"EX-ANTE" ASSET ALLOCATION STRATEGIES FOR GLOBAL INDEX PORTFOLIOS

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ABSTRACT

This thesis addresses the issue of developing optimal "ex-ante" global asset allocation strategies from the viewpoint of a UK investor, without the need to resort in fundamental forecasts of the portfolio inputs. In this context, the main emphasis is placed on the market selection, currency hedging and asset mix decisions as opposed to the individual stock/bond selection within each market. Effectively, the principal focus lies in empirically assessing the extent of inter-temporal instability in the inputs to the global portfolio optimization problem and in developing appropriate multivariate estimation procedures that aim to assist investors in achieving superior out of sample portfolio performance.

The empirical results from application of MANOVA techniques provide strong evidence about the inter-temporal instability of the global index covariance structure and the necessity of controlling estimation risk in index portfolio inputs. Multifactor models based on unobservable factors are shown to be capable of reducing the "noise" from the historical correlation structure, even though statistically superior correlation estimates do not always result in superior out of sample portfolio performance, since the latter has relatively low sensitivity to misestimation of correlations. Instead, Bayesian and Empirical Bayes-Stein type models appear capable of satisfactorily controlling estimation risk in index returns, while very promising results arise from procedures where investors impose prior restrictions on investment weights. Finally "co-integration" analysis reveals significant evidence of common trends and predictable return components in a number of markets, primarily hedged bond indices.

INTRODUCTION

The main aim of this thesis is to investigate how UK investors could develop optimal "ex-ante" international asset allocation strategies, without having to rely on return forecasts that are based on market fundamental analysis. Instead, the emphasis is firmly placed on multivariate statistical techniques that are used to investigate the properties and inter-relationships of international index returns and generate input estimates that might potentially lead to superior out of sample portfolio performance.

One of the key differences between domestic and international investments, largely ignored in academic literature, lies in the relative emphasis among the various performance attributes. An active equity investor with a purely domestic portfolio can attempt to outperform a benchmark index by means primarily of superior stock selection and to a lesser extent sector weighting or market timing. For a large international equity investor, though, individual stock selection becomes a relatively small performance component compared to those arising from country and currency selection.

Consequently, the focal point of all subsequent empirical work that is carried out in the context of this thesis is taken from the viewpoint of an investor who attempts to optimize his/her asset allocation decisions concerning the country (or bond index) and currency selection, while being neutral on the individual stock/bond selection. To this end, particular attention is placed on the limitations of sample historical estimates for asset allocation decisions and on the relative importance of the foreign exchange risk component in international portfolios.

The main theoretical and empirical issues associated with the above aim, as well as the technical tools and innovations used in order to achieve them, form the basis of all subsequent chapters and are briefly summarized below:

Chapter I reviews and inter-relates the main theoretical developments in those areas of international diversification and asset allocation that are of particular interest for the purpose of this thesis: Foreign exchange implications for international portfolios, alternative types of optimization procedures for asset allocation, theoretical and practical problems with international asset pricing, global stock market movements interdependence, the theory and evidence on intertemporal stability of portfolio inputs and, finally discussion of problems related to estimating inputs for the international asset allocation model.

In the first part of Chapter II, the global asset allocation issue is empirically addressed from the viewpoint of a UK investor whose decisions are based on real, as opposed to nominal, Sterling terms. The unhedged and hedged return series were constructed by using monthly data from 15 stock market and 21 bond market indices as well as the corresponding monthly spot and forward rates against Sterling.

A significant innovation, though, from previous studies relates to the fact that in order to arrive at a more accurate measure of actually realizable hedged returns an "ex-ante" hedging strategy has been applied in which the unexpected foreign currency proceeds from foreign investments are converted into Sterling at the uncertain future spot rate.

Subsequently, the second part of Chapter II focuses on an empirical analysis of the different volatility components for overseas investments. For the first time in literature, the relative size of and interactions between local market risk, foreign exchange risk, UK inflation risk and the various covariance risk components are directly measured and inter-related in order to properly understand the total investment risk in Sterling terms. Then, the discussion about total investment risk is complemented by addressing the issue of how foreign exchange risk affects the systematic versus non systematic risk components in Sterling returns from international portfolios.

In the final part of Chapter II, I test the validity of the view expressed by Kritzman (1989) that the existence of persistent trends in monthly currency returns, caused primarily by central bank intervention, can lead to the profitable formulation of "convex" investment strategies: the results from both parametric and non-parametric procedures, applied to a large number of currency pairs, strongly indicate that Kritzman's statement is very dubious as "significant" trends disappear from monthly currency returns after adjusting for bias caused by ignoring the interest differentials.

Chapter III focuses on the critical issue of, whether, the information set contained in historical index returns provides an adequate basis for implementing future global asset allocation decisions. This relates, of course, to the extent of intertemporal instability in the key index portfolio inputs i.e. the mean return vector, volatilities, and the correlation matrix of index returns. Most of the existing literature (the exception being Meric & Meric 1989), is clearly inadequate since it addresses the problem within a univariate context: studying the stability of a single index variance or a single correlation coefficient is hardly appropriate in a portfolio context where stability has to be addressed within a multivariate framework.

In fact, such a framework can be readily created by applying Multivariate Analysis of Variance (MANOVA) techniques, until now totally neglected in finance theory apart from a single test on correlation matrix stability carried by Meric & Meric. In this chapter, a much wider range of testing procedures is being applied to study the stability of the entire index variance-covariance structure and the mean return vector over time intervals of different lengths. This analysis is then supplemented by appropriate univariate procedures, like homogeneity of variance tests, used to provide further information on the individual causes of multivariate instability .

The empirical evidence from the aforementioned tests provides overwhelming support for the view that the index return covariance structure is highly unstable over all time intervals tested, casting therefore serious doubt on the appropriateness of using unadjusted historical covariances as inputs to the portfolio optimization problem.

Certainly, the above results can be seen as indicating that variance-covariance instability is also a longer term phenomenon, i.e. not only confined to the well documented issue of short term volatility "clustering" as captured by the presence of Multivariate Garch effects.

As far as the evidence on the intertemporal instability of the index mean return vector is concerned the results are much less conclusive, but admittedly these tests have reduced power in view of the presence of unstable covariance matrices. Also, it is worth mentioning that despite the fact that portfolio input values tend to vary depending on the choice of sampling period, no evidence was found of significant increase in volatility (or mean returns) over time. In fact, when applying unit root tests for the entire sampling period it is not possible to reject the hypothesis of weak form stationarity for all index returns. Overall, the multivariate tests provide ample evidence on the need of controlling "estimation risk" in historical inputs when formulating ex-ante allocation strategies.

Chapter IV is central to the thesis in the sense that it directly attempts to develop alternative "ex-ante" global market allocation strategies and then measure their realized performance outside the sampling period, using actual data rather than simulation techniques. Effectively the models applied can be separated in two main groups, i.e. those whose primary aim is to forecast the future return correlation structure and use it as inputs in subsequent optimization procedures, and those that attempt to control estimation risk in expected returns and variances or impose additional constraints to the portfolio optimization problem.

The main reference work in previous academic literature that deals with the issue of assessing the out of sample performance of allocation strategies for international index returns with real data is due to Eun & Resnick (1987, 1988), while some useful studies that consider estimation risk and rely primarily on simulations for empirical testing can be found in Jorion (1985, 1986) and Dumas & Jacquillat (1990). These last studies, though, primarily relate to other than stock indices asset classes. Empirical work on correlation matrix forecasts for index returns does not exist, however literature and empirical evidence on correlation forecasts for international stock portfolios includes Eun & Resnick (1984, 1992) and Elton & Gruber (1992).

Even though the empirical work carried out in Chapter IV concerning the out of sample performance assessment of input forecasts and asset allocation strategies has some methodological similarities with the Eun & Resnick (1988) study, it is considerable broader and incorporates a large number of extensions and innovations. The principal features of the testing procedures used and the main improvements on existing studies are outlined below:

i) A Very Wide Model Coverage:

In the Eun & Resnick (1988) study, only four models have been developed and tested against each other, i.e. a simple historical model, an equally weighted model, a minimum variance portfolio and a Jobson-Korkie type model where the shrinkage factor for expected returns is calculated on the basis of a Jorion estimator. In the context of this thesis, no less than fourteen alternative asset allocation models are developed and tested including, in addition to those already mentioned, the following:

- An unobservable factor model based on Maximum Likelihood Factor Analysis and a Likelihood Ratio Criterion
- An unobservable factor model based on Principal Components Analysis and an Eigenvalue Criterion
- Two versions of a Pseudo Single Index type model (single index model adjusted for index returns) with factor betas estimated by means of a Bayesian technique
- An "Overall Mean Correlation" type model
- A Bayesian Klein-Bawa type model with non-informative prior
- An optimization model with imposed upper constraints on investment weights
- An optimization model with imposed minimum (greater than zero) constraints on investment weights
- Three versions of the Jobson-Korkie model that do not depend on Jorion estimators

ii) Broad Asset Mix Considerations:

These aforementioned models have been applied, when appropriate, not only to unhedged and hedged stock index portfolios as in Eun & Resnick, but also to "combined" portfolios, consisting of a stock and bond index asset mix.

iii) "Ex-Ante" Optimized Portfolios based on Alternative Risk Free Rates:

When formulating their optimization strategies, Eun & Resnick selected, reinvested and measured the performance of a portfolio corresponding to an unrealistically hypothesized zero risk free rate. Such an approach is dubious, because it not only affects the choice of which "ex-ante" portfolio on the efficient frontier is to be reinvested, but also can cause a major bias in the value of the Sharpe performance measures. For the aforementioned reason and, in order to mitigate this problem and enhance the robustness of the results, three instead of one "ex-ante" portfolios, corresponding to three different assumed risk-free rates, have been optimized and reinvested in each case.

iv) An "Ex-Ante rather than "Ex-Post" Hedging Strategy:

An inconsistency in the Eun & Resnick approach, related to the hedged returns, is that they formulated their "ex-ante" allocation strategies using an "ex-post" hedging strategy that does make provisions for future spot rate uncertainty. This inconsistency has been remedied in terms of the use of an "ex-ante" hedging strategy that directly copes with this problem. Notice also that in this thesis the "numeraire" currency is Sterling instead of the US Dollar used by most other studies.

v) Testing for both Statistical and Economic Significance:

In Eun & Resnick (1988), only the economic significance of the results has been evaluated by using the out of sample Sharpe Performance Measure as main ranking criterion for the performance of the different models, while in a previous study (1984) the same authors had applied statistical criteria for their forecasts accuracy as well, like the Theil Inequality Coefficient and the decomposition of the Mean Square Forecast Error. For the purpose of this thesis, both economic and statistical criteria (the latter for the correlation matrix forecasts) have been applied at the same time, so that to be able to establish the sensitivity of portfolio performance to improved correlation estimates. Notice also that instead of the standard Sharpe Measure an unbiased estimate due to Jobson & Korkie has been used to adjust for bias due to sampling size.

Overall, the unobservable factor procedures were shown to be able to eliminate "noise" from historical correlations and lead to improved correlation forecasts. Interestingly, though, portfolio performance was found to be highly insensitive to the correlation inputs so that the corresponding benefits in performance were small. On the other hand most other models that control for estimation risk were found to markedly outperform the historical portfolio benchmark.

In fact, the "intuitive" procedures of constraining investment weights performed extremely well, occasionally even better than the Bayesian or Empirical Bayesian procedures. In particular, the virtually neglected strategy of imposing minimum restrictions on investment weights appears to be worthy of much more serious consideration in future research.

Future portfolio performance appears to be considerably more sensitive to estimates of mean returns than estimates of the other inputs, yet the issue of estimating index returns with means other than shrinking individual means towards the grand mean remains unresolved. For this reason, two other approaches were followed in an attempt to establish whether estimates of future index returns with improved forecasting power could potentially be formulated, i.e.

- A "Pseudo-APT" (APT for indices) type Model for both unhedged and hedged returns based on unobservable factors; This generally led to insignificant factor risk premia and failed to provide a meaningful operational framework.

- A "Co-integration" framework combined with Granger-type intertemporal precedence tests for co-integrated variables aiming to establish whether, following a short term shock that might cause two market indices to drift apart, there is a longer term "error correction mechanism" that ensures the two indices will eventually converge back into a predictable equilibrium (Chapter V). The relevant results indicate that among a wide variety of stock and bond index combinations tested, evidence on co-integration is strongest among pairs of hedged government bond indices in Sterling terms, allowing for potentially improved return estimates for investors with relatively long holding horizons.

CHAPTER I

INTERNATIONAL DIVERSIFICATION AND ASSET ALLOCATION: THE THEORY AND EMPIRICAL EVIDENCE

Since the publication of Grubel's pioneering work (1968), there has been an explosion of interest in international asset allocation and the potential benefits from international diversification culminating to a large number of academic publications. Our intention in this chapter is to concentrate on the evolution of the main ideas, while emphasising those developments that are directly linked to issues being explored in subsequent chapters of this thesis.

In Section 1.1 of this chapter, the basic ideas are highlighted as they evolved during the initial period (1968-1980) of theoretical and empirical research in the area. Section 1.2 concentrates on the implications of exchange rate fluctuations for international asset allocation as well as on issues related to hedging the foreign currency exposure in asset portfolios. Section 1.3 covers alternative optimization approaches to asset allocation, which are not based on the standard mean-variance framework. Section 1.4 discusses the main developments concerning international asset pricing in the context of either integrated or segmented markets and explains their failure, until now, to provide much useful guidance for the global asset allocation problem. Then, Section 1.5 addresses the evidence on short term stock market interdependence as well as on the existence and implications of "lead-lag" relationships between the global stock markets is examined and analyzed.

Subsequently, Section 1.6 focuses on the subject of intertemporal stability for the international portfolio inputs which is a necessary prerequisite for the successful application of the traditional Markowitz framework to the problem of international portfolio selection. Finally, Section 1.7 reviews some promising alternatives for estimating the inputs to an international allocation model such as using factor models or "grand mean" type approaches in forecasting the international correlation structure, traditional Bayesian estimates of the variance-covariance matrix, or empirical Bayes-Stein estimators for forecasting expected returns.

1.1. The Foundation Period (1968-1980)

The literature on international portfolios originates from 1968 with the inovative work of Grubel (1968). Its origins, though, are heavily influenced by the pioneering work of Harry Markowitz on Modern Portfolio Theory (1952, 1959) and the various theoretical developments in Capital Asset Pricing Theory whose foundations lay on the early work by William Sharpe (1963, 1964). Alongside with Sharpe (1966), Michael Jensen's (1964, 1969) and Jack Treynor's (1965) early work on performance measurement also had a significant impact on the subsequent literature on international portfolio performance.

The first empirical studies on international portfolios were almost exclusively conducted from the viewpoint of a US investor (i.e. used the US Dollar as "numeraire" currency) and concentrated on "ex-post" benefits that can be achieved from international diversification. These studies attracted considerable interest among both academics and the investment community, because they almost invariably concluded that international portfolios dominated their purely domestic counterparts (primarily a US stock index) in mean-variance space so that investors could achieve a higher risk adjusted portfolio performance.[1]

Grubel (1968) pointed that international portfolio diversification can provide the world with an entirely new type of welfare gain, that does not depend on the gains from trade or the productivity gains from international factor movements. His "ex-post" portfolio consisting of foreign stock indices, clearly dominated the US index for the period tested. However, this could be largely attributed to the high historical return of the South African, Japanese and Australian Markets during the sample period. As Grubel himself observed, an investor that would have failed to include (or even overweight) these stocks when forming her portfolio would have realized substantially reduced diversification benefits.

[1] The question of whether and to which extent these "benefits" could be in fact realized in a meaningful ("ex-ante") sense by an international investor did not explicitly emerge until some years later.

Grubel's work has been subsequently expanded by **Levy & Sarnat (1970)**, who first drew attention to the importance of the correlation structure among country returns, which they viewed as a measure of stock market interdependence. They argued that relatively high observed correlations among industrialized nations indicated lower segmentation and should be attributed to fewer restrictions on capital flows, whereas markets with lower correlation with the rest of the world like South Africa and Japan tended to be more segmented.

Shortly afterwards, **Grubel & Fadner (1971)** were the first to observe that observed correlation coefficients depend on the length of the measurement interval. Their empirical evidence showed that both inter-country and intra-country correlations were lowest when measured over weakly intervals (daily data were not used), a phenomenon they attributed to the fact that short term returns tend to be more influenced by "noise" as opposed to economic fundamentals. They concluded that the longer the investment holding period, the lowest the "ex-post" benefits from international diversification.

Jacquillat & Solnik (1978) drew a comparison between the performance of an internationally diversified portfolio as opposed to investing in stocks of multinational corporations. They found that a portfolio consisting exclusively of multinational firms was approximately 10% less volatile than the US domestic portfolio, while risk reduction levels for purely international portfolios would have been much higher. Consequently, they claimed that multinationals tend to behave to a large extent as domestic stocks and therefore represent a poor substitute for a "pure" international investment.

Dimson, Hodges & Marsh (1980) suggested that when individual stocks, rather than indices, are being considered, then investors can realize most of the diversification benefits by holding only 10-15 stocks from each country. They also pointed to the fact that reliance on "portfolio optimization routines" is likely to cause poor results because of their tendency to assign very high investment weights to assets with high historic performance. By means of this comment, they implicitly acknowledged the significance of "estimation risk" (or parameter uncertainty) for successful international asset allocation.

Errunza's (1977) work is worth mentioning simply because it sparked major interest to the diversification potential of investing into stocks from LDC countries[2], something considered hitherto inconceivable by the US investment community. In this context, he showed that a portfolio consisting exclusively of LDC stocks had a far superior risk-return profile compared to a US stock index.

Mc Donald (1973) was the first to emphasize the portfolio implications of stock market integration, as opposed to segmentation, by pointing out that even if international markets are fully integrated then investors can benefit from a "pure diversification effects" resulting from a reduction in non-systematic portfolio risk. Nevertheless, he acknowledged that higher diversification benefits can potentially accrue in the case of segmented markets since part of the domestic market risk could also be diversified.

Another worthwhile aspect of Mc Donald's paper lies in that he was the first to argue that the conclusions from different studies might well be "numeraire dependent" and international portfolio analysis should, therefore, consider the viewpoint of different national investors. Mc Donald analyzed mutual fund returns denominated in French Francs and concluded that purely domestically invested French mutual funds underperformed their international counterparts.

Lessard (1973) concentrated on the likely implications from the formation of an "Investment Union"[3] between four Latin American countries. He argued that significant benefits could result from such a Union because, on one hand restrictions in capital movements, political crises and lack of synchronization of fiscal and monetary policies will inevitably cause their stock markets movements to be highly independent from those in the rest of the world, while on the other hand (for similar reasons) one should expect a high degree of stock price comovement within these countries[4].

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[2] The international investment management community became strongly aware of the "attractions" of LDC stocks in the mid to late 80's when the so-called country funds mushroomed.

[3] Defined in this context as free movement of capital only within these countries.

[4] Here, we might validly argue that the "thinness" of these markets also contributes substantially to violent market movements in response to political or economic shocks

Lessard applied Principal Components Analysis to quarterly dollar returns from 110 stocks, grouped by country. His conclusion was that the first Principal Component extracted for each one of the four countries separately, explained a much higher percentage of return variability compared to that suggested from similar studies of the USA[5]. He also found that the correlations between the first Principal Components for each country were not significant and consequently concluded that there is "no evidence of systematic relationships between the major movements in the various stock markets".

In a later article, **Lessard (1974)** examined the relative importance of world, national and industry factors in explaining equity returns. He suggested that the existence of a significant national factor component implies that returns must be generated through a multi-factor stochastic process. In this spirit he suggested a two factor return generating process for each security i from country j expressed as

$$R_{ij} = a_i + \beta_i F_w + \gamma_i F_j + \epsilon_i \qquad \textbf{(1.1)}$$

where F_W is the common underlying world factor

F_j is the influence of national index j after the impact of the world market has been removed (by regressing the national index on the world index)

Lessard was the first to argue that a market capitalization weighted world index is an inappropriate proxy for the global market portfolio, because of its very high correlation to the US index (over 50% of world index in 1973). He suggested instead that, for the majority of countries either the first Principal Component or an equally weighted world index explains returns (in terms of R^2) much better[6] than the capitalization weighted index.

[5] This could indicate that the single index model might provide a better description of reality in the case of LDC's, compared to the industrialized markets.

[6] Even today, it is widely argued that market value weighted indices like the Morgan Stanley Capital International or the Goldman Sachs/Financial Times/Wood McKenzie World Index are biased towards the high capitalization countries like Japan and the USA.

Subsequently, Lessard aggregated stocks across both countries and industrial sectors and applied Principal Component Analysis in either case. He found that the first Principal Component explained a much greater percentage of the variance when portfolios are diversified across industries rather than countries and consequently suggested that less is to be gained by diversifying across industries rather than across countries. His results also indicated that the average variance of the national portfolios was substantially higher than the average variance for the industrial portfolios which is in turn much higher than that of the single global portfolio containing all stocks.

Finally, Lessard gave an early warning that instability in the factor structure as well as problems with the factor specification would make it particularly difficult to test CAPM type propositions in an international context[7].

Around the same period, a number of widely quoted studies (Makridakis & Wheelright 1974, Panton, Lessig & Joy 1976, Hilliard 1979, Watson 1980) made the first attempts to study the inter-relationships between different stock markets as well as the inter-temporal stability of the correlation structure in international returns. For this purpose, various multivariate methodologies have been applied:

Makridakis & Wheelright (1974) analyzed daily data from fourteen indices over a thirty-two month period by means of standard correlation analysis and Principal Components. Their results failed to provide support for inter-temporal stability. Their conclusion contrasted to those by **Watson (1980)**, who applied correlation analysis to monthly data from a later period on eight stock indices and concluded that the annual inter-country correlations were stationary over the entire time-horizon. On their part **Panton, Lessig & Joy (1976)** applied cluster analysis on weekly data from twelve stock indices over a ten year period and found some evidence of stability for relatively long term relationships[8].

Hilliard (1979) chose to apply spectral analysis in order to examine the behaviour of international equity markets during a period of market turbulence. He examined daily

[7] A view that was to be vindicated by subsequent research, discussed in Section D of this chapter.

[8] Particularly for correlation coefficients computed over three year intervals

data from ten major stock markets during a particularly turbulent one year period in 1973-74, which included the Arab-Israeli conflict and the first oil shock. Hilliard's results failed to discern any evidence of stock market dependency, nor any significant "lead-lag" relationships with the sole exception of New York leading Amsterdam.

Once people begun to appreciate the potential virtues of international investments, it was inevitable that several attempts would be made to extent domestic asset pricing theory to an international setting. In a pioneering paper, **Solnik (1974)** readily identified major difficulties in developing an International CAPM such as the non existence of a universal risk-free rate, the existence of different national interest rates and the fact that investors face different investment opportunity sets because of exchange risk and therefore are likely to hold different proportions of risky assets depending on their nationality.

Solnik went on to develop a number of alternative international pricing model specifications by assuming that all investors have homogeneous expectations about exchange rate variations and the returns distribution in each country, while their consumption is limited to the home country goods only. In developing his models he made no provision for the possibility of investors hedging the foreign exchange risk through forward contracts or borrowing and lending.

His first and more restrictive specification is simply a direct extension of the single index domestic CAPM, where the domestic index is substituted by the world index and the domestic systematic risk by the international systematic risk. His "ex-ante" specification of a single index International CAPM (ICAPM) can be expressed as:

$$a_i - R_i = \gamma_i (a_m - R_m) \qquad \textbf{(1.2)}$$

where a_i, a_m stand for the expected return on security i (in local currency terms) and the market portfolio respectively.

R_i R_m denote the interest rates in country i and a weighted world average respectively (not generally the same)

γ_i is the international market risk of security i (covariance with the global portfolio).

Nevertheless, Solnik was quick to point out the deficiencies of this model, by stating that the presence of strong national specific influences on stock returns cannot be ignored and recommended instead two alternative specifications capable to take account of national factors. These he termed, in turn, a "nationalistic model" and a "multinational index" model. His nationalistic model, in fact, functionally relates a stock's return to a country index, which in turn is a function of the world index.

The "ex-post" (realized returns) form of this model is specified as[9]

$$\begin{aligned} \tilde{r}_{ki} - a_{ki} &= \beta_{ki}(\tilde{I}_k - a_k) + \eta_{ki} \\ \tilde{I}_k - a_k &= \gamma_k(\tilde{r}_m - a_m) + e_k \end{aligned} \tag{1.3}$$

where $\tilde{r}_{ki} - a_{ki}$ represents the excess return (realized minus expected) return on security i

$\tilde{I}_k - a_k$ is the excess return on national market index k

γ_k is the international systematic risk of country k

Under this specification, Solnik proved that the international systematic risk of a security i equals the product of its national systematic risk times the international country risk i.e.

$$\gamma_{ki} = \beta_{ki}\gamma_k \tag{1.4}$$

In an empirical sense, in order to accept the joint hypothesis that capital markets are perfectly integrated and the "nationalistic model" holds it is necessary that i: the domestic CAPM is valid for all countries ii: all international indices are priced in respect to the world index and iii: equation (1.4) holds.

9 The "ex-ante" (expected returns) form of this model arises if we use the standard relationships

$$\begin{aligned} a_{ki} &= R_k + \gamma_{ki}(a_m - R_m) \quad \forall\ k, i \\ a_k &= R_k + \gamma_k(a_m - R_m) \quad \forall\ k \end{aligned}$$

Tests by Modigliani & Pogue (1974) and Black, Jensen & Scholes (1972) on major European markets found mixed evidence concerning conditions i: & ii: However, Solnik's own tests failed to provide strong support on the validity of condition iii:, which he attributed to the fact that individual stocks within the same country "could have different sensitivity to international events because of the nature of the firm's business".

This last deficiency Solnik tried to remedy in his "multinational index model"[10] where each stock is functionally related, apart from the country index, to the residual from regressing the national index to the world index[11] . This model can be summarized as:

$$\begin{aligned} \tilde{I}_k &= a_k + \gamma_k(\tilde{r}_m - a_m) + e_k \\ \tilde{r}_{ki} &= a_{ki} + \gamma_{ki}(\tilde{r}_m - a_m) + \beta_{ki} e_k + \eta_{ki} \end{aligned} \tag{1.5}$$

which is effectively a two index model with uncorrelated indices (by construction)

Solnik's empirical tests on the three versions of the ICAPM involved daily stock returns from 234 European and 65 American stocks and were far from being conclusive; He found some mild evidence that securities might be priced according to their international systematic risk, even though they have a "large dependence on national factors". He concluded that the "true" systematic risk of a stock is smaller than its domestic undiversifiable risk and consequently the domestic beta cannot be considered as a fully adequate risk measure, even though the undisputable importance of national factors implies it can provide useful information about security risk.

Solnik (1977) became pessimistic about the prospects of performing meaningful empirical tests on international asset pricing and argued that "it is very unlikely that an empirical mean-variance model will ever be able to discriminate between the various views of the world". Subsequent developments, certainly have not been able to refute this view.

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[10] For a complete discussion of the properties and testing procedures related to the "multinational index model" see Solnik 1974 pp: 373-376

[11] This represents the country index impact on the stock, after the global influence has been removed.

Stehle (1977) was one of the first authors to explicitly address the asset pricing implications of international market segmentation. He addressed the issue of whether a valuation model which assumes no barriers to international capital flows would predict rates of return better than a model which assumes complete segmentation. By using a capital market equilibrium model based on multiperiod logarithmic utility functions, Stehle's tested the hypothesis of whether risk premia in the US stock market over a twenty year period were determined as if the US was entirely integrated with nine major international markets, or alternatively the US was segmented from the other markets.

His regression results provided tentative support to the integration hypothesis, even though in terms of statistical significance their is no clear evidence on the validity of either the domestic or the international pricing model (see Scott 1977). Nevertheless, Stehle concluded that there appears to be some evidence that all return variability that is systematic in an international capital market will command a higher mean return, while the return variability that is diversifiable internationally but undiversifiable domestically should not command a positive premium.

Stehle's findings also suggest that low beta securities have outperformed the high beta securities on a beta adjusted basis. He interpreted that phenomenon by referring to the fact that low beta firms tend to have much higher average size compared with high beta firms, so that they are more likely to be involved in international operations and be exposed to global risks.

Black (1974) considered the possibility of non prohibitive cross country investment barriers, in the form of differential taxes on long positions in foreign assets. In this context, Black established that expected returns on foreign assets of similar risk must exceed those of the domestic asset by the foreign tax differential. This was to be the first of a long series of articles dealing with the issue of asset pricing under partial or "mild" segmentation.

1.2: Exchange Rate Implications In Asset Allocation

Biger (1979) was among the first authors to address the implications of foreign exchange risk for international asset allocation. He analyzed quarterly stock market returns from thirteen industrialized countries from the viewpoint of six different national investors. Biger observed that the correlation matrix of returns varied when measured in different numeraires, with the correlations being highest when measured in terms of currencies which were subject to relatively large movements over the sample period, like Sterling. Eventually, he failed to discern significant differences in the composition of the efficient frontier for the different national investors and consequently concluded that exchange rate movements have relatively little impact on asset allocation decisions.

Eun & Resnick (1985) conducted a study in a similar spirit, using data from fifteen stock markets collected exclusively from the period of flexible exchange rates (post 1973). They found that irrespective of the numeraire chosen, the optimal ex-post portfolio for all national investors would be strongly biased towards Japan, Sweden and the Netherlands. However, when comparing the composition of the optimal portfolio in US Dollar terms with that of an optimal portfolio in local currency terms they found very significant differences. They also established that both volatility and betas of national stock markets tended to increase when measured in any currency other than their own.

Madura & Wallace (1985) concentrated on the implications of hedging foreign investments by means of forward contracts[12]. In this context, they showed that the "ex-post" hedged return of a US Dollar denominated foreign investment R_i^H can be defined as

$$R_i^H = (1 + R_i) * (1 + f) - 1 \qquad (1.6)$$

where f is the forward premium/discount.

[12] Portfolio managers can also hedge by means of borrowing an amount equal to the expected currency proceeds from the foreign assets. It is trivial to prove that the two methods will provide identical results provided that Covered Interest Parity holds exactly.

Subsequently, Madura & Wallace analyzed quarterly returns from six major indices and found significant deviations between hedged and unhedged US$ returns for most foreign assets[13]. After establishing that the volatility of the hedged assets was consistently lower than that of their unhedged counterparts, they calculated the efficient frontiers and found that the hedged frontier clearly dominated the unhedged one in mean/variance space.

Finally Madura & Wallace suggested that gains from hedging appear to be large enough to offset the various costs and practical difficulties involved, like transaction costs on the forward contracts, administration costs and problems associated with liquidating stocks prior to the date of expiration of the forward contract. What they failed to mention, though, is that in reality the "ex-post" hedged return can not be realized by the fund managers since the return on the foreign stock over the holding period is unknown, so that there will always be a residual return that will need to be converted in US$ at the uncertain future spot rate.

The aforementioned problem of hedging through forward contracts in a meaningful ex-ante sense was addressed by **Eun & Resnick (1988)** who showed that the actual return from a hedged foreign investment will consist of two components, a deterministic one depending on the asset's expected return and the forward premium/discount and a stochastic one being affected by changes in the spot exchange rate as well as by the ex-post difference between the actual and expected return from the foreign asset in local currency terms. Mathematically, this relationship can be expressed as

$$R_i^H = [1 + \mathscr{E}(R_i)](1 + f_i) + [R_i - \mathscr{E}(R_i)](1 + x_i) - 1 \qquad (1.7)$$

where x_i stands for the realized appreciation/depreciation of the foreign currency against the Dollar.

13 This is equivalent to saying that the ex-post changes in currency values over the sample period were significantly different from the respective forward premia/discounts.

It is surprising, though, that Eun & Resnick made no attempt to test empirically their own ex-ante relationship, preferring to test the simpler ex-post one on the grounds that it provides a "reasonable" approximation. Nevertheless, their work made a number of substantial *contributions* :

- They suggested that foreign exchange volatility affects unhedged asset returns through both a volatility and a covariance component; If this covariance between asset values and exchange rates is positive, as their own empirical results tend to suggest, then the impact of foreign exchange risk compared to the total unhedged risk is magnified.

- The common argument that multi-currency diversification reduces substantially the foreign exchange risk in an international portfolio was put into the test; Their empirical results suggest that correlations among exchange rate changes are much higher than correlations among stock market returns and therefore exchange rate risk is largely non-diversifiable. Notice, though, that their above two conclusions could potentially be numeraire specific, since the US Dollar tends to move in the same direction against all other major currencies[14].

- Finally, they were quick to suggest that foreign exchange fluctuations are responsible for increasing "estimation risk" (or parameter uncertainty), therefore increasing the likelihood of portfolio forecasting errors when estimates are based on historical unhedged returns.

Lee's (1987) main contribution consists in demonstrating that in the process of building multi-currency portfolios, the fund managers should separately determine the optimal asset portfolio and the optimal currency portfolio within the context of a partitioned covariance matrix consisting of the asset and currency return covariances as well as of the cross-covariances; In this case, the difference between the optimization determined asset and currency exposure constitutes the "optimal hedge[15]".

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[14] Because of both the traditional "vehicle currency" role of the Dollar and the fact that ERM currencies tend to be highly correlated with each other

[15] As Lee himself mentions, the term "hedge" here is used to denote altering the exposure from one currency to another, rather than hedging foreign currency into base currency.

The fully hedged position is, therefore, a special case of the Lee model where the (ex-post) currency exposure is equal to zero for all currencies. Lee's model aims to optimize "ex-ante" portfolios and therefore requires estimates of asset and currency returns, forward premia and relevant covariances[16]. To this end for a portfolio consisting of n assets the relevant covariance matrix V will be (2n*2n) since it includes also covariances between various asset and currency returns.

On the basis of Lee's formulation, the expected return of this multicurrency portfolio will be equal to

$$\mathcal{E}(R) = \sum_{i=1}^{n} \left(w_{ai}[\mathcal{E}(r_i) + F_i] + w_{ci}[\mathcal{E}(c_i) - F_i] \right) \tag{1.8}$$

while the portfolio variance to be minimized subject to $\mathcal{E}(R)=k$ can be expressed as

$$var(R_p) = [\vec{w}_{ai}\ \vec{w}_{ci}]\ V\ [\vec{w}_{ai}\ \vec{w}_{ci}]' \tag{1.9}$$

where w_{ai}, W_{ci} denote the asset and currency weights

r_i, c_i stand for local asset and currency returns respectively

n is the number of assets or currencies

V is the (2n*2n) variance-covariance matrix of asset and currency returns and

F_i is the forward premium for asset i

Lee himself applied historical simulations to foreign fixed income portfolios[17] and showed that even for sizable forecasting errors, his optimization portfolio would perform slightly better than the fully hedged and much better than the unhedged portfolio.

Eaker & Grant (1990) further explore the idea of selective or partial hedging, their primary aim being to actively enhance returns, rather than control portfolio risk. The simple strategy used is based on the findings by Meese & Rogoff (1983), whose

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[16] As Lee points out, the active fund manager will not just rely on historical values for all these parameters, but will instead apply her own forecasts, where appropriate.

[17] Lee's model is equally applicable to foreign equities and many other foreign asset classes

empirical evidence suggested that current spot rates have predictive value of future spot rates at least as good as the forward rates.

Consequently, Eaker & Grant empirically tested a strategy consisting of fully hedging the foreign currency exposure when the forward exchange rate is at a premium for the foreign currency, while leaving the position open when the forward rate is at a discount[18]. This "selectively hedged" portfolio strategy was then compared to the standard "fully hedged" one for portfolios consisting of six country indices. Their results showed that the "selective hedge " strategy provided substantially higher mean return for only moderate increase in portfolio volatility and therefore concluded that it should be preferable to all but the very risk averse investors.

Hauser & Levy (1991) expanded the issue of optimal forward hedging for fixed income portfolios so that to relate it to the bonds duration. They showed that even though non-Dollar bond returns are in general positively correlated with exchange rate changes, this correlation is a declining function of the bonds duration. Their results indicate that the variance of six month foreign bonds consists almost entirely of foreign currency risk, whereas for the five year bonds there is a significant contribution of interest rate risk.

Hauser & Levy established that for low risk, low duration, portfolios full hedging or even overhedging is required, because it substantially reduces portfolio volatility. However, for longer duration and higher return portfolios the benefits of hedging become progressively less pronounced and the optimization solution might involve partial hedging or even unhedged positions if the expected return on the foreign currency exceeds the forward premium.

Overall, their findings illustrate the fact that bond investors will tend to gain more from international diversification if they decide to concentrate on longer duration portfolios and that the most efficient way of increasing expected returns is to alter the portfolio duration rather than increase the proportion of foreign assets in the portfolio.

[18] The simple rationale is that when the forward rate is at a discount then not hedging offers greater expected returns since the foreign currency is forecasted to have a higher value than predicted from forward rate.

Gadkari & Spindel (1990) demonstrated that, in an ex-post sense, the decision about full vs partial hedging also depends on the nationality of the investor, i.e. whether she was American or not. This is due to the fact that historical evidence for US investors shows that non-Dollar bond returns had significant positive correlations with the respective exchange rates (e.g. German Bunds with the DM/US$ rate), while this phenomenon did not occur for most European and Japanese investors.

They argue that this phenomenon is likely to continue in the future because of the "reserve currency" and "safe heaven" status of the Dollar and claim that US based investors will continue to derive more benefits from fully hedging their foreign bond portfolios than their foreign counterparts.

Kritzman (1989) argues that profitable asset allocation strategies can be developed as a straight-forward consequence from the fact that spot currency returns are serially dependent, a phenomenon which is likely to persist in the future since it is largely attributed to continuing central bank intervention[19]. According to Kritzman, the typical central bank behaviour is to try to " dampen exchange rate volatility" by intervening when there are large speculative flows causing big shifts in currency values; Since such actions are likely to slow down rather than reverse the currency movements they lead to protracted trends in currency returns, since actual exchange rates behave as a "moving average of the unobservable equilibrium rates".

In order to test his argument about persistent trends in currency returns, Kritzman applied both "runs" tests and a "variance ratio" test to monthly returns from five currency pairs (all involving the Dollar), sampled over the period 1977-1988. Both testing procedures provided overwhelming evidence in support of his arguments about serial dependence, so that he went on recommending an appropriate "convex" investment strategy that would allow the fund managers to earn sizable excess returns on their currency portfolios. Unfortunately, Kritzman's testing procedures contain a serious error, consisting from the fact that when conducting his tests on spot currency returns, he explicitly considered the expected return on all currency pairs to be zero.

[19] This argument is certainly not new, being widely believed to be true by many foreign exchange practitioners in the City.

This might have been a reasonable assumption for daily returns, but certainly interest rate differentials cannot be ignored when expected returns are calculated from monthly data; His findings about significant "trends" could well be due to the existence of sizable and persisting interest differentials between the currencies tested.

Finally, **Sentana, Shah & Wadhwani (1992)** address the empirical issue whether the reduction in foreign exchange volatility for the currencies members of the European Monetary System has in fact reduced the equity risk premium and consequently the cost of capital for cross border European investments.

This issue is clearly theoretically linked to forward hedging because as long as investors can hedge their foreign exchange exposure, it is likely that, within an international asset pricing model context, foreign exchange risk is non-systematic and therefore unlikely to be priced. The authors applied a multi-factor model for intra-European stock returns with time varying volatilities estimated through univariate Garch processes to monthly data from twelve European stock markets collected over a seventeen year period, in order to investigate whether intra-European nominal exchange rate volatility is priced.

Their results show that since the inception of the EMS exchange rate volatility has indeed been reduced, but there is no evidence to sustain the hypothesis that the volatility reduction has led to any significant decrease in the equity risk premia. Nevertheless, the authors accept that exchange rate volatility is a significant real life consideration for businesses and therefore its reduction might affect investment decisions, even though the equity risk premia appear to be unaffected.

Needless to say, in recent years there has been an explosion of interest in forecasting currency returns by means of techniques like "expert systems" and "chaos theory" while modelling exchange rate volatility by means of either the Mean Reversion Model (MRM) or GARCH type procedures. The relevant literature could easily fill several volumes and will not be reviewed here as they are outside the scope of this study.

1.3: Alternative Optimization Approaches to Asset Allocation

The large majority of existing "ex-ante" studies on international asset allocation apply a "Markowitz type" optimization framework, either to unhedged or hedged returns from foreign assets. This is at least partly a direct consequence of the fact that, unlike portfolios with domestic assets only, there is no obvious alternative in terms of an operational international asset pricing model (see Solnik 1977, 1992).

Nevertheless, despite its widespread popularity, the mean-variance framework has had its critics particularly in respect to the fact that it is essentially a single period model, not naturally suited for a multiperiod investment policy requiring frequent portfolio rebalancing (e.g. Crouhy 1987). Other criticisms lie in its dependence to normality or quadratic utility as well as on its reliance on the standard deviation as the appropriate measure of risk for all investors. In this context, a small number of authors relied on different approaches i.e.

i) A "continuous type" model applied by Dumas & Jacquillat in portfolios of currencies only, but equally applicable to international equity or bond portfolios.
ii) A model based on multiperiod investment theory and applied to international portfolios by Grauer & Hakanson.
iii) The work of Leibowitz & Kogelman who recently in a series of articles advocated the use of models where investors use the "shortfall risk" rather than the standard deviation as the appropriate measure of risk.

Dumas & Jacquillat (1990) address the issue of developing appropriate multi-currency portfolio optimization procedures for investors with logarithmic utility functions. In this context they question the validity of using standard mean-variance procedures for the optimization of currency portfolios in discrete time, since "it is not possible for exchange rate changes to be normally distributed"[20].

[20] Dumas & Jacquillat point out that the inverse of a normal variate is not normal but bimodal so that even if, say, the $/DM rate is normal the DM/$ will not be. Also by restricting their analysis to logarithmic investors, they rule out a possible justification of a mean/variance framework by assuming quadratic utility functions.

According to their model formulation the objective function to be maximized in the case of two currencies is equivalent to:

$$\max \int_{-\infty}^{\infty} \log[1 - w + \exp(U)]f(U)\,dU \tag{1.10}$$

where w is the weight of a non numeraire currency deposit in the portfolio and f(U) is a normal density function.

The multicurrency generalization for (n+1) currencies[21] in continuous time can be approximated by[22]

$$\max \sum_{i=1}^{n+1} w_i(\mu_i + 1/2\ \sigma_{ii}) - 1/2 \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} \tag{1.11}$$

whose solution is

$$\begin{gathered} w_i = \sum_{j=1}^{n} s_{ij}(\mu_i + 1/2\ \sigma_{ij} - \mu_{n+1}) \\ w_{n+1} = 1 - \sum_{j=1}^{n} w_j \end{gathered} \tag{1.12}$$

where s_{ij} stands for the elements of the inverse covariance matrix σ_{ij}

Dumas & Jacquillat also acknowledged that in the process of implementing the aforementioned optimization strategy, investors need to be concerned about "estimation risk" as far as the values of the parameters μ, σ_{ij} are concerned. Their own parameter estimates are based on an informative Bayesian prior where all expected returns are equal but unknown, while they tested the aforementioned model on a data set consisting of 955 weekly observations from nine currencies.

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21 Here, the $n+1_{st}$ currency plays the role of the numeraire

22 As a matter of fact Dumas & Jacquillat also test a second approximation due to Ohlson (1972).

Their approach consisted in recalculating the optimization weights on a monthly basis, re-investing the resulting portfolio and calculating the average excess dollar return while using **Cornell's (1979)** "abnormal rate of return[23]" as the appropriate performance benchmark. The Dumas & Jacquillat results showed that even though investors could realize excess returns in comparison to a simple Dollar deposit, they could not achieve statistically significant abnormal returns, something that would have been evidence of inefficiency in the international currency markets.

Grauer & Hakanson (1987) made the known only attempt until now to apply the pure reinvestment version of multiperiod investment theory (developed by Mossin, Hakansson, Leland and Ross) to international portfolios. The model used in an international context, is essentially the same they applied a year earlier to a portfolio consisting of US stocks, US corporate and government bonds and a Treasury Bill.

In brief, If $U_n(w_n)$ is the induced utility of wealth for an investor that has n periods remaining within the given investment horizon, then

$$U_n(w_n) \rightarrow \frac{1}{\gamma} w^r \quad \gamma < 1 \tag{1.13}$$

where r is the single period return and γ is a measure of risk aversion, $1-\gamma$ being the relative risk aversion function.

In this context, different values for γ can range from 1 (risk neutrality), to -75 (extreme risk aversion).

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23 This is a little known performance measure, chosen because it is the only one that does not depend on a particular asset pricing model, something that is particularly wise when one is concerned with international portfolios where asset pricing is problematic. The Cornell measure is based on the difference in any investment period between the actual rate of return on the portfolio under management R_{it} minus the rate of return expected by the market on this portfolio (estimated as a sample average for each security), i.e. it equals

$$\sum w_{it} [R_{it} - \bar{R}_{it}]$$

Notice that in order to use this method one should know the investment weights at each single period. As long each period's abnormal return is known, the investment profile can be summarized in terms of various descriptive statistics i.e. means, standard deviation etc.

The multiperiod decision rule is simply to find the investment weights that satisfy the solution to the following non-linear optimization problem, repeatedly for every single period:

$$Max \ \mathscr{E}[\frac{1}{\gamma}(1+r)^{\gamma}] \qquad (1.14)$$

subject to a set of applicable constraints (see **Grauer & Hakanson 1986)**

For applying their model with quarterly revision period, they used the observable risk free rate and lending rates at the beginning of each quarter and the realized returns for all asset classes for the previous n quarters, calculated on a moving average basis. No attempt was made to control for estimation risk[24].

Their empirical results were calculated for both geared and ungeared portfolios, using annual as well as quarterly revision periods. No less than sixteen different strategies have been tested, corresponding to levels of progressively decreasing risk aversion. The main comparison involved is between US portfolios consisting of four asset classes, with international portfolios consisting of an additional fourteen bond and stock indices.

Grauer & Hakansson conclude that the gains (measured in terms of geometric mean returns) from international diversification have been substantial, particularly so for the high risk aversion strategies[25]. The optimal investment weights for the US strategies are mostly found to be zero and the possibility of creating geared portfolios seems to provide greater benefits when international assets are also included. It is unfortunate, however, that they make no attempt to compare their model's performance against any reasonable benchmark or with any alternative allocation strategies.

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24 Their results clearly indicate that their optimization programs, selected massively the assets with high historical performance so that their optimal portfolios consisted of a very few assets only. It seems that "estimation risk" is in their model as acute a problem as in a standard mean-variance context.

25 In the sense that their international allocation strategies consistently outperformed the US ones, in an "ex-ante" sense.

Leibowitz & Kogelman (1990, 1991) question the validity of using the standard deviation as an appropriate measure of risk from the institutional investor's viewpoint, particularly for pension funds who are concerned with the protection and optimization of their surplus [26]. In this circumstances, they argue, an "asymmetric" measure of risk which emphasizes the "downside" is more appropriate.

As a preferable alternative "risk measure" they recommend the adoption of the "shortfall risk" i.e. the probability that the portfolio return will be lower than a prespecified level (e.g 5%) for a given period. Naturally, this idea is based on the theoretical mean-lower partial moment framework analyzed by **Harlow & Rao 1989** and **Bawa & Lindenberg 1977.** What is new, is that Leibowitz & Kogelman apply it to domestic and international portfolios and manage to draw interesting inferences about its asset allocation implications.

Leibowitz & Kogelman begin by determining the US cash-equity asset mix that provides a no more than x%[27] "shortfall risk" in a given year, level typically specified by the pension fund's trustees. The traditional (mean-variance) approach to benefit from diversification would be to create an international portfolio with the same standard deviation as the domestic one (a "constant volatility" portfolio). As long as the foreign equity is not perfectly correlated with US equity, the constant volatility portfolio will have a higher equity exposure and expected return and lower shortfall risk than the domestic portfolio.

Fund managers, though, can increase expected returns further without violating the domestic shortfall risk; This can easily be achieved by increasing the level of equity exposure until it has the same shortfall risk x% as the US portfolio. The resulting "shortfall portfolio" will have higher expected return and volatility than the international "constant volatility" portfolio, which effectively offers "too much" protection.

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[26] The difference between the value of pension fund's assets minus their accumulated benefit obligations)

[27] Assuming normal distribution of equity returns.

Essentially, the whole idea behind Leibowitz & Kogelman's work is the logical but counter-intuitive notion, that pension fund managers should pursue a more aggressive asset-mix than would be suggested by mean-variance analysis, because of the risk reduction properties of international diversification.

1.4: Integration vs Segmentation and International Asset Pricing

During the 1980's a substantial part of the academic literature on international investments concentrated on whether the global equity markets are integrated or segmented and on the asset pricing implications of tax, regulatory and other barriers that are responsible for segmentation. In effect specific asset pricing models have been developed to deal with specific countries or types of barriers.

At the same time, as all empirical tests consistently rejected all attempted alternative versions of an international CAPM (Jiovannini & Jorion 1989, Thomas & Wickens 1990), a number of papers attempted to extend Ross's Arbitrage Pricing Theory in an international context. These developments are summarized in the remaining of this section:

1.4.1: Capital Market Integration

Following the standard definition by Stulz (1981), capital market integration exists if assets of similar risks located in different countries have the same expected return when expressed in terms of a "numeraire" currency. As Jorion & Schwarz (1986) pointed out segmentation is normally caused by barriers, either by "legal barriers" like taxes and differential treatment of foreign compared to domestic investors, or "indirect barriers" like information asymmetries concerning foreign stocks, differential transaction costs, cross-border variations in accounting and reporting standards, different market practices etc.

Nevertheless, the existence of barriers does not necessarily imply segmentation since asset prices are determined by the "behaviour of the marginal innovative investors that might be able to sidestep controls". Gultekin, Gultekin & Panati (1990) add that segmentation can also result as a consequence of local market inefficiency or investor irrationality. Overall, the implications of capital markets segmentation, as identified by several authors (e.g. Errunza & Losq 1985, Jorion & Schwartz 1986, Wheatley 1988) are wide-ranging: the cost of capital for a given project will depend on the country where the funds are being raised, various standard "irrelevance propositions" in corporate finance break down, the world market portfolio is no longer mean-variance efficient, dual listing of stocks is likely to have price implications, predictions of macroeconomic models will have to be carried on a country by country basis etc.

One major problem with the analysis of segmentation, is that no general testing procedure can be developed; As Solnik identified back in 1977, the only "efficient way to test for segmentation is to specify the type of imperfection which might have created it and study its specific impact on portfolio optimality". Nevertheless, all such tests inevitably are joint tests of the specific asset pricing model and the hypothesis that asset markets are integrated internationally; Consequently they could either wrongly reject integration because simply the specific asset pricing model does not hold, or conversely accept it simply because risk has not been precisely measured (Bosner-Neal, Brauer, Neal, Wheatley 1990).

Also notice that, as correctly identified by Khoury, Dodin & Takada (1987), the existence of high (low) correlation, either contemporaneous or lagged, is neither a necessary nor sufficient condition for market integration (segmentation), since low correlations could theoretically exist in perfectly integrated markets and vice-versa. Nevertheless, increased correlations on returns of similar assets might provide an indication of a higher degree of market integration.

The most important recent developments in this area can be summarized as follows:

Jorion & Schwartz (1986) attempted to test whether the Canadian stock market is integrated with a global North American market. Their approach was slightly more subtle than that of Brennan & Schwartz (1986) in that they took into consideration of the fact that 30% of the Canadian market capitalization, is also listed in the NYSE or AMEX.

Jorion & Schwartz analyzed monthly returns from 1963-1982 on 749 Canadian securities, 98 of which were interlisted[28]. Their first test is effectively a standard two factor model, a priori specified, where the Canadian index is constructed to be orthogonal to the world index. Their test is based on the equation

$$\mathcal{E}(R_i - R_f) = \lambda_0 + \lambda_1 \beta_{iG} + \lambda_2 \beta_{iC} \tag{1.15}$$

where β_{iG}, β_{iC} are the factor betas with the global and Canadian indices respectively

Effectively, if the Canadian market was to be fully integrated with the global index only the global risk should be priced and therefore λ_2 should be zero. Jorion & Schwarz estimated the parameters using maximum likelihood procedures, their results showing that $\gamma_2 \neq 0$ so that the joint hypothesis of integration and validity of the international CAPM had to be rejected[29].

Then they tested a second model of segmented capital markets, which was based on the assumption that the only relevant factor is the systematic risk relevant to the domestic portfolio. Again their results do not provide any evidence in favour of integration. This rejection of integration was equally applies for both interlisted and purely Canadian firms, which exhibited, in fact, very small difference in behaviour as far as pricing is concerned.

Wheatley (1988) chose a discrete time version of the consumption based asset pricing model, in order to test for international equity market integration. According to this model, for every country there is an asset pricing line which connects the expected real return on each asset to the asset's consumption risk which in turn is computed

[28] Jorion & Schwartz follow Stehle (1977) in making the convenient assumption of logarithmic investor utility, since as shown by Adler & Dumas (1975), in this case both the price level as well as PPP considerations become irrelevant about optimal portfolio choice.

[29] Of course, the fact that a national factor appears to be priced, means that a single factor international CAPM can not be valid. Rejection of integration, however, does not exclude the possibility for a Solnik type "nationalistic" model, to be a reasonable alternative.

from a representative individual's real consumption. In this context, the joint hypothesis of stock market integration and the validity of the aforementioned model, is accepted when for all countries foreign equities plot alongside that asset pricing line.

Wheatley's tests were performed on monthly returns from portfolios consisting of US Tbills, ADR's, US corporate bonds and stocks as well as stock indices for 17 countries. His tests failed, in general, to refute the joint hypothesis, even though he admitted that the statistical power of his tests was weak.

Errunza & Losq (1985, 1989) analyzed the problem from the more realistic viewpoint that international equity markets are neither fully integrated, nor totally segmented and, therefore, an approach based on "mild" segmentation is required. In the 1985 paper, they discuss mild segmentation in a two country world: stocks from country one can be bought unrestricted, while stocks from country two are restricted to foreigners, i.e. to investors from country one.

From that basis, they developed an asset pricing model, in which stocks from country one are priced accordingly to integrated markets, while the restricted stocks from country are shown to command a positive risk super-premium. Nevertheless, if these stocks could get an international listing, then this super-premium would inevitably vanish, leading to a situation with lower equilibrium expected return for these stocks.

Their later (1989) paper extended their "mild segmentation" approach from a two country to an N country framework, while at the same time concentrated, apart from the pricing, to the welfare implications for shareholders of investment barriers. In this context they also achieved a much required flexibility, in the sense that they built a model capable of accommodating new investment restrictions, removing old ones or allowing for the issuing of new securities. Their analysis showed significantly different valuation and welfare results compared to those yielded by the two country model.

Eventually, Errunza & Losq showed that in a theoretical context under mild segmentation, the equilibrium price of a security is determined jointly by its international and national risk premiums, while all investors will tend to acquire nationality specific portfolios in conjunction with the best available proxy for the world portfolio.[30]

On the basis of their analysis, partial removal of investment barriers would increase investor welfare, since the aggregate market value of the affected stocks would increase, while in an ex-post sense investors could achieve larger risk reduction through diversification due to a larger investment opportunity set.

Hiettala (1990) build up an asset pricing model that was tailor-made for the legal restrictions applicable in Finland during the 1984-1985, when Finnish citizens could not buy foreign securities, whereas foreign entities could only buy "unrestricted" Finnish shares which formed of up to 20% of total Finnish shares. Despite the fact that unrestricted and restricted shares were identical apart from the right of ownership, increased demand for unrestricted shares by foreigners led to a situation where there were no willing sellers and prices started unofficially to differ, resulting to restricted shares trading at an average premium of 41% over the unrestricted ones, even though these premia were highly volatile from company to company and from month to month.

Hiettala argued that the lower price of unrestricted securities results from the fact that since local investors have no access to foreign securities, they will not be able to diversify the country specific risk in the same way as the foreign investors can so they will have to require a higher expected return. He accepted though, that lower taxes for foreign compared to Finnish investors might also have been an additional factor. In this context, the main emphasis of his paper was to explain the wide cross-sectional variability of the price premia in the Finnish market.

His empirical evidence suggests that the premia depend on the relative perception of riskiness for the specific stock from the viewpoint of foreign investors and that they are positively correlated with domestic betas, as well as with the firm size and a measure of liquidity for the unrestricted shares.

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30 Since segmentation would make it impossible to national investors to hold the world market portfolio.

Gultekin, Gultekin & Penati (1990) study is in the spirit of the Hiettala study, in the sense that it concentrates on a single specific case, i.e. they test for integration between USA and Japan, both before and after the liberization of Japanese capital controls in December 1980, by analyzing weekly stock returns from 1/1/77-31/12/84. They justified the need for specific case by case testing by arguing that any generalized tests of capital market integration are likely to be non-informative, while their main objection towards most existing studies lies in their failure to explain the nature of segmentation.

After rejecting the notion that a single factor asset pricing model can be used to test for integration[31] they decided to use a multifactor framework to test for integration, an intuitive reason for doing so, being that since APT is based on arbitrage conditions of nominal returns it avoids the problem of purchasing power deviations. Their empirical tests on segmentation were based on the standard two stage testing procedures, where in the first stage the factor loading matrix is estimated from the time series of returns, whereas in the second stage cross section regression is applied to calculate the risk premia.

A rather positive aspect of this paper is that the authors acknowledge the fact that any joint tests for integration are very much model dependent and consequently apply a variety of specification and testing procedures, involving both a priori determined factors and factor loadings extracted through factor analysis. Eventually, Gultekin, Gultekin & Panati results showed that before the liberalization of Japanese controls a price differential for risk existed between the Japanese and US capital markets, while in the post 1980 period the assumption of integration cannot in general be rejected. In this respect, they interpreted their findings as implying that government policies rather than individual investor attitudes are the main source of segmentation.

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[31] Apart from the fact that empirical evidence suggest that there are more than one factors with explanatory power in international returns, a single factor pricing model can only be derived under the assumption of uniformally logarithmic investors, or valid PPP, or uncorrelated exchange rates and stock returns. Empirical evidence strongly suggests that the last two assumptions do not hold in the real world, while many investors might display more risk aversion than accommodated by a logarithmic utility function.

A serious problem with all the aforementioned studies on integration have in common, is the fact that they are dependent on an asset pricing model, which is likely not to be valid. There are though, a number of exceptions:

Alexander, Eun & Janakiramanan (1988) test the integration hypothesis by examining whether company announcements of international stock listings affect expected returns. They effectively built on previous work by Stapleton & Subrahmanyan (1977) and Alexander, Eun & Janakiramanan (1987), whereby dual listings ware found to be one possible way[32] to effectively circumvent regulatory barriers. These papers had established that if stock markets are perfectly segmented then dual listings can result in structural changes in equilibrium asset pricing relationships; The equilibrium price of a stock was found to a increase as a result of the foreign listing and therefore its expected return should decline.

Alexander, Eun & Janakiramanan hypothesize that the rapidly increasing trend for dual listings is due to the fact that they help reducing transaction and information costs, as well as mitigate the effects of government restrictions on currency and capital transfers. So they claim that a highly effective test for either complete or partial segmentation is to examine whether a foreign firm's listing in a US stock exchange results in a reduction in its expected return. To that end, they recognize that the reduction will be nationality dependent, countries with low covariance with the US market likely to have the strongest effect.

Their empirical tests covered all foreign firms that became dually listed for the first time between 1969 and 1982, on either NYSE, NASDAQ or AMEX. For the non Canadian stocks they found consistent evidence for the hypothesis that international listings lead to a decline in expected returns, implying a reasonable degree of segmentation. For the Canadian stocks though the decline was much smaller and statistically insignificant, phenomenon which could either mean a lower degree of segmentation or a higher covariance with the US stock market.

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32 Other alternatives are direct foreign investment by firms, foreign portfolio investment, or mergers with foreign firms.

The validity of their results, though, can be cast in doubt because they argue that a decline in expected returns should be accompanied by a sizable increase in the stock price compared to the prelisting period, whereas in fact they admit there is no evidence of it actually happening.[33]

Bonser-Neal, Brauer, Neal & Wheatley (1990) suggest that an alternative way to study the implications of segmentation is to examine the premia/discounts of share price over net asset values for investment trusts that specialize in specific foreign countries but their share price is quoted domestically (country funds). As a general rule, funds from most countries that restrict foreign investment like Taiwan and Korea[34] tend to trade at high average premia, whereas in most cases funds from countries with no restrictions like UK, Germany trade at discounts. However, there are significant exceptions, notably funds from Brazil and Mexico which despite widespread restrictions tend to trade at discounts[35].

The authors approach to testing for segmentation, is to analyze whether changes in investment restrictions affect changes in the premia/discounts. If these restrictions are perceived to be binding then an announcement that they are to be tightened/loosened should raise/decrease the premium on the country fund. Their simple testing methodology lies in regressing premia changes against dummies which are set to equal 1 or -1 when loosening or tightening take place and zero otherwise. If markets are to be segmented, then the coefficients should be significant, while evidence of integration would occur if they are not significantly different from zero.

Overall, the results from this study indicate that during the period May 1981 to January 1989, 80% of country funds experienced a decrease in their price/net asset value ratio either in anticipation or immediately following the announcement of a liberalization in investment restrictions.

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33 A possible justification for this inconsistently is that the price increase might have taken place during the prelisting period, if the dual listing was partly anticipated.

34 In 1985 the price premium on a Korea fund stood at a record 65% over net asset value

35 Not mentioned by the authors, but this is likely to be the impact of high investor perception of country risk

The aforementioned phenomenon turned in fact to be somewhat country specific, the biggest impact related to unanticipated announcements of changes in foreign investment restrictions being attributable to France, Japan, Korea and Mexico. Across all investment funds examined, an announcement of a liberalization was associated with a 6.8% decrease in the ratio, statistically significant even at the 1% level. The conclusion is that announcements of government imposed barriers have been important in segmenting the international capital markets.

1.4.2: International Asset Pricing

As we have already mentioned when discussing the literature on market integration, all joint tests involving any form of International CAPM (ICAPM), failed to provide the slightest evidence in support of the ICAPM. Solnik himself in 1983 argues that in the absence of an optimal world market portfolio investors will hold different portfolios termed "hedge portfolios" and states that " since the composition of these portfolios depends on the covariance of asset returns with state variables, it is hard to identify such portfolios so that to test the theory". Effectively the conclusion based on the domestic CAPM that a well specified market portfolio will be efficient does not exist in an international framework, so that the ICAPM does not yield operational and easily testable conclusions.

From an empirical viewpoint, **Thomas & Wickens (1989)** made a late attempt to provide some hope for the ICAPM, by reviewing the "deficiencies" of several recent empirical studies (e.g. Engel & Rodriguez 1989, Jiovannini & Jorion 1989) that were unanimous in rejecting the ICAPM. In their view some possible causes for this empirical failures, can be attributed to omissions of important assets from the global portfolio, incorrectly calculated rates of return, differential attitudes to risk between investors in different countries and different attitudes to risk among different asset classes.

Thomas & Wickens went at great length in their attempts to remedy these weaknesses by using a much broader global portfolio and a more carefully selected data base, as well as by carrying careful misspecification tests for ARCH effects. Despite these efforts, though, their results fail to provide anything positive for the ICAPM since they found no evidence of a large or even a systematic risk premium[36]. Eventually, Thomas & Wickens had to admit that the use of ARCH type models did not make any improvements in the model specification.

Solnik (1983) was the first to discuss the theoretical problems involved when extending Ross's domestic APT on nominal returns, in an international context. He pointed out that the technical problems posed by currency translation and asset aggregation in the ICAPM do not arise in APT because the factors are not constrained to be portfolios of the original assets.

Consider the standard APT relationship

$$r_i = \mathscr{E}(r_i) + \beta_{1i}\delta_i + \ldots\ldots + \beta_{im}\delta_m + \epsilon_i \tag{1.16}$$

where δ_i are the zero mean common factors

β_{ij} are the sensitivities of the i asset to the j factor

Then Solnik proves that if equation (1.16) is supposed to be the true description of the process underlying the asset returns, then any arbitrage portfolio that is riskless in nominal terms in one numeraire currency, will be riskless for all other numeraires as well. More importantly he shows that the factor structure of returns is also numeraire invariant.

The importance of these "invariance propositions" lies in that, provided they hold, the zero profit arbitrage condition

$$\vec{E} = \lambda_0 + \lambda_1\beta_1 + \ldots\ldots + \lambda_m\beta_m \tag{1.17}$$

will also be numeraire invariant, its only difference from the domestic APT being that it applies to an international set of assets.

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36 They actually found some evidence of risk neutrality.

Empirical tests based on Solnik's model are very scarce, an exception being **Cho, Eun & Senbet (1986)** who tested the joint hypothesis that the Solnik APT is valid and the capital markets integrated. Such a test is only possible by establishing whether the common factors are priced identically across markets. Facing an additional problem compared to that of testing for the domestic APT, Cho, Eun & Senbet had to group the stocks in terms of their country membership, which naturally causes problems of comparing the factor structure across different groups.

Their solution was to adopt intra-battery rather than maximum likelihood factor analysis[37] on monthly returns from 349 stocks representing eleven countries. The three hypotheses they actually tested by applying the Chow test were whether the risk free rate, the risk premia and both risk-free rate and risk premia were the same across two country groups. The hypothesis of equal intercepts was the only one that was not rejected, so that the joint hypothesis had to be rejected.

Clearly, since there is no empirical evidence to support global market integration, there is very little scope for this type of joint tests to provide more useful results in the future. Cho, Eun & Senbet suggest that their results do not exclude the possibility the APT to hold locally or regionally. **Solnik (1991)** admits that due to the extreme difficulties in implementing this type of theory the international investor should follow a more pragmatic approach.

Ikeda (1991) provides a formal proof that Solnik's APT will not yield a riskless portfolio apart from the special case that currency fluctuations follow the same k factor model as asset returns do. He questioned Solnik's approach of specifying the linear factor return generating process in a common numeraire currency as "counter-intuitive" and developed a model where the return generating process is specified in local currency terms instead.

Under Ikeda's no arbitrage conditions when exchange rate risk is hedged through riskless borrowing or lending the expected returns will lie on the same hyperplane

[37] Cho (1984) provides a thorough discussion of inter-battery factor analysis as means of estimating the factor loadings

only if they have been adjusted for the cost of hedging[38]. In his model arbitrage ensures that the covariance of local currency returns with the factor loadings has a linear relationship with currency fluctuations.

Furthermore, Ikeda shows his model to be identical to Solnik's when the currencies follow the same factor structure as the assets. A fundamental difference, though, lies in the fact that while Solnik's APT expresses arbitrage pricing in terms of total factor risk, i.e. both local factor risk and exchange rate risk, Ikeda's APT involves local factor risk of hedged securities.

1.5: Lead-Lag Relationships and Market Interdependence

In this section we will discuss the literature related to the inter-relationships between different markets and their implications. The common aspect of these studies is that they analyze daily returns and take account of different time zones of market opening/closing times so that to establish short term inter-dependencies, possible lead-lag relationships and the dynamic response of the various markets to "shocks" that take place in other parts of the world. A major issue from the viewpoint of asset allocation is whether these inter-dependencies reveal exploitable inefficiencies that would allow the international speculator to earn abnormal rates of return.

Schollhammer & Sand (1987) revived the interest on lead-lag relationships between equity markets, neglected since Hilliard (1979)[39]. The rationale behind their study was that, even though all relevant studies in the 1970's failed to detect any leads-lags, an increasing degree of global equity markets integration and generally rising inter-continental correlations makes it more likely for significant leads-lags to develop. However, they readily identify that due to the different opening-closing times of various markets, leads-lags of up to one day can be perfectly consistent with the efficient market hypothesis.

[38] This is unlike the closed economy APT, where returns always lie on the same hyperplane under the no arbitrage conditions.

[39] Discussed in Section 1

Schollhamer & Sand applied standard ARIMA modelling techniques to prewhiten[40] the series of daily returns, in local currency terms, from thirteen markets sampled over a thirty month period in 1981-1983. They found significant intra and inter-continental correlations, while strong evidence emerged about the US market leading most of the others. Occasionally, however, markets with different opening and closing times were found to both lead and lag each other.

Most of the lead-lag relationships were such as to be consistent with the efficient markets hypothesis, as prices responded quickly to new global information, on the basis of different opening and closing times. Nevertheless, a few lags were of two or more days, providing some mild evidence of market inefficiencies. The most surprising of their findings was that European markets appeared to be more strongly correlated with the USA and Japan than with each other. The Italian, French and Swedish stock markets in particular appeared to be largely unaffected from other European markets.

Overall, the results obtained by Schollhamer & Sand should be regarded with some scepticism, because no provision is made for different dividend yields across countries and more importantly by excluding the impact of foreign exchange fluctuations they make it impossible to answer whether arbitrage operations can be undertaken by the international investor.

Khoury, Dodin & Takada (1987) extended the analysis of correlation, causality and time lags in international markets by applying, Multivariate Time Series (MTS) analysis to 500 daily observations from the US, Canadian, French, German and Japanese markets. The authors first identified and estimated univariate ARIMA models to the five markets, then applied MTS analysis to the stationary (differenced) series using the Akaike information criterion to determine the order of the model and finally interpreted the structure of the identified models.

[40] A difference from all previous studies is that Schollhammer & Sand detrended the series and cross-correlated the white noise residuals, rather than the original data. Nevertheless, their results show little difference between the two different approaches.

Their results showed a very strong lead of the USA over Japan and weaker leads over Germany, France and Canada, while in no occasion the US lagged any of the other markets. Furthermore, analysis of residual correlations at zero lags, confirmed high contemporaneous intra-continental correlations between US-Canada and France-Germany. High contemporaneous correlations and no leads-lags between Japan and the European markets are attributed to common responses in the same day of what happened in the US the previous day.

The more important issue, though, is whether the lag structure allows speculative profits to be made by taking long or short positions in Europe and Japan according to the closing prices in NYSE. If indeed, as they suggest, the real time lead is over a day, this is going to be the case;

In our view, since most of the lead impact takes place within the next day, successful trading will depend on the difference between opening and closing prices in the lagging markets having the same sign as the US change the previous day, a simple fact surprisingly mentioned by no-one.

Also, Khoury, Dodin & Takada's argument that unexploited trading opportunities can largely be attributed to high transaction costs and the restrictions imposed upon institutional investors on shifting large quantities of stock to different markets on a daily basis, no longer makes sense in today's world with the proliferation of stock index futures contracts; Investors can, at very low transaction costs, take long or short positions at the opening of the CAC, DAX or NIKKEI based on the overnight change in the S&P-500 and reverse them at the close. The empirical issue, therefore, is whether such a strategy would have been successful.

O' Hanlon & Papaspirou (1988) combined analysis of leads-lags with an investigation of the day of the week effect, using daily closing index data from nine stock markets for the period 1980-1986. For that respect they regressed the index returns on five daily dummies D_i.e.

$$R_t = \beta_1 D_{1t} + ... + \beta_5 D_{5t} + u_t \tag{1.18}$$

where D_1 takes the value of 1 if Monday and 0 otherwise , D_2...D_5 stand for all days from Tuesday till Friday.

Their regression estimates showed that all nine markets had positive mean returns from Wednesday to Friday inclusive. Six out of nine markets, inclusive the USA, had negative mean Monday returns, whether markets consistently lagging the US like Japan, Australia and France had negative mean returns only on Tuesdays. In addition Japan was shown to be the only market with significantly positive mean Monday return, interpreted partly as a result of lagging the US Friday pattern[41]. Subsequently, O' Hanlon & Papaspirou compared inter-market correlations computed from different days of the week and were unable to reject the hypothesis that they are equal at the 1% confidence level, while their analysis of intra-week correlations provided evidence of significantly negative serial correlation between Monday-Tuesday for Japan, USA and Germany.

Finally O' Hanlon & Papaspirou tested for lead-lag relationships between the US and six other markets by applying Granger causality tests. This was achieved by estimating the unrestricted regression and applying an F test for the hypothesis that $\beta_3 = 0$.

$$R(Y_t) = a + \beta_1 R(Y_{t-1}) + \beta_2 R(Y_{t-2}) + \beta_3 R(X_{t-1}) + e_t \tag{1.19}$$

where R(Y), R(X) stand for returns on the foreign and US market respectively

Subsequently, they repeated the causality tests by interchanging variables so that to test whether the US lagged any of the foreign markets. Their conclusions were similar to previous research, in that they found strong evidence of the US leading all other markets. Nevertheless, they were able to show that the Japanese and Hong Kong markets also led the US up to an extent.

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[41] For Japan, Canada and Australia the F statistic was significant at the 1% level, whereas the same applied for the Japanese and Canadian t-statistic on the Monday dummies.

The most elaborate, to date, development in this area can be attributed to **Eun & Shim (1989)** who applied a nine market vector autoregressive system (VAR) to daily returns from the 1980-1985 period in order to investigate the interactions between markets and establish how "innovations" from one market are dynamically transmitted to others, after taking careful consideration of the exact structure in market time zones[42].

The model used was a moving average representation of VAR which allows the analysis of the system's reaction to random shocks. On the basis of a VAR length of fifteen trading days, they calculated the "residual" returns (i.e. the returns net of expected returns as estimated from the nine market VAR) and the correlation matrix of residual returns, while an orthogonalization procedure[43] allowed them to allocate the residual variance from each market to the appropriate sources.

Following the orthogonalization procedure, their system can be summarily described as

$$\vec{Y}(t) = \sum_{s=0}^{\infty} B(s)\; V\; \vec{u}(t-s) \qquad (1.20)$$

where $Y(t)$ is the 9*1 vector of current and past one step ahead forecast errors (linear combination of residual returns), based alternatively on 5-day, 10-day, 20-day ahead forecasts

B(s) is the 9*9 matrix of conditional expectations of changes in the i^{th} market returns in s periods caused by a unit change in the j^{th} market

V is a 9*9 lower triangular matrix

u is the 9*1 vector of orthogonalized forecast errors with identity covariance matrix

[42] For example, innovations in the German market should be transmitted to the US within the same day, while innovations in the US should affect the German market with a one day lag.

[43] Described at Eun & Sims (Appendix pp: 254-255)

The residual contemporaneous correlations can be interpreted as the extent to which "shocks" that produce abnormal returns in one market at a given day are transmitted to other markets within the same day. Intra-regional correlations were found to be higher than their inter-regional counterparts, phenomenon reflecting primarily time zone differences and possibly a greater extent of economic integration.

In terms of the impact of foreign market innovations, the US was found to be the most exogenous market, all foreign markets collectively explaining only about 11% of its residual variance while on the other hand being the most influential market as well in terms of impact on foreign market residual variances which ranged from 6.5% (Hong Kong) to 42% (Canada). Switzerland, however, was found to be the most interactive market, since its own innovations affected all foreign markets, while being itself highly influenced by them. Surprisingly, Japan proved to be a "follower", since it was seriously influenced by European and US innovations, while having a very small impact on foreign markets.

Finally, Eun & Shim established a dynamic response pattern, by analyzing how the eight remaining markets responded to US innovations. Canada who is in the same time zone and the UK who closes after New York opens, were found to respond primarily within the same day, while the remaining six markets responded largely the next day, the effects dying quickly afterwards. They interpreted these findings as not being inconsistent with informationally efficient markets.

Roll (1992) argued that the well established fact that inter-country correlations computed from daily data are lower than those computed from longer time intervals should not be exclusively attributed to "noise" in daily data but also to technical factors associated with time zone differences and leads-lags. Roll suggested instead that a much better proxy for daily stock market correlations for a country pair {i,j} from different time zones can be obtained in terms of the square root of the adjusted R^2 from the following regression

$$R_{j,t} = g_0 + R_{i,t-1} + g_2 R_{i,t} + g_3 R_{i,t+1} \qquad (1.21)$$

where the three explanatory variables capture the lead, contemporaneous and lagged influence of market i on market j respectively.

Roll's own results indicate that this method leads to materially higher daily correlations compared to those computed from contemporaneous data alone. His conclusions are in fact in agreement with those of **Bailey & Stulz (1990)** who compared daily correlations between the Pacific Basin countries and the US and found that when adjusting of lags this correlation materially increased.

In any case, the most interesting part of Roll's contribution lies in demonstrating that, country specific factors apart, technical factors related to the composition of national stock indices are very important in explaining market volatility and intercountry correlations. Roll's basic argument lies in that some market indices have higher "specific risk" than others because either contain a much smaller number of stocks, or are concentrated in a few dominant industries. Furthermore, wide differences in return volatility across industrial indices have a direct impact on those countries who are "industry specialists"

Roll's empirical evidence suggests that the volatility of index returns is positively related to the "Herfindahl[44]" index of industrial concentration and negatively related to the number of stocks in the index. In addition, he showed that global industrial indices computed exclusively from other countries returns in conjunction to exchange rate movements explain on average over 50% of a given country's daily stock returns, the explanatory power of industrial indices being much higher than that of currencies. Finally, he provided solid evidence that regardless of geographical proximity countries with similarities in their industrial composition tend to have higher daily correlations compared to those from dissimilar industries.

44 The "Herfindahl" index is defined as $\sum_{i=1}^{n} (w_{ij})^2$, where w_{ij} stands for the market value weight of industry i in country j.

1.6: Intertemporal Stability of International Correlations

During the 1980's, a number of authors[45] concentrated on the stability of international correlation structure over time. The intertemporal correlation stability is of great importance, not only because it provides a rough[46] idea about changes in global market integration, but primarily due to the fact that significant changes in the correlation structure reduce the usefulness of historical correlations as inputs of mean-variance optimization models and make useful "ex-ante" allocation strategies difficult to implement.

Surprisingly, even though it is widely accepted (see e.g Jorion 1985, 1986) that the intertemporal instability of the mean return vector has at least as serious implications for the performance of mean-variance efficient portfolios in out of sample periods, no empirical work on the subject has appeared to date other than comparison of empirical Bayes-Stein estimators with historical means (discussed in Part G).

Maldonado & Saunders (1981) tested for stability of monthly returns correlation between the US and four major markets (Japan, Germany, Canada, UK) using data from 1957-1978. At first they computed series of annual correlation coefficients for all 22 years and then tested whether these series were random by applying non parametric runs tests as well as computing the autocorrelation function (ACF) and the Box-Pierce statistic for each series. Their results failed to reject the hypothesis of non randomness, but their validity is arguable due to the very small number of observations for Box-Jenkins purposes.

Their most useful test, was that of longer term equality of correlations between two equal subperiods (approximately ten years each) which is conducted by means of defining a Fisher transformation of the correlation coefficients foe each country pair (i,j) in the two subperiods as

[45] This is an extension of the previously discussed work by Makridakis & Wheelright (1974), Panton, Lessig & Joy (1976) and Watson (1980).

[46] As already mentioned, despite widespread confusion on the issue, increased correlation does not necessarily imply integrated markets.

$$X_k^{ij} = \ln \sqrt{\frac{1+\rho_{ij}}{1-\rho_{ij}}} \tag{1.22}$$

where k = 1, 2

followed by calculating the standardized normal variate Z

$$Z = \frac{X_1^{ij} - X_2^{ij}}{\sqrt{\frac{1}{N_1 - 3} + \frac{1}{N_2 - 3}}} \tag{1.23}$$

where N_k= number of observations for each subperiod

ρ_{ij} = correlation between countries i,j at period k

In this context they rejected the null of equal correlations at the 5% level for three out of four pairs and only accepted it for the USA-Germany correlation.

The Maldonado & Saunders procedures were subsequently criticized by Phillipatos, Christofi & Christofi (1981) and Shaked (1985) on the grounds that they tested for very few correlations, used nominal index returns with no adjustment for dividends, tested only for two consecutive subperiods and inappropriately applied Box-Jenkins procedures. Nevertheless, their study injected new interest in a much deserved area.

Phillipatos, Christofi & Christofi (1981) applied similar procedures to those by Maldonado & Saunders (i.e. runs tests, Box-Jenkins and correlation analysis) but for a much larger number of countries (14) and a slightly different sample period. Furthermore they applied Principal Components Analysis to two consecutive subperiods and compared the variables correlation with the first principal component for each subperiod. Their results indicated a much greater degree of intertemporal stability, than that suggested by Maldonado & Saunders.

Taylor & Tonks (1989) argued that a test such as the one used by Maldonado & Saunders is a static test of "short term" correlation, that does not consider "dynamics" like country specific factors that might be important in the short run but not in the long run. On their part they investigated whether the 1979 abolition of outward exchange controls in the UK reduced the UK market's segmentation and led to increased correlation[47] between the UK stock market returns and those of Germany, USA, Japan and the Netherlands. They applied the Maldonado & Saunders test to Sterling pair-wise correlations for the two subperiods, prior and after the abolition, and failed to reject the hypothesis that correlation were equal.

Shaked (1985) managed to generalize the Maldonado & Saunders procedures so that to be capable to test for equality of correlation coefficients over several subperiods. As a first step he split his twenty year sample period into k subperiods and computed the correlations for all pairs of countries and all subperiods in order to test the hypothesis:

$$\rho_{ij,t} = \rho_{ij,t-1} = \rho_{ij,\,t-2} = \rho_{ij,t-k+1} \tag{1.24}$$

Shaked test is based on the hypothesis that the Z_t's (defined as in Maldonado & Saunders Equation 1.22) for a given pair of countries are all estimates with the same mean correlation and a variance equal to

$$\sigma^2 = \frac{1}{n_t - 3} \tag{1.25}$$

His test of significance is based on a χ^2 test statistic defined as

[47] Taylor & Tonks misinterpret increased correlation as being synonymous to higher level of integration.

$$\sum_{t=1}^{k} w_t(z_t - \bar{z}_w)^2 = \sum_{t=1}^{k} w_t z_t^2 - \frac{\sum_{t=1}^{k} w_t z_t}{\sum_{t=1}^{k} w_t} \sim \chi^2(k-1) \tag{26}$$

where $w_t = \frac{1}{z_t} = n_t - 3$

and[48] $\bar{z}_w = \frac{\sum w_t z_t}{\sum w_t}$

Shaked argued that exchange rate fluctuations in addition to uncertainty in local returns causes the proportion of capital gains as a percentage of total returns to vary substantially across sub-periods, so that price only data are inadequate for analysis of international investments. Furthermore he pointed that when converting all returns into a common "numeraire" currency it is much more appropriate to make decisions in real terms [49] by deflating all returns by the numeraire retail price index. To this end, he applied his tests to real monthly total Dollar returns from sixteen countries. Shaked's tests were performed on holding periods of 2, 2.5, 5 and 10 years respectively, with the χ^2 statistic being calculated for all possible pairwise combinations and holding periods.

His results indicated that stability is highly dependant on the length of the holding period chosen: for the two year subperiods the hypothesis of stability was rejected for 94.2% of the pairs tested, while for the five and ten year subperiods the rejection rate declined dramatically to 31% and 20% respectively. Shaked interpreted that

[48] Notice that an alternative way of expressing the test statistic is

$$\sum_{t=1}^{k} (n_t - 3) z_t^2 - \frac{[\sum_{t=1}^{k} (n_t - 3) z_t]^2}{\sum_{t=1}^{k} (n_t - 3)}$$

[49] This happens because of substantial differences in inflation rates across various numeraires

phenomenon by arguing that short term lagged responses to economic shocks tend to disturb the underlying stability of the returns correlation structure based on common factors.

Odysseos[50] **(1990)** applied both Shaked and Maldonado type tests to total Dollar real monthly returns from ten countries for the more recent period 1982-1990. From a total of 45 pairs for which the Maldonado test was applied, correlation stability was rejected in 32 cases (or 71.11%). Nevertheless, he identified that most of the cases involving rejection were pairwise combinations involving either the US or the UK. His Shaked type tests were applied to two 4-year and eight 1-year subperiods with very few rejections at the 5% confidence level; Stability was rejected for only 6.6% and 2.22% of pairs for the 1-year and 4-year subperiods respectively.

Meric & Meric (1989) acknowledge that a significant deficiency of the aforementioned testing procedures lies in that they concentrate on isolated coefficients, rather than on the entire correlation matrix that is used as input to all optimization programs. Their main testing procedure lies in applying a test of stability between two correlation matrices, based on a variant of the Box-M test defined as:

$$\begin{aligned} M &= k \sum_{i=1}^{g} (n_i - 1) \ln |C_i^{-1} C| \\ C &= \frac{1}{N-g} \sum_{i=1}^{g} (n_i - 1) C_i \sim \chi^2 [v(v+1)(g-1)/2] \\ k &= 1 - \frac{(2v^2 + 3v - 1)}{6(v+1)(g-1)} \left(\sum_{i=1}^{g} \frac{1}{n_i - 1} - \frac{1}{N-g} \right) \end{aligned} \tag{1.27}$$

where C_i is the i^{th} subperiod correlation matrix

g is the number of subperiods to be compared, i = 1, 2,...g

v is the number of variables in the correlation matrix

n_i is the number of observations in the i^{th} subperiod

N is the total number of observations across all subperiods

[50] M.Sc Dissertation at CUBS written under my supervision.

Meric & Meric applied this test on nominal, non dividend adjusted, monthly Dollar returns from 17 countries, covering the 1973-1987 period for time horizons of 1.5, 3, 5, 7.5 years respectively. Nevertheless, they applied the test in a limited way by only testing correlation pairs of consecutive subperiods (i.e. they only tested the case for which g = 2). Their results again indicated that stability was increasing together with the portfolio holding period; Stability was rejected for 7/9 pairs of consecutive 1.5-year periods and 2/4 pairs of 3-year periods, but was always accepted for the 5-year and 7-year subperiods.

Subsequently, Meric & Meric suggested that the same test can be applied to test for stock market seasonality, by formulating series of returns for each calendar month[51] and then testing the hypothesis that the correlation matrices for two consecutive months are the same. Their results indicated the existence of seasonality for consecutive months in the May-September period.

1.7: Estimating the Inputs for the Asset Allocation Optimization Problem

Estimating the portfolio inputs (mean returns, variances, covariances) is perhaps the single most important issue in asset allocation; If "ex-ante" estimates will turn out to be significantly different from the actual ("ex-post") values the portfolio performance will be poor irrespective of the optimization model used. In the absence of a satisfactory international asset pricing model, multivariate statistical theory provides a number of alternative procedures that might prove useful to a portfolio manager i.e.

i) Factor models for forecasting the correlation matrix
ii) Bayesian estimates based on non-informative priors that can be used to estimate the variance-covariance matrix and
iii) Empirical Bayes-Stein estimates with informative priors that can be used to estimate the mean return vector as well.

[51] In this way the number of observations in each series equals the number of years in the sample (15) which is smaller than the number of variables (17). In order to avoid singularity problems the number of variables was reduced to 14. In any case, their sample size is very small and consequently their test results are suspicious.

Surprisingly, despite substantial advances in both multivariate statistics and portfolio theory the relevant literature on international portfolios remains quite scarce, based primarily on Eun & Resnick (1984, 1987, 1988), Jorion (1985) and Dumas & Jacquillat (1990). These developments will be reviewed in the remaining of this section.

Eun & Resnick (1984) argued that forecasting inter-dependency among foreign securities is more difficult than for domestic one's, problem which is compounded from the fact that returns from different countries have to be correlated after being translated to a common "numeraire". For this reason, they thought that forecasting correlations on an individual basis would be arduous, expensive and unlikely, suggesting the need for them to be estimated simultaneously through some form of common factor or Bayesian procedure.

Eun & Resnick concentrated in the correlations between individual stocks, rather than indices, converted in Dollars and without any provisions for hedging foreign exchange risk. Their methodology consisted in analyzing separately two non-overlapping samples of 80 firms each to ensure the robustness of their results, while the "ex-ante" strategies were developed and measured on the basis of splitting their ten year monthly returns matrix into a seven year forecasting period and a three year validation period.

In this context, they tested and compared a large number of alternative testing procedures: the simple "historical" correlation model, a "global mean model" where all correlations are set to be equal to each other and to the historical average, an "industry mean model" where all intra-sector correlations are set to be equal to the average correlation among all stocks from that sector while the inter-sector coefficients are set equal to the average pairwise correlation between each pairs of sectors, a "country mean model" constructed like the industry mean model by substituting sectors with countries, two versions of a global "single index model", a two factor model based on a global and a country index and a two factor model based on a global and an industrial index[52].

[52] Eun & Resnick used regression analysis to orthogonalize the "global index" with the "country" and "industry" indices respectively.

Eun & Resnick did not re-invest any "ex-ante" optimized portfolios, so that the performance of their correlation forecasts was assessed on the basis of statistical criteria alone, namely the Root Mean Square Forecast Error (RMSE[53]) and Stochastic Dominance of the frequency distribution of forecast errors[54]. Their results were generally consistent between the two non-overlapping samples of stocks, their rankings showing the "National Mean" model to be on top followed by the "Historical Model", the two factor "Country Model" and then the remaining nine models. Subsequently, though, Eun & Resnick repeated their analysis after adjusting all models to have the same mean forecast, in which case the "Historical" model's ranking dropped below that of the two factor "Country Model".

Jorion (1985) was the first to discuss the problem of "estimation risk" in an international context. He argued that the benefits from "ex-post" mean-variance studies on international portfolios have been largely illusory, because primarily of the very poor predictory ability of past sample means. According to Jorion, the fact that the portfolio's Sharpe performance measure always deteriorates substantially when measured outside the sample period ("ex-ante") must be attributed primarily to the instability of the sample means and, to a lesser extent, to the instability of variances and covariances. Consequently, controlling estimation risk in the mean return vector is the single most important issue if significant benefits from international diversification are to be realized.

To further support his view about the poor predictive ability of the sample means, Jorion applied a simple univariate test due to **Fama (1976)**, consisting of regressing the realized return for country j at time t against the past moving average[55]:

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[53] The RMSE is defined as

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(F_i - A_i)^2}$$

[54] In practice the stochastic dominance criterion proved almost useless, since in the absence of any strict dominance relationships they were forced to "redefine" it in a rather unconvincing way (see Eun & Resnick 1984, pp: 1317-8)

[55] Jorion argues that choosing the regression model without a constant term is more appropriate, since past averages tend to predict the level rather than the variability of returns.

$$r_{jt} = \beta_j \bar{r}_{j,t-1} + \epsilon_{jt} \tag{1.28}$$

Naturally, If the past average is an unbiased forecast then $\hat{\beta}$ should be close to unity[56]. Jorion applied this procedure using both twelve and sixty month moving averages to seven country indices and found that in most cases $\hat{\beta}$ tended to be close to zero while the R^2 was very low.

Jorion's main contribution is that he advocated the use of Bayes-Stein estimators as significant improvements over the traditional sample means. These estimates predict the future means for all assets, as a weighted average between the asset's past sample mean $\bar{r}_j$ and the overall mean of means $\hat{r}_0$ ("grand mean"), where the weight used $\hat{w}$ is called "shrinkage factor" and has to be estimated from the data.

Therefore, the general form of a Bayes-Stein predicted mean can be expressed as:

$$\mathscr{E}(\bar{r}_j) = \hat{w}\,\hat{r}_0 + (1-\hat{w})\,\bar{r}_j \tag{1.29}$$

Effectively, the closer $\hat{w}$ is to unity (zero) the more predictive ability is placed on the grand mean (sample mean). One serious problem, though is that there is no universally accepted method for estimating the shrinkage factor, the estimates being totally dependant on the chosen of a "suitable" prior. Jorion himself (1984) derived a shrinkage estimate as:

$$\hat{w} = \frac{\hat{\lambda}}{(T+\hat{\lambda})}$$
$$\hat{\lambda} = \frac{(N+2)\,(T-1)}{(\bar{r}-r_0\vec{1})'\,S^{-1}\,(\bar{r}-r_0\vec{1})\,(T-N-2)} \tag{1.30}$$

[56] Fama (1976) has shown that estimating the above equation by OLS is equivalent to minimizing the sum of squared forecast errors.

where **T** is the number of observations

N is the number of variables

S stands for the sample variance-covariance matrix

Subsequently, Jorion applied historic simulations on a variety of investment strategies, based on "passive", "traditional" mean-variance, "minimum variance" and "Bayes-Stein" approaches respectively. His results suggested that for medium sample sizes (up to 60 monthly observations) the minimum variance[57] and Bayes-Stein approaches performed far better than the remaining strategies, the Bayes-Stein outperforming the minimum variance portfolio for larger samples.

Eun & Resnick (1987) examine the problem of estimation risk in the wider context of potential vs realizable gains from international diversification, expressed in terms of fifteen different "numeraire" currencies. Their approach to controlling "estimation risk" is restricted to applying a Jobson-Korkie (1980, 1981) procedure, where the best estimate of the expected return for all stocks is the "grand mean" calculated from the stocks historical mean returns. Naturally, this is just a special case of the Bayes-Stein estimates discussed before, with the shrinkage factor $\hat{w} = 0$.

Eun & Resnick compared the Jobson-Korkie strategy with a "passive" equally weighted strategy and the historical mean-variance portfolio. Their approach consisted in splitting their ten year data period from fifteen countries into a seven year forecasting period and a three year portfolio holding period. The efficient frontier for the three year holding period was calculated on the basis of both the actual (ex-post) inputs and those forecasted on the basis of the three methods.

Then, by arbitrarily assuming a worldwide risk-free interest rate of 5%, they calculated the Sharpe performance (SHP) measure for both domestic and international portfolios in each one of the numeraire currencies. In this context, they define as "gains" from diversification the following differential:

[57] The optimized weights of the minimum variance portfolio do not depend on the sample means, only on the variance covariance matrix.

$$\Delta SHP = SHP(IP) - SHP(DP) \qquad (1.31)$$

where **IP**, **DP** stand for domestic and international portfolios respectively.

On the basis of Equation (1.31) Eun & Resnick measured both actual and potential diversification benefits by calculating the Sharpe measure for portfolios formed on the basis of both the "ex-ante" and the "ex-post" investment weights. In the "ex-post" analysis the ΔSHP was positive for all fifteen numeraires suggesting across the board benefits from international diversification, while naturally all the "ex-ante" strategies led to significantly inferior performance.

Among the "ex-ante" strategies the Jobson-Korkie performed relatively best, while the historical portfolio was by far the worst being outperformed by the national portfolio in ten out of fifteen numeraires. In any case, none of the international "ex-ante" strategies managed to outperform the domestic portfolios for four countries (Japan, Sweden, Singapore, USA). Overall, their results are important because they clearly demonstrate how significantly inflated are the diversification benefits suggested by the large majority of studies in this area.

Eun & Resnick (1988) extended their previous work on performance of "ex-ante" strategies in a number of directions:

First of all, for all unhedged strategies they developed a hedged counterpart; Upon using weekly stock market returns in local currency terms, spot exchange rates and 6-month forward premia (which they converted into weekly) for six currencies over the 1980-1985 period, they created their hedged return series by applying Equation (6). Such an approach is clearly inconsistent in the sense that it mixes an "ex-post[58]" hedging relationship with an "ex-ante" portfolio strategy, even though they claimed that it provides a "reasonable" approximation.

[58] Equation (1.6) is "ex-post" because it ignores uncertainty of future returns; Equation (1.7) should be used instead.

Then, they extended their coverage of estimation risk by testing the Jorion approach to forecasting the mean return vector on the basis of Equations (1.28, 1.29), while when applying the Jobson-Korkie approach they also used their unbiased estimate of the inverse variance-covariance matrix defined as:

$$\hat{\Sigma}^{-1} = \frac{(T-N-2)}{(T-1)} S^{-1} \tag{1.32}$$

where S stands for the sample covariance matrix.

Finally, they extended their investment of "ex-ante" portfolios from a single to multiple out of sample investment holding periods, so that their appropriate performance measure became the average of the Sharpe measures over all holding periods. In doing so they assumed a Dollar numeraire and a weekly risk-free rate of zero.

Their results showed the hedged strategies to dominate the unhedged ones, while controlling estimation risk led to much improved performance in both the hedged and unhedged categories. Among them, however, Jobson-Korkie estimates performed better than the Jorion Bayes-Stein type estimates, which provides an indication that Jorion's choice of prior might not have been successful. One should also be aware, that the dominance of hedged strategies must have been somewhat exaggerated due to their "ex-post" nature.

Dumas & Jacquillat (1990) were highly critical of the methodology applied by Eun & Resnick (1987) on the basis that it is unjustifiable to assume that different national investors have the same risk-free interest rate, as well as inappropriate to compare the performance of tangent portfolios held by investors from different countries[59]. Their argument is justified on the grounds that investors from different countries, but equal risk aversion, would combine in different proportions the local risk-free asset and the local tangent portfolio (proved by Adler & Dumas 1983).

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[59] Notice that this criticism does not apply to the Eun & Resnick (1988) paper, since they only take the viewpoint of a US investor.

Dumas & Jacquillat strongly agree with Jorion about their preference of empirical Bayes-Stein, rather than conventional Bayesian estimators with non-informative priors, on the grounds that the latter for sufficiently large samples tend to converge to historical sample estimates. They disagreed, though, with Jorion about both his choice of prior and the assumptions underlying the derivation of his shrinkage factor:

At first they pointed that the Jorion prior which shrinks the estimated mean return vector towards the "grand mean[60]" is equivalent to shrinking the investor's choice towards the minimum variance portfolio. Nevertheless, since the minimum-variance portfolio is numeraire specific, they argue that it is not a neutral enough prior and suggest instead a prior which assigns equal weights to all assets such as:

$$\begin{aligned} \mu_i &= \mu_{n+1} \quad \forall i \\ \sigma_{ii} &= \sigma_{jj} = 2\sigma_{ij} \quad \forall i,j \end{aligned} \tag{1.33}$$

Essentially, Dumas & Jacquillat suggest a prior that shrinks towards the equally weighted portfolio, rather than the minimum variance portfolio.

Then, they argued that Jorion failed to provide a convincing justification for the assumptions underlying the derivation of his shrinkage factor[61]. In a more practical sense, their own simulation results indicated that optimal performance is achieved when $\hat{w}$ is given values close to zero, while Jorion's approach leads to much higher values for $\hat{w}$ with undesirable consequences. This argument can be given further weight, if we recall the Eun & Resnick (1988) results where the Jobson-Korkie (where $\hat{w}=0$) outperformed the Jorion estimators.

[60] This argument applies even more strongly to the Jobson-Korkie approach which effectively chooses the grand mean as the optimal return forecast for all assets, which in turn leads to the selection of the minimum-variance portfolio for all investors.

[61] In deriving his shrinkage factor, Jorion used a quadratic approximation to the investor's utility-loss function and subsequently assumed without justification that its corresponding matrix equals the returns inverse covariance matrix.

Recently, **Cumby, Figlewski & Hasbrouk (1991)** attempted to provide estimates of variances and correlations within an GARCH type framework and then apply them to international portfolios. Their approach was to built an econometric model in which the asset risk parameters were allowed to vary over time and then apply a number of GARCH variants in order to find which one fitted their data set best. They concluded that the Exponential GARCH (EGARCH) fits variances best, while the simple GARCH is the most appropriate for modelling correlations. Nevertheless, despite choosing the "ex-post" optimal GARCH variants, they were unable to demonstrate any significant improvement in portfolio performance. Apart from that, any attempts to use GARCH methodology for modelling asset returns is meaningful only for very short investment holding periods and, therefore, of very limited use for asset allocation purposes.

Even for daily stock index returns, though, the potential usefulness of GARCH type modelling is far from clear; **Roll (1992)** identified problems of non-normality and heteroscedasticity in the residuals from time series regressions of country index returns on global industry factors, exchange rates and a Monday dummy. In order to solve these problems, he re-estimated the time series model for all countries using a GARCH (1, 1) process to capture non-stationarity in the error variance. Nevertheless, this effort proved fruitless since the pattern of his regression coefficients, t-statistics and adjusted R^2 was almost identical to that of their counterparts from the OLS regression.

CHAPTER II

THE DATA, AN "EX-ANTE HEDGING STRATEGY AND SOME PRELIMINARY RESULTS

2.1: Selecting the Data

2.1.1: Some Important Principles

Since our primary goal is to analyze optimal international asset allocation strategies from the viewpoint of a UK (primarily institutional) investor, we need data that are both reliable and suitable as inputs to real life asset allocation problems. In this context, several considerations apply:

- First of all, the typical asset-mix for institutional portfolios consists of domestic and international stocks, fixed income securities, a limited amount of "cash" and possibly real estate. For this reason, it was deemed necessary to include, apart from stock indices, a number of bond indices denominated in all major currencies[1].

- Secondly, as already discussed in chapter one, the length of the measurement interval for returns is very important because sampling estimates of correlations and asset variances depend on it. Daily (or even more frequent) observations can be of great value to foreign exchange traders, arbitrageurs etc., but are of little value to fund managers who are unable to adjust their asset holdings all too frequently. Our decision to analyze monthly returns is, therefore, the most rational considering the fund managers likely investment horizons.

[1] In recent years, several studies have demonstrated substantial "ex-post" benefits from diversifying into real estate, generally justified on the basis of its historically high long term mean return and low correlation with stocks and fixed income securities. Nevertheless, given the absence of reliable monthly data on property returns for most countries and the difficulties associated with the nature of property investments (high transaction costs, no uniform price, low marketability, problems with diversifying and reallocating real estate) we found no meaningful way to incorporate it into the analysis.

- Another issue of undisputed importance is related to the necessity of measuring returns so that to include both dividends and capital gains. All too many studies have been based on price indices that capture capital gains only, falling short of providing useful guidance to asset allocation. This is particularly important to international portfolios because of the wide discrepancy of average dividend yields across countries. Notice, however, that dividend adjusted returns are likely to have a substantial impact on the mean return vector and, consequently, to "ex-ante" optimization results but are unlikely to have a significant impact on the magnitude of estimated correlations.

- The choice of Sterling as the "numeraire" currency is also likely to have significant implications. While reviewing the literature, we have repeatedly emphasized that global portfolios risk-return profiles are "numeraire" dependent and it is theoretically unsound to compare performances of optimal portfolios held by different national investors. In our view, the most satisfactory and realistic solution is to relate the Sterling based returns from an international portfolio to the real liability structure of an institutional investor (e.g. a pension fund). Since these liabilities increase in line with UK inflation, all foreign market returns should be converted first into Sterling and then deflated by the UK RPI.

- Finally, it is very important to emphasize the implications of analyzing returns from market indices, as opposed to returns from individual stocks[2]. Active managers of international portfolios have a wide range of alternative strategies they can pursue in their attempt to outperform an imposed performance benchmark like stock selection, market timing[3], country selection[4], currency selection or currency timing.

If fund managers decide to invest in foreign stock indices then the possibility of value added from stock selection is eliminated, while if they decide to fully hedge their expected foreign exchange exposure no benefit can be derived from superior currency selection and timing.

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[2] Surprisingly, there is a uniform failure to even mention this issue in existing academic literature.

[3] In this context "market timing" stands for shifts between asset classes within the same country.

[4] The country selection concept can be broadened to include selection of foreign bond indices as well.

In this context, the single most important influence on performance becomes country selection[5]. Notice also that taking or adjusting positions on stock market indices is not only optimal in terms of minimizing portfolio specific risk within each country, but also can be easily and inexpensively implemented by means of investing in appropriate combinations of cash and stock-index futures contracts.

2.1.2. The Data

All monthly index data used in this study, cover a period of nine years or 108 monthly observations from 1-2-82 to 1-2-91[6] and are on a total returns basis. Longer time series were not included because they were not inclusive of dividend yields. In any event, our 108 monthly observations are perfectly adequate for most types of analysis.

Stock market returns have been calculated from the Morgan Stanley Capital International Indices (MSCI), which are market capitalization weighted prices averages[7] from nineteen countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Italy, Japan, Netherlands, Norway, Singapore/Malaysia, Spain, Sweden, Switzerland, UK and USA. These indices account for about 1,500 stocks representing roughly 60% of stock market capitalization for each country. All data have been collected in both local currency and US dollar terms. Furthermore, a market capitalization weighted MSCI world index is available, which can be used as a proxy for the world equity portfolio (not however for the world market portfolio).

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[5] Since this thesis is not concerned with macro-economic fundamental forecasts, we are not going to analyze market timing strategies. In any event there is very little evidence of fund managers who managed to consistently outperform the market through market timing and is only practised to a limited extent by most institutions.

[6] Actually 109 monthly index observations are required in order to produce 108 monthly returns for each data series.

[7] In fact they represent weighted price averages at a given point in time (monthly intervals), adjusted for dividend yields

For the purpose of this study, the monthly bond indices[8] used are the Salomon Brothers Bond Performance Indices (SBBPI). They include relatively long term issues, comprising only bonds with time to maturity over five years. All indices are inclusive of gross interest reinvested and are available in both local currency and US Dollar terms. Government Bond Indices are available in Canadian Dollars, French Francs, Japanese Yen, Dutch Guilders, Swiss Francs, Pound Sterling, US Dollars and Deutschemarks. Eurobond and Foreign Bond Indices[9] were also available for most of these currencies.

Two more types of indices were included as good potential diversification vehicles, i.e. US Dollar Floating Rate Notes (FRN) and US Dollar Zero Coupon Bonds. The attraction of FRNs is their very low sensitivity to interest rate changes, while Zero Coupon Bonds have the maximum duration among all bonds of the same maturity and consequently high price volatility and expected return[10].

Collecting spot and one month forward exchange rates was also necessary in order to convert all returns in Sterling terms and construct the hedged return series. Monthly exchange rates have been collected from Datastream for the same period (1.2.82-1.2.91) for fifteen out of the nineteen currencies. Forward rates for Australia, Hong Kong, Finland and Singapore-Malaysia were not available for the entire nine-year period, so that it proved impossible to create hedged return series for these countries. All exchange rates are closing (middle rates for the first day of each month) and expressed in terms Pounds Sterling per one unit of foreign currency[11].

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[8] As a matter of fact, the MSCI stock indices are updated monthly on the first day of each month, while the SBBPI are updated at the end of the same month. By lagging the stock series by one monthly observation it is possible to calculate monthly bond/stock cross correlations, even though some small error will exist since the sampling periods differ by one day.

[9] Foreign Bond Indices refer to bonds issued domestically at the local currency by a foreign issuer. They are distinct from Eurobonds which are bonds issued internationally through bank syndicates and normally denominated in a currency other than that of the country of issue.

[10] Zero Coupon Bonds with no default risk have always duration equal to their time to maturity.

[11] Notice that in most subsequent analyses we express exchange rates as the Sterling value of one unit of foreign currency so that an increasing exchange rate to denote positive currency returns for UK investors.

Nominal Sterling returns have been deflated by means of the OECD All-Items Retail Price Index for the UK. A technical problem faced was that the monthly RPI is being updated at the middle of each month, whereas stock and bond indices are recalculated at the beginning of the month.

Since it would be clearly inappropriate to use a deflator that leads or lags the price indices by half a month, price levels for the first day of each month were assumed by interpolating between the previous and next observation (eg. assumed price level at 1.12.85 is the average between the price indices for November and December of that year).

2.1.3. Deriving the nominal and real returns

On the basis of the Central Limit Theorem, if returns on an index (when measured over short time intervals) are independent and identically distributed, denoted r_1, r_2.....r_n, then the natural logarithm of the monthly return $\ln(1+R_m)$ will be approximately normal and R_m will be lognormal.

Notice that if no dividends are assumed to be paid within the month[12], then $\ln(P_t/P_{t-1})$ should be also interpreted as the continuously compounded rate of return. Since approximate normality is a desirable statistical property for multivariate statistics, we have followed the standard practice in computing nominal returns as

$$r_N = \ln(1+R_m) = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{2.1}$$

In a very similar way, we can construct a data series of real monthly Sterling returns R_{Rm} by simply deflating the series by means of the RPI index I_t.

$$r_R = \ln(1+R_{Rm}) = \ln\left(\frac{P_t}{P_{t-1}}\right) - \ln\left(\frac{I_t}{I_{t-1}}\right) \tag{2.2}$$

[12] In this case this is not too unrealistic, since Morgan Stanley adjusts the end of month prices for the estimated dividend yield.

2.2. Formulating an "Ex-Ante" Hedging Strategy

2.2.1. Deriving the hedged return series

In this section we consider the viewpoint of a UK investor who wants to eliminate, to the maximum possible extent, his/portfolio's diversifiable risk that can be attributed to exchange rate movements. Assuming that he/she has no better forecast of future spot rate changes other than that implied by the forward premia, her optimal solution will always be to opt for a complete rather than a partial hedge[13].

The problem is, however, that such a hedging strategy is based on uncertain predictions. Assuming she chooses a one month roll-over hedging strategy, the one month forward rate is known, but the monthly return from the foreign assets in local currency has to be forecasted with error. Consequently, this type of hedge will always be imperfect.

Such a hedge can be formalized as follows: the fund manager sells one month forward the expected foreign currency proceeds, while a month later she has to convert the difference between actual and expected return (either positive or negative) back into Sterling at the uncertain future spot rate. The realized Sterling rate of return from this "ex-ante" strategy will be:

$$R_{it}^{£H} = [1 + \mathcal{E}(R_{it}^{L})]\,[1 + \ln(\frac{F_{t-1}}{S_{t-1}})] + [R_{it}^{L} - \mathcal{E}(R_{it}^{L})]\,[1 + \ln(\frac{S_t}{S_{t-1}})] - 1 \quad (3)$$

where $p_i = \ln(F_{t-1}/S_{t-1})$ is the one month forward premium/discount of the foreign currency against Sterling[14] known at time t-1

R_{it}^{L}= Actual monthly return in local currency terms

$x_i = \ln(S_t/S_{t-1})$ is the spot rate change observable at time t

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[13] As already discussed in chapter one, if the investor's forecast of spot rate changes differs from the forward premia then the optimization solution might call for a partial rather than a complete hedge.

[14] Also here F_t is expressed as the Sterling value of one unit of foreign currency.

Essentially, the RHS of equation (1.3) consists of two components: the first one can be interpreted as the known (certain) return at the beginning of each month while the second represents the uncertain part of the hedged returns and can be either positive or negative.

In order to implement this "ex-ante" strategy it is necessary for the fund manager to assign subjective values to $E(R_i)$ each month and for all assets. In this context, it would be entirely inappropriate to derive such forecasts from the entire sample because that would involve imposing "ex-post" knowledge to an "ex-ante" strategy[15].

A rather more consistent approach, i.e. within the "ex-ante spirit", is to be agnostic about individual future stock or bond index mean returns, but instead to use as forecast a "shrinkage" of all sample means towards the "overall mean" as derived from all the stock or bond indices in the sample. With this in mind, we assigned as expected monthly returns for all individual bond and stock indices in our sample the "overall mean" values of 0.835% and 1.272%, as derived from the entire data matrices of monthly bond and stock index returns respectively.

The implementation of the "ex-ante" hedging strategy took place in two steps: at first we calculated the matrices of forward premia and spot rate changes for all observations and currencies[16] and then estimated equation (1.3) for all fifteen stock market indices and twenty-one bond indices separately, resulting to a (108*36) matrix of nominal hedged monthly returns. Finally, the same UK RPI index has been used as deflator in order to derive real monthly hedged returns for all indices.

[15] For example, we can not use the 1982-1991 sample mean for each index since at the time she formulated her strategy the fund manager did not have access to that information.

[16] Both matrices are (108*15)

Table 2.1

Descriptive Statistics for Monthly Sterling Returns

(1) STOCKS	(2) MEAN% NOM-U	(3) STD% NOM-U	(4) MEAN% NOM-H	(5) STD% NOM-H	(6) MEAN% REA-U	(7) STD% REA-U	(8) MEAN REA-H	(9) STD REA-H
Austria	1.04	6.32	1.14	6.24	.57	6.36	.67	6.25
Belgium	1.96	6.27	1.93	5.90	1.50	6.33	1.47	5.97
Canada	.85	6.41	.90	5.16	.38	6.48	.44	5.18
Denmark	1.40	6.02	1.24	5.52	.94	6.05	.78	5.49
France	1.63	6.93	1.49	6.45	1.17	6.91	1.03	6.44
Germany	1.49	7.16	1.56	6.66	1.03	7.20	1.09	6.66
Italy	1.23	7.60	.94	6.97	.76	7.65	.48	7.03
Japan	1.52	7.38	1.51	6.07	1.06	7.42	1.04	6.10
Netherl	1.75	5.83	1.85	5.56	1.28	5.86	1.39	5.58
Norway	1.38	8.21	1.25	7.93	.91	8.22	.79	7.93
Spain	1.53	7.39	1.18	7.05	1.07	7.43	.71	7.08
Sweden	1.81	7.48	1.83	7.36	1.35	7.55	1.37	7.40
Switzer	1.22	5.68	1.47	5.59	.76	5.72	1.01	5.59
UK	1.44	5.67	NA	NA	.97	5.70	NA	NA
USA	1.24	6.32	1.59	4.86	.77	6.36	1.13	4.88
BONDS								
Cana\GB	1.20	4.41	1.27	2.83	.73	4.49	.81	2.83
Cana\EU	.99	3.68	1.07	1.59	.53	3.76	.60	1.61
Fran\GB	1.17	3.09	1.02	2.08	.70	3.18	.55	2.10
Fran\EU	1.15	2.92	1.01	1.48	.69	3.02	.55	1.52
Germ\GB	1.00	3.14	1.06	1.59	.54	3.22	.59	1.63
Germ\EU	.98	3.19	1.04	1.85	.52	3.27	.57	1.87
Japa\GB	1.06	3.96	1.02	1.96	.59	4.03	.56	1.99
Japa\EU	1.05	3.61	1.02	1.48	.59	3.69	.55	1.54
Japa\FB	1.08	3.50	1.04	1.26	.61	3.59	.58	1.32
Neth\GU	.99	3.06	1.07	1.51	.52	3.17	.60	1.56
Neth\EU	1.03	2.76	1.11	1.18	.56	2.86	.65	1.25
Neth\FB	1.06	3.00	1.14	1.45	.60	3.10	.68	1.50
Swit\GB	.61	2.83	.87	1.43	.14	2.92	.41	1.47
Swit/FB	.73	2.90	1.00	1.54	.27	2.99	.53	1.54
UK\GB	1.09	2.90	NA	NA	.62	2.91	NA	NA
UK\EU	1.05	2.14	NA	NA	.59	2.15	NA	NA
US\GB	1.05	4.14	1.36	2.68	.59	4.21	.90	2.70
US\EU	1.03	3.75	1.34	1.65	.56	3.82	.87	1.69
US\FB	1.18	4.10	1.49	2.31	.72	4.17	1.02	2.34
US\FRN	.66	3.64	.98	.84	.19	3.69	.52	.90
US\ZER	1.16	4.63	1.46	3.47	.69	4.69	1.00	3.48

NOM-U, NOM-H denote nominal unhedged and nominal hedged returns respectively in £ terms.
REA-U, REA-H denote real unhedged and real hedged returns respectively in £ terms
All descriptive statistics are expressed as monthly (not annualized) percentages

Table 2.1 presents descriptive statistics for nominal and real Sterling returns both unhedged and hedged[17]:

Mean monthly nominal returns, both hedged and unhedged, exceed their real counterparts by approximately 0.47%, which is the mean monthly change in the UK RPI for the sample period. Our "ex-ante" hedging strategy reduced the volatility of all individual indices, but the reduction was considerably higher for the bond indices. In fact, the volatility of hedged returns from most European stock markets has been only slightly reduced, while the most sizable reduction occurred for the US, Canadian and Japanese stock markets.

As a matter of fact, hedged returns were generally higher than their unhedged counterparts. For example, the overall mean ("grand mean") monthly real hedged bond return was 0.66%, while the unhedged one was only 0.55%[18]. This indicates that over the sample period, Sterling on average appreciated more (depreciated less) than predicted by forward rate expectations. Given that as many as five bond indices in our sample are denominated in US Dollars, this phenomenon should be partially attributed to the effect of a sizable US Dollar depreciation against Sterling over this period.

One of the most interesting findings lies in the fact that in almost all cases Government Bonds exhibit higher mean returns and volatility than Eurobonds. This counter-intuitive phenomenon can by mainly attributed to two factors:
i) Eurobonds are not subject to withholding taxes on coupon payments and, therefore, their effective after tax return could potentially be higher for a non tax exempt investor and
ii) Government bond issues have longer average maturities than Eurobonds and, consequently, tend to have higher price sensitivity to interest rate changes (modified duration).

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[17] Notice that the standard deviation of returns is reported on a monthly and not on an annualized basis. As pointed out by Fama (1975) the annualized equivalent of monthly volatility can be found by multiplying it by the $\sqrt{12}$

[18] Naturally, the UK bond indices are excluded from the calculation of these averages.

In any event, the Dollar Floating Rate Notes (FRN) showed by far the lowest volatility, being almost risk-free when hedged.

Also notice that the correlations between government bonds and Eurobonds or foreign bonds denominated in the same currency were extremely high, in most cases taking values well above 0.9[19]. This means that for asset allocation purposes the benefits from diversification can be fully derived by including just one type of bond index per currency, while in many cases multivariate analysis of bond returns is impossible due to singularity problems[20].

For these reasons, in most of the subsequent analyses the international portfolio asset mix consists of stock indices, government bond indices, Dollar FRN's and zero coupon bonds. The last two types of financial instruments were included because they exhibit distinctive risk-return characteristics and could potentially prove to be useful diversification vehicles. In fact, as expected the zero coupon bonds proved to have the most volatile returns because of their maximum duration.

2.2.2. Is foreign exchange risk completely non-systematic ?

For the foreign exchange risk to be perfectly non-systematic (diversifiable) it would be necessary to show that expected returns of hedged and unhedged returns are equal and that the variance of Sterling hedged returns from a foreign index equals the variance of returns from that index in local currency terms i.e.

$$\begin{aligned} \mathcal{E}(R_i^{£H}) &= \mathcal{E}(R_i^{£U}) \\ var(R_i^{£H}) &= var(R_i^{L}) \end{aligned} \tag{2.4}$$

The former condition implies that hedging is "costless" in terms of trading-off return for risk, while the latter implies that hedging can reduce investment risk to the level of foreign market risk alone.

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19 Sample correlation matrices available from the author on request.

20 For example, it is impossible to factor-analyze the returns matrix, when both Government Bonds and Eurobonds are included.

To see whether these conditions hold, it is worthwhile comparing the realized Sterling returns from an unhedged foreign index $R_{it}^{\pounds U}$ calculated as

$$R_{it}^{\pounds U} = [1 + R_{it}]\,[1 + \ln(\frac{S_t}{S_{t-1}})] - 1 \tag{2.5}$$

to the hedged return given by equation (2.3)

Since $E(R_{it}-E(R_{it}))=0$ then, excluding transaction costs, the expected hedged and unhedged returns will be equal provided that the one month forward premium/discount is an unbiased predictor of the future spot rate. In fact transaction costs should not be ignored, since they not only include bid-ask spreads and brokerage commissions but also administration and management expenses related to hedging. If transaction costs are expressed as a percentage c of annual return then:

$$\mathcal{E}(R_{it}^{\pounds H}) = \mathcal{E}(R_{it}^{\pounds U}) - c \tag{2.6}$$

In fact these costs might partly explain the reluctance of some fund managers to hedge the currency exposure in their portfolios. Notice also that incorporation of such costs would account for the possibility that an optimization determined international asset allocation strategy might favour partial rather than complete hedging.

The overall conclusion is that the exchange rate risk that is assumed by not hedging, should only partially be seen as a non-systematic risk because of the following facts: i) forward rates are not always unbiased predictors of spot rates ii) the existence of transaction costs associated with hedging reduce the effective hedged expected return and iii) any "ex-ante" hedging will inevitably be imperfect since in general $\mathcal{E}(R_{it}) \neq R_{it}$.

The first two facts imply that a price might have to be paid[21] in terms of affecting the expected hedged return, while the last one means that not all foreign exchange risk is diversifiable since the variance of the hedged return in Sterling terms will still be greater than the variance of the unhedged return in local currency terms.

Finally, an important observation based on the second term of the RHS from equation (3) is, that the residual currency exposure depends on the correlation between the unexpected portion of returns and exchange rate changes. If currency returns are positively correlated with foreign index returns in local currency terms, then the systematic portion of foreign exchange risk will be higher.

2.2.3: Decomposing the volatility of unhedged returns

As already observed from our hedged data series, hedging leads to a very substantial decrease in volatility in comparison to the unhedged returns. The next step is to determine from where does this reduction in volatility come from; To answer this question it is necessary to decompose the returns volatility into its constituent components which can be achieved as follows:

Given that the nominal return on an unhedged foreign investment $R_i^{U£}$ is

$$R_i^{U£} = (1+R_i^L)(1+x_i) - 1 = R_i^L + x_i + R_i^L x_i \tag{2.7}$$

where $x_i = \ln(S_t/S_{t-1})$

R_i^L is the return in Local currency terms

then by ignoring the small cross product term $R_i^L x_i$ we can calculate the approximate variance of returns as

[21] Of course, there is always the possibility that the bias in the forward rate might be in the investor's favour.

$$var(R_i^{\pounds U}) \approx var(R_i^L) + var(x_i) + 2cov(R_i^L, x_i) \tag{2.8}$$

This provides us with the decomposition of unhedged volatility into the volatility of foreign returns in local currency, the volatility of changes in the local currency against Sterling and the covariance between local market returns and exchange rate changes. In order to perform the decomposition, the variance of local market returns and exchange rate changes as well as their respective covariance was computed for all stock and bond indices.

Table 2.2 below shows the results from the decomposition of nominal Sterling returns from fourteen stock markets and nine bond indices i.e. seven government bond indices, the US floating rate notes (USN) and the Dollar zero coupon bonds (USZ).

For the nominal stock returns, the percentage contribution of local market variance to total variance is generally high, ranging from a minimum 59.47% for the US to a maximum 97.8% for Austria. These contributions are on general much higher than those found by Eun & Resnick (1988) who carried a similar decomposition for Dollar returns.

In the case of bond returns, though, the contribution of local variance is much smaller indicating that exchange rates are a more important source of risk to a foreign investor than local interest rates. As a matter of fact, the average contribution of local market risk is only 28.9%, with values ranging from only 5.2% for the Dollar FRN's to a maximum 56.4% for the Dollar zero coupon bonds.

The percentage contribution of foreign exchange risk is generally modest for stock returns, exceeding 20% only in the cases of the Canadian, Swiss and US markets. For the bonds the exchange risk contribution was uniformly above 50%, while approaching 90% in some cases (Swiss government bonds and Dollar FRN's).

TABLE 2.2

DECOMPOSITION OF NOMINAL STERLING RETURNS

	1 VAR(£)	2 VAR(L)	3 %MRKT	4 VAR(X)	5 %FOREX	6 COV(X,L)	7 %COV(X,L)	8 %TOTAL
BONDS								
CAN	19.474048	8.0458376	0.4131569	10.613807	0.5450232	0.8150604	0.0404901	0.9986702
FRA	9.5705739	4.3588774	0.4554458	6.991309	0.7305005	−1.778657	−0.175366	1.0105806
GER	9.8511607	2.558481	0.2597137	6.8855431	0.6989575	0.4071289	0.0392336	0.9979048
JAP	15.71166	3.8359426	0.2441462	10.225928	0.6508496	1.6509458	0.1014129	0.9964088
NET	9.3785311	2.3070652	0.2459943	6.5371087	0.6970291	0.534369	0.0532594	0.9962827
SWI	8.0358393	2.0373379	0.2535314	7.1310299	0.8874033	−1.132808	−0.132987	1.0079479
USA	17.121944	7.1243711	0.4160959	11.814793	0.6900381	−1.816567	−0.102713	1.0034209
USN	13.263	0.6849497	0.0516436	11.814793	0.8908085	0.7640968	0.056115	0.9985671
USZ	21.448113	12.088899	0.5636346	11.814793	0.5508546	−2.452358	−0.11161	1.0028791
STOCKS								
AUT	39.958293	39.079711	0.9780125	6.6613152	0.1667067	−5.77938	−0.142901	1.0018181
BEL	39.371754	35.583835	0.903791	7.0524865	0.1791255	−3.263358	−0.081474	1.0014426
CAN	41.131194	25.518448	0.6204159	10.613807	0.2580476	4.9997753	0.1189077	0.9973712
DEN	36.190288	31.287556	0.8645291	6.6575141	0.1839586	−1.752107	−0.047928	1.0005595
FRA	47.988079	41.625877	0.8674212	6.991309	0.1456885	−0.62437	−0.013073	1.0000371
GER	51.323628	44.769907	0.872306	6.8855431	0.1341593	−0.326569	−0.006292	1.0001728
ITA	57.699725	48.75194	0.844925	6.6864557	0.1158837	2.2640785	0.038692	0.9995006
JAP	54.417226	37.038127	0.6806324	10.225928	0.1879171	7.1540402	0.1314665	1.000016
NET	33.946586	31.000168	0.9132043	6.5371087	0.1925704	−3.587364	−0.104589	1.0011856
NOR	67.434792	61.908311	0.918047	5.102516	0.0756659	−0.426558	−0.006312	0.9874008
SPA	54.592897	49.867581	0.9134445	5.3708171	0.0983794	−0.641528	−0.011634	1.0001895
SWE	55.948247	53.916597	0.963687	6.4253295	0.1148442	−4.38887	−0.077054	1.0014774
SWI	32.246305	31.445939	0.9751796	7.1310299	0.2211425	−6.328312	−0.193273	1.0030495
USA	39.890924	23.722398	0.5946816	11.814793	0.2961775	4.356118	0.1077595	0.9986185

VAR(L), VAR(£) = Variance of Returns in Local Currency and Sterling Terms
% MKT = Column 2 / Column 1 (% Contribution of Market Risk)
VAR(X) = Variance of Exchange Rates Changes (against Sterling)
% FOREX = Column 4 / Column 1 (% Contribution of Forex Risk)
% COV(X,L) = % Contribution of Covariance Risk
% TOTAL = % Total Risk Explained

Finally, we diverged from Eun & Resnick in that the contribution of covariance risk was calculated directly and not as a residual[22], so that to investigate how accurate the total variance approximation is when omitting the cross product term. Column (8) in Table 2.2 refers to the combined impact of the three risk components and shows that indeed is very close to 100% in all cases. In this context, those approximations that marginally exceed 100% indicate that the omitted covariance terms are negative.

The most significant of the findings is that the contribution of covariance risk is mostly negative, implying that it mitigates rather than compounds the foreign exchange risk for British investors. In fact, among the fourteen stock markets the covariance contribution was positive only from the Canadian, Italian, Japanese and US markets. A striking example is that of the Austrian market where a covariance contribution of - 14.3% almost totally offsets a foreign exchange contribution of 16.7%. Notice, though, that in the case of bond returns the evidence is mixed, with positive covariance contribution in five out of nine cases.

These results are in complete contrast to those of Eun & Resnick who found positive covariance contribution for every single stock market, implying that foreign currencies tend to uniformally appreciate (depreciate) vis a vis the Dollar when their stock markets are rising (declining). Their findings should be interpreted as being strictly sample and country specific and certainly not applicable in the case of the UK.

Notice that the aforementioned single asset decomposition procedure could easily be generalized to include returns from k foreign market indices, in which case the variance of the Sterling denominated international portfolio can be expressed as

$$var(R_p^{£U}) = \vec{z}' V z \approx \vec{z}' R \vec{z} + \vec{z}' X \vec{z} + 2\vec{z}' S \vec{z} \tag{2.9}$$

[22] i.e. % covariance risk = 1 - % market risk - % forex risk

where z = an k*1 vector of investment weights

V = the k*k $cov(R_i,R_j)$ matrix of Sterling returns

R = the k*k covariance matrix of returns in local currency

X = the k*k matrix of covariances among exchange rate changes

S = a k*k cross covariance matrix between local returns and exchange rate changes

Nevertheless, since we are primarily concerned with real returns it is necessary to expand the previous analysis in order to incorporate the impact of Sterling inflation. When inflation is explicitly considered, the approximate real Sterling return from a foreign investment can be expressed as

$$R_{iR}^{\pounds U} \approx R_i^L + x_i - i_i \tag{2.10}$$

where $i_{\pounds} = \ln (I_t/I_{t-1})$ stands for the change in the UK RPI

By expanding equation (2.10) we can calculate the approximate variance of real returns

$$\begin{aligned} var(R_{iR}^{\pounds U}) &\approx var(R_i^L) + var(x_i) + var(i_i) + \\ &+ 2cov(R_i^L, x_i) - 2cov(x_i, i_i) - 2cov(R_i^L, i_i) \end{aligned} \tag{2.11}$$

In order to decompose the variance of real sterling returns on the basis of equation (2.11), it was necessary to calculate the variance of monthly UK RPI changes as well as its covariance with exchange rate changes and foreign local market returns for all currencies and indices. As can be seen in columns (6 & 7) from Table 2.3 the variance of UK inflation changes is only 0.00001535 and its percentage contribution of total variance is extremely small for all indices.

The covariances between UK inflation changes and returns from the 23 bond and stock indices (columns 10 & 11) are quite low due largely to the very low Sterling inflation variance. Notice, however, that all correlations are negative. Covariances between UK inflation changes and foreign local market returns (Columns 12 & 13) are also very small and practically insignificant.

TABLE 2.3

DECOMPOSITION OF REAL STERLING RETURNS

	1 VAR(£)	2 VAR(L)	3 %MKT	4 VAR(X)	5 %FOREX	6 VAR(I)	7 %INFL	8 COV(X&L)	9 %COV(X&L)	10 COV(X&I)	11 %COV(X&I)	12 %TOTAL
BONDS												
CAN	20.12967397	8.04583756	0.39969637	10.6138069	0.52726643	0.00001535	0.00000076	0.815060423	0.040490091	−0.003908647	−0.000194171	0.967259487
FRA	10.14256182	4.35887742	0.42976099	6.99130897	0.68930405	0.00001535	0.00000151	−1.778656614	−0.175365617	−0.003808381	−0.000375485	0.943325455
GER	10.37705124	2.55848102	0.24655183	6.88554312	0.66353562	0.00001535	0.00000148	0.407128903	0.039233583	−0.003888448	−0.000374716	0.9489478
JAP	16.27944624	3.83594257	0.23563102	10.2259281	0.62814963	0.00001535	0.00000094	1.650945818	0.101412898	−0.004277604	−0.000262761	0.964931728
NET	10.03333186	2.30706521	0.22994009	6.53710867	0.65153917	0.00001535	0.00000153	0.534368958	0.053259372	−0.003882955	−0.000387005	0.934353151
SWI	8.51820171	2.03733795	0.23917465	7.13102991	0.83715204	0.00001535	0.00000180	−1.132808428	−0.132986805	−0.003477946	−0.000408296	0.942933393
USA	17.68584533	7.12437106	0.40282898	11.8147927	0.66803864	0.00001535	0.00000087	−1.816567369	−0.102713064	−0.003714437	−0.000210023	0.967943403
USN	13.6166122	0.6849497	0.0503025	11.8147927	0.86767491	0.00001535	0.00000113	0.764096837	0.056115047	−0.003714437	−0.000272787	0.973820798
USZ	21.97252717	12.0888995	0.55018248	11.8147927	0.5377075	0.00001535	0.00000070	−2.452357564	−0.111610173	−0.003714437	−0.000169049	0.976111457
STOCKS												
AUT	40.44319927	39.0797109	0.96628634	6.66131522	0.16470792	0.00001535	0.00000038	−5.779379926	−0.142901156	−0.003962143	−0.000097968	0.987995512
BEL	40.05403472	35.5838348	0.88839576	7.05248653	0.17607431	0.00001535	0.00000038	−3.263358060	−0.081473891	−0.003564874	−0.000089002	0.982907565
CAN	42.04754803	25.5184481	0.60689503	10.6138069	0.25242392	0.00001535	0.00000037	4.999775334	0.118907655	−0.003908647	−0.000092958	0.978134016
DEN	36.5569662	31.2875559	0.85585756	6.65751408	0.18211342	0.00001535	0.00000042	−1.752106711	−0.047928121	−0.003570774	−0.000097677	0.989945601
FRA	47.76174904	41.6258773	0.87153168	6.99130897	0.14637883	0.00001535	0.00000032	−0.624370008	−0.013072595	−0.003808381	−0.000079737	1.004758498
GER	51.89843881	44.7699074	0.86264459	6.88554312	0.13267342	0.00001535	0.00000030	−0.326569141	−0.006292466	−0.003888448	−0.000074924	0.98895091
ITA	58.51542816	48.7519398	0.83314677	6.68645572	0.11426825	0.00001535	0.00000026	2.264078462	0.038691992	−0.004316118	−0.000073760	0.986033514
JAP	55.1035245	37.0381265	0.67215531	10.2259281	0.18557666	0.00001535	0.00000028	7.154040172	0.129829085	−0.004277604	−0.000077628	0.987483704
NET	34.29959615	31.000168	0.90380563	6.53710867	0.1905885	0.00001535	0.00000045	−3.587363765	−0.104589096	−0.003882955	−0.000113207	0.989692275
NOR	67.57735364	61.9083111	0.91611032	5.10251597	0.0755063	0.00001535	0.00000023	−0.426557818	−0.006312141	−0.001527632	−0.000022606	0.985282101
SPA	55.14046332	49.8675811	0.90437363	5.37081711	0.09740247	0.00001535	0.00000028	−0.641527681	−0.011634427	−0.001899624	−0.000034451	0.9901075
SWE	56.95857335	53.9165973	0.94659318	6.42532951	0.11280707	0.00001535	0.00000027	−4.388870239	−0.077053725	−0.003519874	−0.000061797	0.982284997
SWI	32.74293085	31.4459388	0.96038864	7.13102991	0.21778838	0.00001535	0.00000047	−6.328312317	−0.193272629	−0.003477946	−0.000106220	0.984798642
USA	40.42444171	23.7223977	0.58683303	11.8147927	0.29226854	0.00001535	0.00000038	4.356118007	0.107759509	−0.003714437	−0.000091886	0.986769578

VAR(L), VAR(£) = Variance of Returns in Local Currency and Sterling Terms
% MKT = Column 2 / Column 1 (% Contribution of Market Risk)
VAR(X) = Variance of Exchange Rates Changes (against Sterling)
% FOREX = Column 4 / Column 1 (% Contribution of Forex Risk)
VAR(I) = Variance of Inflation (UK)
% INFL = % Contribution of Inflation Risk
% COV(X,L),(X,I) = % Contribution of Covariance Risk
% TOTAL = % Total Risk Explained

Due to the very low values of all additional variances and covariances we can safely conclude that the decomposition of real returns provides practically identical results to those achieved by decomposing the nominal returns.

2.3. Do Monthly Currency Returns Follow Trends ?

2.3.1. "Convex" allocation strategies and adjusted currency returns

In chapter one, we examined Kritzman's (1989) empirical evidence which "suggested" that monthly currency returns are following protracted trends, a phenomenon they attributed to persistent central bank intervention. If Kritzman's arguments are indeed valid, then fund managers could maximize their portfolio's excess returns by adopting a "convex" payoff investment strategy first suggested by Perold & Sharpe (1988).

The principles of "convex" and "concave" strategies as discussed by Perold & Sharpe are powerful and straightforward. An investor who decides to adopt a "buy and hold" investment strategy for a given currency[23] will face a linear pay-off function, whose slope shall depend on the investors exposure to that type of asset[24]. In contrast to the "buy and hold" approach, a "convex" pay-off function arises from the dynamic strategy which consists in optimally increasing the portfolio weight of the currencies that are appreciating and decreasing it for those that are depreciating. The exact opposite of this strategy results in a "concave" pay-off function.

Perold & Sharpe proved that if returns follow a persistent trend then their "convex" strategy will always outperform the "buy and hold strategy", while the latter will consistently underperform the "concave" strategy in case the currency returns follow a mean reversing stochastic process. Their strategy was in fact duplicated by Kritzman, who on the basis of his data set demonstrated excess portfolio returns of significant magnitude.

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[23] Naturally, the same principles apply to stocks and many other classes of assets.

[24] A "pay-off" function relates the portfolio value to the market value of the asset under consideration.

A key point of contention, though, lies in the fact that Kritzman argued that it is perfectly legitimate to test for trends in spot currency returns by assuming that expected monthly returns from all currencies are equal to zero.

This is in fact incorrect, because monthly interest differentials cannot be ignored[25]. The correct approach should be to create currency return series with expected value of zero, which is achieved as follows:

Consider the continuously compounded version of Covered Interest Parity

$$F_{t-1} = S_{t-1}\, e^{(r_{£}\frac{30}{360} - r_{F}\frac{30}{360})} \tag{2.12}$$

where: r_F, $r_£$ are the continuously compounded foreign and Sterling interest rates respectively

F_{t-1}, S_{t-1} are expressed as the Sterling value of one unit of foreign currency

By dividing both sides of equation (2.12) with S_{t-1} and taking logarithms we can express the monthly premium/discount of the foreign currency against Sterling p_t, known at time t-1 as

$$p_t = \ln(F_{t-1}/S_{t-1}) = (r_{£} - r_F)\,\frac{30}{360} \tag{2.13}$$

implying that the foreign currency will trade at a forward premium if $r_F < r_£$

The monthly return of a UK investor's long position on a foreign currency x_t^* will therefore be

[25] That is to say that for example monthly returns on a long position in SFR will have to be adjusted for the opportunity cost of not holding a higher yielding currency like Sterling.

$$x_t^* = \ln(S_t/S_{t-1}) - \ln(F_{t-1}/S_{t-1}) = \ln(S_t) - \ln(F_{t-1}) = \\ = \ln(S_t/S_{t-1}) - (r_£ - r_F)\frac{30}{360} \tag{2.14}$$

where $\mathscr{E}(x_t^*) = 0$

Equation (2.14) was used to construct adjusted currency return time series for all fourteen currencies. If forward premia are indeed effective in providing unbiased estimates of future spot rates, then the mean adjusted returns should be close to zero. Furthermore, if the adjusted mean currency returns are negative then currency hedging of stock and bond returns will lead to higher mean returns compared to an unhedged strategy and vice-versa.

Values for the unadjusted and adjusted mean currency returns can be found in columns 2 & 3 from Table 2.4: For seven out of fourteen currency pairs the adjusted mean returns are negative, the largest values applying to the USA (-.294% a month) and Switzerland (-.236%). Effectively, as we can observe from Table 2.1, the negative (positive) adjusted means correspond to those indices for which the hedged strategies outperform (underperform) their unhedged counterparts.

It would be useful to compare column (3) from Table 2.4 to columns (2 & 4) from Table 2.1: If the hedging strategy would have been "perfect" ("ex-post") then the approximate difference between unhedged and hedged mean returns should be equal to the adjusted currency mean returns i.e.

$$R_i^{£U} - R_i^{£H} \approx (R_i^L + x_t) - (R_i^L + p_t) = x_t - p_t = x_t^* \tag{2.15}$$

Since our actual hedging strategy has been imperfect the small deviations observed between the results from Table 2.1 ("ex-ante") and those based on equation 2.14 ("ex-post") should be considered as a measure of "ex-ante" bias, caused by the residual exposure of the "unexpected" returns component to the unknown future spot rate. .

Table 2.4

W-W Runs Tests for Adjusted Currency Returns

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	MEAN%	MADJ%	RU	Z	2t-Pr	RU	Z	2t-Pr
CURRENCY								
Austria	.383	-.044	50	- .915	.360	48	-1.341	.180
Belgium	.200	.044	55	.058	.954	53	- .383	.702
Canada	-.011	-.068	42	-2.288	.022*	39	-3.094	.002*
Denmark	.235	.176	53	.383	.702	53	- .383	.702
France	.121	.129	52	- .566	.571	52	- .577	.564
Germany	.384	-.073	54	- .018	.985	56	- .208	.835
Italy	.069	.288	58	.769	.442	60	.971	.332
Japan	.469	.021	50	- .885	.376	52	- .580	.562
Netherl	.357	-.068	54	- .105	.917	54	- .193	.847
Norway	-.025	.108	50	- .848	.396	50	- .848	.396
Spain	.005	.344	50	- .475	.635	58	.614	.539
Sweden	-.021	-.009	48	-1.198	.231	48	-1.350	.177
Switzer	.334	-.236	56	.571	.568	62	1.370	.171
USA	-.035	-.294	46	-1.713	.087	46	-1.740	.082

Columns (2 & 3) list the values of the unadjusted and adjusted sample mean currency returns respectively

Columns (4 & 7) refer to the "runs count" for the mean (0) and median tests respectively

Columns (5 & 8) provide the values of the standardized normal variate Z

Columns (6 & 9) show the significant levels for the 2-tailed probabilities

All "runs" test results refer to the adjusted currency returns

Total number of observations: 108

Nevertheless, the key question to be answered is whether the adjusted currency returns can be still shown to follow "trends" in order to validate Kritzman's conclusions about the profitable use of "convex" asset allocation strategies in currency portfolios. This issue is explored in the remaining sections of this chapter:

2.3.2. Wald-Wolfowitz non-parametric "runs" tests for trend detection

As a first step towards analyzing for the possible existence of trends in the adjusted monthly currency returns, we applied two versions of "runs tests" due to Wald & Wolfowitz (1940). The basic aspects of this type of test can be summarized as follows; Consider a distribution F(x) which has a population median m. We assign a value of 1 to all above median observations and a value of 0 to all below median observations. In this way we can create a sequence of the following form:

00\1\00\111\00\11\00\11\000\11\000\11\0\1\0\1\00\1

A "run" is defined as a maximal consecutive set of 1s or 0s. Every sequence can be subdivided into a number of runs, whose total is called "runs count" c (c = 18 in the example above). Even though we don't need any assumptions about the distribution of the original time series, we need to know the sampling distribution of the "runs count" which is conditional on the observed value of the above median observations p. Wald-Wolfowitz have shown that the sampling distribution of c is approximately normal with mean and variance equal to:

$$\bar{c} = \frac{2p\,(n-p)}{n+1} \tag{2.16}$$

$$\sigma^2 = \sqrt{\frac{2p\,(n-p)\,[\,2p\,(n-p)-n\,]}{n^2\,(n-1)}} \tag{2.17}$$

where **p** = number of above median observations
n = total number of observations.

Consequently we can test for "trends" by means of a standardized normal Z statistic which is defined in terms of the population median m and is expressed as

$$Z = \frac{c - \frac{2p\,(n-p)}{n+1}}{\sqrt{\frac{2p\,(n-p)\,[\,2p\,(n-p)-n\,]}{n^2\,(n-1)}}} \tag{2.18}$$

In practice even though m is usually unknown, it is perfectly valid to apply the above procedure by substituting in its place either the mean μ or the sample median m^* [26] (see Madansky 1988).

In the first of our tests the "runs count" is defined in terms of m^*, while in the second one in terms of the theoretical mean for the adjusted currency returns.

The main results from the two "runs" tests are summarized in Table 2.4[27]. Columns (4-6) list estimates of the "runs count", the Z statistic and the exact two-tail probability level of accepting the null hypothesis of serial independence for the mean (0) test, whereas columns (7-9) list the corresponding statistics for the median test. In general a runs count which is too high implies a possible mean reversing process (Z>0), while a low runs count clearly indicates a possible trend (Z<0).

26 This is true because the following equations can be shown to hold :

$$cov\,(z_i-\bar{z}^*,z_j-\bar{z}^*) = -\frac{s^2}{n}$$

$$cov\,(z_i-\bar{m}^*,z_j-\bar{m}^*) = \frac{1}{n}$$

A simple interpretation to the above equations is that for large enough n the covariances are close to zero so that the sequence of 1s and 0s should be approximately independent (Madansky 1988).

27 All computations were made by using the SPSS-PC "non-parametric" procedures.

Among the fourteen currency pairs only the Sterling/Canadian Dollar "trends" at the 5% significance level, while in the case of the Sterling/US Dollar pair the null could be rejected only at the 10% confidence level. Indeed for these currency pairs the number of "runs" is rather low indicating that application of "convex" allocation strategies might lead to excess portfolio returns. For the remaining adjusted currency pairs there is no evidence whatsoever suggesting the existence of trends in monthly returns. In fact, it appears that most adjusted currency returns fluctuate randomly around a zero mean. Notice also that in all cases the mean and median tests provided qualitatively similar results.

One problem with the "runs tests", however, is that they treat equally all positive and negative returns irrespective of their magnitude. For this reason, it is worthwhile to apply standard parametric Box-Jenkins procedures as well with the aim of detecting possible serial correlation in the adjusted currency return series:

2.3.3. Testing for Serial Correlation : Autocorrelation and Partial Autocorrelation Functions (ACF-PACF) and the Box-Ljung Statistic

The theoretical Autocorrelation Function (ACF) is very useful since it measures how the serial correlation changes with the addition of further lags. Furthermore it is independent of the unit of measurement in the time series and therefore it is scale invariant (see Box & Jenkins 1976). Since in practice the theoretical autocorrelation function is unknown, we have to calculate estimates of the autocorrelations based on our sample of 108 adjusted currency return observations. There exists a number of alternative ACF estimates but the most widely accepted (see Madansky 1988) is to define the kth order autocorrelation as:

$$r_k = \frac{\dfrac{\sum_{i=1+k}^{n} (z_i - \bar{z}^*)(z_{i-k} - \bar{z}^*)}{n-k}}{\dfrac{\sum_{i=1}^{n} (z_i - \bar{z}^*)^2}{n}} \qquad (2.19)$$

where k is the lag length

r_k denotes the sample autocorrelation at lag k

Plotting the estimated autocorrelations as a function of k can be informative because (see Kendall & Stuart 1966), under the hypothesis of independence, we should expect that for large values of n the k_{th} order serial correlation should be approximately normally distributed with $\hat{\mu} = -\frac{1}{n-1}$ and $\hat{\sigma} = \frac{1}{\sqrt{n}}$.

In this context, the identification of a time series model can be assisted by calculating the estimated autocorrelation standard error std[r_k]. If the sample value of, say r_1, is several times the std[r_1] then we will have to conclude that r_1 is non zero. Similarly, we can establish whether r_k becomes effectively zero beyond some lag[28].

The next step is to investigate about whether the currency returns are "white noise" by calculating the Box-Ljung statistic (see Box & Ljung 1978). When the ACF at a given lag m is estimated from a zero number of lags in the autoregressive residuals then the Box-Ljung statistic $\hat{q}_m^*$ can be expressed as

[28] Following Box & Jenkins 1976, by assuming that beyond a certain lag q all theoretical autocorrelations are expected to be zero, then the std[r_k] can be expressed as

$$std[r_k] = \sqrt{\frac{1}{n-1}[1+2\sum_{u=1}^{q} r_u^2]}$$

for all lags k > q. In this context if the adjusted currency series are to be completely random then q = 0 and std[r_k] = $\frac{1}{n-1}$. Then subsequent estimates of std[r_k] will have to be based on the above equation by substituting q = 1.

$$\hat{q}_m^* = n(n+2)\sum_{k=1}^{m}\frac{\hat{r}_k^2}{n-k} \approx \chi^2(m) \qquad (2.20)$$

where $\hat{r}_k^2$ is the estimated autocorrelation coefficient at lag k[29].

Effectively, the estimated value of a Box-Ljung statistic evaluated at say sixteen lags, will have to be compared against a chi-square variable with sixteen degrees of freedom.

One problem with the procedures discussed until now is that they do not allow for an unambiguous identification of the order of an autoregressive process p, particularly in case that p > 1. To this end valuable supplementary information can be provided by thr Partial Autocorrelation Function (PACF). The usefulness of the PACF lies in that it behaves in a different way than the ACF since for a pure autoregressive process of order p the theoretical PACF is zero at all lags beyond p, while the sample PACF beyond lag p is approximately normally distributed with $\hat{\mu} = 0$ and $\hat{\sigma} = \frac{1}{n}$ (see Harvey 1981). This property of the PACF allows for an easier identification of the order of an autoregressive process.

Estimates of the PACF can be obtained by means of using OLS to estimate successive autoregressive processes of order 1, 2,...k

$$\begin{aligned}\hat{x}_{1t} &= \hat{a}_{11} + \hat{f}_{11}x_{t-1}\\ \hat{x}_{2t} &= \hat{a}_{21} + \hat{f}_{21}x_{t-1} + \hat{f}_{22}x_{t-2}\\ \hat{x}_{kt} &= \hat{a}_{k1} + \hat{f}_{k1}x_{t-1} + \dots\hat{f}_{kk}x_{t-k}\end{aligned} \qquad (2.21)$$

and then selecting the values of the additional coefficients $\hat{f}_{11},\dots\hat{f}_{kk}$ fitted at each stage.

29 If the ACF is calculated from a non-zero number of lagged autoregressive residuals z then the Box-Ljung statistic is distributed as $\chi^2(m-z)$.

Using the SPSS-PC Time Series procedures, we estimated the ACF and PACF functions, their corresponding standard errors and the Box-Ljung statistic at sixteen lags for all the adjusted currency pairs. The key findings are summarized in Table 2.5[30]:

In columns (2 & 4) we list the highest observed (absolute) values for the ACF and PACF and their corresponding lags, while in columns (3 & 5) we refer to the specific lag(s) where the ACF and PACF estimates exceeded the two standard error limit. We also recorded (in column 6) the estimated value of the Box-Ljung statistic at the lag where the recorded exact probability level of accepting the hypothesis of serial independence was lowest, as well as the lag(s) where the Box-Ljung statistic was significant at the 5% confidence level:

Adjusted returns from twelve of the fourteen currency pairs appear to be totally random: in nine cases the ACF and PACF estimates for all sixteen lags were within the two standard error limits and in all twelve cases the Box-Ljung statistic was never significant at any lag. In three cases the ACF or PACF slightly exceeded the two standard error limits in either the third or fourteenth lag, phenomenon that is naturally and could easily be attributed to the "law of large numbers".

Returns from two currency pairs, however, i.e. Sterling/French Franc and Sterling/Italian Lira were clearly autoregressive of the first order. In both cases the ACF and PACF were highly positive and exceeded the two standard error limits at the first lag, while the Box-Ljung statistic was also significant at the first lag. Notice that the ACF and PACF were practically zero at the second lag in both currency pairs, while in the case of the Italian Lira they were practically zero in all subsequent lags as well.

30 Complete results and plots of the ACF and PACF are available by the author.

Table 2.5

ACF/PACF for Adjusted Sterling Currency Returns

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	ACF/lg	2 S.E	PACF/lg	2 S.E	B-L/lg	B-L/5%	CON
CURRENCY							
Austria	.156/1	none	.156/1	none	2.709/1	none	RAND
Belgium	-.116/10	none	-.114/10	none	1.271/1	none	RAND
Canada	.163/4	none	.172/4	none	5.857/4	none	RAND
Denmark	-.146/5	none	.136/1	none	2.054/1	none	RAND
France	.205/1	1	.205/1	1	4.668/1	1	1+
Germany	-.124/15	none	-.124/8	none	1.560/1	none	RAND
Italy	.214/1	1	.214/1	1	5.062/1	1	1+
Japan	.132/1	none	.132/1	none	1.930/1	none	RAND
Netherl	-.145/15	none	-.140/15	none	.211/1	none	RAND
Norway	-.189/3	3	-.186/3	none	10.68/5	none	RAND
Spain	-.163/16	none	-.203/3	3	6.237/3	none	RAND
Sweden	-.108/3	none	-.125/16	none	2.656/5	none	RAND
Switzer	-.160/7	none	-.163/8	none	.887/1	none	RAND
USA	-.192/14	14	-.205/14	14	.216/2	none	RAND

Columns (2 & 4) refer to the highest observed ACF and PACF absolute values and their corresponding lag.

Columns (3 & 5) list the lags for which the ACF and PACF exceed the two standard error limits.

Column 6 shows the value of the Box-Ljung statistic at the lag where the recorded probability of accepting the hypothesis of serial independence is lowest.

Column 7 refers to the lags where the computed Box-Ljung statistic is significant at the 5% confidence level.

Column 8 states the overall conclusion about the randomness of the series.

It would also be interesting to contrast the results of the "runs tests" to those conducted in this Section. For eleven out of fourteen pairs, both the non-parametric and parametric procedures suggested total randomness. For the Sterling/Canadian Dollar pair the low number of "runs" led to the conclusion of a significant trend in both "runs tests", while even though its estimated ACF values ware positive for the first, second and fourth lag (.093, .104, .163 respectively), they were never high enough to justify acceptance of autoregression.

The opposite applies to the Franc and Lira which were shown to be random in the "runs tests" and autoregressive in the Box-Jenkins procedures. This is not altogether surprising, since quoting Levene (1952), the "runs tests" are quite powerful in detecting trends but relatively weak in detecting autoregression.

Overall, upon adjusting the currency returns, it is hard to agree with Kritzman that central bank intervention causes persistent trends in the currency return series, at least for the majority of currencies. Nevertheless, his argument that currency returns are much more likely to follow trends, rather than mean-reversion is partly justified by the fact that the ACF was positive at the first lag for all currencies except the Sterling/Swedish Krone pair.

This does not mean, of course, that central bank intervention is unlikely to have significant implications for the foreign exchange markets. In our view, such an influence is unlikely to be captured by monthly currency returns and is much more likely to be visible in high frequency data, measured over daily or even shorter time intervals. The problem here is that such implications would be of great importance to foreign exchange dealers who trade very frequently at extremely low transaction costs, but of much less use to fund managers who are unable to alter their asset portfolios on a continuous basis[31].

[31] An exception to this rule would be a fund whose international equity exposure is synthetically created by appropriate combinations of "cash" and stock index futures.

2.4 Evidence on Serial Correlation for Stock and Bond Indices

This Section concentrates on the question of whether it is possible or not for international investors to utilize information contained in the monthly time series of stock and bond index returns in a way that would allow the formulation of superior "ex-ante" allocation strategies. Naturally, implementation of such strategies can only be possible if there is a significant predictable component in index returns. The most obvious approach would be to concentrate on information included in univariate time series and try to detect whether index returns follow persistent "trends" or "mean reversion" processes that might allow the successful implementation of "convex" or "concave" strategies. This is the same approach used for the interest rate adjusted currency returns in the previous section.

2.4.1: Autocorrelation in Index Returns as opposed to Single Stock Returns

The fact that we are concerned with index returns, rather than individual stock returns, is likely to increase the probability of effectively applying "convex" allocation strategies: Taylor (1986) provided empirical evidence that for individual asset daily returns first-lag autocorrelation coefficients are mostly positive but small and relatively insignificant, whereas daily index returns in general were found to have more significant first order correlations.

This phenomenon, he argued, should mainly be attributed to the fact that individual stock returns are linked through a common market factor resulting to dependent autocorrelation coefficients among them. Furthermore, a number of authors including Roll (1981), Scholes & Williams (1977) and Gibbons & Hess (1981) have provided evidence that first order autocorrelation structures in daily index returns, could also be attributed to infrequent trading of some of the stocks included in the indices.

As far as the second and higher order autocorrelation coefficients of daily returns are concerned, Taylor provided empirical evidence that an unusually high percentage of them is negative, phenomenon which he attributed to the day of the week (mainly Monday) effect.

Schollhammer & Sand (1987) examined for serial correlation in daily returns, expressed in local currency terms, from thirteen stock indices and concluded that most of the largest markets, like Japan, UK, Switzerland and Germany, behaved as random walks while smaller markets like Italy, Norway and the Netherlands were autoregressive.

In a few cases, like Singapore and Canada, the first order coefficients were strongly negative implying daily mean reversion maybe due to profit taking, while for the US they failed to discern a clear pattern. The Schollhammer & Sand results appear to be consistent with previous findings in the sense that deviations from randomness were found primarily in smaller indices which consist of fewer and less frequently traded stocks.

The fact that our data series refer to monthly, rather than daily, index returns should be of great conceptual importance: for monthly returns, the day of the week effect is irrelevant and the problem of infrequent trading of stocks becomes minimal. On the other hand, though, the common factor effect is likely to be at least as strong[32].

2.4.2 Empirical Evidence on Autocorrelation from Index Returns

Testing procedures in this section are identical to those applied in section C3 from chapter two: estimates of the Autocorrelation and Partial Autocorrelation Functions (ACF/PACF), as well as of their standard errors, were calculated for sixteen successive lags and for each one of the fifteen stock and ten bond unhedged real Sterling return data series. Again the Box-Ljung statistics with their corresponding confidence levels were also obtained for all sixteen lags in each case. Plots of the ACF and PACF functions were obtained against the sixteen lags using two standard error limits. The entire procedure was then repeated for the hedged data series.

All these results are being summarized in tables 2.6 & 2.7 below:

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[32] Because in general correlations computed over longer time intervals are considered to be higher than correlations computed on the basis of daily data.

Table 2.6

ACF/PACF For Unhedged Real Sterling Returns

(1)	(2) ACF/lg	(3) 2 S.E	(4) PACF/lg	(5) 2 S.E	(6) B-L/lg	(7) B-L/5%	(8) CON
STOCKS							
Austria	.230/1	1,10	.230/1	1,10	.015/1	1,2	1+
Belgium	.182/1	none	.182/1	none	.055/1	none	INC
Canada	.135/1	8	-.243/8	8	.155/1	none	RND
Denmark	-.265/14	14	-.247/14	14	.146/15	none	RND
France	.154/3	none	.151/1	none	.111/1	none	RND
Germany	-.183/5	none	-.213/5	5	.092/1	none	INC
Italy	.242/1	1,5	.242/1	1-5	.011/1	1-3 5-9,11	1+
Japan	.119/6	none	.104/1	none	.274/1	none	RND
Netherl	-.170/16	none	-.175/16	none	.232/1	none	RND
Norway	.249/1	1	.249/1	1	.009/1	1,2	1+
Spain	.206/1	1,6	.206/1	1	.030/1	1,3	1+
Sweden	.250/1	1	.250/1	1	.008/1	1-3	1+
Switzer	.254/1	1	.254/1	1	.007/1	1-4	1+
UK	-.175/8	none	-.180/8	none	.532/2	none	RND
USA	.149/1	none	.149/1	none	.115/1	none	RND
BONDS							
Cana\GB	.218/4	4	.215/4	4	.072/4	none	INC
Fran\GB	.087/1	none	.087/1	none	.357/1	none	RND
Germ\GB	.172/1	none	.172/1	none	.070/1	none	INC
Japa\GB	.239/1	1	.239/1	1	.012/1	1-3	1+
Neth\GB	.215/1	1	.215/1	1	.023/1	1	1+
Swit\GB	.174/1	none	-.193/7	7	.067/1	none	INC
UK\GB	-.152/3	none	-.160/3	none	.354/4	none	RND
USA\GB	.177/1	none	.177/1	none	.063/1	none	INC
USA\FRN	-.157/14	none	-.178/14	none	.205/2	none	RND
USA\ZER	-.148/10	none	.189/11	none	.148/1	none	RND

Columns (2 & 4) refer to the highest observed ACF and PACF absolute values and their corresponding lag
Columns (3 & 5) list the lags for which the ACF and PACF exceed the two standard error limits
Column 6 shows the value of the B-L statistic at the lag where the probability of accepting the null is lowest
Column 7 refers to the lags where the computed B-L statistic is significant at the 5% confidence level
Column 8 states the overall conclusion about the randomness of the series

Table 2.7

ACF/PACF for Hedged Real Sterling Returns

(1)	(2) ACF/lg	(3) 2 S.E	(4) PACF/lg	(5) 2 S.E	(6) B-L/lg	(7) B-L/5%	(8) CON
STOCKS							
Austria	.204/10	10	.192/10	10	.164/1	none	RND
Belgium	.190/1	1	.190/1	none?	.045/1	1,3	1+
Canada	-.202/8	8	-.207/8	8	.462/1	none	RND
Denmark	-.209/14	14	-.175/12	none	.151/14	none	RND
France	.135/3	none	-.157/6	none	.267/1	none	RND
Germany	-.161/13	none	-.154/13	none	.213/1	none	RND
Italy	.253/5	1,5	.262/5	1,5	.017/5	1,2,5-8	1+
Japan	.153/5	none	.154/5	none	.609/7	none	RND
Netherl	-.111/8	none	-.140/8	none	.363/1	none	RND
Norway	.215/1	1,14	.215/1	1,14	.024/1	1	1+
Spain	-.216/3	3,16	-.201/3	3	.041/3	3	INC
Sweden	.245/1	1	.245/1	1	.010/1	1-2	1+
Switzer	-.148/3	none	-.152/3	none	.217/1	none	RND
UK	-.175/8	none	-.180/8	none	.532/2	none	RND
USA	-.157/9	none	.136/5	none	.405/9	none	RND
BONDS							
Cana\GB	-.112/6	none	-.117/16	none	.281/1	none	RND
Fran\GB	.135/4	none	.153/4	none	.314/1	none	RND
Germ\GB	.209/4	4,6	.195/4	4	.033/8	8	INC
Japa\GB	-.200/4	4	-.218/4	4	.131/8	none	RND
Neth\GB	-.298/6	6,11	-.303/6	6,11	.016/11	6,9-16	INC
Swit\GB	-.200/1	1	-.200/1	1	.035/1	1	1-
UK\GB	-.152/3	none	-.160/3	none	.354/4	none	RND
USA\GB	.102/4	none	.117/4	none	.358/1	none	RND
USA\FRN	-.198/1	1	-.198/1	1	.037/1	1	1-
USA\ZER	.179/11	11	.206/11	11	.654/11	none	RND

Columns (2 & 4) refer to the highest observed ACF and PACF absolute values and their corresponding lag
Columns (3 & 5) list the lags for which the ACF and PACF exceed the two standard error limits
Column 6 shows the value of the B-L statistic at the lag where the probability of accepting the null is lowest
Column 7 refers to the lags where the computed B-L is significant at the 5% confidence level
Column 8 states the overall conclusion about the randomness of the series

The tables above provide information on the highest observed ACF/PACF values and their respective lags, the lag numbers where (if any) the ACF/PACF values exceed the two standard error limits, the value of the B-L at the lag where it is most significant and the lag numbers (if any) where the B-L is significant at the 5% confidence level, and finally the overall conclusion about the randomness of the series. The main conclusions have as follows:

i) In almost all cases the ACF was positive in the first lag. Furthermore, in 9/15 unhedged stock and 6/15 unhedged bond series the first lag exhibits the highest absolute ACF value among all sixteen lags. Notice, however, that this "dominance" of the first lag ACF could be partly attributed to foreign exchange effects and is absent from the hedged return series: as a matter of fact, in only 3/15 and 2/10 cases for hedged stock and bond returns respectively was the highest ACF observed in the first lag.

ii) Deviations from randomness were substantial: at the first lag the ACF function exceeded the two standard error limit in unhedged returns from six stock indices and two bond indices. The PACF also provides additional evidence for first order serial correlation, since in most of these cases it sharply declines after the first lag. Consistently, the B-L statistic is significant at the 5% level in all eight cases under consideration.

Consequently, we can readily conclude that monthly unhedged real Sterling stock index returns from Austria, Italy, Norway, Spain, Sweden and Switzerland exhibit positive first order serial correlation, while the same applies for returns from the Japanese and Dutch government bond indices.

The strongest evidence on non-randomness can be attributed to the Italian stock market, in which case the ACF/PACF function was found to be positive in all the first seven lags, while exceeding the two standard error limits in the first and fourth lags. Also, the PACF and the B-L statistic were highly significant in most lags.

iii) At the same time, unhedged monthly returns from eleven indices appear to be entirely random: these refer to the Canadian, Danish, French, Japanese, Dutch, UK and US stock markets as well as to four bond indices i.e. French and UK Government bonds and Dollar Zero Coupon Bonds and FRN's. Returns from the UK stock market were shown to be highly random and it was one of very few exceptions in not having a positive first order correlation.

iv) In some series the results were either marginally insignificant, or some significant autocorrelations were observed in higher lags which are very difficult to interpret. In those cases, we denoted the evidence as inconclusive. This is the situation with the Belgian and German stocks and the Canadian, German, Swiss and US Government Bond Indices.

v) It becomes clear from our results, that monthly returns from the large stock markets tend to be random, whereas returns from the six smaller European markets tend to exhibit positive first order serial correlation. In this respect, our findings are consistent with previous empirical evidence from daily returns.

What is extremely remarkable, is the fact that monthly returns from smaller markets show at least as much serial correlation compared to evidence from other studies concerned daily returns from the same markets: the standard arguments about thin trading of certain stocks in these indices and day of the week effects cannot be used to explain autocorrelation in monthly data, while the "common factor" phenomenon, even though capable of explaining autocorrelation, cannot readily be used to explain different autocorrelation patterns between small and large markets. My feeling is, therefore, that existing interpretations about what causes autocorrelation in index returns are weak and the issue is certainly worth of further investigation.

vi) Hedging, appears to create important qualitative differences in a number of cases: Only the Belgian, Norwegian, Italian and Swedish stock markets exhibit positive first order serial correlation in the hedged series, whereas only two of the hedged bond series are conclusively non random (i.e. the Dollar FRN's and the Swiss Government Bonds). The most remarkable result, though, refers to the Spanish stock market for which the ACF/PACF are strongly negative and the B-L significant at the third lag.

2.4.3 Empirical Evidence from the Runs Tests

The two types of "runs tests" performed in this section are very similar to those described for the adjusted currency returns in chapter two, with sole difference that for the "mean runs test", the number of runs has been computed on the basis of the sample mean, rather than a zero mean, to account for the fact that monthly stock returns have positive means whereas currency returns when adjusted for interest differentials have expected means of zero. Both types of runs tests were applied to all the unhedged and hedged stock and bond series and the results are summarized in tables 2.8 & 2.9. The main conclusions from our results have as following:

i) Concerning the unhedged stock returns, the number of runs was generally small, resulting in most occasions to negative values for the Z statistic (12/15 times for the mean and 11/15 for the median test). This indicating that international stock markets are much more likely to follow trend rather than mean reversion and is broadly consistent with the evidence from the ACF/PACF previously discussed.

ii) The rejection rate, though, of randomness was smaller than in the previous tests: only in the case of returns from the Austrian and Canadian markets both runs tests are consistent in rejecting the null hypothesis of independence at the 5% confidence level, while in the case of Switzerland it is only rejected on the basis of the median test. As far as unhedged bond returns are concerned, the Z statistic was consistently negative in every single case. Again, the rejection rate was not high, the trend being significant at 5% in the case of returns from the Japanese Government Bonds (both tests) and Dollar FRN's (mean test only).

iii) Hedging again had significant implications in the qualitative assessment of our results concerning stock returns. By removing the impact of exchange rate fluctuations from the equity returns, all evidence of non-randomness disappears. As we can observe from table 2.9 the majority of Z statistics remains negative, but in no case is it possible to reject the null at 5% level. This is not the case concerning the returns from the bond indices, since independence is firmly rejected for the Japanese and Swiss Government Bonds and the Dollar FRN's in both tests.

Table 2.8

W-W Runs Tests for Unhedged Real Sterling Returns

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	RU	MEAN	P	Z	2t-P	RU	MED	Z	2t-P
STOCKS									
Austria	43	.571	53	-2.317	.021	43	.730	-2.320	.020
Belgium	49	1.495	51	-1.131	.258	51	1.809	- .770	.441
Canada	43	.382	54	-2.320	.021	43	.466	-2.320	.020
Denmark	57	.938	53	.390	.258	57	1.009	.387	.670
France	53	1.168	48	- .261	.020	54	1.614	- .193	.847
Germany	53	1.028	50	- .331	.696	55	1.193	.000	1.00
Italy	52	.762	53	- .577	.794	50	1.074	- .967	.334
Japan	51	1.060	51	- .744	.457	49	1.842	-1.160	.246
Netherl	57	1.285	49	.480	.631	55	2.117	.000	1.00
Norway	55	.911	47	.179	.858	49	1.933	-1.157	.247
Spain	50	1.067	53	- .964	.335	52	1.262	- .580	.562
Sweden	52	1.346	53	- .577	.564	52	1.612	- .580	.562
Switzer	46	.755	55	-1.737	.082	44	.716	-2.127	.033
UK	60	.974	48	1.109	.267	61	1.537	1.160	.246
USA	52	.774	49	- .495	.621	46	1.914	-1.740	.082
BONDS									
Cana\GB	54	.731	57	- .162	.872	56	.710	- .197	.844
Fran\GB	52	.702	48	- .457	.648	58	.851	- .580	.562
Germ\GB	54	.538	52	- .179	.858	52	.603	- .581	.562
Japa\GB	42	.593	55	-2.511	.012	42	.455	-2.514	.012
Neth\GB	51	.525	51	- .743	.457	51	.736	- .773	.440
Swit\GB	52	.145	54	- .580	.562	52	.132	- .580	.562
UK\GB	51	.625	56	- .760	.447	51	.479	- .773	.440
USA\GB	46	.590	51	-1.713	.087	48	.816	-1.354	.176
USA\FRN	42	.195	47	-2.379	.017	50	.834	- .967	.334
USA\ZER	48	.691	51	-1.325	.185	50	.914	- .967	.334

Columns (2 & 7) refer to the number of runs for the mean and median test respectively
Columns (3 & 8) list the sample mean and median respectively for each series
Column 4 shows the number of above mean observations for each series
Columns (5 & 9) contain the standardized Z values for the mean and median test respectively
Column (6 & 10) list the exact two-tail significance levels for the mean and median tests

Table 2.9

W-W Runs Tests for Hedged Real Sterling Returns

(1)	(2) RU	(3) MEAN	(4) P	(5) Z	(6) 2t-P	(7) RU	(8) MED	(9) Z	(10) 2t-P
STOCKS									
Austria	49	.672	51	-1.131	.258	47	1.068	-1.547	.122
Belgium	46	1.466	55	-1.737	.082	48	1.391	-1.354	.176
Canada	53	.436	56	- .373	.710	50	.313	- .967	.334
Denmark	57	.775	46	.630	.529	52	1.469	- .580	.562
France	51	1.509	48	- .653	.514	50	2.634	- .967	.338
Germany	52	1.093	49	- .495	.621	54	1.446	- .580	.562
Italy	54	.480	54	- .193	.847	54	.628	- .193	.847
Japan	53	1.043	50	- .331	.741	55	1.540	.000	1.00
Netherl	55	1.388	53	.004	.997	55	1.443	.000	1.00
Norway	49	.786	47	-1.002	.317	47	1.240	-1.547	.122
Spain	49	.712	55	-1.157	.247	51	.477	- .773	.439
Sweden	54	1.366	50	- .137	.891	54	1.813	- .193	.847
Switzer	54	.900	54	- .193	.847	54	.974	- .193	.847
UK	60	.974	48	1.109	.267	61	1.537	1.160	.246
USA	54	1.277	51	- .162	.872	56	1.643	.193	.847
BONDS									
Cana\GB	51	.807	54	- .773	.439	51	.775	- .773	.439
Fran\GB	54	1.034	58	- .137	.891	52	.851	- .580	.562
Germ\GB	48	.593	58	-1.303	.192	50	.481	- .967	.334
Japa\GB	44	.558	51	-2.101	.036*	44	.738	-2.127	.033*
Neth\GB	53	.605	53	- .383	.702	53	.661	- .387	.699
Swit\GB	66	.267	52	-2.144	.032*	66	.334	-2.127	.033*
UK\GB	51	.625	56	- .760	.447	51	.479	- .773	.439
USA\GB	55	1.008	53	.004	.997	57	1.117	.387	.699
USA\FRN	31	.687	51	-4.623	.000*	32	.720	-4.447	.000*
USA\ZER	50	1.178	55	- .964	.335	50	1.109	- .967	.334

Columns (2 & 7) refer to the number of runs for the mean and median test respectively
Columns (3 & 8) list the sample mean and median respectively for each series
Column 4 shows the number of above mean observations for each series
Columns (5 & 9) contain the standardized Z values for the mean and median test respectively
Column (6 & 10) list the exact two-tail significance levels for the mean and median tests

iv) A final observation concerning stock returns is that the median returns are almost universally higher than the mean returns (this applies to 14/15 unhedged and 12/15 hedged series). This provides an indication that most extreme returns tend to be concentrated on the left half of the returns distribution, phenomenon partly caused by the October 1987 observation. This is not applicable in the case of bond returns. Notice also that the results from the runs tests in many cases were qualitatively different from those of the ACF/PACF tests previously discussed. This should not be too surprising, considering the fact that despite their usefulness in detecting trends, the runs tests have low power in detecting autoregression in time series.

2.5: Summary and Conclusions

Foreign exchange risk in international asset portfolios is largely but not completely "non-systematic" because: i) "ex-post" currency hedging strategies that ignore residual currency exposure slightly overestimate the return volatility reduction from hedging ii) transaction costs affect the expected returns from hedged investments and iii) forward rates are not necessarily unbiased predictors of future spot rates.

Decomposition of the volatility of unhedged stock returns in Sterling terms shows that local market volatility is much more important than foreign exchange volatility, fact largely attributed to the negative covariances between most local stock returns and exchange rate changes. The picture is radically different for the non Sterling bonds where foreign exchange fluctuate constitute by far the largest part of return volatility. The conclusions were very similar when we decomposed the real Sterling returns, since both the volatility of changes in the UK RPI and the relevant covariances were of very small magnitude.

An "ex-ante" hedging strategy has been applied in order to capture the impact of uncertainty in future asset returns. In any event the benefit/loss from hedging in terms of mean return closely followed the currency returns adjusted to account for monthly forward premia. In a few cases, including the US Dollar, the mean monthly adjusted currency return was highly negative indicating substantial incremental returns from hedging in Sterling terms.

Both non-parametric and parametric testing procedures have been applied to the adjusted currency return series in order to detect potential trends that might allow profitable application of "convex" investment strategies. Most currency returns were shown to be random, even though the "runs tests" provided evidence of trends in the Sterling/Canadian Dollar monthly currency returns while the Box-Jenkins procedure showed evidence of autoregression for the French Franc and the Italian Lira.

International stock markets generally exhibit positive first order serial correlation and in a number of cases there is some statistically significant evidence of non-randomness in stock index returns. Foreign exchange hedging has significant implications in some cases, mostly notably eliminating evidence of non-randomness from some stock index return series. Nevertheless, significant trends tend to persist in a number of hedged bond indices.

CHAPTER III

MEASURING THE INTERTEMPORAL STABILITY OF INTERNATIONAL PORTFOLIO INPUTS: A MULTIVARIATE ANALYSIS OF VARIANCE APPROACH

This chapter evaluates the extent to which a mean-variance model, in conjunction with the use of historical index returns as portfolio inputs, can provide an adequate basis for successful "ex-ante" asset allocation. For this purpose, Multivariate Analysis of Variance (MANOVA) techniques are applied in order to investigate the issue of intertemporal stability of international portfolio inputs (the variance-covariance matrix and the mean return vector) in addition to providing empirical evidence on stationarity and multivariate normality of international index returns.

3.1 Introduction

As repeatedly discussed in chapter one, due to the absence of an operational international asset pricing model, most of the existing empirical "ex-ante" studies on international portfolios rely heavily on a Markowitz mean-variance framework. Such an approach might potentially be perilous for two reasons:

i) Monthly returns from international indices might seriously deviate from multivariate normality or stationarity.
ii) Even if returns approximately conform to the aforementioned standard assumptions, "ex-ante" strategies formulated on the basis of historical inputs will lead to poor performance if portfolio inputs are not intertemporally stable.

In such an eventuality a mean-variance approach might still be appropriate, but it will be necessary to either provide direct forecasts of the necessary portfolio inputs or, otherwise, control for "estimation risk" at the historical data. In the following sections we focus on the validity of the standard assumptions, by applying a number of alternative normality tests as well as unit root tests for stationarity for all the hedged and unhedged return series from 25 stock and bond indices. The exceptional influence of the October 1987 stock market "crash" is also considered insofar it affects the conclusions about normality.

Subsequently, the analysis of international portfolio input stability, as examined in chapter one, is extended to an appropriate multivariate framework. Techniques from Multivariate Analysis of Variance (MANOVA) are applied to test for stability of the entire sample variance-covariance matrix and the mean return vector both between two consecutive samples and for multi-sample periods. In addition, tests for univariate homogeneity of variance are applied in order to establish which markets are those that cause the deviations. Then a final section summarizes and concludes.

3.2 On Normality and Stationarity of Index Returns

3.2.1: Implications of non-normality in index returns

The fact that our data series consist of real index (rather than single asset) returns in Sterling terms has some important conceptual implications, usually neglected:

i) The normal distribution belongs to the family of stable Paretian distributions. A most useful property associated with this class of distributions is that they are invariant under addition, implying that if returns from n securities are derived from normal distributions then the return on a portfolio formed by any possible combination of these securities will be also normally distributed. The main implication for our purposes is that, if the underlying distribution of the assets that constitute an index is normal then the returns generated by a market capitalization weighted arithmetic index must also be normal.

ii) By virtue of the Central Limit Theorem if n nominal returns from an index measured over a short time interval (say daily or hourly returns) $r_1, r_2, \ldots\ldots r_n$ are identically distributed and independent then the monthly return $\ln(1+R_m)$ will be approximately normal. This becomes obvious if we consider that

$$1 + R_m = (P_t / P_{t-1}) = (1 + r_1) * (1 + r_2) * \ldots.(1 + r_n) \tag{3.1}$$

By taking logarithms from both sides we get

$$\begin{gathered} \ln(1 + R_m) = \ln(P_t / P_{t-1}) = \\ = \ln(1 + r_1) + \ln(1 + r_2) + \ldots.. \ln(1 + r_n) \end{gathered} \tag{3.2}$$

so that the Central Limit Theorem (C.L.T.) can be applied to equation (3.2).

In this context we can state that under the aforementioned assumptions, the simple monthly return R_m is lognormally distributed while the continuously compounded rate of return $\ln(1+R_m)$ is approximately normal.

The previous argument could be extended so that a similar reasoning to be applicable for the real Sterling monthly returns R_{Rm} as well. In this case:

$$\ln(1 + R_{Rm}) = [\ln(1 + r_1) - \ln(1 + i_1)] + \ldots[\ln(1 + r_n) - \ln(1 + i_n)] \quad (3)$$

where $i_1, \ldots. i_n$ denote successive changes in the UK RPI over the chosen time interval.

In fact, all that is needed in order to apply the C.L.T to equation (3.3) is an additional assumption that the i_s are independent and identically distributed as well.

Nevertheless, it has been widely suggested that price sensitive information flows affect asset prices in a way that causes returns measured over short time intervals to be leptokurtic rather than normal. If daily returns are indeed leptokurtic, then monthly returns might or might not be approximately normal: Blattberg & Gonedes (1974) showed that if daily returns are derived from a non-normal stable Paretian distribution then monthly returns will not be approximately normal, while the opposite is true if leptokurtosis can be attributed to the returns following the Student-t distribution.

iii) The exact implications of potential non-normality for asset allocation are still quite unclear: Ross (1982) argued that in the specific case where returns are generated through a single factor process, that mean-variance analysis is justified even without normality or quadratic utility functions. Kroll, Levy and Markowitz (1984) examined a more general case and concluded that non-normality is not sufficient to reject mean-variance analysis, if the probability of extreme returns is low.

Finally, Alexander & Sharpe (1986) thoroughly examine the portfolio implications of lognormality as an alternative to normality, and prove that the CAPM is still valid if risk-free borrowing and lending are permitted, or alternatively investors can engage to unrestricted short selling. In view of the recent explosion of interest on leptokurtic distributions, more developments in this area are certain to follow in the near future.

3.2.2 Testing for non-normality in index returns

In this section we empirically test the hypothesis whether international monthly index returns, in real Sterling terms, are approximately normal against the alternative hypothesis of an asymmetric leptokurtic distribution. To that end, four alternative tests for normality have been applied to both the unhedged and hedged return series: two tests based on "skewness" and "kurtosis" respectively, a "studentized range" test and a non-parametric one sample procedure attributed to Kolmogorov.

Intuitively one might expect that the exceptionally large monthly decline in most global stock indices that took place in October 1987 could alone be the cause of rejecting the normality hypothesis for stock index returns[1], so that it is worthwhile comparing all results based on the original stock data series with those of "adjusted" series from which the October 1987 observation has been omitted. Clearly, no such adjustment is necessary for the bond index returns under consideration.

[1] Because the monthly decline was several standard deviations away from the mean return for virtually all stock indices.

All the key test statistics for skewness, kurtosis[2], studentized range[3] and Kolmogorov's d statistic plus the appropriate critical values have been computed by means of SPSS-PC procedures[4] for all adjusted and unadjusted returns (hedged and unhedged) and can be found in tables 3A.1-3A.3 in an appendix at the end of chapter 3, while table 3.1 summarizes the conclusions from the first three tests concerning the adjusted return series[5]:

In the original unhedged stock series only Denmark, Italy and Japan have skewness which is consistent with that of a normal distribution. All the remaining time series exhibit significant negative skewness, implying that they are asymmetric to the left. Hedged returns fare even worse with Italy alone passing the skewness test. The fact, though, that the skewness sign is consistently negative, provides an indication of the strong influence due to the October 1987 effects.

In fact, by omitting this single observation in the adjusted tests we get a radically different picture, i.e. 11/15 stock indices demonstrate a normal skewness both in the hedged and unhedged cases. As far as the bond indices are concerned, 8/10 unhedged return series portray a normal skewness, the exceptions being the Dollar FRN's and the Japanese Government Bonds. The hedged series appear to be slightly more distorted, with 7/10 having a normal skewness.

The kurtosis test was the one with the highest rejection rate for normality among the four tests performed. Among the original unhedged data series only Denmark and Italy avoid failing the kurtosis test, while Denmark alone survived the test among the hedged series.

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2 Interpolated critical values for the kurtosis statistic (g_2+3) were found to be 3.77 for the upper limit (leptokurtosis) and 2.56 for the lower limit (platykurtosis).

3 The critical values for the upper and lower 2.5 percentage points are 6.15 and 4.25 respectively (interpolated from the studentized range tables compiled by Pearson & Stephens 1964).

4 A more detailed discussion of the above and other procedures for normality testing and their comparative merits can be found in Natsis (1992).

5 As can be easily be observed from the appendix the non-parametric Kolmogorov test had by far the lowest rejection power among the four tests and, therefore, is not included in the summary.

Table 3.1

Normality Tests: Summary Findings Adjusted for October 87

(1)	(2) SKEWN	(3) SKEWN	(4) KURTO	(5) KURTO	(6) ST.RG	(7) ST.RG
STOCKS	**HEDG**	**UNHG**	**HEDG**	**UNHG**	**HEDG**	**UNHG**
Austria	norm	norm	lept	norm	norm	norm
Belgium	asy+	asy+	lept	lept	lept	norm
Canada	norm	norm	lept	norm	norm	norm
Denmark	asy-	norm*	norm*	norm*	norm*	norm*
France	norm	norm	norm	norm	norm	norm
Germany	asy-	asy-	lept	lept	norm	norm
Italy	norm*	norm*	lept	norm*	norm	norm*
Japan	asy-	norm*	lept	lept	lept	norm*
Netherl	norm	norm	norm	norm	norm	norm
Norway	norm	asy-	norm	norm	norm	norm
Spain	norm	norm	lept	lept	lept	lept
Sweden	asy-	asy-	lept	lept	lept	lept
Switzer	norm	norm	lept	norm	norm	norm
UK	norm	norm	norm	norm	norm	norm
USA	norm	asy-	norm	norm	norm	norm
BONDS	**HEDG**	**UNHG**	**HEDG**	**UNHG**	**HEDG**	**UNHG**
Cana\GB	asy+	norm	lept	lept	norm	norm
Fran\GB	asy+	norm	lept	norm	lept	norm
Germ\GB	norm	norm	norm	norm	norm	norm
Japa\GB	norm	asy+	norm	lept	norm	lept
Neth\GB	norm	norm	norm	lept	norm	lept
Swit\GB	norm	asy+	norm	lept	norm	norm
UK\GB	norm	norm	norm	norm	norm	norm
USA\GB	norm	norm	norm	norm	norm	norm
USA\FRN	asy-	asy-	lept	lept	lept	norm
USA\ZER	norm	norm	norm	norm	norm	norm

*** implies that the tests accepted normality even for the original data series (unadjusted for October 1987)**
Adjustment for October 1987 is not applicable for bond data

Adjusting for October 1987 provides again a most important impact on the results, since 10/15 unhedged and 6/15 hedged series have a normal kurtosis, all the remaining being leptokurtic (as one might expect there is no platykurtosis in any of the time series). Concerning the bond returns, in 5/10 among the unhedged and 7/10 among the hedged series normality was accepted, the rest again been leptokurtic.

The "studentized range" test provided similar results with those of the previous tests, even though normality was rejected in a smaller number of cases. For the unadjusted unhedged stock returns normality cannot be rejected only for Italy, Japan, Sweden and Denmark, the last two been the only survivors among their hedged counterparts. In the adjusted series however the number of cases where normality is accepted rises sharply, i.e. 13/15 times in the unhedged and 11/15 in the hedged series. The test also fails to reject normality in 8/10 unhedged, as well as hedged, bond series[6].

Finally, as can be seen from table 3A.3 the non-parametric Kolmogorov test accepted normality in all but one (hedged FRN's) case concerning bond returns as well as for the vast majority of adjusted of adjusted stock index returns.

Our main conclusions can be summarized as follows:

- The October 1987 stock market "crash" had indeed a very strong impact on the stock index returns resulting in all tests showing significant deviations from normality for the vast majority of indices. When that particular observation though is omitted, the results are very different and normality can be accepted in the majority of cases. This becomes clear if we pull together the rejection rates from the four tests:

[6] There are, however, two surprising results that are worth mentioning: i) the studentized range statistic for returns from the Swedish stock index was (uniquely) higher in the "crash adjusted" series, both hedged and unhedged and ii) in the case of unhedged US$ FRN index normality was strongly rejected in both the skewness and kurtosis criteria, but nevertheless accepted on the basis of the "studentized range" test. However, keep in mind that Uthoff (1970) argued that the one situation where the studentized range test has reduced power is when testing against a leptokurtic distribution.

For the unadjusted unhedged stock returns the aggregate rejection rate is 70% or 42/60, while for their hedged counterparts rises to 80% or 48/60 (notice that if we exclude the results from the Kolmogorov test the aggregate rejection rates for unhedged and hedged returns rise to 80% and 91% respectively).

In contrast, for the adjusted stock series the rejection rate is only 20% or 12/60 for unhedged and 35% or 21/60 for hedged stock returns. Notice that the consistently higher rejection rate of normality for hedged returns was to be expected, considering that forward contracts and other derivative securities result in non-linear pay-off structures (see e.g. Bansal, Hsieh & Viswanathan 1992), while as Dumas & Jacquillat (1990) point out it is theoretically impossible for currency returns to follow an exact normal distribution[7].

- The implications of these findings for "ex-ante" asset allocation are far from obvious though: if we accept the view that the October 1987 "crash" was a unique event based on extreme circumstances and unlikely to be repeated then there is adequate justification to rely on the adjusted data series and accept normality as a reasonable approximation. The counter-argument, though, is that "crashes" are likely to happen as inherent to the financial system, so that then one might doubt about how good an approximation normality can be for explaining monthly stock returns. This reservation apart, there does not appear to be ground to reject the approximate validity of mean-variance analysis for international portfolios.

- Finally notice that it is dangerous to generalize the aforementioned conclusions because they are very much index specific: index returns from some stock markets (e.g Sweden, Germany and Belgium) appear to have considerably more deviations from normality than others (e.g. Netherlands, Switzerland, UK and USA), a phenomenon that is hard to explain and justify. Similar considerations apply also to returns from various bond indices.

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[7] These issues have been already discussed in chapter one

3.3: Evaluating the Stationarity Assumption: Unit Root Tests

3.1.3.1: Non-stationarity and Portfolio Theory

Much of Modern Portfolio Theory (MPT) relies on the assumption that asset returns are generated by a stationary distribution. The economic implications of non-stationarity are quite obvious: in the absence of an intertemporally constant mean and variance and independent autocovariance in a return series, any attempt to construct an "ex-ante" efficient portfolio would prove fruitless; or otherwise the composition of the "ex-post" efficient portfolio would provide no useful information towards successful asset allocation.

Furthermore, we should emphasize that the largest part of statistical and econometric theory used in a variety of portfolio applications could be invalid in the presence of non-stationarity: for example in non-stationary series the Cental Limit Theorem is no longer valid and has to be substituted by Functional Limit Theorems, while the sample moments converge to random variables rather than constants (see Dolado & Jenkinson 1987).

Empirical evidence on actual testing for stationarity in asset returns is until now limited: Kon (1984) analyzed daily stock returns and concluded that, since there is evidence of non-stationarity they should be modelled as "a discrete mixture of different normal distributions". Taylor & Tonks (1989) performed unit root tests in five major stock market indices and their corresponding monthly returns, and found returns to be generated from stationary distributions. The spirit of the testing methodology used below is similar to that of Taylor & Tonks, but based on a much more exact testing procedure outlined below:

3.3.2 Dickey-Fuller tests for Stationarity in index Returns

We can start by considering an autoregressive series of the form:

$$\Delta y_t = -\rho y_{t-1} + \epsilon_t \qquad |\rho| < 1 \tag{3.4}$$

Which implies that the series y_t (first logarithmic difference of the original price series) is stationary, or integrated of order zero I(0). If however we had instead that $|\rho| = 0$ the series should be differenced once in order to become stationary and would therefore be integrated of order one I(1), or otherwise stated the time series would have a unit root.

Effectively the null hypothesis to be tested is $H_0:(\rho=0)$[8]. The alternative hypothesis will depend on whether it is appropriate to include a trend and/or a constant in the model. The most accurate approach is to estimate first the most general model (the one with both constant and trend) and then test for the significance of the trend and constant to establish whether it is necessary to re-specify the model in a more restrictive form.

Another problem is that the test statistics will be invalidated in case that the residuals e_t are serially correlated. One way to circumvent that difficulty is to follow an approach recommended by Dickey & Fuller (1981)[9] which consists of capturing the serial correlation by means of adding a lagged polynomial of Δy_t resulting in the Augmented Dickey-Fuller (ADF) statistic which can be defined as follows:

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[8] Sargan and Bhargava (1983), as well as Johansen (1988) have suggested different methodologies for unit root rests based on the Durbin-Watson statistic and a likelihood ratio test respectively.

[9] An alternative approach advocated by Phillips and Perron (1986) is to develop a non-parametric test for the residuals.

$$\Delta y_t = a_1 + a_2 t + \rho y_{t-1} + \sum_{i=1}^{m} \beta_i \Delta y_{t-i} + \epsilon_t \tag{3.5}$$

This particular relationship allows for a constant term (a_1), a trend ($a_2 t$), and as many lags of the first difference of the dependent variable Δy_{t-i} as necessary to achieve uncorrelated residuals.

It is difficult to define "a-priori" the exact number of lags to be included in the polynomial but the B-L serial correlation test can be applied in order to establish whether the chosen number is sufficient. Notice that the limiting distribution of the estimated t is unaltered and consequently it can be used as a valid test statistic when we estimate equation (3.5) by means of OLS.

The empirical cumulative distribution of the aforementioned statistic has been tabulated by Dickey (1975) by means of Monte Carlo methods. Dickey's estimated critical value at the 5% confidence level in the model specification with both constant and trend is -3.45 (while with constant only is -2.89) for a sample of around 100 observations. In case that zero lags of the dependent variable are included in equation (5), then the test statistic is reduced to the ordinary Dickey-Fuller (DF) statistic.

Equation (3.5) was estimated by OLS in order to test the assumption of whether the real Sterling returns (unhedged and hedged) are H_1. In order to assure the robustness of the results against possible serial correlation in the residuals, equation (5) was repeatedly estimated for all time series by including zero, one and three lags of the differenced dependent variable respectively[10].

[10] This involved estimating three different equations for each one of the 50 unhedged and hedged return series, or a total of 150 regressions.

3.4: Testing for Intertemporal Stability of Portfolio Inputs:

3.4.1: Rationale for Inter-temporal Stability Testing

As we have already established in previous sections, it is not possible to refute the assumption of (weak) stationarity for the international index returns. This can be viewed as a necessary but not a sufficient condition for successful "ex-ante" international asset allocation:

A simple interpretation for this phenomenon, is that even though mean returns and portfolio variances might not appear to change substantially over time (so that the unit root tests fail to reject weak stationarity), the sample values of portfolio inputs during the forecasting period might markedly differ from those corresponding to the portfolio holding period. In fact, the shorter the length of the forecasting and portfolio holding periods, the more acute this particular problem, known as portfolio "estimation risk", becomes.

An alternative way to view this problem is to construct mean-variance efficient portfolios based on a number of N observations from the forecasting period and re-invest them for the entire holding period ("ex-ante" portfolios). If the portfolio inputs are relatively stable over time then these "ex-ante" portfolios will produce risk-return combinations that are close to the holding period efficient frontier ("ex-post" portfolios).

If, however, portfolio inputs are unstable from subperiod to subperiod then, the "ex-ante" historical portfolios will severely underperform the "ex-post" one's, while substantial part of international diversification benefits will never be realized by the investors. In such a case a mean-variance asset allocation approach based on historical inputs will only be useful for very long forecasting and investment holding periods.

Originally, the more general model specification with both a constant and a trend has been attempted, but since the trend statistic was almost uniformly insignificant the model was re-specified in a more restrictive form to include a constant but not a trend[11].

The values of the estimated ADF/DF statistics[12], at different numbers of included lags, can be found in table 3.2: It can be readily observed that in every single return series under consideration, the test statistics exceed the critical values so that the null hypothesis of a unit root can be uniformly rejected so that we have to accept that monthly real returns are stationary or I_0.

By being unable to reject the hypothesis that monthly index returns are (weak form) stationary we can, therefore, satisfy a "necessary" condition for the validity of mean-variance analysis for international portfolios. Weak form stationarity, though, is by no means sufficient to guarantee the success of "ex-ante" allocation strategies that are based on historical estimates: as a matter of fact, portfolio input forecasts based on small/medium size samples are likely to be subject to significant "estimation" errors resulting to inferior out of sample portfolio performance measures.

To address this all important issue, in section 3.4 provide multivariate statistical evidence on how intertemporally stable the returns covariance matrix and the mean return vector are when measured over m consecutive samples. Later, in chapters IV and V we investigate how the out of sample performance of "ex-ante" portfolio strategies is affected by unstable portfolio inputs and contrast the statistical and economic performance of input forecasts based on a variety of different methodologies.

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11 Test statistics for the preliminary specification not reported here, but available by the author at request.

12 Notice that in the case we estimate the augmented polynomial(ADF) the only valid statistic is the t statistic of the lagged dependent variable.

Table 3.2

Unit Root Tests for Real Index Returns

(1)	(2) DF(0)	(3) ADF(1)	(4) ADF(3)	(5) DF(0)	(6) ADF(1)	(7) ADF(3)
STOCKS	**UNHEG**	**UNHEG**	**UNHEG**	**HEDG**	**HEDG**	**HEDG**
Austria	-8.102	-6.266	-4.537	-8.973	-6.513	-4.862
Belgium	-8.525	-6.677	-4.993	-8.467	-7.276	-5.635
Canada	-6.982	-6.569	-4.686	-9.622	-7.386	-4.896
Denmark	-9.758	-7.139	-4.362	-9.558	-6.665	-4.040
France	-8.324	-6.487	-4.002	-9.249	-6.785	-4.159
Germany	-8.718	-6.946	-4.055	-9.099	-7.115	-4.538
Italy	-7.980	-5.776	-4.421	-8.186	-6.013	-4.643
Japan	-9.279	-6.588	-4.393	-10.131	-6.775	-5.015
Netherl	-9.201	-6.940	-4.982	-9.465	-7.101	-5.482
Norway	-7.918	-6.867	-5.215	-8.233	-7.176	-5.342
Spain	-8.308	-6.889	-4.661	-8.760	-7.152	-5.023
Sweden	-7.812	-5.761	-4.795	-7.842	-6.058	-4.670
Switzer	-7.894	-5.924	-5.028	-9.065	-6.471	-5.537
UK	-10.430	-8.035	-5.422	-10.430	-8.035	-5.422
USA	-8.831	-6.980	-5.217	-9.608	-7.167	-5.987
BONDS	**UNHEG**	**UNHEG**	**UNHEG**	**HEDG**	**HEDG**	**HEDG**
Cana\GB	-8.664	-6.887	-3.789	-9.251	-7.557	-4.233
Fran\GB	-9.381	-6.929	-4.615	-11.235	-8.018	-4.124
Germ\GB	-8.612	-6.678	-4.630	-9.935	-8.079	-4.207
Japa\GB	-8.024	-5.780	-4.756	-8.976	-6.455	-5.687
Neth\GB	-8.223	-6.508	-4.931	-10.190	-7.142	-4.539
Swit\GB	-8.601	-6.103	-5.204	-12.582	-7.501	-4.782
UK\GB	-9.364	-6.695	-5.663	-9.364	-6.695	-5.663
USA\GB	-8.562	-6.468	-4.128	-9.395	-6.429	-4.500

The null of a unit root is rejected at the 5% confidence level in all cases

Test statistics are based on an ADF model specification with a constant but no trend

ADF(1) and ADF(3) relate to the number of lags included in augmented polynomial

Consequently, an issue of major importance is, to empirically investigate how stable over time portfolio inputs are over alternative sampling periods. Existing literature (covered in detail in chapter one) fails to address the question properly, because empirical evidence almost exclusively relates to univariate tests while the portfolio stability problem is mainly considered for a single correlation coefficient or a single mean only. This can hardly be an appropriate methodology, since portfolio input forecasting is directly related to the multivariate distribution of asset returns. Our approach is to apply procedures from Multivariate Analysis of Variance (MANOVA), originally for two consecutive samples and subsequently for multiple sample periods, in order to:

i) Test for the stability of the entire covariance matrix and the mean return vector

ii) Subsequently identify the "source" of input instability by applying appropriate univariate tests[13].

Finally, in order to be consistent throughout, we investigate whether the volatility and intertemporal instability of Sterling denominated portfolio inputs is affected by the volatility of exchange rates and the correlation between exchange rate and stock market movements. For this purpose, we re-apply the same procedures to the hedged data series and contrast the results with those from the unhedged series, whenever substantially different.

3.4.2 Testing Hypotheses in Two Sample and Multi-Sample MANOVA Procedures

The simplest MANOVA procedures involve comparisons between two independent samples, but they can also be generalized to compare covariance matrices and mean return vectors over multiple independent samples. It is also important to test sample periods of different time lengths, since inter-temporal stability of portfolio inputs might well be a function of the measurement period. For this purpose we divided our index returns matrices (108*15 for stocks and 108*10 for bonds) into m sub-matrices, (m = 2, 3, 4, 6) which contain 54, 36, 27 and 18 observations each respectively.

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[13] Notice that most MANOVA techniques are relatively sensitive to substantial departures from ends to the extent of deviations from multivariate normality, so that careful consideration should be given to the results from the diagnostic tests carried in Part I.

More formally, the types of hypotheses to be tested can be formulated as follows:

Let $\vec{x}_g^{\prime}$ denote the row vector of sample means at subperiod g and V_g the variance-covariance matrix at subperiod g:

Then in the context of the "two sample period" tests the null hypotheses to be tested can be expressed as:

$$\begin{aligned} H_0&: \vec{x}_g^{\prime} = \vec{x}_{g+1}^{\prime} \\ H_0&: V_g = V_{g+1} \end{aligned} \tag{6a}$$

where g = 1, 2,....m

m = total number of subperiods

This approach can naturally be extended to a multi-sample horizon, in which case the null hypotheses are equivalent to testing the assumptions

$$\begin{aligned} H_0&: \vec{x}_1^{\prime} = \vec{x}_2^{\prime} = \ldots\ldots \vec{x}_m^{\prime} \\ H_0&: V_1 = V_2 = \ldots\ldots V_m \end{aligned} \tag{6b}$$

3.4.3 Testing the Stability of the Variance-Covariance Matrix

The main reason why it is important to test for intertemporal stability in the entire variance-covariance matrix of index returns, is because it contains all necessary inputs for measuring whether the volatility of a portfolio, for a given set of weights, remains stable or fluctuates over time. In this sense, it represents a significant improvement from previous studies that either concentrate in testing the stability of a single correlation coefficient (e.g. Shaked 1985, Maldonado & Saunders 1982, Odysseos 1990) or the correlation matrix alone (Meric & Meric 1989)[14]. For this purpose, MANOVA procedures naturally suggest themselves and it is particularly surprising that they are almost entirely neglected in applied portfolio analysis.

[14] Another improvement on Meric & Meric is that of generalizing the procedure in a multi-sample context, while they only tested for correlation matrix equality in the limited case of two consecutive samples.

As a first step, we want to test the hypothesis whether the variance-covariance matrix of index returns is statistically equal over all sample periods (strong form) and then to test whether it is equal during two consecutive time periods only (weaker form). In case the returns variance-covariance matrix is intertemporally stable, it will be unnecessary to look at univariate test statistics. If the null hypothesis (defined by equations 6a & 6b) is being rejected, though, it will be necessary to also test whether individual index returns variances are intertemporally stable in order to determine:

i) Whether the intertemporal instability is primarily caused by shifts in volatility or by changes in the correlation structure of asset returns and
ii) Which individual index return variances tend to cause the multivariate instability.

Subsequently, we can proceed to test whether the mean return vector is statistically different between the two subperiods. Nevertheless, the validity of test statistics for mean equality though, are dependant on the homogeneity of the variance-covariance matrix assumption as well as on that of multivariate normality. In fact, if there are substantial deviations from these assumptions, even the most robust multivariate test criteria (like the Pillai's trace) can provide unreliable results.

The earliest and simplest procedure that can be used to test for the equality of two or more covariance matrices is a chi-square statistic developed by Bartlett (1947). A more powerful and complex test, known as BOX-M test (see Box 1949) has been subsequently developed, which is the one we apply in all subsequent analyses:

In order to compute the BOX-M statistic we need first to calculate the Within Groups Sum of Squares and Cross Product (SSCP) Matrix W and the Pooled Within Groups Covariance Matrix V_W defined as follows:

$$\begin{gathered} W = (X - HX_g)' (X - HX_g) \\ X_g = (H'H)^{-1} H' X \end{gathered} \tag{3.7a}$$

$$V_w = \frac{1}{n-m} W \tag{3.7b}$$

where X_g is an (m*k) partitioned matrix consisting of the g^{th} sample mean return vectors (g=1,...m)

X is the partitioned $(n_1 + n_2 + n_m) * (k)$ data matrix

$n_1 ... n_m$ is the number of observations for each sample

n stands for the total number of observations

H is an (n*m) partitioned data matrix consisting of (m-1) zero vectors and one unit vector

On the basis of the above defined matrices, the Box-M statistic can be calculated as follows:

$$M = (n-m) \log_e |D_w| - \sum_{g=1}^{m} (n_g - 1) \log_e |V_g| \tag{3.8}$$

where n_g is the number of observations in sub-sample g

$|V_g|$ stands for the determinants of the covariance matrices for the m groups

Next we need to define two statistics Y_1, Y_2 as follows

$$Y_1 = \sum_{g=1}^{m} \left(\frac{1}{n_g - 1} - \frac{1}{n-m}\right) \frac{2k^2 + 3k - 1}{6(m-1)(k-1)}$$
$$Y_2 = \sum_{g=1}^{m} \left(\frac{1}{(n_g - 1)^2} - \frac{1}{(n-m)^2}\right) \frac{(k-1)(k+2)}{6(m-1)} \tag{3.9}$$

In order to test about the homogeneity of the covariance matrix on the basis of the Box-M test, we must distinguish between 2 cases:

i) If $Y_2 - Y_1^2 > 0$ then the statistic M/b is distributed as F (h_1, h_2), where the parameter b and the degrees of freedom h_1 and h_2 are defined as

$$h_1 = \frac{(m-1)k(k+1)}{2}$$
$$h_2 = \frac{h_1 + 2}{Y_2 - Y_1^2} \tag{3.10a}$$

ii) If $Y_2 - Y_1^2 < 0$, then the appropriate test statistic is distributed as

$$b = \frac{h_1}{1 - Y_1 - \frac{h_1}{h_2}} \tag{3.10b}$$

$$\frac{h_2 M}{h_1 (b - M)} \approx F(h_1, h_2) \tag{3.11}$$

In this case h_1 is defined exactly as before, but h_2 should now be calculated as

$$h_2 = \frac{h_1 + 2}{Y_1^2 - Y_2} \tag{3.12a}$$

while the parameter b is redefined as

$$b = \frac{h_2}{1 - Y_1 + \frac{2}{h_2}} \tag{3.12b}$$

3.4.4: Box-M Test Results

A number of alternative tests based on the Box-M test statistics have been performed depending on the length of the sample sub-periods: the generalized form of the null hypothesis that the covariance matrix remains the same for all sampling periods has been tested for two, three, four and six consecutive sample sub-periods respectively[15].

Subsequently, we tested the weaker form of the null hypothesis i.e. covariance matrix equality based on two consecutive samples only[16]. The consecutive samples tested were chosen to be of equal time-length corresponding to $n_g = 36$, $n_g = 27$ and $n_g = 18$ respectively.

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[15] Notice, that all observations (107) are needed for this version of the test.

[16] Notice that the number of observations used for this type of test are always $2*n_g$ and therefore less than the total number of observations when analyzing more than two subperiods

Notice also that in order to keep the power of the tests as high as possible it is necessary to minimize deviations from multivariate normality: For this purpose, all test statistics concerning stock returns have been recalculated by omitting the October 1987 observation, or otherwise the multivariate distribution of stock returns has been "normalized" by performing the tests on a "crash adjusted" basis[17]. All BOX-M test results have been obtained by using SPSS-PC MANOVA procedures.

Results from the multisample Box-M tests are summarized in table 3.3, while results from the two sample procedures appear in tables 3.A4, 3.A5 & 3.A6 in an appendix at the end of the chapter. The information recorded refers to the number of subperiods and/or observations, the estimated values for the Box-M statistic, the applicable number of degrees of freedom and the computed value of the F statistic as well as the exact probability level of accepting the null hypothesis of intertemporal covariance stability.

A quick glance at table 3.3 shows that when the covariance matrix stability is tested for more than two subperiods (stronger test form) the null hypothesis of intertemporal stability is uniformly rejected. Notice, however, that when the covariance matrices of "crash adjusted" stock returns (hedged and unhedged) are compared between the two longest subperiods only (4 1/2 years each) then inter-temporal stability can not be rejected at the 5% confidence level. The null, though, is never accepted for the covariance matrices of bond returns irrespective of the time horizon.

[17] Effectively 107 instead of 108 observations have been used, so that one of the sub-samples has in fact one observation less.

Table 3.3

BOX-M Multiperiod Tests for Stability of the Covariance Matrix

(1)	(2) BOX-M	(3) DEGR OF FRED	(4) F	(5) PROB
STOCK UNHDG				
2-Subperiods	180.726	120, 34835	1.282	.021
3-Subperiods	415.778	240, 29002	1.378	.000
4-Subperiods	633.181	360, 22895	1.296	.000
6-Subperiods	1292.08	600, 15344	1.308	.000
CRASH ADJUST				
2-Subperiods	166.64	120, 34156	1.180	.088*
3-Subperiods	401.311	240, 28414	1.327	.001
4-Subperiods	613.385	360, 22414	1.251	.001
6-Subperiods	1270.03	600, 14997	1.277	.000
STOCK HEDGE				
2-Subperiods	167.893	120, 34835	1.191	.076*
3-Subperiods	399.348	240, 29002	1.324	.001
4-Subperiods	604.529	360, 22895	1.237	.002
6-Subperiods	1194.93	600, 15344	1.210	.000
CRASH ADJUST				
2-Subperiods	155.356	120, 34156	1.100	.215*
3-Subperiods	385.113	240, 28414	1.273	.003
4-Subperiods	590.819	360, 22414	1.205	.005
6-Subperiods	1178.22	600, 14997	1.184	.001
BONDS UNHDG				
2-Subperiods	102.113	55, 36284	1.671	.001
3-Subperiods	261.239	110, 29841	2.052	.000
4-Subperiods	316.653	165, 23448	1.585	.000
6-Subperiods	621.692	275, 15654	1.683	.000
BONDS HEDGE				
2-Subperiods	161.235	55, 36284	2.639	.000
3-Subperiods	315.269	110, 29841	2.477	.000
4-Subperiods	428.943	165, 23448	2.148	.000
6-Subperiods	670.299	275, 15654	1.814	.000

Column (2) refers to the computed value of the Box-M statistic
Column (3) refers to the number of degrees of freedom for the corresponding F statistic
Columns (4 & 5) refer to the value of the F statistic and the exact probability of accepting the null
*** Null hypothesis of covariance matrix stability not rejected at 5% confidence level**

Taking into consideration the results from the tests concerning two consecutive samples in tables 3.A4 -3.A6, we can reach the following conclusions:

i) The covariance matrix of bond returns, both hedged and unhedged, is more unstable than that of stock returns. In all but three exceptions the null has been rejected for bond returns, while in the case of unadjusted and adjusted stock returns the null is being accepted in nine and eleven cases respectively.

ii) There does not appear to exist any significant relationship between hedging and inter-temporal stability: Hedged returns have very similar rejection rates to the unhedged one's.

iii) My results suggest that there is considerably more evidence of instability for the variance-covariance matrix compared to those for the correlation matrix (as can be seen by the results due to Meric & Meric 1989) or single correlation coefficients (e.g. Odysseos 1990).

iv) All previous evidence suggests that the longer the time-horizon the more stable the correlation structure; Here the picture is rather ambiguous, since covariance stability of stock returns is rejected for both the three-year interval tests but accepted in many cases with smaller time intervals. Clearly, more future empirical evidence will be required before we can reach any definite conclusions on this issue.

3.4.5: Univariate Tests for Homogeneity of Variance

The next step in establishing why the variance-covariance matrix of returns is more unstable than the correlation matrix, consists of examining the homogeneity of individual asset variances over time between the same sampling sub-periods as those used for the Box-M tests. In this way we can establish which index return variances are unstable over time, so that we can understand what induced us to reject the null in the Box-M type procedures.

An appropriate testing procedure for homogeneity of variance, assuming independent normal samples, has been developed by Bartlett. This test has been traditionally mainly used in experimental designs, but the concept can easily be extended to cover consecutive independent samples from time series data[18].

The null hypothesis to be tested in this context can be expressed as

$$H_0: \sigma_{i1}^2 = \sigma_{i2}^2 = ...\sigma_{im}^2 \tag{3.13}$$

where m stands for the number of samples under comparison

Bartlett's test consists of computing a test statistic that can be approximated by a chi-square distribution with m-1 degrees of freedom. Bartlett's statistic is defined as

$$\chi_0^2 = 2.3026 \frac{q}{c} \tag{3.14}$$

where the parameters q and c are defined as following:

$$q = (N-m)\log_{10} S_p^2 - \sum_{i=1}^{m} (n_i - 1)\log_{10} S_i^2 \tag{3.15a}$$

$$c = 1 + \frac{1}{3(m-1)}\left[\sum_{i=1}^{m} (n_i - 1)^{-1} - (N-m)^{-1}\right] \tag{3.15b}$$

while

[18] This is only possible because we analyze first differences rather than index values. A problem that might arise, particularly for relatively small sample sizes, lies in the possibility that the first observation(s) in each sample to be serially dependent to past observations. In such an event, the use of the Bartlett test would be inappropriate since the samples would not be independent.

$$S_p^2 = \frac{\sum_{i=1}^{m} (n_i - 1)\, S_i^2}{N - m} \tag{3.15c}$$

where S_i^2 is the sample variance from the i^{th} sub-period

Notice that if all sample variances are equal the quantity q will be equal to zero, but if sampling variances greatly differ then both q and the corresponding chi-square will be large causing rejection of the null. More formally, we need to reject H_0 when

$$\chi_0^2 > \chi_{m,m-1}^2 \tag{3.16}$$

where $\chi_{m,m-1}^2$ refers to the upper m percentage points of the $\chi^2(m-1)$ distribution.

Notice that a variant of this test exists, known as Bartlett-Box statistic (see Norussis 1989), whose test statistic's sampling distribution is approximated by the F rather than the chi-square distribution. The Bartlett-Box statistic has been computed by means of SPSS-PC+ MANOVA procedures for the multi-period tests, involving comparisons of the sampling variance over two, three and four subperiods respectively. These results are summarized in Tables 3.4 & 3.4B:

As far the unhedged stock returns are concerned, the major causes of multivariate instability are the German, Swiss, UK and US markets, where the assumption of univariate homogeneity of variance is rejected for all subperiods. Most. of the remaining markets appear to have relatively stable variances. More rejections appear in the hedged stock returns where, in addition to the markets previously mentioned, homogeneity is rejected for Austria, Belgium, France and Japan. On the other hand, however, if we consider the variances of "crash adjusted" stock returns in Table 4.2B, the homogeneity rejection rate diminishes substantially, especially for the unhedged returns.

Table 3.4

Bartlett-Box Multiperiod Homogeneity of Variance Tests

(1)	(2)	(3)	(4)	(5)	(6)	(7)
PERIODS	2-SUBP	2-SUBP	3-SUBP	3-SUBP	4-SUBP	4-SUBP
F (d o f)	1, 33708	1, 33708	2, 24806	2, 24806	3, 19469	3, 19469
STOCKS	UNHG	HEDG	UNHG	HEDG	UNHG	HEDG
Austria	2.701	5.926*	2.466	3.135*	3.315*	4.381*
Belgium	2.344	4.059*	2.950	2.665	2.607	3.571*
Canada	2.007	.431	1.155	3.307*	2.112	3.137*
Denmark	.026	.063	.357	.748	.187	.060
France	2.572	4.494*	3.269*	4.836*	2.369	3.828*
Germany	5.191*	8.529*	4.090*	6.457*	1.930	3.673*
Italy	2.941	1.388	2.925	2.931	.870	.708
Japan	.867	10.63*	.243	4.606*	.748	4.407*
Netherl	.345	.538	3.732*	4.120*	2.381	2.815*
Norway	2.667	2.786	2.330	1.702	2.074	1.947
Spain	.984	1.332	4.175*	3.755*	1.636	1.682
Sweden	1.161	1.792	.806	.338*	.647	1.149
Switzer	7.080*	12.72*	4.017*	11.07*	2.730*	6.431*
UK	14.56*	14.55*	7.181*	7.181*	6.848*	6.848*
USA	6.163*	6.206*	4.711*	4.082*	3.455*	3.725*
BONDS	UNHG	HEDG	UNHG	HEDG	UNHG	HEDG
Cana\GB	.050	3.568	.219	3.607*	.430	2.153
Fran\GB	6.059*	.946	.868	.601	1.884	.486
Germ\GB	5.956*	.284	.842	.099	2.171	.702
Japa\GB	4.872*	6.100*	1.873	8.978*	2.078	4.527*
Neth\GB	7.292*	1.635	1.482	1.645	2.613*	1.437
Swit\GB	7.297*	1.805	1.420	1.079	2.568	1.792
UK\GB	.016	.016	.584	.584	.589	.589
USA\GB	.088	5.091*	.760	4.225*	.937	2.286
USA\FRN	.012	43.63*	5.019*	31.82*	2.819*	27.26*
USA\ZER	1.058	.942	1.152	4.976*	.777	1.726

*** Variance homogeneity rejected at 5% confidence level**

Stock returns are unadjusted for the October 1987 "crash"

Table 3.4B

Bartlett-Box Multiperiod Homogeneity of Variance Tests

Stock Returns Adjusted for October 1987 "Crash"

(1)	(2)	(3)	(4)	(5)	(6)	(7)
PERIODS	2-SUBP	2-SUBP	3-SUBP	3-SUBP	4-SUBP	4-SUBP
F (d o f)	1, 33708	1, 33708	2, 24806	2, 24806	3, 19469	3, 19469
STOCKS	UNHG	HEDG	UNHG	HEDG	UNHG	HEDG
Austria	.073	1.445	.275	1.718	.465	.835
Belgium	.007	.330	.689	.894	.749	.375
Canada	.142	2.367	.781	3.629*	.725	1.806
Denmark	.418	.725	.688	1.756	.434	.316
France	.314	1.275	1.430	2.732	.963	1.703
Germany	2.270	4.256*	2.330	4.110*	2.320*	2.177
Italy	5.319*	2.977	1.828	1.741	1.830	.855
Japan	.521	9.127*	.355	4.613*	3.038*	4.470*
Netherl	1.276	1.338	2.783	2.066	2.216*	.708
Norway	.004	.003	.166	.145	.631	.074
Spain	.042	.015	.903	.478	3.662*	.889
Sweden	.059	.380	.030	.092	3.594*	.884
Switzer	1.820	4.939*	1.165	6.647*	.839	4.330*
UK	3.978*	3.978*	1.624	1.624	1.285	2.901*
USA	.931	.859	.904	.246	.405	.371

* Homogeneity of variance rejected at the 5% confidence level

In the case of bond returns, rejection rates appear to be particularly high in the two subperiod (54 observation each) unhedged case, predominantly for the European bond markets. For the hedged bond returns though, variance homogeneity is most strongly rejected for the Japanese government bonds and the Dollar FRN's.

3.4.6 Intertemporal Stability of the Mean Return Vector

In recent years, a number of authors (e.g. Jorion 1985 & 1989, Odysseos 1990) have emphasized that the mean return vector is more intertemporally unstable than the returns covariance matrix and that estimation errors in the mean return vector will have more substantial implications for international portfolio allocation than estimation errors in the returns covariance matrix[19]. Nevertheless, despite the intuitive logic of this argument, there does not exist to date any empirical multivariate study that formally tests for the instability of the mean return vector when measured over successive time periods.

Unfortunately, existing multivariate testing procedures for the mean vector, generally require approximate multivariate normality and relatively stable variance-covariance matrices. By using the "crash adjusted" rather than the original series we can certainly mitigate the first problem, but our sample covariance matrices have been generally shown to be intertemporally unstable. Inevitably, this leads to reduced power for all the following multivariate procedures and the results, therefore, should be treated with caution.

At first, the null hypothesis of mean vector stability, can be tested for the limited case of two independent samples only, on the basis of equation (3.6a). In this particular case, the choice of an appropriate multivariate criterion is irrelevant, since all known testing procedures (Hotteling T^2, Wilks Lambda, Roy's, Pillais Trace) provide identical results. The Hotteling test statistic used for our purposes below, is being defined as

[19] In the sense that errors in forecasting the mean return vector will lead to a substantial decrease in the portfolio's performance measures, when calculated in an out of sample period, compared to forecasting errors in the variance-covariance matrix.

$$T_2^2 = \frac{n_1 n_2}{n_1 + n_2} (\vec{X}_1 - \vec{X}_2)' \, V_w^{-1} \, (\vec{X}_1 - \vec{X}_2) \tag{3.17}$$

where $\vec{X}_1 - \vec{X}_2$ is the difference vector between the sample centroids from the two independent samples.

V_w is the previously defined pooled-within groups covariance matrix

The usefulness of the Hotteling T_2 lies in the fact that it can be used to test hypotheses by means of a transformation into an approximate F distribution defined as follows:

$$T_2^2 \, \frac{n-k-1}{k(n-2)} \approx F(k, n-k-1) \tag{3.18}$$

Subsequently, we need to generalize the testing procedure for testing the null hypothesis of mean vector stability across several independent samples on the basis of equation (3.6b). In this case, alternative testing procedures need to be applied, since in general their results tend to differ.

At first we apply a testing procedure known as the Wilks Lambda. In order to apply this test we need first to compute two additional MANOVA matrices, also known from discriminant analysis, i.e. the Total Mean Corrected Sum of Squares and Cross Products (SSCP) Matrix T and the Between Groups SSCP matrix A (known also as "Hypothesis" matrix), which are defined as follows:

$$T = (X - \vec{1}\vec{x}')' \, (X - \vec{1}\vec{x}') \tag{3.19a}$$

$$A = (HX_g - \vec{1}\vec{x}')' \, (HX_g - \vec{1}\vec{x}') \tag{3.19b}$$

$$A = W + A \tag{3.19c}$$

where X_g, X, H are defined as previously in equations 3.7a & 3.7b

$\vec{x}$ is the mean return vector computed from the entire sample

Based on the above matrices the Wilks Lambda could be defined as

$$WLK = \prod_{i=1}^{s} \frac{1}{1+\lambda_i} = \frac{|W|}{|T|} \tag{3.20}$$

where s is the number of non zero eigenvalues λ_i of the AW^{-1} matrix

In general most tests of multivariate differences among means are based on the determinant of the AW^{-1} matrix. This determinant can be interpreted as a measure of generalized variance or dispersion of a matrix. In fact it is calculated as the product of the matrix eigenvalues, so the whole procedure is quite similar to that of extracting Principal Components. Apart from the Wilks Lambda, the Pillai's Trace, Roy's Largest Root Criterion and the multivariate generalization of the Hotteling T^2 are based on the aforementioned matrix.

Notice, however, that the distribution of the Wilks statistic can only be calculated exactly in case either the number of variables is even or the number of samples is odd[20] (see Green 1980). In general, numerical approximations are used for computational purposes.

The second procedure to be applied for testing multivariate differences of the mean return vector is the so called Pillai's Trace. It's main usefulness lies in the fact that it tends to be the most robust test statistic, in the presence of deviations from covariance matrix homogeneity (see Norussis 1989). Also the Pillai criterion appears to have the greater power, in the sense of being able to detect multivariate differences where they exist. The Pillai's test statistic can be defined as follows:

[20] In the special case where the number of independent samples is equal to three (m=3) and for any number of dependent variables the test distribution becomes

$$\left(\frac{1-\Lambda^{1/2}}{\Lambda^{1/2}}\right)\frac{n-k-2}{k} \approx F(2k,\ 2n-2k-4)$$

where Λ denotes the diagonal matrix consisting of the AW^{-1} eigenvalues

$$V = \sum_{i=1}^{s} \frac{1}{1 + \lambda_i} \tag{3.21}$$

Finally, the multi-sample generalization of the Hotteling test, known as Hotteling's Trace is defined as:

$$T = \sum_{i=1}^{s} \lambda_i \tag{3.22}$$

Notice that all three aforementioned test statistics, i.e. Wilks, Pillai and Hotteling can be numerically approximated in terms of the F distribution.

3.1.7: Empirical Evidence on Mean Return Vector Stability

Three alternative tests for multi-sample differences among mean return vectors have been performed, namely Hotteling, Pillai and Wilks type tests, using SPSS-PC MANOVA Procedures. The stronger form of the null hypothesis (equation 6b) has been tested for two, three, four and six consecutive samples respectively. Tests concerning hedged and unhedged stock returns have been calculated for both the adjusted and unadjusted data matrices.

Test results are summarized on table 3.5. The first column refers to the number of consecutive samples tested in each case and the type of data matrix used. The subsequent columns list the computed values for the Hotteling T^2, Pillai's Trace and Wilks Lambda Statistics respectively and the approximate values of their corresponding F statistics. These F values are identical for all three tests in the two sub-period case.

For large enough values of the F statistics, we can reject the null of intertemporal stability for the mean returns[21]. Because the null is very rarely rejected at the 5% confidence level, rejection rates at the 10% confidence level are also reported.
As we can readily observe, in all cases involving two subperiods (54 monthly observations each) and most cases involving three or four subperiods, it is not possible to reject the null of centroid stability. The null is often rejected though when testing the six subperiod case (18 observations each) for stock returns.

Finally, we also tested the weaker form of the null hypothesis (equation 6a) concerning the intertemporal stability of the mean return vector over two consecutive periods only for "crash adjusted" hedged and unhedged stock returns. Since in this case all test criteria are identical, only the Hotteling T^2 is being reported.

These results are summarized in table 3.A7: column 1 reports the observation range used for every test, whereas columns 2-4 report the Hotteling statistic, the corresponding F and the exact probability level of rejecting the null. Not in a single case is the null being rejected, even at the 10% confidence level.

Overall the rejection rate appears to be remarkably low, but in any case one should be very cautious before reaching any conclusions: even the Pillai's Trace which is considered to be the most robust of all test statistics, could be unreliable here because of significant violations concerning the assumption of homogeneity of the covariance matrix.

21 To preserve space, exact probability levels for all the F statistics and the number of degrees of freedom are not tabulated but are available by the author at request. Also values for the so called Roy criterion have been computed but omitted: they provide qualitatively very similar results to the other tests.

Table 3.5

Multiperiod Tests for Stability of the Mean Return Vector

(1)	(2) HTLNG	(3) F	(4) PILLAI	(5) F	(6) WILKS	(7) F
STOCK UNHG						
2-Subperiods	.1353	.8300	.1192	.8300	.8808	.8300
3-Subperiods	.2930	.8789	.2521	.8848	.7622	.8820
4-Subperiods	.5674	1.118	.4633	1.120	.5997	1.119
6-Subperiods	1.023	1.178	.7950	1.160	.407	1.170
CRASH ADJUST						
2-Subperiods	.1258	.7637	.1118	.7637	.8862	.7637
3-Subperiods	.3234	.9595	.2730	.9591	.7433	.9594
4-Subperiods	.6163	1.200	.4965	1.203	.5797	1.203
6-Subperiods	1.109	1.263*	.8509	1.244*	.3802	1.255*
STOCK HEDG						
2-Subperiods	.1061	.6508	.0959	.6508	.9040	.6508
3-Subperiods	.2897	.8690	.2500	.8762	.7643	.8727
4-Subperiods	.5986	1.179	.4777	1.161	.5868	1.171
6-Subperiods	1.084	1.249*	.8426	1.243*	.3861	1.247*
CRSH ADJUST						
2-Subperiods	.0971	.5878	.0884	.5878	.9115	.5878
3-Subperiods	.3248	.9635	.2747	.9661	.7421	.9649
4-Subperiods	.6693	1.289	.5201	1.272	.5572	1.282
6-Subperiods	1.171	1.334**	.9003	1.332**	.3597	1.335**
BONDS UNHDE						
2-Subperiods	.1439	1.396	.1258	1.396	.8742	1.396
3-Subperiods	.3144	1.493*	.2688	1.506*	.7480	1.499*
4-Subperiods	.3102	.9684	.2739	.9744	.7474	.9717
6-Subperiods	.6346	1.160	.5327	1.156	.5598	1.159
BONDS HEDGD						
2-Subperiods	.1578	1.531	.1363	1.531	.8637	1.531
3-Subperiods	.1739	.8263	.1581	.8325	.8473	.8295
4-Subperiods	.2830	.8835	.2518	.8888	.7660	.8862
6-Subperiods	.4662	.8521	.4015	.8469	.6493	.8491

Columns (2, 4 & 6) refer to the estimated values of the Hotteling, Pillai and Wilks criteria
**** Null hypothesis of mean return vector stability rejected at the 5% confidence level**
*** Null hypothesis only rejected at the 10% confidence level**

3.5: MANOVA Procedures: Summary & Conclusions

Previous empirical studies have concentrated primarily on testing for intertemporal stability of single correlation coefficients and concluded that to a large extent they appear to be reasonably stable over time. A multivariate test for the entire returns correlation matrix by Meric & Meric led to similar conclusions.

Such procedures cannot answer the question of how much ineffective the "ex-ante" allocation strategies, based on historical Markowitz portfolios, are going to be because they fail to test for stability of the two inputs to that problem, i.e. the index return variance-covariance matrix and the mean return vector.

This problem was empirically addressed in the previous section, where multivariate (MANOVA) techniques have been used to test for the stability of the entire covariance matrix and the centroid vector. These tests were applied to samples from two consecutive time periods (of various lengths), as well as to samples from three or more consecutive time periods. The results, in all cases, provided uniform evidence that the variance-covariance matrix is intertemporally unstable even for medium and long term periods, as opposed to the short term volatility "clustering" as captured by the presence of GARCH effects. Notice that the MANOVA evidence on the instability of the variance-covariance matrix substantially differs from that of comparative tests conducted to establish the degree of stability in the long term correlation structure of index returns, as indicated by previous studies.

Consequently, such a phenomenon should be potentially attributed, at least partially, to the instability of individual index return variances which might be more unstable than the return correlation coefficients. To verify this, univariate "homogeneity of variance tests" have also been performed in order to ascertain which individual index variances tend to be intertemporally unstable; In fact, for several of the individual index series the results failed to support variance homogeneity, phenomenon that at least partially explains the multivariate covariance instability.

It would be also of interest to contrast our tests for weak form stationarity of returns where the null hypothesis of a unit root was uniformly rejected, to the univariate homogeneity tests where several individual variances were shown to be unstable: one possible interpretation to that phenomenon is that variances tend to fluctuate substantially between relatively short term intervals (captured by the homogeneity tests) but that there is no evidence of a persistent increase of volatility over time (so that there is uniform rejection of the hypothesis of a unit root in returns).

Subsequently, a number of alternative MANOVA test procedures have been applied to test the hypothesis of intertemporal instability for the index mean return vector for two or more consecutive samples and time periods of various lengths; In this case the results failed to provide conclusive evidence of the mean return vector instability, but admittedly the tests had relatively low power, being rather sensitive to the assumption violation of covariance matrix homogeneity. Nevertheless, the failure to reject the hypothesis of mean return instability, is consistent with the evidence from the unit root tests, which had failed to detect major shifts in average returns over time.

Failure to find any significant evidence against the stability of mean returns appears to be at odds with the well known fact that the efficient frontier is rather unstable over time, phenomenon that according to Jorion (1986) should be primarily attributed to the instability of mean returns rather than instability of the covariance matrix. My own interpretation is that the Jorion argument might in any event be well justified on the grounds that the efficient frontier is likely to be considerably more sensitive to estimation errors in the index return means rather than estimation errors in the variance-covariance matrix.

In other words, even statistically insignificant differences in the mean return vector between the forecasting and portfolio holding periods might be large enough to cause investors selecting sub-optimal portfolios. This very important issue is extensively addressed in the following chapter.

Table 3A.1

Normality Tests: Higher Moments and Studentized Range

(1)	(2) SKEWN	(3) KURTO	(4) ST.RG	(5) SKEWN	(6) KURTO	(7) ST.RG
STOCKS	**UNHG**	**UNHG**	**UNHG**	**HEDG**	**HEDG**	**HEDG**
Austria	-1.011	4.115	6.86	- .840	4.300	7.43
Belgium	- .607	4.823	7.84	- .551	5.632	8.37
Canada	-1.045	5.673	7.72	-1.021	6.452	8.05
Denmark	- .080	.114	5.24	- .502	.346	4.92
France	- .877	2.412	6.61	- .783	2.460	7.04
Germany	- .816	2.286	6.21	-1.148	3.274	6.22
Italy	.099	.711	5.69	.070	1.117	6.19
Japan	- .203	.788	5.44	- .548	2.525	6.92
Netherl	- .908	4.290	7.23	- .940	5.269	7.63
Norway	-1.380	5.217	6.93	-1.233	4.748	7.14
Spain	- .667	3.234	6.77	- .625	4.348	7.71
Sweden	- .773	2.758	6.14	- .717	2.316	5.92
Switzer	- .983	4.708	7.07	-1.342	5.957	7.34
UK	-1.938	9.471	7.96	-1.938	9.471	7.96
USA	-1.401	5.365	6.94	-1.167	5.677	7.37
BONDS	**UNHG**	**UNHG**	**UNHG**	**HEDG**	**HEDG**	**HEDG**
Cana\GB	.232	.772	5.66	.396	1.517	5.97
Fran\GB	.073	.021	4.89	.392	1.361	6.23
Germ\GB	- .031	.231	5.50	- .005	- .055	5.82
Japa\GB	.616	1.864	6.29	- .211	.395	5.21
Neth\GB	- .116	.829	6.30	.162	.084	5.49
Swit\GB	.547	1.326	5.94	.086	- .056	4.97
UK\GB	- .095	.429	5.78	- .095	.429	5.78
USA\GB	- .169	.324	5.63	.081	- .176	4.61
USA\FRN	- .612	.961	5.73	-2.866	31.189	12.06
USA\ZER	- .028	- .064	4.93	- .027	- .100	4.82

Critical Values at 5% Confidence Level (108 observations):
Skewness: .377, Kurtosis(K+3): Upper Limit 3.72, Lower Limit 2.56
Studentized Range : Upper Limit 6.15, Lower Limit 4.25.

Table 3A.2

Higher Moments and Studentized Range

Returns Adjusted for October 1987 Effects

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	SKEWN	KURTO	ST.RG	SKEWN	KURTO	ST.RG
STOCKS	UNHG	UNHG	UNHG	HEDG	HEDG	HEDG
Austria	-.177	.659	5.29	.070	.872	5.82
Belgium	.450	1.330	5.83	.648	1.916	6.42
Canada	.156	.529	5.61	.350	.735	5.98
Denmark	.044	-.052	4.92	-.389	.146	4.91
France	-.334	.293	5.36	-.192	.220	5.63
Germany	-.434	1.140	5.81	-.715	1.745	5.86
Italy	.263	.563	5.38	.232	1.030	5.95
Japan	-.168	.884	5.54	-.499	2.800	7.12
Netherl	.065	.251	5.32	.229	.414	5.62
Norway	-.421	.402	5.51	-.291	.250	5.67
Spain	-.022	1.218	6.16	.141	2.259	7.22
Sweden	-.446	2.086	6.41	-.443	1.798	6.29
Switzer	.021	.505	5.28	-.250	.775	5.57
UK	-.356	.031	5.18	-.356	.031	5.18
USA	-.383	.101	4.84	.008	.295	5.06

Critical Values at 5% Confidence Level (107 Observations):
Skewness: .377, Kurtosis(K+3): Upper Limit 3.72, Lower Limit 2.36
Studentized Range: Upper Limit 6.15, Lower Limit 4.25.

Table 3A.3

Normality Tests: Kolmogorov one Sample Test

(1)	(2) KOL(d)	(3) ADJ(d)	(4) KOL(d)	(5) ADJ(d)
STOCKS	**UNHG**	**UNHG**	**HEDG**	**HEDG**
Austria	.0847	.0554	.1030(*)	.0884
Belgium	.0689	.0484	.0792	.0662
Canada	.0899(*)	.0592	.1029(*)	.0959(*)
Denmark	.0627	.0573	.0783	.0721
France	.0750	.0618	.0778	.0733
Germany	.0979(*)	.0782	.1161(*)	.0937(*)
Italy	.0780	.0783	.0670	.0654
Japan	.0745	.0707	.1313(*)	.1258(*)
Netherl	.0740	.0595	.0679	.0491
Norway	.0972(*)	.0750	.0732	.0521
Spain	.0603	.0546	.1023(*)	.0804
Sweden	.0948(*)	.0767	.0947(*)	.0798
Switzer	.0782	.0638	.0784	.0415
UK	.0927(*)	.0577	.0927(*)	.0577
USA	.0972(*)	.0880	.0739	.0515
BONDS	**UNHG**	**UNHG**	**HEDG**	**HEDG**
Cana\GB	.0547		.0605	
Fran\GB	.0567		.0575	
Germ\GB	.0638		.0601	
Japa\GB	.0852		.0547	
Neth\GB	.0547		.0639	
Swit\GB	.0583		.0543	
UK\GB	.0552		.0552	
USA\GB	.0522		.0417	
USA\FRN	.0826		.1982(*)	
USA\ZER	.0426		.0399	

Columns (3 & 5) refer to stock returns adjusted for Oct. 87
Critical value for the Kolmogorov d statistic at 5% level: .089 (source: Dallal & Wilkinson 1986)
(*) : normality rejected at 5% confidence level

Table 3.A4

BOX-M Two Sample Test for Stability of the Covariance Matrix

Unadjusted Stock Returns

(1) OBSERVATIONS	(2) BOX-M	(3) D.o.F	(4) F	(5) PROB
STOCK UNHDG				
1-36, 37-72	215.758	120, 15191	1.387	.003
37-72, 73-108	209.108	120, 15191	1.344	.007
1-27, 28-54	212.777	120, 8383	1.221	.051**
28-54, 55-81	209.430	120, 8383	1.202	.067**
55-81, 82-108	249.581	120, 8383	1.432	.001
1-18, 19-36	254.239	120, 3584	1.086	.250**
19-36, 37-54	286.492	120, 3584	1.223	.052**
37-54, 55-72	297.066	120, 3584	1.268	.027
55-72, 73-90	341.069	120, 3584	1.456	.001
73-90, 91-108	334.905	120, 3584	1.430	.002
STOCK HEDGED				
1-36, 37-72	210.546	120, 15191	1.353	.006
37-72, 73-108	197.477	120, 15191	1.269	.025
1-27, 28-54	205.134	120, 8383	1.177	.091**
28-54, 55-81	181.571	120, 8383	1.042	.359**
55-81, 82-108	242.813	120, 8383	1.394	.003
1-18, 19-36	248.972	120, 3584	1.063	.304**
19-36, 37-54	267.109	120, 3584	1.141	.144**
37-54, 55-72	249.791	120, 3584	1.067	.295**
55-72, 73-90	293.218	120, 3584	1.252	.035
73-90, 91-108	309.758	120, 3584	1.323	.012

** Null hypothesis of covariance matrix stability not rejected at 5% confidence level

Table 3.A5

BOX-M two Sample Test for Stability of the Covariance Matrix

Bond Returns

(1) OBSERVATIONS	(2) BOX-M	(3) D.o.F	(4) F	(5) PROB
BONDS UNHDG				
1-36, 37-72	114.981	55, 15823	1.772	.000
37-72, 73-108	129.449	55, 15823	1.995	.000
1-27, 28-54	118.346	55, 8732	1.707	.001
28-54, 55-81	75.856	55, 8732	1.094	.294**
55-81, 82-108	96.701	55, 8732	1.395	.029
1-18, 19-36	150.281	55, 3733	1.856	.000
19-36, 37-54	141.580	55, 3733	1.748	.001
37-54, 55-72	101.733	55, 3733	1.256	.098**
55-72, 73-90	131.801	55, 3733	1.627	.002
73-90, 91-108	115.669	55, 3733	1.428	.021
BONDS HEDGED				
1-36, 37-72	97.423	55, 15823	1.502	.010
37-72, 73-108	159.796	55, 15823	2.463	.000
1-27, 28-54	121.114	55, 8732	1.747	.001
28-54, 55-81	88.119	55, 8732	1.271	.086**
55-81, 82-108	148.261	55, 8732	2.139	.000
1-18, 19-36	129.372	55, 3733	1.597	.004
19-36, 37-54	128.898	55, 3733	1.592	.004
37-54, 55-72	109.059	55, 3733	1.347	.046
55-72, 73-90	111.720	55, 3733	1.379	.034
73-90, 91-108	114.454	55, 3733	1.413	.025

**** Null hypothesis of covariance matrix stability not rejected at 5% confidence level**

Numbers in Column 1 refer to the observation range in the two consecutive samples

Table 3.A6

BOX-M two Sample Test for Stability of the Covariance Matrix

Adjusted Stock Returns

(1) OBSERVATIONS	(2) BOX-M	(3) D.o.F	(4) F	(5) PROB
STOCK UNHDG				
1-36, 37-72	201.524	120, 14735	1.289	.018
37-72, 73-108	205.817	120, 14735	1.317	.012
1-27, 28-54	212.777	120, 8383	1.221	.051**
28-54, 55-81	200.156	120, 8039	1.138	.145**
55-81, 82-108	240.959	120, 8039	1.369	.005
1-18, 19-36	254.239	120, 3584	1.086	.250**
19-36, 37-54	286.492	120, 3584	1.223	.052**
37-54, 55-72	290.878	120, 3351	1.201	.069**
55-72, 73-90	337.579	120, 3351	1.395	.003
73-90, 91-108	334.905	120, 3584	1.430	.002
STOCK HEDGED				
1-36, 37-72	195.673	120, 14735	1.252	.033
37-72, 73-108	193.005	120, 14735	1.235	.042
1-27, 28-54	205.134	120, 8383	1.177	.091**
28-54, 55-81	178.654	120, 8039	1.015	.436**
55-81, 82-108	238.526	120, 8039	1.356	.006
1-18, 19-36	248.972	120, 3584	1.063	.304**
19-36, 37-54	267.109	120, 3584	1.141	.144**
37-54, 55-72	248.193	120, 3351	1.025	.408**
55-72, 73-90	290.748	120, 3351	1.201	.070**
73-90, 91-108	309.758	120, 3584	1.323	.012

Column (1) refers to the range of observations that are included in the two samples

**** Null hypothesis of covariance matrix stability not rejected at 5% confidence level**

Stock returns adjusted for the October 1987 "crash"

Table 3.A7

Hotteling Two Sample Stability test for the Mean Return Vector

Adjusted Stock Returns

(1) OBSERVATIONS	(2) HOTELG	(3) F	(4) PROB
STOCK UNHDG			
1-36, 37-72	.2335	.8565	.613
37-72, 73-108	.3407	1.249	.266
1-27, 28-54	.4780	1.211	.306
28-54, 55-81	.5369	1.324	.237
55-81, 82-108	.4945	1.219	.301
1-18, 19-36	1.084	1.446	.218
19-36, 37-54	.9540	1.272	.303
37-54, 55-72	1.025	1.299	.291
55-72, 73-90	1.065	1.350	.265
73-90, 91-108	.7804	1.040	.458
STOCK HEDGED			
1-36, 37-72	.2704	.9915	.477
37-72, 73-108	.3298	1.209	.293
1-27, 28-54	.5797	1.468	.167
28-54, 55-81	.5491	1.354	.221
55-81, 82-108	.5299	1.307	.247
1-18, 19-36	1.221	1.628	.153
19-36, 37-54	1.271	1.695	.134
37-54, 55-72	.9443	1.196	.351
55-72, 73-90	1.265	1.602	.165
73-90, 91-108	.8703	1.160	.372

Column (1) refers to the range of observations that are included in the two samples
Columns (2, 3 & 4) refer to the estimated value of the Hotelling statistic, the corresponding F statistic and the exact probability level of rejecting the null hypothesis
For the two sample tests the Hotteling, Pillai & Wilks criteria provide identical results

CHAPTER IV

INDEX CORRELATION STRUCTURE, ESTIMATION RISK AND PERFORMANCE OF EX-ANTE ASSET ALLOCATION STRATEGIES: AN EMPIRICAL EVALUATION

4.1. Formulating Alternative Market Allocation Methodologies

4.1.1. Scope for "Ex-Ante" Optimal Index Portfolios

As previously outlined in the thesis introduction, the central aim in Chapter IV is to develop and empirically assess the out of sample performance of alternative asset allocation strategies, all of them formulated on the basis of multivariate techniques. These alternative models are being applied first to global stock index portfolios and subsequently to portfolios allowing for a stock/bond index asset mix.

The same procedures are also being applied to the index return series that have been constructed on the basis of the "ex-ante" hedging strategy, previously discussed in Chapter II, in order to assess in all cases the incremental risk-adjusted performance from removing the foreign exchange risk components from index returns[1]. Effectively, when the expected foreign asset return is fully hedged, the implied global asset allocation strategy is to optimize the out of sample country selection component while neutralizing the "currency selection and "stock selection" performance components.

Given the fact that portfolio inputs derived from index returns were shown in Chapter III to be inter-temporally unstable, optimal portfolios derived from historical data should not be expected to perform particularly well out of sample, while they could well prove inferior to other performance benchmarks, like an equally weighted (passive) portfolio or a market capitalization weighted world index.

[1] Recall from Chapter II that, even though the intention here is to fully, rather than partially, hedge the foreign exchange risk by means of monthly forward contracts, in fact due to the residual risk an "ex-ante" hedging strategy is always imperfect.

The key emphasis of this chapter is on developing estimates of portfolio inputs, that have less "noise" than historical inputs and therefore are likely to have superior forecasting ability when evaluated out of sample. Such "noise" reduction is being attempted through a variety a multivariate techniques, focusing around unobservable factor models and observable factor models, Bayesian and empirical Bayes-Stein types of estimators, "grand mean" type models and models with imposed upper constraints on investment weights. Many of these techniques, notably the factor models, can be effectively used to predict the future correlation matrix of index returns, while the standard and empirical Bayesian techniques can be used to reduce "noise" from the volatility estimates and estimate the mean return vector as well.

4.1.2. Focus on Optimizing the Country (Index) Weights

Since our portfolio inputs are derived from index (rather than individual asset) returns, all our asset allocation strategies are "neutral" on the stock selection component. For a UK investor with an international portfolio of unhedged assets and a stable level of exposure to different asset classes (not a market timer), the "ex-post" measured portfolio performance can be expressed as:

$$
\begin{gathered}
R_i^{£} = (1 + r_{mi})(1 + r_{ci})(1 + r_{si}) - 1 \\
R_p^{£} = \sum_{i=1}^{k} w_i R_i^{£} = \sum_{i=1}^{k} w_i \left[(1 + r_{mi})(1 + r_{ci})(1 + r_{si}) - 1 \right]
\end{gathered}
\qquad (4.1)
$$

where $R_i^{£}$, $R_p^{£}$ represent the Sterling denominated return from the portfolio segment invested in country i and the total international portfolio respectively

w_i stands for the investment weight allocated in country i

r_{mi} denotes the country i stock index return in local currency terms

r_{ci} refers to the currency i return against Sterling

r_{si} shows the excess return from the investment in country i compared to the local market index (stock selection component)

For a large international portfolio, with a relatively small ability to be a superior stock-picker in foreign markets, superior performance could simply arise from optimizing the country weights, while "neutralizing" the stock selection component and possibly (hedged strategy) the currency selection component. The implications of the stock and currency selection "neutralization" on performance can be easily derived by adapting equation 4.1:

If the UK investor fully hedges the foreign exchange exposure from all foreign assets, using monthly forward premia f_i, then $r_{ci}=f_i$ for all currencies (omitting transaction costs and the small residual risk from the hedge) and consequently:

$$R_p^{\pounds H} = \sum_{i=1}^{k} w_i \left[(1 + r_{mi} + f_i)(1 + r_{si}) - 1 \right] \tag{4.2}$$

while by investing in index portfolios, the component $r_{si}=0$ for all countries (omitting indexation track errors)[2] and the portfolio performance can be simplified even further:

$$R_p^{\pounds H} = \sum_{i=1}^{k} w_i \left[(1 + r_{mi} + f_i) - 1 \right] \tag{4.3}$$

Given that the forward premia f_i are relatively stable and predictable, the selection of optimal allocation weights in our hedged "stock selection neutral" strategy will depend only on the local index returns and the forward premia, the latter being relatively stable and predictable.

Similarly, the performance of our unhedged[3] "stock selection neutral" allocation

[2] A very effective, cost effective and practical means of indexing medium size international portfolios is by replicating stock positions through stock index futures contracts, currently available for no less than 12 countries.

[3] Recall that since we are using real returns, deflated by the UK RPI, the performance would also be affected by changes in the inflation rate. In Chapter II, though, we measured the contribution of inflation related volatility to overall volatility as being totally negligible.

strategies can be decomposed as:

$$R_p^{\varepsilon H} = \sum_{i=1}^{k} w_i \, [\, (1 + r_{mi} + r_{ci}) - 1 \,] \tag{4.4}$$

Naturally the component r_{ci} is much more volatile than f_i so that covariance effects between exchange rate changes and local index returns become significant components of portfolio volatility. Consequently, substantial shifts in the location and composition of the unhedged and hedged efficient frontiers over time, should be attributed to the time variability of the aforementioned international portfolio performance components.

4.2. Performance Measurement of "Ex-Ante" Allocation Strategies: Statistical and Portfolio Criteria

The various asset allocation models that have been developed and tested in the remaining of this chapter have been applied, whenever appropriate[4], to four different types of portfolios

- Unhedged Stock Index Portfolios
- Hedged Stock Index Portfolios
- Unhedged Combined Portfolios (stock/bond index asset mix)
- Hedged Combined Portfolios (stock/bond index asset mix)

In order to assure that all the different models have been estimated and their out of sample performance measured on a consistent basis, the following steps have been followed that are common for all models and portfolio types:

i) As a first step appropriate values for the forecasted portfolio inputs have been estimated on the basis of each model. For that purpose, 72 monthly observations from February 1982 - January 1988 were used as a "forecasting period" in order to estimate the inputs.

[4] As will be explained in detail in subsequent sections, some of the models are not applicable for the combined portfolios while the Bayesian Pseudo-Single Index Model has been tested for the unhedged stock index returns only.

ii) Then, quadratic optimizations have been performed[5] for each one of the estimated portfolios in order to derive the "ex-ante" (forecasted) efficient frontiers and to calculate the corresponding vectors of "ex-ante" investment weights. In fact, the optimized portfolio weights have been calculated for three different points on the efficient frontier corresponding to hypothesized[6] real monthly risk-free rates of 0%, 0.2% and 0.4%. In fact, the 0% risk free rate was chosen because it has the property to maximize the Sharpe return to variability ratio, whereas the 0.2 - 0.4% range captures most of the actually observed real Treasury Bill returns during the sampling period. Notice that the three aforementioned "ex-ante" efficient portfolios correspond to progressively reduced levels of investor risk aversion.

iii) The next step consisted of "investing" all three "ex-ante" efficient portfolios for each unhedged/hedged model for portfolio holding periods of 12, 24 and 36 months[7], and then computing the "realized" mean monthly real return and monthly[8] volatility for all risk free rates and holding periods. In fact, the aforementioned 36 monthly observations immediately follow the estimation period covering the time frame from February 1988 to January 1991.

iv) To compare the out of sample performance across different models it is important to select a risk-adjusted performance measure, that is appropriate for international portfolios: given the well documented inadequacy of the international CAPM to provide a sound analytical basis, performance measures like Traynor's and Jensen's are clearly unsuitable for our purposes while the Kornel (1979) performance measure, despite its intuitive appeal, is not applicable in this context[9].

[5] Using the Microsoft Excel Solver

[6] Hypothesized, rather than observed real Sterling risk free rates have been used, because of the fact that the real Treasury Bill rate fluctuated substantially over the estimation period so that it could not be satisfactorily used as a proxy of the real risk free rate.

[7] To reflect on investors with different investment horizons

[8] Not annualized

[9] The standard Mean-Variance framework applied in this thesis is essentially "static", whereas the Kornel measure can be applied only to dynamic allocation strategies where the portfolio inputs are re-estimated with each new observation.

Consequently, I decided to use the Jobson & Korkie (1981) unbiased estimator of a fund's performance that is based on the Sharpe Performance Measure S_p and corrects Sharpe's problems with sample size bias. This measure can be expressed as follows:

$$S_p^* = S_p \frac{N}{N+.75}$$
$$S_p = \frac{R_p - R_f}{\sigma_p} \tag{4.5}$$

where S_p^* is the Jobson-Korkie unbiased portfolio performance estimate

σ_p is the portfolio standard deviation

R_p the portfolio return

R_f the assumed risk-free rate[10]

This Jobson & Korkie version of the Sharpe performance measure was calculated for each model, risk free rate and portfolio holding period in direct comparison with the corresponding performance measures for the historical portfolio that serve effectively as means of assessing the out of sample performance of each model against that of the benchmark. Then two additional criteria have been developed on the basis of the Jobson & Korkie Sharpe measure, namely the "Average Sharpe" and the "Average Rank" criteria that allow the construction of an overall ranking system across all models and portfolio types (see section 4.7)

v) Furthermore, since several of the index allocation models used have been primarily selected because of their ability to provide direct forecasts of the index returns correlation matrix, it is worthwhile to apply statistical tests of significance to assess whether a particular correlation forecasting method can improve on the standard historical correlation estimates. In fact, the parallel use of statistical and portfolio criteria is justified on the grounds that if portfolio performance turns out to be

[10] One of the problems of the Sharpe (as well as the Traynor and Jensen) performance measure is that the ranking can be very sensitive to the choice of a risk free rate. This fact provides further justification to our choice of three as opposed to one assumed risk-free rate.

relatively insensitive to the correlation inputs, then it is perfectly possible to have significant conflicts between the two criteria.

For the aforementioned reasons, the Mean Square Forecast Error (MSFE) was computed and decomposed for each model, on the basis of residual error correlations derived from a six year forecasting and a three year validation period, as well as the Theil Inequality Coefficient (TIC) that permits direct comparisons of each model with the historical benchmark (see Section 4.3.1.3).

As already discussed in the introduction to this thesis[11], the empirical methodology applied to evaluate "ex-ante" index allocation strategies provides a multidimensional extension of previous work by Eun & Resnick (1987, 1988), and to a lesser extent[12] by Eun & Resnick (1984, 1992), Jorion (1985, 1986) and Dumas & Jacquillat (1990). These extensions and improvements relate to a much wider model coverage including several new models, a broader asset mix, use of different risk free rates and implicitly measures of risk aversion, an "ex-ante" rather than an "ex-post" hedging strategy, a combined use of both statistical and portfolio tests of significance, as well as use of an improved performance ranking methodology.

The remaining sections of Chapter IV are structured as follows: Section 4.3 discusses index allocation models and correlation forecasting based on unobservable factor models, Section 4.4 develops and examines the performance of the Pseudo-Single Index Model, Section 4.5 concentrates on traditional Bayesian and empirical Bayes-Stein methodologies for controlling estimation risk in index inputs, in Section 4.6 the focus is on "intuitive" approaches for reducing estimation risk, whereas Section 4.7 presents an overall performance ranking methodology and summarizes the findings:

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[11] This issue is addressed in detail in pages 5-6 of this thesis

[12] These last studies are somewhat conceptually different in that they either:
i) Refer to individual stocks rather than index allocation models or
ii) Utilize simulation techniques rather than actual data to test the performance of the models

4.3 Unobservable Common Factors, Index Correlation Structure and "Ex-Ante" Allocation Strategies

Unobservable factor type models can be of great usefulness for "ex-ante" portfolio analysis. By estimating endogenously the appropriate number of common factors that affect international index returns as well as the sensitivity parameters (betas) associated with each factor, we can obtain some very useful results for asset allocation purposes:

- By extracting unobservable factors that are orthogonal, either by means of Principal Components or Maximum Likelihood Factor Analysis (MLFA), it is possible to "eliminate" noise from the historical index correlation matrix and, therefore, forecast the future correlation matrix on the basis of the common factors. The main idea lies in the fact that asset returns tend to have positive correlation only because they are linked through some common factors. In fact, reproducing the correlation matrix through the significant common factors is reflecting the "fundamental" part of the observed correlations, whereas the remaining portion of correlations, not attributable to common factors, can be deemed as "noise".

Naturally, the success of such an approach in practice will largely depend on the ability to extract the right number of factors; If more factors are extracted than necessary then the reproduced correlation matrix will more closely resemble the historical one but will have reduced forecasting power because it will incorporate unwanted "noise". On the other hand, if fewer, than the "true" number, factors are extracted then real information is eliminated from the correlation matrix and again the forecasting ability of the model will be low[13]. Notice that in order to arrive at such forecasts it is not necessary to properly "identify" the common factors, even though such identification can possibly make more meaningful the interpretation of the results.

[13] We have to emphasize that even though, as a technical result, multifactor models will always more accurately reproduce the correlation matrix the more factors are extracted, the forecasting ability of even a single index model might be greater if enough "noise" is incorporated in the multifactor structure

- In parallel, the unobservable common factor models can provide us with valuable information about the proportion of volatility that can be attributed to common factors, as opposed to the asset's "unique" volatility. Since our analysis refers to market indices, rather than individual assets, the unique variability should be interpreted as the "market specific" component of variability (as opposed to variability attributable to global factors). Effectively, index returns characterized by high unique variability must refer to a market with a high degree of segmentation. Overall, if the first principal component turns out to have very high explanatory power, then there is ground for potentially justifying the assumption that global index returns can best be described in terms of a global "Pseudo Single Index"[14] Model.

Notice, however, that multifactor (as well as single factor) models when estimated from historical data provide mean return and variance estimates that are identical to those of the full-covariance historical model. Investment analysts, therefore, will still be required to provide estimates of these inputs. In this context, the principal usefulness of an appropriate multifactor structure provides us with a means of determining endogenously the expected future correlation structure without the need for additional fundamental information.

The primary reason for which two alternative factor methodologies have been selected, i.e the Principal Components Model (PCM) and the Maximum Likelihood Factor Model (MLFM), is that each one of them has its comparative virtues and limitations:

The PCM, being a mathematical rather than statistical technique, is not dependent on distributional assumptions like multivariate normality and can be best used to explain the variance of the original data since the Components represent linear composites of the original variables that exhibit maximal variance. Furthermore, exact values can be obtained for the orthogonal component scores. On the other hand, the major disadvantage of PCM is that no statistical goodness of fit tests are available, to help us estimate the appropriate number of factors to be included (see Elton & Gruber 1991, Dillon & Goldstein 1984).

---

[14] We use the term "Pseudo Single Index" to denote a relationship between a national stock index and a world stock index

Consider also that when principal components are being extracted from the correlation matrix the variance of all variables is standardized and, therefore, the results could be biased in case there are substantial differences in volatility among the different assets included.

As far as the MLFM is concerned, the main drawbacks lie in the facts that i) it is dependent on the assumption of approximate multivariate normality ii) factor scores can only be calculated by approximation and iii) the factor values, unlike the PCM, are not unique but depend on the number of factors extracted. On the positive side, a Likelihood Ratio test of significance is available to help us establish the appropriate number of factors to be included in the model specification.

4.3.1. A Principal Components Model for Index Correlation Forecasting and Global Asset Allocation

4.3.1.1 Model Suitability and Diagnostic Checking

A necessary first step, occasionally neglected, is to consider the suitability of the returns correlation matrix for factor purposes. A standard diagnostic test, known as the Bartlett's sphericity test can be used in order to reject the hypothesis that the variables included are essentially uncorrelated:

The Bartlett test statistic is calculated as follows:

$$\boldsymbol{BART} = [\,\boldsymbol{n} - 1 - \frac{1}{6}(2\boldsymbol{k} + 5)\,] \;\ln|\boldsymbol{R}| \tag{4.6}$$

where **n, k** represent the number of observations and variables respectively

$|\boldsymbol{R}|$ is the determinant of the correlation matrix product of the k eigenvalues

Subsequently, the computed Bartlett statistic can be compared to a χ^2 with .5 (k^2-k) degrees of freedom in order to establish the exact significant level of accepting the null of an "identity matrix".

In addition, the Keiser-Meyer "measure of sampling adequacy"[15] has been computed (see Norussis 1989) which provides an additional indicator for the suitability of the correlation matrix for factor extraction. All aforementioned (as well as subsequent) statistics have been computed by means of SPSS-PC Factor procedures on the basis of the "forecasting period's" 72 monthly observations and the results are listed below in Table 4.1.

Notice that the "combined" portfolios consist of all fifteen stock indices plus six bond indices (Canadian, German, Japanese, UK and US government bonds and US dollar floating rate notes). Omitting the remaining four bond indices, as previously applied in Chapters II & III, was necessary because they were very highly correlated with other variables and caused singularity of the factor matrix.

Table 4.1

Bartlett Sphericity Tests

PC Model	Bartlett	Signif	Keiser-Meyer
Stock Unhg	769.131	.000	.90549
Comb Unhg	1134.85	.000	.86469
Stock Hedg	889.90	.000	.85732
Comb Hedg	889.90	.000	.91870

Clearly, these diagnostics are highly favourable for the use of the PCM, since the Bartlett statistic is consistently significant even at the 0.1% confidence level, whereas the Keiser-Meyer statistic is always higher that .80, level which according to Keiser is highly satisfactory in terms of justifying a common factor extraction.

[15] The Kaiser-Meyer-Olkin measure of sampling adequacy is a measure based on comparison of the observed to the partial correlation coefficients. Partial correlation coefficients are relate to the "unique" factors and should be close to zero.

4.3.1.2 Communalities and the Components Loading Matrix

The basic aim here is to find linear composites of the original index returns that display scores with maximal variance, subject to being orthogonal to previously computed scores. The Component Loadings Matrix B simply represents correlations of the original variables with each one from the principal components. Effectively it is the matrix of factor betas in a return multifactor model. We can calculate B as

$$B = UD^{.5} \tag{4.7}$$

where: **D** is a Diagonal matrix of the eigenvalues λ_i of R

U is an orthogonal matrix whose columns are the eigenvectors[16] of R,[17]

Here the component loadings or "betas" have an additional interpretation i.e. they represent regression coefficients in a multiple regression where the original variable is the dependent one, while the extracted s<k factors F_i the independent. Such a return multifactor model could be expressed as follows

$$R_i = \alpha + b_{i1} F_1 + b_{i2} F_2 + \ldots\ldots b_{ir} F_s + u_i \tag{4.8}$$

where F_i, i=1...s is the n*1 dimension vector of "scores" for the unobserved components

---

[16] Notice that we are retaining only the first s<k eigenvalues and eigenvectors. This implies that $U'U=I$ while $UU' \neq I$. U is no longer square but of order k*s instead, i.e it is an orthonormal section.

[17] In this context the correlation matrix R is being decomposed as

$$R = U D U'$$

Table 4.2

Estimated Communalities from the Principal Component Models

INDEX	ST/UNH(2)*	CM/UNH(4)	ST/HED(2)	CM/HED(4)
Austria	.718	.730	.657	.669
Belgium	.691	.713	.643	.661
Canada	.786	.794	.678	.764
Denmark	.343	.378	.279	.322
France	.686	.706	.642	.623
Germany	.613	.613	.525	.567
Italy	.690	.550	.689	.715
Japan	.503	.565	.482	.689
Netherl	.756	.757	.713	.709
Norway	.675	.769	.693	.733
Spain	.441	.427	.536	.475
Sweden	.468	.508	.432	.505
Switzerl	.748	.754	.750	.751
UK	.689	.791	.697	.719
USA	.787	.865	.716	.790
CanGB	---	.809	---	.710
GerGB	---	.726	---	.665
JapGB	---	.779	---	.689
UKGB	---	.907	---	.501
USGB	---	.841	---	.719
USFRN	---	.828	---	.769

*** The parenthesis quotes the number of significant components for each model on the basis of the Keizer normalization rule**

Table 4.3

Eigenvalues and Percentage Variance Explained

MODEL	EIGEN 1	EIGEN 2	EIGEN 3	EIGEN 4
ST/UNH	8.41 (56.1%)	1.18 (7.9%)	----	----
CM/UNH	9.48 (45.2%)	2.23 (10.6%)	1.82 (8.7%)	1.27 (6.1%)
ST/HED	8.02 (53.5)	1.11 (7.4%)	----	----
CM/HED	8.15 (38.8%)	2.94 (14.0)	1.36 (6.5%)	1.17 (5.6%)

In this context, the sum of squares of each column of component loadings equals that components variance (or eigenvalue), while the sum of squares of each row of component loadings equals the variable's "communality" or proportion of variance attributable to the common factors (the remaining variance being the unique variability of the index and essentially a measure of market "segmentation").

Notice also that the size of the "eigenvalues" is the standard procedure used, known as the "Keizer normalization rule[18]", for selecting the number of significant components in the PCM. On the basis of this criterion only components with eigenvalues greater than one are retained as significant and used in subsequent analyses. Communalities and eigenvalues for the four different data matrices were estimated using SPSS-PC Factor Procedures and the results are presented in tables 4.2 and 4.3; From these results, the following useful conclusions can be derived:

i) For both the hedged and unhedged stock index returns, two unobservable components were found to be significant. In each case, though, the eigenvalue and percentage variance explained by the first component is very high compared to the second one so that a global single index representation could be a reasonable approximation.

[18] There is admittedly, an inherently "ad-hoc" element to the Keizer rule, so that it is primarily useful for the PCM, among factor methodologies, due to the absence of statistical significance tests.

ii) The addition of just six bond indices leads to an increase in the number of significant components from two to four in each case. Also, for both combined portfolios (particularly the hedged one) the explanatory power of the first component is substantially lower. Both these results suggest "separation" of components between the stocks and bonds. Notice, however, that in the combined portfolios the variance of stock returns is higher from that of bonds, reducing the power of the PCM.

iii) Considering the communality results from the unhedged stock model (table 4.2, column 2), Denmark turns out to be by far the most segmented market with only 34% of its volatility being explained by the common factors, followed by Spain and Sweden. The US, Canadian and Dutch markets turn out to be the most integrated with over 75% of their volatility attributable to the common factors.

iv) Upon hedging the foreign exchange risk, Switzerland becomes the most integrated market followed by the US and the Netherlands, while Denmark is again by far the most segmented followed by Sweden and Japan.

v) Considering the communalities from the combined portfolios, we can observe that for the unhedged bond returns the percentage variance explained is generally very high, but becomes significantly lower for all their hedged counterparts.

vi) Despite having two additional components in the combined portfolios, the percentage volatility explained from the stock returns is mostly quite similar to that from the stock only results. This again indicates possible separation of factors.

All component "betas", i.e. the unrotated component loading matrices from the four portfolios can be found in appendix 4A. A simple observation of the factor loadings corresponding to the first component extracted from the combined hedged portfolio, provides the strongest evidence about the separation of factors between bonds and stocks: in fact all fifteen stock indices have very high correlations with the first component ranging between .54 to .85, while the six bond indices have very low correlations, ranging from -.04 to .25. Exactly the opposite applies for the second component, with whom all stock indices have very low or even negative correlations, while the correlations tend to be very high in the case of the bonds.

When considering the unhedged combined matrices, though, there is a single important difference i.e. the correlations of bond returns with the first principal component are much higher that those of their hedged counterparts. This phenomenon implies that the bond correlations with the first unobserved component are largely attributed to exchange rate movements against Sterling[19].

A standard orthogonal "Varimax" rotation procedure has also been performed in all cases and the resulting "rotated" component loadings matrices are included in the appendix. The main idea of this procedure lies in finding an orthogonal matrix J[20], such as that on the basis of the original loadings matrix B, we can compute a new loadings matrix B^* that has as many near-zero loadings in each column as possible.

In some cases, a Varimax rotation might help improve the interpretability of the unobserved components. In this context, though, the unrotated factors appear to have a more clear pattern so that very little is generally achieved by the rotated solutions.

4.3.1.3 Forecasting the Index Correlation Structure via the PCM

In modern portfolio theory (e.g. APT) we are primarily concerned about standardized factors with zero mean and unit variance. Such a "standardized" components score matrix F_S, can be calculated following the following transformation:

$$F_s = X_s\, U D^{-.5} \tag{4.9}$$

where X_S is the matrix of standardized data (zero mean and unit variance) defined as:

$$X_s = (X - l\bar{x}') D^{-1/2} \tag{4.10}$$

[19] To this end, the UK government bond index has in fact, by far, the lowest correlation with the first principal component.

[20] The relationship between B^* and J can be expressed as follows:

$$J = D^{-.5} U' B^*$$

On the basis of the above transformation[21], and for all pairs of standardized variables (i,j) the following results hold

$$\begin{gathered} cov(i,j) = cor(i,j) \quad \forall\, i,j \\ \sigma_i = \sigma_j = \sigma_{F_s} = 1 \end{gathered} \tag{4.11}$$

The next step is to calculate the covariances or correlations from a multifactor model with orthogonal indices, which can be achieved as follows[22]. First we take expected values from the general multifactor model:

$$\begin{aligned} R_i &= a_i + b_{i1} F_1 + b_{i2} F_2 + \ldots.. + b_{is} F_s + u_i \\ \mathscr{E}(R_i) &= a_i + b_{i1} \bar{F}_1 + b_{i2} \bar{F}_2 + \ldots. + b_{is} \bar{F}_s \end{aligned} \tag{4.12}$$

Then by substituting R_i, $\mathscr{E}(R_i)$ into the definition of the covariance we get

$$\begin{aligned} cov_{ij} = \mathscr{E}[\, & [\, a_i + b_{i1} F_1 + b_{i2} F_2 + \ldots.. + b_{is} F_s + u_i - \\ & - (\alpha_i + \beta_{i1} \bar{F}_1 + b_{i2} \bar{F}_2 + \ldots. + b_{is} \bar{F}_s)\,] * \\ & * [a_j + b_{j1} F_1 + b_{j2} F_2 + \ldots + b_{js} F_s + u_j - \\ & - (\alpha_j + \beta_{j1} \bar{F}_1 + 1 b_{j2} \bar{F}_2 + \ldots. + b_{js} \bar{F}_s)] \end{aligned} \tag{4.13}$$

---

21 The unstandardized components score matrix F (n*s) can be calculated directly as

$$Z = X_s U$$

where $\bar{x}^{\prime}$ is the row mean return (centroid) vector

1 the unit vector and X is the original data matrix

22 This proof is included in Elton & Gruber (1992).

Subsequently, by means of combining all terms with the same betas and then omitting all product terms involving different indices (they equal to zero due to orthogonality) and all terms involving the different residuals u_i u_j (they equal to zero because of standard OLS assumptions), we can express the covariance as

$$cov_{ij} = \sum_{f=1}^{s} b_{if}\, b_{jf}\, \mathscr{E}(F_f - \bar{F}_f) = \sum_{f=1}^{s} b_{if}\, b_{jf}\, \sigma^2_{F_f} \tag{4.14}$$

Finally by combining the previous equations we get that

$$cov_{ij} = cor_{ij} = \sum_{f=1}^{s} b_{if}\, b_{jf} \tag{4.15}$$

where s<k is the number of significant components retained

4.3.1.4 Statistical Performance of the PCM and the Historical Model

Such forecasts of correlation coefficients can be intuitively appealing since they retain only this portion of correlations between indices that can be attributed to common factors. Equation 4.15 has been estimated for each one of the n*(n-1)/2 independent correlation coefficients in each matrix. The resulting estimated correlations for the unhedged and hedged stock index returns can be found in the lower diagonal part of the matrices shown in table 4.4, while the estimates for the unhedged and hedged combined correlation matrix can be found in tables 4.5 and 4.6. The upper diagonal part of these matrices shows the "forecasting errors" i.e. the arithmetic difference between the forecasted and the actual correlations:

$$cor^*_{ij} = cor^f_{ij} - cor^a_{ij} \quad \forall\,(i,j) \tag{4.16}$$

where **$cor^*_{(i,j)}$** denotes the "residual" difference between forecasted and actual correlations

$cor^f_{(i,j)}$ stands for the forecasted correlations on the basis of the first 72 monthly observations (forecasting period)

$cor^a_{(i,j)}$ shows the actually observed correlations during the validation period, computed from the last 36 monthly observations

In order to measure the forecasting ability of the PCM it is necessary to compare the forecasting residuals against those resulting from the historical correlation matrix, which in this case acts as benchmark[23]. The lower diagonal part of the matrices in Tables 4.7 - 4.9 shows the historical correlations, whereas the upper part shows the changes in actually observed correlations between the forecasting and the validation period[24]. For each one of the "residual" matrices, the Mean Square Forecast Error (MSFE) and the Root MSFE have been computed as follows:

$$\begin{aligned} MSFE &= \frac{1}{T}\sum_{i=1}^{T} (cor^{f}_{i,j} - cor^{a}_{i,j})^2 \\ T &= \frac{k*(k-1)}{2} \\ RMSFE &= \sqrt{MSFE} \end{aligned} \tag{4.17}$$

where k = 15 and T = 105 for the stock only matrices

k = 21 and T = 210 for the combined matrices

Also, following Theil (1971) and Eun & Resnick (1984), it is useful to decompose the MSFE into a "bias forecast error", a "variance error" and a "covariance error" as:

$$MSFE = (\bar{F}-\bar{A})^2 + (\sigma_f - \sigma_a)^2 + 2(1-\rho)\sigma_f\sigma_a \tag{4.18}$$

where $\bar{F}, \bar{A}$ are the sample means of the forecast and actual correlations respectively

σ_f, σ_a denote the standard deviation of the forecast and actual correlations respectively

ρ is the estimated correlation coefficient between the forecast and actual correlation series

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[23]In fact "historical correlations" computed over various time intervals are extensively used as inputs by most market practitioners.

[24] Naturally, if correlations were inter-temporally stable then the upper diagonal part would consist of near zero values.

Table 4.4

Forecasted Correlation Matrix of Hedged Stock Returns (Principal Component Analysis)

	SAUT	SBEL	SCAN	SDEN	SFRA	SGER	SITA	SJAP	SNET	SNOR	SSPA	SSWE	SSWI	SUKG	SUSA
SAUT	1	0.06089	0.22755	0.00536	−0.02004	0.06379	−0.2384	−0.18178	0.00215	0.17153	−0.02793	−0.08894	0.11847	0.22549	0.18259
SBEL	0.64519	1	0.0797	−0.04893	−0.18056	−0.14457	−0.15991	−0.07478	0.03704	0.17344	0.04555	0.06708	0.14567	0.2479	0.10319
SCAN	0.64765	0.6161	1	−0.01086	0.08542	0.11888	−0.20995	−0.07289	−0.05161	0.20863	0.02691	0.07504	0.08766	−0.02007	−0.09884
SDEN	0.42626	0.41317	0.43104	1	−0.18749	−0.26609	−0.2127	−0.17596	−0.07829	0.0017	−0.07428	−0.22511	−0.17336	−0.03385	−0.00729
SFRA	0.59556	0.61654	0.52272	0.36891	1	−0.20607	0.01213	0.01711	−0.04164	0.12129	0.02981	−0.08081	−0.03845	0.05322	−0.11058
SGER	0.58599	0.58073	0.56678	0.37721	0.54843	1	−0.26081	−0.07233	−0.0904	0.04781	−0.06825	−0.15761	−0.0729	−0.00901	0.04304
SITA	0.4436	0.49739	0.31205	0.2538	0.60253	0.42929	1	−0.01662	−0.34392	−0.27577	0.06837	−0.31784	−0.34368	−0.1693	−0.1646
SJAP	0.42642	0.46364	0.32931	0.25194	0.53221	0.40477	0.57158	1	−0.05955	0.00797	−0.1659	−0.3553	−0.17073	−0.08821	−0.06134
SNET	0.68215	0.66274	0.68669	0.44641	0.59536	0.6045	0.41588	0.41025	1	0.00337	−0.02564	−0.0317	0.0351	−0.05267	−0.00746
SNOR	0.62463	0.58204	0.67903	0.4224	0.46469	0.54001	0.22413	0.26177	0.67107	1	0.03445	0.08708	0.27295	0.15316	0.34387
SSPA	0.46437	0.50115	0.36621	0.27642	0.56741	0.43875	0.59877	0.5087	0.44946	0.29705	1	−0.34338	−0.10287	−0.07674	−0.05338
SSWE	0.53265	0.52148	0.52814	0.34639	0.47779	0.47419	0.34986	0.3386	0.5541	0.51128	0.36932	1	−0.15675	−0.09169	−0.00626
SSWI	0.69787	0.67557	0.70746	0.45804	0.60115	0.6171	0.41002	0.40847	0.7315	0.69435	0.44853	0.56725	1	0.01454	0.03859
SUKG	0.66839	0.6431	0.68553	0.44085	0.56302	0.58889	0.3679	0.37319	0.70343	0.67756	0.41146	0.54391	0.72294	1	−0.05114
SUSA	0.67039	0.64059	0.69656	0.44461	0.55032	0.58824	0.341	0.35396	0.70874	0.69377	0.39222	0.54624	0.72949	0.70576	1

Forecasted Correlation Matrix of Unhedged Stock Returns (Principal Component Analysis)

	SAUT	SBEL	SCAN	SDEN	SFRA	SGER	SITA	SJAP	SNET	SNOR	SSPA	SSWE	SSWI	SUKG	SUSA
SAUT	1	0.14155	0.12708	0.10441	0.05152	0.11574	−0.08549	−0.07568	0.12194	0.20369	0.05349	−0.04186	0.17214	0.15969	0.14139
SBEL	0.70415	1	0.08949	−0.01945	−0.1519	−0.12776	−0.0736	0.12011	0.02355	0.21325	0.07859	0.11126	0.20103	0.22804	0.15155
SCAN	0.64593	0.61549	1	0.03944	−0.00745	0.0751	−0.2188	0.08425	−0.03412	0.2223	0.03294	0.14348	0.14039	0.185	−0.10238
SDEN	0.49621	0.48535	0.45994	1	−0.15983	−0.25096	−0.12123	0.03301	−0.07949	0.0581	0.00537	−0.11821	−0.11562	0.04488	0.05163
SFRA	0.66642	0.6637	0.48155	0.45267	1	−0.13546	0.03556	0.16	−0.07011	0.1141	−0.02089	−0.06657	0.05347	0.00443	−0.0946
SGER	0.64024	0.63584	0.4831	0.43624	0.64734	1	−0.09973	0.20931	−0.14674	0.02323	−0.00461	−0.11442	−0.02223	−0.07942	0.05563
SITA	0.57871	0.5861	0.306	0.38567	0.66006	0.61277	1	0.13458	−0.2592	−0.12151	−0.02133	−0.22962	−0.20353	−0.20977	−0.12846
SJAP	0.52162	0.52541	0.30885	0.34981	0.576	0.53741	0.58768	1	0.14507	0.16973	−0.15123	−0.21356	0.0102	−0.154	0.13413
SNET	0.72264	0.70185	0.72708	0.50441	0.62639	0.60946	0.5019	0.46477	1	0.09962	0.02954	0.05043	0.09672	0.02714	0.01987
SNOR	0.62189	0.59515	0.7273	0.4408	0.4808	0.47863	0.32589	0.32073	0.68872	1	0.1509	0.12029	0.28094	0.12654	0.32409
SSPA	0.53479	0.53259	0.38664	0.36327	0.55041	0.51939	0.52947	0.46207	0.50274	0.386	1	−0.33631	−0.04405	−0.12085	−0.0187
SSWE	0.57534	0.56076	0.55618	0.40009	0.51173	0.49548	0.42328	0.38774	0.59343	0.52979	0.41069	1	−0.11234	−0.122	0.08678
SSWI	0.73164	0.71533	0.68159	0.50708	0.66547	0.64167	0.56507	0.5131	0.74502	0.65274	0.53405	0.59056	1	0.01694	0.11248
SUKG	0.60489	0.57634	0.7366	0.43063	0.45093	0.45238	0.28653	0.2892	0.68084	0.68104	0.36205	0.5208	0.63824	1	0.1575
SUSA	0.68099	0.65285	0.78332	0.48183	0.534	0.53003	0.37054	0.36143	0.74927	0.72919	0.4287	0.57768	0.71328	0.7335	1

Lower diagonal part of the matrix includes principal component analysis correlations based on 72 observations (Feb 82–Jan 88).
Upper diagonal part includes residuals between forecasted and actual correlations based on 36 observations (Feb 88–Jan 91).

Table 4.5

Forecasted Correlation Matrix of Combined Hedged Returns (Principal Component Analysis)

	SAUT	SBEL	SCAN	SDEN	SFRA	SGER	SITA	SJAP	SNET	SNOR	SSPA	SSWE	SSWI	SUKG	SUSA	CAN	GER	JAP	UKG	USA	USN
SAUT	1	0.0737	0.2024	−0.005	−0.029	0.0723	−0.231	−0.2	0.0013	0.1959	−0.01	−0.083	0.1229	0.2096	0.1673	−0.175	−0.039	−0.482	−0.213	−0.016	−0.062
SBEL	0.658	1	0.044	−0.063	−0.205	−0.118	−0.141	−0.122	0.0391	0.197	0.0415	0.1035	0.1631	0.2276	0.0824	−0.287	−0.417	−0.442	−0.114	−0.171	0.131
SCAN	0.6225	0.5804	1	0.0386	0.1256	0.0578	−0.267	−0.001	−0.063	0.1631	0.0341	−0.017	0.0546	0.0201	−0.03	−0.483	−0.179	−0.419	−0.379	−0.381	−0.328
SDEN	0.4159	0.3995	0.4805	1	−0.198	−0.282	−0.256	−0.206	−0.087	−0.011	−0.114	−0.243	−0.17	−0.017	0.0499	−0.313	−0.172	−0.214	−0.172	−0.232	−0.095
SFRA	0.5367	0.5923	0.5629	0.3588	1	−0.216	−0.067	0.0216	−0.012	0.1401	−0.024	−0.087	−0.032	0.0986	−0.101	−0.244	−0.083	−0.286	−0.003	−0.11	−0.046
SGER	0.5945	0.6076	0.5057	0.3609	0.5387	1	−0.178	−0.107	−0.1	0.0341	−0.053	−0.103	−0.058	−0.049	0.0057	−0.25	−0.408	−0.362	−0.27	−0.106	0.301
SITA	0.4505	0.5159	0.2548	0.2106	0.5239	0.512	1	−0.137	−0.313	−0.252	−0.033	−0.181	−0.285	−0.179	−0.215	−0.116	0.0193	−0.107	0.0345	−0.006	0.2395
SJAP	0.4087	0.4167	0.4011	0.2216	0.5367	0.3702	0.451	1	−0.005	0.0267	−0.214	−0.399	−0.181	−0.002	−0.06	−0.376	0.149	−0.403	−0.135	−0.14	−0.352
SNET	0.6313	0.6648	0.6755	0.4378	0.6251	0.5953	0.4464	0.4646	1	−0.015	0.0235	−0.057	0.0181	−0.058	−0.023	−0.333	−0.237	−0.445	−0.288	−0.187	−0.091
SNOR	0.649	0.6056	0.6335	0.4102	0.4835	0.5263	0.2478	0.2805	0.6522	1	0.1289	0.0461	0.2544	0.1125	0.318	−0.147	−0.153	−0.365	−0.245	0.0215	−0.233
SSPA	0.4326	0.4971	0.3734	0.2372	0.5133	0.4538	0.497	0.4608	0.4987	0.3915	1	−0.314	−0.073	−0.04	−0.072	−0.431	−0.116	−0.473	−0.265	−0.147	0.0982
SSWE	0.5383	0.5579	0.4366	0.3287	0.4715	0.5292	0.437	0.2954	0.5285	0.4703	0.3983	1	−0.144	−0.159	−0.059	−0.237	−0.182	−0.48	−0.423	0.0546	0.139
SSWI	0.7023	0.693	0.6744	0.4612	0.6073	0.632	0.4634	0.3978	0.7145	0.6758	0.4787	0.5804	1	−0.016	0.0273	−0.367	−0.259	−0.57	−0.438	−0.249	−0.076
SUKG	0.6525	0.6228	0.7257	0.4576	0.6084	0.5491	0.3533	0.4593	0.6976	0.6369	0.4486	0.4769	0.692	1	−0.029	−0.493	−0.146	−0.481	−0.539	−0.215	−0.147
SUSA	0.6551	0.6198	0.7655	0.5018	0.5599	0.5509	0.2904	0.3549	0.6929	0.6679	0.3735	0.4933	0.7182	0.7283	1	−0.423	−0.253	−0.61	−0.365	−0.373	−0.175
CAN	−0.012	0.0225	0.1387	0.1597	0.0192	0.0692	0.037	−0.085	0.0038	−0.126	−0.128	0.1108	0.0828	0.0602	0.1855	1	−0.083	0.014	−0.201	−0.044	0.1762
GER	0.0019	0.0396	0.1477	0.0884	0.2301	0.0603	0.254	0.3162	0.0731	−0.208	0.1038	0.0399	0.0341	0.1336	0.1012	0.4155	1	0.1357	0.1016	0.1063	0.1297
JAP	−0.154	−0.107	−0.036	−0.034	0.1055	−0.071	0.1938	0.2484	−0.09	−0.372	0.0246	−0.08	−0.139	−0.038	−0.091	0.3613	0.6534	1	−0.024	0.1443	−0.172
UKG	−0.014	0.0084	0.1613	0.0987	0.1611	0.0251	0.1364	0.2172	0.0512	−0.168	0.0302	0.011	0.0248	0.1233	0.1236	0.4171	0.5655	0.5435	1	0.0755	−0.058
USA	0.0573	0.0966	0.2005	0.1954	0.1162	0.1374	0.1824	0.0185	0.0821	−0.088	−0.043	0.1672	0.1507	0.1351	0.2404	0.7056	0.4834	0.4192	0.4634	1	0.3532
USN	0.187	0.2665	−0.014	0.112	0.126	0.3182	0.4602	−0.054	0.1233	0.0494	0.1251	0.3761	0.2521	0.0306	0.1008	0.3948	0.0621	0.0273	0.0282	0.4029	1

Lower diagonal part of the matrix includes principal component analysis (PCA) correlations based on 72 observations (Feb 82–Jan 88).
Upper diagonal part includes residuals between forecated and actual correlations based on 36 observations (Feb 88–Jan 91).

Table 4.6

Forecasted Correlation Matrix of Combined Unhedged Returns (Principal Component Analysis)

	SAUT	SBEL	SCAN	SDEN	SFRA	SGER	SITA	SJAP	SNET	SNOR	SSPA	SSWE	SSWI	SUKG	SUSA	CAN	GER	JAP	UKG	USA	USN
SAUT	1	0.1241	0.136	0.1105	0.01991	0.1017	-0.097	-0.07	0.1202	0.1984	0.0406	-0.022	0.1648	0.1176	0.1726	0.0381	0.36203	0.03934	-0.28434	0.16109	0.16967
SBEL	0.6367	1	0.0674	-0.037	-0.12983	-0.115	-0.105	0.1079	0.0262	0.2612	0.0883	0.1007	0.213	0.261	0.114	-0.181	-0.18612	0.10955	-0.13213	-0.07846	0.05058
SCAN	0.6549	0.5934	1	0.0393	-0.00091	0.0823	-0.112	0.0809	-0.047	0.1102	0.0555	0.1422	0.1215	0.1563	-0.076	-0.351	-0.20859	-0.13435	-0.14262	-0.25729	-0.21312
SDEN	0.5023	0.468	0.4598	1	-0.15489	-0.244	-0.076	0.0735	-0.073	-0.031	-0.019	-0.135	-0.128	0.0054	0.0513	-0.093	-0.09077	0.14209	-0.10047	-0.04408	0.06501
SFRA	0.6348	0.6857	0.4881	0.4576	1	-0.131	-0.04	0.1421	-0.043	0.1685	-0.053	-0.101	0.0689	0.0874	-0.136	-0.265	-0.00077	0.1107	-0.07779	-0.19687	-0.15809
SGER	0.6262	0.6488	0.4903	0.443	0.65187	1	-0.159	0.2031	-0.127	0.0685	-0.029	-0.127	-0.01	-0.038	0.0389	-0.102	-0.13225	0.31411	-0.27458	-0.03809	0.09809
SITA	0.5667	0.5546	0.4124	0.4309	0.58419	0.5533	1	0.0896	-0.208	-0.085	-0.132	-0.24	-0.207	-0.136	-0.057	-0.04	0.22278	0.37724	-7.0E-05	0.02442	0.03612
SJAP	0.5274	0.5132	0.3055	0.3953	0.55806	0.5312	0.5427	1	0.1618	0.1559	-0.208	-0.222	0.0064	-0.202	0.1269	0.1737	0.56903	-0.08158	-0.47133	0.29454	0.25391
SNET	0.7209	0.7045	0.7143	0.5106	0.65325	0.6293	0.5526	0.4815	1	0.0474	0.04	0.0369	0.0949	0.0215	0.004	-0.201	-0.20328	0.0596	-0.18756	-0.11479	-0.03953
SNOR	0.6166	0.6431	0.6152	0.352	0.53515	0.5239	0.362	0.3069	0.6365	1	0.269	0.1495	0.2991	0.0914	0.248	-0.115	0.09418	0.13036	-0.34678	0.01805	0.01451
SSPA	0.5219	0.5423	0.4092	0.3389	0.51879	0.4955	0.4189	0.4054	0.5132	0.5041	1	-0.321	-0.03	-0.068	-0.004	-0.168	0.06348	-0.07965	-0.44787	-0.0552	0.06832
SSWE	0.5948	0.5502	0.5549	0.3838	0.47744	0.4826	0.4132	0.3796	0.5799	0.559	0.4265	1	-0.115	-0.173	0.1247	0.0355	0.09613	-0.01001	-0.59164	0.19722	0.25955
SSWI	0.7243	0.7273	0.6627	0.4943	0.68087	0.6534	0.5614	0.5093	0.7432	0.6709	0.5477	0.588	1	0.0241	0.0915	-0.118	-0.02829	-0.08715	-0.42188	-0.07483	0.02974
SUKG	0.5628	0.6093	0.7079	0.3912	0.53389	0.4941	0.3604	0.2409	0.6752	0.6459	0.4151	0.4696	0.6454	1	0.0659	-0.196	-0.09434	-0.42792	-0.47679	-0.04701	-0.01387
SUSA	0.7122	0.6153	0.81	0.4815	0.49286	0.5133	0.4421	0.3542	0.7334	0.6531	0.4437	0.6156	0.6923	0.6619	1	-0.267	-0.19238	-0.13594	-0.29021	-0.21709	-0.11851
CAN	0.4451	0.2415	0.51	0.3654	0.18242	0.2372	0.3298	0.2896	0.4087	0.1323	0.1514	0.3531	0.3256	0.1956	0.5903	1	-0.08735	0.18857	-0.04975	-0.08903	-0.0605
GER	0.3394	0.3027	0.0749	0.3225	0.41943	0.3907	0.4987	0.5499	0.2779	-0.038	0.2344	0.1779	0.2359	-0.028	0.1106	0.3242	1	0.49708	-0.00098	-0.1023	0.01718
JAP	0.3525	0.2855	0.0417	0.3061	0.3848	0.3761	0.4885	0.5654	0.248	-0.023	0.2443	0.2074	0.2726	-0.115	0.1182	0.3589	0.72978	1	-0.59316	0.23608	0.23103
UKG	-0.135	-0.088	0.0795	0.0576	0.00331	-0.057	0.015	-0.084	0.0228	-0.301	-0.178	-0.211	-0.086	0.186	-0.076	0.0762	0.10442	-0.08276	1	0.03052	-0.16698
USA	0.4302	0.2735	0.5278	0.3851	0.21193	0.2679	0.3574	0.3212	0.436	0.1636	0.1804	0.3835	0.358	0.2056	0.6166	0.8241	0.3456	0.38808	0.03742	1	-0.07599
USN	0.502	0.2942	0.4868	0.3301	0.18831	0.2652	0.3067	0.3035	0.4069	0.3032	0.2397	0.4498	0.3752	0.1537	0.6273	0.7255	0.22678	0.33723	-0.32638	0.75641	1

Lower diagonal part of the matrix includes principal component analysis correlations based on 72 observations (Feb 82–Jan 88).
Upper diagonal part includes residuals between forecasted and actual correlations based on 36 observations (Feb 88–Jan 91).

Table 4.7

Forecasted Correlation Matrix of Hedged Stock Returns (Historical)

	SAUT	SBEL	SCAN	SDEN	SFRA	SGER	SITA	SJAP	SNET	SNOR	SSPA	SSWE	SSWI	SUKG	SUSA
SAUT	1	0.0213	0.1391	−0.0301	−0.0945	0.1488	−0.316	−0.1298	−0.0116	0.0976	−0.0485	−0.0934	0.1552	0.1359	0.1544
SBEL	0.6056	1	0.0025	−0.0349	−0.1838	−0.1559	−0.1861	−0.0871	−0.0125	0.2174	−0.0292	0.0574	0.0849	0.2428	0.0396
SCAN	0.5592	0.5389	1	−0.0265	0.1414	−0.0602	−0.1643	−0.0417	−0.0998	0.1662	0.0588	0.0103	0.0206	−0.0202	0.0063
SDEN	0.3908	0.4272	0.4154	1	−0.1995	−0.3557	−0.1878	−0.1476	−0.1416	−0.0279	−0.1196	−0.3219	−0.2321	−0.1327	−0.0146
SFRA	0.5211	0.6133	0.5787	0.3569	1	−0.1714	−0.0236	−0.0843	−0.0902	0.1326	−0.0658	−0.1838	−0.0612	0.0356	−0.1124
SGER	0.671	0.5694	0.3877	0.2876	0.5831	1	−0.2855	−0.2027	−0.0478	−0.02	−0.1559	−0.2412	0.095	−0.0913	−0.0915
SITA	0.366	0.4712	0.3577	0.2787	0.5668	0.4046	1	−0.2099	−0.3363	−0.2343	−0.0774	−0.2272	−0.3242	−0.1957	−0.1637
SJAP	0.4784	0.4513	0.3605	0.2803	0.4308	0.2744	0.3783	1	−0.0418	0.0362	−0.2755	−0.424	−0.2526	−0.0602	−0.0021
SNET	0.6684	0.6132	0.6385	0.3831	0.5468	0.6471	0.4235	0.428	1	−0.0233	−0.0424	−0.0935	0.0165	−0.0437	−0.0656
SNOR	0.5507	0.626	0.6366	0.3928	0.476	0.4722	0.2656	0.29	0.6444	1	0.0636	0.1111	0.2179	0.093	0.2575
SSPA	0.4438	0.4264	0.3981	0.2311	0.4718	0.3511	0.453	0.3991	0.4327	0.3262	1	−0.3653	−0.1087	−0.014	−0.0056
SSWE	0.5282	0.5118	0.4634	0.2496	0.3748	0.3906	0.4405	0.2699	0.4923	0.5353	0.3474	1	−0.1788	−0.1081	−0.0765
SSWI	0.7346	0.6148	0.6404	0.3993	0.5784	0.785	0.4295	0.3266	0.7129	0.6393	0.4427	0.5452	1	−0.0475	−0.0301
SUKG	0.5788	0.638	0.6854	0.342	0.5454	0.5066	0.3415	0.4012	0.7124	0.6174	0.4742	0.5275	0.6609	1	−0.0285
SUSA	0.6422	0.577	0.8017	0.4373	0.5485	0.4537	0.3419	0.4132	0.6506	0.6074	0.44	0.476	0.6608	0.7284	1

Forecasted Correlation Matrix of Unhedged Stock Returns (Historical)

	SAUT	SBEL	SCAN	SDEN	SFRA	SGER	SITA	SJAP	SNET	SNOR	SSPA	SSWE	SSWI	SUKG	SUSA
SAUT	1	0.0789	0.1116	0.0778	−0.0426	0.1908	−0.184	−0.0175	0.1032	0.1395	0.0051	−0.0894	0.1978	0.082	0.2019
SBEL	0.6415	1	0.0249	−0.021	−0.1785	−0.1617	−0.1091	0.1203	−0.0169	0.2652	0.0415	0.1037	0.1586	0.2635	0.0945
SCAN	0.6305	0.5509	1	0.0583	0.0288	−0.0361	−0.1362	0.1361	−0.0454	0.1258	0.0424	0.0923	0.115	0.1135	−0.0495
SDEN	0.4696	0.4838	0.4788	1	−0.1966	−0.3029	−0.1434	0.0412	−0.1026	0.041	−0.0881	−0.2377	−0.155	−0.0745	0.0454
SFRA	0.5723	0.6371	0.5178	0.4159	1	−0.1649	0.0218	0.0752	−0.1021	0.1528	−0.0455	−0.1614	0.0367	0.0653	−0.1163
SGER	0.7153	0.6019	0.3719	0.3843	0.6179	1	−0.2128	0.0356	−0.079	0.0141	−0.1391	−0.1777	0.1343	−0.0967	−0.0085
SITA	0.4802	0.5506	0.3886	0.3635	0.6463	0.4997	1	−0.0056	−0.2693	−0.1373	−0.0657	−0.1299	−0.2418	−0.1908	−0.0862
SJAP	0.5798	0.5256	0.3607	0.358	0.4912	0.3637	0.4475	1	0.1517	0.1683	−0.2187	−0.2454	−0.0959	−0.1545	0.1913
SNET	0.7039	0.6614	0.7158	0.4813	0.5944	0.6772	0.4918	0.4714	1	0.0541	−0.0058	−0.0487	0.0886	0.0064	0.0015
SNOR	0.5577	0.6471	0.6308	0.4237	0.5195	0.4695	0.3101	0.3193	0.6432	1	0.1578	0.1539	0.2663	0.0682	0.2132
SSPA	0.4864	0.4955	0.3961	0.2698	0.5258	0.3849	0.4851	0.3946	0.4674	0.3929	1	−0.3513	−0.1397	−0.0331	−0.0002
SSWE	0.5278	0.5532	0.505	0.2806	0.4169	0.4322	0.523	0.3559	0.4943	0.5634	0.3957	1	−0.1418	−0.1266	0.045
SSWI	0.7573	0.6729	0.6562	0.4677	0.6487	0.7982	0.5268	0.407	0.7369	0.6381	0.4384	0.5611	1	−0.0325	0.0603
SUKG	0.5272	0.6118	0.6651	0.3113	0.5118	0.4351	0.3055	0.2887	0.6601	0.6227	0.4498	0.5162	0.5888	1	0.0661
SUSA	0.7415	0.5958	0.8362	0.4756	0.5123	0.4659	0.4128	0.4186	0.7309	0.6183	0.4472	0.5359	0.6611	0.6621	1

Lower diagonal part of the matrix includes historical correlations based on 72 observations (Feb 82 – Jan 88).
Upper diagonal part includes residuals between forecasted and actual correlations based on 36 observations (Feb 88 – Jan 91).

Table 4.8

Forecasted Correlation Matrix of Combined Unhedged Returns (Historical)

	CAN	GER	JAP	UKG	USA	USN	SAUT	SBEL	SCAN	SDEN	SFRA	SGER	SITA	SJAP	SNET	SNOR	SSPA	SSWE	SSWI	SUKG	SUSA
CAN	1	−0.073	0.1716	−0.105	−0.126	−0.165	−0.037	−0.159	−0.329	−0.1	−0.23	−0.095	−0.046	0.1471	−0.216	−0.086	−0.152	0.0353	−0.089	−0.2	−0.3181
GER	0.3388	1	0.4309	−0.06	−0.129	0.0225	0.3068	−0.212	−0.185	−0.058	−0.029	−0.098	0.1191	0.3814	−0.142	0.1389	0.0747	0.0505	0.0368	−0.068	−0.2002
JAP	0.3419	0.4309	1	−0.57	0.1888	0.1903	0.0448	0.1105	−0.07	0.0934	0.0709	0.2066	0.2747	0.0274	0.062	0.1994	−0.173	0.0641	−0.144	−0.347	−0.1013
UKG	0.021	−0.06	−0.06	1	0.0381	−0.108	−0.246	−0.124	−0.18	−0.177	−0.121	−0.277	0.0206	−0.417	−0.208	−0.319	−0.418	−0.501	−0.444	−0.502	−0.3013
USA	0.7868	−0.129	0.3408	0.045	1	−0.093	0.173	−0.046	−0.355	−0.085	−0.147	0.0174	0.0465	0.2455	−0.169	0.0099	0.0102	0.1938	−0.086	−0.028	−0.2386
USN	0.621	0.0225	0.2965	−0.267	0.7394	1	0.1773	0.0492	−0.255	0.0067	−0.159	0.1323	0.0832	0.2461	0.0035	−0.014	0.082	0.2353	0.0195	−0.006	−0.1277
SAUT	0.3703	0.2842	0.358	−0.097	0.4921	0.5096	1	0.0789	0.1116	0.0778	−0.043	0.1908	−0.184	−0.017	0.1032	0.1395	0.0051	−0.089	0.1978	0.082	0.2019
SBEL	0.264	0.277	0.2864	−0.08	0.3057	0.2928	0.6415	1	0.0249	−0.021	−0.178	−0.162	−0.109	0.1203	−0.017	0.2652	0.0415	0.1037	0.1586	0.2635	0.0945
SCAN	0.532	0.0987	0.1064	0.0422	0.4304	0.4454	0.6305	0.5509	1	0.0583	0.0288	−0.036	−0.136	0.1361	−0.045	0.1258	0.0424	0.0923	0.115	0.1135	−0.0495
SDEN	0.3584	0.355	0.2574	−0.019	0.3445	0.2718	0.4696	0.4838	0.4788	1	−0.197	−0.303	−0.143	0.0412	−0.103	0.041	−0.088	−0.238	−0.155	−0.075	0.0454
SFRA	0.2178	0.3908	0.345	−0.04	0.262	0.1879	0.5723	0.6371	0.5178	0.4159	1	−0.165	0.0218	0.0752	−0.102	0.1528	−0.045	−0.161	0.0367	0.0653	−0.1163
SGER	0.2444	0.4249	0.2686	−0.06	0.3234	0.2994	0.7153	0.6019	0.3719	0.3843	0.6179	1	−0.213	0.0356	−0.079	0.0141	−0.139	−0.178	0.1343	−0.097	−0.0085
SITA	0.3243	0.395	0.386	0.0357	0.3795	0.3538	0.4802	0.5506	0.3886	0.3635	0.6463	0.4997	1	−0.006	−0.269	−0.137	−0.066	−0.13	−0.242	−0.191	−0.0862
SJAP	0.263	0.3622	0.6744	−0.03	0.2722	0.2957	0.5798	0.5256	0.3607	0.358	0.4912	0.3637	0.4475	1	0.1517	0.1683	−0.219	−0.245	−0.096	−0.155	0.1913
SNET	0.3934	0.3391	0.2504	0.0028	0.3823	0.4499	0.7039	0.6614	0.7158	0.4813	0.5944	0.6772	0.4918	0.4714	1	0.0541	−0.006	−0.049	0.0886	0.0064	0.0015
SNOR	0.1617	0.0066	0.0459	−0.274	0.1554	0.2752	0.5577	0.6471	0.6308	0.4237	0.5195	0.4695	0.3101	0.3193	0.6432	1	0.1578	0.1539	0.2663	0.0682	0.2132
SSPA	0.1679	0.2456	0.151	−0.148	0.2458	0.2534	0.4864	0.4955	0.3961	0.2698	0.5258	0.3849	0.4851	0.3946	0.4674	0.3929	1	−0.351	−0.14	−0.033	−0.0002
SSWE	0.3529	0.1323	0.2815	−0.12	0.3801	0.4255	0.5278	0.5532	0.505	0.2806	0.4169	0.4322	0.523	0.3559	0.4943	0.5634	0.3957	1	−0.142	−0.127	0.045
SSWI	0.3547	0.351	0.2162	−0.108	0.3472	0.365	0.7573	0.6729	0.6562	0.4677	0.6487	0.7982	0.5268	0.407	0.7369	0.6381	0.4384	0.5611	1	−0.032	0.0603
SUKG	0.1912	−0.002	−0.034	0.1605	0.2248	0.1615	0.5272	0.6118	0.6651	0.3113	0.5118	0.4351	0.3055	0.2887	0.6601	0.6227	0.4498	0.5162	0.5888	1	0.0661
SUSA	0.5389	0.1028	0.1528	−0.087	0.5951	0.6181	0.7415	0.5958	0.8362	0.4756	0.5123	0.4659	0.4128	0.4186	0.7309	0.6183	0.4472	0.5359	0.6611	0.6621	1

Lower diagonal part of the matrix includes historical correlations based on 72 monthly observations (Feb 82−Jan 88).
Upper diagonal part includes residuals between forecasted and actual correlations based in 36 monthly observations (Feb 88−Jan 91).

Table 4.9

Forecasted Correlation Matrix of Combined Hedged Returns (Historical)

	CAN	GER	JAP	UKG	USA	USN	SAUT	SBEL	SCAN	SDEN	SFRA	SGER	SITA	SJAP	SNET	SNOR	SSPA	SSWE	SSWI	SUKG	SUSA
CAN	1	−0.114	0.0003	−0.392	−0.097	0.0787	−0.194	−0.283	−0.463	−0.382	−0.174	−0.261	−0.081	−0.284	−0.336	−0.154	−0.356	−0.2957	−0.3692	−0.5424	−0.4319
GER	0.3846	1	0.0694	0.0443	−0.005	0.1567	−0.068	−0.462	−0.185	−0.209	−0.103	−0.278	−0.065	0.0003	−0.176	−0.116	−0.134	−0.1844	−0.1193	−0.1548	−0.3407
JAP	0.3476	0.0694	1	−0.197	0.115	−0.171	−0.472	−0.424	−0.412	−0.206	−0.335	−0.4	−0.133	−0.41	−0.454	−0.272	−0.578	−0.3393	−0.5955	−0.4625	−0.5895
UKG	0.2255	0.0443	0.3711	1	0.021	0.0464	−0.186	−0.079	−0.39	−0.164	−0.073	−0.26	0.0491	−0.208	−0.236	−0.268	−0.325	−0.4252	−0.4491	−0.5023	−0.4032
USA	0.6524	−0.005	0.3899	0.4089	1	0.2502	0.0377	−0.155	−0.465	−0.256	−0.083	−0.105	−0.02	−0.077	−0.258	0.0154	−0.02	0.0378	−0.2998	−0.2166	−0.3499
USN	0.2973	0.1567	0.0282	0.1323	0.2999	1	−0.09	0.1258	−0.272	−0.071	−0.114	0.2146	0.1652	−0.273	−0.041	−0.215	0.0637	0.0825	−0.118	−0.0904	−0.1317
SAUT	−0.031	−0.027	−0.145	0.0126	0.1109	0.1582	1	0.0213	0.1391	−0.03	−0.095	0.1488	−0.316	−0.13	−0.012	0.0976	−0.049	−0.0934	0.1552	0.1359	0.1544
SBEL	0.0268	−0.005	−0.09	0.0431	0.1126	0.2613	0.6056	1	0.0025	−0.035	−0.184	−0.156	−0.186	−0.087	−0.012	0.2174	−0.029	0.0574	0.0849	0.2428	0.0396
SCAN	0.1587	0.1416	−0.029	0.1507	0.116	0.0423	0.5592	0.5389	1	−0.026	0.1414	−0.06	−0.164	−0.042	−0.1	0.1662	0.0588	0.0103	0.0206	−0.0202	0.0063
SDEN	0.091	0.0509	−0.027	0.1068	0.1712	0.1359	0.3908	0.4272	0.4154	1	−0.199	−0.356	−0.188	−0.148	−0.142	−0.028	−0.12	−0.3219	−0.2321	−0.1327	−0.0146
SFRA	0.0893	0.2103	0.0567	0.0912	0.1426	0.0584	0.5211	0.6133	0.5787	0.3569	1	−0.171	−0.024	−0.084	−0.09	0.1326	−0.066	−0.1838	−0.0612	0.0356	−0.1124
SGER	0.0585	0.1904	−0.109	0.0351	0.1383	0.2318	0.671	0.5694	0.3877	0.2876	0.5831	1	−0.286	−0.203	−0.048	−0.02	−0.156	−0.2412	0.095	−0.0913	−0.0915
SITA	0.1219	0.1694	0.168	0.151	0.1682	0.3859	0.366	0.4712	0.3577	0.2787	0.5668	0.4046	1	−0.21	−0.336	−0.234	−0.077	−0.2272	−0.3242	−0.1957	−0.1637
SJAP	0.0062	0.1675	0.2407	0.1439	0.0808	0.0196	0.4784	0.4513	0.3605	0.2803	0.4308	0.2744	0.3783	1	−0.042	0.0362	−0.276	−0.424	−0.2526	−0.0602	−0.0021
SNET	0.0009	0.1333	−0.098	0.1039	0.012	0.1736	0.6684	0.6132	0.6385	0.3831	0.5468	0.6471	0.4235	0.423	1	−0.023	−0.042	−0.0935	0.0165	−0.0437	−0.0656
SNOR	−0.134	−0.171	−0.279	−0.19	−0.094	0.0677	0.5507	0.626	0.6366	0.3928	0.476	0.4722	0.2656	0.29	0.6444	1	0.0636	0.1111	0.2179	0.093	0.2575
SSPA	−0.053	0.0857	−0.08	−0.03	0.0842	0.0906	0.4438	0.4264	0.3981	0.2311	0.4718	0.3511	0.453	0.3991	0.4327	0.3262	1	−0.3653	−0.1087	−0.014	−0.0056
SSWE	0.0525	0.0379	0.0608	0.0083	0.1504	0.3196	0.5282	0.5118	0.4634	0.2496	0.3748	0.3906	0.4405	0.2699	0.4923	0.5353	0.3474	1	−0.1788	−0.1081	−0.0765
SSWI	0.081	0.1742	−0.165	0.0139	0.1001	0.2096	0.7346	0.6148	0.6404	0.3993	0.5784	0.785	0.4295	0.3266	0.7129	0.6393	0.4427	0.5452	1	−0.0475	−0.0301
SUKG	0.0104	0.1248	−0.02	0.1605	0.1334	0.0872	0.5788	0.638	0.6854	0.342	0.5454	0.5066	0.3415	0.4012	0.7124	0.6174	0.4742	0.5275	0.6609	1	−0.0285
SUSA	0.1766	0.0136	−0.071	0.0858	0.263	0.1443	0.6422	0.577	0.8017	0.4373	0.5485	0.4537	0.3419	0.4132	0.6506	0.6074	0.44	0.476	0.6608	0.7284	1

Lower diagonal part of the matrix includes historical correlations based on 72 monthly observations (Feb 82–Jan 88).
Upper diagonal part includes residuals between forecasted and actual correlations based on 36 monthly observations (Feb 88–Jan 91).

The bias, variance and covariance proportions of the MSFE can be simply found by dividing each one of the terms respectively in the RHS from equation 4.18 by the MSFE. The "bias" proportion should be interpreted as that part of the forecast error that is due to incorrect estimate of the mean correlation, the "variance" proportion to unequal correlation variability between the forecast and validation periods and, finally, the "covariance" proportions relates to misestimation of correlations.

As a final means of comparison between the forecasting performance of the PCM as opposed to the historical correlations benchmark, the Theil Inequality Coefficient (TIC) has being computed as follows:

$$TIC = \sqrt{\sum_{t=1}^{T} \frac{(cor^{f}_{i,j} - cor^{a}_{i,j})^2}{(cor^{h}_{i,j} - cor^{a}_{i,j})^2}} \tag{4.19}$$

where $cor^{h}_{i,j}$ denotes a correlation forecast based on the historical model

In the event that a particular forecasting method provides more accurate estimates than the benchmark, then the TIC<1, while if TIC>1 the forecast is inferior to the benchmark. Table 4.10 summarizes all necessary information about the different models, concerning their MSFE and RMSFE, their "bias", "variance" and "covariance" decomposition and the TIC.

A brief comparison between the Historical and PCA methodologies shows that for both the hedged and unhedged stocks the PCM method lead to reduced MSFE. Also the TIC for the PCM is in both cases smaller than one, indicating clearly that the unobservable components model can indeed eliminate part of the "noise" inherent in historical correlations and, therefore, provide more accurate forecasts.

Table 4.10

Forecast Error Decomposition of HIS and PCM Correlations

MODEL	MSFE	RMSFE	BIAS%	VAR%	COV%	TIC
HIS/ST/U	.0182	.1351	.0058	.0097	.9843	1
HIS/ST/H	.0236	.1536	.1741	.0041	.8216	1
HIS/CO/U	.0290	.1704	---	---	---	1
HIS/CO/H	.0486	.2205	---	---	---	1
PCM/ST/U	.0176	.1329	.0001	.0001	.9998	.9839
PCM/ST/H	.0210	.1450	.0642	.0042	.9315	.9440
PCM/CO/U	.0313	.1770	---	---	---	1.039
PCM/CO/H	.0478	.2186	---	---	---	.9913

As expected, due to the unequal variability of stock and bond returns, the PCM did not perform that well in forecasting correlations from the "combined" matrix. In fact the combined unhedged PCM underperformed the historical model, whereas the combined hedged PCM slightly outperformed it. Overall, one should remain sceptical about the appropriateness of extracting common factors in "mixed" portfolios, that incorporate different asset classes.

Notice, however, that it might be possible to further improve on the PCM correlation estimates, by means of satisfactorily adjusting for "estimation" risk on the components loading matrix, maybe by means of a Bayesian methodology. This is clearly a useful approach, since the forecasted correlations are very sensitive on the inputs of the components loading matrix which is likely to be non-stationary. In a more dynamic setting, the same idea would also apply to the decomposition of a correlation matrix estimated through a multivariate Garch process. All this provides an interesting framework for future research in the area.

4.3.1.5 Out of Sample Portfolio Performance of the PCM and the Historical Model

The next meaningful question to be addressed is whether the superior correlation forecasting ability of the PCM model would lead also to a superior risk-adjusted portfolio performance, when measured out of sample. There is no theoretical basis of ascertaining that this should be the case, since correlations represent one only of the necessary inputs to the portfolio optimization problem, and in fact misestimation of expected returns and variances might cause substantial distortions:

As a first step, it is necessary to investigate the out of sample performance of the historical portfolio, which will act as benchmark for all subsequent methods. On the basis of our findings in Chapter III, the inputs to this model are inter-temporally unstable so that we should expect "ex-ante" portfolios to perform far worse than their "ex-post" counterparts. It is, nevertheless, an open question whether models with alternative input estimates can in fact consistently outperform it.

All necessary inputs for that optimization problem have been directly estimated by using monthly data from February 1982 to February 1988. The optimization problems have been formulated in a way that constraints short sales of the various indices; The imposition of such constraints is made partly in order to make the asset allocation problem to conform with real life investments and partly in order to reduce the "estimation error" implications of having indices with highly negative weights in the optimization solution. This standard quadratic optimization problem has, therefore, been formulated as follows:

$$\begin{aligned} &\textbf{\textit{Minimize}} && \vec{x}' SRS\, \vec{x} \\ &\textbf{\textit{Constrained by:}} && \vec{x}' \vec{E} = r(p) \\ & && x_1, x_2, x_k, \geq 0 \\ & && \sum_{i=1}^{k} x_i = 1 \end{aligned} \qquad (4.20)$$

where $\mathbf{x}$ = vector of asset weights

$\mathbf{S}$ = diagonal matrix of standard deviations

$\mathbf{R}$ = correlation matrix

$\mathbf{E}$ = vector of mean returns

$\mathbf{r_p}$ = required portfolio return

$\mathbf{I}$ = identity vector

$\mathbf{SRS} = \mathbf{V}$ the covariance matrix

The optimization formulation of the PCM is very similar, except that the objective function is now being expressed as:

$$\textbf{\textit{Minimize}} \quad \vec{x}^{\prime} S\, C^{*} S\, \vec{x} \tag{4.21}$$

where each entry i,j of $\mathbf{C}^{*}$ is defined as in equation 4.15

All constrained quadratic optimizations were performed by using the Excel Solver optimization algorithms. For each model, the optimized vector of asset weights $\vec{x}^{*}$ has been computed at three different points of the efficient frontier, corresponding to monthly risk free rates of 0%, 0.2% and 0.4% respectively. These three portfolios were reinvested in three different out of sample portfolio holding periods of one, two and three years respectively.

The exact optimized weights as well as the realized return and volatility for each portfolio are reported in the appendix at the end of Chapter IV. Equation 4.5 has been used to calculate the realized out of sample (Jobson-Korkie adjusted) Sharpe Performance Measure for each reinvestment period and risk-free rate (9 combinations for each model). These results are summarized in Tables 4.11 - 4.14, while for comparison purposes the corresponding statistics for the Historical Model are quoted in parentheses.

As we can observe from these results, the risk adjusted performance varied considerably, when measured over different holding periods and types of portfolios. For the Historical Model, the best adjusted Sharpe measures were achieved by the hedged stock portfolios during the 1 year reinvestment period from February 1988 - January 1989.

Table 4.11

Principal Components Model
Sharpe's Performance Measure
Unhedged Stocks

	n=12	n=24	n=36
Rf = 0	0.28004	0.29960	-0.01082
	(0.44697)	(0.39487)	(0.01155)
Rf = 0.2	0.39136	0.33188	-0.02703
	(0.41161)	(0.30595)	(-0.05681)
Rf = 0.4	0.33712	0.26805	-0.07321
	(0.32752)	(0.26906)	(-0.07603)

Numbers in brackets are based on historical data

Table 4.12

Principal Components Model
Sharpe's Performance Measure
Unhedged Stocks and Bonds

	n=12	n=24	n=36
Rf = 0	0.18969	-0.00923	-0.09005
	(0.04604)	(0.02552)	(-0.08339)
Rf = 0.2	0.07496	-0.12330	-0.17478
	(-0.08260)	(-0.10250)	(-0.19234)
Rf = 0.4	-0.01658	-0.21507	-0.25372
	(-0.21471)	(-0.23563)	(-0.30172)

Numbers in brackets are based on historical data

<u>**Table 4.13**</u>

Principal Components Model
Sharpe's Performance Measure
Hedged Stocks

	n=12	n=24	n=36
Rf = 0	0.80439	0.45258	0.08593
	(0.88576)	(0.57390)	(0.09629)
Rf = 0.2	0.73520	0.38507	0.04287
	(0.81027)	(0.49711)	(0.05361)
Rf = 0.4	0.66600	0.31759	-0.00012
	(0.73328)	(0.40930)	(0.00660)

Numbers in brackets are based on historical data

<u>**Table 4.14**</u>

Principal Components Model
Sharpe's Performance Measure
Hedged Stocks and Bonds

	n=12	n=24	n=36
Rf = 0	1.55333	1.25432	0.17411
	(1.52727)	(1.28410)	(0.17938)
Rf = 0.2	0.96768	0.70569	0.02352
	(0.91933)	(0.72763)	(0.12806)
Rf = 0.4	0.43358	0.06915	-0.12982
	(0.37587)	(0.17562)	(-0.13103)

Numbers in brackets are based on historical data

The pattern for the PCM performance was generally reasonably similar to that of the HIS Model; A comparison between the HIS and PCM Sharpe for the n = 36 period, which exactly corresponds to the time horizon of the correlation forecast, tells us that the superior correlation forecasting ability of the PCM failed to translate into superior out of sample performance. Similar considerations apply also for the shorter time periods of n = 12 and n = 24 month holding horizons. This finding supports the view that portfolio performance is likely to be relatively insensitive to errors in correlation forecasts, compared to errors in estimates of expected returns and (to a smaller extent) volatilities.

4.3.2 A Maximum Likelihood Factor Model (MLFM) for Correlation Forecasting and Global Asset Allocation.

The principal need for applying an alternative factor model, lies primarily to the fact that since no statistical tests of significance are available for the PCM to support the Keizer eigenvalue criterion, it is not always possible to ascertain that the number of factors extracted is the "true" one. Nevertheless, by using the MLFM on the basis of the assumption of multivariate normality, it is possible to obtain goodness of fit statistics for the adequacy of an s factor model.

In the MLFM we assume that a sample is drawn from a multivariate normal distribution with unknown population parameters, i.e. mean return vector and variance-covariance matrix. Then from a specific sample we try to find universe parameters that would have the greatest joint likelihood of producing the statistics we got in the sample. In this context, the estimated values will be the maximum likelihood estimators of the population parameters.

4.3.2.1 Formulating the MLFM

The basic MLFM model in matrix form can be expressed as:

$$\vec{x} = B\vec{f} + \vec{e} \tag{4.22}$$

where: $\vec{x}$ is a (k*1) vector of observed variables

$\vec{f}$ is an (s*1) vector of unobservable common factors, s<k

$\vec{e}$ is a (k*1) vector of unobservable unique factors

B is a (k*s) matrix of factor loadings

In this context we assume that the unique part of each variable is uncorrelated with each other and with the common part. Consequently, it is possible to define a (k*k) diagonal matrix of "communalities" $\Psi = \mathscr{E}(\vec{e}\,\vec{e}^{\prime})$. Also we can define another (k*k) matrix Φ whose entries are 1's in the principal diagonal and the correlations between the factors otherwise. Naturally, when the factors are uncorrelated $\Phi = I$

In the MLFM we assume that the k dimensional vector **x** has a nonsingular k dimensional multivariate normal distribution with mean zero and a variance covariance matrix defined as:

$$\Sigma = B\Phi B^{\prime} + \Psi \tag{4.23}$$

In this context the likelihood function to be minimized (see e.g. Dillon & Goldstein 1984) is given by

$$\begin{aligned} \ln L &= -\frac{1}{2} n \,[\ln|\Sigma| + tr(\Sigma^{-1} V)] = \\ &= -\frac{1}{2} n \,[\ln(|B\Phi B^{\prime}| + \Psi) + tr[(B\Phi B^{\prime} + \Psi)^{-1} V]\,] \end{aligned} \tag{4.24}$$

A problem is that the maximum likelihood estimates B and Φ are not unique, because the elements of the matrices are not independent of one another. By imposing, however, the condition of orthogonality among the factors it is possible to find a conditional minimum of Φ for a given Ψ .

In fact, if we assume that B_0 is the matrix B that satisfies the above equality then the problem lies in finding a matrix $\boldsymbol{\Omega} = \partial F / \partial \boldsymbol{\Psi}$ when evaluated at $B=B_0$. The main problem is that because this problem cannot be solved directly, numerical methods are required to evaluate at each step the function and its derivatives. In fact, alternative algorithms provide different solutions. In this study, the Anderson-Rubin algorithm (see Norussis 1989) has been applied, since it conveniently provides uncorrelated factor scores with zero mean and unit standard deviation.

4.3.2.2 Determining the Number of Unobservable Factors in the MLFM: A Likelihood Ratio Test.

One of the major advantages of the MLFA is that a likelihood ratio test of significance for the number of factors can be derived. The null hypothesis is that all the population variance has been extracted by the hypothesized number of factors. If at a specified probability level the chi-square value is significant, then we can conclude that the residual matrix still has significant variance in it and therefore more factors are needed in order to adequately reproduce the correlations between the original variables.

The likelihood ratio statistic L^* that has been used in this context can be expressed as follows:

$$L^* = (n-1) \ln \frac{|\hat{B}\hat{B}' + \hat{\Psi}|}{|V|} + tr[(\hat{B}\hat{B}' + \hat{\Psi})^{-1} V] - k \qquad (4.25)$$

whose the degrees of freedom are equal to

$$\chi^2 (d.fr) = \frac{1}{2}[(k-s)^2 - k - s] \qquad (4.26)$$

where: **k, s** stand for the number of variables and common factors extracted respectively

Notice that the value of the chi-square goodness of fit statistic is directly proportional to the sample size but the degrees of freedom depend only on the number of common factors and the number of variables. In practice, the number of factors originally extracted needs to be increased until one can obtain a reasonably good fit, i.e a significance level that is not too small.

All MLFM factor loadings and the aforementioned likelihood ratio statistic were computed by using SPSS-PC factor procedures. One significant difference from the PCM model, though, lies in the fact that prior to extracting the factors the data have been adjusted for the October 1987 observation. This was deemed necessary for reasons similar to those related to the MANOVA tests in Chapter III, i.e. in order to minimize the impact of deviations from multivariate normality induced by the stock market "crash".

Since the appropriate number of factors to be extracted is not known a-priori, the procedure was repeated for a different number of factors each time and the significance levels for the likelihood ratio statistic were each time compared: The results are summarized in Table 4.15 below:

When the number of extracted factors for e.g. the unhedged stock index returns is just two, the exact significance level of the Likelihood Ratio Test is .033 indicating that there is over 95% probability that an additional factor is needed. When a third factor has been extracted, then the significance level increased to .4899 so that no extra factors were needed. In all cases, the process of adding extra factors was terminated when the significance level increased beyond 5%.

Table 4.15

A Likelihood Ratio Criterion for Common Factor Selection in the MLFM

PORTF.TYPE	EIGEN >1	ML/FCT	SIGN(L*)	ML/FCT	SIGN(L*)
STOC(UNHG)	2	2	.0330	3	.4899
COMB(UNHG)	4	4	.0027	5	.0623
STOC(HEDG)	2	1	.0064	2	.5154
COMB(HEDG)	4	2	.0004	3	.0574

Eigen > 1 refers to the number of components previously chosen on the basis of the PCM
SIGN(L*) refers to the exact significance level of the Likelihood Ratio Test
Column's (5) entries show the number of common factors finally selected for the MLFM

4.3.2.3 Forecasting the Index Correlation Structure with the MLFM

Given orthogonal factors and standardized factor scores, correlation coefficients can be forecasted from the factor loadings by means of equation 4.15, as was the case with the PCM. The final[25] factor loadings matrices, which are used as inputs for the correlation forecasts, can be found in the Appendix 4A. One key difference from the PCM, though, lies in the fact that the MLFM is known (see e.g. Dillon & Goldberg 1984) to provide lower than the original correlation estimates (mean "bias"), even though it might be capable of optimally estimating deviations from the average correlation coefficient, for samples drawn from multivariate normal distributions.

Preliminary results[26] confirmed indeed that correlation estimates from the MLFM indeed tended to be substantially lower than those arising from both the HIS and PCM. For these reasons, a standard mean adjustment procedure was applied to all unadjusted forecasts in order to equate the average correlation coefficient from the MLFM, to that arising from the HIS model.

25 That correspond to the number of factors finally extracted

26 Not reported for reasons of preserving space

In order to achieve this, a constant k has been added to all correlation forecasts $cor_{i,j}$ defined as follows:

$$k = \frac{1}{T}\sum_{i=1}^{T} cor_{i,j}^{h} - \frac{1}{T}\sum_{i=1}^{T} cor_{i,j}^{m}$$
$$T = \frac{k*(k-1)}{2} \qquad (4.27)$$

where $cor_{i,j}^{m}$ is the unadjusted MLFM correlation forecast for the pair i,j

On the basis of equation 4.27 above, the mean adjusted correlation forecasts $cor_{i,j}^{ma}$ can be defined as

$$cor_{i,j}^{ma} = cor_{i,j}^{m} + k \qquad \forall\ i,j \qquad (4.28)$$

The resulting mean-adjusted MLFM forecasted correlation matrices for the unhedged and hedged stock only and combined index returns can be found in Tables 4.16 - 4.18 below, whereas Table 4.19 provides information about the TIC and the decomposition of the MSFE for the stock index returns.

The pattern for the mean-adjusted MLFM forecasts is reasonably similar to the PCM forecasts. In the stock only matrices, hedged and unhedged, the MLFM outperforms the historical model in forecasting capability, whereas in the combined portfolios its forecasting ability is lower. Notice also, that due to the mean-adjustment towards the historical portfolio, the "bias" component of the error tracks closely that of the historical forecast, being high in the case of hedged stock returns. In all cases, the covariance error component was by far the most substantial as in the previous models.

A more interesting comparison, is that between the two unobserved factor models namely the PCM and the mean-adjusted MLFM; In fact, the MLFM had a higher MSFE than the PCM for both the unhedged and hedged stocks, whereas in the combined portfolios it outperformed the PCM only in the case of hedged returns. In any case the differences between the two methods are quite small, so that no clear conclusions can be drawn about the forecasting superiority of the PCM.

Table 4.16

Forecasted Correlation Matrix of Hedged Stock Returns (Maximum Likelihood)

	SAUT	SBEL	SCAN	SDEN	SFRA	SGER	SITA	SJAP	SNET	SNOR	SSPA	SSWE	SSWI	SUKG	SUSA
SAUT	1	−0.018325	0.1428446	−0.017255	−0.051445	0.1943746	−0.225795	−0.232505	−0.003635	0.0887146	−0.143835	−0.144135	0.1476146	0.1220646	0.1212646
SBEL	0.5659746	1	0.0306146	−0.069985	−0.286715	−0.154555	−0.238725	−0.170055	−0.010435	0.1090746	−0.118995	0.0004746	0.1039946	0.1504046	0.0615646
SCAN	0.5629446	0.5670146	1	0.0379246	0.0900646	−0.130325	−0.086815	0.0537746	−0.068815	0.1668746	0.0513046	0.0896446	−0.027025	−0.008125	0.0175246
SDEN	0.4036446	0.3921146	0.4798246	1	−0.179075	−0.313955	−0.142935	−0.111805	−0.074615	−0.007495	−0.063515	−0.204765	−0.199805	−0.035225	0.0378946
SFRA	0.5641546	0.5103846	0.5273646	0.3773246	1	−0.147025	−0.175805	−0.161385	−0.032675	0.1538246	−0.210255	−0.118905	−0.002045	0.0102146	−0.097415
SGER	0.7165746	0.5707446	0.3175746	0.3293446	0.6074746	1	−0.208275	−0.180295	−0.004025	−0.042255	−0.198945	−0.216395	0.1821646	−0.162245	−0.137555
SITA	0.4562046	0.4185746	0.4351846	0.3235646	0.4145946	0.4818246	1	−0.281665	−0.273355	−0.089185	−0.243625	−0.299175	−0.245435	−0.109335	−0.045015
SJAP	0.3756946	0.3683446	0.4559746	0.3160946	0.3537146	0.2968046	0.3065346	1	−0.050225	0.1370146	−0.397915	−0.345585	−0.181085	−0.045775	0.0478646
SNET	0.6763646	0.6152646	0.6694846	0.4500846	0.6043246	0.6908746	0.4864446	0.4195746	1	−0.057455	−0.095155	−0.055125	0.0684546	−0.111075	−0.004745
SNOR	0.5418146	0.5176746	0.6372746	0.4132046	0.4972246	0.4499446	0.4107146	0.3908146	0.6102446	1	0.0853946	0.0481146	0.1696446	0.0579246	0.3065446
SSPA	0.3484646	0.3366046	0.3906046	0.2871846	0.3273446	0.3030546	0.2867746	0.2766846	0.3799446	0.3479946	1	−0.397805	−0.179445	−0.122395	−0.045695
SSWE	0.4774646	0.4548746	0.5427446	0.3667346	0.4396946	0.4154046	0.3685246	0.3483146	0.5306746	0.4723146	0.3148946	1	−0.203965	−0.133185	0.0075446
SSWI	0.7270146	0.6338946	0.5927746	0.4315946	0.6375546	0.8721646	0.5082646	0.3981146	0.7648546	0.5910446	0.3719546	0.5200346	1	−0.095825	−0.037425
SUKG	0.5649646	0.5456046	0.6974746	0.4394746	0.5200146	0.4356546	0.4278646	0.4156246	0.6450246	0.5823246	0.3658046	0.5024146	0.6125746	1	−0.042575
SUSA	0.6090646	0.5989646	0.8129246	0.4897946	0.5634846	0.4076446	0.4605846	0.4631646	0.7114546	0.6564446	0.3999046	0.5600446	0.6534746	0.7143246	1

Forecasted Correlation Matrix of Unhedged Stock Returns (Maximum Likelihood)

	SAUT	SBEL	SCAN	SDEN	SFRA	SGER	SITA	SJAP	SNET	SNOR	SSPA	SSWE	SSWI	SUKG	SUSA
SAUT	1	0.072432	0.104392	0.120332	0.003702	0.241562	−0.096858	−0.111568	0.168462	0.139582	−0.080978	−0.095778	0.227622	0.045292	0.136692
SBEL	0.635032	1	−0.007898	−0.025388	−0.160688	−0.155418	−0.003088	0.141422	−0.031518	0.107632	0.024712	0.071482	0.139112	0.073192	0.059392
SCAN	0.623292	0.518102	1	0.097782	−0.062318	−0.121658	−0.140328	0.201422	−0.029588	0.121922	−0.025058	0.098062	0.058042	0.066542	0.006942
SDEN	0.512132	0.479412	0.518282	1	−0.151008	−0.254738	−0.060428	0.086302	−0.040548	0.051122	−0.012608	−0.102178	−0.102998	0.011592	0.103102
SFRA	0.618602	0.654912	0.426682	0.461492	1	−0.135358	0.072642	0.144942	−0.035058	0.084712	−0.070228	−0.065768	0.031052	−0.072248	−0.146258
SGER	0.766062	0.608182	0.286342	0.432462	0.647442	1	−0.145878	0.083882	−0.035318	−0.023768	−0.175438	−0.176318	0.215582	−0.202968	−0.059398
SITA	0.567342	0.656612	0.384472	0.446472	0.697142	0.566622	1	0.133882	−0.206198	−0.025908	−0.015978	−0.141158	−0.190028	−0.151138	−0.065298
SJAP	0.485732	0.546722	0.426022	0.403102	0.560942	0.411982	0.586982	1	0.175362	0.249352	−0.166818	−0.151188	−0.019708	−0.093758	0.221352
SNET	0.769162	0.646782	0.731612	0.543352	0.611442	0.720862	0.554902	0.495062	1	0.019042	−0.073738	0.004762	0.156012	−0.105478	0.040682
SNOR	0.557782	0.489532	0.626922	0.433822	0.451412	0.431632	0.421492	0.400352	0.608142	1	0.095512	0.029572	0.191462	−0.096148	0.227772
SSPA	0.400322	0.478712	0.328642	0.345292	0.501072	0.348562	0.534822	0.446482	0.399462	0.330612	1	−0.354758	−0.182548	−0.196838	−0.098018
SSWE	0.521422	0.520982	0.510762	0.416122	0.512532	0.433582	0.511742	0.450112	0.547762	0.439072	0.392242	1	−0.178258	−0.246578	0.037062
SSWI	0.787122	0.653412	0.599242	0.519702	0.643052	0.879482	0.578572	0.483192	0.804312	0.563262	0.395552	0.524642	1	−0.134748	0.068302
SUKG	0.490492	0.421492	0.618142	0.397392	0.374252	0.328832	0.345162	0.349442	0.548222	0.458352	0.286062	0.396222	0.486552	1	0.013692
SUSA	0.676292	0.560692	0.892642	0.533302	0.482342	0.415002	0.433702	0.448652	0.770082	0.632872	0.349382	0.527962	0.669102	0.609692	1

Lower diagonal part of the matrix includes maximum likelihood correlations based on 72 observations (Feb 82–Jan 88).
Upper diagonal part includes residuals between forecasted and actual correlations based on 36 observations (Feb 88–Jan 91).

Table 4.17

Forecasted Correlation Matrix of Combined Unhedged Returns (Maximum Likelihood)

	SAUT	SBEL	SCAN	SDEN	SFRA	SGER	SITA	SJAP	SNET	SNOR	SSPA	SSWE	SSWI	SUKG	SUSA	CAN	GER	JAP	YKG	USA	USN
SAUT	1	0.0353	−0.071	−0.011	−0.147	0.093	−0.22	−0.173	0.0062	−0.037	−0.214	−0.205	0.06299	−0.171	0.0149	0.0905	0.46375	0.1676	−0.203	0.2095	0.12139
SBEL	0.5979	1	−0.024	−0.073	−0.259	−0.202	−0.189	0.1179	−0.044	0.1355	−0.112	−0.002	0.10953	0.0697	0.0167	−0.034	−0.03779	0.3496	0.0459	0.0146	0.06205
SCAN	0.5497	0.502	1	0.0432	−0.084	−0.168	−0.136	0.1472	−0.093	0.062	−0.041	0.0735	−0.01318	0.0705	−0.053	−0.218	−0.07335	0.103	0.0145	−0.177	−0.20609
SDEN	0.4824	0.432	0.4037	1	−0.207	−0.301	−0.125	0.0923	−0.086	−0.011	−0.078	−0.143	−0.1601	−0.04	0.0604	−0.034	−0.05745	0.2536	−0.03	−0.009	0.09199
SFRA	0.5697	0.5562	0.4048	0.4053	1	−0.182	−0.146	0.1221	−0.106	0.0588	−0.246	−0.176	−0.01448	−0.071	−0.217	−0.118	0.08508	0.2906	0.0775	−0.094	−0.11666
SGER	0.7192	0.5615	0.2396	0.3858	0.6008	1	−0.192	0.0369	−0.082	−0.071	−0.222	−0.223	0.16887	−0.25	−0.106	0.0114	0.06663	0.3855	−0.119	0.1143	0.14668
SITA	0.5462	0.4704	0.3884	0.382	0.4781	0.5201	1	0.0535	−0.245	−0.152	−0.249	−0.27	−0.2595	−0.204	−0.063	0.0679	0.23132	0.4594	0.1498	0.1218	0.09538
SJAP	0.5265	0.5232	0.3718	0.4091	0.5381	0.365	0.5066	1	0.1541	0.1573	−0.26	−0.185	−0.07494	−0.203	0.1561	0.2735	0.62325	0.1734	−0.301	0.3422	0.28003
SNET	0.7087	0.6339	0.6679	0.4974	0.5905	0.6741	0.5156	0.4738	1	0.0044	−0.112	−0.03	0.10616	−0.103	−0.026	−0.066	−0.04369	0.2599	−0.04	−0.013	−0.0256
SNOR	0.4825	0.5174	0.567	0.3715	0.4255	0.3847	0.2953	0.3083	0.5935	1	0.0673	0.0218	0.2076	−0.097	0.1633	0.0248	0.27472	0.3595	−0.191	0.0621	−0.02758
SSPA	0.3689	0.3418	0.3131	0.2795	0.3252	0.3021	0.302	0.3537	0.361	0.3024	1	−0.444	−0.20362	−0.44	−0.158	−0.042	−0.04605	−0.163	−0.224	−0.17	0.07524
SSWE	0.5143	0.4477	0.4862	0.3751	0.4024	0.3868	0.3834	0.4168	0.5132	0.4313	0.3031	1	−0.27337	−0.635	−0.037	0.1176	−0.01498	−0.052	−0.321	−0.189	−0.13804
SSWI	0.7243	0.6238	0.528	0.4626	0.5975	0.8328	0.5091	0.428	0.7545	0.5794	0.3745	0.4295	1	−0.551	−0.121	−0.048	−0.19616	−0.271	−0.18	−0.321	−0.23493
SUKG	0.3763	0.418	0.6221	0.3456	0.3754	0.2821	0.2924	0.2398	0.5509	0.4576	0.0431	0.0079	0.06996	1	−0.313	−0.272	−0.00031	−0.156	−0.627	−0.25	−0.07355
SUSA	0.6563	0.518	0.8323	0.4906	0.4118	0.3682	0.4356	0.3834	0.7032	0.5684	0.2891	0.4541	0.47986	0.2826	1	−0.163	−0.14174	−0.177	−0.091	−0.74	−0.64585
CAN	0.5993	0.3891	0.6428	0.4242	0.3295	0.351	0.4381	0.3894	0.5441	0.2721	0.2782	0.4352	0.39615	0.1194	0.6938	1	−0.0211	0.2752	0.0303	−0.049	−0.02417
GER	0.5429	0.451	0.2102	0.3559	0.5053	0.5895	0.5072	0.6041	0.4375	0.1424	0.1249	0.0668	0.11804	0.066	0.1613	0.3904	1	0.5148	0.0549	−0.018	0.13409
JAP	0.5826	0.5255	0.279	0.4176	0.5647	0.4475	0.5707	0.8204	0.4483	0.206	0.1608	0.1656	0.08869	0.1564	0.0769	0.4455	0.7475	1	−0.477	0.3052	0.32263
YKG	0.0475	0.0902	0.2366	0.128	0.1586	0.0986	0.1649	0.0864	0.1705	−0.145	0.0461	0.0593	0.1562	0.0354	0.1229	0.1562	0.16026	0.0334	1	0.1467	0.00652
USA	0.6304	0.3666	0.6081	0.42	0.3149	0.4203	0.4548	0.3689	0.5373	0.2076	0.0652	−0.003	0.11189	0.0021	0.0939	0.8644	0.43028	0.4572	0.1536	1	0.00749
USN	0.5555	0.3057	0.4938	0.3571	0.2297	0.3138	0.366	0.3296	0.4208	0.2611	0.2466	0.0522	0.11057	0.0941	0.1	0.7618	0.34369	0.4288	−0.153	0.8399	1

Lower diagonal part of the matrix includes maximum likelihood correlations based on 72 observations (Feb 82−Jan 88).
Upper diagonal part includes residuals between forecasted and actual correlations based on 36 observations (Feb 88−Jan 91).

Table 4.18

Forecasted Correlation Matrix of Combined Hedged Returns (Maximum Likelihood)

	SAUT	SBEL	SCAN	SDEN	SFRA	SGER	SITA	SJAP	SNET	SNOR	SSPA	SSWE	SSWI	SUKG	SUSA	CAN	GER	JAP	UKG	USA	USN
SAUT	1	−0.147	−0.012	−0.155	−0.217	0.0514	−0.388	−0.404	−0.139	0.0052	−0.296	−0.273	0.0253	−0.036	−0.021	0.0829	0.0843	−0.303	−0.079	0.181	−0.031
SBEL	0.437	1	−0.109	−0.203	−0.432	−0.289	−0.382	−0.321	−0.149	−0.007	−0.263	−0.129	−0.022	0.0093	−0.071	−0.044	−0.304	−0.246	0.0379	0.0053	0.0558
SCAN	0.4085	0.4271	1	−0.091	−0.041	−0.264	−0.223	−0.081	−0.228	0.0076	−0.095	−0.05	−0.175	−0.13	−0.094	−0.24	−0.14	−0.168	−0.268	−0.197	−0.149
SDEN	0.2657	0.2589	0.3508	1	−0.31	−0.443	−0.275	−0.247	−0.216	−0.15	−0.202	−0.338	−0.332	−0.164	−0.087	−0.256	−0.129	−0.059	−0.116	−0.206	−0.082
SFRA	0.3985	0.3647	0.3967	0.2466	1	−0.296	−0.271	−0.262	−0.193	−0.066	−0.331	−0.252	−0.149	−0.11	−0.238	0.0819	0.0077	−0.153	0.1078	0.141	0.0469
SGER	0.5736	0.4359	0.1837	0.2007	0.4585	1	−0.329	−0.305	−0.145	−0.179	−0.319	−0.33	0.0428	−0.3	−0.282	−0.062	−0.174	−0.221	−0.144	0.0445	0.2843
SITA	0.2942	0.2748	0.299	0.1915	0.3198	0.3613	1	−0.373	−0.429	−0.33	−0.359	−0.434	−0.383	−0.227	−0.193	0.1002	0.0834	−0.051	0.1579	0.1355	−0.035
SJAP	0.204	0.2171	0.3208	0.1808	0.2529	0.1719	0.2154	1	−0.216	−0.107	−0.52	−0.49	−0.332	−0.166	−0.098	−0.011	0.1004	−0.397	−0.101	0.1351	−0.162
SNET	0.5409	0.4771	0.5104	0.3092	0.4436	0.5504	0.3308	0.2541	1	−0.174	−0.248	−0.192	−0.065	−0.269	−0.154	−0.025	−0.138	−0.269	−0.161	0.0515	0.0152
SNOR	0.4583	0.4016	0.478	0.2702	0.2771	0.3129	0.1699	0.1472	0.4938	1	−0.106	−0.085	0.0571	−0.126	0.184	0.0902	−0.107	−0.164	−0.118	0.2011	−0.163
SSPA	0.1961	0.1922	0.2441	0.1482	0.2068	0.1879	0.1718	0.1545	0.2269	0.1561	1	−0.54	−0.319	−0.258	−0.195	−0.103	−0.042	−0.343	−0.129	0.1046	0.0906
SSWE	0.3481	0.3252	0.4033	0.2333	0.3061	0.3015	0.2335	0.2044	0.3934	0.3393	0.1727	1	−0.326	−0.269	−0.124	−0.101	−0.079	−0.289	−0.27	0.1397	−0.081
SSWI	0.6047	0.5082	0.4446	0.2991	0.4904	0.7328	0.3712	0.2471	0.6312	0.4785	0.2324	0.3979	1	−0.242	−0.179	−0.13	−0.067	−0.346	−0.281	−0.062	−0.053
SUKG	0.4072	0.4045	0.5752	0.3105	0.3995	0.2982	0.3102	0.2956	0.4866	0.3979	0.2306	0.3662	0.4667	1	−0.165	−0.182	−0.035	−0.215	−0.388	0.0305	0.011
SUSA	0.4667	0.4669	0.7013	0.3644	0.4228	0.2635	0.3129	0.3178	0.5626	0.5339	0.2506	0.4281	0.512	0.592	1	−0.235	−0.195	−0.348	−0.244	−0.239	−0.095
CAN	0.2462	0.2653	0.3826	0.2173	0.3455	0.2575	0.3028	0.2792	0.3117	0.1109	0.1999	0.2472	0.32	0.371	0.3737	1	−0.057	0.0556	−0.238	−0.321	−0.03
GER	0.125	0.1526	0.1862	0.1309	0.3208	0.2941	0.3181	0.2676	0.1717	−0.162	0.1774	0.1429	0.2268	0.2444	0.1589	0.4413	1	0.0577	0.0236	0.1122	0.2779
JAP	0.0248	0.0887	0.2159	0.1202	0.2385	0.0697	0.2499	0.2535	0.0867	−0.171	0.1553	0.1114	0.0841	0.2281	0.1706	0.4029	0.5754	1	−0.106	0.1657	−0.051
UKG	0.1196	0.1599	0.2722	0.1552	0.2718	0.1511	0.2598	0.2505	0.1785	−0.041	0.1667	0.1635	0.1821	0.275	0.2453	0.3793	0.4875	0.4614	1	0.0227	0.0736
USA	0.2542	0.2729	0.3837	0.2209	0.3667	0.288	0.3241	0.2933	0.321	0.0918	0.2088	0.2523	0.3376	0.3805	0.3738	0.4281	0.4893	0.4406	0.4106	1	0.154
USN	0.2176	0.1913	0.1648	0.1245	0.2193	0.3015	0.1857	0.1356	0.23	0.1193	0.1175	0.1557	0.2746	0.1886	0.1805	0.189	0.2103	0.1479	0.1595	0.2037	1

Lower diagonal part of the matrix includes maximum likelihood correlations based on 72 observations (Feb 82–Jan 88).
Upper diagonal part includes residuals between forecasted and actual correlations based on 36 observations (Feb 88–Jan 91).

Table 4.19

Forecast Error Decomposition of MLFM Correlations

MODEL	MSFE	RMSFE	BIAS%	VAR%	COV%	TIC
STO/U	.0180(.0182)	.1357	.0057	.0004	.9998	.9937
STO/H	.0233(.0236)	.1527	.1762	.0030	.8207	.9942
COM/U	.0417(.0290)	.2042	---	---	---	1.198
COM/H	.0442(.0486)	.2103	---	---	---	.9534

The figures in brackets refer to the Historical Model

4.3.2.4 Out of sample Portfolio Performance for the MLFM

As was the case with the PCM, the quadratic optimization procedure includes the historical returns and volatilities as inputs, whereas the correlation matrix is the one forecasted via the MLFM. The constraints applied were the same as previously defined for the HIS and PCM models, whereas the objective function is now defined as:

$$\textit{Minimize} \quad \vec{x}' S\, M^{*} S\, \vec{x} \qquad (4.29)$$

where each entry i,j of $\mathbf{M}^*$ is defined as in equation 4.28

As with the HIS and PCM models, the composition of the optimized portfolios for the MLFM can be found in Appendix 4B, whereas the out of sample risk-adjusted performance for different holding periods and risk-free rates can be seen in Tables 4.20 - 4.23 below:

As was previously the case, portfolio performance is highly volatile depending on the holding period and portfolio type. The MLFM outperformed the HIS model in the case of unhedged stocks for all three risk-free rates in the three year holding horizon that corresponds to the correlation matrix forecasts. It also performed well in respect to the combined hedged portfolio. In most other cases, though, it underperformed the HIS model, while correlation estimates again appear to have a relatively small impact to the overall performance.

Table 4.20

Maximum Likelihood
Sharpe's Performance Measure
Unhedged Stocks

	n=12	n=24	n=36
Rf = 0	0.39878	0.37570	0.02921
	(0.44697)	(0.39487)	(0.01155)
Rf = 0.2	0.34210	0.31568	-0.01587
	(0.41161)	(0.30595)	(-0.05681)
Rf = 0.4	0.28505	0.25505	-0.06075
	(0.32752)	(0.26906)	(-0.07603)

Numbers in brackets are based on historical data

Table 4.21

Maximum Likelihood
Sharpe's Performance Measure
Unhedged Stocks and Bonds

	n=12	n=24	n=36
Rf = 0	0.05786	0.03480	-0.06151
	(0.04604)	(0.02552)	(-0.08339)
Rf = 0.2	-0.05192	-0.06670	-0.14276
	(-0.08260)	(-0.10250)	(-0.19234)
Rf = 0.4	-0.15408	-0.15885	-0.21491
	(-0.21471)	(-0.23563)	(-0.30172)

Numbers in brackets are based on historical data

Table 4.22

Maximum Likelihood
Sharpe's Performance Measure
Hedged Stocks

	n=12	n=24	n=36
Rf = 0	0.82283	0.50901	0.06158
	(0.88576)	(0.57390)	(0.09629)
Rf = 0.2	0.74261	0.42602	0.01414
	(0.81027)	(0.49711)	(0.05361)
Rf = 0.4	0.673213	0.35518	-0.02833
	(0.73328)	(0.40930)	(0.00660)

Numbers in brackets are based on historical data

Table 4.23

Maximum Likelihood
Sharpe's Performance Measure
Hedged Stocks and Bonds

	n=12	n=24	n=36
Rf = 0	1.59084	1.31726	0.18297
	(1.52727)	(1.28410)	(0.17938)
Rf = 0.2	1.02218	0.75597	0.03158
	(0.91933)	(0.72763)	(0.02806)
Rf = 0.4	0.48661	0.24265	-0.12063
	(0.37587)	(0.17627)	(-0.13103)

Numbers in brackets are based on historical data

4.3.3 A "Pseudo-APT" Model with Unobservable Factors for Stock Index Returns

As we have previously established, the two unobservable factor models used, i.e. the PCM and the MLFM were capable in eliminating "noise" from the historical correlation matrix and provide improved correlation forecasts; Nevertheless, in the absence of alternative estimates for the mean return vector and the index volatilities, no significant improvements in the out of sample risk-adjusted portfolio performance were recorded.

In this section, an attempt is being made to establish estimates of expected future stock index returns, by attempting to create a "Pseudo-APT" type framework. The term "Pseudo" in this context is used to denote the fact that our universe of assets does not include individual stocks, but rather market capitalization weighted portfolios aggregated by country.

One practical justification for such an approach, lies in the fact that investors should be able to create easily and inexpensively zero cost arbitrage portfolios by short selling stock index futures contracts when necessary, whereas in the standard APT for individual stocks such portfolios are difficult to create. Nevertheless, the zero risk conditions required by the APT i.e.

$$\begin{aligned} w_i &\approx 1/k \\ k &= \textit{large number} \\ \sum_{i=1}^{k} w_i b_{is} &= 0, \quad \forall \textit{ factors} \end{aligned} \tag{4.30}$$

can only approximately be satisfied, since the "asset" universe is limited to the 15 indices.

Testing of the "Pseudo-APT" can be achieved through conventional methods: At first, the standard multifactor return generating process needs to be defined as:

$$R_i = a_i + \sum_{j=1}^{s} b_{ij} F_j + e_i \qquad (4.31)$$

In this context, as shown in the previous section, the MLFM can be used to simultaneously determine a specific set of factor loadings and uncorrelated factors such as that the covariance of the "residual[27]" returns is the smallest possible. These factor loadings estimates, of course, are identical to those we previously produced on the basis of the likelihood ratio criteria in section 4.3.2

These MLFM first-pass estimates of the factor loadings can be subsequently used to describe the APT that results from the aforementioned return generating process (see Elton & Gruber 1992), i.e.

$$\bar{R}_i = \lambda_0 + \sum_{j=1}^{s} b_{ij} \lambda_j \qquad (4.32)$$

where $\bar{R}_i$ denotes the sample mean index return for index i

b_{ij} is the factor loading of index i with factor j

λ_j denotes the risk-premium associated with factor j

λ_0 is the expected return of an index with zero exposure to all factors

The second step in the "Pseudo-APT" testing procedure is to estimate equation 4.32 above cross-sectionally by means of Generalized Least Squares (GLS) in order to establish whether the risk-premia associated with each factor are statistically significant and, therefore, likely to be priced. For that purpose, recall that in the maximum likelihood procedures used in section 4.3.2 three significant factors were included the unhedged and two for the hedged[28] stock returns.

[27] Residual here denotes that part of the returns that remains unexplained after the influence of the common factors has been removed

[28] The hedged "Pseudo-APT" model should be thought as a hybrid of Ikeda's (1991) International APT previously discussed in Chapter I.

The estimated cross-section equations for the monthly unhedged returns is:

$$E(r_i) = \underset{(1.583)}{1.185} + \underset{(0.017)}{0.012}\, b_{i1} + \underset{(0.123)}{0.122}\, b_{i2} + \underset{(1.794)}{1.246}\, b_{i3}$$

Whereas, the estimated equation for the hedged returns can be written as:

$$E(r_i) = \underset{(2.606)}{1.306} - \underset{(-0.017)}{0.063}\, b_{i1} - \underset{(-0.123)}{0.081}\, b_{i2}$$

As we can see from the t-values reported in brackets above, none of the risk premia appear to be significant at the 5% significance level. The only statistically significant coefficient is the constant in the equation for the hedged returns. From this it is not possible to conclude that we should accept the joint hypothesis that both the "Pseudo-APT" and our particular testing methodology are valid. In fact, there does not appear to be any meaningful way of using this model in subsequent analysis.

In any event it is necessary to emphasize that our testing methodology is not free from several serious flaws; First of all, 15 indices is a relatively small number to assure satisfactory results from the second pass cross-section regression. Then, the magnitude and statistical significance of the risk premia have been estimated in a single point in time; In general, for this type of testing it would be preferable, but also too computationally laborious, to re-estimate average returns, factor loadings and the magnitude and significance of the risk premia for each new observation. This would allow us to establish, for example, the average size of the risk premia over time, or the percentage of the total time interval where the risk-premia associated with each factor were found to be statistically significant.

4.4. Models with Observable Factors: A "Pseudo Single Index" Model (PSIM)

All forecasting and asset allocation procedures, previously discussed in section 4.3, had the common characteristic that correlations were expressed on the basis of unobservable common factors. These models generally demonstrated, that even though more than one factors was found to be significant, the first factor extracted tended to have by far the greater explanatory power. This fact suggests that an observable single index representation might be a potentially useful means of describing the index return generating process.

4.4.1 Rationale of the PSIM Model for Unhedged Stock Index Returns

It is well known that an ordinary international single index model, where individual stock returns are regressed against a world index, is a rather poor tool for describing security returns due to the importance of country and (to a lesser extent) industry specific factors. These additional common influences are reflected in the regression residuals which tend to be highly correlated for stocks that belong in the same country or industry.

This problem could potentially be mitigated when applying PSIM, where stock index returns are being regressed against a world index. In fact, stock index returns should be affected primarily by a global factor and a country specific factor that should be captured by the regression residuals.

A more difficult issue lies in the selection of an appropriate world index against which the stock index returns will be regressed. An obvious choice would be to rely on a market capitalization weighted world index[29]; The problem, though, with such an approach is that movements of such an index depend on a very large extent to USA and Japan, who jointly account for over 80% of world market capitalization, so that the model could provide biased estimates. For this reason, apart from the market weighted index, an equally weighted index of the 15 stock markets is also used.

[29] The Morgan Stanley Capital International World Index

Consequently, the general form of PSIM can be expressed as follows:

$$\begin{aligned} R_{it} &= a_i + b_i I_{Mt} + u_{it} \qquad i = 1....k \\ R_{it} &= c_1 + d_i I_{Et} + e_{it} \qquad i = 1....k \\ I_{Et} &= \frac{1}{k} \sum_{i=1}^{k} R_{it} \end{aligned} \tag{4.33}$$

where a_{it}, c_{it} denote the constant part of index returns

u_{it}, e_{it} denote the residual returns

b_i, d_i are the factor sensitivities with the single common factor

I_M, I_E stand for market-weighted and equally weighted indices respectively

Notice that this model cannot be applied to "combined portfolios", since it would be absurd to assume a single global common factor for both bond and stock index returns. Also it is very difficult to define a meaningful hedged world index that is appropriate proxy for hedged returns generated through an "ex-ante" strategy. For these reasons the model specifications that follow are being applied exclusively to unhedged stock index returns.

4.4.2 Estimating Index Correlations with a Bayesian PSIM

As another alternative to the direct estimation of correlation coefficients, we can assume that index returns move together because of their relation to a single observable common return generating factor. Assuming that the standard OLS assumption of uncorrelated residuals holds, the covariance and correlation between indices i and j, can be expressed as:

$$cov(i,j) = \beta_i \beta_j \sigma_I^2 \tag{4.34a}$$

$$\rho_{ij} = \frac{\beta_i \beta_i \sigma_w^2}{\sigma_i \sigma_j} \tag{4.34b}$$

where σ_I^2 is the variance of the world index (I_M or I_E)

σ_i σ_j standard deviation of indices i and j

The one remaining issue, before applying the aforementioned correlation forecasting model can be applied, is to provide the most accurate possible estimates of the factor loadings b_i, b_j. The most obvious approach is to choose historical betas estimated from standard OLS. Historical betas, though, are known to be non-stationary and subject to sampling errors; Consequently, since the correlation coefficient estimates depend almost entirely on the choice of betas, it is advisable to perform "corrections" on historical betas in order to improve the forecasting ability of PSIM. The beta adjustment procedure applied in the context of this thesis is based on a Bayesian methodology first recommended by Vasicek (1973) as a means of providing beta estimates for US stocks[30].

The main idea behind this Bayesian approach is that estimated betas are in fact weighted averages between the index's own historical beta and the cross-sectional average beta across all indices. Naturally, the greater the uncertainty associated with the measurement of an individual beta, the smaller the beta's weight in the forecast should be. In statistical terms, the measure for "uncertainty" of index i's beta estimate can be defined in terms of its standard error $\sigma^2_{\beta i1}$ which is calculated as follows:

$$\sigma_{\beta iI} = \sqrt{\frac{\sigma^2_{ei}}{\sigma^2_I}} \tag{4.35}$$

where σ^2_{ei}, σ^2_I denote the variance of the regression residuals and the variance of the Index (I_M or I_E) respectively

Then, the beta standard error forms the basis of weighting the betas as follows:

---

30 As a matter of fact, an attempt was made to forecast the index betas by means of yet another technique, originating from Blume (1975). The procedure followed consisted in splitting the forecasting period in two equal halves of three years each and then estimating the sample betas by OLS for each subperiod separately. These betas were subsequently cross-sectionally regressed to derive an equation of best fit that forms the basis of future beta forecasts. Unfortunately the cross-section regression resulted in a very poor fit implying improbable beta estimates, so that the results have not been included in the thesis.

$$\frac{\sigma^2_{\beta 1}}{\sigma^2_{\beta 1} + \sigma^2_{\beta i1}} = w\,(\beta_{i1}) \qquad (4.36a)$$

$$\frac{\sigma^2_{\beta i1}}{\sigma^2_{\beta 1} + \sigma^2_{\beta i1}} = w\,(\beta_1) \qquad (4.36b)$$

where β_1 is the cross-sectional mean of all betas in period 1
β_{i1} is the historical beta of index i estimated from period 1
$\sigma^2_{\beta 1}$ is the cross-sectional variance across all betas from period 1

This results to the following estimation equation

$$\beta_{i2} = \frac{\sigma^2_{\beta 1}}{\sigma^2_{\beta 1} + \sigma^2_{\beta i1}}\,\beta_{i1} + \frac{\sigma^2_{\beta i1}}{\sigma^2_{\beta 1} + \sigma^2_{\beta i1}}\,\beta_1 \qquad (4.37)$$

where β_{i2} denotes the beta forecast for index i for period 2

Effectively, equation 4.37 assures that a beta with high volatility will be adjusted so that to be closer to the overall mean, while a beta with low volatility will have an estimated value close to its historical beta. Notice, though, that here all estimates are based on the cross-sectional variance of the estimated betas, rather than the (smaller) variance of the true (but unobservable) betas as originally syggested by Vasicek. This results to slightly higher estimated weights for each index's own beta as opposed to the global mean beta.

The implementation of the Bayesian model took place in three steps. First OLS was used to estimate the historical betas (β_{i1}) and standard errors ($\sigma^2_{\beta i1}$) of the individual indices against both the equally weighted and market value weighted world index. Then, from the individual betas, the overall mean (β_1) of the betas and the cross-sectional variance of the betas ($\sigma^2_{\beta 1}$) has been calculated. Finally, equation 4.37 was used to forecast the mean unadjusted Bayesian betas (b_{i2}) and subsequently use them as inputs for correlation forecasts and the portfolio optimisation program. Table 4.24 provides summary information about these aforementioned statistics, whereas Table 4.25 shows the forecasted Bayesian correlation matrices based on Equation 4.34b.

Table 4.24

Global Single – Index Forecasted Betas

Unhedged Stocks

Stock	Beta EQWET	SE(BETA) EQWET	B* EQWET	Beta MVWET	SE(BETA) MVWET	B* MVWET
Aut	1.050904	0.082091	1.036251218	1.005763	0.082796	1.004232946
Bel	1.021704	0.08322	1.01533423	0.857992	0.100737	0.907491798
Can	1.026357	0.100143	1.016457526	1.055643	0.088373	1.039412859
Den	0.712941	0.116629	0.841916307	0.622248	0.119209	0.784056939
Fra	1.037794	0.100898	1.023465488	0.849958	0.117779	0.913342835
Ger	1.016976	0.112743	1.009632294	0.751933	0.133909	0.872492356
Ita	1.083118	0.140607	1.038025932	0.844954	0.1561	0.932141588
Jap	0.828579	0.128993	0.914204507	0.927882	0.11281	0.956842124
Net	1.065806	0.079243	1.047801841	0.969307	0.088744	0.978312864
Nor	1.289387	0.133955	1.139377806	1.078245	0.151145	1.035492151
Spa	0.982253	0.140983	0.991904386	0.800882	0.150624	0.909338595
Swe	1.056527	0.130063	1.028058051	0.872186	0.14223	0.938154433
Swi	0.955633	0.070319	0.965781696	0.78559	0.090596	0.850353444
UKG	0.826171	0.093973	0.886362417	0.766667	0.095845	0.842807522
USA	1.04629	0.090092	1.031133315	1.155887	0.058241	1.132235861

Beta = Historical
SE(BETA) = Standard Errors of Betas
B* = Forecasted Beta using Bayesian Method
EQWET = Equally Weighted
MV = Market Value Weighted

Table 4.25

Forecasted Correlation Matrix of Unhedged Stock Returns (Single Index Model: Equally Weighted)

Bayesian Method

	Aut	Bel	Can	Den	Fra	Ger	Ita	Jap	Net	Nor	Spa	Swe	Swi	UKG	USA
Aut	1	0.0810632	0.0799247	0.1486358	−0.018159	0.0419983	−0.162983	−0.077554	0.0538649	0.0974071	0.01739	−0.092029	0.1166829	0.16149	0.0857052
Bel	0.6436632	1	0.0697522	0.0328466	−0.221922	−0.200017	−0.161082	0.1117567	−0.027078	0.1310383	0.0421034	0.0729559	0.1584351	0.2552794	0.1208042
Can	0.5988247	0.5957522	1	0.0793181	0.0631449	0.1160392	−0.06143	0.2559896	−0.155316	−0.028256	0.1073217	0.0729318	0.0847744	0.0097913	−0.307009
Den	0.5404358	0.5376466	0.4998181	1	−0.11444	−0.214654	−0.08943	0.116302	−0.037056	0.0469105	0.0574382	−0.080621	−0.057618	0.1206538	0.0919586
Fra	0.5967407	0.5936783	0.5521449	0.4980598	1	−0.260599	−0.162768	0.0628947	−0.092724	0.1083613	−0.111909	−0.09438	0.011801	0.1129305	−0.051927
Ger	0.5664983	0.5635827	0.5240392	0.4725455	0.5222013	1	−0.274542	0.1261986	−0.183003	−0.004751	−0.08827	−0.150817	−0.071638	−0.000824	0.0729926
Ita	0.5012168	0.4986179	0.46337	0.41747	0.4617317	0.4379579	1	−0.051895	−0.253912	−0.049448	−0.166147	−0.24743	−0.244418	−0.026747	−0.014814
Jap	0.5197456	0.5170567	0.4805896	0.433102	0.4788947	0.4542986	0.4012052	1	0.2062231	0.2619088	−0.21415	−0.180613	0.040605	0.0437865	0.2748262
Net	0.6545649	0.6512219	0.6058837	0.5468439	0.6037764	0.573197	0.5071878	0.5259231	1	−0.067362	0.0314328	−0.011591	0.0358042	−0.039863	−0.096741
Nor	0.5156071	0.5129383	0.4767436	0.4296105	0.4750613	0.4506489	0.3979519	0.4129088	0.5217385	1	0.1608123	0.0077884	0.1673891	−0.071407	0.0930193
Spa	0.49869	0.4961034	0.4610217	0.4153382	0.4593912	0.4357296	0.3846533	0.3991502	0.5046328	0.3959123	1	−0.343605	−0.056553	−0.015724	0.03434
Swe	0.525171	0.5224559	0.4856318	0.4376792	0.4839202	0.4590834	0.4054702	0.4206872	0.531409	0.4172884	0.4033951	1	−0.153737	−0.150709	0.0164791
Swi	0.6761829	0.6727351	0.6259744	0.5650823	0.623801	0.5922621	0.5241819	0.543505	0.6841042	0.5391891	0.5215468	0.5491631	1	0.0128769	0.0527901
UKG	0.60669	0.6035794	0.5613913	0.5064538	0.5594305	0.5309757	0.469553	0.4869865	0.6138367	0.4830927	0.4671756	0.4920913	0.6341769	1	−0.009694
USA	0.6253052	0.6221042	0.5786912	0.5221586	0.5766734	0.5473926	0.4841865	0.5021262	0.6326593	0.4981193	0.48174	0.5073791	0.6535901	0.5863065	1

Forecasted Correlation Matrix of Unhedged Stock Returns (Single Index Model: Market Value)

Bayesian Method

	Aut	Bel	Can	Den	Fra	Ger	Ita	Jap	Net	Nor	Spa	Swe	Swi	UKG	USA
Aut	1	0.0589034	0.145178	0.157144	−0.037329	0.0079662	−0.17103	−0.002521	0.0590095	0.0943426	0.0198087	−0.093598	0.0821274	0.1799437	0.2033568
Bel	0.6215034	1	0.0834598	−0.001011	−0.285537	−0.274935	−0.207101	0.1405561	−0.07285	0.0884794	0.0058852	0.0310302	0.0745544	0.2254255	0.1805555
Can	0.664078	0.6094598	1	0.1178055	0.0773786	0.1141468	−0.041188	0.3586525	−0.114274	−0.002391	0.1376965	0.1007547	0.0879941	0.0614296	−0.157139
Den	0.548944	0.5037889	0.5383055	1	−0.144328	−0.255596	−0.107155	0.1653221	−0.049136	0.0327508	0.048281	−0.093883	−0.102596	0.1209402	0.1720552
Fra	0.5775714	0.5300633	0.5663786	0.4681718	1	−0.328683	−0.203902	0.0912676	−0.133848	0.0704222	−0.143931	−0.131744	−0.064771	0.0866684	0.0050605
Ger	0.5324662	0.4886654	0.5221468	0.4316035	0.4541166	1	−0.324758	0.1395485	−0.237489	−0.052423	−0.130015	−0.198226	−0.159409	−0.040272	0.1097784
Ita	0.4931704	0.4525993	0.4836119	0.3997451	0.4205981	0.3877422	1	−0.019968	−0.28067	−0.074171	−0.185899	−0.271614	−0.301342	−0.041049	0.0420695
Jap	0.5947785	0.5458561	0.5832525	0.4821221	0.5072676	0.4676485	0.4331322	1	0.2597153	0.2991482	−0.173195	−0.141437	0.0606325	0.1058537	0.4252373
Net	0.6597095	0.6054503	0.6469261	0.5347638	0.5626523	0.5187113	0.4804298	0.5794153	1	−0.089798	0.0149632	−0.032924	−0.023245	−0.044703	−0.005631
Nor	0.5125426	0.4703794	0.502609	0.4154508	0.4371222	0.4029769	0.3732293	0.4501482	0.499302	1	0.1441387	−0.013233	0.1138135	−0.081365	0.1572215
Spa	0.5011087	0.4598852	0.4913965	0.406181	0.4273693	0.393985	0.3649005	0.440105	0.4881632	0.3792387	1	−0.359575	−0.10332	−0.02032	0.1023781
Swe	0.5236025	0.4805302	0.5134547	0.4244174	0.446556	0.4116745	0.3812856	0.4598629	0.5100764	0.3962669	0.3874246	1	−0.206807	−0.159455	0.0835546
Swi	0.6416274	0.5888544	0.6291941	0.520104	0.5472286	0.5044912	0.4672582	0.5635325	0.6250549	0.4856135	0.4747798	0.4960928	1	−0.028996	0.1031322
UKG	0.6251437	0.5737255	0.6130296	0.5067402	0.5331684	0.4915282	0.455251	0.5490537	0.6089967	0.4731351	0.4625796	0.4833454	0.5923036	1	0.089849
USA	0.7429568	0.6818555	0.7285614	0.6022552	0.6336605	0.5841784	0.5410695	0.6525373	0.723769	0.5623215	0.5497781	0.5744546	0.7039322	0.685849	1

Lower diagonal part of the matrix includes single−index model correlations based on 72 observations (Feb 82−Jan 88).
Upper diagonal part includes residuals between forecasted and actual correlations based on 36 observations (Feb 88−Jan 91).

Table 4.26 below summarizes the key information about the forecasting performance of the market value weighted and equally weighted Bayesian PSIM strategies:

Table 4.26

Forecast Error Decomposition of Bayesian PSIM Correlations

Unhedged Stock Index Returns

MODEL	MSFE	RMSFE	BIAS%	VAR%	COV%	TIC
MV-W	.0235(.0182)	.1334	.0045	.1068	.8886	1.135
EQ-W	.0173(.0182)	.1318	.0061	.2375	.7563	.9756

The figures in brackets refer to the Historical Model

As can be seen from the MSFE and the TIC the equally weighted Bayesian PSIM model had performed well, whereas the market value weighted PSIM performed very poorly. This fact certainly does little to refute the view that there is substantial bias in market capitalization weighted world indices, caused by the very high weights of a couple of countries, when used as regression proxies. Notice also the very substantial increase in the variance proportion of the forecast error for the equally-weighted method, compared to all previous methods, and the corresponding decline of the covariance proportion.

4.4.3 Out of sample Portfolio Performance of the Bayesian PSIM strategy

As with all previous methods, the correlation matrix being forecasted by means of the Bayesian PSIM methods was used as input for the quadratic optimizations in order to assess the out of sample portfolio performance of the two strategies for the different risk-free rates and portfolio holding periods. The resulting adjusted Sharpe performance measures can be seen in Tables 4.27 and 4.28 below:

Table 4.27

Vasicek's Bayesian PSIM
Sharpe's Performance Measure
Equally Weighted Unhedged Stocks

	n=12	n=24	n=36
Rf = 0	0.46579	0.41875	0.02614
	(0.44697)	(0.39487)	(0.01155)
Rf = 0.2	0.40842	0.35728	-0.01816
	(0.41161)	(0.30595)	(-0.05681)
Rf = 0.4	0.35081	0.29544	-0.06262
	(0.32752)	(0.26906)	(-0.07603)

Numbers in parenthesis are based on historical data.

Table 4.28

Vasicek's Bayesian PSIM
Sharpe's Performance Measure
Market Value Weighted Unhedged Stocks

	n=12	n=24	n=36
Rf = 0	0.42649	0.44184	0.06498
	(0.44697)	(0.39487)	(0.01155)
Rf = 0.2	0.37566	0.38394	0.02028
	(0.41161)	(0.30595)	(-0.05681)
Rf = 0.4	0.32472	0.32518	-0.02501
	(0.32752)	(0.26906)	(-0.07603)

Numbers in parenthesis are based on historical data.

For the three year holding period that corresponds to the correlation forecast horizon, the equally weighted strategy outperformed the historical model in all three portfolio, whereas when the shorter holding horizons are included it outperformed in 7 out of 9 cases. In this context we can argue that the Equally Weighted Bayesian PSIM showed both statistical and economic superiority vis a vis the historical model.

The market value weighted model, provides a very good illustration of the fact that inferior (superior) correlation forecasts do not necessarily imply inferior (superior) portfolio performance. In fact the market value weighted model also outperformed the his model in 6/9 cases, including all three portfolios with a three year horizon, despite the fact that its correlation forecasting ability was poor for the same time period.

This phenomenon is not as paradoxical as it might seem, because the improved correlation forecasts can slightly reduce portfolio risk but can also lead to the selection of portfolio weights that are high for indices with poor returns over the validation period. This further strengthens the view that portfolio performance is relatively insensitive to correlation estimates in the absence of reliable forecasts for expected index returns.

4.5. Index Allocation with "Estimation Risk" Controlled by means of Conventional Strategies: Bayesian and Empirical Bayesian (James-Stein) Estimators

The common theme behind all factor based allocation models discussed in sections 4.3 and 4.4 was that they were capable of providing forecasts of the index correlation matrix, whereas the remaining inputs (variances and the mean return vector) used in the quadratic optimizations were simply the historical sample values. In this section the attempt made is to address the empirical issue of whether part of the "estimation error" or "noise" in the historical returns and variance-covariance matrix can be satisfactorily reduced by means of using standard Bayesian and empirical Bayesian (James-Stein) estimators. Subsequently, in section 4.6 we introduce some more "intuitive" approaches towards controlling estimation risk in index inputs.

4.5.1 An International Index Allocation Model based on Klein & Bawa's Bayesian Approach

4.5.1.1 The Theoretical Framework

The model to be utilized in this section was originally developed by Klein and Bawa (1979) and subsequently adjusted by Bawa and Brown (1979), but never applied in the context of international index returns. This model is still incapable of estimating the mean return vector, but can be used to adjust historical estimates of variances and covariances.

On the basis of the Klein & Bawa model, investors are (realistically) assumed to have "diffuse" or "non-informative" priors, i.e. to have very little prior information about the true values of the means, variances, and covariances before calculating the sample values. Consequently the prior information becomes dominated by the sample information. In this context, Klein & Bawa demonstrated that even when investors do not have quadratic utility functions, in the presence of "estimation risk" a mean variance optimization framework is still appropriate if (N - k) > 0. Then, it is possible to derive appropriate unbiased estimates of variances and covariances as follows:

$$\sigma_i^{2*} = \frac{(N+1)(N-1)}{N(N-k)}\sigma_i^2 \tag{4.38a}$$

$$V^* = \frac{(N+1)(N-1)}{N(N-k)} V \tag{4.39a}$$

where σ_i^2, V denote the historical variance of asset i and the variance-covariance matrix respectively.

Subsequently, Bawa & Brown recommended a slightly modified version of the previous equations, i.e.

$$\sigma_i^{2*} = \frac{(N+1)\,(N-1)}{N(N-k-2)}\,\sigma_i^2 \tag{4.38b}$$

$$V^* = \frac{(N+1)\,(N-1)}{N(N-k-2)}\,V \tag{4.39b}$$

Notice from equation 4.38b & 4.39b above that the historical variance and covariance terms are adjusted by means of a coefficient that is a function of the sample size relative to the number of assets. As the sample size N becomes very large, the adjustment coefficient will be approximately equal to one, implying that the Bayesian forecast will be virtually identical to historical estimates. Conversely, the adjustment coefficient will be large and the two methods will lead to substantially different estimates for small values of (N - k).

Another interesting aspect, is that since the variance and covariance terms are scaled by an equal amount, the correlation coefficients will not be affected by the Klein and Bawa model. To verify this, by denoting the adjustment factor as **a** we can express the Klein & Bawa correlations as

$$\rho_{ij} = \frac{cov_{ij}\,a}{\sigma_i\sqrt{a}\;\sigma_j\sqrt{a}} = \frac{cov_{ij}}{\sigma_i\sigma_j} = \rho_{ij} \tag{4.40}$$

Thus, the adjusted correlation matrix will be identical to the historical correlation matrix.

Notice also that as shown by Alexander & Francis (1986), this Bayesian approach will lead to a parallel rightward shift of the historical efficient frontier, the parallel distance depending on the adjustment coefficient. Essentially the composition of all points on the Bayesian efficient frontier will be identical to those of the historical efficient frontier, due to the fact that the all elements in the two covariance matrices differ by the same constant amount[31]. Nevertheless, because of the rightward shift in the frontier the same portfolios are now perceived by the investors as being more "risky", so that they are choosing more "conservative" portfolios for all risk free rates.

Essentially, the key aspect of the Bayesian method lies in acknowledging that in reality the investment risk consists of both the historical volatility of the asset and an additional risk component associated with estimation errors as a function of sample size. So, for a Bayesian the historical volatility underestimates the risk as perceived by the investors. In this context, for two portfolios have identical sample means and volatilities, the investors will prefer the one whose sample values are based on the larger number of observations.

4.5.1.2 Out of Sample Performance of the Klein & Bawa Bayesian Model

As was the case with the previous models, the Klein & Bawa covariance matrices for hedged, unhedged and combined portfolios were used as input to the optimization program in order to derive optimal "ex-ante" portfolios for three different risk-free rates. These portfolios were subsequently reinvested for three different holding periods each. The adjusted Sharpe measures are listed below in Tables 4.29 - 4.32:

As can be immediately observed from the results, the Klein & Bawa model performed extremely well for the hedged stocks, outperforming the historical benchmark in 9 out of 9 cases. It also outperformed the unhedged stock historical portfolio in 6 out of 6 cases related to the two and three year horizons, the historical model, though, being superior for the one year horizon. Nevertheless, the Klein & Bawa model's performance was rather unimpressive for the unhedged and hedged combined portfolios.

[31] So that the quadratic optimizations will provide the same weights for the two frontiers at all points.

Table 4.29

Klein and Bawa Bayesian Model
Sharpe's Performance Measure
Unhedged Stocks

	n=12	n=24	n=36
Rf = 0	0.43808	0.40988	0.02861
	(0.44697)	(0.39487)	(0.01155)
Rf = 0.2	0.38039	0.34839	-0.01586
	(0.41161)	(0.30595)	(-0.05681)
Rf = 0.4	0.32145	0.28757	-0.05905
	(0.32752)	(0.26906)	(-0.07603)

Numbers in brackets are based on historical data

Table 4.30

Klein and Bawa Bayesian Model
Sharpe's performance measure
Unhedged stocks and bonds

	n=12	n=24	n=36
Rf = 0	0.04812	0.01433	-0.08781
	(0.12036)	(0.01789)	(-0.06642)
Rf = 0.2	-0.08352	-0.11147	-0.19535
	(-0.00653)	(-0.10513)	(-0.17005)
Rf = 0.4	-0.21739	-0.24801	-0.30412
	(-0.10967)	(-0.30326)	(-0.27041)

Numbers in brackets are based on historical data

Table 4.31

Klein and Bawa Bayesian Model
Sharpe's Performance Measure
Hedged Stocks

	n=12	n=24	n=36
Rf = 0	0.89488	0.59490	0.10085
	(0.88576)	(0.57390)	(0.09629)
Rf = 0.2	0.82117	0.52184	0.05946
	(0.81027)	(0.49711)	(0.05361)
Rf = 0.4	0.74907	0.44948	0.01863
	(0.73328)	(0.40930)	(0.00660)

Numbers in brackets are based on historical data

Table 4.32

Klein and Bawa Bayesian Model
Sharpe's Performance Measure
Hedged Stocks and Bonds

	n=12	n=24	n=36
Rf = 0	1.35073	0.99541	0.16576
	(1.52727)	(1.28410)	(0.17938)
Rf = 0.2	0.86029	0.57240	0.01381
	(0.91933)	(0.72763)	(0.02806)
Rf = 0.4	0.39386	0.16404	-0.13897
	(0.37587)	(0.17562)	(-0.13103)

Numbers in brackets are based on historical data

4.5.2 International Index Allocation with Jobson-Korkie Estimators

4.5.2.1 The Theoretical Background

Until now, all asset allocation models used utilized the historical mean return vector as an input to the portfolio optimization problem. Since portfolio performance is likely to be more sensitive to return estimates, as opposed to variances and correlations, it is very useful to consider approaches that utilize alternative estimates for the mean return vector.

Such a multivariate approach can be developed on the basis of the Empirical Bayesian [32] (James-Stein) estimators, first related to finance by Jobson, Korkie and Ratti (1979) and subsequently examined by Jobson & Korkie (1980, 1981), Jorion (1984, 1989) and Dumas & Jacquillat (1990).

As a first step, Jobson & Korkie developed an estimate of the variance-covariance matrix, based on the James-Stein methodology. Their estimate is very similar to the Klein & Bawa Bayesian method, and can be expressed as follows:

$$V^{**} = V \frac{N - 1}{N - k - 2} \tag{4.41}$$

where V^{**} refers to the Jobson-Korkie covariance matrix estimate

V is the historical covariance matrix

Subsequently Jobson & Korkie proceeded to develop, on the basis of James-Stein estimators, an appropriate estimate of portfolio returns: in brief, they showed that for portfolios consisting of one or two assets only there is no alternative estimator, linear or non-linear, with a uniformly smaller expected square error compared to the simple sample mean. Nevertheless, a general form of such an estimator can be derived for portfolios with a minimum of three assets.

[32] "Empirical" Bayesian estimators differ from the traditional Bayesian methods in that it is not required to have investor knowledge of prior distributions.

This Jobson-Korkie estimate of the mean return vector depends on a "shrinkage" coefficient c and can be defined as

$$\vec{E}^{*} = e^{*}\vec{I} + c(\vec{E} - e^{*}\vec{I}) \tag{4.42a}$$

$$e^{*} = \frac{1}{n}\vec{E}'\vec{I} \tag{4.42b}$$

where $\vec{E}^{*}, \vec{E}$ stand for the Jobson-Korkie estimate of the mean return vector and its sample value respectively

$0 \leq c \leq 1$

$\vec{I}$ is the unit vector

e^{*} is the "grand mean" or average of mean returns across all assets

Effectively, the closer $\hat{w}$ is to unity (zero) the more predictive ability is placed on the grand mean (sample mean).

If the Jobson-Korkie mean estimates are expressed in terms of a single asset, then the equation 4.42a becomes:

$$r_i^{*} = e^{*} + c(r_i - e^{*}) \tag{4.43}$$

where r_i is the historical mean return on asset i

r^{*}_{i} is the Jobson-Korkie estimate of future return on asset i

Essentially, the Jobson-Korkie estimator of the mean is nothing but a weighted average between the "grand mean" and a security's own sample mean, the weighting being dependant on the shrinkage factor whose value is not explicitly derived.

Jobson & Korkie's own analysis led them to the conclusion that the best policy would be to set c = 0, thus forecasting for all assets the same expected return, namely the "grand mean" e^{*}. Their simulation results suggested that their estimators became quite reliable when the sample size was around 60 or more monthly observations, as opposed to about 300 for the historical model. They also found their estimation procedure is robust in the sense that the results seem to be unaffected by number of securities and the level of the risk-free rate.

Nevertheless, the issue of determining an appropriate value for c remained unresolved. In fact, no universally accepted method for estimating the shrinkage factor exists, the estimates being totally dependant on the choice of a "suitable" prior. One such possibility is a shrinkage estimate derived by Jorion (1984) as:

$$\hat{c} = \frac{\hat{\lambda}}{(N+\hat{\lambda})}$$
$$\hat{\lambda} = \frac{(k+2)(N-1)}{(\vec{E}-e^{*}\vec{I})' S^{-1} (\vec{E}-e^{*}\vec{I})(N-k-2)} \tag{4.44}$$

where S stands for the sample variance-covariance matrix

Unfortunately, though, as shown subsequently by Dumas & Jacquillat (1990) the Jorion estimator suffers both from an inappropriate choice of prior and in terms of failing to justify the assumptions underlying its derivation[33]. From a practical viewpoint also Dumas & Jacquillat also found that the Jorion estimator leads to very high c values, weighting heavily the sample mean, whereas their own simulation results agree with those of Jobson & Korkie that in general small values of c provide optimal results. This view is further supported by empirical evidence from Eun & Resnick (1988) whose results also provide evidence that the Jobson-Korkie estimators perform better than the Jorion estimators.

4.5.2.2 Formulating the Optimization Problem with Jobson-Korkie Estimators

Because of the aforementioned deficiencies of the Jorion estimator, the three model versions applied in this section, utilize as shrinkage coefficient the Jobson-Korkie suggestion of c=0, as well as two further arbitrarily chosen "small" values of c=.2 and c=.4 respectively. The choice of testing the performance of some arbitrarily defined values of c is based on two main considerations, i.e.

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[33] This issue has already been covered in more detail in Chapter I of the thesis.

i) to apply a method that, in contrast to the standard Jobson-Korkie, gives some positive weight to the index's own sample mean and

ii) to examine how sensitive the out of sample performance of the Jobson-Korkie estimators will be to the choice of c.

Effectively, the mean return vector input for the quadratic optimization problem is been estimated respectively as:

$$\vec{E}^* = e^* \vec{I} \qquad \textbf{(4.45a)}$$

$$\vec{E}^* = e^* \vec{I} + .2(\vec{E} - e^* \vec{I}) \qquad \textbf{(4.45b)}$$

$$\vec{E}^* = e^* \vec{I} + .4(\vec{E} - e^* \vec{I}) \qquad \textbf{(4.45c)}$$

whereas the Jobson-Korkie estimate of the variance-covariance matrix V^{**}, is defined as in equation 4.41.

4.5.2.3 Performance of the three Jobson-Korkie Model Versions

The Jobson-Korkie model versions were naturally applied only to unhedged and hedged index returns, since a "combined" model version would have to be based on the absurd use of a vector with equal expected returns for stock and bond indices together. As with previous models, 72 monthly observations from February 1982 to January 1988 were used as estimating period while the out of sample performance was measured for 1, 2, and 3 year holding periods. The performance of the three strategies is being summarized in Tables 4.33 and 4.34 below:

Table 4.33

Jobson-Korkie Estimators
Sharpe's Performance Measure
Unhedged Stocks

	n=12	n=24	n=36
Rf = 0			
c = 0	0.43324	0.41497	0.03589
c = 0.2	0.44864	0.39037	0.00716
c = 0.4	0.43311	0.40733	0.02949
	(0.44697)	(0.39487)	(0.01155)
Rf = 0.2			
c = 0	0.37544	0.35533	-0.00722
c = 0.2	0.38563	0.33728	-0.02739
c = 0.4	0.37671	0.34773	-0.01420
	(0.41161)	(0.30595)	(-0.05681)
Rf = 0.4			
c = 0	0.31764	0.29569	-0.05032
c = 0.2	0.32712	0.27088	-0.07450
c = 0.4	0.31948	0.28754	-0.05825
	(0.32752)	(0.26906)	(-0.07603)

Numbers in brackets are based on historical data

A comparison among the different performance measures shows that the three Jobson-Korkie versions underperformed the historical model only for the shortest holding period. For the two and three year holding periods, though, the three Jobson-Korkie models performed very well outperforming the historical model in no less than 17/18 cases on aggregate. Another interesting comparison, is to lokk at the relative performance of the three alternative Jobson-Korkie versions: out of the 9 cases under consideration, the c=0 version performed best 6 times, the c=.2 version outperformed the other two on 3 occasions, whereas in not a single occasion was the c=.4 version proven to be the best.

Table 4.34

Jobson-Korkie Estimators
Sharpe's Performance Measure
Hedged Stocks

	n=12	n=24	n=36
Rf = 0			
c = 0	0.91363	0.55516	0.11488
c = 0.2	0.89442	0.63602	0.11321
c = 0.4	0.89474	0.63328	0.11226
	(0.88576)	(0.57390)	(0.09629)
Rf = 0.2			
c = 0	0.84360	0.48578	0.07275
c = 0.2	0.82431	0.56433	0.07252
c = 0.4	0.82422	0.56298	0.07315
	(0.73328)	(0.49711)	(0.05361)
Rf = 0.4			
c = 0	0.77358	0.41641	0.03016
c = 0.2	0.75451	0.49001	0.03038
c = 0.4	0.76680	0.48847	0.03258
	(0.73328)	(0.40930)	(0.00660)

Numbers in brackets are based on historical data

This last finding tends to suggest that the grand mean has better predictive power than an index's own sample mean, while it also tends to support the Dumas & Jacquillat critique of the Jorion shrinkage factor estimate on the basis that it leads in very high values for c resulting in inferior out of sample performance.

In the hedged portfolio, the dominance of the three Jobson-Korkie models over the historical model was almost complete: the c=.2 and c=.4 versions outperformed the historical in every single case, i.e. 9/9 each, whereas the c=0 version only outperformed in 5/9 cases. An intra-version comparison, though, provides a different picture since the c=0 version outperformed the other two on 4 occasions, whereas the c=.2 version on 3 and the c=.4 version on two occasions. Overall, both the hedged and unhedged Jobson-Korkie model versions provide clear evidence that they are capable of outperforming the historical model, particularly so for relatively small values of the shrinkage coefficient.

4.5.3 A Special Case of Jobson-Korkie Estimators: The Minimum Variance Portfolio

The Minimum Variance Portfolio (MVP) is conceptually very similar to the Jobson-Korkie model version where we set c=0; In fact, the only difference between the two models consists in the choice of input covariance matrix, which in the case of the MVP model is the historical rather than the Jobson-Korkie.

Conceptually, the MVP attempts to select the optimized investment weights [34] exclusively on the basis of historical volatilities and correlations. Such an approach could be justified if we assume that there is no valuable information incorporated in the historical mean return vector. Mathematically, the simplified quadratic optimization problem can be expressed as follows:

$$\begin{aligned} &\textbf{\textit{Minimize}} && \vec{x}^{t}\, SRS\, \vec{x} \\ &\textbf{\textit{Constrained by}} && x_1, x_2, x_k, \geq 0 \\ & && \sum_{i=1}^{k} x_i = 1 \end{aligned} \tag{4.46}$$

Notice that the MVP model cannot be applied to combined portfolios, since it is illogical to assume equal expected returns for bond and stock indices. Consequently, the MVP model has been applied only to the unhedged and hedged stock index returns. Naturally, the MVP model results in a single vector of optimized weights, so that out of sample performance differences for the same holding period, should be exclusively attributed to different chosen values for the risk free rate.

The resulting out of sample adjusted Sharpe performance measures are listed below in Tables 4.35 and 4.36, whereas the optimized weight vector for the MVP model can be found in Appendix 4B:

[34] In optimization terms, complete omission of the mean return vector from the constraints provides identical results with selecting equal expected returns for all assets at any level.

Table 4.35

Minimum Variance Portfolio
Sharpe's Performance Measure
Unhedged Stocks

	n=12	n=24	n=36
Rf = 0	0.43628	0.39639	0.02249
	(0.44697)	(0.39487)	(0.01155)
Rf = 0.2	0.37778	0.33615	-0.02052
	(0.41161)	(0.30595)	(-0.05681)
Rf = 0.4	0.31926	0.27591	-0.06353
	(0.32752)	(0.26906)	(-0.07603)

Numbers in brackets are based on historical data

Table 4.36

Minimum Variance Portfolio
Sharpe's Performance Measure
Hedged Stocks

	n=12	n=24	n=36
Rf = 0	0.90280	0.63576	0.11246
	(0.88576)	(0.57390)	(0.09629)
Rf = 0.2	0.83200	0.56382	0.07074
	(0.81027)	(0.49711)	(0.05361)
Rf = 0.4	0.76119	0.49188	0.02901
	(0.73328)	(0.40930)	(0.00660)

Numbers in brackets are based on historical data

From the above results, it becomes clear that for the hedged index returns the MVP strategy completely dominates the historical model, the latter being outperformed in every single case. In the case of the unhedged index returns, the MVP still outperforms the historical model for the two and three year holding horizons but underperforms for the shortest holding period of one year.

A perhaps more interesting comparison is that between the c = 0 version of the Jobson-Korkie model and the MVP model. For the unhedged index returns, the MVP outperformed the Jobson-Korkie for the single year holding period but underperformed for the two and three year periods. This picture, though, is reversed when considering the results from the hedged index return models. Consequently, no clear conclusions can be drawn about the superiority of one over the other model.

4.6. Alternative Approaches in Controlling Estimation Risk from Index Returns

In Section 4.5 we investigated empirically the out of sample performance of "formal" statistical approaches in controlling estimation risk from index returns, based primarily on Bayesian and Empirical Bayesian estimators. In this section, an attempt is being made to apply alternative procedures in controlling estimation risk based either on eliminating differences from historical correlations or by imposing arbitrary selected restrictions on investment weights:

4.6.1 An "Overall Mean" Correlation Model for Index Returns

4.6.1.1 Rationale for an "Overall Mean" Correlation Model

The basic idea behind an "overall mean" correlation model lies in the antipodes of the of the minimum variance model. Instead of equating expected returns for all indices while using the historical volatilities and correlations as portfolio inputs, the "overall mean" approach equates the expected correlation coefficients among all index pairs, while utilizing the historical mean return vector. The intuitive idea behind this model is that deviations from the average correlation are generally random, so that by equating all the correlation coefficients it is more likely to eliminate "noise" from the historical data than to omit useful information:

On the basis of the Overall Mean Correlation (OMC) Model, correlations among any pair of indices can be mathematically expressed as

$$cor_{ij} = p + \varepsilon_{ij} , \qquad i \neq j \tag{4.47}$$

where cor_{ij} is the correlation between indices i and j
p is a constant
ε_{ij} is a mean-zero random variable

In fact **p** is estimated from historical data and set to be equal to the "overall mean" correlation calculated as:

$$p = \frac{1}{T}\sum_{t=1}^{T} cor_{i,j}^{h} \quad \forall\ i \neq j$$
$$T = \frac{k*(k-1)}{2} \tag{4.48}$$

where **k** is the number of indices under consideration

OMC type models have been occasionally applied to correlations among individual stocks (see e.g. Elton & Gruber 1992, Eun & Resnick 1984). In general, the most successful versions where found to be those that took into consideration country or industry groupings for all stocks, in the sense that average correlation coefficients were computed among all stocks that belonged in each industry or country.

Obviously, such a grouping is not possible in the case of index returns since the stock prices are already aggregated by country. One possible alternative considered was to average the correlations among stock indices from countries that belong in the same geographical area, but the idea was abandoned since the Principal Component solutions (rotated and unrotated) show little clear-cut evidence of meaningful regional patterns in stock index returns.

A second alternative examined was whether it is meaningful to group the index returns on the basis of the rotated or unrotated factor loadings derived from principal components (see Appendix 4A). Again, no clear pattern was diagnosed to justify a meaningful grouping.

Table 4.37

Overall Mean Correlation Matrix of Stocks Hedged Returns

	SAUT	SBEL	SCAN	SDEN	SFRA	SGER	SITA	SJAP	SNET	SNOR	SSPA	SSWE	SSWI	SUKG	SUSA
SAUT	1	−0.09379	0.07041	0.06961	−0.12509	−0.03169	−0.19149	−0.11769	−0.18949	0.03741	−0.00179	−0.13109	−0.08889	0.04761	0.00271
SBEL	0.49051	1	−0.04589	0.02841	−0.30659	−0.23479	−0.16679	−0.04789	−0.13519	0.08191	0.03491	0.03611	−0.03939	0.09531	−0.04689
SCAN	0.49051	0.49051	1	0.04861	0.05321	0.04261	−0.03149	0.08831	−0.24779	0.02011	0.15121	0.03741	−0.12929	−0.21509	−0.30489
SDEN	0.49051	0.49051	0.49051	1	−0.06589	−0.15279	0.02401	0.06261	−0.03419	0.06981	0.13981	−0.08099	−0.14089	0.01581	0.03861
SFRA	0.49051	0.49051	0.49051	0.49051	1	−0.26399	−0.09989	−0.02459	−0.14649	0.14711	−0.04709	−0.06809	−0.14909	−0.01929	−0.17039
SGER	0.49051	0.49051	0.49051	0.49051	0.49051	1	−0.19959	0.01341	−0.20439	−0.00169	−0.01649	−0.14129	−0.19949	−0.10739	−0.05469
SITA	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	1	−0.09769	−0.26929	−0.00939	−0.03989	−0.17719	−0.26319	−0.04669	−0.01509
SJAP	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	1	0.02071	0.23671	−0.18409	−0.20339	−0.08869	0.02911	0.07521
SNET	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	1	−0.17719	0.01541	−0.09529	−0.20589	−0.26559	−0.22569
SNOR	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	1	0.22791	0.06631	0.06911	−0.03389	0.14061
SSPA	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	1	−0.22219	−0.06089	0.00231	0.04491
SSWE	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	1	−0.23349	−0.14509	−0.06199
SSWI	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	1	−0.21789	−0.20039
SUKG	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	1	−0.26639
SUSA	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	0.49051	1

Lower diagonal part of the matrix includes overall mean correlations based on 72 observations (Feb 82–Jan 88).
Upper diagonal part includes residuals between overall mean and actual correlations based on 36 observations (Feb 88–Jan 91).

Overall Mean Correlation Matrix of Stocks Unhedged Returns

	SAUT	SBEL	SCAN	SDEN	SFRA	SGER	SITA	SJAP	SNET	SNOR	SSPA	SSWE	SSWI	SUKG	SUSA
SAUT	1	−0.04139	0.00231	0.12941	−0.09369	−0.00329	−0.14299	−0.07609	−0.07949	0.10301	0.03991	−0.09599	−0.03829	0.07601	−0.01839
SBEL	0.52121	1	−0.00479	0.01641	−0.29439	−0.24239	−0.13849	0.11591	−0.15709	0.13931	0.06721	0.07171	0.00691	0.17291	0.01991
SCAN	0.52121	0.52121	1	0.10071	0.03221	0.11321	−0.00359	0.29661	−0.23999	0.01621	0.16751	0.10851	−0.01999	−0.03039	−0.36449
SDEN	0.52121	0.52121	0.52121	1	−0.09129	−0.16599	0.01431	0.20441	−0.06269	0.13851	0.16331	0.00291	−0.10149	0.13541	0.09101
SFRA	0.52121	0.52121	0.52121	0.52121	1	−0.26159	−0.10329	0.10521	−0.17529	0.15451	−0.05009	−0.05709	−0.09079	0.07471	−0.10739
SGER	0.52121	0.52121	0.52121	0.52121	0.52121	1	−0.19129	0.19311	−0.23499	0.06581	−0.00279	−0.08869	−0.14269	−0.01059	0.04681
SITA	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	1	0.06811	−0.23989	0.07381	−0.02959	−0.13169	−0.24739	0.02491	0.02221
SJAP	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	1	0.20151	0.37021	−0.09209	−0.08009	0.01831	0.07801	0.29391
SNET	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	1	−0.06789	0.04801	−0.02179	−0.12709	−0.13249	−0.20819
SNOR	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	1	0.28611	0.11171	0.14941	−0.03329	0.11611
SSPA	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	1	−0.22579	−0.05689	0.03831	0.07381
SSWE	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	1	−0.18169	−0.12159	0.03031
SSWI	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	1	−0.10009	−0.07959
SUKG	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	1	−0.07479
SUSA	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	0.52121	1

Lower diagonal part of the matrix includes overall mean correlations bades on 72 observations (Feb 82–Jan 88).
Upper diagonal part includes residuals between overall mean and actual correlations based on 36 observations (Feb 88–Jan91).

Consequently, only the most general form in which all stock index correlations are equated to the historical average was tested. Notice also that only unhedged and hedged stock index returns were considered, since no single correlation coefficient can be meaningfully imposed in combined portfolios.

4.6.1.2 Forecasting Performance of the Overall Mean Correlation Model

Table 4.37 presents the forecasted correlations on the basis of 72 monthly observations for the unhedged and hedged stock index returns, as well as the residual correlations for the three year out of sample period. The overall mean correlation for the unhedged returns was 0.49051, whereas for the hedged returns slightly higher at 0.52121. Table 4.38 below shows the out of sample forecasting performance of the overall mean model as well as the decomposition of the MSFE and the estimated value for the Theil Inequality Coefficient:

Table 4.38

Forecast Error Decomposition of "Overall Mean" Correlations

MODEL	MSFE	RMSFE	BIAS%	VAR%	COV%	TIC
STO/U	.0189(.0182)	.1374	.0057	.9943	0	1.0162
STO/H	.0187(.0236)	.1367	.2194	.7806	0	0.8909

The figures in brackets refer to the Historical Model

As can be observed from the MSFE and the TIC the unhedged version of the "overall mean" stock index correlation model fails to improve on the historical correlations. Nevertheless, the hedged version appears to be very successful in forecasting index correlations, with the lowest recorded TIC among all models. This potentially implies that upon removal of exchange rate fluctuations, deviations from average correlations tend to become increasingly random.

Table 4.39

Overall Mean Model
Sharpe's Performance Measure
Unhedged Stocks

	n=12	n=24	n=36
Rf = 0	0.36703	0.37500	0.03949
	(0.44697)	(0.39487)	(0.01155)
Rf = 0.2	0.31640	0.31745	-0.00823
	(0.41161)	(0.30595)	(-0.05681)
Rf = 0.4	0.26525	0.26106	-0.05436
	(0.32752)	(0.26906)	(-0.07603)

Numbers in brackets are based on historical data

Table 4.40

Overall Mean Model
Sharpe's performance measure
Hedged stocks

	n=12	n=24	n=36
Rf = 0	0.67588	0.39558	0.09767
	(0.88576)	(0.57390)	(0.09629)
Rf = 0.2	0.61202	0.33139	0.05100
	(0.81027)	(0.49711)	(0.05361)
Rf = 0.4	0.54698	0.26583	0.00342
	(0.73328)	(0.40930)	(0.00660)

Numbers in brackets are based on historical data

Subsequently, the "overall mean" correlation matrices shown on Table 4.37 were used as inputs to the standard quadratic optimization problem. The optimization results can be found in Appendix 4B, whereas the out of sample portfolio performance measures are summarized below in Tables 4.39 and 4.40:

In this case, the optimization results were somewhat inconsistent with the results of the tests of statistical significance, particularly so in the case of the hedged model version. For the three year time horizon that corresponds to the correlation forecasting period the adjusted Sharpe performance measures were very similar between the "overall mean" and the historical models. The hedged overall mean model, though, underperformed its historical counterpart for the shorter holding periods of one and two years.

4.6.2 Asset Allocation with Constrained Investment Weights

4.6.2.1 Rationale for Constraining Investment Weights in Global Index Portfolios

One of the main deficiencies in relying on quadratic optimization routines that utilize inputs derived from unadjusted or adjusted historical returns, lies in the fact that very high or low weights are typically assigned to those assets whose return or volatility tended to have much higher/lower than average values during the sampling period. Furthermore, most of these routines (see optimization results in Appendix 4B) produce portfolios with a small number of assets assigned high weights, since many of the indices with previous lacklustre performance tend to be assigned zero[35] weights during the optimization procedure.

The key point here is that traditional optimization portfolios carry not only substantial estimation risk, but also high undiversifiable risk since they tend to consist of fewer than optimal assets. This problem could be partially avoided, though, by means of restricting investment weights within certain limits, so that to avoid too high/low exposure in a given market.

[35] Notice again that the main reason of imposing non-negativity constraints on index returns is in order to avoid excessive estimation risk, that might arise from substantial short selling of some indices.

Even though the idea that constraining investment weights might potentially improve portfolio performance is far from new (was first noted by Cohen and Pogue 1967), there is very little actual empirical evidence on the issue, apart from Frost & Savarino (1986, 1988) who tested the effect of constraining weights on mean-variance selection strategies for domestic US portfolios. Frost & Savarino imposed maximum weight constraints of 5%, 2%, 1%, 2/3%, and 1/2% respectively in portfolios consisting of 200 stocks. Their simulation results suggested that as constraints were introduced the bias in inputs was substantially reduced and all investors would benefit from a 2% maximum constraint, whereas the more stringent constraints of 1/2%-1% provided more ambivalent results.

Since no empirical evidence of this type exists for international portfolios, neither for single stocks nor indices, it is worthwhile to develop index "constrained weight" models and assess their performance out of sample with actual, rather than simulated, data. For this reason a number of alternative models have been applied and their performance compared to the previously used procedures. In this context, Frost & Savarino's approach has been extended to incorporate, in addition to maximum, minimum restrictions on the index investment weights:

4.6.2.2 A Special Case of Constrained Weights: The "Naive" Model

It is interesting enough, that the standard equally weighted or "naive" portfolio, that is occasionally used as performance benchmark can be considered as a special case of constrained weights. This is in fact the most possibly constrained portfolio for which no optimization is necessary. Despite the obvious fact that this portfolio makes no use of any information derived from data, it has the appealing property of being much more diversified than most of the optimized portfolios.

The equally weighted unhedged and hedged stock portfolios consist of 15 indices, corresponding to weights of 0.0667 for each index. For the combined portfolios of 21 stock and bond indices the corresponding equal weights are 0.0476 for each index. These portfolios have been reinvested for one, two and three year holding periods and the Sharpe performance measures are summarized in Tables 4.41-4.44.

Table 4.41

Naive Model
Sharpe's Performance Measure
Unhedged Stocks

	n=12	n=24	n=36
Rf = 0	0.44721	0.42510	0.02868
	(0.44697)	(0.39487)	(0.01155)
Rf = 0.2	0.39708	0.37030	-0.01401
	(0.41161)	(0.30595)	(-0.05681)
Rf = 0.4	0.34698	0.31550	-0.05670
	(0.32752)	(0.26906)	(-0.07603)

Numbers in parenthesis are based on historical data.

Table 4.42

Naive Model
Sharpe's Performance Measure
Unhedged Stocks and Bonds

	n=12	n=24	n=36
Rf = 0	0.36209	0.34803	0.00205
	(0.04604)	(0.02552)	(-0.08339)
Rf = 0.2	0.30370	0.28457	-0.05058
	(-0.08260)	(-0.10250)	(-0.19234)
Rf = 0.4	0.24530	0.22112	-0.10321
	(-0.21471)	(-0.23563)	(-0.30172)

Numbers in parenthesis are based on historical data.

Table 4.43

Naive Model
Sharpe's Performance Measure
Hedged Stocks

	n=12	n=24	n=36
Rf = 0	0.71954	0.48074	0.09743
	(0.88576)	(0.57390)	(0.09629)
Rf = 0.2	0.66204	0.42100	0.05318
	(0.81027)	(0.49711)	(0.05361)
Rf = 0.4	0.60454	0.36127	0.00892
	(0.72729)	(0.40930)	(0.00660)

Numbers in parenthesis are based on historical data.

Table 4.44

Naive Model
Sharpe's Performance Measure
Hedged Stocks and Bonds

	n=12	n=24	n=36
Rf = 0	0.68912	0.47905	0.10332
	(1.52727)	(1.28410)	(0.17938)
Rf = 0.2	0.61415	0.40149	0.04555
	(0.91933)	(0.72763)	(0.02806)
Rf = 0.4	0.53918	0.32393	-0.01221
	(0.37587)	(0.17562)	(-0.13103)

Numbers in parenthesis are based on historical data.

On the basis of the previous results, it seems that the "naive" investment strategy performed better than the alternative benchmark, i.e. the historical model, when comparing the unhedged portfolios but slightly underperformed in the hedged one's: in fact, the "naive" unhedged stock index model outperformed the historical in 8/9 cases, but only outperformed in 2/9 cases for the hedged stock returns.

The relative performance of the combined strategies, naive vs historical, was widely disparate due to substantial differences in the portfolio composition. Overall, the unhedged combined "naive" portfolio outperformed the historical in 9/9 cases, but only in 4/9 cases for the combined hedged. In the latter case, the relative performance was highly sensitive to the choice of risk free rate due to the very low volatility of the historical combined hedged portfolio.

4.6.2.3 Imposing Minimum or Maximum Weight Restrictions on Index Portfolios

As mentioned previously in section 4.6.2.1, a small number of previous studies addressed the issue of imposing maximum restrictions on investment weights aiming to reduce excessive exposure in individual assets. Surprisingly, there is no mention in the finance literature of the potential usefulness of imposing minimum positive weights to all assets; Such an approach is in fact intuitively attractive, since by guaranteeing the positive participation of all assets in the optimization solution two adjectives can be achieved:

i) Reduce the portfolio diversifiable risk by substantially increasing the investment universe

ii) Reduce the estimation risk of excluding profitable assets due to poor historical performance in the sample period.

Naturally, there is no obvious method to determine what is the optimal level of minimum or maximum restrictions to be imposed on the investment weights. Since the aim of this study is to show whether it is possible to increase the out of sample portfolio performance by imposing a meaningful weight restriction, I arbitrarily selected a maximum weight restriction of two times the "naive" weights and a minimum weight restriction of half the "naive" weights. Formally, the corresponding optimization models are formulated as follows:

Maximum Weight Constraint Model

$$
\begin{aligned}
\textbf{\textit{Minimize}} \quad & \vec{x}^{\prime}\, SRS\, \vec{x} \\
\textbf{\textit{Constrained by:}} \quad & \vec{x}^{\prime} \vec{E} = r(p) \\
& x_1, x_2, x_k, \leq 2 * \frac{1}{k} \\
& \sum_{i=1}^{k} x_i = 1 \\
& x, x_2, x_k, \geq 0
\end{aligned}
\tag{4.49}
$$

where k=15 and k=21 for the stock only and combined portfolios respectively

Minimum Weight Constraint Model

$$
\begin{aligned}
\textbf{\textit{Minimize}} \quad & \vec{x}^{\prime}\, SRS\, \vec{x} \\
\textbf{\textit{Constrained by:}} \quad & \vec{x}^{\prime} \vec{E} = r(p) \\
& x_1, x_2, x_k, \geq \frac{1}{2k} \\
& \sum_{i=1}^{k} x_i = 1
\end{aligned}
\tag{4.50}
$$

Notice that the portfolio inputs used in the above quadratic optimization problems are the same with those from the historical model, since that makes comparison between the two procedures more straightforward. Nevertheless, constrained weight procedures could be imposed to any of the models examined in previous sections.

The adjusted Sharpe performance measures for the various constrained model versions are summarized below in Tables 4.45-4.48, whereas detailed results from all the relevant optimizations can be found in Appendix 4B. The main conclusions from those results concerning the performance of the different constrained strategies in respect to the historical model as well as among themselves have as follows:

Table 4.45

Constrained Weights
Sharpe's Performance Measure
Unhedged Stocks

	n=12	n=24	n=36
Rf = 0			
W <= 0.133	0.43195	0.40973	0.02252
W >= 0.033	0.49852	0.44932	0.03223
	(0.44697)	(0.39487)	(0.01155)
Rf = 0.2			
W <= 0.133	0.37974	0.35336	-0.01863
W >= 0.033	0.44305	0.38980	-0.01150
	(0.41161)	(0.30595)	(-0.05681)
Rf = 0.4			
W <= 0.132	0.32498	0.29534	-0.06068
W >= 0.033	0.38687	0.32721	-0.05769
	(0.32752)	(0.26906)	(-0.07603)

Numbers in brackets are based on historical data

Table 4.46

Constrained Weights
Sharpe's Performance Measure
Hedged Stocks

	n=12	n=24	n=36
Rf = 0			
W <= 0.133	0.70414	0.47205	0.10270
W >= 0.033	0.88731	0.62833	0.09958
	(0.88576)	(0.57390)	(0.09629)
Rf = 0.2			
W <= 0.133	0.63961	0.40591	0.05715
W >= 0.033	0.82330	0.56100	0.05803
	(0.81027)	(0.49711)	(0.05361)
Rf = 0.4			
W <= 0.132	0.57577	0.34004	0.01171
W >= 0.033	0.75656	0.48988	0.01481
	(0.73328)	(0.40930)	(0.00660)

Numbers in brackets are based on historical data

Table 4.47

Constrained Weights
Sharpe's Performance Measure
Unhedged Stocks and Bonds

	n=12	n=24	n=36
Rf = 0			
W <= 0.095	0.19643	0.20605	-0.04435
W >= 0.023	0.28293	0.20573	-0.00604
	(0.04604)	(0.02552)	(-0.08339)
Rf = 0.2			
W <= 0.095	0.12388	0.12932	-0.11168
W >= 0.023	0.19084	0.11548	-0.08270
	(-0.08260)	(-0.10250)	(-0.19234)
Rf = 0.4			
W <= 0.095	0.05574	0.05294	-0.17990
W >= 0.023	0.09480	0.02286	-0.15931
	(-0.21471)	(-0.23563)	(-0.30172)

Numbers in brackets are based on historical data

Table 4.48

Constrained Weights
Sharpe's Performance Measure
Hedged Stocks and Bonds

	n=12	n=24	n=36
Rf = 0			
W <= 0.095	0.71179	0.47128	0.11604
W >= 0.023	0.85680	0.60975	0.14273
	(1.52727)	(1.28410)	(0.17938)
Rf = 0.2			
W <= 0.095	0.59351	0.35683	0.04225
W >= 0.023	0.70679	0.45810	0.04570
	(0.91933)	(0.72763)	(0.02806)
Rf = 0.4			
W <= 0.095	0.47443	0.24314	-0.03614
W >= 0.023	0.55772	0.30909	-0.05081
	(0.37587)	(0.17562)	(-0.13103)

Numbers in brackets are based on historical data

Constraints on Unhedged Stock Index Returns

The new strategy of imposing minimum positive constraints on investment weights had exceptionally satisfactory performance compared to both the historical and the Frost & Savarino type maximum constraint strategies; In fact the minimum positive constraint strategy outperformed both alternatives in every single case under consideration. Notice also that the maximum weight restriction strategy also managed to outperform the historical model for the two and three year holding periods, but underperformed for the single year period.

Constraints on Hedged Stock Index Returns

Again the minimum weight restriction strategy proved to be superior, outperforming the historical in 9/9 cases and the maximum weight restriction strategy in 8/9 cases. Nevertheless, the maximum weight strategy performed quite well in the longest holding period with performance very close to that of the minimum weight strategy. For the one year holding period, though, the performance of the maximum weight strategy was very poor, underperforming even the historical portfolio.

Constraints on the Combined Strategies

Both constrained strategies performed considerably better than the historical model in the hedged and unhedged cases alike. A comparison between the minimum and maximum weight constraint models shows that the former outperformed the latter in 8/9 cases for the hedged combined portfolio and in 6/9 cases for the unhedged combined portfolio.

Overall, it is clear that the new approach of imposing an arbitrary minimum constraint to the index investment weights at a level equal to one half the passive weight performed extremely satisfactorily, so that the whole concept is certainly worthy of further investigation.

4.7 Overall Performance Ranking and Inter-Model Comparisons

After completing all correlation forecasting tests as well as the out of sample performance measurement for all the different procedures that have been developed in the previous sections, it is now possible to summarize all the empirical evidence and rank the alternative methods accordingly:

4.7.1 Performance Ranking of Correlation Matrix Forecasts

Tables 4.49 & 4.50 below summarize all key information about the various alternative methods previously applied to forecast the out of sample correlation matrices. Then the different procedures are ranked against each other on the basis of the MSFE and TIC criteria.

As we can observe from comparing the ranking order across the different categories, the standard historical correlation forecasts performed rather unimpressively. As a matter of fact, historical correlations ranked 4^{th} among the 6 procedures for the unhedged stock index correlations and 4^{th} out of 4 procedures for the hedged correlations. The somewhat better performance of historical correlation forecasts in the combined portfolios, entirely results from the fact that the unobservable factor procedures have here reduced power due to the unequal variance among the variables.

The best forecasts for the unhedged stock index correlations came from the Vasicek/Bayesian Pseudo Single Index Model, while the Overall Mean Model performed best in the case of hedged stock index returns. Overall, though, the Principal Components model was the most consistent performer, ranking second in both categories. On the other hand, it is not easy to recommend any particular procedure in forecasting correlations from mixed portfolios.

Table 4.49

Correlation Forecasting Performance
Unhedged and Hedged Stocks

Method	Unh/Theil	Unh/MSFE	Unh/Rank	Hed/Theil	Hed/MSFE	Hed/Rank
Historical	1.0000	.1082	4	1.0000	.0236	4
PCM	.9839	.0176	2	.9440	.1450	2
MLFM	.9937	.0180	3	.9942	.0233	3
PSIM(EW)	.9756	.0173	1			
PSIM(MW)	1.1350	.0235	6			
Overall Mean	1.0162	.0189	5	.8909	.0187	1

Table 4.50

Correlation Forecasting Performance
Unhedged and Hedged Combined Portfolios

Method	Unh/Theil	Unh/MSFE	Unh/Rank	Hed/Theil	Hed/MSFE	Hed/Rank
Historical	1.0000	.0290	1	1.0000	.0486	3
PCM	1.0390	.0313	2	.9913	.0478	2
MLFM	1.1980	.0417	3	.9534	.0442	1

4.7.2 Summary of the out of Sample Performance Rankings for the various "Ex-Ante" Index Allocation Strategies

In order to be able to make a meaningful comparison of the out of sample performance of the various alternative models two different ranking criteria have been applied:

i) The "Average Adjusted Sharpe Performance Measure" for each model, calculated as the mean value of the nine portfolios (three reinvestment periods and three risk free rates). The problem with this measure lies in the fact that the different measures are not independent, first because the unit of measurement is not uniform since the sharpe measure value depends on the choice of risk free rate and, second because the portfolio holding periods are overlapping.

ii) The "Average Rank Criterion", designed to avoid the problem of taking mean values of heterogeneous measures, which is calculated in two steps:

- First, the Sharpe measures are ranked separately for all nine portfolios, across all models so that the model with the highest Sharpe measured is assigned a value of 1 and the model with the lowest Sharpe a value of 14 for the unhedged, 12 for the hedged and 7 for the combined portfolios[36].

- Then the rank numbers for each model are averaged across all nine portfolios, so that the model with the lowest average rank is the one with the overall superior out of sample performance.

The estimated values for the "Average Sharpe" and the "Average Rank" performance criteria for the four types of portfolios and all different types of models can be found in Tables 4.51-4.54 below:

---

36 These numbers correspond to the number of models applied in each case.

Table 4.51

Overall Performance Ranking

Unhedged Stocks

Max = 1 and Min = 18

Method	Average Rank	Average SPM
Pr. Component	11.00 (13)	0.1996 (14)
Max. Likelihood	11.11 (14)	0.2138 (13)
Vasicek EQW	5.33 (4)	0.2490 (4)
Vasicek MVW	4.33 (3)	0.2597 (2)
Overall Mean	9.66 (10)	0.2087 (12)
Constrained		
W <= 0.132	8.00 (8)	0.2375 (7)
W >= 0.033	2.11 (1)	0.2730 (1)
Klein-Bawa	7.11 (6)	0.2377 (6)
Jobson-Korkie		
c = 0	5.88 (5)	0.2411 (5)
c = 0.2	9.44 (9)	0.2294 (10)
c = 0.4	7.77 (7)	0.2365 (8)
Min. Variance	9.77 (11)	0.2311 (9)
Naive	4.00 (2)	0.2511 (3)
Historical	9.88 (12)	0.2260 (11)

Numbers in parenthesis refer to the final ranking position according to each criterion

Table 4.52

Overall Performance Ranking

Hedged Stocks

Max = 1 and Min = 12

Method	Average Rank	Average SPM
Pr. Component	10.33 (11)	0.3877 (9)
Max. Likelihood	9.44 (10)	0.3973 (8)
Overall Mean	11.11 (12)	0.3310 (12)
Constrained		
W <= 0.132	9.11 (8-9)	0.3676 (11)
W >= 0.033	5.00 (5)	0.4798 (4)
Klein-Bawa	5.11 (6)	0.4678 (5)
Jobson-Korkie		
c = 0	3.22 (4)	0.4622 (6)
c = 0.2	2.66 (2)	0.4866 (3)
c = 0.4	2.88 (3)	0.4876 (2)
Min. Variance	2.55 (1)	0.4888 (1)
Naive	9.11 (8-9)	0.3787 (10)
Historical	7.44 (7)	0.4517 (7)

Numbers in parenthesis refer to the final ranking position according to each criterion

Table 4.53

Overall Performance Ranking
Unhedged Stocks and Bonds
Max = 1 and Min = 7

Method	Average Rank	Average SPM
Pr. Component	5.33 (5)	-.0686 (4)
Max. Likelihood	4.33 (4)	-.0842 (5)
Constrained		
W <= 0.095	2.67 (3)	0.0476 (3)
W >= 0.023	2.33 (2)	0.0738 (2)
Klein-Bawa	6.55 (7)	-.1332 (7)
Naive	1.00 (1)	0.1795 (1)
Historical	5.77 (6)	-.1268 (6)

Numbers in parenthesis refer to the final ranking position according to each criterion

Table 4.54

Overall Performance Ranking
Hedged Stocks and Bonds
Max = 1 and Min = 7

Method	Average Rank	Average SPM
Pr. Component	4.00 (4)	0.5612 (3)
Max. Likelihood	2.33 (1)	0.6121 (1)
Constrained		
W <= 0.095	5.11 (6-7)	0.3303 (7)
W >= 0.023	3.67 (3)	0.4039 (5)
Klein-Bawa	5.11 (6-7)	0.4863 (4)
Naive	4.33 (5)	0.3537 (6)
Historical	3.44 (2)	0.5762 (2)

Numbers in parenthesis refer to the final ranking position according to each criterion

Let us first summarise the results and overall rankings from the unhedged stock index portfolios: the Constrained Minimum Weight Model, which forces all global stock indices to be included in the optimized portfolio with at least half the passive weight, performed remarkably well achieving the top ranking among the 14 models in both criteria, i.e. the "Average Sharpe" and the "Average Rank" criterion. Second best performing model was the "Naive" which is the only other model that ensures participation of all indices in the optimized portfolio. Notice that both aforementioned models performed considerably better than the Constrained Maximum Weight Model, which was only ranked 7^{th} and 8^{th} on the basis of the two criteria respectively.

The two versions of the Bayesian Pseudo-Single Index Model were the next best performing models, followed by the different versions of the Jobson-Korkie Model and the Bayesian Klein-Bawa type approach. The worst performing models were the Historical Model, the Minimum Variance Model, the Overall Mean Correlation Model and the two unobservable multifactor models, i.e. the Principal Components Model and the Maximum Likelihood Factor Model. The last result is particularly surprising, considering the demonstrated ability of the last two models to provide statistically superior estimates of the index correlation matrix. This effect provides strong support to the argument that out of sample portfolio performance is relatively insensitive to the correlation estimates.

Radically different top performance rankings appear, though, in the hedged stock index portfolio. Here the best performing model was the Minimum Variance Model which does not utilize the mean return vector as input, followed by the three Jobson-Korkie model versions, which shrink the sample means towards the "grand mean". Nevertheless, as before, the worst performing models were the Historical, the Maximum Likelihood and the Principal Components Model.

Among the combined unhedged portfolios the best performing was the Naive Model, followed by the two Constrained Weight Models. Again, the Historical, the Klein-Bawa and the two Unobservable Factor Models were the worst performers. The results from the combined hedged portfolios tend to be slightly peculiar, due to the fact that all the unconstrained weight models provided optimization solutions with very high weight for the hedged Dollar FRN's, an index with moderate return but exceptionally low volatility. This low volatility resulted in optimal portfolios with virtually no diversification and realized performance measures that were extremely sensitive to the choice of risk-free rate. As a natural consequence, the rankings follow a very different pattern from the three previous portfolio types, favouring the Maximum Likelihood and Historical Portfolios.

Finally, it is worth making two additional observations concerning the results from different portfolio holding periods and the effects of hedging:

i) For all model versions, particularly so for the stock only portfolios, the performance measures were very much lower for the three year holding horizon, compared to the one and two year holding horizons. This is in fact not surprising, considering the fact that the last year of the portfolio validation period coincided with the Gulf War, an event that caused considerable decline in the value of most global stock markets.

ii) As one might expect, for all hedged models the average Sharpe Measure was higher compared to their unhedged counterparts, a phenomenon which was even more pronounced in the combined models. Nevertheless, a careful inspection of the optimization results (see Appendix 4B) reveals that for most models the volatility of hedged stock returns was substantially lower than that of their unhedged counterparts for the one and two year horizons, but slightly higher for the three year period. This apparently paradoxical result is attributed to the fact that this particular year witnessed strongly negative covariances between foreign stock returns and exchange rate changes against Sterling. This serves well the argument that stock investors should be aware that it is possible for some periods to have increased volatility as a result of hedging. This argument, though, appears to be non-valid for hedged bond returns which seem to be always less volatile than unhedged returns.

APPENDIX 4A

FACTOR LOADINGS MATRICES

FACTOR LOADINGS MATRIX

PRINCIPAL COMPONENT UNHEDGED MODEL* (COMBINED)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4
SAUT	0.85201	-0.01856	0.00683	-0.0652
SBEL	0.79837	-0.18164	0.20638	-0.02526
SCAN	0.77997	-0.22685	-0.31994	0.17894
SDEN	0.59952	0.07846	0.03116	0.1113
SFRA	0.74439	-0.08196	0.37742	0.05392
SGER	0.7313	-0.04238	0.27659	-0.00677
SITA	0.67144	0.17826	0.254	0.05794
SJAP	0.61779	0.27007	0.32714	-0.05842
SNET	0.85093	-0.14723	0.00543	0.10592
SNOR	0.69519	-0.47787	-0.02285	-0.23885
SSPA	0.59766	-0.12481	0.18859	-0.13937
SSWE	0.68509	-0.09253	-0.0844	-0.15329
SSWI	0.84447	-0.11735	0.10276	-0.0138
SUKG	0.67104	-0.51102	-0.05943	0.27681
SUSA	0.83745	-0.14045	-0.37927	0.02035
CAN	0.54779	0.47349	-0.51462	0.14243
GER	0.41668	0.61283	0.40722	0.10736
JAP	0.41901	0.68584	0.35487	-0.08967
UKG	-0.08615	0.02993	0.0499	0.94682
USA	0.58613	0.48249	-0.50463	0.1042
USN	0.58037	0.37286	-0.52533	-0.276

PRINCIPAL COMPONENT HEDGED MODEL* (COMBINED)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4
SAUT	0.80321	-0.15185	0.03032	0.01098
SBEL	0.79858	-0.08696	0.07549	0.1008
SCAN	0.78514	-0.00033	-0.1393	-0.35862
SDEN	0.52614	0.03851	0.04519	-0.20548
SFRA	0.74924	0.0724	-0.18931	0.14424
SGER	0.72881	-0.02419	0.13184	0.13426
SITA	0.59258	0.22448	0.08552	0.55342
SJAP	0.54589	0.12672	-0.4381	0.24669
SNET	0.83247	-0.09979	-0.07074	-0.03339
SNOR	0.73278	-0.4001	0.05405	-0.18285
SSPA	0.5911	-0.06021	-0.15615	0.31361
SSWE	0.6596	-0.00051	0.22586	0.13903
SSWI	0.845468	-0.08716	0.1038	-0.04952
SUKG	0.81328	-0.03822	-0.15014	-0.18468
SUSA	0.81785	-0.0112	0.0099	-0.34789
CAN	0.10667	0.69407	0.36204	-0.29443
GER	0.15361	0.74435	-0.29367	0.03797
JAP	-0.03939	0.75223	-0.3213	0.13697
UKG	0.11646	0.65321	-0.22299	-0.10881
USA	0.20238	0.73492	0.2928	-0.23049
USN	0.25726	0.29528	0.71377	0.32598

* = Number of Components Selected on the Basis of an Eigenvalue Criterion

VARIMAX ROTATED FACTOR MATRIX

PRINCIPAL COMPONENT UNHEDGED MODEL (COMBINED)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4
SAUT	0.71897	0.33744	0.30046	-0.0975
SBEL	0.7875	0.08508	0.2896	-0.04865
SCAN	0.73794	0.47427	-0.07377	0.1388
SDEN	0.46402	0.26659	0.2909	0.08904
SFRA	0.71175	-0.02024	0.44469	0.0379
SGER	0.66563	0.06874	0.40584	-0.0256
SITA	0.49586	0.16775	0.52427	0.0401
SJAP	0.40748	0.1267	0.614647	-0.07185
SNET	0.79181	0.28496	0.20835	0.07367
SNOR	0.8275	0.07232	-0.09515	-0.26492
SSPA	0.58442	0.03729	0.24618	-0.15563
SSWE	0.60628	0.30053	0.13244	-0.18193
SSWI	0.80921	0.18915	0.24815	-0.04239
SUKG	0.83461	0.09712	-0.15212	0.24988
SUSA	0.72782	0.57812	-0.02825	-0.02393
CAN	0.14251	0.85635	0.21228	0.10333
GER	0.06944	0.14875	0.83001	0.10333
JAP	0.02039	0.21633	0.85065	-0.09541
UKG	-0.05644	-0.02029	0.01584	0.95049
USA	0.1698	0.86712	0.23881	0.06399
USN	0.21135	0.81318	0.14913	-0.31595

PRINCIPAL COMPONENT HEDGED MODEL (COMBINED)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4
SAUT	0.74123	-0.0891	0.30626	0.13446
SBEL	0.69137	-0.05333	0.36309	0.22017
SCAN	0.85355	0.15459	0.05374	-0.09551
SDEN	0.5531	0.09975	0.01276	0.08089
SFRA	0.60481	0.15474	0.48207	0.03337
SGER	0.60624	-0.01883	0.34319	0.28549
SITA	0.27631	0.15272	0.66187	0.42116
SJAP	0.37033	0.23718	0.58484	-0.17724
SNET	0.77772	-0.00459	0.3189	0.05011
SNOR	0.79249	-0.30649	0.10608	0.01296
SSPA	0.41205	-0.0191	0.55041	0.05158
SSWE	0.53913	-0.02654	0.28489	0.36455
SSWI	0.80364	-0.03263	0.24753	0.20767
SUKG	0.81306	0.09631	0.21479	-0.05515
SUSA	0.88113	0.10729	0.01853	0.04351
CAN	0.12021	0.62276	-0.34129	0.43831
GER	0.01526	0.78892	0.206	-0.01668
JAP	-0.2005	0.77236	0.22549	-0.04263
UKG	0.05523	0.7043	0.041	-0.03228
USA	0.17408	0.6769	-0.22416	0.42566
USN	0.06103	0.06899	0.10277	0.86606

FACTOR LOADINGS MATRIX

PRINCIPAL COMPONENT UNHEDGED MODEL (STOCKS)

	UNROTATED*		ROTATED**	
	FACTOR1	FACTOR2	FACTOR1	FACTOR2
SAUT	0.84598	0.04976	0.5944	0.60402
SBEL	0.82717	0.0882	0.54468	0.61993
SCAN	0.78758	-0.40786	0.85783	0.22529
SDEN	0.58629	0.00444	0.43208	0.39631
SFRA	0.76977	0.30575	0.36626	0.74289
SGER	0.74218	0.24867	0.38404	0.68204
SITA	0.65392	0.51248	0.14171	0.81864
SJAP	0.59369	0.38919	0.17967	0.68677
SNET	0.86125	-0.1196	0.71926	0.48858
SNOR	0.75432	-0.3266	0.77868	0.2633
SSPA	0.61775	0.24491	0.29423	0.59584
SSWE	0.68275	-0.04524	0.53697	0.42411
SSWI	0.8649	-0.001	0.64247	0.57904
SUKG	0.73749	-0.38192	0.80327	0.21096
SUSA	0.82432	-0.3288	0.8321	0.30858

PRINCIPAL COMPONENT HEDGED MODEL (STOCKS)

	UNROTATED*		ROTATED**	
	FACTOR1	FACTOR2	FACTOR1	FACTOR2
SAUT	0.80956	-0.0412	0.70244	0.40453
SBEL	0.79994	0.05837	0.64035	0.48297
SCAN	0.78776	-0.24072	0.79236	0.22509
SDEN	0.52241	-0.08103	0.48282	0.21531
SFRA	0.75007	0.28311	0.47655	0.64472
SGER	0.72472	0.0171	0.59955	0.40749
SITA	0.57829	0.59614	0.16244	0.8145
SJAP	0.54845	0.42677	0.22924	0.65604
SNET	0.83682	-0.11414	0.76492	0.35805
SNOR	0.75351	-0.35497	0.82557	0.11053
SSPA	0.59533	0.4269	0.26856	0.68158
SSWE	0.65547	-0.04896	0.57721	0.31443
SSWI	0.85485	-0.14146	0.79488	0.34487
SUKG	0.81671	-0.17512	0.7811	0.29591
SUSA	0.81688	-0.22041	0.80582	0.25795

* = Number of Components Selected on the Basis of an Eigenvalue Criterion

** = Components Matrix Produced by Means of an Orthogonal Varimax Rotation

VARIMAX FACTOR LOADINGS MATRIX

MAXIMUM LIKELIHOOD UNHEDGED MODEL* (COMBINED)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5
SAUT	0.62315	0.49129	0.01704	0.0444	0.07556
SBEL	0.46453	0.36612	0.0829	0.30949	0.15305
SCAN	0.14543	0.70556	-0.3494	0.31716	0.03337
SDEN	0.2886	0.38929	0.01272	0.15323	0.03328
SFRA	0.503	0.27136	0.18309	0.31719	0.05288
SGER	0.99944	-0.0106	-0.00142	-0.00158	-0.00063
SITA	0.4228	0.362	0.18203	0.112	-0.06599
SJAP	0.26957	0.47735	0.51912	0.2334	0.04011
SNET	0.57792	0.46804	-0.15481	0.29965	0.08097
SNOR	0.28691	0.30506	-0.22942	0.34736	0.53684
SSPA	0.20347	0.25895	0.09363	0.08936	0.09999
SSWE	0.29011	0.44035	0.00124	0.08465	0.15424
SSWI	0.73478	0.29899	-0.12467	0.18852	0.15182
SUKG	0.1839	0.30378	-0.33592	0.51975	-0.08801
SUSA	0.27445	0.77248	-0.34748	0.06658	0.092
CAN	0.25653	0.73757	-0.15281	-0.20294	-0.17296
GER	0.49157	0.2818	0.45976	0.01258	-0.15883
JAP	0.35233	0.504	0.71156	0.06145	-0.01617
UKG	-0.00344	-0.02034	-0.13256	0.40634	-0.76638
USA	0.32553	0.73671	-0.15948	-0.34201	-0.24238
USN	0.2185	0.68274	-0.08845	-0.48779	0.06987

MAXIMUM LIKELIHOOD HEDGED MODEL* (COMBINED)

	FACTOR1	FACTOR2	FACTOR3
SAUT	0.66075	-0.075	-0.18254
SBEL	0.5715	0.06996	-0.13531
SCAN	0.56327	0.54118	-0.20233
SDEN	0.34012	0.1893	-0.08332
SFRA	0.58055	0.09209	0.09933
SGER	0.78167	-0.50551	0.11365
SITA	0.44031	0.08278	0.16687
SJAP	0.3083	0.24212	0.11781
SNET	0.71614	0.05818	-0.17576
SNOR	0.48941	0.09137	-0.55807
SSPA	0.26128	0.12049	0.04072
SSWE	0.45294	0.15751	-0.11341
SSWI	0.81333	-0.1342	-0.09449
SUKG	0.57527	0.36124	-0.07777
SUSA	0.62848	0.47198	-0.25664
CAN	0.42815	0.29856	0.29716
GER	0.33841	0.17501	0.68719
JAP	0.1829	0.35833	0.59782
UKG	0.27604	0.30535	0.43752
USA	0.45559	0.29357	0.35476
USN	0.29556	-0.03526	0.11125

* = Number of Factors Selected on the Basis of a Likelihood Ratio Criterion

VARIMAX ROTATED FACTOR MATRIX

MAXIMUM LIKELIHOOD UNHEDGED MODEL (COMBINED)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5
SAUT	0.39048	0.3487	0.39287	0.45419	-0.05427
SBEL	0.09736	0.45423	0.40998	0.30263	-0.02089
SCAN	0.45428	0.70678	0.11146	-0.02501	0.15424
SDEN	0.2271	0.32004	0.2836	0.1555	0.03122
SFRA	0.02142	0.3408	0.46986	0.35063	0.05702
SGER	0.10452	0.12113	0.25945	0.95188	-0.00242
SITA	0.22052	0.18655	0.44412	0.27067	0.07687
SJAP	0.10792	0.19811	0.75751	0.03295	-0.01733
SNET	0.27159	0.5802	0.28411	0.42036	0.07457
SNOR	0.01127	0.71211	0.06976	0.18562	-0.30053
SSPA	0.11924	0.19988	0.25736	0.10155	-0.06697
SSWE	0.2801	0.34841	0.27593	0.14889	-0.10428
SSWI	0.20076	0.4648	0.25262	0.61778	-0.04014
SUKG	0.0688	0.61751	0.01443	0.10021	0.34716
SUSA	0.64441	0.60553	0.11707	0.10016	-0.00467
CAN	0.77147	0.22012	0.21023	0.09201	0.08942
GER	0.16874	-0.06156	0.64264	0.32673	0.07625
JAP	0.19286	0.00061	0.91547	0.09221	-0.06874
UKG	-0.03752	0.02365	-0.00105	0.00027	0.87663
USA	0.87273	0.11044	0.19402	0.17202	0.09366
USN	0.81387	0.06561	0.15825	0.0812	-0.25728

MAXIMUM LIKELIHOOD HEDGED MODEL (COMBINED)

	FACTOR1	FACTOR2	FACTOR3
SAUT	0.43014	0.53888	0.01125
SBEL	0.4437	0.38083	0.08901
SCAN	0.7673	0.06637	0.24069
SDEN	0.35722	0.13134	0.11662
SFRA	0.33732	0.38988	0.29932
SGER	0.0741	0.92898	0.10473
SITA	0.21498	0.29377	0.30992
SJAP	0.26434	0.08715	0.30013
SNET	0.541	0.49577	0.09314
SNOR	0.63564	0.27493	-0.28229
SSPA	0.20301	0.12428	0.16668
SSWE	0.4183	0.23565	0.11097
SSWI	0.43394	0.69939	0.10483
SUKG	0.59596	0.20012	0.26882
SUSA	0.79077	0.15681	0.18359
CAN	0.27238	0.15464	0.51249
GER	-0.06412	0.19314	0.75892
JAP	0.00817	-0.04964	0.71882
UKG	0.11434	0.04422	0.58807
USA	0.25428	0.18279	0.56704
USN	0.08833	0.25564	0.1668

APPENDIX 4B

OPTIMISED PORTFOLIO COMPOSITION

FACTOR LOADINGS MATRIX

MAXIMUM LIKELIHOOD UNHEDGED MODEL (STOCKS)

	UNROTATED*			ROTATED**		
	FACTOR1	FACTOR2	FACTOR3	FACTOR1	FACTOR2	FACTOR3
SAUT	0.62061	0.45677	0.0063	0.46548	0.33463	0.51495
SBEL	0.46284	0.43195	0.30889	0.31282	0.55438	0.30171
SCAN	0.14277	0.8482	-0.20364	0.86904	0.15785	0.03379
SDEN	0.2866	0.40557	0.07455	0.36584	0.28793	0.18827
SFRA	0.50169	0.34181	0.4096	0.19397	0.62287	0.33273
SGER	0.99948	-0.00604	-0.00114	0.07309	0.26486	0.96099
SITA	0.42095	0.33746	0.54215	0.13218	0.71833	0.22702
SJAP	0.26624	0.37136	0.37083	0.21774	0.53554	0.10997
SNET	0.57605	0.57683	-0.05679	0.59674	0.30892	0.46504
SNOR	0.28627	0.50748	-0.03469	0.50181	0.22632	0.19404
SSPA	0.2022	0.27114	0.38673	0.11455	0.49693	0.06247
SSWE	0.28794	0.42259	0.18332	0.33932	0.39172	0.16283
SSWI	0.73372	0.40151	-0.02643	0.43632	0.31599	0.64034
SUKG	0.18331	0.50084	-0.0914	0.50969	0.14558	0.10872
SUSA	0.27117	0.7894	-0.17592	0.81433	0.19617	0.16127

MAXIMUM LIKELIHOOD HEDGED MODEL (STOCKS)

	UNROTATED*		ROTATED**	
	FACTOR1	FACTOR2	FACTOR1	FACTOR2
SAUT	0.62374	0.26245	0.39686	0.54812
SBEL	0.48508	0.32669	0.42804	0.39852
SCAN	0.30974	0.73187	0.78299	0.136
SDEN	0.22943	0.31577	0.35951	0.15198
SFRA	0.50927	0.26228	0.37077	0.43666
SGER	0.95331	-0.21229	0.00906	0.97662
SITA	0.36318	0.19816	0.27524	0.30888
SJAP	0.1916	0.29919	0.3348	0.11888
SNET	0.62874	0.40591	0.53773	0.52051
SNOR	0.38882	0.46344	0.53944	0.27379
SSPA	0.18421	0.213	0.24917	0.1312
SSWE	0.32942	0.35941	0.42466	0.23949
SSWI	0.7812	0.23657	0.4073	0.70735
SUKG	0.39186	0.5444	0.61899	0.25843
SUSA	0.39711	0.69994	0.77167	0.22832

* = Number of Factors Selected on the Basis of a Likelihood Ratio Criterion

** = Factor Matrix Produced by Means of an Orthogonal Varimax Rotation

OPTIMISED PORTFOLIO COMPOSITION

HISTORICAL MODEL (STOCKS)

	UNHEDGED Rf=0	Rf=0.2	Rf=0.4	HEDGED Rf=0	Rf=0.2	Rf=0.4
SAUT	0	0	0	0	0	0
SBEL	0	0.0384692	0	0.0167719	0.0221124	0.0377772
SCAN	0	0	0	0	0	0
SDEN	0.240404987	0.1887689	0.2354587	0.2088908	0.202713	0.1848047
SFRA	0	0	0	0	0	0
SGER	0.007919122	0	0.0076975	0.0905823	0.0896704	0.0850357
SITA	0	0	0	0	0	0
SJAP	0.220025048	0.2658631	0.225592	0.3662767	0.3690344	0.3772747
SNET	0	0	0	0	0	0
SNOR	0	0	0	0	0	0
SSPA	0.060009009	0.0720163	0.061562	0.0295505	0.0293878	0.0291405
SSWE	0.026173603	0.0411635	0.0281913	0.0592542	0.0604473	0.0632032
SSWI	0.12705028	0.0830436	0.1228863	0.0320603	0.0283832	0.0216874
SUKG	0.318417946	0.3106754	0.3186115	0.0051602	0.0033286	0
SUSA	0	0	0	0.191453	0.1949229	0.2010756
	OUT OF SAMPLE PERFORMANCE					
1 YR STD	3.131314	3.013174	3.117145	2.6192295	2.628064	2.659337
MEAN	1.487077	1.517775	1.484755	2.465005	2.462524	2.45498
2YRS STD	3.169558	3.108946	3.162208	2.7424113	2.751831	2.784037
MEAN	1.290669	1.180905	1.277417	1.630254	1.617973	1.582455
3YRS STD	4.627309	4.710933	4.639359	4.875362	4.886267	4.92017
MEAN	0.054566	-0.07323	0.039911	0.462145	0.450383	0.41621

HISTORICAL MODEL (COMBINED)

	UNHEDGED Rf=0	Rf=0.2	Rf=0.4	HEDGED Rf=0	Rf=0.2	Rf=0.4
CAN	0	0	0	0	0	0
JAP	0.064181381	0.0709291	0.0840407	0.0342497	0.0344419	0.0403547
UKG	0.482373253	0.4832606	0.4849832	0	0	0
USA	0	0	0	0	0	0
USN	0.235913668	0.2293854	0.2167258	0.8840444	0.8776893	0.8323534
GER	0.150948988	0.1470872	0.139565	0.0629823	0.0667964	0.0905266
SAUT	0	0	0	0	0	0
SBEL	0	0	0	0	0	0.0066119
SCAN	0	0	0	0	0	0
SDEN	0	0	0	0	0	0
SFRA	0	0	0	0	0	0
SGER	0	0	0	0	0	0
SITA	0	0	0	0	0	0
SJAP	0	0	0	0.0174526	0.0193667	0.0280688
SNET	0	0	0	0	0	0.0017951
SNOR	0.052138127	0.0517622	0.0510288	0	0	0
SSPA	0.014444597	0.0175756	0.0236566	0	0	0.0002897
SSWE	0	0	0	0	0	0
SSWI	0	0	0	0	0	0
SUKG	0	0	0	0.001271	0.0017058	0
SUSA	0	0	0	0	0	0
	OUT OF SAMPLE PERFORMANCE					
1 YR STD	1.465549	1.452164	1.427975	0.3078457	0.3083845	0.3253075
MEAN	0.071693	0.072551	0.074236	0.49955	0.501227	0.529917
2YRS STD	1.581231	1.57997	1.579352	0.3551478	0.3596387	0.4010299
MEAN	0.041618	0.032987	0.016223	0.470298	0.469864	0.472633
3YRS STD	1.82865	1.833722	1.846001	1.310588	1.3089197	1.2959062
MEAN	-0.15567	-0.16006	-0.16859	0.240003	0.237502	0.226659

OPTIMISED PORTFOLIO COMPOSITION

PRINCIPAL COMPONENT MODEL (STOCKS)

	UNHEDGED			HEDGED		
	Rf=0	Rf=0.2	Rf=0.4	Rf=0	Rf=0.2	Rf=0.4
SAUT	0	0	0	0	0	0
SBEL	0.0193506	0.022352	0.031219	0.0419771	0.0437081	0.0456957
SCAN	0	0	0	0.0386964	0.0366248	0.0341631
SDEN	0.2557105	0.2533209	0.2464624	0.2192698	0.2176264	0.2157006
SFRA	0	0	0	0	0	0
SGER	0	0	0	0	0	0
SITA	0	0	0	0	0	0
SJAP	0.2132531	0.2151466	0.2206099	0.4103109	0.4111985	0.4122301
SNET	0	0	0	0	0	0
SNOR	0	0	0	0	0	0
SSPA	0.0545457	0.0550448	0.0565485	0	0	0
SSWE	0	0	0	0	0.0003698	0.0003702
SSWI	0.1100934	0.1065197	0.095859	0.0091742	0.0086176	0.0085513
SUKG	0.3470467	0.3476159	0.3493013	0.0329871	0.0337612	0.0346286
SUSA	0	0	0	0.2475845	0.2480937	0.2486604
		OUT OF SAMPLE PERFORMANCE				
1 YR STD	3.057331	3.183204	3.169828	2.338528	2.339593	2.340504
MEAN	1.207224	1.523634	1.535384	2.736197	2.739033	2.742261
2YRS STD	3.150821	3.187633	3.177045	2.907477	2.91038	2.913784
MEAN	1.282441	1.290968	1.278208	1.356997	1.355712	1.354297
3YRS STD	3.911144	4.549856	4.551154	4.585599	4.591028	4.597088
MEAN	-0.05428	0.074437	0.059858	0.402242	0.400912	0.39944

PRINCIPAL COMPONENT MODEL (COMBINED)

	UNHEDGED			HEDGED		
	Rf=0	Rf=0.2	Rf=0.4	Rf=0	Rf=0.2	Rf=0.4
SAUT	0	0	0	0	0	0
SBEL	0.0338524	0.0538675	0.0727499	0	0	0.0075192
SCAN	0	0	0	0	0	0
SDEN	0	0	0	0	0	0
SFRA	0	0	0	0	0	0
SGER	0	0	0	0	0	0
SITA	0	0	0	0	0	0
SJAP	0.0090147	0	0.0345071	0.0243329	0.0280772	0.0350206
SNET	0	0	0	0.0018586	0.0052572	0.0086178
SNOR	0.0304527	0.0200074	0	0	0	0
SSPA	0.0390154	0.044287	0.0487866	0	0	0
SSWE	0.0226168	0.0317747	0.0436195	0	0	0
SSWI	0	0	0	0	0	0
SUKG	0	0	0	0.0034889	0.0024755	0
SUSA	0	0	0	0	0	0
CAN	0	0	0	0	0	0.0026934
GER	0.0130671	0	0	0.060939	0.0673224	0.0803357
JAP	0.1733091	0.194931	0.181402	0.0358469	0.0393776	0.0492207
UKG	0.5224375	0.5234089	0.5240629	0	0	0
USA	0	0	0	0	0	0
USN	0.1562343	0.1317235	0.094872	0.8735336	0.8574902	0.8165926
		OUT OF SAMPLE PERFORMANCE				
1 YR STD	1.393825	1.418767	1.435794	0.3120769	0.314363	0.34083
MEAN	0.280925	0.312995	0.374695	0.515058	0.523216	0.557014
2YRS STD	1.856939	1.758549	1.707886	0.3697445	0.384336	0.430718
MEAN	-0.01627	-0.02361	-0.01187	0.478272	0.479699	0.430718
3YRS STD	2.126993	2.1872	2.399961	1.324875	1.322372	1.320582
MEAN	-0.19553	-0.19025	-0.22161	0.235482	0.231755	0.224985

OPTIMISED PORTFOLIO COMPOSITION

MAXIMUM LIKELIHOOD MODEL (STOCKS)

	UNHEDGED Rf=0	Rf=0.2	Rf=0.4	HEDGED Rf=0	Rf=0.2	Rf=0.4
SAUT	0	0	0	0	0	0
SBEL	0.0814891	0.0836451	0.0853441	0.137741	0.1530702	0.1584447
SCAN	0	0	0	0	0	0
SDEN	0.1954015	0.1933749	0.191709	0.1649584	0.1477313	0.1414189
SFRA	0	0	0	0.0187307	0.0162481	0.01526
SGER	0	0	0	0.0681109	0.0619673	0.0593603
SITA	0	0	0	0.004139	0.0010741	0.0005533
SJAP	0.1192586	0.1208232	0.122026	0.3638995	0.3748061	0.3785785
SNET	0	0	0	0	0	0
SNOR	0	0	0	0	0	0
SSPA	0.0916964	0.0918767	0.0920284	0.0921933	0.0945751	0.0953373
SSWE	0.0012353	0.0015977	0.0019315	0	0.0016889	0.003687
SSWI	0.1853497	0.1831537	0.1813655	0	0	0
SUKG	0.3255694	0.3255286	0.3255956	0.0430817	0.0461922	0.0470472
SUSA	0	0	0	0.1071455	0.1026466	0.1003128
			OUT OF SAMPLE PERFORMANCE			
1 YR STD	3.2560282	3.2526681	3.2500677	2.8721508	2.905433	2.917003
MEAN	1.379608	1.382294	1.384344	2.511007	2.492474	2.486502
2YRS STD	3.2513274	3.2482672	3.2459836	2.910463	2.948978	2.962373
MEAN	1.259699	1.256439	1.253752	1.52776	1.495581	1.485048
3YRS STD	4.4349859	4.4356589	4.4362664	4.86143	4.904209	4.922757
MEAN	0.132258	0.128147	0.124883	0.305626	0.270811	0.257658

MAXIMUM LIKELIHOOD MODEL (COMBINED)

	UNHEDGED Rf=0	Rf=0.2	Rf=0.4	HEDGED Rf=0	Rf=0.2	Rf=0.4
SAUT	0	0	0	0	0	0
SBEL	0	0	0	0.0104336	0.0138021	0.0186169
SCAN	0	0	0	0	0	0
SDEN	0	0	0	0	0	0
SFRA	0	0	0	0	0	0
SGER	0	0	0	0	0	0
SITA	0	0	0	0	0	0
SJAP	0	0	0	0.0162463	0.0202343	0.0259848
SNET	0	0	0	0	0	0
SNOR	0	0	0	0	0	0
SSPA	0.0486689	0.0522853	0.0611512	0.0016961	0.0025583	0.0037751
SSWE	0.0655098	0.0667906	0.0702638	0.0009262	0.0018791	0.0031936
SSWI	0	0	0	0	0	0
SUKG	0.1224643	0.1244653	0.1325943	0	0	0
SUSA	0	0	0	0	0	0
CAN	0	0	0	0	0	0.0018857
GER	0.1656994	0.1607277	0.1564139	0.054944	0.0618155	0.0710263
JAP	0.0716182	0.0786096	0.0877292	0.0301303	0.0317972	0.0336494
UKG	0.4003663	0.3988407	0.3858533	0	0	0
USA	0.0326377	0.0476548	0.093354	0	0	0
USN	0.0930354	0.0706261	0.0126403	0.8856236	0.8679136	0.8418681
			OUT OF SAMPLE PERFORMANCE			
1 YR STD	1.6804842	1.7027319	1.7888686	0.319509	0.328927	0.350639
MEAN	0.103305	0.106072	0.107148	0.540056	0.557236	0.58129
2YRS STD	1.9366843	1.9619872	2.0387172	0.3619738	0.37995	0.410315
MEAN	0.0695	0.065037	0.066038	0.491715	0.496208	0.502675
3YRS STD	2.3517931	2.3882098	2.4840479	1.322827	1.322995	1.327458
MEAN	-0.14768	-0.14804	-0.14496	0.247092	0.242659	0.236438

OPTIMISED PORTFOLIO COMPOSITION

OVERALL MEAN MODEL (STOCKS)

	UNHEDGED			HEDGED		
	Rf=0	Rf=0.2	Rf=0.4	Rf=0	Rf=0.2	Rf=0.4
SAUT	0.0881207	0.0875297	0.0872834	0.0366656	0.0366584	0.0365877
SBEL	0.1369479	0.1410168	0.1426037	0.0762223	0.0769001	0.0795941
SCAN	0	0	0	0.0913964	0.0906559	0.0877349
SDEN	0.1115812	0.107007	0.1053326	0.1037281	0.1025818	0.0979859
SFRA	0.0152382	0.0142917	0.0138498	0	0	0
SGER	0	0	0	0	0	0
SITA	0	0	0	0	0	0
SJAP	0.0333281	0.0378303	0.039472	0.2169487	0.2177212	0.2207808
SNET	0.1061957	0.1083996	0.1092517	0.0372942	0.0377517	0.0396285
SNOR	0	0	0	0	0	0
SSPA	0	0	0	0	0	0
SSWE	0	0	0	0	0	0
SSWI	0.2365985	0.2341581	0.2333219	0.1428121	0.1426607	0.1419988
SUKG	0.2246966	0.2245312	0.2245055	0.080452	0.0805751	0.0811717
SUSA	0.0472931	0.0452356	0.0443794	0.2144808	0.2144953	0.2145176
			OUT OF SAMPLE PERFORMANCE			
1 YR STD	3.6789808	3.6708146	3.6676491	2.9667614	2.968254	2.974375
MEAN	1.434694	1.434033	1.433637	2.130511	2.130175	2.128593
2YRS STD	3.4937129	3.488137	3.4860422	3.0454233	3.047116	3.054094
MEAN	1.351072	1.341909	1.338501	1.242362	1.241353	1.237227
3YRS STD	4.3318278	4.3310345	4.3307528	4.2304314	4.2328702	4.2425664
MEAN	0.174609	0.163673	0.159657	0.421786	0.420393	0.4148

OPTIMISED PORTFOLIO COMPOSITION

KLEIN & BAWA BAYESIAN MODEL (STOCKS)

	UNHEDGED			HEDGED		
	Rf=0	Rf=0.2	Rf=0.4	Rf=0	Rf=0.2	Rf=0.4
SAUT	0	0	0	0	0	0
SBEL	0	0	0	0	0	0.0044598
SCAN	0	0	0	0.0002894	0.0002752	0.0002145
SDEN	0.267269	0.264734	0.26388	0.2291756	0.2269079	0.2248034
SFRA	0	0	0	0	0	0
SGER	0.008379	0.008176	0.008216	0.0842232	0.0837852	0.0825228
SITA	0.003432	0.002651	0.00272	0.0026134	0.0017574	0.0008406
SJAP	0.188663	0.191762	0.192168	0.3596551	0.3617173	0.3624334
SNET	0	0	0	0	0	0
SNOR	0	0	0	0	0	0
SSPA	0.050573	0.051563	0.051686	0.0288392	0.0291112	0.0292422
SSWE	0.013956	0.015529	0.015639	0.0536856	0.054631	0.0547844
SSWI	0.149621	0.148118	0.148154	0.0643131	0.0644292	0.0646023
SUKG	0.318104	0.317463	0.317532	0.015504	0.0155376	0.0143624
SUSA	0	0	0	0.1617012	0.161848	0.1617341
	OUT OF SAMPLE PERFORMANCE					
1 YR STD	3.2163147	3.2072839	3.205567	2.469943	2.465383	2.470352
MEAN	1.497065	1.496274	1.494839	2.5988919	2.596445	2.60131
2YRS STD	3.222136	3.215987	3.215362	2.7190362	2.719451	2.722216
MEAN	1.361973	1.355437	1.353549	1.668107	1.663461	1.661731
3YRS STD	4.567476	4.573267	4.574133	4.839843	4.846378	4.847885
MEAN	0.133418	0.125969	0.124262	0.498282	0.494178	0.492222

KLEIN & BAWA BAYESIAN MODEL (COMBINED)

	UNHEDGED			HEDGED		
	Rf=0	Rf=0.2	Rf=0.4	Rf=0	Rf=0.2	Rf=0.4
CAN	0	0	0	0.0153163	0.0155053	0.0199267
GER	0.142752	0.141317	0.13023	0.1082206	0.1117115	0.1177685
JAP	0.07848	0.082052	0.100324	0.0441774	0.0440183	0.0447904
UKG	0.484252	0.484239	0.487123	0	0	0
USA	0	0	0	0.0052739	0.0075422	0.0109864
USN	0.222099	0.218644	0.200992	0.7686382	0.7602762	0.7382815
SAUT	0	0	0	0	0	0
SBEL	0	0	0	0.0165373	0.0173823	0.020506
SCAN	0	0	0	0	0	0
SDEN	0	0	0	0	0	0
SFRA	0	0	0	0	0	0
SGER	0	0	0	0	0	0
SITA	0	0	0	0	0	0
SJAP	0	0	0	0.0374584	0.0386767	0.0418838
SNET	0	0	0	0.004378	0.0048875	0.0058567
SNOR	0.051339	0.051063	0.050119	0	0	0
SSPA	0.021077	0.022682	0.03121	0	0	0
SSWE	0	0	0	0	0	0
SSWI	0	0	0	0	0	0
SUKG	0	0	0	0	0	0
SUSA	0	0	0	0	0	0
	OUT OF SAMPLE PERFORMANCE					
1 YR STD	1.4379437	1.4314053	1.4013558	0.3953264	0.405761	0.436749
MEAN	0.073523	0.072971	0.076322	0.567352	0.570888	0.582771
2YRS STD	1.579322	1.57921	1.581919	0.4691535	0.478578	0.505982
MEAN	0.023333	0.018463	-0.0046	0.481595	0.482501	0.485598
3YRS STD	1.840407	1.843331	1.865604	1.294564	1.293804	1.297439
MEAN	-0.16498	-0.16759	-0.17919	0.219062	0.218239	0.215945

OPTIMISED PORTFOLIO COMPOSITION

MINIMUM VARIANCE MODEL (STOCKS)

	UNHEDGED			HEDGED		
	Rf=0	Rf=0.2	Rf=0.4	Rf=0	Rf=0.2	Rf=0.4
SAUT	0	0	0	0	0	0
SBEL	0	0	0	0	0	0
SCAN	0	0	0	0.0690158	0.0690158	0.0690158
SDEN	0.2683	0.2683	0.2683	0.2685408	0.2685408	0.2685408
SFRA	0	0	0	0	0	0
SGER	0	0	0	0.1079909	0.1079909	0.1079909
SITA	0	0	0	0	0	0
SJAP	0.203667	0.203667	0.203667	0.3250583	0.3250583	0.3250583
SNET	0	0	0	0	0	0
SNOR	0	0	0	0	0	0
SSPA	0.05194	0.05194	0.05194	0.0288567	0.0288567	0.0288567
SSWE	0	0	0	0.0404008	0.0404008	0.0404008
SSWI	0.131632	0.131632	0.131632	0.0417179	0.0417179	0.0417179
SUKG	0.344458	0.344458	0.344458	0	0	0
SUSA	0	0	0	0.1184189	0.1184189	0.1184189
			OUT OF SAMPLE PERFORMANCE			
1 YR STD	3.2171773	3.2171773	3.2171773	2.658502	2.658502	2.658502
MEAN	1.4913	1.4913	1.4913	2.550109	2.550109	2.550109
2YRS STD	3.219556	3.219556	3.219556	2.695852	2.695852	2.695852
MEAN	1.316078	1.316078	1.316078	1.767473	1.767473	1.767473
3YRS STD	4.554957	4.554957	4.554957	4.695221	4.695221	4.695221
MEAN	0.104605	0.104605	0.104605	0.539044	0.539044	0.539044

OPTIMISED PORTFOLIO COMPOSITION

CONSTRAINT WEIGHTS MODEL W<=.132 (STOCKS)

	UNHEDGED			HEDGED		
	Rf=0	Rf=0.2	Rf=0.4	Rf=0	Rf=0.2	Rf=0.4
SAUT	0	0	0	0	0	0
SBEL	0.0683181	0.0731405	0.0716631	0.0689903	0.0694788	0.0707494
SCAN	0.0887358	0.0850772	0.0866843	0.132	0.132	0.1307855
SDEN	0.132	0.132	0.132	0.132	0.132	0.132
SFRA	0.0310085	0.0316645	0.0320748	0	0	0
SGER	0.0940536	0.0930617	0.0934199	0.1104002	0.1102501	0.1095422
SITA	0.0041293	0	0	0.0591749	0.0589258	0.0584919
SJAP	0.132	0.132	0.132	0.1320001	0.132	0.132
SNET	0	0	0	0	0	0
SNOR	0	0	0	0	0	0
SSPA	0.1028984	0.1045817	0.1043666	0.0670337	0.0671485	0.0673642
SSWE	0.0628464	0.0645654	0.0648123	0.035745	0.0358418	0.0362415
SSWI	0.132	0.132	0.132	0.081596	0.0813774	0.0817001
SUKG	0.132	0.132	0.132	0.0490597	0.0489775	0.0491253
SUSA	0.0200099	0.019909	0.0189789	0.132	0.132	0.132
	OUT OF SAMPLE PERFORMANCE					
1 YR STD	3.436105	3.435406	3.435584	2.909297	2.909883	2.910362
MEAN	1.576985	1.5861	1.586295	2.176601	2.17751	2.180435
2YRS STD	3.346801	3.41475	3.342765	2.933031	2.93305	2.933422
MEAN	1.414145	1.417651	1.41809	1.427805	1.427769	1.428651
3YRS STD	4.65703	4.660345	4.660872	4.299903	4.299693	4.301318
MEAN	0.107039	0.111362	0.11127	0.45082	0.450858	0.451421

CONSTRAINT WEIGHTS MODEL W<=0.095 (COMBINED)

	UNHEDGED			HEDGED		
	Rf=0	Rf=0.2	Rf=0.4	Rf=0	Rf=0.2	Rf=0.4
CAN	0.095	0.095	0.095	0.095	0.095	0.095
JAP	0.095	0.095	0.095	0.095	0.095	0.095
UKG	0.095	0.095	0.095	0.095	0.095	0.095
USA	0.095	0.095	0.095	0.095	0.095	0.095
USN	0.0950001	0.0950001	0.095	0.095	0.095	0.095
GER	0.095	0.095	0.095	0.095	0.095	0.095
SAUT	-1.00E-06	-1.00E-06	0	0	0	0
SBEL	0.0374163	0.0374163	0.0414337	0.0543994	0.0541448	0.0553916
SCAN	0	0	0	0	0	0
SDEN	0.095	0.095	0.0946854	0.0879273	0.0881948	0.0869678
SFRA	0	0	0	0	0	0
SGER	0	0	0	0.0077613	0.0077959	0.0071488
SITA	0	0	0	0	0	0
SJAP	0.0592376	0.0592376	0.0611668	0.095	0.095	0.095
SNET	0	0	0	0.0130197	0.0129084	0.0137639
SNOR	0	0	0	0	0	0
SSPA	0.0578208	0.0578208	0.058192	0.0386734	0.0386496	0.0387252
SSWE	0	0	0	0.0171915	0.0171188	0.0172949
SSWI	0.0855262	0.0855262	0.079522	0.0581871	0.0583236	0.0580105
SUKG	0.095	0.095	0.095	0	0	0
SUSA	0	0	0	0.0578402	0.0578642	0.0576973
	OUT OF SAMPLE PERFORMANCE					
1 YR STD	2.59457	2.59457	2.588411	1.589468	1.589104	1.590506
MEAN	0.541507	0.541507	0.553296	1.202093	1.202111	1.201751
2YRS STD	2.527783	2.527783	2.521616	1.690746	1.69053	1.691803
MEAN	0.537129	0.537129	0.537638	0.821724	0.822098	0.819916
3YRS STD	2.909968	2.909968	2.907171	2.505143	2.505171	2.504245
MEAN	-0.13176	-0.13176	-0.13391	0.308868	0.309165	0.307608

OPTIMISED PORTFOLIO COMPOSITION

CONSTRAINT WEIGHTS MODEL W>=0.033 (STOCKS)

	UNHEDGED Rf=0	Rf=0.2	Rf=0.4	HEDGED Rf=0	Rf=0.2	Rf=0.4
SAUT	0.03333	0.03333	0.03333	0.033	0.033	0.033
SBEL	0.03333	0.03333	0.03333	0.033	0.033	0.033
SCAN	0.03333	0.03333	0.03333	0.033	0.033	0.033
SDEN	0.2061987	0.2065268	0.2026046	0.2493253	0.2511311	0.248953
SFRA	0.03333	0.03333	0.03333	0.033	0.033	0.033
SGER	0.03333	0.03333	0.03333	0.0389905	0.0392634	0.0389334
SITA	0.03333	0.03333	0.03333	0.033	0.033	0.033
SJAP	0.1635625	0.1632359	0.1671396	0.2879974	0.2864499	0.2882892
SNET	0.03333	0.03333	0.03333	0.033	0.033	0.033
SNOR	0.03333	0.03333	0.03333	0.033	0.033	0.033
SSPA	0.03333	0.03333	0.03333	0.033	0.033	0.033
SSWE	0.03333	0.03333	0.03333	0.033	0.033	0.033
SSWI	0.03333	0.03333	0.03333	0.033	0.033	0.033
SUKG	0.2302788	0.2302772	0.2302959	0.033	0.033	0.033
SUSA	0.03333	0.03333	0.03333	0.0606868	0.0601556	0.0608245
			OUT OF SAMPLE PERFORMANCE			
1 YR STD	3.39088	3.39154	3.383705	2.885186	2.885835	2.88506
MEAN	1.796056	1.796528	1.790884	2.720062	2.724406	2.719142
2YRS STD	3.243803	3.245769	3.243803	2.808353	2.807692	2.808503
MEAN	1.503879	1.504727	1.49459	1.819803	1.824315	1.818835
3YRS STD	4.459961	4.45955	4.464552	4.63052	4.628047	4.630934
MEAN	0.146754	0.147639	0.137067	0.470713	0.474169	0.470036

CONSTRAINT WEIGHTS MODEL W>=0.02375 (COMBINED)

	UNHEDGED Rf=0	Rf=0.2	Rf=0.4	HEDGED Rf=0	Rf=0.2	Rf=0.4
CAN	0.02375	0.02375	0.02375	0.02375	0.02375	0.02375
JAP	0.02375	0.02375	0.02375	0.0940081	0.0985655	0.1008481
UKG	0.4573571	0.4567778	0.4572415	0.02375	0.02375	0.02375
USA	0.02375	0.02375	0.02375	0.02375	0.02375	0.02375
USN	0.052848	0.0576844	0.0538132	0.454742	0.4501845	0.4479019
GER	0.0622948	0.0580378	0.0614453	0.02375	0.02375	0.02375
SAUT	0.02375	0.02375	0.02375	0.02375	0.02375	0.02375
SBEL	0.02375	0.02375	0.02375	0.02375	0.02375	0.02375
SCAN	0.02375	0.02375	0.02375	0.02375	0.02375	0.02375
SDEN	0.02375	0.02375	0.02375	0.02375	0.02375	0.02375
SFRA	0.02375	0.02375	0.02375	0.02375	0.02375	0.02375
SGER	0.02375	0.02375	0.02375	0.02375	0.02375	0.02375
SITA	0.02375	0.02375	0.02375	0.02375	0.02375	0.02375
SJAP	0.02375	0.02375	0.02375	0.02375	0.02375	0.02375
SNET	0.02375	0.02375	0.02375	0.02375	0.02375	0.02375
SNOR	0.02375	0.02375	0.02375	0.02375	0.02375	0.02375
SSPA	0.02375	0.02375	0.02375	0.02375	0.02375	0.02375
SSWE	0.02375	0.02375	0.02375	0.02375	0.02375	0.02375
SSWI	0.02375	0.02375	0.02375	0.02375	0.02375	0.02375
SUKG	0.02375	0.02375	0.02375	0.02375	0.02375	0.02375
SUSA	0.02375	0.02375	0.02375	0.02375	0.02375	0.02375
			OUT OF SAMPLE PERFORMANCE			
1 YR STD	1.995885	2.00141	1.996975	1.267591	1.26864	1.269176
MEAN	0.599992	0.605838	0.601158	1.15396	1.152711	1.152086
2YRS STD	2.117985	2.119143	2.118203	1.311677	1.31607	1.318281
MEAN	0.449353	0.45237	0.449955	0.824798	0.821739	0.820207
3YRS STD	2.556503	2.557232	2.556636	2.034636	2.036584	2.03758
MEAN	-0.01577	-0.01591	-0.01579	0.296467	0.295028	0.294307

OPTIMISED PORTFOLIO COMPOSITION

JOBSON & KORKIE MODEL C=0 (STOCKS)

	UNHEDGED			HEDGED		
	Rf=0	Rf=0.2	Rf=0.4	Rf=0	Rf=0.2	Rf=0.4
SAUT	0	0	0	0	0	0
SBEL	0	0	0	0.008036	0.008036	0.008036
SCAN	0	0	0	0	0	0
SDEN	0.2785833	0.2785833	0.2785833	0.293455	0.293455	0.293455
SFRA	0	0	0	0.0405245	0.0405245	0.0405245
SGER	0.0081037	0.0081037	0.0081037	0	0	0
SITA	0.0066224	0.0066224	0.0066224	0	0	0
SJAP	0.1746719	0.1746719	0.1746719	0.3327786	0.3327786	0.3327786
SNET	0	0	0	0	0	0
SNOR	0	0	0	0	0	0
SSPA	0.0459219	0.0459219	0.0459219	0.0416398	0.0416398	0.0416398
SSWE	0.0079418	0.0079418	0.0079418	0.0693702	0.0693702	0.0693702
SSWI	0.1596305	0.1596305	0.1596305	0	0	0
SUKG	0.3185245	0.3185245	0.3185245	0	0	0
SUSA	0	0	0	0.2141959	0.2141959	0.2141959
		OUT OF SAMPLE PERFORMANCE				
1 YR STD	3.2567729	3.2567729	3.2567729	2.688246	2.688246	2.688246
MEAN	1.499151	1.499151	1.499151	2.609553	2.609553	2.609553
2YRS STD	3.25166	3.25166	3.25166	2.795405	2.795405	2.795405
MEAN	1.391521	1.391521	1.391521	1.600397	1.600397	1.600397
3YRS STD	4.545287	4.545287	4.545287	4.650624	4.650624	4.650624
MEAN	0.166516	0.166516	0.166516	0.545398	0.545398	0.545398

JOBSON & KORKIE MODEL C=0.2 (STOCKS)

	UNHEDGED			HEDGED		
	Rf=0	Rf=0.2	Rf=0.4	Rf=0	Rf=0.2	Rf=0.4
SAUT	0	0	0	0	0	0
SBEL	0	0	0	0	0	0
SCAN	0	0	0	0.0794432	0.0794432	0.0737688
SDEN	0.2334766	0.2459398	0.238067	0.2745475	0.2745475	0.2709993
SFRA	0	0	0	0	0	0
SGER	0.0073846	0.0081674	0.0078144	0.1082969	0.1082969	0.1065931
SITA	0	0	0	0.0171738	0.0171738	0.0159166
SJAP	0.227758	0.2137954	0.2226563	0.3133217	0.3133217	0.3170619
SNET	0	0	0	0	0	0
SNOR	0	0	0	0	0	0
SSPA	0.062155	0.0582712	0.0607427	0.0239815	0.0239815	0.0244463
SSWE	0.028974	0.0239157	0.0271271	0.0325936	0.0325936	0.0344462
SSWI	0.1218808	0.1317102	0.1250825	0.0374753	0.0374753	0.0397196
SUKG	0.3183719	0.3182011	0.3185093	0.0010033	0.0010033	0.0010064
SUSA	0	0	0	0.1121632	0.1121632	0.1160417
		OUT OF SAMPLE PERFORMANCE				
1 YR STD	3.1114673	3.1475913	3.1245838	2.684756	2.684756	2.676994
MEAN	1.483177	1.489682	1.485984	2.551387	2.551387	2.546054
2YRS STD	3.159535	3.178416	3.166029	2.705017	2.705017	2.704333
MEAN	1.271939	1.305509	1.284409	1.774212	1.774212	1.766569
3YRS STD	4.644261	4.614477	4.63295	4.647265	4.647265	4.661607
MEAN	0.033945	0.070968	0.04764	0.5371	0.5371	0.534653

OPTIMISED PORTFOLIO COMPOSITION

JOBSON & KORKIE MODEL C=0.4 (STOCKS)

	UNHEDGED			HEDGED		
	Rf=0	Rf=0.2	Rf=0.4	Rf=0	Rf=0.2	Rf=0.4
SAUT	0	0	0	0	0	0
SBEL	0	0	0	0	0	0
SCAN	0	0	0	0.0735105	0.0764712	0.0705741
SDEN	0.2684512	0.2667122	0.2658769	0.2710626	0.2728274	0.269299
SFRA	0	0	0	0	0	0
SGER	0.0032168	0.0067487	0.0067033	0.1064366	0.1073404	0.1055609
SITA	0.004174	0.0038071	0.003935	0.0160325	0.0166052	0.0154434
SJAP	0.1835794	0.1869274	0.1898969	0.3168918	0.3150638	0.3187386
SNET	0	0	0	0	0	0
SNOR	0	0	0	0	0	0
SSPA	0.05099	0.0497585	0.049975	0.0243617	0.0241436	0.0245497
SSWE	0.0127455	0.0122951	0.013087	0.0342437	0.0333807	0.0351235
SSWI	0.1577759	0.1529565	0.1525155	0.0396038	0.038439	0.0407133
SUKG	0.3190671	0.3207945	0.3180105	0.0023539	0.0023467	0.0023563
SUSA	0	0	0	0.115503	0.1133821	0.1176414
		OUT OF SAMPLE PERFORMANCE				
1 YR STD	3.2263075	3.2211846	3.2136817	2.677182	2.68103	2.67344
MEAN	1.484671	1.489306	1.49086	2.545084	2.54786	2.542376
2YRS STD	3.232337	3.228418	3.223098	2.704831	2.705057	2.704691
MEAN	1.35778	1.357711	1.355724	1.766445	1.770483	1.762444
3YRS STD	4.557206	4.56272	4.567842	4.661154	4.654238	4.668129
MEAN	0.137185	0.133855	0.12836	0.534185	0.535507	0.532908

OPTIMISED PORTFOLIO COMPOSITION

NAIVE (STOCKS)

	UNHEDGED			HEDGED		
	Rf=0	Rf=0.2	Rf=0.4	Rf=0	Rf=0.2	Rf=0.4
SAUT	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667
SBEL	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667
SCAN	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667
SDEN	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667
SFRA	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667
SGER	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667
SITA	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667
SJAP	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667
SNET	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667
SNOR	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667
SSPA	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667
SSWE	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667
SSWI	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667
SUKG	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667
SUSA	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667	0.0666667
	OUT OF SAMPLE PERFORMANCE					
1 YR STD	3.756029	3.756029	3.756029	3.273659	3.273659	3.273659
MEAN	1.784729	1.784729	1.784729	2.502756	2.502756	2.502756
2YRS STD	3.539139	3.539139	3.539139	3.246506	3.246506	3.246506
MEAN	1.551501	1.551501	1.551501	1.609522	1.609522	1.609522
3YRS STD	4.589033	4.589033	4.589033	4.427378	4.427378	4.427378
MEAN	0.134347	0.134347	0.134347	0.440333	0.440333	0.440333

NAIVE (COMBINED)

	UNHEDGED			HEDGED		
	Rf=0	Rf=0.2	Rf=0.4	Rf=0	Rf=0.2	Rf=0.4
CAN	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
GER	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
JAP	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
UKG	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
USA	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
USN	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
SAUT	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
SBEL	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
SCAN	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
SDEN	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
SFRA	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
SGER	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
SITA	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
SJAP	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
SNET	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
SNOR	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
SSPA	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
SSWE	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
SSWI	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
SUKG	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
SUSA	0.047619	0.047619	0.047619	0.047619	0.047619	0.047619
	OUT OF SAMPLE PERFORMANCE					
1 YR STD	3.223631	3.223631	3.223631	2.5109466	2.5109466	2.5109466
MEAN	1.24021	1.24021	1.24021	1.83848	1.83848	1.83848
2YRS STD	3.056365	3.056365	3.056365	2.5006197	2.5006197	2.5006197
MEAN	1.096952	1.096952	1.096952	1.235349	1.235349	1.235349
3YRS STD	3.722079	3.722079	3.722079	3.391871	3.391871	3.391871
MEAN	0.007805	0.007805	0.007805	0.357735	0.357735	0.357735

OPTIMISED PORTFOLIO COMPOSITION

BAYESIAN SINGLE-INDEX MODEL (STOCKS)

	UNHEDGED (EW)			UNHEDGED (MV)		
	Rf=0	Rf=0.2	Rf=0.4	Rf=0	Rf=0.2	Rf=0.4
SAUT	0	0	0	0	0	0
SBEL	0.1110981	0.1128098	0.1147639	0.13562	0.1390359	0.1431127
SCAN	0	0	0	0	0	0
SDEN	0.180637	0.1796073	0.1778743	0.1659498	0.1623277	0.1577915
SFRA	0.015883	0.0151626	0.0144635	0.0476012	0.0463924	0.0450686
SGER	0.0200904	0.0191534	0.0183366	0.0654884	0.0641669	0.0624633
SITA	0.0121394	0.0118359	0.0115631	0.0205086	0.0202593	0.0198975
SJAP	0.1609329	0.1626306	0.164135	0.064032	0.0674218	0.0716638
SNET	0.0298644	0.0304905	0.0310944	0.0177883	0.0195057	0.0216671
SNOR	0	0	0	0	0	0
SSPA	0.0602926	0.0605663	0.0614913	0.054183	0.0552036	0.0564701
SSWE	0.0351283	0.0353824	0.0361106	0.0348652	0.035626	0.0366287
SSWI	0.1261816	0.1248651	0.1232976	0.185789	0.1826684	0.1788279
SUKG	0.2477522	0.2474961	0.2468698	0.2081745	0.2073923	0.2064089
SUSA	0	0	0	0	0	0
1 YR STD	3.239199	3.2345015	3.2290463	3.6314997	3.619761	3.6055693
MEAN	1.603074	1.603606	1.603586	1.64558	1.644804	1.643964
2YRS STD	3.2026801	3.1992968	3.1955093	3.4586038	3.449184	3.4380779
MEAN	1.383031	1.378752	1.3736	1.575934	1.565656	1.552935
3YRS STD	4.5190751	4.5185484	4.5188904	4.6091529	4.6054099	4.6012813
MEAN	0.120568	0.116218	0.111144	0.305737	0.295362	0.282506

EW= Equally Weighted
MV = Market Value

In fact, as shown by Taylor & Tonks (1989), a link also exists between the existence of co-integration and the empirical evidence on international market integration-segmentation. A major part of Taylor & Tonks's contribution lies in that they showed that in the "short term" the country specific factors are significant, so that results from the various ICAPM tend to support segmentation, effectively distorting the fact that in the "long term" stock market returns from countries i and j could be much more closely integrated.

5.2 Applying the Econometrics of Co-Integration to International Index Returns

More formally, any short term "distortion" between two market indices i, j can be represented by a random variable ϵ standing for the joint effect of the country specific factors as expressed from the following relationship:

$$\ln P_j = a + b \ln P_k + \epsilon \qquad (5.1)$$

where $\ln P_j$, $\ln p_k$ are the logarithms of the respective price indices

The econometrics of "co-integration" provide a means of testing whether asset markets move stochastically together so that to be in a form of long term equilibrium. All that is required for two variables to be "co-integrated" is to show that:

i) the logarithms of the price indices under consideration are integrated of order one[1] I(1), i.e. have one unit root, and

ii) the random variable ϵ is a unique linear combination between the respective logarithms of the price indices $\ln p_j$, $\ln p_k$ which is stationary or I(0)[2].

The first of the above conditions relates to the fact that the stationary I(0) returns represent the first logarithmic difference of price indices, while the second one is necessary because otherwise $\ln p_j$ and $\ln p_k$ will drift apart without long term equilibrium.

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[1] Implying that they need to be differenced once to become stationary

[2] The uniqueness of the cointegrating vector is applicable in case we test for co-integration between two variables only, practice which we consistently follow throughout this section

CHAPTER V

AN ALTERNATIVE MULTIVARIATE APPROACH FOR IDENTIFYING PREDICTABLE ELEMENTS IN INDEX RETURNS: GRANGER CAUSALITY FOR CO-INTEGRATED INDICES

5.1 Introduction

Despite the fact that most of the procedures in Chapter IV have been found to be capable of improving portfolio performance in relation to the historical benchmark, "ex-ante" optimal portfolios are always bound to substantially underperform the "ex-post" one's primarily because of the difficulties related with estimating future expected index returns. This is particularly important, since portfolio performance has been shown to be more sensitive to misestimation of the mean return vector compared to errors in estimating variances and correlations.

In fact, even though it has been demonstrated that controlling estimation risk in historical returns can add value to global index portfolios, the issue of identifying predictable elements in future index returns has not been addressed beyond the attempt of applying the Pseudo-APT.

The principal aim of this chapter is to examine whether a long term equilibrium relationship exists between some of the indices under consideration, such that would allow for at least one of them to have predictive power over long term movements in the others. Pairwise co-integration analysis is ideally suited for that purpose, since the existence of co-integration would generally imply that following a "shock" that causes temporarily two market indices to drift apart, a long term "Error Correction Mechanism" exists (see Granger 1983) that would cause the indices to be gradually smoothed towards a long term equilibrium in a predictable manner.

The procedure described below, has been applied to all the real stock and bond indices in our sample. Nevertheless, since the primary aim in this section is to establish predictable long term return patterns rather than to generate inputs for portfolio optimization problems, the tests were applied to the entire sample period of 108 monthly observations, rather than only to the 72 observations of the sampling period. Furthermore, in an attempt to identify more subtle patterns in the world's bond markets, the coverage of the bond indices has been extended to include all 21 Salomon Brothers Indices as opposed to the 6 only indices used in the combined portfolios of the previous sections.

5.2.1 Testing Whether Stock Market Indices are Integrated of Order One

The starting point in the testing procedure, consists in establishing whether the logarithms of the indices under consideration[3] are I(1) or not, since in the latter case no further co-integration tests are possible.

For this purpose, equation 3.5 (from chapter 3) has been estimated by OLS for all logarithmic index series by including zero, one and three lags of the differenced dependent variable respectively. In order to make the testing procedure as precise as possible[4], an approach recommended by Dolado & Jenkinson (1987) has been followed which consists of testing first the more general specification, i.e. the hypothesis that the logarithms of the real indices contain both a trend and a unit root[5].

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[3] Real Sterling indices were constructed from the nominal ones by making the appropriate adjustments for the UK RPI.

[4] Taylor & Tonks followed a much less precise testing procedure, since they ignored the trend and did not check the robustness of their results for different number of lags in the augmented polynomial. This was probably due to the fact that the first draft of their paper dated back in 1987, at which time the technical aspects of "co-integration" analysis were not yet well developed.

[5] Dolado & Wilkinson argue that the a priori acceptance of both a trend and a unit root is somewhat counter-intuitive since in logarithmic terms it is equivalent to a continuously increasing or decreasing rate of change.

In this context, since the estimated statistics were found to be consistently lower than the critical values, it was necessary to check about the trend's significance by comparing the corresponding t statistic not with the Dickey but with the ordinary tables. Since the trend statistics were insignificant at the 5% significance level in virtually all cases tested, both hedged and unhedged, the hypothesis that real indices contain both a trend and a unit root had to be rejected and an alternative equation needed to be estimated, i.e.

$$\Delta y_t = a_1 + \rho y_{t-1} + \sum_{i=1}^{m} \beta_i \Delta y_{t-1} + \epsilon_t \tag{5.2}$$

which differs from equation 3.5 only in that the trend had to be omitted.

OLS procedures were used to estimate equation 5.2 for all series repeatedly for zero, one and three lags in the polynomial. Table 5.1 contains the values of the estimated DF/ADF statistics[6]. Notice that since the trend is omitted from the model specification, the critical value become smaller, in this case -2.89 for the 5% confidence level[7].

On the basis of the results for the unhedged series the null hypothesis of a unit root can only be rejected at the 5% level for the Canadian and US Government Bonds as well as for the US and Canadian stock indices (in the last case only the ADF but not the DF statistic was significant). Consequently, these indices should be subsequently excluded from the co-integration tests.

[6] Complete output from unit root regressions for all series available by the author at request. This refers to both the final as well as to the abortive more general specification.

[7] Strictly speaking, it was also necessary to check the t statistic of the constant on the basis of equation 5.2 since if it proves to be insignificant the model needs to be re-specified in a more restrictive form. In this occasion in over 90% of the series the constant was significant at the 10% level and in about 65% of the series significant at the 5% level. Consequently, we concluded that the most appropriate specification is the one with the constant but without the trend.

Table 5.1

Unit Root Tests for Real Asset Indices

(1)	(2) DF(0)	(3) ADF(1)	(4) ADF(3)	(5) DF(0)	(6) ADF(1)	(7) ADF(3)
STOCKS	**UNHEG**	**UNHEG**	**UNHEG**	**HEDGE**	**HEDGE**	**HEDGE**
Austria	-2.271	-2.327	-2.254	-2.315	-2.364	-2.421
Belgium	-1.627	-1.634	-1.829	-1.674	-1.467	-1.816
Canada	-2.216	-2.612	-3.085*	-1.697	-1.968	-2.471
Denmark	-1.513	-1.581	-1.808	-1.222	-1.247	-1.608
France	-1.510	-1.378	-1.441	-1.646	-1.458	-1.496
Germany	-1.841	-1.867	-1.881	-1.685	-1.703	-1.760
Italy	-1.331	-1.315	-1.526	-1.145	-1.194	-1.359
Japan	-1.571	-1.705	-1.783	-1.300	-1.454	-1.588
Netherland	-2.386	-2.654	-2.312	-2.218	-2.501	-2.257
Norway	-1.506	-1.869	-1.934	-1.471	-1.862	-1.984
Spain	- .982	-1.070	-1.017	-1.072	-1.124	-1.062
Sweden	-2.070	-2.037	-2.413	-2.124	-2.110	-2.548
Switzerlan	-1.764	-1.843	-1.952	-1.691	-1.764	-1.819
UK	-1.767	-2.028	-2.151	-1.767	-2.028	-2.151
USA	-2.477	-2.711	-2.810	-1.159	-1.393	-1.445
BONDS	**UNHEG**	**UNHEG**	**UNHEG**	**HEDGE**	**HEDGE**	**HEDGE**
Cana\GB	-3.283*	-3.125*	-3.185*	-2.206	-1.994	-1.930
Fran\GB	-1.519	-1.548	-1.578	-2.087	-2.372	-2.583
Germ\GB	-2.214	-2.208	-1.857	-2.520	-2.638	-2.473
Japa\GB	-1.935	-1.940	-1.892	-1.513	-1.524	-1.526
Neth\GB	-2.170	-2.107	-1.953	-2.651	-2.665	-2.521
Swit\GB	-1.668	-1.896	-1.912	-1.969	-2.423	-2.051
UK\GB	-2.650	-2.506	-2.602	-2.650	-2.506	-2.602
USA\GB	-3.173*	-2.907*	-3.019*	-1.498	-1.366	-1.399
USA\FRN	-2.443	-2.321	-2.445	-2.698	-3.186*	-3.094*
USA\ZER	-3.078*	-2.981*	-2.979*	-1.581	-1.616	-1.517

When the trend is omitted the appropriate critical value at the 5% level is -2.89

*** implies that the null hypothesis of a Unit Root is rejected at the 5% confidence level**

The DF/ADF model is specified to include a constant but no trend

The DF is based on zero lags in the augmented polynomial

ADF(1) and ADF(3) relate to number of lags included in the augmented polynomial

For all the remaining unhedged indices the null could not be rejected at 5%, so that the existence of a unit root is accepted and co-integration tests can be carried out. Notice, however, that there were a few marginal cases where the null would have been rejected at the 10% confidence level; Such marginal cases inevitably create uncertainty about the validity of subsequent results from the corresponding co-integrating regressions.

When considering the hedged index logarithms no DF/ADF statistic we found to be significant at the 5% level, the only exception being the Dollar FRN's. For a number of bond indices, however, the assumption of a unit root would have been rejected at the 10% confidence level.

A final issue to be addressed is related to the question of what is the appropriate number of lags to be included in the augmented polynomial. This is particularly important for those few cases for which the existence of a unit root has been rejected at some lag(s). Statistically, this issue can be handled by saving the residuals from all the OLS regressions at zero, one and three lags and then calculate the Box-Ljung statistic in order to establish whether they are white noise; If the resulting residuals are indeed white noise, then the corresponding ADF/DF statistics at that lag should be deemed to be more or less accurate.

In fact, the ACF and B-L statistic were computed for all the residuals series from the ADF/DF regressions for which the null of a unit root has been rejected even at the 10% level. These results are summarized below in table 5.2:

When examining the residuals from the zero lag regressions, we can observe that both the ACF and the B-L statistic computed at the first lag (columns 4 & 5) are quite high, even though never significant at the 5% level (apart from the hedged Dollar FRN). However, the inclusion of a single lagged dependent variable in the augmented polynomial virtually eliminates all serial correlation in the residuals, both the ACF and Box-Ljung becoming effectively zero in most cases. Similar are the results from the three lag ADF regression:

Therefore, we conclude that the ADF statistics estimated on the basis of a single lag are the most accurate even though the simple DF (no added lags) should also be considered to provide reasonably reliable results.

5.2.2 Testing for Market Index Co-Integration

In principle, when we consider non-stationary time series, almost any linear combination of them will produce a time series with a variance that is asymptotically infinite, the only possible exception being the unique co-integrating vector. Since a simple static OLS regression is known to minimize the variance of the residual error, it is likely that the vector of OLS residuals is the cointegrating vector. Consequently a two step testing procedure naturally suggests itself:

At first, it is necessary to estimate equation 5.2 by OLS for all pairs[8] of series whose natural logarithms were found to be I_1 and save the regression residuals. Since the existence of co-integration requires that the residuals are stationary, the second step in the procedure is to test for unit roots for the residuals using the co-integrating regression defined below as follows:

$$\Delta\hat{\epsilon}_t = c_1 + \rho\,\hat{\epsilon}_{t-1} + \sum_{i=1}^{m} \beta_i\,\Delta\hat{\epsilon}_{t-1} + u_t \tag{5.3}$$

where $\hat{\epsilon}_t$ stands for the estimated residuals from the OLS regression[9]

In this context also the $\hat{t}(\rho)$ can be used to determine whether the estimated residuals are I_0. Notice however that the critical values for the rejection of the null hypothesis of non co-integration (existence of a unit root in the residuals) are higher than the previously used Dickey values. Such critical values have been tabulated by Engle & Yoo (1987), and for a sample size of around 100 are -3.37 and -3.03 at the 5% and 10% significance levels respectively.

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[8] Because of the transitivity property of cointegration, it is in fact unnecessary to test for all pairwise combinations: if I_1 is cointegrated with both I_2 and I_3, then it can be easily proved that I_2 will also be cointegrated with I_3.

[9] Notice, that no trend is included in the co-integrating regression

Table 5.2

Tests of Serial Correlation for the ADF Residuals

(1)	(2) LAG	(3) TYPE	(4) ACF(1lg)	(5) B-L (1lg)	(6) Prob B-L
STOCKS					
Canada	0	Unhg	.140	2.165	.141
Canada	1	Unhg	-.031	.104	.747
Canada	3	Unhg	-.013	.019	.889
UK	0	Unhg	-.020	.043	.836
UK	1	Unhg	-.005	.003	.956
UK	3	Unhg	-.002	.000	.986
USA	0	Unhg	.137	2.098	.147
USA	1	Unhg	-.002	.000	.984
USA	3	Unhg	-.009	.008	.927
BONDS					
Can/GB	0	Unhg	.131	1.919	.166
Can/GB	1	Unhg	.008	.006	.936
Can/GB	3	Unhg	-.014	.022	.881
UK/GB	0	Unhg	.059	.387	.534
UK/GB	1	Unhg	-.000	.000	.998
UK/GB	3	Unhg	-.026	.074	.785
US/GB	0	Unhg	.151	2.519	.112
US/GB	1	Unhg	.000	.000	.996
US/GB	3	Unhg	-.006	.004	.949
US/Zero	0	Unhg	.103	1.180	.277
US/Zero	1	Unhg	-.012	.016	.899
US/Zero	3	Unhg	.062	.000	.984
Net/GB	0	Hedg	-.049	.265	.606
Net/GB	1	Hedg	.000	.000	.999
Net/GB	3	Hedg	-.000	.000	.998
US/FRN	0	Hedg	-.262	7.650	.006*
US/FRN	1	Hedg	-.042	.195	.659
US/FRN	3	Hedg	-.015	.024	.877

Column (2) refers to the number of lags included in the unit root regression

Columns (4 & 5) list the ACF and B-L values for the regression residual series computed at the first lag

Column 6 shows the exact confidence level for the B-L statistic

Notice that in case we are unable to reject the hypothesis that the residuals are non-stationary and therefore reject co-integration, the economic significance lies in that after an initial "shock" the international market indices will be drifting apart towards disequilibrium, so that no meaningful long term relationship might exist among the world's asset markets. Exactly the opposite would be if co-integration is accepted.

5.2.3 Empirical Evidence on Co-integration

In applying the co-integration tests, a number of alternative pair-wise relationships were considered worth investigating empirically:

Inter-Relationships between the World's Stock Markets

The main issue here, is to establish what are the long term implications of distortions caused by country specific influences; If these turn out to be temporary in nature, then the implications for asset allocation will depend on the "speed of adjustment" compared to the investors portfolio holding horizon.

Inter-Relationships between Government Bond Indices from different countries

Since these types of fixed income securities are default-free, the existence of a long term trend among bond indices with similar duration would crucially depend on worldwide interest rate movements.

Inter-Relationships between Fixed Income Indices denominated in the same currency

In the absence of additional disturbing factors (e.g variable risk premia, differential taxation and regulation etc.) one might expect different types of bond indices in the same currency and with similar average maturities (issues over five years) to be in long term equilibrium, in which case this type of diversification would provide minimal benefits for fixed interest investors.

Inter-Relationships between Stock and Bond Indices within the same country

Despite the fact that domestic economic "shocks" have much more severe short term impact on a national stock market then on a bond index and the mean return on stocks is higher, it is still perfectly possible that bond and stock indices within the national boundaries follow a common long term trend based on domestic and global economic developments. Such an eventuality would have implications for the optimal domestic bond/stock for investors with relatively long term investment horizons.

5.2.3.1 Evidence on Co-integration between International Stock Indices

Equations (5.2 & 5.3) were estimated for eighteen different pairs of unhedged real stock market indices (logarithms) and then repeated for their hedged counterparts. Pairwise index combinations were constructed in connection with the German, Japanese and UK indices and not with the US index because the unit root tests showed in to be quite close to the nonstationary boundaries and, therefore, not naturally suitable for co-integration tests[10]. Notice also that because of the transitivity property of co-integration it is unnecessary to test all the remaining pairwise stock index combinations to draw useful inferences about their long term relationships.

For the co-integrating regression, the ADF procedures were applied to the residuals previously estimated from equation (5.2), by including, zero, one and three lags of the dependent variable in the augmented polynomial. The critical statistics from these regressions can be seen in table 5.6:

For the majority of hedged and unhedged pairs tested, no co-integration was found, rejecting therefore the existence of long term equilibrium between international stock markets. There are however notable exceptions, concerning exclusively European markets.

[10] Co-integration tests have been applied only for pairs of indices that the null hypothesis of a unit root cannot be rejected at the 5% confidence level for neither variable. However occasionally one of the variables was close to the nonstationary boundaries, so that the validity of the cointegration results in these cases to be somewhat doubtful.

Table 5.3

Cointegration Tests for Real Stock Indices

(1)	(2) DF(0)	(3) ADF(1)	(4) ADF(3)	(5) DF(0)	(6) ADF(1)	(7) ADF(3)
PAIRS	UNHEG	UNHEG	UNHEG	HEDGE	HEDGE	HEDGE
UK/Canada	- .821^	- .892^	- .734^	-2.602	-2.466	-2.058
UK/France	-3.086**	-3.462*	-3.488*	-3.236**	-3.436*	-3.078**
UK/Japan	-2.651	-2.268	-1.894	-3.040**	-2.776	-2.366
UK/Nether	-3.594*	-3.322**	-2.379	-2.888	-2.644	-1.729
UK/Switze	-2.049	-2.559	-2.647	-2.644	-2.867	-2.752
UK/US	-1.893^	-1.911^	-1.598^	-1.964	-1.529	-1.169
UK/German	-2.181	-2.295	-2.377	-2.244	-2.362	-2.339
Jap/Canada	-1.473^	-1.452^	-1.327^	-2.419	-2.235	-2.294
Jap/France	-1.611	-1.446	-1.772	-1.934	-1.513	-1.679
Jap/Nether	-1.942	-1.871	-1.371	-2.045	-1.990	-1.548
Jap/Switze	-1.658	-1.750	-1.836	-1.843	-1.810	-1.749
Jap/US	-1.953^	-2.017^	-1.729^	-1.468	-1.157	- .937
Jap/German	-1.444	-1.520	-1.753	-1.474	-1.487	-1.711
Ger/Canada	-1.365^	-1.351^	-1.523^	-2.117	-1.995	-2.110
Ger/France	-2.522	-2.758	-2.795	-2.300	-2.494	-2.521
Ger/Nether	-2.045	-2.170	-2.183	-2.043	-2.164	-2.170
Ger/Switze	-3.379*	-3.476*	-2.662	-3.369*	-3.268**	-2.526
Ger/USA	-2.413^	-2.406^	-2.432^	-1.839	-1.754	-1.825

Column (1) lists the pairs of non-stationary stock market indices tested for co-integration

ADF(0), ADF(1) and ADF(3) relate to the number of lags included in the augmented polynomial for the co-integrating regression

*** and ** : The null of no "co-integration" rejected at the 5% and 10% confidence levels respectively**

Critical values for co-integrating regression: 5% = 3.37 10% = 3.03

^ : Implies that the validity of the respective statistics is somewhat doubtful, because one of the variables is close to the non stationary boundaries.

Table 5.4

Cointegration Tests For Real Bond Indices

(1)	(2) DF(0)	(3) ADF(1)	(4) ADF(3)	(5) DF(0)	(6) ADF(1)	(7) ADF(3)
PAIRS	**UNHEG**	**UNHEG**	**UNHEG**	**HEDGE**	**HEDGE**	**HEDGE**
UK/Canada	NA	NA	NA	-3.702^*	-3.541^*	-3.631^*
UK/France	-3.604^*	-3.856^*	-3.635^*	-3.374^*	-3.297^**	-3.032^**
UK/Japan	-2.848^	-2.988^	-2.589^	-4.586^*	-4.604^*	-4.920^*
UK/Nether	-3.178^**	-3.895^*	-3.471^*	-4.029^*	-4.561^*	-4.386^*
UK/Switze	-2.021^	-2.151^	-1.784^	-4.033^*	-3.561^*	-4.042^*
UK/US	NA	NA	NA	-3.023^	-3.091^**	-2.897^
UK/German	-3.037^**	-3.474^*	-3.394^*	-4.094^*	-4.179^*	-4.519^*
Jap/Canada	NA	NA	NA	-2.850	-2.775	-2.575
Jap/France	- .942	- .786	- .666	-2.048	-1.896	-2.124
Jap/Nether	-2.204	-2.087	-2.062	-4.078^*	-3.650^*	-3.937^*
Jap/Switze	-1.604	-1.547	-1.641	-3.032**	-2.405	-2.120
Jap/US	NA	NA	NA	-1.664	-1.815	-1.724
Jap/German	-2.245	-2.048	-1.905	-4.272*	-4.024*	-3.517*
Ger/Canada	NA	NA	NA	-3.181**	-2.822	-2.361
Ger/France	-1.690	-1.794	-1.250	-1.847	-1.826	-1.476
Ger/Nether	-3.961*	-4.741*	-3.871*	-3.660^*	-3.893^*	-3.230^**
Ger/Switze	-1.303	-1.329	- .800	-2.036	-1.382	-1.535
Ger/USA	NA	NA	NA	-1.904	-1.588	- .809

Column (1) lists the pairs of non-stationary bond market indices tested for co-integration

ADF(0), ADF(1) and ADF(3) relate to number of lags included in augmented polynomial

*** and ** : The null of no co-integration rejected at the 5% and 10% confidence levels respectively**

Critical values for co-integrating regression: 5% = 3.37 10% = 3.03

^ : Implies that the validity of the respective statistics is somewhat doubtful, because one of the variables is close to the non-stationary boundaries.

NA Co-integration tests are non applicable, because one of the variables is not I_1

Table 5.5

Cointegration Tests: Stocks/Bonds and Eurobonds/Government Bonds

(1)	(2) DF(0)	(3) ADF(1)	(4) ADF(3)	(5) DF(0)	(6) ADF(1)	(7) ADF(3)
STOCK/GB'S	**UNHEG**	**UNHEG**	**UNHEG**	**HEDGE**	**HEDGE**	**HEDGE**
Canada	NA	NA	NA	-2.899	-2.617	-2.314
France	-2.893	-3.271**	-3.539*	-2.155	-2.565	-3.065**
Germany	-2.042	-2.498	-2.270	-1.803	-2.309	-2.210
Japan	-2.196	-2.131	-1.996	-2.880	-2.616	-2.467
Netherla	-2.179	-2.751	-2.596	-2.183^	-2.645^	-2.605^
Switzerl	-1.638	-1.858	-1.638	-1.805	-1.982	-1.916
UK	-3.671^*	-3.752^*	-3.577^*	-3.680^*	-3.749^*	-3.585^*
USA	NA	NA	NA	-3.397*	-3.504*	-3.506*
EURO/GB'S						
Canada	NA	NA	NA	-4.214*	-3.511*	-2.507
France	-3.896*	-3.016	-2.643	-3.700*	-2.768	-2.377
Germany	-5.097*	-3.424*	-2.282	-4.875*	-3.273**	-2.155
Japan	-3.111**	-2.931	-2.979	-2.387	-2.168	-2.028
Netherla	-2.773	-2.284	-2.280	-2.792^	-2.320^	-2.299^
Switzerl	-1.217	- .951	- .866	-2.493	-1.776	-1.732
UK	-4.435^*	-3.631^*	-4.503^*	-4.487^*	-3.656^*	-4.515^*
US(GB/Euro)	NA	NA	NA	-2.711	-2.466	-2.121
US(GB/FRN)	NA	NA	NA	NA	NA	NA
US(GB/Zero)	NA	NA	NA	-3.507*	-2.932	-2.435

Column (1) lists the pairs of non-stationary indices tested for co-integration

ADF(0), ADF(1) and ADF(3) relate to number of lags included in augmented polynomial

*** and ** : The null of no co-integration rejected at the 5% and 10% confidence levels respectively**

Critical values for co-integrating regression: 5% = 3.37   10% = 3.03

^ : Implies that the validity of the respective statistics is somewhat doubtful, because one of the variables is close to the non stationary boundaries

NA Co-integration tests are non applicable, because one of the variables is not I_1

Table 5.6

Tests for Serial Correlation of Cointegration Residuals

(1)	(2) LAG	(3) TYPE	(4) ACF(1lg)	(5) B-L(1lg)	(6) Prob B-L
STOCK/PAIRS					
UK/France	0	Unhg	.024	.065	.798
UK/France	1	Unhg	.016	.866	.871
UK/France	3	Unhg	-.010	.011	.918
UK/Nether	0	Unhg	-.018	.036	.850
UK/Nether	1	Unhg	-.002	.000	.985
UK/Nether	3	Unhg	.015	.025	.875
Germ/Switz	0	Unhg	.075	.631	.427
Germ/Switz	1	Unhg	.027	.078	.780
Germ/Switz	3	Unhg	-.024	.062	.804
UK/France	0	Hedg	.010	.010	.920
UK/France	1	Hedg	.011	.013	.908
UK/France	3	Hedg	-.001	.000	.989
Germ/Switz	0	Hedg	.029	.095	.758
Germ/Switz	1	Hedg	.009	.010	.921
Germ/Switz	3	Hedg	-.021	.046	.829
BOND/PAIRS					
UK/France	0	Unhg	.132	1.927	.165
UK/France	1	Unhg	-.030	.097	.755
UK/France	3	Unhg	-.011	.013	.908
UK/Nether	0	Unhg	.219	5.319	.021
UK/Nether	1	Unhg	-.041	.183	.669
UK/Nether	3	Unhg	-.003	.001	.972
Germ/Nether	0	Unhg	.173	3.336	.068
Germ/Nether	1	Unhg	.010	.011	.918
Germ/Nether	3	Unhg	.011	.014	.906
Jap/Germ	0	Hedg	-.090	.896	.344
Jap/Germ	1	Hedg	-.007	.006	.940
Jap/Germ	3	Hedg	-.010	.010	.921

Column (2) refers to the number of lags included in the "co-integrating" regression
Columns (4 & 5) list the values of the ACF and the B-L statistics for the "second order" co-integrating residuals computed at the first lag
Column 6 shows the exact confidence level for the B-L statistic

Among unhedged index pairs, the UK market was found to be co-integrated with France and the Netherlands[11], while the German index was found to be co-integrated with its Swiss counterpart. The hedged results are similar with the exception of the UK/Netherlands pair, for which the null of no co-integration can now be accepted.

As before with the ADF/DF regression residuals, here also we need to be concerned with the problem of possible serial correlation in the co-integrating regression residuals (second order residuals) so that to assess the reliability of the ADF statistics when computed at different lags. ACF and B-L serial correlation tests were applied to all second order residuals (estimated by OLS at zero, one and three lags respectively) from pairwise combinations for which co-integration has been accepted.

From the results being summarized in Table 5.6, both the ACF and the B-L statistic were very small, providing strong evidence of white noise residuals, even at zero lags. This means that even the simple DF statistic computed at zero lags is reliable in this context, implying that the statistical conditions for the validity of the co-integration results are satisfied.

5.2.3.2 Evidence on Co-integration Among Government Bond Indices

The next type of co-integration tests performed, concern pairwise combinations of government bond indices from different countries. In this context, the previously discussed estimation procedures were applied to twelve unhedged and eighteen hedged pairs of Government Bond Indices from eight different countries. Six pairs involving the Canadian and US unhedged government bond indices had to be omitted, since we had previously rejected the null of a unit root at the 5% level. The results are presented in table 5.4:

It becomes immediately obvious that international GB indices are overall more integrated than international stock indices. Furthermore when exchange rate risk is removed from the bond series through hedging, the majority of pairs tested exhibit cointegration. Among unhedged pairs tested, only Germany-Netherlands and the UK with France, Netherlands and Germany

[11]Notice however, that in some lags the null of no co-integration can only be rejected at the 10% confidence level.

turn out to be co-integrated[12]. In the case of the hedged indices, most of the UK and several of the Japan and Germany pairwise combinations seem to be in long term equilibrium. Particularly striking is the very high value of the DF/ADF coefficients for all lags of the Germany-Japan hedged pair, which is in sharp contrast with the evidence from their unhedged counterparts.

A perhaps surprising conclusion is that removal of exchange rate risk from the bond indices is likely to reveal hidden inefficiencies in the international bond markets, considering the fact that co-integration implies that at least one of the series can be used as means of forecasting the other. This phenomenon should perhaps be attributed to interdependence of worldwide interest rate movements[13].

Again it was necessary to check about serial correlation in the co-integration residuals, in order to determine the appropriate lag length in the augmented polynomial. Here the situation was slightly different than before, since in two out of four cases tested, the ACF and B-L tests showed serial correlation of the second order residuals when calculated at zero lags of the dependent variable. Nevertheless, a single lag was enough to eliminate all noise from the residuals.

5.2.3.3 Evidence on Co-integration Between Government Bonds and Eurobonds in the Same Currency

The second hypothesis to be tested was whether government bond indices are co-integrated with Eurobonds or other types of bond indices (e.g. Dollar FRN's or zero coupon bonds) in the same currency. Here, the null hypothesis has been tested for six unhedged and eight hedged government bond vs Eurobond pairs[14], plus a Dollar government bond vs a Dollar zero coupon bond hedged pair:

[12] However since the UK GBI is close to the nonstationary boundaries, results on pairs involving the UK should be viewed with caution.

[13] Since GB's are credit risk free and exchange rate risk is removed, returns on hedged indices will depend only on interest rate movements and the modified duration of each index.

[14] In the case of Switzerland, the foreign bond index was used, since there is no Eurobond market in Swiss Francs.

These results can be found in table 5.5: for France, Germany and the UK there is evidence of co-integration between the respective unhedged Eurobond and government bond indices, while in the case of hedged bond pairs, the null of no co-integration is only accepted in the cases of Japan, Netherlands, Switzerland and the US.

5.2.3.4 Evidence on Co-integration Between Stock and Government Bond Indices in the same Country

The final assumption to be tested was whether the various national stock market indices are in long term equilibrium with their domestic government bond counterparts. For that purpose six unhedged and eight hedged country pairs have been tested, the co-integration being rejected in most cases (see also table 3.13). The exceptions were, however, of some importance, since the null of no co-integration was strongly rejected for the hedged US pair (the unhedged was not tested because the series were not I_1) and the UK pair[15].

Overall, the tests reveal that on a respectable number of occasions asset indices follow a common long term trend and some form of potentially exploitable inefficiency exists. The next necessary step is to perform Granger type causality tests for co-integrated series, in order to evaluate which price index movement precedes the other; This issue is addressed in the following section.

5.3 Granger Causality and Co-integrated Market Indices

On the basis of the Granger representation theorem (Engle & Granger 1987), when the null hypothesis of non co-integration is being rejected, the two co-integrated variables must have an "error correction representation" i.e. a VAR based on the first difference of the two variables augmented by an error correction term (see Agenor & Taylor 1990). Miller & Russek 1990 discuss this issue in the context of the standard Granger causality tests as opposed to those adapted particularly for cointegrated variables by using the error correction mechanism. It should be emphasized that "causality" in the Granger sense is a slightly misleading concept, since it is not the movement in one index that "causes" movements in

[15]Clearly, because Sterling is the numeraire currency, the hedged and unhedged results for the UK are identical.

another, but rather causality signifies a form of intertemporal precedence[16].

In general co-integration implies at least one way Granger causality, but two way causality where both indices movements precede each-other is also possible.

In order to test for Granger causality in the context of co-integrated variables denoted by x_t and y_t we need to proceed in two steps:

i) We need to regress x_t on y_t and y_t on x_t and save the residuals e_1 and e_2 respectively.

ii) Then we can apply the adapted Granger causality test by regressing the differenced variables on the error correction term and two n order polynomials of lagged differences i.e.

$$\Delta x_t = c_1 + e_1 + \sum_{i=1}^{n} \alpha_i \Delta x_{t-i} + \sum_{i=1}^{n} \beta_i \Delta y_{t-1} + u_t \tag{5.4a}$$

$$\Delta y_t = c_2 + e_2 + \sum_{i=1}^{n} \gamma_i \Delta x_{t-i} + \sum_{i=1}^{n} \delta_i \Delta y_{t-1} + \varphi_t \tag{5.4b}$$

Equation (5.4a) can be used to test intertemporal precedence of y over x, while equation (5.4b) that of x over y. In this context, two different statistical tests can be of importance i.e.

- A t test for the significance of the error correction terms e_1 and e_2 in the regressions denoted by equations (5.4a & 5.4b) and
- An F test of the joint hypothesis that $e_1=0$, $\delta_i=0$ or $e_2=0$, $\gamma_i=0$. If the null is rejected at the 5% confidence level, then Granger causality exists between x_t and y_t, the direction dependent on whether (5.4a) or (5.4b) has been applied.

Granger Causality tests were applied only for those indices that were found to be co-integrated, so that in all cases it was necessary to include an error correction mechanism. Estimation of equations (5.4a & 5.4b) and computation of the required test statistics was obtained by means of the RATS Econometric Package. In all the tests performed, a number of six lags has been included in both the augmented polynomials.

[16] Maddala 1988 parallels that phenomenon with a weather forecast, which even though it precedes the rain, cannot be responsible for "causing" it.

Table 5.7

Granger Causality for Co-integrated Series

(1) PAIRS	(2) $t(e_1)$	(3) F(7,88)	(4) Sign F	(5) $t(e_2)$	(6) F(7,88)	(7) Sign F
STOCK/UNHG						
UK/France	2.487	1.342	.2402	- .394	3.285	.0037*
UK/Nether	2.717	2.081	.0538	- .828	.737	.6405
Germ/Switz	2.269	3.361	.0032*	- .997	1.316	.2524
STOCK/HEDG						
UK/France	3.184	1.768	.1036	.301	2.075	.0545
Germ/Switz	1.415	3.024	.0067*	-1.569	1.751	.1075
BONDS/UNHG						
UK(Gov-Eur)	4.494	4.076	.0006*	1.749	.792	.5958
Fra(Gov-Eur)	1.298	3.336	.0034*	- .897	1.186	.3192
Ger(Gov-Eur)	1.358	1.263	.2779	- .944	1.725	.1134
UK/Fra(Gov)	2.975	2.387	.0278*	-2.113	2.299	.0336*
UK/Net(Gov)	3.330	2.222	.0397*	-2.702	2.944	.0081*
UK/Ger(Gov)	3.225	2.086	.0533	-2.536	2.717	.0134*
Ger/Net(Gov)	.524	1.607	.1439	- .827	1.516	.1722
BONDS/HEDG						
UK(Gov-Eur)	4.536	4.091	.0006*	1.751	7.528	.6281
Fra(Gov-Eur)	2.415	2.909	.0087*	.384	.839	.5573
Ger(Gov-Eur)	2.035	.988	.4452	-2.177	3.807	.0012*
USA(Gov-Zer)	.896	.733	.6448	2.708	4.508	.0003*
Can(Gov-Eur)	4.168	4.221	.0005*	2.076	3.429	.0027*
UK/Fra(Gov)	3.819	3.896	.0009*	.508	1.735	.1112
UK/Jap(Gov)	7.221	9.310	.0000*	1.365	1.999	.0639
UK/Net(Gov)	6.371	7.723	.0000*	1.177	2.048	.0577
UK/Swi(Gov)	3.969	3.099	.0057	-2.181	1.567	.1557
UK/Ger(Gov)	5.768	3.585	.0000*	.987	2.012	.0624
Jap/Net(Gov)	2.549	2.741	.0127*	-2.382	2.544	.0196*
Ger/Net(Gov)	1.241	1.482	.1837	- .932	1.130	.3516
Jap/Ger(Gov)	2.357	2.420	.0258*	-2.012	3.279	.0038*

Columns (2 & 5) refer to the value of the t statistic of the "error correction term"

Columns (3 & 6) list the computed values of the joint F test and columns (4 & 7) the corresponding exact significance levels

Statistics in Columns (5-7) result from estimating the reverse order causality

The results are summarized in table 5.7. In columns 2-4 the statistics reported correspond to the first quoted index as dependent variable, while the opposite applies for columns 5-7. Columns 2 & 5 refer to the t statistic of the error correction term, while the remaining columns to the computed value and exact significance level of the joint F test. All the F statistics are based on (7,88) degrees of freedom: At first we tested the hedged and unhedged stock market pairs. In all cases there was only one way "causality": the Swiss stock market precedes the German market, while the UK can be used to predict movements in the French market. These results apply to both the hedged and unhedged series. The Dutch market appears also to precede the UK.

A comparison of government bonds with Eurobonds reveals an interesting fact: in two out of three cases (UK & France) movements in the Eurobond indices appear to precede movements in the government bond indices, while in the case of Germany the government bond index precedes its Eurobond counterpart. In one case, notably the Canadian hedged bond indices, there is evidence of two way causality. Notice also that movements in the hedged Dollar zero coupon bond index can be forecasted by means of the Dollar government bond index. Overall, the results from the hedged indices are quite consistent with those from the unhedged ones.

Finally, in the case of co-integrated pairs among different government bond indices there are several cases where we can observe two way causality. However, when considering those hedged pairs involving the UK government bond index, the form of observable causality is that the UK lags rather than precedes movements in the Japanese and the main European indices.

5.4 Summary and Conclusions from Co-integration/Causality Analysis

In this chapter we addressed the empirical question of whether long term asset allocation strategies could benefit from cointegration/causality analysis. In principle, this would be possible if the aforementioned techniques could reveal inter-dependencies (and inefficiencies) among the different market indices, that would allow an improved estimate of the expected mean return vector to be used in the optimization routines.

At first co-integration tests were applied to both hedged and unhedged pairs of stock market indices. Then, similar tests for the existence of a long term equilibrium relationship between markets have been applied to different types of bond indices, as well as to pairwise combinations of stock and bond indices from the same country.

As far as the stock market indices are concerned, the co-integrated pairs were relatively few and almost exclusively related to European markets. Government Bond Index pairs were found to be co-integrated in several more occasions, particularly so when the exchange rate impact has been removed in the hedged series. In some cases, long term equilibrium was also found to exist between government bonds and Eurobonds. Finally in only two cases the stock and bond markets in the same country were found to be co-integrated, but these refer to the UK and the hedged US indices.

Since co-integration implies at least one way inefficiency it is natural to subsequently test for "causality", so that to determine which of the co-integrated variables can be used to forecast the other. For that purpose, an "error correction term" has been added to the standard Granger polynomials to account for the fact that the tests refer to co-integrated variables.

All co-integrated pairs concerning stock indices exhibited one way causality only, as did most of the pairs linking Government Bonds with Eurobonds, the Government Bonds lagging intertemporally in most cases. Finally, evidence on two way causality concentrated mainly in those index pairs that consisted of Government Bond Indices from different countries.

Overall, enough evidence of index co-integration exists to suggest that improved estimates of long term expected returns could potentially be formulated for some indices, if not for the entire mean return vector. In our analysis, no attempt was made to convert our positive evidence about index inefficiency-causality to input estimates for asset allocation purposes. Nevertheless, this could be a potentially useful field for future research.

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