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Citation: Rakocevic, V., Jahromizadeh, S., Gruber, J. K. & Prodanovic, M. (2014). Demand management for home energy networks using cost-optimal appliance scheduling. Paper presented at the 3rd International Conference on Smart Grids and Green IT Systems, 03-04 Apr 2014, Barcelona, Spain.

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DEMAND MANAGEMENT FOR HOME ENERGY NETWORKS USING COST-OPTIMAL APPLIANCE SCHEDULING

Veselin Rakocevic[1], Soroush Jahromizadeh[1], Jorn Klaas Gruber[2], Milan Prodanovic[2]
[1]*School of Engineering and Mathematical Sciences, City University London, United Kingdom*
[2]*Electrical Systems Unit, IMDEA Energy Institute, Madrid, Spain*
veselin.rakocevic.1@city.ac.uk

Keywords: Smart Homes, Optimization for Efficient Energy Consumption, Energy Profiling and Measurement, Energy Demand Management, Economic Models of Energy Efficiency

Abstract: This paper uses problem decomposition to show that optimal dynamic home energy prices can be used to reduce the cost of supplying energy, while at the same time reducing the cost of energy for the home users. The paper makes no specific recommendations on the nature of energy pricing, but shows that energy prices can normally be found that not only result in optimal energy consumption schedules for the energy provider's problem and are economically viable for the energy provider, but also reduce total users energy costs. Following this, the paper presents a heuristic real-time algorithm for demand management using home appliance scheduling. The presented algorithm ensures users' privacy by requiring users to only communicate their aggregate energy consumption schedules to the energy provider at each iteration of the algorithm. The performance of the algorithm is evaluated using a comprehensive probabilistic user demand model which is based on real user data from energy provider E.ON. The simulation results show potential reduction of up to 17% of the mean peak-to-average power estimate, reducing the user daily energy cost for up to 14%.

1 INTRODUCTION

The emergence of *smart* homes enables energy providers to develop sophisticated energy management solutions, in attempt to optimise energy production while providing home users with increased comfort and potential cost reduction. The future smart homes will be equipped with a range of control devices and sensing/actuating systems capable of working together in automatic way to perform some pre-defined functions. Over the past decade, the majority of technical challenges for the home hardware and software solutions have been solved, and a range of commercial products is available. For energy providers, the greatest remaining challenges lie in: (1) development of intelligent resource management algorithms to optimise the energy consumption, both at the single-household level and at the large-scale level; (2) establishing increased level of trust with the user by ensuring that the users' energy consumption data is kept secret. This paper addresses both of these issues by providing an optimal distributed algorithm for home appliance scheduling without the need for sharing detailed information on daily use of home appliances.

The process of resource optimisation in home energy networks has been generating research interest for several decades now, and in the recent years it has been accelerated by the technological advances in sensor networks, smart meters and actuator systems. In an ideal smart home model, the historical consumption data, real-time measurements, pricing, ambient and social aspects are all used as inputs to optimisation algorithms which calculate the optimal home appliance energy consumption schedule. Traditionally, the problem of optimal use of home energy has been approached in two ways: (1) reducing consumption, or (2) shifting consumption. The process of consumption shifting, also called demand management, or load management, has been practices by the industry for several decades, using different forms of load control (Fahrioglu, 2000, Palensky 2011, Siano

2014). The existing solutions use variable pricing to generate incentives to home users to shift their consumption from peak periods, thus reducing the need to start additional generators, which presents a major cost factor for energy providers.

There is a number of research works that take on this challenge, and develop algorithms and network protocols for optimal demand management. For example, Li, Chen and Low (Li, 2011) show that there exist time-varying prices that can align individual optimality with social optimality. In their model, the utility company collects forecasts of total demands from all customers, and then sets the prices to the marginal cost. Each customer updates its demand and charging schedule. Similarly, Pedrasa, Spooner and MacGill (Pedrasa, 2010) present a solution which enables end-users to assign values to desired energy services, and then schedule the resources to maximise the users' benefits. They propose the use of particle swarm optimisation, because of simple implementation. They do not, however, test their solution on large-scale systems and do not prove the optimality of the solution. Zakariazadeh, Jadid and Siano (Zakariazadeh, 2014) propose a multi-objective framework, based on augmented ε-constraint method, to minimize the total operational costs and emissions and to generate Pareto-optimal solutions for the energy and reserve scheduling problem. In the work of Ramchurn et al (Ramchurn, 2011, Vytelingum, 2010) decentralised demand side management is realised through the process of cooperation between the smart meters ('agents'). The meters receive the costs of generating electricity to the consumers, and use learning mechanisms to gradually adapt the agents' deferrable energy load based on the predicted market prices for the next day. Similar approach is also taken by (Mohsenian-Rad, 2010), (Ganu, 2012), and (Ibars, 2010) . In these solutions the end-users are somehow made to voluntarily adjust their consumption. (Mohsenian-Rad, 2010) formulates an energy consumption scheduling game, where the players are the users and their strategies are the daily schedules of their household appliances and loads. Similarly, (Ibars, 2010) bases the solution on a network congestion game, which can be demonstrated to converge in a finite number of steps to a pure Nash equilibrium solution. (Jain, 2013) goes one step further, by applying the concept of bargaining / auctioning of energy resources on the smart grid including electric vehicles.

It is important to stress that most of these works, including ours, rely on consumer's willingness to act. In other words, the end-user benefit is always modelled through cost, and optimality of the scheduling is based on the process of cost minimisation. The mechanism of costing allows the users to react, in their own interest. Dynamic pricing and its drawbacks are analysed in great detail in the past research, e.g. in (Borenstein, 2002) and (Roozbehani, 2010). In a response to this, (Wijaya, 2013) proposes an interesting approach to cut the peak to average energy ratio explicitly from the supply side. The resulting load cuts are then distributed among consumers by the means of a multiunit auction which is done by an intelligent agent on behalf of the consumer.

In this paper, we decompose the provider and the user optimization problem to prove that, if energy prices are set as optimal consistency prices, the energy provider's revenue at optimal energy consumption levels is greater than the variable cost of supplying energy. This motivates the design of a heuristic real-time algorithm where at every timeslot each appliance energy consumption is updated according to the real-time energy price and estimated price of operating each appliance. The home can then use this to calculate the optimal energy consumption schedule. The paper makes no specific recommendations on the nature of energy pricing, but shows that energy prices can normally be found that not only result in optimal energy consumption schedules for the energy provider's problem and are economically viable for the energy provider, but also reduce total users energy costs. The paper shows that optimal dynamic energy prices can be used to pass on the reduction in the cost of supplying energy to the users, when sufficiently scaled down. This provides financial incentives to users to subscribe to the smart home scheme. The presented algorithm ensures users' privacy by requiring users to only communicate their aggregate energy consumption schedules to the energy provider at each iteration of algorithm.

To evaluate the performance of the optimisation algorithm, we use a comprehensive consumer demand model to compute the quantitative benefits of the algorithm. The model is described in section 4; it is based on appliance definition, user profile generation and daily appliance use determination, and is based on real user data from energy provider E.ON and the UK Government Report on home energy use (Zimmermann, 2012). The performance evaluation of the new algorithms is done using simulation, the details of which are given in section 5.

The simulation results show that applying the new optimisation algorithm, it is possible to reduce the mean power peak to average ratio (PAR) between 0.16 and 0.35 with 99% probability. That is between 7.83% and 17.02% of the original time series mean PAR estimate. Furthermore, optimisation reduces average user daily energy cost between 3.54% and 14.72% of its original time series mean estimate with 99% probability.

It is worth noting that reading and understanding the simulation results for the large-scale home energy networks is very difficult, as averaging the benefits of optimal energy supply gives only a part of the full picture. It is for this reason that we believe that the best practical use of the research results presented in this paper is to integrate them in the development of energy consumption visualisation tools for the individual users and for the energy supplier. Visualisation of energy consumption (Goodwin, 2013) will enable better understanding of the pattern of energy use and the consequence of optimisation and optimal appliance schedules.

2 SYSTEM MODEL

We start by presenting a model for energy users and energy provider. The user is modeled as an operator of a set of home appliances which operate over a finite scheduling time horizon. The user's objective is to choose feasible energy consumption schedule so as to minimize the cost of energy. The provider, on the other hand, benefits from selling units of energy to the users. Critically, the objective of the provider is to minimize the cost of supplying the energy consumed by the users by shifting the total energy consumed during the time horizon.

We consider a smart power network comprising a set of N users served by an energy provider who participates in the wholesale energy market. Each user is equipped with a smart meter capable of scheduling energy consumption of appliances, and smart meters are connected to the energy provider via a communication link. In the following sections, we describe how users and the energy provider are modeled.

Users: We assume each user $n \in N$ operates a set of A_n appliances including photovoltaic (PV) appliances, which are operated over a finite scheduling time horizon (e.g. a day) divided into timeslots (e.g. 15 minutes). We denote by T the set of timeslots in the scheduling time horizon. For each user $n \in N$, we denote by $x_{n,a}^t$ the energy consumption scheduled for appliance $a \in A_n$ at time $t \in T$, where negative values of $x_{n,a}^t$ represent power generation. Let $x_{n,a} \triangleq \left(x_{n,a}^t, t \in T\right)$ be the energy consumption schedule vector for appliance $a \in A_n$, and $x_n \triangleq \left(x_{n,a}, a \in A_n\right)$ be the energy consumption schedules for all appliances. We also denote the cardinality of sets by capital letters, e.g. $\mathrm{N} = |N|$ and $\mathrm{T} = |T|$. We assume that each appliance $a \in A_n$ requires a total energy of $E_{n,a}$ during the scheduling horizon, i.e.

$$\sum_{t \in T} x_{n,a}^t = E_{n,a} \;\; \forall a \in A_n \tag{1}$$

In addition, we assume that each appliance $a \in A_n$ can use a minimum power level of $\gamma_{n,a}^{t,min}$ and a maximum power level of $\gamma_{n,a}^{t,max}$ at timeslot $\in T$, i.e.

$$\gamma_{n,a}^{t,min} \leq x_{n,a}^t \leq \gamma_{n,a}^{t,max} \;\; \forall t \in T, a \in A_n \tag{2}$$

Clearly, if appliance $a \in A_n$ is non-controllable then $\gamma_{n,a}^{t,min} = \gamma_{n,a}^{t,max} \triangleq \gamma_{n,a}^t$, $\forall t \in T$ and $\sum_{t \in T} \gamma_{n,a}^t = E_{n,a}$.

User Optimisation Problem: Let p_t be the unit price of energy at time $t \in T$, which is set by the energy provider. We assume that users cannot sell their excess generated energy to the energy provider. Given the energy price vector $\mathbf{p} = (p_t, t \in T)$, the objective of user $n \in N$ is to then choose feasible energy consumption schedules x_n so as to minimize total energy costs, i.e. to solve the following optimization problem:

$$\min_{x_n} \sum_{t \in T} p_t \max\left(\sum_{a \in A_n} x_{n,a}^t, 0\right)$$

$$\text{s.t. (1) and (2)} \tag{3}$$

Evidently, optimal solution of (3) is dependent on the energy prices set by the energy provider.

Energy Provider: The energy provider is characterized by its energy cost function and its optimization objectives. The cost function $C_t(X)$ represents the cost for the energy provider to supply $X \geq 0$ units of energy at time $t \in T$ and is widely assumed to be increasing and strictly convex (see e.g. (Li, 2011) and (Mohsenian-Rad, 2010) As an example, the energy cost function for thermal generators is shown to be quadratic as follows (Mohsenian-Rad, 2010):

$$C_t(X) = a_t X^2 + b_t X + c_t \quad t \in T \tag{4}$$

where $a_t, b_t > 0$ and $c_t \geq 0$.

Optimisation Objectives: Since by constraint (1) users' energy demands during the scheduling horizon are fixed, we define the energy provider's objective as to minimize the cost of supplying the energy consumed by the users by shifting the total energy consumption at each time slot, i.e. to solve the following optimization problem

$$\min_{\mathbf{x}} \sum_{t\in T} C_t\left(\sum_{n\in N} max\left(\sum_{a\in A_n} x_{n,a}^t, 0\right)\right)$$
$$\text{s.t. (1) and (2)} \quad n \in N \qquad (5)$$

where $\mathbf{x} = (x_n, n \in N)$. The optimization problem (5) is convex and can be solved by the energy provider in a centralized fashion, providing that users energy demand constraints are available to the energy provider. Alternatively, (5) can be solved jointly by the energy provider and users using a distributed algorithm. In either way, appropriate energy pricing schemes have to be designed to ensure user participation by providing financial incentives.

3 OPTIMISATION ALGORITHM

Since the objective function in the energy provider's optimisation problem (5) is not strictly convex in x, computation of primal optimal solutions from the dual optimal solutions may not be possible (Boyd, 2004). As here we adopt a dual decomposition approach, we use the generalization of proximal minimization algorithm proposed by (Lin, 2006) that can be applied to the problems with similar form as (5). First, using the auxiliary vector $\boldsymbol{z} \triangleq (z_n, n \in N)$, where $z_n \triangleq (z_{n,a}, a \in A_n)$, $z_{n,a} \triangleq (z_{n,a}^t, t \in T)$, we transform the optimization problem (5) into the following equivalent form

$$min_{x,z} \sum_{t\in T}\left(C_t\left(\sum_{n\in N}\sum_{a\in A_n} x_{n,a}^t\right) + \frac{1}{2c_t}\sum_{n\in N}\sum_{a\in A_n}\left(x_{n,a}^t - z_{n,a}^t\right)^2\right)$$
$$\text{s.t. (1) and (2) } n \in N \qquad (6)$$

where $c_t > 0\ t \in T$. Let $\mathbf{x}^*$ be the optimal solution of (5). Then $\mathbf{x} = \mathbf{x}^*$ and $\mathbf{z} = \mathbf{x}^*$ is the optimal solution of (6). The optimization problem (6) can then be solved using the algorithm as presented in (Lin, 2006):

Algorithm A: Fix K ≥ 1. At jth iteration:

1. Fix z = z(j) and estimate the solution of the dual problem of (6) by applying gradient method on dual variable ρ for K iterations.
2. Let $\rho(j+1,0) = \rho(j,K)$. Let $x(j)$ be the primal variable associated with the dual variable $\rho(j,K)$. Set

$$z_{n,a}^t(j+1) = z_{n,a}^t(j) + \beta_t\left(x_{n,a}^t(j) - z_{n,a}^t(j)\right)$$
$$\forall t \in T, a \in A_n, n \in N \ , \qquad (7)$$

where $0 < \beta_t \leq 1, \forall t \in T$.

We now focus on development of a distributed algorithm for step 1 of algorithm **A** at jth iteration. Note that optimization problem (6) is strictly convex when z is fixed. Introducing the auxiliary variable

$$y_t = \sum_{n\in N}\sum_{a\in A_n} x_{n,a}^t \quad \forall t \in T \qquad (8)$$

The optimization problem becomes

$$min_{y,x} \sum_{t\in T}\left(C_t(y_t) + \frac{1}{2c_t}\sum_{n\in N}\sum_{a\in A_n}\left(x_{n,a}^t - z_{n,a}^t(j)\right)^2\right)$$
$$\text{s.t. (1), (2) } \forall n \in N \text{ and (8)} \qquad (9)$$

The Lagrangian after relaxation of constraint (8) is $L(\boldsymbol{\rho},\mathbf{y},\mathbf{x}) = \sum_{t\in T}\left(C_t(y_t) + \frac{1}{2t}\sum_{n\in N}\sum_{a\in A_n}\left(x_{n,a}^t - z_{n,a}^t(j)\right)^2 + \rho_t\left(\sum_{n\in N}\sum_{a\in A_n} x_{n,a}^t - y_t\right)\right)$, where $\boldsymbol{\rho}$ is the vector of consistency prices. The dual problem is then

$$\max_\rho g(\boldsymbol{\rho}) \qquad (10)$$

where

$$g(\boldsymbol{\rho}) = \min_{y,x} L(\boldsymbol{\rho},\boldsymbol{y},\boldsymbol{x}) \ ,$$
$$\text{s.t. (1) and (2) } \forall n \in N \qquad (11)$$

Since (9) is strictly convex, the dual function (11) is differentiable and its gradient is given by (Bertsekas, 1999):

$$\nabla g(\boldsymbol{\rho})_t = \sum_{n\in N}\sum_{a\in A_n} x_{n,a}^t(\boldsymbol{\rho}) - y_t(\boldsymbol{\rho}) \ \ \forall t \in T \qquad (12)$$

where $\left(y(\boldsymbol{\rho}), x(\boldsymbol{\rho})\right)$ is the solution of (11) given $\boldsymbol{\rho}(j,k)$. The dual problem (10) can then be solved using gradient method as follows

$$\rho_t(j,k+1) = \rho_t(j,k) +$$
$$+\alpha_t\left(\sum_{n\in N}\sum_{a\in A_n} x_{n,a}^t(j,k) - y_t(j,k)\right), \forall t \in T \qquad (13)$$

where $\left(y(j,k), x(j,k)\right)$ denote the solution of (11) given $\boldsymbol{\rho}(j,k)$. Let the primal-dual pair (z^*, ρ^*) denote the stationary point of algorithm A defined by

$$z^* = \arg\max_{x} L(\boldsymbol{\rho}^*, \mathbf{z}^*, \mathbf{y}^*, \mathbf{x})$$
$$\text{s.t. (1), (2) } \forall n \in N$$

$$y_t^* = \sum_{n\in N}\sum_{a\in A_n} z_{n,a}^{t,*} \quad \forall t \in T$$

By KKT optimality conditions (Boyd, 2004) for any stationary point $(\boldsymbol{z}^*, \boldsymbol{\rho}^*)$ $\boldsymbol{z}^*$is the optimal solution of (6). It is shown in (Lin, 2006) that when α_t in (13) is small enough, algorithm A converges to a stationary point $(\boldsymbol{z}^*, \boldsymbol{\rho}^*)$.

The dual function $g(\rho)$ can be decomposed into two subproblems $g(\rho) = g_1(\rho) + g_2(\rho)$, where

$$g_1(\rho) = \min_y \sum_{t \in T} (C_t(y_t) - \rho_t y_t) \quad (14)$$

and

$$g_2(\rho) = \min_x \sum_{t \in T} \sum_{n \in N} \sum_{a \in A_n} \left(\rho_t x_{n,a}^t + \frac{1}{2c_t} \left(x_{n,a}^t - -z_{n,a}^t(j) \right)^2 \right), \text{ s.t. (1), (2) } \forall n \in N \quad (15)$$

Subproblem (6) is an unconstrained convex minimisation problem and due to the strict convexity of $C_t, t \in T$, has a unique solution. Let $y(\rho)$ be the unique solution of (14). Then

$$\rho_t = C_t'(y_t(\rho_t)) \quad \forall t \in T \quad (16)$$

Thus

$$y_t(\rho_t) = C_t'^{-1}(\rho_t) \ \ \forall t \in T \quad (17)$$

Equation (17) can be computed by the energy provider for each timeslot $t \in T$ independently, given the associated consistency price ρ_t. Subproblem (15) can be decomposed into optimisation problems for individual users:

$g_2(\rho) = \sum_{n \in N} g_{2,n}(\boldsymbol{\rho})$, where

$$g_{2,n}(\boldsymbol{\rho}) = \min_{x_n} \sum_{t \in T} \sum_{a \in A_n} \left(\rho_t x_{n,a}^t + \frac{1}{2c_t} \left(x_{n,a}^t - z_{n,a}^t(j) \right)^2 \right) \text{ s.t. (1), (2)} \quad (18)$$

It can be noted that at the stationary point of algorithm $\boldsymbol{A}$ the quadratic term in the objective function of (18) is zero and (18) is equivalent to the user optimization problem (3) with $\mathbf{p} = \boldsymbol{\rho}^*$. Hence, optimal consistency prices can be interpreted as energy prices that encourage users to opt for optimal energy consumption schedules for the energy provider's problem (5), in order to minimise their energy costs under these prices. We will show later in the next section that reduction in the cost of energy supply as a result of solving (5) can be passed on to the users, if energy prices $\mathbf{p}$ are based on adequately scaled down optimal consistency prices $\boldsymbol{\rho}^*$.

The user optimization problem (18) can be further decomposed into optimisation problems for individual appliances as

$g_{2,n}(\boldsymbol{\rho}) = \sum_{a \in A_n} g_{2,n,a}(\boldsymbol{\rho})$, where

$$g_{2,n,a}(\boldsymbol{\rho}) = \min_{x_{n,a}} \sum_{t \in T} \left(\rho_t x_{n,a}^t + \frac{1}{2c_t} \left(x_{n,a}^t - z_{n,a}^t(j) \right)^2 \right) \text{ s.t. (1), (2)} \quad (19)$$

Using dual decomposition, (19) can be decoupled into appliance optimization problem for each time slot. The Lagrangian after relaxation of constraint (1) is

$$L(\sigma_{n,a}, \mathbf{x}_{n,a}) = \sum_{t \in T} \left(\rho_t x_{n,a}^t + \frac{1}{2c_t} \left(x_{n,a}^t - z_{n,a}^t(j) \right)^2 \right) + \sigma_{n,a} \left(E_{n,a} - \sum_{t \in T_{n,a}} x_{n,a}^t \right)$$

where $\sigma_{n,a}$ is the Lagrange variable associated with constraint (1) or price of operating appliance $a \in A_n$. The dual problem is then

$$\max_{\sigma_{n,a}} h_{2,n,a}(\sigma_{n,a}) \quad (20)$$

where

$$h_{2,n,a}(\sigma_{n,a}) = \min_{\mathrm{x}_{n,a}} L(\sigma_{n,a}, \mathrm{x}_{n,a}) \text{ s.t. (2)} \quad (21)$$

The dual function (21) can be decoupled into appliance optimization problems for each time slot:

$h_{2,n,a}(\sigma_{n,a}) = \sum_{t \in T} h_{2,n,a}^t(\sigma_{n,a}) + \sigma_{n,a} E_{n,a}$, where

$$h_{2,n,a}^t(\sigma_{n,a}) = \min_{x_{n,a}^t} (\rho_t - \sigma_{n,a}) x_{n,a}^t + \frac{1}{2c_t} \left(x_{n,a}^t - z_{n,a}^t(j) \right)^2 \text{ s.t.(2)} \quad (22)$$

Let $x_{n,a}^t(\sigma_{n,a})$ be the solution of (22). Then

$$x_{n,a}^t(\sigma_{n,a}) = \left[c_t(\sigma_{n,a} - \rho_t) + z_{n,a}^t(j) \right]_{\gamma_{n,a}^{t,min}}^{\gamma_{n,a}^{t,max}} \quad (23)$$

We consider two measures of performance, namely, peak- to-average ratio (PAR) and average user daily energy cost, to evaluate the benefits of optimization to the energy provider and users, respectively. PAR is defined as the ratio of daily peak to average load, and used here as a measure of variation of aggregate daily energy consumption. It is defined by

$$PAR = \frac{T \max_{t \in T} (\sum_{n \in N} \sum_{a \in A_n} x_{n,a}^t)}{\sum_{n \in N} \sum_{a \in A_n} E_{n,a}} \quad (24)$$

The average user daily energy cost is defined as the daily cost of supplying energy divided by the number of users, and used to measure the minimum possible daily energy cost that can be passed on to a user on average:

$$\frac{\sum_{t\in T} C_t(\sum_{n\in N}\max(\sum_{a\in A_n} x_{n,a}^t,0))}{N} \quad (25)$$

Considering this solution, the proposed approach for solving the energy provider's optimization problem (5) can then be summarized as the following distributed algorithm:

Algorithm A: Fix $K \geq 1$. At j^{th} iteration:

1. Fix $z = z(j)$ and run algorithm **S** for K iterations.
2. Let $\rho(j+1,0) = \rho(j,K)$. Let $x(j)$ be the primal variable associated with the dual variable $\rho(j,K)$. Set

$$z_{n,a}^t(j+1) = z_{n,a}^t(j) + \beta_t\left(x_{n,a}^t(j) - z_{n,a}^t(j)\right)$$
$$\forall t \in T_{n,a}, a \in A_n, n \in N \quad (26)$$

Where $0 < \beta_t \leq 1, \forall t \in T$.

Algorithm S: At k^{th} iteration:

1. Given the consistency prices $\rho(j,k)$, each user $n \in N$

computes:

- the price of operating each appliance $\sigma_{n,a}(j,k)$, $a \in A_n$, by solving (20)
- appliance energy consumption schedule $x_{n,a}^t(j,k)$, $a \in A_n$, $t \in T_{n,a}$, using (23), and communicates its aggregate energy consumption schedule $\sum_{a\in A_n} x_{n,a}^t(j,k)$, $t \in T_{n,a}$, to the energy provider.

2. Given the consistency prices $\rho(j,k)$, the energy provider computes:

- the auxiliary variable $y_t(j,k)$, $t \in T$, using (17),
- updates the consistency price $\rho_t(j,k)$, given the aggregate energy consumption schedules for all users $\sum_{n\in N}\sum_{a\in A_n} x_{n,a}^t(j,k)$ and $y_t(j,k)$, $t \in T$, according to the gradient algorithm (13).

Note that the proposed algorithm ensures users' privacy by requiring users to only communicate their aggregate energy consumption schedules to the energy provider at every iteration of algorithm **S**. Note also that, with the exception of computation of $\sigma_{n,a}(j,k)$, $a \in A_n$, all the computations can be further decoupled across individual timeslots. This motivates the heuristic real-time algorithm presented in the following sections where at every timeslot each appliance energy consumption is updated according to the real-time energy price and estimated price of operating each appliance $\sigma_{n,a}(j,k)$, $a \in A_n$.

As discussed in the previous section, optimal energy consumption schedules for the energy provider's problem (5) can be attained if energy prices are set as optimal consistency prices ρ^*, i.e. setting $\mathbf{p} = \rho^*$, energy consumption schedules that are minimizers of the users optimization problem (3) are also minimizers of the energy provider's problem (5). Moreover, as stated in the following theorem, the energy provider's revenue based on energy prices ρ^* is greater than the variable cost of supplying energy, at optimal energy consumption levels.

Theorem 1. If energy prices are set as optimal consistency prices ρ^*, the energy provider's revenue at optimal energy consumption levels is greater than the variable cost of supplying energy, i.e.

$$\rho^* y_t^* > C_t(y_t^*) - C_t(0), \forall t \in T \quad (27)$$

Proof. Since we assumed that C_t is increasing and strictly convex, it follows from the first order condition for strict convexity (Boyd, 2004) that

$$C_t'(y_t^*)y_t^* > C_t(y_t^*) - C_t(0) \quad \forall t \in T \quad (28)$$

replacing (16) in the above inequality then yields (27).

However, we are interested in energy pricing scheme that not only results in optimal energy consumption schedules for the energy provider's problem (5) and covers the variable cost of supplying energy, but also reduces or ideally minimizes users energy costs, in order to ensure users participation in the smart home scheme.

To examine the existence of such scheme note that by the mean value theorem (Bertsekas, 1999) there exists $\widehat{y_t} \in [0, y_t^*)$ for all $t \in T$ such that

$$C_t'(\widehat{y_t})y_t^* = C_t(y_t^*) - C_t(0) \quad \forall t \in T \quad (29)$$

It follows from (28) that there exists 0 < t < 1, for all $\in T$, such that

$$\gamma_t C_t'(y_t^*) = C_t'(\widehat{y_t}) \quad \forall t \in T \quad (30)$$

Let $\gamma_{max} = max_{t\in T}\{\gamma_t\}$. Then,

$$C_t'(y_t^*) > \gamma_{max} C_t'(y_t^*) \geq C_t'(\widehat{y_t}) \ \forall t \in T \quad (31)$$

So,

$$\sum_{t\in T} C_t'(y_t^*)y_t^* > \gamma_{max}\sum_{t\in T} C_t'(y_t^*)y_t^*$$
$$\geq \sum_{t\in T} C_t'(\widehat{y_t})y_t^*$$
$$= \sum_{t\in T} C_t(y_t^*) - C_t(0) \quad (32)$$

The term on the right side of the above equality is the minimum daily cost of supplying energy and hence is the lower bound on the viable total users energy costs at optimal energy consumption levels. Note that users optimization problem (3) with energy prices $\rho = \gamma_{max}\rho^*$ is equivalent to the case when $\rho = \rho^*$ and thus result in optimal energy consumption schedules for the energy provider's problem (5). The above inequality states that there exist energy prices that lead to optimal energy consumption schedules for the energy provider's problem (5), economically viable for the energy provider and result in lower viable total users energy costs than with energy prices ρ^*, but not necessarily the minimum viable level. This implies that, unless the current energy consumption schedules are very close the levels that optimize the energy provider's problem (5), energy prices can normally be found that not only result in optimal energy consumption schedules for the energy provider's problem (5) and are economically viable for the energy provider, but also reduce total users energy costs. In the case of quadratic cost function (4), it follows from (29) that $\hat{y}_t = \frac{1}{2} y_t^*$, $t \in T$. If $b = 0$ in (4), then $\gamma_t = \gamma_{max} = \frac{1}{2}$, for all $t \in T$. Thus, in this case energy prices $\rho = \frac{1}{2}\rho^*$ results in minimum total users energy costs at optimal energy consumption levels, while still economically viable for the energy provider.

Notice that if the objective function in the energy provider's problem (5) is scaled by a positive constant $\gamma > 0$, the resulting optimization problem is equivalent to (5) and hence the minimum cost of supplying energy, and by (16), optimal consistency prices ρ^* are also scaled by $\gamma > 0$.

4 CONSUMER DEMAND MODEL

To evaluate the algorithm performance in detail, it is necessary to use a comprehensive household consumer energy demand model. The model – developed specifically for this project - generates artificial consumption data, both for a single household and an entire neighbourhood. The model has been developed on the basis of real home user data generated at the E.ON testbed facility in the UK in 2012.

The model generates the consumption data of the households in the following three main steps (Figure 1): (1) determining the household configuration (i.e. which appliances can be found in a household); (2) computing the daily use of each appliance (i.e. how many times is an appliance used on a certain day); (3) calculating the exact energy demand of each appliance (i.e. at what time is the appliance used on a certain day)

The different steps of the consumer energy demand model are based on probabilistic approaches using basic appliance definitions for the generation of the consumption data. The general model structure uses some basic appliance definitions to generate the synthetic consumption data in three main steps: (1) Basic appliance definition; (2) User profile generation; (3) Daily appliance use determination.

The most common household appliances can be classified according to a reduced number of simplified power level patterns (Yao, 2012, Richardson, 2010, Carpaneto, 2007). In the proposed model three different power level patterns for the approximation of the demand curve have been considered (Figure 2). Pattern 1 represents continuously running appliances with a constant power level, such as fridges or freezers. Pattern 2 allows the approximation of occasionally operated appliances with possible non-zero energy consumption in standby operation such as washing machines or TVs. Finally, pattern 3 is used to approximate the power curve.

The three simplified power level patterns were used in the development of a classification scheme based on different usage types. These usage types take into account factors such as frequency, duration and time of use of the considered appliances and allow a classification closely related to the customer habits.

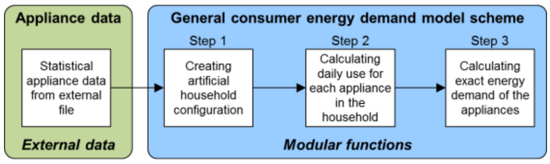

Figure 1. General structure of the developed consumer energy demand model

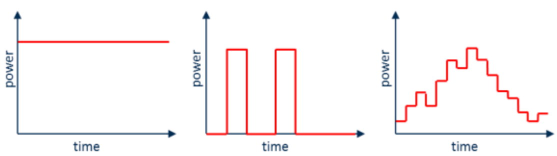

Figure 2. Classification of household appliances by power level patterns

The consumer energy demand model determines in the first step the configuration of one or several households. For most appliance types, the number of devices is computed using a probabilistic approach. However, exceptions have been considered for a few appliances. The computation of the number of devices of a certain appliance type is based on a binomial distribution in order to obtain certain variation around a desired average value.

Finally, an important aspect of the model is consideration of exceptions. In our case, special care was taken: (1) to accurately represent lighting, (2) to exclude appliances which exist with gas and electricity connections; (3) to limit the sum of electric and gas space heaters to one device per household (This limitation is also used in the case of water heating appliances). For a detailed description of the developed usage types and a complete list of the considered appliances the reader is referred to (Gruber, 2012).

The household consumer energy demand model is used in the remainder of the paper to simulate the representative households in order to evaluate the performance of the optimal algorithms presented in section 3. Figure 3 shows the link between consumer demand model and aggregated demand optimisation algorithm, at each simulation replication. The appliance total daily energy requirements $\mathrm{E} \triangleq \left(E_{n,a}, a \in A_n, n \in N\right)$ is computed from the daily energy consumption time series x generated by the proposed consumer demand model.

5 IMPLEMENTATION AND SIMULATION RESULTS

Having defined the model and the theoretical optimisation algorithm in the previous sections, in this section we focus on the actual implementation of the algorithm.

Using the controllability and power level data from the basic appliance definition, minimum and maximum power levels $\gamma^{min} \triangleq \left(\gamma_{n,a}^{t,min}, a \in A_n, n \in N, t \in T\right)$ and, $\gamma^{max} \triangleq \left(\gamma_{n,a}^{t,max}, a \in A_n, n \in N, t \in T\right)$ are set equal to the energy consumption time series for non-controllable appliances, and to the minimum and maximum power level for fully controllable appliances. Here, we refer to appliances with no operational timing constraints as fully controllable appliances. For partially controllable appliances the values of these parameters are set according to their specific constraints, as will be explained later in the simulation results. Given the values of parameters $\boldsymbol{E}, \gamma^{min}, \gamma^{max}$, the optimal energy consumption schedules x^* are computed using the algorithm described in Section 3. Finally, daily estimates of mean performance measures are computed for the original energy consumption time series and its optimisation, given the values of x and x^*, respectively.

The simulation experiment involved generation of energy consumption time series and its optimization for 100 users for 12 independent weeks during a typical winter season. For each week, energy consumption time series and its optimized version were generated separately for every weekday with sampling time of 15 minutes. The Peak to Average Ratio (PAR) and the average user daily energy cost were subsequently measured for each weekday and used to estimate their mean values for the week. In the optimisation model, the operation time of washing appliances were assumed to be flexible throughout the day and hence treated as a control variable.

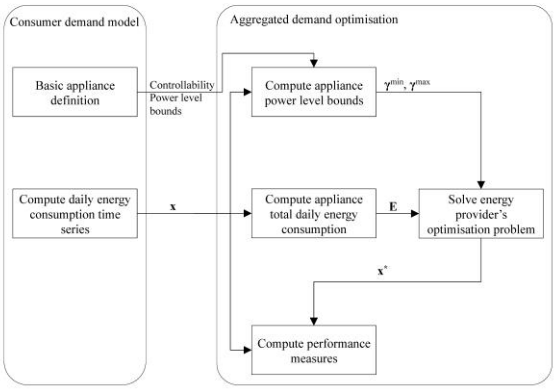

Figure 3. Interface between consumer demand model and aggregated demand optimization results

Table 1: PAR Values.

PAR values	weekday	weekend
original	2.16	1.78
optimised	2.05	1.65

Furthermore, the power level of heating and cold appliances were assumed to be adjustable within the range of 10% of their original time series values and treated as additional control variables. The daily energy requirement of these types of appliances was assumed to be fixed and equal to their daily usage generated by the consumer demand model. Power levels and operation times of the remaining appliances were set equal to their original time series values. The energy cost function was assumed to be of the form (4) with parameters $a_t = 0.1, b_t = 0, c_t = 0,$ for all $t \in T$.

The simulation results indicate that the original time series peak loads during the late afternoon/early night at weekday are significantly higher than weekend as indicated by their respective PAR values of 2.16 and 1.78. In these examples, optimisation reduces the load variation resulting in PAR values of 2.05 and 1.65 for the example weekday and weekend, respectively (Table 1).

Figure 4 shows the resulting aggregate energy consumption time series and its optimisation for a typical weekday (similar results for weekend exist, the figure is omitted because of the space constraints). Figure 5 gives a better visualization of the benefit of optimization, using the load duration curve to show the gains made in the peak demand using out optimization algorithm. The load duration curve shows the energy consumption data by 15-minute intervals, sorted in descending order. Results presented in Figures 4 and 5 show average values for 100 households, with individual household gains greatly depending on the household model.

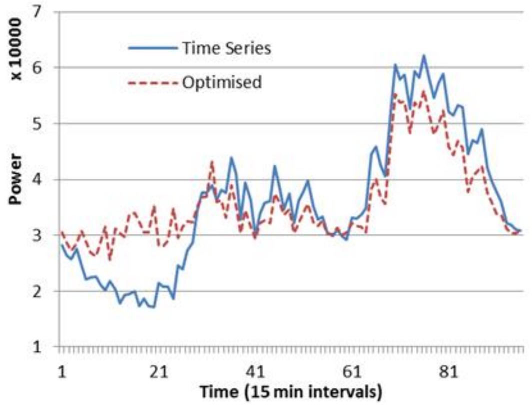

Figure 4. Aggregate energy consumption time series and its optimisation for 100 users for a typical weekday

The overall simulation results indicate that optimisation reduces mean PAR between 0.16 and 0.35 with 99% probability. That is between 7.83% and 17.02% of the original time series mean PAR estimate. Notice that this is despite the fact that the optimisation objective was to minimise the quadratic energy cost function, rather than to minimise the PAR explicitly. Furthermore, optimisation reduces average user daily energy cost between 3.54% and 14.72% of its original time series mean estimate with 99% probability, for all γ>0 multiples of parameters $a_t, t \in T$.

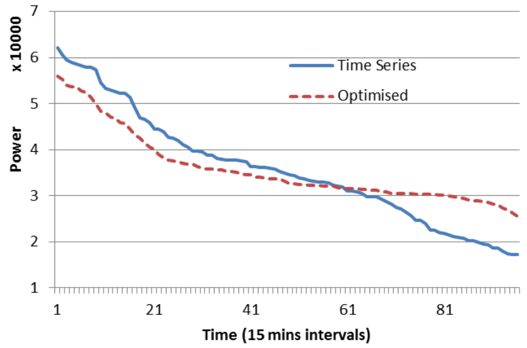

Figure 5. Load Duration Curve for aggregate energy consumption for 100 users on a typical weekday

As it was mentioned in the Introduction section, averaging the benefits of optimal energy supply rarely gives the full picture. The optimisation presented in this paper can be used in the design and development of user visualisation tools. These tools can be used by home users to understand better the benefits of optimal appliance schedule at their home. For more details about the potential use of data visualisation in energy networks the reader is referred to (Goodwin, 2013).

6 CONCLUSION

This paper looks into the problem of optimal use of energy in homes. The paper uses problem decomposition to show that optimal dynamic home energy prices can be used to reduce the cost of supplying energy, while at the same time reducing the cost of energy for the home users. We provide a proof that if energy prices are set as optimal consistency prices, the energy provider's revenue at optimal energy consumption levels is greater than the variable cost of supplying energy. This is then used to design a heuristic real-time algorithm for demand management using home appliance scheduling. The performance of the algorithm is evaluated using simulation, where a comprehensive model of home energy consumption is used.

In terms of the future work, the focus will be on two issues: (1) the detailed performance evaluation of the presented algorithm, using concrete pricing idea, a larger variety of objective functions, including peak minimisation and optimisation of user comfort/discomfort, and realistic models of user reaction; (2) utilising the linear time complexity (O(n)) of our algorithm, which makes it suitable for performing simulation on very large sets of data (entire city or country) using cluster/cloud computing in a very short time for interactive energy data analysis and visualisation. In our future work, we will aim to experiment with the efficiency of the algorithm for large-scale optimisation and visualisation of household energy use, to understand better the nature of the energy price from the user point of view.

ACKNOWLEDGEMENTS

The authors would like to thank E.ON International Research Initiative for providing support for this work. Also, we would like to thank Sara Jones, Jason Dykes, and their teams from the School of Informatics, City University London, for their cooperation on the project.

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