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Aeroelastic stability analysis of high aspect ratio aircraft wings

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Abstract

Free vibration and flutter analyses of two types of high aspect ratio aircraft wings are presented. The wing is idealised as an assembly of bending-torsion coupled beams using the dynamic stiffness method leading to a nonlinear eigenvalue problem. This problem is solved using the Wattrick-Williams algorithm yielding natural frequencies and mode shapes. The flutter analysis is carried out using the normal mode method in conjunction with generalised coordinates and two-dimensional unsteady aerodynamic theory of Theodorsen. This is essentially a complex eigenvalue problem in terms of both air-speed and frequency. The flutter determinant is solved by an iterative procedure covering a wide range of air-speeds and frequencies. The computed natural frequencies, mode shapes, flutter speeds and flutter frequencies are compared and contrasted for the two type of aircraft wings and some conclusions are drawn.

1 Introduction

Fluid-structure interaction is an important consideration in the design of many engineering systems, especially when the structure is comparatively slender, such as aircraft wings, bridges and tall buildings. The nonlinearities generated by fluid flow and structure can have serious implications and may require special treatments (e.g., see [1] and [2]). The problem can be more severe for dynamical systems for which the interaction between fluid and structure may assume more complex and nonlinear forms. One classical example of such problems is flutter of an aircraft wing which involves interaction between aerodynamic, inertia and elastic forces. Flutter is probably the most important of all aeroelastic problems particularly for high aspect ratio slender wings. Justifiably, it is an important aspect in aeronautical design and is the central theme of this research. In the context of present paper, aeroelastic stability essentially means flutter stability.

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In general, for an aircraft with high aspect ratio, the first several free vibrational modes are in essence those of the wing because the natural frequencies of the fuselage and tailplane are usually much higher. Therefore, the free vibration and flutter analyses of wings are generally given precedence over other parts of the aircraft such as the fuselage and the tailplane. To this end, designers often use cantilever boundary condition of the wing to obtain reasonably accurate results.

The aeroelastic stability analyses of aircraft wings are generally carried out using the finite element method (FEM), which is no-doubt a versatile structural analysis tool. However, the FEM is computationally expensive and sometimes inadequate particularly at high frequencies and also when greater accuracies are required. Therefore, the FEM is not so suitable at the conceptual design stage when repetitive computation of flutter speeds and flutter frequencies is necessary by varying a wide range wing parameters.

In this paper, an in-house code called CALFUN (Calculation of Flutter Speed Using Normal Modes) is used to carry out the free vibration and flutter analyses of wings belonging to two sailplanes and two transport airliners. CALFUN (Banerjee [3]; Banerjee [4]; Lillico et al. [5]; Banerjee et al. [6]) is a FORTRAN program with the first version reported as early as in 1984 (Banerjee [3]), which provided both the free vibration and flutter analysis of a wide range of high aspect ratio, slender structures using normal modes and unsteady aerodynamics in two dimensional flow. The main advantages of CALFUN over conventional finite element codes (Lottati [7]; Neill et al. [8]; Bartholomew and Wellen [9]; Guo et al. [10]; Li et al. [11]) stem from its computational efficiency and uncompromising accuracy due to the application of the dynamic stiffness method when solving the nonlinear eigenvalue problem to obtain the natural frequencies and mode shapes. This is particularly useful in optimisation studies where repetitive sensitivity analysis with respect to various wing parameters becomes essential. For the free vibration analysis, the FEM is an approximate method based on assumed shape functions, and the method sometimes introduces considerable errors in modal analysis, particularly at high frequencies. By contrast, the dynamic stiffness method (DSM), which is similar and analogous to FEM but based on the exact shape functions obtained from the closed form analytical solution of the structural element in free vibration, is always accurate. The results from DSM are therefore called exact because they are independent of the number of elements used in the analysis. CALFUN is DSM based and it can carry out both the free vibration and flutter analysis of either the wing or the whole aircraft configuration.

It is worth noting that the flutter determinant of an aircraft wing is formulated within CALFUN as a complex-eigenvalue problem, where the flutter determinant is a highly nonlinear function of air-speed and frequency amongst other parameters. In order to search for the flutter point, an appropriate solution strategy is devised within CALFUN to guarantee that the flutter point is located accurately to identify the flutter speed and the flutter frequency.

The program CALFUN has undergone considerable changes and improvements over the years and one of the significant current features is that it can handle the whole aircraft configuration while complimentary to this, CALFUN can analyse a metallic (Banerjee [3]; Banerjee [12]) as well as a composite aircraft (Banerjee [6]). Thus, CALFUN has been successfully used in solving the aeroelastic optimisation problems of composite wings (Lillico et al. [5]). For the purpose of present study, only the wing structure with cantilever boundary condition is investigated with the help of an earlier version of CALFUN.

Against the above background, the main aim of this investigation is to provide both the free vibration and flutter results of two different categories of aircraft, of which one is sailplane and the other is transport airliner. It is important to note that the mass and stiffness properties of these two categories of aircraft, namely, the sailplane and the transport airliner are markedly different although both can be classified as high aspect ratio aircraft. For each category, two aircraft are chosen to serve as examples. This investigation enables a detailed insight that can be gained from the results computed for the two categories of aircraft with contrasting data. This work is expected to pave the way for further research into the aeroelastic behaviour of high aspect ratio wings including response and optimisation studies.

2 Method of Analysis

2.1 The free vibration analysis

In an aircraft, the wings are principal load-carrying structures which provide the necessary lift for the air vehicle (Bisplinghoff et al. [13]). For sailplanes and transport airliners, the wings are designed to have relatively high aspect ratio to generate sufficient lift. When compared with the fuselage, the bending and torsional stiffnesses of the wing are much lower. Understandably, the wings are considered to be the most vital and sensitive parts of an aircraft. In the current study, attention is thus confined to the free vibration and flutter analysis of aircraft wings. Without much loss of generality the wing can be treated as cantilevered on the side wall of the fuselage. It is typical of an aircraft wing that its bending and torsional deformations are generally coupled due to non-coincident mass and shear centres.

The wing is idealised as an assembly of bending-torsion coupled beam elements which are governed by the following differential equations in free vibration (Banerjee [14])

$$\begin{cases} EIh'''' + m\ddot{h} - mx_\alpha\ddot{\psi} & = 0\,, \\ GJ\psi'' + mx_\alpha\ddot{h} - I_\alpha\ddot{\psi} & = 0\,, \end{cases} \tag{1}$$

where h and ψ are the transverse displacement and torsional rotation respectively; EI and GJ are the bending and torsional stiffnesses; m is mass per unit length; I_α represents the mass moment of inertia per unit length and x_α denotes the distance between the mass and elastic axes of the elements.

Next the dynamic stiffness matrices for every beam element (Banerjee [14]; Banerjee [15]) are formulated and assembled based on the partial differential equation (1). Note that when a non-uniform aircraft wing is idealised as a collection of bending-torsion coupled beam elements, every element will have different mass (m), inertia (I_α), and stiffness properties (EI, GJ). The distance between mass axis and elastic axis (x_α) can also vary from element to element.

In addition to the above idealisation, the engine mass and inertia mounted on a transport aircraft wing can be also taken into account. The presence of the engine on the wing can influence its modal behaviour significantly. In the present study, the engine is idealised as concentrated lumped mass and inertia located at a particular node on the wing.

Once the global dynamic stiffness matrix for the whole wing according to dynamic stiffness matrix method is constructed leading to a nonlinear eigenvalue problem. The Wittrick-Williams algorithm is applied as a solution technique to compute the natural frequencies and mode shapes of the wing. From a mathematical point of view, the natural frequencies are essentially the eigenvalues and the modes shapes are the eigenfunctions of the elastodynamic system which in this case, is an aircraft wing. By solving this eigenvalue problem, we obtain a range of natural frequencies and the corresponding mode shapes. It is only the first few natural frequencies and mode shapes that matter in flutter analysis. It was found that the first six modes were sufficient in the flutter analysis that is carried out in this paper.

2.2 Flutter analysis

In carrying out the flutter analysis, normal mode method together with generalised coordinates and unsteady aerodynamics of Theodorsen type is utilised to compute the flutter speed and flutter frequency. A selective first few natural modes which included the fundamental bending and torsion modes are used. The solution of the flutter stability determinant is a complex eigenvalue problem since the determinant of the system is complex and its elements depend on two variables, namely, the air speed V and the flutter frequency ω. There are a variety of methods to solve the flutter determinant (Bisplinghoff and Ashley [16]; Dowell [17]; Fung [18]; Wright and Cooper [19]), but the emphasis is generally placed to reduce the computation time when solving the problem. In CALFUN, the Theodorsen's method (Theodorsen [20]) which is one of the options is applied. The flow is considered to be incompressible, and strip theory based on Theodorsen expressions for unsteady aerofoil motion (Theodorsen [20]) is employed to obtain the aerodynamic matrices. Then, both the dynamic stiffness and aerodynamic matrices are expressed in term of the generalised co-ordinates to formulate the flutter problem as below

$$\left\{[K_D(\omega)] - \frac{\rho V^2}{2}[QA]_R + i\omega[D] + i\frac{\rho V^2}{2}[QA]_I\right\}\{q\} = 0\,, \tag{2}$$

where $[K_D(\omega)]$ is the frequency dependent dynamic stiffness matrix and $[D]$ is the damping matrix of the cantilever wing structure; $[QA]_R$ and $[QA]_I$ are the real and imaginary parts of the generalised aerodynamic matrix. In equation (2), ω is the circular or angular frequency of the wing in its oscillatory motion.

The flutter frequency ω_f and flutter speed V_f can be found when both the real and imaginary parts of the flutter determinant of equation (2) are identically zero.

3 Results and discussion

3.1 Geometry and other properties of aircraft

As mentioned earlier, two categories of aircraft, namely, sailplanes and transport airliners are analysed for their free vibration and flutter characteristics. In each category, designated as **S** for sailplane and **T** for transport airliner: two aircraft models **S1** and **S2** for sailplanes, while **T1** and **T2** for transport airliners, whose main geometrical configurations and particulars are given in Table 1. It is clear that the two aircraft in the same category share quite similar, but not identical properties. However, the properties of one category are very different from the other. So it is expected that the free vibration and flutter behaviour of the same category of aircraft may have similar features, whereas that of different categories will be dissimilar.

It is noted that there is one engine on each wing for the transport aircraft **T1** whereas two engines on each wing for the transport aircraft **T2**. By contrast, the sailplanes **S1** and **S2** have no engines (see Table 1).

Table 1 Particulars of the four aircraft

Geometrical parameters	Sailplane		Transport aircraft	
	S1	**S2**	**T1**	**T2**
Span(m)	15	22	29.24	40.4
Wing area(m^2)	10.05	15.44	90.00	162.1
Aspect ratio	22.39	31.35	9.5	10.08
Wing root chord (m)	0.85	0.90	5.35	4.88
Wing tip chord (m)	0.35	0.36	1.42	2.54
Sweep angle (°)	0	0	27.6	0
No. of engines in each wing	0	0	1	2

3.2 Natural frequencies and mode shapes

The first six natural frequencies for the two categories of the aircraft were computed using CALFUN (Banerjee [3]). These are shown in Table 2 for all of the aircraft. The natural frequency values are labelled with B, T or C which stand for the bending dominated (B), torsional dominated (T), and bending/torsional coupled (C) modes, respectively. The mode shapes corresponding to the natural frequencies of the two types of aircraft are illustrated in Figs. 1 and 2 respectively. Note that the bending displacements are shown by solid lines, whereas the torsional rotations are shown by dashed lines.

Table 2 Natural frequencies of two types of aircraft wings

Frequencies (rad/s)	Sailplane		Transport aircraft	
	S1	**S2**	**T1**	**T2**
ω_1	13.512(B)	10.657(B)	19.710(B)	11.524(B)
ω_2	42.686(B)	42.594(B)	55.288(B)	33.085(B)
ω_3	95.025(B)	109.837(B)	100.248(B)	45.420(C)
ω_4	165.137(T)	111.651(T)	120.907(C)	87.857(B)
ω_5	171.375(C)	201.303(B)	197.742(C)	97.761(T)
ω_6	281.683(B)	261.204(T)	248.250(T)	121.521(T)

(B)– Bending dominated; (T)–Torsional dominated; (C)– Bending/torsional coupled

3.2.1 Sailplanes S1 and S2

It can be seen from Table 2 that natural frequencies for the two sailplanes are different, but quite similar. An inspection of the two sets of mode shapes in Fig. 1 suggests that the first three modes of the two sailplanes are bending dominated whereas the fourth one of each of the two sailplanes is a pure torsional mode. It should be noted that the sailplane wings are made up of two parts, the inner wing and the outer wing, and they are connected by a solid metallic rod. As a consequence, the mass and inertia distributions near the junction between the inner and the outer wings will be nonuniform. This is reflected in some of the mode shapes shown in Fig. 1.

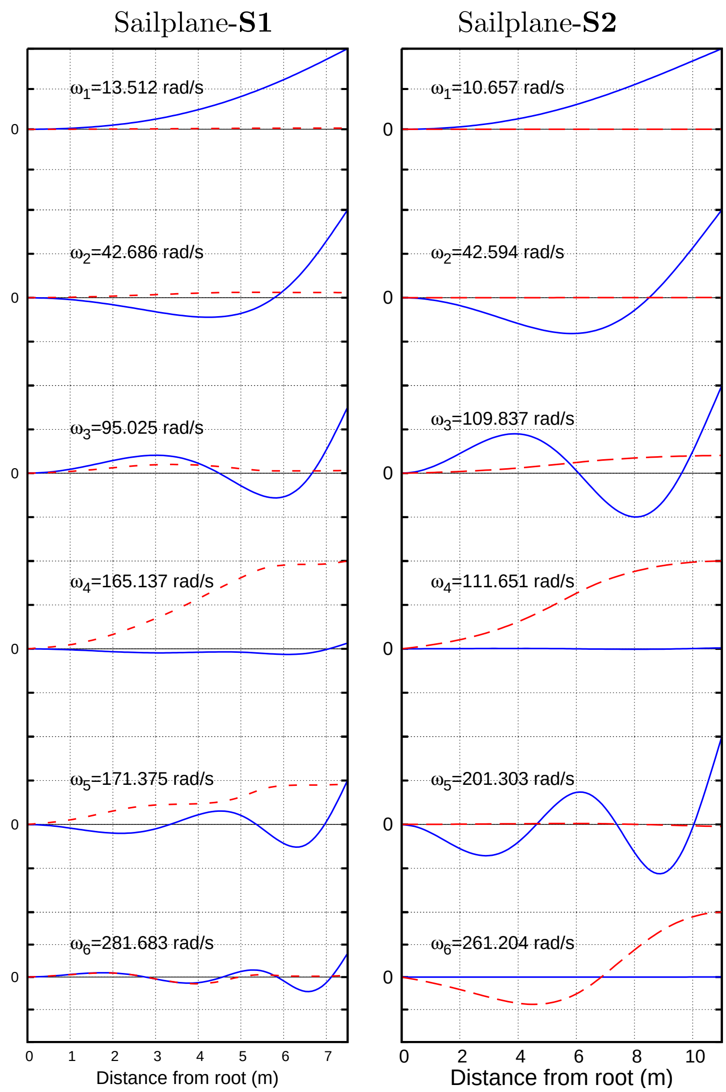

Fig. 1 Natural frequencies and mode shapes of sailplane wings. ——— bending displacement; – – – torsional displacement.

3.2.2 Transport airliners T1 and T2

The results in Table 2 show that the natural frequencies of transport airliner **T1** are higher than those of **T2**. One of the reasons for this difference can be attributed to the fact that the transport airliner **T2** has a much higher aspect ratio than **T1**. It is worth-noting that there are more coupled modes in this category of aircraft than the previous one. This is mainly due to the significant separation between the mass and elastic axes, and also due to the presence of the engine(s) on the wing. The mode shapes for **T1** and **T2** shown in Fig. 2 reveal some interesting features. The first three modes of **T1** are primarily bending modes, whereas the fourth, fifth and sixth modes are coupled in bending and torsion. The coupling between the bending and torsional motions in these three latter modes is mainly due to the outboard engine and the elastic axis locations. As for transport airliner **T2**, the first, second and fourth modes are bending dominated, whilst the other

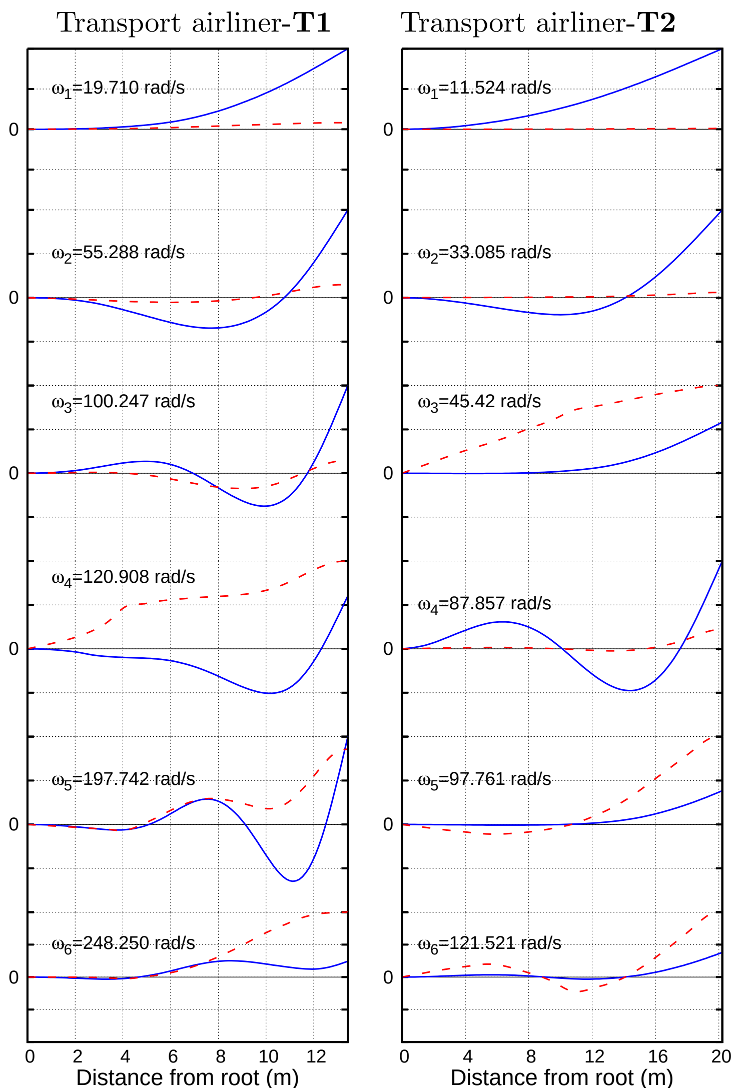

Fig. 2 Natural frequencies and mode shapes of transport airliner wings. ——— bending displacement; – – – torsional displacement.

three are coupled modes exhibiting relatively more torsional deformation than bending.

3.3 Flutter analysis

After carrying out the modal analysis to establish the first six natural frequencies and mode shapes of the two types of the aircraft wings in the previous section, the next step is to use the results to conduct a detailed flutter analysis. In order to achieve this, the software CALFUN is used.

Since the flutter determinant is a highly complex function involving the air speed V and frequency $\boldsymbol{\omega}$, it was necessary to search for the zero of the flutter determinant both in terms of its real and imaginary parts. The search is carried out in a two dimensional plane using air speed V and frequency $\boldsymbol{\omega}$ as variables to ensure that the real and imaginary parts of the flutter determinant and hence the whole flutter determinant are zeros. From a computational point of view, a range of airspeeds and frequencies are chosen. Then for a fixed airspeed (V), the real and imaginary parts of the flutter determinant are computed for a range of frequencies, and next, the process is repeated for a range of airspeeds until the whole flutter determinant is zero.

Essentially, the loci of the roots of the real and imaginary parts of the flutter determinant are plotted to locate the flutter point. A typical example of this plot for sailplane **S2** is shown in Fig. 3, where the roots of the real and imaginary parts of the flutter determinant are shown by the solid and dashed lines, respectively. Clearly, the intersecting point of the contour plots gives the flutter speed and flutter frequency because at this point both the real and imaginary parts of the flutter determinant are zeros. For the sailplane **S2**, the flutter speed V_f and flutter frequency ω_f at the intersecting point gives $V_f = 71.02\ m/s$ and $\omega_f = 53.67\ rad/s$ respectively, see Fig. 3.

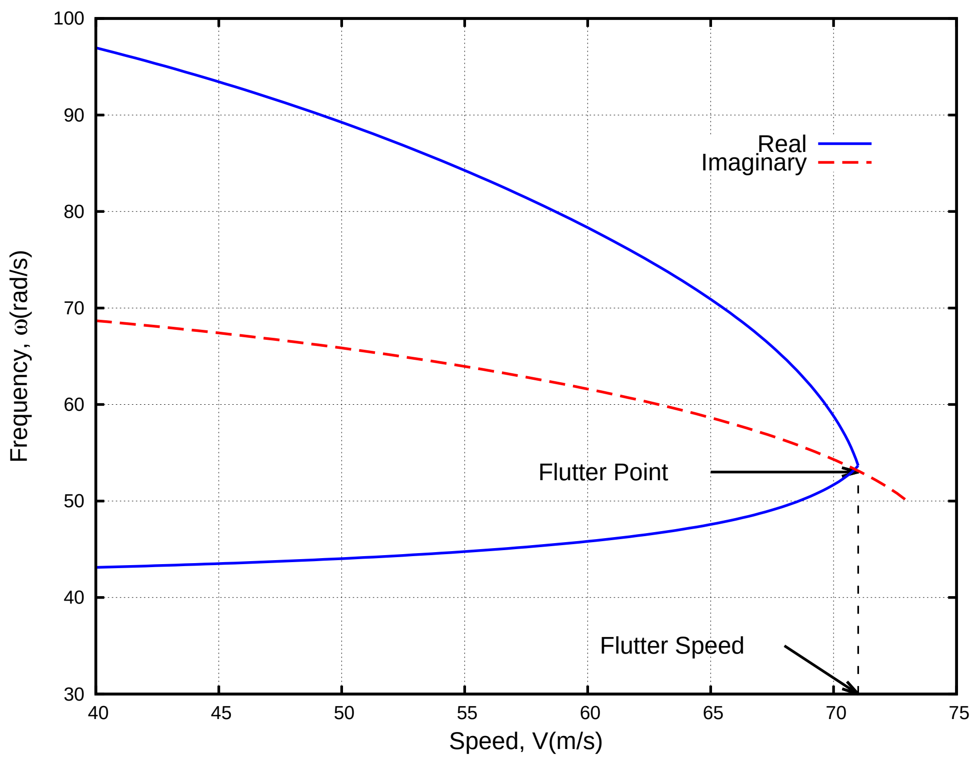

Fig. 3 Zeros of real and imaginary parts of flutter determinant of sailplane **S2**.

Using the above procedure, the flutter speed and the flutter frequencies of all the four aircraft are computed and shown in Table 3. As can be seen from the results in Table 3, the flutter speeds of **S1** and **S2** are quite similar (77.02 m/s for **S1** and 71.02 m/s for **S2**) although the flutter frequencies are somehow different (76.51 rad/s for **S1** and 53.67 rad/s for **S2**).

With regard to the results of the two transport airliners, **T1** has a flutter speed of 406.25 m/s whereas that of **T2** is 251.10 m/s. The corresponding flutter frequencies are 78.39 rad/s and 28.70 rad/s respectively. Given the different nature and dissimilar mass and stiffness properties of the two transport airliners, these markedly different results are not unexpected. In order to confirm the existence of flutter, a further confirmatory check was performed in that the real and imaginary parts of the flutter determinant were computed within the neighbourhood of the flutter

Table 3 The flutter speeds and frequencies of the two types of aircraft wings by using CALFUN

Critical values for flutter	Sailplane		Transport aircraft	
	S1	**S2**	**T1**	**T2**
Flutter speed $V_f(m/s)$	77.02	71.02	406.25	251.10
Flutter frequencies $\omega_f(rad/s)$	76.51	53.67	78.39	28.70

speed and flutter frequency. This was carried out just before and just after the flutter speed. To illustrate the procedure, the sailplane **S2** and the transport airliner **T2** which have flutter speeds 71.02 m/s and 251.10 m/s respectively, are used as examples. Fig. 4 shows the values of the real and imaginary parts of the flutter determinant of the sailplane **S2** for speeds 65 m/s and 75 m/s. Clearly, the figure shows that there is a swap-over between the real and imaginary parts of the determinant values between the two speeds, indicating the existence of the flutter somewhere in between. Likewise, similar confirmatory check was performed on transport airliner **T2** at speed 245 m/s and 260 m/s respectively (Note that the flutter speed of transport airliner **T2** which has been computed at 251.10 m/s, falls within this narrow range). The results are shown in Fig. 5, where again the flip-over between the values of the real and imaginary parts of the flutter determinant over the appropriate frequency range is distinctively apparent. As flutter is a very complex phenomenon, these confirmatory checks were necessary to give confidence concerning the accuracy of flutter speed and flutter frequency.

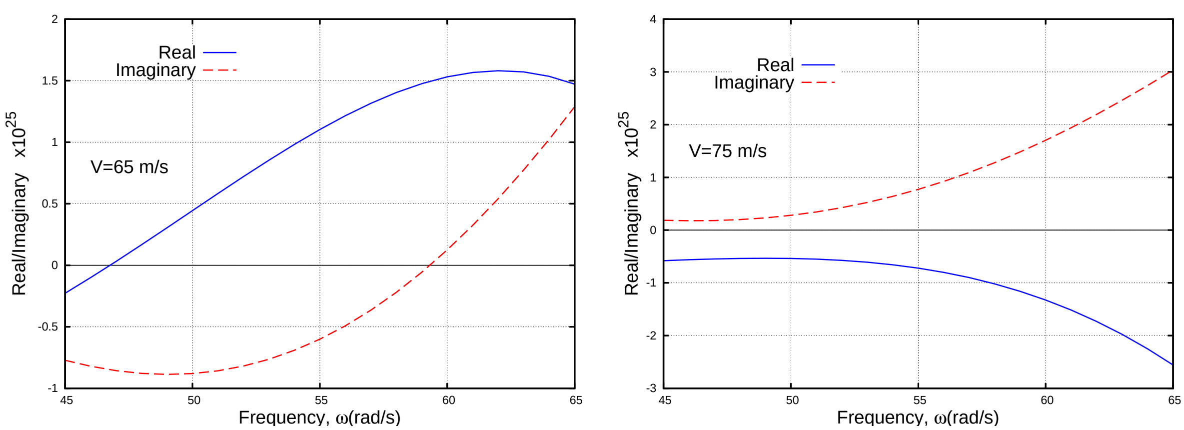

Fig. 4 Plot of real and imaginary parts of the flutter determinant in the neighbourhood of flutter speed for sailplane **S2**.

4 Conclusions

The free vibration and flutter investigations of high aspect ratio aircraft wings of two contrasting categories which include sailplanes and transport airliners have been carried out using the well-established computer program CALFUN. The first six natural frequencies and mode shapes for four aircraft wings with two examples taken from each category have been computed and discussed. Following the modal investigation, flutter analysis has been carried out and the flutter speeds and

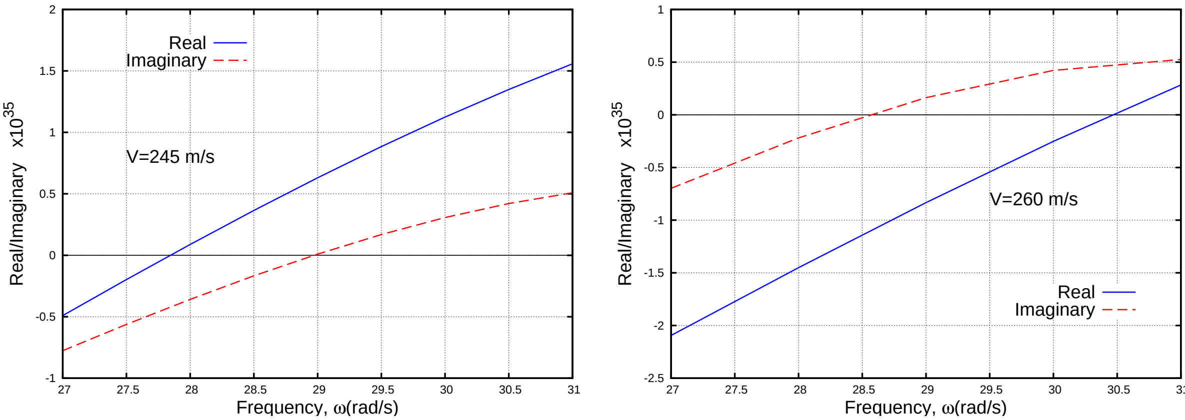

Fig. 5 Plot of real and imaginary parts of the flutter determinant in the neighbourhood of flutter speed for transport airliner **T2**.

flutter frequencies are presented for all of the four aircraft wings. The results have been compared and contrasted. The research carried out in this paper provides prospects for further research, particularly for composite wings for their free vibration, flutter and response behaviour. Another useful extension would be replacing the Theodorsen type two dimensional unsteady aerodynamic theory by more sophisticated unsteady aerodynamics theory to cover transonic speed aeroelasticity. It is in this context, this preliminary research is expected to be most useful.

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