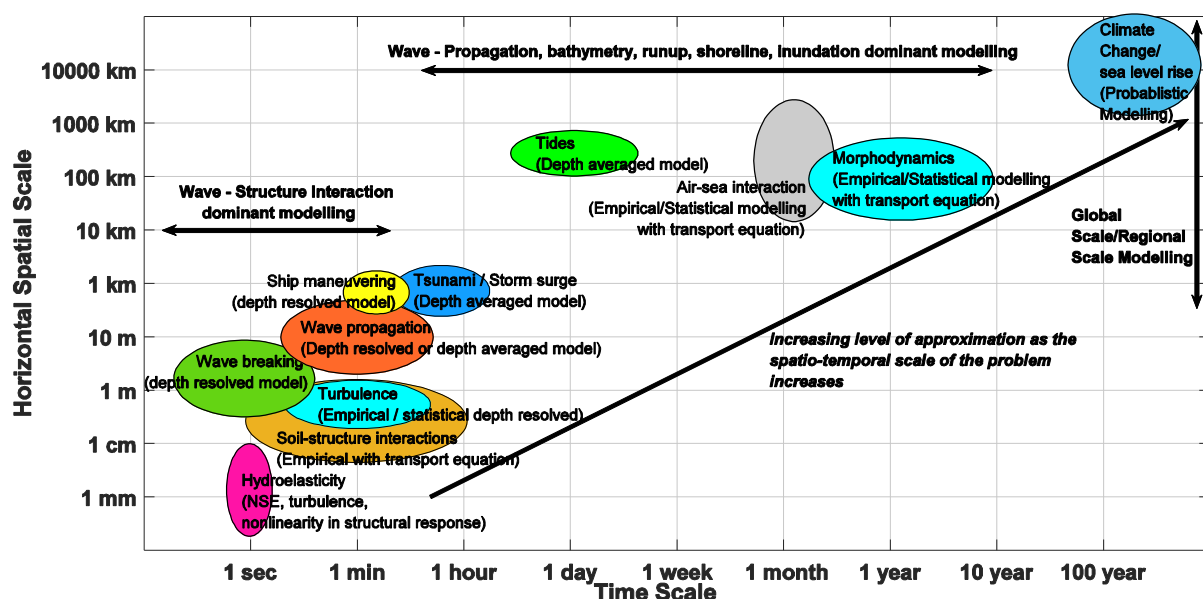


Figure 1. Bubble plot variation to represent the spatio-temporal scales of various physical processes in ocean engineering.

Figure 1 showcase various processes ranging from climate change, sea-level rise, morphodynamics, tides, tsunami/storm surges, wave propagation over varying bathymetry from offshore to their interactions with structures. Each of these cases has a different horizontal spatial scale from 1mm to more than 10,000 km and time scale from less than 1s to 100 years. Further, which type of modelling is dominant or one is required to carry out is also represented in Figure 1, along with global scale or regional scale modelling. The mathematical modelling approximations based on depth averaging can be seen as predominant for increasing large scale problems. This is based on the assumptions of the vertical flow structure. When the vertical flow motion is considered weak or insignificant, then depth averaged horizontal velocities can be adopted. Such classifications of mathematical models are called as depth averaged models and depending upon the approximations adopted in the horizontal velocities different models are available. This will be discussed in the later part of this paper. When the time scale and horizontal spatial scale is small, then wave-structure interaction is dominant, in those cases depth resolving models are normally adopted. This is solved based on Navier-Stokes equations (NS) with various simplified approximations. Depending upon the application (such as porous-structure, vegetation interactions, hydroelasticity or sediment transport) either microscopic or macroscopic modelling can be adopted within the NS framework to model the structure interaction process. Further, the physical process involved in the ship manoeuvring is in km and minutes in prototype, however for numerical modelling they will be normally carried out in the reduced scale using depth resolving models. Hence, for some applications even the physical process is in large scale, the numerical simulations are normally carried out in scaled ones due to computational aspects.

In the past decades, one of the major reasons for resorting to the different physical approximations to model the different scales of the problem was to reduce the computational time. However, this leads to compromise on the physics of the problem. Figure 2 shows three different broader classifications namely depth averaging, depth resolving and hybrid models. In this broader classification, different governing equations for modelling based on approximations are available, which are currently in practise within the numerical modelling community. The basic modelling task in each case is to solve the continuity and momentum equations for the fluid dynamics problem. However, the modelling complexity increases based on the physical problem to address, type of structure (coastal, offshore or marine) and type of sea-state considered.

In coastal engineering, the majority of the structures (e.g. breakwater, sea-wall, pile structures) are fixed or stationary. Then the physical problem to represent is the wave transformation process (i.e., wave shoaling, diffraction, refraction, reflection, wave-overtopping and wave-breaking) and its interaction with the structures. In case of offshore engineering, the structures may be fixed (e.g. offshore wind turbine foundations in <50m deep water) or floating (e.g. oil production platforms, floating offshore wind turbines, floating solar arrays). In the latter case, the modelling complexity increases because the fluid flow and structure motion(s) are coupled and thus need to be solved in conjunction; failure to do so would over-predict the hydrodynamic loads. Nonetheless, the overall excursion of an

offshore structure is small when compared to marine structures such as a ship or submarine. For a marine structure, the numerical modelling needs to account for large displacements (e.g. ship maneuvering in waves) thus necessitating large domains and, if the sea-state is violent, also hydroelasticity plays a role (e.g. hull-slamming in violent sea-states). Nonetheless, these scenarios may not always necessitate the NS equations; potential-flow models as long as the hydrodynamic loads and resulting body motions are properly accounted for the underlying phenomenon. For instance, models based on the Boussinesq equations are quite popular for modelling wave tranquillity and recently, for ship-generated waves.

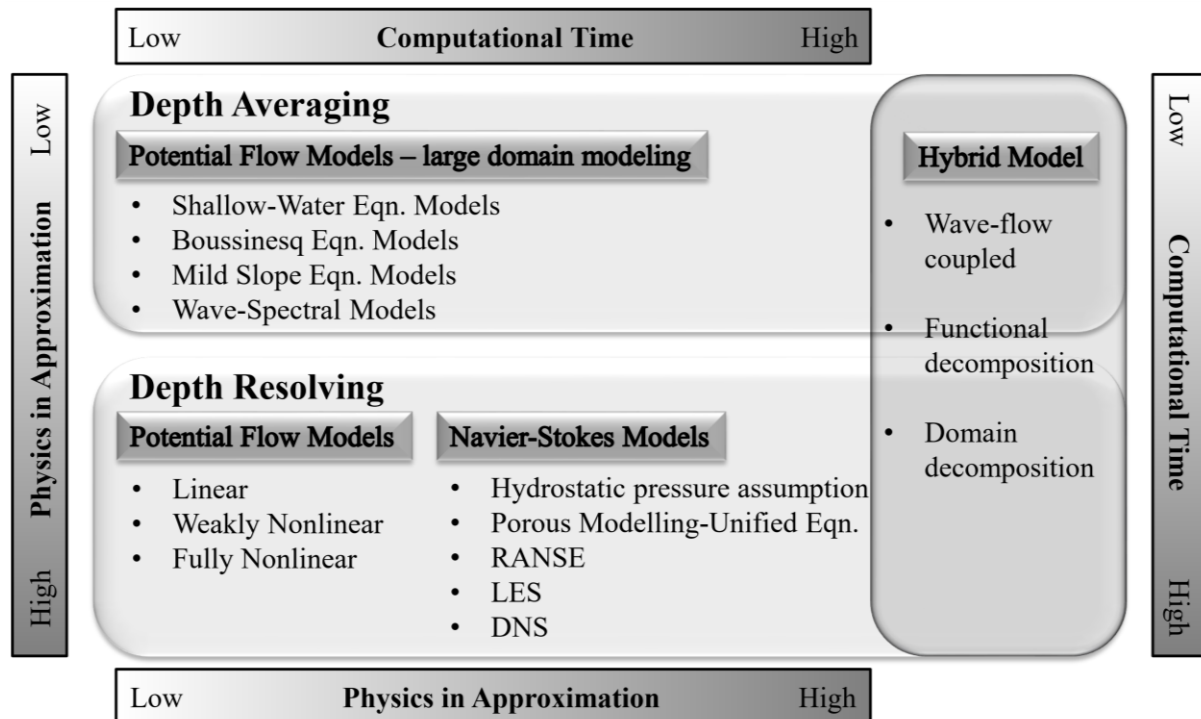


Figure 2. Different modelling strategies characterized by the level of physics approximation and resulting computational cost.

Various numerical methods are currently in practice to solve the mathematical model. The numerical methods are broadly classified into strong and weak forms. The traditional methods such as Finite Different, Finite Element method, Boundary Element method, Finite Volume method and modern methods such as particle/mesh free methods are being employed in the ocean engineering problems. Mostly, the choice of the numerical methods depends upon the developers and one is not superior to the others as one might expect. Each of these numerical methods has their own advantages and disadvantages, and the overall goal is to reduce or minimize the disadvantages using numerical treatments/algorithms/schemes.

In the following sections, these mathematical models are discussed with their governing equations to handle physical problems, assumptions, implementation strategies and adopted numerical methods along with their applications. The existing numerical efforts carried out worldwide are provided along with the detailed discussion on numerical model development actualized and supported in-part by the Newton Fellowship.

3 Depth Resolving Mathematical Models

We review the depth-resolving models. These models are mostly used for the wave-structure interaction problems to estimate the wave loads, wave damping characteristics and motion / structural responses. The problems that are based on small spatial and time scale are normally handled by the depth-resolving approach which models the physical process using a high (spatio-temporal) resolution thus leading to high computational costs.

3.1 Full Incompressible Navier Stokes Equations

The Navier-Stokes equations include the equations governing the conservation of mass (termed “equation of continuity” (EOC) for incompressible flows which is in turn a reasonable assumption for WSI) and conservation of momentum. The term “full” indicates the *absence* of simplifying assumptions such as irrotationality, depth-averaging, Reynolds-averaging, two-dimensionality, axisymmetry, single-phase nature of the flow (density is spatio-temporally constant) etc.

3.1.1 Governing Equations

The full Navier-Stokes equations (NSE) governing fluid motion are written here in differential form for the instantaneous velocity field $\vec{V}$:

$$\underbrace{\vec{\nabla} \cdot \vec{V} = 0}_{\text{continuity}} \quad \text{and} \quad \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\text{time}} + \underbrace{(\vec{V} \cdot \vec{\nabla}) \vec{V}}_{\text{advection}} = - \underbrace{\frac{1}{\rho^*} \vec{\nabla} p}_{\text{pressure}} + \underbrace{\frac{1}{\rho^*} \vec{\nabla} \cdot (\mu^* \vec{\nabla} \vec{V})}_{\text{diffusion}} + \underbrace{\vec{g}}_{\text{gravity}} \quad (1)$$

where, p is the total pressure, ρ^* and μ^* are the mixture density and viscosity respectively and $\vec{g}$ is the gravitational acceleration vector. Equation (1) represents the “instantaneous” Navier-Stokes equations (Anghan *et al.*, 2019) indicating that $\vec{V}$ is neither time-averaged (RANS) nor spatially-filtered (LES). The mixture properties ρ^* and μ^* account for the presence of multiple contiguous phases in the domain. Here, advantage is derived from the fact that the phases can be considered as being “individually incompressible” (Saincher and Banerjee, 2018) for most applications which precludes the necessity of solving (say) N sets of the NSE for N phases. This results in the so-called “single-fluid formulation” wherein the entire computational domain is assumed to be filled with a single, albeit, variable-property fluid (Saincher and Sriram, 2022a). It should also be noted that within the single-fluid framework, equation (1) is “non-conservative” (Saincher and Sriram, 2023) meaning ρ^* is on the right-hand-side with the pressure and diffusion terms. On the other hand, the formulation would be termed “conservative” if ρ^* were on the left-hand-side with the time and advection terms. The positioning of ρ^* in the governing equations is immaterial for a single-phase treatment of the NSE (for instance cf. Sriram *et al.*, 2014). The same, however, would have far-reaching consequences for a multiphase framework especially for violent flows involving wave-breaking and/or slamming loads; a conservative formulation is recommended in these cases (Saincher and Sriram, 2023). However, an important limitation of the conservative formulation is that it may lead to the formation of unrealistically large velocities at the interface (Tryggvason *et al.*, 2007) and thus is deemed unnecessary for more benign WSI scenarios. In context to equation (1), it is also worth mentioning that the total pressure p is comprised of static, hydrostatic as well as dynamic contributions; p is not the true pressure but rather a pseudo pressure which satisfies the EOC. The advantage with WSI and ocean

engineering problems in general is that the simulation begins from quiescent/calm water conditions which allows for a very accurate “guess” of the initial pressure field using the hydrostatic law. This results in a dynamic pressure field that is very close to the true (say experimentally measured) dynamic pressure, once the hydrostatic contribution has been removed (Saincher and Sriram, 2022a ; 2022b).

3.1.2 Solving the Navier-Stokes Equations

For a given flow problem, the solution variables of interest include the velocity $\vec{V}$ and pressure p . It is characteristic of the incompressible Navier-Stokes equations to *not* have a separate equation for pressure. Owing to this, a majority of incompressible NSE flow solvers are based on a predictor-corrector approach which was pioneered by Chorin (1967); the same is illustrated in Figure 3.

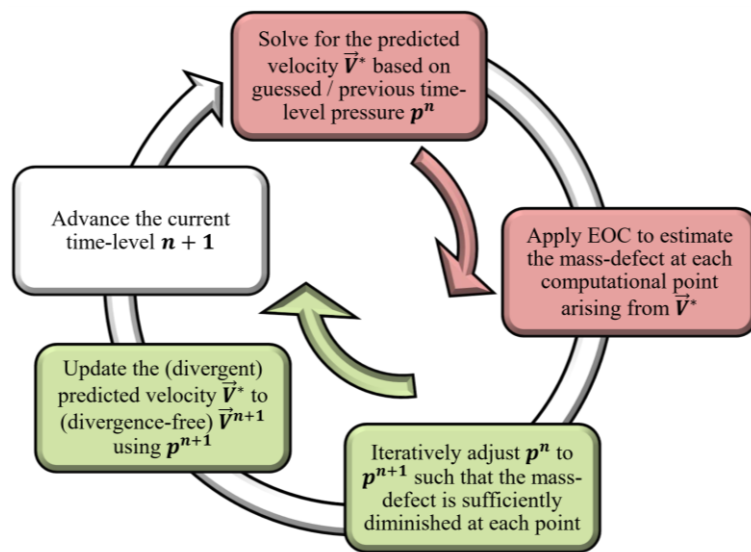


Figure 3. A typical predictor-corrector loop characteristic of projection methods pioneered by Alexandre Chorin in 1967.

At the beginning of the solution, both $\vec{V}^{n+1}$ and p^{n+1} at the current time-level are unknown and the momentum equations are solved for a predicted velocity field $\vec{V}^*$ wherein either:

- the pressure term $\left(-\frac{1}{\rho^*} \vec{\nabla} p\right)^n$ from the previous time-level is considered (Saincher and Banerjee, 2015) or,
- the pressure term is not considered at all which was the case with Chorin’s original method (normally adopted in Meshfree methods, see, Sriram and Ma, 2021).

At this point, the incompressibility condition $\vec{\nabla} \cdot \vec{V}^{n+1} = 0$ is invoked at the current time-level and the same is split into a mass defect $\vec{\nabla} \cdot \vec{V}^*$ and divergence correction $\vec{\nabla} \cdot \vec{V}'$ contributions. This marks the end of the “predictor-step” (highlighted in red in Figure 3). Following this, the property $\vec{V} = -\frac{\Delta t}{\rho} \vec{\nabla} p$ is invoked to establish a relationship between either $\vec{\nabla} \cdot \vec{V}^*$ and p^{n+1} (Sriram and Ma, 2021) or between $\vec{\nabla} \cdot \vec{V}^*$ and the pressure correction p'

(Saincher and Banerjee, 2015). In either case, one ends up with a pressure Poisson equation (PPE) which needs to be iteratively solved for p^{n+1} (or p'). This is oftentimes the most computationally-intensive step in a flow solver. Following solution of the Poisson equation, $\vec{V}^{n+1}$ can be obtained using $\vec{V} = -\frac{\Delta t}{\rho} \nabla p$ which marks the end of the “corrector-step” (highlighted in green in Figure 3). The splitting of the solution into predictor and corrector steps is also known as the “projection method” since the pressure is used to project $\vec{V}^*$ onto a space of divergence-free velocity-field which is essentially the Helmholtz decomposition.

Various flow solvers (or so-called “pressure-velocity coupling” schemes) such as SIMPLE, PISO, PIMPLE essentially have the same predictor-corrector constitution but differ with regards to how $\vec{V}^*$ is calculated as well as the number of predictor-corrector cycles per time-step. In fact, regardless of whether $\vec{V}^*$ is computed fully-explicitly or semi-implicitly (because a fully-implicit treatment of the advection term is not possible), the solver still belongs to the SIMPLE class of algorithms (Ferziger *et al.*, 2020). However, some authors also call the fully-explicit category of algorithms “semi-explicit” (Dave *et al.*, 2018 ; Sharma, 2022) owing to the implicit nature of solution of the PPE. It is important to note that, for a given order of time-discretization, the solutions obtained from a fully-explicit or semi-implicit predictor step should be identical. Nonetheless, the semi-implicit treatment would accord further stability to the solution.

In context to WSI, a forward Euler time-discretization and fully-explicit evaluation of $\vec{V}^*$ has been extensively used by the authors (Saincher and Sriram, 2022a ; 2022b ; 2023). From the authors’ experience, explicit (forward Euler) time discretization is recommended for waves owing to the hyperbolic nature of solution propagation and a fully-explicit evaluation of $\vec{V}^*$ was found to be sufficient for relatively benign WSI scenarios especially ones that didn’t involve slamming loads. In fact, it is demonstrated in Saincher *et al.* (2023a ; 2023b) that a fully-explicit evaluation of $\vec{V}^*$ works even in slamming conditions for modest mesh resolutions. Thus, the CFD user ought to make an informed decision whilst selecting the pressure-velocity coupling scheme keeping in mind the trade-off between numerical stability (better for semi-implicit treatment) and computational efficiency (better for fully-explicit treatment). Unfortunately, users of commercial CFD solvers seldom have fully-explicit pressure-velocity coupling available to them and thus alternatively opt for (say) the PISO solver with a Non-Iterative Time Advancement (NITA) option available in ANSYS® FLUENT.

When the predictor and corrector steps are considered in conjunction, say for a 3D flow problem, a single time-step would have one iterative solution loop (for p) in case of a fully-explicit solver and four (for U, V, W, p) in case of a semi-implicit solver. However, our experience suggests that the computational effort required for solving p may sometimes be greater than the three velocity components U, V, W combined. This is primarily because of differences in the rate of convergence which is in turn dependent on the type of boundary conditions involved. The boundary conditions are predominantly Dirichlet in case of

velocities which results in predominantly Neumann conditions for the pressure thus leading to an increase in the computational effort for solving the PPE.

3.1.3 Boundary conditions – Wave/Current Generation and Absorption

A prerequisite to accurate WSI simulations in ocean engineering applications is high fidelity wave generation as well as reflection-free absorption of waves/currents in the computational domain. The task of absorption is generally more challenging for WSI simulations involving regular and irregular waves when compared to focusing waves primarily due to the larger number of wave cycles/periods involved in the former case. The task of absorption also becomes complex if currents co-exist with waves. The various methods of wave/current generation and absorption in NSE-based NWTs are mapped against their numerical characteristics in Table 2.

Table 2. Type of wave/current generation and absorption strategies in NSE-based NWTs.

Wavemaker		U	V	W	p	η
Inflow-boundary		Dirichlet	Dirichlet	Dirichlet	Dirichlet	Dirichlet
Mass-source function		--	--	--	--	Source-term in EOC
Momentum-source function		Source-term in momentum equation			--	--
Internal inlet		--	--	Dirichlet	--	--
Relaxation zone		Dirichlet	Dirichlet	Dirichlet	Dirichlet	Dirichlet
Moving wall	Flap / Piston type	Prescribed motion	--	Prescribed motion	--	--
	Segmented type					
Wave-absorber		U	V	W	p	η
Outflow boundary		Orlanski / Continuity / Sommerfeld radiation boundary condition				
Sponge-layer		Sink terms in momentum equation			--	--
Relaxation zone		Solution gradually ramped from/to wave theory to/from numerical model				
Moving wall (active absorption)		Prescribed motion	--	Prescribed motion	--	--
Adaptive passive absorption		Adaptively predicted using on-board elevation		Neumman	Neumman	--

With reference to Table 2, the development of “numerical wavemakers” for NSE models was pioneered by Lin and Liu (1998; 1999) wherein the inflow-boundary and mass-source function techniques were proposed. As seen from Table 2, the inflow technique involves a Dirichlet prescription of the wave-induced orbital velocities (predicted from a suitable wave theory) as well as the free-surface elevation at the domain boundary. The present research group has proposed a modified inflow technique to improve the volume-conservation properties of inflow-boundaries, particularly for scenarios involving strong Stokes drift such as steep wave generation in near-shallow water (Saincher and Banerjee, 2017).

In conjunction with inflow boundaries, the mass-source function technique was also developed which involved the modification of the EOC through the inclusion of a time-varying source term that is in turn proportional to the wave elevation. Wave generation is achieved through periodic ejection/ingestion of water-volume from/into the source region and this offers some advantages over inflow-boundaries. For instance, the only wave characteristic to be input is the time-varying free-surface elevation $\eta(t)$ and thus wave-

records from the field could be reproduced. Also, waves reflected-off of domain boundaries would not interfere with the wave generation. Nonetheless, the source region itself has several design variables requiring parameterization and, in this context, the authors have proposed guidelines to decide the geometry, placement and strength of the source region based on the relative depth and wave-steepness (Saincher and Banerjee, 2017). As listed in Table 2, other similar methods have also been proposed such as the internal inlet (Hafsia *et al.*, 2009) and momentum-source function (Choi and Yoon, 2009) techniques. Some researchers have also attempted to directly model piston/flap-type wave-paddle motions into their NWTs using embedded boundary treatment for the solid (cf. fast-fictitious-domain (FFD) based modelling of wave-paddles in Anbarsooz *et al.* (2013)).

However, currently, the most popular technique of numerical wave generation is the so-called “relaxation zones” developed by Jacobsen *et al.* (2012) for OpenFoam[®]. Here, the solution is spatio-temporally “ramped-up” from wave-theory to NSE before the structure/region of interest and again “ramped-down” from NSE to calm-water conditions after the structure/region of interest. Thus, relaxation zones not only prevent upstream reflection of waves from the far-end of the NWT but also downstream re-reflection of waves reflected-off of the structure. It is also worth mentioning that relaxation zones in and of itself is a more general concept that has been implemented in hybrid potential theory-NSE models (Agarwal *et al.*, 2022b) as well as in hybrid spectral theory-NSE models (Aliyar *et al.*, 2022).

Apart from relaxation zones, other methods of wave absorption have also been implemented for NSE-based NWTs. For instance, Lin and Liu (1999) employed a radiation/outflow boundary condition for wave absorption at the far-end of the NWT. Outflow boundaries generally implement the Sommerfeld condition (Dave *et al.*, 2018): $\frac{\partial \phi}{\partial t} + C \frac{\partial \phi}{\partial n} = 0$ where ϕ is the property to be effluxed from the boundary, t is time, C is the phase velocity and n points normal to the boundary. The prescription of C is relatively straightforward for “flow problems” making outflow boundaries suitable for tsunamis, tidal flows, scour etc. which involve a dominant current component. Sommerfeld conditions are also suitable for absorbing small-amplitude waves. However, these pose a challenge for absorbing steep waves particularly because C is spatio-temporally variable along the boundary. It has been shown in Dave *et al.* (2018) that improper prescription of C leads to severe (inward) reflections even for free-shear flows.

Self-adaptive wavemaker theory has also been used popularly in both the physical and numerical wave tanks. This method utilizes wavemaker (moving wall) whose motion is specified to generate both generating the incident waves and an additional wave to cancel the undesirable wave (e.g. the reflected wave from other part of the tank). More details may be found in Yan *et al.*, 2016). In addition, the same concept of ‘adaptive absorber’ was also used in our recent work on developing a passive wave absorber (Yan *et al.*, 2020). This boundary behaves similarly to the inflow boundary, however the fluid velocity condition is specified by considering its relation with the wave elevation recorded at the boundary. This method does not require the use of the relaxation zone for wave absorption and thus results in

a considerable improvement of the computational efficiency. A recent application can be found in Xiao et al (2024).

3.1.4 Free surface capturing/tracking

A majority of ocean engineering problems involve waves and/or other flows such as bores, hydraulic jumps, etc. which necessitates computing the topology of the free-surface. In reality, the free-surface marks a discontinuity between two media (say air and water) and thus acts as an interface. The numerical algorithms for computing the interfacial topology can be broadly classified into interface-tracking and interface-capturing techniques. In the former category of algorithms, the free-surface is modelled as a boundary and is tracked by updating the mesh as the solution progresses. In the latter category, the free-surface evolves spatio-temporally within a fixed domain wherein the interface is identified by an indicator function. Interface-capturing algorithms are obviously more advantageous (especially for violent flows involving complex interfacial deformation such as overturning and aeration) and thus have been extensively employed in NSE-based flow solvers; the same have been listed in Table 3.

As evidenced from Table 3, the interface-capturing algorithms can be further classified based on the technique used for interface identification (“*reconstruction*”) and advection. The interface identification techniques differ based on the type of indicator function used (volume fraction or level-set function) as well as whether the identification itself is geometric in nature or not. The level-set method and high-resolution schemes such as CICSAM are algebraic in nature in that they do not involve explicit geometrical computations of the placement (or advection) of the interface within the domain. In comparison, geometric methods such as PLIC-VOF and MOF are higher fidelity in that the interfacial coordinates are geometrically computed subject to conservation of the primary phase volume in each cell.

Volume conservation is *intrinsic* for geometric VOF methods and also for single-phase meshfree methods such as IMLPG_R. This is not the case for algebraic VOF schemes or the level-set method where additional numerical treatment is necessary to achieve volume conservation. This has been comprehensively demonstrated by the present authors (Saincher and Sriram, 2022a) and others (Anghan *et al.*, 2021 ; Arote *et al.*, 2021) wherein a material redistribution algorithm originally developed for geometric VOF (Saincher and Banerjee, 2015) has been shown to dramatically improve the conservation properties of algebraic VOF schemes.

Similarly, interfacial diffusion is *intrinsic* for algebraic VOF as well as level-set methods. This could be mitigated to some extent using operator-split/direction-split advection as doing so would eliminate multi-fluxing errors (Saincher and Sriram, 2022a). Whilst algebraic VOF techniques are indeed capable of capturing large-scale interfacial segregation in WSI problems (Saincher *et al.*, 2023a), small-scale droplets and bubbles would still diffuse upon separation from the parent phase. This diffusion seldom contributes to the hydrodynamics in a WSI simulation and, in fact, provides numerical stability to the solution. Conversely, droplets/bubbles separating from the parent phase would *never* dissipate in geometric VOF and thus excessive interfacial fragmentation might, in fact, lead to solver instability.

Table 3. Various interface-capturing algorithms developed for NSE-solvers; cf. nomenclature for the abbreviations.

Authors	Algorithm	Interface identification	Interface advection	Interface diffusion	Volume conservation	Mesh
O'Shea <i>et al.</i> (2014)	NPFA	Geometric VOF	Unsplit Eulerian	Zero	Intrinsic	Cartesian
Sriram <i>et al.</i> (2014)	IMLPG_R	MPNDAF	Lagrangian	Zero	Intrinsic	--
Saincher and Banerjee (2015)	Redistribution-based PLIC-VOF	Geometric PLIC-VOF	Operator-split Eulerian	Zero	Intrinsic	Cartesian
Bihs <i>et al.</i> (2016)	REEF3D	Level-set	Unsplit Eulerian	Intrinsic	Extrinsic	Cartesian
Zinjala and Banerjee (2016)	LEAS-MOF	Geometric MOF	Lagrangian-Eulerian	Zero	Intrinsic	General Polygonal
Zinjala and Banerjee (2017)	RMOF	Geometric MOF	Lagrangian-Eulerian	Zero	Intrinsic	General Polygonal
Anghan <i>et al.</i> (2021)	MSTACS	Algebraic VOF	Unsplit Eulerian	Intrinsic	Extrinsic	Cartesian
Arote <i>et al.</i> (2021)	SAISH	Algebraic VOF	Unsplit Eulerian	Intrinsic	Extrinsic	Cartesian
Saincher and Sriram (2022a)	OS-CICSAM	Algebraic VOF	Operator-split Eulerian	Intrinsic	Extrinsic	Cartesian

3.1.5 Turbulence Modelling

Ocean engineering problems involve flow of sea-water which has a kinematic viscosity of $\nu \sim 1e - 06 \text{ m}^2/\text{s}$. The corresponding Reynolds number $\text{Re} = \mathcal{V} \cdot \mathcal{L}/\nu$ would typically be $O(10^6)$ even if the characteristic velocity ($\mathcal{V}$) and length ($\mathcal{L}$) are $O(1)$, that is, at model-scale. This is generally the case since the Froude-law is invoked for scaling based on the fact that gravity is the dominant restoring force in ocean engineering applications. As a consequence, most scenarios being simulated are *not* laminar and some form of modelling may be required to account for the additional viscous effects near the structure. Some of the typical applications necessitating turbulence modelling include:

- Wave/tsunami interactions with vegetation: turbulence-induced viscous effects arising from flow separation need to be accounted for to correctly estimate energy attenuation.
- Response of floating bodies: failing to account for viscous effects within the boundary layer may result in over-prediction of the motion response.
- Resistance of marine vessels in waves/calm water: failing to account for viscous effects within the boundary layer may result in under-prediction of resistance.

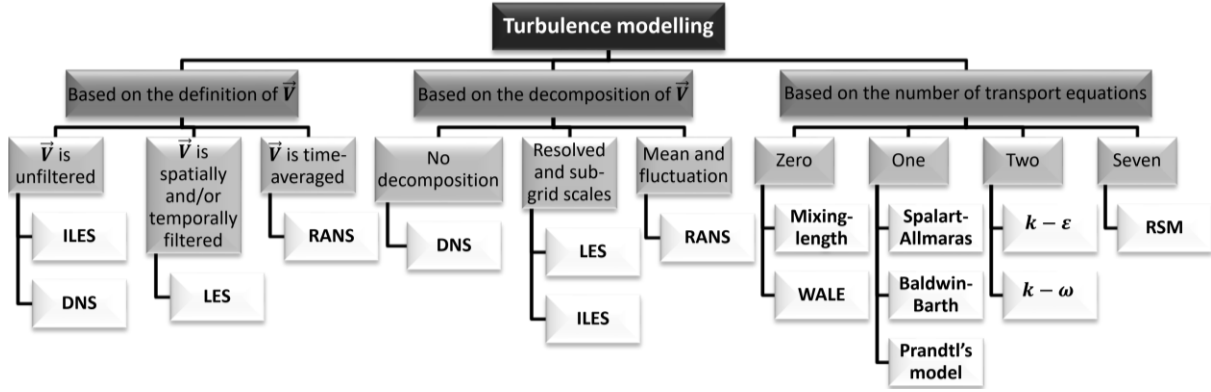
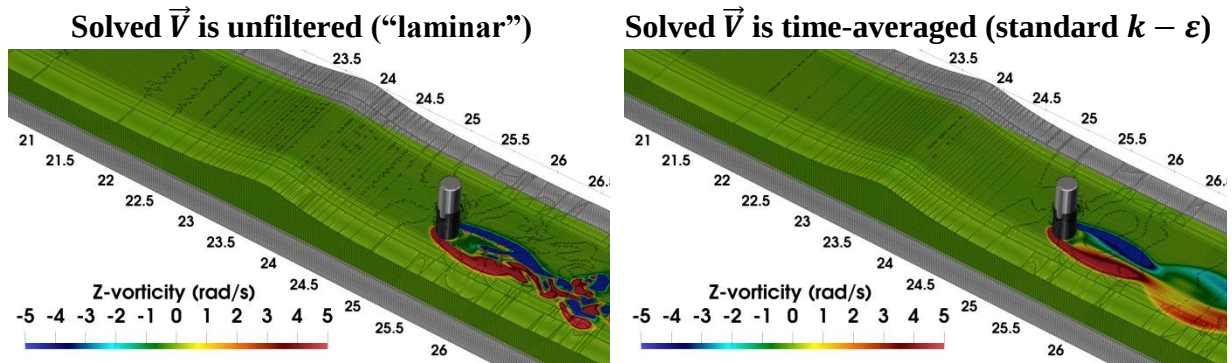


Figure 4. An illustration of the different means to categorize various strategies to model turbulence in depth-resolving models; cf. nomenclature for the abbreviations.

Several popular methods have been developed for modelling turbulence in NSE-based solvers; some have been integrated with self-developed codes by the present research group. There exist different means of classifying turbulence modelling strategies for depth-resolving methods; the same are depicted in Figure 4. In conjunction with Figure 4, the momentum equation (1) is also re-written:

$$\underbrace{\frac{\partial \vec{V}}{\partial t}}_{\text{time}} + \underbrace{(\vec{V} \cdot \nabla) \vec{V}}_{\text{advection}} = - \underbrace{\frac{1}{\rho^*} \nabla p'}_{\text{pressure}} + \underbrace{\frac{1}{\rho^*} \nabla \cdot ((\mu^* + \mu_t) \nabla \vec{V})}_{\text{diffusion}} + \underbrace{\vec{g}}_{\text{gravity}} \quad (2)$$

where, p' is the modified pressure which includes the normal components of the Reynolds or SGS stress tensor and μ_t is the turbulent viscosity; the terms modified/introduced by turbulence modelling have been highlighted in bold. In context to equation (2) and Figure 4, $\vec{V}$ can be unfiltered, spatio-temporally filtered or time-averaged. The filtering and time-averaging operations are essentially decompositions of the unfiltered velocity and thus, once performed, information about the instantaneous velocity field is invariably lost. For instance, the $\vec{V}$ field obtained following solution to the RANS equations is time-averaged and thus, (temporal) fluctuations in $\vec{V}$ do not represent fluctuations in the instantaneous field. Only the effect of the true fluctuating field on $\vec{V}$ is modelled through the eddy viscosity μ_t .



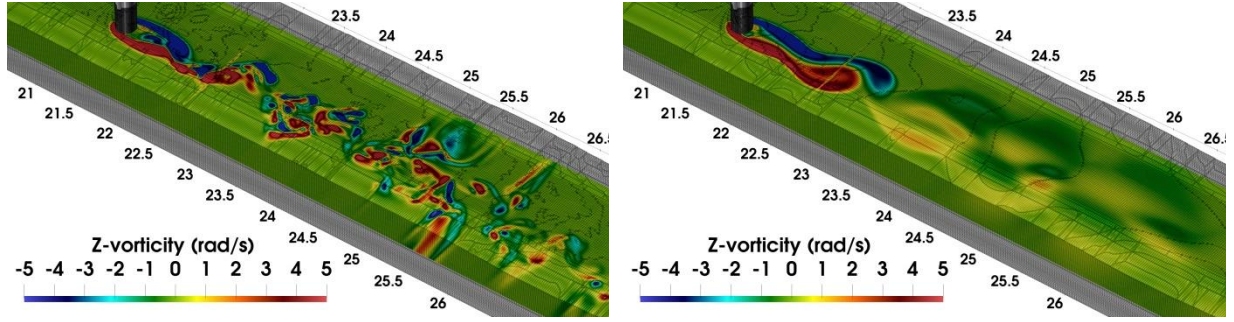


Figure 5. The vorticity field ($\vec{\nabla} \times \vec{V}$) generated by a moving cylinder interacting with a focusing wave (Saincher and Sriram, 2022b); note the change in the nature of the solution based on the definition of $\vec{V}$. The cylinder moves from bottom-right to top-left.

This important aspect is illustrated in Figure 5 wherein vortices shed by a moving cylinder interacting with focusing waves are shown (adapted from Saincher and Sriram (2022b)). The same problem has been simulated first using unfiltered NSE (a “laminar” solver) and then using time-averaged NSE (a RANS solver based on standard $k - \varepsilon$). The aforementioned loss of information regarding the true fluctuating velocity field is readily apparent from Figure 5; the vorticity field is “instantaneous” in both cases. It should also be noted that the so-called “laminar” solver is a misnomer as it simply refers to solving the NSE without any turbulence modelling. In this regard, the laminar approach is neither DNS (since no attempt is made to resolve the Kolmogorov scales) nor ILES (since no attempt is made to adjust the discretization errors so that they would mimic SGS modelling (Rodi *et al.*, 2013)). Having said that, figure 5 indicates that the laminar solver captures more “turbulence” than the actual turbulence model!

The necessity and nature of turbulence modelling depends on the nature of the problem itself and oftentimes the fidelity of the solution/simulation (against experiments) depends on the expertise of the CFD practitioner (this is later discussed at length in §6.1 on comparative numerical studies). One is not only required to assess the *need* of a turbulence model but also the *impact* of a particular model on the solution. Considering a wave-floating structure interaction problem as an example, a need for turbulence modelling may arise due to an over-prediction of the angular acceleration of the body by a laminar model. If RANS-based turbulence modelling is introduced to supplement the viscous damping in the near-field of the body, the same may also negatively impact the simulation through unwanted damping of the incident waves. In such a case, the unwanted damping could be mitigated by:

- Stabilizing the unbounded growth of μ_t using limiters (Larsen and Fuhrman, 2018).
- Increasing advection using higher-order upwind schemes (Saincher and Sriram, 2022b).
- Increasing advection using conservative NSE formulations (Saincher and Sriram, 2023).
- Switching to a less empirical model such as WALE (zero-equation model with a single model constant) for computing μ_t (Rodi *et al.*, 2013).

The above discussion indicates that there exist multiple solutions to a given problem and there is a general consensus that the simplest models also prove to be the most robust. Taking into account the strongly empirical nature of turbulence modelling in general (RANS in particular), a modestly accurate albeit robust model applicable to several problems should be preferred over a heavily calibrated model that works perfectly albeit only for a single problem. Further, turbulence model should be employed for the practical problems in need and not for all scenarios.

3.1.6 Numerical Methods

In addition to the algorithms used for pressure-velocity coupling, interface capturing and turbulence modelling, the flow solver is also comprised of spatio-temporal discretization schemes as well as linear equation systems solvers. Both categories of algorithms directly impact the accuracy and stability of the flow solver.

Some of the popular discretization schemes that have been widely implemented for ocean engineering problems are now discussed in context to the momentum equation (1) and Table 4. Discretization of the time-term can be carried out either using linear multi-step methods (LMMs) or Runge-Kutta (RK) methods. These two categories of methods can be further classified into explicit and implicit schemes. Explicit LMMs are also known as the Adams-Bashforth Methods (ABMs) whilst implicit LMMs are known as Adams-Moulton Methods (AMMs). As the name suggests, LMMs build accuracy by storing the flow solution across multiple time-levels such that a first-order LMM would require an existing flow-field solution from one time-level, a second-order LMM would necessitate solutions from two time-levels and so on. Owing to the requirement of an existing flow-field solution, LMMs $> \mathcal{O}(1)$ aren't "self-starting" and some complexities exist in implementing these methods for variable time-steps. Moreover, the region of stability of LMMs shrinks with increasing order of accuracy (Drikakis and Rider, 2005). Nonetheless, a key advantage of LMMs is that the per-time-step computation effort does not increase with increasing order of accuracy; only the storage requirements increase.

On the other hand, RK methods divide a single time-step into a number of intermediate steps with all intermediate velocity fields made divergence-free; only the most recently known velocity field is necessary for a given intermediate step. Given this characteristic, RK methods are self-starting and automatically account for variable time-steps. However, the fact that the predictor-corrector loop (cf. Figure 3) is executed multiple times within a time-step introduces a unique set of merits and shortcomings. The chief merit is the numerical stability which, unlike LMMs, increases with increasing order of the method. Another merit over LMMs is that storage requirements do not increase with increasing order. The chief shortcoming associated with RK methods is that each intermediate step entails a computationally expensive solution of the elliptic PPE or EOPC; per-time-step computation effort thus increases with increasing order. Referring to Table 4, it is seen that a number of NSE algorithms implement explicit time-integration (ABM or TVD-RK) which is suitable given the hyperbolic nature of wave propagation. In cases where a greater amount of numerical stability is desired, say conservative NSE formulations for violent WSI (Benoit *et*

al., 2023) authors opt for AMM rather than TVD-RK. This is probably because the additional linear equation systems encountered for AMM (one system for each component of $\vec{V}$) is parabolic and less expensive to solve than the elliptic PPE/EOPC encountered multiple times within a time-step in the case of TVD-RK. It is also possible that very high-order AMMs might lead to dispersive (phase) errors in wave-propagation.

Table 4. A summary of the various discretization methods and linear equation system solvers implemented for NSE algorithms applied to ocean engineering problems reported in the literature; cf. nomenclature for abbreviations.

Authors	Time	Momentum Advection		Pressure	Diffusion	PPE/EOPC
		Scheme	Treatment			
Sriram <i>et al.</i> (2014)	ABM1	Lagrangian	--	SFDI	SFDI	GMRES
Bihs <i>et al.</i> (2016)	TVD-RK3	WENO	Non-conservative	???	???	BiCGStab
Xie and Stoesser (2020)	AMM1	Second-order TVD	Conservative	CD2	CD2	ADI / BiCGStab
Agarwal <i>et al.</i> (2021b)	ABM1	Lagrangian	--	SFDI	SFDI	BiCGStab
Anghan <i>et al.</i> (2022)	ABM2	Blended FOU-FiOU	Non-conservative	CD2	CD4	GSSOR
Saincher and Sriram (2022b)	ABM1	Blended FOU-FiOU	Non-conservative	CD2	CD2	GSSOR
Benoit <i>et al.</i> (2023)	AMM1	Slope-limited SOU	Conservative	CD2 with third-order numerical smoothing		GMRES
Saincher and Sriram (2023)	ABM1	Blended FOU-FiOU	Conservative	CD2	CD2	GSSOR

In addition to time-integration, the numerical schemes chosen for momentum advection, pressure and diffusion terms as well as the linear systems solver chosen for solving the pressure field also play a key role deciding the robustness and accuracy of NSE solvers. In context to the discretization of the pressure and diffusion terms, second-order central differencing (CD2) suffices for most scenarios and is thus the most widely used (cf. Table 4). However, recent studies involving DNS of marine outfalls have instead implemented fourth-order central differencing (CD4) for higher resolution treatment of the diffusion term (cf. Anghan *et al.*, 2022). It should also be noted that CD4 treatment of the pressure gradient does not dramatically improve the accuracy of a solver and should rather be avoided to save computational effort (Tafti, 1996).

Numerical formulations of the NSE inherently contain some form of numerical diffusion. In context to ocean engineering applications, this diffusion gets manifested as a gradual reduction in wave-height (Saincher and Banerjee, 2017). Whilst the numerical diffusion can be arrested through mesh refinement, a more computationally efficient way to do this (especially for mesh-based Eulerian solvers) is by increasing the order of advection discretization. However, computational efficiency does not translate to a straightforward implementation, especially for multiphase solvers. Implementation of a high order advection scheme in its “pure form” leads to severe dispersion errors in regions of sharp velocity gradients which, in case of waves, are at the air-water interface; the consequence is

unphysical deformation of the generated waves. This can be corrected by either using inherently bounded schemes such as WENO (Bihs *et al.*, 2016) or blended schemes where (say) only 50% of the advected momentum is estimated using the high-order scheme, the rest being estimated using FOU (Saincher and Sriram, 2022b). For more violent scenarios involving wave-breaking and/or wave-slamming, a higher order treatment of advection may not be sufficient and rather the correct amount of advection being attributed to each fluid-phase needs to be ensured. This is where conservative NSE formulations come into picture wherein ρ^* is shifted to the left-hand-side of equation (1) with the time and advection terms. It has been recently demonstrated by the authors that conservative NSE solvers are necessary for correctly capturing the topology of waves overturning over a long distance; such as solitary waves breaking over a beach / shallow water (Saincher and Sriram, 2023). It is worth mentioning that conservative NSE formulations strongly and consistently couple mass and momentum transport (cf. the discussion on mass inconsistency in Saincher and Sriram (2023)) and thus momentum advection is more strongly governed by material transport (owing to the 1:800 density ratio between air and water) rather than the momentum advection scheme itself (Bussmann *et al.*, 2002). This makes conservative NSE a suitable alternative to high-order advection schemes for arresting wave-damping in non-violent / moderately violent WSI scenarios.

For rigid structures, one can incorporate this in the computational domain and solve the interaction problems as shown in Figure 5. For elastic and floating structures, a separate equation motion will be solved to understand the fluid-structure interaction process, see, Sriram and Ma (2012), Rijas *et al.* (2019), Vineesh and Sriram (2022). In the case of the modelling, porous/vegetation structure interactions one can adopt microscopic or macroscopic approaches. The macroscopic approach is the commonly adopted due to the computational advantages as well as in terms of requirement for physical process (see, Divya and Sriram (2020)). For modelling the wave porous/vegetation structure, in the governing equations additional resistance terms such as the linear drag coefficient representing the laminar flow, non-linear drag coefficient representing the turbulent flow, coefficient for the transitional flow and virtual mass coefficient for inertia terms were incorporated. The numerical studies on the wave porous structure can be carried out in two different ways, i) Coupling of pure fluid and porous flow equations, in which the fluid flow was solved for Navier Stokes equation and porous flow with different porous flow model, after which the interface was coupled by matching the flow properties. It can be explicit, implicit or iterative in nature. ii) Based on unified or single governing equations to model both porous structure and fluid flow. In microscopic approach a detailed flow will be addressed (see Xie and Stoesser (2023)).

Apart from mesh based approach in solving the NS, mesh free or particle methods are quite popular and developments are being carried out. Many recent review papers exist such as Luo *et al.* (2021), Sriram and Ma (2021), Lind *et al.* (2020) and references therein. However, the acceptability of the meshfree methods or particle methods for industry and practical applications in the projects are not matured compared to mesh based methods. The consolidation of the work carried out in mesh free method based on Meshless Local Petrov

Galerkin Method (MLPG) by the authors through Newton fellowships are reviewed in detail in Sriram and Ma (2021) and shall not be repeated here for the sake of brevity. Further, an important relation between the widely popular Smoothed Particle Hydrodynamics, Moving Particle Semi-Implicit Method and MLPG was established. However, as this special issue contributes to the Newton fellowships, a flow chart of development has been reproduced for completeness as shown in Figure 6.

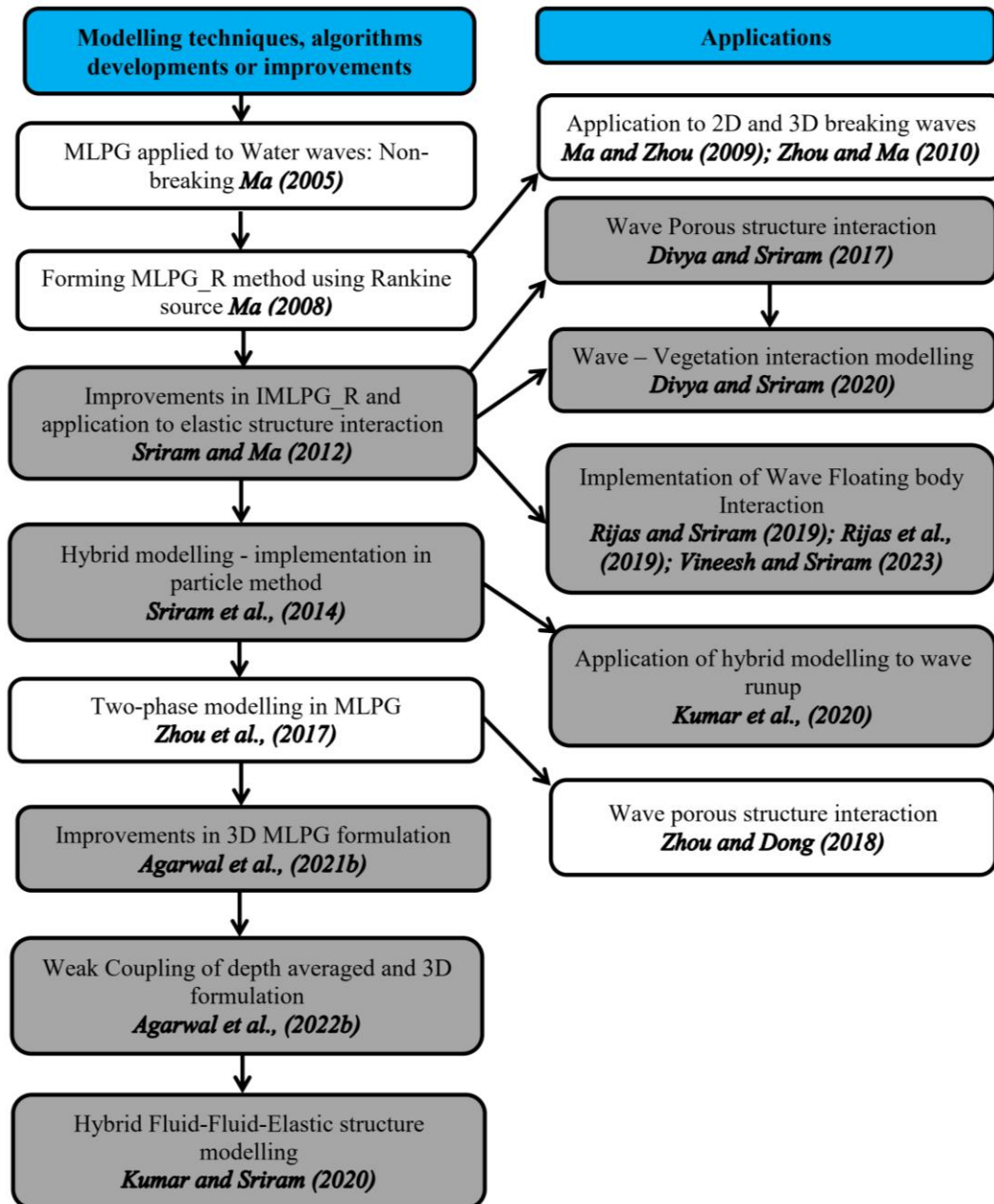


Figure 6. Summary of the history of the development of MLPG and its application in Ocean Engineering (revised and updated from Sriram and Ma, 2021). Grey shaded boxes are development with partial or full support from the Newton fellowship.

3.3 Potential Flow Theory

The fully nonlinear potential flow theory is matured in the today's context and has been used by researchers and industry. The methodology is pioneered by Longuet-Higgins and Cokelet (1976) using mixed Eulerian and Lagrangian approach. This simulation of nonlinear waves can be carried out either by fully discretising the domain and then solving the Laplace equation using numerical approaches (like FEM, BEM and so on) or by obtaining the solution of the Laplace equations using spectral, Eigen function or Fourier methods. In the former case, the computational time would be quite expensive when one extend the method to 3D, and the advantage is one can solve the structure interaction of any arbitrary shape. In the later, the computational time is not that much expensive and mostly used for simulating the fully nonlinear waves. Dommermuth and Yue (1987), West *et al.* (1987) proposed an attractive fast convergence, high accuracy and fast resolution properties based on higher order spectral (HOS) method. These fast methods of computations will be of much useful for calculating the long-time evolution of the nonlinear waves and can be used as an input for the numerical models based on NS. The detailed review on these models can be referred in Kim *et al.* (1999), Ma (2008) and the references therein. Normally, these models are quite effective in reproducing the extreme steep non-breaking waves, however once the wave overturns or breaks, the simulation crashes. In order to overcome these effects and carry out the simulations for long durations, recently researchers used empirical treatment such as eddy viscosity models to incorporate breaking effects (Tian *et al.* (2010), Barthelemy *et al.* (2018), Sieffert and Ducrozet (2018), Hasan *et al.* (2019)). The wave generations discussed in Table 2 can also be incorporated in these models, mostly using moving wall, relaxation zone and/or prescribing inlet wave characteristics.

4 Depth-averaged Mathematical Models

These models are developed before 1980s and completely evolved in use for practical problems. These models are used to address the larger spatial and temporal process in the ocean engineering (cf. Agarwal *et al.*, 2022a). Considering the assumption of horizontal velocity, i.e., the depth averaged these models are evolved in the past, such as Shallow water equations, various forms of Boussinesq equations (cf. Brocchini, 2013 for a detailed review). These are widely used in the industry for waves and current circulations. These models are based on irrotational and inviscid assumptions; however turbulence and wave breaking have also been treated using the empirical approaches (Shi *et al.*, 2012). These models are also used for coupling with other models/NS equations to minimize the computational time (cf. Agarwal *et al.*, 2022b and references therein). The waves are generated in the models using the different approaches as discussed in Table 2, except moving wall approach.

5 Coupled models

In the previous section, we discussed about different models that are available to treat the problem at hand. However, it is ideal to have one particular model to handle all class of problem. One of the approaches to carry out this is coupling different modelling tools that are developed over the period of years leading to multi-scale modelling in ocean engineering. There are two approaches; one is domain decomposition and the other functional decomposition. The domain decomposition (DD) strategy divides the computational domain

into parts and applies different mathematical models in each part. In most DD-based WSI problems, the computational domain is decomposed into a viscous inner sub-domain and a potential outer sub-domain. The information (velocity, pressure and surface elevation) will be transferred through either relaxation zones or a sharp interface. Also, based on how the information between the solvers is being transferred, it can be either one-way coupling (weak coupling) or two-way coupling (strong coupling). In one way coupling, information is transferred only from the potential solver into the viscous solver, but in two way coupling, the information is transferred in both ways, from depth-averaged or depth resolving irrotational models to full NS and vice-versa. The two-way coupling is advantageous since it allows for a significantly smaller computational region for the viscous solver. However, it necessitates an iterative process or an implicit approach between the two models on a shared interface, which might increase the computational costs. The advantage of one-way coupling is that no such iterations are needed, but it needs a longer viscous domain to avoid the reflection from outer boundaries. This method is suitable, wherein, one needs to analyse the kinematics of the breaking waves in deep water or depth induced breaking in the shallow water region. Grilli and co-workers (1999, 2003, 2004) coupled the 2D HOBEM-FNPT with NS model based on SL-VOF.

Extension of the FEM code with the NS model is being carried out by Clauss and co-workers (2004, 2005). For NS model they have tested with the commercial software such as FLUENT, CFX and COMET. They tested their coupling approach by studying the deep water wave breaking (breaking of freak waves) and comparing with experimental measurements. Yan and Ma (2009) coupled the QALE-FEM with the commercial software STAR-CD to study the wind effects on breaking waves. Hildebrandt et al. (2013) coupled the FEM with the commercial software ANSYS to model the wave impacts with tripod structure. Narayanaswamy *et al.* (2010) and Kassiotis *et al.* (2011) used one way coupling of the Boussinesq model with the SPH method for solitary wave simulations without feedback from the SPH to the Boussinesq model, a fixed overlapping zone is being considered to transfer the information. Recently, this was improved by Agarwal *et al.*, (2022b) by coupling Boussinesq with MLPG (Meshless Local Petrov Galerkin) method.

However, if one needs to analyse the wave structure interactions in the presence of floating bodies or fixed structure, then strong coupling of the two models are required, wherein the radiated waves will propagate from NS model to depth-averaged or depth-resolved irrotational models. In strong coupling, computational domain will be divided into two parts, in one part the generation and propagation of waves is being considered and in the other part structure/breaking region will be present. The modelling of first part of the domain will be carried out using depth-averaged or depth resolved irrotational models and then the boundary conditions (velocity and pressures) are fed into the NS model at the same time steps, to study the remaining part of the domain. Then the velocity from NS model is again feed back to the depth averaged or depth resolved irrotational model domain for the next time steps. Thus, in general, the strong coupling needs to couple the models both in space and time domains. For the coupling in space domain, the following four methods have been found to be employed, as pointed out by Sriram *et al.* (2014): (a) fixed boundary interface, (b) moving boundary interface, (c) fixed overlapping zone and (d) moving overlapping zone.

One of the pioneering works in this was carried out by Grilli and co-workers (2005, 2010) wherein, they extended the model from weak coupling to strong coupling for studying the 3D breaking waves by coupling 3D HOBEM-VOF. Later, Grilli and co-workers (2007, 2008, 2009) coupled the NWT and NS based on Large Eddy Simulation (LES) to study the forced sediment transport simulations. Later studies from Greco (2001), Colicchio *et al.* (2006), Greco *et al.* (2007), Sitanggang (2008) highlighted in the feasibility and the breakthrough kind of work with detailed study was carried out Sriram *et al.*, (2014) exploring the full capability. However, until now mostly these strong coupling are realised only in the 2D problems, and, to the best of the authors' knowledge, the strong coupling in 3D is yet to be attempted. Typical examples of simulations performed based on one-way coupling using the codes developed by the authors and their co-workers: IITM-RANS3D, HOS-NWT-foamstar (using depth resolved potential and viscous models) and FEBOUSS-MLPG (using depth averaged potential and viscous models) are reported in Figures 7-9.

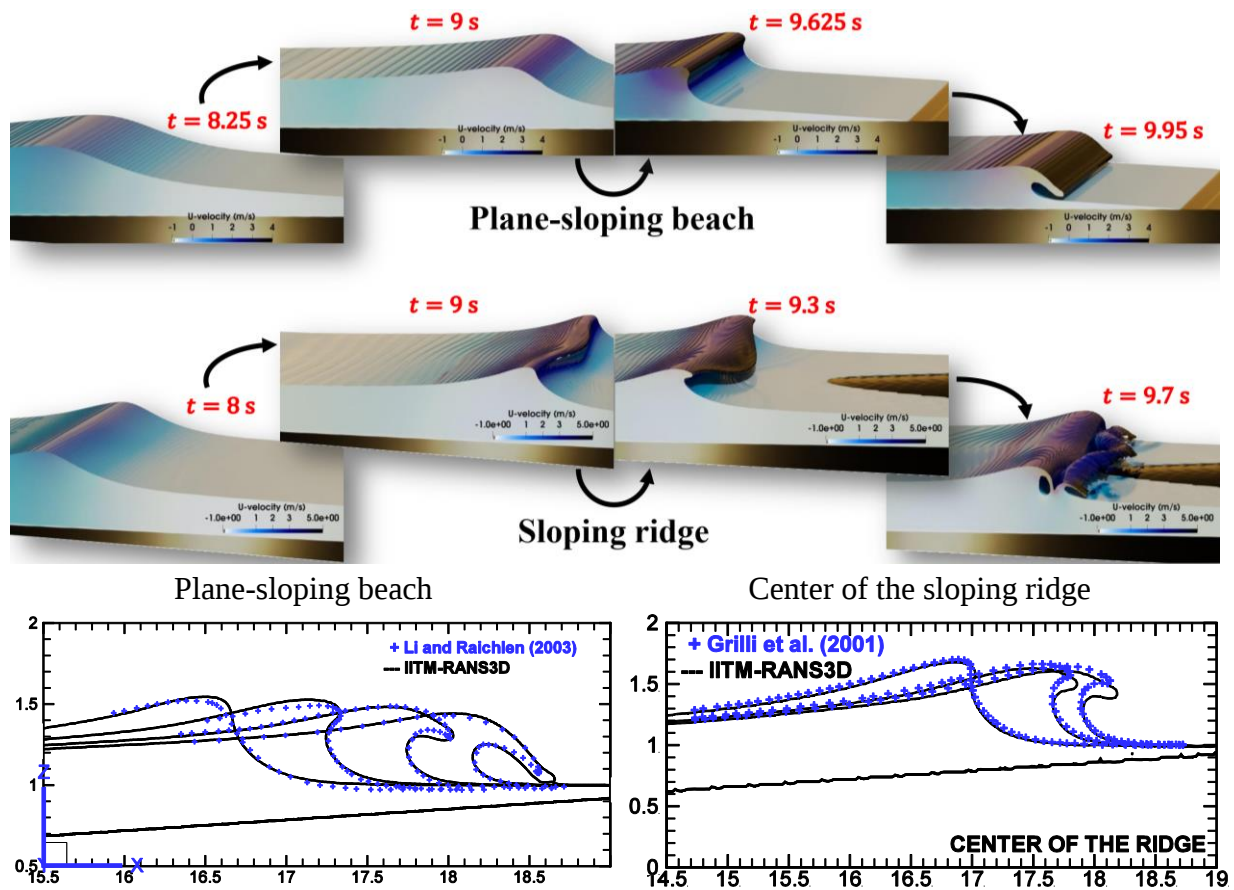


Figure 7. Simulation of solitary wave-breaking over a 1:15 plane-sloping beach and sloping ridge using weakly coupled IITM-FNPT2D and IITM-RANS3D: (top) topology of the overturning wave visualized using iso-volumes of VOF and coloured using streamwise velocity, (bottom) validation of the breaking topology against literature.

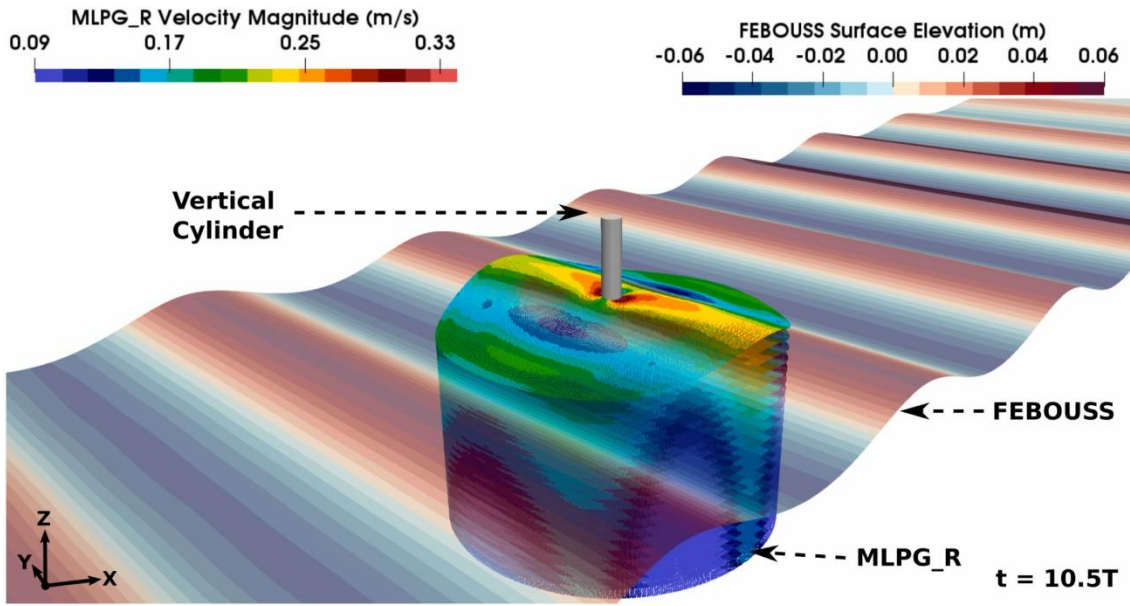


Figure 8. Simulation of directional regular waves (aligned at 30° to the x -axis) interacting with a fixed cylinder using weakly coupled FEBUOUS and MLPG_R using 3D cylindrical coupling interfaces (Agarwal et al., 2022b).

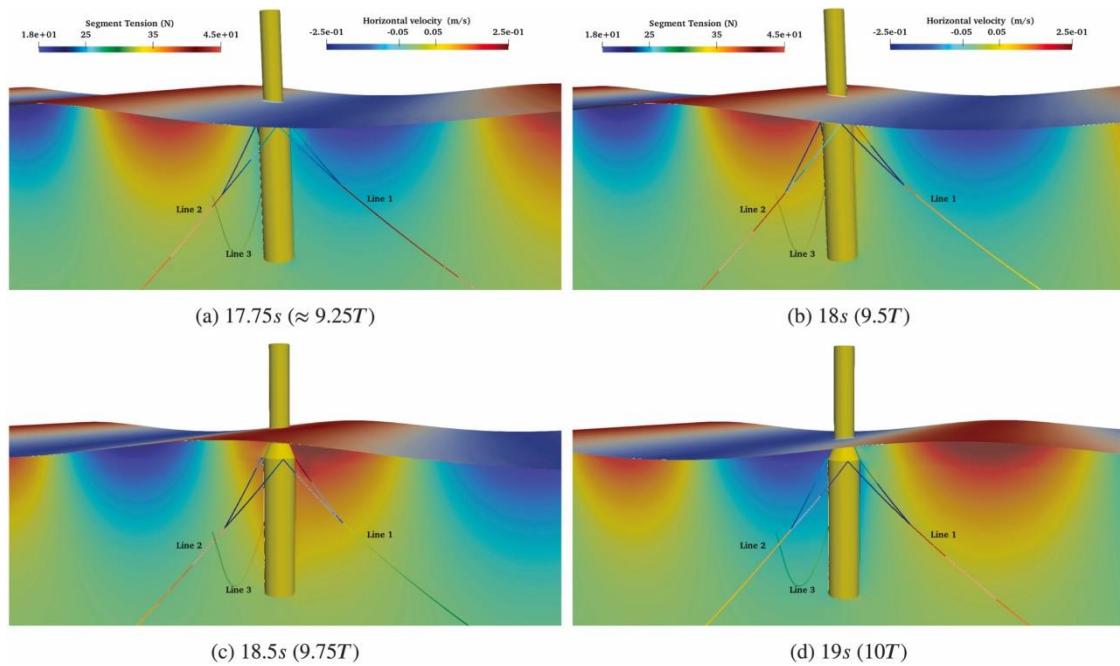


Figure 9. Simulation of regular waves interacting with a moored floating spar using a coupled model employing HOS-NWT, foamStar and MoorDyn (Aliyar et al., 2022).

One more popular hybrid model is qaleFOAM, which has been developed based on the experience from QALE-FEM. This model adopts the domain decomposition approach, which combines a two-phase Navier–Stokes (NS) model with a model based on the fully nonlinear potential theory (FNPT). In a region around the structures and/or the breaking waves (NS domain), the open-source NS solver OpenFOAM/interDyMFoam is applied. In the rest of the computational domain (FNPT domain), the FNPT-based quasi arbitrary Lagrangian–Eulerian finite element method (QALE-FEM) is adopted. The qaleFOAM was originally developed

for modelling the turbulent flow near offshore structures subjected to extreme waves (Li *et al.*, 2018). It has now extended and applied to model wide range of wave-structure interaction problems, such as the wave resistance (e.g. Gong *et al.*, 2020), violent wave impact on sea walls (Li *et al.*, 2023), survivability and performance of floating wind turbines (Yu *et al.*, 2023; Yuan *et al.*, 2023) and wave energy converters (Yan *et al.*, 2020), wave-driven drift of floating objects (Xiao *et al.*, 2024). Recently blind tests and numerical comparative studies have confirmed it's superiority over single-model methods including the potential theory and the NS solvers. The details will be discussed below. However, one of the theoretical issues in these DD coupling is that the researchers coupled irrotational flow model with the rotational flow models. Particularly for strong coupling, there is a mathematical discontinuity in the velocity field and they overcome this with numerical approaches (Sriram and Ma, 2021). This needs to be overcome in the future modelling efforts, see Yang and Liu (2022) for the development of the multi-layer model based on rotational flow.

The fundamental concept for the functional decomposition (FD) is to use the Helmholtz decomposition to separate the velocity field into the rotational and irrotational parts to investigate the free surface flow (Dommermuth, 1993). There are two categories under this decomposition, based on whether the structure is considered in the potential solver or not. In the first category, the WSI problem is split into a potential component and a viscous part. The complete problem is initially solved by a potential solver, and then rectified by adding the viscous correction (Kim *et al.*, 2005; Edmund *et al.*, 2013; Rosemurgy *et al.*, 2016; Robaux and Benoit, 2021). One drawback of this strategy is that the potential solver must first solve the entire problem before applying viscosity correction. As a result, challenges such as higher-order waves, stability issues in the steep waves and breaking induced by presence of structure with complex interactions are still constraints in this classification. Recently, Robaux (2020) published a thorough description of nonlinear wave's interactions with a horizontal cylinder with a rectangular cross section employing potential solver, CFD solver, and HPC-OpenFOAM coupled DD and FD based solvers. In comparison to the full CFD simulation, both coupling approaches, in particular the FD-based approach, need a minimal amount of computational time while providing an accurate representation of the loads and associated hydrodynamic coefficients. In the second category, the total unknown is decomposed into the incident part and the complementary part. Only the incident flow is modelled in the incident part (wave only), leaving all the interaction with structure calculated by the viscous solver as the complementary part. The common name among researchers for this classification is SWENSE (Spectral Wave Explicit Navier Stokes Equations), proposed by (Ferrant *et al.*, 2002) and actively developed by (Gentaz, 2004; Li *et al.*, 2018; Kim, 2021). The NS equation modified into the SWENSE is solved to yield the complementary fields. The advantage of this method is that the wave models directly provide incident wave solutions, minimising the problem's complexity and cost. For a detailed derivation of single-phase and two-phase SWENSE, refer to Luquet *et al.* (2007) and Li *et al.* (2018) respectively. The applications in single-phase SWENSE over the years can be read in (Luquet *et al.*, 2007; Monroy *et al.*, 2010). Recently, the two-phase SWENSE method (Li *et al.*, 2021) has been implemented on top of *foamStar* and is called as *foamStarSWENSE*, and the only difference is that in this solver, the NS equations in *foamStar* are replaced by SWENSE. Recent

developments of *foamStarSWENSE* such as efficient regular and irregular wave generation in the solver and higher-order forces estimation on a vertical cylinder, buoy and floating spar can be referred to in Choi (2019), Kim (2021), Li *et al.* (2018) as well as in Aliyar *et al.* (2022).

6 Benchmarking the Numerical Models (comparative studies)

In the past researchers developed numerical models and validated with their own experimental simulations, the data sharing and comparison between different numerical models, its accuracy and performance in terms of computational efficiency are not attempted. In the field of ocean engineering, when the concept of numerical wave tank was developed inline with the numerical wind tunnels that are quite popular in those times, Clément (1999) and Tanizawa and Clément (2000) carried out such exercise for fully nonlinear potential flow theory. Recently, major initiatives were undertaken by Ransley *et al.* (2019, 2020), Sriram *et al.* (2021), Agarwal *et al.* (2021a) and Saincher *et al.* (2023a). These studies highlighted some of the commonly adopted guidelines by the researchers pertaining to WSI simulations:

- The fidelity of regular/focusing wave generation deteriorates away from the wavemaker irrespective of the nature of the numerical model and no single wave-generation method may be regarded as superior over others. Far from the wavemaker, the models generally deviate by 5 – 10% in terms of primary energy content which is acceptable. However, the deviation across models may be as high as 50% in terms of the sub- and super-harmonic wave components.
- The performance of a solver should not be judged based on phase-shifts but rather on the peak values of the surface-elevation / hydrodynamic pressures / loads captured by the model.
- The inclusion of turbulence modeling does not necessarily improve the accuracy of a simulation. This rather depends on the problem at hand. Further, for the same problem, different turbulence models may lead to the same/similar results (cf. Saincher *et al.*, 2023a) which indicate that the expertise of the user should also be factored-in whilst using turbulence models, especially RANS-based models which are strongly empirical.
- Hybrid modeling invariably improves the computational efficiency of the solver and should be adopted for large-scale WSI problems (Agarwal *et al.*, 2021a ; Saincher *et al.*, 2023a).
- The state of the art in modelling large domain problems for transient waves appeared to be based on hybrid numerical modelling using weakly coupled algorithms (or one-way coupling); this strategy was adopted by most of the participants.
- In simulating the same WSI problem at different scales, no general correlation could be obtained between computational effort and the scale of the problem. For instance, amongst the ten models compared for breaking waves interacting with a recurved seawall, the hybrid codes qaleFOAM and IITM-RANS3D were simultaneously the fastest and slowest at two different scales of the problem (Saincher *et al.*, 2023a).

In these studies it was also noted that the experimental error/uncertainty should be taken into consideration during validation. The inclusion of the experimental uncertainty would make

the above guidelines less stringent. However, a conservative approach is beneficial in order to maintain a reduced error margin when adopting the said guidelines in practice.

7 The Future

The machine learning (ML) techniques is becoming popular in assisting the fluid simulation, e.g. to reconstruct the fluid field from data (Raissi *et al.*, 2020), to predict the turbulence related parameters (Ling *et al.*, 2016; Zhang *et al.*, 2015; Kutz, 2017), and to approximate time-independent flow field governed by NS models, such as the projection-based Pressure Poisson Equation (PPE, e.g. Yang *et al.*, 2016; Xiao *et al.*, 2018; Tompson *et al.*, 2017; Dong *et al.*, 2019; Ladicky *et al.*, 2015; Wessels *et al.*, 2020, Li *et al.*, 2022). Recently, both the convolution neural network (CNN, Zhang *et al.*, 2023a) and graphic neural network (GNN, Zhang *et al.*, 2023b, 2023c) have been coupled with the incompressible smoothed particle hydrodynamics (ISPH) model to accelerate the numerical simulations. In these work, high-fidelity time-domain numerical results are produced using stand-alone ISPH simulation on wave propagation and impact on fixed structure. The CNN or GNN are used to train a machine learning algorithm to predict the pressure in the future step based on the numerical results at the current time step including the velocity, velocity divergence and pressure. After the algorithm is trained, it will be used to replace the PPE solver in the classic ISPH. Both the CNN-supported and GNN-supported ISPH models have been applied to modelling wave propagation, impact on seawall and interaction with other structures. Figure 10 and Figure 11 illustrate some numerical results from the GNN-supported ISPH, which does not only show the capacity of the ML-supported ISPH but also demonstrate its promising accuracy. Further evidence on numerical accuracy and CPU speeding-up can be demonstrated in Figure 12 for the cases with solitary wave propagation. In this figure, the error is defined by the L2-norm of the time history of the wave crest; ISPH and ISPH-CQ adopt the linear and 2nd-order PPE solvers, respectively. As shown in Figure 12(a), both the convergence and accuracy of the ISPH-GNN are bounded by the corresponding values of the ISPH and ISPH-CQ, implying a promising computational accuracy. Figure 12(b) illustrates excellent CPU time speeding-up ratios against directly solving the PPE using the 2nd order solver. For the solitary wave propagation using 80k particles, the GNN can speed up the simulation by 80 times.

The existing work related to AI and ML may be quantified as hybrid model combining a CFD solver with the ML algorithms, e.g. Zhang *et al.* (2023b) combining ISPH with graph neural network for simulating free surface flows. Data are needed to train the ML algorithms. Recently, researchers started solving the fluid mechanics and fluid-structure interaction problems using the AI library for discretising the required partial differential equation (AI4PDE, see, e.g. Chen *et al.*, 2024). This work does not need to train the neural network but directly modifying the filters of the neural network. Limited benchmarking rest has demonstrated its promising computational accuracy and efficiency. The applications of the AI/ML to existing hybrid model, such as the qaleFOAM, have yet found to the best of our knowledge. Based on our preliminary work on CNN/GNN supported ISPH, its feasibility to the hybrid modelling is confirmed.

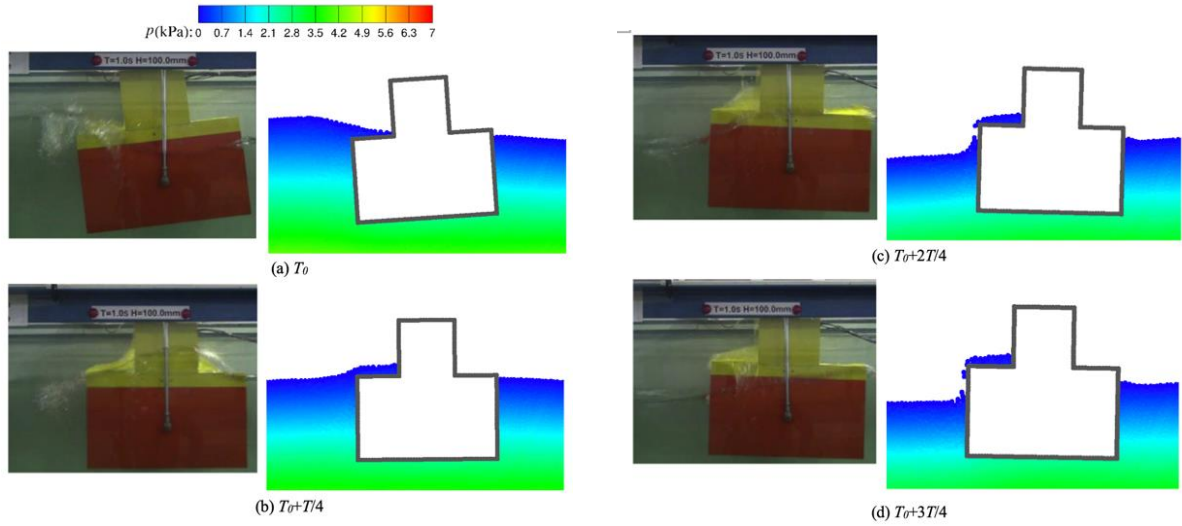


Figure 10. Comparisons of the floater movement progress during green water impact between laboratory photos (Zheng et al., 2016) (left) and ISPH_GNN simulations (right) at different instants (duplicated from Zhang et al., 2023c).

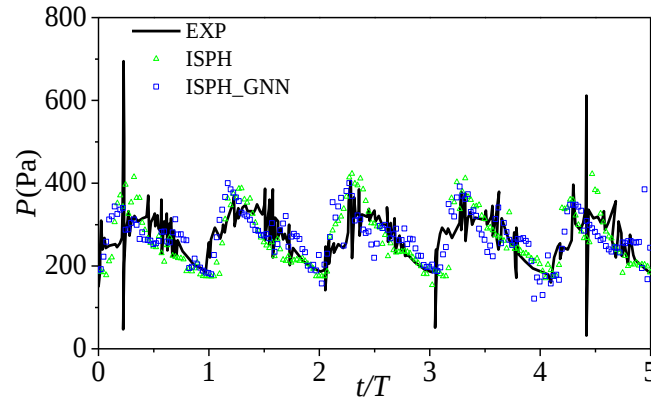


Figure 11. Time histories of the impact pressure on deck at P1 (duplicated from Zhang et al., 2023c).

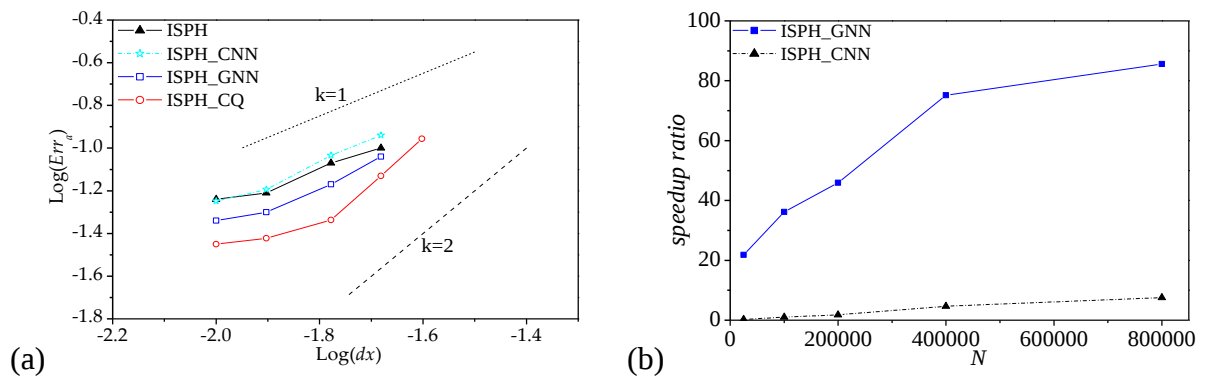


Figure 12. Averaged errors of numerical results corresponding to different particle spacing in the solitary wave propagation (a) and the CPU speeding ratio (b) against solving PPE directly (solitary wave height = 0.28*water depth).

The challenges in the hybrid modelling can be fully or partially solved by the AL/ML technologies. These include: (1) replacing the NS solver by the ML-supported version; (2)

intelligently decomposing the computational domain in an adaptive way, i.e. to minimising the NS domain in the run-time depends on the development of the viscosity/turbulence effect and breaking wave occurrence; (3) intelligently choosing the appropriate models, such as RANS or LES; (4) in the function-decomposition approach, using the ML algorithms for solving the compromised equations instead of solving them directly; (5) dynamic load balancing in the cases with parallel computing.

Another important aspect in the blue economy theme is the renewable energy. The development of offshore wind farms based on Floating Offshore Wind Turbine (FOWT) arrays is one of the popular, potential and realizable area. In order to reduce the CAPEX and installation costs, shared mooring systems have been proposed for FOWT arrays where anchors and a part of the mooring line are shared between turbines. This introduces challenges that manifest differently in shallow and deep water. The deep-water mooring system is susceptible to motions of the FOWT platform being amplified leading to large displacements in the mooring line and peak anchor loads. Chain catenary moorings in shallow water experience snap loads due to their susceptibility to violent wave-current-structure interactions during extreme events and individual loads superimposing nonlinearly with the structural response. In order to develop a comprehensive understanding of the mechanisms leading to snap loads and peak anchor forces in shared mooring systems of a FOWT farm, high fidelity multi-scale solver is required. To achieve this, the existing FNPT (Fully Nonlinear Potential Theory), RANS (Reynolds Averaged Navier-Stokes) and Large Eddy Simulation (LES) codes can be coupled via a zonal approach to yield a high-fidelity multi-scale solver for wave-current-structure interaction. A FEM-based structural solver will be integrated to accurately predict the coupled fluid-structure interaction of several mooring lines and to facilitate the modelling of elastic materials. A critical aspect of the model development would be scaling-up the code for prototype-scale FOWT arrays whilst retaining computational efficiency and accuracy. This could be achieved using AI and ML-based prediction of turbulence-generation near the floating platforms, as this is expected to be the most computationally intensive aspect of the modelling (traditionally handled using hybrid RANS-LES). Thus, a continuous research efforts in the field of computational hydrodynamics is required. This is in fact supported by the Newton Fellowship (recently awarded to the second author from the authors research group in 2023) wherein the existing understanding in hybrid modelling as well as AI/ML-based prediction of turbulence shall be carried forward.

Acknowledgement

The first author would like to acknowledge the Newton International Fellowship for supporting the development of the numerical model in his group. Through the alumni funds of Newton Fellowship, the research group students (Dr. Rijas, Dr. Shagun Agarwal, Dr. Ravindar, Dr. Manoj Kumar, Mrs. Divya R, Dr. Shaswat Saincher) visited and benefited from the collaboration with UK university.

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