On the effectiveness of natural hedging for insurance companies and pension plans

Jackie Li [a,*], Steven Haberman [b]

[a] *Department of Mathematics and Statistics, Curtin University, Australia*

[b] *Faculty of Actuarial Science and Insurance, Cass Business School, City University London, UK*

ABSTRACT

Natural hedging is one possible method to reduce longevity risk exposure for an annuity provider or a pension plan. In this paper, we provide an assessment of the effectiveness of natural hedging between annuity and life products, using the correlated Poisson Lee-Carter model, Poisson common factor model, product-ratio model, and historical simulation. Our analysis is based on the mortality experience of UK assured lives, pensioners, and annuitants, and the national population of England and Wales. We consider a range of different scenarios, and find that the level of risk reduction is significant in general, with an average of around 60%. These results have important implications for those insurers, reinsurers, and pension plan sponsors who are seeking ways to hedge their unwanted risk exposures.

Keywords: Longevity risk, natural hedging, Poisson Lee-Carter model, Poisson common factor model, product-ratio model, historical simulation, Value-at-Risk, risk reduction

* Corresponding author. Tel.: +618 92662399.

E-mail address: kakijackie.li@curtin.edu.au (J. Li).

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1. Introduction

Continual improvement in life expectancy poses a significant challenge to annuity providers and pension plan sponsors. These financial entities encounter the so-called longevity risk, which is the risk of paying more than expected due to an unanticipated decline in mortality. In principle, there are three possible approaches to manage longevity risk. The first is traditional reinsurance, in which the risk is transferred to a reinsurer for a premium. This option, however, is practically non-existent, as reinsurers appear to have limited appetite for longevity risk (Creighton et al., 2005). The second is capital market solutions, which include insurance securitization (e.g. Cowley and Cummins, 2005) and mortality-linked securities and derivatives (e.g. Coughlan et al., 2007b). In 2010, the Life and Longevity Markets Association (LLMA) was established in the UK by several insurance companies, reinsurance companies, and investment banks to promote the development of a liquid 'life market', where longevity- and mortality-linked securities and liabilities could be traded amongst insurers, reinsurers, and investors who want to diversify across an uncorrelated market sector. For instance, total pension liabilities of about 14 billion pounds were covered by longevity swaps during 2009 to 2011 in the UK (LCP, 2012). Despite these recent advances, the life market is still in its infancy stage and is far from reaching its full potential for providing diversification opportunities and enhancing market efficiency.

The third approach is natural hedging, which exploits the opposite movements in the values of annuities and life insurances when mortality changes. If mortality improves more than expected, an annuity book incurs losses while a life book makes profits. The situation is reversed if mortality improvement turns out to be lower than expected. This 'natural' offsetting effect allows an insurer to reduce uncertainty by selling both lines of products. For certain large insurance companies, it may be advantageous to adjust the business composition and utilise this hedging effect to lower the capital requirement. But for other institutions, this approach may be impractical and uneconomic, e.g. a pure annuity writer or a pension plan. Moreover, insurance companies in many countries have issued products with participation features (Cummins and Venard, 2007; Ernst & Young, 2013). For example, life insurers in Germany must distribute at least 90% of their annual profits to certain policyholders under the regulations. In the UK, bonuses are added to the policyholder accounts at the discretion of the insurers. For these cases, using the profits from one book to cover the losses from another book may not be feasible. Cox and Lin (2007) suggested that instead of implementing such an internal natural hedging scheme, different parties can enter into a mortality swap to perform an external hedge. Under this swap, an annuity provider (or a pension plan) pays floating cash flows to a life insurer based on the actual number of deaths in the life insurer's portfolio. In return, the life insurer pays floating cash flows to the annuity provider based on the actual number of survivors in the annuity provider's portfolio. Since this swap is not structured on a standardised index, it is an over-the-counter transaction and there is counterparty risk that one party may default.

A major concern about natural hedging is the existence of basis risk, which arises from mismatching between the characteristics of the two portfolios involved (e.g. Coughlan et al., 2011). Firstly, there is *age* basis risk, as the ages of an annuity portfolio are generally higher than those of a life insurance portfolio. Secondly, there is *population* basis risk, because the two groups of lives may come from different socioeconomic classes and may be subject to different underwriting requirements. Moreover, *maturity* basis risk is also present if the two portfolios have very different cash flow patterns, which depend on the types of the underlying products. Nevertheless, it should be noted that the presence of basis risk does not necessarily mean that natural hedging is ineffective. In fact, basis risk should be properly assessed via joint modelling (see Section 2) and can be managed through careful structuring of the product mix (see Sections 3 and 4).

The feasibility of natural hedging has received some attention in the literature. Milevsky and Promislow (2001) proposed that the option to annuitise in US variable annuities can be priced by constructing a replicating portfolio consisting of insurance, annuities, and bonds. They argued that the risk in annuities can be hedged by life policies. Dowd et al. (2006) suggested that survivor swaps can be used to help insurers rebalance their positions and exploit natural hedging opportunities. Gründl et al. (2006) considered a shareholder value maximisation framework and showed that natural hedging is optimal when equity is scarce. Bayraktar and Young (2007) assumed the same hazard rate for all buyers and deduced that pure endowment serves as a hedge to life insurance. Coughlan et al. (2007a) presented a case study and used historical simulation to show that the level of risk reduction is sizable, assuming the two populations are similar demographically. They also found that the result is more pronounced for a longer horizon, as noise becomes less important and the association between ages tends to increase over time. Based on historical improvement rates and mortality shocks, Cox and Lin (2007) demonstrated that a more balanced business mix reduces cash flow volatility. They further discovered that annuity writers who have more balanced business tend to charge lower premiums. Tsai et al. (2010) proposed a conditional Value-at-Risk (VaR) minimisation approach to optimise the product mix. Wang et al. (2010) set forth an immunisation model for determining the optimal product mix and devised mortality duration and convexity measures. Gatzert and Wesker (2012) included assets and liabilities in a dynamic framework and computed the optimal product mix with respect to default risk measures. Zhu and Bauer (2012) considered a non-parametric mortality model and found that higher order variations in death rates may affect the effectiveness of natural hedging. Cox et al. (2013) applied the ‘MV+CVaR’ approach to optimise the mean-variance trade-off of a portfolio of annuities and life insurance. Lin and Tsai (2013) and Tsai and Chung (2013) derived closed-form formulae of mortality duration and convexity to determine the weights of life insurance and annuity products in a portfolio. Chan et al. (2014) suggested that natural hedging often cannot work perfectly in practice because a certain part of longevity risk remains.

Despite these interesting findings, none of the work above models the mortality of annuitants (or pensioners) and assured lives jointly in an explicit manner and takes full account of basis risk. Without proper co-modelling of the two populations, it is difficult to produce a reasonable estimate of the hedging effectiveness. Recently, Wang et al. (2013)

used dependent standard Brownian motions to co-model the Lee-Carter mortality indices of the two populations, in order to find the optimal allocation. However, they took the observed death rates of Taiwan life insurance policies with heavy principal repayment as a proxy for annuitants' rates, which may not be truly reflective of actual annuity experience. They also considered only one way of joint modelling while the hedging effectiveness may depend considerably on the model assumed. In this paper, we attempt to fill these gaps in the current literature. First, we test a number of choices for modelling the two populations jointly, including the correlated Poisson Lee-Carter model, Poisson common factor model, product-ratio model, and historical simulation. This comparison is made to see if the performance of natural hedging varies significantly under different simulated environments. In particular, to our knowledge, we are the first one to adopt the product-ratio model in actuarial applications. Second, we use actual mortality experience of assured lives, pensioners, annuitants, and general population in England and Wales. The corresponding results are thus expected to be more reflective of the real situations insurers and pension plans are actually facing. Furthermore, we examine a range of scenarios and product features, in order to find out how sensitive our results are with respect to certain changes in the settings and assumptions. It is important to check the robustness of the results and implications.

This paper is organised as follows. In Section 2, we provide an outline of some mortality models for handling annuity and life portfolios jointly. In Section 3, we discuss the effectiveness of natural hedging in terms of the level of risk reduction, based on industry mortality data and different ways of joint modelling. In Section 4, we make a number of changes to the initial settings and assumptions and investigate the resulting effects on the hedging performance. Finally, we state our concluding remarks in Section 5.

2. Joint modelling

Hitherto, most of the mortality modelling work has either treated each population separately (e.g. by gender, country) or simply considered the whole population in aggregate. There has been relatively little attention paid to modelling two related populations simultaneously in a systematic manner. In order to allow for (age and population) basis risk properly, we experiment with the following joint modelling techniques. A more detailed study of joint mortality models can be found in Li et al. (2014).

The first one we consider here is the first approach proposed by Carter and Lee (1992), in which the Lee and Carter (1992) model is fitted to each population separately and then dependence between the two mortality indices is measured in some way. Since the Lee-Carter mortality index is modelled by a random walk with drift in many applications, a bivariate random walk with drift can be similarly assumed for the case of having two concurrent indices. Mathematically, this approach can be expressed as

$$\ln m_{x,t,i} = \alpha_{x,i} + \beta_{x,i}\,\kappa_{t,i}; \qquad \text{(Lee-Carter model)}$$

$$\mathrm{K}_t = \Theta + \mathrm{K}_{t-1} + \Delta_t, \qquad \text{(bivariate random walk with drift)} \qquad (1)$$

in which $i = 1, 2$ refer to the two populations under study, $m_{x,t,i}$ is the central death rate at age x in year t, $\alpha_{x,i}$ is the overall mortality schedule, $\kappa_{t,i}$ is the mortality index with $\beta_{x,i}$ as the sensitivity measure, $\mathrm{K}_t = (\kappa_{t,1}, \kappa_{t,2})'$, Θ is the vector drift term, and Δ_t is the vector error term of the random walk. Alternatively, the third approach in Carter and Lee (1992) is to model the two mortality indices jointly as a co-integrated process (e.g. Li and Hardy, 2011). In a similar vein, Cairns et al. (2011) and Dowd et al. (2011) co-modelled the period and cohort parameter series of the age-period-cohort (APC) model between a large population and a small subpopulation, using time series models and gravity models respectively. Yang and Wang (2013) treated the Lee-Carter residual terms as correlated between several countries and also fit a vector error correction model (VECM) to the multiple mortality indices. Zhou et al. (2013, 2014) assumed the same age-specific sensitivity for two populations and co-modelled the two Lee-Carter mortality indices with time series processes. Since these alternatives apply similar procedures, i.e. fitting the same model to each population and co-modelling the two (or more) associated parameter series, we adopt only the Lee-Carter model with bivariate random walk for analysing the hedging performance. Note that instead of assuming homoscedastic error terms as in the initial Lee-Carter method, we estimate the parameters by treating the number of deaths as a Poisson variable and following the iterative updating scheme in Brouhns et al. (2002). We refer to this choice as the correlated Poisson Lee-Carter model (cPLCM) below.

On the other hand, the second approach in Carter and Lee (1992) is to estimate a single, common mortality index for both populations. While this way may give greater consistency between the two populations and is parsimonious, it does not appear to be a suitable option for measuring the hedging effectiveness, as it implicitly assumes that the two groups' mortality levels are perfectly associated and will lead to an underestimation of basis risk. As an extension, Li and Lee (2005) proposed the augmented common factor model, which is composed of a common factor for both groups of lives as a whole, as well as an additional factor specifically assigned for each group. The overall approach is stated as

$\ln m_{x,t,i} = \alpha_{x,i} + B_x\, K_t + \beta_{x,i}\, \kappa_{t,i}$; (augmented common factor model)

$K_t = \mu + K_{t-1} + e_t$; (random walk with drift)

$\kappa_{t,i} = \delta_{0,i} + \delta_{1,i}\, \kappa_{t-1,i} + \omega_{t,i}$, (AR(1) process) (2)

where $B_x\, K_t$ is the common factor for both groups, $\beta_{x,i}\, \kappa_{t,i}$ is the additional factor for group i, K_t is the mortality index of the common factor with B_x as the sensitivity measure, and $\kappa_{t,i}$ is the time component of the additional factor with $\beta_{x,i}$ as the sensitivity measure. The mortality index K_t is assumed to follow a random walk with the drift term μ and the error term e_t. The time component $\kappa_{t,i}$ is assumed to follow an autoregressive model of order one, AR(1), with $\delta_{0,i}$ and $\delta_{1,i}$ as the parameters and $\omega_{t,i}$ as the error term. (The terms $m_{x,t,i}$ and $\alpha_{x,i}$ have the same meanings as before.) The use of weakly stationary AR(1) processes ensures that the projected (central estimate) ratio of death rates (i = 1 vs. i = 2) tends to a constant at each age, i.e. the forecasts are 'coherent'. This model explicitly differentiates between the joint effect and specific effect, and we include it in our hedging assessment. We follow Li (2013) and use the Poisson assumption to estimate the parameters, and call this Poisson common factor model (PCFM). Delwarde et al. (2006) and Debón et al. (2011) also incorporated a common factor for the combined population and specific factors for each population. In a similar way, Hatzopoulos and Haberman (2013) constructed a common model for a number of countries in aggregate together with a sex difference (age-period) model and a residual model for each sex and country. Alternatively, Russolillo et al. (2011) adopted a three-way structure which multiplies the common factor above with a group-specific or country-specific parameter.

Finally, we test the product-ratio model (PRM) proposed by Hyndman et al. (2013) in the field of demography. This model exploits the fact that the sum of and the difference between two random components will roughly be uncorrelated if the two components have approximately equal variances. For example, suppose A and B are two random variables, and it can be deduced that $\mathrm{Cov}(A+B\,, A-B) = \mathrm{Var}(A) - \mathrm{Var}(B) \approx 0$ if $\mathrm{Var}(A) \approx \mathrm{Var}(B)$. We apply the PRM to the central death rates as

$p_{x,t} = \sqrt{m_{x,t,1}\, m_{x,t,2}}$; (square root of product of death rates)

$r_{x,t} = \sqrt{m_{x,t,1}/m_{x,t,2}}$; (square root of ratio of death rates)

$\ln p_{x,t} = \eta_x + \phi_x(1)\lambda_t(1) + \phi_x(2)\lambda_t(2) + ... + \phi_x(g)\lambda_t(g)$; (product model)

$\ln r_{x,t} = \xi_x + \psi_x(1)\gamma_t(1) + \psi_x(2)\gamma_t(2) + ... + \psi_x(h)\gamma_t(h)$; (ratio model)

$\lambda_t(j) = \tau_0(j) + \tau_1(j)\lambda_{t-1}(j) + \nu_t(j)$; (AR(1) process)

$\gamma_t(j) = \iota_0(j) + \iota_1(j)\gamma_{t-1}(j) + \varepsilon_t(j)$. (AR(1) process) (3)

The quantities $p_{x,t}$ and $r_{x,t}$ are the square roots of the products and ratios of the central death rates. Assuming the two groups' log death rates have roughly equal variances, which is often the case when the two sets of data have about the same volume, $\ln p_{x,t}$ and $\ln r_{x,t}$ are more or less uncorrelated. We can then apply principal component analysis (PCA) to each of them separately to estimate the parameters (η_x, $\phi_x(j)$, $\lambda_t(j)$ of product model; ξ_x, $\psi_x(j)$, $\gamma_t(j)$ of ratio model). Otherwise, if the two data volumes are very different, treating the product and ratio models separately would be imprecise and serve at best as a rough approximation. We follow Hyndman et al. (2013) in setting $g = h = 6$, as they found that having more than six factors makes almost no difference to the resulting forecasts but having too few reduce forecast accuracy. In our analysis, the parameter $\tau_1(1)$ is set to one, i.e. $\lambda_t(1)$ is assumed to follow a random walk with drift. The other time components $\lambda_t(j)$ for $j \geq 2$ and $\gamma_t(j)$ for $j \geq 1$ are assumed to be AR(1) processes, with $\tau_0(j)$, $\tau_1(j)$, $\iota_0(j)$ and $\iota_1(j)$ as the parameters and $\nu_t(j)$ and $\varepsilon_t(j)$ as the error terms. Using weakly stationary AR(1) processes for the ratio model makes sure that the projected ratio of death rates ($i = 1$ vs. $i = 2$) converges at each age. Note that when only one set of data is available for a certain age range, the product model is applied directly to the death rates available while the ratio model is dropped for that age range. Earlier, Plat (2009) took a fairly similar approach, in which the ratio of death rates (portfolio vs. population) is expressed as a function of age-time factors. Jarner and Kryger (2011) also modelled the log ratio of death rates (small population vs. reference population) with a number of age-time factors. Ngai and Sherris (2011) applied a simple linear model to the ratio of death rates (annuitants vs. population). In a recent work, Hatzopoulos and Haberman (2013) formulated the log ratio of death rates (females vs. males) in terms of age and time effects. Villegas and Haberman (2014) used age and period parameters for the log ratio of death rates (modelling the experience of socio-economic groups relative to reference population).

There are a few more things to note before we present our results in the next two sections. Firstly, all the error terms above are assumed to be uncorrelated with one another and across time. Secondly, the length of our industry data is not very long and an AR(1) process would suffice in general. Other time series models with higher orders or more complicated structures may be used if there is more data. In the Appendix, we provide further analysis on the various time series produced from the three models using the data as discussed in the next section. Moreover, while we focus on discrete-time mortality models, there are a couple of continuous-time models that have been tested for modelling related populations jointly. Cox et al. (2006) described each population's mortality as a combination of a Brownian motion and a compound Poisson process and assumed the Brownian motions of different populations are correlated. Barbarin (2008) adopted the Heath-Jarrow-Morton (HJM) framework to model longevity bond prices and allow for basis risk. Dahl et al. (2008, 2011) used Cox-Ingersoll-Ross (CIR) processes for modelling mortality intensities of an insurance portfolio and a large population. Lastly, historical simulation (e.g. Coughlan et al., 2011) can also be used for analysing the hedging performance. Although the data volume is limited, we apply this method in Section 4.

3. Hedging effectiveness

In this section, we provide an assessment of the potential effectiveness of natural hedging based on empirical mortality data. These data include the experience of male assured lives, pensioners of insured pension schemes, and annuitants collected from the Continuous Mortality Investigation (CMI). These three groups are subject to different underwriting requirements and represent three distinct sets of experience. The pensioners and annuitants data are only available for the period of 1983-2006, so we take this as our sampling period. We use the age ranges of 30-95 for assured lives, 60-95 for pensioners, and 65-95 for annuitants, as the data outside these ranges are sparse and unsuitable for modelling and projection. Figure 1 plots the central death rates of three age groups over the period. In general, the assured lives have the lowest mortality, followed by the annuitants and then the pensioners. Because the annuitants data volume is much smaller, their death rates appear to be more volatile. It can be seen that the declining trends of assured lives, pensioners, and annuitants are roughly in line with one another, which suggests some degree of statistical dependence between them. In particular, for ages 65-74, the correlations in the rates of change of death rates are 0.13 between the assured lives and pensioners and 0.04 between the assured lives and annuitants; for ages 75-84, these correlations are 0.10 and 0.12; and for ages 85-94, the correlations are –0.11 and 0.08. Half of these figures are statistically significant at the 10% significance level and one is significant at the 5% level. It appears necessary to co-model these populations in a systematic fashion and take basis risk into account properly.

Consider a pension plan sponsor or an insurer selling annuities who wants to reduce its risk level by implementing either an internal or external hedge. Suppose there are only two products: a whole-life pension payable from age 65 or a life annuity issued at age 65, and a life insurance written at age 35, all on 1 January 2007. The pension (annuity) size is $1,400 p.a. ($1,300 p.a.) payable at the end of each year, and the sum assured of the life insurance is $100,000 payable at the end of the year of death. These amounts are set in such a way that the expected present values on 1 January 2007 of the two 'offsetting' products are approximately equal, so that the effects of different portfolio compositions can readily be compared, given a fixed total number of policies. All the premiums are paid at the inception of the contract and so there are no future contributions outstanding. The interest rate is assumed to be 3% p.a. flat. The limiting age is set as 95, based on the age ranges of the CMI data being investigated. This simplifying assumption is adequate for our purposes and avoids the necessity of choosing a realistic limiting age and introducing a corresponding 'topping out' technique for interpolating between age 95 and the limiting age (e.g. Renshaw and Haberman, 2003). We set up this rather hypothetical situation in order to highlight the hedging effect between a pure survival benefit and a pure death benefit for the whole age range. In reality, there is a growing number of centenarians; also, term life insurance and endowment assurance are more popular than whole life insurance in several countries.

To start with, we fit the cPLCM, PCFM, and PRM (models (1) to (3)) discussed in Section 2 to the CMI data. We find that the mean absolute percentage error (MAPE) values of the fitted log death rates are all less than 5%, and that there are no significant effects present in the residuals along age, calendar year, and cohort year. These results indicate that the three models provide a reasonable description of the CMI data. For the assured lives and pensioners data, the Bayesian Information Criterion (BIC) values for the three models are 22,603, 23,256, and 28,592 respectively. For the assured lives and annuitants data, the BIC values are 19,281, 19,753, and 24,804. These values suggest that the cPLCM provides the best fit (in terms of goodness-of-fit and also parameter parsimony) to the data. One should note, however, that the BIC is only one of the many statistical criteria, and that a good fit to past data does not necessarily mean that the corresponding projections into future are sensible. Figures 2 to 5 illustrate the parameter estimates of these models for the assured lives and pensioners data. As shown, the major time components are highly linear, which suggest that a random walk with drift is suitable for projection. The other time components demonstrate some cyclical or irregular patterns and can be modelled and projected with an AR(1) process. For the combination of assured lives and annuitants data, the time components (not shown here) have similar characteristics. (See Appendix for more details.)

Suppose there are a total of 100,000 policies. We consider 101 portfolio compositions of pension (or annuity) and life products, in which the weight of life policies w increases by 1% each time from 0% to 100%. For each product mix, we use the fitted models to generate 5,000 scenarios of future death rates, and adopt the binomial distribution to sample the number of survivors in each future year, which determine the cash flows of the portfolio. We then compute 5,000 samples of the total present value of the portfolio for each case. Similar to Coughlan et al. (2011), we define the level of longevity risk reduction as 1 – Risk ($w = j$) / Risk ($w = 0\%$) for j = 0% to 100%, in which the risk metrics considered include the standard deviation, variance, and 95% and 99% Value-at-Risk (VaR) (minus the mean) of the total present value of the portfolio.

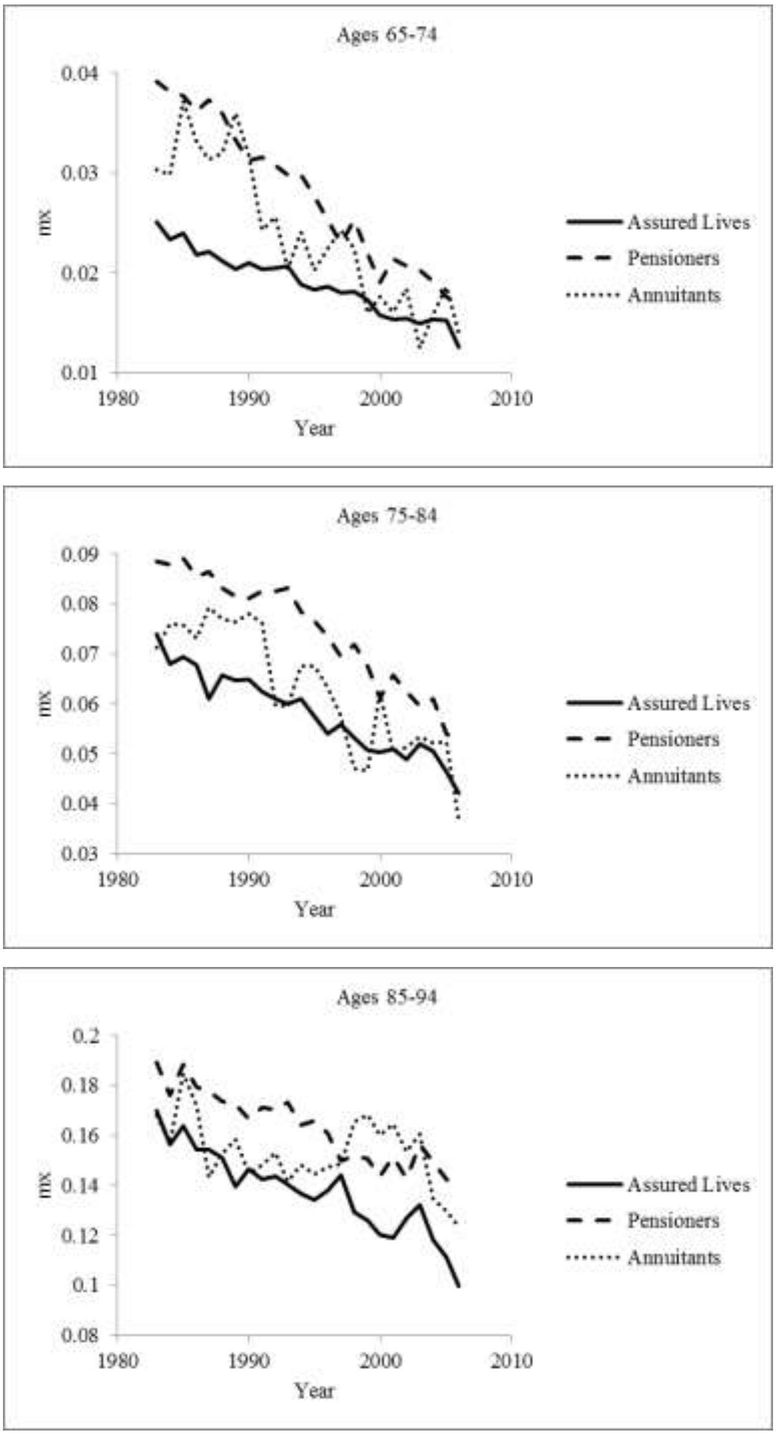

Fig. 1. Central death rates of male assured lives, pensioners, and annuitants in UK, 1983-2006.

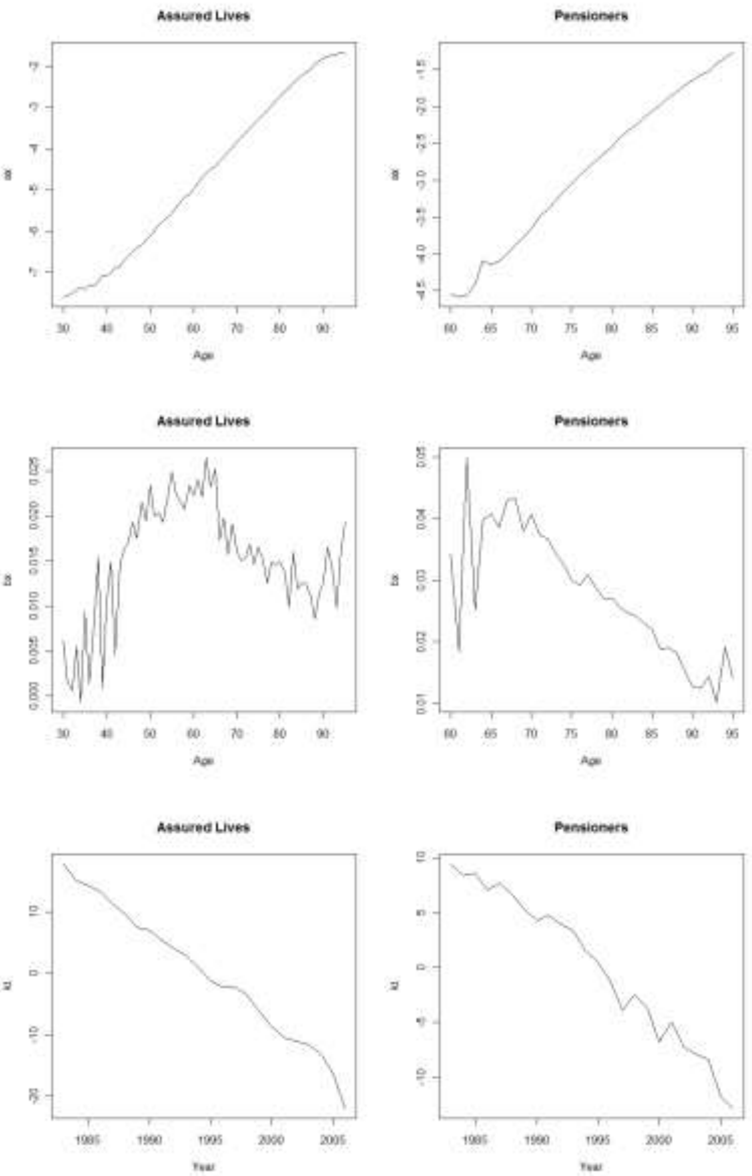

Fig. 2. Parameter estimates of $\alpha_{x,i}$ (top), $\beta_{x,i}$ (middle), and $\kappa_{t,i}$ (bottom) of cPLCM for assured lives (left) and pensioners (right).

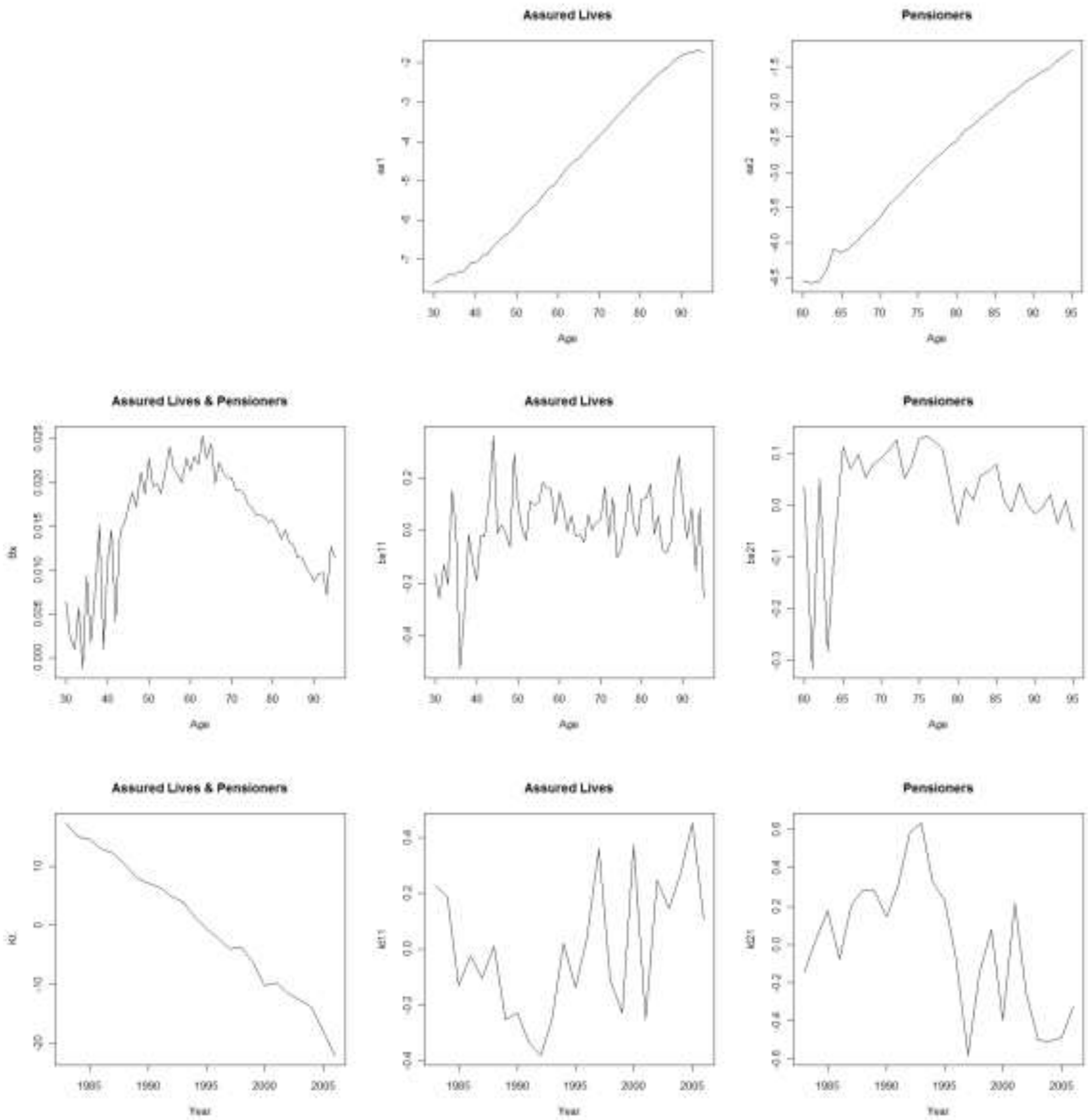

Fig. 3. Parameter estimates of $\alpha_{x,i}$ (top), B_x or $\beta_{x,i}$ (middle), and K_t or $\kappa_{t,i}$ (bottom) of PCFM for common effect (left), assured lives' effect (middle), and pensioners' effect (right).

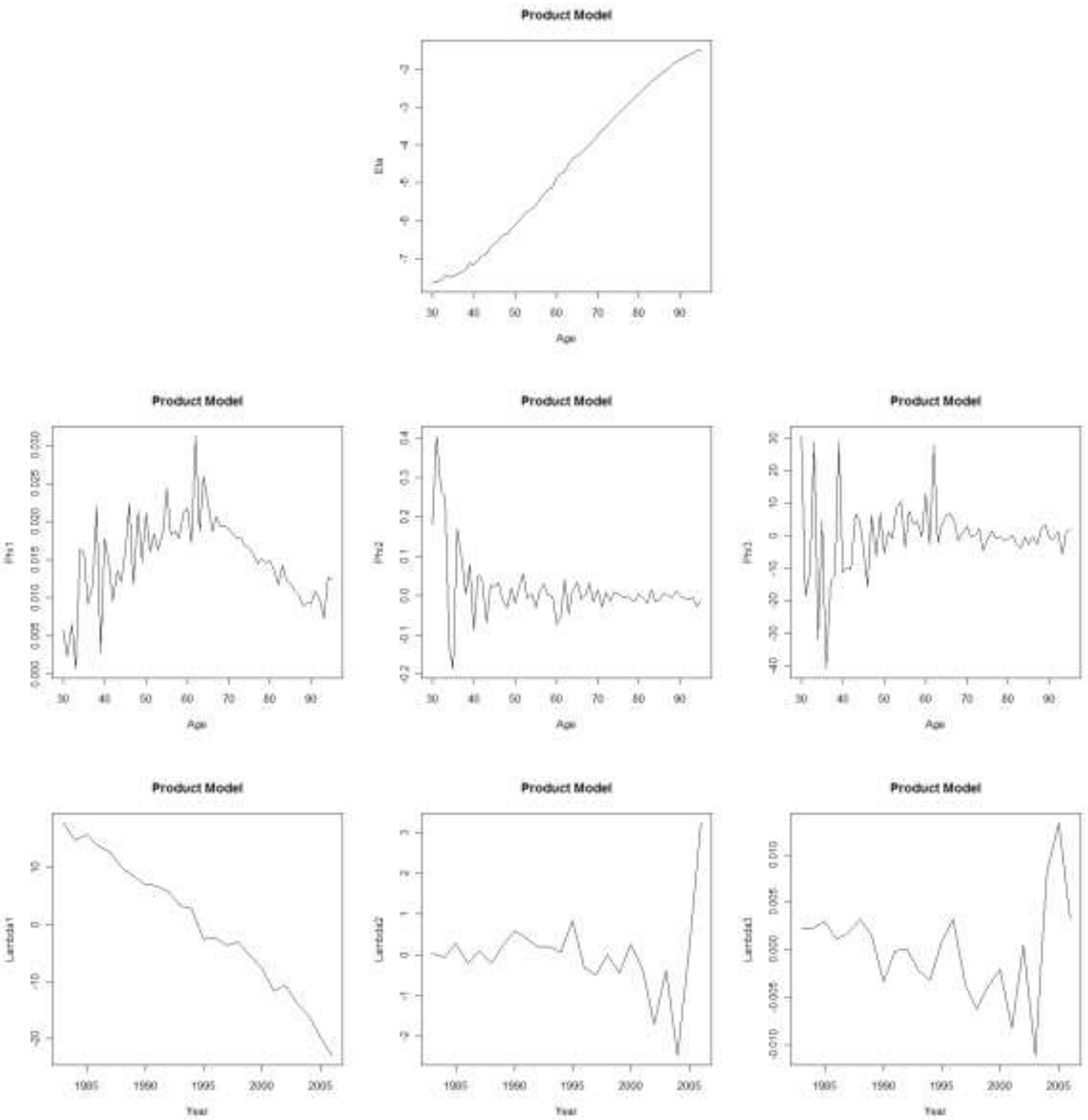

Fig. 4. Parameter estimates of η_x (top), $\phi_x(j)$ (middle), and $\lambda_t(j)$ (bottom) of product model of PRM for j = 1, 2, 3 (left to right of last two rows).

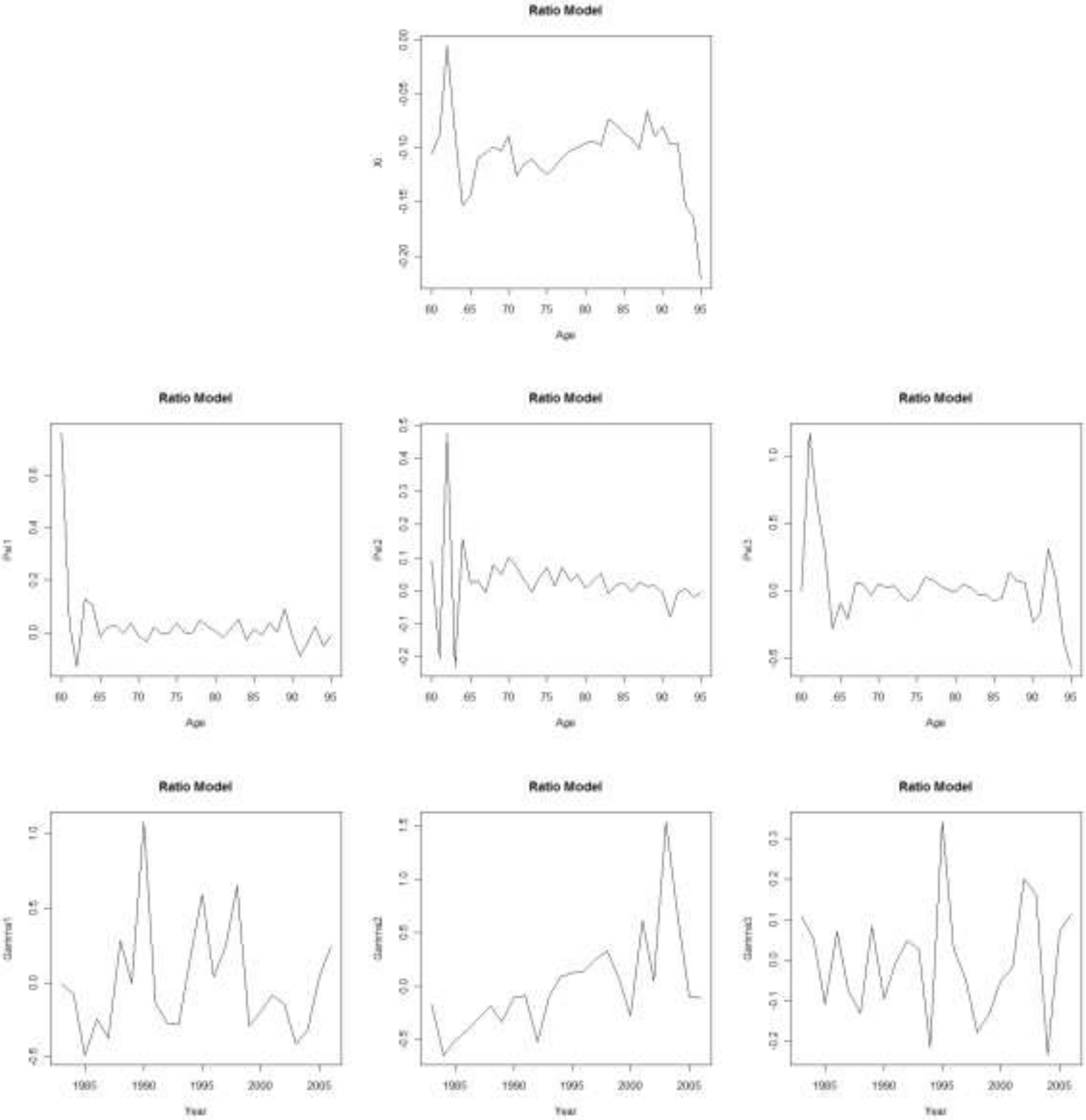

Fig. 5. Parameter estimates of ξ_x (top), $\psi_x(j)$ (middle), and $\gamma_t(j)$ (bottom) of ratio model of PRM for j = 1, 2, 3 (left to right of last two rows).

Table 1

Maximum levels of longevity risk reduction for a portfolio of 100,000 policies at 3% interest rate.

Pension and life products				
Model	SD	Variance	95% VaR	99% VaR
cPLCM	49%	74%	44%	45%
PCFM	62%	86%	61%	59%
PRM	62%	85%	58%	59%
Annuity and life products				
Model	SD	Variance	95% VaR	99% VaR
cPLCM	64%	87%	56%	53%
PCFM	53%	78%	54%	54%
PRM	54%	78%	49%	48%

Table 1 lists the maximum levels of longevity risk reduction for different models and risk metrics, based on the simulated samples of the portfolio compositions under consideration. There are a number of insightful observations. First, the risk reduction is clearly less than 100%, because basis risk exists between the two lines of products. The level of maximum reduction ranges from 44% to 87%. In this study, the ages, mortality experience, and cash flow patterns of the two groups are different, which reflect the likely situation in reality. Second, the risk reduction is larger for the variance than the other risk metrics. This result has an important implication about the construction of a hedge in practice. As the choice of risk metric has a significant impact on the assessment of the hedging effectiveness, one has to be careful in choosing an appropriate risk metric when setting the hedging objectives, which in turn directs the selection, structuring, and calibration of hedging instruments. Third, the results of the PCFM and PRM are quite close to each other, but those of the cPLCM are somewhat different. This effect may arise from the fact that both the PCFM and PRM involve stationary time series processes and produce coherent forecasts in the long run, whereas the cPLCM works with a bivariate random walk with drift and has a tendency to generate divergent values. The necessity of having coherent forecasts depends on the time horizon. While there can be some extent of divergence or fluctuation in the short term, it is natural to expect that the death rates of different groups will move more consistently in the long term. Our projection period is 60 years and hence it seems that the PCFM and PRM are more suitable in this sense. Lastly, though the level of risk reduction varies for different models and risk metrics, it is around 60% overall, which is still a sizable amount. The natural hedging strategy is feasible if the benefits of risk reduction outweigh the costs of implementing it, such as adjusting the business composition, entering into mortality swaps, and employing expertise to design the hedging programme. It can be used simultaneously with traditional reinsurance and other longevity-linked securities. Even if this strategy turns out to be too costly for a particular institution, the effect of the existing business composition still needs to be taken into account properly in the reserving and capital calculations, e.g. under Solvency II requirements.

Figure 6 shows the coefficient of variation, and the 95% and 99% VaR (minus the mean) as a percentage of the mean, of the total present value of each portfolio composition of pension and life products. Under the PCFM and PRM, the 'optimal' product mix is to have a weight of about 45% in life policies; under the cPLCM, the optimal proportion of life policies is around 65%. These results give some important insights into the calibration exercise. First of all, the optimal mix is model-dependent, and the similarities and differences in the results between the three models are in agreement with the fact that both the PCFM and PRM furnish coherent forecasts while the cPLCM does not. Although the model choice can be subjective to some extent, it may be argued (as above) that the PCFM and PRM are more appropriate for a long projection period. Moreover, it is interesting to see that even when a 'wrong' model is selected to determine the optimal mix, the resulting hedging effectiveness may still be considerable. For example, if the optimal proportion is 'incorrectly' taken as 65% based on the cPLCM but the real situation is actually reflected by the PRM, it can be seen from Figure 6 that the risk level is still reduced by about 40%. Overall, the major implication in this hypothetical study is that an optimal allocation does exist under some fairly realistic conditions and assumptions. It suggests that a financial institution with a more balanced business mix would be subject to lower variability in its liabilities compared to one with less diversified compositions. In practice, it is advisable to test several different models with varying features and obtain a more comprehensive view of the matter; and as noted above, one has to examine carefully the benefits and various costs of building such a hedge. Similar patterns are observed for the portfolios of annuity and life products (the detailed results of which can be provided upon request).

Note that the PRM requires an assumption that the two populations' log death rates have approximately equal variances, so that the product and ratio models can be treated separately. We find that the variances of the log death rates are 0.81, 0.82, and 1.38 respectively for the assured lives, pensioners, and annuitants data, for ages 65-95 during 1983-2006. (In particular, for ages 65-74, the variances are 0.15, 0.18, and 1.00; for ages 75-84, the variances are 0.13, 0.12, and 0.29; for ages 85-94, the variances are 0.07, 0.07, and 0.14; during 1983-1994, the figures are 0.75, 0.69, and 0.92; and during 1995-2006, the figures are 0.84, 0.90, and 1.77.) The annuitants data have a larger variance because the volume is much smaller. As such, the PRM is suitable for modelling the assured lives and pensioners data, but treating the product and ratio models independently for the combination of assured lives and annuitants data would represent an arbitrary assumption and serve instead as a practical approximation. Note also that the values of the risk metrics produced by the cPLCM are larger than those by the other two models when the proportion of life policies is small, while the situation is opposite when the life product dominates the portfolio (see Figure 6). Moreover, despite the similarity in the risk reduction results between the PCFM and PRM as discussed above, the values of the risk metrics obtained from the former are consistently smaller than those from the latter. These differences highlight the presence of model uncertainty and the necessity to compare results from a range of models in practical work.

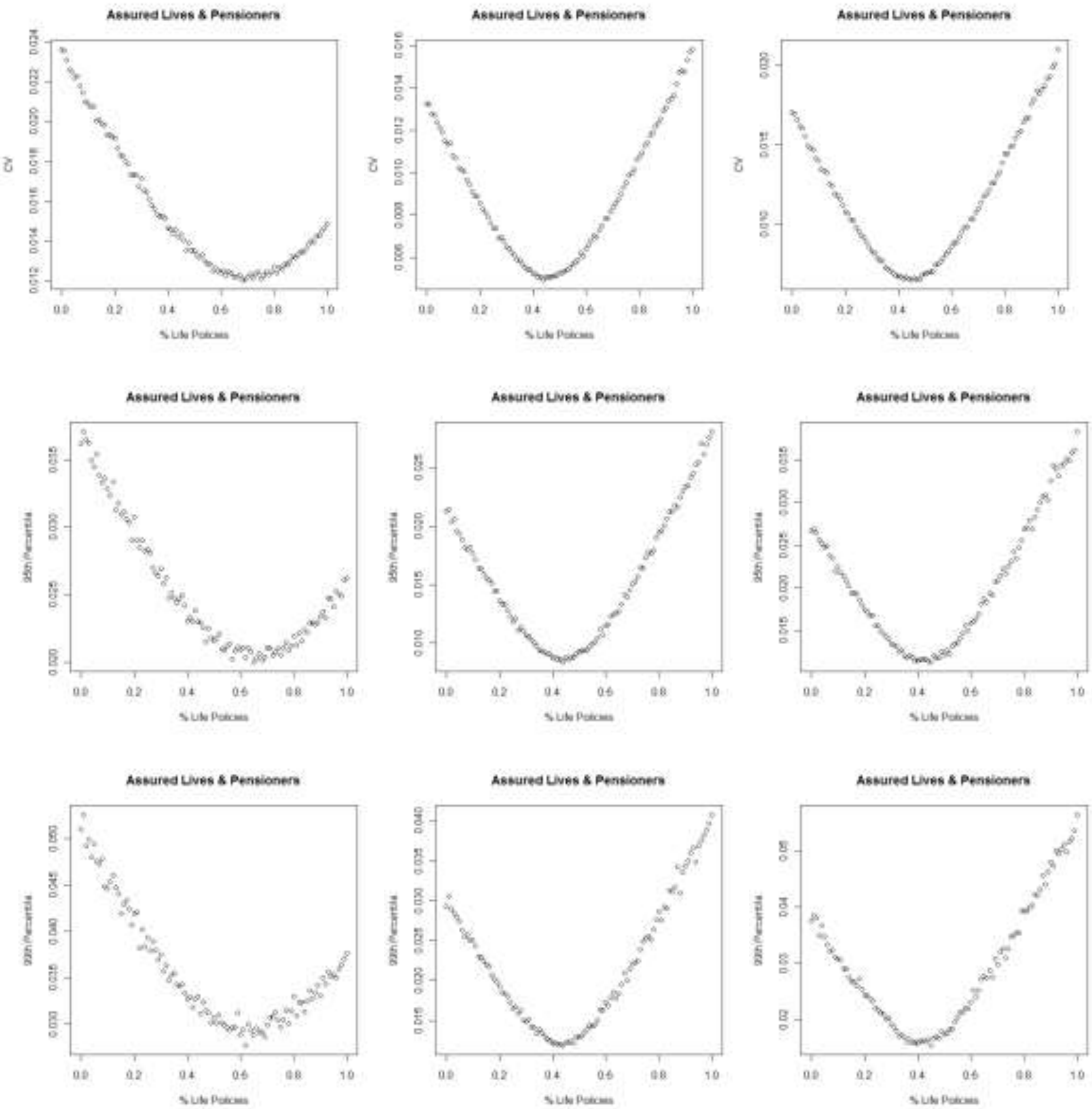

Fig. 6. Coefficient of variation (top) and 95% (middle) and 99% (bottom) VaR of total portfolio present value for different compositions of pension and life products (0% to 100%) under cPLCM (left), PCFM (middle), and PRM (right).

4. Sensitivity analysis

In this section, we consider a number of changes in the initial settings and assumptions and examine the resulting effects on the hedging performance. These changes include the portfolio size, interest rate, population experience, product features, and simulation procedure. Through these sensitivity testings, we can have a better idea of how robust the hedging results are under different circumstances.

If the portfolio size is small, there is a risk that the overall portfolio experience turns out to be too different to the underlying true mortality levels. This discrepancy may have a material impact on the hedging effectiveness. Table 2 shows that when the number of policies is reduced from 100,000 to 10,000, the risk reduction is generally not much smaller; but Table 3 demonstrates that when there are only 1,000 policies, the average risk reduction is lowered from 60% to less than 40%. These results suggest that when the portfolio size is too small (say, below 1,000 policies), the effect of natural hedging can be rather limited.

Next we consider two other interest rates of 2% p.a. and 4% p.a. The pension and annuity sizes are adjusted to ensure that the expected present values of the two hedging sides are roughly equal. Tables 4 and 5 present the levels of risk reduction for these two new interest rates. At 2% p.a., the risk reduction is clearly larger and the average level is above 70%. The situation is reversed at 4% p.a. and the risk reduction is about 50% on average. This phenomenon may be due to the fact that the cash flows of both sides are discounted less at a lower interest rate and so are more variable, under which the offsetting effect becomes more obvious.

As mentioned earlier, Cox and Lin (2007) proposed a mortality swap to construct an external hedge, in which the pension plan sponsor or annuity provider pays floating cash flows linked to the number of deaths in the life portfolio, and the life insurer pays floating cash flows linked to the number of survivors in the pension plan or annuity portfolio. This swap is not a standardised contract, and so there is counterparty risk that one side may default. As the life market develops and becomes more mature, it may be possible to build a special purpose vehicle (e.g. Cairns et al., 2008) that transacts with each party separately without the two parties dealing with each other directly. For example, on one side, the pension plan sponsor pays floating cash flows to the special purpose vehicle based on the number of deaths of the reference population, and the special purpose vehicle returns floating cash flows based on the number of survivors in the pension plan. On the other side, a similar transaction is conducted between the life insurer and the special purpose vehicle. To explore this possibility, we replace the assured lives data with England and Wales population data (taken as reference population) collected from the Human Mortality Database (HMD, 2013) and repeat the simulations. Table 6 shows that the corresponding risk reduction is on average 8% smaller than the figures in Table 1. This decrease implies higher population basis risk, which appears to arise from the heterogeneity of the population data. In reality, the payments made by the pension plan sponsor to the special purpose vehicle may be simplified as observed death rate minus forward death rate (like a *q*-forward), in which proper calibration is necessary.

Table 2

Maximum levels of longevity risk reduction for a portfolio of 10,000 policies at 3% interest rate.

		Pension and life products		
Model	SD	Variance	95% VaR	99% VaR
cPLCM	47%	71%	45%	42%
PCFM	55%	80%	54%	52%
PRM	58%	83%	55%	56%
		Annuity and life products		
Model	SD	Variance	95% VaR	99% VaR
cPLCM	63%	86%	55%	52%
PCFM	46%	71%	44%	46%
PRM	51%	76%	46%	45%

Table 3

Maximum levels of longevity risk reduction for a portfolio of 1,000 policies at 3% interest rate.

		Pension and life products		
Model	SD	Variance	95% VaR	99% VaR
cPLCM	34%	57%	32%	31%
PCFM	27%	47%	26%	24%
PRM	34%	56%	30%	31%
		Annuity and life products		
Model	SD	Variance	95% VaR	99% VaR
cPLCM	53%	78%	45%	41%
PCFM	21%	37%	17%	19%
PRM	40%	64%	34%	32%

Table 4

Maximum levels of longevity risk reduction for a portfolio of 100,000 policies at 2% interest rate.

Pension and life products				
Model	SD	Variance	95% VaR	99% VaR
cPLCM	66%	89%	64%	63%
PCFM	72%	92%	69%	69%
PRM	71%	92%	68%	68%
Annuity and life products				
Model	SD	Variance	95% VaR	99% VaR
cPLCM	78%	95%	72%	71%
PCFM	65%	87%	63%	64%
PRM	65%	88%	58%	60%

Table 5

Maximum levels of longevity risk reduction for a portfolio of 100,000 policies at 4% interest rate.

Pension and life products				
Model	SD	Variance	95% VaR	99% VaR
cPLCM	36%	58%	29%	31%
PCFM	57%	81%	56%	55%
PRM	57%	81%	53%	53%
Annuity and life products				
Model	SD	Variance	95% VaR	99% VaR
cPLCM	52%	77%	43%	39%
PCFM	45%	69%	44%	43%
PRM	45%	70%	41%	42%

Table 6

Maximum levels of longevity risk reduction for a portfolio of 100,000 policies at 3% interest rate.

Pensioners vs England & Wales population				
Model	SD	Variance	95% VaR	99% VaR
cPLCM	41%	65%	36%	33%
PCFM	50%	75%	48%	45%
PRM	59%	83%	55%	55%
Annuitants vs England & Wales population				
Model	SD	Variance	95% VaR	99% VaR
cPLCM	50%	75%	40%	38%
PCFM	46%	71%	47%	49%
PRM	51%	76%	46%	47%

As discussed previously, there are three types of basis risk present: age basis risk, population basis risk, and maturity basis risk. We now attempt to 'reduce' the first two risks by adjusting the product features and examine the corresponding effects. First, suppose all the life policyholders are currently aged 65, the same as the starting age of the pensions and annuities. Second, assume the pensioners (or annuitants) follow the experience reflected by the assured lives data instead (i.e. joint modelling is not needed; and simply the Poisson Lee-Carter model is used). For the first case, Table 7 illustrates that setting the same age range leads to an increase in the risk reduction of around 17% on average, compared to the figures in Table 1. For the second case, Table 8 reveals that the average risk reduction is about 7% larger than previously. These results suggest that age basis risk seems to have a more significant impact than population basis risk for the data and products being considered here.

We also consider the case where premiums are charged periodically for the life policies. Suppose the premiums are payable annually in advance as long as the policyholder is alive for a maximum of 10 years, calculated based on the principle of equivalence. Comparing Table 9 with Table 1, it can be seen that there is basically not much difference in the risk reduction results between charging the premiums in instalments and outright.

Table 7

Maximum levels of longevity risk reduction for a portfolio of 100,000 policies at 3% interest rate.

Pension and life products (current age = 65)				
Model	SD	Variance	95% VaR	99% VaR
cPLCM	68%	90%	66%	65%
PCFM	89%	99%	88%	88%
PRM	88%	99%	88%	88%
Annuity and life products (current age = 65)				
Model	SD	Variance	95% VaR	99% VaR
cPLCM	79%	96%	75%	73%
PCFM	73%	93%	75%	75%
PRM	54%	79%	41%	39%

Table 8

Maximum levels of longevity risk reduction for a portfolio of 100,000 policies at 3% interest rate.

Pension and life products (based on assured lives data only)				
Model	SD	Variance	95% VaR	99% VaR
PLCM	63%	86%	61%	62%

Table 9

Maximum levels of longevity risk reduction for a portfolio of 100,000 policies at 3% interest rate.

	Pension and life products (periodic premiums)			
Model	SD	Variance	95% VaR	99% VaR
cPLCM	48%	73%	44%	43%
PCFM	62%	86%	61%	62%
PRM	62%	85%	58%	59%
	Annuity and life products (periodic premiums)			
Model	SD	Variance	95% VaR	99% VaR
cPLCM	64%	87%	57%	54%
PCFM	53%	78%	53%	52%
PRM	55%	79%	49%	49%

So far, we have adopted model-based approaches to perform simulations. On the other hand, one can also take a model-free approach such as historical simulation, which does not assume any model setting but uses repeated sampling from the historical data (e.g. Liu and Braun, 2010; Coughlan et al., 2011; Li and Ng, 2011; D'Amato et al., 2012). One such technique we test here is the block bootstrap method, which divides the data into overlapping (or non-overlapping) blocks of equal size, draws random samples of blocks (with replacement), and then sequentially lines up these sampled blocks to form pseudo data (e.g. Liu and Braun, 2010). By sampling data blocks instead of individual data points, the underlying serial dependence can be preserved in the simulation. Moreover, to allow for the mortality experience of the two populations simultaneously, we group together the two populations' rates of change in death rates at each age-time cell as an individual (bivariate) data point for resampling, so that the relationships between the populations can be incorporated into the simulated samples. Regarding sample autocorrelations and cross-correlations, we find that while most statistically significant correlations are within the first two lags, some significant ones are spread over much longer lags. Hence we experiment with two block sizes of 5 (i.e. no significant correlations beyond lag 5) and 10 (i.e. no significant correlations beyond lag 10), the results of which are given in Table 10. The data period is too short for this bootstrap method to produce extreme percentile measures, so only the standard deviation and variance are computed. It is interesting to see that by using the bootstrap method instead of the three models, the risk reduction is about 20% smaller for the pensions but 10% larger for the annuities. These differences re-emphasise the importance of examining different modelling approaches in practice, so as to acquire a more balanced assessment of the hedging effects.

Table 10

Maximum levels of longevity risk reduction for a portfolio of 100,000 policies at 3% interest rate.

	Pension and life products	
Bootstrap	SD	Variance
Block size = 5	38%	61%
Block size = 10	37%	60%
	Annuity and life products	
Bootstrap	SD	Variance
Block size = 5	76%	94%
Block size = 10	62%	85%

In summary, we demonstrate that there is an optimal composition for each case being considered. Based on this simulation study, under certain realistic conditions of the mortality experience, interest rate, and basis risk, and when the portfolio size is reasonably large, the maximum variance reduction ranges from around 60% to 95%, with an average of about 80%. For reduction in the standard deviation and extreme measures, the range is about 30% to 80% and the average is above 50%. It is clear that the potential natural hedging effect can be significant and it is important to have a proper allowance for this effect in reserve and capital calculations. A financial institution with a more diversified portfolio would be subject to lower variability in aggregate under random movements in mortality levels.

5. Concluding remarks

In this paper, we assess the potential effectiveness of natural hedging between annuity and life products. We apply the correlated Poisson Lee-Carter model, Poisson common factor model, product-ratio model, and historical simulation to actual mortality experience of assured lives, pensioners, annuitants, and general population in England and Wales. Particularly, this is the first attempt to adapt the product-ratio model from the demographic literature to an actuarial issue. Besides the initial settings, we also consider a variety of scenarios and product features and perform sensitivity analysis. In general, we find that there is an optimal mix for each case under consideration, and our simulations suggest that the level of risk reduction would be too significant to be overlooked in practical work such as reserving and capital allocations. Financial institutions with mortality-linked liabilities would benefit from holding more diversified portfolios. Nevertheless, our numerical results appear to be model-dependent, which pinpoints that an actuary should use a number of different models in order to obtain a better view of the hedging effects.

Finally, there are a few more things to note. First, while we have focused on static hedging, it would be interesting to study dynamic hedging in future research and investigate how maturity basis risk can be addressed. Second, the participants of the CMI data may have changed over time, which could lead to more heterogeneity and so higher basis risk than otherwise. Third, the data used are for the industry in aggregate, whereas an individual insurer or pension plan may have very different mortality experience. Fourth, though we have considered different models and conditions, it would be useful to test other joint modelling techniques as well as other kinds of products in future work. Some important financial concepts such as mortality duration and convexity, multivariate risk-neutral valuation (e.g. Kogure et al., 2014), and counterparty risk can also be further explored for natural hedging. Lastly, as the life market continues to develop and becomes more liquid, it can be expected that natural hedging would gradually become more economic and feasible, especially for smaller financial entities.

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Appendix

As discussed earlier, the major time components computed from the three models appear to be highly linear, which imply that a random walk with drift is suitable for projection. We further apply the augmented Dickey-Fuller test to the major time components, in which the null hypothesis of unit root is not rejected at the 5% significance level. These results provide further support on using the random walk. We also apply the Engle-Granger test to the two mortality indices of the cPLCM. For the assured lives and pensioners data, the test results suggest that the two indices are not cointegrated. For the assured lives and annuitants data, however, the test results are mixed regarding the presence of cointegration, which calls for future research to compare modelling the two indices as cointegrated and as a bivariate random walk.

Under the Box and Jenkins (1976) approach, the partial autocorrelation function (PACF) can be used to identify the order of an AR process. The tables below use the symbols +, –, and • to indicate whether the sample PACF value at a certain lag is larger than twice the estimated standard error, smaller than negative twice the standard error, or is statistically insignificant. Considering there are only 24 years of data and ensuring the convergence of the projected ratio of death rates, these statistics suggest that a weakly stationary AR(1) process would largely be appropriate for the various parameter time series generated from the PCFM and PRM. Although there are a few significant values at higher lags, the corresponding AR processes are either non-stationary or leading to highly unstable central projections, and do not suit the purpose of our analysis.

Assured lives and pensioners

lag	1	2	3	4	5	6	7	8
kappa1	•	•	•	•	•	•	•	•
kappa2	+	•	•	•	•	•	•	•
lambda2	•	•	•	–	•	•	•	•
lambda3	•	•	•	•	•	•	•	•
lambda4	•	•	•	•	•	•	–	•
lambda5	–	•	•	•	•	•	•	•
lambda6	•	•	•	•	•	•	•	•
gamma1	•	•	•	•	•	•	•	•
gamma2	+	•	•	•	•	•	•	•
gamma3	•	•	•	•	•	•	•	•
gamma4	•	•	•	•	•	•	•	•
gamma5	•	•	•	•	•	•	•	•
gamma6	•	•	•	–	•	•	–	•

Assured lives and annuitants

lag	1	2	3	4	5	6	7	8
kappa1	•	•	•	•	•	•	•	•
kappa2	+	•	•	•	•	•	•	•
lambda2	•	•	•	–	•	–	•	•
lambda3	•	•	•	–	•	•	•	•
lambda4	•	•	–	•	•	•	•	•
lambda5	•	–	•	•	•	•	•	•
lambda6	•	•	•	•	•	•	•	•
gamma1	•	•	•	–	•	•	•	•
gamma2	+	•	+	•	•	•	•	+
gamma3	–	•	•	•	•	•	•	•
gamma4	•	–	•	•	•	•	•	•
gamma5	•	•	•	•	•	•	•	•
gamma6	•	•	•	•	•	•	•	•

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