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# Integrated modelling and optimisation framework for multi-stage screw compressors utilising Gaussian process regression and Bayesian methods

Abhishek Kumar<sup>1,2</sup> · Ahmed Kovacevic<sup>1</sup> · Sathiskumar Anusuya Ponnusami<sup>1</sup>

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## Abstract

High-pressure sectors like mining and construction require multi-stage screw compressors that can operate reliably at pressures over 16 bar. Single-stage compressors frequently encounter constraints such as elevated temperatures, rotor bending deformation, imperfect cooling effect of the injected oil, condensate, and diminished bearing longevity, rendering them inadequate for these specifications. This paper introduces a comprehensive modelling and optimisation approach for multi-stage screw compressors, integrating a physics-based chamber model with machine learning via Gaussian process regression. The framework employs Bayesian optimisation to methodically refine stage-specific parameters, enhancing performance and dependability while ensuring computing economy. The innovation is in its capacity to precisely forecast the performance of both individual and final stages, experimentally validated with a two-stage air screw compressor for water-well applications, attaining an error margin below 5%. A case study illustrated the framework's efficacy by decreasing specific power usage by 2% via the optimisation of fluid injection parameters. This approach represents a significant advancement in compressor technology, providing a scalable and efficient solution for designing and optimising multi-stage screw compressors in high-pressure applications.

**Keywords** Bayesian optimisation · Chamber model · Compressor performance modelling · Fluid injection parameters · Gaussian process regression (GPR) · Multi-stage screw compressors

## List of symbols

$\phi$	Wrap angle of screw rotors (°)
GAPI	Interlobe clearance (μm)
GAPA	Axial clearance (μm)
VI	Built-in volume ratio
$T_{oil}$	Oil injection temperature (°C)
$D_{oil}$	Oil port diameter (mm)
$Q$	Volume flow rate (m <sup>3</sup> /min)
$W_{tip}$	Tip speed of the male rotor (m/s)
$P_{suc}$	Suction pressure (bar)

RMSE	Root mean squared error
GPR	Gaussian process regression
ANN	Artificial neural network
SCORG	Screw compressor rotor grid generation
HP	High pressure
$T_{suc}$	Suction temperature (°C)
$\eta_{ad}$	Adiabatic efficiency (%)
GAPR	Radial clearance (μm)
$L/D$	Relative length of rotors
$P_{oil}$	Oil injection pressure (bar)
$\theta$	Oil injection angle (°)
$P$	Power consumption (kW)
SPC	Specific power consumption (kW/m <sup>3</sup> /min)
$P_{dis}$	Discharge pressure (bar)
$\eta_{vol}$	Volumetric efficiency (%)
$R^2$	Coefficient of determination
ML	Machine learning
GUI	Graphical user interface
LP	Low pressure
RPM	Rotations per minute
$P_{suc}$	Suction pressure (bar)

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✉ Abhishek Kumar  
 abhishek.kumar.2@city.ac.uk

<sup>1</sup> Centre for Compressor Technology, Department of Engineering, City St George's, University of London, Northampton Square, EC1V 0HB London, England, UK

<sup>2</sup> Research and Development Division, Kirloskar Pneumatic Company Limited, Hadapsar Industrial Estate, Pune, Maharashtra 411 013, India

## 1 Introduction

Compressed air is an essential utility in numerous industries, driving applications ranging from pneumatic equipment to high-pressure systems in fields such as mining, construction, and oil and gas. The selection of compressor technology frequently depends on operational criteria, including pressure range, efficiency, and reliability. Oil-injected screw compressors have emerged as the preferred option at pressures up to 12 bar and small-to-medium power inputs, gradually supplanting piston compressors owing to their compact design, diminished maintenance requirements, and enhanced energy efficiency [1, 2].

Nonetheless, in applications necessitating elevated discharge pressures, the functionality of screw compressors is somewhat limited. Single-stage oil-injected screw compressors encounter intrinsic constraints that restrict their efficacy at high pressures [3]. Critical considerations encompass the pressure differential between suction and discharge, which exerts stress on the compressor components, and the constraints of oil cooling, which fails to adequately address the heightened thermal load. Moreover, bearing life emerges as a significant issue, as antifriction thrust bearings endure substantial axial stresses under high pressures. These issues lead to condensate accumulation in the oil separator, hence undermining performance [1, 4].

Although two-stage screw compressors may achieve a pressure range of around 30 bar without intercooling, they are less competitive than multi-stage piston compressors at discharge pressures above 40 bar. This results from the superior mechanical efficiency and heat management capabilities of piston compressors in demanding applications [5]. For discharge pressures exceeding this threshold, compound compressors-comprising a single-stage screw compressor succeeded by a multi-stage piston compressor-are frequently utilised, capitalising on the advantages of both technologies to fulfil operational requirements [1, 6].

Screw compressors are frequently underutilised for pressure ranges up to 30 bar due to difficulties in the design and optimisation of both individual and final stage characteristics. Challenges encompass cooling constraints, bearing longevity, and the careful adjustment of stage-specific factors necessary for optimal performance. Multi-stage compressors mitigate these challenges by segmenting the compression process into several phases. This segmentation enhances cooling between stages, diminishing overall temperature rise and enhancing efficiency relative to single-stage compressors [7, 8].

Intercoolers are frequently utilised in reciprocating compressors to cool the air expelled from the first stage prior to its entry into the suction of the second stage. This cooling substantially decreases the necessary work input

and improves efficiency. In two-stage screw compressors, an intermediate pipe is frequently utilised instead of a conventional intercooler. This intermediate pipe links the discharge of the first stage to the suction of the second stage and includes oil injection ports to cool the air-oil combination prior to its entry into the second stage. This cooling system emulates the advantages of an intercooler by lowering the temperature of the compressed mixture, thus enhancing thermodynamic efficiency.

To fully achieve the potential efficiency of this design, the fluid injection parameters must be optimised. Essential characteristics like oil injection port diameter, oil injection pressure, oil injection angle, and oil injection temperature are crucial for effective cooling, loss reduction, and decreased power consumption [9–12]. Optimising these parameters gives superior heat exchange, enhanced sealing, and diminished thermal stresses, hence improving the performance and dependability of the two-stage screw compressor.

Other critical parameters influencing the efficiency of multi-stage screw compressors are the geometrical characteristics, including the wrap angle, length-to-diameter ratio, built-in volume ratio, and the intermediate pressure and temperature of the individual stages. These parameters significantly affect volumetric efficiency and the compressor's specific performance [13–15]. Achieving high-performance screw compressors necessitates optimisation of these parameters [5]. The rational selection of intermediate pressure and temperature requires sophisticated mathematical models and optimisation methodologies [16].

Recent advancements in screw compressor design have focussed on lightweight composite rotors, leading to considerable reductions in weight and manufacturing costs [17, 18]. For instance, employing carbon fibre epoxy for composite screw rotors has demonstrated a 52% weight reduction compared to aluminium rotors. Furthermore, the development of innovative rotor profiles, such as those introduced by Sakun and Amosov, has significantly improved delivery rates while minimising leakage [16]. Optimal rotor geometries play a crucial role in enhancing efficiency, reducing deflection, and minimising bearing loads and contact forces [19, 20].

Geometric modelling has been further refined through optimisation frameworks that address key features like rotor clearances, blowhole height, and polar moment of inertia. Tang's research [21] highlighted that optimised clearance distributions contribute to enhanced thermodynamic performance, while Fujiwara's studies on rotor profiles for oil-injected compressors established a strong correlation between predicted and experimental efficiencies [22]. Hauser's application of NonUniform Rational B-Splines (NURBS) for rotor profile optimisation has demonstrated accelerated optimisation speeds and improved performance metrics compared to traditional approaches [23–26].

Bayesian Optimization (BO) has been increasingly applied in the design and optimization of rotary compressors. Lu et al. [27] demonstrated a two-stage BO approach for optimising the geometric and port parameters of a limaçon rotary compressor, resulting in significant performance improvements. Additionally, Zheng et al. [28] conducted an experimental and modelling study on high-speed rotary compressors, providing valuable data that can inform BO strategies for performance enhancement. These studies underscore the growing relevance of BO in compressor technology and support the broader applicability of the methods employed in this work.

In this research, the focus is on the development of advanced modelling frameworks and optimisation techniques to enhance the efficiency and performance of screw compressors, with particular emphasis on multi-stage configurations. This study introduces an innovative hybrid modelling framework that integrates traditional chamber models with machine learning techniques, specifically Gaussian process regression (GPR), for thermodynamic analysis. GPR, a nonparametric Bayesian machine learning approach, is employed for its ability to make precise predictions, drawing from applications in diverse fields such as performance prediction and material structure modelling [29–31]. The framework further leverages Bayesian Optimisation to facilitate stage-specific optimisation of compressor parameters, applicable to both single-stage and multi-stage screw compressors.

The validation of the proposed framework was performed through the prototyping of a two-stage air screw compressor designed for water-well applications, achieving a maximum discharge pressure of 25 bar. Experimental results demonstrated the framework's predictive accuracy with an error margin within 5%. Following this validation, the framework was utilised to optimise the fluid injection parameters of the same two-stage compressor, resulting in a 2% improvement in specific power consumption, showcasing the practical effectiveness of the methodology.

This paper is structured as follows: Section 2 outlines the comprehensive modelling framework, detailing its architecture and boundary conditions. Section 3 focuses on the experimental validation of the framework, which involved constructing a two-stage screw compressor to evaluate the framework using both the chamber model and GPR. Section 4 discusses the optimisation of compressor fluid injection parameters, employing Bayesian Optimisation to illustrate the computational efficiency of the framework in multi-stage settings. Finally, Sect. 5 concludes the paper by summarising the findings and identifying potential avenues for future research.

## 2 Modelling framework

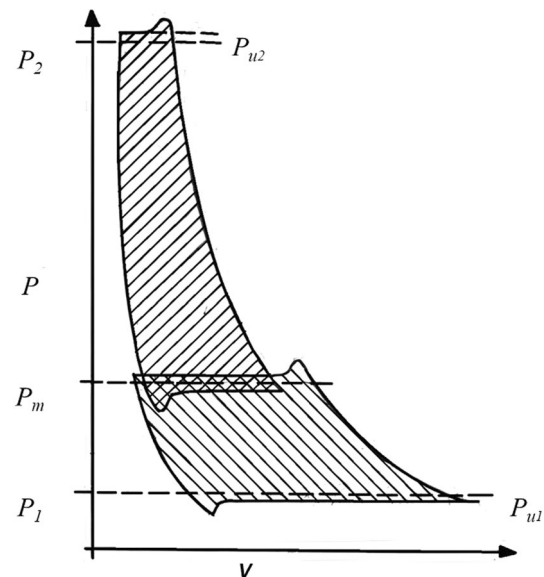
### 2.1 Calculation of optimum intermediate pressure and built-in volume ratio

To achieve maximum efficiency and minimise power consumption, it is essential to accurately calculate the intermediate pressure and built-in volume ratio based on the specified initial and final pressure requirements. These parameters significantly influence the thermodynamic performance and overall energy efficiency of multi-stage screw compressors. This section outlines the calculation approach incorporated into the proposed modelling framework for multi-stage screw compressors.

An indicator diagram of a two-stage compression system is presented in Fig. 1. The diagram illustrates the relationship between pressure ( $P$ ) and volume ( $V$ ) during the compression process, spanning from the initial suction pressure ( $P_{u1}$ ) to the final discharge pressure ( $P_{u2}$ ). The intermediate pressure ( $P_m$ ) acts as the dividing point between the low-pressure (LP) and high-pressure (HP) stages, ensuring balanced work distribution and thermodynamic efficiency.

This section is divided into two key parts:

- Section 2.1.1 discusses the methodology for calculating the optimal intermediate pressure, which balances the work between the stages and minimises energy losses.



**Fig. 1** Indicator diagram of a two-stage compression screw compressor with overlapping compression phases at intermediate pressure  $P_m$ . The diagram demonstrates the relationship between pressure ( $P$ ) and volume ( $V$ ) across both stages, highlighting the compression process from the initial suction pressure  $P_{u1}$  to the final discharge pressure  $P_{u2}$

- Section 2.1.2 describes the approach to determine the optimal built-in volume ratio, ensuring that the screw compressor achieves the desired compression ratio while maintaining high efficiency.

The indicator diagram serves as a visual representation of the compression process, emphasising the critical role of intermediate pressure in balancing the thermodynamic workload between stages. Furthermore, optimising the built-in volume ratio enhances volumetric and adiabatic efficiency, contributing to improved overall system performance. These calculations form the foundation of the proposed modelling framework, aligning with the objective of creating a robust and efficient tool for multi-stage screw compressor design and optimization.

### 2.1.1 Intermediate pressure

In a multi-stage compressor, the intermediate pressure between stages plays a crucial role in distributing the work evenly across the stages. For two-stage screw compressors, the intermediate pressure  $P_m$  is calculated based on the suction pressure  $P_1$  and discharge pressure  $P_2$  [32]. The general formula for intermediate pressure in a multi-stage system with  $z$  stages is given as:

$$P_m = (P_1 \cdot P_2)^{\frac{1}{z}} \quad (1)$$

In the case of a two-stage compression system, this expression simplifies to:

$$P_m = \sqrt{P_1 \cdot P_2} \quad (2)$$

This equation represents the geometric mean of the suction and discharge pressures. It ensures that the compression process is balanced between the stages, preventing excessive pressure gradients that could lead to increased energy losses or overheating of the system [32].

As shown in the pressure-volume (PV) diagram in Fig. 1, the intermediate pressure  $P_m$  corresponds to the point where the first stage of compression ends and the second stage begins. This intermediate pressure acts as the transition boundary, enabling the compressor to continue compression at a lower temperature due to interstage cooling. Selecting the optimal intermediate pressure not only enhances energy efficiency but also ensures reliable performance across all stages of the compressor.

### 2.1.2 Built-in volume ratio (VI)

The built-in volume ratio (VI) is another key parameter that affects the pressure ratio achieved by a screw

compressor, depending on its geometric design. It is defined as the ratio of the volumes at the suction and discharge points:

$$VI = \frac{V_s}{V_d} \quad (3)$$

where  $V_s$  is the suction volume, and  $V_d$  is the discharge volume. The VI ratio directly influences the compressor's ability to achieve the desired compression ratio, and optimising it ensures better thermodynamic performance. A well-designed VI ratio reduces energy consumption, improves volumetric efficiency, and minimizes temperature rise during compression.

The PV diagram (Fig. 1) illustrates how the compression process occurs over two stages. The intermediate pressure  $P_m$  serves as the dividing point, while the built-in volume ratio controls the pressure ratio across each stage. Proper calculation of these parameters helps the compressor operate efficiently, reducing mechanical stresses and avoiding energy losses during the compression process.

Thus, the careful calculation of both the intermediate pressure and built-in volume ratio is vital for ensuring the optimal performance of multi-stage screw compressors, enhancing their operational efficiency, and prolonging their service life.

## 2.2 Boundary conditions

The selection of appropriate boundary conditions is critical for accurately modelling multi-stage screw compressors, ensuring that the system's behaviour is represented effectively. These boundary conditions encompass parameters such as inlet and outlet temperatures, mass flow rates, pressure ratios for each stage, and oil injection points, as depicted in Fig. 2.

### 2.2.1 Conservation of mass

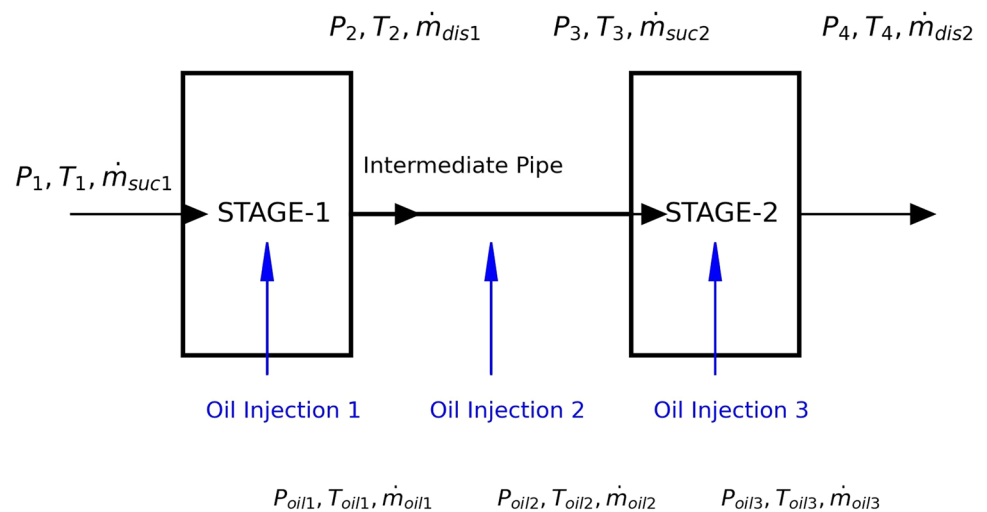
The principle of mass conservation is key in fluid dynamics and dictates that mass cannot be created or destroyed. In the context of a two-stage screw compressor, this means the total mass flow exiting Stage-1 must match the mass flow entering Stage-2, considering both air and oil injections. Therefore, the corrected mass flow balance equations are:

$$\dot{m}_{suc1} + \dot{m}_{oil1} = \dot{m}_{dis1}, \quad \dot{m}_{dis1} = \dot{m}_{suc2}, \quad \dot{m}_{suc2} + \dot{m}_{oil2} = \dot{m}_{dis2} \quad (4)$$

This ensures that the total mass flow rate, including oil injection, is accurately conserved through the suction and discharge points of both stages, as illustrated in Fig. 2.



**Fig. 2** Schematic representation of a two-stage oil-injected air screw compressor



### 2.2.2 Intermediate pressure and temperature

The intermediate pressure  $P_3$  at the suction of Stage-2 is a critical parameter that directly impacts the efficiency and performance of the two-stage screw compressor. Typically,  $P_3$  equals the discharge pressure ( $P_2$ ) from Stage-1, but in practice, a pressure drop  $\Delta P$  is often observed due to pipe friction, heat exchange, or other factors:

$$P_3 = P_2 - \Delta P \quad (5)$$

The pressure drop  $\Delta P$  between the stages is highly dependent on several factors, including the final discharge pressure, piping configuration, and fluid flow rate. At higher discharge pressures, the pressure drop tends to be larger due to the increased flow resistance in the piping system and heat dissipation. For instance, in scenarios where the final discharge pressure exceeds 21 bar, the pressure drop can reach values greater than 0.5 bar, while at lower discharge pressures (e.g., below 18 bar), the pressure drop might be less than 0.5 bar. Thus, the optimization of the intermediate pressure  $P_3$  must account for the variability of  $\Delta P$ , especially as higher discharge pressures induce more significant pressure drops.

Similarly, the intermediate temperature  $T_3$  at the suction of Stage-2 is typically equal to the discharge temperature ( $T_2$ ) from Stage-1, but a temperature drop  $\Delta T$  may occur due to cooling losses:

$$T_3 = T_2 - \Delta T \quad (6)$$

Both the pressure drop  $\Delta P$  and temperature drop  $\Delta T$  need to be considered during the optimization process to minimize energy losses and ensure efficient compression between stages. Therefore, the optimization of intermediate pressure  $P_3$ , accounting for variations in  $\Delta P$ , plays a crucial role in achieving balanced stage operation, especially at varying discharge pressures. The framework allows users to

iteratively adjust  $P_3$  during the optimization process, ensuring that the system adapts to different operating conditions and maximises performance.

In summary, by refining the intermediate pressure  $P_3$  and incorporating the variability of the pressure drop  $\Delta P$ , the model ensures a more accurate prediction of compressor performance, leading to a more robust optimisation process.

### 2.2.3 Oil injection and its impact

In addition to the mass flow of the working fluid, oil is injected at different points to provide cooling and lubrication. The oil injections  $\dot{m}_{oil1}$ ,  $\dot{m}_{oil2}$ ,  $\dot{m}_{oil3}$  need to be accounted for in both mass conservation and the thermodynamic modelling of the compressor, as they influence both cooling efficiency and the overall mass balance.

### 2.2.4 Summary of boundary conditions

The boundary conditions for a two-stage screw compressor are centred on the following:

- Conservation of mass for each stage, ensuring the mass entering and exiting is balanced.
- Intermediate pressure and temperature adjustments that account for the losses occurring between stages.
- Oil injection at various points, contributing to both the mass balance and the thermodynamic efficiency of the system.

Within the simulation framework, the boundary conditions ensure that the system adheres to mass and energy conservation principles during optimization. The mass flow rates of the individual stages are maintained equal by dynamically adjusting the rotational speeds of each stage compressor. This approach preserves the continuity of mass flow

across the stages, preventing discrepancies in performance calculations.

Additionally, the discharge temperature of the first-stage compressor is treated as the suction temperature for the subsequent stage. This transition accounts for potential temperature drops ( $\Delta T$ ) caused by heat losses during transfer through the intermediate pipe, reflecting realistic thermal behavior. These boundary conditions are executed seamlessly in the background of the framework, providing robust checks and balances that align with practical compressor operation. While these parameters serve as constraints within the model, the optimization process focuses on stage-specific performance enhancements. Their implementation guarantees physical feasibility and consistency across different operating scenarios.

These boundary conditions are essential for accurate modelling, optimization, and design of multi-stage screw compressors, ensuring that the system operates effectively and efficiently.

### 2.3 Model architecture

The developed modelling framework for multi-stage screw compressors is designed to optimise design and operational parameters while delivering critical performance insights. The framework integrates control variables, geometric features, and fluid injection parameters to facilitate both single and multi-objective optimization.

Key input parameters such as suction pressure ( $P_{\text{suc}}$ ), discharge pressure ( $P_{\text{dis}}$ ), suction temperature ( $T_{\text{suc}}$ ), rotor speed ( $N$ ), and working fluid type define the system's boundary conditions. Geometric parameters, including length-to-diameter ratio ( $L/D$ ), wrap angle ( $\phi$ ), built-in volume ratio ( $VI$ ), and rotor profile clearances-axial clearance ( $GAPA$ ), inter-lobe clearance ( $GAPI$ ), and radial clearance ( $GAPR$ )-significantly influence the compressor's capacity, efficiency, and leakage characteristics. Furthermore, fluid injection parameters like oil injection pressure ( $P_{\text{oil}}$ ), temperature ( $T_{\text{oil}}$ ), angle ( $\theta$ ), and port diameter ( $D_{\text{oil}}$ ) are optimised to enhance heat transfer, sealing, and lubrication. The significance of each optimisation parameter used in this framework has been extensively and separately described in the cited papers by Kumar et al [13–15, 33].

Performance metrics are central to evaluating the framework's efficacy. These include:

- **Volume flow rate ( $Q$ ):** Reflects the gas volume delivered by the compressor.
- **Volumetric efficiency ( $\eta_{\text{vol}}$ ):** Indicates the effective utilisation of the compressor's capacity.
- **Compressor power ( $P$ ):** The total power required for operation, accounting for mechanical and thermal losses.

- **Specific power consumption ( $SPC$ ):** Represents the energy consumption per unit of delivered gas.
- **Adiabatic efficiency ( $\eta_{\text{ad}}$ ):** Evaluates the thermodynamic efficiency of the compression process.

Before presenting Fig. 3, a brief explanation of the methods used is provided for clarity.

Gaussian process regression (GPR) is a nonparametric, probabilistic machine learning technique that models data using a Gaussian distribution and makes predictions based on Bayesian inference [34]. It is particularly effective in handling small datasets and provides uncertainty quantification in predictions, which is essential for performance modelling of compressors.

Bayesian Optimisation (BO) is a global optimisation method that efficiently explores the design space by using surrogate models (such as GPR) to balance exploration and exploitation. BO iteratively selects sampling points based on acquisition functions, which makes it ideal for optimising multi-stage screw compressor parameters where computational costs and system complexity are high [35].

These methods complement each other in the proposed framework: GPR predicts compressor performance, while BO refines parameter settings to enhance efficiency and performance.

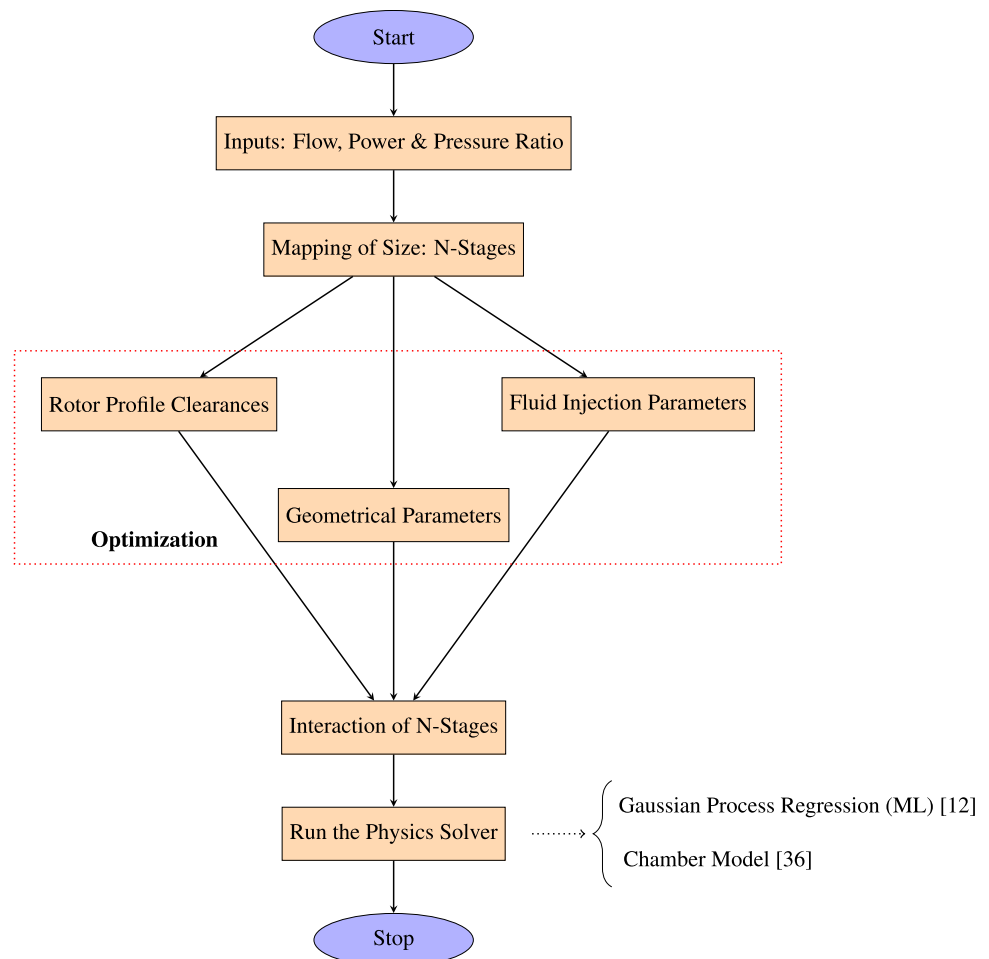
The framework employs a streamlined approach to map these input parameters across multiple compressor stages, aligning with optimisation goals. Figure 3 illustrates the framework's workflow, beginning with input variables such as flow, power, and pressure ratio. It progresses through stage-specific interactions, incorporating optimisation of rotor profile clearances, geometric characteristics, and fluid injection properties. These parameters are refined using either the chamber model or Gaussian process regression (GPR) techniques [12, 36].

**Framework overview:** The framework begins by taking essential input parameters such as flowrate, power ratings, and pressure ratio, typically provided as per customer requirements for compressed air applications. Based on these inputs, it maps the size combination of stages by selecting the optimal configuration from the existing sizes available at Kirloskar [2], using boundary conditions that ensure mass conservation, pressure, and temperature compliance. Once the best two-stage combination is determined, the framework allows for the optimisation of individual parameters for each stage, including rotor profile clearances ( $GAPA$ ,  $GAPI$ ,  $GAPR$ ), fluid injection parameters (such as oil injection pressure, temperature, angle, and port diameter), and geometrical parameters (wrap angle, built-in volume ratio, etc.).

After the optimisation process, boundary conditions are re-evaluated to ensure total mass conservation and system feasibility. The framework provides flexibility by employing



**Fig. 3** Flowchart depicting the model architecture for multi-stage screw compressor design, showing the progression of inputs, mapping, optimization, and physics solver stages



both a conventional chamber model-based solver for detailed physical accuracy and a Gaussian process regression (GPR) model for faster performance estimation. This integrated approach balances precision and computational efficiency, enabling the design and optimisation of multi-stage screw compressors that meet diverse operational and customer requirements.

This systematic and adaptable framework ensures enhanced compressor performance, delivering key outputs such as reduced power consumption, improved volumetric efficiency, and optimal specific power consumption. By integrating diverse parameters and metrics, the framework addresses varied application requirements, providing a robust tool for optimising multi-stage screw compressors.

### 3 Experimental validation

#### 3.1 Experimental setup

A two-stage oil-flooded air screw compressor block was designed and manufactured by Kirloskar Pneumatic

Company Limited (KPCL), Pune, India, to validate the accuracy of the multi-stage modelling framework. This compressor is designed specifically for water-well applications, functioning within a discharge pressure range of 21 to 25 bar absolute. The compressor was integrated with a diesel engine to meet the standard needs of portable applications, thereby ensuring both compactness and enhanced portability. The experimental test setup was configured in accordance with CAGI and PNEUROP standards, and all testing procedures followed ISO 1217 guidelines.

Test setup requirements:

1. *Inlet temperature*: Normal: 40° C, Highest: 50°C, Lowest: 0°C
2. *Discharge temperature*: Normal: 90–95°C, Shutdown: 105°C (manual by-pass)
3. *Oil temperature*: Normal: 62°C, Highest: 70°C
4. *Oil type*: Mineral oil (ISO VG 68)
5. *Unloading condition*: Capacity is regulated down to zero via inlet valve throttling
6. *Regulating pressure range*: Discharge pressure (Pd)  $\pm 5\%$

Compressor operating conditions:

1. *Bearing life*: 18,000 h (L10h) @ 1200 CFM/23.41 bar (a); 15,000 h (L10h) @ 1200 CFM/25.13 bar (a)
2. *Seal life*: 2 years or 10,000–12,000 h
3. *Pressure test*:  $350 \text{ psig} * 1.1 * 1.5 = 557 \text{ psig}$  (38.41 bar g/39.41 bar a)
4. *Operating speed*: 1470 RPM (Electrical)/1500–1900 RPM (Engine), with two different gear ratios

The two-stage oil-flooded air screw compressor block employs oil injection during the first stage, the intermediate pipe, and the final stage to improve thermodynamic efficiency (Refer to Fig. 2). The compressor operates via a bull gear that is linked to a diesel engine, providing power to both the low-pressure (LP) and high-pressure (HP) stages. The outlet nozzle's discharge pressure is regulated through a ball valve. The ball valve, when fully opened, achieves a discharge pressure of approximately 12.75 bar (a).

Figure 4a, b illustrate the isometric view of the compressor block 3D model and the post-manufacturing view, respectively. The final test configuration utilised for framework validation is illustrated in Fig. 5.

## 3.2 Test results and validation

### 3.2.1 Experimental results

The experimental configuration was carefully crafted to comply with the ISO 1217 standard for assessing displacement compressors, guaranteeing accurate and consistent outcomes. The prototype of the two-stage oil-flooded air screw compressor was subjected to a series of performance

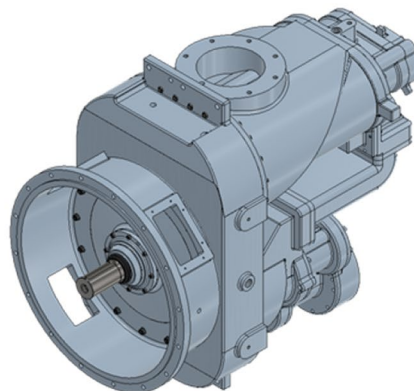
evaluations under steady-state settings. The tests were performed at designated operating points until stable parameters, such as pressure, temperature, and rotating speed, were attained. Data collection commenced immediately once the system attained temperature and mechanical equilibrium, ensuring dependable and precise performance measurements.

Each test was performed for a minimum of 10 min to obtain valid performance results. Measurements were conducted at regular intervals and averaged over several samples to reduce transient variations. Additionally, all data was standardised to uniform settings, specifically regarding pressure ratios and rotor speeds, to facilitate significant comparisons and verify adherence to industry standards.

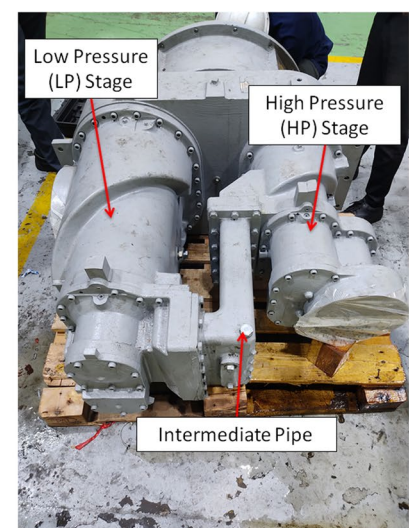
The performance test results are displayed in Table 1, emphasising critical metrics such power consumption, rotational speed, interstage pressure, and compressor capacity. The results offer a thorough assessment of the compressor's performance under diverse testing situations, establishing a reliable foundation for the validation of the multi-stage modelling framework.

Table 1 presents the performance testing data for the two-stage oil-flooded screw compressor at various discharge pressures. Notably, the power consumption of the low-pressure (LP) compressor remains nearly constant across different interstage pressures. This is due to the LP stage operating within a fixed suction pressure and speed boundary, where variations in interstage pressure are minor and do not significantly impact its workload. The LP compressor's performance is predominantly influenced by its suction conditions and rotor geometry, which remain unchanged in the tested scenarios. Consequently, the observed power remains steady, while the high-pressure (HP) stage absorbs the load

**Fig. 4** Comparison of the 3D design model and the post-manufacturing prototype of the two-stage oil-flooded air screw compressor block. The 3D model represents the conceptual design, while the prototype reflects the practical implementation of the design



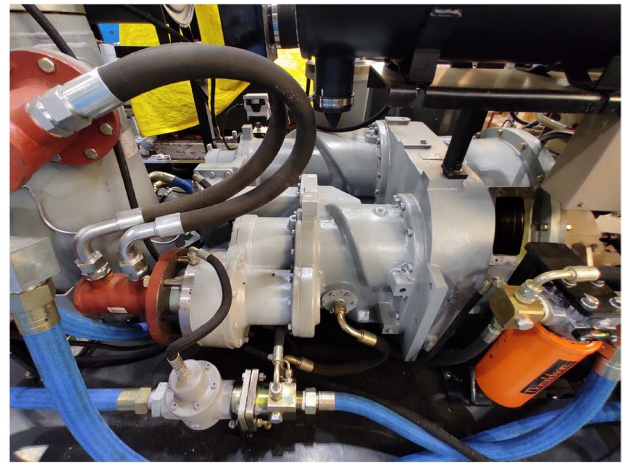
(a) Isometric view of the 3D model of the two-stage oil-flooded air screw compressor block, showcasing its design and layout.



(b) Post-manufacturing schematic of the two-stage oil-flooded air screw compressor block, illustrating the final build of the prototype.



(a) Side view of the test setup



(b) Assembled view of the compressor system



(c) Compressor block connected to diesel engine



(d) Close-up of the air separator and piping system

**Fig. 5** Test setup for the two-stage oil-flooded air screw compressor block**Table 1** Performance testing data for two-stage oil-flooded air screw compressor

Parameters	21.54 bar (a)	23.31 bar (a)	25.07 bar (a)
Inlet Pressure (bar a)	0.95	0.95	0.95
Interstage Pressure (bar a)	5.07	5.20	5.37
Discharge Pressure (bar a)	21.54	23.31	25.07
Power, LP (kW)	143.18	143.18	143.18
Power, HP (kW)	146.91	155.11	164.06
Total Power (kW)	290.09	298.29	307.24
Engine RPM	1900	1900	1900
LP, RPM	3026	3026	3026
HP, RPM	3713	3713	3713
Capacity (cu ft/min)	1151	1150	1150

variation resulting from increased final discharge pressures. This behavior aligns with the typical operational dynamics of two-stage screw compressors.

### 3.2.2 Validation of chamber model predictions

This part concentrated on corroborating the chamber model's predictions with experimental data. The input parameters supplied to the tool encompassed suction pressure, final discharge pressure, flow requirements, suction temperature, and the working fluid. Validation was conducted for three specific discharge pressures: 21.54 bar, 23.31 bar, and 25.07 bar. After inputting these circumstances into the program, simulations were conducted, and essential output characteristics, including power consumption and efficiency, were computed.

The validation primarily concentrates on the power consumption predictions generated by the chamber model, given



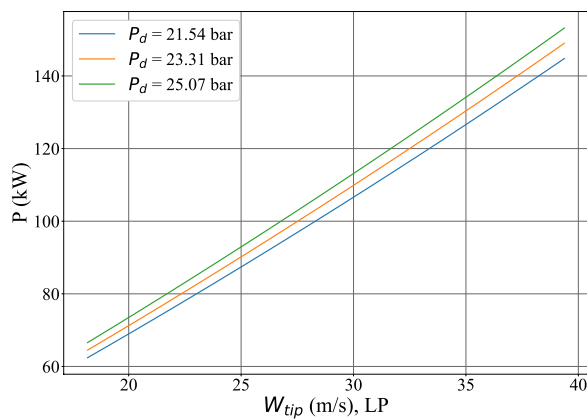
the experimental data solely encompassed power and flow measurements at varying discharge pressures. Figure 6a, b depicts the correlation between tip speed and power consumption for the low-pressure (LP) and high-pressure (HP) stages over three discharge pressures.

The total power consumption forecasted by the chamber model was ultimately juxtaposed with the experimental outcomes. It is important to note that the total power depicted in Fig. 6c represents the shaft power of the two-stage screw compressor, which includes mechanical losses due to shaft seals, bearing friction, and oil drag specific to oil-flooded compressors. Electrical losses, such as those arising from the motor or drive system, are not included in this measurement. The chamber model computes shaft power directly from thermodynamic and mechanical interactions, providing a realistic performance assessment. The comparison indicated that the forecasts fell within the allowable deviation

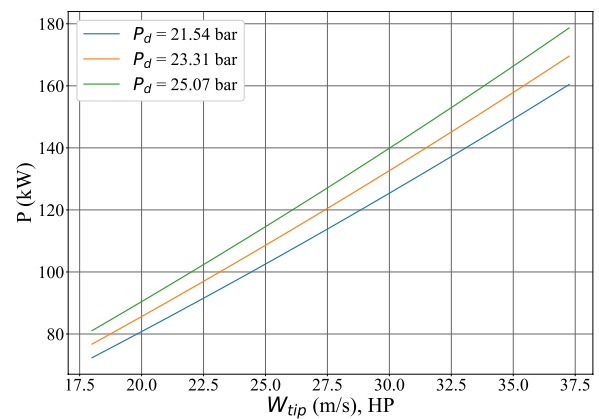
range of 5%, as demonstrated in Fig. 6c. This illustrates the precision of the chamber model in forecasting power usage at different discharge pressures.

### 3.2.3 Validation of machine learning model predictions

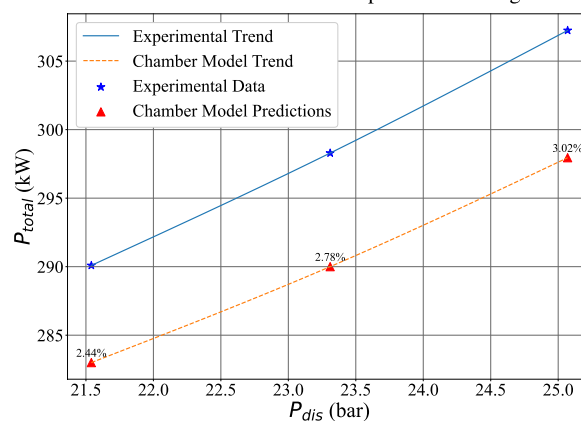
This section focuses on validating the modelling framework using the machine learning solver, specifically the Gaussian Process Regression (GPR) model. The GPR predictions are compared against the experimentally validated chamber model predictions, as detailed in the preceding section, to assess the accuracy and reliability of the machine learning approach. Previous research assessed various machine learning models for their predictive efficacy on screw compressor data, with Gaussian process regression (GPR) surpassing other models owing to its intrinsic capacity to capture uncertainty and deliver reliable predictions in settings with limited



(a) Low-pressure (LP) stage: Tip speed vs. power consumption predictions using the chamber model.



(b) High-pressure (HP) stage: Tip speed vs. power consumption predictions using the chamber model.



(c) Percentage error between experimental data and chamber model predictions for total power consumption, demonstrating the accuracy of the chamber model within acceptable error margins.

**Fig. 6** Chamber model predictions and experimental validation. **a, b** Illustrate the tip speed versus power consumption relationship for the low-pressure (LP) and high-pressure (HP) stages, respectively. **c** The

percentage error between experimental data and chamber model predictions for total power consumption

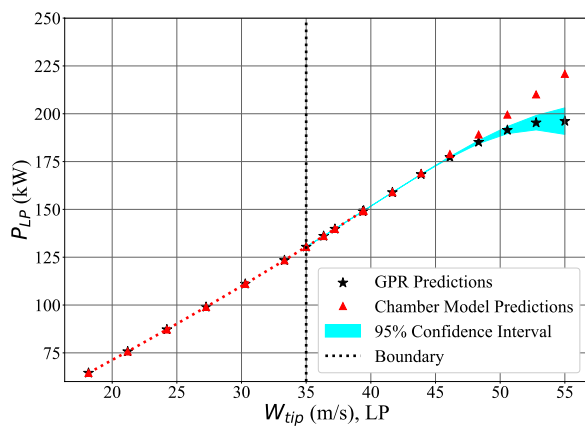
data [12]. The GPR model was selected for its adaptability in estimating intricate interactions in multi-stage compressors, rendering it ideal for scenarios where comprehensive physical models are either inaccessible or costly to compute.

The program has been trained on data from 10 distinct screw compressors produced by Kirloskar Pneumatic Company Limited (KPCL), employing a minimum of 19,200 data points for each unit. The experimental validation of single-stage machine learning predictions is elaborated in the literature [12]. Hyperparameter adjustment was performed using Bayesian optimisation to improve the accuracy of the GPR model. This procedure enhances the kernel functions and other parameters of the GPR model, guaranteeing effective generalisation to novel data and the generation of superior predictions. In the GPR model, 80% of the available data was allocated for training, while the remaining 20% was

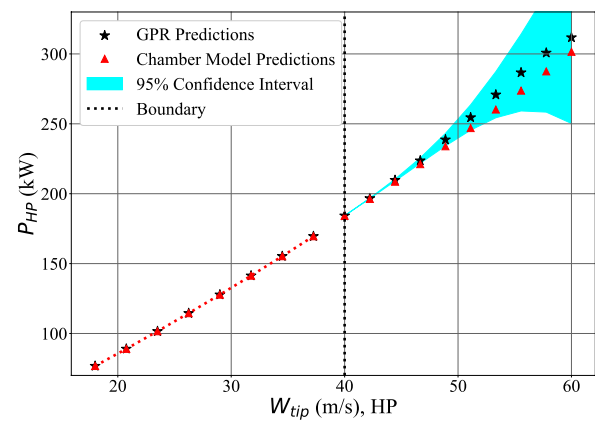
designated for testing, thereby establishing a rigorous evaluation framework.

Upon selecting the GPR solver within the modelling framework, forecasts for power and flow rates of individual stages were derived, and overall power consumption was computed (Refer Fig. 7a, b). The results were subsequently compared with the predictions of the chamber model, which was empirically confirmed in the preceding section. The comparison evaluates the precision and dependability of machine learning predictions in both familiar (territorial) and new (extra-territorial) areas.

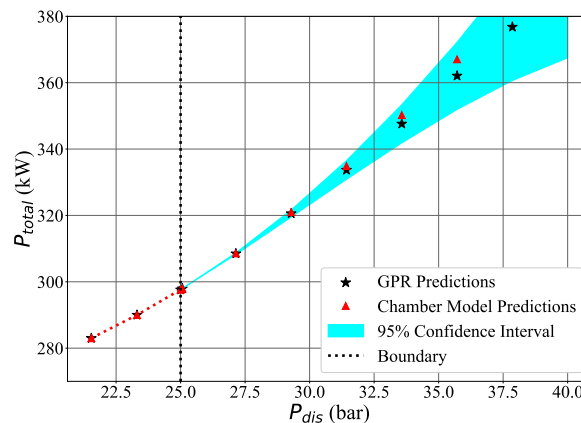
Figure 7c depicts the total power forecasts at various discharge pressures. The black star markers denote the GPR model predictions, whereas the red triangles signify predictions from the chamber model. The dotted boundary delineates the territorial region (where the model was trained and



(a) Low-pressure (LP) stage: Tip speed vs. power consumption predictions using the Gaussian Process Regression (GPR) model.



(b) High-pressure (HP) stage: Tip speed vs. power consumption predictions using the Gaussian Process Regression (GPR) model.



(c) Compressor performance predictions using the GPR model for various discharge pressures in both territorial and extra-territorial regions, with uncertainties. The predictions are compared to those from the physics-based chamber model.

**Fig. 7** Gaussian process regression (GPR) model predictions for compressor performance. **a, b** Tip speed versus power consumption for the low-pressure (LP) and high-pressure (HP) stages, respectively. **c**

Illustrates overall performance predictions for various discharge pressures, highlighting GPR predictions' accuracy and robustness

evaluated) from the extra-territorial region, encompassing entirely unknown data points that were excluded from both the training and testing datasets.

Although the GPR model faced unfamiliar data in the extra-territorial zone, the predictions consistently fell within the 95% confidence range, illustrating the model's resilience and applicability. The uncertainty boundaries, represented as coloured regions surrounding the GPR predictions, highlight the model's capacity to quantify prediction uncertainty. The comparison with the chamber model further corroborates the GPR solver's precision and underscores its capability for forecasting compressor performance under diverse operating situations.

#### 4 Performance optimization of two-stage screw compressor

The optimization process was conducted using a Bayesian optimization approach within the developed modelling framework, refining critical fluid injection properties for both stages of the two-stage screw compressor. The primary objectives were to minimize the total power consumption, maximise the volumetric flow rate ( $Q$ ), and reduce the specific power consumption (SPC). Through an iterative optimisation approach, each parameter was adjusted to improve compressor performance, as shown in Fig. 8a, b.

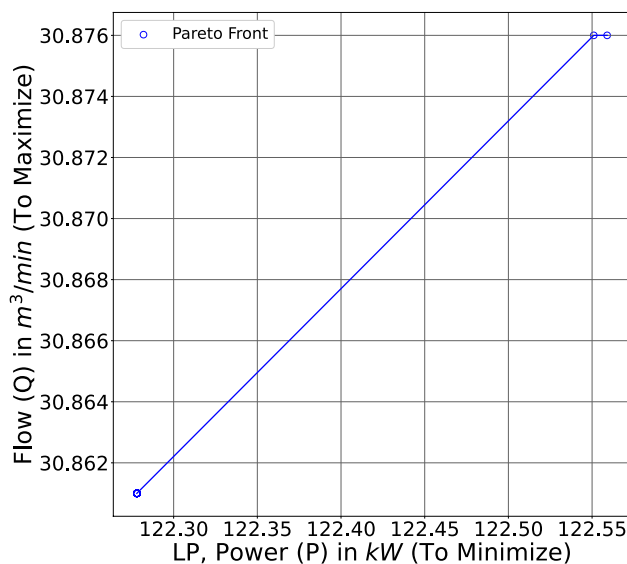
The main input parameters and working fluid properties used for optimization were:

- Suction Pressure for LP Stage: Atmospheric pressure (bar)
- Final Discharge Pressure: 23 bar
- Suction Temperature: 30 °C
- Number of Stages: 2
- Working Fluid: Air
- Initial Speed of LP Stage Male Rotor: 3000 RPM

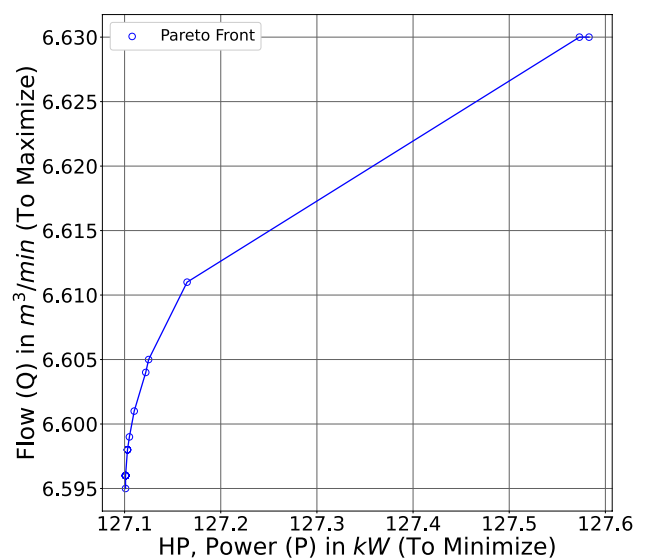
##### 4.1 Optimisation of fluid injection parameters

The fluid injection parameters for both stages were selected as key variables due to their direct impact on the compressor's thermal and mechanical performance. In this case, oil serves as the injected fluid to cool the compression chamber, seal leakage gaps, and lubricate rotors and bearings. For screw compressors, oil contributions are typically allocated in the following proportions: cooling (100 parts), sealing (10 parts), and lubrication (1 part). Optimising injection quantities and conditions is crucial, as excessive or insufficient oil injection can lead to additional power losses [37]. The fluid injection parameters considered in this study are detailed as follows:

- Oil Injection Pressure ( $P_{oil}$  in bar): This controls the flow rate of the injected oil. Higher oil flow rates improve heat exchange between the gas and oil, which can reduce discharge temperatures and enhance volumetric and adiabatic efficiency. However, excessive oil flow increases power losses due to higher resistance.



(a) Low-pressure stage optimization



(b) High-pressure stage optimization

**Fig. 8** Pareto Fronts illustrating the trade-offs between power and flow rate for **a** low-pressure stage and **b** high-pressure stage achieved through fluid injection parameter optimization



- Oil Injection Temperature ( $T_{oil}$  in °C): Lower injection temperatures improve heat exchange but can add strain on the cooling system. Optimal oil temperature helps maintain balance between cooling efficiency and system load.
- Oil Injection Angle ( $\theta$  in °): Proper positioning of the oil injection port within the compression chamber is essential for maximising heat transfer. An injection point closer to the discharge port limits residence time for heat transfer, while a point closer to the suction side enhances cooling efficiency.
- Oil Injection Port Diameter ( $D_{oil}$  in mm): This influences oil flow rate and pressure losses. An optimal port diameter minimizes frictional losses while ensuring adequate cooling, preventing thermal deformation.

## 4.2 Optimised parameters and performance results

The optimised parameters for both stages, along with the corresponding performance improvements, are presented in Table 2. Significant improvements were achieved through optimised fluid injection, which positively influenced the compressor's overall efficiency. The results demonstrate that total power consumption was reduced by 1.2%, with a nearly 2% reduction in specific power consumption (SPC), enhancing the system's energy efficiency. Additionally, the volumetric flow rate ( $Q$ ) saw a slight increase of approximately 0.8%, indicating an improvement in the compressor's capacity.

It is important to note that before defining operating conditions for optimisation, understanding their physical significance is essential. This aspect has been thoroughly addressed in the comprehensive doctoral work by Kumar [33], as well as recent publications by Kumar et al. [12, 14, 15]. Within this framework, optimisation is performed within feasible operating ranges determined by both manufacturing

considerations and the trade-off between minimising power consumption and maximising flow rate. When the operating conditions are chosen from these physically feasible ranges, the optimisation reliably converges to an optimal point, ensuring consistency in results. Furthermore, the framework's reliability is reinforced by its validation on multiple single-stage screw compressors prior to application on a two-stage screw compressor. This multi-stage implementation demonstrates the robustness and scalability of the proposed framework.

## 4.3 Interpretation of results

The optimised oil injection pressure of 19 bar, together with oil port widths of 12 mm for the low-pressure stage and 11 mm for the high-pressure stage, enhanced the flow rate and decreased the discharge temperature. Injection at roughly 40 °C facilitated efficient heat exchange without overburdening the cooling system. Moreover, oil injection angles of 122° for the LP stage and 76° for the HP stage ensured adequate residence time for heat transmission, with injection sites strategically positioned to enhance chamber temperature management.

These optimisations collectively yielded a 2% enhancement in system efficiency. The modified injection parameters reduced frictional losses and enhanced cooling and sealing effects, illustrating that meticulous regulation of fluid injection characteristics is essential for performance improvements.

This optimisation case study underscores the efficacy of the modelling tool in enhancing power and flow parameters. Nevertheless, in practical implementations, factors such as customer-specified flow requirements or power limitations imposed by the motor or drive system must be taken into account to ensure conformity with operational demands and system efficiency requirements.

**Table 2** Optimised fluid injection parameters and performance analysis for two-stage screw compressor

Optimised fluid injection parameters			
Parameter	Range (LP & HP)	Optimised value (LP)	Optimised value (HP)
$P_{oil}$ (bar)	18–23	19	19
$T_{oil}$ (°C)	40–90	40	44
$D_{oil}$ (mm)	4–20	12	11
$\theta$ (°)	30–150	122	76
Performance analysis of optimisation			
Performance parameter	Original value	Optimised value	% Change
Total Power (kW)	252	249	1.2%
SPC (kW/m <sup>3</sup> /min)	8.218	8.057	2.0%
$Q$ (m <sup>3</sup> /min)	30.664	30.903	0.8%

## 5 Conclusions

This study presents a comprehensive modelling framework for multi-stage screw compressors, enabling the optimisation of parameters for each stage. Experimental validation indicates that the framework functions within an error margin of 5%, satisfying the acceptance criteria for both the chamber model and the machine learning-based solver. This paper elucidates critical boundary conditions for multi-stage compressor modelling, emphasising the importance of intermediate-stage pressure and the impact of pressure reductions between stages on compressor efficacy.

This methodology facilitates faster and more computationally efficient performance predictions for both single-stage and multi-stage compressors compared to traditional thermodynamic solvers. The prototype created for this investigation was enhanced by fine-tuning fluid injection settings, leading to a power consumption reduction of around 2%, highlighting the efficacy of the optimisation strategy.

Future endeavours will concentrate on augmenting the data set with supplementary compressor sizes and employing sophisticated machine-learning techniques to improve the accuracy and generalisability of the Gaussian process regression (GPR) solver. Moreover, integrating real-world constraints into the optimisation process, such as customer-specific flow requirements or power limitations due to motor or engine capacity, would enhance the framework's actual usability for industrial purposes.

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**Data availability** Data will be made available on request.

## Declarations

**Conflict of interest** The authors declare no known competing financial interests or personal relationships that might have influenced the work reported in this paper.

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## References

1. Rinder L (1999) Oil-injected screw compressors for high-pressure applications. In: IMECHE conference transactions, Vol. 6. Mechanical Engineering Publications, pp 37–48
2. Kumar A, Kovacevic A, Stosic N (2025) Lifecycle cost analysis and performance evaluation of multi-stage screw compressors. *Proc Inst Mech Eng Part C J Mech Eng Sci* 09544062241312875
3. Sato H (2001) Development of high-pressure screw compressor. In: International conference on compressors and their systems 9–12 September 2001, pp 111–119 (2001)
4. Kovačević A, Stošić N, Smith IK (2006) Numerical simulation of combined screw compressor-expander machines for use in high pressure refrigeration systems. *Simul Model Pract Theory* 14(8):1143–1154
5. Vinogradov A, Volodichev S, Kanyshv G, Konstantinov V, Kuryshkin N (1986) Calculation of the geometrical parameters of screw compressors. *Chem Pet Eng* 22(10):493–495
6. Stosic N, Smith I, Kovacevic A (2005) *Screw compressors: mathematical modelling and performance calculation*. Springer, Berlin
7. Turner A, Nguyen P (2020) Comparing reciprocating and screw compressors in high-pressure applications. *Mech Eng Rev* 50:92–101. <https://doi.org/10.1016/j.mechrev.2020.92>
8. Garcia L, Thomas M (2019) Limitations and solutions in multi-stage screw compressors: a technical review. *Compress Syst Eng* 47(3):195–208. <https://doi.org/10.1007/s11464-019-3207-6>
9. Etefagh AH, Razfar MR (2024) Bayesian optimization of 3d bioprinted polycaprolactone/magnesium oxide nanocomposite scaffold using a machine learning technique. *Proc Inst Mech Eng Part B J Eng Manuf* 238(10):1448–1462
10. Fleming JS, Tang Y, Anderson H (1994) Optimisation techniques applied to the design of a refrigeration tin screw compressor
11. Guo C, Tang Y (2003) Influence of process parameters on screw rotor profiles. *Mach Sci Technol* 7(1):105–118
12. Kumar A, Patil S, Kovacevic A, Ponnusami SA (2024) Performance prediction and Bayesian optimization of screw compressors using Gaussian process regression. *Eng Appl Artif Intell* 133:108270
13. Kumar A, Kovacevic A, Anusuya Ponnusami S (2024) Optimisation of industrial oil-flooded screw compressors: a comparative analysis of conventional and soft computing approaches
14. Kumar A, Kovacevic A, Ponnusami S, Patil S, Abdan S, Asati N (2022) On performance optimisation for oil-injected screw compressors using different evolutionary algorithms. In: IOP conference series: materials science and engineering, vol 1267. IOP Publishing, p 012021
15. Kumar A, Kovacevic A, Stosic N (2024) Impact of rotor geometry and fluid injection on screw compressor performance. In: IOP conference series: materials science and engineering, vol 1322. IOP Publishing, p 012008
16. Stosic N, Hanjalic K (1997) Development and optimization of screw machines with a simulation model-part I: profile generation
17. Do Suh J et al (2001) Manufacture of composite screw rotors for air compressors by RTM process. *J Mater Process Technol* 113(1–3):196–201

18. Kumar A, Patil K, Kulkarni A, Patil S (2023) Investigating alternative rotor materials to increase displacement and efficiency of screw compressor while considering cost and manufacturability. In: International conference on compressors and their systems. Springer, Berlin, pp 115–125
19. You C, Tang Y, Fleming JS (1996) Optimum rotor geometrical parameters in refrigeration helical twin screw compressors
20. Kumar A, Bikramaditya N (2024) Advancing sustainability: a comprehensive study on energy-efficient screw compressors for biogas applications
21. Tang Y, Fleming J (1994) Clearances between the rotors of helical screw compressors: their determination, optimization and thermodynamic consequences. *Proc Inst Mech Eng Part E J Process Mech Eng* 208(2):155–163
22. Fujiwara M, Osada Y (1995) Performance analysis of an oil-injected screw compressor and its application. *Int J Refrig* 18(4):220–227
23. Hauser J, Bruemmer A, Kauder K (2008) Rotor profile generation and optimization of screw machines using nurbs
24. Hauser J, Brümmer A (2011) Geometrical abstraction of screw compressors for thermodynamic optimization. *Proc Inst Mech Eng C J Mech Eng Sci* 225(6):1399–1406
25. Kaya B, Oysu C, Ertunc HM, Ocak H (2012) A support vector machine-based online tool condition monitoring for milling using sensor fusion and a genetic algorithm. *Proc Inst Mech Eng Part B J Eng Manuf* 226(11):1808–1818
26. Li H, Zhang Q, Qin X, Yuantao S (2020) Raw vibration signal pattern recognition with automatic hyper-parameter-optimized convolutional neural network for bearing fault diagnosis. *Proc Inst Mech Eng C J Mech Eng Sci* 234(1):343–360
27. Lu K, Phung TH, Sultan IA (2021) On the design of a class of rotary compressors using bayesian optimization. *Machines* 9(10):219
28. Zheng C, Zhao W, Lyu B, Gao K, Cao H, Zhong L, Gao Y, Liao R (2025) Performance analysis for a rotary compressor at high speed: Experimental study and mathematical modeling. *Appl Therm Eng* 263:125275
29. Papananias M, McLeay TE, Mahfouf M, Kadirkamanathan V (2023) A probabilistic framework for product health monitoring in multistage manufacturing using unsupervised artificial neural networks and gaussian processes. *Proc Inst Mech Eng Part B J Eng Manuf* 237(9):1295–1310
30. Liu H, Lin H, Mao X, Jiang X, Liu Q, Li B (2019) Surface roughness optimal estimation for disc parts turning using gaussian-process-based bayesian combined model. *Proc Inst Mech Eng C J Mech Eng Sci* 233(11):4032–4046
31. Zhao X, Zhang D, Zhang R, Xu B (2022) A comparative study of gaussian process regression with other three machine learning approaches in the performance prediction of centrifugal pump. *Proc Inst Mech Eng C J Mech Eng Sci* 236(8):3938–3949
32. O'Neill PA (1993) Industrial compressors: theory and equipment. Butterworth-Heinemann
33. Kumar A (2025) Developing a framework for modelling and analysing of multi-stage screw compressors. Ph.D. thesis, City St George's, University of London
34. Williams CK, Rasmussen CE (2006) Gaussian Processes for Machine Learning, vol 2. MIT Press, Cambridge
35. P. I. Frazier, A tutorial on bayesian optimization. *arXiv preprint arXiv:1807.02811* (2018)
36. Kovacevic A, Rane S, Analysis P (2024) Scorg—screw compressor rotor grid generation. Software Package for Design and Analysis of Positive Displacement Machines. <https://pdmanalysis.co.uk/scorg/>. Accessed 16 Sept 2024
37. Basha N (2021) Numerical analysis of oil injection in twin-screw compressors. Ph.D. thesis, City, University of London

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