Disruption Mitigation and Pricing Flexibility

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Abstract

We study how a firm's pricing flexibility to potentially increase its prices during a disruption impacts its decisions on the level of reserve inventory or reserve capacity to carry in anticipation of disruptions. While the reserve inventory provides a certain quantity of inventory that the firm can continue to sell during a disruption, reserve capacity allows the firm to maintain production during a disruption at a certain production rate. Specifically, we consider a firm producing a single product and that is exposed to random disruptions. The firm first decides on the level of reserve inventory or reserve capacity to carry in anticipation of disruptions. When a disruption does occur, it may then choose to adjust its price to better align demand with the supply restrictions during the disruption. For both reserve inventory and reserve capacity cases, we characterize the firm's optimal pricing decisions during disruptions, and the level of reserve inventory or reserve capacity to hold in anticipation of disruptions. We find that a certain level of pricing flexibility may be required to justify investing in holding reserve inventory or reserve capacity. Further, we also show that while a firm's pricing flexibility during a disruption and reserve capacity decisions are substitutes, pricing flexibility and reserve inventory may act either as substitutes or complements, that is, a firm may be incentivized to carry a lower or higher amount of reserve inventory if it possesses further flexibility to increase its prices during a disruption. Finally, through numerical studies, we also investigate the sensitivity of reserve inventory or reserve capacity decisions when the firm may or may not have pricing flexibility with respect to factors such as disruption rates, disruption durations, and carrying costs for reserve inventory or reserve capacity.

1. Introduction

Two common operational measures utilized by firms to mitigate the impact of supply chain disruptions are holding *reserve inventory* or *reserve capacity* (e.g., Sheffi (2020)). We refer to reserve inventory as a specific quantity of inventory that the firm holds in order to continue to serve demand in the event of a disruption. Reserve capacity, on the other hand, is an additional capacity that allows a firm to continue to maintain production during a disruption at a certain production rate.

While reserve inventory is a useful lever in industries where inventory holding costs are relatively low, perishability is slow, and inventory devaluation is low, reserve capacity is generally a more useful lever in industries where inventory holding costs are high, lead times are short, and contract manufacturing is common (e.g., Sodhi and Tang (2012)). For example, while the chocolate maker Hershey utilizes reserve inventories by holding chocolate inventory buffers of up to six months to protect against sudden disruptions, the automotive firm Toyota as well as the telecom equipment maker Cisco keep excess reserve capacity to allow for emergency production in case of a disruption (Sodhi and Tang (2012), Sheffi (2020)).

Despite efforts by firms to mitigate the impact of supply disruptions, when supply disruptions do occur, customer demand often exceeds available supply, which may in turn result in higher prices. For example, according to J.P. Morgan Research, global automotive production decreased by 26% during the first nine months of 2021, at the height of the chip shortage during the pandemic (J.P.Morgan (2023)). During the same period, average new car prices in the US increased by 7.5% according to authors' calculations based on data from the U.S. Bureau of Labor Statistics (U.S. Bureau of Labor Statistics (2024)). A similar dynamic was also observed during the 2011 earthquake in Japan. The disaster affected production at Honda and Nissan. Consequently, Nissan Rogue's price rose by 3% after the earthquake (while the average industry price increase was 0.3%). Honda Fit's price rose by nearly 6% (while the average increase in the prices of compact cars was 2.3%) (Shunk (2011)). In the semiconductor industry, according to the Wall Street Journal, supply shortages during the pandemic prompted the chip manufacturer TSMC to raise prices by 10% for high-end semiconductors and by 20% for less advanced chips (Jie et al. (2021)). Such price increases have also been observed in food industry as well. For example, coffee prices soared 44% from June to September 2010 due to bad weather in South America threatening crops (Forbes

(2011)) and U.S. egg prices hit a record high due to bird flu in 2015 (Reuters (2015)). Further, we also observe a similar relationship between supply limitations and price levels at the aggregate level in the economy. According to the Federal Reserve Bank of San Francisco, the Global Supply Chain Pressure Index, which is a measure that incorporates supply disruptions and delays, and PCE headline inflation show a correlation of 0.53 and supply pressures have accounted for approximately 60% of the increase in US inflation beginning in early 2021.

Our goal in this paper is to provide a better understanding of the interplay between a firm's pricing decisions during a disruption and its utilisation of reserve inventory or reserve capacity to mitigate the impact of disruptions. Furthermore, we also specifically explore how a firm's disruption *pricing flexibility*, i.e., the extent the firm can increase prices during a disruption to better align demand with restricted supply, influences the extent to which it will utilize the operational measures of reserve inventory or reserve capacity.

To do so, we consider a firm producing a single product that is exposed to random supply disruptions, resulting in a production stop for a random time length. We model a two-stage problem where in the first stage the firm decides on the optimal amount of reserve inventory or reserve capacity to carry in order to meet demand during an anticipated disruption. When a disruption occurs, the firm decides, in the second stage, on the optimal price to charge during the disruption, given the available reserve inventory or reserve capacity it has previously decided to carry (if any). We specifically address the following questions: (1) When faced with a disruption, how does a firm decide whether and to what extent it should increase its price during a disruption, while taking into account any reserve inventory or reserve capacity it has been carrying in anticipation of the disruption? (2) How should a firm decide on the level of reserve inventory or reserve capacity to carry in anticipation of a disruption taking into account that it can optimally adjust its price during a disruption? (3) How does the extent a firm can increase its price during a disruption, i.e., the pricing flexibility it possesses impacts its initial decision on the amount of reserve inventory or reserve capacity to carry? The latter question enables us to provide additional insights in case there are any limitations to the extent a firm can increase its prices during a disruption (e.g., prior price commitments or a government regulation). Such upper price bounds have been utilized in the pricing literature in various contexts (Federgruen and Heching 1999, Gong et al. 2014).

For the case of utilising reserve inventory as a disruption mitigation strategy, we find that depending on the length of the disruption in relation to the amount of reserve inventory the firm is holding, it may be optimal for the firm to maintain its current pricing level for relatively short disruptions, or implement a price surcharge to suppress demand in order to spread the use of the reserve inventory throughout the disruption period. As would be expected, we find that higher reserve inventory levels reduces the magnitude of price increases during disruptions. Regarding the impact of pricing flexibility on reserve inventory levels however, we find that a higher pricing flexibility can induce the firm to carry either more or less reserve inventory. Specifically, a certain level of pricing flexibility may be required to justify holding any reserve inventory. Furthermore, if a firm does decide to carry reserve inventory, the level of inventory it chooses to carry may either decrease or increase with its pricing flexibility, depending on whether it would target to address demand during only a short disruption period or for a longer disruption. Therefore, we find that pricing flexibility and reserve inventory may be complements or substitutes.

Regarding disruption pricing with reserve capacity, we again similarly find that as the firm has more reserve capacity going into the disruption, it will apply lower price surcharges. However, as opposed to the pricing policy in the presence of reserve inventory, we now find that the firm's disruption price for a given reserve capacity level is independent of the length of the disruption. In terms of the impact of pricing flexibility on the firm's optimal reserve capacity choice, we again find that a certain level of pricing flexibility may be required to justify any investment in reserve capacity. However, compared to the reserve inventory case, we now find that once the firm does decide to invest in reserve capacity, if it has more pricing flexibility, it will choose to carry a lower amount of reserve capacity. Therefore, if it is optimal to carry reserve capacity, pricing flexibility and reserve capacity act only as substitutes.

In addition to the optimal policy characterizations, we also conduct a numerical study to further investigate the sensitivity of reserve inventory or reserve capacity decisions when the firm may or may not have pricing flexibility to increase prices during a disruption with respect to various problem parameters such as disruption rates, disruption duration, and carrying costs for reserve inventory or reserve capacity.

Overall, by showing that reserve inventory and reserve capacity decisions may have different complementarity and substitutability relationships with respect to the pricing flexibility a firm may have during a disruption, we believe our work provides interesting and important insights into the interplay of these disruption mitigation strategies and price increases during disruptions.

2. Literature Review

The research topic of managing disruptions in supply chains has become an important research domain over the last years. One of the foundational articles in this area is by Tomlin (2006), who focuses on a setting where a buyer can source from either an unreliable supplier or a reliable, and thus more expensive, one. The reliable supplier is assumed to have a reserve capacity or volume flexibility that allows the buyer to increase the order quantity up to a certain limit when the unreliable supplier is disrupted. The author finds that there is significant value in using the reserve capacity of the reliable supplier if disruptions tend to be less frequent but long. By contrast, more frequent and shorter disruptions tend to be best mitigated by using reserve inventory. This setting has been extended along various dimensions, such as sourcing from multiple suppliers (Dada et al. 2007), differentiating between recurrent supply uncertainty and disruption risk (Chopra et al. 2007), including operational aspects such as delivery delays (Qi 2013), or expanding to multiple echelons (Lücker et al. 2021). We refer to the literature review by Snyder et al. (2016) for a more complete discussion of various extensions and more generally also to consider other risk mitigation levers such as increasing the reliability of suppliers. Across these papers, prices are assumed to be fixed and therefore the firms do not influence disruption period demand through price changes.

Pricing decision as a tool to better match supply and demand has been studied extensively in the absence of disruptions. A typical setup considers a firm that orders inventory from a supplier and sells the goods to customers who are price-sensitive, i.e., demand depends on the product's price. Van Mieghem and Dada (1999) study the value of pricing flexibility when a firm makes capacity and inventory decisions while facing uncertain demand. The authors consider a two-stage problem where in the first stage inventory is decided and in the second stage the price is set when the uncertainty is resolved. Federgruen and Heching (1999) study the joint optimal decision of inventory replenishment and dynamic pricing under demand uncertainty. The authors quantify significant benefits of adopting dynamic pricing relative to a static price case. Some scholars have relaxed the common assumption in the pricing literature that supply always arrives in full. In this literature stream, a firm may be exposed to yield uncertainty where only a part of an order arrives or whether a specific order arrives or does not arrive. Li and Zheng (2006) study a firm ordering goods from a supplier with random yield while facing price-sensitive, stochastic demand. The authors determine optimal joint ordering and pricing policies. Feng (2010) considers a similar setup where the capacity level is subject to random variations. This work has then been extended by Chen et al. (2013) where the firm has access to multiple unreliable suppliers. The authors identify the optimal set of order quantities and prices. Further, Gong et al. (2014) study a similar setting where one supplier is a more expensive quick-response supplier whereas another supplier is a cheaper supplier with a longer leadtime. Crucially, in all these papers, prices are set before the yield is known. This is a different setting from ours, where prices are potentially changed once a disruption starts.

In contrast to the above-mentioned literature, there are also a few papers that consider a setting with yield uncertainty where prices are set after the yield information is revealed (also referred to as responsive-pricing). In these settings, order quantities are placed with unreliable suppliers and prices are set to maximize profits after inventories are received (Tang and Yin 2007). Shan et al. (2022) consider a setup where a retailer orders supply from two competing suppliers with correlated disruption risks. A study by Li et al. (2013) also focuses on the correlation of the reliability between two competing suppliers, but identifies optimal decisions taken by the buyer. Li et al. (2017) consider a setting where a buyer can diversify orders among various suppliers. The authors find that order diversification and responsive pricing may be complements as well as substitutes. Dong et al. (2023) extend this work by modelling supply uncertainty in the form of proportional random yield. Geng et al. (2023) study a decentralized supply chain where a buyer purchases goods from a supplier with uncertain yield. The buyer may use responsive pricing while using different supplier payment schemes. Using a game-theoretic approach, the authors develop insights on designing procurement contracts that help mitigate supply risks. Kouvelis et al. (2021) elaborate on the role of risk-aversion when a firm sources from a supplier with uncertain yield (while using responsive pricing). Some papers explicitly study responsive pricing when there is a risk of supply chain disruptions (rather than yield uncertainty). Gheibi and Fay (2021) consider a retailer that sources two vertically differentiated products from two different suppliers- one reliable and one unreliable. The authors develop various insights on the optimal joint pricing strategy of both products and the optimal procurement strategy from both suppliers (compare also Alptekinoğlu et al. (2023) for a two-product setup). The authors of both papers do not consider the role of reserve inventory or reserve capacity to protect from stockouts. Xing et al. (2022) consider emergency supply and responsive pricing. Emergency supply refers to an alternative supplier with infinite capacity that comes at zero cost upfront. This is different from a constraint reserve capacity where for each unit of capacity an upfront reservation fee per unit of time must be paid. Finally, Feng et al. (2022) draw insights from analyzing the behaviour of customers during a disruption when pricing flexibility is used. Our work contributes to the above literature as we study the joint use of reserve inventory and responsive pricing as well as the joint use of reserve capacity and responsive pricing.

3. Model Preliminaries

We consider a firm that produces a single product and that employs *reserve inventory* or *reserve capacity* to mitigate the effects of random disruptions. We assume the firm also possesses some *pricing flexibility* to increase its price during a disruption to better align demand with its limited supply during a disruption. Our main objective in this study is to provide insights on the impact of pricing flexibility during a disruption on a firm's reserve inventory or capacity decisions that it takes in anticipation of disruptions.

The reserve inventory is a fixed quantity of I units that the firm decides to hold during the non-disrupted time periods with a holding cost rate of h per unit and per unit time. The firm can then sell from the reserve inventory to partially or fully meet the demand during a disruption. We let u denote the replenishment/production cost per unit for the reserve inventory.

The reserve capacity, on the other hand, is the level of ancillary replenishment/production capability that the firm has obtained access to during a disruption and can provide a steady production of a units per unit time during the disruption at a cost of $c_a \geq u$ per unit. Further, we let c denote the capacity reservation cost per reserve capacity production rate.

To model the firm's pricing decisions, we assume a linear price-demand relationship with a demand rate of $d(p) = b_0 - b_1 p$, where p is the price the firm charges, b_0 is the demand intercept, and b_1 is the price sensitivity coefficient. (We assume $b_0, b_1 > 0$.) For future reference, the profit maximizing price for the non-disrupted period, $p_o := \arg\max_p (p-u)\,(b_0 - b_1 p)$, can be found as $p_o = \frac{b_o}{2b_1} + \frac{u}{2}$, which we will refer to as the *base price*. We also let $r(p_o) := (p_o - u)\,d(p_o)$ denote the base profit rate per unit time.

The general timeline of the firm's decisions for both the reserve inventory and reserve capacity setting is as follows: First, the firm decides on the level of reserve inventory, I, or reserve capacity, a, to invest in before experiencing the disruption. When a disruption occurs, the firm may adjust its price, taking into account its reserve inventory or capacity level as well as the length of the disruption, k. In order to capture the uncertainty in the disruption time, we consider a two-point distribution where the disruption length can be either short or long. Specifically, we assume that the disruption will have a length of k_s with probability q or a length of $k_l > k_s$ with probability $1 - q$. This framework facilitates the characterization of the optimal solution. As is commonly assumed in the disruption risk literature (Ang et al. 2017, Lücker et al. 2021, Dada et al. 2007), we assume that once a disruption occurs, the firm is able to foresee whether the disruption is of short or long nature for pricing purposes.

The notation for the main decisions and model parameters are provided in Table 1.

Decision variable	
I	Reserve inventory
a	Reserve capacity production rate
p	Price charged during the disruption
Parameters	
p_0	List price in undisrupted periods
b_0	Demand intercept
b_1	Price sensitivity coefficient
h	Inventory holding cost per unit and per unit time
u	Replenishment/production cost per unit for the reserve inventory
c	Reserve capacity reservation cost per capacity production rate
c_a	Unit production cost of the reserve capacity
$k \in k_s, k_l$	Disruption length, short or long
q	Probability that disruption is short
$1/\beta$	Expected disruption duration
$1/\alpha$	Expected time of not being disrupted

Table 1: Decision variables and parameters of the model

In the next section, we focus on pricing flexibility with reserve inventory. Subsequently, we will focus on pricing flexibility with reserve capacity.

4. Responsive Pricing with Reserve Inventory

In the following, we will first describe the firm's optimal pricing problem during a disruption given reserve inventory levels and then formulate the initial problem of setting the optimal reserve inventory levels taking into account the optimal pricing decisions that the firm will apply during a subsequent disruption.

4.1 Problem Formulation

4.1.1 Pricing During a Disruption

We start by describing the firm's pricing problem during a disruption, given a reserve inventory level. Suppose the firm enters a disruption period of length k with a reserve inventory of I units. The firm's optimal price setting problem can be stated as:

$$\tilde{\Pi}_d(I,k) = \max_p \quad (p-u)\min\{I,(b_0-b_1p)\,k\} \tag{1}$$

where $\tilde{\Pi}_d(I,k)$ denotes the optimal disruption profit. As the formulation given in (1) indicates, the objective function is piecewise in disruption price. Specifically, we have the following two cases:

(i) $(b_0-b_1p)\,k < I$. In this case, the reserve inventory exceeds disruption demand at price p. Therefore, inventory is partially used to meet the disruption period demand. The firm's disruption profit function becomes: $(p-u)\,(b_0-b_1p)\,k$.

(ii) $I \leq (b_0-b_1p)\,k$. In this case, the reserve inventory is used fully. The profit function thus becomes $(p-u)\,I$, obtained through selling the entire reserve inventory.

4.1.2 Setting Reserve Inventory Levels

Next, we formulate the initial problem of setting optimal reserve inventory levels taking into account the optimal pricing decisions that the firm will apply during a subsequent disruption.

During the non-disrupted stage, and in anticipation of a future disruption, the firm sets a reserve inventory level that it may utilize during a subsequent disruption. We aim to maximize long-run expected profit per unit time. We assume that after every disruption, the supply chain returns to the undisrupted stage before the next disruption occurs. This allows us to define a renewal cycle as a period of no disruption followed by a period with one disruption. The duration of a renewal cycle is then defined as the expected time duration of not being disrupted plus the expected time duration of one disruption. Specifically, let α and β denote, respectively, the disruption rate and the recovery rate. Hence, the expected time of not being disrupted is $\frac{1}{\alpha}$ while the expected time duration of one disruption is $\frac{1}{\beta}$. Thus, the expected renewal cycle length is given by $\frac{1}{\alpha}+\frac{1}{\beta}$. Based on this definition of the renewal cycle we use the well-known renewal-reward theorem to calculate the long-run expected profit per unit time. The long-run expected profit $\mathbb{E}[\Pi(I)]$ is the ratio of the expected profit per cycle and the expected renewal cycle length. In order to determine the expected profit per cycle, we first introduce the expected profit when there is no disruption, $\mathbb{E}[\Pi_0(I)]$, and the expected profit during a disruption, $\mathbb{E}[\Pi_d(I)]$.

The expected profit when there is no disruption, $\mathbb{E}[\Pi_0(I)]$, includes the base profit rate $r(p_o)$ minus the costs associated with holding the reserve inventory. Specifically, if the firm holds I units of reserve inventory at a cost of h per unit per unit time, the expected profit when there is no disruption can be stated as:

$$\mathbb{E}[\Pi_0(I)] = \frac{r(p_o) - h\,I}{\alpha}. \tag{2}$$

The expected profit when there is a disruption, $\mathbb{E}[\Pi_d(I)]$, takes into account the optimal pricing decision corresponding to (1). Recall that the disruption will have a length of k_s with probability q or a length of $k_l \geq k_s$ with probability $1-q$. Therefore, we have $1/\beta = q\,k_s + (1-q)\,k_l$. Consequently, the expected profit when there is a disruption is given by

$$\mathbb{E}[\Pi_d(I)] = q\,\tilde{\Pi}_d(I, k_s) + (1-q)\,\tilde{\Pi}_d(I, k_l) \tag{3}$$

where $\tilde{\Pi}_d(I, k)$, $k \in \{k_s, k_l\}$ is described in (1).

For expositional clarity of our problem formulation, we do not account for the holding cost of any remaining inventory during a disruption period as for any reasonable values for the parameters α and β, we stay in a disrupted period for a relatively shorter duration compared to the non-disrupted period. This assumption has also been used in works such as Lücker et al. (2021). We note that we do account for the cost to bring the inventory back to the desired reserve inventory target at the end of a disruption (specifically, through the unit replenishment/production cost u in (1)). Our complementary numerical analysis indicates that this assumption does not qualitatively impact our main insights.

The long-run expected profit is thus given by

$$\mathbb{E}[\Pi(I)] = \frac{\mathbb{E}[\Pi_0(I)] + \mathbb{E}[\Pi_d(I)]}{\frac{1}{\alpha} + \frac{1}{\beta}} \tag{4}$$

and the firm's first stage reserve inventory problem can be stated as $\max_I \mathbb{E}[\Pi(I)]$.

4.2 Optimal Disruption Period Pricing under Reserve Inventory

In this subsection, we provide insights on the optimal solution to the two-stage problem. We start by providing the characterization of the optimal disruption period price $p^*(I,k)$ for a given reserve inventory level I, and a disruption of length k that results from the pricing problem described in (1).

Proposition 1 *For a given reserve inventory level I, and a disruption length of k, the optimal disruption price, $p^*(I,k)$ is as follows:*

$$p^*(I,k) = \begin{cases} \frac{b_o}{2b_1} + \frac{u}{2}, & \text{if } k < \frac{2I}{b_0 - ub_1} \\ \\ \frac{b_0 - I/k}{b_1}, & \text{if } k \geq \frac{2I}{b_0 - ub_1} \end{cases} \tag{5}$$

The proof of Proposition and all subsequent results are provided in the Appendix.

The result summarized in (5) indicates that the optimal disruption price may take two different forms depending on the disruption length.

For short disruptions, i.e., if $k < \frac{2I}{b_0 - ub_1}$, it is optimal for the firm to charge $p^*(I,k) = \frac{b_o}{2b_1} + \frac{u}{2}$, i.e., to continue applying the list price p_o. In this case, the disruption price is independent of the disruption length.

If the disruption length k is longer, satisfying $k \geq \frac{2I}{b_0 - ub_1}$, then it is optimal for the firm to charge $p^*(I,k) = \frac{b_0 - I/k}{b_1}$, which is the price that will suppress demand such that the reserve inventory will be depleted just at the end of the disruption period. In this case, the disruption price is increasing in the disruption length.

We also note the following sensitivity results regarding the optimal price during a disruption. (We use the terms increasing and decreasing in the weak sense of nondecreasing and nonincreasing, respectively.)

Proposition 2 *The optimal price, $p^*(I,k)$, is decreasing in the reserve inventory level I, and increasing in the disruption length k. In addition, the optimal price is increasing in the demand intercept b_o, decreasing in the price sensitivity coefficient b_1, and increasing in the reserve inventory usage (replacement) cost u.*

Proposition 2 describes how the optimal price applied during a disruption changes with the problem parameters. First, we find that if the firm enters a disruption with a larger amount of reserve inventory, then it would charge a lower price during the disruption. (We note that the term lower is in the weak sense that the firm may also keep the same price, but it would never be optimal for the firm to increase its price.) Regarding the disruption length, if the disruption length is longer, then it is also optimal for the firm to charge a higher price during the disruption. The firm also charges a higher price if the demand intercept is stronger or if the demand is less sensitive to price increases. Finally, if the inventory replacement cost for the reserve inventory is higher, then it would be optimal for the firm to increase the price it charges during the disruption.

4.3 Impact of Pricing Flexibility during a Disruption on Optimal Reserve Inventory Levels

Next, we study how the level of pricing flexibility a firm has during a disruption impacts its reserve inventory decision. To do so, we first introduce an exogenous upper bound on the price that a firm can charge during the disruption, denoted by $\bar{p}$ such that $p_o \leq \bar{p} \leq b_0/b_1$. That is, the firm's optimal price selection for the disruption period is $\min\{p^*(I,k), \bar{p}\}$. Such upper bounds have been utilized in the literature in various pricing contexts to provide more granular insights (see for example, Federgruen and Heching (1999), Gong et al. (2014), Ceryan et al. (2018)). We then investigate how an increase in this price upper bound impacts the firm's reserve inventory decisions.

Recall from Proposition 1 that when the firm enters a disruption period of length k with a reserve inventory level of I, it will continue to apply the base price p_o if $k < \frac{2I}{b_0 - ub_1}$. For longer disruptions, i.e., for $k \geq \frac{2I}{b_0 - ub_1}$, and in the presence of a price bound $\bar{p}$, it will charge $\min\{\frac{b_0 - I/k}{b_1}, \bar{p}\}$. Intuitively, as the firm increases its inventory level, it may shift its pricing policy from charging $\bar{p}$ to $\frac{b_0 - I/k}{b_1}$, and finally to p_o. As the disruption length can be either short or long, a particular reserve inventory position may lead the firm to charge different prices for different disruption lengths. Therefore, depending on the disruption length, these price transitions can occur at different inventory levels. We find that the firm's objective function of expected profit per unit time results in a piecewise unimodal function with respect to the reserve inventory. We provide the main findings below.

Proposition 3 *(a) If $\bar{p} < \frac{h}{\alpha} + u$, the firm does not invest in reserve inventory, i.e., $I^* = 0$. If $\bar{p} \geq \frac{h}{\alpha} + u$, the firm selects a reserve inventory level of $I^* \geq k_s(b_0 - b_1 \bar{p})$, i.e., at least to cover demand during a short disruption at the price bound.*
(b) The firm's optimal reserve inventory level may increase or decrease with or be independent of the price bound $\bar{p}$.

There are two main implications of Proposition 3. First, part (a) indicates that the price bound the firm is allowed to charge up to ($\bar{p}$) must be at least at some sufficiently high level for it to justify the firm to hold reserve inventories. In other words, this implies that there may need to be at least some pricing flexibility for the firm to hold reserve inventories. As can be intuitively expected, the required minimum price bound is increasing in the inventory holding cost (h), the length of the non-disrupted stage the firm will be holding this inventory for (i.e., $1/\alpha$), and the production cost of the item (u). We also find that the inventory level the firm selects will, at the minimum, fully meet the demand at the price bound for a short disruption length.

Second, part (b) of Proposition 3 indicates that the optimal inventory level may be increasing or decreasing with the price bound or be independent of the price bound. The explanation is as follows: In instances where the firm's optimal inventory selection is such that it charges $\frac{b_0 - I/k}{b_1}$ if the disruption is short and $\bar{p}$ if the disruption is long, a marginal increase in $\bar{p}$ prompts the firm to increase the inventory level as the disruption period profit it gains from this additional inventory outweighs the additional holding cost. As a side note, in this case, the firm's disruption profit is linearly increasing in inventory for long disruptions (as each additional inventory can be sold at $\bar{p}$) and concavely increasing in inventory (i.e., with diminishing returns) for short disruptions (as each additional inventory will now result in a slightly lower price to be sold throughout the short disruption duration). In other instances, the firm's optimal inventory selection may be such that it targets to fully meet either the short disruption demand at the price bound or the long disruption demand at the price bound. For these instances, the firms optimal inventory level decreases with the price bound. Lastly, the optimal inventory decisions may be such that the firm applies p_o during a short disruption and either $\frac{b_0 - I/k}{b_1}$ or p_o in a long disruption, for which the firm's reserve inventory decisions are independent of the price bound. Overall, Proposition 3 indicates that pricing flexibility and reserve inventories may be complements or substitutes.

Having analyzed the reserve inventory setting, we next focus on the reserve capacity setting.

5. Responsive Pricing with Reserve Capacity

The reserve capacity is the level of auxiliary production capability that the firm can obtain access to during a disruption and can provide a steady production of a units per unit time during the disruption at a cost of c_a per unit. As we have previously mentioned, the firm also pays a reservation cost of c per rate for this capacity per unit time.

The timeline of the firm's decisions is as follows: First, the firm decides on the level of reserve capacity, a, to invest in before experiencing the disruption. When a disruption occurs, the firm may adjust its price taking into account its reserve capacity level as well as the length of the disruption, $k \in \{k_s, k_l\}$.

We will first describe the firm's pricing problem during a disruption given reserve capacity and then formulate the initial problem of setting optimal reserve capacity levels taking into account the optimal pricing decisions that the firm will apply during a subsequent disruption.

5.1 Problem Formulation

5.1.1 Pricing During a Disruption

In this subsection, we describe the firm's pricing problem during a disruption, given a reserve capacity level. Suppose the firm enters a disruption period of length k with a reserve capacity of a units per unit time. The firm's optimal price setting problem can be stated as:

$$\tilde{\Pi}_d(a,k) = \max_p \quad (p - c_a)\min\{a\,k, (b_0 - b_1 p)\,k\} - c\,a\,k \tag{6}$$

where $\tilde{\Pi}_d(a,k)$ denotes the optimal profit generated during the disruption. The first part of (6) is the profit from selling while producing goods using the reserve capacity. The remaining term $c\,a\,k$ refers to the cost for reserving the reserve capacity during the disruption time k. As the formulation given in (6) indicates, the objective function is again piecewise in disruption price. Specifically, we have the following two cases:

(i) $(b_0 - b_1 p)\,k < a\,k$. In this case, the reserve capacity is partially used to meet disruption period demand at price p. The firm's corresponding disruption profit function is given by $(p - c_a)\,(b_0 - b_1 p)k - c\,a\,k$.

(ii) $a\,k \le (b_0 - b_1 p)\,k$. In this case, the reserve capacity is used fully. The profit function thus becomes $(p - c_a)\,a\,k - c\,a\,k$, where the profit obtained through utilizing the entire reserve capacity.

5.1.2 Setting Reserve Capacity Levels

In this subsection, we formulate the initial problem of setting the optimal reserve capacity level taking into account the optimal pricing decisions that the firm will apply during a subsequent disruption.

During the non-disrupted stage, and in anticipation of a future disruption, the firm sets a reserve capacity level that it may utilize during a subsequent disruption. We aim to maximize long-run expected profit per unit time, as in the previous section. Considering the same renewal cycle as in the previous section, the long-run expected profit is given by

$$\mathbb{E}[\Pi(a)] = \frac{\mathbb{E}[\Pi_0(a)] + \mathbb{E}[\Pi_d(a)]}{\frac{1}{\alpha} + \frac{1}{\beta}} \tag{7}$$

where $\mathbb{E}[\Pi_0(a)] := \frac{r(p_o) - c\,a}{\alpha}$ denotes the expected profit when there is no disruption and $\mathbb{E}[\Pi_d(a)] := q\,\tilde{\Pi}_d(a,k_s) + (1-q)\,\tilde{\Pi}_d(a,k_l)$ denotes the expected profit when there is a disruption, similar to the formulation in Section 4.1.2. Therefore, the firm's first stage reserve capacity problem can be stated as $\max_a \mathbb{E}[\Pi(a)]$.

5.2 Optimal Disruption Period Pricing under Reserve Capacity

We begin our characterization of the optimal policy by first providing the characterization of the optimal disruption period price for a given reserve capacity level.

Proposition 4 *For a given reserve capacity level of a, the optimal disruption price, $p^*(a)$ is as follows:*

$$p^*(a) = \begin{cases} \frac{b_0 - a}{b_1}, & if a \leq \frac{b_0 - c_a b_1}{2} \\ \\ \frac{b_o}{2b_1} + \frac{c_a}{2}, & if a > \frac{b_0 - c_a b_1}{2} \end{cases} \tag{8}$$

The optimal price $p^*(a)$ is decreasing in reserve capacity production rate a and is independent of the disruption length, k. In addition, the optimal price is increasing in the demand intercept b_o and reserve capacity production cost c_a, and decreasing in the price sensitivity coefficient b_1.

The independence of the pricing decision with respect to the disruption length allows us to make a further observation that facilitates our subsequent analysis. First, we observe that the firm picks the price that always fully utilizes its reserve capacity level as long as this price does not fall below the price of $\frac{b_o}{2b_1} + \frac{c_a}{2}$. On the other hand, the price level $\frac{b_o}{2b_1} + \frac{c_a}{2}$ implies a partial use of reserve capacity. As reserve capacity is costly, and its use does not depend on the disruption length, we can note that there is no economic benefit for the firm to have invested in excess reserve capacity to only partially use it since any reduction in the underutilized reserve capacity would improve its profit. In other words, the firm would only invest in a reserve capacity level that it intends to fully utilize during a disruption. Therefore, for the remainder of this section and to facilitate our analysis, we can limit our attention to reserve capacity levels that would be fully utilized during a disruption, corresponding to the price expression $\frac{b_0 - a}{b_1}$ for any given such capacity level.

5.3 Impact of Pricing Flexibility during a Disruption on Optimal Reserve Capacity Levels

As in the previous section, we will next consider how the level of pricing flexibility a firm has during a disruption impacts its reserve capacity decisions. We will use the exogenous upper bound on the price that a firm can charge during the disruption as introduced in the previous section: $\bar{p}$. That is, the firm's optimal price selection for the disruption period is $\min\{p^*(a), \bar{p}\}$. We then investigate how an increase in this price upper bound impacts the firm's reserve capacity decisions. Similar to the previous section, we investigate how an increase in the upper price bound $\bar{p}$ impacts the firm's reserve capacity decisions.

Recalling again the optimal pricing decisions outlined in Proposition 4 and our subsequent discussion on the sufficiency of considering instances where reserve capacity is utilized fully, we observe that if the firm enters a disruption with a reserve capacity level of a, then it will charge $\min\{\frac{b_0-a}{b_1}, \bar{p}\}$. That is, the firm will charge the price bound $\bar{p}$ for low levels of reserve capacity (i.e., $a < b_0 - b_1\,\bar{p}$) and charge $\frac{b_0-a}{b_1}$ for higher reserve capacity levels (i.e., $a \geq b_0 - b_1\,\bar{p}$). We have the following result regarding the firm's optimal reserve capacity decision.

Proposition 5 *(a) If $\bar{p} < c\frac{\alpha+\beta}{\alpha} + c_a$, the firm does not invest in reserve capacity, i.e., $a^* = 0$. If $\bar{p} \geq c\frac{\alpha+\beta}{\alpha} + c_a$, the firm selects a reserve capacity level $a^* \geq b_0 - b_1\,\bar{p}$, i.e., at least to cover demand rate at the price bound.*
(b) Once it is optimal for a firm to invest in reserve capacity, the optimal reserve capacity level may decrease with or be independent of the price bound $\bar{p}$.

Part (a) of Proposition 5 indicates that, similar to our previous finding regarding the reserve inventory decisions, we find that the firm will invest in reserve capacity only if there is sufficient pricing flexibility, i.e., if $\bar{p} \geq c\frac{\alpha+\beta}{\alpha} + c_a$. Specifically, the firm may not find it economical to invest in reserve capacity if the price bound is relatively low, cost of reserving the reserve capacity, c, is high, the per unit production fee through this capacity, c_a, is high, the expected non-disrupted stage is long, or the expected disruption is short. Further, if the firm decides to invest in reserve capacity, it does so to be able to at least cover demand during disruption at a demand rate corresponding to the price bound $\bar{p}$.

Proposition 5 part (b) shows that the optimal capacity investment may decrease with the price bound or may be independent of the price bound. Specifically, if the optimal reserve capacity decision exactly targets to cover demand during disruption at a demand rate corresponding to the price bound $\bar{p}$, then the required capacity decreases as $\bar{p}$ increases. If, on the other hand, the firm's optimal capacity selection exceeds this rate, then the optimal capacity selection is found to be independent of the price bound.

In summary, Proposition 5 indicates that the firm may require some pricing flexibility to justify investing in reserve capacity and would then select a (weakly) lower capacity level as it has further pricing flexibility. Therefore, compared to the reserve inventory case described in the preceding section, we find that once a firm decides to invest in reserve capacity, the reserve capacity level and disruption period pricing flexibility act only as substitutes. In other words, the main difference between the relationship between the optimal reserve capacity and pricing flexibility as compared to the relationship between the optimal reserve inventory and the price flexibility is that once a firm decides to invest in reserve capacity, any increase in pricing flexibility may only decrease the optimal reserve capacity level whereas an increase in pricing flexibility may both increase and decrease the optimal reserve inventory levels.

6. Numerical results

A key objective of this numerical section is to illustrate how reserve inventory and capacity are used differently when there is pricing flexibility relative to the case when there is no pricing flexibility.

6.1 Reserve inventory

In this subsection, we study numerically the optimal reserve inventory as a function of various model parameters. Specifically, we use a base case with the following parameters: demand intercept, $b_0 = 20$, price sensitivity coefficient, $b_1 = 2$, probability of a disruption, $\alpha = 0.1$, short disruption length, $k_s = 1$, long disruption length, $k_l = 3$, probability of a short disruption, $q = 0.5$ (with probability of a long disruption $1 - q = 0.5$), regular inventory replenishment cost per unit, $u = 2$, and reserve capacity holding cost per unit and per time, $h = 0.1$. We first would like to note that, these parameter values lead to a base price of $p_0 = 6$ and a corresponding base demand rate of 8 units per unit time. Next, we plot the optimal reserve inventory level while varying one of the above parameters for the case of having pricing flexibility and the case of not having pricing flexibility.

In Figure 1, we plot the optimal reserve inventory as a function of the inventory holding cost, h. We observe that we generally hold less inventory when there is pricing flexibility during a disruption

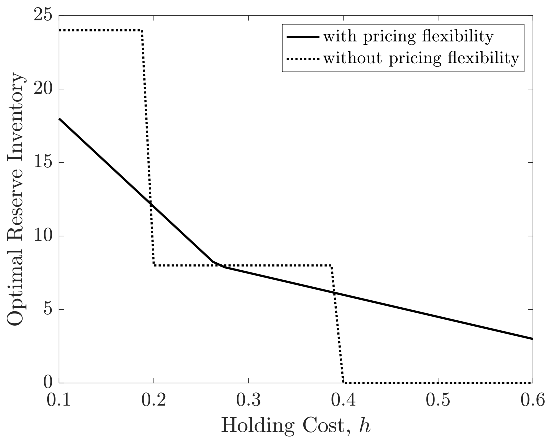

Figure 1: Optimal reserve inventory as a function of inventory holding cost

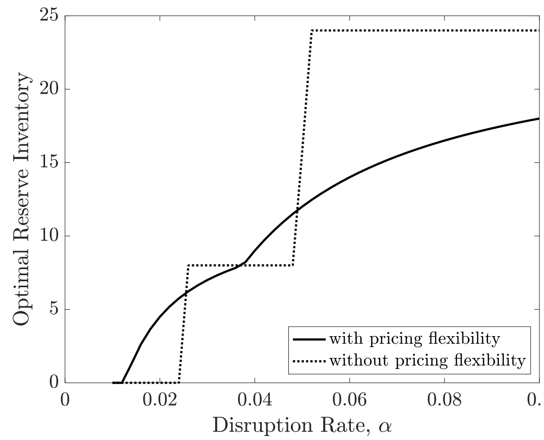

Figure 2: Optimal reserve inventory as a function of disruption rate α

relative to no pricing flexibility when the inventory holding cost is relatively cheap. The optimal price during the disruption is typically higher than the list price. Thus, when the firm has the ability to increase the price during the disruption, the higher price results in reduced demand during the disruption which in turn results in less inventory carried. As the inventory holding cost gets more expensive (e.g., $h \approx 0.2$ in this case), we observe that the amount of inventory carried drops from $I = 24$ to $I = 8$ for the case without pricing flexibility. This drop occurs because it becomes too expensive to carry sufficient inventory to serve all demand during the long disruption and the firm prefers to carry enough inventory to only serve all demand during for a short disruption length. Likewise, we observe a decrease in optimal inventory levels for increasing inventory holding costs when there is pricing flexibility. The decline in inventory, however, is now more gradual because demand during a disruption can be reduced by increasing the price. When inventory holding costs are relatively expensive, we observe that we hold more inventory with pricing flexibility relative to no pricing flexibility at optimality. In the absence of pricing flexibility, inventory may be too expensive to carry relative to the profit achieved during the disruption. However, with pricing flexibility, it may be optimal to carry some inventory as this can be sold at a high price during a disruption, yielding a higher disruption profit.

In Figure 2, we plot the optimal reserve inventory as a function of the disruption rate, α. We observe that we generally hold more inventory if there is pricing flexibility during disruptions relative to no pricing flexibility when the disruption rate is relatively low. In the absence of pricing flexibility, carrying inventory may not be optimal for low disruption rates, given that the inventory is unlikely to be used. However, with pricing flexibility, it may be optimal to carry some inventory even at relatively low disruption rates. In case of a disruption, a higher profit can be achieved because the optimal price during the disruption is higher than the list price when there is limited supply to meet disruption demand. Thus, the expected profit generated during a disruption may be sufficiently high to justify carrying some inventory. As the disruption rate increases, more inventory is carried. With pricing flexibility, we observe that we hold less inventory relative to no pricing flexibility at optimality. With pricing flexibility, the firm maximizes long-term profit by saving on inventory holding costs (by holding less inventory) while experiencing only a modest decline in profit during the disruption, given the option to increase prices when supply is limited during a disruption.

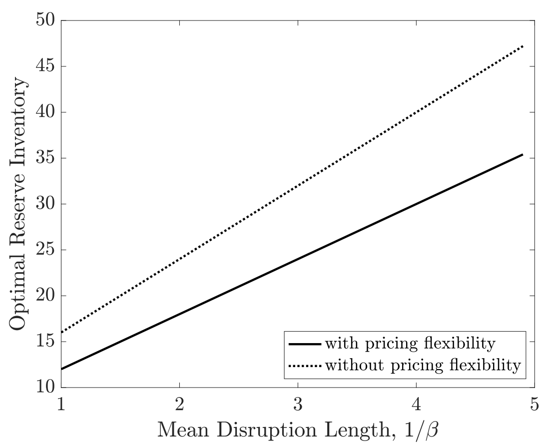

Figure 3: Optimal reserve inventory as a function of mean disruption time

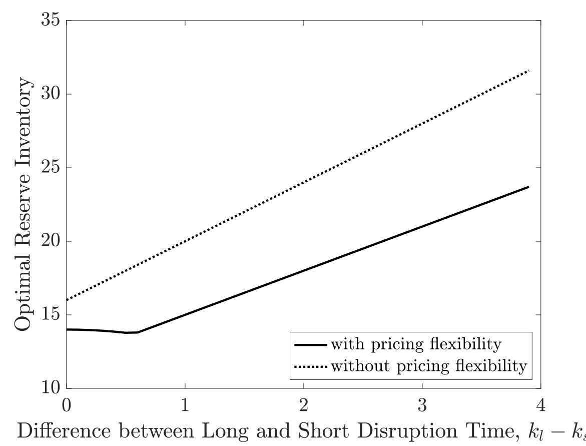

Figure 4: Optimal reserve inventory as a function of the range of disruption times

Next, in Figure 3, we plot the optimal reserve inventory as a function of the mean disruption time, $1/\beta$, while keeping the difference between disruption lengths constant at $k_l - k_s = 2$. We observe that the longer the disruption lasts in expectation, the more inventory we carry. We also observe that the inventory increase is higher for the no-flexibility case relative to having pricing flexibility. The reason is that with pricing flexibility, the firm can save inventory holding costs by carrying less inventory while experiencing only a modest profit decline during the disruption because of having the option to increase the price during the disruption. As the expected disruption time gets longer, more demand can be suppressed during the disruption by charging a disruption price above the list price, which is reflected in a smaller slope for the case with disruption pricing flexibility.

Lastly, in Figure 4, we plot the optimal reserve inventory as a function of the difference in disruption time between the long and short disruptions while keeping the mean disruption time constant at $\frac{1}{\beta} = 2$ as in the base case. As we can see, when the firm has pricing flexibility during the disruption period, it needs less inventory overall compared to the instance where there is no pricing flexibility across various disruption length spreads. For most of our parameter ranges, the optimal reserve inventory increases with the difference between the long and short disruption time. This is because at these parameter values, when there is no pricing flexibility, the firm finds it worthwhile to keep inventory to meet demand even for the long disruption. (Note that as the spread between the two disruption lengths increases, this implies that the long disruption gets longer.) In the presence of pricing flexibility, when the spread between the disruptions is small, a slight increase in this spread (i.e., a slight decrease in the short disruption length and a slight increase in the long disruption length) may be met by the firm by mainly price adjustments and therefore reducing the reserve inventory requirement. As the long disruption gets longer, the firm may need to increase its reserve inventory level to meet the demand during the long disruption even with the price adjustments.

6.2 Reserve capacity

In this subsection, we study numerically the optimal reserve capacity as a function of various model parameters. We use the same base case parameters as applicable as we have described previously in the preceding subsection. Specifically, we set the demand intercept at $b_0 = 20$, price sensitivity coefficient at $b_1 = 2$, probability of a disruption at $\alpha = 0.1$, a short disruption length of $k_s = 1$, a long disruption length of $k_l = 3$, and a probability of a short disruption of $q = 0.5$ (with probability of a long disruption $1 - q = 0.5$). In addition, we now define the cost to hold reserve capacity at $c = 0.5$ per unit production rate and per unit time, as well as a cost to produce using this reserve capacity at $c_a = 2.5$ per unit. We again plot the optimal reserve capacity level while varying one of the above parameters for the case of having disruption period pricing flexibility and the case of not having pricing flexibility.

In Figure 5, we plot the optimal reserve capacity as a function of the reserve capacity holding

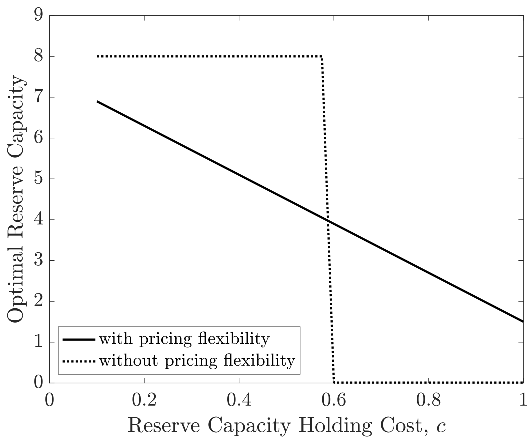

Figure 5: Optimal reserve capacity as a function of reserve capacity holding cost c

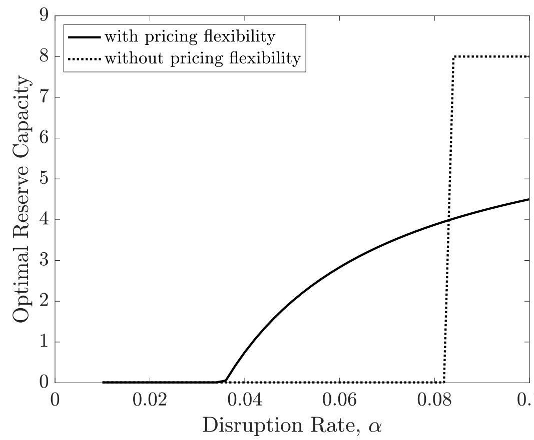

Figure 6: Optimal reserve capacity as a function of disruption rate α

cost, c. The pattern and the rationale is similar to the one in Figure 1: For low holding costs of the reserve capacity, less capacity is used when there is pricing flexibility than when there is no pricing flexibility. As the reserve capacity holding cost increases, the optimal reserve capacity decreases. Interestingly, in the absence of pricing flexibility, the reserve capacity drops from $a = 8$ to $a = 0$ at $c \approx 0.6$. Intermediate levels of reserve capacity ($0 < a < 8$) are never optimal because short and long disruptions require the same amount of reserve capacity to serve all demand during a disruption. In contrast, the reserve capacity reduces continuously as the reserve capacity cost increases when there is pricing flexibility. The reason is that it is optimal to charge a higher disruption period price when the reserve capacity becomes more expensive, which suppresses disruption demand, and in return, also suppresses the capacity required to satisfy this reduced demand rate.

In Figure 6, we plot the optimal reserve capacity level as a function of the disruption rate, α. The pattern and the rational is again similar to our discussion corresponding to Figure 2. For low disruption rates, more capacity is used when there is pricing flexibility than when there is no pricing flexibility. In contrast, for high disruption rates, the pattern reverses. Observe that intermediate levels of reserve capacity ($0 < a < 8$) are never optimal when there is no pricing flexibility because short and long disruptions require the same amount of reserve capacity to serve all demand during a disruption and without pricing flexibility, the firm cannot suppress this demand rate.

In Figure 7, we plot the optimal reserve capacity as a function of the mean disruption time while keeping the spread of the disruption times constant at $k_l - k_s = 2$. We observe that the level of

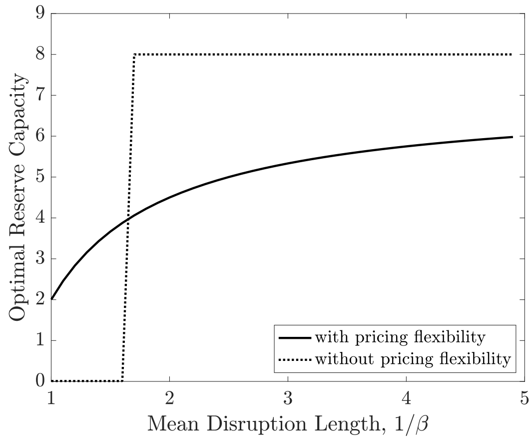

Figure 7: Optimal reserve capacity as a function of mean disruption time

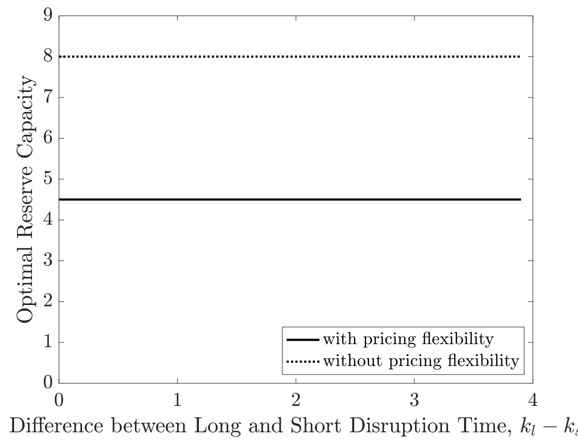

Figure 8: Optimal reserve capacity as a function of the difference between long and short disruption time

reserve capacity increases with the expected disruption time. In the absence of pricing flexibility, no reserve capacity is used at optimality when the expected disruption time is short. This is because the expected disruption profit is too small to justify holding reserve capacity during undisrupted times. Once the expected disruption time increases, the reserve capacity level jumps from $a = 0$ to $a = 8$, because more profit can be generated during the disruption. As the expected disruption length increases further, the level of reserve capacity remains constant in the expected disruption length. This is because the reserve capacity level is sufficient to fully meet all demand during either the short or long disruption. When there is pricing flexibility, the reserve capacity level increases without jumps as the expected disruption time increases. As the expected disruption time gets longer, more profit can be generated during the disruption, justifying carrying larger amounts of reserve capacity. The firm charges a lower price during the disruption as the disruption time increases, which results in more demand during the disruption. Thus, more reserve capacity is carried when the expected disruption time is long.

In Figures 8, we plot the optimal reserve capacity level as a function of the difference between long and short disruption time while keeping the mean disruption time constant. We observe that in the presence and absence of pricing flexibility, the level of reserve capacity is independent of the difference in disruption time. This is because the reserve capacity level is sufficient to fully meet all demand during a disruption (at either the list price or a higher price when pricing flexibility is available). Thus, the variance in disruption time does not influence the decision how much reserve capacity to use. Here, the reserve capacity behaves differently from reserve inventory as illustrated in Figure 4.

7. Conclusions

In this paper, we mainly studied how the extent a firm can adjust its prices during a disruption influences its investment in reserve inventory or reserve capacity to mitigate the impact of disruptions.

To do so, we considered a firm producing a single product and that may experience random disruptions from time to time. The firm first decides on the level of reserve inventory or reserve capacity to hold in anticipation of disruptions. When a disruption does occur, it may then adjust its price to better align demand with the supply restrictions during the disruption. We first characterized the firm's optimal pricing decisions during disruptions. For the reserve inventory setting, we showed that optimal disruption period price decreases with the level of reserve inventory and increases with the length of disruption. For the reserve capacity setting, we find that the optimal disruption period price decreases with the level of reserve capacity but is independent of the disruption length.

For either setting, we find that a certain level of pricing flexibility may be required to justify investing in holding reserve inventory or reserve capacity. Perhaps our most important finding is that while a firm's pricing flexibility during a disruption and reserve capacity decisions are susbtitutes, pricing flexibility and reserve inventory may act either as substitutes or complements. In other words, we find that a firm would reduce its reserve capacity if it possesses further flexibility to increase its prices during a disruption but may prefer to hold either a lower or higher amount of reserve inventory if it has further pricing flexibility to increase its prices during a disruption. Finally, we also conduct numerical studies to further explore the sensitivity of reserve inventory and reserve capacity decisions with respect to factors such as disruption rates, disruption durations, and carrying costs for reserve inventory or reserve capacity for a firm that may or may not possess pricing flexibility.

Our focus in this research has been limited to consider the two operational levers of reserve inventory and reserve capacity separately. As we have described earlier in the Introduction section, the settings under which these tools are preferred are generally different, e.g., whereas lower holding costs favor reserve industry, higher holding costs may favor reserve capacity, in addition to other factors such as frequencies and lengths of disruptions. Though the analysis of either case leads to piecewise unimodal profit functions and makes it very difficult to characterize a joint application of these two operational levels in the presence of pricing flexibility, we believe future work that may provide additional insights in the joint application of these levers and their relative values or joint application with other operational levers such as supplier diversification may also be valuable.

Appendix: Proofs of Propositions

Proof of Proposition 1:

We first note that the objective function for the disrupted period given in (1), i.e., $(p - u)\min\{I, (b_0 - b_1 p)\, k\}$ is piecewise in the decision variable p. Specifically, for $p <= \frac{b_0 - I/k}{b_1}$, the objective function is $(p-u)I$ and the profit is linearly increasing in p. Therefore, the firm will set at least the price $p = \frac{b_0 - I/k}{b_1}$. For $p > \frac{b_0 - I/k}{b_1}$, the objective function is $(p-u)(b_0 - b_1 p)\, k$, which is concave in p. The first order condition results in an optimal price of $p^* = \frac{b_0}{2b_1} + \frac{u}{2}$. Consequently, we also find that for $k < \frac{2I}{b_0 - ub_1}$, we have the interior solution at $p^* = \frac{b_0}{2b_1} + \frac{u}{2}$, and for $k < \frac{2I}{b_0 - ub_1}$, the optimal price is at the boundary at $p^* = \frac{b_0 - I/k}{b_1}$.

Proof of Proposition 2:

Proof of Proposition 2 directly follows from the optimal pricing policy described in Proposition 1. Regarding the sensitivity with respect to the reserve inventory level, I, the optimal price decreases in I for the boundary solution and is independent of I for the interior solution. Further, as I increases, the boundary price also decreases. The sensitivity results for other parameters are derived similarly.

Proof of Proposition 3:

We first note that in the presence of a price bound, $\bar{p}$, the optimal pricing decision provided in Proposition 1 can be extended to be written as:

$$p^*(I,k) = \begin{cases} p_0 = \frac{b_o}{2b_1} + \frac{u}{2}, & \text{if } k < \frac{2I}{b_0 - ub_1} \\ \frac{b_0 - I/k}{b_1}, & \text{if } \frac{2I}{b_0 - ub_1} \le k \text{ and } \frac{b_0}{b_1} - \frac{I}{b_1 k} \le \bar{p} \\ \bar{p}, & \text{if } \frac{b_0}{b_1} - \frac{I}{b_1 k} > \bar{p} \end{cases} \tag{9}$$

Hence, based on the reserve inventory level, the firm may find it optimal to apply different prices depending on whether the disruption is short or long, i.e., $k = k_s$ or $k = k_l$. Specifically, as the firm has more reserve inventory, it may transition from applying $\bar{p}$, $\frac{b_0 - I/k}{b_1}$, and p_0 for various regions of the reserve inventory and these regions may be different for $k = k_s$ or $k = k_l$.

In particular, we distinguish two cases regarding the ordering of these various regions: (a) $k_s(b_o - ub_1)/2 < k_l(b_0 - b1\bar{p})$ and (b) $k_s(b_o - ub_1)/2 > k_l(b_0 - b1\bar{p})$. Equating these cases at the boundary, we can obtain a critical price cap value denoted by p^c such that we have case (a) if $\bar{p} < p^c$, and case (b) otherwise. We first consider case (a).

In case (a), for reserve inventory level, I, between 0 and $k_s(b_0 - b_1\bar{p})$, the firm charges $\bar{p}$ regardless of the length of the disruption. For $k_s(b_0 - b_1\bar{p}) < I < k_s(b_o - ub_1)/2$, it is optimal to charge $\frac{b_0 - I/k}{b_1}$ for short disruptions and continue to charge $\bar{p}$ for long disruptions. For $k_s(b_o - ub_1)/2 < I < k_l(b_0 - b_1\bar{p})$, the firm should charge p_0 for short disruptions and still charge $\bar{p}$ for long disruptions. For $k_l(b_0 - b_1\bar{p}) < I < k_l(b_o - ub_1)/2$, the firm continues to charge p_0 for short disruptions and now charge $\frac{b_0 - I/k}{b_1}$ forlong disruptions. Finally, if $I > k_l(b_o - ub_1)/2$,the firm charges p_0 for both short and long disruptions. (As a side note, holding more reserve inventory than needed the base price and for a long disruption is never optimal.)

Hence, while the firm considers increasing the reserve inventory level, it possibly transitions through five different regions. The expected profit for each of these regions depends on the optimal prices outlined above and is therefore piecewise as given below:

$$\mathbb{E}[\Pi(I)] = \begin{cases} \frac{-hI/\alpha+q_s(\bar{p}-u)I+q_l(\bar{p}-u)I}{1/\alpha+1/\beta}, & \text{if } I < k_s(b_0 - b_1\bar{p}) \\ \frac{-hI/\alpha+q_s(\frac{b_0-I/k}{b_1}-u)I+q_l(\bar{p}-u)I}{1/\alpha+1/\beta}, & \text{if } k_s(b_0 - b_1\bar{p}) < I < k_s(b_o - ub_1)/2 \\ \frac{-hI/\alpha+q_s(p_0-u)k_s(b_o-ub_1)/2+q_l(\bar{p}-u)I}{1/\alpha+1/\beta}, & \text{if } k_s(b_o - ub_1)/2 < I < k_l(b_0 - b_1\bar{p}) \\ \frac{-hI/\alpha+q_s(p_0-u)k_s(b_o-ub_1)/2+q_l(\frac{b_0-I/k}{b_1}-u)I}{1/\alpha+1/\beta}, & \text{if } I > k_l(b_0 - b_1\bar{p}) \end{cases} \tag{10}$$

We note that considering all the profit functions given in (11) and their transitions, it can easily be verified that the overall profit function is continuous and piecewise unimodal in the reserve inventory level. We omit the details for brevity.

For the first region in which $I < k_s(b_0 - b_1\bar{p})$, the profit function is linear in I. Specifically, for $(\bar{p}-u) \leq h/\alpha$, profit is decreasing in I and therefore it is optimal not to hold any reserve inventory. Consequently, the firm should only hold reserve inventory if $(\bar{p} - u) > h/\alpha$, and in this case, it should hold at least $I = k_s(b_0 - b_1\bar{p})$, i.e., to fully cover demand at the price level equaling the price cap and during a short disruption. In the second region where $k_s(b_0 - b_1\bar{p}) < I < k_s(b_o - ub_1)/2$, the optimal reserve inventory level I^* can be obtained from the first order condition. We find that I^* increases in b_0 and α, and decreases in u and h in this region. In the third region, the profit is again linear in I. If the boundary $I^* = k_s(b_o - ub_1)/2$ is the solution, I is independent of $\bar{p}$. If the boundary $I^* = k_l(b_o - b_1\bar{p})$, then I is decreasingt in $\bar{p}$. For either case, we have I^* increasing in b_0 and decreasing in b_1. Finally, in the fourth region, I^* can again be solved through the first order condition and we again find that I^* is increasing in b_0 and α, and decreasing in b_1 and u.

Case (b) is very similar to case (a) except we now have $k_l(b_0 - b_1\bar{p}) < k_s(b_o - ub_1)/2$ and for $k_l(b_0 - b_1\bar{p}) < I < k_s(b_o - ub_1)/2$, it is optimal charge $\frac{b_0-I/k}{b_1}$ for both short and long disruptions. As the remaining analysis is very similar to case (a), we omit the details for brevity.

Proof of Proposition 4:

The proof of this proposition is very similar to the proof of Proposition 1 and it directly follows from the profit function provided in (6). We first note that the objective function for the disrupted period given in (6) is again piecewise in the decision variable p. Specifically, for $p \leq \frac{b_0-a}{b_1}$, the objective function is $(p-c_a)ak - c\,a\,k$ and the profit is linearly increasing in p. Therefore, the firm will set at least the price $p = \frac{b_0-a}{b_1}$. For $p > \frac{b_0-a}{b_1}$, the objective function is $(p-c_a)(b_0-b_1p)\,k - c\,a\,k$, which is concave in p. The first order condition results in an optimal price of $p^* = \frac{b_0}{2b_1} + \frac{c_a}{2}$. Consequently, we find that for $a < \frac{b_0-c_ab_1}{2}$, the optimal price is at the boundary at $p^* = \frac{b_0-a}{b_1}$ and for $a > \frac{b_0-c_ab_1}{2}$, we have the interior solution at $p^* = \frac{b_0}{2b_1} + \frac{c_a}{2}$. The sensitivity results follow immediately from the optimal price expressions.

Proof of Proposition 5:

The proof of Proposition 5 follows a similar structure to the proof of Proposition 3. Specifically, we again have a piecewise and unimodal profit function, this time, given by:

$$\mathbb{E}[\Pi(a)] = \begin{cases} \frac{-ca/\alpha - ca/\beta + q_s(\bar{p}-c_a)ak_s + q_l(\bar{p}-c_a)ak_l}{1/\alpha+1/\beta}, & \text{if } a < b_0 - b_1\bar{p} \\ \\ \frac{-ca/\alpha - ca/\beta + q_s(\frac{b_0-a}{b_1}-u)ak_s + q_l(\frac{b_0-a}{b_1}-c_a)ak_l}{1/\alpha+1/\beta}, & \text{if } a \geq b_0 - b_1\bar{p} \end{cases} \tag{11}$$

The first region is linear in a. Specifically, if $\bar{p} < c\frac{\alpha+\beta}{\alpha} + c_a$, then it is linearly decreasing in a and therefore, it is not optimal for the firm to invest in reserve capacity. For $\bar{p} \geq c\frac{\alpha+\beta}{\alpha} + c_a$, the firm will invest in reserve capacity rate of at least as much as the boundary $(b_0 - b_1\bar{p})$, i.e., to cover the demand rate at the price cap. Therefore, in this region, the optimal capacity rate is decreasing with the price bound, $\bar{p}$. The profit function in the second region is concave. Through the first order condition, we find that the optimal reserve capacity investment increases in b_0 and decreases in c_a and c as is independent of the price bound, $\bar{p}$.

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