A Metric for the Asymmetry in Matched-Pair Data for Buyer-Supplier Dyads

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Abstract

Although various difference-based methods are utilized to analyze asymmetry in buyer-supplier matched-pair data within the literature, these approaches are ad hoc and do not always address differences across multiple dimensions. Furthermore, they do not provide a significance test. This paper extends the concept of the paired t-test for dyad-level differences by developing a Mahalanobis distance-based metric in multiple dimensions, along with a significance test. The metric and the significance test can be used in empirical research to identify dyads in a dataset that are significantly asymmetric at any selected confidence level. In practice, the method can identify those suppliers for a buyer that have significantly mismatched expectations relative to other suppliers. The paper utilizes simulated datasets to compare the proposed metric with other distance-based metrics that lack a significance test. Finally, the paper applies a retail dataset to demonstrate (1) the utility of the metric in identifying significantly asymmetric dyads and (2) the use of the same distance concept to consolidate multiple items in any buyer or supplier construct into a single score for the construct, rather than using factor scores. The latter approach is lossless, in contrast to factor analysis. Using distance-based metrics with this retail dataset in a structural equation model suggests that asymmetry can negatively affect relationship-specific operational performance for buyers and suppliers. This study contributes a robust methodological framework, offering a structured basis for future research in the measurement of dyadic asymmetry.

Keywords: Buyer-supplier relationships; dyadic asymmetry; matched-pair data analysis; Mahalanobis distance; relationship performance.

1 Introduction

Empirical analysis of dyadic data is integral to the buyer-supplier literature [Ellram & Murfield, 2019]. Analyzing these relationships differs from merely examining suppliers, as seen in supplier selection within the analytical literature (e.g., [Bhutta, 2003]). Of particular interest are the performance of the relationship and its antecedents [O'Toole & Donaldson, 2002, Yang et al., 2009, Autry & Golicic, 2010, Liu et al., 2012]. An antecedent proposed is the asymmetry in the relationship (for example, [Klein et al., 2007, Liu et al., 2012, Villena & Craighead, 2017, Vanpoucke et al., 2022]). However, methodological guidance on the use of dyadic data for asymmetry has received little attention in the supply chain literature. Researchers have employed diverse and ad hoc methods without being able to test whether the asymmetry of any dyad in a random sample is statistically significant or justify its suitability for regression or other statistical models.

This paper aims to narrow this gap by proposing a method to measure asymmetry or any other differences between the matched buyer and supplier data in dyadic datasets, along with a statistical test for significance. It also demonstrates the application of the method to a dyadic retail data set, including testing the link between asymmetry and relationship performance using structural equation modeling (SEM).

The paired t-test can measure asymmetry when there is only one dimension of interest for the dyads, such as buyer and supplier log-sales, provided the paired variables are normally distributed. In these cases, we calculate the difference between the values of the buyer and the supplier to perform the paired t-test, identifying which dyads are significantly different (for a chosen significance level) and therefore asymmetric. In addition, we can use the difference in a regression or an SEM model to examine the relationship between asymmetry (measured as the difference) and relationship performance.

This paper examines the difference, measured by the Mahalanobis distance, across multiple paired dimensions between the buyer and the supplier, along with a significance test. The normality assumption underlying the t-test thus becomes a multinormal assumption. Just as the paired t-test is robust against any violation of the normality assumption, the proposed statistic is also resilient against breaches of the multinormal assumption, as demonstrated with a real-life dataset. I also illustrate how we can utilize the Mahalanobis distance to provide scores for multi-item constructs instead of factor scores to test the link between asymmetry and relationship performance.

This paper aims to methodologically enrich the buyer-supplier stream within the empirical literature on supply chain management in two ways. *First*, it provides a multidimensional difference-based statistic with a significance test to measure asymmetry in dyads, such as perceptual gaps, asset-specific differences, or mismatched expectations from the relationship. Current methods for identifying such gaps are ad hoc and lack a statistical significance test, whereas this paper generalizes the paired t-test to multiple dimensions, including dyadic data for buyer-supplier pairs.

Second, the paper also presents a more speculative contribution by proposing using the same distance-based metric as a single lossless measure for multi-item constructs instead of factor scores from factor analysis for regression or SEM applications. Given matched-pair buyer and supplier data, (1) the asymmetry between the buyer and supplier items across multiple dimensions (possibly from various constructs) could be measured and checked for significance, and (2) the buyer and supplier constructs could be represented by the distance-based metric that captures data from the constructs' items. Thus, for a dyadic dataset, the buyer and supplier constructs, and the asymmetry (or asymmetries) between the buyer and supplier in dyads could all be represented as distances for use in regression or SEM.

In the remainder of this paper, Section 2 discusses using dyadic datasets in the buyer-supplier relationship literature, including various ways to measure asymmetry and their limitations, to outline the research gap of interest. Section 3 proposes a modified Mahalanobis distance between two random vectors (the buyer's and the supplier's responses to multiple paired questions) and a test for the statistical significance of the asymmetry in any dyad for a chosen confidence level. Section 4 uses simulated datasets to identify asymmetric dyads employing the proposed test. We compare the Mahalanobis distance with the Manhattan and Euclidean distances to demonstrate the importance of having a significance test, as proposed. Section 5 illustrates (1) the proposed metric and (2) its variant for buyer-only and supplier-only variables in constructs with a real-life dyadic retail dataset to identify significantly asymmetric dyads and explain the performance of the relationship using SEM. In Section 6 we conclude with implications for further research.

2 Limitations in Measuring Asymmetry in the Empirical Buyer-Supplier Literature

When collecting and analyzing dyadic data to test theories about buyer-supplier relationships, researchers must remember that considerations differ from those for analyzing a single party. As an analogy, consider a survey of married couples with matched-pair data obtained from husbands and wives about their relationship. We can then examine their relationship by exploring the differences between their responses to matched-pair questions.

Empirical research using dyadic data analysis should account for the following key issues in matched-pair buyer–supplier datasets [Kenny et al., 2006]:

1. *Unit of analysis:* In matched-pair data, the unit of analysis is the *dyad*, not the individual buyer or supplier.

2. *Types of variables:* In a dyad, certain variables in the data set of a researcher in dyadic data pertain only to the buyer, while others relate solely to the supplier. In addition, some variables are paired to reflect the comparable characteristics of buyers and suppliers or their responses to the same questions about their relationship.

3. *Non-independence of variables:* Paired variables between the buyer and supplier are generally not independent. This non-independence can result from various sources, including compositional effects (e.g., how the dyad was formed), partner effects (the behavior of one party influences the outcome of the other), mutual influence (each party's outcomes affect the other), or common fate (both share the same industry or supply chain environment) [Kenny et al., 2006, pp. 4–5, 25–52]. Ignoring this non-independence or analyzing only one side of the dyad can lead to incorrect inferences. Hence, it is essential to explicitly incorporate data from both sides of the dyad, avoiding oversimplified approaches like averaging, and to respect the non-independence of the data.

The empirical buyer–supplier literature does not consistently follow these principles. Many researchers have focused solely on either buyers or suppliers while making broader dyadic claims. For example, Terpend et al. [2008] found that only 6 out of 151 empirical buyer–supplier studies published between 1986 and 2005 collected responses from both buyers and suppliers. A notable example is Kumar et al. [1995], who explored interdependence asymmetry, total interdependence, interfirm conflict, trust, and commitment, but surveyed only car dealers (buyers). These dealers shared a relatively small number of auto companies as suppliers, implying their responses were not independent. Similarly, Saeed et al. [2005] drew conclusions about dyads based solely on surveys from vice presidents of buying firms.

Even when both buyer and supplier responses are collected, researchers have taken different approaches. One group treats buyers and suppliers as two independent populations for comparison (see Table 1, first row). Others apply various ad hoc techniques to analyze matched pairs, measuring asymmetry one dyad at a time, either on a single dimension or using aggregated measures across multiple dimensions (Table 1, second row).

Table 1: Approaches to asymmetry in the literature analyzing dyadic buyer-supplier data

Buyers and suppliers as separate groups	*Paired (dyadic) data in one dimension*	*Paired (dyadic) data in multiple dimensions*
- Chow tests: Heide & Miner [1992], Ambrose et al. [2010]; - PLS models: Cheung et al. [2010]; - SEM models: Johnston & Kristal [2008]; - Correlations: Campbell [1997], Spekman et al. [1997], Krause et al. [2007]; - T-tests: Ellram & Hendrick [1995]; - Separate SEM models: Whipple et al. [2015]	- Paired t-tests [Forker and Stannack, 2000, Barnes et al., 2007, Liu et al., 2009, Oosterhuis et al., 2013] - Modified differences (absolute, splined): [Nyaga et al., 2013, Gulati & Sytch, 2007, Brinkhoff et al., 2015, Villena & Craighead, 2017, Montes-Sancho et al., 2022]	- Degree-symmetry score [Straub et al., 2004, Klein et al., 2007, Liu et al., 2012]; - **Mahalanobis distance (this paper)**

Buyers and suppliers as separate groups. Comparing buyers and suppliers does not require paired data, nor is asymmetry measured at the dyad level. Studies in this area employ mainly separate regression models [Heide & Miner, 1992, Ambrose et al., 2010], PLS models [Cheung et al., 2010], or SEM models [Johnston & Kristal, 2008] for buyers and suppliers, and then compare the corresponding coefficients. Another approach has been to correlate the responses of the buyers and suppliers [Campbell, 1997, Spekman et al., 1997, Krause et al., 2007]. Alternatively, paired t tests can be used to compare specific attributes of the buyer and the supplier [Ellram & Hendrick, 1995].

Measuring asymmetry using a single paired variable. With a single matched variable per dyad, researchers often calculate the difference and apply a paired t-test. Even when multiple paired variables are available, these are sometimes analyzed sequentially to identify which relationships exhibit significant differences at the dyad level [Forker and Stannack, 2000, Barnes et al., 2007, Liu et al., 2009, Oosterhuis et al., 2013]. Some researchers modify the difference, for example, taking the absolute value or using a 'splined' form (setting negative differences to zero) to include asymmetry as a variable in regression or other models [Gulati & Sytch, 2007, Villena & Craighead, 2017]. In these cases, as many asymmetry variables are created as there are matched-pair variables of interest [Nyaga et al., 2013, Montes-Sancho et al., 2022].

Measuring and testing asymmetry using multiple paired variables. Some studies reduce several paired variables to one by averaging buyer and supplier values and taking their difference as if the construct were one-dimensional. Others compute a composite of the absolute differences, known as a "degree-symmetry" score [Straub et al., 2004, Klein et al., 2007, Liu et al., 2012]. Liu et al. [2012] explain the calculation as follows:

1. Sum all items for a construct and standardize the buyer and supplier values between 0 and 1, denoted as C_1 (buyer) and C_2 (supplier).
2. Compute the average $C_{\text{Deg}} = (C_1 + C_2)/2$ to represent dyad magnitude.
3. Calculate symmetry as $C_{\text{Sym}} = \min(C_1, C_2)/\max(C_1, C_2)$.
4. Average the two: $CDS = (C_{\text{Deg}} + C_{\text{Sym}})/2$.

However, neither Straub et al. [2004] nor Klein et al. [2007] provide statistical properties of this score for inferential use, nor do they justify its use in PLS or SEM. The *CDS* score has several limitations:

- It is not a pure indicator of asymmetry; it conflates average magnitude with relative symmetry.
- Its use in regression is not well-defined.
- No statistical significance test is associated with it, so thresholds for 'high' or 'low' asymmetry are subjective.

Alternative approaches. Not all researchers treat asymmetry as a difference between the buyer and the supplier in a dyad. Even when using dyadic datasets, one approach is to measure asymmetry separately for the buyer and the supplier [Lumineau et al., 2022, Vanpoucke et al., 2022].

Another challenge is the use of difference-based metrics in regression and similar statistical models [Edwards, 2001, 2002]. Edwards & Parry [2018] proposed polynomial regression with degree-2 equations (as an approximation of whatever function of the difference is intended to capture asymmetry) along with *response surface methodology* to interpret the results. The difficulty here lies in interpreting these models clearly. However, some scholars continue to use both difference-based and polynomial regression approaches, even if not explicitly for asymmetry analysis (e.g., Vanpoucke et al., 2022). In general, the use of asymmetry measures in regression and related statistical models remains an area that needs further investigation.

Research gap. The 'gap' in the literature that this article seeks to address is a difference-based asymmetry measure using paired data in multiple dimensions and a significance test. In addition, this measure should be applied to regression or other statistical models. There may be asymmetry measures that do not depend on the difference between the buyer's and the supplier's responses in a dyad; similarly, other quantities of interest in addition to the asymmetry may depend on this difference. The focus here is on the differences in matched pairs in multiple dimensions. *Therefore, this paper aims to develop:*

1. *A statistically justified way to collapse the difference in multiple dimensions between paired constructs for buyer-supplier dyads to a scalar quantity (to be used, e.g., as a measure of asymmetry) with a test of significance to determine if this scalar is significantly different from zero and*

2. *A similar scalar (score) for reducing buyer-only and supplier-only variables (or items for constructs) to test hypothesized links between constructs or variables in statistical models.*

This approach focuses specifically on differences in matched pairs in multiple dimensions to enable rigorous and testable modeling of asymmetry in dyadic relationships in regression and SEM models.

3 An Innovative Metric for Asymmetry in Dyads

This paper considers any selected subset of paired items, potentially aligned with the same factor construct, as a vector in the correlated dimensions that the individual items represent, with asymmetry serving as a scalar function of the vector difference between the two vectors representing the paired buyer and supplier items. Consequently, *we measure the asymmetry between the buyer and the supplier in any dyad as a distance function of the vector difference between the buyer and supplier vectors representing their responses to any subset of items.* By examining different subsets of paired variables, such as items of buyer and seller constructs, multiple difference-based measures ("asymmetry") can be generated for each dyad.

3.1 Theoretical Preliminaries

We assume a dataset comprising data on buyer–supplier dyads, where each observation includes responses from both the buyer and the supplier to a matched set of paired questions, each pair consisting of one question directed at the buyer and a corresponding one to the supplier, or to otherwise comparable items (e.g., the size of either party). The buyers' and suppliers' responses are organized in ascending order and are assumed to be of interval type; that is, numerically higher responses indicate judgments of more desirable outcomes for the dyad. For example, on a 7-point scale, a difference between scores of 7 and 5 is equivalent in magnitude to that between 3 and 1. Although we do not assume that these discrete

Likert-scale responses are normally distributed, we can still treat them as if they were for practical analytical purposes [Johnson and Creech, 1983].

Alternatively, there could be continuous variables from secondary sources, such as the logarithm of the annual revenues for the buyer and the supplier, respectively. Variables might be classified as "between dyads" (e.g. duration of the relationship, same for both parties), "within-dyad" (e.g., reward allocation in case of supply chain savings), or "mixed" (again, e.g., annual revenues) [Kenny et al., 2006].

As noted above, for any dyad, we represent the buyer's and supplier's responses to a selected subset of questions as vectors. We are interested in the difference between these two (random) vectors to indicate asymmetry in the dyad. The dimensions of the two vectors are correlated when the buyer's (and also the seller's) responses are correlated. The Mahalanobis distance serves as an appropriate scalar function (in providing a score) to measure the length of a vector in correlated dimensions. Researchers are familiar with its use as a distance function between a fixed point and a distribution to identify outliers in regression. Here, we consider a modification that analyzes the distance between two random vectors drawn from a multidimensional distribution. The Mahalanobis distance function, as proposed here, collapses the n-dimensional difference vector (or any other vector) into a scalar, providing a metric for the asymmetry between the buyer and the supplier.

There are many other distance functions: Manhattan distance (L1), which is the sum of the absolute values of the item values; Euclidean (L2) distance, calculated as the square root of the sum of squares of the individual values, which serves as another measure that provides the magnitude of a vector; and Minkowski distance, which generalizes L1 and L2 (Table 2). However, these distance functions do not account for (1) the different variances in the items (although we can normalize the items using their means and standard deviations), (2) the correlations between the different items or dimensions, or (3) multiple scales [Berry et al., 2010]. Mahalanobis distance overcomes these limitations by generalizing the Euclidean distance from orthogonal axes to correlated ones [De Maesschalck et al., 2000] and by scaling the value of each dimension by the standard deviation of the corresponding item.

Let there be m dyads in the sample. Consider the k'th dyad, $1 \leq k \leq m$. Let x_k be the n-dimensional random vector representing the responses for the buyer and y_k the n-dimensional random vector representing the supplier's responses with asymmetry in the k'th dyad being a function of x_k and y_k (Table 3).

The buyer responses in the vector $\mathbf{x_k}$ can be correlated, as can the supplier responses $\mathbf{y}_k$, especially if the responses are to items that are related, e.g., by being in the same construct. Moreover, the corresponding items, assumed to be paired, within $\mathbf{x}_k$ may be correlated with those in $\mathbf{y}_k$. The joint vector $[\mathbf{x}_k\ \mathbf{y}_k]$ is assumed to be drawn from a multivariate normal distribution with mean $[\mu_\mathbf{x}\ \mu_\mathbf{y}]$ and covariance:

$$\Sigma = \begin{bmatrix} \Sigma_x & cI_n \\ cI_n & \Sigma_y \end{bmatrix}$$

where I_n is the identity matrix of size n. Therefore, the buyer vectors $\mathbf{x}_k$, for $1 \leq k \leq m$, are a random sample from a multivariate normal distribution with mean vector $\mu_\mathbf{x}$ and covariance matrix $\mathbf{\Sigma_x}$. Similarly, the supplier vectors $\mathbf{y}_k$, from $k = 1$ to m, are a random sample from a multivariate normal distribution with mean vector $\mu_\mathbf{y}$ and covariance matrix $\mathbf{\Sigma_y}$.

Table 2: Comparison of Distance Metrics in n-Dimensional Non-Orthogonal Space

Aspect	**Mahalanobis distance (proposed)**	**Euclidean distance (L2)**	**Manhattan distance (L1)**
Definition	Distance between two points while accounting for the correlation between dimensions; incorporates the covariance structure.	The straight-line distance between two vectors in n-dimensional space, assuming the dimensions are orthogonal	The 'city-block' distance between any two points, assuming the dimensions are orthogonal
Calculation		Square root of the sum of squares of the differences in each dimension	Sum of the absolute value of the differences in each dimension
Covariance structure	Taken into account	Ignored	Ignored
Test of significance (difference = 0?)	Yes, presented in this paper	N/A	N/A

Furthermore, paired elements within $\mathbf{x}_k$ and $\mathbf{y}_k$ are correlated across dyads in the population with covariance $\mathbf{c} = [c_1, c_2, \ldots, c_n]$, or item-wise correlation $\boldsymbol{\rho} = [\rho_1, \rho_2, \ldots, \rho_n]$ between paired items of buyers and suppliers n.

3.2 Measuring Asymmetry as a Difference with Mahalanobis Distance

It follows that the difference vector $\mathbf{x}_k - \mathbf{y}_k$ is also drawn from a multivariate normal distribution with mean $(\mu_{\mathbf{x}} - \mu_{\mathbf{y}})$ and covariance $\Sigma_x + \Sigma_y + 2\mathbf{c}I_n$. The centered vector difference between the buyer and supplier response vectors in the n questions is $(\mathbf{x}_k - \mathbf{y}_k)$ when $\mu_{\mathbf{x}} = \mu_{\mathbf{y}}$, or $(\mathbf{x}_k - \mathbf{y}_k - \delta)$ when $\mu_{\mathbf{x}} = \mu_{\mathbf{y}} + \delta$. In addition, the weights $\mathbf{w} = [w_1, w_2, \ldots, w_n]$ can be used to apply item-level weights to these differences. The rest of this paper assumes $w_i = 1$ for $i = 1, \ldots, n$, with the covariance matrix adjusted accordingly.

Asymmetry in the k^{th} dyad can now be measured using Mahalanobis distance:

$$G_k = \sqrt{(\mathbf{x}_k - \mathbf{y}_k - \delta)^\top \Sigma_{x-y}^{-1} (\mathbf{x}_k - \mathbf{y}_k - \delta)}$$

The population covariance matrices Σ_x, Σ_y, and Σ_{x-y} can be estimated from a random sample of dyads. Let the sample means of buyer and supplier responses across all dyads be $\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$, respectively. Then S_x and S_y are consistent estimators of Σ_x and Σ_y, computed as:

$$S_x = \frac{1}{m-1} \sum_{k=1}^{m} (\mathbf{x}_k - \bar{\mathbf{x}})(\mathbf{x}_k - \bar{\mathbf{x}})^\top$$

Table 3: A dyadic dataset showing data structure with n paired questions (among other variables) for $1..m$ dyads, each with the buyer's responses x_k and the supplier's responses y_k, with asymmetry or any other relationship-specific variable as a function $f(x_k, y_k)$ for the k'th dyad.

Dyad	**Buyer responses**	**Supplier responses**	**Asymmetry or relationship-specific variable**
1	$x_{11},\ x_{12},\ \ldots,\ x_{1n}$	$y_{11},\ y_{12},\ \ldots,\ y_{1n}$	$f(x_1, y_1)$
2	$x_{21},\ x_{22},\ \ldots,\ x_{2n}$	$y_{21},\ y_{22},\ \ldots,\ y_{2n}$	$f(x_2, y_2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	$x_{k1},\ x_{k2},\ \ldots,\ x_{kn}$	$y_{k1},\ y_{k2},\ \ldots,\ y_{kn}$	$f(x_k, y_k)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	$x_{m1},\ x_{m2},\ \ldots,\ x_{mn}$	$y_{m1},\ y_{m2},\ \ldots,\ y_{mn}$	$f(x_m, y_m)$

and

$$S_y = \frac{1}{m-1}\sum_{k=1}^{m}(\mathbf{y}_k - \bar{\mathbf{y}})(\mathbf{y}_k - \bar{\mathbf{y}})^\top$$

Similarly, S_{x-y} can be estimated directly from the data or derived from S_x, S_y, and estimates of $\mathbf{c}$. A commonly recommended minimum sample size for stable estimates of S_x, S_y, and S_{x-y} is 30 dyads [Johnson and Wichern, 2013].

The Mahalanobis distance-based measure of asymmetry in the k^{th} dyad is estimated, with S replacing Σ, as:

$$G_k = \sqrt{(\mathbf{x}_k - \mathbf{y}_k - \delta)^\top S_{x-y}^{-1}(\mathbf{x}_k - \mathbf{y}_k - \delta)}$$

We can also use Mahalanobis distance to map the responses of either party in the dyad as distances from the origin if we want to replace all the buyer-only variables corresponding to a buyer construct with a single value; likewise, for supplier-only constructs (Figure 1). Unlike dimensionality-reducing techniques such as principal components or factor analysis, information in all the dimensions is used for the Mahalanobis distance.

Mahalanobis distance can also be used to measure each party's response vector length from the *origin*. Doing so collapses any subset of buyer-only (or supplier-only) items, say those related to a construct, into a single scalar value analogous to a factor score (see Figure 1). All the relevant dimensions are used, so the method is lossless, which contrasts with dimensionality-reducing methods such as principal components or factor analysis that discard some variance.

Let the buyer and supplier be denoted by B and S, respectively. Then, for the k'th dyad, the Mahalanobis distances of buyer's and the supplier's respective response vectors are given by:

$$D_k^B = \sqrt{\mathbf{x}_k^\top \Sigma_x^{-1} \mathbf{x}_k}$$

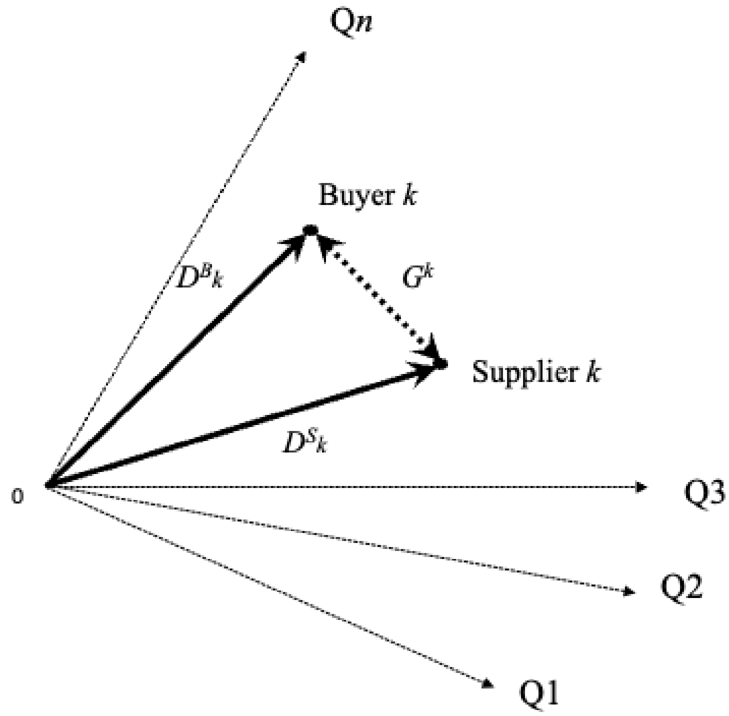

Figure 1: The vectors representing the buyer's and the supplier's responses to the selected group of paired questions $Q_1, Q_2, \ldots, Q_n$ for the k'th dyad with Mahalanobis distance D_k^B (buyer) and D_k^S (supplier) from the fixed-point origin, respectively, with the difference between these two vectors having Mahalanobis distance G_k. The dimensions corresponding to Q_1 to Q_n are correlated and, therefore, not orthogonal.

and

$$D_k^S = \sqrt{\mathbf{y}_k^\top \Sigma_y^{-1} \mathbf{y}_k}$$

These scalar quantities, D_k^B and D_k^S, represent the magnitude of the buyer's and the supplier's responses for the k'th dyad over a set of items, respectively. If the items correspond to a latent construct for the buyer (or supplier), the distances serve as multivariate factor score. When using estimates, S_x and S_y replace the population covariances Σ_x and Σ_y. To make comparisons more consistent, we can use the pooled covariance matrix:

$$S_{\text{pooled}} = \frac{1}{2}(S_x + S_y)$$

This pooled estimator replaces Σ_x and Σ_y when calculating Mahalanobis distances across dyads in the above equations.

3.3 A Statistical Test of Significance for the Proposed Metric

In any sample of dyads, some dyads may exhibit greater asymmetry than others. Therefore, we need a way to determine whether the k'th dyad is *significantly* asymmetric, i.e., whether the buyer's and supplier's responses in that dyad come from the same population. Specifically, we are testing the null hypothesis regarding the equality of the population means for both the buyer's and the supplier's responses in the k^{th} dyad, i.e., the mean difference vector $(\mathbf{x}_k - \mathbf{y}_k)$ equals the population mean difference $\delta = \mu_{\mathbf{x}} - \mu_{\mathbf{y}}$. We can then obtain

Proposition 1. *Let G_k be the Mahalanobis distance-based measure of asymmetry for the k of $1, \ldots, m$ dyads with S_{x-y} as a consistent estimator of the covariance matrix Σ_{x-y}. The test statistic*

$$G_k^2 = (\mathbf{x}_k - \mathbf{y}_k - \delta)^\top S_{x-y}^{-1} (\mathbf{x}_k - \mathbf{y}_k - \delta)$$

then has a chi-square distribution with n degrees of freedom, that is, $G_k^2 \sim \chi^2(n)$ as $m \to \infty$, where n is the length of the asymmmetry vector $(\mathbf{x}_k - \mathbf{y}_k)$, i.e., the number of pairs of elements.

Proof. Let the eigen-decomposition of the population covariance matrix Σ_{x-y} be

$$\Sigma_{x-y} = U\Lambda U^\top = U\Lambda^{\frac{1}{2}}(U\Lambda^{\frac{1}{2}})^\top$$

where Λ is a diagonal matrix of eigenvalues and U is an orthogonal matrix whose columns are the corresponding eigenvectors. Then, $U^{-1} = U^\top$ and $\Sigma_{x-y}^{-1} = U\Lambda^{-1}U^\top$. The vector $(\mathbf{x}_k - \mathbf{y}_k - \delta)$ is multivariate normal with mean 0 and covariance matrix Σ_{x-y}. Therefore, we can write

$$\mathbf{x}_k - \mathbf{y}_k - \delta = U\Lambda^{1/2}\mathbf{Z}$$

where $\mathbf{Z}$ is an n-dimensional standard normal random vector with zero mean and I_n covariance.

Substituting into the test statistic:

$$G_k^2 = (\mathbf{Z}^\top \Lambda^{1/2} U^\top)(U\Lambda^{-1}U^\top)(U\Lambda^{1/2}\mathbf{Z}) = \mathbf{Z}^\top \mathbf{Z}.$$

Hence, G_k^2 is the sum of squares of n independent standard normal random variables, i.e. $G_k^2 \sim \chi^2(n)$.

Since S_{x-y} is a consistent estimator of Σ_{x-y}, we have $S_{x-y} \to \Sigma_{x-y}$ as $m \to \infty$, and so the distribution of G_k^2 converges to the chi-square distribution with n degrees of freedom. □

Proposition 1 allows us to test whether the asymmetry between the buyer and the supplier in any dyad k is statistically significant. For a chosen significance level α (e.g., 0.05), we reject the null hypothesis when the test statistic G_k^2 exceeds the $(1 - \alpha)$ percentile of the χ^2 distribution with n degrees of freedom.

3.4 Robustness to Assumption Violations

In practice, the paired t-test is quite robust against violations of normality. Additionally, researchers treat Likert scale variables as normally distributed and continuous. Section 5 illustrates the test statistic G_k^2 to identify significantly asymmetric dyads in a matched-pair dataset from the retail sector. This robustness is reflected in the test statistic G_k^2, which has a chi-square distribution despite violating assumptions regarding the data and its joint distribution.

If the distribution of any variable is highly skewed, the researcher can transform the data using the logarithm. The researcher could seek a larger sample size if the test statistic G_k^2 does not follow a chi-squared distribution. As with a paired t-test, a sample size exceeding 30 is always advisable. SEM typically requires large datasets for testing hypotheses, but collecting large sets of matched-pair data, with each observation corresponding to a dyad, is challenging. With dimensionality reduction, this approach offers a solution by requiring fewer observations for SEM. As we shall see in Section 5, using Mahalanobis distance to reduce dimensionality can decrease the number of observations needed compared to using SEM with latent variables.

Implicitly, the covariance matrix is assumed to be invertible, as in many statistical methods. Not being able to invert the covariance matrix suggests practical problems related to the choice of variables. If one variable is fully explained by a subset of the others, leading to multicollinearity, we would need to drop it, just as we would in regression. It may also be that some variable is either missing a considerable number of values or has exactly the same value across all rows; in such cases, it would be better to drop it. In addition to these practical measures, there are statistical tests – determinant, rank, and condition number – to evaluate the covariance matrix before attempting to invert it. There is another potential violation in that the responses across dyads (observations) in a real dataset may not be independent, as assumed. In a buyer-supplier matched pair dataset like the one used in Section 5, some large buyers may belong to multiple dyads as they purchase from various suppliers. Similarly, some large suppliers may sell to many buyers in the dataset. For the test statistic, such a lack of independence across dyads may not matter if it has the expected distribution, as seen in Section 5. However, we should exercise caution when using regression or SEM. One workaround is to cluster the standard errors of the regression coefficients [White, 1980, Wooldridge, 2002, 2003].

3.5 Other Measures of Distance

The Manhattan (L1) and Euclidean (L2) distances can also be scaled by their standard deviation to address one of the shortcomings of these approaches compared to the Mahalanobis distance. (As with Mahalanobis distance, we can adjust the difference by δ.) The normalized Euclidean distance is obtained by ignoring the correlation terms of the covariance matrix S_{x-y} (unlike the Mahalanobis distance):

$$G'_k = \sqrt{(\mathbf{x}_k - \mathbf{y}_k - \delta)^\top \mathrm{diag}(S_{x-y})^{-1} (\mathbf{x}_k - \mathbf{y}_k - \delta)}$$

The test statistic $G_k'^2$ corresponding to this measure would follow a χ^2 distribution with n degrees of freedom only if the real covariance matrix were diagonal. However, the real covariance matrix is unlikely to be diagonal for paired items in a dyadic dataset.

Similarly, the Manhattan or L1 distance can be normalized as:

$$G''_k = |x_k - y_k|' \text{diag}(S_{x-y})^{-1} \begin{bmatrix} 1 \\ 1 \\ \dots \\ 1 \end{bmatrix}$$

However, there is no known distribution for the sum of half-normal distributed variables (under our assumption that $\mathbf{x}_k$ and $\mathbf{y}_k$ are drawn from multivariate normal distributions with the same mean), so there is no significance test. Here, we use the same χ^2 distribution with n degrees of freedom to test significance with the (normalized) L1 metric in our tests solely for comparison, even though it is not a valid test metric. Simulations described in the next section suggest this is a workable assumption, with the distribution appearing as non-central chi-squared.

Numerical example. Consider two paired items, Q_1 and Q_2, in a matched-pair dataset with the same mean values (so $\delta = 0$). In the fifth dyad ($k = 5$), the buyer's response to the two questions is $x_5 = [4 \quad 7]$ and the supplier's response is $y_5 = [7 \quad 4]$, so the difference is $x_5 - y_5 = [-3 \quad 3]$.

We can obtain the covariance matrix S_{x-y} by taking $x_k - y_k$ across all dyads, $k = 1$ to m. Let this covariance matrix be:

$$S_{x-y} = \begin{bmatrix} 3.42 & 2.55 \\ 2.55 & 2.8 \end{bmatrix}, \quad S_{x-y}^{-1} = \begin{bmatrix} 0.91 & -0.83 \\ -0.83 & 1.11 \end{bmatrix}$$

Also, ignoring covariances,

$$\text{diag}S_{x-y} = \begin{bmatrix} 3.42 & 0 \\ 0 & 2.80 \end{bmatrix}, \quad (\text{diag}S_{x-y})^{-1} = \begin{bmatrix} 0.292 & 0 \\ 0 & 0.357 \end{bmatrix}$$

Now we can compute all three distances for the fifth dyad:

- Mahalanobis distance

$$G_5 = \sqrt{[-3 \quad 3] \begin{bmatrix} 0.91 & -0.83 \\ -0.83 & 1.11 \end{bmatrix} \begin{bmatrix} -3 \\ 3 \end{bmatrix}} = 5.76$$

- Euclidean distance

$$G'_5 = \sqrt{[-3 \quad 3] \begin{bmatrix} 0.292 & 0 \\ 0 & 0.357 \end{bmatrix} \begin{bmatrix} -3 \\ 3 \end{bmatrix}} = 2.42$$

- Manhattan distance

$$G''_5 = [3 \quad 3] \begin{bmatrix} \frac{1}{\sqrt{3.42}} \\ \frac{1}{\sqrt{2.80}} \end{bmatrix} = 3.42$$

Only G_k^2 has a known distribution, i.e., a chi-square distribution for any k with $n = 2$ degrees of freedom. Here $G_5^2 = 5.76^2 = 33.15$, which is greater than 5.99, the 95$^{\text{th}}$ percentile of the chi-square distribution with two degrees of freedom. As such, the "asymmetry" by way of the Mahalanobis distance is significant at the 5% level.

4 Comparing with Other Distance Metrics Using Simulated Dyadic Datasets

As with any test statistic, Type I and Type II errors will arise when performing the corresponding significance test. We can control Type I errors, which represent false positives (incorrect identification of a dyad as asymmetric, that is, wrongly convicting the innocent), by selecting a confidence level of 1%, 5%, and so on. However, we cannot control Type II errors, which signify false negatives (failing to identify a dyad as asymmetric, i.e., allowing the guilty to escape punishment). In this context, we compare false positive (Type I) and false negative (Type II) errors using inference metrics based on the Mahalanobis distance, the normalized Euclidean distance, and the normalized Manhattan distance with three random datasets. As with any significance test, the focus is on Type I errors, which are the only ones we can control.

4.1 Creating the Random Datasets

To compare different metrics including the one proposed for Type I and Type II errors, I first generated three datasets of matched buyer-supplier pairs in 10 dimensions as follows:

I. Create covariance matrices

I first created three 10×10 covariance matrices Σ_1, Σ_2, and Σ_3, setting the diagonal terms $\sigma_{ii} = 1$ to ensure unit variances.

- For Σ_1, all off-diagonal terms were set to a positive value $\sigma_{ij} = 0.4$ for all $i \neq j$, where i and j index the row and column from 1 to $n = 10$.
- For Σ_2, off-diagonal values were mixed in sign: $\sigma_{ij} = +0.4$ if $(i + j)$ was even, and $\sigma_{ij} = -0.4$ otherwise.
- Σ_3 is an identity matrix with all off-diagonal terms set to zero.

All three matrices are positive-definite, allowing us to use Cholesky decomposition (e.g., Golub & Van Loan, 1996) to obtain the Cholesky matrices H_1, H_2, and H_3, respectively.

II. Generate buyer and supplier responses for each dyad

There were three simulation runs for 1000 dyads using each of the three matrices, H_1, H_2, and H_3, taking the sample covariance matrix S to the same as the population correlation matrix Σ. The supplier and buyer responses are assumed to share the same (population) covariance matrix.

For each dyad k, the 'buyer' response was generated as:

$$\mathbf{x}_k = HZ,$$

and a 'supplier' response as:

$$\mathbf{y}_k = HZ',$$

where Z and Z' are vectors generated with $n = 10$ independent standard normal variables. This procedure was repeated for each dyad, with $k = 1$ to 1000, using H_1, H_2, and H_3, respectively, to obtain three datasets, each with 1,000 dyads.

III. Introducing Asymmetry in the Dataset

Next, asymmetry was introduced in only the first 100 dyads (of the total 1,000 dyads) in each dataset by shifting the buyer vector by $+1.25$ and the supplier vector by -1.25 for each of the ten dimensions.

Thus, we have three datasets with known covariance structure between buyer and supplier responses, where dyads 1–100 are significantly asymmetric by construction (mean difference $\neq 0$), and dyads 101–1000 are not asymmetric, as their differences are from a population with mean zero.

4.2 Identifying Significantly Asymmetric Dyads with Type I/II Errors

With the three datasets and known asymmetric dyads in place, I tested the significance of each dyad using the test statistics corresponding to the proposed metric, the Euclidean distance, and the Manhattan distance. The following test was performed for each dyad to identify which ones are significantly asymmetric. For a chosen significance level of α, say 0.05, I rejected the null hypothesis when the proposed test statistic G_k^2 using Mahalanobis distance (similarly, $G_k'^2$ for Euclidean or $G_k''^2$ for Manhattan) exceeds the $(1-\alpha)$ percentile value of the χ^2 distribution with n degrees of freedom. The procedure involved the following three steps:

1. **Calculate the distance metrics:** All three metrics—based on the Mahalanobis distance G_k, the normalized Euclidean distance G_k', and the normalized Manhattan distance G_k'', respectively—were computed for each dyad in all the datasets.

2. **Significance test:** Any dyad in each dataset is considered *significantly asymmetric* if the asymmetry statistic (G_k^2, $G_k'^2$, or $G_k''^2$) exceeds the critical value of the chi-squared distribution with 10 degrees of freedom at the 10%, 5%, or 1% significance level.

3. **Calculate false positives and false negatives:** Using the fact that the first 10% of all the dyads in each dataset (subject to randomness in the generated data) were constructed to be asymmetric, I counted:

   - *False positives* — when non-asymmetric dyads were incorrectly identified as asymmetric (Type I error – "innocent, wrongly convicted") from dyads 101–1000
   - *False negatives* — when actually asymmetric dyads were not identified (Type II error – "guilty, not convicted") from dyads 1–100.

   This procedure was repeated to identify false positives and false negatives for each metric across all nine simulation runs.

4.3 Results

The simulation results (**Table 4**) show that only the proposed metric using the Mahalanobis distance has the same percentage of Type I errors as the chosen significance level, while this is not the case with the other two metrics. Therefore, using a desired confidence level, the proposed metric using the Mahalanobis distance can effectively control the false positive error. In contrast, the other two metrics exhibit not only larger Type I errors, but also errors that are uncontrollable, which is not surprising, since the test metric does not have a chi-square or any other known distribution. Thus, *the Mahalanobis distance enables us to test significance when applying a difference-based view of asymmetry in multiple variables* (Table 4).

Our key findings are:

- Only the Mahalanobis distance correctly controls Type I error (false positives).

- Euclidean and Manhattan metrics "identify" more dyads as asymmetric than appropriate, but the identification is based on the chi-square distribution (as with the Mahalanobis distance) even though we don't have a known distribution for these distances.

As such, the Mahalanobis distance-based metric offers a valid test for asymmetry across multiple dimensions and is superior to alternatives that lack significance testing foundations. We cannot say anything about the *power* of the test using Euclidean or Manhattan distance because the inference test based on these metrics does not have the correct proportion of Type I errors. Therefore, comparing the Type II errors of these two metrics to Type I errors is incorrect.

There seem to be more false negatives (Type II errors) with the proposed Mahalanobis-distance-based metric, especially when the covariance matrix has only positive correlations. However, such claims require further work with power-related tests.

5 Application of the Metric on a Retail Buyer-Supplier Dyadic Dataset

Recall that the goal of this paper is to propose (1) a statistically justified method to combine matched buyer and supplier variables (constructs) to create relationship-specific variables, such as asymmetry for the relationship, and (2) a consistent and statistically robust way to use buyer and supplier variables in addition to relationship-specific variables in regression or SEM.

Using a retail dyadic dataset, I now illustrate both applications for buyer-supplier relationships in the grocery sector. The dataset, used previously by Sodhi and Son [2009] and Son et al. [2016], has responses to matched-pair questions from the buyer and the supplier in 74 buyer-supplier relationships. These relationships involve 12 buyers, including large discount stores, supermarket chains, and Internet shopping outlets, and 70 suppliers, including some large international players, such as Tesco on the buyer side and Coca-Cola, Kimberley-Clark and Nestlé on the supplier side. I also discuss how to account for the 74 dyads consisting of only 12 buyers, as many dyads have the buyer in common. The

Dataset	Signif.level	Total identified			True positives			False negat.			False posit.		
		Proposed	Euclidean L2	Manhattan L1	Proposed	Euclidean L2	Manhattan L1	Proposed	Euclidean L2	Manhattan L1	Proposed	Euclidean L2	Manhattan L1
1a. Positive	10%	123	199	175	47	86	82	53	14	18	76	113	93
1a. Positive	5%	68	162	129	29	82	75	71	18	25	39	80	54
1a. Positive	1%	20	114	71	13	71	57	87	29	43	7	43	14
1b. Positive	10%	153	211	189	56	90	86	44	10	14	97	121	103
1b. Positive	5%	86	167	135	41	81	77	59	19	23	45	86	58
1b. Positive	1%	30	123	76	24	75	60	76	25	40	6	48	16
1c. Positive	10%	145	212	198	49	87	87	51	13	13	96	125	111
1c. Positive	5%	77	182	142	34	84	82	66	16	18	43	98	60
1c. Positive	1%	21	126	83	11	76	61	89	24	39	10	50	22
2a. Mixed	10%	187	215	179	92	98	97	8	2	3	95	117	82
2a. Mixed	5%	135	168	129	88	97	90	12	3	10	47	71	39
2a. Mixed	1%	73	119	74	66	87	66	34	13	34	7	32	8
2b. Mixed	10%	167	203	182	93	96	96	7	4	4	74	107	86
2b. Mixed	5%	116	164	122	86	96	91	14	4	9	30	68	31
2b. Mixed	1%	73	113	66	69	87	63	31	13	37	4	26	3
2c. Mixed	10%	208	212	185	95	99	98	5	1	2	113	113	87
2c. Mixed	5%	156	172	134	93	98	96	7	2	4	63	74	38
2c. Mixed	1%	97	126	78	81	93	71	19	7	29	16	33	7
3a. Zero	10%	186	186	140	97	97	94	3	3	6	89	89	46
3a. Zero	5%	138	138	84	94	94	78	6	6	22	44	44	6
3a. Zero	1%	87	87	28	78	78	28	22	22	72	9	9	0
3b. Zero	10%	172	172	135	93	93	91	7	7	9	79	79	44
3b. Zero	5%	126	126	83	91	91	77	9	9	23	35	35	6
3b. Zero	1%	81	81	34	76	76	34	24	24	66	5	5	0
3c. Zero	10%	176	176	133	97	97	96	3	3	4	79	79	37
3c. Zero	5%	126	126	91	90	90	82	10	10	18	36	36	9
3c. Zero	1%	83	83	33	78	78	33	22	22	67	5	5	0

Table 4: Simulation results on datasets with positive, mixed, and zero correlations using different significance levels: Total significantly asymmetric dyads identified; true positives identified; false negatives (Type II error) having failed to identify significantly asymmetric dyads; and false positives (Type I error) having falsely identified dyads as asymmetric even though they were not. Asymmetry is measured by (a) the proposed metric using Mahalanobis distance, (b) the (normalized) Euclidean distance, and (c) the (normalized) Manhattan distance. Number of dyads $m = 1,000$ with 100 asymmetric (with the buyer and supplier data drawn from a distribution with other means).

matched-pair questions capture buyer and supplier perceptions regarding their relationship, so I interpret the differences between their responses as asymmetry for this illustration.

The survey used two questionnaires, one for the supplier and one for the buyer, featuring paired questions that required responses on a seven-point Likert-type scale. The questionnaire included ten questions adapted from past studies and five additional questions for relational-specific operational performance (Appendix, Table A1): Questions 1–3 focused on the information technology (IT) capabilities of the partners and the level of relationship-specific IT assets; Questions 4, 5 (originally reverse coded), and 6 examined trust versus the buyer's power; Questions 7 and 8 investigated corporate cultural similarities between the partners; and Questions 9 and 10 assessed the level of commitment from either party to their relationship. A summary of the responses from the buyers and suppliers is presented in the Appendix (Table A2).

5.1 Identifying Significantly Asymmetric Relationships

For this illustration, I used all the ten paired questions Q1–Q10 to obtain a single measure of asymmetry. I computed Mahalanobis distances for each dyad. Ten of the 74 dyads had $G_k^2 > 18.31$, the 95th percentile of $\chi^2(10)$, and therefore were significantly asymmetric at the level 5% (Table 5).

Robustness against violations of assumptions. This dataset violates the assumptions of normality and multi-normality underlying our proposed measure and test of significance. The responses on a 7-point Likert scale [DeVellis, 2012] are not distributed normally. However, using statistics based on normal-distribution assumptions is common when the data are responses on a Likert scale [Johnson and Creech, 1983]. Here, univariate tests of normality [D'Agostino et al., 1990, Royston, 1991] indicate that the measure for asymmetry based on Q1–Q10 was consistent with a normal distribution, as were the distance measures for relationship attributes and the operational performance for the supplier. However, for the buyer, both distance measures were skewed. Additionally, tests of multivariate normality using Mardia, Henze-Zirkler, and Doornik-Hansen methods on the buyers' relationship attributes and the suppliers' attributes rejected multi-normality. Despite these assumption violations, the measure for testing asymmetry, G_k^2, had a χ^2 distribution, albeit with slightly fewer degrees of freedom than the number of questions. More research is needed to investigate robustness against different nonnormal data distributions.

5.2 Use in Structural Equation Modeling (SEM)

The literature indicates that the strength of the relationship influences the relationship-specific operational performance for both the buyer and the supplier. One could consider two separate models, one for the buyer and one for the supplier [Whipple et al., 2015], combining the two sides using the average or another composite approach, or simply neglecting one side. However, in these instances, we cannot utilize dyad-specific variables, such as asymmetry. Alternatively, we could explicitly incorporate the buyer and supplier constructs within the same model alongside their corresponding paired constructs, then introduce relationship-specific variables such as asymmetry to connect relevant paired constructs across the two

Table 5: Significantly asymmetric dyads at 5% and 1% levels

Buyer	Supplier in dyad k (test statistic G_k^2)
1	**1 (34.12**)**; 2 (11.73); 3 (11.51); 4 (7.54)
2	5 (13.94); **6 (19.74*)**; 7 (14.27); **8 (23.48**)**; 9 (5.72); **10 (29.55**)**; 11 (14.79)
3	12 (8.74); **13 (19.93*)**; 14 (13.13); 15 (6.74); 16 (4.68); 17 (5.09); 18 (5.19); 19 (14.09); 20 (3.77)
4	21 (1.56); 22 (2.11); 23 (5.21); 24 (12.02); 25 (4.18); 26 (4.19); 27 (3.98); 28 (6.57); 29 (2.26); 30 (10.51); 31 (3.79); 32 (7.88); 33 (3.51)
5	34 (2.52); 35 (3.55); 36 (9.86); 37 (3.04); 38 (2.55); 39 (5.25); 40 (3.39); 41 (8.06); 42 (2.43); 43 (11.18)
6	44 (15.08); 45 (7.92); 46 (10.91); 47 (7.85); 48 (6.26); **49 (21.96*)**
7	50 (15.76); 51 (13.55)
8	52 (8.87)
9	53 (7.25); 54 (8.37); 55 (12.87); 56 (5.24); 57 (2.34); 58 (8.47); 59 (7.31)
10	60 (11.44); **61 (19.68*)**; 62 (16.92); 63 (1.96); 64 (15.06); **65 (29.17**)**; 66 (16.27); 67 (5.37); 68 (15.34)
11	69 (10.34)
12	**70 (20.20*)**; 71 (11.24); 72 (15.34); **73 (21.26*)**; 74 (13.32)

Note: * $p < 0.05$ (critical value: $G_k^2 > 18.31$), ** $p < 0.01$ (critical value: $G_k^2 > 23.21$).

models. The researcher must also account for covariances between the paired constructs as necessary.

Five measures were calculated using the Mahalanobis distance for this minimal model for illustration, which has no control variables:

- $D_k^{B,R}$: Buyer relationship attributes
- $D_k^{S,R}$: Supplier relationship attributes
- $D_k^{B,O}$: Buyer operational performance
- $D_k^{S,O}$: Supplier operational performance
- G_k^R: Asymmetry in relationship attributes

There are the two distance measures, $D_k^{B,R}$ and $D_k^{S,R}$, to capture the multi-attribute relationship values for the buyer and supplier, respectively, and two distance measures, $D_k^{B,O}$ and $D_k^{S,O}$, to capture the multi-attribute operational performance of the buyer and supplier, respectively. These represent the distances of the (random) vectors from the origin to reduce the dimensionality of the buyer and supplier variables. The fifth metric is the asymmetry measure G_k^R, based on relationship attributes, which indicates the distance between two (random) vectors (see Figure 2 and Table 6).

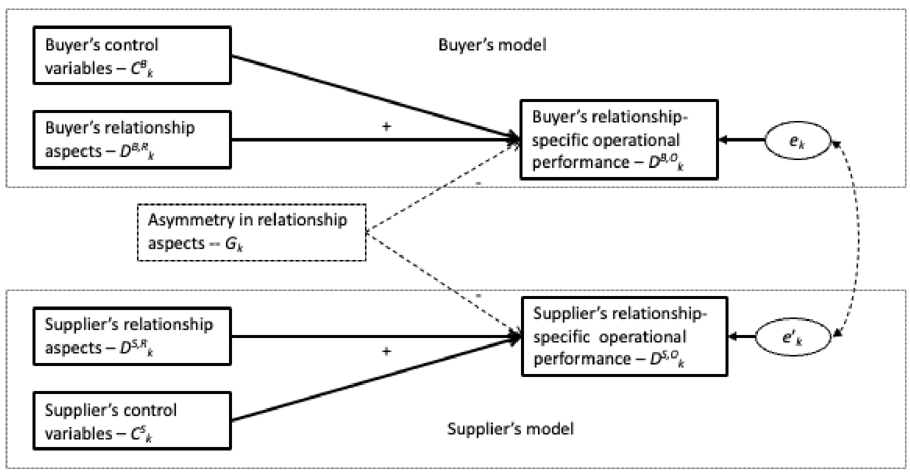

Figure 2: An SEM model to test the relationship between buyer (supplier) relationship performance and buyer (supplier) relationship attributes. There is one model for the buyer and one for the supplier, and connecting the two models are the buyer-supplier asymmetry and the correlated residual errors.

The literature indicates a link between asymmetry and relationship-specific performance for either party. The hypothesized link can be tested by two simultaneous equations for the relationship-specific operational performance, one for the buyer B and the other for the supplier S in the k^{th} dyad:

$$D_k^{B,O} = \alpha_B + \beta_B D_k^{B,R} + \gamma_B G_k^R + \epsilon_k$$
$$D_k^{S,O} = \alpha_S + \beta_S D_k^{S,R} + \gamma_S G_k^R + \epsilon'_k$$

with co-varying residuals ϵ_k and ϵ'_k and $k = 1$ to m. The coefficients of the second term in the first equation $D_k^{B,R}$ help us to check whether the relationship attributes of the buyer are positively related to the relationship-specific operational performance of the buyer. The same holds for the supplier with $D_k^{S,R}$ in the second equation. The third set of terms in the two equations represents asymmetry, allowing us to check whether G_k^R has negative coefficients to test whether asymmetry has a detrimental effect on the relationship-specific operational performance for either party. The fourth set of terms, C_k^B and C_k^B, represents

Table 6: Mapping of Relationship Attributes and Performance Metrics

	Relationship attributes	**Operational performance**
Buyer in the k^{th} dyad	$D_k^{B,R} = \sqrt{x_k^\top (S_{\text{pooled}}^R)^{-1} x_k}$	$D_k^{B,O} = \sqrt{x_k^\top (S_{\text{pooled}}^O)^{-1} x_k}$
Supplier in the k^{th} dyad	$D_k^{S,R} = \sqrt{x_k^\top (S_{\text{pooled}}^R)^{-1} y_k}$	$D_k^{S,O} = \sqrt{x_k^\top (S_{\text{pooled}}^O)^{-1} y_k}$
Asymmetry in the k^{th} dyad	$G_k^R = \sqrt{(x_k - y_k - \delta)^\top S_{x-y}^{-1} (x_k - y_k - \delta)}$	

control variables such as buyer and supplier revenues, omitted here. Finally, asymmetry and the correlated aspects of the relationship may affect the relationship-specific operational performance of the buyer in the same way as it does the supplier. Thus, we expect the residuals e and e' for relationship-specific performance for the buyer and the supplier to co-vary and we need to accommodate this covariance in the model (Figure 2).

The appendix lists the variables relating to the attributes of the buyers and suppliers within each dyad in the retail dataset. These include matched-pair questions Q11–Q15 regarding relationship-specific performance for the buyer and the supplier (Appendix: Tables A3 and A4). For further details on the data, see Sodhi and Son [2009].

Model fit. I fitted the two simultaneous equations for the relationship-specific operational performance of the buyer and the supplier using the SEM module in Stata with the method *mlvl* for handling the missing data on the revenues of some suppliers in the dataset. The χ^2 value of the model compared to the saturated model is low at 0.334 (p=0.846), indicating a fit close to the saturated model. In contrast, the baseline model versus the saturated model has a χ^2 value of 36.27 (p=0.000), indicating that this model is much better than the baseline model. Regarding the model's fit, *RMSEA* (root mean square error of approximation) is 0.000, the probability that $RMSEA < 0.05$ is 0.870, *CFI* (comparative fit index) is 1.000, and *CD* (coefficient of determination) is 0.349.

Buyer clusters in the dyads. Recall that the dyads in the dataset come from 12 retailers and 70 suppliers. Therefore, we could observe correlations across several dyads that share the same retailer among the 12 retailers [Handley and Gray, 2015]. In other words, the retailer serves as a level for these dyads. Consequently, when running SEM in Stata, we can specify a clustering variable to account for such level effects or correlations and to adjust for cluster correlation [Rogers, 1993]. This procedure clusters the standard errors of the regression coefficients [White, 1980, Wooldridge, 2002, 2003].

The resulting p values of the coefficients changed only slightly (Table 7, column (b)), indicating that clustering of dyads by retailer does not have a significant effect; the dyads appear to be independent of each other.

Table 7: SEM Results: Effects of Buyer/Supplier Attributes and Asymmetry on Operational Performance

Standardized variable	**Coefficient Estimate**	**Std. error**	**(a) P> $\|z\|$**	**(b) P> $\|z\|$**
Buyer's operational performance in the relationship: $D_k^{B,O}$				
Constant	2.0430**	0.7761	0.008	0.118
Buyer Attribute $D_k^{B,R}$	0.4816**	0.0852	0.000	0.001
Asymmetry G_k^R	−0.2409*	0.1022	0.018	0.086
Supplier's operational performance in relationship: $D_k^{S,O}$				
Constant	3.8523**	1.1057	0.000	0.000
Supplier Attribute $D_k^{S,R}$	0.3245**	0.0975	0.001	0.001
Asymmetry G_k^R	−0.2906**	0.1035	0.005	0.018
Variance				
$e.D_k^{B,O}$	0.7739	0.0772	–	–
$e.D_k^{S,O}$	0.8522	0.0711	–	–
Covariance				
$e.D_k^{B,O}$ & $e.D_k^{S,O}$	0.2925**	0.1065	0.006	0.009
Likelihood ratio	χ^2	$p > \chi^2$		$p > \chi^2$
Model vs. saturated	0.334	0.846		N/A
Baseline vs. saturated	36.273**	0.000		N/A

Note: * $p < 0.05$ ** $p < 0.01$ *** $p < 0.001$; number of observations = 74.

5.3 Interpretation of Results

The results in this illustrative example align with the literature concerning the relationship between asymmetry and relationship-specific operational performance. The SEM model, when compared to the saturated model, is insignificant, whereas the baseline model, in comparison to the saturated model, is highly significant. Thus, the model is useful (Table 7).

1. **Buyer and supplier relationship attributes matter for performance.** The operational performance specific to the buyer relationship $D_k^{B,O}$ is positively and significantly related to the attributes of the buyer relationship $D_k^{B,R}$, and similarly, the performance of the supplier $D_k^{S,O}$ is positively and significantly related to the attributes of the supplier relationship $D_k^{S,R}$.

2. **Asymmetry impacts relationship-specific performance negatively.** Both the buyer's and the supplier's relationship-specific operational performance are negatively and significantly related to the level of asymmetry between them, as measured by our proposed measure G_k.

3. **Residuals are positively covariant.** Residuals for the customer and supplier relationship-specific operational performance models are significantly and positively covariant, suggesting that asymmetry affects their results in a similar direction.

4. **Interaction terms are unnecessary.** Interaction effects ($D_k^B \times G_k$ and $D_k^S \times G_k$) were not significant. Although these have been proposed as a proxy for asymmetry [see Edwards, 2001, p. 270], our Mahalanobis distance-based metric directly captures asymmetry without the need for interactions.

5. **Control variables are partly informative.** Retailer revenues were significant when tested as control variables; however, supplier revenues could not be tested due to missing data. The revenue difference between the two also appeared significant in some specifications, though values were missing for many suppliers.

6. **Asymmetry-based splits show no performance differences.** To test whether significantly asymmetric dyads biased results, the sample was split using significance thresholds 10% and 20% for asymmetry. No significant differences in buyer performance were found in the 10% split ($t = 0.5306$, $p = 0.3023$) or the 20% split ($t = 0.6424$, $p = 0.2369$). A MANOVA confirmed that there was no significant difference in buyer-supplier performance in these splits, supporting the robustness of including all dyads in SEM.

These results are consistent with the findings of Liu et al. [2012], among others, who used the 'degree symmetry' metric to evaluate dyadic relationships.

6 Discussion

This paper has focused on how to statistically approach buyer-supplier dyadic data in multiple dimensions by reviewing the variety of methods currently in use. It has identified a research gap concerning a metric for an attribute such as asymmetry across multiple matched dimensions between the buyer and the supplier in each dyad, such that:

1. We have a statistical significance test for it, and

2. We can use it meaningfully in regression, SEM, or other statistical models.

I proposed a Mahalanobis distance-based metric and a significance test, assuming a multivariate normal distribution for the vectors. In this context, asymmetry serves as a distance function that captures the difference between the paired buyer and supplier vectors. I conducted simulations to demonstrate the superiority of the proposed metric over other distance metrics, particularly in controlling the Type I error. The simulations also indicated that the significance test performs as expected, unlike statistics based on Euclidean or Manhattan distance.

I also used a real-life dyadic dataset from the retail sector to identify significantly asymmetric dyads. With this dataset, I demonstrated how we could apply the metric in SEM with buyer and supplier constructs and asymmetry, all represented by measures based on the Mahalanobis distance. The results align with the literature, suggesting that the metric could help study buyer-supplier relationships using dyadic data sets.

The primary contribution of this paper to the empirical dyadic literature on buyer–supplier relationships is the provision of a testable measure of asymmetry. This measure reduces the difference vector between the buyer's and the supplier's responses in n dimensions to a scalar. The square of the metric follows a χ^2 distribution with n degrees of freedom, allowing us to test whether the difference (asymmetry) is significantly different from zero at a chosen significance level. Thus, we can identify which buyer–supplier relationships are significantly asymmetric.

A secondary and more speculative contribution is using Mahalanobis distance as a lossless method in place of factor scores to reduce multiple correlated variables (e.g., items in a construct) to a single scalar. However, developing SEM models with Mahalanobis distance-based scores instead of factors requires further investigation.

6.1 Further Research in Measuring Asymmetry

Asymmetry is a vast area of study, and this paper proposes a specific measure based on pairwise differences, augmented with a significance test. Other approaches include comparing buyers with suppliers as separate groups (see Table 1) or using polynomial regression [Edwards & Parry, 2018]. More research is needed to assess the suitability of each method depending on the context.

1. **Power and outlier handling:** The power of the significance test should be evaluated, including under conditions where some variables are negatively correlated. In addition, the robustness of the metric against masking (obscuring outliers) and swamping (false outlier identification) needs assessment [Hadi, 1992].

2. **Assumption violations:** We must understand how robust the metric is to violations of (a) multivariate normality and (b) the equality of population covariance matrices for buyers and suppliers.

3. **Lossless dimension reduction:** The secondary contribution of this paper - the replacement of constructs with Mahalanobis distances from the origin - must be justified empirically and practically. Can Mahalanobis distance truly replace factor scores without loss of interpretability?

6.2 Implications for Using the Asymmetry Metric in SEM

We also need to compare the use of various asymmetry metrics in SEM and other models using simulated and real dyadic datasets. Re-testing datasets from existing studies using the proposed metric would be a logical starting point.

Incorporating dyad-specific variables (such as asymmetry) opens several new possibilities, especially if groups of matched-pair items are treated as vectors. For instance, asymmetry could be evaluated even in control variables. Our finding that firm revenue differences are significantly related to performance outcomes aligns with Villena & Craighead [2017].

However, caution is needed:

- Edwards [2001, 2002] raised concerns about using difference-based metrics in regression and SEM.

- Sample size requirements must be considered [Shah and Goldstein, 2006].

- The power and practical feasibility of applying our test in SEM models must be validated [Roh et al., 2013].

Although this paper focused on dyads, the proposed asymmetry measure could also be used in triads, for example, in evaluating a supplier's relationships with two buyers or a buyer's relationships with two suppliers. This could inform research such as that by Yang et al. [2022], who explored triadic agency problems.

6.3 Implications for Practice

The proposed metric has practical utility beyond our retail case study:

- An OEM managing multiple suppliers can identify "gaps" in expectations versus delivery, using the metric to guide continuous improvement or supplier replacement.
- Large manufacturers can use matched-pair surveys and the asymmetry metric to categorize suppliers by significance levels of asymmetry and communicate priorities accordingly.
- The metric can help with sustainability auditing. Firms could assess gaps in sustainability expectations using matched items and eliminate or assist suppliers accordingly [Montes-Sancho et al., 2022, Bhutta, 2003, Ho et al., 2010].
- Instead of radar charts or unweighted differences, a manager could use the single-score asymmetry metric to prioritize renegotiation with highly asymmetric suppliers. This accounts for the differing variance between dimensions, providing more accurate and objective prioritization.

Thus, the Mahalanobis distance-based asymmetry metric introduced in this paper is theoretically justified and practically applicable in buyer–supplier research.

In conclusion, this paper contributes a robust, testable, and practically useful method to measure asymmetry in buyer-supplier dyads. Replacing several related items with their Mahalanobis distance from the origin in SEM or regression may also be interesting as a lossless method instead of performing factor analysis and then using the factor scores for constructs. The article opens new directions for research and offers managers a powerful tool to diagnose and improve supply chain relationships.

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Appendix

Table A1: Survey Questions on Buyer–Supplier Relationship Dimensions

Q#	**Posed to (about)**	**Question***
Q1	Both (information sharing)	My company shares standardized information externally with <the name of the supplier/buyer>
Q2	Both (information sharing)	My company shares customized information externally with <the name of the supplier/buyer>
Q3	Both (information sharing)	My company invests in technology designed to facilitate information exchange with <the name of the supplier/buyer>
Q4	Both (trust/power)	The <name of the supplier/buyer> is one of our prime suppliers/buyers.
Q5	Buyer (trust/power)	My company avoids exercising power to <the name of the supplier>. **Supplier:** My company feels that the <name of the buyer> leads the business relationship by exercising power.
Q6	Both (trust/power)	The business relationship with the <name of the supplier/buyer> is based on trust.
Q7	Both (corporate culture)	The <name of the supplier/buyer> places as much importance on meeting their commitment as we do.
Q8	Both (corporate culture)	The <name of the supplier/buyer> is as willing to bring about change as we are.
Q9	Both (commitment)	My company expects that the supply chain partnership with <name of the supplier/buyer> will continue long term.
Q10	Both (commitment)	My company intends to establish a closer partnership with <the name of the supplier/buyer>.

*Customized with the buyer's or supplier's name. Q5 to the supplier was reverse-coded for the statistical analysis. As indicated, all questions are paired with slightly different wording in Q5.

Table A2: Summary of responses by buyers (Q1b–Q10b) and suppliers (Q1s–Q10s) to questions Q1–Q10 on relationship attributes

(a) Buyers' relationship attributes

Item	**Mean**	**SD**	Q1b	Q2b	Q3b	Q4b	Q5b	Q6b	Q7b	Q8b	Q9b
Q1b	4.66	1.967	–								
Q2b	4.38	1.710	0.84	–							
Q3b	3.14	1.808	0.28	0.35	–						
Q4b	5.09	1.305	0.36	0.28	0.43	–					
Q5b	5.28	1.360	0.60	0.45	0.26	0.27	–				
Q6b	5.31	1.344	0.57	0.48	0.01	0.23	0.72	–			
Q7b	5.34	1.174	0.60	0.53	0.33	0.52	0.52	0.55	–		
Q8b	5.49	1.241	0.59	0.42	0.24	0.52	0.48	0.52	0.83	–	
Q9b	5.62	1.107	0.66	0.61	0.24	0.43	0.62	0.76	0.65	0.58	–
Q10b	5.68	1.074	0.51	0.46	0.23	0.47	0.60	0.73	0.61	0.60	0.82

(b) Suppliers' relationship attributes

Item	**Mean**	**SD**	Q1s	Q2s	Q3s	Q4s	Q5s	Q6s	Q7s	Q8s	Q9s
Q1s	4.04	1.724	–								
Q2s	4.01	1.634	0.81	–							
Q3s	3.00	1.783	0.46	0.54	–						
Q4s	5.30	1.362	0.09	0.12	0.03	–					
Q5s	4.46	1.377	0.02	0.13	0.26	0.22	–				
Q6s	5.34	1.337	0.44	0.41	0.22	0.35	-0.10	–			
Q7s	4.97	1.414	0.35	0.40	0.28	0.37	0.10	0.69	–		
Q8s	5.12	1.414	0.42	0.44	0.24	0.40	0.06	0.60	0.65	–	
Q9s	5.96	1.091	0.08	0.15	-0.04	0.39	0.16	0.40	0.37	0.29	–
Q10s	6.07	1.163	0.01	0.04	-0.13	0.33	0.13	0.27	0.32	0.24	0.88

(c) Difference in buyers' and buppliers' relationship attributes

Item	**Unadj. Mean**	**SD**	Q1d	Q2d	Q3d	Q4d	Q5d	Q6d	Q7d	Q8d	Q9d
Q1d	0.62	1.85	–								
Q2d	0.36	1.82	0.74	–							
Q3d	0.14	2.14	0.23	0.32	–						
Q4d	-0.20	1.74	0.21	0.17	0.05	–					
Q5d	0.82	2.28	0.26	0.22	0.22	0.26	–				
Q6d	-0.03	1.86	0.37	0.32	0.11	0.37	0.32	–			
Q7d	0.36	1.76	0.33	0.38	0.17	0.52	0.24	0.60	–		
Q8d	0.36	1.83	0.28	0.22	0.15	0.60	0.16	0.44	0.72	–	
Q9d	-0.34	1.59	0.18	0.32	-0.08	0.44	0.34	0.56	0.47	0.33	–
Q10d	-0.39	1.59	0.05	0.24	0.01	0.54	0.29	0.46	0.48	0.38	0.83

Note: Items Q1–Q10 were posed as matched pairs to buyers and suppliers. Correlations are below the diagonal. Means and SDs are unadjusted. Differences computed as buyer minus supplier values.

Table A3: Summary of responses by buyers (Q11b–Q15b) and suppliers (Q11s–Q15s) to questions Q11–Q15 regarding relationship-specific performance

Q#	**Posed to (about)**	**Question**
Q11	Buyer (lead time) Supplier (lead time)	**Buyer:** Reduction of lead time from order placement to the <name of the supplier> to the receipt of the order. **Supplier:** Reduction of lead time from receipt of an order from <name of the buyer> to the fulfillment of the order.
Q12	Both (responsiveness)	Supply chain responsiveness.
Q13	Both (cost)	Cost reduction of all SCM activities related to <name of the supplier/buyer>.
Q14	Both (forecast accuracy)	Increased forecasting accuracy.
Q15	Both (inventory)	Reduced inventory level.

Note: Questions Q11–Q15 were paired for buyers and suppliers. Q11 has different wording for buyer and supplier to reflect lead time on their respective ends of the supply chain.

Table A4: Summary of responses to questions Q11–Q15 on relationship-specific performance

(a) Buyer Relationship-Specific Performance

Item	**Mean**	**SD**	Q11b	Q12b	Q13b	Q14b
Q11b	5.45	1.406	–			
Q12b	5.45	1.356	0.95	–		
Q13b	5.39	1.422	0.92	0.91	–	
Q14b	5.43	1.325	0.89	0.91	0.89	–
Q15b	5.45	1.416	0.88	0.89	0.86	0.88

(b) Supplier Relationship-Specific Performance

Item	**Mean**	**SD**	Q11s	Q12s	Q13s	Q14s
Q11s	5.36	1.171	–			
Q12s	5.23	1.161	0.63	–		
Q13s	5.05	1.104	0.63	0.79	–	
Q14s	5.08	1.382	0.66	0.75	0.73	–
Q15s	4.62	1.497	0.60	0.62	0.62	0.71

Note: Responses reflect buyer and supplier evaluations of relationship-specific performance on five matched items (Q11–Q15). Correlation coefficients are reported below the diagonal.