The valuation of Guaranteed Lifelong Withdrawal Benefit Options in Variable Annuity contracts and the impact of mortality risk

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Abstract

In the light of the growing importance of the Variable Annuities market, in this paper we introduce a theoretical model for the pricing and valuation of Guaranteed Lifelong Withdrawal Benefit Options (GLWB) embedded in Variable Annuity products. As the name suggests, this option offers a lifelong withdrawal guarantee; therefore, there is no limit on the total amount that is withdrawn over the term of the policy, because if the account value becomes zero while the insured is still alive he continues to receive the guaranteed amount annually until death. Any remaining account value at the time of death is paid to the beneficiary as a death benefit We offer a specific framework to value the GLWB in a market consistent manner under the hypothesis of a static withdrawal strategy, according to which the withdrawal amount is always equal to the guaranteed amount. The valuation approach is based on the decomposition of the product into living and death benefits. The model makes use of the standard no-arbitrage models of mathematical finance, that extend the Black-Scholes framework to insurance contracts, assuming the fund follows a Geometric Brownian Motion and the insurance fee is paid, on an ongoing basis, as a proportion of the assets. We develop a sensitivity analysis, which shows how the value of the product varies with the key parameters, including the age of the policyholder at the inception of the contract, the guaranteed rate, the risk free rate and the fund volatility. We calculate the fair fee, using Monte Carlo simulations under different scenarios. We give special attention to the impact of mortality risk on the value of the option, using a flexible model of mortality dynamics, which allows for the possible perturbations by mortality shock of the standard mortality tables used by practitioners. Moreover, we evaluate the introduction of roll-up and step-up options and the effect of the decision to delay withdrawing. Empirical analyses are performed and numerical results are provided.

Keywords: Guaranteed Lifelong Withdrawal Benefit option, mortality risk, sensitivity analysis, stochastic mortality model, Variable Annuity.

JEL classification: G22

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1. Introduction

Variable Annuities (VA) were introduced in the 1970s in the United States. This market has increased considerably in size in the past decade, when bullish financial markets and low interest rates have tempted investors to look for higher returns. VA, whose benefits are based on the performance of an underlying fund, are very attractive, because they provide a participation in the stock market and also a partial protection against the downside movements of interest rates or the equity market. The typical VA is a unit-linked deferred annuity contract. Since the 1990s, two kinds of embedded guarantees have been offered in such policies (see Hanif et al (2007)): the Guaranteed Minimum Death Benefit (GMDB) and the Guaranteed Minimum Living Benefit (GMLB). There are three main products which guarantee some living benefits: the two earliest forms, the Guaranteed Minimum Accumulation Benefit (GMAB) and the Guaranteed Minimum Income Benefit (GMIB), offer the policyholder a guaranteed minimum at the maturity T of the contract; however, with the GMIB, this guarantee only applies if the account value is annuitized at time T. In 2002, a new type of GMLB was issued: the Guaranteed Minimum Withdrawal Benefit (GMWB), which gives the insured the possibility to withdraw a pre-specified amount annually, even if the account value has fallen below this amount. In 2004, each of the 15 largest Variable Annuity providers in the US offered this guarantee and 69% of the Variable Annuities sold included a GMWB option (Lehman Brothers (2005)); in 2007 the percentage was 86% (see Ledlie at al. (2007)).

In this paper, we analyze the latest GMWB option: the Guaranteed Minimum Withdrawal Benefit for Life or Guaranteed Lifelong Withdrawal Benefit option (GLWB). As the name suggests, it offers a lifelong withdrawal guarantee; therefore, there is no limit to the total amount that is withdrawn over the term of the policy, because, if the account value becomes zero while the insured is still alive, he can continue to withdraw the guaranteed amount annually until death. The first VA with a withdrawal benefit guaranteed for the life was introduced in the US market in 2003. Since

2006, nine out of ten VA products have offered guaranteed living benefits; GLWB options captured some GMIB markets and represented the 35% of the whole market in early 2006 (see Advantage Compendium 2008). Despite these developments, until now, the actuarial literature has tended to focus on GMWBs, paying little attention to the differences with the GLWB options. In this regard, our purpose is to offer a specific framework to value the GLWB in a market consistent manner under the hypothesis of a static withdrawal strategy and describe the impact of the main risk factors over time, especially the impact of mortality risk. Our work uses the standard no-arbitrage models of mathematical finance, in line with the tradition of Boyle and Schwartz (1977) that extended the Black-Scholes framework to insurance contracts. The main difference is that, for the options embedded in VA products, the fee is deducted on an ongoing basis as a proportion of the market value of the underlying assets, while in the Black and Scholes approach the option premium is paid up-front.

Our approach follows the recent actuarial literature on the valuation of VA products: Ballotta and Haberman (2003); Bauer et al. (2008);, Chen et al. (2008), Coleman et al. (2006); Holz et al (2006), Liu (2010), Milevsky and Posner (2001), Milevsky and Promislow (2001), Milevsky and Salisbury (2002), Milevsky and Salisbury (2006). We consider a static strategy in which the policyholder withdraws the guaranteed amount each year. Milevsky and Salisbury (2006) highlight that "most, if not all, insurance companies assume this type of behavior on the part of policyholders". There are some important reasons for our making this choice. First of all, VAs providers can influence the behavior of policyholders through imposing penalty charges on the amount of withdrawal that exceeds the guaranteed amount. In addition, we have to consider that these options are being introduced in pension plans, in order to ensure a constant income during retirement and provide protection against market downside risk. Moreover, the value of a lifetime GMWB is only slightly higher to an arbitrageur with optimal behavior than it is to the more simplistic deterministic withdrawal behavior of a typical investor (Xiong et al. 2010). Under the static hypothesis, we decompose the payoff into life benefits and death benefits and offer a valuation formula. The sensitivity of the price of the product to changes in the key parameters is analyzed. The aim of this work is to offer a guideline for the insurance companies to price and manage this new product, which is affected by financial and mortality risk due the long maturity. In particular, we study the impact of mortality risk on the value of a GLWB using a stochastic approach, which we believe is necessary in order to avoid underestimation or overestimation of the expected present value of insurance and annuity contracts. Indeed, recently unanticipated changes over time in the mortality rates have been observed. In this note, we propose a simplified version of the stochastic model suggested by Cox and Lin (2005) and developed by Ballotta et al (2006). We present an application using Italian data.

The remainder of the paper is organized as follows. In section 2, we describe the product. Section 3 develops the model for the pricing of a GLWB option. In section 4, we carry out a sensitivity analysis, in which we analyze the effects of changes in the parameters of the pricing model on the GLWB value. In Section 5, we study the impact of mortality risk on the value of the contract using a simulation-based stochastic mortality model. Concluding remarks are offered in section 6.

2. Product description

The GLWB is the latest variant of the GMWB option recently introduced in US, Asia and Europe. Products with a GMWB option give the policyholder the possibility to withdraw annually a certain percentage g of the single premium W_0,that is invested in one or several mutual funds. The guarantee consists of the entitlement to withdraw amounts until an amount equal to the premium paid even if the account value falls to zero. Instead, if the account value does not vanish, at maturity the policyholder can take out or annuitize any remaining funds. Products with a GLWB option offer a lifelong guarantee: the maximum amount to be withdrawn is specified but the total amount is not limited and the insured can annually request a portion of the premium paid while he is still alive, even if the fund value drops to zero. Any remaining account value at the time of death is paid to the beneficiary as a death benefit. The insurer charges a fee for this guarantee, which is usually a pre-specified annual percentage of the account value.

Many additional features can be added to this base contract:

- in the case of a *Roll-up* option, the annual guaranteed withdrawal amount is increased by a fixed percentage every year during a certain time period but only if the policyholder has not started withdrawing money. Therefore, *Roll-ups* are commonly used as a disincentive to withdraw during the first years;

- when the contract contains a *Step-up* option, at pre-specified points in time (step-up dates), the guaranteed annual withdrawal amount is increased if, at the step up date, the percentage g of the account value exceeds the contractual guaranteed amount $G=gW_0$. Therefore, Step-ups occur if the fund has a high performance and the account value has not been decreased too greatly by previous withdrawals;

- in the case of a deferred version of the contract, the policyholder cannot withdraw money during the deferment period; however, at the end of this period, if the fund value has fallen below the guaranteed amount, the VA provider has to add enough money to the fund in order to reproduce the guaranteed level.

In order to explain the operation of GMWB and GLWB options, we provide a simple numerical example. Let W_t the market value of the underling fund at time t and let 100 the initial investment, so W_0=100. In a typical GMWB contract without Roll-up or Step-up the amount guaranteed is gW_0. Let g, the guaranteed rate, be equal to 7% and then G, the guaranteed amount, is equal to 7 in this particular case, given the single premium. The guarantee continues until the amount of 100 has been withdrawn, so the minimum period is 100/7=14.28 years. The withdrawal dates are fixed by the contract and between two dates the guaranteed amount G is equal to 0. If during this period W_t collapses to zero the investor will withdraw 7 per year until T=14.28. Instead, if the account value does not vanish, at the expiration date the policyholder can take out or annuitize any remaining funds. Moreover, in any given year, the policyholder is entitled to withdraw an amount less than 7 and thereby extend the life of the guarantee or an amount greater than 7 and hence reduce it.

In a typical GLWB contract, there is no limit for the total amount that is withdrawn over the term of the policy, because, if the account value becomes zero while the insured is still alive, he will continue to withdraw 7 until death. Any remaining account value at the time of death is paid to the beneficiary as a death benefit. If the policyholder withdraws an amount less or greater than 7, this does not have an effect on the life of guarantee but does have an effect on the death benefit which is linked to the account value.

3. The model

Let Γ be the future lifetime random variable expressed in continuous time, $F_x(t)$ be its *cdf* and $f_x(t)$ be its *pdf*; therefore, for an individual aged x the probability of death before time t is

$$F_x(t) = P(\Gamma \le t) = {}_t q_x = 1 - {}_t p_x = 1 - \exp\left\{ -\int_0^t \mu(x+s)ds \right\} \qquad (1)$$

where μ denotes the force of mortality.

Let us introduce two probability spaces $(\Omega, \Im^{'}, P^{'})$ and $(\Omega, \Im^{''}, P^{''})$, where $\Im^{'}$ and $\Im^{''}$ are the σ-algebras containing respectively the financial events and the demographic events. It is reasonable to assume the independence of the randomness in mortality and that in interest rates, and so we denote the space $(\Omega, \Im, P)$ generated by the proceeding two spaces. Let P be the real probability measure.

Let W_t be the fund value at time t, where the single premium paid by the policyholder is invested. Following the standard assumptions in the literature and considering a static withdrawal strategy, we model the evolution of the fund as:

$$\begin{aligned} dW_t &= (\mu - \delta) W_t dt - G dt + \sigma W_t dZ_t \\ W_0 &= \omega_0 \end{aligned} \qquad (2)$$

where Z_t is a standard Brownian motion, μ is the drift rate, σ is the fund volatility, δ is the insurance fee paid ongoing as fraction of assets, G is the withdrawal from the fund at time t and is equal to the guaranteed amount. Equation (2) holds while $W_t>0$.

This dynamic model for the underlying investment is consistent with the actuarial literature on pricing insurance guarantees (Chen and Forsyth (2008), Gerber and Shiu (2003), Milevsky and Salisbury (2006), Windcliff et al. (2001)). Following the literature, we assume that exists a risk neutral probability space $(\Omega, \Im, \{F_t\}_{t\geq 0}, Q)$, with a filtration $\{F_t\}_{t\geq 0}$ and a risk neutral measure, under which payment streams can be valued as expected discounted value using the risk-neutral valuation formula (c.f. Bingham and Kiesel (2004)); existence of this measure implies the existence of an arbitrage free market. If we consider this new space, the evolution of the fund is:

$$dW_t = (r - \delta) W_t dt - G dt + \sigma W_t d\tilde{Z}_t^Q \qquad (3)$$

where r is the risk free rate and $\tilde{Z}_t^Q = Z_t + \frac{\mu - r}{\sigma} t$ is a Brownian motion under Q.

The GLWB offers a lifelong guarantee: the maximum amount to be withdrawn is specified but the total amount is not limited and the insured can annually request a portion of the premium paid while he is still alive, even if the fund value drops to zero. Any remaining account value at the time of death is paid to the beneficiary as death benefit.

We consider a simple form of product, without a Roll-up option, Step-up or deferred period. Let V_0 be the discounted value at t=0 of the GLWB; it is the sum of the discounted values of the living and death benefits:

$$V_0 = LB_0 + DB_0 \quad (4)$$

LB_0 is the well-known discounted value of a life annuity; DB_0 can be calculated considering the payoff that the beneficiary will receive at the random time of death τ:

$$DB_\tau = Max(W_\tau ;0) \quad (5)$$

Since the time of maturity is stochastic and τ and Wτ are independent, the discounted value at t=0 of the death benefit is given by the following conditional expectation :

$$DB_0 = E\ \{E\{e^{-r\tau} DB_\tau \,|\tau\}\} \quad (6)$$

Conditioning on τ = T, the death benefit can be calculated by Ito's lemma as in Milevsky and Salisbury (2006); the solution to equation 5 is:

$$DB_T = e^{(\mu-\delta-\frac{\sigma^2}{2})T+\sigma Z_T} \max\left[\left(\omega_0 - G\int_0^T e^{-(\mu-\delta-\frac{\sigma^2}{2})t-\sigma Z_t} dt\right);0\right] \quad (7)$$

Using a standard technique in the literature, the no-arbitrage time-zero value of the death benefit occurring at time T is:

$$DB_0(\tau = T) = E_0^Q(DB_T) = E_0^Q\left[e^{(r-\delta-\frac{\sigma^2}{2})T+\sigma\tilde{Z}_T} \max\left[\left(\omega_0 - G\int_0^T e^{-(r-\delta-\frac{\sigma^2}{2})t-\sigma\tilde{Z}_t} dt\right);0\right]\right] \quad (8)$$

where the expectation is taken under the risk neutral measure Q.

If we consider both the expectations in the equation (6), we obtain:

$$DB_0 = \int_0^{\omega-x} f_x(t) E_0^Q(DB_t) dt \quad (9)$$

In the discrete case we have:

$$DB_0 = \sum_{t=0}^{\omega-x} {}_t p_x \quad q_{x+t} E_0^Q \left[DB_t \right] \tag{10}$$

for a policyholder aged x at inception of the contract. Thus, the discounted value at t=0 of the death benefit is a weighted average of the values of ω-x+1 different death benefits, where the weight for year t is the deferred probability of death in year t, i.e. the probability of survival until t and death between t and t+1.

The policyholder is also entitled to withdraw the guaranteed amount each year while he is alive; therefore, the actual value of the GLWB option if the policyholder assumes a static strategy is:

$$V_0 = \sum_{t=0}^{\omega-x} \left[{}_t p_x g W_0 e^{-rt} + {}_{t/1} q_x E_0^Q (DB_t) \right] \tag{11}$$

where ${}_{t/1}q_x = {}_t p_x \quad q_{x+t}$ is the deferred probability of death.

4. Numerical results and sensitivity analysis

In this section, we carry out a sensitivity analysis, in which we analyze the relationship between the GLWB value and- the age of policyholder at the inception of the contract; - the rate g; - the risk free rate r; - the fund volatility σ. Finally, we evaluate the impact of the roll-up and step-up options on the GLWB.

Initially, we price a simple form of GLWB, without a Roll-up option, Step-up option or deferred period. The amount invested in the fund at t=0 is W_0=100; the policyholder receives the amount guaranteed gW_0 while he is alive; moreover, at the date of death the beneficiary will receive any remaining account value. We offer an evaluation of GLWB for the Italian market and, for this reason, we consider a mortality table based on the experience of the Italian male population: the SIM2002. The maximum age considered is 110. Figure 1 shows the deferred probabilities of death for policyholders aged 50, 60, 70 and 80, where $q_{x+t\ t} p_x$ is the probability for a policyholder aged x to survive to time t and die between t and $t+1$

Figure 1 shows an unusual feature in that each curve has two modes; this bimodal feature can be found also in a recent life table for Belgian males (see Pitacco et al. (2009)).

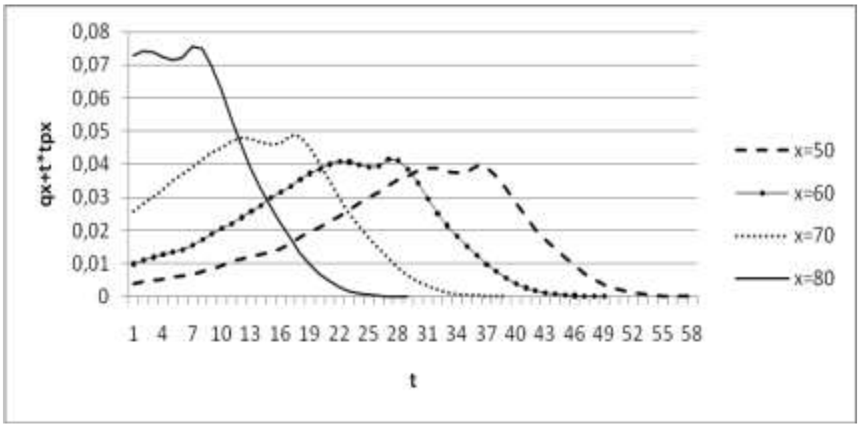

Figure 1: the deferred probabilities of death

We calculate the GLWB value for different policyholders with ages from 50 to 80 at inception. Since this product has not been introduced yet in Italy, we refer to typical parameter values from the US market. Thus,we set g=4%, r=5%, σ=20% and δ=0.70 b.p. Currently, the typical GMWB for a life rider fee ranges from 0.6%-0.9% (Xiong et al., 2010); the guaranteed rate increases with the age of policyholder at the inception of the contract and usually it is equal to 4% for a policyholder aged 50-59 and 5% for a policyholder aged 60-69. Finally, according to Morningstar Principia Pro, the average of the sub-account volatility for the universe of variable annuity products is 18%, the 25th percentile is 16% and the 90th percentile is 25%.

In order to study the way in which the GLWB value varies when the age of policyholder at the inception of the contract increases, we decompose the product into the sum of the living benefit and the death benefit and observe their relationship with age at inception. In the following, we refer to the value of death benefit, living benefit and GLWB value as the discounted value at the inception of the contract of death benefit, living benefit and GLWB option. Given the guaranteed rate, as the age at inception increases, the value of living benefit decreases while that of the death benefit increases, because the number of possible withdrawals decreases and the death benefit is discounted for a shorter period. The overall effect is represented in Figure 2: as age increases, the discounted value of the GLWB decreases, in line with the hypothesis being considered. For this reason, VA providers are able to offer higher guaranteed rates to older policyholders.

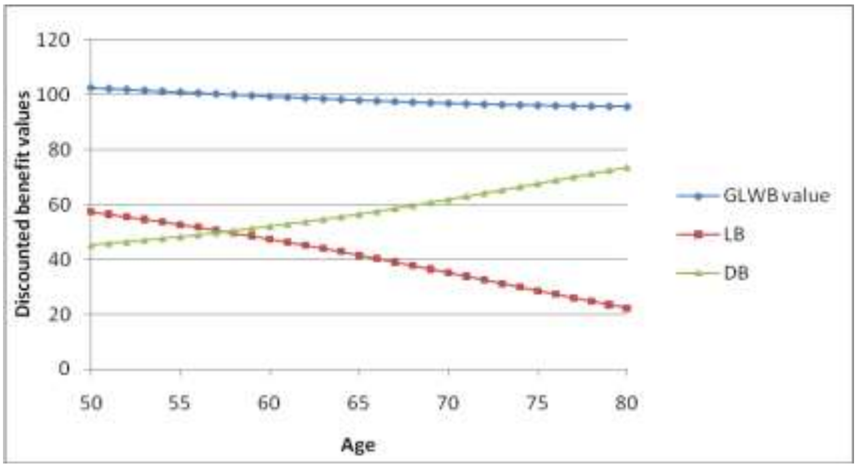

Figure 2: the discounted values at t=0 of the GLWB and of the death and living benefits.

For younger policyholders, the living benefit is of greater importance; we note that the number of withdrawals is high, the account value at the time of death is relatively small because the amount withdrawn deducted from the fund value is significant and the death benefit is discounted for a long period. Instead, as the age of the policyholder at the inception of the contract increases, the living benefit becomes less significant and the death benefit becomes more significant.

Next, we continue the analysis and consider the relation between the GLWB value for policyholders with different ages at inception and the parameters g, r and σ.

As the rate of guaranteed withdrawal increases, there are two effects: for each age at inception, on the one hand, the actual value of the living benefit increases because the amount withdrawn increases; on the other hand, the amount of the death benefit decreases because the amount deducted from the fund increases (see Figure 3).

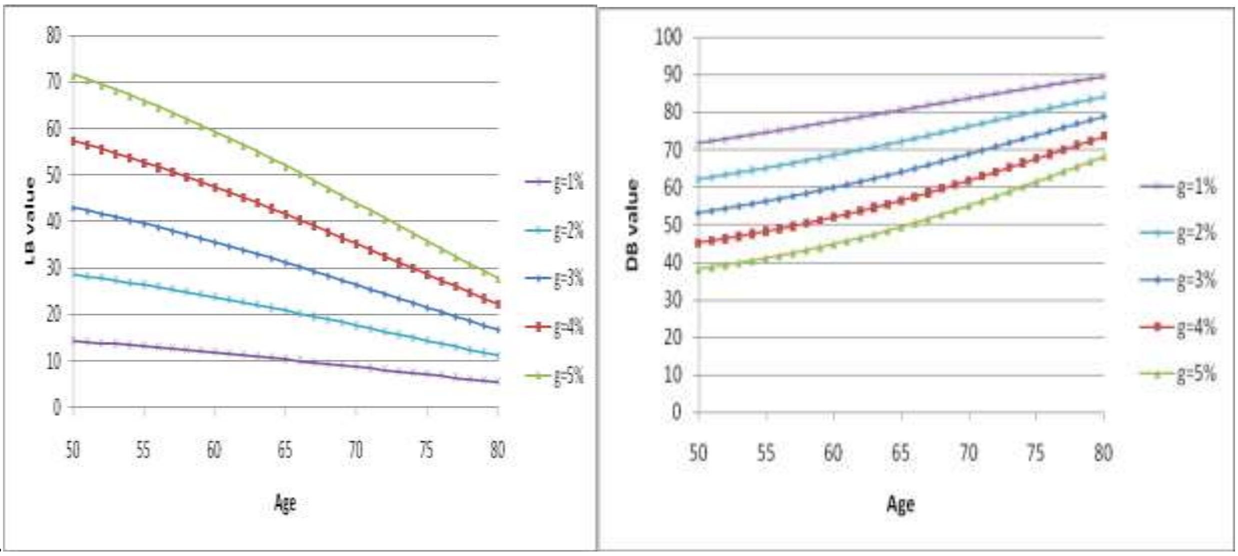

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Figure 3: the relation between g and the discounted value of the living benefit (LH) and death benefit (RH)

Overall, the relation between the GLWB value and the guaranteed rate of withdrawal is shown in Figure 4. The relationship between g and the value of the living benefit prevails at each age so that as g increases the product becomes more attractive. However, the impact is stronger for younger policyholders, for whom the living benefit is of greater importance, while for older policyholders the influence of g is moderate. Taking into account this consideration, from the point of view of VA providers, the guaranteed rate offered to older customers should be used as a key feature to make the product more attractive without having a strong influence on the payoff,

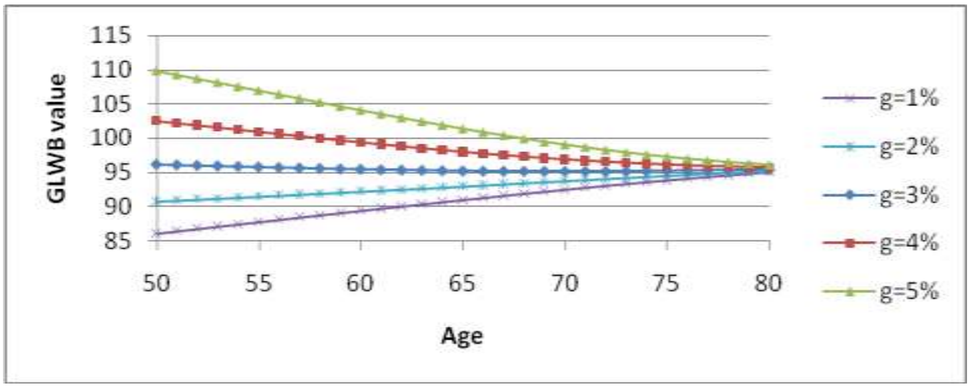

Figure 4: the relation between the GLWB value and g

The third step of our sensitivity analysis is the study of the relation between the GLWB value and the risk free rate r. As r increases, the discounted value of each withdrawal decreases; so the value of the living benefit decreases at each time point. Instead, the value of the death benefit increases, (see Figure 5). In fact, the risk free rate influences both the evolution of the fund and the discounted value of the death benefit and there is a “balancing out effect” Further, there is a third effect: in eq. 8 the term in brackets increases as r increases.

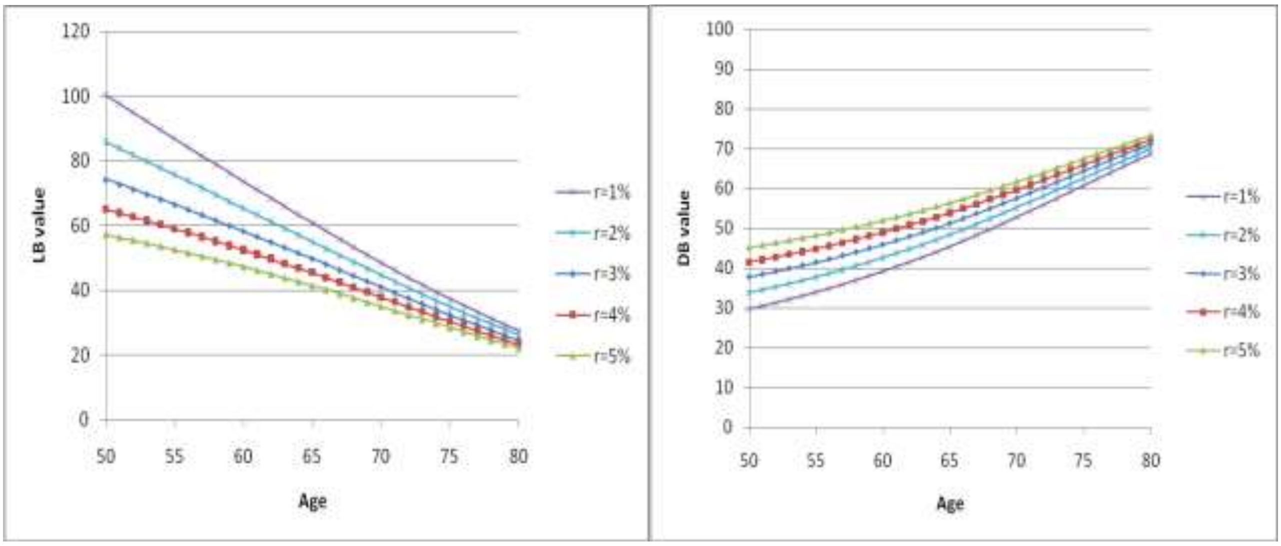

Figure 5: the relation between r and the discounted value of the living benefit (LH) and death benefit (RH)

Overall, the relation between the GLWB value and the risk free rate is shown in Figure 6: as the risk free rate increases, the GLWB value decreases and the impact is stronger for younger policyholders, who hold the product for longer period. In particular, they gain more advantage from reductions in the risk free rate.

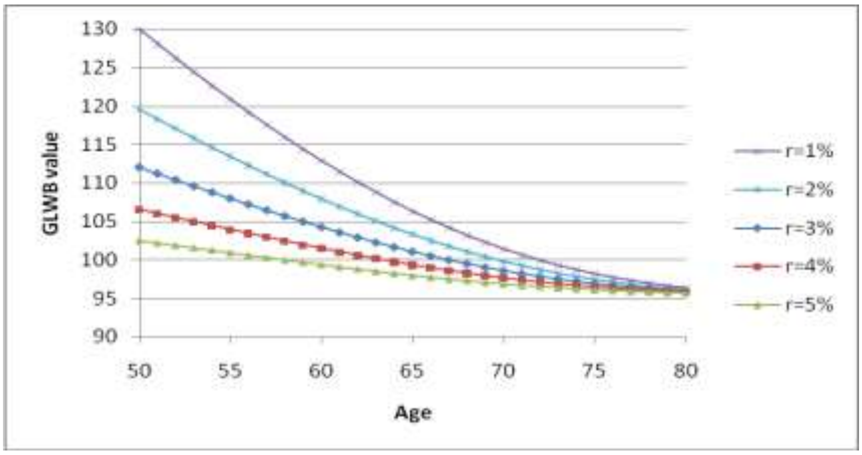

Figure 6: the relation between the GLWB value and r

The fourth step of the sensitivity analysis is the study of the relation between the GLWB and σ. Figures 7 shows the results.

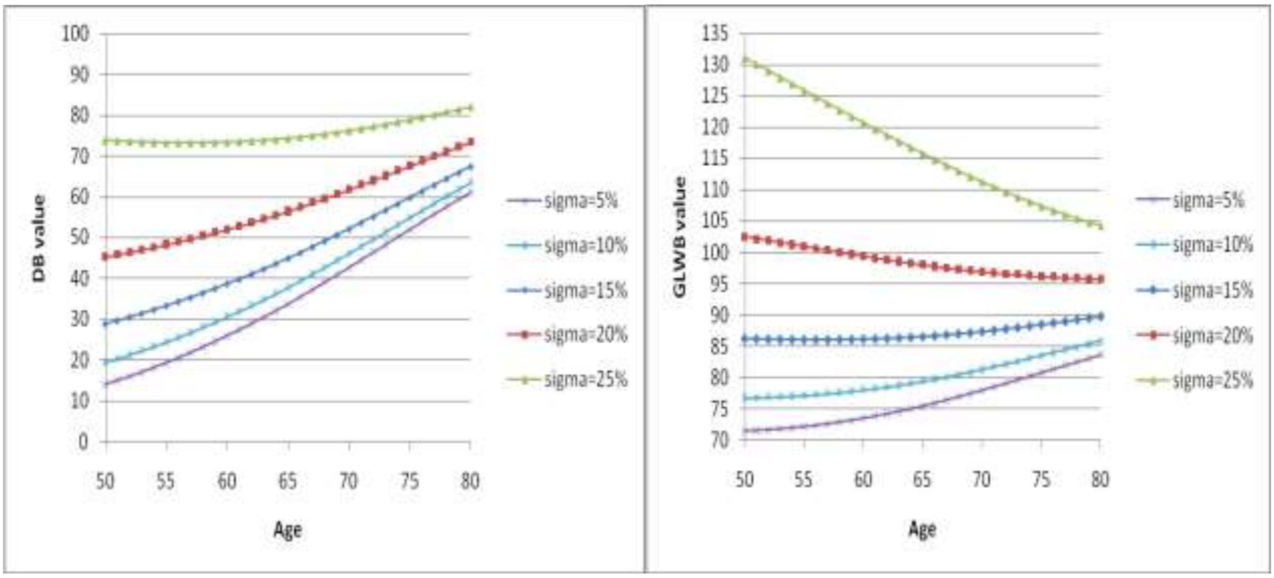

Figure 7: the relation between σ and the discounted value of the death benefit (LH) and the GLWB (RH)

As the fund volatility increases, assuming that there are no step-up options, the value of the living benefit does not change because the withdrawals do not depend on the fund value while the value of the death benefit increases (see equations 2 and 5). In particular, this effect is stronger for younger policyholder because they hold the product for a longer period during which the fund value evolves. Consequently, as the volatility increases the curve is raised and is raised more for younger policyholders, so that if volatility is low, then age has a significant impact on the DB value, while if volatility is high, age has much less influence on the DB value. Overall, the GLWB value increases with age at inception when volatility is low, but decreases with age when it is high. This effect occurs because GLWB is the sum of death benefits and living benefits, and the latter are not influenced by volatility and their value is a decreasing function of age. When volatility is low, the death benefit increases consistently with age and so it compensates for the decreasing trend in the living benefit; instead, when volatility is high the increase in the death benefit is less marked and it is not sufficient to compensate for the decrease in the living benefit.

Until now, we have considered a given charge; however, VA providers price the GLWB option requiring different fees. In order to offer a realistic market consistent valuation, we calculate the fair insurance fee based on the pricing model developed in the previous section. As in Piscopo (2009), giving the value of the other parameters, the fair insurance fee can be obtained by equating the single premium ω_0 to the value at time 0 of the all future cash flows.

We consider a policyholder aged 60 at the inception of the contract; we carry out Monte Carlo simulations under different scenarios, generating for each of them 10,000 paths of evolution of the fund. Once the interest rate, the volatility and the guaranteed rate have been fixed, we search for the fair value of the fee using an iterative procedure: if the time-zero cost of the whole product turns out to be higher than ω_0 then we increase the fee in order to decrease the cost in order to get closer to the value of ω_0; and vice-versa, if the time-zero cost of the whole product turns out to be smaller than ω_0 . Table 1 displays the fair insurance fee under different guaranteed rate and sub-account volatility:

g/σ	16%	18%	20%
4%	36,97	48,25	59,99
5%	68,14	85,32	102,77
6%	126,4	151,61	176,76

Table 1: The impact of the guaranteed rate and sub-account volatility on the fair insurance fee for a policyholder aged 60.

As expected, given the guaranteed rate, the fair fee increases with increases in the volatility, because guarantees are more expensive when volatility increases, and, given the volatility, the fair fee increases with increases in the guaranteed rate, because the product is more valuable to the policyholder.

We conclude this section with a consideration of the valuation of roll-up and step-up options.

In the case of a *Roll-up* option, the annual guaranteed withdrawal amount is compounded at the *roll-up rate* during a certain time period but only if the policyholder has not started withdrawing money. The valuation of this option needs to consider the interaction of different factors. On the one hand, a roll-up rate greater than the risk free rate leads to an increase in the discounted value of future guaranteed withdrawals if they are postponed. On the other hand, the insurer becomes older during the deferment period and the expected number of withdrawals until death decreases with a consequential effect on both the living and death benefits. In the light of this consideration, it is not possible to provide general statements regarding the profitability of the option, since it is necessary to evaluate, on a case by case basis, the product in line with the chosen values of the key parameters: the roll-up rate, the deferment period, the expectancy of life of the policyholder and so on. By way of an example, we modify the basic contract analyzed above by introducing different roll-up options. As in the previous analysis, we set g=4%, r=5%, σ=20% and δ=0.70 b.p. We compare DB, LB and GLWB values between contract 1, which guarantees lifelong withdrawals of 4% of the initial premium without offer a roll-up, and contracts 2 and 3, which provide an annual increase of the guaranteed withdrawal amount of 7% as long as no withdrawals are made for 5 and 10 years respectively (the so-called waiting period). Figures 8-9 show the results for different ages at inception. If the policyholder decides to delay the withdrawals, the value of the death benefits provided by contracts 2 and 3 is greater than that provided by contract 1 during the waiting period and is lower afterwards. Further, we need to take into account the interaction of this effect with the probability of death: a younger policyholder is less likely to die during the waiting period, when the death benefit is higher, than an older policyholder. As regards the living benefits, contracts 2 and 3 provide greater guaranteed withdrawal after the waiting period, but during the waiting period there are no withdrawals. The probability of survival is higher during the waiting period and consequently the first set of cash flows has a greater weight in the valuation formula. Overall, in the example provided, contracts 2 and 3 are less valuable than contract 1.

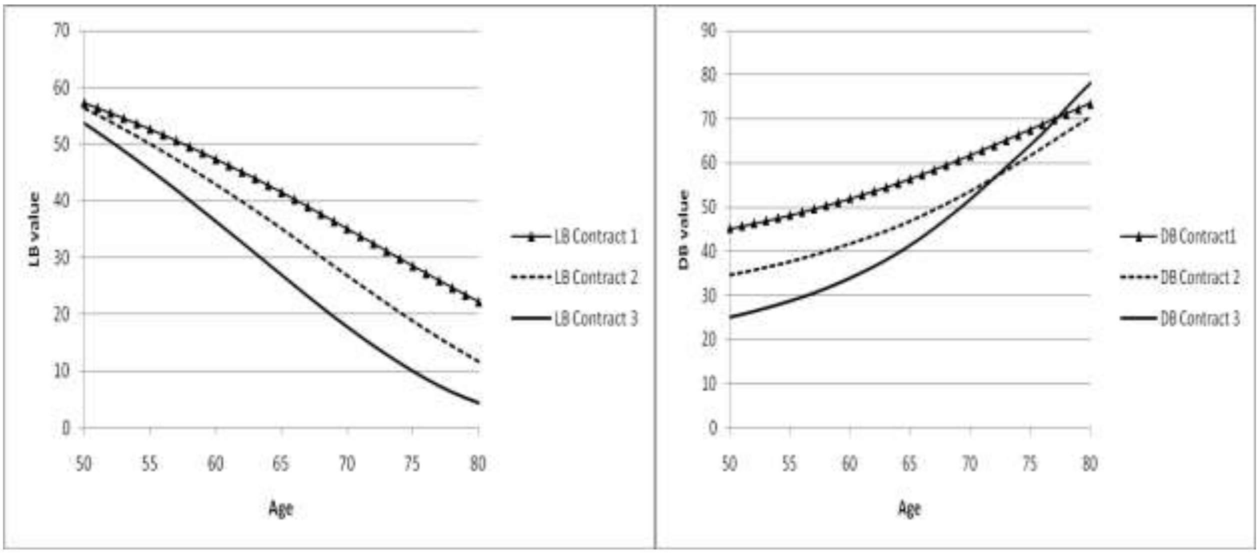

Figure 8: Living benefit (LH) and death benefit (RH) without and with Roll-up Option

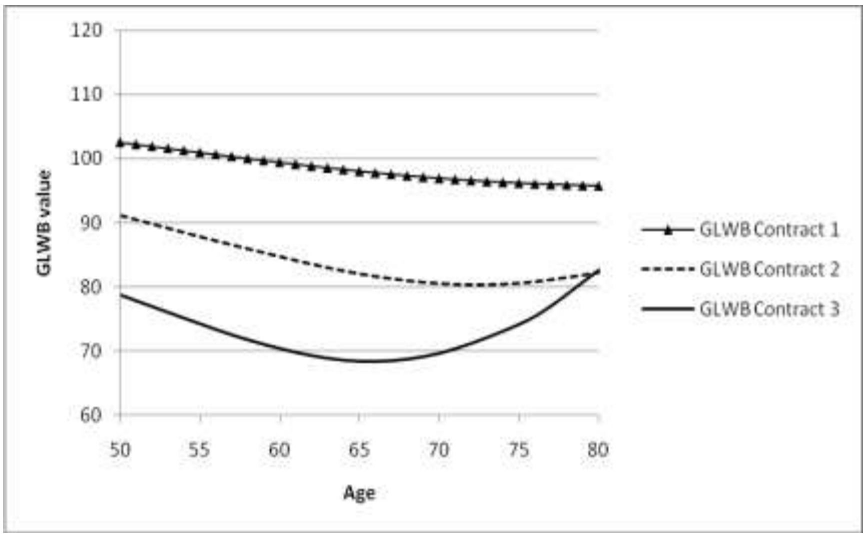

Figure 9: GLWB value without and with Roll-up Option

A similar discussion applies to the case of *Step-up* options. When the contract contains a *Step-up* option, at pre-specified points in time (*step-up dates*), the guaranteed annual withdrawal amount is increased if the percentage g of the account value exceeds the contractual guaranteed amount. The policyholder can start to withdraw immediately or decide to delay the withdrawals. As previously, we consider a specific example and compare DB, LB and GLWB values between contract 1 and contract 4, which provides an annual *Step-up* option, with and without a deferment period. Figures 10-11 show the results for different ages at inception. If the policyholder does not decide to delay the withdrawals, the value of the living benefit increases with respect to the product without a *step-up*, because the step-up option provides a floor equal to the guaranteed amount and also a potentially higher return. The option is more valuable for younger insureds than for older ones, because they can profit from it for a longer period. In contrast, the value of the death benefit decreases because the amount withdrawn becomes significant. Overall, in this example, the *step-up* option makes the GLWB more valuable for younger policyholders. In the case where the insured decides to delay the withdrawals by 5 or 10 years, the valuation is similar to the *Roll-up*, with the difference that in the *Step-up* case the guaranteed amounts after the waiting period are not constant but depend on the fund performance.

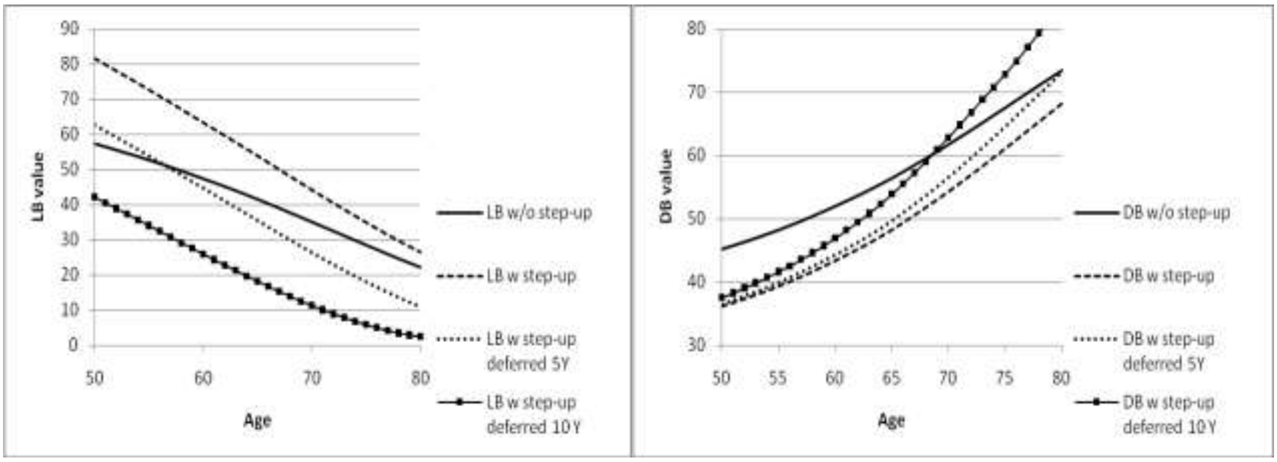

Figure 10: Living benefit (LH) and death benefit (RH) without and with Step-up Option

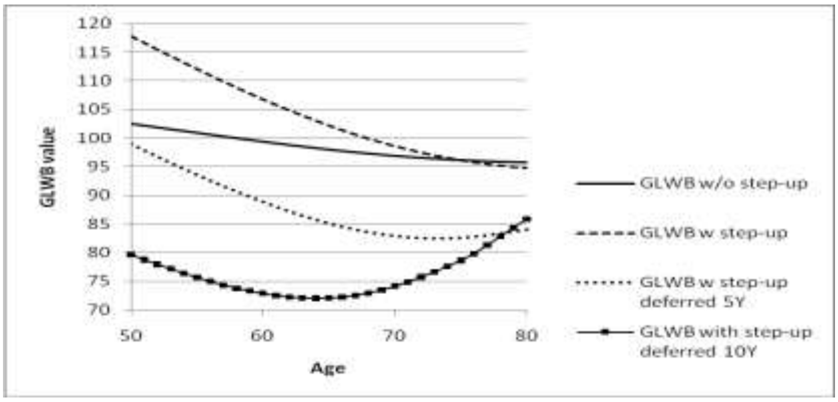

Figure 11: GLWB value without and with Step-up Option

As previously, it is not possible to outline general statements on the profitability of the option, since it is necessary to evaluate, on a case by case basis, the products according to the values of the parameters selected. In particular, in the case of the step-up option, the volatility of the fund account influences considerably the profitability of the products, since the cash flows are strongly dependent on the fund performance.

5. The impact of mortality risk on the GLWB value: a stochastic approach

In the previous section, we have considered the price of the GLWB taking into account a fixed pre-specified mortality table, viz. SIM2002. Recently, it has become evident that a stochastic mortality approach is necessary in order to avoid underestimation or overestimation of the expected present value of life insurance contracts with a significant mortality component (see Olivieri (2001), Pitacco (2004), Pitacco et al (2009)). In this section, we propose a simplified version of the stochastic model suggested by Cox and Lin (2005) and developed by Ballotta et al (2006).

Our approach starts with the survival table used before and develops an adjusted mortality table, which takes into account possible mortality shocks. In this regard, the number of survivors at age x+t, l(x+t), is approximately distributed as a normal random variable with mean equal to l(x) ${}_tp_x$ and variance equal to l(x) ${}_tp_x(1- {}_tp_x)$. However, the latest actuarial literature highlights the feature that the empirical data show perturbations in the survival probabilities due to random shocks. Cox and Lin (2005) express the mortality improvement shock ε_t as a percentage of the force of mortality μ_{x+t}. Without considering shocks in mortality, the survival probability for an age x at year t is p_{x+t} with

$$p_{x+t} = \exp(-\mu_{x+t})$$

under certain assumptions. Accordingly, we simulate the survival probabilities adjusted for shocks as follows:

$$p'_{x+t} = \exp(-\mu_{x+t})^{1-\varepsilon_t} = p_{x+t}^{(1-\varepsilon_t)} \quad (12)$$

Ballotta et al (2006) assume that ε_t follows a beta distribution with parameter a and b and the sign of the shocks depends on the random number k(t) simulated from the uniform distribution U(0,1). In particular, they set:

$$\varepsilon(t) \quad \text{if} \quad k(t) < c$$

$$-\varepsilon(t) \text{ if } \quad k(t) \geq c \quad (13)$$

where c is a parameter which depends on the user's expectation of the future mortality trend.

The importance of assigning a random sign to ε_t is that, in this way, the model captures not only the long period variations in mortality rates, but also the short period fluctuations due to exceptional circumstances.

In our application, by way of an illustration, we consider two contrasting cases for the value of c: c = 1 and c = 0. In the first case, there will be improvements in life expectancy at each date; in other words, all shocks are expected to be positive. Conversely, in the second case further improvements of an already high expectancy of life are impossible and all shocks are expected to be negative. In our application, we simulate the value of p'_x for a policyholder aged x = 50 at inception of the contract under the two different hypotheses and then we calculate the expected number of survivors l'(x+t+1) as follows:

$$l'(x+t+1) = l(x+t) * p'(x+t) \quad (14)$$

We are then able to calculate the other mortality functions that we need.

In order to analyze the impact of different variations in mortality probabilities, we carry out the following calculation procedures: we fix a = 0.5 and b = 4.5, so that shocks have expected value equal to 0.10 and standard deviation equal to 0.12. This hypothesis is consistent with the mean and standard deviation of annual percentage mortality improvements experienced by Italian males aged from 50 to 75 between the years 1955, 1965, 1975, 1985, 1995 and 2005. We simulate 10.000 paths of evolution of mortality using the Monte Carlo method and consider the alternative hypotheses c =

0 and c = 1. Then, we calculate the price of the GLWB option and compare the results under the different scenarios.

At first, we report the results relating to only one path simulated under the hypothesis c=0 and c=1, in order to reflect upon the impact of an improvement or a worsening on the deferred probabilities of death; afterwards, we show more general results of our simulations.

In figure 12, we compare the unadjusted survival function for a policyholder aged 50 based on SIM2002 mortality table with those simulated under the hypothesis c = 1 and c = 0 for a particular path. In the first case, we expect that there will be only improvements in life expectancy and, consequently, the adjusted function lies above the unadjusted survival function. Instead, in the second case we expect there will be only deteriorations and the adjusted function lies below the unadjusted survival function.

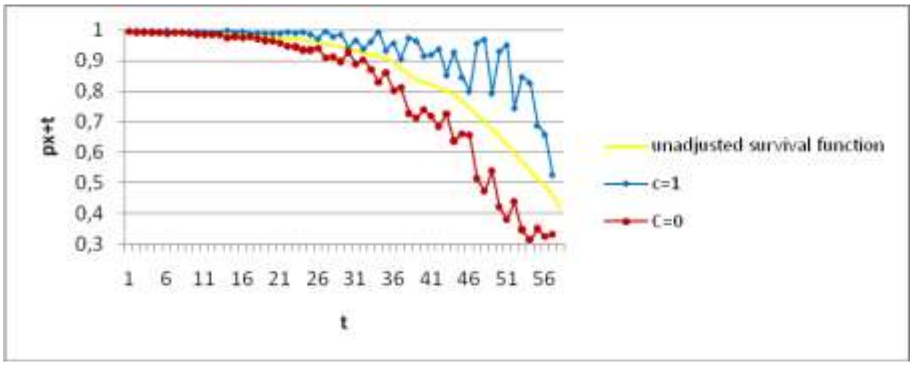

Figure 12: The simulated effects of improvements and worsening on the survival functions (c=1: mortality improving; c=0, mortality deteriorating)

In order to quantify the impact of mortality risk on the GLWB value we have to consider the projected deferred probabilities of death $q_{x+t\ t}p_x$, i.e. the probability for a policyholder aged x to survive at year t and die between t and $t+1$. In Figure 13, we compare the unadjusted mortality probability $q_{50+t\ t}p_{50}$ for $t=1,\ldots,60$ and the adjusted probabilities under the hypotheses c = 1 and c = 0 for a particular single simulated path of mortality; polynomial smoothed functions are also shown in order to identify the general shape of the curves. We have to keep in mind that the probability function changes because of two different effects: if c = 1 the survival probability increases and the mortality probability decreases at every time point; in contrast, if c=0, the mortality probability increases and the survival probability decreases. The consequences are that, under the hypothesis c=0, the adjusted probability function is translated so that the left tail becomes fatter and the right tail less fat than for the unadjusted probability function. In contrast, if c=1, the adjusted probability function is translated so that the left tail becomes less fat and the right tail more fat than for the unadjusted probability function.

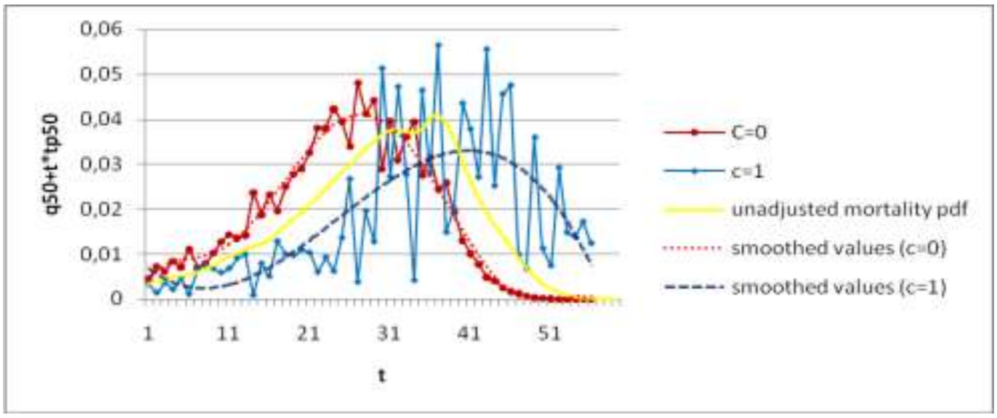

Figure 13: The simulated effects of improvements and worsening on the deferred mortality probability functions (c=1: mortality improving; c=0, mortality deteriorating)

We have simulated 10,000 values of ε_t for each t from a beta distribution, and then we have calculated the 50^{th} and the 95^{th} percentiles of the adjusted mortality pdf and have re-priced the GLWB, taking into account both the living and death benefits. The results for different ages at inception are shown in Figures 14-15. .

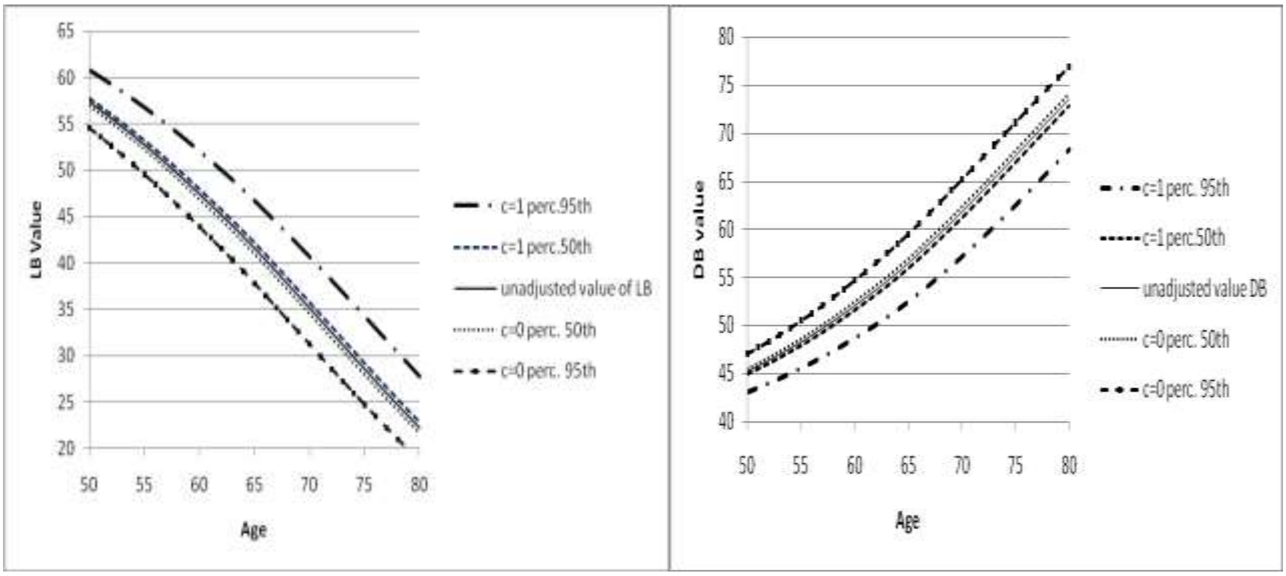

Figure 14: Living benefit (LH) and death benefit (RH) under different mortality hypothesis (c=1: mortality improving; c=0, mortality deteriorating)

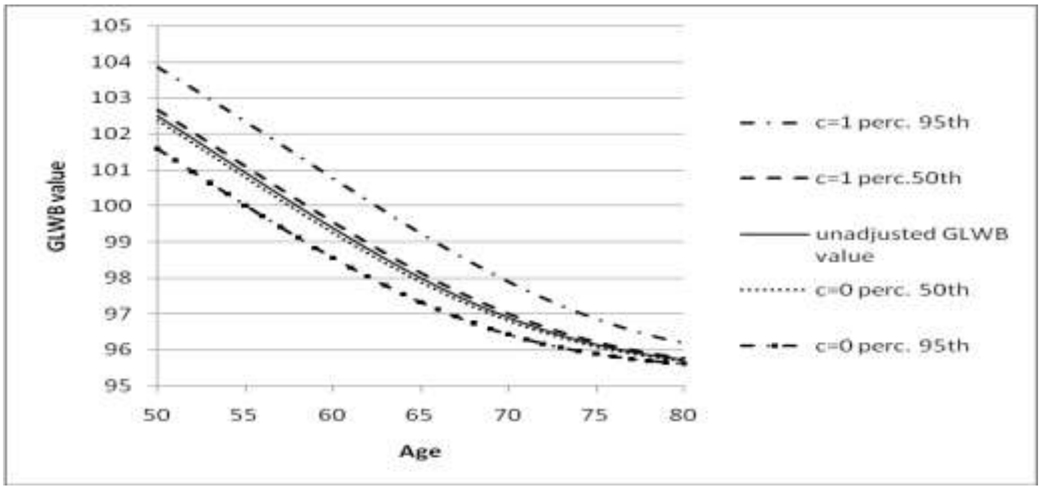

Figure 15: GLWB value under different mortality hypothesis (c=1: mortality improving; c=0, mortality deteriorating)

Under the assumption that c=0, the probability of survival at each date is lower than for the unadjusted probability function; and so, because the number of withdrawals decreases, the adjusted curve of the value of the living benefit lies below that of the unadjusted living benefit as shown in Figure 14. In contrast, under the assumption c=1, the probability of survival at each date is higher than for the unadjusted probability function; and so, the curve for the value of the living benefit lies above that for the unadjusted living benefit. The reverse effect influences the death benefit. Under the assumption of improvements in life expectancy, the death benefit decreases at each age for two reasons: on the one hand, it is received further off in the future and so it has to be discounted for a longer period; on the other hand, as the policyholder lives longer, the number of withdrawals increases and consequently the amount of fund available at the time of death decreases.Overall, the former effect regarding the living benefit prevails over the latter regarding the death benefit and the GLWB value increases as the life expectancy increases. The effect of mortality risk on the GLWB is described in Figure 15, where the simulated option values at the 50^{th} and 95^{th} percentiles are reported under the assumptions c=0 and c=1 . We highlight that, as the age of the policyholder at inception of the contract decreases, the impact of mortality risk on GLWB increases.

6. Conclusion

In this paper, we have described the latest financial innovation introduced in the VA market: the GLWB Option. We have dealt with the problem of valuation of this option and we have developed a theoretical model for its pricing. We have taken a static approach that assumes individual investors behave passively in utilizing the embedded guarantee. This hypothesis is plausible because the insurer can induce the policyholder to adopt this behavior by introducing a penalty charge on the amount of withdrawal that exceeds the guaranteed amount. Moreover, these options are being introduced in pension plans, in order to ensure a constant income during the retirement and provide protection against market downside risk. In the light of these considerations, our aim has not been to offer a policyholder behavioural analysis but rather to provide a conceptual valuation scheme of the product and offer some initial estimates for the fair fee. The model has been illustrated with numerical results and a sensitivity analysis, which have shown how the value of the option varies with the key parameters, including the age of the policyholder at the inception of the contract, the guaranteed rate, the risk free rate and the fund volatility. We have considered the basic form of the product as well as more sophisticated guarantees, such as the roll-up and step-up option. Despite a common belief to the contrary, we have shown that these features do not always make the GLWB more valuable; however, we have highlighted the necessity to evaluate, on a case by case basis, the products according to the values of the key parameters such as the roll-up rate, the deferment period, the frequency of step-up dates, the life expectancy of the policyholder and the evolution of the financial market because of the complex interaction of both financial and demographic factors. Our results are case sensitive and the choice of parameter assumptions is fundamental. Behind a more realistic quantification of impact of different risk sources, our aim has been to offer a general framework for valuating GLWB options.

Owing to the long time horizon of their commitment, GLWB providers are exposed to important financial and mortality risk. In this paper,we have given special attention to the study of the impact of mortality on the value of the GLWB. Useful instruments for controlling mortality risk are the projected mortality tables, which can be produced with different stochastic models. In this regard, we have chosen the Cox-Lin model (2005) as one which includes a stochastic mortality trend as well as shocks. The mortality model that we consider has the potential of both future mortality improvement and deterioration and it is useful to offer a complete description of the potential impact of mortality risk on the product under consideration. In order to understand in which way the GLWB value varies when there is an improvement or a worsening in life expectancy, we have decomposed the product into the sum of the living benefit and the death benefit and have observed the impact of mortality shocks on them. Our results show that the product is sensitive to mortality risk. Since the fluctuation in the GLWB value depends on the interaction of the factors considered in the paper, it is necessary to implement a simulation approach to measure and manage mortality risk. We highlight the importance of the choice of the model and parameter assumptions, while not necessarily trying to identify the most correct mortality model and financial assumption.

In the light of the analysis conducted here, we identify areas where there is scope for further work. One problem left open is the definition of an efficient risk management strategy for the GLWB option. The valuation formula shows that this product is affected by financial risk, due to the changes in the fund value and in the level of interest rates over time, and by mortality risk. The hedging of financial risk is difficult because of the long maturity of these contracts. Also, our study highlights that the mispricing due to neglecting mortality improvements or worsening is noticeable. An integrated analysis of the interaction of demographic and financial risk would be an important step to deliver a more detailed scheme to identify, measure and manage global risks for GLWB providers

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