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Glass slippers and glass ceilings: An analysis of marital anticipation and female education.

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ABSTRACT

This paper studies how marital anticipation affects female schooling in the presence of gender wage inequality and private benefits of education. Gender wage inequality induces a marital division of labor that creates (i) a marginal disincentive to girls' schooling and (ii) a tradeoff between consumption and education facing females in marriage markets. We show that in the presence of the last effect, an increase in the market wage can have negative consequences for the education of females who specialise in housework.

Keywords: Female education, labor market discrimination, marriage.
JEL Classification: I20, J12, J16, O12.

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1 Introduction.

Females in developing countries have traditionally received less education than males.[1] There is also evidence of anti-female bias in child nutrition and healthcare (see, *e.g.* Khanna *et. al.* [2003]). A common economic explanation is that these biases represent optimal parental responses to gender inequalities in returns to labour and human capital (see Rosenzweig and Schultz [1984] for a seminal investigation of this hypothesis). Faced with lower returns to females, parents shift resources towards males.[2]

Gender biases in survival tend to get eliminated as household incomes rise above poverty (Deaton [1989] and Rose [1999]). For biases in education, cross- and single-country studies using aggregate data suggest that these too might gradually disappear with increases in per-capita incomes. For example, in a comprehensive cross-country study, Mammen and Paxson [2000] find that average years of female schooling rise monotonically with per-capita income.

However, aggregate data can mask composition effects across different income strata within a country. For example, in low and lower-middle income countries, where a significant proportion of females receive either no or sub-primary levels of education, gains at low levels of achievement would lead to an increase in average years, even if there is stagnation at higher levels.[3] Indeed, even Mammen and Paxson's fitted

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[1]See Dreze and Kingdon [2000], Grootaert [1998], Ilahi [1999], Ilahi and Sedlacek [2000], Ray [2000]. As an exception, Munshi and Rosenzweig [2004] find that among lower-caste Marathas, girls are more likely than boys to receive a modern English education, as opposed to a traditional Marathi one. The arguments of this paper are broadly consistent with both types of findings.

[2]In this paper, we shall assume that there is gender wage inequality, while noting that the empirical evidence for this claim is mixed. Kingdon [1998] and Nasir [2002] found evidence for lower returns to girls' schooling in India and Pakistan respectively, whereas Behrman and Deolalikar [1995] and Aslam [2009] found the opposite to be true in, respectively, Indonesia and Pakistan.

[3]In South Asia, among the age group of 15 years and more, Afghan females have an average of 1.5 years of schooling with 80% receiving none; Indian females have an average of 4 years with 45% receiving none; Pakistani females have 4.3 years with 51% receiving none, Nepal has 3.5 years with 50% receiving none. Apart from Sri Lanka, which is well known for its socioeconomic progressiveness, Bangladesh is the only major country in which female achievement exceeds primary level (at 5.6

regression lines suggest that the *marginal* impact of per-capita income declines at low combinations of per-capita income and educational achievement – until roughly 1000 USD of income and 2 years of schooling – and only begins to show a steady increase after this threshold has passed. Moreover, they also show that over roughly this same interval, the educational gender gap widens.

In a paper that analysed the impact of economic growth on gender inequality across different levels of education, Dollar and Gatti [1999] found that while economic growth generally reduced gender inequality, in the 'important' area of secondary education, females tended to lag behind males until a threshold level of per-capita income of about 2000 USD had been passed.[4]

Gender gaps in enrolment are listed by level of education for a selection of low and lower middle income countries belonging to Africa, South Asia and Southeast Asia respectively. The first sub-column under level of schooling pertains to levels and the second sub-column to the average annual rate of increase in gender parity since 1999.[5]

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years) and "only" 35% of females receive no education (Barro and Lee [2011]).

[4]Dollar and Gatti [1999] also split their sample in half on the basis of PPP-adjusted per-capita income. They found that there was no statistical relationship between female enrolment and per-capita income, controlling for other factors, for the poorer sub-sample but a significant positive one for the richer one.

[5]Average rates of change in gender parity were taken to smooth out the year-to-year noise, provided that there were not too many gaps in the data series, as was the case for Afghanistan in tertiary education.

Female:Male Enrolment Ratio, 2011						
	Primary		Secondary		Tertiary	
Country	Level	Average Annual Growth[a]	Level	Average Annual Growth[a]	Level	Average Annual Growth[a]
Burkina Faso	93	2.33	78	2.11	50	2.83
Guinea	87	2.27	64	4.4	36	8.4
Ghana	100	0.65	91	0.98	62	3.55
Afghanistan	71	5.97	55	8.605	23	N/A
Bangladesh	N/A	N/A	117	1.33	70	2.26
India[b]	100	2.25	92	2.48	73	1.53
Pakistan	82	1.88	73	-0.65	91	0.75
Cambodia	95	0.72	85[c]	5.07	61	6.04
Indonesia	102	0.49	100	0.47	87	-0.013
Lao PDR	74	3.75	94	0.83	86	1.72

One feature of the data is that, Pakistan and Lao PDR excluded, disparity increases with level of education. Second, apart from countries that already had high gender parity in primary enrolment by 1999-2000 (Cambodia = 87%, Ghana = 93% and Indonesia = 97%) the other countries have shown increases at the average rate of 1.9%-3.8% per annum in this ratio. This suggests that primary education is more or less moving towards equality albeit the Muslim South Asian countries of Pakistan and Afghanistan are lagging.

In the case of secondary and tertiary enrolment, there appears to be a wider dispersion in both levels and trends. Pakistan and Indonesia have the highest parity ratio in tertiary education but the data for both show significant ups and downs in this ratio, resulting in trends that are either low or negative.[6] Also note that the countries that had the lowest gender parity in higher education at the end of the millennium (Cambodia = 32% in 2000, Burkina Faso = 30% in 1999, Guinea = 19% in 2003) showed the highest growth rate but that is more a matter of having started with low

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[6]In particular, Pakistan's reported gender gap in tertiary education is quite anomalous in light of its poor performance in other areas relating to gender parity in economic affairs, such as secondary education and labour force participation, discussed later.

denominators in the growth rate calculation

It is also worth noting that countries that are showing a reversal of the gender gap in secondary education, Bangladesh and Indonesia, are countries in which concerted efforts have been made to expand educational opportunities and social protection programmes over the last couple of decades.[7] Even in these countries, however, the gap between male and female tertiary enrolments remains considerable. All in all, the above data suggest that with respect to the above selection of low and lower middle income countries gender disparity in enrolments continues to be significant at all levels for a few countries, at secondary levels for a larger group and at tertiary levels for almost all.

Since within a country higher levels of education tend to be the preserve of progressively higher-income households, who also tend to have smaller family sizes, one would expect gender gaps to be narrower at this level, even if the country itself is a lower income country in which overall enrolments are low. That it does not, weakens the hypothesis that gender gaps are due to rationing by income- and credit -constrained households and implies that there might be other factors that impede the progress of female education as incomes grow, both cross-sectionally within a country and inter-temporally as its economy grows.

In this paper, we analyse the effect that the anticipation of marriage has on the gender gap in schooling. We show that marriage exacerbates labour market inequality by inducing a household division of labour that encourages females to spend time doing housework. In our model, formal education has the conventional monotonic, linear effect on market earnings, but a non-monotonic, inverse U-shaped effect on household skills. This creates the potential for an anti-female bias in education. We show that the bias is multiplicative in that marriage can lead to an even stronger anti-education

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[7]However, Haq and Rahman [2008] report that while secondary school enrolments are higher for girls than for boys in Bangladesh, retention and completion rates show an opposite pattern.

effect than if the same female was expected to remain single. It is also discontinuous in the gender wage gap, *i.e.* a progressive reduction in wage inequality does not progressively eliminate the education gap.

We further compare female educational outcomes across two scenarios concerning marriage formation. In the first, a solitary pair of male and female are exogenously assigned to marry, with their optimising decisions on education and labour supply following from this inevitability. In this case, each of them receives a share of rivalrous household resources that is also set exogenously. In the second scenario, marriage formation requires the couple's mutual consent. In this case, the division of rivalrous resources becomes an important component of the decision to marry, and is endogenously determined via pre-nuptial negotiation between the couple.

We refer to the first scenario as *exogenous marriage formation* and the second interchangeably as *consensual marriage formation* or in brief *consensual marriage*.[8,9]

In the context of consensual marriage, we show that each partner's share of rivalrous resources depends on their respective bargaining power and that this dependence can create additional constraints on female education. The crucial assumption here is that agents derive a degree of private utility from education, in addition to its human capital-enhancing benefits and any contribution it makes to marital companionship. Under exogenous marriage formation a female's optimal education level is determined

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[8]The term 'exogenous marriage' has been used by some authors to refer to what is more precisely known as 'exogamy': marriage outside one's own kinship group (see, *e.g.*, Herlihy [1995]); to avoid confusion we use 'exogenous' to condition the term 'marriage formation', which has been used by other authors, *e.g.*, de Moor and van Zandan [2010], to refer to the mechanism through which couples are assigned to marriage. To our knowledge no semantic confusion arises between the terms 'consensual marriage formation' and 'consensual marriage' so these will be used interchangeably.

[9]The term 'consensual' has been used by de Moor and van Zandan [2010] in describing what they formally call the 'European Marriage Pattern' (EMP). EMP is characterised by the following conditions: (i) marriage formation is based on mutual consent, as opposed to parental or clan determination; (ii) on marrying a couple forms a new household distinct from their respective parental ones; (iii) the internal household relationships are based on implicit and explicit contracts between husband and wife and between parents and children and (iv) these contracts in turn depend on power balances between household members which are in turn influenced by socioeconomic, ideological and institutional factors. Our model of consensual marriage meets all the above conditions.

without taking into account the effect it will have on her spouse's utility. Under consensual marriage creation, by contrast, both partners will take into account how the other's welfare is affected by their own choices. Take the case where marriage leads the female to specialise in housework. From the male's point of view, her optimum education will be the one that maximises her housework skills but the female herself would prefer a higher level, depending on the strength of her private taste for education. In anticipation of pre-marital bargaining, the female's parents could restrict her education in order to make her more attractive to her partner and strengthen her bargaining power over the rivalrous resources created by their union.

We also show that if the market wage goes up, with no change in the degree of discrimination, her level of education might fall. This is because an increase in the market wage can, under plausible circumstances, increase the value of the male's outside option more than it does the female's. Of course, if the wage is high enough so that a married female supplies positive labour, the need to contribute through housekeeping skills becomes less important and her education can respond positively to further increases in the market wage.

The restrictive impact of consensual marriage on female education need not apply to all levels of schooling. Rather we assume that up to a point, exposure to formal schooling complements hands-on experience in the acquisition of domestic skills. There is an optimal combination of the two at which domestic skills are maximised. It is only after this level has been passed that a conflict will arise between a female's education and her domestic skills. The potential for such a conflict is greater at high levels of schooling than at low ones.

Lahiri and Self [2007] have also argued for the disincentive effect of marriage on female education Their argument is based on patrilocal living arrangements that lead parents to discount daughters' education on the grounds that their income after

marriage will contribute to their in-laws' household, while sons will contribute to their natal households.[10]

By contrast, Behrman *et. al.* [1999] argue that the prospect of marriage can encourage female education. This is because women play a role in providing home schooling to children. Thus more educated women are more desirable on the marriage market as they are likely to be more effective home teachers. One implication of their model is that with economic growth, the demand for sons' education will go up, leading to an increase in demand for educated brides and mothers, even if their own labour market participation is low. Using data sets from India, they find evidence in support of their hypotheses.

Behrman *et. al.* seem to negate our main argument, especially since our paper shares important features with theirs: namely, gender wage inequality, specialisation in household duties and marital selection based on female education. Yet the papers are not mutually incompatible. For one thing, Behrman *et. al.* focus on agricultural settings in which both the level of education needed to enhance farm productivity and the mother's own education level are implicitly at the primary level. Our paper is more applicable to urban middle class settings in which men earn enough to allow their wives to specialise in home production. In these settings, a primary level of education is a foregone conclusion for both genders.

Moreover, if we interpret the household production function of our paper as incorporating the task of home schooling then Behrman *et. al.'*s argument implies that as wages increase, the optimum level of education to maximise household skills also increases. Indeed, if this effect is strong enough then it can lead to an increase in female education at even post-primary levels. Having acknowledged that possibility,

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[10]Lahiri and Self [2008] extend this line of argument to explain the anti-female bias in survival ratios. They argue that in the presence of costly health care, both labor market discrimination and an inter-household externality can lead to such a bias.

our paper identifies a channel through which a private taste for education interacting with pre-nuptial bargaining will, given a fixed technology for the production of household skills, create a countervailing effect against female education. Which of the two effects dominates is an empirical matter but the point is that these two channels are not mutually exclusive.[11]

Chiappori *et. al.* [2009] also provide a rationale for the pro-education effect of marriage. This result follows from the effects of marital sorting on schooling choice when schooling not only enhances labour market returns but also the share of the marital surplus accruing to the relevant spouse. While that paper's analysis of the sorting process is more detailed than ours, it's model restrict education to a binary 0-1 choice with no variation across levels of education. The authors also treat the marital surplus as a black box which increases in the combined education of the two spouses, without decomposing it into a rivalrous and non-rivalrous goods as we do. Thus the possibility that the marriage surplus might be non-monotonic in the combined schooling of the two spouses is not present in their model while in ours, it is precisely that possibility that creates a tension between a female's education and her marriageability and might lead her to lower her educational level in order to enhance her household skills and increase her bargaining power on the marriage market.

In economic theory, formal education is assumed to enhance marketable human capital. If one applies this interpretation strictly, education should be measured not just by years of schooling but by years weighted by a metric of marketable skills which differs according to subject specialisation. Marital matching might then operate through the couple's respective fields of study and not just their years in education. In that case, less 'formal' schooling might mean less exposure to subjects that enhance mar-

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[11]Indeed, even in our given technology for producing household skills, dropping the private desire for education and adding explicit costs to schooling could result in a female receiving too little education and then our model could yield predictions as to the effects of economic growth on female education that are similar to those of Behrman *et. al.*

ketable human capital and more time studying subjects that enhance domestic skills. Folbre and Badgett [2003] and Fisman *et. al.* [2006] have, in separate experiments, linked gender differences in fields of study and occupational choice with differences in how males and females rate each other in terms of attractiveness. While the former authors found that both men and women who reported studying for or holding non-stereotypical occupations were rated as less attractive by members of the other gender, the latter found that females are more likely to select male partners on the basis of higher intelligence and ambition while males are more likely to use physical attractiveness as a criterion. Moreover men did not value women's intelligence or ambition when it appeared to exceed their own. Although the context of our paper is different from the above two, its arguments shed some light on their findings.

The rest of the paper is organised as follows. Section 2 describes the model. Section 3 analyses the labor market and educational decisions of single individuals. Section 4 analyses the analogous decisions for individuals whose marriage is determined exogenously. Section 5 analyses the decision to marry in the context of a single male and single female and Section 6 analyses specific examples and presents numerical results for the preceding sections. Section 7 considers a marriage market in which each agent has a choice over partners. Section 8 offers concluding remarks.

2 The model.

There are two households, labelled X and Y respectively. Each has one offspring, intrinsically identical save for gender: X is male and Y is female. Each offspring proceeds through two stages of life: childhood and adulthood. In each stage of life they have a time endowment equal to unity.

In the childhood stage, the parent in each household decides the allocation of the

child's time between formal schooling and domestic training. This allocation deter-
mines the combination of labour market versus household skills that the child grows
up with.

2.1   Labour market skills and household skills.

Labelling time spent in schooling as $s \in [0,1]$, and the level of human capital as e,
we follow the literature in assuming a linear relationship, $e = s$.[12]

Household skills are denoted by α. Unlike market skills, the optimal development
of household skills requires positive inputs of both formal schooling and domestic
hands-on training. At one extreme, a child who spends all his or her time performing
domestic chores might not pick up the basic literacy and numeracy skills needed to
run a household. At the other extreme, a childhood spent entirely in formal schooling
would lead to no direct training in domestic chores.

The above properties are captured by a function: $\alpha = A(1-s)$ that satisfies:

($\mathcal{A}$-1): $A(\cdot)$ is continuously differentiable and strictly concave for all $s \in [0,1]$.

($\mathcal{A}$-2): $A(1-s) \geq 0 \; \forall s \in [0,1]$; $A(0) \equiv \alpha_0$; $A(1) \equiv \alpha_1$.

($\mathcal{A}$-3): $\exists$ a unique $s^* \in [0,1)$ such that $A(1-s^*) \equiv \alpha^* = \max\{A(1-s)\} \; \forall \; s \in [0,1]$.

Together with ($\mathcal{A}$-1), ($\mathcal{A}$-3)guarantees that household skills are maximised only when
a strictly positive amount of domestic training is undertaken along with formal edu-
cation.

The level of formal education that maximises household skills might increase with
the state of development of the society and the social class to which the household
belongs. At higher levels of development and in higher social strata, not only literacy

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[12]The literature usually assumes a proportionate relationship, *e.g.* $e = \sigma s$, $\sigma > 0$. Our purpose
is served by normalising σ to unity.

Figure 1: Household Skills and Market Skills
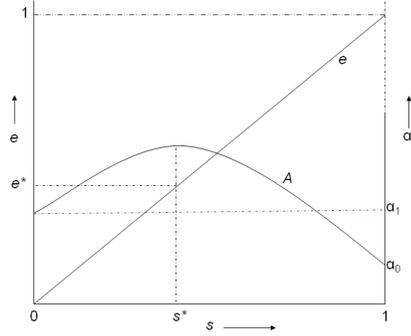
and numeracy but also exposure to the arts, history and literature might be important for the development of domestic skills. What is important is that there is some point at which further development of market skills comes into conflict with the formation of household skills.[13]

Figure 1 illustrates the combinations of household skills and formal education that result from any given choice of s. The horizontal axis measures time in schooling, s, which lies between zero and unity. The vertical axis on the left measures formal education, e, which lies between zero and unity, increasing linearly with s. The vertical axis on the right measures housekeeping skill, α. Time spent in housework as a child decreases as s rises. Accordingly, at $s = 1$, $\alpha = \alpha_0$. At $s = 0$, $\alpha = \alpha_1$ and at $s = s^*$, $\alpha = \bar{\alpha}$.

At any value of s, α and e are uniquely related. This relationship may be expressed as :

$$\alpha = A\left(1 - e\right)$$

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[13]Formally, the results of this paper merely require that household skills are a continuous function of $1 - s$ and that they are not maximised at $1 - s = 0$, i.e. there is some region of a child's time allocation over which formal education and domestic training come into conflict in the formation of such skills. Dale [2009] takes a similar view of the relationship between formal education and household skills, although unlike us, assumes that both formal education and hands-on training contribute up to a point to the development of both market and household skills.

where $\alpha = \alpha_0$ when $e = 1$, $\alpha = \alpha_1$ when $e = 0$ and $\alpha = \bar{\alpha}$ when $e = e^*$.

In the second period, the adult decides whether to marry or not, moves out accordingly and makes a time-allocation decision between market work (ℓ) and housework $(1 - \ell)$. This decision is made conditional on the individual's marital status and market wage. The latter in turn depends linearly on an underlying wage ω_i and the individual's human capital e_i through the linear function:

$$w_i = \omega_i e_i \tag{1}$$

where w_i represents the hourly wage, and ω_i is an underlying return to human capital.

Wage inequality implies that ω is not the same for both genders. A parametric manner to express this is that there is an underlying market wage ω and that $\omega_X = \omega$ while $\omega_Y = \phi\omega$, $\phi \leq 1$.

2.2 Preferences and constraints.

We assume that childhood utility is separable from adult utility and does not depend on the choice between schooling and household chores. Each adult has a utility function:

$$U_i = u(c_i) + h_i + be_i \tag{2}$$

$i = X, Y$; $u(\cdot)$ is a concave function of the adult's consumption of a rivalrous market good, satisfying $u'(0) = \infty$; h_i is the utility from consumption of a household good.

The last term in the utility function represents a non-pecuniary private benefit that an adult derives from being educated. The possibility that education confers private benefits on top of marketable ones is often ignored in the labour market and develop-

ment literatures but is not without precedent in the broader literature on education.[14]
These benefits could be direct: education confers pride and satisfaction for its own
sake, or indirect: education enables individuals to seek out information which leads
to better choices.

An example is the link between education and health explored in the health literature.
As summarised in Cutler and Lleras-Muney [2006], a large body of evidence exists
for a positive relationship between health and education, even after accounting for
the effects of education on income. Their argument is that education leads to better
decision-making and information-seeking and thus helps individuals maintain good
health.

This assumption plays an important role in our analysis of female education through-
out the paper but especially in comparing outcomes across different types of marital
institution. We shall show that the presence of a private benefit from education leads
to an externality in the case of married agents: each agent values their partner's
education less than the partner does. The relevance of this externality is related to
the labour market participation of each spouse. For example, if a married male spe-
cialises in market work while a married female specialises in housework, the male's
private preference for education will be inconsequential to the female, but not the
other way around. This asymmetry will affect the marital decision-making process
for each prospective spouse.

For a single individual, market consumption is:

$$c_i = w_i \ell_i e_i; \quad i = X, Y; \tag{3}$$

where ℓ denotes the fraction of adult time spent in market work.

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[14]See, for example, early attempts to empirically disentangle these two effects by Schaafsma [1976]
and Lazear [1977].

If married, the consumption of the market good will be subject to a market budget constraint:

$$c_X + c_Y = w_X \ell_X e_X + w_Y \ell_Y e_Y. \tag{4}$$

The utility from consumption of the household good depends on the amount of effort put into household production according to a concave function. For a single individual, the utility is given by

$$h_i = h\left(\alpha_i(1 - \ell_i)\right) \tag{5}$$

where α_i is the individual's level of household skills and $(1 - \ell_i)$ is adult time spent in household production. We assume that $h' > 0$, $h'' < 0$ and that $h'(0) = \infty$.[15]

For married households, we assume that there is (i) a single production function per household; (ii) the ability-adjusted effort levels of each spouse are mutual perfect substitutes and (iii) the household good is a pure public good.[16]

$$h_X = h_Y = h\left(\alpha_X(1 - \ell_X) + \alpha_Y(1 - \ell_Y)\right) \tag{6}$$

2.3 Decision making.

We first impose the state of being married or remaining single and within each state we solve for the agents' time allocations, taking as given their levels of schooling; then in solving for the latter we take into account its effect on adult time allocation.

We assume that regardless of whether their union is formed exogenously or consensually, couples decide on their respective time allocation in a unitary fashion. This

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[15]The concavity of $h(\cdot)$ can be interpreted in two ways: either that the household production technology is itself concave or that the utility of household goods is concave, or both.

[16]In another paper, Jafarey [2008], the assumption that the household good is a pure public good is relaxed and a more complete analysis is undertaken of the time and educational decisions of married couples under both wage equality and wage discrimination.

assumption is made partly to elaborate on Becker's [1973] approach by exploring the role of wage inequality in creating the mutual comparative advantage that each partner brings to marriage, which then leads to a household division of labour on the basis of efficient time-use by each partner. It is also made partly because assuming non-cooperative decisions on time use could by itself lead to some of the asymmetries in outcomes that characterise our results. Basu [2006] has already shown that with non-unitary decision-making, the allocation of household resources depends crucially on each members contribution to household income. In addition, Rainer [2008] has shown that under conditions of wage discrimination, intra-household bargaining leads to a magnification of gender inequalities in time allocation. In that sense, the results of this paper show that such a magnification effect can arise even when the time allocation of each spouse is jointly welfare-maximising. Furthermore, Behrman *et. al.* [1999] show that when education increases spousal bargaining power, prospective husbands become biased against female education for that reason. Our arguments pursue a different channel for such effects so assuming unitary decision-making ties our hands against the predicted outcome.

Moving back from the time allocation decision, we consider the marital decision, comparing outcomes under exogenous marriage formation with those under consensual formation. We have already noted the importance of side payments in the latter case. Since there are no bequests or outside assets in our model through which transfers such as dowries could be financed, the share of each partner's consumption in the rivalrous market good becomes the main source of side payments. We assume that a enforceable contract can be written over these shares through a pre-nuptial agreement between the spouses. We also assume that the exact division is the one that equalises the gains from marriage relative to an appropriate outside option for each spouse. We initially assume that each prospective spouse's only outside option is to remain single, in other words, there is no choice over marriage partners. We later

extend this to a setup in which each partner faces a rival in the marriage market.

In the last step of our analysis, the education decision itself is analysed. In principle this should take into account the impact of each child's education on his or her future time use in each state of adulthood, along with its effect on his or her post-nuptial share in consumption in case of consensual marriage. In practice, this is done in all states only for a female; for a male we impose a corner solution on both labour supply and education in all but the state of remaining single.

In discussing the various cases, we shall refer to the "single self" of a married agent as the same agent had he or she not got married and likewise, the "married self" of a single agent as that agent had he or she got married.

3   Single agents.

On reaching adulthood, the agent's human capital e has been fixed, as has α (agent subscripts are suppressed since the problem is qualitatively identical for both). The agent maximises utility with respect to labor market participation, ℓ.

Plugging the adult budget constraints into the utility function, the maximisation problem is expressed as:

$$\max_{\ell} = u(\omega e \ell) + h(A(e)(1 - \ell)) + be$$

which has first-order condition:

$$u'(\cdot)\omega e - A(e)h'(\cdot) = 0 \tag{7}$$

For the single agent, the first-order condition will hold as an equality since speciali-

sation is ruled out by Inada conditions.

The interpretation is analogous to the one in the standard case of endogenous labor supply when leisure counts for its own sake. Here, the trade-off becomes one between market and home labor. A small increase in market labor increases utility from the market good by $u'(\cdot)\omega e$ but reduces that from the home good by $h'(\cdot)\alpha$. At the optimum, the two effects cancel out.

The resulting solution can be expressed as $\ell(\omega, e)$. As is well known, a non-monotonic relationship between labor supply and the market wage is possible. A necessary and sufficient condition to rule this out is:

($\mathcal{A}$-4): $u'(c) + u''(c) \cdot c > 0$

Under ($\mathcal{A}$-4) it can be established that $\ell_\omega > 0$ (see Lemma 3, Appendix). Since changes in e can also affect equation (7) through the home production function, ($\mathcal{A}$-4) is by itself not sufficient to rule out $\ell_e \leq 0$ but imposing a further sufficient condition ensures that $\ell_e > 0$:

($\mathcal{A}$-5): $h'(\alpha(1 - \ell)) + h''(\alpha(1 - \ell))\alpha(1 - \ell) > 0$

($\mathcal{A}$-4) and ($\mathcal{A}$-5) are in line with conventional restrictions imposed to prevent 'backward bending' labor supply and allow us to set benchmarks for comparing time allocation across genders and marital states.

The decision on education is taken by a parent during childhood.[17] We assume the parent takes into account the implications of education on adult outcomes. The problem can be expressed as:

$$\max_{e} = u(\omega e \ell(e)) + h(A(e)(1 - \ell(e))) + be$$

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[17]This is mainly for expositional purposes. We could equally have the child taking it themselves, so long as we maintain the assumption that during childhood the child does not internalise the welfare of the prospective spouse.

The first-order condition is:

$$u'(\cdot)\omega\ell + b + h'(\cdot)(1-\ell)A'(e) \geq 0 \qquad (8)$$

The first-order condition can only be satisfied at $e \geq \hat{e}$. But there is no incentive to choose $e < \hat{e}$ since $\hat{e}$ is the amount of education where household skill α is maximised. If the first-order condition is satisfied with equality, $e \leq 1$. If, as a strict inequality, $e = 1$. The intuition is that a small increase in education will increase the utility from consumption, at given wages and market labor supply by an amount $u'(\cdot)\omega\ell$ and the private utility from education by b, while the utility from home production will fall, at given levels of home work and production, due to a fall in home skills α by an amount $A'(e)$. These effects cancel out at the optimum.

Proposition 1 below establishes the effect of wage inequality between otherwise identical agents

Proposition 1: Suppose that ($\mathcal{A}$-4) and ($\mathcal{A}$-5) hold, then an increase in the wage, ω will, for given underlying characteristics, lead to higher education e and more labour supply ℓ.

Proof: See Appendix.

The intuition behind Proposition 1 is that a higher wage tilts the first-order condition with respect to education towards acquiring more market skills and less household skills. For given education, it also tilts the balance in favour of supplying more market labour (due to ($\mathcal{A}$-4)) and less household work. Under ($\mathcal{A}$-5) the latter will be reinforced by the increase in education. Thus on the whole, market labour will also go up. Applied to gender wage inequality, Proposition 1 implies that a single male will acquire more education and supply more labour than an intrinsically identical single female. Because wage inequality is the only source of asymmetry between

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the single male and the single female, as the wage gap narrows so will the gap in outcomes, until a benchmark of wage equality and symmetric outcomes is reached.

Form hereon, whenever the context does not by itself make clear whether we are referring to the cases of single agents, exogenously formed marriages or consensually formed ones, superscripts will be used to distinguish them: single selves' outcomes are labeled e_i^s and ℓ_i^s; while married selves' outcomes are labeled as either e_i^a and ℓ_i^a, or e_i^m and ℓ_i^m, $i = X, Y$, depending on whether marriage formation is exogenous $(^a)$ or mutually consensual (m).

4   Exogenous marriage formation.

As explained before, we assume unitary decision-making in regards to the time use of married adults. The household utility function can be expressed as:

$$V = \{u(c_X) + h_X + be_X\} + \{u(c_Y) + h_Y + be_Y\}$$

Only the first four terms of the utility function are affected by the choice of labor market participation. Given the exogenous nature of marriage in this section, we further assume for the sake of saving on notation that the market good is shared equally: $c_X = c_Y$.[18]

The time allocation problem now reduces to:

$$\max_{\{\ell_X, \ell_Y\}} V = 2u\left(\frac{\omega e_X \ell_X + \phi\omega e_Y \ell_Y}{2}\right) + 2h\left(A(e_X)(1-\ell_X) + A(e_Y)(1-\ell_Y)\right)$$

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[18]This allocation would result from unitary household decision-making, given symmetric utility functions over the consumption of the market good and a utilitarian rule which assigned equal weights to each partner's utility.

which has first-order conditions:

$$u'(\cdot)\omega_i e_i - 2A(e_i)h'(\cdot) \geq 0;$$

which is more usefully rearranged as:

$$\frac{\omega_i e_i}{A(e_i)} \geq \frac{2h'(\cdot)}{u'(\cdot)}; \qquad (9)$$

$i = X, Y$.

In choosing education, we assume that the parents care only for the welfare of their own offspring. We also assume that parents take into account the effect of their own child's education on own adult labor supply but not the spouse's. Each parent maximises the following function:

$$\max_{\{e_i\}} u\left(\frac{\omega_i e_i \ell_i(e_i) + \omega_j e_j \ell_j}{2}\right) + h\left(A(e_i)(1 - \ell_i(e_i)) + A(e_j)(1 - \ell_j)\right) + be_i; \quad i, j = (X, Y)$$

The first-order condition is (terms involving the effect of e on adult labor supply drop out when evaluated at the optimum):

$$0.5u'(\cdot)\omega_i \ell_i + h'(\cdot)(1 - \ell_i)A'(e_i) + b \geq 0; \qquad (10)$$

$i = X, Y$. It is clear from equation (9) that when $\phi < 1$, both spouse's first-order conditions cannot simultaneously hold as equalities. This means that an interior solution for labour supply cannot simultaneously hold for both spouses: either one will specialise in market work, or the other in housework or both. Which spouse does what can be determined by considering the implied inequalities in (9). Suppose that when $\phi < 1$, $\omega_X e_X/A(e_X) \geq \omega_Y e_Y/A(e_Y)$. In that case. three possible combinations of spousal time use exist (assuming interior solutions for both market work and

housework at the level of the household):

$$
\begin{array}{lll}
\text{(i)} & \dfrac{\omega_X e_X}{A(e_X)} > \dfrac{\omega_Y e_Y}{A(e_Y)} = \dfrac{2h'(\cdot)}{u'(\cdot)}; \\[2ex]
\text{(ii)} & \dfrac{\omega_X e_X}{A(e_X)} > \dfrac{2h'(\cdot)}{u'(\cdot)} > \dfrac{\omega_Y e_Y}{A(e_Y)}; & \qquad (11) \\[2ex]
\text{(iii)} & \dfrac{\omega_X e_X}{A(e_X)} = \dfrac{2h'(\cdot)}{u'(\cdot)} > \dfrac{\omega_Y e_Y}{A(e_Y)}.
\end{array}
$$

In case (i) $\ell_X = 1$ while $\ell_Y \geq 0$; in case (ii) $\ell_X = 1$ while $\ell_Y = 0$; in case (iii) $\ell_X \leq 1$ while $\ell_Y = 0$. Either the male specialises in market work, or the female in housework or both.

Proposition 2 below in fact establishes that when $\phi < 1$, then $e_X > e_Y$ in turn implying that $\omega_X e_X / A(e_X) \geq \omega_Y e_Y / A(e_Y)$ (since education is always chosen along the decreasing portion of the household skills curve, $A(e)$).

Proposition 2: In the presence of wage discrimination $(\phi < 1)$ the higher paid spouse chooses an education level at least as high as that chosen by the lower paid spouse $(e_X > e_Y)$.

Proof: See Appendix.

While Proposition 2 covers all three cases concerning the division of tasks by a married couple, we shall henceforth neglect case (iii) in favour of the cases of our main interest.

How do the labor market effort and educational levels of the married female compare with that of her single self?[19] Conditioning on a given level of female education, the comparison of her market labour is stated in Proposition 3.

Proposition 3: Given the same level of female education e_Y in both possible states regarding marital status, a married female supplies less market labour than a single

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[19]A comparison between the married male and his single self is made in (Jafarey [2008]) where the output of household production is allowed to be partly rivalrous. In this context, it is shown that so long as the male wage is sufficiently high or the public good nature of housework is not too large, then the married male will in all three cases of marital specialisation do more market work and receive more education than his single self at a comparable wage.

one.

Proof: See Appendix.

The intuition behind Proposition 3 is that even if the married female were to have the same level of education as her single self, it would be efficient from the marital household's point of view to make use of her spouse's comparative advantage in providing market income by allocating her to do more housework than she would as a single female.

In comparing education choice between married and single females, a straightforward comparison of equation (8) the first-order condition for a single female's education, and equation (10), the analogous condition for a married female, suggests that at given values of e_Y and ℓ_Y, the LHS of the former equation exceeds that of the latter. This is because for a married female, the marginal impact of education on market income is discounted relative to that of her single self.[20] Other terms in the two equations are identical for both selves.[21] Thus, at given levels of market participation, a married female will receive less education than her single self. In fact, especially in light of Proposition 3, we expect that a married female's education will be further discounted because of lower market participation than her single self.

This intuition is formalised in Proposition 4.

Proposition 4: When married to a male who specialises in market work, a married female will attain less education and supply less market labour than her single self.

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[20]This is also true for a married male relative to his single self. The difference is that when a married male specialises in market work, his first-order condition for education will hold as an inequality regardless of the discount.

[21]Jafarey [2008] considers a setting in which the household good is partly rivalrous. In this case, the last term in equation (10) will, all else equal, be smaller than its counterpart in equation (8), indicating that some of the cost of greater education as reflected in lower household productivity is borne by the spouse. This creates a pro-education externality which can lead a married female's education to exceed that of her single self, especially if the private benefit from education is high enough.

Proof: See Appendix.

Proposition 4 notwithstanding, the next section will go on to argue that consensual marriage might restrict a married woman's education even more than exogenous marriage formation.

Finally note that the labor supply of married adults reacts discontinuously to gender wage discrimination. Equation (11) has established the existence of specialisation in time use when $\phi < 1$. On the other hand, if there was gender wage equality, *i.e.* $\phi = 1$, then the first-order conditions for both spouses would be symmetric and it can be shown that $\ell_X = \ell_Y$ and $e_X = e_Y$.[22] This shows that due to the possibility of specialisation, even a small amount of wage discrimination can discontinuously induce corner solutions in the time use and education levels of one or both married agents. This is not true for their single selves, since their labor market and education levels vary continuously and identically with their respective wages . Thus as $\omega_Y \longrightarrow \omega_X$, the time allocation and education of single agents converge towards each other.

5 Consensual marriage.

We now endogenise marriage formation while continuing to base our analysis on a solitary pair of male and female. In Becker's [1973] analysis, it is implicit that potential partners differ in their respective labour and homemaking characteristics, so that married couples gain by exploiting these differences on the basis of mutual comparative advantage. With gender wage inequality, asymmetry creeps in through differences in time allocation and gets reinforced by education choices, even if potential partners are intrinsically identical. As a result of both the wage and the educational gap, the male partner may end up with a comparative advantage in market work and

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[22]See Jafarey [2008].

the female in housework.

With consensual marriage, a selection criterion needs to be satisfied for both partners:

$$U_i^m \geq U_i^s \quad i = X, Y.$$

where U^m stands for the utility from entering into consensual marriage and U^s is the utility from staying single.

We assume that the couple are able to realise any potential gain from marriage by making a binding pre-nuptial agreement on post-marital outcomes. Since neither has a direct preference over how to use their time, we assume that they agree to let their respective duties be determined by a unitary decision-making process. But since they do care about their own consumption of the market good, they can bargain over their share of it. Let μ denote agent Y's share of the market good, so that $(1 - \mu)$ is agent X's share.[23] In the context of this model, where there are no external assets, bequests or other means to make side transfers, this is the only form in which such payments can be made in order to realise the possible gains from marriage. In addition, we assume that the pre-nuptial bargain results in consumption shares that split the gains from marriage equally between the two partners. Because of the existence of only one possible spouse, the outside option for each agent is the maximum utility they could derive from remaining single. The next section extends this to a set-up in which the outside option includes a choice of partners.

Given μ, the time allocation of the married couple follows analogously to the case of exogenous marriage. The difference is that the endogenous consumption share replaces the exogenously determined 50:50 split that was employed in the case of

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[23]An alternative mechanism would be to allow an agreement on the weights ψ and $1 - \psi$, that would be assigned to their respective levels of utility in the unitary household optimisation problem. For the functional forms assumed later in this paper, it can be shown that equivalence exists between the two forms.

exogenous marriage formation. The first order conditions for the optimal time allocation lead to the following analogous cases.

$$\text{(i)} \quad \frac{\omega_X e_X}{\alpha_X} > \frac{\omega_Y e_Y}{\alpha_Y} = \frac{2h'(\cdot)}{\mu u'_Y(\cdot) + (1-\mu)u'_X(\cdot)};$$

$$\text{(ii)} \quad \frac{\omega_X e_X}{\alpha_X} > \frac{2h'(\cdot)}{\mu u'_Y(\cdot) + (1-\mu)u'_X(\cdot)} > \frac{\omega_Y e_Y}{\alpha_Y};$$

$$\text{(iii)} \quad \frac{\omega_X e_X}{\alpha_X} = \frac{2h'(\cdot)}{\mu u'_Y(\cdot) + (1-\mu)u'_X(\cdot)} > \frac{\omega_Y e_Y}{\alpha_Y}.$$

Three cases are again possible: in case (i) $\ell_X = 1$ while $\ell_Y \geq 0$; in case (ii) $\ell_X = 1$ while $\ell_Y = 0$; in case (iii) $\ell_X \leq 1$ while $\ell_Y = 0$. Since our focus is on those cases in which a married male specialises in market work and his education is at its maximum we shall ignore (iii).

In modeling the choice of female education, we now take into account not only its impact on the time-use decision, but also its effect on her share of the consumption good μ arising from pre-nuptial bargaining. This problem is formally stated as (superscripts m denote consensually determined marriages):

$$\max_{\{e_Y^m, \mu\}} U_Y^m = u\left(\mu\omega(1 + \phi e_Y^m \ell_Y^m(e_Y^m, \mu))\right) + h\left(A(e_Y^m)(1 - \ell_Y^m(e_Y^m, \mu))\right)$$
$$+ b e_Y^m \tag{12}$$

$$s.t.$$

$$U_Y^m - U_Y^s \leq \left\{u\left((1-\mu)\omega(1 + \phi e_Y^m \ell_Y^m(e_Y^m, \mu))\right) + h\left(A(e_Y^m)(1 - \ell_Y^m(e_Y^m, \mu))\right)\right.$$
$$\left. + b\right\} - U_X^s \tag{13}$$

where $U_i^s = u(\omega_i e_i^s \ell_i^s) + h(A(e_i^s)(1 - \ell_i^s) + b e_i^s; \ i = \{X, Y\}$. The constraint simply states that the female's gain from marriage is no greater than the male's. Relative to the case of exogenously formed marriage, this additional constraint involving μ

creates the possibility that consensual marriage can, under some circumstances, further discourage female education. Noting that the utility from consumption of the household good enters both sides of the marital selection constraint, the Lagrangean associated with the above problem can be written as

$$\mathcal{L} = U_Y^m + \lambda \Big[[u((1-\mu)\omega(1+\phi e_Y^m \ell_Y^m(e_Y^m,\mu))) + b - U_x^s] - [u(\mu\omega(1+\phi e_Y^m \ell_Y^m(e_Y^m,\mu))) - b e_Y^m - U_Y^s] \Big]$$

which has first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial e_Y^m} = \Gamma \omega \phi \ell_Y^m [1 + \xi_e] + h'(\cdot)\left[A'(\cdot) - \ell_Y^m \left(A'(\cdot) + \frac{A(\cdot)}{e_Y^m} \xi_e \right) \right]$$
$$+ (1-\lambda)b \geq 0; \tag{14}$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = \Gamma \frac{\ell_Y^m}{\mu} \phi \omega \xi_\mu$$
$$+ [(1-\lambda)u'(c_Y^m) - \lambda u'(c_X^m)] [\omega(1 + + \phi e_Y^m \ell_Y^m)] \geq 0; \tag{15}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = u(c_X^m) - u(c_Y^m) + b(1 - e_Y^m) - \Psi^s \geq 0. \tag{16}$$

where

$$\Gamma = (1-\lambda)\mu u'(c_Y^m) + \lambda(1-\mu)u'(c_X^m)$$
$$\xi_e = \frac{e_Y^m}{\ell_Y^m} \frac{\partial \ell_Y^m}{\partial e_Y^m}$$
$$\xi_\mu = \frac{\mu}{\ell_Y^m} \frac{\partial \ell_Y^m}{\partial \mu}$$
$$\Psi^s = U_X^s - U_Y^s$$

If a married female specialises in housework, and assuming interior solutions for e_Y^M

and μ, the first-order conditions reduce to:

$$\frac{\partial \mathcal{L}}{\partial e_Y^m} = h'(\cdot)\left[A'(\cdot)\right] + (1-\lambda)b = 0; \tag{17}$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = \left[(1-\lambda)u'(c_Y^m) - \lambda u'(c_X^m)\right]\omega = 0; \tag{18}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = u(c_X^m) - u(c_Y^m) + b(1-e_Y^m) - \Psi^s = 0. \tag{19}$$

Intuitively, the first equation states that a small increase in a prospectively married female's education will lower her marginal household productivity (since $A'(\cdot) < 0$ at the optimum) and also lead to a relaxation of her marital selection constraint relative to that of her prospective spouse (by the amount λb) but it will directly benefit her by the amount b. At the optimum point, the marginal benefit equals the marginal cost. The second equation describes the net benefit to a prospectively married female of a small increase in her own share of the market good. It increases her utility directly by $u'(c_Y^m)$ but reduces it indirectly by relaxing her marital selection constraint relative to that of her spouse by the amount $\lambda(u'(c_Y^m) + u'(c_X^m))$. At the optimum the net benefit must be zero. Note that an implication of the first-order condition for μ is that:

$$\lambda = \frac{u'(c_Y^m)}{u'(c_Y^m) + u'(c_X^m)} < 1$$

Turning to comparative statics, our main interest is in the effects of the underlying market wage ω on outcomes under different states and institutions involving marriage. Intuition suggests that while such an increase will increase the outside option for both males and females, the former's will increase more than the latter's due to wage discrimination. This will result, all else equal, in the male's marital selection constraint tightening relative to the female's. At the same time, at given market shares, both partners will also experience an increase in their respective marital utility from consumption of the market good. All else equal again, this will relax both

selection constraints. Overall, what happens to the respective selection constraints will determine the impact of an increase in the market wage on the endogenous variables.

In the Appendix we prove the following result

Proposition 5: Suppose that at an original equilibrium, the following conditions are satisfied

($\mathcal{A}$-6):

$$\frac{\partial \Psi^s}{\partial \omega} - [(1-\mu)\frac{\partial u'(c_X^m)}{\partial \omega} - \mu \frac{\partial u'(c_Y^m)}{\partial \omega}] \geq 0.$$

($\mathcal{A}$-7):

$$(1-\mu)u''(c_X^m)\frac{u'(c_Y^m)}{u'(c_X^m)} - \mu u''(c_Y^m) = 0$$

then an increase in the market wage, ω, will lead to a decrease in both a prospectively married female's (i) education, e_Y^m and (ii) her share of the market good μ.

Proof See Appendix.

Assumption ($\mathcal{A}$-6) covers two types of marginal effects of higher market wages on male-female differences in welfare: the first effect is on the difference between their respective outside options (both of which will increase); the second is on the difference between their respective utilities from given shares of the market good (again, at given shares both will also increase).[24] Since $\mathcal{A}$-6 comprises the difference between these two effects, a plausible condition under which it would hold is for the first effect to be positive and larger in magnitude than the second, unless the second effect is negative to begin with.

Because of concave utility, it cannot be guaranteed that the first effect in ($\mathcal{A}$-6) will hold; however it will be more likely to hold if wages are initially low and/or

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[24]Differentiating the RHS of equation (19) with respect to ω, the two terms affected are Ψ^s and $[u(c_X^m) - u(c_Y^m)]$ leading to the expression in $\mathcal{A}$-6 and the above interpretation.

gender wage discrimination is high. Since the result of Proposition 5 are themselves more likely to arise when wages are low enough (but not so low that the male does not specialise in market work) and discrimination is high enough that the female specialises in housework, it appears plausible that the first effect in $\mathcal{A}$-6 will hold.

As for the second effect, it can go either way, depending on whether the female share is initially greater or less than 0.5 and on the specific functional forms used. Indeed, at a benchmark of $\mu = 0.5$, the second effect would be exactly zero. This means that if initial consumption shares are relatively equal, the second effect will be small in magnitude whichever sign it takes. Since in our interpretation of the gains from marriage, one of the sources of female gains is the increase in consumption of the market good compared to what they would get if single, it seems plausible that any male-female gap in marital utility from consumption will be muted in comparison to the gap in outside options and that, since the second effect in $\mathcal{A}$-6 depends on the initial size of the first gap, it too will be muted.[25]

($\mathcal{A}$-7) restricts each agent's Engel curve for the market good to be linear. The implication is that an increase in the underlying wage shall, at the original consumption share, leave the ratio between the marginal utility of their respective consumption levels unaffected. From a theoretical point of view, it rules out income effects on marital shares which might influence outcomes either way. For our part, we would not want our main results to depend on such effects.[26]

Under the joint impact of these conditions, the female both accepts a lower share of the market good and presents herself on the marriage market with greater house-

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[25]In the numerical examples that are studied in the next section, the equilibrium value of μ is below 0.5 at all wages except the relatively low value, $\omega = 50$, suggesting that the second effect in ($\mathcal{A}$-) is positive in most cases. Nonetheless $\mathcal{A}$-6 appears to hold at all the relevant range of wages in which the female specialises in housework.

[26]Without ($\mathcal{A}$-7) a stronger version of ($\mathcal{A}$-6) would suffice to establish the part of Proposition 5 that relates to education. However, since ($\mathcal{A}$-7) appears sensible given our purposes and since imposing it allows us to use the weaker condition ($\mathcal{A}$-6) we have chosen this route.

keeping skills. Indeed the latter prevents her share of the market good falling even further. We now turn to specific functional forms and numerical simulations to obtain further insights into the circumstances under which these effects occur.

6   Specific forms and numerical simulations:

We start by defining specific functional forms for key aspects of the model. Utility from the market good is

$$u(c_i) = \sqrt{c_i} \quad i = X, Y.$$

The relationship between household skills and education is

$$\alpha_i = A(e_i) = \eta(e_i - e_i^2) \quad i = X, Y.$$

where $\eta > 0$ is a constant parameter. Note that $\alpha_0 = \alpha_1 = 0$ for this formulation so that strictly positive inputs of both formal education and home training are required for a child to have any household skills at all.

Production of the household good is assumed to be equal to the efficiency-weighted input of household labour. The utility from this good is given by

$$h_i = \beta\sqrt{\alpha_i(1 - \ell_i)};$$

when agent i remains single, $i = X, Y$ and

$$h_X = h_Y = \beta\sqrt{\alpha_X(1 - \ell_X) + \alpha_Y(1 - \ell_Y)} \quad i = X, Y.$$

when married. $\beta > 0$ indicates a relative preference for the household good.

The main purpose for using functional forms is to conduct numerical comparative

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static analysis and identify regions of parameter space which differentiate the two main regimes of our interest in terms of marital time use in our model. In particular we wish to characterise outcomes as functions of the underlying wage ω. However, we shall first state some straightforward analytical results.

6.1 Analytical solutions:

The following reduced-form solutions can be derived for single agents:

$$\ell_i^s = \frac{\omega_i e_i^s}{\omega_i e_i^s + \beta^2[\eta(e_i^s - e_i^{s2})]}$$

$$e_i^s = \left[\frac{\omega_i + \beta^2 \eta}{2\beta^2 \eta}\right] \left[1 + \sqrt{\frac{b^2}{b^2 + \beta^2 \eta}}\right] \quad i = X, Y$$

It is easy to verify that these solutions yield comparative static effects that are consistent with the general analysis of the single agent-case.

It is also possible to derive explicit solutions in some cases for females married by exogenous arrangement when males specialise in market work. The solution for ℓ_Y^a is

$$\ell_Y^a = \frac{\phi^2 \omega_X e_Y^{a2} - 2\beta^2 \alpha_Y}{\phi^2 \omega_X e_Y^{a2} + 2\beta^2 \alpha_Y \phi e_Y^a};$$

When $\ell_Y^a = 0$, a solution for e_Y^a can also be found.

$$e_Y^a = 0.5 \left[1 + \sqrt{\frac{b^2}{b^2 + \beta^2 \eta}}\right]$$

which reduces to $e_Y^a = 0.5$ if $b = 0$. $e^a = 0.5$ is the level of formal education at which α is maximised in this example. Thus for $b > 0$ a female's education will exceed the level which optimises her household skills, creating a potential clash between her own educational preference and one that might be optimum from her spouse's perspective.

6.2 Numerical results:

We conducted numerical simulations in order to compare outcomes across the cases of single agents, agents in exogenously formed marriage and agents in consensual marriage. Keeping other parameters fixed, endogenous variables were mapped against values of the underlying market wage starting at $\omega = 50$.

The benchmark values for other parameters are as follows.

Parameter	b	η	β	ϕ
Value	2	20	4	0.5

Except ϕ these values were not chosen on empirical grounds. In a detailed survey of the Indian labour force Bhalla and Kaur [2011] estimate the unadjusted ratio of female to male wages as approximately 0.58, and the value of ϕ is an approximation of that.

Figure 2 depicts the results for the benchmark case. The numerical values on which Figure 2 is based are listed in the Appendix, Table 8. This should be consulted in conjunction with Figure 2.

The top left panel of Figure 2 depicts male and female labour supply and education levels for single agents. The other three panels depict only outcomes for married females, since males are by construction restricted to market work in these examples. The top right panel shows the female's education and labour supply when marriage formation is exogenous. The bottom left panel depicts the corresponding outcomes along with her share of the market good when marriage is consensual. The bottom right panel magnifies the details of the female education choice in the latter case. In each panel, the horizontal axis measures the market wage.

We also studied a variant of the benchmark model with almost no labour market inequality: $\phi = 0.95$ and other parameters as before. The results are presented in

Figure 2: Single, exogenous and consensual marriage.
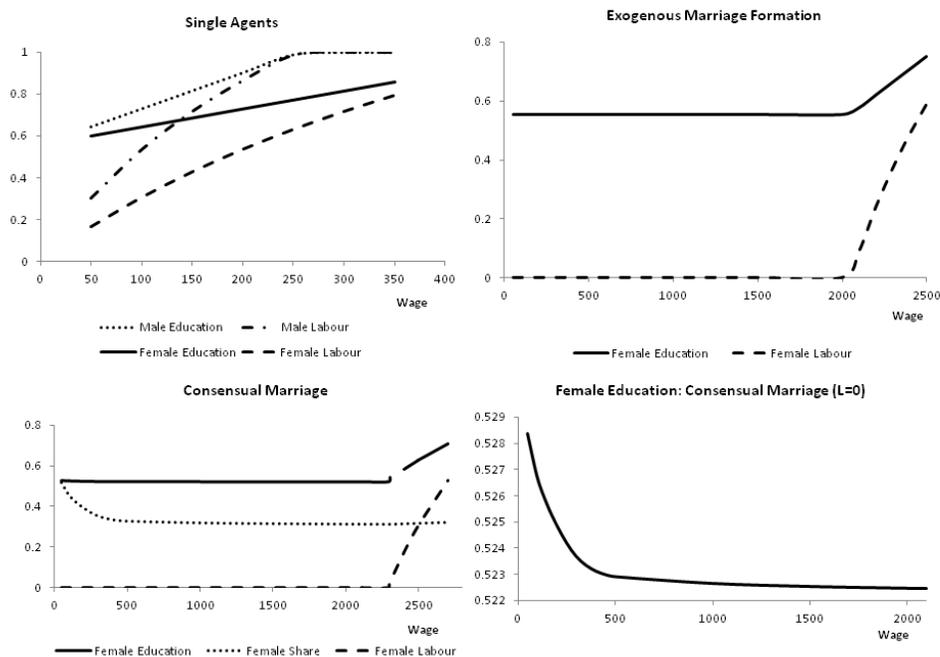
Appendix, Table 9. The results were qualitatively similar to the benchmark case and the quantitative differences were in line with expectation: a reduction in the degree of wage inequality will, all else equal, promote female education and labour market participation in all marital states and induce a married female's entry into the labour force at a lower wage.

Some noteworthy features of the results:

1) The education and labour supply of the female's single self is greater than that of both her married selves at each wage. At low wages, mutual specialisation occurs under each marital arrangement. when this happens, the education level of a consensually married female tends to lie below that of her exogenously married self, as anticipated in the previous section.

2) So long as a married female supplies no labour, her education steadily decreases with the market wage when marriage is consensual. The relationship turns upward

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once her labour supply becomes positive. This is the key effect of this section and is consistent with Proposition 5. In the case of exogenous marriage formation her educational level remains constant throughout the specialisation regime. This is because in an exogenously formed marriage, her education depends on the wage only if she supplies market labour,while in consensual marriage her education affects her relative bargaining power even when she specialises in housework

3) As the underlying wage increases, a housewife's share of the market good follows the same declining pattern as her education. This is also in line with Proposition 5.

4) The threshold wage at which a consensually married female enters the labour market is approximately $\omega = 2300$ in the benchmark version. In the case of a female in an exogenously formed marriage, this threshold arises at a slightly lower level of the underlying wage: $\omega = 2100$.

5) Once the threshold level of ω is crossed, the female's education level appears to jump up discontinuously and from there to increase with further increases in the wage. These results should be treated as illustrative, emerging as they do from numerical simulations and specific functional forms. The intuition behind the latter effect is that once a female begins to provide positive amounts of labour, she is able to meet the male's selection constraint, as represented on the right-hand side of equation (13), partly by earning market income. Thus, in her educational decision, the balance shifts towards acquiring greater market skills. Note also that the female share in consumption begins to increase once this threshold has been crossed (see Table 8). The apparent discontinuity is likely to be driven by functional forms along with the discrete nature of the manner in which wages are increased throughout the simulation.[27]

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[27]Comparing equation (14), the first-order condition for female education for a married female who supplies market labour and equation (17), the analogous first-order condition for one who does not, there are terms in the latter which drop out in the former. In particular, note the unambiguously positive term $\Gamma\phi\omega[1+\xi_e]\ell_Y^M$ which gets activated when female labour supply is positive. This term

6) Although not depicted in the diagrams, the utility levels of the two spouses move opposite to each other when comparisons are made between exogenous marriage formation (with equal shares) and consensual marriage. For the benchmark case, Appendix Table 8 shows that female utility is greater under exogenous marriage formation than under consensual marriage. However when the degree of wage discrimination is low, ($\phi = 0.95$) her utility from consensual marriage may be higher or lower than from exogenously formed marriage: the former is higher at low wages but the latter becomes higher at high wages. For the male the comparisons go the other way. These results suggest that despite the discontinuity in the effect of even a small degree of wage inequality on the marital division of labour, a reduction in inequality can enhance the female's bargaining power within the household by increasing her outside option to a level close to that of her prospective partner.

The key result of this section is that consensual marriage can lead to a negative pressure on female education, and this pressure might increase with increasing market wages when the female specialises in housework. The reason is that when marriage is subject to mutual selection constraints, an increase in the market wage can disproportionately increase the male's outside option relative to the female's. Faced with this, the female hedges between accepting a lower consumption share and choosing less education, thus moving closer to the absolute maximum in terms of housekeeping skills. This effect is reversed once she starts to contribute to the household's market income. At the same time, a reduction in wage discrimination (or an increase in female wages alone) increases a consensually married female's bargaining power even when she does no market work and this increases her education and share of the

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captures the benefit of female education for both her and her spouse's utility from consuming the market good (represented by Γ), both directly, at given labour market supply and by the positive inducement that education has on labour supply (represented by ξ_e). This effect seems to drive the sharp increase in education near the threshold. Of course, in principle this does not imply discontinuity at values of ω *arbitrarily* close to the threshold but in the simulations we did not control ω that finely, especially given that out main interest lies with the overall comparative static effects.

consumption good at given male wages.

7 A competitive marriage market:

We have thus far assumed that there is only one agent of each gender. This precludes the possibility of agents having a choice over partners. As Chiappori *et. al.* [2009] have shown, marital choice matters for both the intra-household allocation of rivalrous resources and each agents' *ex ante* investment in schooling. Choice expands each agent's set of outside options and opens each pre-nuptial negotiation to competition from potential rivals. Since we are analysing the interaction between marital selection and female education it is instructive to see the effect of competition on this interaction.

We extend the model by introducing an additional male and an additional female, one or both of whom differ from the original pair in at least one attribute. All attributes are, as before, common knowledge. Index agents belonging to the same gender as 1 and 2 and suppose that the matching process is complete, *i.e.* both types of one gender may match with both types of the other. As in the previous section, we restrict our analysis to cases in which there is mutual specialisation in time use by married couples. Hence, the total amount of the market good available to a married couple depends only on the exogenous male wage and its total output of the household good depends only on the wife's homemaking skills.

Let (X_i, Y_j) denote a couple formed by male X_i and female Y_j, $i, j = \{1, 2\}$. Let $e_{Y_j}^{ij}$ be Y_j's optimal educational choice when married to X_i and μ^{ij} be Y_j's equilibrium share of the consumption good in that match. The pair $(e_{Y_j}^{ij}, \mu^{ij})$ is defined as the household profile of (X_i, Y_j) and it completely determines the utility levels of both partners in (X_i, Y_j). Thus $U_{X_i}^{ij} = U_{X_i}(e_{Y_j}^{ij}, \mu^{ij})$ and $U_{Y_j}^{ij} = U_{Y_j}(e_{Y_j}^{ij}, \mu^{ij})$. Finally, let

$\{(X_i, Y_j), (X_h, Y_k)\}$ denote a *matching pattern* which assigns each individual to one and only one couple, $X_i, X_h \in \{X_1, X_2\}, i \neq h$; $Y_j, Y_k \in \{Y_1, Y_2\}, j \neq k$.

A *stable* matching pattern is an assignment $\{(X_i, Y_j), (X_h, Y_k)\}$ that, given its associated profiles $\{(e^{ij}_{Y_j}, \mu^{ij}), (e^{hk}_{Y_k}, \mu^{hk})\}$ satisfies

(A) : $\quad U^{ij}_{X_i} \geq U^s_{X_i}; \quad U^{ij}_{Y_j} \geq U^s_{Y_j}$

$\quad\quad U^{hk}_{X_h} \geq U^s_{X_h}, \quad U^{hk}_{Y_k} \geq U^s_{Y_k}$

and

(B) : $\quad \nexists$ *any* feasible profiles $(\tilde{e}^{ik}_{Y_k}, \tilde{\mu}^{ik}), (\tilde{e}^{hj}_{Y_j}, \tilde{\mu}^{hk})$

$\quad\quad$ associated with alternative couples (X_i, Y_k) or $(X_h, Y_j)\}$ that satisfy

(i) $\quad U^{ik}_{X_i}(\tilde{e}^{ik}_{Y_k}, \tilde{\mu}^{ik}) \geq U^{ij}_{X_i}$ and $U^{ik}_{Y_k}(\tilde{e}^{ik}_{Y_k}, \tilde{\mu}^{ik}) \geq U^{hk}_{Y_k}$; or

ii $\quad U^{hj}_{X_h}(\tilde{e}^{hj}_{Y_j}, \tilde{\mu}^{hj}) \geq U^{hk}_{X_h}$ and $U^{hj}_{Y_j}(\tilde{e}^{hj}_{Y_j}, \tilde{\mu}^{hj}) \geq U^{ij}_{Y_j}$

$\quad\quad$ with ">" for at least one of the inequalities.

(A) ensures that each agent willingly enters into marriage. (B) ensures that there are no feasible deviations from the candidate assignment and its associated profiles that result in at least one partner in the deviating couple being made better off, with the other no worse off, than under the candidate assignment.[28] If (B) fails, then at least one couple would mutually benefit by breaking away from their assigned partners.

Unlike matching models of non-transferable utility, such as Adachi [2003], in our model each agent's ranking of potential partners is endogenous, depending on both the absolute size and the division of the surplus that their union is capable of generating. In particular, the division of the rivalrous good depends on each partner's outside

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[28]This definition is adapted from Definition 5.2 in Adachi [2003] who shows that equilibrium matching patterns in search theoretic models of marriage markets converge, as search frictions go to zero, to stable matching patterns in frictionless marriage markets of the type studied in Gale and Shapley [1962]. In Adachi's case, preference orderings are exogenous. In our case, they depend on household profiles. Accordingly in our definition, the profiles generated by each household under the candidate assignment must be optimal subject to each couple's mutual constraints; however the profile of a blocking couple, denoted by ~ need not be optimal, only to satisfy constraints.

options, but these in turn depend on the surpluses and shares possible with rival candidates. Thus preferences and outcomes are mutually dependent in our model.

We therefore follow a step-wise procedure in solving the matching problem. First, we construct a matrix of household profiles and payoffs by arbitrarily matching each male with each female. At this stage, we ignore the possibility of competition from rival candidates. This effectively makes the outside option of each partner exogenous and equal to their utility from remaining single at this stage.[29]

The resulting utility levels are then used to generate a preliminary ranking of each agent's preferences over three outcomes: marriage to partner 1, marriage to partner 2 and staying single. At this point, a number of different possible combinations of rankings can arise. Rather than an exhaustive analysis of these combinations, we focus on two combinations that are both relevant to the aims of this paper and lend themselves to examples based on simple modifications of the specification introduced in the previous section.

7.1 Preferred couple:

The first combination is one in which the preliminary ranking results in one male and one female being unanimously preferred over their respective rival. Without loss of generality, suppose that these are X_1 and Y_1 respectively. The rankings for this case are:

$$U_{Y_j}^{1j} \geq U_{Y_j}^{2j} \geq U_{Y_j}^s \tag{20}$$

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[29]Although this scenario is introduced only as a first step in identifying a stable matching pattern, to avoid semantic confusion we shall henceforth refer to it as 'non-competitive matching', while the equilibrium matching pattern will be referred to as 'competitive matching' because it determines outside options endogenously via competitive offers between rival candidates. Note that both scenarios involve consensual marriage formation and the non-competitive scenario is the same as the one that was analysed in the case of a solitary couple.

for *both* $j = \{1, 2\}$ *and*

$$U_{X_i}^{i1} \geq U_{X_i}^{i2} \geq U_{X_i}^{s} \tag{21}$$

for *both* $i = \{1, 2\}$. In this case, both females (weakly) prefer X_1 and both males (weakly) prefer Y_1. We call X_1 and Y_1 the "preferred" couple.

We now come to the second stage of our solution procedure and introduce competition from potential rivals. For example, X_2 might offer Y_1 and/or Y_2 might offer X_1 a share of the consumption good that differs from the one they would have negotiated if they had been matched without external competition. Y_2 might additionally deviate from the education level that she would bring to a match with X_1 under non-competitive marriage.

In other words, if there exists an alternative profile $(e_{Y_1}^{21}, \tilde{\mu}^{21})$ that satisfies:

$$U_{Y_1}(e_{Y_1}^{21}, \tilde{\mu}^{21}) = U_{Y_1}^{11} \tag{22}$$

$$U_{X_2}(e_{Y_1}^{21}, \tilde{\mu}^{21}) \geq U_{X_2}^{22}; \tag{23}$$

(X_1, Y_1) will be blocked by (X_2, Y_1). In addition if there is an alternative profile $(\tilde{e}_{Y_1}^{12}, \tilde{\mu}^{12})$ that satisfies

$$U_{X_1}(\tilde{e}_{Y_1}^{12}, \tilde{\mu}^{12}) = U_{X_1}^{11} \tag{24}$$

$$U_{Y_2}(\tilde{e}_{Y_1}^{12}, \tilde{\mu}^{12}) \geq U_{Y_2}^{21}; \tag{25}$$

(X_1, Y_1) will be blocked by (X_1, Y_2). For $\{(X_1, Y_1), (X_2, Y_2)\}$ to be stable, no alternative profiles should exist that satisfy either equations (22) and (23) or equations (24) and (25).

In the third stage, we describe a sufficient condition to rule out such profiles. Suppose

that there exists a profile $(\hat{e}^{11}_{Y_1}, \hat{\mu}^{11})$ such that

$$U_{X_1}(\hat{e}^{11}_{Y_1}, \hat{\mu}^{11}) \geq \bar{U}^{12}_{X_1}$$

where

$$\bar{U}^{11}_{X_1} \equiv \max_{\{\tilde{e}^{12}_{Y_1}, \tilde{\mu}^{12}\}} U^{12}_{X_1}$$

$$s.t.$$

$$U_{Y_2}(\tilde{e}^{12}_{Y_2}, \tilde{\mu}^{12}) = U^{22}_{Y_2}$$

and

$$U_{Y_1}(\hat{e}^{11}_{Y_1}, \hat{\mu}^{11}) \geq \bar{U}^{21}_{Y_1}$$

where

$$\bar{U}^{21}_{Y_1} \equiv \max_{\{\tilde{e}^{21}_{Y_1}, \tilde{\mu}^{21}\}} U^{21}_{Y_1}$$

$$s.t.$$

$$U_{X_2}(\tilde{e}^{21}_{Y_1}, \tilde{\mu}^{21}) = U^{21}_{X_2}.$$

then (X_1, Y_1) cannot be blocked by either X_2 or Y_2. Since both (X_2, Y_2) prefer each other to remaining single the assignment $\{(X_1, Y_1), (X_2, Y_2)\}$ is stable.

The above condition requires that (X_1, Y_1) are both no worse off under $(\hat{e}^{11}_{Y_1}, \hat{\mu}^{11})$ than either would be under the most favourable feasible profile that could be negotiated with an alternative partner. This is a rather strong restriction and no such profile may exist, leaving their union vulnerable to competing offers. If a profile does exist, it might not be unique, especially when there is slack in one or both inequalities of the above restriction.

This leads to the fourth and final stage of the solution, which is to pin down the profile $(\hat{e}^{11}_{Y_1}, \hat{\mu}^{11})$ by defining each partner's outside option as $\bar{U}^{12}_{X_1}$ and $\bar{U}^{21}_{Y_1}$ respectively, rather

than $U^s_{X_1}$ and $U^s_{Y_1}$.[30] This ensures that, so long as the potential gains to X_1 and Y_1 from marrying each other are large enough, the resulting profile meets equations (22)-(25)

Since X_2 and Y_2 have no prospect of a rival bid from their alternative partners, their only outside options are $U^s_{X_2}$ and $U^s_{Y_2}$ respectively. In other words their equilibrium profile will be the same as the one that is calculated in the first stage.

For completeness sake, we propose a simple adaptation of the Gale-Shapley matching algorithm (see Gale and Shapley [1962]) for implementing a stable match. Start by allowing both X_1 and X_2 to propose engagement to their most preferred female, namely Y_1. Since Y_1 prefers X_1 on the basis of his subjective attributes, she accepts his proposal and rejects X_2. The latter then proposes to Y_2 who accepts. At this stage both couples enter into pre-nuptial negotiation during which time any of the four could break off the engagement. In particular, X_1 and Y_1 negotiate keeping in mind that the other has an outside option worth $\bar{U}^{21}_{Y_1}$ and $\bar{U}^{12}_{X_1}$ respectively. This generates the profile $(\hat{e}^{11}_{Y_1}, \hat{\mu}^{11})$. X_2 and Y_2, on the other hand, negotiate under the outside option of remaining single and their profile remains $(e^{22}_{Y_2}, \mu^{22})$.

We now consider an example in which the two males and two females are differentiated only by their respective taste for the household good, β. Let $\beta_{X_1} = \beta_{Y_1} = 4$ and $\beta_{X_2} = \beta_{Y_2} = 3$. The second type of each gender has a relatively lower preference for the household good. The following table lists the profiles generated by non-competitive matching.[31] The first entry in each cell is female education under the given match and the second is female share in consumption. X_1's absolutely higher utility from home production makes him a less demanding partner than X_2 in terms of consumption share but more demanding in terms of housekeeping skill; in matching with him each

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[30]Needless to say, the pre-nuptial bargain itself only determines $\hat{\mu}^{11}$ but by backward induction, this will be taken into account by Y_1 in determining $\hat{e}^{11}_{Y_1}$.

[31]All parameters except for β are at their benchmark value and the market wage $\omega = 50$.

Table 1: Profile under non-competitive matching.

	Y_1	Y_2
X_1	(0.52836,0.51620)	(0.53358,0.54641)
X_2	(0.53063,0.45445)	(0.53660,0.484595)

female receives a higher consumption share but enjoys less education than if she marries X_2. Analogously, Y_1's absolutely higher utility from home production makes her choose higher housekeeping skill (less education) and accept a lower share of the consumption good than Y_2. Males prefer Y_1 while females prefer X_1, indicating that in this case, the effect of a higher consumption share is stronger than the effect from lower education.

The following table lists the payoffs associated with the above profiles. The first entry in each cell is the male's payoff and the second is the female's. As expected, both

Table 2: Utility levels from non-competitive matching.

	Y_1	Y_2
X_1	15.8482, 15.0670	15.6864, 12.9871
X_2	13.9184, 14.7555	13.7667, 12.6858

males enjoy higher utility when matched with Y_1 (column 1 dominates column 2 for male payoffs). Both females also enjoy higher utility when matched with X_1 (row 1 dominates row 2 for female payoffs) indicating that the consumption share effect from marriage to X_1 dominates the anti-education effect.

Both partners in (X_1, Y_1) *ex ante* prefer each other to their alternative partners. However, their match must be robust to counteroffers from their less preferred partners. Let us see if this is the case. Constraining Y_2 to a payoff of 12.6858, which is what she gets from marrying X_2, she can offer X_1 a profile consisting of her own education equal to 0.53136 and her consumption share equal to 0.48572 (both are less than they would receive in a non-competitive match with him). This combination

results in X_1's utility level rising to 15.9976 which is greater than what he gets from the non-competitive match with Y_1. Any agreement between Y_2 and X_1 which offers Y_2 slightly more than 12.6858 and X_1 slightly less than 15.9976 will block a match between X_1 and Y_1, at least under the profile generated by their non-competitive marriage. To rule out such a blocking counteroffer, X_1's and Y_1's pre-nuptial contract is re-calibrated against their outside options being to marry rival candidates.

In the case of X_1 we have already seen that constraining Y_2's utility to 12.6858, results in $\bar{U}_{X_1}^{12} = 15.9976$. Likewise constraining X_2's utility to 13.7667, we can calculate $\bar{U}_{Y_1}^{21} = 14.9166$. The latter is actually less than what Y_1 receives from a non-competitive match with X_1. When matching under the threat of alternative partners, it is only X_1 who could be tempted away. Thus we expect that when pre-nuptial bargaining takes place under the competitive outside options rather than the non-competitive ones, X_1's utility should rise while Y_1's should fall. Indeed, we find that the equal gains solution applied to the outside options $\bar{U}_{X_1}^{12} = 15.9976$, $\bar{U}_{Y_1}^{21} = 14.9166$ leads to $U_{X_1}^{11} = 15.9983$, $U_{Y_1}^{11} = 14.9173$. What is important here is that this pair of payoffs is jointly robust to blocking offers from X_2 or Y_2.

The couple (X_2, Y_2) is based on less preferred candidates so their outside option remains being single and their profile remains as with non-competitive marriage. The table below summarises the payoffs in each match.

Table 3: Payoffs from competitive matching.

	Y_1	Y_2
X_1	15.9983, 14.9173	N.A.
X_2	N.A	13.9184, 14.7555

We defer until later a comparison of female education and consumption shares between non-competitive matching and that which takes place in a competitive marriage market.

7.2 Preferred male:

We next consider a case in which the preliminary rankings lead to one male being preferred by both females while males are indifferent between females. Without loss of generality let the preferred male be X_1.

$$U_{Y_j}^{1j} \geq U_{Y_j}^{2j} \geq U_{Y_j}^s \tag{26}$$

for *both* $j = \{1, 2\}$ *and*

$$U_{X_i}^{i1} = U_{X_i}^{i2} \geq U_{X_i}^s \tag{27}$$

for *both* $i = \{1, 2\}$.

The general strategy for finding a stable pattern in this case is analogous to the one used in the previous sub-section so we shall skip the details and proceed straight to an example.

Suppose that $\beta_{X_1} = \beta_{Y_1} = \beta_{Y_2} = 4$ while $\beta_{X_2} = 3$. In this example, only males differ, the two females are identical. Under non-competitive matching each household profile will depend only on the male partner. At the same time, under competitive matching, females will be expected to receive equal treatment.

The table below reports household profiles and utility levels from non-competitive matching at a market wage $\omega = 50$: for $j = 1, 2$.

Table 4: Profiles from non-competitive matching.

	Y_1 and Y_2
X_1	(0.52836,0.51620)
X_2	(0.53063,0.45445)

For analogous reasons as in the previous case, both females would prefer marrying X_1. By construction, the males are indifferent between the females.

Table 5: Payoffs from non-competitive matching.

	Y_1 and Y_2
X_1	15.8482, 15.0670
X_2	13.9184, 14.7555

Competition for X_1 will result in the females being pinned down by their second-best option, which is to marry X_2. In equilibrium, regardless of who marries X_1, both females will attain the utility that they get from matching with X_2. If this were not the case, for example, if the female who marries X_1 receives a higher utility than the one who marries X_2, then the latter female could attract away X_1 by offering him a slightly higher consumption share and/or slightly higher homemaking skills than he receives from his present match.

The equilibrium payoffs are represented below (for the sake of comparison with previous sections we have resolved the tie so that Y_1 marries X_1).

Table 6: Payoffs from competitive matching.

	Y_1	Y_2
X_1	16.1503, 14.7555	N.A.
X_2	N.A	13.9184, 14.7555

It might appear at first glance that the equal gains solution to marital matching does not apply in this case. In fact, it does. Consider that the highest possible utility that X_1 could extract from marrying Y_2 is the same as what he gets from marrying Y_1. Thus $\bar{U}_{X_1}^{12} = U_{X_1}^{11}$. Similarly the highest utility that Y_1 can obtain from marrying X_2 is the same as she gets from her marriage to X_1. Thus $\bar{U}_{Y_1}^{21} = U_{Y_1}^{11}$. In fact in equilibrium, both X_1 and Y_1 receive no more or less than what they get under their respective outside options so that their own match generates neither gains nor losses.

For X_2 the only credible outside option is to stay single as under both exogenous and equilibrium pairing, he is indifferent between the two females. In turn the female who

marries him (Y_2) also has as a residual outside option only the utility of remaining single. Thus their equilibrium outcomes remain the same as in exogenous matching.

7.3 The effect of rising market wages:

We have so far focused on the effect of competition in a marriage market on the equilibrium pattern of matches. In the above examples, the market wage was held constant at $\omega = 50$. An important aim of this paper is to study how changes in the market wage induce changes in female education in the context of consensual marriage. We now study this question in the context of the above examples of marital matching. We focus on outcomes for the couple (X_1, Y_1) as both partners' intrinsic characteristics are identical to those of the solitary couple studied in the examples of the previous section.

In the table below we compare, at values of ω that range from 50 to 200, Y_1's education, her share of the market good and X_1's and Y_1's utility levels under non-competitive marriage and both examples of competitive matching that were studied in the preceding sub-sections of this section. Although not reported it has been verified that (X_1, Y_1) remain a couple under a stable matching pattern at each wage.[32]

Table 7: Comparing outcomes and payoffs for the couple (X_1, Y_1).

ω	NON-COMPETITIVE MARRIAGE				PREFERRED COUPLE				PREFERRED MALE			
	$e_{Y_1}^{11}$	μ^{11}	$U_{Y_1}^{11}$	$U_{X_1}^{11}$	$e_{Y_1}^{11}$	μ^{11}	$U_{Y_1}^{11}$	$U_{X_1}^{11}$	$e_{Y_1}^{11}$	μ^{11}	$U_{Y_1}^{11}$	$U_{X_1}^{11}$
50	0.52836	0.51620	15.0670	15.8482	0.52753	0.48640	14.9173	15.9983	0.53063	0.45445	14.7555	16.1503
100	0.52669	0.45649	16.7413	18.3038	0.52551	0.41456	16.4223	18.5841	0.52825	0.37153	16.0818	18.8576
150	0.52566	0.41972	17.9184	20.2621	0.52423	0.37001	17.4321	20.6548	0.52664	0.31899	16.9020	21.0386
200	0.52486	0.39191	18.8363	21.9613	0.523563	0.347667	18.3201	22.3565	0.52568	0.28995	17.599	22.8492

As compared to a non-competitive consensual match with X_1, Y_1 receives less utility when their match takes place in a competitive market. In particular, even when she is a preferred female, her utility is less than it is under non-competitive conditions. To

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[32]At wages above $\omega = 400$, the distinction between the two males becomes too small in our example to make a difference between exogenous and equilibrium outcomes.

some extent, the comparison is based on the choice of parameter values; given these, Y_2 appears to be a more effective rival in competing for X_1, than X_2 is in competing for Y_1. This works to X_1's advantage, raising the value of his outside option relative to that of Y_1. By contrast, the fact that Y_1 receives even less utility from marrying X_1 when Y_2 is her equal appears to be a more general result and not just a matter of parameter values.

Although Y_1 is worst off in utility terms in that case, her education is actually higher in that case than in the other two cases. These comparisons hold true at each wage in the interval under consideration. This outcome might also well be sensitive to parameter values but it nonetheless illustrates an interesting possibility, that competitive marriage formation can lead even females who are highly ranked by males in the marriage market to trade off education in the pursuit of higher consumption and higher overall utility in marrying highly-ranked males. This reinforces the intuition and concerns underlying this paper.

To return to our main focus, in all three cases, Y_1's education and share of the consumption good decrease with the market wage. This result was already derived for the case of non-competitive matching in the previous section; the examples from competitive matching reaffirm its robustness. The intuition for all three cases remains the same: in the presence of gender wage inequality, an increase in the market wage can increase male bargaining power relative to that of females, especially when the latter specialise in housework when married. They opt for lower education and extract a lower share of the rivalrous good from their husbands, even though their overall utility rises due to higher absolute consumption of both market and household goods.

Some final points about the robustness of these results. The key driving force of the model has been that, all else equal, females prefer a higher level of education than what a prospective spouse might consider optimal. This is an idiosyncratic

phenomenon that affects otherwise equivalent couples, and might not apply across socioeconomic classes, in which the phenomenon of hypergamy has been receiving attention.[33] In our own model, the male's preference for female education is determined by its effect on household productivity and, in case of a wife who supplies market labour, on her earnings. A wife's education does not directly enter a husband's utility function or act as a signal of her social standing. Such considerations could be incorporated without affecting our key arguments, so long as the female's private taste for education was strong enough to put her own ideal above that of her spouse.

What would change the results is introducing factors which discourage females from pursuing even that level of education that a suitor might prefer. Such factors could involve schooling costs and/or differences in effort or ability needed to convert education into marketable and/or household skills. These factors could work against the female's private taste for education and induce outcomes in which females enter the marriage market with what their potential spouses consider sub-optimal levels of education. In that case, a strengthening of male bargaining power might induce females to increase their education, while marital competition might lead to a reversal of behaviour by a female whose bargaining power decreases in the presence of a rival.

It might also be argued that were agents to be endowed with marketable assets or bequests, a bride-price and/or dowry could constitute a more efficient, lump-sum mechanism for ensuring that the marital selection constraints were jointly satisfied. This is indeed true since it would give the female an additional dimension for meeting the male selection constraint and might relax the need for her to increase her household skills in order to meet his outside options as the market wage increases. However, unless a bride's endowment is arbitrarily large, if the pre-nuptial bargain requires a

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[33]See, for example, recent papers by Abramitzky *et. al.* [2011], Banerjee *et. al.* [2013], Lafortune [2013].

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large enough transfer to the groom, part of it might still need to be conditioned on variables such as female household skills.

8 Conclusions:

This paper has compared female education and time-use under three states: (i) remaining single, (ii) exogenous marriage formation and (iii) consensual marriage. Our analysis suggests that single females receive more education and work more than married ones. Additionally, consensually married females who are likely to specialise in housework face an additional constraint regarding their housekeeping skills and this can lower their education even more than exogenous marriage formation does.

Our model also suggests that neither does reducing labor market discrimination necessarily lead to a continuous decline in the gender gap affecting married females' labour force participation and education, nor does an increase in market wages unambiguously lead to an increase in married females' education. Indeed our numerical examples illustrated a range of market wages over which married couples mutually specialised in time use, and female education declined with the market wage. Once wages had risen enough, females stops specialising in housework and further wage increases had positive effects on their education.

From an empirical point of view, whether or not an across-the-board increase in market wages will discourage female education depends on many factors, of which our analysis identifies only one. This effect could be confounded by other factors, of which evolving social attitudes, innate preferences among males regarding their wives educational and social status and the efforts by governments and international organisations to lower gender imbalances in labour force and educational participation, will be important.

In our paper this effect appears to be restricted to females who specialise in house-work. As the extent of such specialisation varies from region to region, it will be more relevant to South Asia, where labour force participation by women is particularly low, than to all developing regions. In the latest available figures from the World Bank, Pakistan's female labour force participation in 2011 was only 23%, Afghanistan's 16%, India's 29% and Bangladesh's 57% (World Bank, 2012).

It is also worth noting the observed U-shaped relationship between female labour force participation and household income [Mammen and Paxson [2000]). In other words it is at middle levels of income that females are least likely to work outside the home. Thus although a special case of our model, female specialisation in housework seems to fit the overall context of our analysis quite well. Indeed, although a recent study of female labour force participation in India by Bhalla and Kaur [2011] finds no evidence for a U-shaped relationship between family income and female labour force participation in the aggregate data, it does find a strong negative effect of high male education and earnings on work by a wife. This provides some support for the relevance of mutual specialisation as an outcome in middle-class households.

The literature on statistical discrimination points to a circular relationship between a disadvantaged group's labor low market participation and the tendency of employers to statistically discriminate against them (see Aigner and Cain [1977]). Our model suggests that marriage plays a compounding role in this relationship.

R E F E R E N C E S

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APPENDIX

Proof of Proposition 1: The first-order conditions for single agents are given by equations (7) and (8) respectively:

$$u'(\cdot)\omega e - \alpha h'(\cdot) = 0$$

$$u'(\cdot)\omega \ell + b + h'(\cdot)(1-\ell)A'(e) = 0$$

Totally differentiating the above with respect to ℓ, e and ω, and arranging in matrix form we obtain:

$$\begin{bmatrix} u''\omega^2 e^2 + h''\alpha^2 & \omega\Phi - A'\chi \\ \omega\Phi - A'\chi & u''\omega^2\ell^2 + h''(A'(1-\ell))^2 + h'(1-\ell)A'' \end{bmatrix} \begin{bmatrix} \partial\ell \\ \partial e \end{bmatrix} = \begin{bmatrix} -e\Phi\partial\omega \\ -\ell\Phi\partial\omega \end{bmatrix}$$

where

$$\begin{aligned}
\Phi &= u' + u''c > 0 \text{ by } (\mathcal{A}\text{-}4) \\
\chi &= h' + \alpha(1 - \ell)h'' > 0 \text{ by } (\mathcal{A}\text{-}5)
\end{aligned}$$

Therefore

$$\omega\Phi - A'\chi > 0 \text{ since } A' < 0$$

The determinant of the left-hand side is

$$\Delta = (u''\omega^2 e^2 + h''\alpha^2) \cdot (u''\omega^2\ell^2 + h''(A'(1-\ell))^2 + h'(1-\ell)A'') - (\omega\Phi - A'\chi)^2$$

which, although not unambiguous in sign, needs to be positive for the maximand to be concave (principal minors need to alternate in sign).

Using Cramer's Rule, we then obtain the following comparative static effects

$$\begin{aligned}
\frac{\partial \ell}{\partial \omega} &= \frac{-e\Phi(u''\omega^2\ell^2 + h''(A'(1-\ell))^2 + h'(1-\ell)A'') + \ell\Phi(\omega\Phi - A'\chi)}{\Delta} \\
\frac{\partial e}{\partial \omega} &= \frac{-\ell\Phi(u''\omega^2 e^2 + h''\alpha^2) + e\Phi(\omega\Phi - A'\chi)}{\Delta}
\end{aligned}$$

which are both positive under $(\mathcal{A}\text{-}4)$ and $(\mathcal{A}\text{-}5)$. $\square$

Proof of Proposition 2: We have already established that, given the combination of $\omega_X > \omega_Y$ and $e_X \geq e_Y$, there is specialisation in the time use of the married couple. Three cases arise: (i) $\ell_X = 1$, $\ell_Y \in (0,1)$; (ii) $\ell_X = 1$, $\ell_Y = 0$ and (iii) $\ell_X \in (0,1)$, $\ell_Y = 0$.

Since $\ell_X = 1$ in cases (i) and (ii), $e_X = 1$, so the inequality follows trivially. In case (iii), with $\ell_X < 1$, note that $\ell_Y = 0$ so the optimal value of education for agent Y satisfies

$$h'(\cdot)(1 - \ell_Y)A'(e_Y) + b = 0$$

Plugging the value of e_Y that satisfies the above first-order condition, into X's first-order condition

$$0.5u'(\cdot)\omega_X\ell_X + h'(\cdot)(1 - \ell_X)A'(e_Y) + b > 0$$

Since $\omega_X > \omega_Y$, $\ell_X > \ell_Y = 0$, $(1 - \ell_X) < (1 - \ell_Y) = 1$ and the other terms are identical for both spouses, it is clear the above equation is strictly positive when evaluated at e_Y. Thus $e_X > e_Y$. $\square$

Proof of Proposition 3: The proposition is obvious when $\ell_Y^a = 0$ so the only case in which it needs to be proven is when $\ell_Y^a > 0$. If this is the case then $e_X^a = 1$ and $1 - \ell_X^a = 0$.

Now suppose that in fact $\ell_Y^a \geq \ell_Y^s$. Then $1 - \ell_Y^a \leq 1 - \ell_Y^s$. By the concavity of $h(\cdot)$, it must then be the case that $2\alpha_Y h'(\alpha_Y(1 - \ell_Y^a)) \geq \alpha_Y h'(\alpha_Y(1 - \ell_Y^s))$. From the respective first-order conditions for the labor supply of agent Y's married and single

self (both of which hold as equalities in this case), it follows from comparing terms that

$$u'\left(\frac{w_X + w_Y e_Y \ell_Y^a}{2}\right) \geq u'\left(w_Y e_Y \ell_Y^s\right)$$

where e_Y is an arbitrary level of education for Y. By the concavity of $u(\cdot)$, this implies that

$$\frac{w_X + w_Y e_Y \ell_Y^a}{2} \leq w_Y e_Y \ell_Y^s$$

But since our working hypothesis is that $w_Y e_Y \ell_Y^a \geq w_Y e_Y \ell_Y^s$, the above can only be true if $w_X \leq w_Y e_Y \ell_Y^s$ which is not possible since $w_X > w_Y$ and $1 \geq e_Y^s$. Thus $\ell_Y^a < \ell_Y^s$. □

The following lemmas are useful for proving Proposition 4.

Lemma 1: Under ($\mathcal{A}$-4) and ($\mathcal{A}$-5), the first-order conditions for labor and education respectively for both single and married females imply that, given that a female supplies positive labour in both states

$$\left.\frac{\partial \ell_Y^k}{\partial e_Y^k}\right|_{\ell^{k*}} > 0$$

$$\left.\frac{\partial \ell_Y^k}{\partial e_Y^k}\right|_{e^{k*}} > 0$$

for $k = \{s, a\}$; $|_{\ell^{k*}}$ indicates a derivative along the first-order condition for labor while $|_{e^{k*}}$ indicates a derivative along the first-order condition for education.

Proof: Differentiating the relevant first-order conditions for ℓ_Y and e_Y, keeping in mind that the exercise applies only for interior solutions in the case of married agents, we get the following

$$\left.\frac{\partial \ell_Y^s}{\partial e_Y^s}\right|_{\ell^{s*}} = -\frac{(u'' \cdot c_Y^s + u')\omega_Y - A' \cdot (h''\alpha \cdot (1 - \ell_Y^s) + h')}{u'' \cdot (\omega_Y)^2 (e_Y^s)^2 + \alpha^2 h''} \tag{A.1}$$

$$\left.\frac{\partial \ell_Y^s}{\partial e_Y^s}\right|_{e^{s*}} = -\frac{u'' \cdot (\omega_Y)^2 + (A')^2 (1 - \ell_Y^s)^2 h'' + A''h' \cdot (1 - \ell_Y^s)}{(u'' \cdot c_Y^s + u')\omega_Y - A' (h'' \cdot \alpha \cdot (1 - \ell_Y^s) + h')} \tag{A.2}$$

$$\left.\frac{\partial \ell_Y^a}{\partial e_Y^a}\right|_{\ell^{a*}} = -\frac{(u'' \cdot (c_Y^a - 0.5\omega_X) + u')\omega_Y - 2A' \cdot (h''\alpha \cdot (1 - \ell_Y^a) + h')}{u''(\omega_Y)^2 (e_Y^a)^2 + \alpha^2 h''} \tag{A.3}$$

$$\left.\frac{\partial \ell_Y^a}{\partial e_Y^a}\right|_{e^{a*}} = -\frac{u'' \cdot (\omega_Y)^2 + (A')^2 (1 - \ell_Y^a)^2 h'' + A''h' \cdot (1 - \ell_Y^a)}{(u'' \cdot (c_Y^a - 0.5\omega_X) + u')\omega_Y - A' \cdot (h''\alpha(1 - \ell_Y^a) + h')} \tag{A.4}$$

Since the denominators of equations (A.1) and A.3) and the numerators of equations (A.2) and (A.4) are unambiguously negative, ($\mathcal{A}$-4) and ($\mathcal{A}$-5) are sufficient for each of the above derivatives to be positive. □

It follows from Lemma 1 that e_Y^k and ℓ_Y^k, $k = \{s, a\}$ are monotonically related along each first-order condition. This further suggests that along each first-order condition we can define functional relationships between the two variables. For example, along

the first-order condition for labour we can define $\ell_Y^k(e_Y^k)$ as the function which relates female labour supply to her educational level. We could define an analogous functional dependence of ℓ_Y^k on e_Y^k along the first-order condition for education but for expositional convenience we shall define the inverse function in this case: $e_Y^k(\ell_Y^k)$.

Lemma 2: Concavity of the maximisation problem requires that

$$\left.\frac{\partial \ell_i^k}{\partial e_i^k}\right|_{e^{k*}} > \left.\frac{\partial \ell_i^k}{\partial e_i^k}\right|_{\ell^{k*}}$$

for $i = X, Y$ and $k = s, a$.

Proof: For single agents, the Hessian matrix formed by the own- and the cross-partial derivatives of each first-order condition with respect to each choice variable is two-dimensional. For married agents, there are four endogenous variables in principle. However, because *at least* one spouse is always in a corner with respect to labor supply, the Hessian is also two-dimensional, if it is defined at all.

The determinant of the Hessian is

$$\left.\frac{\partial \ell_i^k}{\partial e_i^k}\right|_{e^{k*}} - \left.\frac{\partial \ell_i^k}{\partial e_i^k}\right|_{\ell^{k*}}$$

where, in the married agent's case, i is the spouse whose labor supply is interior.

It is easily verified that the diagonal elements of the Hessian are negative so the principal minors alternate in sign as (required for concavity) if and only if the above expression is positive. □

An implication of Lemmas 1 and 2 is that equilibrium is stable.

Lemma 3: At given ℓ_Y, $e_Y^a < e_Y^s$.

Proof:

Recall equation (8), the first-order condition for education of Y's single self. This can be rewritten as

$$u'\left(\omega_Y e_Y^s \ell_Y\right)\omega_Y \ell_Y + b = -h'(A(e_Y^s)(1-\ell_Y))(1-\ell_Y)A'(e_Y^s) \tag{A.5}$$

where ℓ_Y is arbitrary and e_Y^s is an equilibrium value that satisfies the above equation.

Similarly recall equation (10). the analogous first-order condition for Y's married self. Recall that the context of this proposition is one in which $e_X^a = 1$ and $\ell_X^a = 1$; thus equation (10) can be rewritten as

$$0.5u'\left(\frac{\omega_X + \omega_Y \ell_Y e_Y^a}{2}\right)\omega_Y \ell_Y + b = -h'(A(e_Y^a)(1-\ell_Y))(1-\ell_Y)A'(e_Y^a) \tag{A.6}$$

Let $\Lambda = -h'(A(e)(1-\ell))(1-\ell)A'(e)$ for a generic agent at either marital status.

Differentiating Λ with respect to e:

$$\frac{\partial \Lambda}{\partial e} = -h''(\cdot)(1-\ell)^2(A(e))^2 - h'(\cdot)(1-\ell)A''(e) > 0$$

since both h'' and A'' are negative. This establishes that the RHS of equation (A.6) is increasing in e_Y^a.

Now let e_Y^s solve equation (A.5) and evaluate equation (A.6) at this value of e_Y. The LHS of equation (A.5) must be strictly greater than that of equation (A.6). This is both because her married self's marginal utility from market consumption is multiplied by 0.5 and because it would in any case be lower than that of her single self for given ω_Y and ℓ_Y. Note that at given values of these variables, Y's market consumption will be higher and her marginal utility lower, when married to the higher paid agent X than when single.

Furthermore, by inspection, the RHS of equation (A.5) and equation (A.6) are equal. Hence, evaluated at e_Y^s, the LHS of Equation (A.6) is smaller than the RHS. In other words, the marginal cost of married Y's education is strictly greater than its marginal benefit; and since the LHS of equation is decreasing while the RHS is increasing in e_Y, all else equal, this implies that $e_Y^a < e_Y^s$ for given values of ℓ_Y. $\qquad\square$

Proof of Proposition 4: Suppose that an equilibrium exists in the case of exogenous marriage formation. Note that equilibrium values (e_Y^{a*}, ℓ_Y^{a*}) may be written as $\ell_Y^{a*} = \ell_Y^a(e_Y^{a*})$ and $e_Y^{a*} = e_Y^a(\ell_Y^{a*})$ as argued after the proof of Lemma 1.

Given Lemma 3, it must be the case that $e_Y^s(\ell_Y^{a*}) > e_Y^{a*}$. Denote this value of e_Y^s as $e_Y^{s'}$. Then, from Lemma 1 it must be the case that $\ell_Y^{s'} = \ell_Y^s(e_Y^{s'}) > \ell_Y^{a*}$. From Lemma 1 again, it will further be the case that that there exists $e_Y^{s''} = e_Y^s(\ell_Y^{s'})$ satisfying $e_Y^{s''} \geq e_Y^{s'}$ and by further induction $\ell_Y^{s''} = \ell_Y^s(e_Y^{s''}) > \ell_Y^{s'}$ and so on. Lemma 2 ensures that this process converges monotonically to a pair (ℓ_Y^{s*}, e_Y^{s*}) such that $\ell_Y^{s*} = \ell_Y^s(e_Y^{s*})$ and $e_Y^{s*} = e_Y^s(\ell_Y^{s*})$. By induction, the series of inequalities along the way ensure that $\ell_Y^{s*} > \ell_Y^{a*}$ and that $e_Y^{s*} > e_Y^{a*}$. $\qquad\square$

Proof of Proposition 5: Totally differentiating equations (17)-(19) with respect to e_Y^m, μ and λ we obtain a 3×3 matrix system

$$\begin{bmatrix} G & 0 & -b \\ 0 & (1-\lambda)u''(c_Y^m) + \lambda u''(c_X^m) & -J \\ -b & -\omega J & 0 \end{bmatrix} \begin{bmatrix} \partial e^m \\ \partial \mu \\ \partial \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ M\partial\omega \\ \Psi_\omega \partial\omega \end{bmatrix}$$

where:

$$
\begin{aligned}
G &\equiv (A')^2 h'' + h' A'' < 0 \\
H &\equiv (1-\lambda)u''(c_Y^m) + \lambda u''(c_X^m) < 0 \\
J &\equiv (u'(c_X^m) + u'(c_Y^m)) > 0 \\
M &\equiv \lambda(1-\mu)u''(c_X^m) + (1-\lambda)\mu u''(c_Y^m) = 0 \text{ under } (\mathcal{A}\text{-}7) \\
\Psi_\omega^s &\equiv \frac{\partial \Psi^s}{\partial \omega} \\
\Psi_\omega &\equiv \Psi_\omega^s - \left[(1-\mu)\frac{\partial u'(c_X^m)}{\partial \omega} - \mu \frac{\partial u'(c_Y^m)}{\partial \omega} \right] > 0 \text{ under } (\mathcal{A}\text{-}6)
\end{aligned}
$$

The determinant of the left-hand side matrix is:

$$
\mathcal{D} = - \left[\omega G J + b^2 H \right] > 0
$$

Using Cramer's rule we obtain (note that under $(\mathcal{A}\text{-}7)$, M=0 and drops out):

$$
\begin{aligned}
\frac{\partial e_Y^m}{\partial \omega} &= \frac{b \Psi_\omega H}{\mathcal{D}} < 0 \\
\frac{\partial \mu}{\partial \omega} &= \frac{G J \Psi_\omega}{\mathcal{D}} < 0
\end{aligned}
$$

which establishes the result. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ $\square$

Numerical Tables:

<p style="text-align:center">Table 8: Benchmark Case: $\phi = 0.5$</p>

ω	e_X^s	ℓ_X^s	e_Y^s	ℓ_Y^s	e_Y^a	ℓ_Y^a	μ	e_Y^m	ℓ_Y^m	U_X^s	U_Y^s	U_X^a	U_Y^a	U_X^m	U_Y^m
50	0.642	0.304	0.599	0.163	0.55556	0	0.516	0.52836	0	11.56	10.78	15.89	15.00	15.85	15.07
100	0.729	0.536	0.642	0.304	0.55556	0	0.456	0.52669	0	13.13	11.56	17.96	17.07	18.30	16.74
500	1	1	0.990	0.987	0.55556	0	0.326	0.52290	0	24.36	17.81	26.70	25.81	29.29	22.75
1000	1	1	1	1	0.55556	0	0.317	0.52264	0	33.62	24.36	33.25	32.36	37.06	27.80
1500	1	1	1	1	0.55556	0	0.314	0.52253	0	40.73	29.39	38.28	37.39	43.02	31.67
2000	1	1	1	1	0.55556	0	0.312	0.52246	0	46.72	33.62	42.51	41.62	48.04	34.94
2100	1	1	1	1	0.57813	0.088	0.311	0.52245	0	47.83	34.40	43.25	42.41	48.97	35.55
2300	1	1	1	1	0.66493	0.370	0.311	0.54367	0.023	49.96	35.91	44.64	43.97	50.75	36.70
2500	1	1	1	1	0.75174	0.587	0.316	0.62700	0.308	52.00	37.36	46.03	45.53	52.51	37.87

<p style="text-align:center">Table 9: Variant: $\phi = 0.95$</p>

ω	e_X^s	ℓ_X^s	e_Y^s	ℓ_Y^s	e_Y^a	ℓ_Y^a	μ	e_Y^m	ℓ_Y^m	U_X^s	U_Y^s	U_X^a	U_Y^a	U_X^m	U_Y^m
50	0.642	0.304	0.638	0.290	0.55556	0	0.58581	0.53031	0	11.56	11.48	15.89	15.00	15.48	15.40
100	0.729	0.536	0.720	0.515	0.55556	0	0.55541	0.52945	0	13.13	12.97	17.96	17.07	17.60	17.44
200	0.903	0.865	0.885	0.838	0.55556	0	0.53148	0.52878	0	16.25	15.94	20.89	20.00	20.61	20.30
400	1	1	1	1	0.55556	0	0.51545	0.52834	0	22.00	21.49	25.03	24.14	24.85	24.35
500	1	1	1	1	0.55556	0	0.51193	0.52824	0	24.36	23.79	26.70	25.81	26.55	25.99
600	1	1	1	1	0.58251	0.10412	0.50934	0.52817	0	26.49	25.87	28.16	27.33	28.09	27.47
800	1	1	1	1	0.74744	0.57763	0.49929	0.65779	0.41831	30.28	29.57	30.80	30.30	30.95	30.24
1000	1	1	1	1	0.91237	0.87995	0.49237	0.81387	0.74446	33.62	32.82	33.44	33.26	33.80	33.00