Keith Pilbeam and Kjell Noralf Langeland

CITY UNIVERSITY LONDON

Forecasting Exchange Rate Volatility: GARCH models versus Implied Volatility Forecasts

Abstract

This study investigates whether different specifications of univariate GARCH models can usefully forecast volatility in the foreign exchange market. The study compares in-sample forecasts from symmetric and asymmetric GARCH models with the implied volatility derived from currency options for four dollar parities. The data set covers the period 2002 to 2012. We divide the data into two periods one for the period 2002 to 2007 which is characterised by low volatility and the other for the period 2008 to 2012 characterised by high volatility. The results of this paper reveal that the implied volatility forecasts significantly outperforms the three GARCH models in both low and high volatility periods. The results strongly suggest that the foreign exchange market efficiently prices in future volatility.

Please note this is the public version of the paper, the final paper appears in International Economics and Economic Policy. **Corresponding author Keith Pilbeam K.S.Pilbeam@city.ac.uk**

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1. Introduction

The foreign exchange market is by far the largest and most liquid financial market in the world. As reported by the Bank for International Settlement in April 2013 the average daily turnover was $5.0 trillion. The foreign exchange market is made up primarily of three inter-related parts; spot transactions, forward transactions and derivative contracts. As with other financial markets currency markets can be volatile and exhibit periods of volatility clustering as traders react to new information.

Improving the forecasting of volatility in the foreign exchange market is important to multinational firms, financial institutions and traders wishing to hedge currency risks. Volatility is usually defined as the standard deviation or variance of the returns of an asset during a given time period. Traders of foreign currency options attempt to make profits by buying options if they expect volatility to rise above that implied in currency option premiums and writing options if they expect volatility to be lower than that currently implied by option premiums.

This paper examines the efficiency of the foreign exchange market in pricing option volatility by comparing the forecasts given the implied volatility from currency option prices with volatility forecasts from three different univariate GARCH models. If the foreign exchange market is efficient, then the implied volatility forecasts should outperform the GARCH forecasts. In addition, as Engle and Patton (2001) the whole point of GARCH forecasting models is that they should help in forecasting future volatility and as such see whether they can beat implied volatility forecasts is an interesting topic in itself. Our period of study which covers the period 2002 – 2012 is particularly interesting, since it also incorporates the period of the financial crisis which also resulted in a noticeable increase in turbulence in the foreign exchange market.

The paper is organized as follows section 2 gives a review on the three univariate GARCH models we use for our empirical forecasting exercise. Section 3 gives a more detailed introduction to the models being used and the estimation of volatility. Section 4 looks at the features of the data set and its properties. In section 5 we present the results of the study and section 6 concludes.

2. Review of the use of GARCH Models

In the last decade, forecasting exchange rate volatility has been a very popular topic in economic journals, see for example Busch *et al* (2012). Using different time periods, data frequency and the exchange rate pairs research has used a wide range of volatility models. Conditional variance models, such as ARCH and GARCH are the most often used to forecast volatility. In this study, we use both symmetric and asymmetric GARCH models (1). The symmetric model we use is the GARCH (1,1) of Bollerslev (1986) and Taylor (1986) the GARCH(1,1) model is far more widely used than ARCH due to the fact that it is more parsimonious and avoids over fitting (2) and is consequently less likely to breach the non-negativity constraint. We also look at two asymmetric models the EGARCH of Nelson (1991) and GJR-GARCH of Glosten, Jagannathan, and Runkle (1993). The EGARCH model has two key advantages over the GARCH (1,1). Firstly, the model measures the log returns, and therefore even if the parameters are negative, the conditional variance will be positive. Secondly, the model allows for asymmetries can capture the so called *leverage effect* (3). The second asymmetric model we use if the GJR-GARCH model of Glosten *et al* (1993). The GJR is an extension of GARCH with an additional term that is added to capture possible asymmetries. (4).We compare the forecasts of these models to the *implied volatility* series provided on bloomberg.

Bollerslev (1986) showed that the GARCH model outperformed the ARCH model. However, Baillie and Bollerslev (1991) used the GARCH model to examine patterns of volatility in the US forex market and results were generally poor. In the two decades after the arrival of ARCH and GARCH, several approaches building on GARCH has been created. EGARCH was introduced by Nelson (1991), NGARCH by Higgins and Bera (1992), GJR-GARCH by Glosten, Jagannathan and Runkle (1993), TGARCH by Zakoian (1994), QGARCH by Sentana (1995), and many more are available see for example Bollerslev (2008). In an interesting study, Hansen and Lunde (2005) finds that none of the models in the GARCH family outperforms the simple GARCH (1,1) which might be surprising since the GARCH (1,1) does not rely upon a leverage effect. While Nelson`s EGarch has several advantages over the linear GARCH model authors such as Brownlees and Gallo (2010) find that while at some horizons EGARCH produces the most accurate forecast, but at other horizons EGARCH is outperformed by the linear GARCH model. Donaldson and Kamstra (1997) used GJR-GARCH (1,1) to forecast international stock return volatility, and found that this model yielded better forecasts than the GARCH(1,1) and EGARCH(1,1). However using ARCH, GARCH, GJR-GARCH and EGARCH, Balaban (2004) found that the standard GARCH models was overall the most accurate forecast for monthly U.S. dollar-Deutsche mark exchange rate volatility.

Dunis *et al* (2003) examine the medium-term forecasting ability of several alternative models of currency volatility with respect to 8 currency pairs and find that no particular volatility model outperforms in forecasting volatility for the period 1991-99. Andersen and Bollerslev (2002) show how volatility at even very short term horizons as low as 5 minutes can have an information content in explaining intra-day and even daily volatility. In a similar vein, Ghysels *et al* (2005) suggest that mixing data at different time horizons can have a useful information content in forecasting future volatility. In a recent study, Ronaldo (2008) shows that there are intra-day patterns in exchange rate volatility depending upon the official opening and closing times of the domestic and foreign currency hours of business, with the domestic currency tending to weaken during the opening hour as domestic residents sell the domestic currency to obtain the foreign currency.

Regardless of the widespread literature on volatility model evaluation, we are nowhere close to finding the optimal model for providing the most favourable performance in forecasting volatility. However, this study is concerned with the efficiency with which the foreign exchange market is efficient in pricing currency options. If it prices these efficiently, then one would expect that implied volatility will outperform the econometric models such as provided by the GARCH models.

3. Alternative GARCH specifications

In this section, we will look at three GARCH models that we use in this study; namely the GARCH(1,1), EGARCH(1,1) and GJR(1,1).

The full GARCH (p,q) model is given by:

$$y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + u_t, \qquad (1)$$

$$h_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \qquad (2)$$

In the GARCH model the conditional variance depends upon the *q* lags of the squared error and the *p* lags of the conditional variance. From the equation (2) we see that the fitted variance called h_t (σ_t) is a weighted function the information about the volatility from the previous period's, the fitted variance from the model during the previous period and the long-run variance (α_0) (5). It should be noted that the GARCH model is symmetric because of the sign of the disturbance being ignored. Since we are using the GARCH(1,1) the conditional variance of the model is:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad (3)$$

where σ_t^2 is the conditional variance because it is a one period ahead estimate for the variance calculated on any past information thought to be relevant. While the conditional variance depends on past observations the *unconditional variance* of GARCH model is constant and more concerned with the long-term behaviour of the time series. The unconditional variance is given by:

$$Var(\mathrm{u}_t) = \frac{\alpha_0}{1-(\alpha_1+\beta)} \qquad (4)$$

The coefficient measures the extent to which extent to which a volatility shock today feeds through into next period's volatility, in other words it corresponds to the *long term volatility*. As long as $\alpha_1 + \beta < 1$, the unconditional variance is constant (6).

The exponential GARCH model is one of many approaches to the standard GARCH model. There are several ways to express the conditional variance equation in the EGARCH model. We use the following specification:

$$\ln(\sigma_t^2) = \omega + \beta ln(\sigma_{t-1}^2) + \gamma \ \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] \quad (5)$$

In this equation ω represents the long term average value. The parameter γ allows for asymmetries, since if the relationship between volatility and returns are negative, γ will be negative implying that good news generates less volatility than bad news (7). The unconditional variance of EGARCH is given by:

$$\ln(\sigma_t^2) = E(\frac{\omega}{1-\beta}) \quad (6)$$

The GJR-GARCH variant also includes a leverage term to model asymmetric volatility. In the GJR model, large negative changes are more likely to be followed by large negative changes than positive changes. The GJR model is only a simple extension of the GARCH model, with an additional term added to capture possible asymmetries. The GJR-GARCH specification is given by:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1} \quad (7)$$

Where $I_{t-1} = 1 \ if \ u_{t-1} < 0$ otherwise $I_{t-1} = 0$

If there is a leverage effect, we will observe that $\gamma > 0$. It can also be observed that the non-negativity constraint that has to be imposed requires that $\alpha_0 > 0, \alpha_1 > 0,$ $\beta \geq 0, and \ \alpha_1 + \gamma \geq 0$ and explains why this model is less likely to breach the non-negativity constraint than the standard GARCH model. The model is still tolerable if $\gamma < 0$, given that $\alpha_1 + \gamma \geq 0$ holds. Even though the GJR model has the same purpose as the EGARCH model, the way this models act is different. As can be seen from equation (5), the leverage coefficient of the EGARCH is directly connected to the actual innovations. However for the GJR-GARCH as given by equation (7) we see that the leverage coefficients are connected through an indicator variable (I). As such, when an asymmetric shock occurs, the leverage effect for the GJR model should be positive, while the leverage effect should be negative for the EGARCH model. Hence, the two models are different even though they are designed to capture the same effects.

The unconditional variance for the GJR-GARCH model is given by:

$$var(u_t) = \frac{\omega}{1-(\alpha+\frac{\gamma}{2}+\beta)} \qquad (8)$$

When estimating the parameters in the GARCH models we employ the maximum likelihood since its estimates are more efficient than the OLS because the distribution converges to the true value of the parameter at faster rate and generally the maximum likelihood is finds the most likely values of parameters given the actual data. In both asymmetric and symmetric GARCH models this technique is commonly used for finding the parameters. Next we have to specify the appropriate equations for mean and variance. If we have an autoregressive process with one lag and a GARCH(1,1) model, the mean and variance equation will be:

$$y_t = \mu + \varphi y_{t-1} + u_t \qquad u_t \sim N(0, \sigma^2) \qquad (9)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad (10)$$

Given the mean and variance equation, we can now specify the log-likelihood function (LLF) to have to be maximised under a normality assumption for the error terms:

$$L = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\log(\sigma^2) - \frac{1}{2}\frac{\sum_{t=1}^{T}(y_t - \mu - \varphi y_{t-1})^2}{\sigma^2} \qquad (11)$$

In the presence of conditional heteroscedasticity we have to make a few adjustments to $\sim y_t$ we change the assumptions of the error terms to $\sim N(0, \sigma_t^2)$ so that the variances are varying with time. In the log likelihood function for the GARCH model we substitute the second term, $\frac{T}{2}\log(\sigma^2)$, with $\frac{1}{2}\sum_{t=1}^{T}\log(\sigma_t^2)$. In addition, we replace σ^2 in equation (11) with σ_t^2. When there is heteroscedasticity in the error terms, the calculation of LLF are more complicated and we used MATLAB to do the calculations. In line with many earlier studies, we ended up with constant mean GARCH(1,1) model, hence the conditional variance is dependent upon one moving average lag and one autoregressive lag. We performed a Ljung-Box-Pierce Q-test in order to verify that it is not any correlation in the raw returns up to 20 lags.

Following Andersen *et al* (2001 and 2003) to calculate the realized volatility we used the following calculation (8):

$$Realized\ Volatility = \sqrt{252 * \left(\sum_{i=1}^{N-1} \frac{R_i^2}{N_e - 1}\right)} \qquad (12)$$

$$R_i = ln\left(\frac{P_t}{P_t - 1}\right) ,Daily\ return\ on\ exchange\ rates\ from\ P_t\ \ to\ P_{t-1}$$

$$N = Number\ of\ trading\ days\ in\ the\ period$$

$$P_t = Underlying\ reference\ price\ at\ time\ t$$

$$P_{t-1} = The\ underlying\ reference\ price\ the\ the\ time\ period\ preceding\ time\ t$$

We use the Root Mean Square Deviation (RMSE) to measure the accuracy of the forecasts as given by equation (13)

$$RMSE(\hat{\theta}) = \sqrt{MSE(\hat{\theta})} = \sqrt{\frac{1}{N}\sum_{t=1}^{n}\left(\left(\hat{\theta}_i - \theta_i\right)^2\right)} \qquad (13)$$

Where:

$\hat{\theta}$ = the predicted value of the data and $\theta = the\ actual\ value\ of\ the\ data$

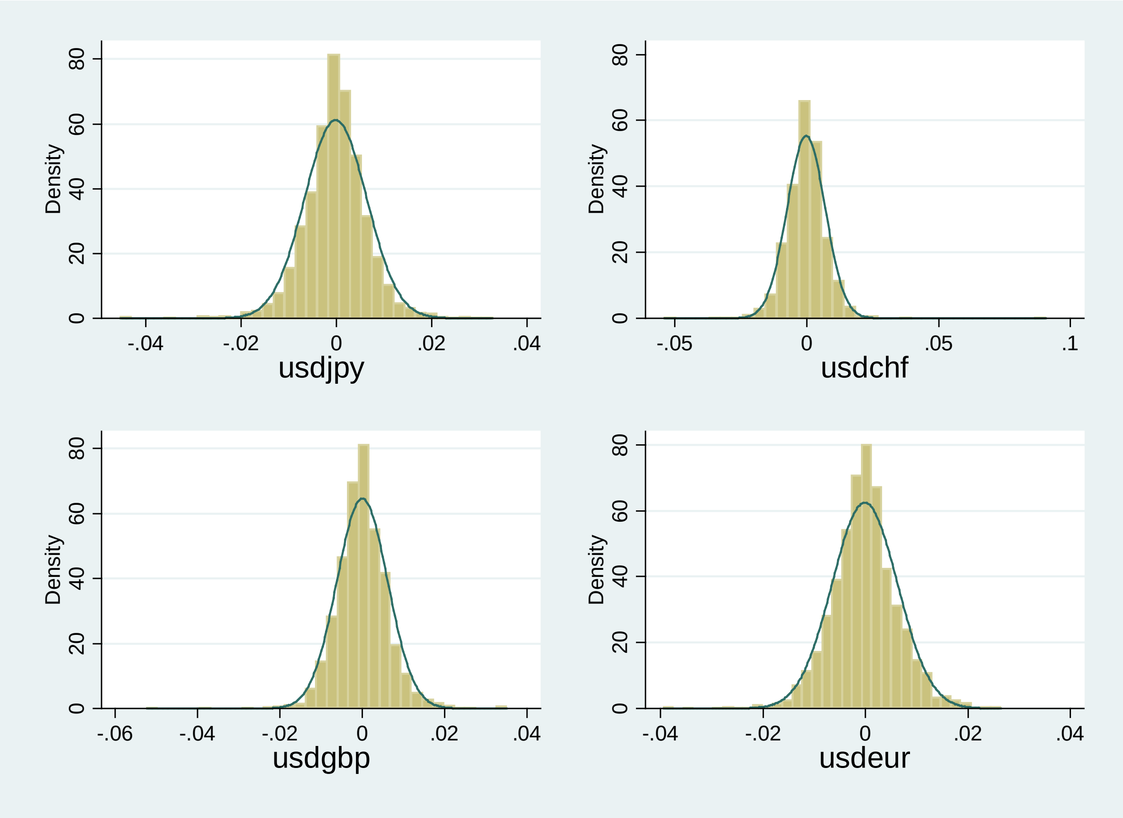
The RMSE has the advantage of being measured in the same unit as the forecasted variable.

In this study we generate forecasts within the sample. So, for the in-sample forecasting, all observations within the period will be used to estimate the models, and the results will be compared to the actual value (realized volatility). Using in sample forecasts means maximises the chance that the GARCH models will beating the implied volatility forecast. In this section we will look at the data sampled for this study.

4. Data

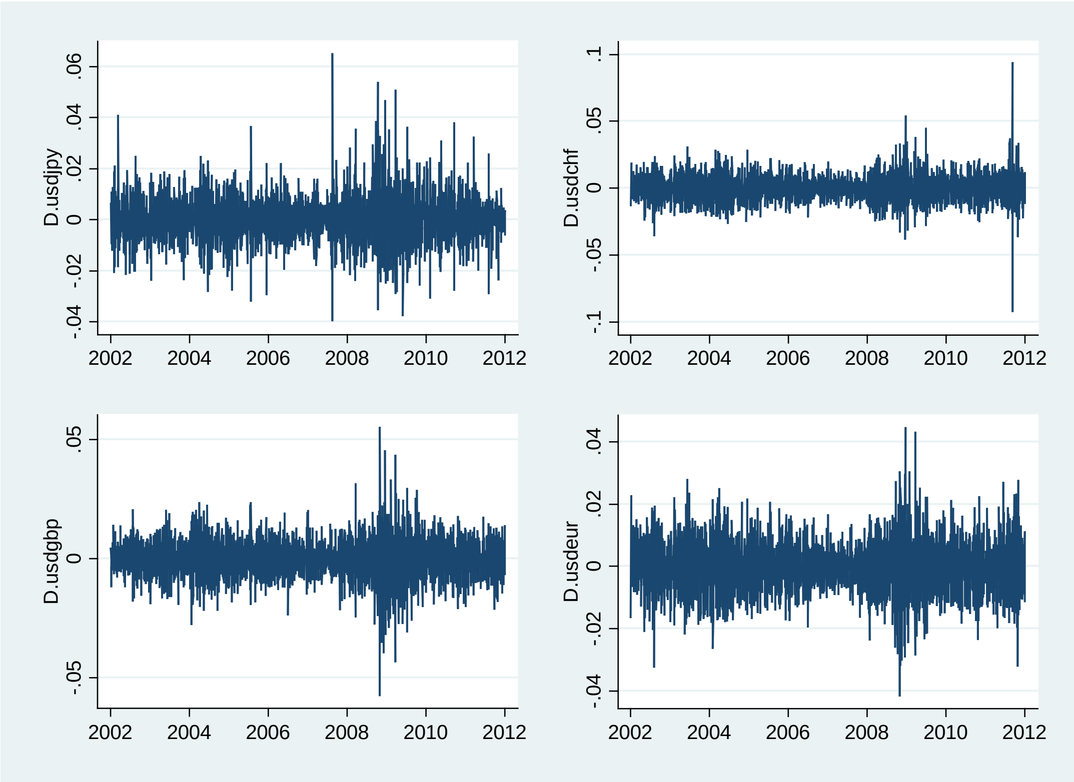
We have collected daily closing prices for four currency pairs the euro, pound, swiss franc and yen against the dollar. The data has been collected from 1/1-2002 to 30/12-2011. Each currency pair had 2609 observations, Figure 1 shows the empirical distribution of returns. We will use a histogram to illustrate the density of returns and a curve from normal distribution is overlaid.

Figure 1 The distribution of daily exchange rate returns



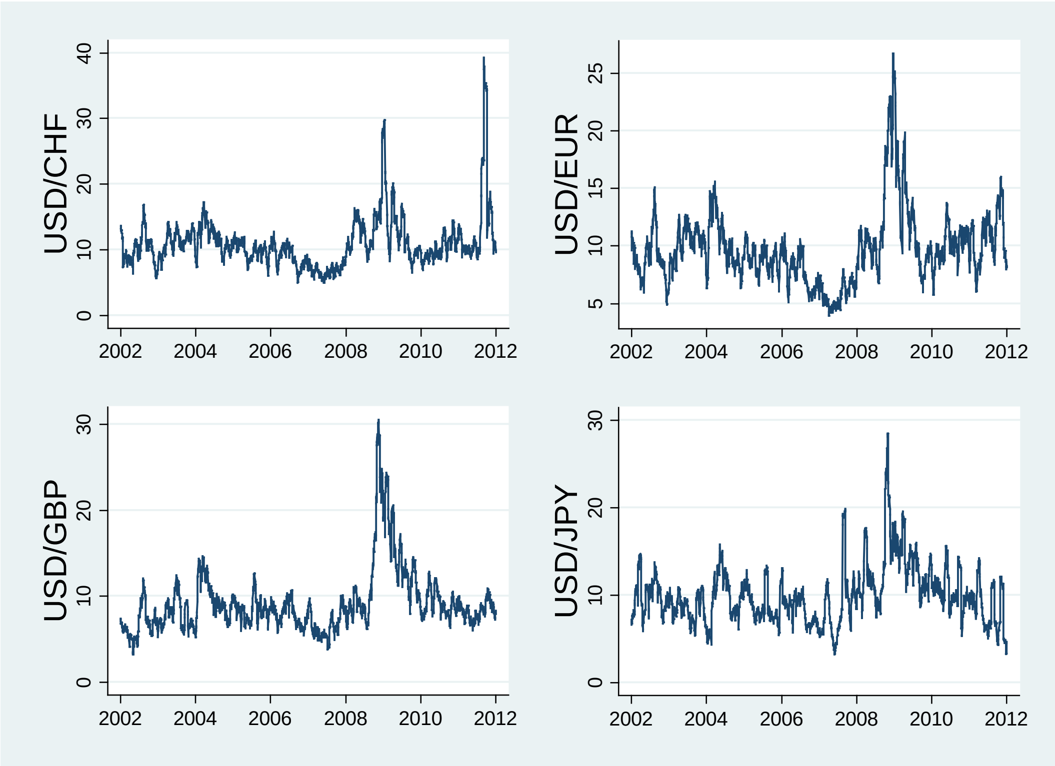
From Figure1 we see that the returns approximate to a normal distribution. Figure 2 shows that daily log of returns during the time period under study and there are clear periods of volatility clustering.

Figure 2 Log of Daily returns



In Figure 2 we can see that that the series are stationary with most of the returns being located around zero. However these some spikes in the first order difference in periods with high volatility. To compare the proposed models we will use the realised volatility which is shown in figure 3:

Figure 3 Realized volatility



Source Bloomberg

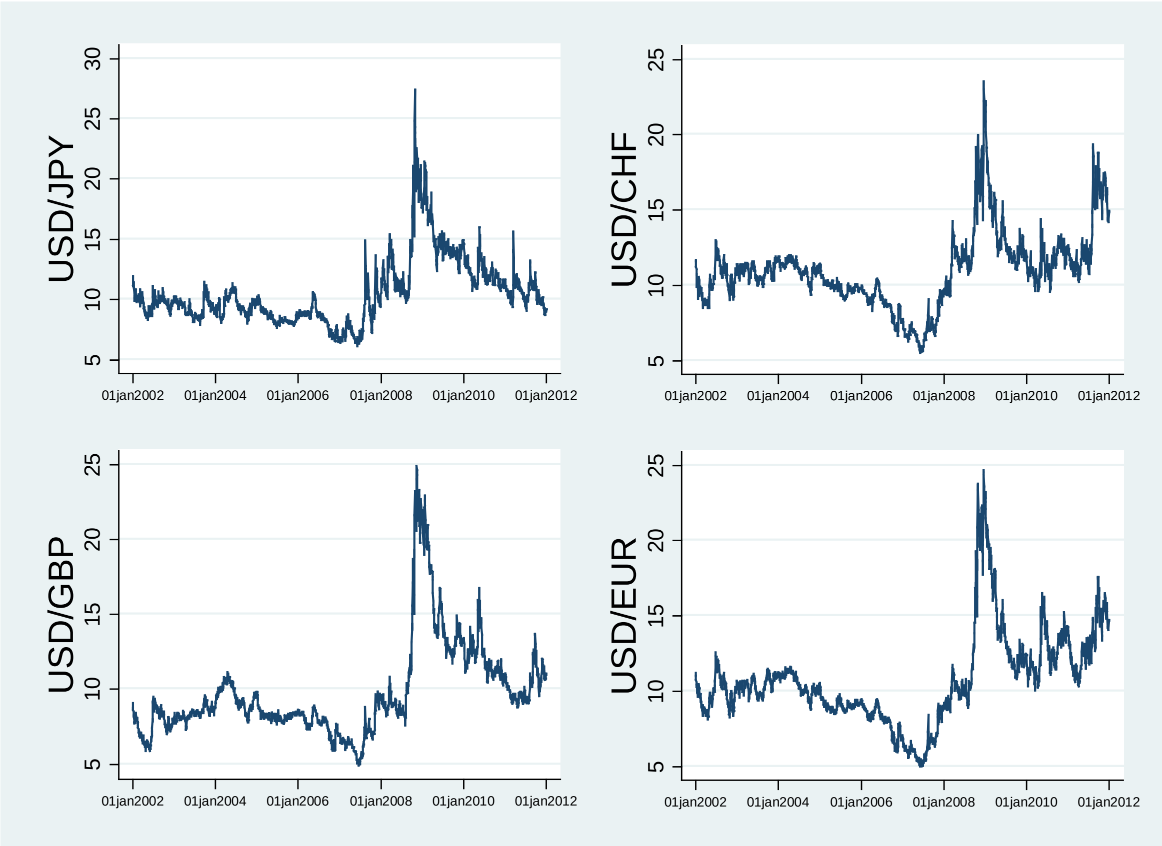
The credit crunch as observed caused a spike in the volatility in all of the exchange rate pairs starting in 2008, the properties of the realised volatility are outlined in Table 1

Table 1: Properties of the realized volatility

	Observations	Mean	Std.Dev	Min	Max
JPY	2609	9.99	3.35	3.24	28.49
CHF	2609	10.95	3.96	4.92	39.30
GBP	2609	9.22	3.77	3.21	30.54
EUR	2609	9.74	3.20	3.93	26.72

From table 1 we see that the swiss franc – dollar parity has both the highest mean of volatility and highest standard deviation, this currency pair also has by far has the largest spread in volatility, much caused by the two spikes in volatility.

Figure 4 Implied Volatility



Source: Bloomberg

Figure 3 plots the data on implied volatility. We can see the similarity between the realized and implied volatility. However we see that in case of the swiss franc – dollar parity, the estimated peaks look different for the realized and implied volatility. We can look closer at the properties by putting the data statistics in a table.

Table 2: Properties of the implied volatility

	Observations	Mean	Std.Dev	Min	Max
USDJPY	2609	10.50	2.77	6.10	27.39
USDCHF	2609	10.96	2.54	5.52	23.52
USDGBP	2609	9.76	3.25	4.93	24.95
USDEUR	2609	10.71	3.10	5.12	24.65

5. Empirical Results

In section, we present results of the in-sample forecast for both the full period and the pre and post commencement of the financial crisis periods. For the GARCH (1,1) model we have four unknown parameters to estimate, namely, $C, \alpha_0, \alpha_1, \beta$. Estimates were made using MATLAB and are reported it Table 3:

Table 3: Value of GARCH (1,1) parameters for period 2002-12

GARCH(1,1)				
Variance Equation	C	$\alpha_0 * e^{007}$	α_1	β
USD/JPY	-0.0001	6.4891	0.0301	0.9533
	(-1.01)	(6.726)	(8.08)	(177.59)
USD/CHF	-0.0003	2.3697	0.0392	0.9576
	(-2.08)	(-2.31)	(9.64)	(177.77)
USD/GBP	-0.0001	2.5964	0.0397	0.9531
	(-1.30)	(3.45)	(8.46)	(163.49)
USD/EUR	-0.003	2.6433	0.0333	0.9620
	(-2.30)	(2.87)	(7.45)	(206.59)

Notes: t-statistics are in parentheses

α_1 is the *ARCH parameter* and significant for all exchange rates at the 1% significance level, β is the *GARCH parameter* is also significant for all exchange rate pairs on a 1% level. Given the estimated parameters for the variance equations, *ex-post* forecasts were carried out (10).

In the EGARCH model an additional parameter has to be estimated in comparison with the standard GARCH model. Since this model is asymmetric, γ has to be estimated in order to capture the leverage effect. The results are reported in Table 4.

Table 4: Value of EGARCH(1,1) parameters and significance for period 2002-12

EGARCH(1,1)					
Variance Equation	C	ω	α_1	B	γ
USD/JPY	-0.0002	-0.2178	0.0927	0.9781	-0.0545
	(-2.07)	(-6.62)	(8.82)	(303.11)	(-9.45)
USD/CHF	-0.004	-0.0145	0.0745	0.9982	-0.0269
	(-2.92)	(-0.98)	(10.61)	(663.10)	(-5.34)
USD/GBP	-0.0001	-0.0773	0.0938	0.9923	0.0058
	(-1.09)	(-3.61)	(9.20)	(481.07)	(1.91)
USD/EUR	-0.0002	-0.0398	0.0743	0.9959	0.0040
	(-2.22)	(-2.37)	(7.45)	(601.41)	(1.67)

Note t-statistics in parentheses

We can see from **Table 4** that the leverage parameter is not significant at 1% level for pound-dollar and euro-dollar pairs. However the leverage parameter is significant for both the USD/JPY and USD/CHF. Both the ARCH and GARCH parameters are significant on a 1% level for all exchange rate pairs and as in the standard GARCH(1,1) the GARCH parameters are strongly significant (10).

As with EGARCH, modelling the GJR-GARCH model requires estimation of, γ to capture the leverage effect, the results are reported in **Table 5**

Table 5: GJR-GARCH (1,1) parameter estimates for period 2002-12

GJR(1,1)					
Variance Equation	C	α_0*e^{007}	α_1	β	γ
USD/JPY	-0.0002	7.8472	0.0122	0.9449	0.0475
	(-1.50)	(6.62)	(2.62)	(145.24)	(6.44)
USD/CHF	-0.0003	2.7445	0.0156	0.9634	0.0365
	(-2.73)	(2.17)	(3.37)	(202.14)	(5.28)
USD/GBP	-0.0001	2.3673	0.0441	0.9546	-0.0110
	(-1.12)	(3.24)	(7.66)	(166.22)	(-1.81)
USD/EUR	-0.0002	2.1432	0.0369	0.9629	-0.009
	(-2.08)	(2.94)	(6.53)	(207.74)	(-1.73)

Note t-statistics in parantheses

From Table 5 we can see that the leverage parameter is significant at the 1% level for yen-dollar and swiss franc dollar parities but only significant at the 10% level for yen-pound-dollar and euro- dollar parities. The GARCH and ARCH parameters are significant at the 1% level. We see that the leverage parameter is negative for pound-dollar and euro-dollar parity, indicating that the relationship between returns and volatility is negative. However, the leverage parameter is allowed to be negative as long as $\alpha_1 + \gamma \geq 0$, which is the case. In addition the following constraints that $\alpha_0 > 0, \alpha_1 > 0,\ \beta \geq 0$ are all are satisfied in our case.

The results reported in Tables 3 to 5 show clear support for the significance of GARCH effects in modelling exchange rate volatility. The results for the leverage parameter in the EGARCH and GJRGARCH models are, however somewhat mixed, as there are significant results for only the Japanese Yen and Swiss franc parities. It is important not to worry about the change in the sign of the leverage coefficient as between the EGARCH and GJR models. In the EGARCH model given by equation (5) the model the γ parameter estimates are directly connected to the actual innovations. While in the GJR model, given by equation (7), the leverage coefficient is connected through an indicator variable (I). So when an asymmetric shock occurs, the leverage effect should be negative for the EGARCH model and positive for the GJR model.

In table 6 we report the in sample volatility forecasts derived from the parameters estimates reported in in tables 3 to 4 for the whole sample period 2002-2012

Table 6: Root Mean Squared Error of GARCH and Implied Volatility Forecasts for the period 2002-2012

USD/JPY				
	GARCH(1,1)	EGARCH(1,1)	GJR(1,1)	Implied Volatility
RMSE	0.0837	0.0835	0.0820	0.0459
Rank	4	3	2	1
USD/CHF				
	GARCH(1,1)	EGARCH(1,1)	GJR(1,1)	Implied Volatility
RMSE	0.0888	0.0943	0.0917	0.0530
Rank	2	4	3	1
USD/GBP				
	GARCH(1,1)	EGARCH(1,1)	GJR(1,1)	Implied Volatility
RMSE	0.0766	0.0778	0.0763	0.0384
Rank	3	4	2	1
USD/EUR				
	GARCH(1,1)	EGARCH(1,1)	GJR(1,1)	Implied Volatility
RMSE	0.0776	0.0787	0.0778	0.0409
Rank	2	4	3	1

In table 6 we report the in sample forecasting accuracy of the different models using RMSE criterion. It can be clearly seen that the implied volatility forecast is clearly superior to all three GARCH models. The EGARCH(1,1) is ranked 4th in three out of four exchange rates. This despite the fact tha EGARCH(1,1) allows for "good" and "bad" news to have different impacts on volatility. The standard GARCH(1,1) performs better than EGARCH model and is ranked second in two out of four exchange rates. Overall, the GARCH forecasts are heavily outperformed by the implied volatility forecasts which suggests that the foreign exchange markt is efficient. Indeeed, the implied volatility are significantly below the GARCH estimates for all four currencies studied.

It is well known from previous studies that in periods of high volatility, the GARCH models tend to significantly underestimate volatility. As such, it is important to compare the forecasts in periods with high and low volatility. For this reason, we have divided the data into two sub periods, the pre-financial crisis period 2002-7 and the post-financial crisis period 2008-12. The results of the in sample forecasting exercise is shown in Tables 7 and 8.

Table 7: Root Mean Squared Error for one-step-ahead in-sample forecasts 2002-2007

USD/JPY				
	GARCH(1,1)	EGARCH(1,1)	GJR(1,1)	Implied Volatility
RMSE	0.0890	0.0895	0.0873	0.0516
Rank	3	4	2	1
USD/CHF				
	GARCH(1,1)	EGARCH(1,1)	GJR(1,1)	Implied Volatility
RMSE	0.0943	0.0939	0.0961	0.0413
Rank	3	2	4	1
USD/GBP				
	GARCH(1,1)	EGARCH(1,1)	GJR(1,1)	Implied Volatility
RMSE	0.0774	0.0779	0.0778	0.0375
Rank	2	4	3	1
USD/EUR				
	GARCH(1,1)	EGARCH(1,1)	GJR(1,1)	Implied Volatility
RMSE	0.0849	0.0848	0.0850	0.0373
Rank	3	2	4	1

As can be seen in table 7 that there is not a dominant in-sample forecaster among the GARCH models. However, the GJR model is now the least accurate forecast in total. This is the opposite compared to the full period in-sample forecast in section. The implied volatility forecast still outperforms all three GARCH models suggesting that the foreign exchange market efficiency hypothesis .

We turn our attention to the period 2008-2012 which is related to periods with greater uncertainty and instability in the financial markets. The volatility is generally at a much high level, and volatility clustering appears in all the exchange rate series .The in-sample forecasts for the period 2008 to end 2012 are reported in Table 8.

Table 8: Root Mean Squared Error for one-step-ahead in-sample forecasts 2008-2012.

JPY				
	GARCH(1,1)	EGARCH(1,1)	GJR(1,1)	Implied Volatility
RMSE	0.1612	0.1600	0.1478	0.0849
Rank	4	3	2	1
CHF				
	GARCH(1,1)	EGARCH(1,1)	GJR(1,1)	Implied Volatility
RMSE	0.1714	0.1893	0.1783	0.1173
Rank	2	4	3	1
GBP				
	GARCH(1,1)	EGARCH(1,1)	GJR(1,1)	Implied Volatility
RMSE	0.1523	0.1562	0.1439	0.0779
Rank	3	4	2	1
EUR				
	GARCH(1,1)	EGARCH(1,1)	GJR(1,1)	Implied Volatility
RMSE	0.14643	0.150025	0.146787	0.0856
Rank	2	4	3	1

Table 8 shows the performance of the GARCH models during the period after the commencement of the financial crisis which was characteried by higher exchange rate volatility. During this time-period the GARCH models are significantly less accurate and the implied volatility forecasts are also less satisfactory compared to the pre risis period. Npnethless the implied volatility forecasts are significantly better than the GARCH models suggesting continued foreign exchange market pricing of options even in periods of high volatility. However is should be remembered as Nelson (2009) points out that implied volatility forecasts can themselves be far from optimal.

In sum, we can see that the GARCH models do not fit the data well in periods of higher volatility. We see how more accurate the models fit the data in the period before the credit crunch. However we observe that this is also the case for the implied volatility. In the first period, the implied volatility is a very good predictor, however, in the high volatility period the implied volatility forecast performs significantly less well in predicting the true volatility.

6. Conclusions

This study shows that GARCH models are not particularly useful in forecasting foreign exchange volatility in periods of either low or high volatility. This can be seen in that none of our three models come close to fitting the data as well as the implied volatility. By contrast, the implied volatility forecasts outperform the GARCH models by a significant amount in both the low and high volatility periods.

Our results also confirm that GARCH models perform significantly better in periods characterised by low volatility than during periods of high volatility. Interestingly the implied volatility forecast also performs noticeably poorer in periods of high volatility but despite this it continued to significantly outperform the GARCH model forecasts. The results of this study therefore strongly suggest that the foreign exchange market efficiently prices foreign currency options so that unusual excess profits cannot be made using past volatility such as used by GARCH models to forecast future volatility. Indeed, the GARCH models do not come close to being a competitor to the superior information contained in implied volatility.

Typically one of the objectives of foreign exchange rate policy has been to iron out “excessive volatility”, but if the foreign exchange market is efficiently pricing in volatility as our results tend to suggest, then from a policy perspective this suggests that the need to intervene to iron out exchange rate volatility is reduced. By efficiently pricing in future prospective volatility currency options provide a means for companies to effectively hedge volatility.

Another policy implication of the market efficiently pricing in volatility is that speculators provide a stabilising influence on the foreign exchange market. Since speculators can buy or sell volatility through currency options which are efficiently priced, then policy makers should worry less about their role in determining exchange rates. Indeed there is a danger that the introduction of a “Tobin tax” on foreign exchange transactions could interfere with the efficiency and price discovery process in the foreign exchange market.

Areas for further research could involve the use of alternative models such as the AP-GARCH specification. Another possibility would be to move away from *univariate* models to the use of *multivariate* GARCH models, see for example, Silvennoinen (2008) incorporating independent macroeconomic variables, such as interest rates, fiscal indicators, current account balances, money supplies and government expenditure. Another approach could be that taken by Bildirici, and Ersin (2011) who suggest supplementing GARCH models with the use of neural networks to improve their forecasting ability.

In this study, we have analysed volatility using daily closing prices to perform *ex-post* forecasts. It would be interesting to use higher frequency data to see whether the results reported in this paper extend to forecasting intra-day volatility. Authors such a**s** Chen *et al* (2011) have shown that combining a variety of GARCH models and use of intra day data can provide useful information for forecasting daily volatility.

Footnotes

(1) A symmetric model means that when a shock occurs, we will have a symmetric response of volatility to both positive and negative shocks. Asymmetric models on the other hand, allow for an asymmetric response with empirical results show that negative shocks will lead to higher volatility than a positive shock.

(2) Overfitting happens when the statistical model describes a random error or noise instead of the underlying relationship, causing biasedness in parameter estimates.

(3) The leverage effect where it is typically interpreted as a negative correlation between lagged negative returns and volatility.

(4) As with the EGARCH the GJR-GARCH model captures the leverage effect but the way that it acts is not the same as for the EGARCH, The GJR-GARCH does not measure log returns, so in this model we still need to impose non-negative constraints.

(5) We observe that the difference between ARCH and GARCH is the last term that makes the model less likely to break the non-negativity constraint.

(6) If the restriction does not hold we will have non-stationarity in the variance, if $\alpha_1 + \beta = 1$, we have a unit root in the variance.

(7) If $\gamma = 0$, the model is symmetric. There is no need to be concerned about the conditional variance being negative since $ln(\sigma_t^2)$ is modelled.

(8) Bollerslev *et al* (2001) argue that this type of volatility is an unbiased and very efficient estimator of return volatility.

(9) It should be noted that the parameters $(\alpha + \beta)$ were less but close to unity, suggesting that the shocks are highly persistent and die out only gradually.

(10) It should be noted that the parameters are “forced” to be positive since we are measuring the natural log of returns. In theory, the “EGARCH benchmark model” has an AR(1) mean equation, but in our case the parameters proved to be more significant using a constant mean equation.

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