# Dynamic Trading and Asset Prices: Keynes vs. Hayek *

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**Abstract**

We investigate the dynamics of prices, information and expectations in a competitive, noisy, dynamic asset pricing equilibrium model with long-term investors. We argue that the fact that prices can score worse or better than consensus opinion in predicting the fundamentals is a product of endogenous short-term speculation. For a given, positive level of residual payoff uncertainty, if liquidity trades display low persistence rational investors act like market makers, accommodate the order flow, and prices are farther away from fundamentals compared to consensus. This defines a "Keynesian" region; the complementary region is "Hayekian" in that rational investors chase the trend and prices are systematically closer to fundamentals than average expectations. The standard case of no residual uncertainty and liquidity trading following a random walk is on the frontier of the two regions and identifies the set of deep parameters for which rational investors abide by Keynes' dictum of concentrating on an asset "long term prospects and those only." The analysis also explains momentum and reversal in stock returns and how accommodation and trend chasing strategies differ from these phenomena.

*Keywords:* Efficient market hypothesis, long and short-term trading, average expectations, opaqueness, momentum, reversal.

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# 1 Introduction

Are asset prices aligned with the consensus opinion on the fundamentals in the market? A (somewhat) simplistic version of the Efficient Market Hypothesis (EMH), provides an affirmative answer to this question. According to this view competition among rational investors will drive prices to be centered around the consensus estimate (average expectations) of underlying value given available information. This proposition has generated much debate among economists and in the finance profession.[1]

In his General Theory, Keynes pioneered the vision of stock markets being prey of short-run speculation with prices loosely linked to fundamentals. This view tends to portray a stock market dominated by herding, behavioral biases, fads, booms and crashes (see, for example, Shiller (2000)), and goes against the tradition of considering market prices as aggregators of the dispersed information in the economy advocated by Hayek (1945) and followed by the modern tradition of the rational expectations analysis. According to the latter view prices reflect, perhaps noisily, the collective information that each trader has about the fundamental value of the asset (see, for example, Grossman (1989)), and provide a reliable signal about assets' liquidation values.

There is thus a tension between the Keynesian and the Hayekian views of financial market dynamics, with the first emphasizing short-run speculation and the latter long run informational efficiency. Keynes, however, distinguished between enterprise, or the activity of forecasting the prospective yield of assets over their whole life (where the investor focuses on the "long-term prospects and those only"), and short term speculation.[2] While the former corresponds to the simplistic version of the EMH (that is, that prices should equal average expectations of value plus noise), Keynes thought that in modern stock markets the latter would be king. Recurrent episodes of bubbles have the flavor of Keynes' speculation with prices that seem far away from average expectations of fundamentals in the market.

In this paper, we address the tension between the Keynesian and the Hayekian visions in a dynamic finite horizon market where investors, except for liquidity traders, have no behavioral bias and hold a common prior on the liquidation value of the risky asset. We therefore allow for the possibility that investors concentrate on the long-term prospects in a rich noisy dynamic rational expectations environment where there is residual uncertainty on the liquidation value of the asset (so that the collective information of rational investors is *not* sufficient to recover the ex-post liquidation value) and where liquidity trading follows a general process.

We find that in most cases in the rational expectations equilibrium, investors find it profitable to engage in short term speculation and this implies that the simplistic version of the

[1]See, e.g., "Something big in the city," Jonathan Ford, *Financial Times*, November 15, 2008. Professional investors attribute considerable importance to the consensus estimate as a guide to selecting stocks. Bernstein (1996) reports how in 1995 Neil Wrigth, chief investment officer of ANB Investment Management & Trust, introduced a strategy "explicitly designed to avoid the Winner's Curse." Such a strategy was based on the composition of a portfolio from stocks with a narrow trading range, "an indication that [these stocks] are priced around consensus views, with sellers and buyers more or less evenly matched. The assumption is that such stocks can be bought for little more than their consensus valuation."

[2]Keynes, Ch. 12, General Theory (1936).

EMH does not hold.[3] Furthermore, the fact that prices can be systematically farther away or closer to fundamentals compared to consensus, thereby scoring worse or better than consensus in predicting the fundamentals, is a manifestation of endogenous short-term speculation. In a *static* market investors speculate on the difference between the price and the liquidation value, and prices are aligned with their average expectations about this value. Thus, in this context the price is just a noisy measure of investors' consensus opinion. In a *dynamic* market, investors speculate *also* on short-run price differences. With heterogeneous information, this may misalign prices and investors' average expectations, potentially leading prices either closer or farther away from the fundamentals compared to consensus. Two key deep parameters, the level of residual payoff uncertainty and the degree of persistence of liquidity trades, determine whether prices predict fundamentals better than consensus. When there is no residual uncertainty on the asset liquidation value and liquidity trading follows a random walk then prices are aligned with consensus like in a static market. This is one of the boundary cases where rational investors do not have incentives to speculate on short run price movements. For a given, positive level of residual uncertainty, low persistence deteriorates the predictive power of prices vis-à-vis consensus; conversely, high liquidity trades' persistence has the opposite effect. This partitions the parameter space into a Keynesian region, where prices are farther away from fundamentals than average expectations, and a Hayekian region where the opposite occurs. The boundary of these regions reflects Keynes' situation where investors concentrate on the long-term prospects and where the simplistic version of the EMH holds. In the Keynesian region short run price speculation based on market making motives (reversion of liquidity trades) predominates, while in the Hayekian region short run price speculation based on information (trend chasing) predominates. As a consequence we can characterize accommodation and trend chasing strategies in a model with rational investors and study how do they map to momentum (recent performance tends to persist in the near future) and reversal (a longer history of performance tends to revert).

The intuition for our results is as follows. In a dynamic market, the relationship between price and fundamentals depends both on the quality of investors' information *and* on their reaction to the aggregate demand. Suppose an investor observes a positive signal and faces a high demand for the asset. Upon the receipt of good news he increases his long position in the asset. On the other hand, his reaction to high asset demand is either to accommodate it, counting on a future price reversal – thereby acting as a "market-maker"– or to follow the market and further increase his long position anticipating an additional price rise (in this way "chasing" the trend). The more likely it is that the demand realization reverts over time, e.g., due to liquidity traders' transient demand, the more likely that the investor will want to accommodate it. Conversely, the more likely it is that the demand realization proxies for a stable trend, e.g., due to the impact of fundamentals information, the more likely that the investor will

[3]It should be no surprise that in a *noisy* rational expectations equilibrium prices may be systematically closer or farther away from the fundamentals compared with investors' average expectations. This result depends on the relative weights that in equilibrium traders put on private and public information and, obviously, could not arise in a *fully revealing* equilibrium where the price coincides with the liquidation value.

want to follow the market.[4] In the former case, the investor's long-term speculative position is partially offset by his market making position. Thus, the impact of private information on the price is partially sterilized by investors' market making activity. This, in turn, loosens the price from the fundamentals in relation to average expectations. Conversely, in the latter case, the investor's reaction to the observed aggregate demand realization reinforces his long-term speculative position. Thus, investors' trend chasing activity enhances the impact of private information on the price and tightens the price to the fundamentals in relation to average expectations.[5]

Low liquidity trades' persistence strengthens the mean reversion in aggregate demand, and tilts investors towards accommodating the aggregate demand. This effect is extreme when the stock of liquidity traders' demand is independent across periods.[6] The impact of residual uncertainty over the liquidation value, on the other hand, enhances the hedging properties of future positions, boosting investors' signal responsiveness and leading them to speculate more aggressively on short-run price differences. Thus, depending on the persistence of liquidity traders' demand, prices predict fundamentals worse or better than consensus, respectively yielding the Keynesian and the Hayekian regions. Conversely, when liquidity traders' demand is very persistent (i.e., when liquidity trades increments are i.i.d.) and absent residual uncertainty, investors act as in a static market, and prices are aligned with consensus. This, together with the boundary between the Keynesian and the Hayekian regions, identifies the set of parameter values for which investors concentrate on the asset long term prospects, shying away from short term speculation. It is for this set of parameter values that the simplistic version of the EMH holds.

Interestingly, the Keynesian and Hayekian regions can be characterized in terms of investors' consensus opinion about the systematic behavior of future price changes. Indeed, in the Hayekian region, investors chase the market because the consensus opinion is that prices will systematically continue a given trend in the upcoming trading period. In the Keynesian region, instead, investors accommodate the aggregate demand because the consensus opinion is that prices will systematically revert. We illustrate how expected price behavior under the latter metric *does not* always coincide with a prediction based on the unconditional correlation of returns. Indeed, as in our setup the evolution of aggregate demand is governed both by a transient (liquidity trades) and a persistent (fundamentals information) component, depending on the patterns of private information arrival momentum and reversal can arise. However, due to the usual signal extraction problem investors face in the presence of heterogeneous information, these phenomena are compatible with both the Hayekian and Keynesian equilibrium regions.

[4] In this case, indeed, the aggregate demand is likely to proxy for upcoming good news that are not yet completely incorporated in the price. There is a vast empirical literature that documents the transient impact of liquidity trades on asset prices as opposed to the permanent effect due to information-driven trades. See e.g. Wang (1994), and Llorente et al. (2002).

[5] Other authors have emphasized the consequences of investors' reaction to the aggregate demand for an asset. For example, Gennotte and Leland (1990) argue that investors may exacerbate the price impact of trades, yielding potentially destabilizing outcomes, by extracting information from the order flow.

[6] Indeed, assuming that the stock of liquidity trades is i.i.d. implies that the *gross* position liquidity traders hold in a given period $n$ completely reverts in period $n+1$. This lowers the risk of accommodating the aggregate demand in any period, as investors can always count on the possibility of unwinding their inventory of the risky asset to liquidity traders in the coming round of trade.

Furthermore, as reversal and momentum can happen at the frontier between the Keynesian and Hayekian regions, the simplistic version of the EMH *does not* preclude return predictability.

Our paper contributes in several aspects to the research investigating asset pricing in the presence of heterogeneously informed agents. First, it contributes to the literature that analyzes dynamic trading by competitive, long-term investors in the presence of asymmetric information. Most of the results in this literature deal with the case in which the information structure is nested (i.e. where investors' information sets can be completely ordered in terms of informativeness). Wang (1993) presents results in continuous-time. The effect of a non-nested information structure (and thus heterogeneously informed agents) is analyzed by Grundy and McNichols (1989), Brown and Jennings (1989), and Cespa (2002) in the context of a two-period model. Vives (1995) and He and Wang (1995) study $N$-period models. The former investigates the effect of different patterns of information arrival on price efficiency, when prices are set by competitive, risk-neutral market makers. The latter analyze the patterns of volume in a market with risk-averse investors. In this paper we also assume that all investors are risk averse and provide a novel characterization of the equilibrium that arises in this context.[7]

Second, our paper also contributes to the literature analyzing asset pricing anomalies within the rational expectations equilibrium paradigm. Biais, Bossaerts and Spatt (2008), in a multi-asset, noisy, dynamic model with overlapping generations show that momentum can arise in equilibrium. Vayanos and Woolley (2008) present a theory of momentum and reversal based on delegated portfolio considerations. We add to this literature by showing how momentum and reversal relate to the ability of the price to track fundamentals compared to investors' consensus.

Finally, our paper is related to the literature emphasizing the existence of "limits to arbitrage." De Long et. al (1990) show how the risk posed by the existence of an unpredictable component in the aggregate demand for an asset can crowd-out rational investors, thereby limiting their arbitrage capabilities. In our setup, it is precisely the risk of facing a reversal in liquidity traders' positions that tilts informed investors towards accommodating the aggregate demand. In turn, this effect is responsible for the fact that asset prices can score worse than consensus in predicting the fundamentals.

The paper is organized as follows: in the next section we present the static benchmark, showing that in this framework the simplistic version of the EMH is satisfied, and the price scores as well as investors' consensus at predicting fundamentals. In Section 3 we introduce the three-period model and in Section 4 we prove equilibrium existence for the case with two trading rounds and argue that prices may be better or worse estimators of fundamentals whenever, in the presence of heterogeneous information, investors speculate on short term returns. In Section 5 we extend our analytical results to the market with three trading rounds, via numerical simulations. In Sections 6 and 7 we introduce the Keynesian and Hayekian regions, characterize investors' behavior and price properties in these regions and draw the implications of our model for return regularities. The final section provides concluding remarks. Most of the proofs are

[7]See Vives (2008), Ch. 8 and 9 for a survey of the relevant literature.

relegated to the appendix.

## 2 A Static Benchmark

Consider a one-period stock market where a single risky asset with liquidation value $v + \delta$, and a riskless asset with unitary return are traded by a continuum of risk-averse, informed investors in the interval $[0, 1]$ together with liquidity traders. We assume that $v \sim N(\bar{v}, \tau_v^{-1})$, $\delta \sim N(0, \tau_\delta^{-1})$. Investors have CARA preferences (denote with $\gamma$ the risk-tolerance coefficient) and maximize the expected utility of their wealth: $W_i = (v + \delta - p)x_i$.[8] Prior to the opening of the market every informed investor $i$ obtains private information on $v$, receiving a signal $s_i = v + \epsilon_i$, $\epsilon_i \sim N(0, \tau_\epsilon^{-1})$, and submits a demand schedule (generalized limit order) to the market $X(s_i, p)$ indicating the desired position in the risky asset for each realization of the equilibrium price.[9] Assume that $v$ and $\epsilon_i$ are independent for all $i$, and that error terms are also independent across investors. Liquidity traders submit a random demand $u$ (independent of all other random variables in the model), where $u \sim N(0, \tau_u^{-1})$. Finally, we make the convention that, given $v$, the average signal $\int_0^1 s_i di$ equals $v$ almost surely (i.e. errors cancel out in the aggregate: $\int_0^1 \epsilon_i di = 0$).[10] The random term $\delta$ in the liquidation value is orthogonal to all the random variables in the model and thus denotes the residual uncertainty affecting the final pay off about which no investor possesses information. This allows to interpret $\delta$ as a proxy for the level of opaqueness that surrounds the value of fundamentals.[11]

We denote by $E_i[Y]$, $\text{Var}_i[Y]$ the expectation and the variance of the random variable $Y$ formed by an investor $i$, conditioning on the private and public information he has: $E_i[Y] = E[Y|s_i, p]$, $\text{Var}_i[Y] = \text{Var}[Y|s_i, p]$. Finally, we denote by $\bar{E}[v] = \int_0^1 E_i[v]di$ investors' average opinion (the "consensus" opinion) about $v$.

In the above CARA-normal framework, a symmetric rational expectations equilibrium (REE) is a set of trades contingent on the information that investors have, $\{X(s_i, p) \text{ for } i \in [0, 1]\}$ and a price functional $P(v, u)$ (measurable in $(v, u)$), such that the following two conditions hold:

(I) Investors in $[0, 1]$ optimize

$$X(s_i, p) \in \arg\max_{x_i} E\left[-\exp\left\{-W_i/\gamma\right\} | s_i, p\right]. \tag{1}$$

(II) The market clears:

$$\int_0^1 x_i di + u = 0. \tag{2}$$

[8] We assume, without loss of generality with CARA preferences, that the non-random endowment of rational investors is zero.

[9] The unique equilibrium in linear strategies of this model is symmetric.

[10] See Section 3.1 in the Technical Appendix of Vives (2008) for a justification of the convention.

[11] For example, the component $v$ relates to the "traditional" business of the firm, so that an analyst or an expert can obtain information about it. The component $\delta$, instead, originates from decisions and actions that insiders make and regarding which the market is totally clueless.

Given the above definition, it is easy to verify that a unique, symmetric equilibrium in linear strategies exists in the class of equilibria with a price functional of the form $P(v,u)$ (see, e.g. Admati (1985), Vives (2008)). The equilibrium strategy of an investor $i$ is given by

$$X(s_i, p) = \frac{a}{\alpha_E}(E_i[v] - p),$$

where

$$a = \frac{\gamma\tau_\epsilon}{1+\kappa}, \tag{3}$$

denotes the responsiveness to private information, $\kappa \equiv \tau_\delta^{-1}\tau_i$, $\tau_i \equiv (\mathrm{Var}_i[v])^{-1}$, and $\alpha_E = \tau_\epsilon/\tau_i$ is the optimal statistical (Bayesian) weight to private information.[12] Intuitively, investors have a private signal about $v$, but the liquidation value also features a random term $\delta$ about which they are uninformed. The larger is $\tau_\delta^{-1}$ (and thus $\kappa$), the less relevant is investors' private information to predict $v+\delta$. Imposing market clearing the equilibrium price is given by

$$p = \bar{E}[v] + \frac{\alpha_E}{a}u. \tag{4}$$

We will use the above framework to investigate conditions under which the ability of the equilibrium price to track the fundamentals potentially differs from that of investors' consensus. That is whether $\mathrm{Cov}[p,v]$ differs from $\mathrm{Cov}[\bar{E}[v],v]$. In view of (4), it is easy to see that in a static market both covariances are equal. Indeed, as liquidity trades are orthogonal to the ex-post liquidation value,

$$\begin{aligned}\mathrm{Cov}[p,v] &= \mathrm{Cov}\left[\bar{E}[v] + \frac{\alpha_E}{a}u, v\right]\\ &= \mathrm{Cov}\left[\bar{E}[v], v\right].\end{aligned} \tag{5}$$

A simplistic version of the Efficient Market Hypothesis (EMH) would say that competition among rational investors will drive prices to be centered around the consensus estimate of the underlying value, given available information. In view of (4) and (5), we can therefore conclude that in a static setup the equilibrium price satisfies the simplistic version of the EMH, thereby providing as good a forecast of the ex-post liquidation value as investors' consensus.

Rearranging (4) we obtain

$$p = \alpha_P\left(v + \frac{u}{a}\right) + (1-\alpha_P)E[v|p], \tag{6}$$

where

$$\alpha_P = \frac{a(1+\kappa)}{\gamma\tau_i}, \tag{7}$$

captures the weight assigned by the price to the noisy signal coming from the aggregate demand. The above comparison can then be given two alternative, equivalent representations:

[12] The responsiveness to private information is given by the unique, positive real solution to the cubic $\phi(a) = a^3\tau_u + a(\tau_v+\tau_\epsilon+\tau_\delta) - \gamma\tau_\delta\tau_\epsilon = 0$. To prove uniqueness note that $\phi(0) = -\gamma\tau_\delta\tau_\epsilon < 0$, $\phi(\gamma\tau_\epsilon) = a(a^2\tau_u+\tau_v+\tau_\epsilon) > 0$, implying that a real solution $a^*$ exists in the interval $(0, \gamma\tau_\epsilon)$. Finally, since $\phi'(a)|_{a=a^*} > 0$, the result follows.

**Lemma 1.** *In the static market, the following three conditions are equivalent:*

$$Cov[p, v] < Cov\left[\bar{E}[v], v\right] \tag{8}$$

$$\alpha_P < \alpha_E \tag{9}$$

$$|E\left[p - v|v\right]| > \left|E\left[\bar{E}[v] - v|v\right]\right|. \tag{10}$$

*Proof.*

Owing to normality, $E_i[v]$ represents an investor $i$'s best predictor of the liquidation value and can be expressed as a weighted average of the investor's private and public information:

$$E_i[v] = \alpha_E s_i + (1 - \alpha_E)E[v|p].$$

Furthermore, due to our convention:

$$\bar{E}[v] = \alpha_E v + (1 - \alpha_E)E[v|p]. \tag{11}$$

Using (6), and computing the relevant covariance we obtain:

$$\begin{aligned}
\text{Cov}[p, v] &= \text{Cov}\left[\alpha_P\left(v + \frac{\theta}{a}\right) + (1 - \alpha_P)\, E[v|p], v\right] \\
&= \alpha_P \text{Cov}[v, v] + (1 - \alpha_P)\text{Cov}\left[v, E[v|p]\right] \\
&= \alpha_P \frac{1}{\tau_v} + (1 - \alpha_P)\left(\frac{1}{\tau_v} - \frac{1}{\tau}\right),
\end{aligned} \tag{12}$$

where $\tau \equiv \text{Var}[v|p]^{-1} = \tau_v + a^2\tau_u$. Similarly, using (11) we can compute the covariance between the consensus opinion and the fundamentals:

$$\text{Cov}\left[\bar{E}[v], v\right] = \alpha_E \frac{1}{\tau_v} + (1 - \alpha_E)\left(\frac{1}{\tau_v} - \frac{1}{\tau}\right), \tag{13}$$

Subtracting (13) from (12) yields

$$\text{Cov}\left[p - \bar{E}[v], v\right] = \frac{\alpha_P - \alpha_E}{\tau}, \tag{14}$$

which implies that $\text{Cov}[p, v]$ differs from $\text{Cov}[\bar{E}[v], v]$ whenever the weight assigned by the price to private information ($\alpha_P$) differs from the optimal statistical weight ($\alpha_E$). Finally, using once again (6) and (11) we have

$$p - v = (1 - \alpha_P)(E[v|p] - v) + \alpha_P \frac{1}{a} u, \text{ and } \bar{E}[v] - v = (1 - \alpha_E)(E[v|p] - v),$$

implying

$$E[p - v|v] = (1 - \alpha_P)(E[E[v|p]|v] - v), \text{ and } E\left[\bar{E}[v] - v|v\right] = (1 - \alpha_E)(E[E[v|p]|v] - v),$$

and from these equations the equivalence between (9) and (10) follows.

□

We thus see that the covariance condition (8) is equivalent to the optimal statistical weight on private information ($\alpha_E$) differing from the weight that the price assigns to the noisy signal about the private information contained in the aggregate demand ($\alpha_P$). This is in turn equivalent to say that the price is either systematically farther away from (closer to) the fundamentals compared to consensus: $|E[p-v|v]| > |E[\bar{E}[v]-v|v]|$ ($|E[p-v|v]| < |E[\bar{E}[v]-v|v]$), a condition similar to the one used by Allen et al. (2006) to analyze the impact of Higher Order Expectations on asset prices.

From the definition of $\alpha_P$ and $\alpha_E$, we can verify that

$$\alpha_P = \alpha_E \Leftrightarrow a = \frac{\gamma\tau_\epsilon}{1+\kappa},$$

which, given (3), is clearly satisfied. Therefore, we can reinterpret our previous conclusion and say that in a static market the price *(i)* assigns the optimal statistical weight to the fundamentals, *(ii)* is as farther away from the fundamentals as investors' consensus, and *(iii)* the simplistic version of the EMH is satisfied.[13]

**Remark 1.** The model introduced above captures the idea that, collectively taken, rational investors do not know the ex-post liquidation value and is therefore qualitatively equivalent to a market in which investors receive a signal with a common error term (like the one studied by Grundy and McNichols (1989)). To see this, maintaining the informational assumptions of our model, suppose that the ex-post liquidation value is given by $v$ whereas investor $i$ receives a signal $s_i = v+\delta+\epsilon_i$. Then, it is easy to see that in this model there exists a unique equilibrium in linear strategies in which $X(s_i,p) = (a/\alpha_E)(E_i[v]-p)$.[14] $\square$

**Remark 2.** In the paper we refer to $u$ as the demand of liquidity traders. Those are akin to investors who receive a shock to their endowment and use the market to optimally hedge against such a shock. It is worth noting that even in a static model the presence of such hedgers may generate multiplicity of linear partially revealing equilibria (see, e.g., Ganguli and Yang (2009) and Manzano and Vives (2010)). This would further complicate the analysis of the dynamic market that we carry out in the following sections.

Note, however, that our specification for the demand of liquidity traders is consistent with the following model. Replace liquidity traders with a measure 1 sector of risk-averse, competitive hedgers who receive a random shock to their endowment. A hedger $i$ receives a shock $u_i = u+\eta_i$ where $\eta_i$ is a normally distributed white-noise error, uncorrelated with all the other random variables in the model. If we denote by $\gamma^U$ the risk-tolerance of hedgers, then letting $\gamma^U \to 0$ implies that each hedger gets rid of $u_i$ in the market place. Owing to the convention that $\int_0^1 \eta_i di = 0$, a.s., this in turn implies that the position hedgers hold is given by $\int_0^1 u_i di = u$, yielding the random component of the aggregate demand that we assume in our model. This

[13] If $E[u]$ is non null, e.g. if $E[u] = \bar{u} > 0$, we have to replace the price $p$ by the price net of the expected liquidity trades component $\hat{p} = p - \bar{u}\mathrm{Var}_i[v+\delta]/\gamma$. Using this definition it is immediate to verify that also when $\bar{u} > 0$, $\alpha_P = \alpha_E$.

[14] In this model $\alpha_E \equiv ((\tau_\epsilon + a^2\tau_u)\tau_v + \tau_\delta(\tau_\epsilon + a^2\tau_u + \tau_v))^{-1}\tau_\delta\tau_\epsilon$, and $a$ is the unique real solution to the cubic $\varphi(a) \equiv a^3\tau_u + a(\tau_\delta+\tau_\epsilon) - \gamma\tau_\delta\tau_\epsilon = 0$. As in our model, $a \in (0, \gamma\tau_\epsilon)$. With an improper prior about the liquidation value, $\tau_v = 0$ and the two models yield exactly the same result. When $\tau_v > 0$, in the model with a common error in the signal, investors' responsiveness to private information is always higher than in the model considered here. To see this note that $a$ is given by the unique solution to $\phi(a) = a^3\tau_u + a(\tau_v+\tau_\epsilon+\tau_\delta) - \gamma\tau_\delta\tau_\epsilon = 0$, whereas in the presence of a common error in the signal it is given by the solution to $\varphi(a) \equiv a^3\tau_u + a(\tau_\epsilon+\tau_\delta) - \gamma\tau_\delta\tau_\epsilon = 0$. Now $\phi(0) = \varphi(0) = -\gamma\tau_\delta\tau_\epsilon < 0$, and $\phi'(0) = \tau_v+\tau_\epsilon+\tau_\delta > \varphi'(0) = \tau_\epsilon+\tau_\delta$, which together with $\phi''(a) = \varphi''(a) = 6a\tau_u$, implies that the unique solution to $\phi(a) = 0$ always lays to the left of the unique solution to $\varphi(a) = 0$.

is in line with Medrano and Vives (2004), who argue that upon receiving a shock to their endowment, infinitely risk-averse hedgers unwind their exposure to the market, yielding the random component of the aggregate demand for the stock that characterizes the model with noise traders. □

In the following sections we will argue that in a dynamic market, long term investors often engage in speculating on short-run price movements, so that their strategies depart from the solution of the static setup. This in turn implies that the simplistic version of the EMH fails to hold, and prices can score better (or worse) than consensus in forecasting the fundamentals.

# 3 A Three-Period Framework

Consider now a three-period extension of the market analyzed in the previous section. We assume that any investor $i \in [0,1]$ has CARA preferences and maximizes the expected utility of his final wealth $W_{i3} = (v+\delta-p_3)x_{i3}+\sum_{n=1}^{2}(p_{n+1}-p_n)x_{in}$.[15] The random term $\delta$ is independent of all the random variables in the model. In period $n$ an informed investor $i$ receives a signal $s_{in} = v + \epsilon_{in}$, where $\epsilon_{in} \sim N(0, \tau_{\epsilon_n}^{-1})$, $v$ and $\epsilon_{in}$ are independent for all $i, n$ and error terms are also independent both across time periods and investors. Denote with $s_i^n \equiv \{s_{it}\}_{t=1}^n$ and $p^n \equiv \{p_t\}_{t=1}^n$, respectively, the sequence of private signals and prices an investor observes at time $n$. It follows from Gaussian theory that the statistic $\tilde{s}_{in} = (\sum_{t=1}^n \tau_{\epsilon_t})^{-1}(\sum_{t=1}^n \tau_{\epsilon_t} s_{it})$ is sufficient for the sequence $s_i^n$ in the estimation of $v$.

The stock of liquidity trades is assumed to follow an AR(1) process: $\theta_n = \beta\theta_{n-1}+u_n$, where $u_n \sim N(0, \tau_u^{-1})$ is orthogonal to $\theta_{n-1}$, and $\beta \in [0,1]$. To interpret, suppose $\beta < 1$, then at any period $n > 1$ market clearing involves the $n-1$-th and $n$-th period aggregate demands of informed investors (respectively, $x_{n-1} \equiv \int_0^1 x_{in-1}di$, and $x_n \equiv \int_0^1 x_{in}di$), a fraction $1-\beta$ of the demand coming from the $n-1$-th generation of liquidity traders who revert their positions, and the demand of the new generation of liquidity traders. Considering the equilibrium conditions for the first two trading dates, and letting $\Delta x_2 \equiv x_2 - x_1$, $\Delta\theta_2 \equiv \theta_2 - \theta_1 = u_2 + (\beta-1)\theta_1$, this implies

$$x_1 + \theta_1 = 0$$
$$\Delta x_2 + \Delta\theta_2 = 0 \Leftrightarrow x_2 + \beta\theta_1 + u_2 = 0.$$

Thus, assuming that liquidity trading follows an AR(1) process allows to take into account the possibility that only part of the trades initiated by liquidity traders at time $n$ actually reverts at time $n+1$. The lower (higher) is $\beta$, the higher (lower) is the fraction of period $n$ liquidity traders who will (will not) revert their positions at time $n+1$, and thus won't (will) be in the market at time $n+1$. Equivalently, for $0 \leq \beta < 1$, a high, positive demand from liquidity traders at time $n$ is unlikely to show up with the same intensity at time $n+1$, implying

[15] We assume, as before without loss of generality, that the non-random endowment of investors is zero.

that $\mathrm{Cov}[\Delta\theta_n, \Delta\theta_{n+1}] < 0$.[16] Intuitively, a low $\beta$ is likely to occur when the time between two consecutive trades is large. Conversely, a high $\beta$ depicts a situation in which the time between two consecutive transactions is small, so that investors make repeated use of the market to satisfy their trading needs.[17]

**Remark 3.** In line with Remark 2 in the dynamic case considered here it is also possible to show that that the specification for the demand coming from liquidity traders is consistent with a model in which the risk aversion of rational hedgers grows unboundedly. □

Informed investors submit a demand schedule (generalized limit order) to the market $X_n(\tilde{s}_{in}, p^{n-1}, p_n)$ indicating the desired position in the risky asset for each realization of the equilibrium price. We will restrict attention to linear Perfect Bayesian equilibria in which in period $n$ an investor trades according to $X_n(\tilde{s}_{in}, p^n) = a_n\tilde{s}_{in} - \varphi_n(p^n)$, where $\varphi_n(\cdot)$ is a linear function of the price sequence $p^n$. Note that this equilibrium will be symmetric given the symmetry in the preferences of the traders and in the information structure. Let us denote with $z_n$ the intercept of the $n$-th period net aggregate demand $\int_0^1 \Delta x_{in} di + u_n$, where $\Delta x_{in} = x_{in} - x_{in-1}$. The random variable $z_n \equiv \Delta a_n v + u_n$ represents the informational addition brought about by the $n$-th period trading round, and can thus be interpreted as the informational content of the $n$-th period order-flow (where, with a slight abuse of notation, we set $\Delta a_n \equiv a_n - \beta a_{n-1}$).

Extending the notation adopted in the previous section, we denote by $E_{in}[Y] = E[Y|s_i^n, p^n]$, $E_n[Y] = E[Y|p^n]$ ($\mathrm{Var}_{in}[Y] = \mathrm{Var}[Y|s_i^n, p^n]$, $\mathrm{Var}_n[Y] = \mathrm{Var}[Y|p^n]$), respectively the expectation (variance) of the random variable $Y$ formed by an investor conditioning on the private and public information he has at time $n$, and that obtained conditioning on public information only. We also denote by $\tau_n \equiv (\mathrm{Var}_n[v])^{-1}$, and by $\tau_{in} \equiv (\mathrm{Var}_{in}[v])^{-1}$. Finally, $\alpha_{E_n} = \sum_{t=1}^n \tau_{\epsilon_t}/\tau_{in}$ is the optimal statistical weight to private information at time $n$, and we make the convention that, given $v$, at any time $n$ the average signal $\int_0^1 s_{in} di$ equals $v$ almost surely (i.e. errors cancel out in the aggregate: $\int_0^1 \epsilon_{in} di = 0$).

# 4 The Two-Period Market

In this section we restrict attention to the case with two trading periods. This allows us to provide a fully analytical characterization of the market with heterogeneous information.

[16]Alternatively, the AR(1) assumption for liquidity traders' demand can be interpreted as a way to parsimoniously model the existence of a positive feedback in these traders' strategies. To see this, consider a 2-period version of our model, then for $\beta > 0$, $\mathrm{Corr}[\theta_2, \theta_1] = \beta/(1+\beta^2)^{1/2} > 0$. For two normal random variables, positive correlation is equivalent to the monotone likelihood ratio property. Therefore, we can conclude that if $\beta > 0$ the probability of observing a higher $\theta_2$ increases in $\theta_1$.

[17]The literature that has dealt with dynamic trading models featuring an AR(1) process for liquidity posits relatively high values for $\beta$. For example, in their analysis of a dynamic FX market, Bacchetta and van Wincoop (2006) model the aggregate exposure to the exchange rate as an AR(1) process and in their numerical simulations assume $\beta = 0.8$ (Table 1, p. 564). This assumption is somehow validated by empirical analysis. In a recent paper, Easley et al. (2008) analyze the order arrival process using the daily number of buys and sell orders for 16 stocks over a 15-year time period. Their findings point to a highly persistent process for uninformed investors.

## 4.1 The Equilibrium

Suppose $N = 2$. The following proposition characterizes equilibrium prices and strategies:

**Proposition 1.** *Let $\sum_{t=1}^{n} \tau_{\epsilon_t} > 0$, there exists a linear equilibrium of the 2-period market in which*

$$p_n = \alpha_{P_n} \left( v + \frac{\theta_n}{a_n} \right) + (1 - \alpha_{P_n}) E_n[v], \quad n = 1, 2, \tag{15}$$

*where $\theta_n = u_n + \beta\theta_{n-1}$. An investor's first period strategy is given by:*

$$X_1(s_{i1}, z_1) = \frac{a_1}{\alpha_{E_1}} (E_{i1}[v] - p_1) + \frac{\alpha_{P_1} - \alpha_{E_1}}{\alpha_{E_1}} \frac{a_1}{\alpha_{P_1}} (p_1 - E_1[v]), \tag{16}$$

*while at time 2:*

$$X_2(\tilde{s}_{i2}, z^2) = \frac{a_2}{\alpha_{E_2}} (E_{i2}[v] - p_2), \tag{17}$$

*where $\alpha_{P_2} = \alpha_{E_2}$, $a_2 = (1 + \kappa)^{-1} \sum_{t=1}^{2} \tau_{\epsilon_t}$, $\kappa \equiv \tau_\delta^{-1} \tau_{i2}$, and expressions for $a_1 > 0$ and $\alpha_{P_1} > 0$ are provided in the appendix (see equations (A.49), (A.50)).*

Proposition 1 extends Vives (1995), restating a result due to He and Wang (1995), providing an alternative, constructive proof. According to (15), at period $n$ the equilibrium price is a weighted average of the market expectation about the fundamentals $v$, and a monotone transformation of the $n$-th period aggregate demand intercept. A straightforward rearrangement of (15) yields

$$\begin{aligned} p_n - E_n[v] &= \frac{\alpha_{P_n}}{a_n} \left( a_n \left( v - E_n[v] \right) + \theta_n \right) \\ &= \Lambda_n E_n[\theta_n], \end{aligned} \tag{18}$$

where $\Lambda_n \equiv \alpha_{P_n} / a_n$. According to (18), the discrepancy between $p_n$ and $E_n[v]$ is due to the contribution that liquidity traders are expected to give to the $n$-th period aggregate demand.[18] Thus, $\Lambda_n$ is a measure of market depth. The smaller is $\Lambda_n$ and the smaller is the anticipated (and realized) contribution that the stock of liquidity trading gives to the aggregate demand and to the price.

Substituting (18) into (16), we obtain

$$X_1(s_{i1}, z_1) = \frac{a_1}{\alpha_{E_1}} (E_{i1}[v] - p_1) + \frac{\alpha_{P_1} - \alpha_{E_1}}{\alpha_{E_1}} E_1[\theta_1].$$

Thus, an investor's first period strategy is the sum of two components. The first component captures the investor's activity based on his *private* estimation of the difference between the fundamentals and the first period equilibrium price. This can be considered as "long-term"

[18] The expression for $E_1[\theta_1]$ in (18) follows immediately from the fact that

$$\begin{aligned} a_1 v + \theta_1 &\equiv E_1 \left[ a_1 v + \theta_1 \right] \\ &= a_1 E_1[v] + E_1[\theta_1]. \end{aligned}$$

speculative trading, aimed at profiting from the liquidation value of the asset. The second component captures the investor's activity based on the extraction of order flow, i.e. *public*, information. This trading is instead aimed at exploiting short-run movements in the asset price determined by the evolution of the future aggregate demand. Upon observing this information, and depending on the sign of the difference $\alpha_{P_1} - \alpha_{E_1}$, investors engage either in "market making" (when $\alpha_{P_1} - \alpha_{E_1} < 0$, thereby accommodating the aggregate demand) or in "trend chasing" (when $\alpha_{P_1} - \alpha_{E_1} > 0$, thus following the market).[19]

To fix ideas, consider the following example. Suppose that $E_1[\theta_1] > 0$. An investor's reaction to this observation depends on whether he believes it to be driven by liquidity trades or fundamentals information. Indeed,

$$E_1[\theta_1] = a_1(v - E_1[v]) + \theta_1. \tag{19}$$

Hence, $E_1[\theta_1] > 0$ can either signal a short-term demand pressure coming from liquidity traders ($\theta_1 > 0$) or rather a fundamental value realization that is above public expectation ($v > E_1[v]$). In the former (latter) case, the forward looking attitude implied by rational behavior, would advise the investor to accommodate (join) the aggregate demand in the expectation of a future price reversion (further increase).[20] Suppose $\alpha_{P_1} < \alpha_{E_1}$, then informed investors count on the reversal of liquidity traders' demand in the next period and take the other side of the market, acting as market makers. They thus short the asset expecting to buy it back in the future at a lower price. If, on the other hand, $\alpha_{P_1} > \alpha_{E_1}$, informed investors anticipate that the role of "positive" fundamentals information looms large in the $n$-th period aggregate demand and that this is most likely affecting the sign of $E_1[\theta_1]$. As a consequence, they buy the asset, expecting to re sell it once its price has incorporated the positive news, effectively chasing the trend.

Finally, note that according to (17), in the second period investors concentrate in "long term speculation." Indeed, at $n = 2$, investors anticipate that the asset will be liquidated in the next period and thus that its value will *not* depend on the information contained in that period's aggregate demand. As a consequence, they choose their position only taking into account their information on the fundamentals, acting like in a static market.

**Remark 4.** Multiple equilibria may in principle arise. For some parameter values, it is easy to find different equilibria. For instance, if liquidity trades increments are i.i.d., and investors only receive private information in the first period (i.e., if $\beta = 1$ and $\tau_{\epsilon_2} = 0$), there always exists an equilibrium where $a_1 = a_2 = (1+\kappa)^{-1}\gamma\tau_{\epsilon_1}$, whereas for large values of $\tau_\delta$ another equilibrium where $a_1 = (\gamma\tau_u)^{-1}(1 + \kappa + \gamma^2\tau_{\epsilon_1}\tau_u) > a_2 = (1+\kappa)^{-1}\gamma\tau_{\epsilon_1}$ may also arise (in line with what happens in a model where investors receive a signal containing a common error term – see Remark 1). The first equilibrium disappears when $\beta < 1$. In the absence of residual uncertainty (i.e., if $\tau_\delta^{-1} = 0$), $\kappa = 0$, and the equilibrium with $a_1 = a_2 = \gamma\tau_{\epsilon_1}$ is unique (see Section 4.3). □

[19] He and Wang (1995) point out that in a market with long term investors the weights that prices and average expectations assign to fundamentals can differ.

[20] In other words, owing to the traditional signal extraction problem, it is entirely possible that the sign of $E_1[\theta_1]$ is due to the presence of a positive demand coming from informed traders.

As argued above, the difference $\alpha_{P_1} - \alpha_{E_1}$ plays a crucial role in shaping investors' reactions to public information and thus their trading behavior. In our static benchmark, on the other hand, the same difference also determines how "close" the price is to the fundamentals compared to the average expectations investors hold about it. This fact is also true in a dynamic market. Indeed, since

$$\bar{E}_n[v] \equiv \int_0^1 E_{in}[v]di = \alpha_{E_n} v + (1 - \alpha_{E_n}) E_n[v], \tag{20}$$

and using (15), a straightforward extension of the argument used in section 2 allows to obtain the following

**Lemma 2.** *At any linear equilibrium of the 2-period market, the following three conditions are equivalent:*

$$Cov[p_n, v] < Cov\left[\bar{E}_n[v], v\right] \tag{21}$$

$$\alpha_{P_n} < \alpha_{E_n} \tag{22}$$

$$|E\left[p_n - v|v\right]| > \left|E\left[\bar{E}_n[v] - v|v\right]\right|. \tag{23}$$

*Proof.* For the first part of the proof, computing the covariance between $p_n$ and $v$ yields

$$\text{Cov}[v, p_n] = \alpha_{P_n} \frac{1}{\tau_v} + (1 - \alpha_{P_n}) \left( \frac{1}{\tau_v} - \frac{1}{\tau_n} \right), \tag{24}$$

and carrying out a similar computation for the time $n$ consensus opinion

$$\text{Cov}\left[\bar{E}_n[v], v\right] = \alpha_{E_n} \frac{1}{\tau_v} + (1 - \alpha_{E_n}) \left( \frac{1}{\tau_v} - \frac{1}{\tau_n} \right), \tag{25}$$

where $\tau_n \equiv (\text{Var}[v|p^n])^{-1} = \tau_v + \tau_u \sum_{t=1}^n \Delta a_t^2$. We can now subtract (25) from (24) and obtain

$$\text{Cov}\left[p_n - \bar{E}_n[v], v\right] = \frac{\alpha_{P_n} - \alpha_{E_n}}{\tau_n}, \tag{26}$$

implying that whenever the price scores worse than consensus in forecasting the fundamentals, the weight the former assigns to public information is larger than the optimal statistical weight.

To prove the equivalence between (22) and (23), we use the expressions for the prices and consensus in the 2-period market (respectively equations (15) and (20)) and obtain

$$p_n - v = (1 - \alpha_{P_n})(E_n[v] - v) + \alpha_{P_n} \frac{1}{a_n} \theta_n, \text{ and } \bar{E}_n[v] - v = (1 - \alpha_{E_n})(E_n[v] - v),$$

implying

$$E[p_n - v|v] = (1 - \alpha_{P_n})(E[E_n[v]|v] - v), \text{ and } E\left[\bar{E}_n[v] - v|v\right] = (1 - \alpha_{E_n})(E[E_n[v]|v] - v),$$

Thus, if $\alpha_{P_n} > \alpha_{E_n}$ the price is closer to the fundamentals compared to the consensus opinion, while the opposite occurs whenever $\alpha_{P_n} < \alpha_{E_n}$.

□

We can now put together the results obtained in Proposition 1 and Lemma 2: if upon observing the first period aggregate demand investors expect it to be mostly driven by liquidity trades, they accommodate the order flow. As a consequence, their behavior drives the price away from the fundamentals compared to the average market opinion. If, instead, they deem the aggregate demand to be mostly information driven, they align their short term positions to those of the market. This, in turn, drives the price closer to the fundamentals, compared to investors' average expectations.

Alternatively, when investors speculate on short term returns the first period equilibrium price and the consensus opinion have different dynamics:

$$p_1 = \bar{E}_1[v] + \frac{\alpha_{P_1} - \alpha_{E_1}}{a_1} E_1[\theta_1] + \frac{\alpha_{E_1}}{a_1}\theta_1. \tag{27}$$

Indeed, as the price originates from market clearing, it reflects *both* determinants of investors' demand, i.e. their long term forecast *and* their short term speculative activity. Conversely, as the consensus opinion is *only* based on investors' long term expectations, it *does not* reflect the impact of short term speculation.

**Remark 5.** If we use (27) and the fact that according to Lemma 2

$$\text{Cov}[p_n - \bar{E}_n[v], v] = \frac{\alpha_{P_n} - \alpha_{E_n}}{\tau_n},$$

then we can write the first period equilibrium price as follows:

$$p_1 = \bar{E}_1[v] + \frac{\text{Cov}[p_1 - \bar{E}_1[v], v]}{\text{Var}_1[v]} E_1\left[\frac{\theta_1}{a_1}\right] + \alpha_{E_1}\frac{\theta_1}{a_1}.$$

The above expression is akin to a traditional asset pricing equation in which the asset price loads on two factors. Since investors are informed about the "long term" fundamentals, it reflects their consensus view. However, given that predictability induces investors to also speculate on short term returns, a systematic risk premium is required to compensate them for holding the expected liquidity stock (scaled by informed investors' responsiveness to private information) across trading dates in the presence of differential information. This latter component can be viewed as a "short term" fundamentals. Note that the coefficient that multiplies

$$E_1\left[\frac{\theta_1}{a_1}\right],$$

is like a "beta":

$$\frac{\text{Cov}[p_1 - \bar{E}_1[v], v]}{\text{Var}_1[v]},$$

in that for given conditional precision (or variance, in the denominator), the larger (in absolute value) the covariance between price departures from consensus and fundamentals, the higher the risk associated either to liquidity trades or adverse selection that is borne by investors (in fact according to Proposition 1, in equilibrium these departures signal the presence of short term trading). Note also that this "beta" can be positive or negative depending on the difference $\alpha_{P_1} - \alpha_{E_1}$. □

To establish the direction of inequality (23) we thus need to determine what is the force that drives an investor's reaction to the information contained in the aggregate demand. Prior to that we consider a special case of our model in which investors have symmetric information.

## 4.2 Symmetric Information and Short Term Speculation

In this section we assume that investors have symmetric information, setting $\tau_{\epsilon_n} = 0$, for all $n$. This considerably simplifies the analysis and allows us to show that in the absence of private information short term speculation *does not* lead prices to be systematically closer or farther away from the fundamentals compared to investors' average expectations. We start by characterizing the equilibrium in this setup, and then analyze its properties.

**Proposition 2.** *In the 2-period market with symmetric information, there exists a unique equilibrium in linear strategies, where prices are given by*

$$p_n = \bar{v} + \Lambda_n \theta_n, \tag{28}$$

*where*

$$\Lambda_2 = \frac{1+\kappa}{\gamma\tau_v} \tag{29}$$

$$\Lambda_1 = \Lambda_2 \left(1 + \frac{(\beta-1)\gamma^2\tau_u\tau_v}{1+\kappa+\gamma^2\tau_u\tau_v}\right), \tag{30}$$

*and* $\kappa = \tau_v/\tau_\delta$. *Risk averse investors trade according to*

$$X_n(p^n) = -\Lambda_n^{-1}(p_n - \bar{v}), \quad n = 1, 2. \tag{31}$$

When $\tau_\epsilon = 0$, at any period $n$ investors have no private signal to use when forming their position. As a consequence, the aggregate demand only reflects the stock of liquidity trades. According to (31), this implies that speculators always take the other side of the market, buying the asset when $p_n < \bar{v} \Leftrightarrow \theta_n = \Lambda_n^{-1}(p_n - \bar{v}) < 0$, and selling it otherwise. Indeed, in the absence of private information, risk averse investors face no adverse selection problem when they clear the market. The discrepancy between the equilibrium price and the unconditional expected value reflects the risk premium investors demand in order to accommodate the demand of liquidity traders. Even in the absence of adverse selection risk, in fact, investors anticipate the possibility that the liquidation value $v$ may be lower (higher) than the price they pay for (at which they sell) the asset.

If $\beta < 1$, in the first period risk averse investors also speculate on short term asset price movements providing additional order flow accommodation. This can be seen rearranging (31) in the following way:

$$X_1(p_1) = \Lambda_2^{-1}(\bar{v} - p_1) - \left(\Lambda_1^{-1} - \Lambda_2^{-1}\right)(p_1 - \bar{v}).$$

As a result, for $\beta \in (0, 1)$, market depth decreases *across* trading periods:

$$0 < \Lambda_1 < \Lambda_2,$$

and it decreases in $\beta$ in the first period:

$$\frac{\partial \Lambda_1}{\partial \beta} > 0,$$

as one can immediately see from (30). The intuition for these results is that if $\beta < 1$, as liquidity trades' increments are negatively correlated, prior to the last trading round investors have more opportunities to unload their risky position. This reduces the risk they bear, and lowers the impact that the liquidity shock has on the price. If $\beta = 1$ liquidity trades' increments are i.i.d.. Therefore, speculators cannot count on the future reversion in the demand of liquidity traders and their extra order flow accommodation disappears. As a consequence, depth is constant across periods: $\Lambda_1 = \Lambda_2 = (\gamma\tau_v)^{-1}(1+\kappa)$.[21]

As one would intuitively expect, short term speculation arises insofar as investors can map the partial predictability of liquidity trades' increments into the anticipation of short term returns. The following proposition formalizes this intuition:

**Corollary 1.** *In the market with symmetric information, in the first period investors speculate on short term asset price movements if and only if, provided $\theta_1 > 0$ ($\theta_1 < 0$), they expect the next period return to revert: $E_1[p_2 - p_1] < 0$ ($E_1[p_2 - p_1] > 0$).*

*Proof.* Using (28) we can easily obtain

$$E_n[p_2 - p_1] = (\beta\Lambda_2 - \Lambda_1)\,\theta_n.$$

Using (30) we then obtain

$$\begin{aligned}(\beta\Lambda_2 - \Lambda_1)\,\theta_1 &= \Lambda_2(\beta - 1)\frac{1+\kappa}{1+\kappa+\gamma^2\tau_u\tau_v}\theta_1 \\ &= \Lambda_2\Lambda_1^{-1}(\beta - 1)\frac{1+\kappa}{1+\kappa+\gamma^2\tau_u\tau_v}(p_1 - \bar{v}).\end{aligned} \tag{32}$$

Since for $\beta \in [0,1)$, the term multiplying $\theta_1$ in (32) is negative, $E_1[p_2 - p_1] < 0 \Leftrightarrow \theta_1 > 0$. If $\beta = 1$ investors do not speculate on short term returns, and $\Lambda_1 = \Lambda_2 = (\gamma\tau_v)^{-1}(1+\kappa)$. This, in turn, implies that $E_1[p_2 - p_1] = 0$, proving our claim. □

Both in the market with homogeneous information and in the one with heterogeneous information investors speculate on short term returns. However, while in the latter market this possibly leads to the fact that prices are worse predictors of the fundamentals compared to consensus, in the presence of symmetric information this never happens:

**Corollary 2.** *With symmetric information at $n = 1, 2$, the price is as far away from the fundamentals as investors' average expectations.*

*Proof.* According to (28), the equilibrium price can be expressed as the sum of investors' average expectations and $\theta_n$ which is by assumption orthogonal to $v$. Hence,

$$E[p_n - v|v] = E[\bar{v} + \Lambda_n\theta_n - v|v] = \bar{v} - v.$$

Given that investors do not have private information, the price only reflects $\theta_n$, and $E_{in}[v] = E[v] = \bar{v}$. Hence,

$$E\left[\bar{E}_n[v] - v|v\right] = \bar{v} - v.$$

[21] This matches the result that He and Wang obtain when looking at the case of homogeneous information when signal are fully informative on $v$, i.e. with $\tau_{\epsilon_n} \to \infty$.

Thus $E[\bar{E}_n[v] - v|v] = E[p_n - v|v]$, which proves our result. $\square$

As risk-averse investors have no private information to trade with, their orders do not impound fundamentals information in the price. As a consequence, as shown in Proposition 2, at any period $n$ investors are able to extract the realization of liquidity traders' demand $\theta_n$ from the observation of the aggregate demand, implying that the price perfectly reflects $\theta_n$. As the latter is orthogonal to $v$, and in the absence of heterogeneous signals $\bar{E}_n[v] = \bar{v}$, both prices and speculators' consensus opinion about fundamentals stand at the same "distance" from $v$.

The last result of this section draws an implication of our analysis for the time series behavior of returns, showing that returns display reversal if liquidity trades increments are correlated:

**Corollary 3.** *In the first period, returns exhibit reversal if and only if $\beta < 1$.*

*Proof.* This follows immediately from the fact that $\mathrm{Cov}[p_2 - p_1, p_1 - \bar{v}] = \Lambda_1(\beta\Lambda_2 - \Lambda_1)\tau_u^{-1} < 0$. $\square$

With homogeneous information, reversal occurs because if $\beta < 1$ not all the liquidity traders revert their position in the second period, so that the impact of their demand partially evaporates over time.[22]

Summarizing, in the model with no private information investors speculate on short term asset price movements if and only if they can exploit the predictability of future liquidity trades' increments. However, prices are as far away from fundamentals as the consensus opinion. Furthermore, corollaries 1 and 3 imply that at any time $n = 1, 2$, and for all $(\beta, 1/\tau_\delta) \in [0, 1) \times \mathbb{R}_+$ the short term, contrarian strategy based on the realization of $\theta_n$ univocally maps into return reversal.

## 4.3 The Effect of Heterogeneous Information

As explained in Section 4.1, the assumption $\beta < 1$ implies that liquidity trades' increments are negatively correlated, and introduces a mean reverting component in the evolution of the aggregate demand. In the market with no private information analyzed in Section 4.2, as the position of liquidity traders is perfectly observable, this leads investors to speculate on short term returns, providing additional order flow accommodation. When investors have private signals, the aggregate demand also features a component that reflects fundamentals information. As a consequence, the stock of liquidity trades *cannot* be perfectly retrieved, and informed investors face an adverse selection problem. Thus, when they observe the aggregate demand, they *estimate* the stock of liquidity trades and choose the side of the market on which to stand, based on which component (liquidity shocks or fundamentals information) they trust will influence the evolution of the future aggregate demand. Mean reversion in liquidity trades increments pushes investors to take the other side of the market (see Section 4.2). In this section we will argue that with heterogeneous information, if $\tau_\delta^{-1} > 0$ investors scale up their

[22]In the second period, one can check that $\mathrm{Cov}[v - p_2, p_2 - p_1] = -\Lambda_2(\Lambda_2 + \beta(\beta\Lambda_2 - \Lambda_1))\tau_u^{-1} < 0$ for all $\beta$. In this case negative covariance always arises because the third period price is exogenous.

signal responsiveness prior to the last trading round. This, in turn, implies that prior to the last trading round informed investors are more inclined to attribute a given aggregate demand realization to the impounding of fundamentals information, and are pushed to follow the market. Both effects eventually bear on the magnitude of the weight the price assigns to the fundamentals.

We start by assuming away residual uncertainty. In this case it is possible to show that a unique equilibrium in linear strategies exists (He and Wang (1995) and Vives (1995)):

**Corollary 4.** *In the absence of residual uncertainty, a unique equilibrium in linear strategies exists where* $a_n = \gamma(\sum_{t=1}^{n} \tau_{\epsilon t})$, *and*

$$\alpha_{P_1} = \alpha_{E_1}\left(1 + (\beta - 1)\frac{\gamma\tau_1\tau_u(\gamma\tau_1 + \beta a_1)}{(1 + \gamma\tau_u\Delta a_2)^2\tau_{i1} + (\gamma\tau_1 + \beta a_1)^2\tau_u}\right) < \alpha_{E_1}, \tag{33}$$

*if and only if* $\beta < 1$.

According to the above result, if $\tau_\delta^{-1} = 0$, investors' responsiveness to private information matches the static solution and, when $\beta < 1$, prices score worse than consensus in predicting the fundamentals. Intuitively, if $\tau_\delta^{-1} = 0$, as investors respond to private signals as in a static market, when $\beta < 1$ in the first period the only source of predictability of the future aggregate demand comes from the mean reverting nature of liquidity traders' demand. Investors' short term behavior is then akin to the one they display in the market with symmetric information. Thus, upon observing $p_1 > E_1[v] \Leftrightarrow E_1[\theta_1] > 0$ ($p_1 < E_1[v] \Leftrightarrow E_1[\theta_1] < 0$), investors accommodate the expected positive demand (supply) of liquidity traders, selling (buying) the asset in the anticipation of a future price reversion. As these price movements do not reflect fundamentals information, this drives the price away from the terminal pay off compared to the consensus opinion.

Corollary 4 argues that, absent residual uncertainty, investors' sole motive to speculate on price differences is the possibility to profit from the mean reversion of liquidity trades. This suggests that shutting down this prediction channel should eliminate any short term speculative activity:

**Corollary 5.** *In the absence of residual uncertainty,* $\alpha_{P_1} = \alpha_{E_1}$ *if and only if* $\beta = 1$.

*Proof.* This follows immediately by replacing $\beta = 1$ in (33). □

If $\tau_\delta^{-1} = 0$, and $\beta = 1$, liquidity trades' increments are i.i.d. and in the first period investors have no way to exploit the predictability of future periods' aggregate demand. As a consequence, they concentrate their trading activity on long term speculation, and $\alpha_{P_1} = \alpha_{E_1}$.

We now introduce residual uncertainty. In this case, it is possible to prove that:

**Corollary 6.** *In the presence of residual uncertainty, at any linear equilibrium the weight the price assigns to the fundamentals in the first period is given by*

$$\alpha_{P_1} = \alpha_{E_1}\left(1 + (\beta\rho_1 - 1)\Upsilon_1\right), \tag{34}$$

*where*

$$\rho_1 = \frac{a_1(1+\kappa)}{\gamma\tau_{\epsilon_1}} > 1, \tag{35}$$

*and the expressions for* $a_1 > 0$ *and* $\Upsilon_1 > 0$ *are provided in the appendix (see equations* (A.49) *and* (A.51)*).*

According to (34), residual uncertainty generates an effect that offsets the one that liquidity trades' mean reversion has on $\alpha_{P_1}$. The intuition is as follows. As argued in section 4.1 in the last trading round agents concentrate on the long term value of the asset, speculating as in a static market. This implies that their responsiveness to private information is given by

$$a_2 = \frac{\gamma \sum_{t=1}^{2} \tau_{\epsilon_t}}{1+\kappa}.$$

The above expression generalizes (3) and shows that in a *static* market with residual uncertainty, the weight investors assign to private information is the risk-tolerance weighted sum of their private signal precisions, scaled down by a factor $1+\kappa$, which is larger, the larger is $\tau_\delta^{-1}$. Residual uncertainty also affects an investor's signal responsiveness in the first period, and this is reflected by the parameter $\rho_1$, which captures the deviation from the long term private signal responsiveness due to the presence of residual uncertainty (see (35)). As stated in Corollary 6 in the presence of residual uncertainty $\rho_1 > 1$. Thus, in the first period, investors react to their private signals *more* aggressively than if they were just about to observe the liquidation value:

$$a_1 > \frac{\gamma\tau_{\epsilon_1}}{1+\kappa}.$$

Indeed, while residual uncertainty makes investors less confident about their signals, the presence of an additional trading round increases the opportunities to adjust suboptimal positions prior to liquidation. This, in turn, boosts investors' reaction to private information compared to the static solution. Furthermore, this also implies that a given aggregate demand realization is more likely to be driven by informed investors, contributing to explain the component capturing trading based on order flow information in investors' strategies:

**Corollary 7.** *In the presence of residual uncertainty, at any linear equilibrium in the first period* $\alpha_{P_1} < \alpha_{E_1}$ *if and only if* $\beta\rho_1 < 1$.

*Proof.* Follows immediately from (34). □

To fix ideas, suppose $\beta = 1$ and assume that in the first period investors observe $p_1 > E_1[v]$ (i.e., $E_1[\theta_1] > 0$). Given that the demand of liquidity traders displays no predictable pattern, a short term position based on shorting the asset in the anticipation of buying it back at a lower price one period ahead is suboptimal. At the same time, the fact that $\rho_1 > 1$ implies that informed investors react more aggressively to their private signal than in a static market. This generates additional informed trading which may be responsible for the observed price realization. Informed investors thus go long in the asset in the anticipation of a further price increase in the coming period. If $\beta < 1$, the mean reversion effect of liquidity trades kicks in and investors' decisions as to the side of the market in which to position themselves needs to trade off this latter pattern against the one driven by fundamentals information.

## 5 The Three-Period Market

In this section we extend our analysis to the case with three trading dates. This allows to check the robustness of our findings and, as we argue in Section 7, use our model to study return regularities with endogenous prices.

In the model with three trading dates, the equilibrium has the same form of the two-period case. With symmetric information, natural extensions of Proposition 2, and Corollaries 1, 2, and 3 hold. In particular, in the appendix (see the proof of Proposition A.2) we show that

$$\mathrm{Cov}[p_3 - p_2, p_2 - p_1] = \left(\frac{\beta\Lambda_3 - \Lambda_2}{\tau_u}\right)\left(\Lambda_2\left(1+\beta^2\right) - \beta\Lambda_1\right) < 0 \Leftrightarrow \beta < 1. \tag{36}$$

In the model with heterogeneous information when $\tau_\delta^{-1} > 0$, we are unable to provide an existence proof and have to resort to numerical simulations to study the properties of the model. Numerical simulations were conducted assuming different patterns of private information arrival. In particular, we assumed that investors receive information of a constant precision at every trading round (so that $\tau_{\epsilon_n} = \tau_\epsilon > 0$, for all $n$), that private information only arrives in the first two periods (i.e., $\tau_{\epsilon_1} = \tau_{\epsilon_2} > 0$, while $\tau_{\epsilon_3} = 0$), and that private information is only received in the first period (i.e., $\tau_{\epsilon_1} > 0$, while $\tau_{\epsilon_n} = 0$ for $n = 2, 3$). The values of the chosen parameters are as follows: $\tau_v, \tau_u, \tau_{\epsilon_n} \in \{1, 4\}$, $\gamma \in \{1/4, 1/2, 1\}$, and $\beta \in \{0, 0.001, 0.002, \ldots, 1\}$, $\tau_\delta^{-1} \in \{0.1, 0.2, \ldots, 5\}$, for each pattern of private information arrival. While the values of the risk tolerance coefficient reflect realistic assumptions, the values of the precisions have been chosen to verify the robustness of our conclusions. As we argue in Remark 4, with residual uncertainty, multiple equilibria can arise, but we find that the qualitative properties of the model do not change across equilibria.[23]

More in detail (see Proposition A.1), for $n = 1, 2, 3$, prices satisfy (15), investors' strategies in the first two periods are given by

$$X_n(\tilde{s}_{in}, z^n) = \frac{a_n}{\alpha_{E_n}}(E_{in}[v] - p_n) + \frac{\alpha_{P_n} - \alpha_{E_n}}{\alpha_{E_n}} E_n[\theta_n],$$

whereas in the third period they are akin to those of the static market. Furthermore, a 3-period extension of Corollary 2 holds, while the weights assigned by prices to aggregate private information in the first two periods are given by

$$\alpha_{P_1} = \alpha_{E_1}\left(1 + (\beta\rho_1 - \rho_2)\Upsilon_1^1 + (\beta\rho_2 - 1)\Upsilon_1^2\right) \tag{37}$$

$$\alpha_{P_2} = \alpha_{E_2}\left(1 + (\beta\rho_2 - 1)\Upsilon_2^1\right), \tag{38}$$

where

$$\rho_n = \frac{a_n(1+\kappa)}{\gamma\sum_{t=1}^{n}\tau_{\epsilon_t}}, \tag{39}$$

$\kappa = \tau_\delta^{-1}\tau_{i3}$, and the expressions for $\Upsilon_n^k$, $a_n$ are provided in the appendix for $k, n \in \{1, 2\}$ (see equations (A.26), (A.44), (A.45), and (A.6), (A.21), (A.46), respectively). A straightforward

[23]Simulations were conducted using Mathematica. Additional simulations have been done to extend the space of parameter values for the precisions, assuming $\tau_v, \tau_u, \tau_{\epsilon_n} \in \{.1, .2, \ldots, 2\}$, $\beta \in \{0, .1, \ldots, 1\}$ and $\gamma \in \{1, 3\}$, $\tau_\delta \in \{1, 10\}$.

extension of Proposition 1 shows that the parameter $\Upsilon_2^1$ is positive while numerical simulations suggest that $\Upsilon_1^1 > 0$ and $\Upsilon_1^2 > 0$, and that $\rho_1 \geq \rho_2 > 1$.[24] Finally, when $\tau_\delta^{-1} = 0$, an extension of Corollary (4) allows us to prove existence and uniqueness of the equilibrium in the 3-period case:

**Corollary 8.** *When $N = 3$ and $\tau_\delta^{-1} = 0$, there exists a unique equilibrium in linear strategies where at any period $n = 1, 2$, (a) $a_n = \gamma \sum_{t=1}^{n} \tau_{\epsilon t}$; (b) $\alpha_{P_n} < \alpha_{E_n}$ if and only if $\beta < 1$; (c) $\alpha_{P_n} = \alpha_{E_n}$ if and only if $\beta = 1$.*

# 6 Prices and Consensus Opinion: Keynes vs. Hayek

Summarizing the results we obtained in the previous sections (analytically and numerically), the systematic discrepancy between prices and the consensus opinion in the estimation of the fundamentals, depends on the joint impact that liquidity trades' mean reversion and informed investors' response to private information have on short term speculative activity. According to Corollary 4, lacking residual uncertainty, liquidity trades' mean reversion pushes informed investors to act as market makers. This pulls the price away from the fundamentals compared to the average market opinion. When residual uncertainty is introduced, Corollary 7 together with our numerical results imply that the decision to "make" the market or "chase" the trend arises as a solution to the trade off between the strength of liquidity trades' mean reversion and that of informed investors' response to private information. Finally, when liquidity trades' increments are i.i.d., Corollaries 5 and 8 respectively imply that lacking residual uncertainty investors concentrate on long term speculation only, while introducing residual uncertainty they tend to chase the market. This, in turn, leads to a price that is either as far away from, or closer to the fundamentals compared to investors' average opinion. Table 1 summarizes this discussion.

| | | Liquidity trades' persistence | | |
|---|---|---|---|---|
| | | $\beta = 0$ | $0 < \beta < 1$ | $\beta = 1$ |
| Residual uncertainty | $\tau_\delta^{-1} = 0$ | $\alpha_{P_n} < \alpha_{E_n}$ | $\alpha_{P_n} < \alpha_{E_n}$ | $\alpha_{P_n} = \alpha_{E_n}$ |
| | $\tau_\delta^{-1} > 0$ | $\alpha_{P_n} < \alpha_{E_n}$ | $\alpha_{P_n} \lesseqgtr \alpha_{E_n}$ | $\alpha_{P_n} > \alpha_{E_n}$ |

Table 1: A summary of the results for $n = 1, 2$.

Our summary suggests that in both periods and for $\tau_\delta^{-1} \geq 0$, there must exist a $\beta$ such that $\alpha_{P_n} = \alpha_{E_n}$, and investors forgo short term speculation. Our numerical simulations confirm this insight as shown in figures 1 and 2. The figures plot the locus $\Omega_n \equiv \{(\beta, 1/\tau_\delta) \in [0, 1] \times \mathbb{R}_+ | \alpha_{P_n} =$

[24] This is consistent with our intuition for $\rho_1 > 1$ in Section 4.3. Indeed, the more extra trading dates an investors has, the more opportunities he has to correct a speculative position based on private information.

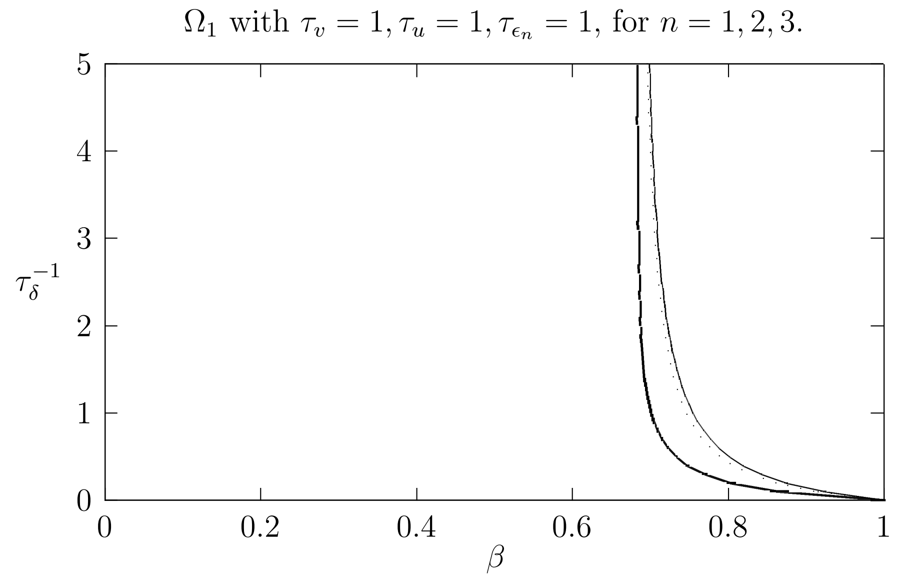


Figure 1: The Keynesian and Hayekian regions for $n = 1$ with "constant" arrival of information: $\tau_{\epsilon_n} = \tau_\epsilon$ for $n = 1, 2, 3$. The bold, dotted, and thin curves are associated respectively to $\gamma = 1$, $\gamma = 1/2$, and $\gamma = 1/4$. The area to the left of each curve identifies the set of parameter values where prices score worse than consensus at predicting the fundamentals (i.e., the Keynesian region). Conversely, the area to the right of each curve identifies the set of parameter values for which the opposite occurs (the Hayekian region).

$\alpha_{E_n}\}$, $n = 1, 2$, assuming that investors receive a private signal in every trading period of the same precision. At any period $n$, the set $\Omega_n$ divides the parameter space $(\beta, 1/\tau_\delta)$ into a Keynesian region (to the left of the locus) where prices are worse predictors of the fundamentals compared to consensus, and a Hayekian region (the rest) where the opposite occurs. Formally, the Keynesian region is thus given by the set

$$\{(\beta, 1/\tau_\delta) \in [0, 1] \times \mathbb{R}_+ | \alpha_{P_n} < \alpha_{E_n},\ n = 1, 2\}.$$

Conversely, the Hayekian region is given by

$$\{(\beta, 1/\tau_\delta) \in [0, 1] \times \mathbb{R}_+ | \alpha_{P_n} > \alpha_{E_n},\ n = 1, 2\}.$$

With no residual uncertainty ($\tau_\delta^{-1} = 0$) and i.i.d. liquidity trades increments ($\beta = 1$), $\Omega_n = (1, 0)$ (Corollaries 5 and 8). The introduction of residual uncertainty, on the other hand, may have a non-monotone effect on $\Omega_n$. Observing the figures for small (large) values of $\tau_\delta^{-1}$ the Hayekian region widens (shrinks). This is especially true for high levels of risk tolerance. The intuition is as follows. For small levels of residual uncertainty, the fact that speculators can re trade in a dynamic market has a first order impact on $\rho_n$ as the possibility to readjust one's position more than compensates for the increase in risk due to the augmented residual uncertainty over the liquidation value. As $\tau_\delta^{-1}$ grows larger, the possibility to retrade has an increasingly weaker effect on an investor's dynamic responsiveness, as private signals become less and less relevant to forecast the fundamentals. Investors thus scale back their responsiveness and more liquidity trades persistence is needed to make investors forgo short term speculation.[25]

[25] According to the figures above as $\tau_\delta^{-1}$ grows unboundedly investors' private signal responsiveness shrinks

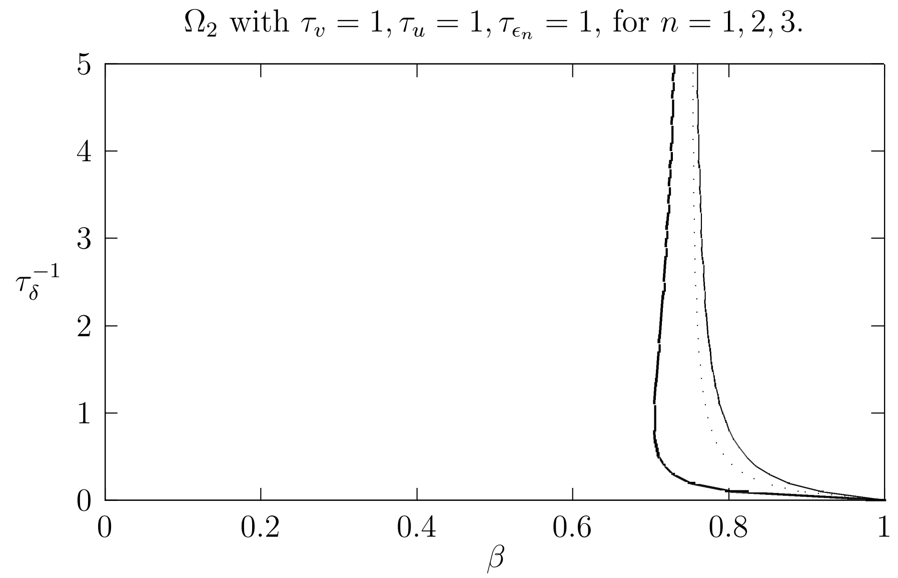


Figure 2: The Keynesian and Hayekian regions for $n = 2$ with "constant" arrival of information: $\tau_{\epsilon_n} = \tau_\epsilon$ for $n = 1, 2, 3$. The bold, dotted, and thin curves are associated respectively to $\gamma = 1$, $\gamma = 1/2$, and $\gamma = 1/4$. The area to the left of each curve identifies the set of parameter values for which the price scores worse than consensus at predicting the fundamentals (i.e., the Keynesian region). Conversely, the area to the right of each curve identifies the set of parameter values for which the opposite occurs (the Hayekian region).

According to our simulations, when $\tau_{\epsilon_n} = \tau_\epsilon > 0$ for all $n$, at any trading period the Hayekian (Keynesian) region widens (shrinks) whenever the impact of investors' response to private information on aggregate demand realizations is strong. This occurs for large values of $\gamma$, $\tau_\epsilon$, and $\tau_u$. When, on the other hand, $\tau_v$ is large, investors enter the market with sufficiently good prior information, and the trading process is unlikely to have a strong informational impact on the price. In this case, the Hayekian (Keynesian) region shrinks (widens). Interestingly, when investors only receive information in the first and second period we find that $\alpha_{P_2} < \alpha_{E_2}$. Similarly, our numerical simulations show that if $\tau_{\epsilon_2} = 0$, the same happens in the first period as well, implying that the Hayekian region disappears in both period 1 and 2, and $\Omega_n = \{(1, \tau_\delta^{-1}),\ \text{for } \tau_\delta^{-1} > 0\}$. The intuition is as follows: from our previous analysis the reason why informed investors may want to side with the market is that they believe that fundamentals information drives the aggregate demand realization. However, with this pattern of information arrival, investors do not receive any new signal after the first (or second) trading round. As a consequence, in the presence of a mean reverting demand from liquidity traders, siding with the market exposes informed investors to a considerable risk of trading in the expectation of a price increase (decrease) in the second and third period and instead being faced with a price decrease (increase).[26]

but the Hayekian region *does not* disappear. In the 2-period model it is easy to see that when $\tau_\delta^{-1} \to \infty$, $\Omega_1$ becomes a constant. Indeed, in this case $\Omega_1 = \{(\beta, 1/\tau_\delta) \in [0,1] \times \mathbb{R} | \beta\rho_1 = 1\}$, and $\lim_{\tau_\delta^{-1} \to \infty} \rho_1 = (\tau_v + \tau_{\epsilon_1})^{-1}(\tau_v + \tau_{\epsilon_1} + \tau_{\epsilon_2}) > 1$ is a constant that only depends on deep parameters. Therefore, $\beta\rho_1 = 1$ can be explicitly solved, yielding $\beta = (\tau_v + \tau_{\epsilon_1} + \tau_{\epsilon_2})^{-1}(\tau_v + \tau_{\epsilon_1}) < 1$. In the three-period model our numerical simulations show that a similar effect is at work.

[26]The figures in the text refer to a set of numerical simulations that were conducted assuming $\tau_v, \tau_u, \tau_{\epsilon_n} \in \{1, 4\}$, $\gamma \in \{1/4, 1/2, 1\}$, and $\beta \in \{0, 0.001, 0.002, \ldots, 1\}$, $\tau_\delta^{-1} \in \{0.1, 0.2, \ldots, 5\}$, for each pattern of private

According to the above discussion, the set $\Omega_n$ captures the set of deep parameter values granting the existence of an equilibrium in which investors only focus on an asset long-term prospects. The exclusive focus on an asset long term prospects arises either in the absence of any systematic pattern in the evolution of the aggregate demand (as argued in Corollaries 5 and 8) or when the forces backing trend chasing are exactly offset by those supporting market making (as shown in figures 1 and 2). In both cases, along the region $\Omega_n$, long term investors can only devote their attention to forecasting the fundamentals, shying away from the exploitation of the profits generated by short-term price movements. As a consequence, the price ends up being as close to the fundamentals as the market average opinion and the simplistic version of the EMH holds.

Corollary 1 argues that in the presence of symmetric information it is possible to map observed price departures from the public expectation at a given period $n$ (i.e., $p_n - E_n[v]$), into a position which is coherent with investors' expectations about the future evolution of the market price. The following corollary shows that an equivalent result also holds in the market with heterogeneous information, characterizing the consensus opinion about the evolution of future prices in the Hayekian and Keynesian regions:

**Corollary 9.** *In the presence of residual uncertainty, at any linear equilibrium*

$$E[p_2 - E_2[v]|v] > 0 \Leftrightarrow E\left[\bar{E}_2[p_3 - p_2]|v\right] > 0,$$

*if and only if* $\alpha_{P_2} > \alpha_{E_2}$. *If* $\tau_\delta^{-1} = 0$

$$E[p_n - E_n[v]|v] > 0 \Leftrightarrow E\left[\bar{E}_n[p_{n+1} - p_n]|v\right] < 0.$$

Thus, in the Hayekian (Keynesian) region, a systematic positive price departure from the public expectation about the fundamentals at time 2 "generates" the consensus opinion that prices will systematically further rise (decrease) in the third period. In the first period numerical simulations confirm that a similar result holds: $E[p_1 - E_1[v]|v] > 0 \Leftrightarrow E[\bar{E}_1[p_2 - p_1]|v] > 0$. If $\tau_\delta^{-1} = 0$ informed investors' response to their private information matches that of the static solution. Hence, provided $\beta < 1$, only the Keynesian equilibrium can arise and a systematic positive discrepancy between prices and public expectations creates the consensus opinion that prices will systematically revert. Finally, along the region $\Omega_n$, the market consensus opinion is that the next period price won't change in any systematic way. As a consequence, $E[\bar{E}_n[p_{n+1} - p_n]|v] = 0$, and investors concentrate on the asset long term prospects.

# 7 Reversal and Momentum

A vast empirical literature has evidenced the existence of return predictability based on a stock's past performance. DeBondt and Thaler (1986) document a "reversal" effect, whereby stocks with low past returns (*losers*) tend to outperform stocks with high past returns (*winners*) over medium/long future horizons. Jegadeesh and Titman (1993), instead, document a

information arrival.

"momentum" effect, showing that recent past winners tend to outperform recent past losers in the following near future.[27] At a theoretical level, it has proved difficult to reproduce these anomalies within the context of models with rational investors.[28] In contrast, a large number of theories have been proposed in the behavioral finance literature which allow for departures from full rationality and deliver return anomalies.[29]

In our framework, as we argued in Section 4.2, when investors have no private information, liquidity trades' low persistence implies that returns are negatively correlated, and thus exhibit reversal. In this section we consider the 3-period extension of the model with heterogeneous information summarized in Section 5, and analyze its implications for returns' correlation. The introduction of a strongly persistent factor affecting asset prices (i.e., fundamentals information) contrasts the impact of the transient component represented by the stock of liquidity trades. As a consequence, and except for the case in which $\beta = 0$, momentum and reversal can arise in both the Keynesian and the Hayekian equilibrium.

Using (18), we concentrate on the covariance between second and third period returns, as this fully depends on endogenous prices:

$$\begin{aligned}\mathrm{Cov}[p_3 - p_2, p_2 - p_1] = {} & \mathrm{Cov}\left[E_3[v] - E_1[v], \Lambda_3 E_3[\theta_3] - \Lambda_1 E_1[\theta_1]\right] \\ & + \mathrm{Cov}\left[\Lambda_2 E_2\left[\theta_2\right] - \Lambda_1 E_1[\theta_1], \Lambda_3 E_3[\theta_3] - \Lambda_2 E_2[\theta_2]\right]. \end{aligned} \tag{40}$$

Explicitly computing the covariances in (40) and rearranging yields:

$$\begin{aligned}\mathrm{Cov}[p_3 - p_2, p_2 - p_1] = {} & \left(\frac{\beta\Lambda_3 - \Lambda_2}{\tau_u}\right) \times \\ & \left(\Lambda_2\left(1+\beta^2\right) - \beta\Lambda_1 + \frac{a_2\tau_u(1-\alpha_{P_2})}{\tau_2} - \frac{\beta a_1 \tau_u (1-\alpha_{P_1})}{\tau_1}\right). \end{aligned} \tag{41}$$

The latter expression shows that in a market with heterogeneous information the covariance of returns is generated by two effects. The first one is captured by

$$\left(\frac{\beta\Lambda_3 - \Lambda_2}{\tau_u}\right)\left(\Lambda_2\left(1+\beta^2\right) - \beta\Lambda_1\right),$$

[27]More in detail, DeBondt and Thaler (1986) classify all the NYSE-traded stocks according to their past three-year return in relation to the corresponding market average in the period spanning January 1926 to December 1982 in stocks that outperform the market ("winners") and stocks that underperform it ("losers"). According to their results, in the following three years, portfolios of losers outperform the market by 19.6% on average while portfolios of winners underperform the market by 5% on average. Jegadeesh and Titman (1993), classify NYSE stocks over the period from January 1963 to December 1989 according to their past six-month returns. Their results show that the top prior winners tend to outperform the worst prior losers by an average of 10% on an annual basis. Research on momentum and reversal is extensive (see Vayanos and Woolley (2008) and Asness, Moskowitz and Pedersen (2008) for a survey of recent contributions).

[28]Notable exceptions are Biais, Bossaerts and Spatt (2008) and Vayanos and Woolley (2010).

[29]The behavioral finance literature on return anomalies is vast. Barberis, Shleifer, and Vishny (1997) show that if investors incorrectly interpret the probability law that drives an asset earning (which in reality follows a random walk), returns display predictable patterns. Daniel, Hirshleifer, and Subrahmanyam (1998) show that when investors suffer from overconfidence about the precision of their private signals and biased self-attribution, both reversal and momentum patterns in return arise. Hong and Stein (1999) model a market with two categories of investors: "newswatchers," who make decisions only based on private information, and "momentum traders," who instead invest only using heuristic rules based on public signals. If the process through which information diffuses across the newswatchers population is slow, both reversal and momentum can arise. See Barberis and Thaler (2003) for a comprehensive survey of the literature.

which coincides with the expression given for the third period returns' covariance in the model with homogeneous information (see equation (36)). As we argued in Section 4.2, this component reflects the impact of the liquidity shocks affecting the first and second period aggregate demand. The second component is given by

$$\left(\frac{\beta\Lambda_3 - \Lambda_2}{\tau_u}\right)\left(\frac{a_2\tau_u(1-\alpha_{P_2})}{\tau_2} - \frac{\beta a_1\tau_u(1-\alpha_{P_1})}{\tau_1}\right),$$

and captures the impact of the fundamental information shocks affecting the first and second period aggregate demand.

Inspection of (41) shows that if $\beta = 0$, then $\mathrm{Cov}[p_3 - p_2, p_2 - p_1] < 0$, implying that if liquidity trades' increments are strongly negatively correlated (i.e., the stock of liquidity trades is transient, and i.i.d), returns can only exhibit reversal. Hence, when $\beta = 0$ equilibria are Keynesian (in that the price is a worse predictor of the liquidation value compared to consensus) *and* display negative returns' autocorrelation.

As $\beta$ increases away from zero, depending on the patterns of private information arrival, momentum can arise. To see this, we start by assuming away residual uncertainty and set $\beta = 1$, so that any pattern in the correlation of returns must depend on the time distribution of private information. In this situation, as argued in Corollaries 5 and 8, the equilibrium is unique and we have $\alpha_{P_n} = \alpha_{E_n} = \tau_{in}^{-1}\sum_{t=1}^{n}\tau_{\epsilon t}$, $a_n = \gamma\sum_{t=1}^{n}\tau_{\epsilon t}$, and $\Lambda_n = 1/\gamma\tau_{in}$, implying that, provided investors receive information at all trading dates, and differently from what happens in the market with homogeneous information, $\Lambda_{n+1} < \Lambda_n$.[30] As a consequence, $\Lambda_3 < \Lambda_2$ and, similarly to the case with homogeneous information, the impact of a given liquidity shock "evaporates" across trading periods. Note, however, that as now market depth depends on the patterns of information arrival, the presence of heterogeneous information makes it possible for the impact of the first period liquidity shock to overpower that of the liquidity shock arriving in the second period. Indeed, as one can verify:

$$\begin{aligned}\mathrm{Cov}[p_3 - p_2, p_2 - p_1] > 0 &\Leftrightarrow 2\Lambda_2 - \Lambda_1 + \frac{a_2\tau_u(1-\alpha_{P_2})}{\tau_2} - \frac{a_1\tau_u(1-\alpha_{P_1})}{\tau_1} < 0\\ &\Leftrightarrow \tau_{\epsilon_2} > \frac{\tau_{i1}}{1+\gamma\tau_u a_1},\end{aligned}$$

and given that $(1+\gamma\tau_u a_1)^{-1}\tau_{i1} > \tau_{\epsilon_1}$, we can conclude that with no residual uncertainty and i.i.d. liquidity trades' increments, returns are positively correlated provided that investors receive private information at all trading dates (i.e., $\tau_{\epsilon_n} > 0$, for all $n$), *and* the quality of such information shows sufficient improvement across periods 1 and 2. In this situation, the impact of the first period liquidity shock is always stronger than the one coming from $u_2$, building a positive trend in returns.[31] Furthermore, a large second period private precision strengthens the impact of fundamentals information, eventually yielding $\mathrm{Cov}[p_3 - p_2, p_2 - p_1] > 0$.

When $\beta < 1$ (keeping $\tau_\delta^{-1} = 0$), liquidity trades' persistence is lower and this helps to generate a negative covariance. As a result, the value of $\tau_{\epsilon_2}$ which is needed for the model

[30] In the market with homogeneous information if $\beta = 1$, $\Lambda_n = (\gamma\tau_v)^{-1}(1+\kappa)$, for $n = 1, 2, 3$.

[31] Formally, $2\Lambda_2 - \Lambda_1|_{\tau_{\epsilon_2} = (1+\gamma a_1\tau_u)^{-1}\tau_{i1}} < 0$.

to display momentum, increases. Adding residual uncertainty lowers investors' responsiveness to private information. This, in turn, implies that for any $\beta$, the value of $\tau_{\epsilon_2}$ that triggers momentum further increases (see Figure 3).

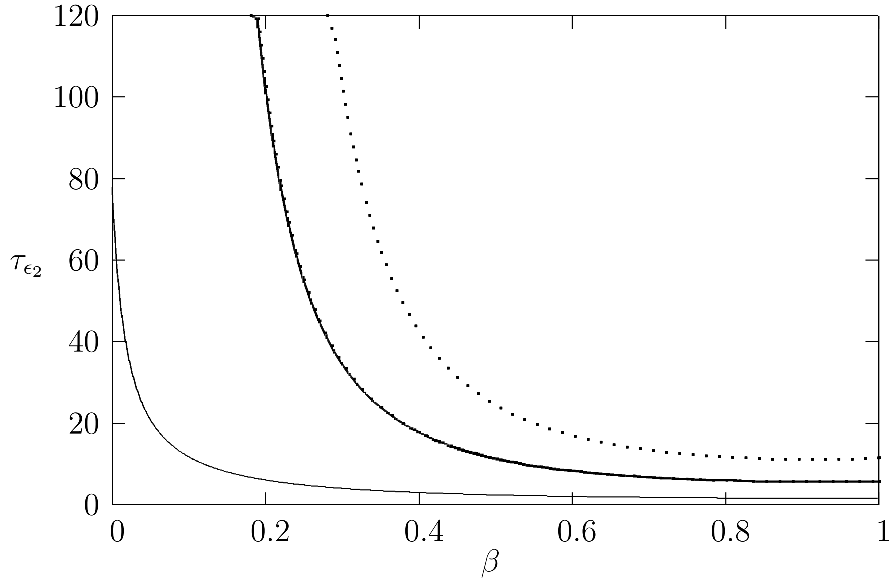


Figure 3: The figure displays the set $\{(\beta, \tau_{\epsilon_2}) \in [0,1] \times \mathbb{R}_+ | \text{Cov}[p_3 - p_2, p_2 - p_1] = 0\}$, partitioning the parameter space $[0,1] \times \mathbb{R}_+$ into two regions: points above the plot identify the values of $(\beta, \tau_{\epsilon_2})$ such that there is momentum. Points below the plot identify the values of $(\beta, \tau_{\epsilon_2})$ such that there is reversal. Parameters' values are $\tau_v = \tau_u = \tau_{\epsilon_1} = \tau_{\epsilon_3} = 1$. The thin, thick and dotted line respectively correspond to $\tau_\delta^{-1} = 0$, $\tau_\delta^{-1} = .2$, and $\tau_\delta^{-1} = .3$.

Summarizing, when $\beta = 0$ as argued in section 6 the Keynesian equilibrium realizes. There we obtain that prices score worse than consensus at predicting the fundamentals. Investors accommodate a positive expected liquidity demand, as the consensus opinion is that prices systematically revert. Furthermore, returns are negatively correlated. As $\beta$ grows larger, for intermediate values of the residual uncertainty parameter, the Hayekian equilibrium may occur, with prices that are closer to the fundamentals compared to the consensus opinion. Upon observing a positive realization of the expected liquidity demand, investors chase the trend, as in this case the consensus opinion is that prices will systematically increase. In this equilibrium, momentum obtains provided that the quality of investors' private information improves sufficiently across trading dates. Momentum and reversal are therefore compatible with both types of equilibria.[32]

Inspection of Figure 3 suggests that for a given $\tau_{\epsilon_2}$, higher values of $1/\tau_\delta$ require a larger liquidity trades' persistence for $\text{Cov}[p_3 - p_2, p_2 - p_1] = 0$. Numerical simulations confirm this insight, showing that the set of parameter values $(\beta, 1/\tau_\delta)$ for which $\text{Cov}[p_3 - p_2, p_2 - p_1] = 0$, and where therefore return predictability based on the observation of prices is not possible, has the shape displayed by the thick line in Figure 4. Points above (below) the thick line represent combinations of $(\beta, 1/\tau_\delta)$ such that the third period returns display reversal (momentum),

[32]Therefore, as momentum can arise also in the Keynesian region, a price runup is entirely compatible with a situation in which prices are farther away from the fundamentals compared with the consensus opinion.

so that $\text{Cov}[p_3 - p_2, p_2 - p_1] < 0$ ($\text{Cov}[p_3 - p_2, p_2 - p_1] > 0$). It is useful to also draw the set $\Omega_2 = \{(\beta, 1/\tau_\delta) \in [0,1] \times \mathbb{R}_+ | \alpha_{P_2} = \alpha_{E_2}\}$ for the chosen parameter configuration. This partitions the parameter space $[0,1] \times \mathbb{R}_+$ into four regions. Starting from the region $HR$ in which prices are worse predictors of fundamentals compared to consensus and returns display reversal and moving clockwise, we have the region $HM$ in which prices are closer to fundamentals compared to consensus and momentum occurs; the region $KM$ where prices are worse at estimating fundamentals compared to consensus and momentum occurs; the region $KR$ in which prices are worse predictors of fundamentals compared to consensus and reversal occurs.[33]

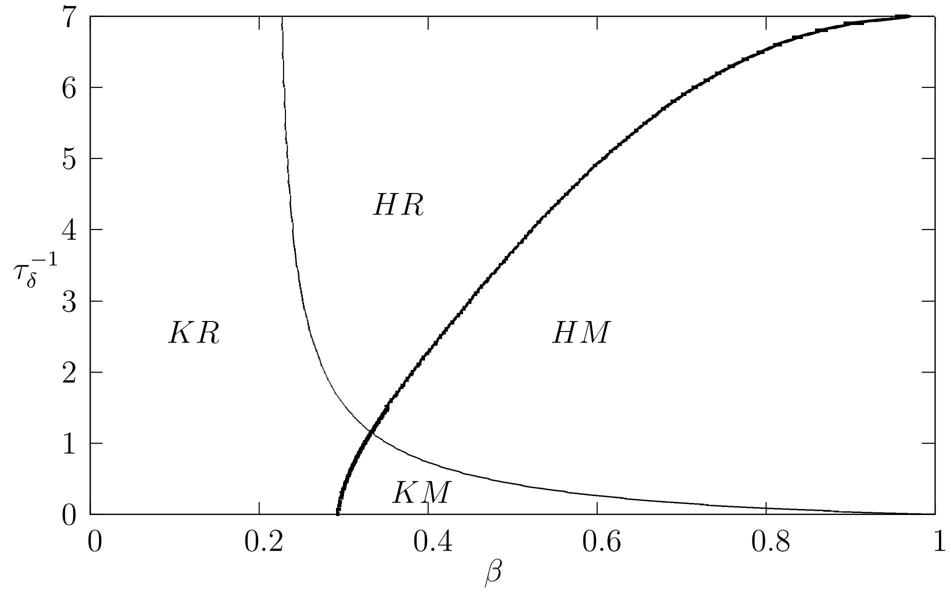


Figure 4: The figure displays the set $\Omega_2 = \{(\beta, 1/\tau_\delta) \in [0,1] \times \mathbb{R}_+ | \alpha_{P_2} = \alpha_{E_2}\}$ (thin line) and the set $\{(\beta, 1/\tau_\delta) \in [0,1] \times \mathbb{R}_+ | \text{Cov}[p_3 - p_2, p_2 - p_1] = 0\}$ (thick line). Parameters' values are $\tau_v = 1/25$, $\tau_u = 1/0.011^2$, $\gamma = 1/2$ and $\tau_{\epsilon_1} = 1/144$, $\tau_{\epsilon_2} = \tau_{\epsilon_3} = 4/144$.

According to Corollary 9, in the Hayekian (Keynesian) region investors' short term strategies reflect the consensus opinion about the systematic behavior of future prices. For instance, in the region to the right of $\Omega_2$ (i.e., the region $H$), a systematic positive discrepancy between $p_2$ and $E_2[v]$ creates the consensus opinion that the third period price will increase above $p_2$. This rationalizes informed investors' decisions to ride the market upon observing $p_2 - E_2[v] > 0$. As Figure 4 clarifies, in this region the consensus opinion about the systematic future price behavior *does not* always coincide with the forecast based on unconditional correlation. Indeed, suppose that at time 2 investors observe $p_2 > p_1 > E_2[v]$. For $(\beta, 1/\tau_\delta) \in HR$, unconditional correlation predicts that the short term increase in prices across the first two periods will be followed by a reversal, in stark contrast with the prediction based on the consensus opinion. To understand the reason for this difference, it is useful to refer to the case with homogeneous information. In that case, upon observing the *realization* of a positive demand from liquidity traders $\theta_2 > 0$, investors speculate on short run price differences by taking the other side of the market. Furthermore, unconditional correlation predicts a price reversal. Indeed, with

[33]In the figure we use parameters' values in line with Cho and Krishnan (2000)'s estimates based on S&P500 data. Thus, we set $\tau_v = 1/25$, $\tau_u = 1/0.011^2$, $\gamma = 1/2$ and $\tau_{\epsilon_1} = 1/144$, $\tau_{\epsilon_2} = \tau_{\epsilon_3} = 4/144$.

homogeneous information the only factor moving prices is represented by liquidity traders' demand which is transient. Therefore, both a positive liquidity stock and a price increase are deemed to be temporary. In the presence of heterogeneous information, on the other hand, fundamentals information, which is persistent, also affects prices. This contrasts the mean reverting impact of liquidity trading, creating a signal extraction problem, and implying that investors have to base their short term strategies on the realization of the *expected* stock of liquidity trades, $E_2[\theta_2]$, filtered out of the observed aggregate demand. In this situation, it is natural that the anticipation of future price behavior crucially depends on the information set on which such a forecast is based.

The latter result is reminiscent of Biais, Bossaerts, and Spatt (2008) who study the empirical implications that a multi-asset, dynamic, noisy rational expectations equilibrium model has for optimal trading behavior. One of their findings points to the existence of a discrepancy between momentum strategies based on unconditional correlation and the optimal, price contingent strategies that investors adopt in their model.[34]

# 8 Conclusions

In this paper we have investigated the relationship between prices and consensus opinion as estimators of the fundamentals. We have shown that whenever heterogeneously informed, long term investors find it optimal to exploit short term price movements, prices can either be systematically farther away or closer to the fundamentals compared to the consensus opinion. This gives rise to a Keynesian and a Hayekian region in the space of our deep parameters (i.e., the persistence of liquidity trades and the dispersion of residual uncertainty affecting the asset liquidation value). In the Hayekian (Keynesian) region a systematic positive price departure from the public expectation about the fundamentals "generates" the consensus opinion that prices will systematically further rise (decrease) in the upcoming period. On the boundary between the two regions, on the other hand, the market consensus opinion is that the next period price won't change in any systematic way. As a consequence, investors concentrate on "the asset long term prospects and those only," abiding by Keynes's dictum.

Our paper provides a number of empirical implications. According to our results, for a given level of residual uncertainty, investors tend to use accommodating strategies when liquidity trading is strongly mean reverting. Conversely, they are trend chasers when liquidity trading is close to random walk and there is a continuous flow of private information. The latter parameter region widens when investors are more risk tolerant, receive better private information and a lower level of liquidity trading affects prices.

Furthermore, as in our setup the evolution of prices is governed by a transient and a persistent component, depending on the quality of private information, our model can generate empirically documented return regularities. Interacting the space of parameter values yielding momentum and reversal with the Keynesian and Hayekian regions, we have illustrated that

[34] Biais, Bossaerts, and Spatt (2008) also find that price contingent strategies are empirically superior to momentum strategies.

the set of deep parameters yielding the two phenomena are different. As we argued, the consensus opinion can be taken as a measure of the market view of an asset fundamentals which, differently from the market price, is free from the influence of short term speculation dynamics. Therefore, our theory gives indications as to when a price runup (momentum) should be associated with a situation in which prices are a better or worse indicator of the liquidation value compared to consensus. Low residual uncertainty in the liquidation value together with a high liquidity trades' persistence are likely to characterize situations of the first type. On the other hand, low liquidity trades' persistence (again coupled with low residual uncertainty) can be responsible for prices growing increasingly apart from fundamentals compared to the market consensus opinion.[35]

Overall, our analysis points to the fact that the predictability of the aggregate demand evolution leads long-term investors to speculate on short-term returns, in turn implying that the simplistic version of the EMH is likely to fail. We identify two factors which may explain this result: the persistence of liquidity trades and the opaqueness of fundamentals. Indeed, as we have shown, low liquidity trades persistence together with opaque fundamentals make the evolution of the aggregate demand, and thus of the asset returns, predictable. This lures investors towards the exploitation of these regularities, partially diverting them from the activity of evaluating the fundamentals. As a result, the equilibrium price ends up reflecting both components of investors' strategies (long and short term speculation), decoupling its dynamic from that of the consensus opinion. In these conditions, we have also argued that reversal occurs, and prices score worse than consensus in predicting the fundamentals. Momentum, instead, needs high liquidity trading persistence, and a transparent environment to arise. Hence, insofar as a high $\beta$ proxies for a high trading frequency, we can conclude that any technological arrangement conducive to an increase in trading frequency together with improved disclosure is likely to promote positive return correlation and prices being better predictors of fundamentals compared to consensus.

A number of issues are left for future research. Our analysis has concentrated on the case in which investors have long horizons. Indeed, short term speculation in our setup arises endogenously whenever investors find it optimal to exploit regularities in the evolution of future returns.[36] Furthermore, while our paper gives a very detailed characterization of the conditions leading to departures from the simplistic version of the EMH, it does not assess the welfare

[35]From an empirical point of view, our "Hayekian" and "Keynesian" regions can potentially be identified expost by estimating the covariance of prices and consensus with the fundamentals. This enables to characterize when situations in which the market view is at odds with prices are a signal that consensus should be trusted as a better indicator of ex-post liquidation value. Indeed, as we show in Section 7, the fact that momentum and reversal can occur in both the Hayekian and Keynesian regions implies that in some cases we should trust price runups to be strong indicators of value (compared to consensus), whereas in other cases, this is not true. Of course, the testability of these implications relies on the availability of reliable information on consensus estimates which is not easy to obtain because of incentive issues of market professionals which are likely to induce biases (see Vissing-Jorgensen (2003) and the references cited therein). More recently, however, survey data based on investor beliefs which circumvent incentive issues start being collected (see, e.g., Vissing-Jorgensen (2003) and Piazzesi and Schneider (2009)).

[36]Cespa and Vives (2011) analyze the implications of forcing on investors a short term horizon and show that in our general framework this is conducive to multiple equilibria with either Keynesian or Hayekian features.

consequences that this may have for market participants. In particular, in the Keynesian equilibrium informed investors explicitly take advantage of liquidity traders, exploiting the low persistence of their demand shocks. A model in which the noise in the price is due to rational traders entering the market to hedge a shock to their endowment would allow to analyze the welfare properties of this equilibrium. Furthermore, it would also allow to see whether in response to informed investors' activity, liquidity patterns can change over time, thereby inducing a time-varying degree of liquidity trades' persistence, and ultimately affecting the sign and magnitude of the discrepancy between prices and average expectations in the estimation of fundamentals.

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# A Appendix

To characterize the equilibrium and prove its existence we assume that investors recover the sequence $z^n \equiv \{z_t\}_{t=1}^n$ of informational additions from the aggregate demand and use it to make inferences about the ex-post liquidation value. The following lemma establishes that at any linear equilibrium, working with this sequence is equivalent to working with $p^n \equiv \{p_t\}_{t=1}^n$:

**Lemma A.1.** *In any linear equilibrium the sequence of informational additions $z^n$ is observationally equivalent to $p^n$.*

*Proof.* Consider a candidate equilibrium in linear strategies $x_{in} = a_n \tilde{s}_{in} - \varphi_n(p^n)$. In the first period imposing market clearing yields $\int_0^1 a_1 s_{i1} - \varphi_1(p_1)di + \theta_1 = a_1 v - \varphi_1(p_1) + \theta_1 = 0$ or, denoting with $z_1 = a_1 v + \theta_1$ the informational content of the first period order-flow, $z_1 = \varphi_1(p_1)$, where $\varphi_1(\cdot)$ is a linear function. Hence, $z_1$ and $p_1$ are observationally equivalent. Suppose now that $z^{n-1} = \{z_1, z_2, \ldots, z_{n-1}\}$ and $p^{n-1} = \{p_1, p_2, \ldots, p_{n-1}\}$ are observationally equivalent and consider the $n$-th period market clearing condition: $\int_0^1 X_n(\tilde{s}_{in}, p^{n-1}, p_n)di + \theta_n = 0$. Adding and subtracting $\sum_{t=1}^{n-1} \beta^{n-t+1} a_t v$, the latter condition can be rewritten as follows:

$$\sum_{t=1}^{n} z_t - \varphi_n(p^n) = 0,$$

where $\varphi_n(\cdot)$ is a linear function, $z_t = \Delta a_t v + u_t$ denotes the informational content of the $t$-th period order-flow, and $\Delta a_t = a_t - \beta a_{t-1}$. As by assumption $p^{n-1}$ and $z^{n-1}$ are observationally equivalent, it follows that observing $p_n$ is equivalent to observing $z_n$. □

To prove proposition 1 we first present a general argument that characterizes the equilibrium prices and strategies in the 3-period market. We then specialize the equilibrium system of equations to the 2-period market and show that it always possesses a real solution.

**Proposition A.1.** *Let $\sum_{t=1}^n \tau_{\epsilon_t} > 0$, at any linear equilibrium of the 3-period market the equilibrium price is given by*

$$p_n = \alpha_{P_n}\left(v + \frac{\theta_n}{a_n}\right) + (1 - \alpha_{P_n})E_n[v], \quad n = 1, 2, 3, \tag{A.1}$$

*where $\theta_n = u_n + \beta\theta_{n-1}$. For $n = 1, 2$, an investor's strategy is given by:*

$$X_n(\tilde{s}_{in}, z^n) = \frac{a_n}{\alpha_{E_n}}(E_{in}[v] - p_n) + \frac{\alpha_{P_n} - \alpha_{E_n}}{\alpha_{E_n}} \frac{a_n}{\alpha_{P_n}}(p_n - E_n[v]), \tag{A.2}$$

*while at time 3:*

$$X_3(\tilde{s}_{i3}, z^3) = \frac{a_3}{\alpha_{E_3}}(E_{i3}[v] - p_3), \tag{A.3}$$

*where $\alpha_{E_n} = \sum_{t=1}^n \tau_{\epsilon_t}/\tau_{in}$, and expressions for $\alpha_{P_n}$ and $a_n$ are provided in the appendix (see equations* (A.7), (A.25), (A.43), *and* (A.6), (A.21), (A.46), *respectively). The parameters $\alpha_{P_n}$ and $a_n$ are positive for $n = 2, 3$. Numerical simulations show that $\alpha_{P_1} > 0$ and $a_1 > 0$.*

*Proof.* To prove our argument, we proceed backwards. In the last trading period traders act as in a static model and owing to CARA and normality we have

$$X_3(\tilde{s}_{i3}, z^3) = \gamma \frac{E_{i3}[v] - p_3}{\mathrm{Var}_{i3}[v + \delta]}, \tag{A.4}$$

and

$$p_3 = \alpha_{P_3}\left(v + \frac{\theta_3}{a_3}\right) + (1 - \alpha_{P_3})\, E_3[v], \tag{A.5}$$

where

$$a_3 = \frac{\gamma \sum_{t=1}^{3} \tau_{\epsilon_t}}{1+\kappa}, \tag{A.6}$$

$$\alpha_{P_3} = \frac{\sum_{t=1}^{3} \tau_{\epsilon_t}}{\tau_{i3}}, \tag{A.7}$$

$\kappa = \tau_\delta^{-1}\tau_{i3}$. An alternative way of writing the third period equilibrium price is

$$p_3 = \lambda_3 z_3 + (1-\lambda_3 \Delta a_3)\hat{p}_2, \tag{A.8}$$

where

$$\lambda_3 = \alpha_{P_3}\frac{1}{a_3} + (1-\alpha_{P_3})\,\frac{\Delta a_3 \tau_u}{\tau_3}, \tag{A.9}$$

captures the price impact of the net informational addition contained in the 3rd period aggregate demand, while

$$\begin{aligned}\hat{p}_2 &= \frac{\alpha_{P_3}\tau_3\beta(\sum_{t=1}^{2}\beta^{2-t}z_t) + (1-\alpha_{P_3})a_3\tau_2 E_2[v]}{\alpha_{P_3}\tau_3\beta a_2 + (1-\alpha_{P_3})a_3\tau_2} \\ &= \frac{\gamma\tau_2 E_2[v] + \beta(1+\kappa)(z_2+\beta z_1)}{\gamma\tau_2 + \beta a_2(1+\kappa)},\end{aligned} \tag{A.10}$$

$z_n = \Delta a_n v + u_n$, and $\Delta a_n = a_n - \beta a_{n-1}$.

## Second Period

Substituting (A.4) in the second period objective function, a trader in the second period maximizes

$$E_{i2}\left[U\left(\pi_{i2}+\pi_{i3}\right)\right] = -E_{i2}\left[\exp\left\{-\frac{1}{\gamma}\left((p_3-p_2)x_{i2} + \frac{x_{i3}^2 \mathrm{Var}_{i3}[v+\delta]}{2\gamma}\right)\right\}\right]. \tag{A.11}$$

Let $\phi_{i2} = (p_3-p_2)x_{i2} + x_{i3}^2\mathrm{Var}_{i3}[v+\delta]/(2\gamma)$. The term $\phi_{i2}$ is a quadratic form of the random vector $Z_2 = (x_{i3}-\mu_1, p_3-\mu_2)'$, which is normally distributed (conditionally on $\{\tilde{s}_{i2}, z^2\}$) with zero mean and variance covariance matrix

$$\Sigma_2 = \begin{pmatrix} \mathrm{Var}_{i2}[x_{i3}] & \mathrm{Cov}_{i2}[x_{i3},p_3] \\ \mathrm{Cov}_{i2}[x_{i3},p_3] & \mathrm{Var}_{i2}[p_3] \end{pmatrix}, \tag{A.12}$$

where

$$\begin{aligned}\mathrm{Var}_{i2}[x_{i3}] &= \frac{(\Delta a_3(1+\kappa)-\gamma\tau_{\epsilon_3})^2\tau_u + \tau_{i2}((1+\kappa)^2+\gamma^2\tau_u\tau_{\epsilon_3})}{\tau_{i2}\tau_u(1+\kappa)^2}, \\ \mathrm{Cov}_{i2}[x_{i3},p_3] &= \lambda_3\left(\frac{\gamma\tau_{\epsilon_3}\Delta a_3\tau_u - (1+\kappa)(\tau_3+\sum_{t=1}^{2}\tau_{\epsilon_t})}{\tau_{i2}\tau_u(1+\kappa)}\right), \\ \mathrm{Var}_{i2}[p_3] &= \lambda_3^2\left(\frac{\tau_3+\sum_{t=1}^{2}\tau_{\epsilon_t}}{\tau_{i2}\tau_u}\right),\end{aligned}$$

and

$$\mu_1 \equiv E_{i2}[x_{i3}] = \frac{a_3(1-\lambda_3\Delta a_3)}{\alpha_{E_3}}(E_{i2}[v]-\hat{p}_2) \tag{A.13}$$

$$\mu_2 \equiv E_{i2}[p_3] = \lambda_3\Delta a_3 E_{i2}[v] + (1-\lambda_3\Delta a_3)\hat{p}_2. \tag{A.14}$$

Writing in matrix form:

$$\phi_{i2} = c_2 + b_2' Z_2 + Z_2' A_2 Z_2,$$

where $c_2 = (\mu_2 - p_2)x_{i2} + \mu_1^2 \text{Var}_{i3}[v+\delta]/(2\gamma)$, $b_2 = (\mu_1 \text{Var}_{i3}[v+\delta]/\gamma, x_{i2})'$, and $A_2$ is a $2 \times 2$ matrix with $a_{11} = \text{Var}_{i3}[v+\delta]/(2\gamma)$ and the rest zeroes. Using a well-known result from normal theory we can now rewrite the objective function (A.11) as

$$E_{i2}\left[U\left(\pi_{i2}+\pi_{i3}\right)\right] = \tag{A.15}$$
$$-\left|\Sigma_2\right|^{-1/2}\left|\Sigma_2^{-1}+2/\gamma A_2\right|^{-1/2} \times \exp\left\{-1/\gamma\left(c_2 - \frac{1}{2\gamma}b_2'\left(\Sigma_2^{-1}+2/\gamma A_2\right)^{-1}b_2\right)\right\}.$$

Maximizing the above function with respect to $x_{i2}$ yields

$$x_{i2} = \Gamma_2^1(\mu_2 - p_2) + \Gamma_2^2 \mu_1, \tag{A.16}$$

where

$$\Gamma_2^1 = \frac{\gamma}{h_{2,22}},\;\; \Gamma_2^2 = -\frac{h_{2,21}\text{Var}_{i3}[v+\delta]}{\gamma h_{2,22}},$$

and $h_{2,ij}$ denotes the $ij$-th term of the symmetric matrix $H_2 = (\Sigma_2^{-1} + 2/\gamma A_2)^{-1}$:

$$h_{2,12} = -\frac{\lambda_3\tau_{i3}^2(1+\kappa)(1-\lambda_3\gamma\tau_{\epsilon_3}/(1+\kappa))}{D_2/\gamma^2} \tag{A.17}$$

$$h_{2,22} = \frac{\lambda_3^2\tau_{i3}((1+\kappa)(\tau_3+\sum_{t=1}^2\tau_{\epsilon_t})+\tau_{\epsilon_3})}{D_2/\gamma^2}, \tag{A.18}$$

and

$$\frac{D_2}{\gamma^2} = \tau_{i3}\left(\tau_{i3}\left(\lambda_3^2\tau_{i2} + (1-\lambda_3\Delta a_3)^2\tau_u\right) + \tau_{i2}\tau_u\kappa\right). \tag{A.19}$$

Substituting (A.13) and (A.14) into (A.16) and rearranging yields

$$X_2(\tilde{s}_{i2}, z^2) = \frac{a_2}{\alpha_{E_2}}\left(E_{i2}[v] - \hat{p}_2\right) - \frac{\gamma}{h_{2,22}}\left(p_2 - \hat{p}_2\right), \tag{A.20}$$

where $a_2$ denotes the 2nd period trading aggressiveness:

$$a_2 = \frac{\gamma(\sum_{t=1}^2\tau_{\epsilon_t})\tau_{i3}(1+\kappa)(1+\gamma\tau_u\Delta a_3)}{(1+\kappa+\gamma\tau_u\Delta a_3)(\tau_{\epsilon_3}+(\tau_3+\sum_{t=1}^2\tau_{\epsilon_t})(1+\kappa))}. \tag{A.21}$$

Imposing market clearing yields

$$\int_0^1 \frac{a_2}{\alpha_{E_2}}\left(E_{i2}[v]-\hat{p}_2\right)di - \frac{\gamma}{h_{2,22}}\left(p_2 - \hat{p}_2\right) + \theta_2 = 0,$$

which after rearranging implies

$$\frac{\gamma\tau_2(\beta\rho_2-1)}{\gamma\tau_2+\beta a_2(1+\kappa)}E_2[\theta_2] = \frac{\gamma}{h_{2,22}}(\hat{p}_2 - p_2), \tag{A.22}$$

where $\rho_2 \equiv a_2(1+\kappa)/(\gamma\sum_{t=1}^2\tau_{\epsilon_t})$. As a consequence, a trader $i$'s second period strategy can be written as follows:

$$X_2(\tilde{s}_{i2}, z^2) = \frac{a_2}{\alpha_{E_2}}\left(E_{i2}[v] - p_2\right) + \frac{(\gamma + h_{2,21})(\beta\rho_2 - 1)\tau_2}{\gamma\tau_{i3}}E_2[\theta_2]. \tag{A.23}$$

Using (A.20) we can obtain an expression for the second period equilibrium price that clarifies the role of the impact of expected noise traders' demand. Indeed, imposing market clearing yields

$$\frac{a_2}{\alpha_{E_2}}\left(\bar{E}_2[v] - p_2\right) + \frac{(\gamma+h_{2,21})(\beta\rho_2-1)\tau_2}{\gamma\tau_{i3}}E_2[\theta_2] + \theta_2 = 0,$$

where $\bar{E}_2[v] \equiv \int_0^1 E_{i2}[v]di$. Isolating $p_2$ and rearranging we obtain

$$p_2 = \alpha_{P_2}\left(v + \frac{\theta_2}{a_2}\right) + (1-\alpha_{P_2})E_2[v], \tag{A.24}$$

where

$$\alpha_{P_2} = \alpha_{E_2}\left(1 + (\beta\rho_2 - 1)\Upsilon_2^1\right) \tag{A.25}$$

denotes the weight that the second period price assigns to $v$, and

$$\Upsilon_2^1 = \frac{\gamma\tau_2\tau_u(\gamma\tau_2 + \beta a_2(1+\kappa) + \gamma\tau_{i2}\kappa)}{D_2} > 0. \tag{A.26}$$

Using (A.24) and (A.25) in (A.23) yields:

$$X_2(\tilde{s}_{i2}, z^2) = \frac{a_2}{\alpha_{E_2}}(E_{i2}[v] - p_2) + \frac{\alpha_{P_2} - \alpha_{E_2}}{\alpha_{E_2}}\frac{a_2}{\alpha_{P_2}}(p_2 - E_2[v]). \tag{A.27}$$

Finally, note that in period 2 as well we can obtain a recursive expression for the price that confirms the formula obtained in (A.8). Indeed, rearranging (A.24) we obtain

$$p_2 = \lambda_2 z_2 + (1 - \lambda_2\Delta a_2)\hat{p}_1, \tag{A.28}$$

where

$$\lambda_2 = \alpha_{P_2}\frac{1}{a_2} + (1-\alpha_{P_2})\frac{\Delta a_2\tau_u}{\tau_2}, \tag{A.29}$$

measures the price impact of the new information contained in the second period aggregate demand (since $\int_0^1 x_{i2}di + \theta_2 = a_2 v + \theta_2 - \varphi_2(p_1, p_2) = z_2 + \beta z_1 - \varphi_2(p_1, p_2)$), and

$$\hat{p}_1 = \frac{\alpha_{P_2}\tau_2\beta z_1 + (1-\alpha_{P_2})a_2\tau_1 E_1[v]}{\alpha_{P_2}\tau_2\beta a_1 + (1-\alpha_{P_2})a_2\tau_1}. \tag{A.30}$$

An alternative expression for $\lambda_2$ is as follows:

$$\lambda_2 = \underbrace{\frac{1+\kappa+\gamma\tau_u\rho_2\Delta a_2}{\gamma\rho_2\tau_{i2}}}_{\lambda_2^S} + \frac{(\beta\rho_2 - 1)(1+\kappa)\tau_u(\gamma\tau_2 + \beta a_2(1+\kappa) + \gamma\tau_{i2}\kappa)(\tau_2 - a_2\Delta a_2\tau_u)}{\rho_2\tau_{i2}D_2}, \tag{A.31}$$

where $\lambda_2^S$ denotes the "static" measure of the price impact of trade. The above expression thus highlights how noise trade predictability and the presence of residual uncertainty affect the static measure of the price impact of trade.

## First Period

To solve for the first period strategy, we now plug (A.16) into the argument of the exponential in (A.15):

$$\begin{aligned} c_2 - \frac{1}{2\gamma}b_2'\left(\Sigma_2^{-1} + 2/\gamma A_2\right)^{-1}b_2 &= (E_{i2}[p_3] - p_2)x_{i2} + \frac{\mathrm{Var}_{i3}[v+\delta]}{2\gamma}(E_{i2}[x_{i3}])^2 \\ &- \frac{1}{2\gamma}\begin{pmatrix} \frac{\mathrm{Var}_{i3}[v+\delta]}{\gamma}E_{i2}[x_{i3}] & x_{i2}\end{pmatrix}\begin{pmatrix} h_{2,11} & h_{2,12} \\ h_{2,21} & h_{2,22}\end{pmatrix}\begin{pmatrix} \frac{\mathrm{Var}_{i3}[v+\delta]}{\gamma}E_{i2}[x_{i3}] \\ x_{i2}\end{pmatrix}. \end{aligned}$$

Carrying out the matrix multiplication and simplifying yields

$$c_2 - \frac{1}{2\gamma}b_2'\left(\Sigma_2^{-1} + 2/\gamma A_2\right)^{-1}b_2 = \frac{1}{2\gamma}\left(h_{2,22}x_{i2}^2 + \frac{\gamma^2(1+\kappa)^2\tau_{i2}\tau_u}{D_2}(E_{i2}[x_{i3}])^2\right),$$

implying that

$$E_{i2}\left[U(\pi_{i2}+\pi_{i3})\right] = -\left|\Sigma_2\right|^{-1/2}\left|\Sigma_2^{-1}+(2/\gamma)A_2\right|^{-1/2}\times$$
$$\exp\left\{-\frac{1}{2\gamma^2}\left(h_{2,22}x_{i2}^2+\frac{\gamma^2(1+\kappa)^2\tau_{i2}\tau_u}{D_2}(E_{i2}[x_{i3}])^2\right)\right\}.$$

The first period objective function now reads as follows:

$$E_{i1}\left[U\left(\pi_{i1}+\pi_{i2}+\pi_{i3}\right)\right] = -E_{i1}\left[\exp\left\{-\frac{1}{\gamma}\left((p_2-p_1)x_{i1}\right.\right.\right. \tag{A.32}$$
$$\left.\left.\left.+\frac{1}{2\gamma}\left(h_{2,22}x_{i2}^2+\frac{\gamma^2(1+\kappa)^2\tau_{i2}\tau_u}{D_2}(E_{i2}[x_{i3}])^2\right)\right)\right\}\right].$$

Note that since

$$E_{i2}[x_{i3}] = \frac{\gamma\tau_2}{1+\kappa}(E_{i2}[v]-E_2[v])-\beta E_{i2}[\theta_2],$$

we have

$$E_{i2}[v]-E_2[v] = \frac{(1+\kappa)(E_{i2}[x_{i3}]+\beta E_{i2}[\theta_2])}{\gamma\tau_2},$$

and replacing the latter in the expression for $x_{i2}$ yields

$$E_{i2}[x_{i3}] = \frac{x_{i2}+(1-\beta\rho_2)E_{i2}[\theta_2]}{\rho_2}. \tag{A.33}$$

Thus, denoting by $\phi_{i1}$ the argument of the exponential in (A.32) we obtain:

$$\phi_{i1} = (p_2-p_1)x_{i1}+\frac{1}{2\gamma}\left(h_{2,22}x_{i2}^2+\frac{\gamma^2(1+\kappa)^2\tau_{i2}\tau_u}{D_2}\left(\frac{x_{i2}+(1-\beta\rho_2)E_{i2}[\theta_2]}{\rho_2}\right)^2\right).$$

Finally, as one can verify, letting $\nu_1=\alpha_{E_2}$, $\nu_2=-(\lambda_2\tau_{i2})^{-1}(\tau_2-a_2\Delta a_2\tau_u)$, and $\nu_3=1$, we have

$$\nu_1 x_{i2}+\nu_2 p_2+\nu_3 E_{i2}[\theta_2] = \frac{1}{\lambda_2\tau_{i2}}(\Delta a_2\tau_u\beta z_1-\tau_1 E_1[v]) \equiv c(z_1), \tag{A.34}$$

implying that

$$E_{i2}[\theta_2] = c(z_1)-\alpha_{E_2}x_{i2}+\frac{\tau_2-a_2\Delta a_2\tau_u}{\lambda_2\tau_{i2}}p_2.$$

Given a trader's information set at time 1, $c(z_1)$ is a constant. Hence, the uncertainty that a trader $i$ faces at time 1 is reflected in $\phi_{i1}$ through $p_2$ and $x_{i2}$ only:

$$\phi_{i1} = (p_2-p_1)x_{i1}+\frac{1}{2\gamma}\left(h_{2,22}x_{i2}^2+\frac{\gamma^2(1+\kappa)^2\tau_{i2}\tau_u}{\rho_2^2 D_2}\times\right. \tag{A.35}$$
$$\left.\left((1-(1-\beta\rho_2)\alpha_{E_2})x_{i2}+c(z_1)(1-\beta\rho_2)+\frac{(\tau_2-a_2\Delta a_2\tau_u)(1-\beta\rho_2)}{\lambda_2\tau_{i2}}p_2\right)^2\right).$$

The term $\phi_{i1}$ is a quadratic form of the random vector $Z_1\equiv(x_{i2}-\mu_1,\, p_2-\mu_2)$, which is normally distributed conditionally on $\{s_{i1},z_1\}$ with mean zero and variance-covariance matrix

$$\Sigma_1 = \begin{pmatrix} \mathrm{Var}_{i1}[x_{i2}] & \mathrm{Cov}_{i1}[x_{i2},p_2] \\ \mathrm{Cov}_{i1}[x_{i2},p_2] & \mathrm{Var}_{i1}[p_2] \end{pmatrix},$$

where $\mu_1\equiv E_{i1}[x_{i2}]$,

$$\mu_1 = \frac{(1-\lambda_2^S\Delta a_2)a_2}{\alpha_{E_2}}(E_{i1}[v]-\hat{p}_1)+\frac{a_2\tau_1(\alpha_{P_2}-\alpha_{E_2})}{\alpha_{P_2}\alpha_{E_2}\tau_2}(\hat{p}_1-E_1[v]), \tag{A.36}$$

and $\mu_2 \equiv E_{i1}[p_2]$,

$$\mu_2 = \lambda_2 \Delta a_2 E_{i1}[v] + (1 - \lambda_2 \Delta a_2)\hat{p}_1, \tag{A.37}$$

while

$$\mathrm{Var}_{i1}[x_{i2}] = \frac{(\Delta a_2 \sum_{t=1}^2 \tau_{\epsilon_t} - a_2 \tau_{\epsilon_2})^2 \tau_u + \tau_{i1}((\sum_{t=1}^2 \tau_{\epsilon_t})^2 + a_2^2 \tau_u \tau_{\epsilon_2})}{(\sum_{t=1}^2 \tau_{\epsilon_t})^2 \tau_{i1} \tau_u}$$

$$\mathrm{Cov}_{i1}[x_{i2}, p_2] = \lambda_2 \left( \frac{a_2 \Delta a_2 \tau_u \tau_{\epsilon_2} - (\tau_2 + \tau_{\epsilon_1})(\sum_{t=1}^2 \tau_{\epsilon_t})}{(\sum_{t=1}^2 \tau_{\epsilon_t}) \tau_{i1} \tau_u} \right)$$

$$\mathrm{Var}_{i1}[p_2] = \lambda_2^2 \left( \frac{\tau_2 + \tau_{\epsilon_1}}{\tau_{i1} \tau_u} \right).$$

Writing in matrix form:

$$\phi_{i1} = c_1 + b_1' Z_1 + Z_1' A_1 Z_1,$$

where

$$c_1 = (\mu_2 - p_1)x_{i1} + a_{11}\mu_1^2 + a_{22}\mu_2^2 + m_3 c(z_1)^2 + 2(m_1 \mu_1 c(z_1) + m_2 \mu_2 c(z_1) + a_{12}\mu_1 \mu_2),$$

$b_1 = (2(a_{11}\mu_1 + a_{12}\mu_2 + m_1 c(z_1)), 2(a_{22}\mu_2 + a_{12}\mu_1 + m_2 c(z_1)) + x_{i1})'$, and

$$A_1 = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix},$$

with

$$a_{11} = \frac{h_{2,22}}{2\gamma} + \frac{a_{22}}{\nu_2^2} \left( \frac{1 - (1 - \beta\rho_2)\alpha_{E_2}}{1 - \beta\rho_2} \right)^2 \quad a_{12} = -\frac{a_{22}}{\nu_2} \left( \frac{1 - (1 - \beta\rho_2)\alpha_{E_2}}{1 - \beta\rho_2} \right)$$

$$a_{22} = \frac{(\nu_2(1 - \beta\rho_2))^2}{2\gamma} \left( \frac{\gamma^2 (1+\kappa)^2 \tau_{i2} \tau_u}{\rho_2^2 D_2} \right),$$

and

$$m_1 = \frac{a_{22}}{\nu_2^2} \frac{1 - (1 - \beta\rho_2)\alpha_{E_2}}{1 - \beta\rho_2}, \ m_2 = -\frac{a_{22}}{\nu_2}, \ m_3 = \frac{a_{22}}{\nu_2^2}.$$

Along the lines of the second period maximization problem we then obtain

$$E_{i1}\left[U\left(\pi_{i1} + \pi_{i2} + \pi_{i3}\right)\right] = \tag{A.38}$$
$$-|\Sigma_1|^{-1/2} |\Sigma_1^{-1} + 2/\gamma A_1|^{-1/2} \exp\left\{ -1/\gamma \left( c_1 - \frac{1}{2\gamma} b_1' \left( \Sigma_1^{-1} + 2/\gamma A_1 \right)^{-1} b_1 \right) \right\}.$$

Maximizing (A.38) with respect to $x_{i1}$, solving for $x_{i1}$ and rearranging yields

$$X_1(s_{i1}, p_1) = \Gamma_1^1 E_{i1}[p_2 - p_1] + \Gamma_1^2 E_{i1}[x_{i2}] + \Gamma_1^3 E_{i1}\left[x_{i3}\right], \tag{A.39}$$

where

$$\Gamma_1^1 = \frac{\gamma}{h_{1,22}}, \ \Gamma_1^2 = -\frac{h_{1,12} h_{2,22}}{\gamma h_{1,22}}$$

$$\Gamma_1^3 = -\frac{\gamma^2 (1+\kappa)^2 \tau_u ((\beta\rho_2 - 1)\tau_{i2}\nu_2 h_{1,22} + \tau_{i3}(1 - \lambda_3 \Delta a_3) h_{1,12})}{D_2 h_{1,22}}.$$

and the terms $h_{1,ij}$ denote the $ij$-th elements of the symmetric matrix $H_1 = (\Sigma_1^{-1} + 2/\gamma A_1)^{-1}$:

$$h_{1,22} = \frac{\tau_{i3}}{\Phi_1 D_2 \rho_2^2 \tau_{\epsilon_2}} \left( (1+\kappa)^2 ((\tau_2 + \tau_{\epsilon_1})(\kappa \tau_u + \lambda_3^2 \tau_{i3}) + (1 - \lambda_3 \Delta a_3)^2 \tau_{i3} \tau_u) \right. \tag{A.40}$$
$$\left. + \lambda_3^2 \rho_2^2 \tau_{\epsilon_2} \left( (1+\kappa) \left( \tau_3 + \sum_{t=1}^{2} \tau_{\epsilon_t} \right) + \tau_{\epsilon_3} \right) \right),$$

$$h_{1,12}=\frac{(1+\kappa)^2}{\Phi_1 D_2\rho_2^2\tau_{\epsilon_2}\gamma^2\tau_{i2}\lambda_2(\sum_{t=1}^2\tau_{\epsilon_t})}\Bigg(\left(a_2\Delta a_2\tau_u\tau_{\epsilon_2}-(\tau_2+\tau_{\epsilon_1})\left(\sum_{t=1}^2\tau_{\epsilon_t}\right)\right)D_2 \tag{A.41}$$
$$+(\tau_2-a_2\Delta a_2\tau_u)(\beta\rho_2-1)\tau_u\tau_{i3}(1-\lambda_3\Delta a_3)\gamma^2\tau_{\epsilon_2}\sum_{t=1}^2\tau_{\epsilon_t}\Bigg),$$

and

$$\Phi_1=|\Sigma_1|^{-1}\Bigg(1+\frac{2a_{22}}{\gamma}\Bigg(\mathrm{Var}_{i1}[p_2]+\mathrm{Var}_{i1}[x_{i2}]\frac{\tau_{i3}^2(1-\lambda_3\Delta a_3)^2}{\nu_2^2\tau_{i2}^2(1-\beta\rho_2)^2}$$
$$-2\frac{\tau_{i3}(1-\lambda_3\Delta a_3)}{\nu_2\tau_{i2}(1-\beta\rho_2)}\mathrm{Cov}_{i1}[p_2,x_{i2}]\Bigg)+\frac{h_{2,22}\mathrm{Var}_{i1}[x_{i2}]}{\gamma^2}\Bigg)+2\frac{a_{22}h_{2,22}}{\gamma^3}.$$

Substituting (A.36) and (A.37) into (A.39) and imposing market clearing, yields

$$p_1=\alpha_{P_1}\left(v+\frac{\theta_1}{a_1}\right)+(1-\alpha_{P_1})E_1[v], \tag{A.42}$$

where

$$\alpha_{P_1}=\alpha_{E_1}\left(1+(\beta\rho_1-\rho_2)\Upsilon_1^1+(\beta\rho_2-1)\Upsilon_1^2\right), \tag{A.43}$$

denotes the weight that the first period price assigns to the fundamentals and

$$\Upsilon_1^1=\left(\frac{\gamma}{h_{1,22}}-\frac{a_1}{\alpha_{E_1}}\right)\frac{h_{1,22}\rho_1\tau_1\alpha_{E_2}}{\gamma a_2(1-\lambda_2\Delta a_2)}, \tag{A.44}$$

$$\Upsilon_1^2=\left(1-\frac{h_{1,22}}{\gamma}\left(\frac{\gamma}{h_{1,22}}-\frac{a_1}{\alpha_{E_1}}\right)\right)\frac{\gamma\beta\tau_1\tau_u}{h_{1,22}(1-\lambda_2\Delta a_2)D_2}\times$$
$$\left(\frac{\gamma\tau_{i3}\lambda_3(1+\kappa)^3(1+\gamma\tau_u\Delta a_3)\Phi_2}{\rho_2^2 D_2\lambda_2}-h_{1,12}h_{2,22}(\tau_{i3}(1-\lambda_3\Delta a_3)+\tau_{i2}\kappa)\right) \tag{A.45}$$
$$-\left(\frac{\gamma}{h_{1,22}}-\frac{a_1}{\alpha_{E_1}}\right)\frac{h_{1,22}\rho_1\tau_1\sum_{t=1}^2\tau_{\epsilon_t}}{\gamma a_2\tau_2(1-\lambda_2\Delta a_2)}\frac{\tau_2\tau_u(a_2(1+\kappa)+\gamma\tau_2\beta\rho_1)(\tau_{i3}(1-\lambda_3\Delta a_3)+\tau_{i2}\kappa)}{\tau_{i2}D_2},$$

and $a_1$ denotes a trader $i$'s first period private signal responsiveness:

$$a_1=\frac{\alpha_{E_1}}{h_{1,22}}\Bigg(\lambda_2\Delta a_2(\gamma-2(h_{1,22}a_{22}+h_{1,12}a_{12}))+ \tag{A.46}$$
$$-2(h_{1,22}a_{12}+h_{1,12}a_{11})\left(\frac{a_2\tau_1}{\sum_{t=1}^2\tau_{\epsilon_t}}+\beta a_1\right)\Bigg).$$

Using (A.42) in (A.39) and rearranging yields

$$X_1(s_{i1},z_1)=\frac{a_1}{\alpha_{E_1}}(E_{i1}[v]-p_1)+\frac{\alpha_{P_1}-\alpha_{E_1}}{\alpha_{E_1}}\frac{a_1}{\alpha_{P_1}}(p_1-E_1[v]). \tag{A.47}$$

Note that (A.47), together with (A.42) show that the expressions for equilibrium prices and traders' strategies have a recursive structure. Finally, note that as obtained in periods 2 and 3, we can express the first period equilibrium price as follows

$$p_1=\lambda_1 z_1+(1-\lambda_1 a_1)\bar{v},$$

where

$$\lambda_1=\alpha_{P_1}\frac{1}{a_1}+(1-\alpha_{P_1})\frac{a_1\tau_u}{\tau_1}.$$

This completes our proof.

□

## Proof of Proposition 1

To prove existence we specialize the expressions for the equilibrium parameters to the 2-period case. Therefore in the second period we obtain

$$a_2 = \frac{\gamma \sum_{t=1}^{2} \tau_{\epsilon_t}}{1+\kappa}, \tag{A.48}$$

where $\kappa \equiv \tau_\delta^{-1}\tau_{i2}$, and $\alpha_{P_2} = \alpha_{E_2}$. In the first period, we have

$$a_1 = \frac{\gamma \tau_{\epsilon_1}\tau_{i2}(1+\kappa)(1+\gamma\tau_u\Delta a_2)}{(1+\kappa+\gamma\tau_u\Delta a_2)(\tau_{\epsilon_2}+(\tau_2+\tau_{\epsilon_1})(1+\kappa))}, \tag{A.49}$$

and

$$\alpha_{P_1} = \alpha_{E_1}\left(1+(\beta\rho_1-1)\Upsilon_1\right), \tag{A.50}$$

where

$$\Upsilon_1 = \frac{\tau_1\tau_u(\gamma\tau_1+\beta a_1(1+\kappa)+\gamma\tau_{i1}\kappa)}{\gamma\tau_{i2}(\tau_{i2}(\lambda_2^2\tau_{i1}+(1-\lambda_2\Delta a_2)^2\tau_u)+\tau_{i1}\tau_u\kappa)}. \tag{A.51}$$

Note that (A.48) together with (A.49) defines a system of non-linear equations. Let us denote with $f(a_1,a_2)=0$ the equation defining $a_2$, and with $g(a_1,a_2)=0$ the equation defining $a_1$. Both $f(\cdot)$ and $g(\cdot)$ are continuous. In particular, it is easy to check that $f(a_1,a_2) = (\tau_\delta + \sum_{t=1}^{2}\tau_{\epsilon_t})(a_2^3\tau_u - 2a_2^2 a_1\tau_u + a_2(\tau_1 + a_1^2\tau_u)) - \gamma\tau_\delta(\sum_{t=1}^{2}\tau_{\epsilon_t}) = 0$ is a nondegenerate cubic in $a_2$, given that $(\tau_\delta + \sum_{t=1}^{2}\tau_{\epsilon_t})\tau_u > 0$, and always admits a real solution for any $a_1$. Furthermore, since $\partial f/\partial a_2 = (\tau_\delta + \sum_{t=1}^{2}\tau_{\epsilon_t})(3a_2^2\tau_u - 4a_2a_1\tau_u + \tau_1 + a_1^2\tau_u)$ and the discriminant associated to this quadratic equation in $a_1$ can be shown to be negative, we have that $\partial f/\partial a_2 \neq 0$ and the solutions to the cubic equation are continuous in $a_1$.[37] Hence, denoting by $a_2(a_1)$ a (real) solution to the cubic, we have that

$$\lim_{a_1\to 0} a_2(a_1) = \bar{a}_2 > 0, \qquad \lim_{a_1\to\infty} a_2(a_1) = 0.$$

We can now verify that a real solution always exists to the equation $g(a_1, a_2(a_1)) = 0$. Indeed,

$$\lim_{a_1\to 0} g(a_1,a_2(a_1)) = \gamma\tau_\delta\tau_{\epsilon_1}(1+\bar{a}_2\gamma\tau_u)\left(\tau_v + \bar{a}_2^2\tau_u + \sum_{t=1}^{2}\tau_{\epsilon_t} + \tau_\delta\right) > 0,$$
$$\lim_{a_1\to\infty} g(a_1,a_2(a_1)) = -\infty,$$

and the result follows.

□

As in the case with heterogeneous information, we first present the argument for the 3-period market with symmetric information and then specialize the equilibrium expressions to the 2-period case.

**Proposition A.2.** *In the 3-period market with no private information, there exists a unique equilibrium in linear strategies, where prices are given by*

$$p_n = \bar{v} + \Lambda_n\theta_n, \tag{A.52}$$

*where*

$$\Lambda_3 = \frac{1+\kappa}{\gamma\tau_v} \tag{A.53}$$

$$\Lambda_2 = \Lambda_3\left(1+\frac{(\beta-1)\gamma^2\tau_u\tau_v}{1+\kappa+\gamma^2\tau_u\tau_v}\right) \tag{A.54}$$

$$\Lambda_1 = \Lambda_2\left(1+\frac{(\beta-1)\gamma^2\tau_u\tau_v((1+\kappa)(1-\beta)+\gamma^2\tau_u\tau_v)}{(1+\kappa+\gamma^2\beta^2\tau_u\tau_v)(1+\kappa)+\gamma^2\tau_u\tau_v(1+\kappa+\gamma^2\tau_u\tau_v)}\right), \tag{A.55}$$

*and $\kappa = \tau_v/\tau_\delta$. Risk averse speculators trade according to*

$$X_n(p^n) = -\Lambda_n^{-1}(p_n - \bar{v}), \quad n = 1,2,3. \tag{A.56}$$

[37] Indeed, as one can check $\Delta \equiv 16a_1^2\tau_u^2 - 12\tau_u(\tau_1 + a_1^2\tau_u) = -(8a_1^2\tau_u + 12\tau_v)\tau_u < 0$.

*Proof.* Suppose that $\tau_{\epsilon_1} = \tau_{\epsilon_2} = \tau_{\epsilon_3} = 0$. Then, since in equilibrium

$$
\begin{aligned}
a_3 &= \frac{\gamma \sum_{t=1}^{3} \tau_{\epsilon_t}}{1+\kappa} \\
a_2 &= \frac{\gamma(\sum_{t=1}^{2} \tau_{\epsilon_t})\tau_{i3}(1+\kappa)(1+\gamma\tau_u\Delta a_3)}{(1+\kappa+\gamma\tau_u\Delta a_3)(\tau_{\epsilon_3}+(\tau_3+\sum_{t=1}^{2}\tau_{\epsilon_t})(1+\kappa))},
\end{aligned}
$$

we immediately obtain $a_2 = a_3 = 0$. Note that this is in line with what one should assume in a linear equilibrium where traders possess no private information. Indeed, at any candidate linear equilibrium a trader's strategy at time $n$ is given by $X_n(p^n) = \varphi(p^n)$, where $\varphi(\cdot)$ is a linear function. Imposing market clearing, in turn implies that $\varphi(p^n) = \theta_n$, so that at any linear equilibrium the price only incorporates the supply shock ($a_n = 0$) which is therefore perfectly revealed to risk averse speculators.

This, in turn, implies that $\tau_n = \tau_{in} = \tau_v$,

$$E_n[v] = E_{in}[v] = \bar{v},$$

and that $\alpha_{P_n} = \alpha_{E_n} = 0$. Now, we can go on and characterize the strategies that traders adopt, using the expressions that appear in proposition 1 in the paper:

$$X_3(p^3) = \frac{\gamma\tau_v}{1+\kappa}(\bar{v} - p_3) \tag{A.57}$$

$$X_2(p^2) = \frac{\gamma\tau_v}{1+\kappa}(\bar{v} - p_2) + \frac{(\beta-1)\gamma^3\tau_v^2\tau_u}{(1+\kappa)(1+\kappa+\beta\gamma^2\tau_u\tau_v)}(p_2 - \bar{v}). \tag{A.58}$$

The second component of the latter expression, in particular, comes from the fact that

$$\lim_{\tau_{\epsilon_n}\to 0} \frac{\alpha_{P_1} - \alpha_{E_1}}{\alpha_{E_1}} \frac{a_1}{\alpha_{P_1}} = \frac{(\beta-1)\gamma^3\tau_v^2\tau_u}{(1+\kappa)(1+\kappa+\beta\gamma^2\tau_u\tau_v)}.$$

Imposing market clearing we obtain:

$$p_2 = \bar{v} + \frac{(\beta-1)(1+\kappa)\gamma\tau_u}{1+\kappa+\gamma^2\tau_v\tau_u} E_2[\theta_2] + \frac{1+\kappa}{\gamma\tau_v}\theta_2. \tag{A.59}$$

Given that $a_2 = 0$, $z_2 = u_2$, and since traders at time 2 have also observed $z_1 = \theta_1$, the second period stock of noise $\theta_2 = \beta\theta_1 + u_2$ can be exactly determined, and

$$E_2[\theta_2] = \theta_2.$$

Hence, as argued above, traders perfectly anticipate the noise shock and accommodate it, and the price only reflects noise. But then this implies that

$$p_2 = \bar{v} + \frac{(\beta-1)(1+\kappa)\gamma\tau_u}{1+\kappa+\gamma^2\tau_v\tau_u}\theta_2 + \frac{1+\kappa}{\gamma\tau_v}\theta_2. \tag{A.60}$$

As a last step we need to characterize the first period equilibrium. Substituting the second period optimal strategy in the corresponding objective function and rearranging, at time 1 a trader chooses $x_{i1}$ to maximize

$$-E_{i1}\left[\exp\left\{-\frac{1}{\gamma}\left((p_2 - p_1)x_{i1} + \frac{(1+\kappa)(1+\kappa+\gamma\beta^2\tau_v\tau_u)}{2\gamma\tau_v(1+\kappa+\gamma^2\tau_v\tau_u)}\theta_2^2\right)\right\}\right].$$

According to (A.59) $p_2$ only depends on $\theta_2$. Hence, in the first period the argument of the trader's objective function is a quadratic form of the random variable $\theta_2$ which is normally distributed:

$$\theta_2|\theta_1 \sim N\left(\beta\theta_1, \tau_u^{-1}\right) \Rightarrow (\theta_2 - \beta\theta_1)|\theta_1 \sim N\left(0, \tau_u^{-1}\right),$$

and we can apply the usual transformation to compute the above expectation, obtaining that the function maximized by the trader is given by

$$(\bar{v} - p_1)x_{i1} + \beta\theta_1(m_1 x_{i1} + m_2\beta\theta_1) - \frac{1}{2\gamma(\tau_u + (2/\gamma)m_2)}(m_1 x_{i1} + 2m_2\beta\theta_1)^2,$$

where

$$m_1 = \frac{(1+\kappa+\gamma^2\beta\tau_u\tau_v)(1+\kappa)}{(1+\kappa+\gamma^2\tau_u\tau_v)\gamma\tau_v}, \quad m_2 = \frac{(1+\kappa+\gamma^2\beta^2\tau_u\tau_v)(1+\kappa)}{(1+\kappa+\gamma^2\tau_u\tau_v)2\gamma\tau_v}.$$

Computing the first order condition and solving for $x_{i1}$ yields

$$\begin{aligned} X_1(p_1) &= \frac{\gamma\tau_v}{1+\kappa}(\bar{v}-p_1) \\ &+ \frac{(\beta-1)((1+\kappa+\gamma^2\tau_u\tau_v)(1+\kappa+\gamma^2\tau_u\tau_v(1+\beta))+(1+\kappa)^2(1-\beta))}{(1+\kappa)(1+\kappa+\beta\gamma^2\tau_u\tau_v)((1+\kappa+\gamma^2\beta\tau_u\tau_v)^2+\beta\gamma^4\tau_u^2\tau_v^2(1-\beta))}(p_1-\bar{v}). \end{aligned} \tag{A.61}$$

Imposing market clearing and explicitly solving for the price

$$p_1 = \bar{v} + \Lambda_1\theta_1, \tag{A.62}$$

where

$$\Lambda_1 = \left(\frac{\gamma\tau_v}{1+\kappa} - \frac{(\beta-1)((1+\kappa+\gamma^2\tau_u\tau_v)(1+\kappa+\gamma^2\tau_u\tau_v(1+\beta))+(1+\kappa)^2(1-\beta))}{(1+\kappa)(1+\kappa+\beta\gamma^2\tau_u\tau_v)((1+\kappa+\gamma^2\beta\tau_u\tau_v)^2+\beta\gamma^4\tau_u^2\tau_v^2(1-\beta))}\right)^{-1},$$

which can be rearranged to obtain (A.55).

Using the expression for the equilibrium price we can compute the the covariance between second and third period returns which appears in Section 5:

$$\begin{aligned} \mathrm{Cov}[p_3-p_2, p_2-p_1] &= \left(\Lambda_2\left(\beta\Lambda_3-\Lambda_2\right)+\beta\left(\beta\Lambda_3-\Lambda_2\right)\left(\beta\Lambda_2-\Lambda_1\right)\right)\tau_u^{-1} \\ &= \left(\frac{\beta\Lambda_3-\Lambda_2}{\tau_u}\right)\left(\Lambda_2\left(1+\beta^2\right)-\beta\Lambda_1\right). \end{aligned}$$

Using (A.53), (A.54), and (A.55) we can then sign the above expression. In particular, it is easy to see that $\Lambda_1 < \Lambda_2$. Therefore, a necessary and sufficient condition for $\mathrm{Cov}[p_3-p_2, p_2-p_1] < 0$ is that $(\beta\Lambda_3-\Lambda_2) < 0$, and direct computation shows that

$$\beta\Lambda_3 - \Lambda_2 = (\beta-1)\Lambda_3\frac{1+\kappa}{1+\kappa+\gamma^2\tau_u\tau_v} < 0,$$

for all $\beta \in [0,1)$.

□

## Proof of Proposition 2

The expressions in (29) and (30) are readily obtained by moving the time indexes in (A.53) and (A.54) one period down.

□

## Proof of Corollary 4

When $\tau_\delta^{-1} = 0$, $\kappa = 0$ and from (A.48), and (A.49) $a_n = \gamma(\sum_{t=1}^n \tau_{\epsilon t})$. As a consequence $\rho_1 = 1$ and from (A.50)

$$\alpha_{P_1} = \alpha_{E_1}\left(1+(\beta-1)\frac{\tau_1\tau_u(\gamma\tau_1+\beta a_1)}{\gamma\tau_{i2}^2(\lambda_2^2\tau_{i1}+(1-\lambda_2\Delta a_2)^2\tau_u)}\right),$$

so that $\alpha_{P_1} \le \alpha_{E_1}$ if and only if $\beta \le 1$. □

## Proof of Corollary 6

The first part follows immediately from (A.50) and (A.51). For the second part notice that in any equilibrium $a_1 > 0$, hence if $1 + \gamma\tau_u\Delta a_2 > (<)0$, then also $1 + \kappa + \gamma\tau_u\Delta a_2 > (<)0$.[38] Notice also that if $\Delta a_2 < 0$ then $1 + \kappa + \gamma\tau_u\Delta a_2 < 0$. To see this last point, compute $\Delta a_2$ using (A.48) and (A.49):

$$\Delta a_2 = \tag{A.63}$$
$$\frac{\gamma}{D}\left(\tau_{\epsilon_2}(1+\kappa+\gamma\tau_u\Delta a_2)(\tau_{\epsilon_2}+\tau_{\epsilon_1}+\tau_2(1+\kappa)) - \tau_{\epsilon_1}\kappa(1+\kappa)\gamma\tau_u\Delta a_2\left(\tau_2+\sum_{t=1}^{2}\tau_{\epsilon_t}\right)\right),$$

where $D = (1+\kappa+\gamma\tau_u\Delta a_2)(\tau_{\epsilon_2} + (\tau_2+\tau_{\epsilon_1})(1+\kappa))(1+\kappa)$. Suppose that $\Delta a_2 < 0$ but that $(1+\kappa+\gamma\tau_u\Delta a_2) > 0$, then given (A.63) this is impossible.

To prove our claim start by assuming that $\Delta a_2 > 0$, then using (A.49) we can directly check whether $\rho_1 < 1$ since as one can see

$$a_1 < \frac{\gamma\tau_{\epsilon_1}}{1+\kappa} \Leftrightarrow \tau_{\epsilon_2}(1+\gamma\tau_u\Delta a_2+\kappa) + (1+\kappa)\left(\tau_2+\sum_{t=1}^{2}\tau_{\epsilon_t}\right)\gamma\tau_u\Delta a_2 < 0,$$

which is clearly impossible. If, on the other hand $\Delta a_2 < 0$, given what we have said above for $\rho_1 < 1$ we need

$$\tau_{\epsilon_2}(1+\gamma\tau_u\Delta a_2+\kappa) + (1+\kappa)\left(\tau_2+\sum_{t=1}^{2}\tau_{\epsilon_t}\right)\gamma\tau_u\Delta a_2 > 0,$$

which is again impossible. Therefore, at any linear equilibrium $\rho_1 > 1$.

□

## Proof of Corollary 8

Note that for $\tau_\delta^{-1} = 0$, $\kappa = 0$, and (A.6) (A.21) imply $a_n = \gamma(\sum_{t=1}^{n}\tau_{\epsilon t})$, for $n = 2, 3$. Hence, $\rho_2 = 1$ and (A.31), (A.25) respectively become:

$$\lambda_2 = \frac{1+\gamma\tau_u\Delta a_2}{\gamma\tau_2 + a_2} + \frac{(\beta-1)\tau_u(\gamma\tau_2+\beta a_2)(\tau_2 - a_2\Delta a_2\tau_u)}{\tau_{i2}D_2} \tag{A.64}$$

$$\alpha_{P_2} = \alpha_{E_2}\left(1 + \frac{(\beta-1)\gamma\tau_2\tau_u(\gamma\tau_2+\beta a_2)}{D_2}\right), \tag{A.65}$$

so that

$$\Upsilon_2^1 = \frac{\gamma\tau_2\tau_u(\gamma\tau_2+\beta a_2)}{D_2} > 0.$$

In the first period tedious algebra allows to show that

$$h_{1,12} = -\frac{\lambda_2\tau_{i2}^2(1-\lambda_2\gamma\tau_{\epsilon_2})}{D_1}, \quad h_{1,22} = \frac{(\lambda_2\tau_{i2})^2}{D_1}, \tag{A.66}$$

where

$$D_1 = \tau_{i2}^2\left(\lambda_2^2\tau_{i1} + (1-\lambda_2\Delta a_2)^2\tau_u + \frac{(\beta-1)^2(\tau_2 - a_2\Delta a_2\tau_u)^2\tau_u h_{2,22}}{D_2}\right). \tag{A.67}$$

Substituting (A.36), (A.37), and (A.66) in (A.39) and rearranging yields:

$$X_1(s_{i1}, p_1) = \frac{a_1}{\alpha_{E_1}}(E_{i1}[v] - p_1) + \frac{\gamma}{h_{1,22}}(1-\tau_{i1}h_{1,22})(\hat{p}_1 - p_1) \tag{A.68}$$
$$-\frac{\gamma\tau_u\tau_1\beta(\beta-1)^2(\tau_2 - a_2\Delta a_2\tau_u)(\gamma\tau_{i3}\lambda_3)^2}{\lambda_2(1-\lambda_2\Delta a_2)D_2^2}E_1[\theta_1].$$

[38] For suppose $a_1 < 0$, then $\Delta a_2 > 0$ and both $1 + \gamma\tau_u\Delta a_2 > 0$ and $1 + \gamma\tau_u\Delta a_2 + \kappa > 0$, implying $a_1 > 0$, a contradiction.

Using (A.66), we can now simplify (A.46) to obtain

$$\begin{aligned}a_1 &= \frac{\tau_{\epsilon_1}}{\lambda_2\tau_{i1}\tau_{i2}}\Bigg(\frac{D_1\Delta a_2}{\tau_{i2}} - (\gamma\tau_1+\beta a_1)(\Delta a_2\tau_u(1-\lambda_2\Delta a_2)-\lambda_2\tau_{i1})+\\ &\quad - \frac{(1-\beta)(\tau_2-a_2\Delta a_2\tau_u)\Delta a_2\tau_u(\gamma(1-\beta)(\tau_2-a_2\Delta a_2\tau_u)-(\gamma\tau_2+\beta a_2)(1-\lambda_2\gamma\tau_{\epsilon_2}))}{D_2}\Bigg)\\ &= \gamma\tau_{\epsilon_1},\end{aligned} \tag{A.69}$$

since, as one can verify,

$$\begin{aligned}\frac{D_1}{\tau_{i2}} &= \lambda_2\tau_{i1}(1+\gamma\Delta a_2\tau_u)+(1-\lambda_2\Delta a_2)\tau_u(\gamma\tau_1+\beta a_1)+\\ &\quad + \frac{(1-\beta)(\tau_2-a_2\Delta a_2\tau_u)\Delta a_2\tau_u(\gamma(1-\beta)(\tau_2-a_2\Delta a_2\tau_u)-(\gamma\tau_2+\beta a_2)(1-\lambda_2\gamma\tau_{\epsilon_2}))}{D_2}.\end{aligned}$$

Finally, imposing market clearing yields

$$\begin{aligned}\frac{(\beta-1)\tau_1}{1-\lambda_2\Delta a_2}\Bigg(&\frac{\alpha_{P_2}(\beta-1)(1-\alpha_{E_2})+\alpha_{P_2}-\alpha_{E_2}}{\alpha_{E_2}\tau_2(\beta-1)}\\ &+\frac{\beta(\beta-1)\gamma\tau_u(\tau_2-a_2\Delta a_2\tau_u)(\gamma\tau_{i3}\lambda_3)^2}{\lambda_2 D_2^2}\Bigg)E_1[\theta_1] = \frac{\gamma}{h_{1,22}}(\hat{p}_1-p_1).\end{aligned} \tag{A.70}$$

We can now substitute (A.70) in (A.68). Imposing market clearing and rearranging allows to obtain an expression for the first period price as (A.42), where

$$\begin{aligned}\alpha_{P_1} = \alpha_{E_1}\Bigg\{&1+\frac{(\beta-1)\gamma\tau_1(1-h_{1,22}\tau_{i1})}{1-\lambda_2\Delta a_2}\frac{\alpha_{P_2}}{a_2}+\frac{(\beta-1)\tau_1}{1-\lambda_2\Delta a_2}\times\\ &\left(h_{1,22}\tau_{i1}\frac{\beta(1-\beta)\gamma\tau_u(\tau_2-a_2\Delta a_2\tau_u)(\gamma\tau_{i3}\lambda_3)^2}{\lambda_2 D_2^2}+(1-h_{1,22}\tau_{i1})\frac{\alpha_{P_2}-\alpha_{E_2}}{\alpha_{E_2}\tau_2(\beta-1)}\right)\Bigg\}.\end{aligned} \tag{A.71}$$

Finally, for $\alpha_{P_2}$, using (A.65), the result stated in the corollary is immediate. For $\alpha_{P_1}$, inspection of (A.71) shows that $\alpha_{P_1} < \alpha_{E_1}$ if and only if $\beta < 1$ since the sum of the terms multiplying $\beta - 1$:

$$\begin{aligned}\Upsilon_1^1+\Upsilon_1^2 = &\frac{\gamma\tau_1(1-h_{1,22}\tau_{i1})}{1-\lambda_2\Delta a_2}\frac{\alpha_{P_2}}{a_2}+\frac{\tau_1}{1-\lambda_2\Delta a_2}\times\\ &\left(h_{1,22}\tau_{i1}\frac{\beta(1-\beta)\gamma\tau_u(\tau_2-a_2\Delta a_2\tau_u)(\gamma\tau_{i3}\lambda_3)^2}{\lambda_2 D_2^2}+(1-h_{1,22}\tau_{i1})\frac{\alpha_{P_2}-\alpha_{E_2}}{\alpha_{E_2}\tau_2(\beta-1)}\right),\end{aligned}$$

can be verified to be always positive.

□

## Proof of Corollary 9

For the first part of the corollary, consider the following argument. From the first order condition of the trader's problem in the second period

$$x_{i2} = \gamma\frac{E_{i2}[p_3-p_2]}{h_{2,22}} - \frac{h_{2,21}(1+\kappa)}{\gamma h_{2,22}}E_{i2}[x_{i3}].$$

Imposing market clearing, using (A.13) and (A.14), and rearranging yields

$$\begin{aligned}&\frac{\tau_2(\beta\rho_2-1)}{h_{2,22}\tau_{i3}(1-\lambda_3\Delta a_3)}\left(h_{2,22}-\frac{\lambda_3\Delta a_3(1+\kappa)}{\rho_2\tau_{i2}}\right)E_2[\theta_2]-\frac{h_{2,21}(1+\kappa)(1-\alpha_{E_2})(1-\beta\rho_2)}{\gamma h_{2,22}\rho_2\tau_{i2}\tau_{i3}}E_2[\theta_2]\\ &\qquad+\left(1+\frac{\alpha_{E_2}}{a_2}\left(\frac{h_{2,21}(1+\kappa)a_3(1-\lambda_3\Delta a_3)}{\gamma h_{2,22}\tau_{i3}\alpha_{E_3}}-\frac{\gamma\lambda_3\Delta a_3}{h_{2,22}}\right)\right)\theta_2 = 0.\end{aligned}$$

The first line in the above equation respectively captures the impact that the expected change in price and the expected third period position have on traders' aggregate second period strategy. Rearranging the term multiplying $\theta_2$ in the second line yields

$$\begin{aligned} 1+\frac{\alpha_{E_2}}{a_2}\left(\frac{h_{2,21}(1+\kappa)a_3(1-\lambda_3\Delta a_3)}{\gamma h_{2,22}\tau_{i3}\alpha_{E_3}}-\frac{\gamma\lambda_3\Delta a_3}{h_{2,22}}\right) &= 1+\frac{\alpha_{E_2}}{a_2}\left(-\frac{a_2}{\alpha_{E_2}}\right)\\ &= 0. \end{aligned}$$

The above result implies that for any realization of $E[E_2[\theta_2]|v]=(a_2/\alpha_{P_2})E[p_2-E_2[v]|v]$,

$$\frac{\tau_2(\beta\rho_2-1)}{h_{2,22}\tau_{i3}(1-\lambda_3\Delta a_3)}\left(h_{2,22}-\frac{\lambda_3\Delta a_3(1+\kappa)}{\rho_2\tau_{i2}}\right)E\left[E_2[\theta_2]|v\right]$$

and

$$-\frac{h_{2,21}(1+\kappa)(1-\alpha_{E_2})(1-\beta\rho_2)}{\gamma h_{2,22}\rho_2\tau_{i2}\tau_{i3}}E\left[E_2[\theta_2]|v\right],$$

must have opposite sign. Given that $h_{2,21}$ can be verified to be negative, this implies that if (and only if) $\beta\rho_2>1$, $E[\bar{E}_2[p_3-p_2]|v]$ is positive. If $\kappa=0$, then a similar argument shows that at time 2 $E[p_2-E_2[v]|v]<0\Leftrightarrow E[\bar{E}_2[p_3-p_2]|v]>0$ for $\beta<1$.

In the absence of residual uncertainty, at time $n=1$, using (A.70) and rearranging the market clearing equation yields

$$\begin{aligned} \frac{h_{1,22}}{\gamma}\frac{(\beta-1)\tau_1}{1-\lambda_2\Delta a_2}\Bigg(&\frac{\alpha_{P_2}(\beta-1)(1-\alpha_{E_2})+\alpha_{P_2}-\alpha_{E_2}}{\alpha_{E_2}\tau_2(\beta-1)}\\ &+\frac{\beta(\beta-1)\gamma\tau_u(\tau_2-a_2\Delta a_2\tau_u)(\gamma\tau_{i3}\lambda_3)^2}{\lambda_2 D_2^2}\Bigg)E_1[\theta_1]=\hat{p}_1-p_1. \end{aligned} \tag{A.72}$$

Averaging out noise in the above expression, in this case the sign of $E[\bar{E}_1[p_2-p_1]|v]$ depends on the sign of the sum of the term multiplying $E[E_1[\theta_1]|v]$ in the above expression and

$$\lambda_2\Delta a_2\left(\frac{\alpha_{E_1}}{a_1}-\frac{\beta\alpha_{P_2}}{a_2(1-\lambda_2\Delta a_2)}\right), \tag{A.73}$$

which after rearranging can be shown to be always negative provided $\beta<1$.

□