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Review Paper

Review of Mathematical Models in Performance Calculation of Screw Compressors

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Abstract

The mathematical modelling of screw compressor processes and its implementation in their design began about 30 years ago with the publication of several pioneering papers on this topic, mainly at Purdue Compressor Conferences. This led to the gradual introduction of computer aided design, which, in turn, resulted in huge improvements in these machines, especially in oil-flooded air compressors, where the market is very competitive. A review of progress in such methods is presented in this paper together with their application in successful compressor designs. As a result of their introduction, even small details are now considered significant in efforts to improve performance and reduce costs. Despite this, there are still possibilities to introduce new methods and procedures for improved rotor profiles, design optimisation for each specified duty and specialized compressor design, all of which can lead to a better product and new areas of application. A review of methods and procedures which lead to modern screw compressor practice is presented in this paper. This paper is intended to give a cross section through activities being done in mathematical modelling of screw compressor process through last five decades. It is expected to serve as a basis for further contributions in the area and as a challenge to the forthcoming generations of scientists and engineers to concentrate their efforts in finding future and more extended approaches and submit their contributions.

Keywords: Screw Compressor, Mathematical Model, Performance Calculation

1. Introduction

During the past half a century, for many applications, traditional reciprocating compressors have been displaced by those of the twin screw type. The main reasons for this are the improved rotor profiles, which have reduced internal leakage, and machine tools, which can manufacture the most complex shapes, to tolerances of 3 micrometers, at an acceptable cost.

Although, advances have been made in analytical procedures, which are gradually being adopted by designers to predict compressor performance more reliably, these improved methods of analysis can create, as yet unrealised, opportunities for further improving the performance and reducing the cost of screw machines.

Rotor profile enhancement is still a means of further improving screw compressors and rational procedures are now being developed both to replace earlier shapes and also to vary the proportions of any selected profile to obtain the best result for the application for which the compressor is required. In addition, improved modelling of flow patterns within the machine can lead to better porting design. Also, more accurate determination of bearing loads and how they fluctuate enable better choices of bearings to be made. Finally, if rotor and casing distortion, as a result of temperature and pressure changes within the compressor, can be estimated reliably, machining procedures can be devised to minimise their adverse effects.

Up to date modelling and analytical procedures, now being developed to address all these possibilities, are reviewed here together with examples of how their utilisation has led to improved designs and new applications.

1.1 Manufacturing and use of screw compressors

Screw compressors in normal commercial usage today have main rotors whose outer diameters vary between 75 mm and 620 mm. These deliver between 0.6 m^3/min and 600 m^3/min of compressed gas. The normal pressure ratios attained in a single stage are 3.5:1 for dry compressors and up to 15:1 for oil flooded machines. Normal stage pressure differences are up to 15 bars, but maximum values sometimes exceed 40 bars. Typically, for oil flooded air compression applications, the volumetric efficiency of these machines now exceeds 90% while specific power inputs, which are both size and performance dependent, have been reduced to values which were regarded as unattainable only a few years ago.

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In order to operate effectively a sealing line must be formed between the rotors and between the rotors and the casing. A small gap, the so called "blowhole", occurring between the cusp of the casing and the rotors, extends along the length of the casing to form a path through which the gas being compressed leaks. The aim is to select a rotor profile which maximises the flow while minimising the blowhole area, the sealing line length and the contact forces between the male and female rotors. Although this principle requirement for screw compressors has been known for over 100 years, it is only since the development of the SRM "A" profile by Shibbie, 1973, which met these criteria far better than any of its predecessors, that screw compressors began to be commercially viable.

1.2 Information available in screw compressors

Despite the rapid growth in screw compressor usage, the scientific basis of their design is still limited. Several screw compressor textbooks were published in Russia in the early nineteen sixties. *Sakun 1960* gives a full analysis on rotor profiles based on the envelope method. *Andreev, 1961* gives more information on manufacturing and tooling for screw compressor rotors, while *Golovnitsov, 1964* introduced contemporary procedures in a more general book on rotary compressors. *Amosov et al 1977* in his handbook made a contribution by giving a reproducible presentation of how to generate the SRM asymmetric profile, the Lysholm profile, as well as a presentation of the early Russian SKBK profile. In two German textbooks, *Rinder, 1979* used a profile generation method based on gear theory to reconstruct the SRM asymmetric profile while *Konka, 1988* published some detailed engineering aspects of screw compressors. There are two-three textbooks on screw compressors published in English, *O'Neil, 1993* and *Arbon, 1994*. More recently, *Xing, 2000* published a comprehensive textbook on screw compressors in Chinese. More recently, the authors published two books on screw compressors, *Stosic et al, 2005 and 2006*. A few compressor manufacturers' handbooks on screw compressors and a number of brochures give useful information, but they are either classified or of a very limited accessibility.

Literally thousands of patents have been awarded for screw compressors during the past thirty years and SRM alone has more than 1000. They cover various aspects of screw compressor design but are mainly for rotor profiles. SRM patents *Nilson, 1952* for the symmetric, *Shibbie, 1979*, for the asymmetric, *Astberg, 1982* for the "D" and *Ohman, 1999* for the "G" profiles are examples of state of the art screw compressor profile generation. Other successful profile patents are those of *Bammert, 1979*, *Hough et al, 1984*, *Edstroem, 1974*, *Kasuya et al, 1983* and *Bowman, 1983*. More recently, several successful patents such as *Lee, 1988* and *Chia-Hsing C, 1995* have been granted to relatively small companies. A fresh approach to profile generation based on the use of a rack for the primary curves was published in *Rinder, 1987* and *Stosic, 1996*. Although all patented profiles were generated by well defined procedures, so little was published about the methods on which they were based that it was difficult to reproduce them.

Most of the known characteristics of screw compressors, such as oil flooding, the shaping of the suction and discharge ports to follow the rotor tip helices, axial force compensation, unloading, the slide valve and the economizer port, were also patented, mainly by SRM. However other companies were equally keen to file patents. It appears that, in the field of screw compressors, patent experts are as important as engineers.

Three conferences dominate the screw compressor area, Purdue Compressor Engineering Conference, Dortmund VDI Tagung 'Schraubenmaschinen' and the IMechE Conference on Compressors and their Systems, London. Modern screw compressor practice started with calculation of the compressor process, based on the solution of differential equations derived from the conservation of mass and energy and temperature and pressure relationships derived from equations of state. This was strongly supported in Purdue publications, early examples of which are *Benson et al, 1972, MacLaren et al, 1974* and *Prakash et al, 1974* in reciprocating compressors. In screw compressors, *Fujiwara et al, 1974* and *1984*, *Fukazawa et al, 1980*, as well as *Sangfors, 1982* and *1984*, *Bain et al, 1982*, then *Singh et al, 1984* and *1990, Dagang et al, 1986* and later *Edstroem, 1992* all made contributions. *Stosic et al* introduced a solution of the energy and mass differential equations in their primitive form in *1988*. The Dortmund proceedings contain some interesting screw compressor papers. *Rinder, 1984* presents a rack method of rotor profile generation, based on classical gearing procedure, which is fully reproducible. *Sauls, 1998* gives more details on profiling and manufacturing control. *Kauder and Harling, 1994* showed a typical example of a successful university research applied to solve real engineering problems. *Edstroem 1989* published an interesting paper at the IMechE Conference, which was followed by other papers, such as those of *Venumadhav et al and McCreath et al, 2001* and more recently *Stosic et al, 2005, 2007* and *Delash et al, 2009*.

There are surprisingly few journal publications on screw compressors in the technical literature. An early exceptions was that of *Margolis 1978*. Others followed in the Journal of the International Institution of Refrigeration, such as those of *Stosic et al 1992* and *Fujiwara and Osada, 1995*, and then in the IMechE proceedings by *Tang et al, 1994*, and *Fleming et al 1999* and *Hanjalic and Stosic, 1997* in the ASME Journal of Fluids Engineering. All of them either introduced or contributed to a mathematical model based on solution of the differential equations of mass and internal energy in terms of their primitive variables, which is indeed the backbone of the modern approach to the analysis of screw compressors. This was followed by *Stosic 1998* and more recently, *Stosic 2004*, as well as *Nouri et al, 2007* in various engineering journals. This led to the presentation of more information on screw compressors in journals than in all previous years together.

2. Screw Compressor Process and ITS Mathematical Modelling

Screw compressor combines thermodynamics and flow processes. Both of them are dependent on screw compressor geometry. A combination of all is a prerequisite for calculation of the screw compressor performance. As such, it represents a complex process which may be solved by use of mathematical models, either one or multidimensional.

The here described algorithm of the thermodynamics and flow processes in a screw compressor is based on the mathematical model, represented by a set of equations which describe the physics of the complete process in a compressor. The equation set consists of the conservation equations for energy and mass continuity together with a number of algebraic equations defining various phenomena, which accompany the fluid suction, compression and discharge. The mathematical model employs the differential kinematic relationship which describes the instantaneous operating volume and its change with the rotation angle or time, as well as the equations of conservation of the mass and energy for the adopted control volume. These are applied for each phase of the process that the fluid is subjected to: suction, compression and discharge. With all the described effects being accounted for in differential forms the model does not render itself to an analytical treatment. Various simplifications of the equations that have often been applied in the past in order to achieve a more efficient and economical numerical solution of the equation set, are in this case less significant, and the model allows to observe the consequences of neglecting some of the terms in the equations and to draw meritorious conclusions on the justification of various simplifications.

The solution of the equation set is performed numerically by employing the Runge-Kutta 4th order method, with appropriate initial and boundary conditions.

The model accounts for a number of "real-life" effects which may influence to a large extent the final performances of a real compressor. These provisions ensure a sufficient degree of generality of the model and its suitability for a wider application. The following effects have been accounted for:

- the working fluid in the compressor can be any gas or liquid-gas-mixture of known equation of state and known relations for internal thermal energy and enthalpy, i.e. any ideal or real gas or liquid-gas mixture of known properties,
- the model has a provision to account for heat transfer between the gas and the compressor screws or its casing; the model of heat exchange is specified in an approximate form, but still capable of reproducing reasonably well the real heat transfer effects;
- the model accounts for inevitable leakage of the working medium which can occur in any stage of the process through the clearances between the two rotors or between the rotors and stationary parts of the compresor;
- the model works independently of the specification of compressor geometry, hence any geometry can be specified, or computed by on the basis of specification of basic geometrical parameters;
- any liquid, oil, water, or refrigerant can be injected during any of the compressor process stages: suction, expansion or discharge, for lubrication, sealing or cooling purposes; the model allows the injection to affect all of the processes in the compressor.
- the model accounts also for the gas solubility in the injected fluid;
- the model includes the thermodynamic equations of state and change of state of the fluid and the constitutive relationship which completes and closes the equation set.

Certain assumptions had to be introduced in order to ensure an efficient computation; the assumptions do not impose any specific limitations to the model nor cause a significant departure from reality:

- the fluid flow in the model is assumed to be quasi-one-dimensional
- kinetic energy of the fluid is neglected in comparison with its internal energy
- gas or gas-liquid mixture inflow or outflow through the compressor suction or discharge orts was assumed isentropic,
- the leakage of the fluid through the clearances is assumed to be adiabatic

2.1 Equations governing screw compressor process

The working space (chamber) of a screw machine is a typical example of an open thermodynamic system in which the mass flow varies with time. The control volume is a working chamber together with the suction and discharge plenums, for which the differential equations of conservation laws for energy and mass are written. These are derived using Reynolds Transport Theorem.

A feature of the model is the use of the unsteady flow energy equation to compute the effect of profile modifications on the thermodynamic and flow processes in a screw machine in terms of rotational angle, or time. Internal energy rather than enthalpy is then the derived variable and this was found to be computationally more convenient. This practice was found computationally beneficial in evaluating the properties of real fluids, as compared with the conventional methods using enthalpy as the dependent variable. All the remaining thermodynamic and fluid properties within the machine cycle are derived from it and the computation is carried out through several cycles until the solution converges.

The following forms of the conservation equations have been employed in the model:

The conservation of internal energy:

$$\omega\left(\frac{dU}{d\theta}\right)=\dot{m}_{in}h_{in}-\dot{m}_{out}h_{out}+Q-\omega p\frac{dV}{d\theta}$$

where θ is angle of rotation of the main rotor, h=h(θ) is specific enthalpy, $\dot{m}=\dot{m}(\theta)$ is mass flow rate p=p(θ), fluid pressure in the working chamber control volume, $\dot{Q}=\dot{Q}(\theta)$, heat transfer between the fluid and the compressor surrounding, $\dot{V}=\dot{V}(\theta)$ local volume of the compressor working chamber.

In the above equation the index in denotes inflow and the index out the fluid outflow.

The fluid total inflow enthalpy consists of the following components:

$$\dot{m}_{in} h_{in} = \dot{m}_{suc} h_{suc} + \dot{m}_{l,g} h_{l,g} + \dot{m}_{oil} h_{oil}$$

where indices l,g denote leakage gain suc, suction conditions, and oil denotes oil.

The fluid total outflow enthalpy consists of:

$$\dot{m}_{out} h_{out} = \dot{m}_{dis} h_{dis} + \dot{m}_{l,l} h_{l,l}$$

where indices l,l denote leakage loss and dis denotes the discharge conditions with $\dot{m}_{dis}$ denoting the discharge mass flow rate of the gas contaminated with the oil or other liquid injected.

The right hand side of the energy equation consists of the following terms which are modelled:

- The heat exchange between the fluid and the compressor screw rotors and casing (and through it to the surrounding) due to the difference in temperatures of gas and the casing and rotor surfaces is accounted for by the heat transfer coefficient evaluated from the expression Nu=0.023 $Re^{0.8}$. For the characteristic length in the Reynolds and Nusselt number the difference between the outer and inner diameters of the main rotor was adopted. This may not be the most appropriate dimension for this purpose, but the characteristic length appears in the expression for the heat transfer coefficient with the exponent of 0.2 and has a little influence as long as it remains within the same order of magnitude as other characteristic dimensions of the machine and as long as it characterizes the compressor size. The characteristic velocity for the Re number is computed from the local mass flow and the cross-sectional area. Here the surface over which the heat is exchanged, as well as the wall temperature, depend on the rotation angle θ of the main rotor.
- The energy gain due to the gas inflow into the working volume is represented by the product of the mass intake and its averaged enthalpy. As such the energy inflow varies with the rotating angle. During the suction period gas enters the working volume bringing the averaged gas enthalpy which dominates in the suction chamber. However, during the time when the suction port is closed, a certain amount of the compressed gas leaks into the compressor working chamber through the clearances. The mass of this gas, as well as its enthalpy are determined on the basis of gas leakage equations. The working volume is filled with gas due to leakage only when the gas pressure in the space around the working volume is higher, otherwise there is no leakage, or it is in the opposite direction, i.e. from the working chamber towards other plenums.
- Total inflow enthalpy is further corrected by the amount of enthalpy brought into the working chamber by the injected oil.
- The energy loss due to the gas outflow from the working volume is defined by the product of the mass outflow and its averaged gas enthalpy. During the delivery this is the compressed gas entering the discharge plenum while in case of expansion due to inappropriate discharge pressure, this is the gas which leaks through the clearances from the working volume into the neighbouring space at a lower pressure. If the pressure in the working chamber is lower than one in the discharge chamber and if the discharge is open, the flow will be in reverse direction, i.e. from the discharge plenum into the working chamber. The change of mass has a negative sign and associated enthalpy is equal to the averaged gas enthalpy in the pressure chamber.
- The thermodynamic work supplied to the gas during the compression process is represented by the term $p\frac{dV}{d\theta}$. This term is evaluated from the local pressure and local volume change rate. The latter is obtained from the relationships defining the screw kinematics which yield the instantaneous working volume and its change with rotation angle. In fact the term dV/dφ can be identified with the instantaneous interlobe area, corrected for the captured and overlapping areas.
- If oil or other fluid is injected into the working chamber of the compressor, the oil mass inflow and its enthalpy should be included in the inflow terms. In spite of the fact that oil mass fraction in the mixture is significant, its effect upon the volume flow rate is only marginal because oil volume fraction is usually very small. The total fluid mass outflow includes also the injected oil, the greater part of it remains mixed with the working fluid. Heat transfer between the gas and oil droplets is described by the first order differential equation.

The mass continuity equation

$$\omega \frac{d\dot{m}}{d\theta} = \dot{m}_{in} h_{in} - \dot{m}_{out} h_{out}$$

The mass inflow rate consists of:

$$\dot{m}_{in} = \dot{m}_{suc} + \dot{m}_{l,g} + \dot{m}_{oil}$$

The mass outflow rate consists of:

$$\dot{m}_{out} = \dot{m}_{dis} + \dot{m}_{l,l}$$

Each of the mass flow rates satisfies the continuity equation $\dot{m} = \rho w A$ where w[m/s] denotes fluid velocity, ρ - fluid density and A - the flow cross-section area.

The instantaneous density ρ=ρ(θ) is obtained from the instantaneous mass *m* trapped in the control volume and the size of the corresponding instantaneous volume V as ρ=m/V.

2.2 Flow through admission and discharge ports

The cross-section area A is obtained from the compressor geometry and it was considered as a periodical function of the angle of rotation θ. The suction port area is defined by:

$$A_{suc} = A_{suc,0} \sin\left(\pi \frac{\theta}{\theta_{suc}}\right)$$

where index suc means the starting value of θ in the moment of the suction port opening, and $A_{suc,0}$ denotes the maximum value of the suction port cross-section area. The reference value of the rotation angle θ is assumed at the suction post closing so that suction ends at θ=0, if not specified different.

The discharge port area is likewise defined by:

$$A_{dis} = A_{dis,0} \sin\left(\pi \frac{\theta - \theta_c}{\theta_e - \theta_s}\right)$$

where index e denotes the end of discharge, c denotes the end of compression and $A_{dis,0}$ stands for the maximum value of the discharge port cross-sectional area.

Suction and discharge port fluid velocities

$$w = \mu\sqrt{2(h_2 - h_1)}$$

where μ is the suction/discharge orifice flow coefficient, while indices 1 and 2 denote the conditions downstream and upstream of the considered port. The provision is supplied in the computer code to account for a reverse flow if $h_2 < h_1$.

2.3 Flow through leakage paths

The leakages in a screw machine amount to a substantial part of the total flow rate and play therefore an important role because they influence the process both by affecting the compressor mass flow rate or compressor delivery, i.e. volumetric efficiency and the thermodynamics of the compression work efficiency. For a practical computation of the leakage effects upon the compressor process it is convenient to distinguish two types of the leakages, according to their direction in regard to the working chamber: gain and loss leakages. The gain leakages come from the discharge plenum and from the neighbouring working chamber which has a higher pressure. The loss leakages leave the chamber towards the discharge plenum and to the neighbouring chamber with a lower pressure.

Computation of the leakage velocity follows from the approximate consideration of the fluid flow through the clearance. The process is essentially adiabatic Fanno-flow. In order to simplify the computation, at some stages the conditions of constant temperature, T=const or constant enthalpy, h=const are applied. This departure from the prevailing adiabatic conditions has only a marginal influence if the analysis is carried out in a differential form, i.e. for the small changes of the rotational angle, as followed in the present model. The present model treats only the gas leakage. No attempt is made to account for leakages of a gas-liquid mixture, while the effect of the oil film can be incorporated by an appropriate reduction of the clearance gaps.

An idealized clearance gap is assumed to have a rectangular shape and the mass flow of leaking fluid is expressed by the continuity equation:

$$\dot{m}_l = \mu_l \rho_l w_l A_g$$

where ρ and *w* are density and velocity of the leaking gas, $A_g = l_g\delta_g$ the clearance gap cross-sectional area, l_g leakage clearance length, sealing line, δ_g leakage clearance width or gap, μ=μ(Re,Ma) the leakage flow discharge coefficient.

Four different sealing lines are distinguished in screw compressor: leading tip sealing line formed by the main and gate rotor forward tip and casing, trailing tip sealing line formed by the main and gate reverse tip and casing, front sealing line between the discharge rotor front and the hosing, interlobe sealing line between the rotors.

All sealing lines have appropriate clearance gaps forming leakage areas. Additionally, tip leakage areas are accompanied with the blow-hole areas.

According to the type and position of leakage clearances five different leakages can be identified: losses through the trailing tip sealing and front sealing and gains through the leading and front sealing. The fifth, ‘through-leakage’ does not directly affect the process in the working chamber, but it is passing through it from the discharge plenum towards the suction port.

The computation of the leakage gas velocity follows from the momentum equation, which accounts for the fluid-wall friction:

$$w_l dw_l + \frac{dp}{\rho} + f\frac{w_l^2}{2}\frac{dx}{D_g} = 0$$

where f(Re,Ma) is the friction coefficient which is dependent on Reynolds and Mach numbers, D_g is the effective diameter of the clearance gap, $D_g \approx 2\delta_g$ and dx is the length increment. From the continuity equation and assuming that $T \approx const$ to eliminate gas density in terms of pressure, the equation can be integrated in terms of pressure from the high pressure side at position 2 to the low pressure side at position 1 of the gap to yield:

$$\dot{m}_l = \rho_l w_l A_g = \sqrt{\frac{p_2^2 - p_1^2}{a^2\left(\zeta + 2\ln\frac{p_2}{p_1}\right)}}$$

where $\zeta = f\,L_g/D_g + \Sigma\xi$ characterizes the leakage flow resistance, with L_g clearance length in theleaking flow direction, f friction factor and local resistance coefficient. ζ can be evaluated for each clearance gap as a function of its dimensions and shape and flow characteristics. a is a speed of sound.
The full procedure requires the model to include the friction and drag coefficients in terms of Reynolds and Mach numbers for each type of clearance.

Likewise, the working fluid friction losses can also be defined terms of the local friction factor and fluid velocity related to the tip speed, density, and elementary friction area. At present the model employs the value of ζ in terms of a simple function for each particular compressor type and use it as an input parameter.

These equations are incorporated into the model of the compressor and employed to compute the leakage flow rate for each clearance gap at the local rotation angle θ.

2.4 Injection of oil and other liquids

Injection of oil or other liquids for lubrication, cooling or sealing purposes, modifies substantially the thermodynamic process in a screw compressor. The following paragraph outlines the procedure for accounting for the effects of oil injection. The same procedure can be applied to treat the injection of any other liquid. Special effects, such as gas or its condensate mixing and resolving in the injected fluid or vice versa should be accounted for separately if they are expected to affect the process. A procedure for incorporating these phenomena into the model will be outlined later.

A convenient parameter to define the injected oil mass flow is the oil-to-gas mass ratio, m_{oil}/m_{gas}, from which the oil inflow through the open oil port which is assumed to be uniformly distributed, can be evaluated as

$$\dot{m}_{oil} = \frac{\dot{m}_{oil}}{\dot{m}_{gas}}\dot{m}\frac{z_1}{2\pi}$$

where the oil-to-gas mass ratio is specified in advance as an input parameter.

In addition to lubrication, major purpose for injecting oil into a compressor is the cooling of the gas. To enhance the cooling efficiency the oil is atomized into a spray of fine droplets by which the contact surface between the gas and oil is increased. The atomization is performed by using specially designed nozzles or by a simple high-pressure injection. A distribution of droplets sizes can be defined in terms of oil-gas mass flow and velocity ratio for a given oil-injection system. Further destiny of each distinct class of the oil droplets, until they hit the rotor or casing wall can be followed by solving the dynamic equation for each droplet size in Lagrangian frame, accounting for inertial, gravity, drag, and other forces. The solution of droplet energy equation in parallel with the momentum equation should yield the amount of heat exchange with the surrounding gas.

In the present model we follow a simpler procedure in which the heat exchange with the gas was determined from the differential equation for the instantaneous heat transfer between the surrounding gas and an oil droplet. Assuming that the droplets retain a spherical form, with a prescribed Sauter mean droplet diameter d_S, the. Heat transfer between the droplet and the gas can be expressed in terms of a simple cooling law $Q_o = h_o A_o\left(T_{gas} - T_{oil}\right)$, where A_o is the droplet surface, $A_o = d_S^2\,\pi$, d_S is the mean Sauter diameter of the droplet and h_o is the heat transfer coefficient on the droplet surface, determined from the empirical expression $Nu=2+0.6Re^{0.6}\,Pr^{0.33}$ The exchanged heat must balance the rate of change of heat taken or given away by the droplet in the unit time, $Q_o=m_o c_{oil} dT_o/dt = m_o c_{oil}\omega dT_o/d\theta$, where c_{oil} is the oil specific heat and the index o denotes oil droplet. The rate of change of oil droplet temperature can be expressed now as:

$$\frac{dT_o}{d\theta} = \frac{h_o A_o\left(T_{gas} - T_o\right)}{\omega m_o c_{oil}}$$

The heat transfer coefficient h_o is obtained from:

$$Nu = 2 + 0.6\ Re^{0.6}\ Pr^{0.33}$$

A integration of the equation in the two time/angle steps yields the new oil droplet temperature at each new time/angle step:

$$T_o = \frac{T_{gas} - kT_{o,p}}{1+k}$$

where $T_{o,p}$ is the oil droplet temperature at the previous time step and k is the non-dimensional time constant of the droplet, $k=\tau/\Delta t = \omega\tau/\Delta\theta$, with $\tau = m_o c_{oil}/h_o A_o$ being the real time constant of the droplet. For the given mean Sauter diameter d_S the non-dimensional time constant takes a form

$$k = \frac{\omega m_o c_{oil}}{h_o A_o \Delta\theta} = \frac{\omega d_S c_{oil}}{6 h_o \Delta\theta}$$

The obtained droplet temperature is further assumed to represent the average temperature of the oil, i.e. $T_{oil} \approx T_o$, which is further used to compute the enthalpy of the gas-oil mixture.

The above described approach is based on the assumption that the oil-droplet time constant τ is smaller than the droplet travelling time through the gas before it hits the rotor or casing wall, or reaches the compressor discharge port. This means that the heat exchange is completed within the droplet travelling time through the gas during the compression. This prerequisite is fulfilled by appropriate atomization of the injected oil which produces sufficiently small droplet sizes, which gives a small droplet time constant, as well as by choosing adequate nozzle angle, and, to some extent, the initial oil spray velocity. A separate computation of the droplet trajectory on the basis of the solution of droplet momentum equation for different droplet mean diameters and initial velocities, for more details refer *Stosic et al., 1992* indicate that for most screw compressor currently in use, except, perhaps for the smallest ones, with typical tip speed velocities between 20 and 50 m/s, this condition is well satisfied for oil droplets with diameters below 50 μm.

Because the inclusion of a complete model of droplet dynamics would unnecessary complicate the computer code and the outcome will always be dependant on the design and angle of the oil injection nozzle, the present computation code uses the above described simplified approach which was found to be fully satisfactory for a range of different compressors. The input parameter is only the mean Sauter diameter of the oil droplets, d_S and oil properties - density, viscosity and specific heat.

2.5 Solution Procedure for Compressor Thermodynamics

To summarize, the description of the thermodynamic processes in a screw machine is completed by the differential equations for the lobe volumes, which define $V(\theta)$ and $dV/d\theta$, by the differential equation of the internal thermal energy, and by the differential equations describing the working chamber mass balance. Boundary pressures and temperatures in the suction and discharge chambers are known. In addition the algebraic equations of state and specific internal energy and specific enthalpy, are sufficient to obtain the mass flows through the suction and discharge ports and through the clearances, the mass in the working chamber, the pressure and temperature of the fluid in the working chamber and the mass and the temperature of the injected oil.

If the fluid states described by the pressure and temperature in the pressure and suction plenums are considered to be variable with the rotation angle, it is necessary to couple the differential equations for energy and mass flow rates. The total number of differential equations is increased now by another two for each plenum.

All the differential equations are solved by means of the Runge-Kutta fourth order procedure. As the initial conditions were arbitrary selected, the convergence of the solution is achieved by applying cyclic boundary conditions when the difference between the two consecutive compressor cycles reached a sufficiently small monitoring value prescribed in advance.

The instantaneous bulk density ρ is obtained from the instantaneous mass trapped in the control volume and the size of the corresponding instantaneous volume V as $\rho=m/V$.

The equations of energy and continuity are solved to obtain $U(\theta)$ and $m(\theta)$. Together with $V(\theta)$, the specific internal energy and specific volume $u=U/m$ and $v=V/m$ are now known. We can then calculate T and p, or x.

For the ideal gas, the internal thermal energy of the gas-oil mixture is given by:

$$U = (mu)_{gas} + (mu)_{oil} = \frac{mRT_{gas}}{\gamma - 1} + (mcT)_{oil} = \frac{pV}{\gamma - 1} + (mcT)_{oil}$$

where R is the gas constant and γ is adiabatic exponent.

Hence, the pressure or temperature of the fluid in the compressor working chamber can be explicitly calculated by help of the equation for the oil temperature T_{oil}:

$$T = (\gamma - 1)\frac{(1+k)U - (mcT)_{oil}}{(1+k)mR + (mc)_{oil}}$$

If k tends 0, i.e. for high heat transfer coefficients or small oil droplet size, the oil temperature approaches fast the gas temperature. In this case T and p are calculated explicitly.

For the case of a real gas the situation is more complex, because the temperature and pressure can not be calculated explicitly. However, since the internal energy can be expressed as a function of the temperature and specific volume only, the calculation procedure can be simplified by employing the internal energy as a dependent variable instead of enthalpy, as often is the practice. The equation of state $p=f_1(T,V)$ and the equation for specific internal energy $u=f_2(T,V)$ are usually decoupled. Hence, the temperature can be calculated from the known specific internal energy and the specific volume obtained from the solution of differential equations, whereas the pressure can be calculated explicitly from the temperature and the specific volume by means of the equation of state.

These equations are usually uncoupled, with T obtained by numerical solution of the equation set, where p is obtained explicitly from the equation of state.

In the case of a phase change for a wet vapour during the compression or expansion process, the specific internal energy and volume of the liquid-gas mixture are:

$$u = (1-x)u_f + xu_g \qquad v = (1-x)v_f + xv_g$$

where u_f, u_g, v_f and v_g are the specific internal energy and volume of liquid and gas and they are functions of saturation temperature only. The equations require an implicit numerical procedure which is usually incorporated in property packages. As a result, temperature T and dryness fraction x are obtained. These equations are in the same form for any kind of fluid, and they are essentially simpler than any others in derived form. In addition, the inclusion of any additional phenomena into the differential equations of internal energy and continuity is straightforward.

2.6 Calculation of thermodynamic properties of working fluids

Thermodynamic properties of pure fluids and their mixtures are obtained by use of appropriate property equations incorporated into the related software. These are IIR (International Institution of Refrigeration) Routines and the own THERPROP data bank and subroutine package developed at City University London, as well as NIST (National Institute of Standards) property routines. The thermodynamic properties of non-polar and weakly polar fluids are estimated with the Lee-Kesler vapour pressure equation and equation of state. Polar fluid properties were estimated with the Martin-Hou vapour equation with suitable liquid phase correlations together with the Cox-Antoine vapour equation. These methods are highly accurate and revealed maximum differences of the order of ±1% in locally computed property values when compared with independent calculations carried out by other investigators using alternative procedures such as the Starling version of the Benedict Webb and Rubin equation.

2.7 Calculation of compressor performance parameters

Numerical solution of the mathematical model of the physical process in the compressor provides a basis for a more exact computation of all desired integral (bulk) characteristics with a satisfactory degree of accuracy and in that respect is superior as compared to the more empirical integral approach. The most important of these properties are the compressor mass flow rate $\dot{m}$ [kg/s], the indicated power P_{ind} [kW], specific indicated power P_s [kJ/kg], volumetric efficiency η_v, adiabatic efficiency η_a, isothermal efficiency η_t and other efficiencies, and the power utilization coefficient, indicated efficiency η_i.

The instant fluid mass trapped in the working volume is determined as a difference between the total fluid mass inflow and outflow:

$$m = m_{in} - m_{out}$$

where m_{in} and m_{out} are obtained from the integration of the corresponding differential equations over the cycle. During the rotation of the compressor shaft, due to the different shaft speed only the number of working volumes of the main screw lobes z_1 contribute towards the process, so that the total mass participating is m z_1. Hence the actual fluid mass flow $\dot{m}$ [kg/s] is

$$\dot{m} = mz_1 n / 60$$

where n is the number of revolution per minute of the main rotor.

The volume delivery $\dot{V}$ is defined with reference to the suction conditions and is usually expressed in [m^3/min]:

$$\dot{V} = 60m / \rho_0$$

From the known maximum volume of the working chamber, the theoretical mass flow is:

$$\dot{m}_t = \frac{\left(F_{1n} + F_{2n}\right) L n z_1 \rho}{60}$$

where F_{1n} and F_{2n} are the cross-section areas of the lobes in the front plane of the main screw and the gate screw, respectively, and L is the length of the screw. The volumetric efficiency is readily obtained as:

$$\eta_v = \frac{\dot{m}}{\dot{m}_t}$$

It is worth noting that effects of leakage, gas heat exchange, the gas retention in the pockets on the pressure side of asymmetric lobe profiles, are all included in the volumetric coefficient through the differential treatment of the governing equations of the mathematical model.

The indicated work transferred to the screw rotors during the suction, expansion and discharge processes is represented by the area of the indicated p-V diagram.

$$W_{ind} = \int_{cycle} V dp$$

Within the indicated work, flow loses during the suction, expansion and discharge, leakages and heat exchange, as well as the influence of injected oil have been included in the same way into the differential equations of the model.

The indicated work in a single compressor working chambers is further used for the computation of the compressor indicated power:

$$P_{ind} = \frac{W_{ind} z_1 n}{60}$$

In addition to the indicated power, it is useful to know the specific indicated power:

$$W_{sind} = \int_{cycle} \frac{V}{m} dp$$

where m is the mass of the fluid contained in the working chamber V in the considered instant of time.

The indicated work can be compared with the theoretical adiabatic or isothermal work to yield the corresponding efficiency:

$$\eta_t = \frac{W_t}{W_{ind}} \qquad \eta_a = \frac{W_a}{W_{ind}}$$

Here the theoretical isothermal and adiabatic works are determined from the common theoretical expressions. For an ideal gas, the theoretical isothermal and adiabatic works are respectively:

$$W_t = RT_1 \ln \frac{p_2}{p_1} \qquad W_a = \frac{\gamma}{\gamma - 1} R\left(T_2 - T_1\right)$$

where 1 denotes the beginning, and 2 the end conditions of the compression process.

Specific indicated power is obtained from the known indicated power and delivery:

$$P_{\sin d} = \frac{P}{\dot{V}}$$

2.8 Calculation of pressure forces acting on screw compressor rotors

Screw compressor rotors are subjected to severe pressure loads. The rotors, as well as rotor bearings must satisfy rigidity and elasticity requirements to ensure appropriate and reliable compressor operation.

In the position 1, there is no contact between rotors. Since A and B are on the circle, overall forces F_1 and F_2 act towards rotor axes and they are radial forces only. There is no torque caused by pressure forces in this position.

In the position 2, there is only one contact between rotors, point A. Forces F_1 and F_2 are eccentric, they comprise two components, radial and circumferential ones. The latter cause the torque. Due to the force position, the torque on the gate rotor is significantly smaller then the main rotor torque.

In order to explain a calculation of the pressure loads, several cases are considered. Let the pressure p(θ) be known for any instantaneous angle of rotation θ, with a reasonable angle increment, say 1 degree. The figure presents the radial and torque forces in a rotor cross section. The example is given for 5/6 "N" rotors. The pressure p acts in the corresponding interlobes normally to line AB. A and B are either on the sealing line between rotors or on the rotor tips. Since they belong to the sealing line, they are fully defined from the rotor geometry.

In the position 3, both contact points are on the rotors, the overall and radial forces are equal for both rotors. Similarly to the previous case, they also cause the torque.

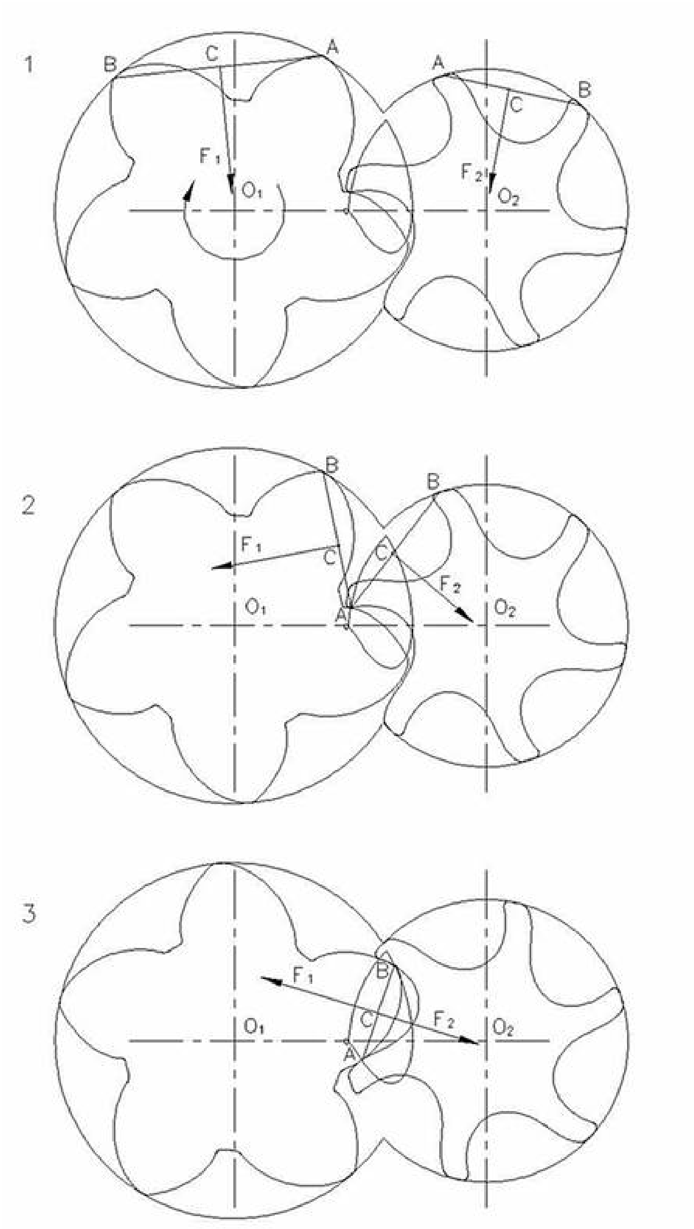

Fig. 1 Pressure forces acting on screw compressor rotors

2.9 Radial, axial rotor loads, torque and bearing reactions

Let x direction be parallel to the line between rotor axes O_1 and O_2 y is perpendicular to x. Radial force components are:

$$R_x = -p\int_A^B dy = -p\left(y_B - y_A\right)$$

$$R_y = -p\int_A^B dx = -p\left(x_B - x_A\right)$$

Torque is:

$$T = p\int_A^B x dx + p\int_A^B y dy - 0.5p\left(x_B^{\,2} - x_A^{\,2} + y_B^{\,2} - y_A^{\,2}\right)$$

The above equations are integrated along the profile for all profile points. Then they are integrated for all angle steps to complete one revolution employing a given pressure history p=p(θ). Finally, a sum for all rotor interlobes is made taking into account the phase shift, as well as axial shift between the interlobes.

Since the gate rotor has a larger lead angle then the main one, proportional to the gearing ratio z_2/z_1, where z are rotor lobe numbers, appropriate summation usually leads to the larger radial forces of the gate rotor despite the gate rotor may be smaller then the main one.

The axial force is a product of the pressure and interlobe cross section. In some regions the interlobes overlap each other. The gate rotor covers a small part of the main rotor interlobe, while the main rotor covers a majority of the gate interlobe. This phenomenon causes that the main axial force is disproportionally larger than the gate one. A correction is allowed for axial forces which takes into account a fact that the pressure in rotor front gaps also acts in axial direction by using an average of pressures in two neighbouring interlobes to act on the lobe in question.

Rotor axial forces act like to minimize the discharge bearing radial force increasing the suction bearing forces. This is generally a convenience, because the suction bearing forces are usually smaller than the discharge ones. Bearing in mind that the main rotor axial force is larger than the gate one, this effect is more beneficial for the main rotor.

Since the radial force R and its axial position z_1 are calculated in advance for every rotation angle θ, δ calculated from the expression above is also a function of the rotation angle.

2.10 Rotor deflections

If the rotors are loaded by radial force R with the bearing reactions R_D and R_S on the discharge and suction rotor sides over a span z_2. A rotor elastic line function is given by differential equation:

$$\frac{d^2\delta}{dz^2} = \frac{M}{EI}$$

where δ=δ(z) is a rotor bending deflection, while M is a bending moment function, E is a elasticity module and I is a rotor polar momet of inertia, calculated from the rotor geometry, by means of numerical integration.

Integration of the above equation over the rotor span between two radial bearings gives bending deflections in function of the rotor axial coordinate z which has its own maximum value. This is calculated for every increment of the rotation angle.

3. Recent Advances in Screw Compressor Development

Mathematical modelling facilitated a more accurate calculation of the compressor performance and effects of different parameters influencing the process may be accounted for. One of these allows tight clearances are achievable nowadays, thus internal compressor leakage rates have become small. Hence, further improvements in screw compressor design are possible only by the introduction of more refined analytical principles.

The main requirement is to improve the rotor profiles so that the internal flow area through the compressor is maximised while the leakage path is minimised and internal friction due to relative motion between the contacting rotor surfaces is made as small as possible. This is achieved through several steps described in this chapter.

3.1 Proper choice of rotor configuration

Increasing the number of rotor lobes enables the same built-in volume ratio to be attained with larger discharge ports. Larger discharge ports decrease the discharge velocity and therefore reduce the discharge pressure losses, thereby increasing the compressor overall efficiency. Hence screw compressors tend to be built with more lobes than the traditional 4-6 combination and 5-6 and 6-7 configurations are becoming increasingly popular. Also, the greater the number of lobes, the smaller the pressure difference between the two neighbouring working chambers. Thus, interlobe leakage losses are reduced. Furthermore, more lobes combined with a large wrap angle ensure multiple rotor contacts which reduce vibrations and thus minimize noise. However, more lobes usually mean less rotor throughput, which implies that rotors with more lobes are somewhat larger than their counterparts with fewer lobes. Also the leakage to delivery ratio is worse with more rotor lobes. Therefore, such compressors are less efficient. Additionally, more lobes increase the manufacturing cost.

3.2 Considerations of rotor sealing line length and blow-hole Area

Since screw compressors tend to rotate relatively slowly, rotor profiles must have the smallest possible blow-hole area if leakages are to be minimised. However, reduction of the blow-hole area is associated with increase in the sealing line length. It is therefore necessary to find the optimum profile shape which minimises the sum of both the blow-hole and sealing line leakage areas.

3.3 Rotor proportions

A general feature of screw compressors is that the pressure difference through them causes high rotor loads and this is especially the case for low temperature refrigeration compressors, where these are large. Therefore, to maintain their rigidity and minimise deflection, rotor profiles usually have a relatively small male rotor addendum in order to increase the female root diameter. This sometimes leads to very shallow and clumsy rotors. An alternative possibility is to increase the female rotor lobe thickness. This greatly increases its moment of inertia and thereby reduces the rotor deflection more effectively.

3.4 Choice of rotor wrap angle

Increasing the rotor wrap angle is generally associated with reducing the interlobe sealing line and hence, with reduced leakage between the rotors. Contemporary trends in screw compressor design are therefore towards larger wrap angles. However, on occasion, this has led to exceeding the limiting values and thereby reducing the compressor displacement.

3.5 Progress in compressor bearings

In some compressor designs, multiple cylinder roller bearings or multipoint ball bearings are located at the high pressure end of the rotors to withstand the large radial forces reliably over a long operating life, for example, *Meyers, 1997*. Frequently, two bearings are also employed for axial loads. Since only one axial bearing actually takes the load, the role of the other is mainly to prevent rotor bounce in the axial direction.

3.6 Rotor clearance distribution and contact at the flat lobe side

Oil flooded compressors have direct contact between their rotors. In well designed rotors, the clearance distribution will be set so that this is first made along their, so called, contact bands, which are positioned close to the rotor pitch circles. Since the relative motion between the contacting lobes in this region is almost pure rolling, the danger of their seizing, as a result of sliding contact, is thereby minimised. The traditional approach is to maintain a high, so called, positive gate rotor torque, which ensures round flank contact, *Edstroem, 1992*. What is not widely appreciated is that there are significant advantages to be gained by maintaining a negative gate rotor torque to ensure that contact, when it occurs, will be on the flat lobe face. The reason for this can be understood by examination of the sealing line lengths that for the flat flank is much longer than that of the round flank. Thus, minimising the clearance on the flat flank will reduce the interlobe leakage more than minimising the round flank clearance. Also, negative gate torque is achieved by making the gate rotor lobes thicker and the main rotor lobes correspondingly thinner. The displacement is thereby increased. Thus both these effects lead to higher compressor flows and efficiencies.

3.7 Account of thermal expansion of the rotors and housing

Although the temperature range over which screw compressors operate is not large, the effects of thermal expansion are highly significant if the small clearances required between the rotors and between the rotors and the housing are to be maintained under working conditions. Thus, the rotor clearances obtained under manufacturing conditions must be estimated while taking account of thermal distortion that will occur when the compressor reaches its operating temperature and pressure and calculation must allow for unequal expansion of the rotors in different coordinate directions. An example of this is given in Fig. 1, where the left diagram shows the estimated clearance distribution when the rotors are cold, while, the centre and right diagrams show the clearances after the rotors reach their working temperatures. Additional information about screw compressor clearance management and other means of improving efficiency may be found in *Stosic et al, 2004*.

3.8 Introduction of bearing centre displacement

One additional design aspect, which though important, is not widely appreciated is that the pressure loads will tend to push the rotors apart from their design position in the casing, as a result of the clearances within the bearings. If these are not taken fully into account, the resulting displacement will cause contact between the rotor tips and the casing, when the rotor clearances are small and the pressure loads are high. To counter this, the bearing centre distance must be smaller than that of the rotor housing bores. To maintain the rotor interlobe clearance as small as possible, the bearing centre distance must be even further reduced.

Also, if the bearing centres are set to be the same as those of the rotors, the clearance between the rotors and housing will be smaller at the low pressure side of the rotors and larger at the high pressure side. Since leakage is caused by the pressure difference, this displacement creates the least favourable rotor position for efficient compressor operation. The bearing centre distances must therefore be arranged to maintain a uniform clearance between the rotors and the housing.

3.9 Optimisaton of the compressor process

Analysis of compressor behaviour shows that there are conflicting requirements for desirable machine characteristics. This implies that only simultaneous optimisation of all the variables involved in the design process will lead to the best possible compressor performance. A full multivariable optimisation of screw compressor geometry and operating conditions should be performed to establish the most efficient compressor design for a given duty. This can be achieved by the use of a computer software package, based on a Box constraint simplex method, which provides the general specification of the rotor and compressor characteristics in terms of several key parameters and which can generate various rotor and compressor shapes. For example, see *Stosic et al, 2003*.

4. Examples of Compressor Designs

Many screw compressor manufacturers have followed the more up to date practices described. Eight examples of these are taken from the recent publications, in which some or all of the features described in the previous chapter have been taken into account. In addition, three projects are presented in this paper for the first time.

4.1 Rotor Retrofit for Efficient Screw Compressors

Since the market for oil-flooded screw air and refrigeration compressors is highly competitive, new designs are continually being introduced which are more efficient and cost effective than their predecessors. However, because of the high cost of development of new machines, manufacturers seek to maintain their existing designs for as long as possible. Closer study of many of the older designs has shown that in the majority of cases, all that is required to bring them up to date is to change the rotor profile to one of a more recent type. An example of this is given by *Stosic et al, 2000,* which describes the retrofit of new rotors into an existing family of oil-flooded compressors instead of A rotors. The old and new rotors are compared in Fig. 3.

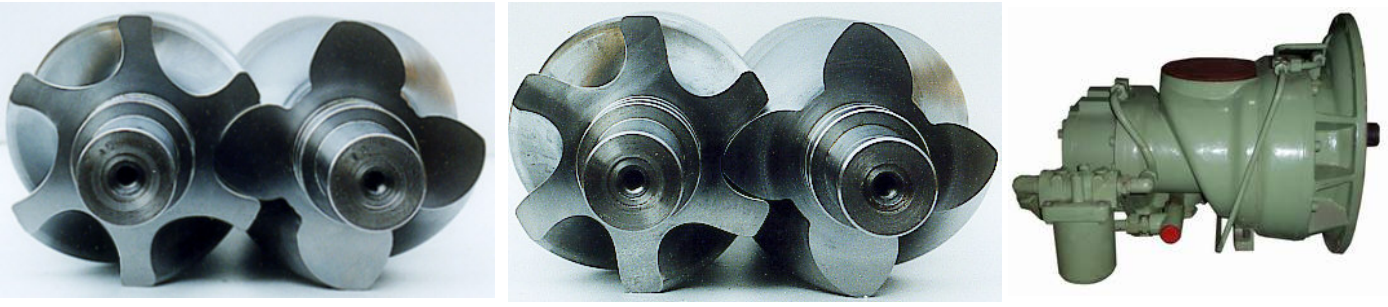

Fig. 2 Rotor retrofit in oil-flooded screw compressors, old rotors left, new rotors, right, Stosic et al, 2000

More recently, SRM 'A' rotors were replaced by new rotors in classic open refrigeration compressors, as described by *Zhang et al, 2006*. A similar exercise where the symmetric externally synchronized 4/6 rotors in a semihermetic refrigeration compressor were replaced by new rotors in direct contact, was described in detail by *Delash et al, 2009*.

4.2 Screw Compressor for Delivery of Dry Air

McCreath et al, 2001 published a paper which describes two high efficiency oil-free screw compressors designed for dry air delivery. Their design is based on rack generated 3/5 rotor profiles, shown in Fig. 2.

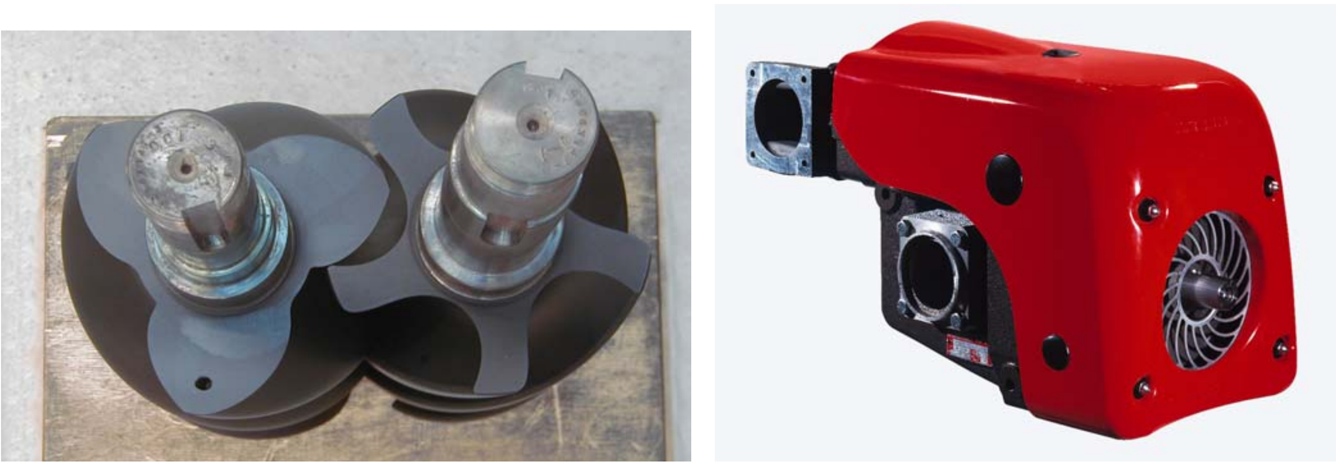

Fig. 3 Rotor profile for delivery of dry air, McCreath et al, 2001

4.3 Design of Oil-Flooded Air Compressors

The design of a family of efficient oil-flooded twin screw air compressors is described by *Venumadhav et al, 2001*. Rack generated rotors with a 4/5 configuration were applied to 5 screw compressors of 73, 102, 159, 225 and 284 mm rotor diameter, respectively, to cover air delivery of 0.6 to 60 m^3/min at delivery pressures between 5 and 13 bar. The compressor family is being gradually introduced by manufacturing prototypes, pre-production compressors and finally, production units. The compressor prototypes tests showed that the volumetric and adiabatic efficiencies of the prototypes were high when compared to published data on the best compressors currently manufactured.

The principles of modern screw compressor practice described in the previous chapter were used in design of a family of two-stage oil flooded screw compressors, Fig. 3. The measurements preformed on the family confirmed the highest efficiency ever reported in the open literature.

Fig. 4 Two-stage oil flooded screw compressor plant

Fig. 5 Oil flooded air end

Later Stosic et al, 2006 published a design of an oil flooded compressor which exhibited the highest efficiency of a single stage oil flooded screw compressor. More recently, the principles of modern compressor practice were applied to an air screw compressor presented in Fig. 4.

4.4 Design of Refrigeration Compressors

Early works in refrigeration compressors resulted in a design presented in Fig. 5, Zhang et al 2006, later Broglia et al 2006 published their work, Fig. 6 and modern practice is applied for efficiency improvements in the refrigeration compressors presented in Fig. 7.

Fig. 6 Refrigeration compressor, Zhang et al, 2006

Fig. 7 Refrigeration compressor, Broglia, 2007

Fig. 8 Refrigeration compressor

5. Acknowledgement

The authors wish to thank all the companies who contributed to the projects published in the papers cited in the previous chapter and additionally to Gardner Denver, Quincy, IL U.S.A, Rotorcomp, Munich Germany and Bitzer, Rottenburg Ergenzingen, Germany for their contribution to the projects publicized in this paper for the first time. Material presented in the paper is used as lecture notes at postgraduate courses at City University London. More details in mathematical modelling of screw compressors are given in *Stosic et al, 2005.*

6. Conclusions

Although the screw compressor is now a well developed product, greater involvement of engineering science in the form of computer modelling and mathematical analysis at the design stage, makes further improvements in efficiency and reduction in size and cost possible. Also, advances in bearing technology and lubrication, must continually be included to obtain the best results. Let the material presented in this review paper, which gives a cross section through activities being done in mathematical modelling of screw compressor process through last five decades, serve as a starting point for further contributions and as a challenge to the forthcoming generations of scientists and engineers to concentrate their efforts in finding better and more efficient approaches to mathematical modelling and to submit their contributions.

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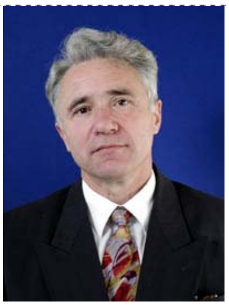

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