The Beauty Contest and Short-Term Trading*

Journal of Finance, forthcoming

Giovanni Cespa† and Xavier Vives‡

January 2015

Abstract

Short-termism need not breed informational price inefficiency even when generating Beauty Contests. We demonstrate this claim in a two-period market with persistent liquidity trading and risk-averse, privately informed, short-term investors and find that prices reflect average expectations about fundamentals and liquidity trading. Informed investors engage in "retrospective" learning to reassess inferences (about fundamentals) made during the trading game's early stages. This behavior introduces strategic complementarities in the use of information and can yield two *stable* equilibria that can be ranked in terms of liquidity, volatility, and informational efficiency. We derive implications that explain market anomalies as well as empirical regularities.

Keywords: price speculation, multiple equilibria, average expectations, public information, momentum and reversal, disagreement, volume, volatility

JEL Classification numbers: G10, G12, G14

---

*A previous version of this paper was circulated under the title "Expectations, Liquidity, and Short-Term Trading". For helpful comments we thank Elias Albagli, Snehal Banerjee, Max Bruche, Giacinta Cestone, Hans Degryse, Marcelo Fernandes, Itay Goldstein, Peter Hoffmann, Harrison Hong, Peter Kondor, Jean-Paul Laurent, Olivier Loisel, Stephen Morris, Richard Payne, Ioanid Roşu, Kristien Smedts, Gunther Wuyts, Lian Yang, Kathy Yuan, and participants at numerous seminars and conferences. Cespa acknowledges financial support from ESRC (grant no. ES/J00250X/1). Vives acknowledges financial support from the European Research Council under the Advanced Grant project Information and Competition (no. 230254) and from project ECO2011-29533 of the Spanish Ministry of Science and Innovation at the Public-Private Sector Research Center at IESE.

†Cass Business School, City University London, CEPR, and CSEF. 106, Bunhill Row, London EC1Y 8TZ, UK. e-mail: giovanni.cespa@gmail.com

‡IESE Business School, Avinguda Pearson, 21 08034, Barcelona, Spain.

> It might have been supposed that competition between expert professionals [...] would correct the vagaries of the ignorant individual left to himself. It happens, however, that the energies and skill of the professional investor and speculator are mainly occupied otherwise. For most of these persons are [...] largely concerned, not with making superior long-term forecasts [...] but with foreseeing changes in the conventional basis of valuation a short time ahead of the general public. (Keynes, *The General Theory of Employment, Interest and Money*, 1936)

Does short-termism breed informational price inefficiency? We find that this need not be the case—even though Beauty Contests arise—provided that liquidity shocks are *persistent*. We examine this question in a two-period market where short-term, informed, competitive, risk-averse agents trade on private information and to accommodate liquidity supply while facing persistent demand from liquidity traders.

Traders' "myopia" ranks high on the regulatory agenda, which testifies to policy makers' concern about the possibly detrimental effects of such myopia on the market.[1] Debate over this issue has a long tradition in economic analysis. Indeed, short-term trading is the very basis of Keynes's dismal view of financial markets. According to his "Beauty Contest" analysis, traders' investment decisions are driven by anticipation of their peers' changing whims and not by actual knowledge of the companies they trade. As a result, competition among informed traders does not necessarily counteract the actions of uninformed traders.[2] It has been claimed that this type of behavior introduces a particular form of informational inefficiency whereby traders tend to put a disproportionately high weight on public information in their forecast of asset prices (see Allen, Morris, and Shin (2006)). Furthermore, the anticipation of short-term price movements may induce market participants to act in a way that amplifies such movements (Shin (2010)) and may contribute to crashes. We show that the Beauty Contest analogy for financial markets tells just part of the story because, when liquidity traders' demand shocks are persistent, prices reflect average expectations not only of the fundamental value but also of liquidity trading.

In this paper we present a two-period model of short-term trading with asymmetric information, in which investors observe an exogenous public signal—in the tradition of dynamic noisy rational expectations models (see, e.g., Singleton (1987), Brown and Jennings (1989)). We find that if liquidity trading is persistent then there is strategic complementarity in the use of private information, and we provide sufficient conditions for that complementarity to generate multiple extremal *stable* equilibria that can be ranked in terms of price informativeness, liquidity, and volatility. In particular, we find that there are two extremal equilibria: a "high information" equilibrium (HIE) and a "low information" equilibrium (LIE). At the HIE, prices are good signals of the underlying fundamentals, volatility is low, and liquidity is high; the LIE displays the *opposite* properties in terms of informational efficiency, volatility, and liquidity.

In a model identical to ours but with *transient* liquidity trading (and without an exogenous public signal), Allen et al. (2006) find that prices (i) are driven by higher-order expectations (HOEs) about fundamentals, (ii) underweight private information (with respect to the optimal statistical weight), and (iii) are farther away from fundamentals than investors' consensus. The same result obtains in our setup when liquidity trading is transient. A similar result also holds at the LIE when liquidity trading is persistent. However, at the HIE we find that the price is more strongly tied to fundamentals (as compared with investors' consensus) and overweights average private information (as compared with the optimal statistical weight).[3] Therefore, the Beauty Contest feature of asset prices does not necessarily imply that prices are worse estimators of fundamentals compared to consensus; neither does it imply that prices exhibit inertia or react slowly to changes in the fundamentals. Hence we can establish the limits of this Beauty Contest analogy for financial markets and refute the view that short-term trading *always* amplifies demand shocks or necessarily leads to uninformative prices or "excess" volatility.[4] We also identify the circumstances under which informed traders stabilize the market by counteracting the actions of liquidity traders (at the HIE). Finally, we deliver sharp predictions on asset pricing that are consistent with the received empirical evidence (including noted anomalies).

A crucial hypothesis of our model is that liquidity trading displays persistence. This hypothesis can be viewed as a reduced-form assumption for the performance–flow relationship's effect on the holdings of mutual funds. Coval and Stafford (2007) show that mutual funds faced with aggregate redemption orders will curtail their positions and engage in "fire sales". This dynamic generates a temporary and allegedly uninformed price pressure that reduces fund performance. As shown by evidence on the performance–flow relationship (Chevalier and Ellison (1997), Sirri and Tufano (1998)), poor performance in turn breeds investors' redemptions, thus engendering further fire sales. The implication is that uninformed orders can display persistence. Building on this intuition, Lou (2012) tests a capital flow–based explanation for some well-known empirical asset pricing regularities and finds that mutual funds' shareholdings display strong persistence at a quarterly frequency.[5] Campbell and Kyle (1993) follow a different approach and disentangle the properties of the noise process from the properties of returns; these authors find that liquidity traders' positions are highly persistent at an annual frequency. In sum, the persistence of liquidity trading appears to be a natural and plausible assumption that is backed by empirical evidence.

The mechanism responsible for complementarities is as follows. Suppose a risk-averse, short-term trader has a private signal on the firm's fundamentals. His willingness to speculate on that signal is directly related to how well he can estimate the next period's price and, significantly, such willingness is also *inversely* related to the trader's uncertainty about the liquidation price. Indeed, the more volatile the price at which the investor unwinds, the riskier his strategy and the less willing to exploit his private signal the trader becomes. Yet an average increased response to private information today, increases the price informativeness and reduces the residual variance of tomorrow's price conditional on today's price. This pushes up the response to the private signal today and may induce strategic complementarities in the responses to the private signals. For this to happen the variance reduction effect must be strong enough to overwhelm the usual substitution effect according to which a more informative price today decreases the weight put by the trader on his private signal in the estimation of tomorrow's price. Therefore, a trader's willingness to act on private information not only depends on his uncertainty about the liquidation price but also affects that uncertainty. We argue that this two-sided loop may account for the existence of multiple stable equilibria that can be ranked in terms of liquidity, volatility, and informational efficiency.

The variance reduction effect is potentially strong when liquidity trading is persistent.[6] The crux of our argument revolves around a particular type of inference from the information (as reflected by prices) that arises in this case. If there is persistence then second-period investors can *retrospectively* assess their first-period inferences about the fundamentals—that is, based on the new evidence gathered in the second period. We therefore term this effect "retrospective inference". In a market whose investors are both risk averse and asymmetrically informed, it is well-well known that the price impact of trades stems from the sum of an "inventory" component and an "inference" component.[7] In a static market the two terms are positive, but in a dynamic market it is possible for retrospective inference to render the inference component *negative*. That effect diminishes the price impact of trades, reducing the volatility of the asset price and boosting the response of traders to private information. The intuition is as follows.

Suppose that second-period informed investors observe a large demand for the asset.

If in the first period these investors traded aggressively on their private information, then the first-period price is informative about the fundamentals. Therefore, the bulk of the price adjustment to fundamentals must have already occurred in the first period. This reduces the likelihood that demand is being driven by informed trading and thereby makes it more likely that it is driven by liquidity trading. If there is persistence, then this result implies that a high demand for the asset from liquidity traders *also affected the first-period aggregate demand*—which further implies (for a given price realization) a lower expectation of the fundamentals. So in this case, a large aggregate demand realization leads second-period investors to revise downward their expectation of the liquidation value. This implies in turn that the inference component of the price impact offsets the inventory component, reducing the latter's significance. The result is diminished first-period investor uncertainty about the unwinding price, which boosts the response of investors to private signals. We have therefore a self-fulfilling loop of strong reaction to first period private signals which leads to a high information equilibrium. The same logic leads to a low information equilibrium when one assumes that first-period investors trade *weakly* on their private signals.

We show that if retrospective inference is present but the variance reduction effect moderately strong, then two extremal and stable equilibria arise: the LIE and the HIE. In the HIE, volatility is low, liquidity is high, and prices closely reflect the underlying fundamentals; in the LIE, volatility is high, liquidity is low, and prices reflect poorly underlying fundamentals. If retrospective inference and the variance reduction effect are very strong then the HIE becomes unstable, in which case the only likely equilibrium to arise is the LIE.

Our analysis shows that the strength of the retrospective inference loop depends on traders' reliance on prices as a source of information for their decisions. Suppose, for example, that private signals are much more precise than exogenous public information; then prices are relatively more informative, the loop is very strong, and the HIE is unstable. When the public signal precision increases, the retrospective inference loop weakens and thereby stabilizes the HIE. Finally, when precision of the public signal increases further, the HIE disappears, because the variance reduciton effect is weakened, and uniqueness occurs at the LIE. Thus, our paper shows that public information plays an active role in determining the type of market equilibrium when there is short-term trading.

Our model predicts that the LIE arises under extreme values of public signal precision (which could be proxied by the number of analysts following a given security). The low-information equilibrium is characterized by: (i) a positive inference component of the price impact; (ii) momentum or reversal, depending on the strength of persistence; (iii) high expected returns from providing liquidity; (iv) prices that are far from the semi-strong efficient price; (v) strong short-horizon return predictability from order imbalances; and (vi) low volume accompanied by high levels of disagreement. For intermediate values of public signal precision, the HIE can also arise. The high-information equilibrium features (i) a negative inference component of the price impact, (ii) mild momentum, (iii) low expected returns from liquidity provision, (iv) prices that are close to the semi-strong efficient price, (v) weak short-horizon return predictability from order imbalances, and (vi) high volume accompanied by low levels of disagreement.

It is important to note that the autocovariance of short-term returns is always positive in the HIE but is positive in the LIE only when liquidity trading is sufficiently persistent. Since the predictability of liquidity trading is an essential ingredient in multiple equilibria, it follows that momentum is a consequence of persistence. That being said, the patterns of prices and price informativeness implied by the two equilibria are markedly different. In particular, momentum along the HIE (resp., LIE) is a sign that prices are rapidly (resp., slowly) converging toward the full-information value.

Our model has the following implications:

- When high trade volume is associated with informative prices and with low levels of disagreement, then this discriminates in favor of the HIE, not only in relation to the LIE but also in relation to alternative theories based on differences of opinions (DO) models, in which disagreement is associated with high volume.

- A negative covariance between conditional volatility of returns and volume does not rule out the applicability of rational expectations models (as suggested by Banerjee (2011), for example) since it is consistent with ours in some scenarios.

- A negative inference component of the price impact identifies the HIE;[8] the observation of a reversal at short horizons identifies the LIE (under the maintained hypothesis that our model holds).

- If for "fragile" stocks – in the sense of Greenwood and Thesmar (2011) – there is a low transaction volume when there is high disagreement, then this would be evidence for a LIE. Similarly, a high transaction volume for non-fragile stocks about which there are low levels of disagreement, constitutes evidence for a HIE.

The rest of the paper is organized as follows. After discussing the related literature, we analyze the static benchmark. In Section III we study the two-period model, relate it to the Beauty Contest literature, and present the multiplicity result. We then derive asset pricing implications and use our model to interpret some aspects of the recent financial crisis. The paper's final section summarizes our results and discusses their empirical implications. Most formal proofs are relegated to the paper's Appendix. An Online Appendix offers a detailed robustness analysis of the model.

I Related literature

Our results are related to—and have implications for—three strands of the literature.

First, our paper is related to the literature that investigates the relationship between the effect of short-term investment horizons on prices and the reaction of investors to their private signals (see Singleton (1987), Brown and Jennings (1989), Froot, Scharfstein, and Stein (1992), Dow and Gorton (1994), Vives (1995), Cespa (2002), Vives (2008), and Albagli (2011)). If prices are semi-strong efficient (as in Vives (1995)), then traders do not require compensation for increasing their exposure to the asset and so the inventory component of the price impact disappears. As a consequence, the retrospective inference loop breaks down and a unique equilibrium obtains. Brown and Jennings (1989) analyze a model in which prices are *not* semi-strong efficient, investors have a short-term horizon, and liquidity trading can be correlated. Their work provides a rationale for "technical analysis" that shows how, absent semi-strong efficiency, the *sequence* of transaction prices is more informative—about the final payoff—than is the *current* stock price. We argue that, in the absence of semi-strong efficiency, if liquidity trading is correlated then second-period investors can retrospectively evaluate their first-period inferences. This opportunity for reassessment generates strategic complementarities in the use of private information and can also lead to multiple equilibria.

Other authors find that multiple equilibria can arise in the presence of short-term traders. In this regard, part of the literature assumes an *infinite horizon* economy. Under that assumption, multiplicity arises from the bootstrap nature of expectations in the steady-state equilibrium of an overlapping generations (OLG) model in which investors live for two periods. Spiegel (1998) studies the model with symmetric information.[9] Watanabe (2008) extends the model of Spiegel (1998) to account for the possibility that investors have heterogeneous short-lived private information.[10] Other authors generate multiple equilibria in *finite*-horizon economies. Zhang (2012) shows that short-term trading generates multiple equilibria that can be ranked in terms of price informativeness. However, multiplicity in that paper arises at the information *acquisition* stage, whereas we find multiplicity in the *response* to private information. Furthermore, public information in the LIE crowds out the production of private information, which is the opposite of what happens in our case. Along similar lines, Avdis (2012) finds that short-term trading can generate multiple equilibria in information acquisition. Finally, Chen, Huang, and Zhang (2012) analyze a model with short-term trading and in which traders receive signals of different precisions. These authors show that, even with *transient* liquidity trading, multiple equilibria can arise in the response to private information. This is so because the uncertainty reduction effect of an increase in the response to private information is boosted by the dispersion of private precisions.[11]

The second stream of literature to which our paper relates is the work that addresses the influence of higher-order expectations on asset prices (see Allen et al. (2006), Nimark (2007), Bacchetta and van Wincoop (2008), Kondor (2012)). Bacchetta and van Wincoop (2006) study the role of HOEs in the foreign exchange market. They show that such expectations worsen the signal extraction problem that investors face when observing exchange rate fluctuations that originate from trades based on hedging motives and fundamentals information. In our setup this deterioration occurs at the LIE; at the HIE, in contrast, investors' strong reaction to private information makes signal extraction less of a problem. Our results contrast with the implications of DO models (Kandel and Pearson (1995), Banerjee (2011)) as we have stated in the introduction.

Finally, this paper is related to the literature on limits to arbitrage. In that regard, our multiplicity result is reminiscent of De Long et al. (1990), but in a model with rational traders and a finite horizon. Thus, our paper naturally relates to the strand of this literature that views limits to arbitrage as the analysis of how "non-fundamental demand shocks" impact asset prices in models with rational agents (Gromb and Vayanos (2010), Vayanos and Woolley (2013)), emphasizing the role of liquidity shocks persistence. Our model also predicts that momentum is related to a high volume of informational trading, which is in line with the evidence presented in Llorente et al. (2002). Some have claimed that limits to arbitrage capital are responsible for crashes and meltdowns (see Duffie (2010); see also Khandani and Lo (2011) for the August 2007 quant meltdown). In our model, meltdowns can be explained in terms of a transition from the high- to the low-information equilibrium.

II The static benchmark

Consider a one-period stock market in which a single risky asset (of liquidation value v) and a riskless asset (of unitary return) are traded by a continuum of risk-averse, informed investors in the interval $[0,1]$ and also by liquidity traders. We assume that $v \sim N(\bar{v}, \tau_v^{-1})$. Investors have constant absolute risk aversion (CARA) preferences (we use γ to denote the risk-tolerance coefficient) and maximize the expected utility of their wealth: $W_i = (v-p)x_i$.[12] Before the market opens, each informed investor i obtains private information on v, receiving a signal $s_i = v + \varepsilon_i$, $\varepsilon_i \sim N(0, \tau_\varepsilon^{-1})$, and submits a demand schedule (generalized limit order) $X(s_i, p)$ to the market, indicating the investor's desired position in the risky asset for each realization of the equilibrium price.[13] Assume that v and ε_i are independent for all i and that error terms are also independent across investors. Liquidity traders submit a random market order u (independent of all other random variables in the model), where $u \sim N(0, \tau_u^{-1})$. Finally, we adopt the convention that the average signal $\int_0^1 s_i \, di$ is equal to v almost surely. In other words, errors cancel out in the aggregate: $\int_0^1 \varepsilon_i \, di = 0$.[14]

In the CARA-normal framework just described, a symmetric rational expectations equilibrium (REE) is a set of trades contingent on the information that investors have, $\{X(s_i, p) \text{ for } i \in [0,1]\}$, and on a price functional $P(v,u)$ (measurable in (v,u)) such that investors in $[0,1]$ optimize (given their information) and the market clears:

$$\int_0^1 x_i \, di + u = 0.$$

Given this definition, it is easy to verify that a unique and symmetric equilibrium in linear strategies exists in the class of equilibria with a price functional of the form $P(v,u)$ (see, e.g., Admati (1985), Vives (2008)). The equilibrium strategy of CARA investor i is given by

$$X(s_i, p) = \gamma \frac{E[v|s_i, p] - p}{\mathrm{Var}[v|s_i, p]}.$$

Letting $\tau_i \equiv (\mathrm{Var}[v|s_i, p])^{-1}$ and denoting by $\alpha_E = \tau_\varepsilon / \tau_i$ the optimal statistical (Bayesian) weight given to private information in $E[v|s_i, p]$, we have that $\gamma \tau_i = (a/\alpha_E)$, where

$$a = \gamma \tau_\varepsilon, \tag{1}$$

denotes the responsiveness to private information of investor i. Note that a is independent of the variance of the price or of liquidity trading. This is because $\mathrm{Var}[v|s_i, p]$ cancels in the numerator and denominator of $X(s_i, p)$. If market clearing is imposed then the equilibrium price is given by

$$p = \int_0^1 E_i[v]\, di + \frac{\alpha_E}{a} u \tag{2}$$

$$= E[v|p] + \Lambda E[u|p], \tag{3}$$

where $E[u|p] = a(v - E[v|p]) + u$ and

$$\Lambda = \frac{\mathrm{Var}_i[v]}{\gamma}. \tag{4}$$

Equations (2) and (3) show that the price can be represented in two different ways. According to the representation in (2), the price reflects not only the consensus opinion that investors hold about the liquidation value but also the effect of demand from liquidity traders (multiplied by their risk tolerance–weighted uncertainty over the liquidation value). Indeed, owing to CARA and normality, in a static market an investor's demand is proportional to the expected gains from trade, $E[v|s_i, p] - p$. Because the price aggregates the demand of all investors, it reflects the consensus opinion $\int_0^1 E_i[v]\, di$ that is shocked by the orders of liquidity traders.

According to (3), the equilibrium price reflects investors' estimates of the fundamentals v (the so-called semi-strong efficient price $E[v|p]$) *and* of liquidity traders' demand u; risk-averse investors demand compensation for accommodating liquidity traders' orders. Under asymmetric information, that demand is not perfectly observable from the price. Therefore, such compensation is increasing in an investor's estimate $E[u|p]$ scaled by Λ. It follows from (4) that, for a given $E[u|p]$, greater investor uncertainty about the liquidation value or about the risk involved is associated with higher Λ and greater compensation. Thus Λ captures the "inventory" component of market liquidity.[15] Liquidity traders' orders have an additional effect on the price through their impact on the semi-strong efficient price $E[v|p]$. This effect induces an inference component that adds to the inventory component, implying that the (reciprocal of) market liquidity can be written as

$$\lambda \equiv \frac{\partial p}{\partial u} = \Lambda + (1 - \alpha_E)\frac{a\tau_u}{\tau},$$

where $\tau = 1/\mathrm{Var}[v|p] = \tau_v + a^2\tau_u$.[16] We remark that, all else equal, in a static setup the inference component magnifies the price impact of liquidity traders' orders.

III A two-period market with short-term investors

Consider now a two-period extension of the market analyzed in the previous section. At date 1 (resp., 2), a continuum of short-term investors in the interval $[0,1]$ enters the market, loads a position in the risky asset, and then unwinds that position in period 2 (resp., 3). Investor i has CARA preferences (we denote by γ the coefficient for common risk tolerance) and maximizes the expected utility of her short-term profit $\pi_{in} = (p_{n+1} - p_n)x_{in}$ for $n = 1, 2$ with $p_0 = \bar{v}$ and $p_3 = v$.[17] The short-term horizons of investors can be justified on the grounds of incentive reasons related to performance evaluation or of the difficulties associated with financing long-term investment when there are capital market imperfections (see Holmström and Joan Ricart i Costa (1986) and Shleifer and Vishny (1990)). An investor i who enters the market in period 1 receives a signal $s_i = v + \varepsilon_i$ that she recalls in the second period, where $\varepsilon_i \sim N(0, \tau_\varepsilon^{-1})$ and where v and ε_i are independent for all i. We assume further that in the second period a signal $s_P = v + \eta$ is publicly disclosed to the market, where $\eta \sim N(0, \tau_\eta^{-1})$ is independent of both v and ε_i for all i. The public signal introduces an additional source of public information on the fundamentals (i.e., besides the equilibrium prices) whose informativeness is *exogenous* to the trading process. The real-world counterpart of this assumption is any public announcement about the asset's value (e.g., an earnings announcement, analysts' consensus forecast of earnings). Once again we adopt the convention that, given v, the average private signal $\int_0^1 s_i\, di$ equals v almost surely and so errors cancel out in the aggregate: $\int_0^1 \varepsilon_i\, di = 0$.

We restrict our attention to equilibria in linear demand functions. We denote by $X_1(s_i, p_1) = a_1 s_i - \varphi_1(p_1)$ and $X_2(s_i, s_P, p_1, p_2) = a_2 s_i + b s_P - \varphi_2(p_1, p_2)$ an investor's desired position in the risky asset for each realization of the equilibrium price at (respectively) dates 1 and 2. The constants a_n and b denote, respectively, the weight an investor gives to private information at date n and the weight she gives to the public signal. The function $\varphi_n(\cdot)$ is a linear function of the equilibrium prices.[18]

The position of liquidity traders is assumed to follow a first-order autogressive or AR(1) process:

$$\begin{aligned} \theta_1 &= u_1, \\ \theta_2 &= \beta\theta_1 + u_2; \end{aligned} \tag{5}$$

here $\beta \in [0,1]$ and $\{u_1, u_2\}$ is an independent and identically distributed (i.i.d.) random process (independent of all other random variables in the model) with $u_n \sim N(0, \tau_u^{-1})$. Other authors have adopted this assumption for liquidity traders, including Singleton (1987), Campbell and Kyle (1993), He and Wang (1995), Biais, Bossaerts, and Spatt

(2010), and Cespa and Vives (2012). If $\beta = 1$, then $\{\theta_1, \theta_2\}$ follows a random walk and we are in the usual case of independent liquidity trade increments: $u_2 = \theta_2 - \theta_1$ is independent of u_1 (Kyle (1985), Vives (1995)). If $\beta = 0$, then liquidity trading is i.i.d. across periods; this is the case considered by Allen, Morris, and Shin (2006).

Persistent liquidity trading is taken as a primitive of the model, but such trading can be shown to arise endogenously in a model where liquidity traders are replaced by rational hedgers with different investment horizons (see Cespa and Vives (2014)).[19]

Persistence in liquidity trading can be given several possible interpretations, depending on the frequency of observations. At a daily or intra-daily frequency, to assume persistence is a simple way to capture the need of liquidity traders to break down a large order into a series of smaller orders and thereby minimize price impact; as such, this assumption is consistent with several empirical findings (e.g., Griffin, Harris, and Topaloglu (2003), Chordia and Subrahmanyam (2004)). At a lower frequency, liquidity trading persistence can be seen as a reduced-form assumption capturing the performance–flow relationship's effect on the holdings of mutual funds. Coval and Stafford (2007) show that mutual funds faced with outflows (resp., inflows) engage in fire sales (resp., purchases), creating contemporaneous, uninformed, and temporary negative (resp., positive) price pressure.[20] Coupling this result with the evidence that capital flows in and out of mutual funds are strongly related to past performance (Chevalier and Ellison (1997), Sirri and Tufano (1998)), negative (resp., positive) shocks to the capital of mutual funds can affect those funds' trades and thus have a negative (resp., positive) effect on their performance through feeding back to the funds' capital outflows (resp., inflows). Overall, we see our model as fit to describe the patterns that arise at a low frequency (monthly–quarterly).

Empirical evidence of liquidity trading persistence has been obtained in two ways.[21] Campbell and Kyle (1993) disentangle the properties of the noise process from the properties of returns. These authors examine annual aggregate returns of index data in the United States and then attempt to fit the properties of the time series with different models that feature noise. They find that, at a yearly level, noise traders' positions are highly persistent (from 95% to 97% of the noise remains after one year). An alternative strategy is to look at the 13F filings of mutual funds that face negative or positive net inflows and are thus led to trade for non-informational purposes (Coval and Stafford (2007)). This is the approach taken by Lou (2012), who finds that mutual funds' shareholdings display strong persistence at a quarterly frequency.

We denote by $E_{i1}[Y] = E[Y|s_i, p_1]$ and $\text{Var}_{i1}[Y] = \text{Var}[Y|s_i, p_1]$ the expectation and variance of the random variable Y formed by a date-1 investor using private and public information; $E_1[Y] = E[Y|p_1]$ and $\mathrm{Var}_1[Y] = \mathrm{Var}[Y|p_1]$ denote the same case but when only public information is used. Similar definitions for date-2 investors yield $E_{i2}[Y] = E[Y|s_i, s_P, p_1, p_2]$ and $\mathrm{Var}_{i2}[Y] = \mathrm{Var}[Y|s_i, s_P, p_1, p_2]$ as well as $E_2[Y] = E[Y|s_P, p_1, p_2]$ and $\mathrm{Var}_2[Y] = \mathrm{Var}[Y|s_P, p_1, p_2]$. The variables τ_n and τ_{in} denote the precision of investor forecasts of v based (respectively) only on public information and on both public and private information; thus, $\tau_n = (1/\mathrm{Var}_n[v])$ and $\tau_{in} = (1/\mathrm{Var}_{in}[v])$. Letting $\alpha_{E_n} = \tau_\varepsilon/\tau_{in}$, we have $E_{in}[v] = \alpha_{E_n} s_{in} + (1 - \alpha_{E_n})E_n[v]$.

We now derive the informational content of prices in a linear equilibrium. Consider a candidate linear (symmetric) equilibrium where $x_{i1} = a_1 s_i - \varphi_1(p_1)$ and $x_{i2} = a_2 s_i + b s_P - \varphi_2(p_1, p_2)$ for $\varphi_n(\cdot)$ a linear function. Denote by $z_1 \equiv a_1 v + \theta_1$ the noisy informational addition about v generated by informed investors in period 1 (i.e., the "informational content" of the first-period order flow). Similarly, put $\Delta a_2 \equiv a_2 - \beta a_1$ and denote investors' second-period informational addition by $z_2 \equiv \Delta a_2 v + u_2$. We now show that, at a linear equilibrium, p_1 is observationally equivalent (o.e.) to z_1 and that, given s_P, the sequence $\{z_1, z_2\}$ is o.e. to $\{p_1, p_2\}$. If we let $x_n \equiv \int_0^1 x_{in}\, di$ and impose market clearing in the first period, then (by our convention) the implication is that

$$x_1 + \theta_1 = 0 \iff a_1 v + \theta_1 = \varphi_1(p_1). \tag{6}$$

In the second period, the market-clearing condition is

$$\begin{aligned} x_2 + \beta\theta_1 + u_2 = 0 &\iff x_2 - \beta x_1 + u_2 = 0 \\ &\iff a_2 v + b s_P - \varphi_2(p_1, p_2) - \beta(a_1 v - \varphi_1(p_1)) + u_2 = 0 \\ &\iff \Delta a_2 v + u_2 = \varphi_2(p_1, p_2) - \beta\varphi_1(p_1) - b s_P; \end{aligned} \tag{7}$$

in the second line we have used (6), and by $\Delta a_2 = a_2 - \beta a_1$ we denote the β-weighted net trading intensity of second-period informed investors. From (6) and (7) it is easy to see that z_1 is o.e. to p_1 and that, given s_P, $\{z_1, z_2\}$ is o.e. to $\{p_1, p_2\}$. Hence $E_1[v] = \tau_1^{-1}(\tau_v \bar{v} + a_1 \tau_u z_1)$, $E_2[v] = \tau_2^{-1}(\tau_1 E_1[v] + \tau_\eta s_P + \Delta a_2 \tau_u z_2)$, $\mathrm{Var}_1[v] \equiv \tau_1^{-1} = (\tau_v + a_1^2 \tau_u)^{-1}$, $\mathrm{Var}_2[v] \equiv \tau_2^{-1} = (\tau_1 + \tau_\eta + (\Delta a_2)^2 \tau_u)^{-1}$, $E_{in}[v] = \tau_{in}^{-1}(\tau_n E_n[v] + \tau_\varepsilon s_i)$, and $\mathrm{Var}_{in}[v] \equiv \tau_{in}^{-1} = (\tau_n + \tau_\varepsilon)^{-1}$.

We shall now relate price formation to the Keynes's notion of a Beauty Contest. Then we look at the retrospective inference mechanism that we associate with persistent liquidity trading and characterize equilibria, their stability properties, and trading strategies. In this way we demonstrate the limits of the Beauty Contest analogy for financial markets. We end Section III with a robustness analysis.

A Prices and Beauty Contests

Here we give an expression for the equilibrium price that highlights the dependence of that price on investors' higher-order expectations about fundamentals (cf. Allen, Morris, and Shin (2006)). When there is persistence, liquidity traders' orders at time n also affect the demand for the asset at time $n+1$. So in a two-period model, first-period investors use their private information *also* to infer the demand of liquidity traders from the first-period price. As a result, the latter is driven by investors' HOEs about fundamentals *and* by their average expectations about liquidity trading. This result, in turn, has implications for the informational properties of the price.

Let us denote by $\bar{E}_n[v] \equiv \int_0^1 E_{in}[v]\,di$ the consensus opinion about the fundamentals at time n, where $\bar{E}_n[v] = \alpha_{E_n} v + (1-\alpha_{E_n})E_n[v]$. Starting from the second period, if we impose market clearing then $\int_0^1 X_2(s_i, s_P, p_1, p_2)\,di + \theta_2 = 0$. Because of CARA and normality, we have $X_2(s_i, s_P, p_1, p_2) = \gamma \mathrm{Var}_{i2}[v]^{-1}(E_{i2}[v] - p_2)$. Substituting this expression into the market-clearing equation and solving for the equilibrium price now yields

$$p_2 = \bar{E}_2[v] + \frac{\mathrm{Var}_{i2}[v]}{\gamma}\theta_2. \tag{8}$$

Similarly, imposing market clearing in the first period we have $\int_0^1 X_1(s_i, p_1)\,di + \theta_1 = 0$; solving for the equilibrium price then yields

$$p_1 = \bar{E}_1[p_2] + \frac{\mathrm{Var}_{i1}[p_2]}{\gamma}\theta_1. \tag{9}$$

Substituting (8) and then rearranging, we obtain

$$\begin{aligned} p_1 &= \bar{E}_1\left[\bar{E}_2[v] + \frac{\mathrm{Var}_{i2}[v]}{\gamma}\theta_2\right] + \frac{\mathrm{Var}_{i1}[p_2]}{\gamma}\theta_1 \\ &= \bar{E}_1[\bar{E}_2[v]] + \beta\frac{\mathrm{Var}_{i2}[v]}{\gamma}\bar{E}_1[\theta_1] + \frac{\mathrm{Var}_{i1}[p_2]}{\gamma}\theta_1. \end{aligned} \tag{10}$$

According to (10), three terms constitute the first-period price: investors' second-order average expectations over the liquidation value ($\bar{E}_1[\bar{E}_2[v]]$), the risk-adjusted effect of the first-period stock of liquidity trades (θ_1), and investors' average expectations over first-period liquidity trades ($\bar{E}_1[\theta_1]$). Those latter expectations arise because p_2 depends on θ_2, which in turn is correlated with θ_1 when $\beta > 0$. Thus, investors in period 1 are also interested in estimating θ_1.

Expression (10) implies that, because of persistence in liquidity trading, the weight of the price on investors' average information is the sum of two terms. The first term captures the effect of HOEs on v, and the second reflects the effect of investors' average expectations concerning θ_1. We have that

$$\begin{aligned} \bar{E}_1[\bar{E}_2[v]] &= \bar{\alpha}_{E_1} v + (1 - \bar{\alpha}_{E_1}) E_1[v]; \\ \bar{E}_1[\theta_1] &= a_1(1 - \alpha_{E_1})(v - E_1[v]) + \theta_1. \end{aligned}$$

Here $\bar{\alpha}_{E_1} = \alpha_{E_1}(1 - (\tau_1/\tau_2)(1 - \alpha_{E_2}))$, and a_1 denotes first-period investors' average responsiveness to private information.[22] So given (10), the implication is that the *total* weight of the price on average private information is given by

$$\alpha_{P_1} = \bar{\alpha}_{E_1} + \beta \frac{\mathrm{Var}_{i2}[v]}{\gamma} a_1(1 - \alpha_{E_1}). \tag{11}$$

Observe that $\bar{\alpha}_{E_1} < \alpha_{E_1}$ for any β. Therefore, if liquidity trading is transient ($\beta = 0$) then the first-period price places more weight on public information than does the optimal statistical weight. This finding is in line with Allen et al. (2006), who prove that if information is heterogeneous then prices reflect investors' HOEs about the final payoff. In this case, the law of iterated expectations does not hold and investors' forecasts overweight public information (in the sense that $\alpha_{P_1} = \bar{\alpha}_{E_1} < \alpha_{E_1}$). The reason is that investors anticipate the average market opinion while knowing that this opinion depends also on the public information observed by other investors. In Section III.E we show that the price is then systematically farther away (than the consensus) from fundamentals. If liquidity trading is persistent, however, then p_1 also reflects investors' average expectations about θ_1. Hence another term is added to $\bar{\alpha}_{E_1}$ in the expression for α_{P_1} given by (11). We argue in Section III.E that this additional term can reverse the conclusion we derived under transient liquidity trading.

B Retrospective inference and equilibrium prices

We start by giving a general description of the equilibrium price function providing three equivalent expressions for prices that highlight different properties of the model.

PROPOSITION 1: *At a linear equilibrium, the price is given by*

$$p_n = \alpha_{P_n}\left(v + \frac{\theta_n}{a_n}\right) + (1 - \alpha_{P_n})E_n[v], \tag{12}$$

where $\theta_n = u_n + \beta\theta_{n-1}$ *and where* a_n *and* α_{P_n} *denote the responsiveness to private information exhibited at period* n *by investors and by the price, respectively (see equations* (A.3a), (A.3b), (A.8), *and* (A.9)*). We have that* $\alpha_{P_2} = \alpha_{E_2} < 1$.

According to (12), at period n the equilibrium price is a weighted average of the market expectation about the fundamentals v (the semi-strong efficient price) and a noisy market signal about v that depends on the response to private information. Rearranging this expression yields

$$\begin{aligned} p_n - E_n[v] &= \frac{\alpha_{P_n}}{a_n}(a_n(v - E_n[v]) + \theta_n) \\ &= \Lambda_n E_n[\theta_n] \end{aligned} \tag{13}$$

for $\Lambda_n \equiv \alpha_{P_n}/a_n$, which implies that there is a discrepancy between p_n and $E_n[v]$. As in the static market (see (3)), this discrepancy reflects a premium that is proportional to the expected stock of liquidity trading that investors accommodate at date n. The result is our second equilibrium price expression as follows.

COROLLARY 1.1: *At a linear equilibrium, the price incorporates a premium above the semi-strong efficient price:*

$$p_n = E_n[v] + \Lambda_n E_n[\theta_n], \tag{14}$$

where $\Lambda_2 = \mathrm{Var}_{i2}[v]/\gamma$ *and*

$$\Lambda_1 = \frac{\mathrm{Var}_{i1}[p_2]}{\gamma} + \beta\Lambda_2. \tag{15}$$

A comparison of (15) with (4) reveals that short-term trading affects the inventory component of liquidity. In a static market, investors who absorb the demand of liquidity traders are exposed to risk stemming from the randomness of v. In a dynamic market, however, short-term investors at date 1 face the risk due to the randomness of the next-period price (i.e., the price at which they unwind). To the extent that liquidity trading displays persistence, informed second-period investors absorb part of the first-period liquidity traders' positions; this dynamic contributes to first-period investors' uncertainty about p_2, yielding (15).

We can use (14) to show that, as in the static benchmark, under asymmetric information the price is affected not only by the inventory component Λ_n but also by an inference component. This claim is formalized in our next corollary.

COROLLARY 1.2: *Let $a_0 = 0$. At a linear equilibrium,*

$$p_1 = \lambda_1 z_1 + (1 - \lambda_1 a_1)\bar{v} \quad \text{and} \tag{16a}$$

$$p_2 = \lambda_2 z_2 + \frac{\tau_\eta}{\tau_{i2}} s_P + \frac{\gamma \tau_1 E_1[v] + \beta z_1}{\gamma \tau_{i2}}. \tag{16b}$$

Here λ_n denotes the price impact of trades in period $n = 1, 2$:

$$\lambda_n \equiv \frac{\partial p_n}{\partial u_n} = \frac{\alpha_{P_n}}{a_n} + (1 - \alpha_{P_n}) \frac{\Delta a_n \tau_u}{\tau_n}. \tag{17}$$

According to (17), the inference component of liquidity at $n = 2$ is captured by

$$(1 - \alpha_{P_2}) \frac{\Delta a_2 \tau_u}{\tau_2}. \tag{18}$$

As shown in (7), a dynamic market differs from the static benchmark in that the sign of this component depends on the β-weighted *net* position of informed investors; thus the net trading intensity $\Delta a_2 = a_2 - \beta a_1$. As a result, the effect of private information in the second period depends on the *change* in informed investors' positions, where this change is measured by $\Delta a_2 = a_2 - \beta a_1$. The implication is that the sign of the inference component depends on the magnitude of a_1 as compared with a_2/β.[23] Therefore, the first-period response to private information affects the informational innovation extracted from p_1 and also (when $\beta > 0$) the one extracted from p_2—that is, both z_1 and z_2—as well as the sign of the inference component in λ_2.

When $\beta > 0$ the door is open to a negative inference component in the second period when a_1 is large since in this period investors *retrospectively* reassess the first period inference about the fundamentals based on the new evidence gathered. Suppose that second-period informed investors observe high demand for the asset (i.e., z_2 high). If a_1 is large then the first-period price is quite informative about v (since $z_1 = a_1 v + \theta_1$). Hence (i) most of the price adjustment to fundamentals information must have occurred in the first period and (ii) demand is likely to be driven by liquidity trading. Since $\beta > 0$, it follows that high demand for the asset from liquidity traders *also affected the first-period aggregate demand.* But for a given price realization p_1, that would imply a lower expectation of the fundamentals.[24] Indeed, if a_1 is so large that $\Delta a_2 < 0$, then a large aggregate demand realization leads second-period investors to revise downward their expectations of the liquidation value because a large $z_2 = \Delta a_2 v + u_2$ is bad news about v when $\Delta a_2 < 0$. In this case the inference component in λ_2 is *negative* and offsets the inventory component. If instead a_1 is small, the first period price is poorly informative about v, and a second period large realization z_2 is more likely to come from informed investors. For $\beta > 0$ this reinforces the belief that low liquidity traders' demand affected the first period demand for the asset and, for a given price realization p_1, implies a higher expectation of the fundamentals (since $\Delta a_2 > 0$). In this case, the inference component is *positive* and adds to the inventory component, increasing λ_2.

In summary, when liquidity trading displays persistence, second period investors can *retrospectively* reassess the first period inference about the fundamentals, based on the new evidence gathered in the second period, affecting the inference component of second period price impact of trade (λ_2). We thus term this effect "retrospective inference".[25]

C Equilibrium characterization

Short horizons create a dependence of a_1 on the residual variance of p_2 given p_1, generating an uncertainty reduction effect when a_1 increases. The reason is that short-term investors in the first period trade according to

$$X_1(s_i, p_1) = \gamma \frac{E_{i1}[p_2] - p_1}{\mathrm{Var}_{i1}[p_2]} \implies a_1 = \gamma \frac{\text{Weight to } s_i \text{ in } E_{i1}[p_2]}{\mathrm{Var}_{i1}[p_2]}. \tag{19}$$

By this implication, a_1 is directly related to the relevance of the private signal to forecast p_2 and inversely related to investors' uncertainty about p_2. Two effects are present that, contrary to the static case, do not cancel each other. On the one hand, we have the usual substitution effect whereby a higher average a_1 leads to a more informative price in period 1 (higher τ_1 and τ_{i1}), and therefore a lower weight to private information τ_ϵ/τ_{i1} in $E_{i1}[p_2]$, which tends to depress a_1. On the other hand, a higher average a_1 and larger τ_1 leads to a lower $\mathrm{Var}_{i1}[p_2]$, which tends to raise a_1. This is a source of strategic complementary in the responses to private information. With short-horizons, this uncertainty reduction effect also works through λ_2. This is because, first period investors are interested in forecasting p_2 and, for any public signal, the extent to which p_2 differs from p_1 depends on λ_2 (see (16b)). Therefore, a higher λ_2 (in absolute value) will, ceteris paribus, increase first-period investors' uncertainty about p_2 and depress a_1. Recall that when $\beta > 0$ it is possible that $\Delta a_2 < 0$ when a_1 is high enough, implying that the inference component in

λ_2 offsets the inventory component. In this case, the second-period price impact of trade is small, which diminishes first-period investors' uncertainty about p_2 and so boosts their response to private signals. Hence, the variance reduction (second) effect dominates the substitution (first) effect and a high level of a_1 can be sustained in equilibrium. At the same time, a lower level of a_1 is also self-fulfilling. Indeed, a low a_1 generates a low τ_1 and a high residual variance for p_2 given p_1 which makes the low a_1 self-fulfilling. In this case we have $\Delta a_2 > 0$ (even if $\beta > 0$) and the second period price impact of trade (λ_2) is high, augmenting the residual variance of p_2.

Summarizing, investors' willingness to speculate on private information not only depends on their uncertainty about the liquidation price, but also affects that uncertainty. This two-sided loop in the determination of a_1 gives rise to strategic complementarities in the use of private information, which can yield multiple equilibria. The following proposition characterizes linear equilibria and provides suffcient conditions for multiple equilibria to exist. Those conditions imply that the variance reduction effect is strong enough.

PROPOSITION 2: *Suppose* $\tau_\eta > 0$.

- *If* $\beta > 1/2$, $\gamma^2\tau_\varepsilon\tau_u > 2(2\beta-1)/(3-2\beta)$, *and* $\tau_\eta \leq \hat{\tau}_\eta$ *(for some* $\hat{\tau}_\eta > 0$ *defined in the appendix, see* (A.13c)*), there always exist at least three linear equilibria where* $a_2 = \gamma\tau_\varepsilon$ *and* $a_1 \in \{a_1^*, a_1^{**}, a_1^{***}\}$, *with* $a_1^* \in (0, a_2)$,

$$a_1^{**} \in \left(\frac{1+\gamma\tau_u a_2}{\gamma\beta\tau_u}, \frac{2+3\gamma\tau_u a_2}{2\gamma\beta\tau_u}\right),\ a_1^{***} > \frac{2+3\gamma\tau_u a_2}{2\gamma\beta\tau_u},$$

  implying $a_1^* < a_2 < a_1^{**} < a_1^{***}$. *When*

$$a_1 = \begin{cases} a_1^*, & \text{then} \quad a_2 - \beta a_1^* > 0, \quad \text{and} \quad \lambda_2^* > 0, \\ a_1^{**}, a_1^{***}, & \text{then} \quad a_2 - \beta a_1^{**} < 0, \quad \text{and} \quad \lambda_2^{**} < 0. \end{cases} \tag{20}$$

  Along these equilibria:

$$\tau_n^* < \tau_n^{**} < \tau_n^{***},\ n = 1, 2. \tag{21}$$

- *If* $\beta = 0$, *there exists a unique equilibrium with* $a_2 = \gamma\tau_\varepsilon$,

$$\frac{\gamma^2 a_2\tau_u(a_2^2\tau_u + \tau_\varepsilon + \tau_\eta)}{1+\gamma^2\tau_u(a_2^2\tau_u + 2\tau_\varepsilon + \tau_\eta)} < a_1^* < a_2,$$

  and $\lambda_2^* > 0$.

According to this proposition, multiplicity requires that private information be strongly reflected in prices ($\gamma^2\tau_\varepsilon\tau_u > 2$ is sufficient), persistence (β) is high, and public precision (τ_η) is low. All of these conditions strengthen the retrospective inference and the described uncertainty reduction effect loop. A contrario, for example, a higher precision of the public signal makes p_2 more dependent on s_P and less on p_1 and therefore the impact of τ_1 on the residual variance of p_2 is lessened.

In view of (21), we refer to the three equilibria described in Proposition 1 as (respectively) the low-, intermediate-, and high-information equilibrium: LIE, IIE, and HIE. In the Appendix we show that first-period equilibrium responsiveness is obtained as a fixed point of the following function:

$$\psi(a_1) = \gamma\frac{(\lambda_2\Delta a_2 + \tau_\eta/\tau_{i2})\alpha_{E_1}}{(\lambda_2\Delta a_2 + \tau_\eta/\tau_{i2})^2/\tau_{i1} + \lambda_2^2/\tau_u + \tau_\eta/\tau_{i2}^2}. \tag{22}$$

Numerical analysis shows that this function crosses the 45-degree line at most three times, which suggests that the three equilibria described in Proposition 2 are the only ones that can arise (see Figure 1). Our numerical results show further that these equilibria can be ranked in terms of second-period price impact (λ_2), inventory component of liquidity (Λ_n), and conditional volatility ($\text{Var}_1[p_2]$). The following result gives more details.

[Figure 1 about here.]

NUMERICAL RESULT 1: *When multiple equilibria arise, these inequalities hold:*

$$\lambda_2^* > |\lambda_2^{**}| > |\lambda_2^{***}|; \tag{23a}$$

$$\Lambda_n^* > \Lambda_n^{**} > \Lambda_n^{***}; \tag{23b}$$

$$\text{Var}_1[p_2]^* > \text{Var}_1[p_2]^{**} > \text{Var}_1[p_2]^{***}. \tag{23c}$$

Along the HIE, $\Delta a_2 < 0$ and the inference component of liquidity is negative. This finding is consistent with second-period traders revising downward their first-period assessment of the payoff in the presence of a positive demand shock (i.e., engaging in retrospective inference). This result is consistent also with the findings of some spread decomposition models (e.g., Huang and Stoll (1997), Van Ness, Van Ness, and Warr (2001), Henker and Wang (2006)) in which the adverse selection component of the spread can be *negative*.[26] Thus, our model provides a theoretical justification for this empirical finding.

In Figure 2 we display the effects of a change in the values of public and private signal precision, persistence, and liquidity traders' demand precision on the best response (22).[27] As the graphs show, uniqueness always occurs at the LIE and requires high public precision or low private precision, persistence, or liquidity traders' precision. Intuitively, in all of these cases the endogenous public signal (the price) becomes relatively less informative than the exogenous public signal (s_P), leading second-period investors to rely less on price information. This dynamic weakens strategic complementarity by softening the self-reinforcing uncertainty reduction loop resulting from retrospective inference and thereby yields a unique equilibrium. It is worth noting that it can be checked that the best response $\psi(\cdot)$ is downward sloping when $\beta = 0$.

[Figure 2 about here.]

In Figure 3 we show that the effect of an increase in public signal precision on a_1 depends on the equilibrium that arises. Along the HIE (resp., LIE), a larger τ_η leads to a decrease (resp., increase) in a_1. The reason for this result is that a more precise public signal reduces traders' reliance on price information when forecasting the fundamentals. Thus, along both equilibria, the effect of retrospective inference is weaker in the second period. In the HIE (LIE) this weakened effect increases (decreases) first-period investors' uncertainty about p_2, leading to a decrease (increase) in a_1.[28]

If the public signal is totally uninformative then, for $\beta > 0$, the retrospective inference and variance reduction loop becomes extremely strong. In this case, the best response (22) becomes discontinuous at the IIE (which therefore disappears; see Figure 3) and so we *always* obtain two equilibria that can be computed in closed form. This claim is formalized in our next corollary.

[Figure 3 about here.]

COROLLARY 2.1: *Suppose* $\tau_\eta = 0$.

- *If* $\beta > 0$ *then there always exist two linear equilibria, where* $a_2 = \gamma\tau_\varepsilon$ *and* $a_1 \in \{a_1^*, a_1^{***}\}$ *for* $a_1^* < a_2 < a_1^{***}$ *(see* (A.17) *and* (A.18) *for explicit expressions). If*

$$a_1 = \begin{cases} a_1^* & \textit{then } a_2 - \beta a_1^* > 0 \textit{ and } \lambda_2^* > 0 \ \ (\textit{LIE}), \\ a_1^{***} & \textit{then } a_2 - \beta a_1^{***} < 0 \textit{ and } \lambda_2^{***} < 0 \ \ (\textit{HIE}). \end{cases}$$

Furthermore, $|\lambda_2^{***}| < \lambda_2^*$, $\Lambda_n^{***} < \Lambda_n^*$, *prices are more informative, and* $\mathrm{Var}_1[p_2]$ *is lower along the HIE.*

- *If $\beta = 0$, then there exists a unique equilibrium with $a_2 = \gamma\tau_\varepsilon$ and*

$$a_1^* = \frac{\gamma a_2^2 \tau_u}{1 + \gamma a_2 \tau_u} < a_2.$$

REMARK 1: *It is possible to show (as in Vives (1995)) that a unique equilibrium arises when prices are set by a sector of competitive and risk-neutral market makers. In this case the market makers do not require compensation (for inventory risk) in order to clear the market, and prices are semi-strong efficient. The uncertainty reduction effect of an increase in a_1 is present when $\beta > 0$ but weakened, which ensures uniqueness.*[29] *It is also possible to show that the equilibrium is unique when investors have no private information ($\tau_\varepsilon = 0$). In this case our model is akin to Grossman and Miller (1988) and investors trade only to accommodate liquidity traders' orders. Prices are therefore invertible in the latter's demand and retrospective inference does not arise, with price informativeness depending only on prior precsion.*

REMARK 2: *We can draw a parallel between our model and models in which investor actions have a feedback effect on the asset's value. In papers that feature such models (e.g., Ozdenoren and Yuan (2008), Bond, Goldstein, and Prescott (2010), Dow, Goldstein, and Guembel (2011), Goldstein, Ozdenoren, and Yuan (2013)), complementarities-driven multiplicity of equilibria arises also from the effect of the price on the asset's value. In our paper, the price at $n = 2$ (i.e., p_2) represents the asset's value from the perspective of investors at $n = 1$, and their trading also affects p_2. This dynamic corresponds to the feedback effect from prices to values in a one-period feedback model. Bond et al. (2010) show that, if agents use market prices when deciding on corrective actions (as when the board considers firing the CEO in response to a low stock price), then prices adjust to reflect this use and may thus become less revealing. In Ozdenoren and Yuan (2008), prices are informative about both the fundamentals and the likelihood of coordination among informed investors. Multiple equilibria arise when the price is more informative of the coordination motive than of the fundamentals. In their paper, the feedback effect's strength depends on the sensitivity of asset value to investment in the risky asset. The parallel in our model is the degree of persistence in liquidity trading. In both models, multiplicity tends to arise when the feedback effect is strong. Much as in our model, in Ozdenoren and Yuan (2008) there are multiple equilibria also when the precision of private information is high and base liquidity trading low. Yet unlike their study, in which an increase in public precision leads to a higher coordination motive and multiple equilibria,* *our model (with no coordination motive) yields the opposite result.*

D Stability

In this section we use the best response (22) to perform a stability analysis of the equilibria. Toward that end, consider the following argument. Assume that the market is at an equilibrium point $\bar{a}_1$ and so $\bar{a}_1 = \psi(\bar{a}_1)$. Suppose now that a small perturbation to $\bar{a}_1$ occurs. As a consequence, first-period investors modify the weight they give to private information; then the aggregate weight becomes $\bar{a}_1' = \psi(\bar{a}_1')$. If the market returns to the original $\bar{a}_1$ then—according to the best-reply dynamics with the best-response function $\psi(\cdot)$—the equilibrium is stable; otherwise, it is unstable. Hence we can say that, in a stable (unstable) equilibrium, if investors other than i put a lower weight on their signals then (i) the price is noisier and (ii) investor i reacts by increasing less (more) than proportionally the weight on his own signal and so contributing less (more) than proportionally to restoring price informativeness. Formally, we have the following definition.

DEFINITION 1 (Stability): *An equilibrium is stable (unstable) if and only if its corresponding value for a_1 is a stable (unstable) fixed point for the best-response function $\psi(\cdot)$—that is, iff its corresponding value for a_1 satisfies the inequality $|\psi'(a_1)| < 1$.*

For $\tau_\eta > 0$, if multiple equilbria arise then the IIE is always unstable.[30] On the contrary, in our simulations the LIE is always stable. Finally, the behavior of the HIE is more complex. In particular, for the HIE to be stable we require that private signals not be "too" precise when compared with the public signal. In the extreme case when $\tau_\eta = 0$, we can formally analyze the best-response mapping and obtain the following result.

COROLLARY 2.2: *Suppose $\tau_\eta = 0$. Then* (i) $\psi'(a_1) < 0$ *and* (ii) *the LIE (resp., HIE) is stable (resp., unstable) with respect to the best-response dynamics:*

$$|\psi'(a_1^{***})| > 1 > |\psi'(a_1^{*})| \,. \tag{24}$$

Intuitively, if private information is much more precise than public information then the retrospective inference and the variance reduction loop become very strong. In that case we approach a situation close to that described in Corollary 2.1, which makes the HIE always unstable.

In Figure 4 we set $\beta = 1$, $\gamma = 1/2$, and $\tau_u = \tau_v = 10$; we also partition the parameter space $\{\tau_\varepsilon \times \tau_\eta \mid \tau_\eta \in \{0, .01, \ldots, 10\}, \tau_\varepsilon \in \{.01, .02, \ldots, 10\}\}$ into five regions depending on whether multiple equilibria or instead a unique equilibrium obtains, whether the HIE is stable, and whether responses to private information are strategic substitutes or strategic complements.[31]

[Figure 4 about here.]

REMARK 3: *What is the effect of a shock to parameter values on the market's equilibrium? The answer to this question depends on whether or not the HIE is stable (see the Online Appendix). When the HIE is stable we can easily generate non-monotonic effects of exogenous parameter changes (e.g., in private signal precision or in risk tolerance) on the response to private information, the conditional volatility of returns, and the informational efficiency of prices. These results are possible because a parameter change may induce traders to coordinate at a different equilibrium. For instance, it is possible that a large decrease in private signal precision produces an increase in the equilibrium response to private information and informational efficiency—as well as a decline in the conditional volatility of returns, with the equilibrium shifting from the initial LIE to the HIE. When the HIE is unstable, numerical simulations show that the effect of even a mild shock to parameter values depends on how persistent the demand of liquidity traders is. When $\beta \in (0,1)$, the equilibrium converges to the LIE; when $\beta = 1$, the market oscillates between two nonequilibrium values.*

E Equilibrium strategies and limits of the Beauty Contest analogy

We characterize first investors' strategies.

COROLLARY 2.3: *At a linear equilibrium, the strategies of an informed investor are given by*

$$X_1(s_i, p_1) = \frac{a_1}{\alpha_{E_1}}(E_{i1}[v] - p_1) + \frac{\alpha_{P_1} - \alpha_{E_1}}{\alpha_{E_1}} E_1[\theta_1], \tag{25}$$

$$X_2(s_i, s_P, p_1, p_2) = \frac{a_2}{\alpha_{E_2}}(E_{i2}[v] - p_2). \tag{26}$$

In the event of multiple equilibria, if

$$a_1 = \begin{cases} a_1^* & \textit{then } \alpha_{P_1} < \alpha_{E_1},\ (\partial x_{i2}/\partial p_2) < 0,\ \mathrm{Cov}_{i1}[v - p_2, p_2 - p_1] < 0; \\ a_1^{**}, a_1^{***} & \textit{then } \alpha_{P_1} > \alpha_{E_1},\ (\partial x_{i2}/\partial p_2) > 0,\ \mathrm{Cov}_{i1}[v - p_2, p_2 - p_1] > 0. \end{cases} \tag{27}$$

If $\beta = 0$ *then* $\alpha_{P_1} < \alpha_{E_1}$, $(\partial x_{i2}/\partial p_2) < 0$, *and* $\mathrm{Cov}_{i1}[v - p_2, p_2 - p_1] < 0$.

According to (26), in the second period an investor behaves as if he were in a static market. So in the first period this investor loads his position while anticipating the second-period price and scaling it down according to his uncertainty regarding p_2 (see (19)). In this case, the investor's strategy can be expressed as the sum of two components (see (25)). The first component captures the investor's activity based on his *private* estimation of the difference between the fundamentals and the equilibrium price. Such activity is akin to "long-term" speculative trading that aims to take advantage of the investor's superior information on the asset's liquidation value (since p_2 is correlated with v). The second component captures the investor's activity based on the extraction of *public* information (i.e., order flow). This trading instead aims to time the market by exploiting short-run movements in the asset price related to the evolution of aggregate demand.

Along the HIE, the price is closer to fundamentals. As a result, when observing

$$E_1[\theta_1] = a_1(v - E_1[v]) + \theta_1 > 0$$

the investor infers that this realization is mainly driven by fundamentals information; he therefore goes long in the asset, "chasing the trend". This behavior reflects his anticipation that second-period investors will bid the price up when he unwinds his position, as implied by the sign of $\text{Cov}_{i1}[v - p_2, p_2 - p_1]$. Along the "Keynesian" LIE, prices are driven more by liquidity trading and so the trader acts instead as a "contrarian" investor.

This observation also suggests that the *aggregate* trading behavior of informed investors differs across the two equilibria. In the LIE, investors trade less aggressively on private information and thus exploit more aggressively the predictability of liquidity traders' demand. The opposite occurs in the HIE, where aggregate demand is driven by trading that is relatively more informed. We can demonstrate these claims formally by evaluating informed investors' first-period aggregate position $\int_0^1 X_1(s_i, p_1)\, di = X_1(v, p_1)$,

$$X_1(v, p_1) = \frac{a_1}{\alpha_{E_1}}(\bar{E}_1[v] - p_1) + \frac{\alpha_{P_1} - \alpha_{E_1}}{\alpha_{E_1}} E_1[\theta_1], \tag{28}$$

and then computing the following covariances:

$$\text{Cov}\left[\frac{a_1}{\alpha_{E_1}}(\bar{E}_1[v] - p_1), \theta_1\right] = -\left(\frac{a_1^2}{\tau_1} + \frac{\alpha_{P_1}}{\alpha_{E_1}} \frac{\tau_v}{\tau_1 \tau_u}\right) < 0 \tag{29a}$$

$$\text{Cov}\left[\frac{\alpha_{P_1} - \alpha_{E_1}}{\alpha_{E_1}} E_1[\theta_1], \theta_1\right] = \frac{\alpha_{P_1} - \alpha_{E_1}}{\alpha_{E_1}} \frac{\tau_v}{\tau_1 \tau_u} \begin{cases} < 0 & \text{LIE} \\ > 0 & \text{HIE} \end{cases} \tag{29b}$$

The aggregate long-term speculative position is always negatively correlated with the noise shock (see (29a)). For the short-term aggregate position, this correlation is observed only in the LIE (see (29b)). In other words, for a given noise shock realization, investors in the HIE speculate against it according to the long-term component of their strategy while apparently trading along with it according to the short-term component. The intuition is that, along the HIE, investors trade so aggressively on their private information that they more than offset the initial price deviation from fundamentals generated by θ_1. For example, if $\theta_1 > 0$ then investors short the stock so aggressively that its price undershoots the fundamentals.[32] In equilibrium, then, investors find it profitable to purchase shares based on order flow information. Along the LIE, investors trade less aggressively on private information and so the covariances of both the long- and short-term components of investors' demand with liquidity traders' demand have concordant signs.

REMARK 4: *It is interesting that, along the HIE, the asset is a Giffen good in the second period (see* (27)*). Differentiating x_{i2} with respect to p_2, we can break down the effect of a price increase into a substitution effect and an "information" effect:*

$$\frac{\partial X_2(s_i, s_P, p_1, p_2)}{\partial p_2} = \frac{a_2}{\alpha_{E_2}} \left(\underbrace{\frac{\partial E_{i2}[v]}{\partial p_2}}_{\textit{Information effect}} - \underbrace{1}_{\textit{Substitution effect}} \right). \tag{30}$$

Along the HIE, investors rely strongly on prices, which are extremely informative about liquidation value. In this case, the substitution effect is swamped by the information effect.[33] *Along the LIE, the opposite happens and the asset is a normal good in the second period.*[34] *Giffen goods often arise when the learning from uninformed investors prevails, in which case the aggregate information effect dominates the substitution effect (see, e.g., Barlevy and Veronesi (2003), Yuan (2005), Vives (2008)). This could happen also in the presence of feedback effects when prices are informative both about the fundamentals and about the likelihood of coordination among informed investors, since the feedback effect would then strengthen the information effect (as in Ozdenoren and Yuan (2008)).*

Our last result in this section relates the two equilibria to the reliance of price on public information.

COROLLARY 2.4: *Suppose $\tau_\eta > 0$.*

1. Let $\beta \in (0,1]$, and assume there are multiple equilibria. In that case, if

$$a_1 = \begin{cases} a_1^* & \textit{then } \alpha_{P_1} < \alpha_{E_1} \textit{ and } \mathrm{Cov}[p_1, v] < \mathrm{Cov}[\bar{E}_1[v], v], \\ a_1^{***} & \textit{then } \alpha_{P_1} > \alpha_{E_1} \textit{ and } \mathrm{Cov}[p_1, v] > \mathrm{Cov}[\bar{E}_1[v], v]. \end{cases}$$

2. If $\beta = 0$, then $\alpha_{P_1} < \alpha_{E_1}$ and $\mathrm{Cov}[p_1, v] < \mathrm{Cov}[\bar{E}_1[v], v]$.

If liquidity trading is persistent then, along the HIE, investors escalate their response to private information. In this case the extra weight added to $\bar{\alpha}_{E_1}$ (see (11)) is large enough to draw the price closer to fundamentals (than is the consensus), in contrast with Allen et al. (2006). In view of the results obtained in Section III.D, this equilibrium is stable provided that private information is not too much more precise than the exogenous public signal. Along the LIE, the price is farther away from fundamentals compared to consensus. This equilibrium, which shares the same properties of the one found by Allen et al. (2006), is always stable.

REMARK 5: *According to Numerical Result 1 and Corollary 2.1, the inventory component of liquidity is larger in the LIE than in the HIE. That difference suggests an alternative interpretation of Corollary 2.4: When prices are farther away from (resp., closer to) fundamentals as compared with the consensus, inventory risk is high (resp., low). From an empirical standpoint, the implication is that the inventory component of liquidity is increasing in the extent of the difference between how accurately fundamentals are assessed by asset prices versus the consensus.*

F Robustness

In this section we perform some robustness exercises. First, we extend our model to encompass the possibility that residual uncertainty affects the asset payoff. Second, we show that a very similar pattern of equilibrium multiplicity arises in a model with long-term traders who face residual uncertainty about the final payoff. (Our analysis of both these extensions is given in the Online Appendix.) Finally, we discuss the effect of extending the number of trading rounds and allowing investors to receive more than one private signal.

F.1 The effect of residual uncertainty

Assume that investors face residual uncertainty over the final liquidation value. That final payoff is written as $\hat{v} = v + \delta$, where $\delta \sim N(0, \tau_\delta^{-1})$ is a random term orthogonal to all random variables in the market and about which no investor is informed. Our inclusion of the random term δ allows one to study the effect of an increase in the residual uncertainty that characterizes the investing environment in periods of heightened turbulence. It is intuitive that, when investors face residual uncertainty, they put less weight on their own signals because prices and private information are less useful in predicting the asset payoff. This dynamic is likely to weaken the retrospective inference and variance reduction loop and may even eliminate the HIE. Yet our analysis shows that, in general, residual uncertainty neither eliminates the HIE nor makes it unstable.

Even when there is residual uncertainty, the expressions for prices and investors' strategies do not change (i.e., the expressions (14), (25), and (26) still hold). However, the equilibrium obtains as the solution of a system of two highly nonlinear equations and is therefore more difficult to solve. Numerical analysis establishes that: (i) an equilibrium akin to the LIE always arises; (ii) for low values of $1/\tau_\delta$ there can be as many as five equilibria, and at least one of these will be a HIE; (iii) if residual uncertainty is high then a unique equilibrium obtains and only LIE-type equilibria survive. In our baseline simulation, we set $\tau_v = 1$ and find that, for $\tau_\delta < 40$, the HIE vanishes. Although at first blush a small level of residual uncertainty (e.g., $\sigma_\delta^2 \leq 1/40$) may seem to cast doubt on the HIE's relevance, it is possible to show that this parameterization is in line with calibrated asset pricing models. For instance, Wang (1994) models the asset payoff as a dividend process $F_{t+1} = \rho F_t + \omega_{t+1}$, where F_t is a persistent component and ω_{t+1} an orthogonal random error term that corresponds to our residual uncertainty term. Here the coefficient ρ parameterizes the effect of past fundamentals on current ones. In this framework, the fraction of variance coming from residual uncertainty is $1 - \rho^2$ (i.e., the ratio of σ_ω^2 to the steady-state variance of F_{t+1} or $\sigma_\omega^2/(1-\rho^2)$), which in our framework corresponds to $\sigma_\delta^2/(\sigma_\delta^2 + \sigma_v^2)$. It is easy to show that, for $\rho \in (.98, .99)$, residual uncertainty in the dynamic model is of comparable importance to that implied by the parameter $\tau_\delta = 40$. So if, for example, we take our model to represent trading patterns that occur at a quarterly frequency, then the previous statement implies (roughly) a critical value of $\rho = .95$ at a yearly frequency; that value is commonly used for calibration in asset pricing models.[35]

F.2 Long-term investors

Consider again the market with residual uncertainty, but now suppose that investors have a long horizon and so maximize the expected utility of their final wealth. For simplicity we will address the case in which the public signal is useless ($\tau_\eta = 0$). We find that multiple equilibria are possible in this scenario as well, and the reasons are similar to those given for the case of short-term investors.

A long-term investor in the first period speculates on short-term returns and takes into account the hedging possibilities of second-period trading. The equilibrium strategy of investor i in the first period is actually a linear combination of $(E_{i1}[p_2]-p_1)$ and $E_{i1}[x_{i2}]$ (Cespa and Vives (2012)).[36] If traders do *not* expect prices to change, then their optimal period-1 position would be just as in a static market. The risk of holding such a position would be due only to the liquidation value's unpredictability.[37]

However, if a change in prices *is* expected then traders optimally exploit short-run price differences. Two factors increase the risk of their period-1 position: the partial unpredictability of the price change; and the impossibility of determining their exact future position. Nevertheless, the opportunity to trade again in the future does serve as a hedge against potentially adverse price movements. In equilibrium, this option reduces risk and thereby—in the *absence* of residual uncertainty—exactly offsets the price risk conditional on private information. So when there is *no* residual uncertainty, traders' strategies incorporate static responses to private information and there is a unique equilibrium. In this case, the optimal strategy of an informed trader is static (i.e., buy and hold): in the first period, informed traders receive their private signal and take a position; in the second period, there is no informed trading. Although investors may nonetheless speculate on price differences, they do so only for market-making purposes (i.e., to profit from the mean reversion of liquidity trading). When there is residual uncertainty, investors in the second period scale down their trading activity as the final payoff becomes harder to forecast. This weakens the hedging effect of the re-trading opportunity and renders strategies truly dynamic, leading investors to speculate on short-term price movements based on their private information.[38] These consequences reinforce the retrospective inference and variance reduction loop and lead to multiple equilibria, since the market's liquidity in the second period strongly affects a trader's reaction to private information in the first period.

In summary, with long-term and risk-averse investors there may be multiple equilibria when there is either residual uncertainty (He and Wang (1995), Cespa and Vives (2012)) or a common shock to private signals (Grundy and McNichols (1989)). We may have situations then with a negative price impact in the second period. This arises because in those cases informed traders have incentives to use their private information to speculate on short-term price movements and long-term traders may behave as short-term ones.

F.3 More private signals, more trading rounds

The model can be extended to encompass the possibility that investors trade for more than two periods and/or receive additional private signals during each round of trading. However, the analysis becomes more complicated without any effect on the qualitative results. In particular, we can still show that multiple equilibria (with the stated properties) arise provided $\beta > 0$.

IV Asset pricing implications

In this section we investigate the asset pricing implications of our analysis. First, we show that liquidity trading persistence can generate positive autocovariance of returns irrespective of whether beliefs are heterogeneous (as in Banerjee, Kaniel, and Kremer (2009)) or whether investors' preferences exhibit a behavioral bias (as in Daniel, Hirshleifer, and Subrahmanyam (1998)). Second, we look at the expected volume of informational trading and then compare the model's predicitons with predictions under "differences of opinion" models. Third, we discuss identification problems in models with multiple equilibria and derive a set of empirical implications that allow one to distinguish the HIE from the LIE. Finally, we show how our model can provide a narrative of some episodes related to the recent financial crisis.

A Return autocovariance

We start by computing the return autocovariance at different horizons.

COROLLARY 2.5: *Suppose $\tau_\eta > 0$. At equilibrium, the following statements hold.*

(i) *For all $\beta \in [0, 1]$,* $\mathrm{Cov}[p_2 - p_1, p_1 - \bar{v}] < 0$.

(ii) *For $\beta \in (0, 1]$,* $\mathrm{Cov}[v - p_2, p_1 - \bar{v}] < 0$; *for $\beta = 0$,* $\mathrm{Cov}[v - p_2, p_1 - \bar{v}] = 0$.

(iii) *For $\beta \in (0, 1]$, if there are multiple equilibria then—along the HIE—we have* $\mathrm{Cov}[v - p_2, p_2 - p_1] > 0$. *If $\beta = 0$, then* $\mathrm{Cov}[v - p_2, p_2 - p_1] < 0$.

This result states that, along the HIE, momentum always occurs at short horizons (i.e., near the end of the trading horizon) whereas the reversal of returns occurs at long horizons.[39]

Parts (i) and (ii) of the corollary hold because a given estimated first-period imbalance $E_1[\theta_1]$ has the opposite effect on $p_1 - \bar{v}$ as it does on both $p_2 - p_1$ and $v - p_2$.[40] For part (iii), a covariance decomposition (and the normality of returns) yields:

$$\begin{aligned}
&\mathrm{Cov}[v - p_2, p_2 - p_1] \\
&\qquad = \mathrm{Cov}[E_1[v - p_2], E_1[p_2 - p_1]] + \mathrm{Cov}_1[v - p_2, p_2 - p_1] \\
&\qquad = \beta\Lambda_2 \frac{\mathrm{Var}_{i1}[p_2]}{\gamma} \mathrm{Var}[E_1[\theta_1]] + \frac{(1 + \gamma\tau_u\Delta a_2)(\beta a_1 \Delta a_2 \tau_u - \tau_1) + \gamma\tau_\eta\tau_u\beta a_1}{(\gamma\tau_{i2})^2\tau_1\tau_u}.
\end{aligned} \tag{31}$$

The first term in this decomposition captures the covariation in forecasts of conditional returns, $E_1[v - p_2]$ and $E_1[p_2 - p_1]$—in other words, the covariance "explained" by p_1. The second term captures the conditional covariation of returns, the "residual" covariance. All else equal, if trading is persistent then the anticipated effect of the first-period imbalance on the second- and third-period expected returns is of the same sign; hence the first term is always positive when $\beta > 0$. Suppose that investors in the first period estimate a selling pressure from liquidity traders. If $p_1 < p_2$ then the outcome $p_2 < v$ is more likely than is $p_2 \geq v$ because liquidity traders' sales of the asset are likely to persist in the second period.[41] For the second term, factoring out the effect of first-period information suggests that the joint covariation of returns around their expectations could be driven either by liquidity trading or by fundamentals information. In the HIE, prices are driven by informed traders; hence the second effect predominates and there is positive covariance of returns around their means.[42] Conversely, prices in the LIE are more driven by liquidity trades and so returns tend to covary around their means in opposite directions.

When $\tau_\eta = 0$, all of the results obtained in Corollary 2.5 continue to hold. We can also prove that, for β sufficiently high, momentum occurs at short horizons also along the LIE (numerical simulations confirm this result for the case $\tau_\eta > 0$). Formally, we have the following statement.

COROLLARY 2.6: *Suppose $\tau_\eta = 0$. Then, along the LIE, for $\tau_v < \hat{\tau}_v$ there exists a value $\hat{\beta}$ such that for all $\beta > \hat{\beta}$,* $\mathrm{Cov}[v - p_2, p_2 - p_1] > 0$. (The expression for $\hat{\tau}_v$ is given in the Appendix; see (A.28).)

Along the LIE, momentum is a sign of strong liquidity trading persistence and is due to the effect of the covariance explained by p_1 in (31). This finding is consistent with prices in that equilibrium being driven by liquidity trades, so here the predictability of returns is a sign of poor informational efficiency. Indeed, it is possible to show that, for β sufficiently large, momentum arises also in a model with no private information.[43] Along the HIE, however, momentum occurs for *any* value of $\beta \in (0, 1]$. This means that even though (a very mild) persistence is required, momentum in this case is *not* due to liquidity trading. To the contrary, the HIE properties illustrated in Proposition 2 suggest that momentum here is rather a sign of rapid price convergence to the fundamentals.

According to the "time series momentum" evidence, which spans a vast class of financial instruments, the lagged 12-month excess return on a given asset is a good predictor of that same asset's "one year ahead" return. In addition, Moskowitz, Ooi, and Pedersen (2012) document the following patterns associated with time-series momentum: (i) hedgers (who in our setup can be proxied by liquidity traders) have stable positions for extended time periods and so induce a persistent price pressure, yielding continuation of returns; (ii) speculators benefit from time-series momentum by going long in an asset to exploit its anticipated price trend (at the expense of hedgers); (iii) the spot price of the asset underlying the futures reacts slowly to information. These patterns are consistent with behavior along the LIE. Indeed, along the LIE, the covariance of the long- and short-term components of investors' demand with liquidity traders' demand has the same (negative) sign; see equations (29a) and (29b). The implication is that investors fully exploit the predictability of liquidity trades—while mildly offsetting their price impact with their information so that prices are driven by liquidity trades. Under these conditions, it pays to exploit liquidity traders' predictability *precisely because* so little of the information about fundamentals affects aggregate orders.

B Expected volume and return predictability

We now address the implications of our results for the expected volume of informational trading.

We start by computing the expected traded volume in the market with heterogeneous information *net of* the expected volume when there is no private information.[44] We have

$$\begin{aligned} V_1 &\equiv \int_0^1 E[|X_1(s_i,p_1)|]\,di - \int_0^1 E[|X_1(p_1)|]\,di \\ &= \int_0^1 \sqrt{\frac{2}{\pi}\mathrm{Var}[X_1(s_i,p_1)]}\,di - \int_0^1 \sqrt{\frac{2}{\pi}\mathrm{Var}[X_1(p_1)]}\,di \\ &= \sqrt{\frac{2}{\pi}}\left(\sqrt{a_1^2\tau_\varepsilon^{-1}+\tau_u^{-1}} - \sqrt{\tau_u^{-1}}\right) \end{aligned} \tag{32}$$

and

$$\begin{aligned} V_2 &\equiv \int_0^1 E[|X_2(s_i,p_1,p_2) - X_1(s_i,p_1)|]\,di - \int_0^1 E[|X_2(p_1,p_2) - X_1(p_1)|]\,di \\ &= \int_0^1 \sqrt{\frac{2}{\pi}\mathrm{Var}[\,X_2(s_i,p_1,p_2) - X_1(s_i,p_1)]}\,di - \int_0^1 \sqrt{\frac{2}{\pi}\mathrm{Var}[\,X_2(p_1,p_2) - X_1(p_1)]}\,di \\ &= \sqrt{\frac{2}{\pi}}\left(\sqrt{(a_1-a_2)^2\tau_\varepsilon^{-1} + (1+(\beta-1)^2)\tau_u^{-1}} - \sqrt{(1+(\beta-1)^2)\tau_u^{-1}}\right). \end{aligned} \tag{33}$$

Using $V_1 + V_2$ to measure the total volume of informational trading, we obtain the following result.

COROLLARY 2.7 (Expected volume of informational trading): *Suppose* $\tau_\eta > 0$. *Then, at equilibrium, for all* $\beta \in (0,1]$ *the expected total volume of informational trading is higher along the HIE. When* $\beta = 0$, *only the equilibrium with a low total volume of informational trading survives.*

PROOF. Rearranging Corollary 2.3's expressions for investor strategies yields $x_{in} = a_n\varepsilon_{in} - \theta_n$ for $n = 1, 2$. For a normally distributed random variable Y, we have

$$E[|Y|] = \sqrt{\frac{2}{\pi}\mathrm{Var}[Y]}.$$

Since $a_1^{***} > a_1^*$, it now follows from (32) that V_1 is larger along the HIE. According to (33), $V_1 + V_2$ is an increasing function of a_1 for $a_1 > a_2$, a condition that is satisfied along the HIE. Finally, by Corollary 2, if $\beta = 0$ then $a_1^* < a_2$. □

The intuition for Corollary 2.7 is straightforward: because investors in the HIE step up their response to private signals, the position change due to private information is higher along such equilibrium. Taken together, Corollaries 2.5 and 2.7 imply that a high volume of informational trading in the second period predicts a continuation of returns *regardless* of how persistent liquidity trading is—a conclusion that accords with the evidence presented by Llorente et al. (2002). Yet a low volume of informational trading can also be associated with momentum as long as liquidity trading is persistent enough. However, momentum in this case signals slow price convergence to the liquidation value. In short, momentum is compatible with both a high volume and a low volume of informational trading, but the implications of continued returns for price informativeness are markedly different in the two situations.

The extant literature typically associates volume realizations with investors' divergence of opinions about the asset payoff (see, e.g, Kandel and Pearson (1995)). In our setup, disagreement is measured as follows:

$$\text{Disag} = \text{Var}\big[E_{in}[v] - \bar{E}_n[v]\big] = \text{Var}[\alpha_{E_n}\varepsilon_i] = \frac{\tau_\varepsilon}{\tau_{in}^2}. \tag{34}$$

From this expression and Proposition 2, we conclude that disagreement is low (high) along the HIE (LIE). This statement and Corollary 2.7 now imply the following.

COROLLARY 2.8: *Total volume is high (resp., low) in the equilibrium with low (resp., high) levels of disagreement.*

This corollary accommodates recent empirical evidence on the relationship between the convergence of opinions and the trading volume around earnings announcements. Indeed, Giannini, Irvine, and Shu (2013) find that large volume is actually compatible with convergence of opinions—a result that runs counter to what most of the literature on transaction volume implies.

Their finding can be explained as follows. From expressions (32) and (33) it is clear that volume is increasing in a_1^2/τ_ε. We now use (19) and obtain

$$\frac{a_1^2}{\tau_\varepsilon} = \left(\gamma\frac{\lambda_2\Delta a_2}{\text{Var}_{i1}[p_2]}\right)^2 \text{Var}[\alpha_{E_1}\varepsilon_i]. \tag{35}$$

The first component on the right-hand side of equation (35) captures the effect of investors' perceived risk on volume; the second term coincides with our definition of disagreement. Along the HIE (LIE), we know that disagreement is low (high) but we also know that investors face little (considerable) risk concerning the price at which their positions unwind. The latter effect prevails in our setup, which implies that high trading volume is a good proxy for *low* disagreement and *low* perceived risk. Our model predicts in addition that, when volume is high and opinions are converging, prices should serve as a reliable signal of the underlying fundamentals.

REMARK 6: *Up to know we have compared volume, disagreement and volatility across equilibria. We can also explore how these magnitudes change within an equilibrium as one deep parameter of the model changes. Our simulations show that the relationship between volume, disagreement, and volatility as the precision of private information* τ_ε *changes is quite rich. In particular, along the LIE we find that an increase in* τ_ε *leads to higher total volume and lower conditional volatility. In some simulations this effect is due to the increase in the first period expected volume (typically, with a low liquidity trading persistence, second period expected volume is positively related to the conditional volatility of returns). We also find that when liquidity trading is highly persistent (*β *close to 1), expected volume is in both periods negatively related to the conditional volatility of returns. At both dates, and as it should be expected, disagreement is hump-shaped in the precision of private signals: it is close to* 0 *for* τ_ε *close to* 0*, increases for small values of* τ_ε*, and finally converges back to* 0 *for* τ_ε *large. First period volume is increasing in* τ_ε*, while second period volume can be hump-shaped (for* β *low) or increasing in* τ_ε *(for* β *high). The volatility of returns (conditional and unconditional) is instead always decreasing in* τ_ε*. It is possible, for example, that volume in period* 2 *increases while conditional volatility decreases, and that volume in period* 1 *increases while disagreement decreases.*

Finally, our findings on volume are also related to Banerjee (2011) and Kondor (2012). The former paper compares differences of opinion (DO) models to rational expectations (RE) models. One prediction is that in a DO model, differently from a RE one, the conditional volatility of returns is negatively related to expected volume. This relation also occurs in our RE setup with strategic complementarities, even though investors agree on a common prior, in two distinct scenarios. First, we compare the patterns of volume and volatility across the HIE and the LIE. In this case, coupling Corollary 2.7 and Numerical Result 1, implies that when the economy starts at the HIE a switch from the HIE to the LIE, e.g. because of a small increase in public signal precision, moves the market from a combination of high total volume and low conditional volatility, to one with low total volume and high conditional volatility. Second, we study the patterns of total volume and volatility along the LIE as private signal precision increases. As we have seen in the remark above we can have first and even second period volume increasing while conditional volatility decreases as private precision increases. Hence, we conclude that observing a negative correlation between trading volume and the conditional volatility of returns is insufficient grounds for rejecting a RE model. We have also seen that, along the LIE, a more volatile market can be associated with more or less disagreement, depending on the value of private precision. The potential positive relationship between disagreement and volatility is consistent with empirical evidence in Gallant et al. (1992) and Kandel and Pearson (1995).[45] Kondor (2012) studies a two-period trading model with three-factor fundamentals and short-term traders who have factor-specific information. In his model, a public signal on the fundamentals generates disagreement because it leads traders to compare their factor-specific private information to the public signal and thereby gauge the magnitude of the factor about which they have no private information. The author shows that an increase in the informativeness of public signals can, by facilitating disagreement, generate large volume as well as prices that are not only more informative but also more volatile (conditional on the public signal). Our model can produce similar patterns for price volatility.[46] However, in our setup the association between a burst in trades and higher price volatility results from a public signal that is precise enough to stabilize the HIE.[47]

C Stock price fragility

According to Greenwood and Thesmar (2011), a *fragile asset* is one that is vulnerable to non-fundamental demand shocks. These authors find that ownership concentration increases asset fragility but also that the active trades of hedge funds and mutual funds can mitigate fragility by counteracting the effect of mutual funds' flow-induced trades. The idea is that if the mispricing induced by non-fundamental demand shocks becomes large enough, then hedge funds will step in to correct it by taking offsetting positions. Along the HIE, this hypothesis is in line with the mechanism outlined in (29a) and (29b). Greenwood and Thesmar (2011) find that the distribution of offsetting trades is extremely heterogeneous across stocks, with some stocks characterized by strong offsetting effects and with other stocks for which this effect is relatively weak. Our analysis implies that the former stocks should be those for which the HIE is likely to prevail and hence should be associated with the combination of high trading volume and converging opinions. Conversely, we predict that stocks associated with low volume and diverging opinions should be more fragile.

D Identifying equilibria

We know from Jovanovic (1989) that models governed by multiple equilibria of structural parameters have severe identification problems and also that "the set of distributions on observable outcomes that are consistent with a given structure can be quite large." Positive identification results are obtained in a range of papers that use simple models with multiple equilibria (e.g., discrete entry or binary games; see the accounts in Ackerberg et al. (2007) and Berry and Tamer (2006)). Identification of structural parameters is achieved through equilibrium refinements, shape restrictions, informational assumptions, or the specification of equilibrium selection mechanisms. Alternatively, inference can be based on the identified features of the models with multiple equilibria, which are sets of values of the structural parameter vector (see, e.g., Ciliberto and Tamer (2009) and the refinement in Henry and Galichon (2011)).

In the model developed here, with two stable equilibria there is no equilibrium refinement or equilibrium selection mechanism to be used. Given the presence of strategic complementarities in our model, a promising approach is the one of Echenique and Komunjer (2009) based on monotone comparative statics (MCS) results. Unfortunately, the type of multiplicity that arises in our paper fails to satisfy MCS, which this literature requires for identification (see the Online Appendix).

Equilibria can nonetheless be identified based on their respective implications for specific market observables. According to Proposition 2, the inference component of the spread is negative along the HIE and positive along the LIE. At the same time, by Numerical Result 1 and Corollaries 2.7 and 2.8 we know that, in the HIE, the conditional volatility of returns is low, trading volume is high, and these patterns occur with low disagreement. Finally, Corollary 2.5 implies that in the HIE there is mild positive autocorrelation of returns at short horizons.

More insights on identifying the specific features of each equilibrium can be obtained by contrasting the qualitative properties of the HIE and LIE via numerical simulations. In what follows we fix parameters' values that yield a stable HIE, and positive return autocovariance at the LIE and at the equilibrium with no private information (i.e., $\tau_\varepsilon = 0$). We then extract 1,000 i.i.d. normal shocks for v, u_1, u_2, and η, and average across the prices and shocks. The results of this exercise are presented in Figure 5, where we plot the positions of liquidity traders at dates 1 and 2 followed by the simulated price paths along the low-information equilibrium, the high-information equilibrium, and the equilibrium with no private information.[48] Our results further show that compared to the HIE, along the LIE (i) short term returns are more strongly positively autocorrelated as well as more easily predictable based on order flow information, and (ii) the expected returns from liquidity provision investors obtain at both trading dates are higher.[49]

[Figure 5 about here.]

Panel (a) in the figure displays the position of liquidity traders, which in this simulation is (on average) positive at both dates. According to panel (b), if there is no private information then prices mirror liquidity traders' demand. Indeed, since short-term traders are risk averse, they require compensation—for satisfying the positive demand of liquidity traders—that is proportional to their perceived uncertainty about the payoff. That compensation drives p_1 above $\bar{v}$ in this equilibrium. In panel (c) of Figure 5 we see that prices display a qualitatively similar behavior along the LIE. Yet because traders are informed, $a_n > 0$ and so part of liquidity traders' shock is accommodated by offsetting speculative orders (in this numerical example, sell orders). These orders transmit information and thereby diminish investors' perceived uncertainty about the payoff, which implies that the price adjustment needed to accommodate θ_n is lower than in the $\tau_\varepsilon = 0$ case. In the LIE, then, the price path reflects the liquidity traders' position and is therefore hump-shaped. Along the HIE (panel (d)), in contrast, the first-period price coincides almost exactly with the semi-strong efficient price and with the full-information value. In this equilibrium, traders aggressively speculate (sell) against the liquidity (buy) shock θ_n based on their private information. This behavior drives p_1 below $\bar{v}$ and close to v, accelerating price adjustment. These trades are highly informative and thus dramatically reduce investors' perceived uncertainty about the payoff, which explains why the price nearly matches $E_n[v]$ even though the market's risk-bearing capacity is limited ($\gamma < \infty$). In this equilibrium, short-term trading offsets the impact of liquidity traders' orders on prices, and the price path is inversely hump-shaped.

Combining these observations with Proposition 2 and our previous results in Section IV suggests a way to identify the HIE and the LIE from the data. Our model predicts that the LIE arises for *extreme* values of public signal precision (which could be proxied, e.g., by the number of analysts following a given security). This equilibrium is characterized by (i) a positive inference component of the price impact, (ii) momentum or reversal depending on the strength of trading persistence, (iii) high expected returns from liquidity provision, (iv) prices that are far from the semi-strong efficient price, (v) high short-horizon return predictability from order imbalances, and (vi) low volume accompanied by high levels of disagreement. Our model predicts that the HIE may arise for *intermediate* values of public signal precision. This equilibrium is characterized by (i) a negative inference component of the price impact, (ii) mild momentum, (iii) low expected returns from liquidity provision, (iv) prices that are close to the semi-strong efficient price,

(v) low short-horizon return predictability from order imbalances, and (vi) high volume accompanied by low levels of disagreement. See Table I, which summarizes the model's empirical implications.

[Table 1 about here.]

The equilibrium predictions of Table 2 help us when using market data to distinguish among equilibria and also when seeking to discriminate among different behavioral theories. The main implications of our findings can be listed as follows.

1. High volume associated with informative prices and low disagreement argues for the HIE in relation not only to the LIE but also to alternative theories based on differences of opinion and in which disagreement is associated with high volume.

2. A negative covariance between conditional volatility and volume need not disqualify RE models (as suggested by Banerjee (2011), for example) because that finding is consistent with our results in some scenarios.

3. A negative inference component of the price impact identifies the HIE and the occurrence of return reversals at short horizons identifies the LIE, with the maintained hypothesis that our model holds.

4. If for fragile (non-fragile) stocks—in the sense of Greenwood and Thesmar (2011)—there is a low (high) volume transacted when there is high (low) disagreement, then that would constitute evidence for our LIE (HIE). In other words, fragility should be associated with low trade volume and divergence of opinions.

E Meltdowns

Finally, our model can also shed some light on episodes of sudden liquidity dry-ups as exemplified in severe form by the recent financial crisis. We offer an information-based explanation that complements the standard one given in terms of insufficient arbitrage capital.

The quant meltdown of August 2007. In the second week of August 2007, several hedge funds started unwinding their holdings (arguably for non-informational reasons). Khandani and Lo (2011) show that the price impact of trades spiked during the event.

Their conclusion is that a lack of arbitrage capital (together with the increased importance of high-frequency trading for market making) was largely responsible for the meltdown.

In our model, a large increase in price impact is consistent with an increase in the volume or volatility of liquidity trading and a switch from the HIE to the LIE (since then the HIE may disappear; see Figure 2(d)). In fact, the LIE prevails also when the demand of liquidity traders becomes larger and more volatile. In that case, the retrospective inference and variance reduction loop weakens and the high-liquidity equilibrium disappears. The alternative view is that even if capital had been abundant (with CARA utilities there is no room for endowment effects), a similar meltdown could have occurred if informational conditions were like those that we find.

The financial crisis and public information. Several authors have argued that, during the 2007–2008 crisis, the selling pressure of investors drove asset values downward and below the fundamentals (though this was followed by a rebound; Cella, Ellul, and Giannetti (2013)). A possible reason for the occurence of such large corrections is the lack of (or "slow moving") arbitrage capital (Duffie (2010)), which exhausted the risk-bearing capacity of liquidity suppliers. Our theory provides an alternative explanation based on the absence of informational conditions that would have allowed for a milder correction. In particular, the dearth of reliable public information (proxied here by a steep reduction in τ_η) may have reduced the market's risk-bearing capacity, relegating most of the economy to the LIE.[50] As argued in the Online Appendix, if the market coordinates at a HIE that poor public information has rendered unstable, then an additional mild shock to public information leads this market to the LIE. Indirect evidence of such a transition is that "contrarian" liquidity providing strategies were extremely profitable during the financial crisis (as documented by Nagel (2012))—in line with our prediction of the LIE derived in Section IV.D. With regard thereto our paper makes the additional prediction that, in the cross section, the assets that underwent the most extreme corrections were those for which the public information was poorest.[51]

V Conclusions

In this paper we argue that the persistence in liquidity traders' positions has a significant effect on the response of risk-averse, short-term investors to their private signals. When the orders of liquidity traders are correlated across trading dates, investors reassess the evidence (about the fundamentals) obtained at the early trading stage based on the new information gathered in the market. Such "retrospective" inference can generate strategic complementarities in the use of private information that in turn can yield multiple stable equilibria, and these equilibria can be ranked in terms of price informativeness, liquidity, and volatility.

Our analysis reveals that, if uninformed orders are predictable, then the effect of investors' short horizons on market observables depends on the quality of public information. When public information is not much more precise than private signals, the retrospective inference channel is not too strong; in this case, a *stable* equilibrium arises that is characterized by low volatility, high liquidity, high price informativeness, high volume, and low levels of disagreement. This equilibrium exists alongside another equilibrium in which prices are more volatile and less informationally efficient, the market is thinner, volume is low, and disagreement is high. When public information is either very precise or very poor, the low-volatility equilibrium disappears or becomes unstable while the high-volatility equilibrium survives. Thus our analysis indicates that there could be a nonlinear effect of public information on market observables. Furthermore, our results can guide the empirical literature investigating the effect of investors' horizons on market patterns as a means of identifying the stock characteristics associated with the high- or low-volatility equilibrium.

Our paper also clarifies the role of higher-order expectations in asset pricing. With liquidity trading persistence, prices are driven by average expectations about fundamentals and about liquidity trading. This dynamic, in contrast to the Beauty Contest results of Allen et al. (2006), can draw prices either systematically farther away from or closer to fundamentals—along the LIE and the HIE, respectively—as compared with the consensus of investors. We show that when public information is either very precise (compared with private signals) or very poor, prices are farther away from fundamentals compared to consensus. However, a public signal of intermediate precision makes the HIE stable, thereby drawing prices closer (than consensus) to the fundamentals. We also link the HIE and LIE to the magnitude of the inventory component of liquidity, to the price impact, and to the returns from liquidity supply. Thus, our analysis establishes the limits of the Beauty Contest analogy for financial markets and provides empirical implications to assess the effect of HOEs on asset prices.

Table 2 summarizes the empirical implications of our model. We provide the observables that can discriminate among equilibria (under our model's hypothesis) and among behavioral theories as well. High trading volume associated with low conditonal volatility and low disagreement discriminates in favor of our HIE against not only the LIE but also alternative, DO theories wherein disagreement is associated with high volume. Furthermore, a negative covariance between conditional volatility and volume does not preclude the validity of rational expectations models because it is consistent with our model. We find also that fragility (in the sense of Greenwood and Thesmar (2011)) should be associated with low transacted volume and divergence of opinions. In addition, our paper provides an alternative interpretation for empirically documented regularities in the patterns of return autocorrelation. The literature has only recently begun to investigate the relationship between empirical regularities, such as the momentum effect, and the role of HOEs in asset prices (see, e.g., Verardo (2009)). Our paper offers empirical predictions in this regard, uncovering the existence of two types of momentum with very different informational properties. These findings can guide further research in the empirical analysis of asset pricing anomalies.

Finally, our results have implications for the forces behind market meltdowns and enable us to offer—as an alternative to explanations based on limits to arbitrage capital—an accounting for the financial crisis in terms of a transition from the high- to the low-information equilibrium in response to sudden changes in the volume of liquidity trading or in the precision of public information.

REFERENCES

Ackerberg, Daniel, C. Lanier Benkard, Steven Berry, and Ariel Pakes, 2007, Econometric tools for analyzing market outcomes, in J.J. Heckman, and E.E. Leamer, eds., *Handbook of Econometrics*, volume 6 of *Handbook of Econometrics*, chapter 63 (Elsevier).

Admati, Anat R., 1985, A noisy rational expectations equilibrium for multi-asset securities markets, *Econometrica* 53, 629–657.

Admati, Anat R., and Paul Pfleiderer, 1988, A theory of intraday patterns: volume and price variability, *Review of Financial Studies* 1, 3–40.

Albagli, Elias, 2011, Amplification of uncertainty in illiquid markets, Working Paper.

Allen, Franklin, Stephen Morris, and Hyun Song Shin, 2006, Beauty contests and iterated expectations in asset markets, *Review of Financial Studies* 19, 719–752.

Avdis, Efstathios, 2012, Information trade-offs in dynamic financial markets, Working Paper.

Bacchetta, Philippe, and Eric van Wincoop, 2006, Can information heterogeneity explain the exchange rate determination puzzle?, *American Economic Review* 96, 552–576.

Bacchetta, Philippe, and Eric van Wincoop, 2008, Higher order expectations in asset pricing, *Journal of Money, Credit and Banking* 40, 837–866.

Banerjee, Snehal, 2011, Learning from prices and the dispersion in beliefs, *Review of Financial Studies* 24, 3025–3068.

Banerjee, Snehal, Ron Kaniel, and Ilan Kremer, 2009, Price drift as an outcome of differences in higher order beliefs, *Review of Financial Studies* 22, 3707–3734.

Barlevy, Gadi, and Pietro Veronesi, 2003, Rational panics and stock market crashes, *Journal of Economic Theory* 110, 234–263.

Berry, Steven, and Elie Tamer, 2006, Identification in Models of Oligopoly Entry, in Richard Blundelll, Whitney Newey and Torsten Persson eds.: *Advances in Economics and Econometrics* (Cambridge University Press).

Biais, Bruno, Peter Bossaerts, and Chester Spatt, 2010, Equilibrium asset pricing and portfolio choice under asymmetric information, *Review of Financial Studies* 23, 1503–1543.

Bond, Philip, Itay Goldstein, and Edward Simpson Prescott, 2010, Market-based corrective actions, *Review of Financial Studies* 23, 781–820.

Brown, David P., and Robert H. Jennings, 1989, On technical analysis, *Review of Financial Studies* 2, 527–551.

Campbell, John Y, and Albert S Kyle, 1993, Smart money, noise trading and stock price behaviour, *Review of Economic Studies* 60, 1–34.

Cella, Cristina, Andrew Ellul, and Mariassunta Giannetti, 2013, Investors' horizons and the amplification of market shocks, *Review of Financial Studies* 26, 1607–1648.

Cespa, Giovanni, 2002, Short-term investment and equilibrium multiplicity, *European Economic Review* 46, 1645–1670.

Cespa, Giovanni, 2005, Giffen goods and market making, *Economic Theory* 25, 983–997.

Cespa, Giovanni, and Xavier Vives, 2012, Dynamic trading and asset prices: Keynes vs. Hayek, *Review of Economic Studies* 79, 539–580.

Cespa, Giovanni, and Xavier Vives, 2014, The welfare impact of high frequency trading, Working Paper.

Chen, Qi, Zeqiong Huang, and Yun Zhang, 2012, The effects of public information with asymmetrically informed short-horizon investors, Working Paper (http://ssrn.com/abstract=1944328).

Chevalier, Judith, and Glenn Ellison, 1997, Risk taking by mutual funds as a response to incentives, *Journal of Political Economy* 105, 1167–1200.

Chordia, Tarun, Richard Roll, and Avanidhar Subrahmanyam, 2008, Liquidity and market efficiency, *Journal of Financial Economics* 87, 249–268.

Chordia, Tarun, and Avanidhar Subrahmanyam, 2004, Order imbalance and individual stock returns: Theory and evidence, *Journal of Financial Economics* 72, 485–518.

Ciliberto, Federico, and Elie Tamer, 2009, Market structure and multiple equilibria in airline markets, *Econometrica* 77, 1791–1828.

Coval, Joshua, and Erik Stafford, 2007, Asset fire sales (and purchases) in equity markets, *Journal of Financial Economics* 86, 479–512.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 1998, Investor psychology and security market under- and overreactions, *Journal of Finance* 53, 1839–1884.

De Long, J Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann, 1990, Noise trader risk in financial markets, *Journal of Political Economy* 98, 703–738.

Dennert, Jürgen, 1991, Insider trading and the cost of capital in a multi-period economy, Financial Market Group Discussion Paper 128, London School of Economics.

Dow, James, 2004, Is liquidity self-fulfilling?, *Journal of Business* 77, 895-908.

Dow, James, Itay Goldstein, and Alexander Guembel, 2011, Incentives for information production in markets where prices affect real investment, Working Paper.

Dow, James, and Gary Gorton, 1994, Arbitrage chains, *Journal of Finance* 49, 819–49.

Duffie, Darrell, 2010, Presidential address: Asset price dynamics with slow-moving capital, *Journal of Finance* 65, 1237–1267.

Easley, David, Soeren Hvidkjaer, and Maureen O'Hara, 2002, Is information risk a determinant of asset returns?, *Journal of Finance* 57, 2185–2221.

Echenique, Federico, and Ivana Komunjer, 2009, Testing models with multiple equilibria by quantile methods, *Econometrica* 77, 1281–1297.

Foucault, Thierry, Marco Pagano, and Ailsa Röell, 2013, *Market Liquidity* (Oxford University Press).

Froot, Kenneth A., David S. Scharfstein, and Jeremy C. Stein, 1992, Herd on the street: Informational inefficiencies in a market with short-term speculation, *Journal of Finance* 47, 1461–1484.

Gallant, A. Ronald, Peter E. Rossi, and George Tauchen, 1992, Stock prices and volume, *Review of Financial Studies* 5, 199–242.

Ganguli, Jayant Vivek, and Liyan Yang, 2009, Complementarities, multiplicity, and supply information, *Journal of the European Economic Association* 7, 90–115.

Giannini, Robert, Paul Irvine, and Tao Shu, 2013, The convergence and divergence of investors' opinions around earnings news: Evidence from a social network, Working Paper.

Goldstein, Itay, Emre Ozdenoren, and Kathy Yuan, 2013, Trading frenzies and their impact on real investment, *Journal of Financial Economics* 109, 566–582.

Goldstein, Itay, and Liyan Yang, 2014, Information diversity and complementarities in trading and information acquisition, *Journal of Finance, forthcoming*.

Gorton, Gary, and Andrew Metrick, 2010, Haircuts, *Federal Reserve Bank of St. Louis Review* 507–517.

Greenwood, Robin, and David Thesmar, 2011, Stock price fragility, *Journal of Financial Economics* 102, 471–490.

Griffin, John M., Jeffrey H. Harris, and Selim Topaloglu, 2003, The dynamics of institutional and individual trading, *The Journal of Finance* 58, 2285–2320.

Gromb, Denis, and Dimitri Vayanos, 2010, Limits of arbitrage: The state of the theory, *Annual Review of Financial Economics* 2, 251–275.

Grossman, Sanford J, and Merton H Miller, 1988, Liquidity and market structure, *Journal of Finance* 43, 617–37.

Grossman, Sanford J., and Joseph E. Stiglitz, 1980, On the impossibility of informationally efficient markets, *American Economic Review* 70, 393–408.

Grundy, Bruce D., and Maureen McNichols, 1989, Trade and the revelation of information through prices and direct disclosure, *Review of Financial Studies* 2, 495–526.

Haldane, Andrew G., and Richard Davies, 2011, The short long, Working Paper, Bank of England.

Hamm, Sophia J. W., 2014, The effect of ETFs on stock liquidity, Working Paper.

He, Hua, and Jiang Wang, 1995, Differential information and dynamic behavior of stock trading volume, *Review of Financial Studies* 8, 919–972.

Henker, Thomas, and Jian-Xin Wang, 2006, On the importance of timing specifications in market microstructure research, *Journal of Financial Markets* 9, 162–179.

Henry, Marc, and Alfred Galichon, 2011, Set identification in models with multiple equilibria, *Review of Economic Studies* 78, 1264–1298.

Holmström, Bengt, and Joan Ricart i Costa, 1986, Managerial incentives and capital management, *Quarterly Journal of Economics* 101, 835–860.

Huang, Roger D, and Hans R Stoll, 1997, The components of the bid-ask spread: a general approach, *Review of Financial Studies* 10, 995–1034.

Jovanovic, Boyan, 1989, Observable implications of models with multiple equilibria, *Econometrica* 57, 1431–1437.

Kandel, Eugene, and Neil D. Pearson, 1995, Differential interpretation of public signals and trade in speculative markets, *Journal of Political Economy* 103, 831–872.

Khandani, Amir E., and Andrew W. Lo, 2011, What happened to the quants in august 2007? evidence from factors and transactions data, *Journal of Financial Markets* 14, 1–46.

Kondor, Peter, 2012, The more we know, the less we agree on the price, *Review of Economic Studies* 79, 1175–1209.

Kyle, Albert, 1985, Continuous auctions and insider trading, *Econometrica* 53, 1315–1336.

Llorente, Guillermo, Roni Michaely, Gideon Saar, and Jiang Wang, 2002, Dynamic volume-return relation of individual stocks, *Review of Financial Studies* 15, 1005–1047.

Lou, Dong, 2012, A flow-based explanation for return predictability, *Review of Financial Studies* 25, 3457–3489.

Madhavan, Ananth, Matthew Richardson, and Mark Roomans, 1997, Why do security prices change? a transaction-level analysis of NYSE stocks, *The Review of Financial Studies* 10, 1035–1064.

Moskowitz, Tobias J., Yao H Ooi, and Lasse H. Pedersen, 2012, Time series momentum, *Journal of Financial Economics* 104, 228–250.

Nagel, Stefan, 2012, Evaporating liquidity, *Review of Financial Studies* 25, 2005–2039.

Nimark, Kristoffer P., 2007, Dynamic higher order expectations, Working Paper.

Ozdenoren, Emre, and Kathy Yuan, 2008, Feedback effects and asset prices, *Journal of Finance* 63, 1939–1975.

Pagano, Marco, 1989, Endogenous market thinness and stock-price volatility, *Review of Economic Studies* 56, 269–288.

Shin, Hyun Song, 2010, *Risk and Liquidity* (Oxford University Press).

Shleifer, Andrei, and Robert Vishny, 1990, Equilibrium short horizons of investors and firms, *American Economic Review* 80, 148–153.

Singleton, Kenneth J., 1987, Asset prices in a time-series model with disparately informed, competitive traders, in W. Barnett, and K. Singleton, eds., *New approaches to monetary economics* (Cambridge University Press).

Sirri, Erik R., and Peter Tufano, 1998, Costly search and mutual fund flows, *Journal of Finance* 53, 1589–1622.

Spiegel, Matthew, 1998, Stock price volatility in a multiple security overlapping generations model, *Review of Financial Studies* 11, 419–447.

Van Ness, Bonnie, Robert A. Van Ness, and Richard S. Warr, 2001, How well do adverse selection components measure adverse selection?, *Financial Management* 30, 5–30.

Vayanos, Dimitri, and Paul Woolley, 2013, An institutional theory of momentum and reversal, *Review of Financial Studies* 26, 1087-1145.

Verardo, Michela, 2009, Heterogeneous beliefs and momentum profits, *Journal of Financial and Quantitative Analysis* 44, 795–822.

Vives, Xavier, 1995, Short-term investment and the informational efficiency of the market, *Review of Financial Studies* 8, 125–160.

Vives, Xavier, 2008, *Information and Learning in Markets: The Impact of Market Microstructure* (Princeton University Press).

Wang, Jiang, 1994, A model of competitive stock trading volume, *Journal of Political Economy* 102, 127–168.

Watanabe, Masahiro, 2008, Price volatility and investor behavior in an overlapping generations model with information asymmetry, *Journal of Finance* 63, 229–272.

Yuan, Kathy, 2005, Asymmetric price movements and borrowing constraints: A rational expectations equilibrium model of crises, contagion, and confusion, *Journal of Finance* 60, 379–411.

Zhang, Jinfan, 2012, Learning, short-horizons, and asset pricing, Working Paper, Yale University.

Appendix A. Appendix

Proof of Proposition 1

To prove our argument, we proceed by backwards induction. In the last trading period traders act as in a static model and owing to CARA and normality we have

$$X_2(s_i, s_P, z_1, z_2) = \gamma \frac{E_{i2}[v] - p_2}{\mathrm{Var}_{i2}[v]}, \tag{A.1}$$

and denoting by $\bar{E}_2[v] \equiv \int_0^1 E_{i2}[v]\, di$,

$$\begin{aligned} p_2 &= \bar{E}_2[v] + \frac{\mathrm{Var}_{i2}[v]}{\gamma}\theta_2 \\ &= \alpha_{P_2}\left(v + \frac{\theta_2}{a_2}\right) + (1 - \alpha_{P_2})\, E_2[v], \end{aligned} \tag{A.2}$$

where

$$a_2 = \gamma \tau_\varepsilon \tag{A.3a}$$

$$\alpha_{P_2} = \alpha_{E_2}. \tag{A.3b}$$

Rearranging (A.2) we obtain

$$\begin{aligned} p_2 &= \frac{\alpha_{P_2}}{a_2}(a_2 v - \beta a_1 v + \beta a_1 v + \theta_2) + (1 - \alpha_{P_2}) E_2[v] \\ &= \left(\underbrace{\frac{\alpha_{P_2}}{a_2} + (1 - \alpha_{P_2}) \frac{\Delta a_2 \tau_u}{\tau_2}}_{\lambda_2} \right) z_2 + \frac{\tau_\eta}{\tau_{i2}} s_P + \frac{\gamma \tau_1 E_1[v] + \beta z_1}{\gamma \tau_{i2}} \\ &= \lambda_2 z_2 + \frac{\tau_\eta}{\tau_{i2}} s_P + \frac{\beta \alpha_{P_2}}{a_2} z_1 + (1 - \alpha_{P_2}) \frac{\tau_1}{\tau_2} E_1[v], \end{aligned} \tag{A.4}$$

which provides an alternative expression for p_2 which separates the impact on second period "news" from the information contained in the first period price and the public signal.

In the first period owing to CARA and normality, an agent i trades according to

$$X_1(s_i, z_1) = \gamma \frac{E_{i1}[p_2] - p_1}{\mathrm{Var}_{i1}[p_2]}, \tag{A.5}$$

where, using (A.4),

$$E_{i1}[p_2] = \left(\lambda_2 \Delta a_2 + \frac{\tau_\eta}{\tau_{i2}}\right) E_{i1}[v] + \frac{\gamma \tau_1 E_1[v] + \beta z_1}{\gamma \tau_{i2}}, \tag{A.6}$$

$$\mathrm{Var}_{i1}[p_2] = \left(\lambda_2 \Delta a_2 + \frac{\tau_\eta}{\tau_{i2}}\right)^2 \frac{1}{\tau_{i1}} + \frac{\lambda_2^2}{\tau_u} + \frac{\tau_\eta}{\tau_{i2}^2}. \tag{A.7}$$

Replacing (A.6) and (A.7) in (A.5) yields

$$\begin{aligned} X_1(s_i, z_1) &= \gamma \frac{(\lambda_2 \Delta a_2 + \tau_\eta/\tau_{i2}) E_{i1}[v]}{\mathrm{Var}_{i1}[p_2]} + \frac{\gamma}{\mathrm{Var}_{i1}[p_2]} \left(\frac{\beta \alpha_{P_2}}{a_2} z_1 + (1-\alpha_{P_2}) \frac{\tau_1}{\tau_2} E_1[v]\right) - \frac{\gamma}{\mathrm{Var}_{i1}[p_2]} p_1 \\ &= a_1 s_i + \frac{a_1 \tau_1}{\tau_\varepsilon} E_1[v] + \frac{\gamma}{\mathrm{Var}_{i1}[p_2]} \left(\frac{\beta \alpha_{P_2}}{a_2} z_1 + (1-\alpha_{P_2}) \frac{\tau_1}{\tau_2} E_1[v]\right) - \frac{\gamma}{\mathrm{Var}_{i1}[p_2]} p_1, \end{aligned}$$

where

$$a_1 = \gamma \frac{(\lambda_2 \Delta a_2 + \tau_\eta/\tau_{i2}) \alpha_{E_1}}{(\lambda_2 \Delta a_2 + \tau_\eta/\tau_{i2})^2/\tau_{i1} + \lambda_2^2/\tau_u + \tau_\eta/\tau_{i2}^2}. \tag{A.8}$$

Imposing market clearing: $x_1 + \theta_1 = 0$, which implies

$$a_1 v + \theta_1 + \frac{a_1 \tau_1}{\tau_\varepsilon} E_1[v] + \frac{\gamma}{\mathrm{Var}_{i1}[p_2]} \left(\frac{\beta \alpha_{P_2}}{a_2} z_1 + (1-\alpha_{P_2}) \frac{\tau_1}{\tau_2} E_1[v]\right) = \frac{\gamma}{\mathrm{Var}_{i1}[p_2]} p_1.$$

Finally, solving for the equilibrium price and collecting terms yields

$$p_1 = \underbrace{a_1 \left(\frac{\mathrm{Var}_{i1}[p_2]}{\gamma} + \frac{\beta \alpha_{P_2}}{a_2}\right)}_{\alpha_{P_1}} \left(v + \frac{\theta_1}{a_1}\right) + (1-\alpha_{P_1}) E_1[v]. \tag{A.9}$$

□

Proof of Corollary 1.1

In the second period, rearranging (A.2), $p_2 = E_2[v] + \Lambda_2 E_2[\theta_2]$, where $\Lambda_2 = \mathrm{Var}_{i2}[v]/\gamma$. In the first period, from (A.9) we have

$$\alpha_{P_1} = a_1 \left(\frac{\mathrm{Var}_{i1}[p_2]}{\gamma} + \beta \frac{\mathrm{Var}_{i2}[v]}{\gamma}\right).$$

By definition of the inventory component obtained in (13), $\Lambda_1 = \alpha_{P_1}/a_1$. This implies

$$\Lambda_1 = \frac{\mathrm{Var}_{i1}[p_2]}{\gamma} + \beta \frac{\mathrm{Var}_{i2}[v]}{\gamma}.$$

$\square$

PROOF OF COROLLARY 1.2

For the second period price, see (A.4). For the first period price, we rearrange (A.9) to obtain

$$p_1 = \left(\underbrace{\frac{\alpha_{P_1}}{a_1} + (1 - \alpha_{P_1}) \frac{a_1 \tau_u}{\tau_1}}_{\lambda_1} \right) z_1 + (1 - \alpha_{P_1}) \frac{\tau_v}{\tau_1} \bar{v}. \tag{A.10}$$

$\square$

PROOF OF PROPOSITION 2

To prove existence it suffices to note that in the first period, the equilibrium responsiveness to private information is defined by the fixed points of the following function

$$\psi(a_1) = \gamma \frac{(\lambda_2 \Delta a_2 + \tau_\eta/\tau_{i2})\alpha_{E_1}}{(\lambda_2 \Delta a_2 + \tau_\eta/\tau_{i2})^2/\tau_{i1} + \lambda_2^2/\tau_u + \tau_\eta/\tau_{i2}^2}. \tag{A.11}$$

By inspection $\phi(a_1) \equiv a_1 - \psi(a_1) = 0$ is a quintic in a_1, and therefore always possesses a real root. Note that at equilibrium $a_1 > 0$, otherwise $\lambda_2 \Delta a_2 > 0$, which in view of (A.11) yields a contradiction.

Suppose that $\beta > 0$. To prove multiplicity we proceed as follows. Note that

$$\phi(0) = -\gamma^2 \tau_\varepsilon \tau_u (a_2 + \gamma(\tau_\eta + a_2^2 \tau_u))(\tau_\varepsilon + \tau_\eta + a_2^2 \tau_u + \tau_v) < 0 \tag{A.12a}$$

$$\phi(a_2) > 0. \tag{A.12b}$$

Therefore, there exists an equilibrium $a_1^* \in (0, a_2)$. Next, evaluating $\phi(\cdot)$ at

$$a_1 = \frac{1 + \gamma \tau_u a_2}{\gamma \beta \tau_u},$$

yields

$$\phi\left(\frac{1 + \gamma \tau_u a_2}{\gamma \beta \tau_u}\right) > 0,$$

while evaluating it at

$$a_1 = \frac{2 + 3\gamma \tau_u a_2}{2\gamma \beta \tau_u},$$

yields

$$\phi\left(\frac{2+3\gamma\tau_u a_2}{2\gamma\beta\tau_u}\right) < 0,$$

provided

$$\beta > \frac{1}{2} \tag{A.13a}$$

$$\gamma^2\tau_\varepsilon\tau_u > \frac{2(2\beta-1)}{3-2\beta} \tag{A.13b}$$

$$\tau_\eta \leq \hat{\tau}_\eta \equiv \frac{(1/4\beta - 3/8)\gamma a_2^3\tau_u^2 + 1/2a_2^2\tau_u(\beta - 1/2)}{1+\gamma a_2\tau_u(3/2-\beta)}. \tag{A.13c}$$

Therefore, provided (A.13a), (A.13b), and (A.13c) are satisfied, a second equilibrium a_1^{**} exists in the interval

$$\left(\frac{1+\gamma\tau_u a_2}{\gamma\beta\tau_u}, \frac{2+3\gamma\tau_u a_2}{2\gamma\beta\tau_u}\right).$$

Given that $\phi(\cdot)$ is a quintic, it must have an odd number of roots, which implies that when (A.13a), (A.13b), and (A.13c) are satisfied at least another equilibrium a_1^{***} must exist in the interval

$$\left(\frac{2+3\gamma\tau_u a_2}{2\gamma\beta\tau_u}, \infty\right).$$

Given the location of the roots we can conclude that $0 < a_1^* < a_2 < a_1^{**} < a_1^{***}$. Furthermore, we have

$$1+\gamma\tau_u\Delta a_2 = \begin{cases} > 0 & \text{for } a_1 = a_1^* \\ < 0 & \text{for } a_1 \in \{a_1^{**}, a_1^{***}\} \end{cases}$$

which implies that $\lambda_2^* > 0$, while $\lambda_2^{**} < 0$, and $\lambda_2^{***} < 0$. Finally, we prove that price informativeness increases across the three equilibria. For τ_1 this is immediate, since it increases in a_1. For τ_2 as one can verify, given that $a_1^* < a_2 < a_1^{**}$ we have $\tau_2^* < \tau_2^{**}$. Furthermore, for $a_2 > (1+\gamma\tau_u a_2)/(\gamma\beta\tau_u)$,

$$\frac{\partial\tau_2}{\partial a_1} > 0,$$

which implies $\tau_2^{**} < \tau_2^{***}$.

Suppose now that $\beta = 0$. Then, $\phi(\cdot)$ becomes a cubic in a_1:

$$\begin{aligned}\phi(a_1) = {} & a_1^3\tau_u((1+\gamma a_2\tau_u)^2 + \gamma^2\tau_u\tau_\nu) - a_1^2\gamma^2 a_2\tau_u^2(a_2^2\tau_u + \tau_\varepsilon + \tau_\nu) \\ & + a_1(3a_2\tau_u(a_2(1+\gamma a_2\tau_u) + \gamma\tau_\nu) + \tau_v((1+\gamma a_2\tau_u)^2 + \gamma^2\tau_\nu\tau_v) + \tau_\varepsilon + \gamma^2\tau_u(\tau_\nu + a_2^2\tau_u)^2) \\ & - \gamma^2 a_2\tau_u(a_2^2\tau_u + \tau_\varepsilon + \tau_\nu)(\tau_v + a_2^2\tau_u + \tau_\varepsilon + \tau_\nu),\end{aligned} \tag{A.14}$$

with a negative discriminant. This implies that with $\beta = 0$ there exists a unique equilibrium in linear strategies with first period responsiveness a_1^*. To locate the equilibrium, note that

$$\phi\left(\frac{\gamma^2 a_2\tau_u(a_2^2\tau_u + \tau_\varepsilon + \tau_\nu)}{1+\gamma^2\tau_u(a_2^2\tau_u + 2\tau_\varepsilon + \tau_\nu)}\right) = -\frac{\gamma^2 a_2\tau_\nu\tau_u(a_2^2\tau_u + \tau_\varepsilon + \tau_\nu)}{1+\gamma^2\tau_u(a_2^2\tau_u + 2\tau_\varepsilon + \tau_\nu)} < 0 \tag{A.15a}$$

$$\phi(a_2) = a_2(\tau_v + \tau_\varepsilon(1 + \gamma^2\tau_u(\tau_\nu + \tau_\varepsilon(3 + 2\gamma a_2\tau_u) + \tau_v))) > 0. \tag{A.15b}$$

Therefore,

$$a_1^* \in \left(\frac{\gamma^2 a_2\tau_u(a_2^2\tau_u + \tau_\varepsilon + \tau_\nu)}{1+\gamma^2\tau_u(a_2^2\tau_u + 2\tau_\varepsilon + \tau_\nu)}, a_2\right).$$

Furthermore, since

$$\psi'(a_1) \propto 2a_1\tau_u(\gamma^2 a_2\tau_u(a_2^2\tau_u + \tau_\varepsilon + \tau_\nu) - a_1(1+\gamma^2\tau_u(a_2^2\tau_u + 2\tau_\varepsilon + \tau_\nu)),$$

we also have that for $\beta = 0$, the weights to private information in the first period are strategic substitutes.

□

Proof of Corollary 2.1

For any $\beta \in [0,1]$, in the second period an equilibrium must satisfy $a_2 = \gamma\tau_\varepsilon$. In the first period, assuming $\tau_\eta = 0$, and using (A.8), at equilibrium a_1 equilibrium must satisfy

$$\begin{aligned}\phi_1(a_1) &\equiv a_1\lambda_2(\tau_2 + \tau_\varepsilon) - \gamma\tau_\varepsilon\Delta a_2\tau_u \\ &= a_1(1+\gamma\tau_u\Delta a_2) - \gamma^2\tau_\varepsilon\Delta a_2\tau_u = 0.\end{aligned} \tag{A.16}$$

The above equation is a quadratic in a_1 which for any $a_2 > 0$ and $\beta > 0$ possesses two positive, real solutions:

$$a_1^* = \frac{1+\gamma\tau_u a_2(1+\beta) - \sqrt{(1+\gamma\tau_u a_2(1+\beta))^2 - 4\beta(\gamma\tau_u a_2)^2}}{2\beta\gamma\tau_u} \tag{A.17}$$

$$a_1^{**} = \frac{1+\gamma\tau_u a_2(1+\beta) + \sqrt{(1+\gamma\tau_u a_2(1+\beta))^2 - 4\beta(\gamma\tau_u a_2)^2}}{2\beta\gamma\tau_u}, \tag{A.18}$$

with $a_1^{**} > a_1^*$. This proves that for $\beta > 0$ there are two linear equilibria.

Inspection of the above expressions for a_1 shows that $\beta a_1^* < a_2$, while $\beta a_1^{**} > a_2$. The result for λ_2, $\mathrm{Var}_1[p_2]$ follows from substituting (A.17) and (A.18), respectively in λ_2 and $\mathrm{Var}_1[p_2]$. To see that prices are more informative along the HIE note that in the first period $\mathrm{Var}[v|z_1]^{-1} = \tau_1 = \tau_v + a_1^2\tau_u$. In the second period, the price along the HIE is more informative than along the LIE if and only if

$$\frac{(1+\beta^2+\gamma a_2\tau_u((1-\beta^2)+\beta(1+\beta^2)))\sqrt{(1+\gamma a_2\tau_u(1+\beta))^2 - 4\beta(\gamma a_2\tau_u)^2}}{\gamma^2\beta^2\tau_u} > 0,$$

which is always true. Given that $\tau_{i2} = \tau_2 + \tau_\varepsilon$, this also implies that $\Lambda_2^{***} < \Lambda_2^*$. Finally, substitution of (A.17) and (A.18) in $\mathrm{Var}_{i1}[p_2]$ shows that $\mathrm{Var}_{i1}[p_2]^{***} < \mathrm{Var}_{i1}[p_2]^*$. In view of (15) this implies that $\Lambda_1^{***} < \Lambda_1^*$.

When $\beta \to 0$, along the HIE we have

$$\lim_{\beta\to 0} \frac{1+\gamma\tau_u(a_2+\beta\gamma\tau_\varepsilon) + \sqrt{1+\gamma\tau_u(2(a_2+\beta\gamma\tau_\varepsilon)+\gamma\tau_u(a_2-\beta\gamma\tau_\varepsilon)^2)}}{2\beta\gamma\tau_u} = \infty,$$

while along the LIE, using l'Hospital's rule,

$$\lim_{\beta\to 0} \frac{1+\gamma\tau_u(a_2+\beta\gamma\tau_\varepsilon) - \sqrt{1+\gamma\tau_u(2(a_2+\beta\gamma\tau_\varepsilon)+\gamma\tau_u(a_2-\beta\gamma\tau_\varepsilon)^2)}}{2\beta\gamma\tau_u} = \frac{\gamma a_2^2\tau_u}{1+\gamma a_2\tau_u}.$$

From (A.19) it then follows that in this case $\alpha_{P_1} < \alpha_{E_1}$. Finally, defining

$$a_{10}^* = \frac{\gamma a_2^2\tau_u}{1+\gamma a_2\tau_u},$$

and taking the limit of λ_2 as $\beta \to 0$ when $a_1 = a_{10}^*$ yields

$$\lim_{\beta\to 0} \lambda_2^* = \frac{1+\gamma\tau_u a_2}{\gamma(\tau_v + (a_{10}^*)^2\tau_u + a_2^2\tau_u + \tau_\varepsilon)} > 0,$$

whereas $\lim_{\beta\to 0}\lambda_2^{***} = 0$. □

Proof of Corollary 2.2

Starting from the LIE, we need to verify that $|\psi'(a_1^*)| < 1$, or that when $a_1 = a_1^*$,

$$\gamma\beta a_2\tau_u < (1 + \gamma\tau_u\Delta a_2)^2.$$

Substituting (A.17) on R.H.S. of the above inequality and rearranging yields

$$\begin{aligned}&|\psi'(a_1^*)| < 1 \iff \\ &\quad -2(1 + a_2\gamma\tau_u(1-\beta))(1 + a_2\gamma\tau_u(1-\beta) + \sqrt{(1 + \gamma\tau_u a_2(1+\beta))^2 - 4\beta(\gamma\tau_u a_2)^2}) < 0,\end{aligned}$$

which is always satisfied. For the HIE, we need instead to verify that $|\psi'(a_1^{***})| > 1$, or that when $a_1 = a_1^{***}$,

$$\gamma\beta a_2\tau_u > (1 + \gamma\tau_u\Delta a_2)^2.$$

Substituting (A.18) on R.H.S. of the above inequality and rearranging yields

$$\begin{aligned}&|\psi'(a_1^{***})| > 1 \iff \\ &\quad 2(1 + a_2\gamma\tau_u(1-\beta))(-(1 + a_2\gamma\tau_u(1-\beta)) + \sqrt{(1 + \gamma\tau_u a_2(1+\beta))^2 - 4\beta(\gamma\tau_u a_2)^2}) > 0,\end{aligned}$$

which is always satisfied, since the first factor in the product on the R.H.S. of the above expression is positive, while manipulating the second factor shows that

$$\sqrt{(1 + \gamma\tau_u a_2(1+\beta))^2 - 4\beta(\gamma\tau_u a_2)^2} > (1 + a_2\gamma\tau_u(1-\beta)) \iff 4a_2\beta\gamma\tau_u > 0.$$

□

Proof of Corollary 2.3

In the second period, the result follows from the fact that since at equilibrium $a_2 = \gamma\tau_\varepsilon$,

$$\frac{a_2}{\alpha_{E_2}} = \frac{\gamma}{\text{Var}_{i2}[v]}.$$

In period 1 we have

$$X_1(s_i, z_1) = \gamma\frac{(\lambda_2\Delta a_2 + \tau_\eta/\tau_{i2})E_{i1}[v]}{\text{Var}_{i1}[p_2]} + \frac{\gamma}{\text{Var}_{i1}[p_2]}\left(\frac{\beta\alpha_{P_2}}{a_2}z_1 + (1-\alpha_{P_2})\frac{\tau_1}{\tau_2}E_1[v]\right) - \frac{\gamma}{\text{Var}_{i1}[p_2]}p_1.$$

Adding and subtracting $(a_1/\alpha_{E_1})p_1$ from the right hand side of the above expression yields

$$X_1(s_i, z_1) = \frac{a_1}{\alpha_{E_1}}(E_{i1}[v] - p_1) + \left(\frac{a_1}{\alpha_{E_1}} - \frac{\gamma}{\text{Var}_{i1}[p_2]}\right)p_1 + \frac{\gamma}{\text{Var}_{i1}[p_2]}\left(\frac{\beta}{\gamma\tau_{i2}}z_1 + \frac{\tau_1}{\tau_{i2}}E_1[v]\right).$$

The second and third terms in the above expression can be rewritten to obtain

$$\begin{aligned}\left(\frac{a_1}{\alpha_{E_1}} - \frac{\gamma}{\text{Var}_{i1}[p_2]}\right)p_1 + \frac{\gamma}{\text{Var}_{i1}[p_2]}\left(\frac{\beta}{\gamma\tau_{i2}}z_1 + \frac{\tau_1}{\tau_{i2}}E_1[v]\right) &= \frac{\beta(1-\alpha_{P_1}) - \gamma\tau_1\alpha_{P_1}}{a_1\tau_{i2}\text{Var}_{i1}[p_2]}E_1[\theta_1] \\ &= \frac{\alpha_{P_1} - \alpha_{E_1}}{\alpha_{E_1}}E_1[\theta_1].\end{aligned}$$

Note, also, that setting $\rho \equiv a_1/a_2$, we can express

$$\alpha_{P_1} = \alpha_{E_1}\left(1 + \frac{(\beta\rho - 1)\tau_1}{\tau_{i2}}\right). \tag{A.19}$$

This implies that for $a_1 = a_1^*$, $\alpha_{P_1} < \alpha_{E_1}$, whereas the opposite holds for $a_1 \in \{a_1^{**}, a_1^{***}\}$. To differentiate x_{i2} with respect to p_2, we first express the information contained in a trader's forecast in terms of p_2. To this end we use (A.4) and write

$$z_2 = \frac{1}{\lambda_2}\left(p_2 - \frac{\tau_\eta}{\tau_{i2}}s_P - \frac{\beta\alpha_{P_2}}{a_2}z_1 - (1-\alpha_{P_2})\frac{\tau_1}{\tau_2}E_1[v]\right).$$

Substituting the above in $E_{i2}[v]$, and differentiating x_{i2} with respect to p_2 yields

$$\frac{\partial x_{i2}}{\partial p_2} = -\frac{\gamma\tau_{i2}}{1 + \gamma\tau_u\Delta a_2}.$$

For $a_1 = a_1^*$, we know that $\Delta a_2 > 0$, so that the information effect reinforces the substitution effect and the asset is a normal good. Conversely, when multiple equilibria arise, for $a_1 \in \{a_1^{**}, a_1^{***}\}$, $1 + \gamma\tau_u\Delta a_2 < 0$, implying that the asset is a Giffen good.

Finally, to compute the conditional covariance we have

$$\begin{aligned}\text{Cov}_{i1}[v - p_2, p_2 - p_1] &= \text{Cov}_{i1}[v - p_2, p_2] \\ &= \text{Cov}_{i1}[v, p_2] - \text{Var}_{i1}[p_2].\end{aligned} \tag{A.20}$$

Using (16b), we obtain

$$\mathrm{Cov}_{i1}[v, p_2] = \frac{1}{\tau_{i1}}\left(\lambda_2 \Delta a_2 + \frac{\tau_\eta}{\tau_{i1}}\right).$$

On the other hand, from (A.11) we have

$$\mathrm{Var}_{i1}[p_2] = \frac{1}{\tau_{i1}}\left(\lambda_2 \Delta a_2 + \frac{\tau_\eta}{\tau_{i2}}\right)^2 + \frac{\lambda_2^2}{\tau_u} + \frac{\tau_\eta}{\tau_{i2}^2}.$$

Substituting these expressions in (A.20) and rearranging yields

$$\mathrm{Cov}_{i1}[v - p_2, p_2 - p_1] = -\frac{1}{\gamma \tau_{i1} \tau_{i2}}\left(\frac{\lambda_2(\tau_{i2} - \tau_\eta)}{\tau_u} + \frac{\Delta a_2 \tau_\eta}{\tau_{i2}}\right).$$

According to Proposition 2, when $a_1 \in \{a_1^{**}, a_1^{***}\}$, $\Delta a_2 < 0$ and $\lambda_2 < 0$. Therefore, $\mathrm{Cov}_{i1}[v - p_2, p_2 - p_1] > 0$. Conversely, along the LIE, the opposite occurs.

□

Proof of Corollary 2.4

We have already established in Corollary 2.3 that along the HIE (LIE) $\alpha_{P_1} > \alpha_{E_1}$ ($\alpha_{P_1} < \alpha_{E_1}$). Now, using (12) the covariance between p_1 and v is given by

$$\mathrm{Cov}[v, p_1] = \alpha_{P_1}\frac{1}{\tau_v} + (1 - \alpha_{P_1})\left(\frac{1}{\tau_v} - \frac{1}{\tau_1}\right), \tag{A.21}$$

and carrying out a similar computation for the first period consensus opinion

$$\mathrm{Cov}\left[\bar{E}_1[v], v\right] = \alpha_{E_1}\frac{1}{\tau_v} + (1 - \alpha_{E_1})\left(\frac{1}{\tau_v} - \frac{1}{\tau_1}\right). \tag{A.22}$$

We can now subtract (A.22) from (A.21) and obtain

$$\mathrm{Cov}\left[p_1 - \bar{E}_1[v], v\right] = \frac{\alpha_{P_1} - \alpha_{E_1}}{\tau_1}, \tag{A.23}$$

implying that the price at time 1 over relies on public information (compared to the optimal statistical weight) if and only if the covariance between the price and the fundamentals falls short of that between the consensus opinion and the fundamentals. □

Proof of Corollary 2.5

To compute $\mathrm{Cov}[p_2 - p_1, p_1 - \bar{v}]$ we first note that

$$\mathrm{Cov}[p_2 - p_1, p_1 - \bar{v}] = \mathrm{Cov}[p_2, p_1] - \mathrm{Var}[p_1]. \tag{A.24}$$

Next,

$$\mathrm{Cov}[p_2, p_1] = \mathrm{Cov}[E_1[p_2], E_1[p_1]] + \mathrm{Cov}_1[p_2, p_1] = \mathrm{Cov}[E_1[p_2], p_1].$$

Computing

$$\begin{aligned}\mathrm{Cov}[E_1[p_2], p_1] &= \mathrm{Var}[E_1[v]] + (\Lambda_1 + \beta\Lambda_2)\mathrm{Cov}[E_1[v], E_1[\theta_1]] + \beta\Lambda_2\Lambda_1\mathrm{Var}[E_1[\theta_1]] \\ &= \frac{a_1^2\tau_1}{\tau_1\tau_v} + (\Lambda_1 + \beta\Lambda_2)\frac{a_1}{\tau_1} + \beta\Lambda_2\Lambda_1\frac{\tau_v}{\tau_1\tau_u},\end{aligned}$$

and

$$\mathrm{Var}[p_1] = \frac{a_1^2\tau_u}{\tau_1\tau_v} + \Lambda_1^2\frac{\tau_v}{\tau_1\tau_u} + 2\Lambda_1\frac{a_1}{\tau_1}.$$

Substituting these expressions in (A.24) and rearranging yields

$$\mathrm{Cov}[p_2 - p_1, p_1 - \bar{v}] = -\frac{\mathrm{Var}_{i1}[p_2]}{\gamma}\left(\Lambda_1\frac{\tau_v}{\tau_1\tau_u} + \frac{a_1}{\tau_1}\right) < 0.$$

Consider now $\mathrm{Cov}[v - p_2, p_1 - \bar{v}]$, decomposing the covariance yields

$$\begin{aligned}\mathrm{Cov}[v - p_2, p_1 - \bar{v}] &= \mathrm{Cov}[E_1[v - p_2], p_1] + \underbrace{\mathrm{Cov}_1[v - p_2, p_1 - \bar{v}]}_{=0} \\ &= -\beta\Lambda_2\left(\mathrm{Cov}[E_1[\theta_1], E_1[v]] + \Lambda_1\mathrm{Var}[E_1[\theta_1]]\right) \\ &= -\beta\Lambda_2\left(\frac{a_1}{\tau_1} + \Lambda_1\frac{\tau_v}{\tau_1\tau_u}\right) \\ &= -\beta\Lambda_2\lambda_1\frac{1}{\tau_u},\end{aligned}$$

which is always negative for $\beta \in (0, 1]$, and null for $\beta = 0$. Finally, to compute $\mathrm{Cov}[v - p_2, p_2 - p_1]$ we decompose again the covariance

$$\mathrm{Cov}[v - p_2, p_2 - p_1] = \mathrm{Cov}[E_1[v - p_2], E_1[p_2 - p_1]] + \mathrm{Cov}_1[v - p_2, p_2 - p_1].$$

Computing, $E_1[v - p_2] = -\beta\Lambda_2 E_1[\theta_1]$, and $E_1[p_2 - p_1] = (\beta\Lambda_2 - \Lambda_1)E_1[\theta_1]$. Therefore,

$$\mathrm{Cov}[E_1[v - p_2], E_1[p_2 - p_1]] = \beta\Lambda_2 \frac{\mathrm{Var}_{i1}[p_2]}{\gamma} \mathrm{Var}[E_1[\theta_1]]. \tag{A.25}$$

Next, we obtain

$$\mathrm{Cov}_1[v - p_2, p_2 - p_1] = \frac{(1 + \gamma\tau_u\Delta a_2)(\beta a_1 \Delta a_2 \tau_u - \tau_1) + \gamma\tau_\eta\tau_u\beta a_1}{(\gamma\tau_{i2})^2\tau_1\tau_u}. \tag{A.26}$$

When $a_1 \in \{a_1^{**}, a_1^{***}\}$, $\Delta a_2 < -(\gamma\tau_u)^{-1}$, and the above expression is always positive, which implies that along the HIE $\mathrm{Cov}[v - p_2, p_2 - p_1] > 0$. □

PROOF OF COROLLARY 2.6

To prove this result, we impose $\tau_\eta = 0$ in (A.25) and (A.26), obtaining

$$\mathrm{Cov}[v - p_2, p_2 - p_1] = -\frac{\lambda_2}{\gamma\tau_{i2}\tau_u}\left(1 - \beta\Delta a_2 \frac{\tau_{i1} - \tau_v}{a_1\tau_{i1}}\right). \tag{A.27}$$

Looking at (A.27) we again verify that along the HIE there is momentum. This is true because in that equilibrium $\lambda_2 < 0$ and $\Delta a_2 < 0$. Along the LIE momentum can occur, *depending on the persistence of liquidity trades*. To see this, note that since in this equilibrium $\lambda_2 > 0$ and $\Delta a_2 > 0$, from (A.27) momentum needs

$$1 - \beta\Delta a_2 \frac{\tau_{i1} - \tau_v}{a_1\tau_{i1}} < 0,$$

which can be rearranged as an (implicit) condition on the magnitude of β:

$$\frac{a_1\tau_{i1}}{\Delta a_2(\tau_{i1} - \tau_v)} < \beta < 1.$$

If $\beta = 0$, the above condition is never satisfied. Indeed, in this case there exists a unique equilibrium in which $\Delta a_2 = a_2 > 0$. Therefore, when $\beta = 0$ returns always display reversal. If $\beta = 1$, the condition is satisfied if

$$a_1\tau_v + a_1(\tau_\varepsilon + a_1^2\tau_u) < \Delta a_2\tau_u(\tau_\varepsilon + a_1^2\tau_u).$$

Isolating τ_v in the above expression yields:

$$\tau_v < \hat{\tau}_v \equiv \frac{(\Delta a_2 - a_1)(\tau_\varepsilon + a_1^2 \tau_u)}{a_1}, \tag{A.28}$$

which, since a_1 does not depend on τ_v (see (A.17)), gives an explicit upper bound on τ_v. Hence, if $\tau_v < \hat{\tau}_v$, there exists a $\hat{\beta}$ such that for all $\beta \geq \hat{\beta}$, when $\tau_\eta = 0$, momentum occurs between the second and third period returns along the LIE. $\square$

Appendix B. Online Appendix

Appendix A. Non-monotone comparative statics and dynamic adjustment

What is the effect of a shock to parameters' values on the equilibrium of the market? The answer to this question depends on whether the HIE is stable or not.

Starting from the case in which the HIE is stable, Figure 2 in the paper (panel (b)) implies that a decrease in private signal precision or in risk tolerance can have a non-monotone effect on a_1 and thus on the conditional volatility of returns, and informational efficiency of prices. To see this, consider first the case of private signal precision. Suppose that $\tau_v = \tau_u = 10$, $\tau_\varepsilon = 9.5$, $\gamma = 1/2$, $\tau_\eta = 3$, and $\beta = 1$. With these parameter values, the LIE and HIE are respectively $a_1^* = 4.001$, and $a_1^{***} = 5.606$, and correspond to the intersection of the orange best response function with the 45-degree line in Figure 6 (panel (a)). Suppose the market coordinates on the LIE. Suppose now that the precision of the private signal decreases to $\tau_\varepsilon = 7$. The new best response is depicted by the dashed curve in the figure. Again we have three equilibria with the LIE and HIE given respectively by $a_1^{*,NEW} = 2.906$, and $a_1^{***,NEW} = 4.124$.[52] Which equilibrium does the market coordinate on? With adaptive dynamics, we can see that this will be the HIE. Thus, in this case a *decrease* in private signal precision determines an *increase* in the response to private information and informational efficiency, and a decrease in the conditional volatility of returns (along the initial LIE, $\mathrm{Var}_1[p_2] = 0.00035$, while along the new HIE, $\mathrm{Var}_1[p_2] = 0.00013$). Non-monotonicity requires however a sufficiently large reduction in private precision.[53] Indeed, in panel (b) we repeat the same exercise, assuming that τ_ε is lowered to 8. In this case, adaptive dynamics implies that the new equilibrium along the dashed best response is the LIE. Panel (c) and (d) show that similar effects arise with a reduction in risk tolerance.

[Figure 6 about here.]

Consider now the case in which the HIE is unstable. In this situation numerical simulations show that starting from the HIE, the effect of a shock (even a very mild one) to parameters' values depends on the persistence of liquidity traders' demand. In detail: when $\beta \in (0, 1)$, the equilibrium converges to the LIE; when $\beta = 1$, the market oscillates between two non-equilibrium values. In Figure 7, panel (a) we plot the best response for $\tau_v = \tau_u = 10$, $\tau_\varepsilon = 20$, $\gamma = 1/2$, $\tau_\eta = 1/2$, and $\beta = .9$. With these parameter values, the LIE and HIE are respectively $a_1^* = 9.04$, and $a_1^{***} = 12.25$, and the slope of the best response at these two points is given by $\psi'(a_1^*) = -.41$ and $\psi'(a_1^{***}) = -2.12$. As shown in the figure, perturbing the HIE an iterated application of the best response leads the market to coordinate on the LIE. Consider now panel (b) where we plot the best response for $\tau_v = \tau_u = 10$, $\tau_\varepsilon = 15$, $\gamma = 1/2$, $\tau_\eta = 1/2$, and $\beta = 1$. With these parameter values, the LIE and HIE are respectively $a_1^* = 6.64$, and $a_1^{***} = 9.37$, and the slope of the best response at these two points is given by $\psi'(a_1^*) = -.43$ and $\psi'(a_1^{***}) = -1.9$. In this case, iterating the application of the best response perturbing the HIE (after about 380 iterations) leads the market to oscillate between the non-equilibrium values 7.99 and 10.75. Thus, the implication is that, provided $\beta < 1$, if the market is at the HIE a small shock to parameter values leads it to the LIE.

[Figure 7 about here.]

Appendix B. The effect of residual uncertainty

In this section we perform a robustness exercise and assume that investors face residual uncertainty over the final liquidation value. Therefore, we model the final payoff as $\hat{v} = v + \delta$, where $\delta \sim N(0, \tau_\delta^{-1})$ is a random term orthogonal to all the random variables in the market, and about which no investor is informed. The addition of the random term δ allows to study the effect of an increase in the residual uncertainty that surrounds investors' environment in periods of heightened turbulence, and shows that a price crash can occur within our framework. Intuitively, when investors are faced with residual uncertainty, they put less weight on their signals, since prices and private information are less useful to predict the asset payoff. This is likely to weaken the retrospective inference and variance reduction loop, eliminating the HIE. Our analysis shows, however, that in general residual uncertainty does not eliminate the HIE, nor makes it unstable.

With residual uncertainty, the expressions for prices and investors' strategies do not change (that is, expressions (8), (19), and (20) in the paper hold). However, the equilibrium obtains as the solution of a system of the following highly non-linear equations:

$$a_2 = f_{a_2}(a_1, a_2) \equiv \frac{\gamma \tau_\varepsilon}{1+\kappa} \tag{B.1a}$$

$$a_1 = f_{a_1}(a_1, a_2) \equiv \frac{\gamma^2 \tau_{i2}(\Delta a_2(1+\kappa+\gamma\tau_u\Delta a_2) + \gamma\tau_\eta)\tau_\varepsilon\tau_u}{(\Delta a_2(1+\gamma\tau_u\Delta a_2+\kappa)+\gamma\tau_\eta)^2\tau_u + \tau_{i1}((1+\gamma\tau_u\Delta a_2+\kappa)^2+\gamma^2\tau_\eta\tau_u)}. \tag{B.1b}$$

Inspection of the cubic (B.1a) shows that it possesses a unique real solution, which can therefore be substituted in (B.1b) to solve the equilibrium as a fixed point of a best response in a_1, $f_{a_1}(a_1, a_2(a_1))$. In Figure 8 we show the plot of such a best response mapping for the following parameterization: $\tau_u = \tau_v = 1$, $\tau_\varepsilon = 4$, $\tau_\eta = 2/3$, $\gamma = 1$, $\beta = 9/10$, and $\tau_\delta \in \{40, 70, 75, 100\}$. When $\tau_\delta = 100$, we have 5 equilibria, only two of which are stable with respect to best response dynamics. Furthermore, the equilibrium with a higher value of a_1 displays a negative inference component of liquidity, as shown by the first row of Table II.

[Figure 8 about here.]

This suggests that, when retrospective inference is not too strong, the presence of residual uncertainty *per-se* is not sufficient neither to make the HIE disappear, nor to make it unstable. Of course, as residual uncertainty increases, the strength of the loop weakens even more and the HIE tends to disappear as shown by panels (b), (c), and (d) in the figure, where we plot the best response for $\tau_\delta = 75$, $\tau_\delta = 70$, and $\tau_\delta = 40$. Table II collects the results of our calculations for the different values of τ_δ.

[Table 2 about here.]

In our baseline simulation, we set $\tau_v = 1$, and find that for $\tau_\delta < 40$, the HIE vanishes. While at first blush this small level of residual uncertainty may seem to question the relevance of the HIE, it possible to show that this parameterization is not uncommon in calibrated asset pricing pricing models. For instance, Wang (1994) models the asset payoff as a dividend process $F_{t+1} = \rho F_t + \omega_{t+1}$, where F_t is a persistent component, and ω_{t+1} an orthogonal random error term, which corresponds to our residual uncertainty term. The coefficient ρ in this case parameterizes the impact of past fundamentals on current ones. The fraction of variance coming from residual uncertainty in this framework is $1-\rho^2$ (that is, the ratio between σ^2_ω and the steady-state variance of F_{t+1}, which is $\sigma^2_\omega/(1-\rho^2)$), which in our framework corresponds to $\tau_\delta^{-1}/(\tau_\delta^{-1} + \tau_v^{-1})$. It is easy to show that with a value of $\rho \in (.98, .99)$, residual uncertainty in the dynamic model has a comparable importance as the one implied by the parameter $\tau_\delta = 40$. Thus, if for example we take our model to represent trading patterns that occur at a quarterly frequency, this roughly implies a critical value of $\rho = .95$ at a yearly frequency, which is not uncommon as a calibration in asset pricing models.[54]

Appendix C. Long-term investors

Consider the market with residual uncertainty but assume that now investors have a long horizon and maximize the expected utility of final wealth. For simplicity we will deal with the case where the public signal is useless ($\tau_\eta = 0$). In this case multiple equilibria are also possible and the reasons are similar to those of the case with short-term investors.

A long-term investor in the first period speculates on short-term returns and takes into account the hedging possibilities of second period trading. The equilibrium strategy of investor i in the first period is in fact a linear combination of $(E_{i1}[p_2] - p_1)$ and $E_{i1}[x_{i2}]$ (Cespa and Vives (2012)).[55] Were traders not to expect a change in prices, then their optimal period 1 position would be like the one of a static market, and the risk of holding such a position would only be due to the unpredictability of the liquidation value.[56] If a change in prices is expected, traders optimally exploit short-run price differences. Two factors add to the risk of their period 1 position, as traders suffer from the partial unpredictability of the price change, and from the impossibility of determining exactly their future position. However, the opportunity to trade again in the future also grants a hedge against potentially adverse price movements. This, in equilibrium, yields a risk-reduction which when there is *no residual uncertainty* exactly offsets the price risk conditional on private information. As a consequence, *with no residual uncertainty,* traders' strategies have a static nature in their response to private information.[57] Still investors may speculate on price differences but only for market making purposes to profit from the mean reversion of liquidity trading. With residual uncertainty strategies are truly dynamic and informed investors speculate on short-term price movements based on their private information.

We have that in equilibrium the responsiveness to private information (when informed traders do not receive a new signal in the second period as in our base model, see Cespa and Vives (2012)) is given by:

$$a_1 = \frac{\gamma\tau_\varepsilon(1 + \gamma\tau_u\Delta a_2)}{1 + \kappa + \gamma\tau_u\Delta a_2},$$
$$a_2 = \frac{\gamma\tau_\varepsilon}{1 + \kappa}.$$

When $\kappa = 0$ then $a_1 = a_2 = \gamma\tau_\varepsilon$. With long-term investors, and under the assumptions of the model, the feed-back loop that generates multiplicity is broken because the optimal strategy of an informed trader is static (buy-and-hold): in the first period informed traders receive their private signal, take a position and then in the second period there is no informed trading, the informed traders just make the market absorbing the demands of liquidity traders.[58]

When $\kappa > 0$, $a_1 = \rho\gamma\tau_\varepsilon/(1+\kappa)$ with $\rho > 1$ at any equilibrium. The endogenous parameter ρ captures the deviation from the long term private signal responsiveness due to the presence of residual uncertainty. Thus, prior to the last trading round, investors react to their private signals more aggressively than if the liquidation value were to be realized in the next period. Indeed, while residual uncertainty makes investors less confident about their signals, the presence of an additional trading round increases the opportunities to adjust suboptimal positions prior to liquidation. This, in turn, boosts investors' reaction to private information. Residual uncertainty implies that informed investors speculate on short-term price movements based on their private information. This makes possible multiple equilibria. Indeed, faced with uncertain impending liquidation a long-term trader in period 2 is not going to use much his private signal. This makes the trader behave in the first period more like a short-term trader since he will try to unwind his first period holdings in the market at time 2, and carry little of that inventory to the liquidation date. In this case the liquidity of the second period market becomes much more important to determine the trader's reaction to private information in the first period and multiple self-fulfilling expectational loops are possible as in the case with short-term traders. Again the possibility of multiple equilibria is linked to having a negative price impact in the second period ($\lambda_2 < 0$) due to a large response to private information in the first period (generating $\Delta a_2 < 0$). For example, three equilibra arise with $\tau_\delta = 200, \tau_\varepsilon = \tau_v = \tau_u = \gamma = 1$ and $\beta = .2$ and only in the low a_1 equilibrium we have $\Delta a_2 > 0$ and stability. In general three equilibria are obtained for high β and high τ_δ. Multiple equilibria may arise also when there is a common shock in the private signal (Grundy and McNichols (1989)).

In summary, with long-term risk averse investors and either residual uncertainty (He and Wang (1995), and Cespa and Vives (2012)), or a common shock in the private signals (Grundy and McNichols (1989)) there may be multiple equilibria. We may have situations then with a negative price impact in the second period. This arises because in those cases informed traders have incentives to use their private information to speculate on short-term price movements and long-term traders may behave as short-term ones.

Appendix D. Numerical simulations

We give here the details of the numerical simulation discussed in Section IV–D. We assume that $\tau_v = 0.01$, $\tau_u = 0.1$, $\tau_\varepsilon = 1$, $\gamma = 1/2$, $\tau_\eta = 0.001862$, and $\beta = 1/2$. With these values, a stable HIE obtains ($\{a_1^*, a_1^{***}\} = \{0.012, 41.25\}$ with $\{\psi'(a_1^*), \psi'(a_1^{***})\} = \{-0.012, -0.92\}$) and we obtain momentum along the LIE and in the unique equilibrium with no private information ($\tau_\varepsilon = 0$). Next we set $\bar{v} = 40$ and extract 1,000 i.i.d. normal shocks for v, u_1, u_2, and η, and average across the prices and shocks. Table III displays the numerical results of this exercise.

[Table 3 about here.]

Notes

[1]While the market presence of traditionally long-term investors such as institutions has steadily increased during the last two decades, their holding period has decreased substantially (see OECD, `http://www.oecd.org/daf/fin/financial-markets/48616812.pdf`). Haldane and Davies (2011) examine a large panel of UK- and US-listed companies over the period from 1980 to 2009; they find compelling evidence of investors' short-term bias, which is even more pronounced in the last 10 years of their sample.

[2]"Or, to change the metaphor slightly, professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practise the fourth, fifth and higher degrees." (Keynes, *The General Theory of Employment, Interest and Money*, 1936).

[3]In a related paper, we show that a similar conclusion holds in a model with long-term investors (see Cespa and Vives (2012)).

[4]Part of the debate over the consequences of short-term trading revolves precisely around its alleged negative effect on the informativeness of asset prices (see, e.g., the "Kay review of UK equity markets and long term decision making," available at `http://lawcommission.justice.gov.uk/docs/kay-review-of-equity-markets-final-report.pdf`).

[5]Our model should be understood to fit low frequency (monthly–quarterly) patterns.

[6]That is, liquidity traders' position are correlated across trading dates.

[7]The former captures the price variation due to the the change in asset exposure that investors experience when clearing the market; the latter captures the price change due to investors' inferences from aggregate demand for the asset.

[8]This finding has an empirical counterpart: some spread decomposition models find that, consistent with our HIE prediction, the spread's inference component can be negative.

[9]In the absence of private information, our model is akin to a finite-horizon version of Spiegel (1998); hence, in this case we can show that a unique equilibrium obtains.

[10]In his case, too, the analysis concentrates on the steady-state equilibrium; his results are therefore not directly comparable to ours. Furthermore, information on fundamentals is short lived in Watanabe (2008) whereas in our model it is long lived; this difference substantially changes the nature of the inference problem faced by first-period investors. In related research, Dennert (1991) concentrates on the steady-state solution in his OLG extension of Grossman and Stiglitz (1980). Private information is short lived in this setup also.

[11]In a static trading model, Ganguli and Yang (2009) discuss complementarities in information acquisition.

[12]We assume without loss of generality that, with CARA preferences, the nonrandom endowment of

informed investors is zero.

[13]The unique equilibrium in linear strategies of this model is symmetric.

[14]This convention is justified in Section 3.1 of the Technical Appendix to Vives (2008).

[15]When risk-averse investors accommodate an expectedly positive demand of liquidity traders, the former require compensation for risking that the liquidation value ends up being higher than the public expectation (conversely, if $E[u|p] < 0$ then investors insist on a price that is *lower* than $E[v|p]$—to cover the risk that $v < E[v|p]$). Such compensation is increasing in the uncertainty faced by investors (as captured by Λ) and in the the extent of their anticipated exposure to the liquidity traders' shock (i.e., their expected inventory $E[u|p]$).

[16]The adverse selection effect is a consequence of the signal extraction problem that dealers face in this market: since $a > 0$, if investors on average have good news then they buy the asset and so $E[v|p]$ increases, reflecting that information. However, this effect cannot be distinguished from the buying pressure of liquidity traders, which also has the effect of increasing $E[v|p]$.

[17]As before, we assume (w.l.o.g.) that the nonrandom endowment of investors is zero.

[18]The equilibria in linear strategies of this model are symmetric.

[19]More specifically, suppose that we replace the first-period liquidity traders with a set of hedgers in the interval $[0, 1]$ and that each of these hedgers receives an idiosyncratic and normally distributed endowment shock θ_{i1} that is independent of the model's other random variables. All hedgers take a position in the asset at date 0. However, a fraction β of them (denoted HL) have a long-term horizon and hold their positions until the liquidation date; the complementary fraction (denoted HS) has a short-term horizon, and these hedgers liquidate their positions at date 2. Under this model we can show that, in a linear equilibrium, the persistence coefficient corresponds to the β-weighted relative responsiveness to the endowment shock displayed by HL (i.e., as compared with the average response of both HL and HS). Since the responsiveness of hedgers is endogenous and since information is asymmetric, it follows that that in this case a participation externality (similar to the one in Admati and Pfleiderer (1988), Pagano (1989), and Dow (2004)) arises. Namely, hedgers' decisions to trade in the first period depend on market liquidity, which in turn depends on hedgers' decisions to trade. This new loop can generate multiplicity with different levels of hedging activity.

[20]Coval and Stafford argue in more detail that fire sales occur in mutual funds that follow specialized investment strategies and that exhibit considerable overlap in their holdings.

[21]Easley, Hvidkjaer, and O'Hara (2002) develop a methodology to estimate the probability of informed trading from transaction data.

[22]When computing $\bar{E}_1[\theta_1]$ we have $z_1 = E_{i1}[z_1] = a_1 E_{i1}[v] + E_{i1}[\theta_1]$, and $E_{i1}[\theta_1] = z_1 - a_1 E_{i1}[v] = a_1(v - E_{i1}[v]) + \theta_1$. Computing average expectations yields $\bar{E}_1[\theta_1] = a_1(v - \bar{E}_1[v]) + \theta_1$. Finally, using $\bar{E}_1[v] = \alpha_{E_1} v + (1 - \alpha_{E_1})E_1[v]$ yields the expression in the centered formula above.

[23]We have that $\alpha_{P_2} = \alpha_{E_2}$ and so $1 - \alpha_{P_2} \in (0, 1)$.

[24] This is because, for given p_1 and $z_1 = a_1 v + \theta_1$, a higher value for θ_1 provides stronger evidence that the fundamentals v is low.

[25]Retrospective inference plays an anlaogous role as the "inference augmentation" effect in Goldstein and Yang (2014).

[26]It is worth noting that Huang and Stoll (1997) assumes uninformed market orders to be generated by an AR(1) process, and the estimated parameter of that process turns out to be positive and close

to 1. Negative *inference* components also occur in different spread decomposition models. For example, Hamm (2014) investigates the effect of exchange-traded funds on the liquidity of underlying stocks and estimates a spread decomposition model based on Madhavan, Richardson, and Roomans (1997). In several of her findings, the inference component of the spread is negative—leading to the exclusion of such "out-of-range" observations. See also Foucault, Pagano, and Röell (2013), Ch. 5.2.2.

[27]Public precision has little effect on the equilibrium ex ante, while the degree of risk tolerance has an effect similar to that of private precision.

[28]Along the HIE, second-period investors facing a large positive demand for the asset will adjust their estimate of the fundamentals *downward*, which implies that the inference component of λ_2 is negative. Other things equal, a more precise public signal reduces the absolute value of the inference component, which works to increase first-period investors' uncertainty about p_2. In the LIE, the converse statement holds.

[29]It is easy to see that in equilibrium $a_1 = \gamma/\left(\tau_\epsilon^{-1} + \tau_2^{-1}\right)$ and therefore we have always that $\Delta a_2 > 0$ and $\lambda_2 = \tau_u \Delta a_2/\tau_2 > 0$.

[30]This claim follows immediately from the inequality $\psi(0) > 0$; hence the best-response mapping cuts the 45-degree line from below at the IIE, which implies that $\psi'(a_1^{**}) > 1$.

[31]As shown in Figure 4, the responses of traders to private information in the HIE and the LIE are strategic complements or substitutes depending on the parameters. For given τ_ε, the higher is τ_η the more likely it will be for at least one of the two equilibria to display strategic complementarities. For instance, when $\beta = 1$, $\gamma = 1/2$, $\tau_u = \tau_v = 10$, $\tau_\varepsilon = 8$, and $\tau_\eta = 2.5$ we obtain $(a_1^*, a_1^{***}) = (3.32, 4.8)$ and $(\psi'(a_1^*), \psi'(a_1^{***})) = (-0.4, -0.23)$. Increasing τ_η to 3 yields $(a_1^*, a_1^{***}) = (3.34, 4.73)$ and $(\psi'(a_1^*), \psi'(a_1^{***})) = (-0.32, 0.03)$. Finally, if we further increase τ_η to 5 (resp., to 8), then the HIE disappears and we obtain $a_1^* = 3.44$ with $\psi'(a_1^*) = -0.05$ (resp., $a_1^* = 3.59$ with $\psi'(a_1^*) = 0.25$).

[32]We can prove that (29a) is larger (in absolute value) at the HIE than at the LIE when $\tau_\eta = 0$, and our numerical simulations indicate that this difference holds also when $\tau_\eta > 0$.

[33]See Admati (1985) and Cespa (2005) for discussions about the existence of Giffen assets due to information effects in the context of a multi-asset REE model.

[34]To see this, observe that $\partial x_{i2}/\partial p_2 = (a_2/\alpha_{E_2})(\Delta a_2 \tau_u/\lambda_2 \tau_{i2} - 1) = -\gamma\tau_{i2}/(1+\gamma\Delta a_2 \tau_u)$. Along the HIE (resp., LIE), as shown in Proposition 2, $1+\gamma\Delta a_2\tau_u < 0$ (resp., $1+\gamma\Delta a_2\tau_u > 0$). These inequalities prove the result.

[35]We are indebted to an anonymous referee for suggesting this calibration exercise.

[36]This strategy is of the form $x_{i1} = \Gamma_2^1(E_{i1}[p_2] - p_1) + \Gamma_2^2 E_{i1}[x_{i2}]$, where Γ_2^1 and Γ_2^2 are equilibrium parameters and $E_{i1}[x_{i2}] = \Lambda_2^{-1}(1 - \lambda_2\Delta a_2)(E_{i1}[v] - \hat{p}_1)$.

[37]If the asset price is not expected to change in response to today's information, then the market is not expected to receive any new private information and so the model collapses to one in which traders hold the risky asset for two periods. In that case, those positions naturally coincide with the ones they would hold in a static market.

[38]Indeed, a long-term trader who faces uncertain impending liquidation will find his private signal to be of little use in period 2. As a result, such a trader will behave in the first period more like a short-term trader: he will try to unwind his first-period holdings in the market at time 2 and thus carry little of that inventory to the liquidation date.

[39]Numerical simulations in a three-period model show that, along the HIE, both $\mathrm{Cov}[v - p_3, p_3 - p_2]$

and $\mathrm{Cov}[p_3 - p_2, p_2 - p_1]$ are positive.

[40]One can easily verify that $\mathrm{Cov}[v-p_2, p_1-\bar{v}] = \mathrm{Cov}[E_1[v-p_2], E_1[p_1-\bar{v}]] = -\beta\Lambda_2 \mathrm{Cov}[E_1[\theta_1], p_1] < 0$ and $\mathrm{Cov}[p_2 - p_1, p_1 - \bar{v}] = \mathrm{Cov}[E_1[p_2 - p_1], E_1[p_1 - \bar{v}]] = (\beta\Lambda_2 - \Lambda_1) \mathrm{Cov}[E_1[\theta_1], p_1] < 0$.

[41]Suppose that $E_1[\theta_1] < 0$. Then our pricing equation (14) implies a reduction in p_1 so that a risk-averse investor will be awarded higher expected returns, inducing her to absorb the shock. Indeed, we have $E_1[p_2 - p_1] = (\beta\Lambda_2 - \Lambda_1)E_1[\theta_1] > 0$. If $\beta > 0$ then selling pressure is likely to persist, which implies that prices will be depressed again at date 2 and will thus ensure positive expected returns: $E_1[v - p_2] = -\beta\Lambda_2 E_1[\theta_1] > 0$. In this way, the persistence of liquidity trades offsets the mean reversion effect resulting from the first period's short-term investors' unwinding at date 2.

[42]Proposition 2 shows that in this case $1+\gamma\tau_u\Delta a_2 < 0$, which by (31) implies that $\mathrm{Cov}_1[v-p_2, p_2-p_1] > 0$.

[43]This claim can be demonstrated by computing $\lim_{\tau_\varepsilon \to 0} \mathrm{Cov}[v - p_2, p_2 - p_1] = (\beta - \gamma^2\tau_v\tau_u)/\gamma^4\tau_u^2\tau_v^3$, which implies that momentum in this case arises when $\beta > \gamma^2\tau_v\tau_u$.

[44]In a market with no private information, investors absorb only the orders of liquidity traders. In equilibrium, then, their positions reflect only liquidity traders' demand and so $E[|x_1|] = ((2/\pi)\tau_u^{-1})^{1/2}$. This approach is taken by He and Wang (1995), among others.

[45]The relationshio also contrasts with Banerjee (2011) who predicts a negative relation between disagreement and volatility in the high volatility equilibrium of his RE model.

[46]The result is consistent with the positive association between volume and conditional volatility found in Gallant at al.(1992).

[47] For example, set $\tau_u = \tau_v = 10$, $\tau_\varepsilon = 1$, $\tau_\eta = .2$, $\gamma = 1/2$, and $\beta = 1$; then we obtain two stable equilibria, $a_1^* = .29$ and $a_1^{***} = .86$. By Corollary 2.7, volume is higher along the HIE. Furthermore, calculating price volatility yields $\mathrm{Var}[p_1]^{***} = .05$ and $\mathrm{Var}[p_2|s_P]^{***} = .07$, as compared with $\mathrm{Var}[p_1]^* = .02$ and $\mathrm{Var}[p_2|s_P]^* = .04$. Thus, price volatility is larger in the HIE compared to the LIE in the first (resp. second) period.

[48]The green line in panels (b), (c), and (d) of the figure is the (log of the) average of the semi-strong efficient prices obtained in the simulations—that is, $\ln((1/1000)\sum_{j=1}^{1000} E_{nj}[v])$, $n = 1, 2$. The horizontal line in the plots for the price paths is the (log of the) average value of the fundamentals (in this simulation, $\ln(38.08) \approx 3.64$).

[49]The former finding is consistent with the evidence presented by Chordia, Roll, and Subrahmanyam (2008), who find that short-horizon return predictability from order flows declines when the market is more liquid. The details of the simulation are available in the Online Appendix.

[50]For instance, Gorton and Metrick (2010) report that repo depositors during the crisis "did not know which securitized banks were most likely to fail."

[51]Nagel (2012) finds that the stocks of small, illiquid, and highly volatile companies generated the largest contrarian returns during the financial crisis.

[52] In both cases the HIE is stable case since $(\partial\psi/\partial a_1)|_{a_1=a_1^{***}} = -.24$, and $(\partial\psi/\partial a_1)|_{a_1=a_1^{***,NEW}} = .35$.

[53] Denoting respectively by ψ and ψ^{NEW} the orange and dashed best responses, non monotonicity requires that that $\psi(a_1^*) > \psi^{NEW}(a_1^{**,NEW})$. If the reduction in private precision is such that $\psi(a_1^*) > \psi^{NEW}(a_1^{***,NEW})$, the new equilibrium along the dashed best response is still the HIE, but monotonicity is restored since $a_1^{***,NEW} < a_1^*$.

[54]We are indebted to an anonymous referee for suggesting this calibration exercise.

[55]It is of the form $x_{i1} = \Gamma_2^1(E_{i1}[p_2] - p_1) + \Gamma_2^2 E_{i1}[x_{i2}]$ where Γ_2^1 and Γ_2^2 are equilibrium parameters and $E_{i1}[x_{i2}] = \Lambda_2^{-1}(1 - \lambda_2 \Delta a_2)(E_{i1}[v] - \hat{p}_1)$.

[56]Intuitively, if given today's information the asset price is not expected to change, no new private information is expected to arrive to the market and the model collapses to one in which traders hold for two periods the risky asset. Their position, then, naturally coincides with the one they would hold in a static market.

[57]In fact, when $\beta = 1$ we have that $x_{i1} = E[x_{i2}|s_i, p_1]$. When $\beta < 1$ traders speculate also on price changes based on public information.

[58]If informed traders were to receive a second signal in the second period then there would be informed trading in this period but still the strategies would be static and price impact would still be positive. The reason is that the trading intensity in the second period will always be larger than the one in the first period, $a_2 = \gamma(\tau_{\varepsilon_2} + \tau_{\varepsilon_1}) > a_1 = \gamma\tau_{\varepsilon}$, because private information about the liquidation value accumulates over time. With long-term traders we can not have negative price impacts when the joint information of traders reveals the liquidation value.

	$\beta = 0$	$\beta \in (0, 1]$	
		LIE	HIE
Reliance on public information	High	High	Low
Liquidity	Low	Low	High
Price impact (period 2)	$+$	$+$	$-$
	Strong	Strong	Weak
Price informativeness	Low	Low	High
Risky asset (period 2)	Normal	Normal	Giffen
Expected total volume of informational trading	Low	Low	High
Return correlation at long horizons	$-$	$-$	$-$
Return correlation at short horizons	$-$	$\pm$	$+$
Disagreement	High	High	Low
Conditional volatility	High	High	Low

Table I: The Beauty Contest revisited.

τ_δ	$\{(a_1^*, \psi'(a_1^*)), a_2^*, \lambda_2^*\}$	$\{(a_1^{**}, \psi'(a_1^{**})), a_2^{**}, \lambda_2^{**}\}$	$\{(a_1^{***}, \psi'(a_1^{***})), a_2^{***}, \lambda_2^{***}\}$	$\{(a_1^{****}, \psi'(a_1^{****})), a_2^{****}, \lambda_2^{****}\}$	$\{(a_1^{*****}, \psi'(a_1^{*****})), a_2^{*****}, \lambda_2^{*****}\}$
100	$\{(.8, -.29), 1.86, .43\}$	$\{(5.02, 3.8), 1.42, -.007\}$	$\{(6.2, -.28), 1.23, -.02\}$	$\{(19.6, 5.8), .25, -.0002\}$	$\{(21.9, -46.6), .21, -.02\}$
75	$\{(.78, -.29), 1.82, .44\}$	$\{(5.2, 4.11), 1.26, -.005\}$	$\{(7.5, .31), .88, -.01\}$	$\{(12.06, 2.45), .45, -.005\}$	$\{(15.38, -26.78), .3, -.0004\}$
70	$\{(.77, -.29), 1.81, .44\}$	$\{(5.28, 4.1), 1.21, -.005\}$	$\{--, --\}$	$\{--, --\}$	$\{(14.03, -22.55), .33, -.0005\}$
40	$\{(.72, -.3), 1.69, .47\}$	$\{--, --\}$	$\{--, --\}$	$\{--, --\}$	$\{--, --\}$

Table II: Equilibrium values and values of the derivative of the best response mapping at equilibrium for the plots of Figure 8.

	LIE	HIE	Equilibrium with $\tau_\varepsilon = 0$
$\mathrm{Cov}[v - p_2, p_2 - p_1]$	735	7×10^{-6}	4.8×10^{8}
$\mathrm{Cov}_1[v - p_2, p_2 - p_1]$	-37	7×10^{-6}	$-284,200$
$\mathrm{Cov}[z_1, p_2 - p_1]$	-801	-1.7×10^{-6}	-5.7×10^{6}
$E_1[v - p_1]$	-81	-0.005	$-568,700$
$E_1[p_2 - p_1]$	-80	-1.7×10^{-7}	$-568,600$

Table III: Autocovariance of returns (unconditional and conditional), return predictability from order flows, and expected returns from liquidity provision in the numerical example.

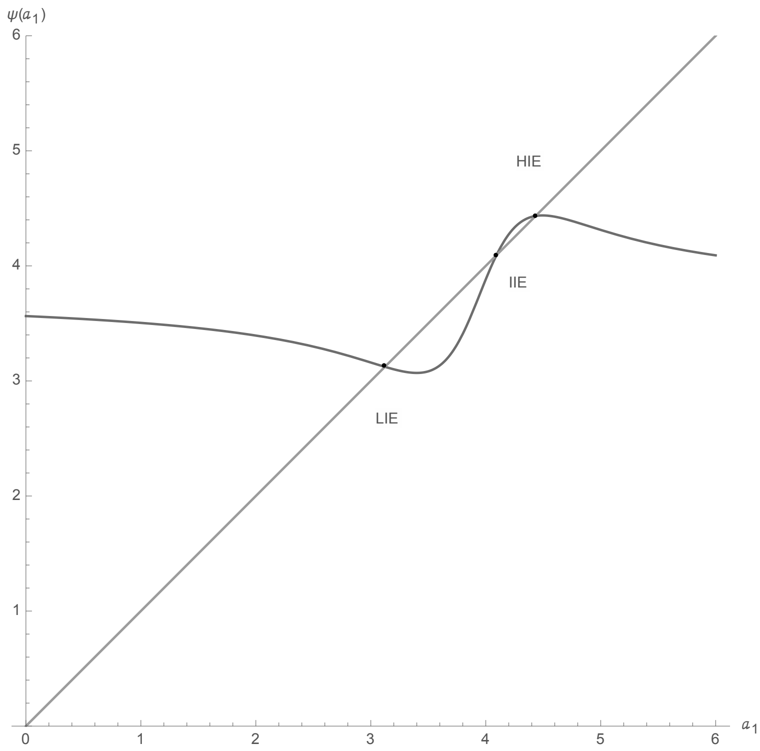


Figure 1: The best-response mapping (22) for $\beta = 1$, $\gamma = 1/2$, $\tau_u = \tau_v = 10$, $\tau_\eta = 3$, and $\tau_\varepsilon = 7.5$.

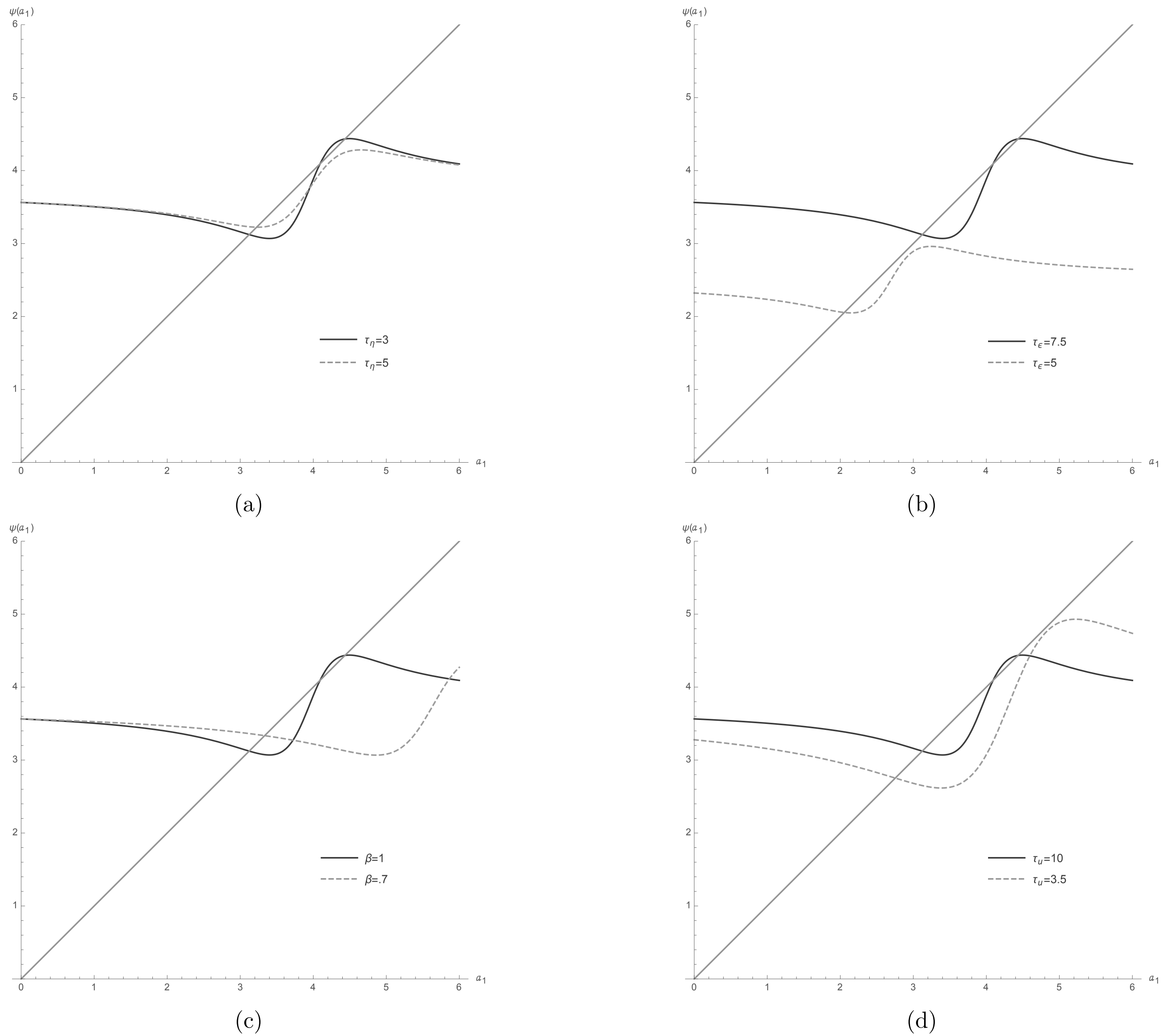


Figure 2: Comparative statics. We plot the best response (22) for the parameters value of Figure 1 (in orange) and show the effect of a change in the values of τ_η, τ_ε, β, and τ_u (dashed, blue curve). In panel (a) we increase public signal precision to $\tau_\eta = 5$; in panel (b) we decrease private signal precision to $\tau_\varepsilon = 5$; in panel (c) we decrease liquidity traders' persistence to $\beta = .7$, and in panel (d) we decrease the precision of liquidity traders' demand to $\tau_u = 3.5$.

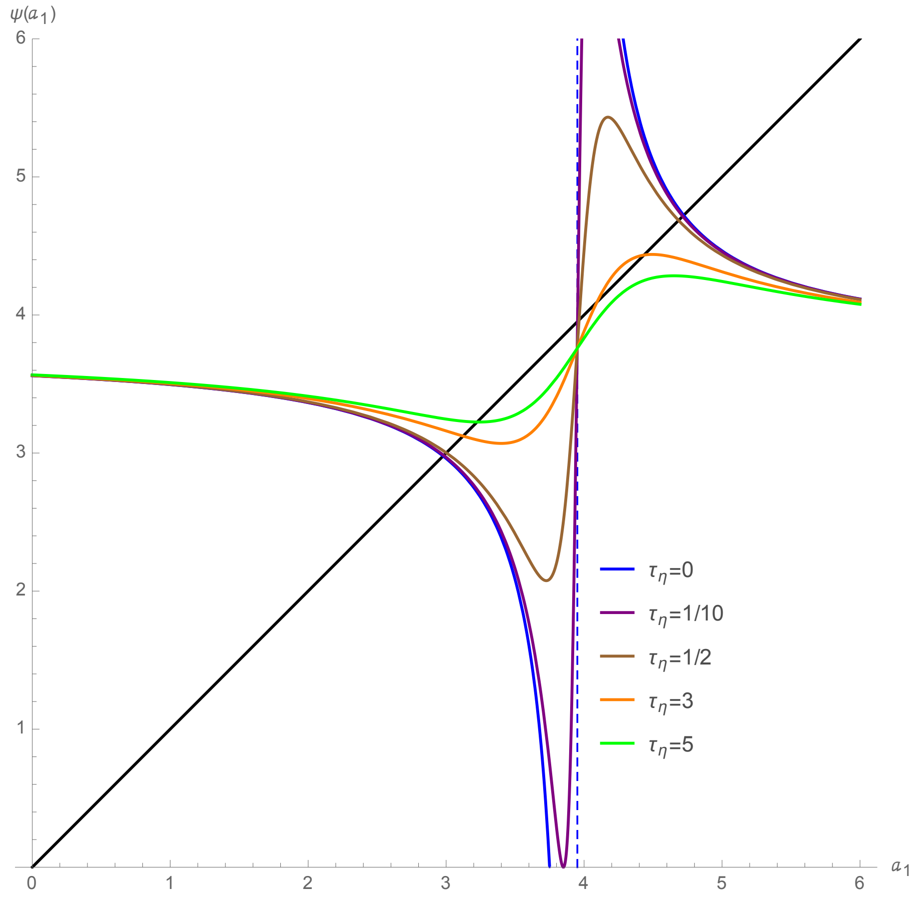


Figure 3: The best-response mapping (22) when the public signal's precision ranges within the set $\{0, 1/10, 1/2, 3, 5\}$. The other parameter values are $\beta = 1$, $\gamma = 1/2$, $\tau_u = \tau_v = 10$, and $\tau_\varepsilon = 7.5$. When $\tau_\eta = 0$, note that $\psi(\cdot, \tau_\eta)$ diverges at the point $\hat{a}_1 \equiv (1+\gamma\tau_u a_2)/(\gamma\beta\tau_u)$ and the IIE disappears. For $\tau_\eta > 0$, we have $\psi(\hat{a}_1, .1) = 3.75864$, $\psi(\hat{a}_1, .5) = 3.75862$, $\psi(\hat{a}_1, 3) = 3.7585$, and $\psi(\hat{a}_1, 5) = 3.7584$.

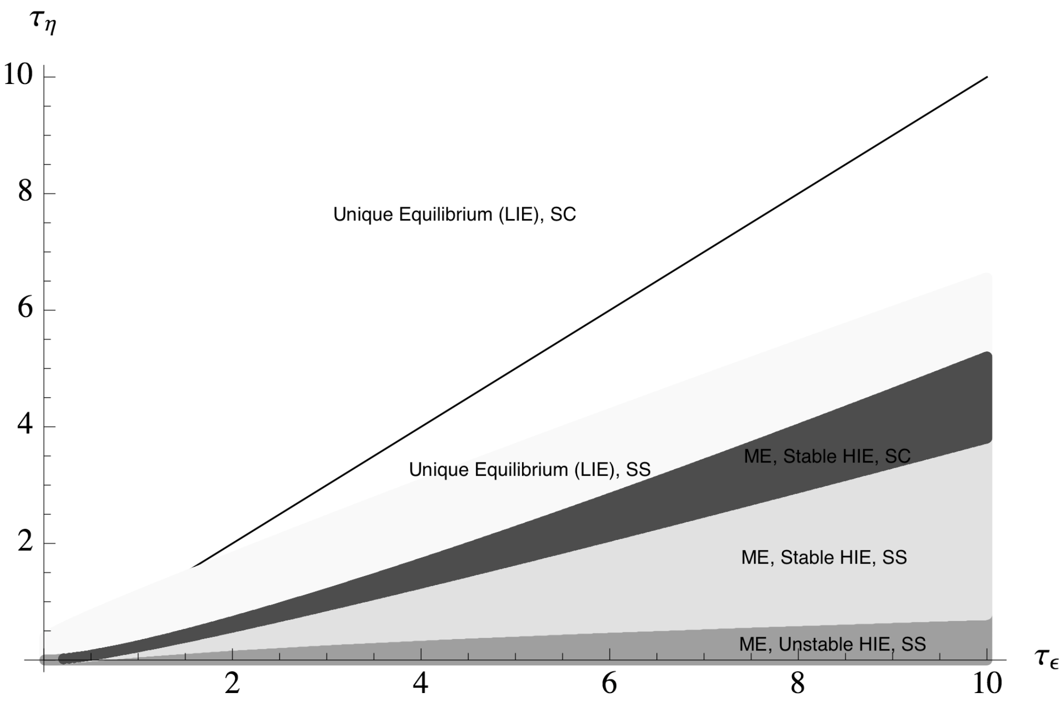
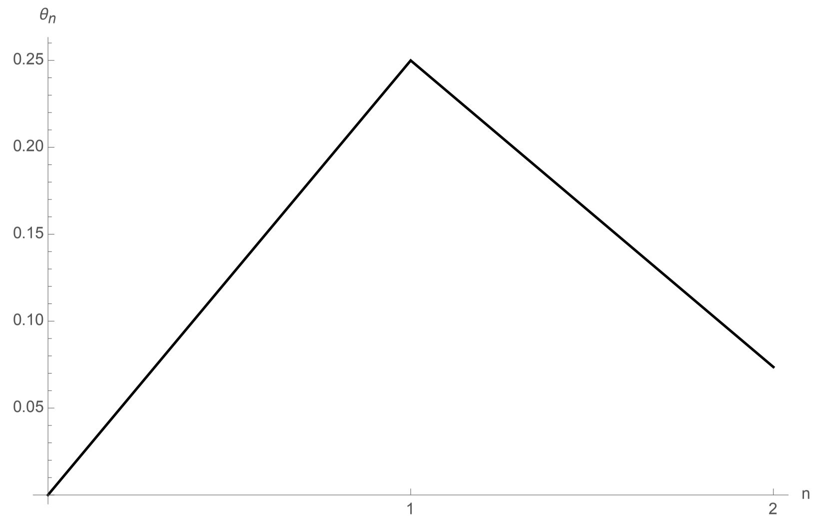
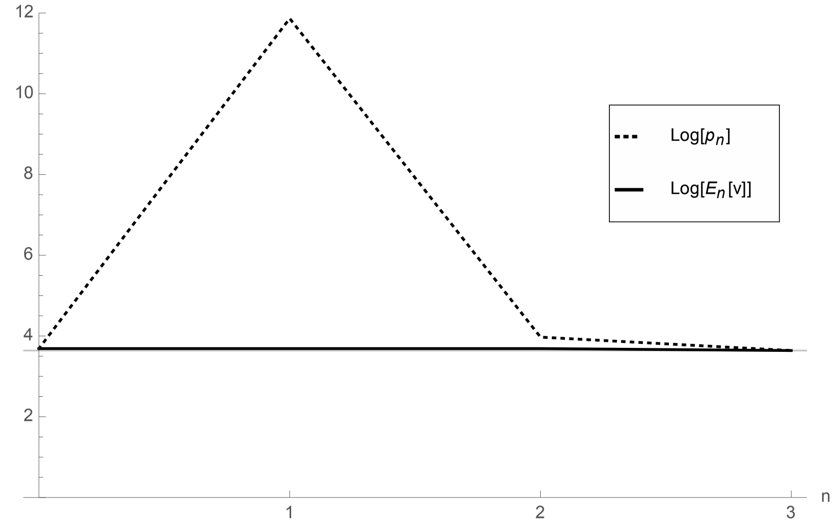


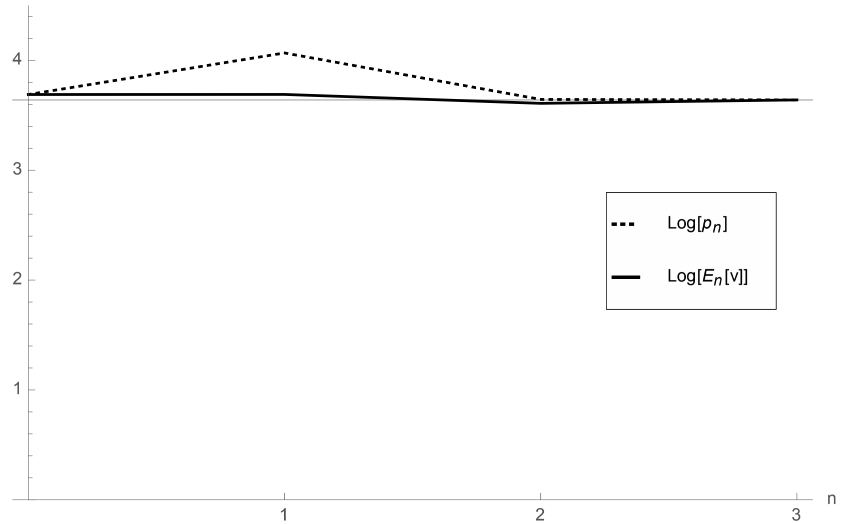
Figure 4: The equilibrium set for $\beta = 1$, $\gamma = 1/2$, $\tau_u = \tau_v = 10$, $\tau_\eta \in \{0, .01, \ldots, 10\}$, and $\tau_\varepsilon \in \{.01, .02, \ldots, 10\}$. The black line denotes the set $\tau_\eta = \tau_\varepsilon$. For values of $(\tau_\varepsilon, \beta)$ in the white and yellow regions, the equilibrium is unique, whereas multiple equilibria (ME) obtain when $(\tau_\varepsilon, \beta)$ are in the other regions, where the HIE can be stable or unstable depending on the difference between τ_η and τ_ε.



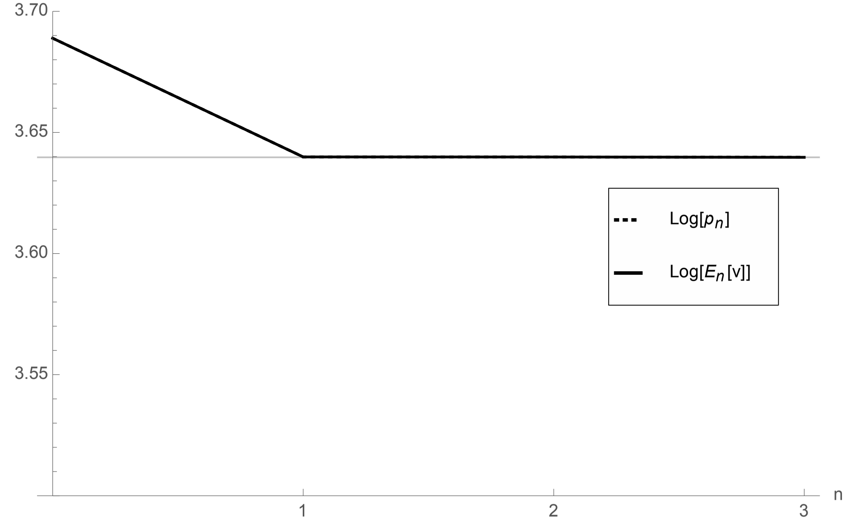
(a)



(b)



(c)



(d)

Figure 5: Liquidity traders' demand (panel (a)) and the path of prices and semi-strong efficient prices along the equilibrium with no private information, the LIE, and the HIE (respectively, panels (b), (c), and (d)). Parameter values are as follows: $\bar{v} = 40$, $\tau_v = .01$, $\tau_u = .1$, $\tau_\varepsilon = 1$, $\gamma = 1/2$, $\tau_\eta = .001862$, and $\beta = 1/2$.

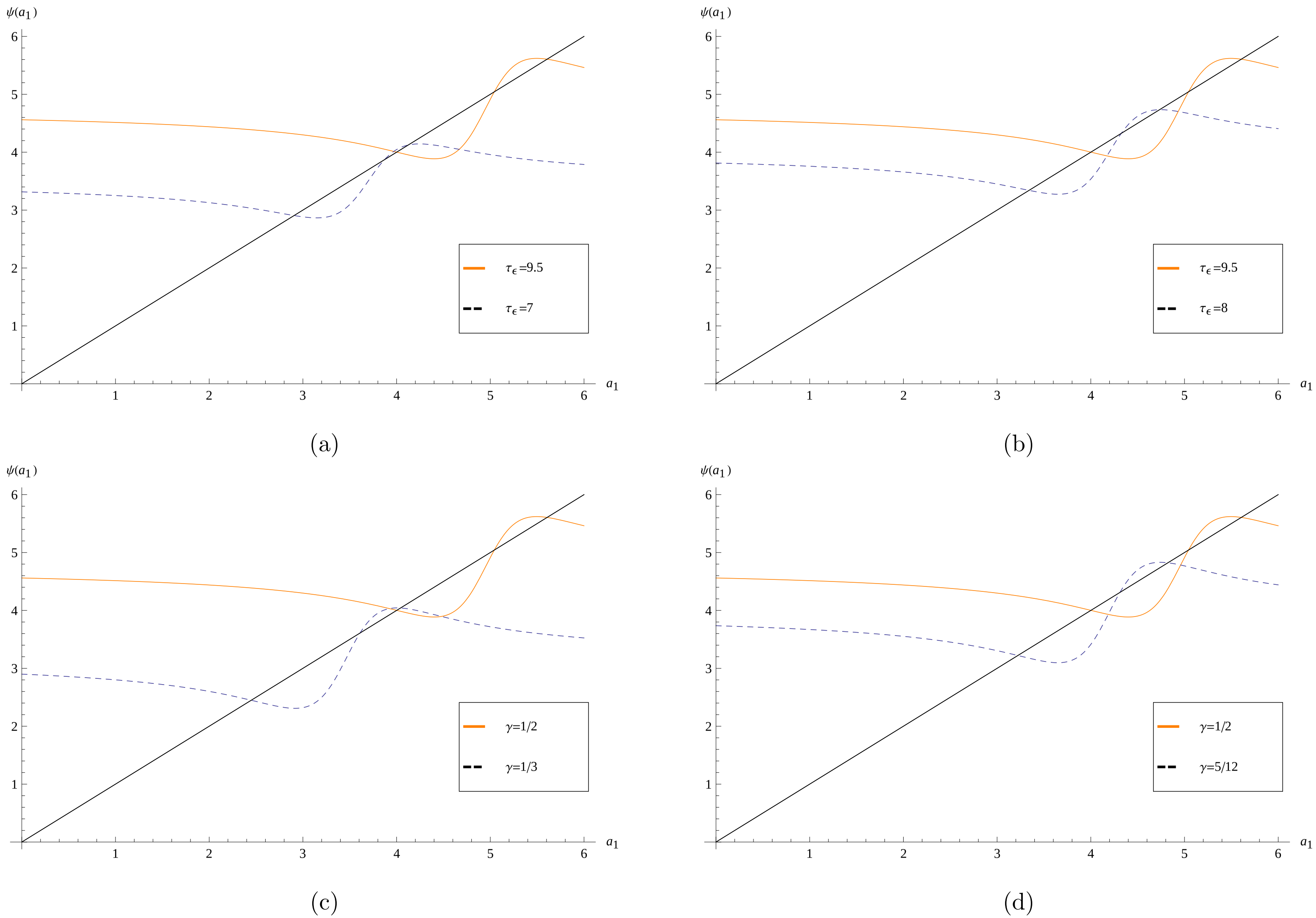
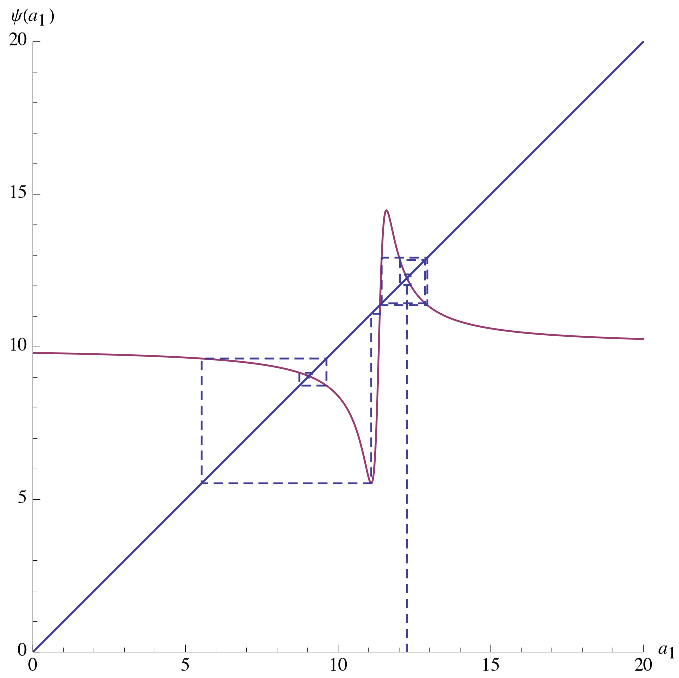
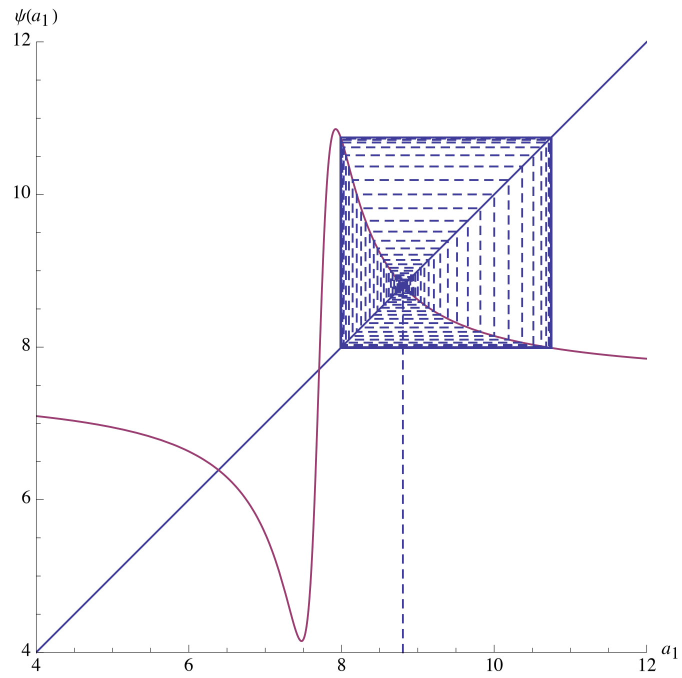


Figure 6: Adjustment to a shock when the HIE is stable. We plot the best response (22) for $\tau_v = \tau_u = 10$, $\gamma = 1/2$, $\tau_\eta = 3$, $\beta = 1$, and $\tau_\varepsilon = 9.5$ (orange). In panel (a) and (b) we analyze the effect of a reduction in the precision of the private signal to $\tau_\varepsilon = 7$ (dashed best response, panel (a)) and $\tau_\varepsilon = 8$ (dashed best response, panel (b)). In panel (c) and (d) we analyze the effect of a reduction in risk tolerance to $\gamma = 1/3$ (dashed best response, panel (c)) and $\gamma = 5/12$ (dashed best response, panel (d)).



(a)



(b)

Figure 7: Adjustment to a shock when the HIE is unstable. In panel (a) we plot the best response (22) for $\tau_v = \tau_u = 10$, $\tau_\varepsilon = 20$, $\gamma = 1/2$, $\tau_\eta = 1/2$, and $\beta = .9$. With these parameter values, the LIE and HIE are respectively $a_1^* = 9.04$, and $a_1^{***} = 12.25$, and the slope of the best response at these two points is given by $\psi'(a_1^*) = -.41$ and $\psi'(a_1^{***}) = -2.12$. In panel (b) we plot the best response (22) for $\tau_v = \tau_u = 10$, $\tau_\varepsilon = 15$, $\gamma = 1/2$, $\tau_\eta = 1/2$, and $\beta = 1$. With these parameter values, the LIE and HIE are respectively $a_1^* = 6.64$, and $a_1^{***} = 9.37$, and the slope of the best response at these two points is given by $\psi'(a_1^*) = -.43$ and $\psi'(a_1^{***}) = -1.9$.

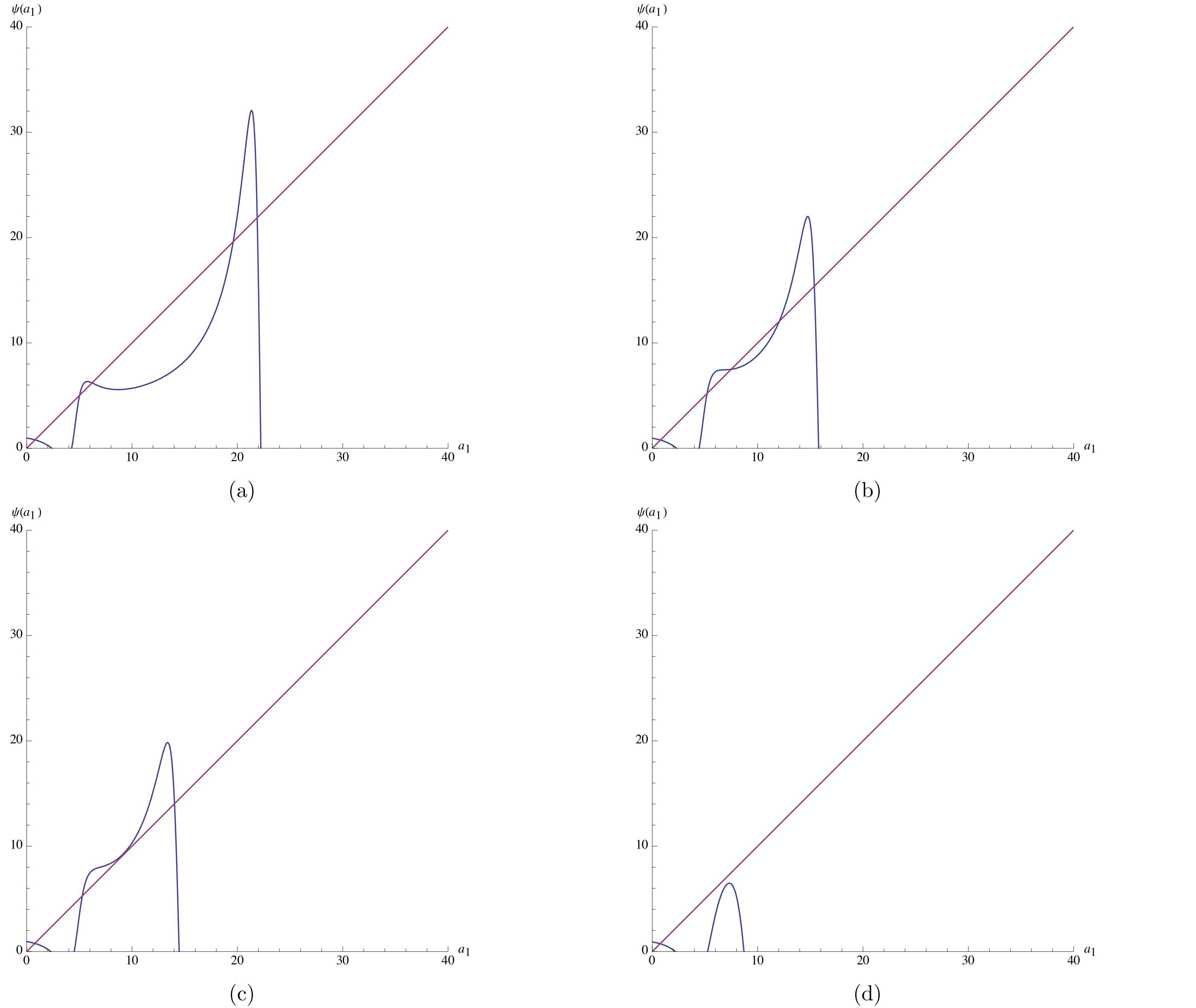


Figure 8: The best response mapping in the model with residual uncertainty. Parameters' values are $\tau_u = \tau_v = 1$, $\tau_\varepsilon = 4$, $\tau_\eta = 2/3$, $\gamma = 1$, and $\beta = 1$. When $\tau_\delta \in \{75, 100\}$, five equilibria obtain, only two of which are stable (respectively, panel (b), and (a)). When $\tau_\delta = 70$, only the LIE is stable (panel (c)). When $\tau_\delta = 40$, only the LIE survives (panel (d)).