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Networked relationships in the e-MID Interbank market: A trading model with memory

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Abstract

In this paper, we introduce a model of interbank trading with memory. The memory mechanism is used to introduce a proxy of trust in the model. The key idea is that a lender, having lent many times to a borrower in the past, is more likely to lend to that borrower again in the future than to other borrowers, with which the lender has never (or has infrequently) interacted. The core of the model depends on only two parameters, which are common to all lenders: one is w and it is representing the attractiveness of borrowers, the other is Q and it represents the memory of lenders in their assessment of counter parties. The stronger the w parameter, the more random the matching results between lenders and borrowers. The stronger the Q parameter, the more stable trading relationships become. Model outcomes and real money market data are compared through a variety of measures that describe the structure and properties of trading networks. These include number of statistically validated links, bidirectional links, and 3-motifs. The model reproduces well features of preferential trading patterns empirically observed in a real market.

Keywords: Interbank market, Network formation, Statistically validated

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networks, Java/Mason
JEL: G15, G21

1. Introduction

Well functioning interbank markets effectively channel liquidity from institutions with surplus funds to those in need and thus play a key role in banks liquidity management and the transmission of monetary policy. Before the 2007-2008 financial crisis, liquidity and credit risks were perceived as negligible in these markets. Nonetheless the collapse of interbank lending has been a central feature of the subprime financial crisis. Liquidity hoarding and trust evaporation have been identified as two important determinants of the interbank market drying up during the crisis (Heider et al. (2009); Acharya and Merrouche (2013)). Haldane (2009) has advocated that the interbank market freeze is a manifestation of the behaviour under stress of a complex, adaptive network, the complexity arising from the interconnectedness of players via mutual exposures to each other, and the adaptation from the attempts of agents to optimise interdependent strategies in the presence of (Knightian) uncertainty. Several authors have since called for the adoption of network analysis to understand the mechanisms leading to the formation of trading relationships.

Recently some theoretical studies have considered the problem of network formation in a financial system (Babus (2007), Allen and Babus (2008)) and also, from the perspective of network formation games (Jackson and Wolinsky (1996), Dutta et al. (2005), Bloch and Jackson (2007), Goyal and Vega-Redondo (2007)). The presence of a network underlying the bilateral credit interactions occurring, for example, in an interbank market has a role in the setting of both linkages that insure against liquidity risk and linkages that can channel contagion risk.

The empirical network literature has aimed at characterising the observed topology of the interbank market checking for regularities and stylised facts (Boss et al. (2004), Iori et al. (2008), Iazzetta and Manna (2009), Bech and Atalay (2010), Craig and Von Peter (2010), Brauning and Fecht (2011), Martinez-Jaramillo et al. (2014), Iyer and Peydro (2011)).

Simulation studies have attempted to quantify more directly the impact of the network structure on the propagation of contagion addressing a number of complementary issues including: the relationship between the network structure of the interbank market and its resilience to different kind of shocks (Gai

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et al. (2011), Iori et al. (2006), Battiston et al. (2012), Lenzu and Tedeschi (2012), Georg (2013), Ladley (2013)); the effects of assets fire-sale (Nier et al. (2007)); roll-over risk and portfolio overlaps (Anand et al. (2013), Caccioli et al. (2012)); impact of regulatory taxes (Thurner and Poledna (2013), Poledna and Thurner (2014)) feedback loops between the macroeconomy and the financial sector (Grilli et al. (2012)). An actor-oriented model of network formation in the Interbank market is proposed in Finger and Lux (2014).

Broadly speaking, two complementary approaches have been adopted. In the agent-based models case, the interbank exposure networks and default events emerge endogenously from the behavioural rules followed by the economic agents (Iori et al. (2006), Georg (2013), Ladley (2013)). In the stress test experiments case, the exposure network is taken as exogenously given, either calibrated to real market data or generated according to preset specifications (Upper (2011).Gai and Kapadia (2010), Caccioli et al. (2012)). In this second approach, even when detailed information on banks bilateral exposures is available, the analysis is typically restricted to few snapshots of the banking system. For this reason a probabilistic approach has been advocated consisting in generating an ensemble of random networks, of which the empirical network can be considered a typical sample. This allows to analyze not only the vulnerability of one particular network realization retrieved from the real data, but of many alternative realistic networks, compatible with a set of constrains (Lu and Zhou (2011), Halaj and Kok (2013)).

In the present study we introduce a model of preferential formation of bilateral credit relationships in a centralized interbank market with heterogeneous market participants. The market heterogeneity, in number of credit transactions, is assumed to be exogenously given. The existence of stable credit relationships is associated with the detection of over-expression of bilateral transactions with respect to a null hypothesis or random matching, taking into account banks' heterogeneity. Our model contributes to both streams of the literature. In itself it is a simple agent based model, but in assuming a predefined constrain, that can be calibrated from real data, it can be used as a tool for generating scenarios for stress-test experiments.

While our model has a general validity we test its performance against the e-MID market. The e-MID is the only electronic market for interbank deposits in the euro area and the USA. In a centralized interbank market, such as the e-MID, banks publicly quote their offers to lend or borrow money at a given maturity. The quotes can be anonymous but before finalizing the loan

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contract the lender has the right to know the identity of the borrower and can refuse to finalize the transaction. While early studies on the e-MID market (Iori et al. (2008)) have revealed a fairly random network at the daily scale, a non-random structure has been uncovered for longer aggregation periods. Monthly and quarterly aggregated data show that since the 1990s a high degree of bank concentration occurred (Iazzetta and Manna (2009)), with fewer banks acting as global hubs for the whole network. The hubs tend to cluster together and a significant core-periphery structure has been observed (Finger et al. (2013)). Lasting interbank relationships, which remained stable throughout the financial crisis, have been observed by Affinito (2011) and Temizsoy et al. (2014)[1]. A networked structure of the e-MID market was observed in Hatzopoulos et al. (2013) by using a methodology based on the detection of statistically validated networks (Tumminello et al. (2011)) that allows the researcher to control for bank heterogeneity. In the cited study the networked nature of the e-MID market was highlighted by selecting repeated credit interactions (specifically, overnight loan contracts) between pairs of banks that were statistically incompatible with a null hypothesis of random pairing of the loans, which took into account the transaction heterogeneity of the banks. In other words, the underlying trading network of banks was assumed to be primarily driven by the heterogeneity of the banks whereas the networked nature of some bilateral relationships was associated with a dynamical over-expression of the number of bilateral transactions.

Overall empirical studies of the e-MID data have identified important properties of the market but have also shown that the e-MID interbank network remained surprisingly stable during the subprime crisis (Fricke and Lux (2012)) with a structural break only appearing after the Lehman default. These findings were confirmed, at the intraday scale, by Abraham et al. (2013) who have uncovered regularities in the network growth process that did not change during crisis. This indirectly suggests that the under-

[1]In these papers the strength of lending relationships is measured by the concentration of lending/borrowing activity between banks. More precisely, for every lender and borrower a preference index is computed, equal to the ratio of total funds that a lender (borrower) has lent to (borrowed from) a borrower (lender) during a given period, over the total amount of funds that the lender (borrower) has lent in (borrowed from) the interbank market during the same period. This measure nonetheless does not take into account the heterogeneity of the banking system and the fact that large banks may have no alternative than to trade with each other if they need to exchange large volumes.

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lying mechanisms driving the link formation in this market are stable over time, making it a good benchmark to test our model.

Our model outcomes and e-MID data are compared through a variety of measures including the number of statistically validated links, bidirectional links, and 3-motifs. Comparison with the data is performed by separately considering the role of lender aggressors from the role of the borrower aggressors, where the aggressor is the party proposing the setting of the loan contract. In both cases the number of networked relationships observed in simulations agrees well with the number observed in real data. The fraction of bidirectional links is small ($< 10\%$) in both networks obtained from real data and simulations, indicating a low degree of reciprocity in the system. Specifically, on average, model outcomes present a slightly smaller number of bidirectional links than real data showing that the introduction of a tunable level of reciprocity is required to take into account the degree of reciprocity observed in real data.

In addition to the analysis of the mean number of networked relationships, and of the fraction of bidirectional links we also analyze the local nature of the relationships by studying the triads present in simulations and real data both for the original and statistically validated networks. Triads, or 3-motifs, i.e. isomorphic classes of subnetworks of 3 nodes, have been recently investigated in empirical studies of the Interbank trading networks of the Netherlands (Squartini et al. (2013)) and Italy (Bargigli et al. (2013)). We show that real data presents over-expression and under-expression of some specific 3-motifs in some of the investigated three-maintenance periods whereas the over-expression or under-expression is almost absent for statistically validated networks. Again our basic model is generating networks characterized by preferential links but the preferential relationships is introduced only at the level of a specific pair of bank.

The article is organized as follows. In Section 2 we present our trading model of an interbank market. In Section 3 we describe the e-MID interbank dataset. In Section 4 we present the output of the model for different choices of the control parameters and we compare the trading networks obtained from simulations and from real data. Finally, in Section 5 we draw our conclusions.

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2. A trading model with reinforcing memory

Our model relies on the assumption that if a bank B_j has lent many times to a bank B_i ($i, j = 1, \cdots, n$, where n is the number of banks) in a past time window then it's likely that it will keep doing so in the future, unless external conditions change. Our model attempts to incorporate the intrinsic heterogeneity of banks with respect to their trading activity as a lender or a borrower in a given time period. Specifically, we introduce a characteristic time window, T_M, which will constitute the time step of our model and over which external conditions do not change. In particular we assume that at the beginning of each trading period T_M, banks are exogenously allocating a number of transactions to execute within that time period. While in reality banks receive liquidity shocks continuously, it is not unreasonable to assume that they can anticipate their liquidity needs over future periods. Settlement of security transactions for example occurs on a transaction date plus one, two or three days, so banks know in advance about large payments that have to be executed. The BCBS Principles for Sound Liquidity Risk Management and Supervision explicitly require banks to *"... be able to measure expected daily gross liquidity inflows and outflows, anticipate the intraday timing of these flows where possible, and forecast the range of potential net funding shortfalls that might arise at different points during the day"*. Under the new Liquidity Coverage Ratio banks are required to be able to meet (and hence anticipate) their liquidity needs for a 30 calendar day liquidity stress scenario. We acknowledge that our simplifying assumption implies an aspect on non-causality in our model especially when the number of transactions occurring during a period of investigation is rather large. We will discuss again about this limitation at the end of this section.

The choice of the marginal distributions from which the number of transactions that each bank, B_i, is allocated as a lender (borrower) in *lender-aggressor* transactions, $B_i^{l,la}$ ($B_i^{b,la}$), and as a lender (borrower) in *borrower-aggressor* transactions, $B_i^{l,ba}$ ($B_i^{b,ba}$), is in principle arbitrary (in our simulations we calibrate it to e-MID data as explained in section 4).

Within T_M each bank will execute all the transactions that have been assigned to it at the beginning of the time step. An event parameter t defines the order at which transactions sequentially occur, *i. e.* the event t indicates that t transactions already occurred in T_M. As the model shall distinguish between borrower-aggressor and lender-aggressor transactions, at each transaction t in T_M, the outcome of a binary random variable, x_{lb}, determines if

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the next transaction is a lender-aggressor or a borrower-aggressor transaction. The variable x_{lb} takes value "lender-aggressor" with probability $p_l(t)$, proportional to the number of lender-aggressor transactions that remain to be executed in time window T_M, (and value "borrower-aggressor" otherwise). Once the decision about the type of transaction is made, the model indicates how the counterparts of the transaction should be selected. We first focus on a lender-aggressor transaction. To mimic a lender-aggressor transaction we assume that an order to borrow is placed by a bank B_i that is randomly selected with probability

$$p_b(B_i, t) \propto B_i^{b,la}(t). \tag{1}$$

Quantity $B_i^{b,la}(t)$ is the number of transactions that bank B_i has left to do in time-window T_M, as a borrower, in the lender-aggressor transactions, after a total number of t transactions already occurred in T_M. Once the borrower is selected, we randomly select a bank B_j to be the counterpart of the transaction, the lender, with probability

$$p_l(B_j, t|B_i) \propto B_j^{l,la}(t) \left[w + N_{B_j \to B_i}(t)\right], \tag{2}$$

where $B_j^{l,la}(t)$ is the number of transactions that bank B_j has left to do in time-window T_M as a lender in the lender-aggressor transactions after t total transactions. The quantity $N_{B_j \to B_i}(t)$ is the total number of transactions in which B_j lent money to B_i over the past Q time-windows, in spite of the type of transaction[2]. Finally, w is a parameter, here assumed equal for all the banks, which represents a common level of attractiveness of borrowers. This parameter dominates the dynamics at the beginning of a simulation, when $N_{B_j \to B_i}(t)$ is equal to zero or it is very small. The parameter w acts as a randomization factor. In fact a large value of w would prevent the memory mechanism from working effectively, and the result will be a random network without preferential links[3]. The two equations above imply that the

[2]The memory mechanism that we introduced in the model is based on the number of transactions between banks in the past Q time windows . However, such a mechanism could easily be adapted to take into account volumes: it is sufficient to replace, in all the equations above, $N_{B_j \to B_i}(t)$ with the volume $V_{B_j \to B_i}(t)$ that bank B_j lent to bank B_i in the past Q time windows.

[3]The parameter w could be made node specific, such as in fitness models, and/or time varying, and used to model bank reputation. However, this possibility is out of the scope of the present paper and will be explored elsewhere.

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probability that a (lender-aggressor) transaction occurs from A (the lender) to B (the borrower) after t transactions is:

$$\begin{aligned} p(B_j \to B_i, t) &= p_b(B_i, t)\, p_l(B_j, t|B_i) \\ &= \frac{B_i^{b,la}(t)}{\sum_{k=1}^{n} B_k^{b,la}(t)} \cdot \frac{B_j^{l,la}(t)\ \left[w + N_{B_j \to B_i}(t)\right]}{\sum_{q=1}^{n} B_q^{l,la}(t)\ \left[w + N_{B_q \to B_i}(t)\right]}. \end{aligned} \quad (3)$$

The way in which we model borrower-aggressor transactions is slightly different, because the lender has always the possibility to refuse to trade with a specific borrower. The model works as follows. A lender, B_j, is randomly selected with probability

$$p_l(B_j, t) \propto B_j^{l,ba}(t), \quad (4)$$

that is, with probability proportional to the number of transactions $B_j^{l,ba}$, that B_j, has left to do as lender in borrower-aggressor transactions after t transactions of time window T_M occurred. Then a borrower, B_i, is selected with probability

$$p_b(B_i, t) \propto B_i^{b,ba}(t), \quad (5)$$

proportional to the number of transactions $B_i^{b,ba}(t)$, that B_i, has left to do as a borrower in borrower-aggressor transactions after t transactions of time window T_M. So far, the selection of lender and borrower occurred independently. However, once the borrower is selected, the lender, B_j has a certain probability to accept borrower B_i as counterpart in the transaction. This probability is set to be proportional to the attractiveness w plus the degree of trust that lender B_j associates with bank B_i. Therefore the probability that B_j lends to B_i at event t of time window T_M, conditioned to the fact that B_j has been selected as the lender is:

$$p_b(B_i, t|B_j) \propto B_i^{b,ba}(t)\ \left[w + N_{B_j \to B_i}(t)\right]. \quad (6)$$

If the two banks do not end up trading then another borrower should be randomly selected, and so on, until the lender B_j finds an acceptable counterpart. In our simulations, we work in the space of transactions[4], and, from

[4]In fact time is just an event time increasing as an integer variable describing successive transactions

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the perspective of transactions, the process of searching a suitable counterpart of lender B_j is equivalent to randomly selecting a borrower B_i with probability

$$p_b(B_i, t|B_j) = \frac{B_i^{b,ba}(t)\ \left[w + N_{B_j \to B_i}(t)\right]}{\sum_{m=1}^{n} B_m^{b,ba}(t)\ \left[w + N_{B_j \to B_m}(t)\right]}. \quad (7)$$

Therefore, in the case of borrower-aggressor transaction, we obtain that the probability that a transaction occurs between a lender B_j and a borrower B_i is given by:

$$\begin{aligned} p(B_j \to B_i, t) &= p_l(B_j, t)\, p_b(B_i, t|B_j) \\ &= \frac{B_j^{l,ba}(t)}{\sum_{q=1}^{n} B_q^{l,ba}(t)} \cdot \frac{B_i^{b,ba}(t)\ \left[w + N_{B_j \to B_i}(t)\right]}{\sum_{m=1}^{n} B_m^{b,ba}(t)\ \left[w + N_{B_j \to B_m}(t)\right]}. \quad (8) \end{aligned}$$

The objective of our model is mainly to show how an intuitive mechanism, such as a memory mechanism, which determines the probability that two banks trade at a given time, may give rise to preferential patterns of trading that have been observed in real data (see Hatzopoulos et al 2013). The simplicity of the model has a cost in terms of an absence of causality especially when the number of transactions occurring during a period of investigation and network analysis is rather large. In Appendix A we discuss how to generalize the model to solve the problem of the causality for any number of transactions occurring in a period of investigation and network analysis.

3. Data

Interbank markets can be organized in different ways: physically on the floor, by bilateral interactions, or on electronic platforms. In Europe, interbank trades are executed in all these ways. The only electronic market for Interbank Deposits in the euro area and USA is called e-MID. It was founded in Italy in 1990 for Italian Lira transactions and became denominated in Euros in 1999. When the financial crisis started, the market players were 246, belonging to 16 EU countries: Austria, Belgium, Switzerland, Germany, Denmark, Spain, France, United Kingdom, Greece, Ireland, Italy, Luxembourg, Netherlands, Norway, Poland, and Portugal. According to the European Central Bank e-MID accounted, before the crisis, for 17% of total turnover in unsecured money markets in the Euro Area. A recent report on money

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markets (European Central Bank, 2011), recorded around 10% of the total overnight turnovers. Trading in e-MID starts at 8 a.m. and ends at 6 p.m. Contracts of different maturities, from one day to a year can be traded but the overnight segment (defined as the trade for a transfer of funds to be effected on the day of the trade and to return on the subsequent business day at 9 : 00 a.m.) represents more than 90% of the transactions. One distinctive feature of the platform is that it is fully transparent. Trades are public in terms of maturity, rate, volume, and time. Buy and sell proposals appear on the platform with the identity of the bank posting them (the quoter may choose to post a trade anonymously but this option is rarely used). Market participants can choose their counterparts. An operator willing to trade can pick a quote and manifest his wish to close the trade while the quoter has the option to reject an aggression. The database is composed by the records of all transactions registered in the period from 25-Jan-1999 to 7-Dec-2009. Each line contains a code labeling the quoting bank, i.e. the bank that proposes a transaction, and the aggressor bank, i.e. the bank that accepts a proposed transaction. The rate the lending bank will receive is expressed per year; the volume of the transaction is expressed in millions of Euros. The banks are reported together with a code representing their country and, for Italian banks, a label that encodes their size, as measured in terms of total assets. A label indicates the side of the aggressor bank, i.e. whether the latter is lending/selling ("Sell") or borrowing/buying ("Buy") capitals to or from the quoting bank. Other labels indicate the dates and the exact time of the transaction and the maturity of the contract. We consider the dataset obtained by considering only the overnight ("ON") and the overnight long ("ONL") contracts. The latter is the version of the ON when more than one night/day is present between two consecutive business day. This is the same dataset already investigated by Hatzopoulos et al. (2013). In the present study, as in Hatzopoulos et al. (2013), we limit our investigation to transactions occurring only between Italian banks.

The period of time in which credit institutions have to comply with the minimum reserve requirements is called the reserve maintenance period. During each reserve maintenance period minimum reserve levels are calculated on the basis of banks' own balance sheet. Each reserve maintenance period is usually equivalent to one calendar month, i.e. about 23 trading days. In the investigations we present below, we have aggregated the maintenance periods in groups of three. In fact, these aggregated periods better capture the natural economic cycles that are usually organized on a nearly 3-monthly

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basis. We therefore will consider 44 three-maintenance periods ranging from 25-Jan-1999 to 07-Dec-2009[5].

4. Numerical Simulations

This section is divided into two subsections. In the first subsection we run the model[6] under different choices for the value of the two parameters w and Q, and study their effect on the resulting interaction networks. For these simulation exercises, that we call Mode I, we sample the number of transactions each banks executes, in a time period T_M, from predetermined marginal distribution functions.

In the second subsection we compare the simulated and empirical networks and assess the ability of our model to generate realistic scenarios, given a set of constraints, that could, for example, be used for stress test exercises. For these simulation exercises, that we call Mode II, the number of transactions each bank executes in a time period T_M is taken as an input from the e-MID market data.

4.1. Simulations of the model: Mode I

While the model is defined for arbitrary distributions for the number of transactions that each bank is allocated, as a lender and borrower, in *lender*-aggressor transactions, $B_i^{l,la}$ and $B_i^{b,la}$, and in *borrower*-aggressor transactions, $B_i^{l,ba}$ and $B_i^{b,ba}$, we take here the functional choice that is more consistent with e-MID data. Empirical evidence for the e-MID market shows that, when aggregating over different time scales, banks can act both as a lender and a borrower and that the trading activity can vary considerably across banks. To best account for this heterogeneity in our model we rely upon the results obtained in Fricke and Lux (2013), and assume that $B_i^{l,la}$, $B_i^{b,la}$, $B_i^{l,ba}$ and $B_i^{b,ba}$ follow Log-normal distributions, with parameters specific to

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[5]It should be noted that the first three-maintenance period covers the time period from 25-Jan-1999 to 23-Mar-1999, thus involving a number of trading days which is reduced of a factor of about one third with respect to all the other three-maintenance periods.

[6]Results presented in this section were obtained through an implementation of the model written by using Mathematica. The model has also been developed using Java and the MASON library for multi-agent modeling, and is being currently tested. The implementation within the Java/Mason framework will allow to include and integrate our model into the interbank sector of the CRISIS macro-financial software library. A description of the implementation in Java/Mason is provided in Appendix B.

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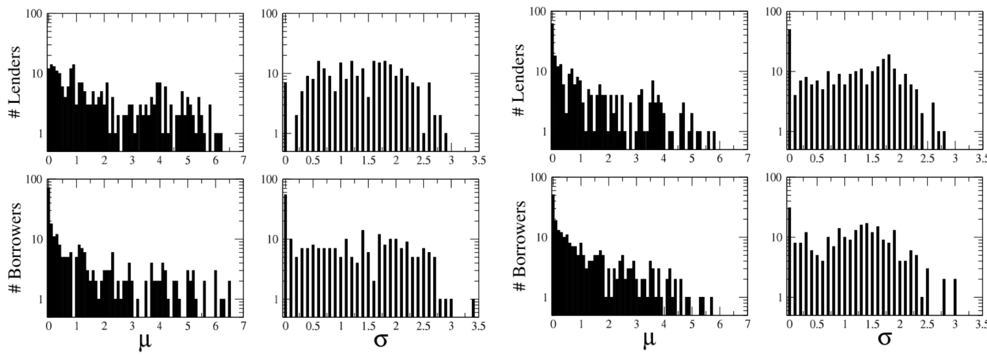

Figure 1: Distributions of the parameters of the Log-normal distributions, $B^{l,la}$ and $B^{b,la}$, for the Lender-Aggressor database (left) and $B^{l,ba}$ and $B^{b,ba}$ for the Borrower-Aggressor (right) database (μ and σ are the log-scale and the shape parameter of the lognormal distribution respectively).

each bank, and to the bank's role (quoter/aggressor, lender/borrower). The parameters were chosen in order to match, for each bank, the mean and variance of the logarithm of the number of transaction of the e-MID data over the 44 three maintenance periods. The distributions of the parameters of the 4 lognormal marginal distributions, across the 254 banks[7] included in the simulation, are plotted in figure 1. We note that all the distributions for the μ parameter are fat tailed and they reveal a high degree of heterogeneity in the system.

It is worth pointing out that our model is not short-term stationary although we have verified that it is long-term stationary. In fact, the link-formation probability depends on the number of transactions between each pair of banks over a given past time-window. This fact, together with the presence of an arbitrary time zero, gives rise to a transient in each simulation. However, according to all the quantities that we measured, such as, for instance, the number of links in the original and statistically validated networks, the transient is rather short, being of the order of one time window or less.

In the simulations we report below we considered three possible values for the reputation parameter w: $w = 0.1$, $w = 1.0$ and $w = 10$. For each of these parameters we considered four possible values for the memory Q parameter:

[7]There are 254 active Italian banks on the e-MID market

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$Q = 1$, $Q = 2$, $Q = 4$ and $Q = 8$. Simulation runs span over $T_M = 100$ time periods.

In Fig. 2 we show the number of links in the original (top panel) and Bonferroni (bottom panel) networks for $Q = 1, 2, 4, 8$ and $w = 0.1, 1.0, 10$. The Bonferroni networks have been obtained from the original networks by retaining only those links whose existence is not consistent with a random null hypothesis taking into account the heterogeneity of nodes' degree (see Appendix C for more details on the statistical validation method).

The main result of this section is to show that our model is capable to generate a networked market whenever the parameter w is small enough. Indeed Figure 2 shows that small values of the parameter w imply large numbers of validated links. This is in agreement with the interpretation of w as a randomization parameter that, when large, can impair the effectiveness of the memory mechanism. Small values of w allow the memory mechanism to determine and enhance even small deviations from random matching, and such deviations are easily detected by the statistical validation method. On the contrary, larger values of w may easily destroy small deviations from random matching, which result in a smaller number of validated links. The figure also shows that the number of the networked over-expressed connections is weakly dependent on the memory parameter Q.

The parameter Q nonetheless impacts the degree of persistence of the simulated networks. In fact, the probability that two banks trade depends on the their trading history in the previous Q time windows of the simulation. This memory mechanism introduces a certain degree of similarity in the network of transactions obtained at different time intervals T_M. It is possible to quantify this similarity through the weighted Jaccard index between any two of these networks, net_1 and net_2, obtained at different time intervals:

$$J_W(net_1, net_2) = \frac{\sum_{i,j} Min[w^1_{i,j}, w^2_{i,j}]}{\sum_{i,j} Max[w^1_{i,j}, w^2_{i,j}]}, \tag{9}$$

where $w^1_{i,j}$ is the weight of link $i \to j$ in the first network, net_1, and $w^2_{i,j}$ is the weight of link $i \to j$ in the second network, net_2. For an unweighted network $J_W(net_1, net_2)$ reduces to usual Jaccard index:

$$J_U(net_1, net_2) = \frac{|E_1 \cap E_2|}{|E_1 \cup E_2|}, \tag{10}$$

that is the number of directed links that belong to both network over the number of links in the union network.

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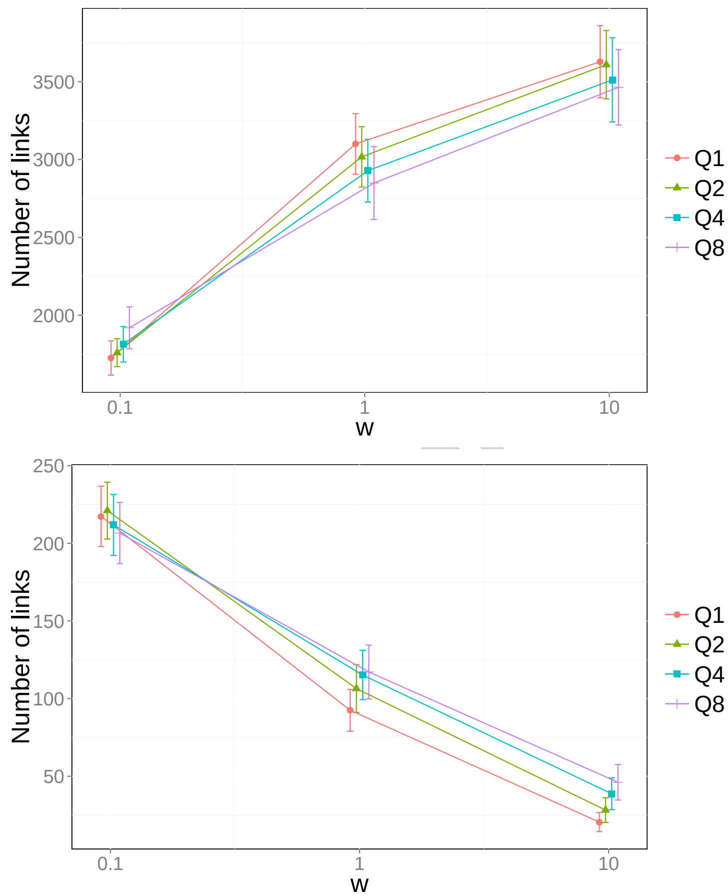

Figure 2: Number of Links in the Lender aggressor transaction networks for the original simulated networks (top) and Bonferroni networks (bottom). $Q = 1, 2, 4, 8$ and $w = 0.1, 1.0, 10$. The values of w are slightly shifted in the figure to improve readability for different values of Q.

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Figure 3 shows the contour plots of the weighted Jaccard index for every pair of networks for the lender-aggressor transactions resulting from a 100 time-step simulation (results are similar for the borrower-aggressor transactions). The left panels show results for simulated networks with $Q = 1$ (first row), $Q = 2$ (second row), $Q = 4$ (third row) and $Q = 8$ (fourth row). The value of the parameter w has been fixed to 1 through all the simulations under discussion[8]. The right panels show the weighted Jaccard index for the corresponding statistically validated networks.

Looking at the left panels of Fig. 3 it is clear that, as we increase the memory parameter, the area where the Jaccard index is higher (the green area) becomes wider and wider: moving from $Q = 1$ to $Q = 8$, while keeping w fixed, it becomes less likely for a new credit relationship to appear and therefore the network structure persists over a longer time-span. This pattern is even clearer when we look at the validated networks on the right panels. Here we notice that the similarity among consecutive time periods is slightly higher than the one in the corresponding original network. For longer lags, however, the similarity in the validated networks decreases faster, as the wider red areas in the contour plots show. At first sight this may look as a contradiction, as one could expect that validated networks, preserving only the most (statistically) relevant and stable links, should show a stronger similarity, especially across time. Actually, the analysis in Hatzopoulos et al. (2013) revealed that the same trend can be found in the E-Mid empirical networks. Furthermore Hatzopoulos et al. (2013)) shows that a rewiring procedure (equivalent to running our model purely with random matching and no memory mechanism) does not destroy the pattern of persistence in the original networks demonstrating that high values for the weighted Jaccard index in the original networks are to a great extent consequence of the strong constrains on which links can form imposed by heterogeneity in the banks' activity.

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[8]We have repeated the analysis for all values of $w = 0.1, 1.0, 10$ but the results do not change. Here we report only on the case $w = 1$ to limit the length of the paper.

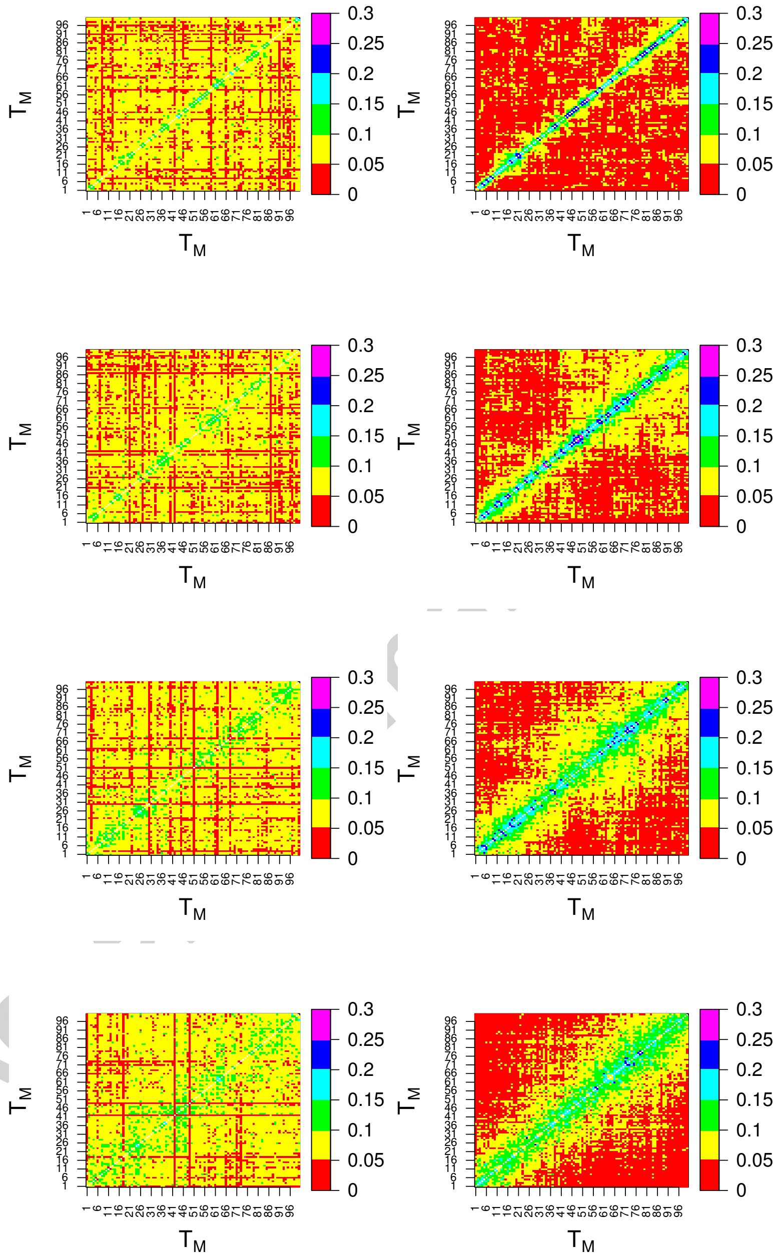

Figure 3: Weighted Jaccard index matrices for the lender-aggressor simulated networks (left panels) and the corresponding validated networks (right panels). The networks are obtained from simulation with values for the memory parameter, from top to bottom, $Q = 1$, $Q = 2$, $Q = 4$ and $Q = 8$. The *randomization* parameter w is equal to 1 in all the simulations showed.

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4.2. Simulations of the model: Mode II

In this subsection we run simulations using the transactions realised by each bank in the e-MID market as an input of the model. More precisely the quantities $B_i^{l,la}$, $B_i^{b,la}$, $B_i^{l,ba}$, and $B_i^{b,ba}$ $(i = 1, \cdots, n)$, which are used in the model to set the trading frequency of bank A as a lender and as a borrower, in the two types of transactions—borrower-aggressor and lender-aggressor—are those observed in real data. Moreover, we set the time interval T_M over which the number of transactions is planned equal to one three-maintenance period. With this choice, we have 44 three-maintenance periods over which the simulation runs.

This way the model can be used as a tool to generate stress test scenarios where the total number of transactions executed by each bank is preserved. The approach typically used by regulators in stress-test exercises is to take a snapshot of the banking system, at a given date, and test its stability under different stress assumptions. Rather than run stress test exercises only on the realized exposure network, it would be desirable to generate a statistical ensemble of scenarios, that preserve some features of the data, over which stress test exercises can be executed. We choose here to preserve the number of transactions executed by each banks. The underlying idea is that while the number of transactions a bank needs to execute is determined exogenously by its liquidity needs, the exact counter-party with which a bank end up trading could be the outcome of a random match rather than a deliberate choice. Rather than simply randomize the links via a configuration model that preserves the strength of each node (strength measured in units of number of transactions) our model allows to also preserve stable relationships among banks by tuning the memory parameter.

As we have shown in the previous subsection, the number of links in the transaction network is very sensitive to the w parameter but only weakly dependent on the Q parameter. This permits to choose the values of the two parameters separately. We first tune the w parameter by comparing the number of links in the simulated and real networks. Once the value of w has been selected, as the one that better reproduces the empirical data, we fix it and choose the parameter Q that minimizes the Frobenius distance between the weighted Jaccard index matrices in simulated and real transaction networks.

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4.3. Tuning of randomization parameter w

The various panels in Fig. 4 show the number of links in the trading networks obtained by running our model, for different values of the parameters w and Q, and the number of links in the original networks of overnight transactions among Italian banks (the horizontal red lines, with corresponding confidence interval). The comparison in Fig. 4 is done separately, for the lender-aggressor data, for each year by averaging over 4 consecutive 3-maintenance periods, for a total of 11 years.

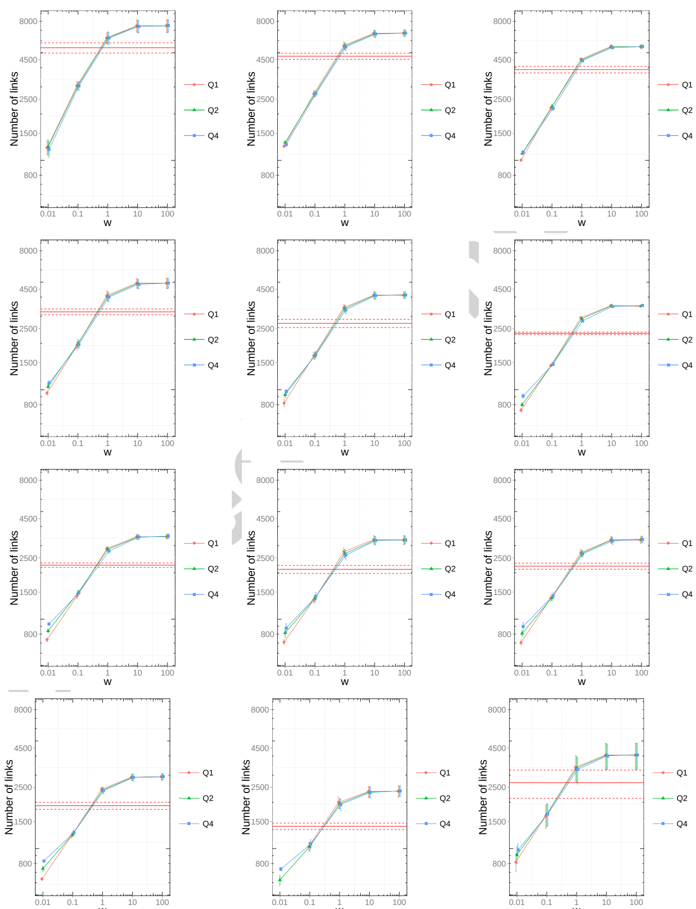

Figure 4: Log-log plot of the number of links in lender-aggressor simulated networks, for different values of w and Q, and real network (solid red line). The comparison is done separately for each year by averaging over 4 consecutive 3-maintenance periods. The last plot (bottom, right) shows the overall result when averaging over the periods 11 till 30. Dotted red lines indicate the intervals at plus or minus one standard deviation.

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We obtain a good agreement, in each year, between simulations and real data for values of w between 0.1 and 1, with w closer to 1 at the beginning of the analysed period of time and closer to 0.1 during the financial crisis. This variation is a consequence of changes in both the composition of the e-MID market in terms of participating banks and their strategies over the 11 years periods. To achieve a proper calibration of the model we should thus allow w to be time dependent and capture the existence of different "states" of the market, such as with respect to market liquidity. However this is beyond the scope of the current paper. Rather here we identify a time interval over which the number of links in the e-MID adjacency matrices remained approximately constant (see Hatzopoulos et al. (2013) for more details on the evolution of the e-MID market over time), that is the time between periods 11 and 30, and use this subsample of the data to determine the parameter w. As shown in the last panel (bottom, right) of Figure 4 a value of $w = 1$ generates a number of links that is consistent, within the error bars, with the real data. This is also the case for the borrower-aggressor dataset. We thus choose $w = 1$ and we keep it constant throughout the 44 time windows of the simulations.

4.3.1. Tuning of memory parameter Q

The tuning of the Q parameter is done by comparing the matrix of weighted Jaccard index between for each pair of networks obtained from real data, over the 44 three-maintenance periods, and the corresponding matrix obtained from simulations of the model. The choice of Q we make, among the values analysed, is the one that minimises the Frobenius distance between the two matrices. The Frobenius distance between two matrices A and B, with the same dimension n, is defined as:

$$F(A,B) = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}(a_{ij} - b_{ij})^2} \tag{11}$$

As reported in Table 1 the best fit for the original network is obtained for $Q = 2$, both for lender aggressor and borrower aggressor transactions. The two datasets of borrower-aggressor and lender-aggressor transactions are analyzed independently one of the other. The results obtained from our simulations indicate that the tradeoff between memory and randomness is in-

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dependent of the type of transactions, lender-aggressor or borrower-aggressor, in which banks are involved.

Table 1: Frobenius distance between the matrices of weighted lagged Jaccard indices between networks of real data and simulations (with $w = 1$) of lender aggressor and borrower aggressor transactions.

	Lender aggressor		Borrower aggressor	
Memory	Original	Bonferroni	Original	Bonferroni
Q=1	0.91	1.61	0.66	1.57
Q=2	0.45	1.75	0.36	1.41
Q=4	0.73	1.91	0.52	1.49

In Fig.5, we report the matrix of the weighted Jaccard index obtained from the data (left panels), and the corresponding matrix obtained from simulations of the model with $w = 1$ and $Q = 2$ (right panels) across the corresponding 44 time windows . Top (bottom) panels correspond to lender-aggressor (borrower-aggressor) transactions. The figure confirms that the pattern of the weighted Jaccard index across the 44 three-maintenance periods analyzed for real data is similar to the one observed for simulations.

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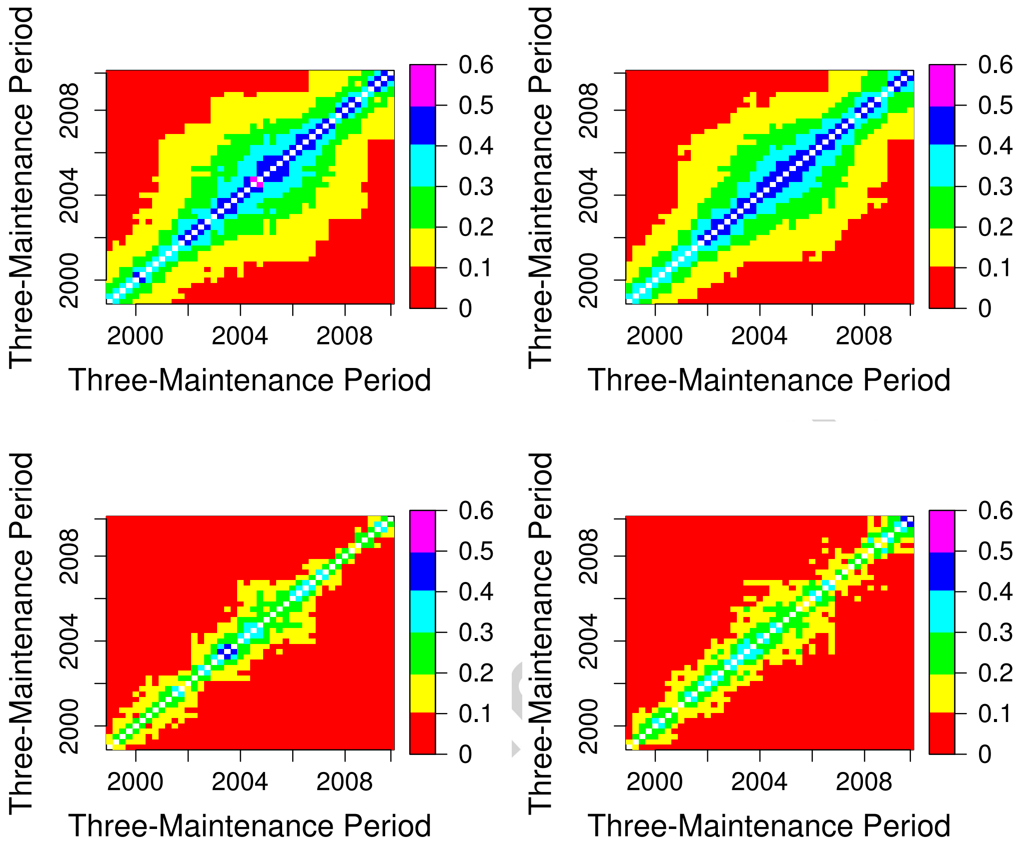

Figure 5: Matrix of weighted Jaccard index between all pairs of transaction networks of three-maintenance periods. The left panel shows the weighted Jaccard index between original networks. The right panel shows the weighted Jaccard index between the corresponding simulated networks for $w = 1$ and $Q = 2$. Lender-aggressor (top) and borrower-aggressor (bottom).

4.3.2. Comparison of validated network

After tuning the parameters of the model with respect to the number of links of the original networks, in this subsection we check if, with this choice of parameters, the model is also able to reproduce realistic validated networks.

In Figures 6 and 7, the number of links in the original and Bonferroni networks, obtained by running and analyzing simulations of our model for $w = 1$ and $Q = 2$, are compared with the number of links in the original

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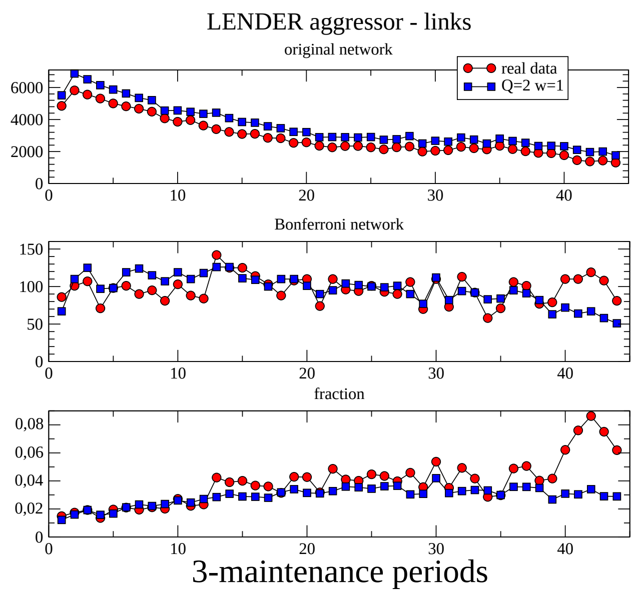

Figure 6: Number of links in the original (top) and Bonferroni (central) network associated with lender-aggressor transactions simulated according to the presented model, for $w = 1$ ad $Q = 2$. The red circles refer to the empirical data. The bottom panel shows the ratio between number of links in the Bonferroni and original network.

and Bonferroni empirical networks for each of the 44 periods of trading in the e-MID market. We observe that the agreement between the model and the data is even better at the level of Bonferroni networks, a part from the last 6 three-maintenance periods that correspond to the post-Lehman default. This suggests that after the Lehman shock trust evaporated form the market and banks strengthened their preferential relationships, a feature the simulation analysis cannot reproduce as we have assumed a constant value for w. By comparing the bottom panel of Figures 6 and 7, one notices that the deviation from real data and the values expected from simulations starts to be observed sooner for borrower aggressor networks than for lender aggressor networks. This observation suggests that the preferential trading links of borrower aggressors were under stress earlier that the preferential trading links set by lender aggressors.

In Figure 8, we plot the weighted Jaccard index matrix for validated networks associated with real data (left panels) and validated networks associated with simulations of our model for $w = 1$ and $Q = 2$ (right panels).

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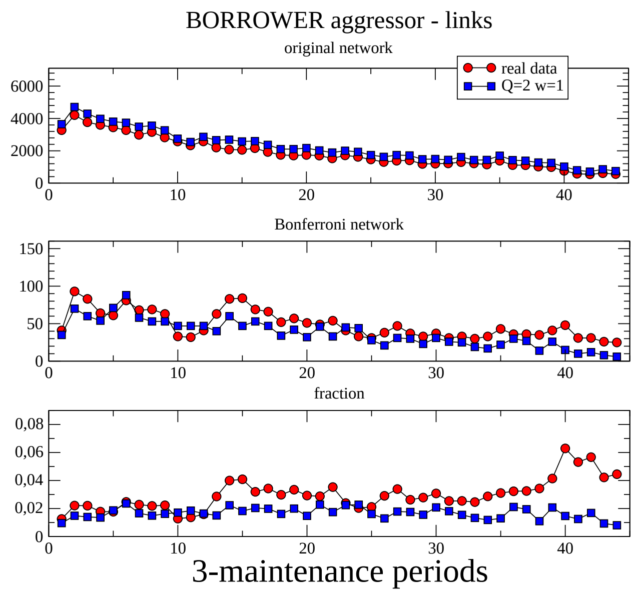

Figure 7: Number of links in the original (top) and Bonferroni (central) network associated with borrower-aggressor transactions simulated according to the presented model, for $w = 1$ ad $Q = 2$. The red circles refer to the empirical data. The bottom panel shows the ratio between number of links in the Bonferroni and original network.

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The model and the data present a similar pattern of persistency. In fact Table 1 reveals that $Q = 2$ is the value, among those analysed, for which the Frobenious distance between real and simulated weighted Jaccard matrices is smaller, in the case of the borrower-aggressor transactions. While a value of $Q = 4$ corresponds to the smallest Frobenious distance in the case of lender-aggressor transaction, a value $Q = 2$ is only marginally worse.

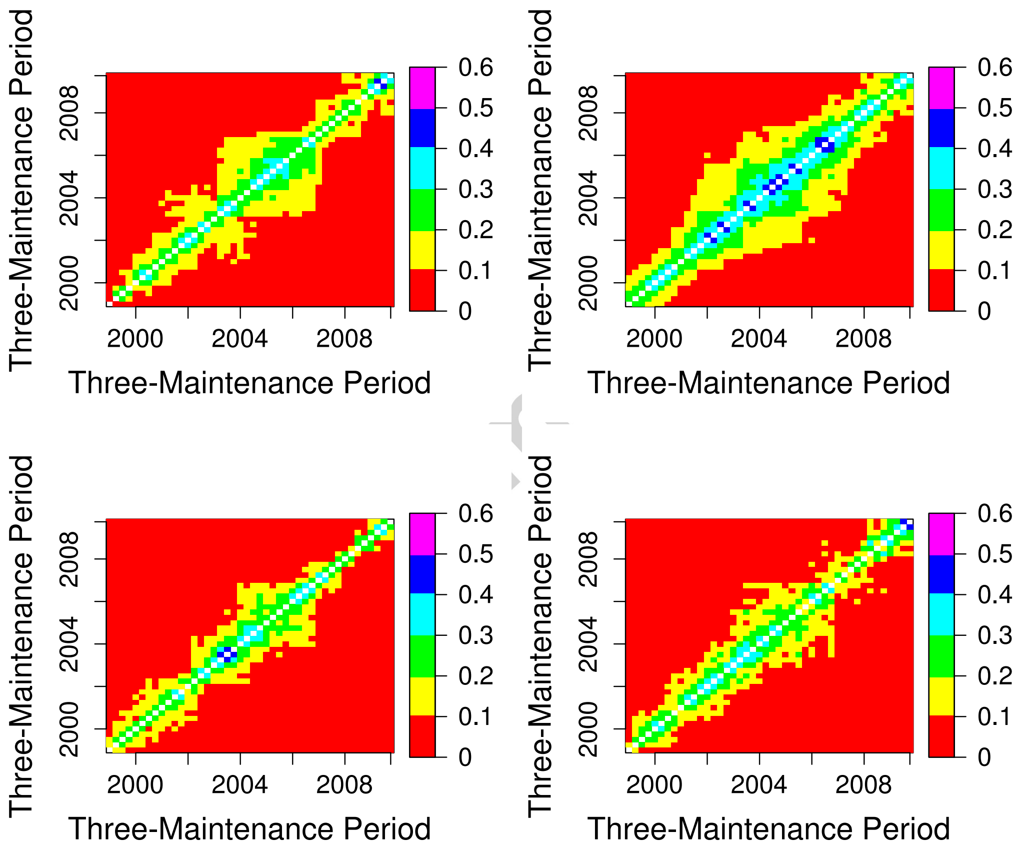

Figure 8: Matrix of weighted Jaccard indices between validated networks associated with real data (left panels) and between validated networks associated with simulations of our model for $w = 1$ and $Q = 2$ (right panels). Lender-aggressor (borrower-aggressor) transactions are considered in the top panels (bottom panels).

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4.3.3. Network prediction and model performance

In this section we ask how good or bad the model is to produce realizations that are pointwise (i.e. link by link) and not only structurally similar to the real data. To do that we set our model parameters to the values that, according to our previous analysis, provide the original and statistically validated networks which are most similar to the real data we have from the E-MID, that is, $Q = 2$ and $w = 1$. We also set the time window of investigation to one three-maintenance period, in such a way that our real database is divided in 44 subsets, of roughly three months each. We use our model in the following way. Suppose our objective is to compare the networks observed in real data in a given time window, e.g., the fifth one, from 24-Dec-1999 to 23-Mar-2000. We imagine to have access to precise information about the individual bank overall-liquidity needs in this period, and, accordingly, we set the quantities $B_i^{l.la}$, $B_i^{b,la}$, $B_i^{l,ba}$, and $B_i^{b,ba}$ $(i = 1, ..., n)$ in the fifth time window from real data. Furthermore, we use the real networks observed in the previous two time windows (Q=2), the third and the fourth one, to construct the memory terms $N_{B_j \to B_i}$, $(i, j = 1, ..., n)$, with $i \neq j$, that is used in our model. Then we obtain a realization of our model for the the fifth three-maintenance period, that we shall compare with the real network observed at the E-MID. We repeat the analysis for all the three-maintenance periods in the dataset starting from the third one, due to the memory requirement of our model. We perform the comparison separately for lender-aggressor and borrower-aggressor transaction, and for the original and statistically validated networks. The comparison could be performed according to two standard measures of performance of a classification test, specificity and sensitivity. Let's consider a real weighted network R, where a weight $w_{i,j}^R$ is the number of transactions that occurred from bank i (the lender) to j (the borrower) in a given time window, and the corresponding network obtained with our model, S, with weights $w_{i,j}^S$ that indicate the number of predicted transactions from i to j. We indicate the total number of transactions that occur in R (S) with $E_R = \sum_{i,j=1}^n w_{i,j}^R$ ($E_S = \sum_{i,j=1}^n w_{i,j}^S$). Finally, we indicate the number of transactions that occur in both R and S

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with $E_{R,S} = \sum_{i,j=1}^{n} Min(w_{i,j}^R, w_{i,j}^S)$. Using this notation we have that

$$\begin{aligned} \text{Sensitivity} &= \frac{\text{TP}}{\text{TP}+\text{FN}} = \frac{E_{R,S}}{E_{R,S}+(E_R-E_{R,S})} = \frac{E_{R,S}}{E_R}; \qquad (12)\\ \text{Specificity} &= \frac{\text{TN}}{\text{TN}+\text{FP}} = \frac{U-E_R-E_S+E_{R,S}}{[U-E_R-E_S+E_{R,S}]+(E_S-E_{R,S})} = \\ &= \frac{U-E_R-E_S+E_{R,S}}{U-E_R}, \qquad (13) \end{aligned}$$

where TP, TN, FP, and FN are the number of true positive, true negative, false positive, and false negative detections of real transactions in model simulations respectively and U is the total number of all the potential transactions among banks that can occur in a given time window. Such a number of transactions does not have a natural upper bound, which indicates that specificity of model predictions here tends to 1, in spite of the model used to anticipate empirical observations. So, we decided to disregard specificity in the reminder of this section, and to include another measure of performance of model predictions. Specifically, we considered the weighted Jaccard index between real and simulated networks. Using the notation introduced above, the weighted Jaccard index reads:

$$WJI = \frac{\text{TP}}{\text{TP}+\text{FP}+\text{FN}} = \frac{E_{R,S}}{E_R+E_S-E_{R,S}}. \qquad (14)$$

This equation shows that the only difference between WJI and sensitivity is the denominator, which is larger in the WJI index than in the sensitivity, unless the model provides exact predictions. Conceptually, the advantage of including the WJI in the present analysis, besides sensitivity, is that it also includes information about the number of False positives to assess the level of performance of a given model. In this section, we shall investigate the improvement of performance that the memory term in our model introduces with respect to the case in which we just assume to have precise information about each individual bank overall-liquidity needs in a three-maintenance period. Therefore we also report results obtained by neglecting the memory term in our model, which is equivalent to assume a random pairing of banks in a time window, weighted according their specific liquidity needs.

Table 2 reports the average values of sensitivity and WJI associated with the different sets (borrower and lender aggressor), models (memory Q=2 and no-memory), and networks (original and Bonferroni networks). Averages

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Table 2: Performance of model prediction. L-A indicates lender-aggressor transactions. B-A indicates borrower-aggressor transactions.

	Original network		Bonferroni network	
Data type	<Sensitivity> (SEM)	<WJI> (SEM)	<Sensitivity> (SEM)	<WJI> (SEM)
L-A (Q=2)	0.67(0.004)	0.51(0.005)	0.39(0.011)	0.26(0.008)
L-A (no memory)	0.55(0.004)	0.38(0.004)	0(0)	0(0)
B-A (Q=2)	0.61(0.004)	0.44(0.004)	0.37(0.015)	0.27(0.012)
B-A (no memory)	0.48(0.006)	0.31(0.004)	0(0)	0(0)

are calculated over 42 three-maintenance periods, from the 3rd to the 44th, and the standard error of the mean (SEM) is reported in parentheses. The results reported in the table indicate that our model clearly outperforms the model with no memory, which is solely based on the (exact) prediction of liquidity needs of each bank. The result that sensitivity and WJI of the no-memory model in the Bonferroni network are both equal to 0 is due to the fact that the Bonferroni network associated with this model is empty, that is, no preferential pattern is anticipated by this model.

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4.3.4. Bidirectional links

Our model does not involve any mechanism of reciprocity. Indeed the memory term $N_{A \to B}(t)$ only counts the number of times in which bank A lent to bank B in the past, and, therefore, does not include information about the number of times in which A borrowed money from B in the past. This lack of reciprocity implies that a bidirectional link, either statistically validated or not, may appear in an outcome of the model only by chance. To check if our hypothesis of no reciprocity is consistent with real data, we compared the number of bidirectional links, in the original and Bonferroni network. Table 3 shows that the average number of bidirectional links observed in real data is rather small, in both the original and Bonferroni network. Specifically the fraction of bidirectional links is always smaller than 8% in real data. Such a small value justifies neglecting reciprocity in the basic setting of our model. However, according to Table 3 the percentage of bidirectional links in empirical data is always larger than the percentage of bidirectional links obtained from simulations, with parameter $w = 1$ and $Q = 2$, in both lender-aggressor and borrower-aggressor datasets. Such results suggest that it may be worth considering the possibility of introducing a reciprocity mechanism

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Table 3: Bidirectional links in real data and simulations with $w = 1$ and Q=2

	Original network			Bonferroni network		
Data type	Mean	Std.	Perc.	Mean	Std.	Perc.
Lender aggr. (data)	210.8	111.9	7.6%	1.64	1.59	1.7%
Lender aggr. (sim.)	223.4	83.6	6.8%	0.02	0.15	0.02%
Borrower aggr. (data)	91.1	67.2	4.7%	0.45	0.85	1.2%
Borrower aggr. (sim.)	92.0	57.8	4.0%	0.02	0.15	0.04%

as a refinement of our model. This could be done by weighting the memory term $N_{A\to B}(t)$ with $N_{B\to A}(t)$:

$$N^{\lambda}_{A\leftrightarrow B}(t) = \lambda\, N_{A\to B}(t) + (1-\lambda)\, N_{B\to A}(t),$$

where λ is a parameter ranging between 0 and 1. The quantity $N^{\lambda}_{A\leftrightarrow B}(t)$ can be used in place of $N_{A\to B}(t)$ in all the equations of the model in order to introduce a degree of reciprocity, which is controlled by parameter λ. Intuitively, the value of λ should be quite close to 1, in order to replicate the (rather small) average number of bidirectional validated links observed in real data.

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4.3.5. 3-motifs

So far we have investigated the capability of the model in reproducing the number of links between pairs of banks. In fact, one can also investigate the behavior over time of higher order network structures. Specifically, we will consider isomorphic 3-motifs, i.e. network structures formed by triplets of nodes, see (Milo et al. (2002)). In Fig. 9 we show the 13 different types of isomorphic 3-motifs. There are different ways to label 3-motifs. In the present paper we use the labeling of FANMOD program, see (Wernicke et al. (2006)). FANMOD algorithm is one of the most widespread algorithm for detection and analysis of isomorphic motifs. Specifically, it detects 3-motifs (and also motifs with a larger number of nodes) and compares their percentage with the ones obtained with different null hypotheses by repeating the analysis on rewired versions of the investigated network. The program allows three different rewiring procedures. The one we have selected for the present investigation is the one setting constant the value of in-degree and

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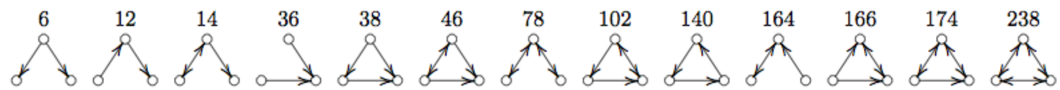

Figure 9: The 13 isomorphic directed 3-motifs. The numeric code is the one used by the FANMOD program.

out-degree of each note without any other additional constraint (this option is called NR in the FANMOD package). A p-value is directly associated with a motif by the tool FANMOD, by comparing the frequency of the given motif in the network with its frequency in a number of networks (we have set this number to 1,000) obtained by randomly rewiring the original one. There are different options for the rewiring. In the present analysis, we have chosen the rewiring done by keeping constant the in-degree and out-degree of each bank. From the p-value one can determine if a motif is over-expressed, under-expressed, or normally expressed.

We have compared the original and statistically validated networks from real data and from simulations according to the fraction of 13 different types of 3-motifs that can be present in a network. Figure 10 shows the number of three-maintenance periods in which each 3-motif type turns out to be over-expressed, under-expressed, normally expressed, and not present in the original networks obtained from real data of lender-aggressor (left panels) and from lender-aggressor transactions generated by our model with $w = 1$ and $Q = 2$. Figure 10 shows that the number of three-maintenance periods in which each motif is normally expressed in the original network obtained from simulations is larger than the corresponding number obtained from real data. Interestingly motif 140, which represents circular transactions is under-expressed in real network indicating that liquidity does not normally flow from one bank to the other and back to the first. In fact most of the motifs described by closed triangles (motifs 38 and 46 are an exception) are under-expressed suggesting that (on aggregate) banks tend to act as either lender or borrower in the system and do not actively engage in transactions on both side of the market.

Similar results are observed in Fig. 11 where the same investigation is repeated for the networks obtained from borrower -aggressor transactions. Interestingly, the main difference between Fig. 10 and Fig. 11 concerns motifs 36 that is under-expressed in the lender-aggressor network and normally expressed in borrower-aggressor network. This 3-motif describes credit

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provided to one bank from two banks. The 3-motifs analysis shows that such relationships can be different when the transactions are originated by borrower-aggressors or lender-aggressors.

We have also performed the analysis on the statistically validated networks. Also in this case we detect some over-expressed 3-motif. However the comparison with simulated data is problematic for validated networks because the number of 3-motifs is limited and many 3-motifs are not detected at all. Therefore we do not provide here a comparison between real data and simulated data for statistically validated networks.

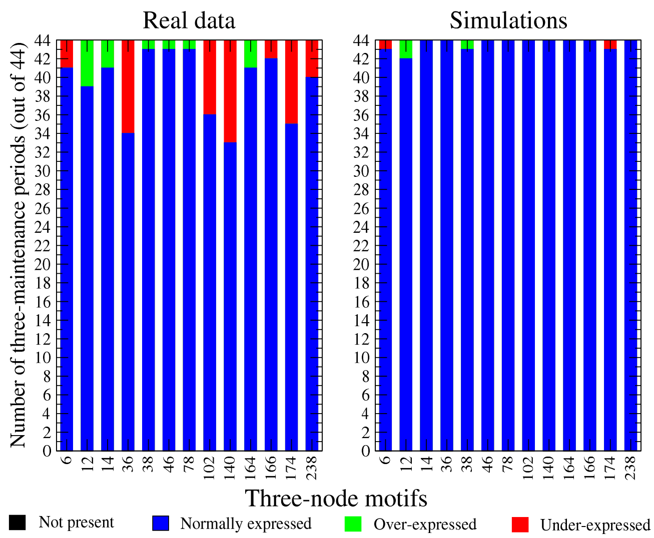

Figure 10: Number of three-maintenance periods (out of 44) in which each three-motif type (indicated in the horizontal axis) is over-expressed, under-expressed, normally expressed, and not present in the original (left panel) networks associated with real data of *lender-aggressor* and with corresponding *lender-aggressor* transactions from simulations of the model with $w = 1$ and $Q = 2$ (top left). Over-expressions and under expressions are obtained by performing a multiple hypothesis test correction. The over/under expression of a three motif indicates that the corresponding p-value provided by FANMOD was smaller than $0.01/(13 \cdot 44)$, where 13 is the number of three-motif types and 44 is the number of three-maintenance periods investigated.

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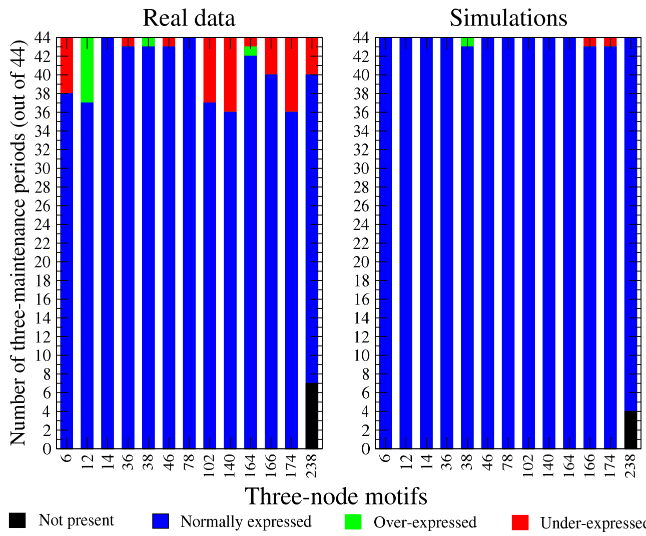

Figure 11: Number of three-maintenance periods (out of 44) in which each three-motif type (indicated in the horizontal axis) is over-expressed, under-expressed, normally expressed, and not present in the original (left panel) networks associated with real data of *lender-aggressor* and with corresponding *borrower-aggressor* transactions from simulations of the model with $w = 1$ and $Q = 2$ (top left). Over-expressions and under expressions are obtained by performing a multiple hypothesis test correction. The over/under expression of a three motif indicates that the corresponding p-value provided by FANMOD was smaller than $0.01/(13 \cdot 44)$, where 13 is the number of three-motif types and 44 is the number of three-maintenance periods investigated.

5. Conclusions

We have introduced a simple model with memory to describe the formation of a networked structure of an interbank market. Such a structure presents preferential patterns of trading between banks incompatible with the null hypothesis of random pairing of banks. The null hypothesis takes into account the heterogeneity of banks, and so does the model. Within a time interval of the model, the memory mechanism assumes that the probability that a lender and a borrower end up trading at a given time step depends on their trading frequency (their heterogeneity) times the sum of two terms. The first one is the number of times in which the borrower borrowed from the lender in the past, and the other term, w, represents an overall attrac-

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tiveness of borrowers. High values of parameter w, e.g. $w > 10$, disfavor the appearance of preferential patterns of trading. On the contrary, low values of parameter, e.g. $w < 0.1$, favor the formation of many preferential patterns of trading. If the number of transactions per bank remains constant over time then such preferential patterns will remain essentially the same, and the market will tend to freeze in a network in which most of the transactions occur between banks that have heavily traded in the past. On the other hand, if the number of transactions per bank in a given time period is obtained as in mode I, that is, by independently sampling from a Lognormal distribution, then preferential patterns will change over time. So, low values of w allow to model a status of the market in which "trust" (and "distrust") dominates the process of bank pairing. A high degree of agreement between model and real data of the e-MID market, in terms of number of preferential links observed over time, is attained by setting $w \approx 1$.

The parameter Q allows one to set the level of memory of banks. Four values of Q have been considered, $Q = 1, 2, 4$ and 8 time periods.The comparison between model outcomes obtained with these values of Q indicates that the model is not significantly affected by this parameter, in terms of the number of observed preferential connections.

The best agreement with e-MID data was achieved for values of $w = 1$ and $Q = 2$. Model outcomes and real data have also been compared in terms of number of bidirectional links observed in the original and Bonferroni networks. The presence of bidirectional links is small ($< 10\%$) in both real data and simulations, indicating a low degree of reciprocity in the system. However, on average, model outcomes present a smaller percentage of bidirectional links than real data. A simple method to introduce a tunable level of reciprocity has been proposed, but not investigated, in consideration of the low number of bidirectional links observed in real data. Finally, we have compared real data and simulations in terms of the presence of 3-motifs in the original networks. Our results suggest that the observed frequency of 3-motifs in the networks associated with model realizations is, on average, more similar to the one expected in a random network than to the one observed in real data. This result suggests that the moderate presence of over-expression and under-expression of triadic structures in real data is not captured by our model of random pairing with memory. However, the parameter Q influences the degree of persistence of preferential patterns of trading activity across different time periods. The model proposed can be extended in a number of dimensions. Besides incorporating, as already discussed, a

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reciprocity mechanism and a time and node dependent parameter w (thus changing its role from a randomization parameter to a fitness parameter) we could also allow for preferential trading within small groups of banks. This would allow to take into account the existence of banking groups, but also of trust propagation effects: at times of greater uncertain, banks may prefer to trade with banks that are trusted counter parties of their own trusted counter parties.

Furthermore it would be interesting to introduce uncertainty in the model by assuming a random arrival of liquidity shocks. In this way we could overcome the problem of non-causality occurring when the number of transaction in an estimating period is too large to realistically assume that a bank can foresee all of them . In Appendix A we have sketched how to generalize our model to this setting. Finally the model could be extended by allowing banks to trade strategically in anticipation of their future liquidity needs and not only respond to their immediate ones.

Appendix A. Extension of the model

The objective of our model is mainly to show how an intuitive mechanism, such as a memory mechanism, which determines the probability that two banks trade at a given time, may give rise to preferential patterns of trading that have been observed in real data (see Hatzopoulos et al 2013). If the memory term is neglected in the model equations, then no statistically validated links are generated (see subsection 4.3.3), in spite of the way in which bank-liquidity needs are modeled. In the model, we assume to know in advance bank liquidity needs in a given time window, specifically, the exact number of lender aggressor and borrower aggressor transactions that each bank will do in the next time window. Such an assumption is a strong one, particularly if the time window is long. In principle, it could be relaxed and allow the information about the number of trades to be executed to be revealed over time. Specifically, Eq. 3 and 8 of the paper could have been replaced by

$$p(B_j \to B_i, t) = p_i^{b,la}(t) \cdot p_j^{l,la}(t) \cdot \frac{w + N_{B_j \to B_i}(t)}{\sum_{q=1}^{n} w + N_{B_q \to B_i}(t)}, \tag{A.1}$$

$$p(B_j \to B_i, t) = p_j^{l,ba}(t) \cdot p_i^{b,ba}(t) \cdot \frac{w + N_{B_j \to B_i}(t)}{\sum_{m=1}^{n} w + N_{B_j \to B_m}(t)}, \tag{A.2}$$

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respectively, where $p_i^{b,la}(t)$ is the probability that bank i acts as a borrower in a lender aggressor transaction after t transactions occurred, $p_j^{l,la}(t)$ is the probability that bank j acts as a lender in a lender aggressor transaction after t transactions occurred, $p_i^{b,ba}(t)$ is the probability that bank i acts as a borrower in a borrower aggressor transaction after t transactions occurred, and $p_j^{l,ba}(t)$ is the probability that bank i acts as a lender in a borrower aggressor transaction after t transactions occurred. These probabilities only include information about the instantaneous liquidity needs of banks, and the causality concern would have been fully addressed. Furthermore, using these equations would also have the advantage of removing the dependency of the model from the time window T_M. However, in this case, it would have been necessary to introduce a model to account for the co-evolution of the $4 \times n$ probabilities above, in order to run any simulation, and a multivariate model of bank-liquidity needs was out of the scope of the present paper. Furthermore, it would also have been very difficult to fit such probabilities to E-MID transaction data, in order to calibrate the model. It is to note that if the probabilities above are assumed to be approximately constant over a time window T_m, then quantities reported in Eq. 3 and 8 of the paper represent proxies of the four probabilities reported above:

$$p_i^{b,la}(t) \approx \frac{B_i^{b,la}(t)}{\sum_{k=1}^{n} B_k^{b,la}(t)}; \quad p_j^{l,la}(t) \approx \frac{B_j^{l,la}(t)}{\sum_{k=1}^{n} B_k^{l,la}(t)};$$
$$p_i^{b,ba}(t) \approx \frac{B_i^{b,ba}(t)}{\sum_{k=1}^{n} B_k^{b,ba}(t)}; \quad p_j^{l,ba}(t) \approx \frac{B_j^{l,ba}(t)}{\sum_{k=1}^{n} B_k^{l,ba}(t)};$$

So, the model presented in the paper aims at untangling a model of bank-liquidity needs and a model of trading between banks, which is the one we focus in our paper. However, we envision for the future the possibility to model bank-liquidity needs and to present a more comprehensive model that incorporates both these aspects of an inter-bank market.

Appendix B. Details about Java Implementation

A version of the model has been developed using Java and the **MASON** library for multi-agent modeling (Luke et al. (2005)). The implementation within the Java/Mason framework will allow to include and integrate our model into the interbank sector of the CRISIS macro-financial software library and in this appendix we briefly outline the structure of our Java/Mason

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software. Besides the *Model* and *Scheduler* **MASON** classes, our model defines three new classes: *Transaction*, *Bank* and *Market*. The class *Transaction* basically represents a credit line between a *Bank* which acts as a lender and a *Bank* which acts as a borrower. The class *Bank* has data members to hold the number of transactions each bank wants to do as borrower and as lender (two variables for the lender-aggressor scheme and two for the borrower-aggressor one) and four ArrayLists to store the *Transaction* objects describing the bank's credit lines with all the other banks. It also contains methods to call when a new *Transaction* occurs: they will update the cumulative and marginal transactions between the two agents and correspondingly decrease the number of the transaction they can still perform until the end of the simulated three-maintenance period. The class *Market* is an abstract class and is currently extended by two child classes: *LA_Market* for the lender-aggressor setting and *BA_Market* for the case of borrower-aggressors. While these two classes implement methods to choose the aggressor agent of the transaction and its counterpart following equations 1 and 2 (or equations 5 and 7 for the borrower-aggressor scheme), the parent abstract class defines a method to perform a single transaction, which automatically picks up a couple of agents with the correct probability and updates all of their data members (cumulative and marginal transactions, and transactions to be performed).

Appendix C. Statistically validated networks

We perform a statistical test by comparing the empirically observed number of transactions between each pair of banks against a random null hypothesis taking into account the trading heterogeneity of the system. As discussed in Hatzopoulos et al. (2013), we estimate a p-value for the empirically detected values in terms of a probabilistic description based on the hypergeometric distribution. The probabilistic description provides analytical results that are only approximated because the probabilistic description does not avoid the possibility that a bank can lend money to itself.

Hereafter, for the sake of completeness, we briefly outline the used approach. For each link in the network, we perform a statistical test to check whether two banks preferentially traded in a given time interval. Our test is done by using a recently proposed method (Li et al. (2014)) that is a directional variant of the method presented in Tumminello et al. (2011). The statistical test is implemented as follows. For each time interval, we define

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N_T as the total number of trades among banks in the system and focus on two banks i and j to check whether i preferentially lent money to j. Let us call n_l^i the number of times bank i lent money to any other bank, and n_b^j the number of times bank j borrowed money from any other bank. Assuming that n_{lb}^{ij} is the number of times bank i lent money to j then the probability of observing such n_{lb}^{ij} trades, assuming that j and i select counter-parties randomly, is given by the hypergeometric distribution

$$H(n_{lb}^{ij}|N_T, n_l^i, n_b^j) = \frac{\binom{n_l^i}{n_{lb}^{ij}} \binom{N_T - n_l^i}{n_b^j - n_{lb}^{ij}}}{\binom{N_T}{n_b^j}}. \tag{C.1}$$

We use this probability to associate a p-value with the observed number n_{lb}^{ij} of trades from bank i to bank j as $p(n_{lb}^{ij}) = \sum_{X=n_{lb}^{ij}}^{\min[\mathrm{n_l^i}, \mathrm{n_b^j}]} H(X|N_T, n_l^i, n_b^j)$, that is the probability of observing by chance a number of trades from i to j equal to n_{lb}^{ij} or larger. This p-value is calculated by taking the sum of probabilities over the right tail of the hypergeometric distribution. Therefore the p-value can be used to disprove or verify the null hypothesis of random selection of a counter-party

The hypergeometric distribution can be used to describe variable n_{lb}^{ij} because the problem can be mapped into an urn model (Feller (1968), Hatzopoulos et al. (2013)). Note that mapping the problem of randomizing bank loans onto an urn model is done at the cost of removing the constraint that a bank cannot lend money to itself. This means that our analytical solution for the random system is an approximation of what we would obtain by randomly rewiring data and enforcing the condition of no self loans.

To avoid a large number of false positive validated links, due to the large number of statistical tests, it is advisable to consider a method to control the family-wise error rate. This control can be done by applying the so-called Bonferroni correction. This correction requires that the univariate level of statistical significance, e.g. $p_u = 0.01$, is corrected in presence of multiple tests. In the present study, we set the Bonferroni correction as discussed in Hatzopoulos et al. (2013).

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