PENSION FUND VALUATION

by

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CONTENTS

Tables and Illustrations

The following is the graphs for all the cash flow projections described in Appendix 3. The graphs contain the Standard Contribution rate, the adjustment to contribution rate and the Recommended Contribution rate for each year of projection.

Appendix 3.1

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Declaration

Abstract

The thesis discusses various actuarial aspects of the management of a pension fund, in particular, those related to valuation of the pension fund. The investigation covers three main areas; the funding mechanism, investment and matching and control of the fund.

In the part dealing with the funding mechanism, a model is introduced in order to assist analyses of the mechanism of a pension fund. The model notionally separates the fund into individual pots and a common pool, and notional moves of the assets between them, X-functions, are defined. Using this model, various events in the pension fund are analysed. Particularly, the model is shown to be useful for explaining the financial impact of withdrawals and the problem of cross-subsidy.

In the next part, investment and matching are discussed referring to a collection of papers and books written by actuaries and economists. Two different types of matching are defined according to the definition of risk, which are named V-matching and S-matching. Based on this discussion on matching, the meanings of the use of a particular portfolio for valuation purposes are analysed.

Finally, various means of controlling a pension fund are discussed in the light of control theory. A particular focus is set on the choice of the valuation basis as a means of control, and an extensive series of long term cashflow projections are carried out to explore the optimum way to choose the valuation basis under various scenarios of changing experience. The projections are carried out separately for three different aspects of the experience; the real rate of investment return, the dividend growth rate and the dividend yield, and the withdrawal rates. The results suggest that the use of averages of past experience over a long period suits best for different circumstances, and that delayed changes in the valuation basis after the corresponding changes in experience are useful for identifying more clearly the trend in the actual experience.

Notation

Notation used in the thesis is summarised as follows. Notation in each part is shown separately.

Part II

e:	age at which the member joined the scheme, also the time when the scheme started
x:	current age, also the current time
r:	normal retirement age
v:	$= 1/(1+i)$
i:	interest rate assumed in the basis
BN(s):	value of the leaving service benefit for the member at time s, or value of normal retirement benefit for the member if s=r; This is the size of the individual pot.
SF(t):	Standard fund calculated at time t
S(x):	salary at age x
k:	constant (used as the accrual rate of benefits)
$\bar{a}_x$:	value of unit annuity at age x
SUR(t):	surplus arising at time t on conventional basis
$SUR'(t)$:	rate of surplus arising at time t on conventional basis
SURa(t):	surplus arising at time t on cash accumulation basis
$SURa'(t)$:	rate of surplus arising at time t on cash accumulation basis
B(t;s):	the total benefits to be paid to the members at time s estimated at time t on the valuation basis
B(s):	actual total benefits paid to the members at time s
SC(t;s):	Standard contribution to be paid at time s and calculated at time t
SC(x):	Standard contribution for an individual member at age x
P(t):	the balance of the fund after deducting all the individual pots
R(t):	the required contribution at time t
X(t):	X function; $X(t) = SC(t) - R(t)$

PART III

m(x,t): mean surplus of investment strategy x at time t

s(x,t): measurement of risk (e.g. standard deviation) of investment strategy x at time t

Part IV

The time is measured by the number of years passed from the origin. At the beginning of a scheme year, the time is expressed by an integer. To express the timing accurately, for a particular year T, which starts from time T and ends at time T+1, T-0, T and T+0 are defined as follows.

T-0: The end of year T-1, but before any end year cash flows; benefit payments and dividend receipts.

T: The end of year T-1, when the valuation takes place.

T+0: The beginning of year T after receiving contributions.

D(t): Amount of the dividend received at time t

M(t): Market value of the assets held at time t

C(t): Contributions received at time t.

B(t): Benefits paid at time t.

I(t): the rate of return achieved during the period (t-1,t) on the market value basis.

MV(t): Market value (ex-dividend) of a specific asset at time t excluding dividends at time T

Div(t): Dividend receivable at time t

i"(t): Market interest rate at time t. The long term interest rate in the market

g"(t): Market expectation of dividend growth at time t

d"(t): Market dividend yield at time t. For the precise definition, see section A1.3.3.

p"(t): Market expectation of long term future price inflation at time t

g'(t): Actual dividend growth from t-1 to t

p'(t): Actual price inflation rate from t-1 to t

e'(t): Increase in general level of salary from t-1 to t

ss'(t): Promotional salary increase experienced between t-1 and t

w'(t): Rate of withdrawals experienced between t-1 and t

VM(t): Assessed value placed on M(t)

VD(t): Assessed value placed on D(t)

VC(t): Assessed value placed on C(t)

c(t): Conversion factor of the market value to the assessed value: i.e,

$$VM(t) = c(t) * M(t)$$
$$VD(t) = c(t) * D(t)$$
$$VC(t) = c(t) * C(t)$$

i(t): Interest rate assumption for the valuation

g(t): Dividend growth assumption for the valuation

d(t): Dividend yield assumption for the valuation p(t): pension increase rate assumption in payment and deferment for the valuation

e(t): salary inflation rate assumption for the valuation

ss(t): promotionary salary increase assumption for the valuation

w(t): withdrawal rate assumption for the valuation

I'(t): the rate of return achieved during the period (t-1,t) on the valuation basis.

VM^(t): assessed value on an alternative basis

g^(t): the alternative dividend growth rate assumption

d^(t): the alternative dividend yield assumption

VSp(t): the valuation surplus arising from the change of the basis from the original one to the alternative one.

f(y): rate of reduction in the retirement pension for a member who withdrawed at age y

l(y): survival function at age y

dw(y): expected number of withdrawals at age y-0 (with l(x)=1)

V(y): value of liabilities at age y exact

Where there is no fear of confusion, (t) may be omitted. It should be noted that e was used in Part II as the entry age, but in Part IV, e(t), or e, means the rate of salary inflation. However, there is only one meaning of e in each part, and this notation is not considered to cause any confusions.

PART I

Chapter 1 Introduction

1.1 Actuarial approaches

Scientific activities which actuaries are engaged in may be summarised by the following definition of the actuarial scientific method by Education Strategy Working Party of Institute of Actuaries(1990);

> The actuarial scientific method is to formulate practical conclusions and advice (including analysis of risk and long-term applications) within finance, insurance, demography and related fields through:
>
> (a) selecting and collecting information;
>
> (b) structuring, presenting and analysing information, including investigating secular and other trends;
>
> (c) formulating assumptions and constructing models to process information for divers purposes, including long-term future applications; and
>
> (d) monitoring and appraising emerging experience against the model and the assumptions.

Because of its nature, research in actuarial science may share these activities. Also, the methodology of carrying out these activities can be the subjects of research. Firstly, (a), (b) and (d) are all related to the processing of data. The methodology of processing and analysing data can be investigated as part of statistics. So, we shall call such an approach a 'statistical approach'. Also, the processing and analysing data itself can be a method of actuarial research. We shall call this a 'numerical approach'. Secondly, (c) relates to establishing and developing models, which are usually expressed mathematically. Such activities themselves can be regarded as scientific research. We shall call this approach a 'mathematical approach'. Although these three approaches are by no means exhaustive, these would be regarded as typical approaches of research in actuarial science. Now, the characteristics of each approach are briefly discussed in turn.

i) Statistical approach

Through this approach, the methodology of collection and analysis of data would be investigated. It is an applied area of statistics. Research in this category would contribute not only to improving the accuracy and efficiency of data processing and analysis, but also to give a background to the construction of theoretical models, which are discussed in iii) below. For instance, a stochastic investment model may be created allowing each variable in the model to take a range of values stochastically. (For example, see Wilkie(1986)) The creation of such a model would be regarded as a combination of a statistical approach and a mathematical approach. The perspective provided by statistics has been increasing its practical importance with the advance of computer technology.

ii) Numerical approach

One of the aims of this approach would be to supply that information which is useful in setting numerical assumptions within a particular model. This could be part of actuarial practice; e.g. past salary rise experience of a pension scheme may be investigated when the salary increase assumption is set. However, often, more extensive research into past trends is required, e.g. long-term trends in average equity performance, and these may be left as the subject of research.

Also, this approach may be taken to investigate the fitness of models and assumptions made in the light of actual experience. (see (d) in the list by Education Strategy Working Party of Institute of Actuaries(1990).)

iii) Mathematical approach

This approach begins with creation of a model. The model should express the salient features of the operation of a particular financial or other system by a set of mathematical formulae. Through the analysis of the model, some solutions to a particular problem may be suggested, or a theory may be developed.

The classical actuarial mathematics applied to the area of life assurance is an example. The mechanism of life assurance is simplified to consist of the functions of receiving premiums, building up policy values, earning interest on the funds held and paying contingent benefits on the deaths occurring. In this framework, various commutation functions are developed on the basis of the given assumptions, and premium rates and the associated reserves are expressed in terms of formulae which contain these commutation functions.

The funding methods for pension funds are also the products of this approach. The model simplifies the mechanism of a real pension fund by expressing all the events in terms of investment returns, salary growth rates, withdrawal rates and other financial and demographic elements. The funding methods are developed to ensure the payments of benefits promised if the future events are correctly predicted.

At the same time, in order to cover the effect of the inevitable errors of predictions, methods of amortising surpluses/deficits have been developed as part of the same model.

Developing such theories greatly assist practice because they can be a basis for consideration of a particular practical problem.

Sometimes, the theories or the models which are created through this approach may directly influence the real world, in the same way as economic theories. For example, Anderson(1959) introduced the concept of profit testing. This has been applied to calculate appraisal values of life assurance companies, which influence the acquisition price of life assurance companies. (see Sherlock et al(1994)) Also, the market prices of financial options may be influenced by the actuarial or statistical models which are used by the traders. One of the well known models is due to Black & Scholes(1973).

Mortality investigations may involve all three approaches. Firstly, a statistical approach may be taken to investigate the methods of constructing mortality tables. Secondly, establishing a theory of mortality, e.g. Gompertz' Law, would be classified as a mathematical approach. Finally, it is obvious that the creation of mortality tables is an example of a numerical approach. The mortality tables created would supply mortality assumptions with respect to various actuarial models.

1.2 Outline of research carried out in the thesis

This thesis deals with the further development of theory in the field of the funding of occupational pensions, which is one of the main areas of actuarial application.

Although purely mathematical discussions are seen in only a limited part of this thesis, the main objective is to develop theoretical views with the aid of a mathematical model. The extraction of information from data and the investigation of the treatment of data are not considered. Extensive cash flow projections using the model have been made, but they have only been used to assist the theoretical understanding of the fundamental mechanisms. Hence, according to the classification in section 1.1, this work belongs to the mathematical approach.

In the field of pension funds, the valuation of a pension fund is central to the actuarial applications. This thesis deals with various aspects of valuations and related topics. Therefore, the title, 'Pension Fund Valuation', has been chosen.

The thesis consists of five parts. Part I consists of two chapters, and provides an introduction to the main part of the thesis.

In Parts II, III and IV, the main discussions are developed. They deal with different areas independently. So, each part may be read separately. However, each part should be read in the order given to allow the reader to follow the arguments.

All the Appendices relate to Part IV, but are placed after Part V because of the relative size of each part.

Part II deals with the mechanism of funding.

In Chapter 3, a mathematical model is created, and various aspects of funding, including funding methods, are analysed using the model.

In Chapter 4, the problem of 'cross-subsidy' is discussed in the light of the model developed in Chapter 3.

Part III deals with investments and the valuation of assets, and consists of one chapter, Chapter 5.

The main aim of this part is to consider the implications of notional matching in the valuation, which is widely applied in the UK practice.

To achieve this, it is felt necessary to develop discussions on investment strategy and matching, and a theoretical framework of investment and matching has been developed. Based on this framework, the meaning of notional matching in the context of a pension fund valuation is discussed. Also, a brief discussion on a stochastic approach to valuations is made.

Part IV deals with the contol of a pension fund.

Pension funds may be established with certain objectives: stability of the contribution rate, security of the pension rights already accrued, and so on. The purpose of managing pension funds is to control the pension fund through applying actuarial techniques and other means in order to achieve these objectives. Chapter 6 discusses various means by which a pension fund can be controlled. The discussion is based on control theory. To exercise controls, valuations have an important role, and controls through the choice of the valuation basis are discussed in detail.

As an extention to this, a series of cash flow projections have been made on the basis of a mathematical model to investigate the optimal approach for setting the valuation basis.

Chapter 7 explains the model and projections carried out. Some details are left to Appendices 1 and 2.

Chapter 8 gives an analysis of the results, and draws conclusions from the projections. Full results of the projections are shown in Appendix 3.

Part V contains Chapter 9 which provides a conclusion to the thesis and indicates areas for future research.

Chapter 2 Background

2.1 Introduction

To begin discussions on various aspects of the valuation of a pension fund, brief summaries of valuation processes and funding methods are given in this chapter to clarify the subject.

2.2 Pension Scheme

A pension scheme is established to provide income after retirement and protection for the family in the case of various contingencies including death and disability.

A pension scheme may be arranged by an individual, or for a group of people. This thesis does not deal with the individual arrangements, but only deals with collective or group arrangements.

A collective pension scheme may be related to a group of employees in the same company/group of companies, and the scheme is referred to as an occupational pension scheme. The employers usually pay a significant portion of the costs, and the pension arrangements may be recognised as a part of the remuneration or employee benefits provided.

One of the important features of collective arrangements is the existence of rules specifying the contributions to be received and the benefits to be provided; for example, a normal retirement age is set to define the commencement of the payment of retirement income.

There are two main types of occupational pension schemes according to the differences in the rules: defined contribution schemes and defined benefit schemes.

A defined contribution scheme requires separately recognised contributions for individual members, and the benefits are provided out of the individual contributions accumulated with investment proceeds. Such a scheme may be regarded as a group of individual arrangements.

A defined benefit scheme specifies the form and amount of benefits to be paid at specific events; e.g. retirements and deaths. The amounts of benefits are often expressed in terms of salaries at retirements or deaths, and such schemes are also known as final salary schemes. Contributions are usually paid in accordance with a specific funding plan. If the fund is judged to be insufficient to provide the benefits under the rules, extra contributions are required. On the other hand, any excess funds may be removed by refunds or reductions in contributions. Alternatively, such excesses may be utilised to improve benefits. Under defined benefit schemes, the separation of the fund to individuals is not made, and the financing arrangements are made collectively.

There are hybrid schemes which have both aspects. The benefits may be calculated as the larger of two rules: one being of the defined contribution type, and the other of the defined benefit type. Alternatively, a scheme may consist of a defined contribution part and a defined benefit part.

In this thesis, only defined benefit schemes are considered.

The pension benefits provided by defined benefit schemes may be related to the salary at the time of exit from the scheme, i.e. retirements or withdrawals. The benefits may also be related to the length of the period of active membership in the scheme. Hence, at a particular point during the active membership, the benefits may be separated into the benefits already accrued from the past service, i.e. past service benefits, and the benefits expected to accrue from the future service, i.e. the future service benefits.

2.3 Valuation

The costs of providing benefits under defined benefit schemes are met by contributions, which are usually paid into the scheme in accordance with a plan for funding. A plan for funding is made for various reasons as follows;

-to ensure the payments of promised benefits
-to ensure that the contributions progress smoothly over time
-to monitor the progress of funding.

A plan for funding usually consists of two elements: a funding method and a method of amortisation. A funding method specifies the Standard Fund, which is the target level of funding, and the Standard Contribution Rate, which would be requested if the value of the assets is equal to the Standard Fund. Details of typical funding methods are described in section 2.4.

The differences between the Standard Fund and the value of assets is planned to be removed by adjusting the contribution rates, through the method of amortisation.

The plan for funding is applied to obtain a specific Recommended Contribution Rate. This process of calculation is called a valuation. A valuation includes various stages;
-making assumptions on the future course of events (they are called collectively the 'valuation basis'.)
-estimating the cost of providing the promised benefits
-setting the Recommended Contribution Rate according to the funding method and the method of amortisation chosen.

The valuation is also used to monitor the progress of the funding plan, and is performed regularly. (In the UK, it is a statutory requirement to perform valuations at least once every 3 years and 6 months: see section 24.7 of Fenton et al(1993).

2.4 Funding methods

The The Institute of Actuaries and the Faculty of Actuaries(1988) has given standardised definitions of commonly used funding methods. In the following, descriptions of typical funding methods are given in line with these definitions.

Mathematical expressions may vary according to the nature of the cash flows assumed and whether a discrete or continuous approach is adopted. Trowbridge(1952) has given the definitions on a discrete approach in order to classify funding methods according to the level of the fund at its stationary state. Dufresne(1986) has given both discrete and continuous definitions for typical funding methods.

However, here, no mathematical definitions are given, but general descriptions are given. Thus, the descriptions below are applicable to both cases, discrete and continuous.

The following represent only a selection of typical funding methods, and are not exhaustive. In particular, the following can be modified by allowing future new entrants, or the use of a control period, but these extentions are not described here. However, the descriptions below are not limited to consider the funding methods only in terms of normal retirement benefits but also include withdrawal benefits.

2.4.1 Current Unit Method

The Standard Fund is the present value of benefits if all the existing members leave the service as at the valuation date. The value of the benefits may include pension increases in deferment and in payment.

The Standard Contribution Rate is determined as follows. Firstly, the Standard Fund as at one year after the valuation date in respect of existing members is calculated assuming withdrawal rates and salary rises. Secondly, this Standard Fund as at one year in the future is then discounted for a year taking into account the assumed investment return during the year. Thirdly, the present value of the Standard Contribution during the following year is calculated as this discounted value less the Standard Fund calculated as at the valuation date. Finally, the Standard Contribution Rate is determined in a manner that is consistent with the timing and frequency of contribution payments, which may be singly, quarterly, monthly, continuously, or on any other basis.

If the current value of the assets is equal to the current Standard Fund and all the assumptions are realised, the Standard Contribution is the amount required to be paid in so as to make the assets equal to the Standard Fund in one year's time.

The Standard Contribution Rate may be split into two elements: the value of benefits accruing during the next 12 months, and the increase in the value of the benefits already accrued arising from the salary rises during the year.

The Recommended Contribution Rate is determined by adjusting the Standard Contribution Rate taking into account the difference between the value of the assets and the Standard Fund. The adjustment aims to remove any surpluses/deficits according to a particular plan. So, if the value of assets is higher than the Standard Fund, the adjustment is a reduction in the contribution rate, and vice versa.

2.4.2 Projected Unit Method

The Standard Fund is the present value of the past service benefits including a full allowance for future salary increases. The value of benefits may also include pension increases in deferment and in payment.

The Standard Contribution Rate is determined to finance, in full, the cost of benefits accruing in the year following the valuation date in accordance with the frequency of contribution payments. Again, future salary increases are allowed for fully until the final exits from the scheme.

If the value of assets were equal to the Standard Fund at the valuation date and if the experience were to follow the assumptions made in the following year, the contribution income at the Standard Contribution Rate would make the value of assets one year later exactly equal to the Standard Fund as at one year after the valuation date.

Similarly to the Current Unit Method, the Recommended Contribution Rate is determined by adjusting the Standard Contribution Rate.

2.4.3 Entry Age Method

This method assumes an Entry Age, e, say.

The Standard Contribution Rate is calculated as the level contribution rate which is sufficient to meet all future benefits for a new entrant aged e under the valuation basis. Full allowances for future salary increases is made until the member's exit from the scheme.

The Standard Fund is the present value of past and future benefits for all the existing members with full allowance for future salary increases less the present value of future contribution income at the Standard Contribution Rate in respect of all the existing members.

If the value of assets were equal to the Standard Fund, if the Standard Contribution Rate were applied, and if all the assumptions made were realised in the future, then the value of assets should always be equal to the Standard Fund calculated then.

Similarly to the Current Unit Method, the Recommended Contribution Rate is determined by adjusting the Standard Contribution Rate.

2.4.4 Attained Age Method

The Standard Fund is the same as for the Projected Unit Method.

The Standard Contribution Rate is calculated as the level contribution rate which is sufficient for all future benefits for all the existing members. This allows fully for future salary increases, but this does not cover the past service benefits.

Similarly to the funding methods already described above, the Recommended Contribution Rate is determined by adjusting the Standard Contribution Rate.

If the value of assets were equal to the Standard Fund, (which is the same as that of the Projected Unit Method), if the standard Contribution Rate were applied, and if all the assumptions made were realised, then the value of assets in the future time is likely to be higher than the Standard Fund calculated because the Standard Contribution Rate is usually higher than that for the Projected Unit Method.

2.4.5 Aggregate Method

This method does not have a Standard Fund nor a Standard Contribution Rate.

The Recommended Contribution Rate is calculated as the level contribution rate sufficient to meet all past and future benefits for all the existing members allowing for the existing assets held.

This method can be identified as the Entry Age Method or the Attained Age Method with the surpluses/deficits being spread over the working lifetime of existing members.

2.4.6 Initial Funding

The Standard Contribution is calculated at the entry of each new entrant as the present value of all future benefits for the new entrant. The Standard Contributions should be sufficient to provide all the benefits in full for the new entrant if all the assumptions are realised.

The Standard Fund is the present value of all the past and future benefits for all the existing members as at the valuation date. The difference between the Standard Fund and the value of assets held at the valuation date may be eliminated by a one off or a series of special contributions or refunds.

2.4.7 Pay-As-You-Go

Under this method, contributions are required whenever benefits need to be paid. The amount of contributions are equal to the benefit outgoes.

This method defines a system of finance, but no planning for funding is made. That is, the timing and amount of contributions cannot be planned in advance. Thus, a Standard Fund and a Standard Contribution Rate cannot be defined.

2.4.8 Terminal Funding

The Standard Contribution is not calculated regularly at specified valuation dates, but calculated whenever a member leaves the active service and any benefits are vested on his/her behalf. The amount of the Standard Contributions is equal to the present value of the newly vested benefits on the valuation basis. The Standard Contributions should be sufficient to provide the vested benefits in full if all the assumptions are realised.

The timing of vesting is the normal retirement age for the members who retire normally. However, the timing for withdrawing members who are entitled to deferred pensions may depend on the precise definition of the word 'vesting'. One is to take it as the normal retirement age when the pension payments start. Another may be the time of withdrawals.

The Standard Fund is the present value of all the vested benefits, and the difference between the Standard Fund and the value of assets held at the valuation date may be eliminated by a one off or a series of special contributions or refunds.

If all the benefits are provided in the form of lump sums, this method becomes equivalent to the Pay-As-You-Go Method.

PART II

Chapter 3 Analysis of Funding Method

3.1 Introduction

An approach for the analysis of funding methods for a pension fund is described in this chapter with some applications.

A pension fund receives contributions and investment proceeds, and pays benefits.

If it is a money purchase scheme, the contributions are allocated to individual members. The fund is also divisible, and investment proceeds are distributed to individual pots. The value of benefits is equal to the amount of money in the pot. The only exception would be the pooling for death benefits within the fund.

However, in the case of a defined benefit scheme, the contribution rate may be set collectively, and the fund may not be readily divisible to individual pots. The benefits are determined by the scheme rules, and the value may be irrelevant to the level of funding.

Here, a model based on the idea of notional divisions of the fund and contributions is introduced to examine the work of various funding methods in a defined benefit scheme.

A model, the 'global model', is described in section 3.2. In section 3.3, a mathematical model, the 'individual model', is independently developed in a more mathematical manner but with strict restrictions. The two models presented are reconciled in section 3.4, to show that the individual model is a special case of the global model. Various applications of the theory are discussed afterwards in this chapter and in Chapter 4.

It would have been more logically straightforward if the global model could be expressed mathematically. However, this approach was avoided because of the possible complexity and the possible difficulty to apply the model. Instead, the mathematical expressions are fully developed under restricted assumptions in order to give the minimum means to analyse the funding method mathematically, and the applications to more general topics are intended to be done using the global model.

Developing mathematical expressions further is left to future research.

3.2 Global model

A model of a pension fund is described in this section. The model assumes an individual pot for each member and a common fund as the rest of the fund. In the following, definitions and the explanation of how the model works are given.

3.2.1 Notional division

Each individual's pot is defined as the fund the value of which is equal to the value of benefits to be paid if the member leaves the scheme now.

The balance of the pension fund is defined as the 'common pool'. This amount can be negative.

All the assets and cash flows are valued on the valuation basis in this model, so that the model can be used for the analysis of the funding methods. The effect of the difference between the value of the assets and the market value may be analysed separately outside this model.

Contributions, the values of which are recognised on the valuation basis, are divided into the following:

-individual's portion, SC
This is defined as the standard contribution rate which is calculated individually using each member's age, sex and class according to the funding method and the valuation basis used, plus any earmarked contributions for the member.
-the balance
This is the actual contributions made less the total of the individuals' portions. This can be negative.

Here, standard contribution rates are needed for this model, so the Aggregate method has to be excluded from the range of applications of this model.

The sum of the individuals' contributions should be in theory equal to the standard contribution for the fund because the standard contribution rate is calculated by taking the average of individual rates. However, there can be differences because of the following;

-rounding
-asset values given on the valuation basis are different from market values
-the population changes while the same contribution rate is applied for the period until the next valuation
-adjustments to a simple average taking into account expected future changes in population, and so on.

Investment proceeds are allocated into all the individual pots and the common pool in proportion to the sizes of the pots and the pool. When a part of the assets are notionally attached to a particular part of the pension fund liabilities, the allocation of the investment proceeds is done within the pots and/or the pool corresponding to these particular liabilities.

Benefits are paid to/for individuals, and the amounts are recognised invdividually. Hence individual portions are readily available without any notional divisions. Here, the value is recognised on the valuation basis. The benefits paid are deducted from individual pots.

3.2.2 Movements of fund

3.2.2.1 Change in an individual pot

The size of an individual pot changes because of the following:

-accrual of benefits
As the member completes more service, more benefits will accrue according to the scheme rules. This makes the size of the individual pot grow because its size is defined as the value of the leaving service benefit at the time.

-investment income anticipated
The individual pot continuously grows at the rate of interest assumed for the valuation.

-payment of benefits
Here, for simplicity, all the benefits are assumed to be bought out when the members exit the scheme. After the benefits have been bought out, the individual pot becomes 0.

It may be possible to include pensioners and deferred pensioners in the model by recognising the benefit payments as they occur. Such an expansion of the model might give useful insights, although this route was not taken in this research.

-salary growth
Where the scheme benefit is salary related, and when the salary is increased, the individual pot expands.

-extra benefits
Benefits may be enhanced when the member exits the scheme because of death, early retirement, etc. Or simply, extra benefits may be granted on a discretionary basis at exit, during the service, or at entry (allowance for past service credits).

The individual pot is increased when such extra benefits are granted, although the pot will become 0 immediately after such increases because the members exit the scheme.

-change of basis

When the valuation basis is changed, the value of the leaving service benefit, which is the amount of the individual pot, is recalculated.

3.2.2.2 X function

As described in section 3.2.1 above, when contributions are paid into the fund, the contributions for individuals are allocated to their pots, and the balance is credited to the common pool.

Similarly, investment incomes are allocated to individual pots and the common pool. Benefits are paid out of individual pots.

Surpluses/shortfalls can arise in the individual pots because these cash flows may not keep the size of the individual pot as defined in section 3.2.1 with changes stated in section 3.2.2.1. The surpluses/shortfalls are met by transfers to/from the common pool, immediately. The value of the assets allocated to each individual pot is thus kept equal to the size of the individual pot at any time.

The amount of transfer from an individual pot to the common pool is expressed as X. A negative X would mean transfers in the opposite direction.

When contributions are paid into the individual pot, the amount allocated is dependent on the funding method used, and may not be equal to the amount required for the growth of the individual pot, R.

So, the excessive contributions are redirected to the common pool immediately. Where the individual contribution is not enough, the amount necessary to fill the gap is transferred from the common pool.

Similarly, when investment incomes are allocated to the individual pot, transfers between individual pots and the common pool occur when the growth of an individual pot is not matched by the investment proceeds allocated to the pot. This may occur when the investment returns are not the same as assumed under the valuation basis.

If there is a difference between the assessed values of the assets and the market values when benefits are paid out, the cost of buying out the benefits on the valuation basis can be different from the value of the individual pot. When the individual pot is not sufficient for the cost of benefit, the balance is supplemented by a negative X function. When there is a surplus in the individual pot after the payment of the benefit, the surplus is brought back to the common pool by a positive X function. So, the individual pot becomes 0 after the payment of the benefit and the positive X function.

If the benefit is related to salaries, a growth in salary expands the individual pot. The shortfall is filled by a negative X.

When a larger benefit value than the leaving service benefit is granted at exit, or when any extra benefit credits are given to the member at entry or during the membership, the individual pot expands, and the shortfall needs to be filled by a negative X.

When the valuation basis for valuation is changed, the value of leaving service benefits may be changed. (Neither the dividend growth assumption nor the salary growth assumption is likely to affect this.) The changes in the sizes of the individual pots will require a positive/negative X function.

3.2.3 Insurance within the fund

There can be possibilities for several different types and levels of benefits because of different reasons for the exit from the membership. For example, the values of death benefits, early retirement benefits, etc. may be different from those of early leavers' benefits.

The individual standard contributions may be calculated including the allowances for the extra costs for such higher benefits than the leaving service benefit. But in this model, the contributions corresponding to these extra costs are not retained in the individual pot, the size of which is always the value of the leaving service benefit at the time. Any extra contributions are transferred to the common pool as a positive X function. When such higher level of benefits are paid, the extra costs are transfered from the common pool to the individual pot as a negative X function to meet the balance.

This may be interpreted as an insurance arrangement within the fund; individual pots as the insured and the common pool as the insurer.

3.2.4 Standard common pool

As mentioned in section 3.2.1 above, the model can only be applied to the funding methods with standard contribution rates, and standard funds are always defined for such funding methods.

Here, the "standard common pool" is defined as the standard fund less the total of individual pots.

The following diagram summarises the working of the model.

Case I   X > 0

```
                                              ______________
                                   X         |              |
                          ---------------->  | Common Pool  |
                     SC          |           |______________|  <---  Investment
Contribution  --------------->   |                                    Proceeds
                                 |            ______________
                                 |   R       | Individual   |
                          ---------------->  | Pot          |
                                             |______________|  <---  Investment
                                                                      Proceeds
```

Case II   X = 0

```
                                              ______________
                                             |              |
                                             | Common Pool  |
                     SC                      |______________|  <---- Investment
Contribution  ----------------------->                                Proceeds
                                  |           ______________
                                  |  R=SC    | Individual   |
                                  -------->  | Pot          |
                                             |______________|  <---- Investment
                                                                      Proceeds
```

Case III   X < 0

```
                                              ______________
                                             |              |
                                   <-------  | Common Pool  |
                     SC              |       |______________|  <---- Investment
Contribution  --------------------->  | -X                            Proceeds
                              |       |       ______________
                              |       ------> | Individual   |
                              --------------> | Pot          |
                                    SC        |______________|  <---- Investment
                                                                      Proceeds
```

Diagram 3.2.5   X-function Model

3.3 Individual model

Now, we shall develop a more mathematical model, the 'individual model', under a very restricted situation independently from the global model. The main descriptions on the model are made in sections 3.3.3 to 3.3.5. Sections 3.3.6 and 3.3.7 discuss the recognition of surpluses. As examples, the model is applied to various funding methods in section 3.8.

The relationship between the global model and the individual model will be discussed in section 3.4.

Here, when we consider a particular member, obviously, such divisions are not allowed regarding the past service. However, for the future service, probabilities of exits may be allowed for.

When the principles of funding methods are discussed in section 3.3.2, however, the member is treated as divisible into any smaller portions, i.e. a non integer number of members is allowed.

3.3.1 Notation

The following are used in the discussion from section 3.3.3 onwards for the discussion of the individual model. So, they are defined in conjucction with a single member.

e:	age at which the member joined the scheme, also the time when the scheme started
x:	current age, also the current time
r:	normal retirement age
v:	$= 1/(1+i)$
i:	interest rate assumed in the basis
BN(s):	value of the leaving service benefit for the member at time s, or value of normal retirement benefit for the member if s=r; This is the size of the individual pot.
SF(t):	Standard fund calculated at time t
SC(x):	Standard contribution at age x
S(x):	salary at age x
k:	constant (used as the accrual rate of benefits)
$\bar{a}_x$:	value of unit annuity at age x
SUR(t):	surplus arising at time t on conventional basis
$SUR'(t)$:	rate of surplus arising at time t on conventional basis
SURa(t):	surplus arising at time t on cash accumulation basis
$SURa'(t)$:	rate of surplus arising at time t on cash accumulation basis

The following are specifically used in section 3.3.2 for a general discussion of the funding method. Here, the membership can be more than one, and the notation is defined for a collective membership.

B(t;s):	the total benefits to be paid to the members at time s estimated at time t on the valuation basis

B(s): actual total benefits paid to the members at time s

SC(t;s): Standard contribution to be paid at time s and calculated at time t

The same notation SF(t) is used for the Standard fund calculated at time t.

3.3.2 Funding method

Before starting the construction of the model, the term "funding method" must be defined clearly. This is a general discussion, and the assumptions in section 3.3.3 do not apply although the notation in section 3.3.1 is used.

A funding method decides the way to accumulate assets to meet future benefit payments. So, the following definitions are given.

Definition 3.3.2.1

A funding plan is a series of contributions to be paid in the future which satisfies:

1. {Value of assets held} + {Present value of future contributions} = {Present value of benefits to be paid}

under a set of assumptions, the "basis", for the future experience. Here, "benefits to be paid" are projected according to the scheme rules.

2. The value of the assets is not expected to become negative at any time in the future if the assumptions are realised.

Definition 3.3.2.2

A funding method is a way to set a funding plan.

A funding method may use the concepts of the Standard Fund and the Standard Contribution rate in the process of setting a funding plan. However, there can be funding methods which do not have the Standard Fund or the Standard Contribution rate; e.g. the Aggregate method.

The Standard Fund and the Standard Contributions are a pair of concepts which must satisfy the following Condition 3.3.2.2.3.

Condition 3.3.2.2.3 ---

{Standard Fund} + {Present value of future Standard Contributions}
= {Present value of benefits to be paid}

under a basis.

When these concepts are employed, the value of the assets is compared with the Standard Fund, and any surplus or shortfall is planned to be amortised by adjustments to the Standard Contributions, which gives a funding plan.

The Standard Fund can be defined at any level. However, in some cases, the Standard Fund and the Standard Contribution have the relationship as Condition 3.3.2.2.4 below.

Condition 3.3.2.2.4 ---

Take any two times a and b, s.t. $a < b$.
Assume the same basis is used at both times, and the experience between a and b exactly follows the basis.
Then,

$$SC(b,t) = SC(a,t) \quad \text{for all } t > b$$

This condition is held if the Current Unit method, the Projected Unit method and the Entry Age method. However, the Attained Age method does not satisfy this condition.

Proposition 3.3.2.2.5 ---

If condition 3.3.2.2.4 is held, assuming the same basis is used at both times and the experience between a and b exactly follows the basis,

$$SF(b) = SF(a) * (1+i)^{b-a} + \int_a^b (SC(a,s)-B(s)) * (1+i)^{b-s} ds$$

Proof

From Condition 3.3.2.2.3,

$$SF(a) = \int_a^{\infty} [B(a;t)-SC(a;t)] v^{t-a} dt$$

$$= \int_a^b [B(a;t)-SC(a;t)] v^{t-a} dt + \int_b^{\infty} [B(a;t)-SC(a;t)] v^{t-a} dt$$

From the assumptions,

$$B(a;t) = B(t) \quad \textit{for any } t \quad \textit{such that } a < t < b$$
$$B(a;t) = B(b;t) \quad \textit{for any } t \quad \textit{such that } t > b$$

So,

$$SF(a) = \int_a^b [B(t)-SC(a;t)]v^{t-a}dt + \int_b^\infty [B(b;t)-SC(a;t)]v^{t-b} * v^{b-a}dt$$

Applying Condition 3.3.2.2.3 for time b,

$$SF(a) = \int_a^b [B(t)-SC(a;t)]v^{t-a}dt + v^{b-a} * SF(b)$$

Multiplying both sides by $(1+i)^{b-a}$,

$$(1+i)^{b-a} * SF(a) = \int_a^b [B(t)-SC(a;t)](1+i)^{b-t}dt + SF(b)$$

This is equivalent to the equation in Proposition.

QED

3.3.3 Assumptions

The assumptions on which the individual model is constructed are described in this section. These assumptions together with the functions defined in section 3.3.4 form the individual model. Further descriptions on the individual model is made in section 3.3.5.

-One memeber
One member joins the scheme when the scheme is established. Thereafter, no new entrants are assumed.

-Single mode of exit
Only one type of exit from the scheme, i.e. withdrawals, is allowed except retirements at the normal retirement age.

This assumption may be interpreted that the benefits does not depend on how the exit from the scheme occurs. For example, the value of the death benefit is assumed to be equal to that of the leaving service benefit. No extra benefits are assumed to be granted on any occasions.

Similar to section 3.2.3, multiple mode of exits may be dealt with as insurance within the fund. However, such a feature has been omitted from the individual model for the simplicity of the mathematical arguments.

-Lump sum benefit

The benefits are paid as lump sums on exits. That is, the withdrawal beneifts are always transfer values, and the retirement benefits are bought out.

-Funding method

A funding method which has the Standard Fund and the Standard Contribution rate is used. So, from section 3.3.2, Condition 3.3.2.2.3 holds. Also, Condition 3.3.2.2.4 holds.

-Fixed valuation basis

The same valuation basis is used all the time. Some withdrawals may be assumed in the valuation basis.

-The experience

The experience is assumed to follow the valuation basis exactly except withdrawals. Because there is only one member, the actual withdrawal rate can only be 100% or 0%.

The value of the assets is equal to the market value at all times.

-Initial assets

The value of the initial assets is 0.

-Actual contribution rate

The Standard Contribution rate is used. Any excesses/shortages in the assets at the member's exit is recognised as surpluses/losses to the scheme.

-Continuous movements

Contributions and investment proceeds are received continuously.

Salaries grow continuously, and the benefits accrue continuously.

Funding methods in this section are consistent with the continuous nature of the items above for the calculation of the Standard Contribution rate and the Standard Fund. As a part of the valuation assumptions, withdrawals are assumed to occur continuously, and the benefits to be paid out continuously.

3.3.4 Definitions of P, X, R

Now, some key functions are defined together with the discussion of the properties of these functions.

3.3.4.1 P(x)

Definition3.3.4.1.1---

$$P(x) = \int_e^x SC(t)(1+i)^{x-t}dt - BN(x)$$

Because everything is assumed to follow the basis, the first part of the right-hand side of the definition represents the accumulated assets in the pension fund through the member's standard contributions. Hence, P(x) is the balance of the fund after deducting all the individual pots. (In this case only one individual pot exists.)

Now, from Condition 3.3.2.2.3, at entry,

{Present value of expected benefits} = {Present value of expected contributions}

This is expressed as

$$\int_e^r BN(t)\mu_t \frac{l_t}{l_e} v^{t-e}dt + BN(r)\frac{l_r}{l_e}v^{r-e} = \int_e^r SC(t)\frac{l_t}{l_e}v^{t-e}dt$$

The right hand side can be changed to

$$\int_e^r SC(t)\frac{l_t}{l_e}v^{t-e}dt = \int_e^r \frac{l_t}{l_e}\frac{d}{dt}\left(\int_e^t SC(s)v^{s-e}ds\right)dt$$

..........3.3.4.1.1.1

$$= \frac{l_r}{l_e}\int_e^r SC(s)v^{s-e}ds + \int_e^r \frac{\mu_t l_t}{l_e}v^{t-e}\int_e^t SC(s)v^{s-t}dsdt$$

..........3.3.4.1.1.2

Hence,

$$\int_e^r \mu_t \frac{l_t}{l_e}v^{t-e}\left[\int_e^t SC(s)(1+i)^{t-s}ds - BN(t)\right]dt + \frac{l_r}{l_e}v^{r-e}\left[\int_e^r SC(s)(1+i)^{r-s}ds - BN(r)\right] = 0$$

From the definition of P(x),

$$\int_e^r \mu_t \frac{l_t}{l_e} v^{t-e} P(t)dt \ + \ \frac{l_r}{l_e} v^{r-e} P(r) \ = \ 0$$

The expected present value of P at entry is shown to be 0.

Hence, the following theorem holds.

Theorem3.3.4.1.2--

If P(t) > 0 for some t where $\mu_t > 0$

there exists s such that P(s) < 0 and $\mu_s > 0$.

Here, it should be noted that $\mu_r > 0$ $\quad$ $if\ l_r > 0$.

For example, let us think about a scheme where benefits are given only on retirements at age r. That is,

$$BN(t) = \begin{cases} 0 & for\ t \neq r \\ BN(r) & for\ t=r \end{cases} \qquad \text{............3.3.4.1.2.1}$$

Assume that the Standard Contribution is a level amount from the entry at age e to the retirement at age r. i.e. SC(t) = c, where c: constant.

The following equation should hold,

$$BN(r)\frac{l_r}{l_e}v^{r-e} = \int_e^r c\frac{l_t}{l_e}v^{t-e}dt$$

So, BN(r) is expressed as,

$$BN(r) = \int_e^r c\frac{l_t}{l_r}(1+i)^{r-t}dt \qquad \text{............3.3.4.1.2.2}$$

Calculating P(t),

$$P(t) = \begin{cases} \int_e^t c(1+i)^{t-s}ds & for\ \ t<r \\ \int_e^r c(1+i)^{r-s}ds \ - \ BN(r) & for\ \ t=r \end{cases} \qquad \text{............3.3.4.1.2.3}$$

When e < t < r, obviously, P(t) > 0.

So, the following holds;

Proposition3.3.4.1.3

Assume

BN(r) satisfies 3.3.4.1.2.1,

SC(t) = c (constant), and

there exists t_0 *which satisfies* $\mu_{t_0} > 0$.

Then, P(r) < 0.

Proof

Calculate BN(r) using 3.3.4.1.2.2.

Because $l_t \geq l_r$ for all t

and $l_t > l_r$ for $t < t_0$,

$$BN(r) = \int_a^r c\frac{l_s}{l_r}(1+i)^{r-s}ds < \int_a^r c(1+i)^{r-s}ds$$

Applying this to 3.3.4.1.2.3, P(r) < 0.

QED

Now, think about a special case where

$$P(t) \equiv 0 \qquad \textit{for all } t$$

i.e.

$$BN(t) = \int_a^t SC(s)(1+i)^{t-s}ds \qquad \textit{for all } t$$

By differentiating this,

$$\frac{d}{dt}BN(t) = \frac{d}{dt}(1+i)^t \int_a^t SC(s)(1+i)^{-s}ds$$

$$= \delta BN(t) + SC(t)$$

So, the growth of BN is completely met by two elements; the standard contribution income and the assumed level of investment income. That is, the standard contributions can be expressed in terms of BN as follows;

$$SC(t) = \frac{d}{dt}BN(t) - \delta BN(t)$$

3.3.4.2 R(x) and X(x)

Now, we shall define functions R(x) and X(x). Although the X function is already defined in the global model (see section 3.2.2.2), the definition is independently given here in the context of the individual model.

Definition 3.3.4.2.1

The required contribution at time t, R(t), is defined as a series of amounts satisfying the following;

$$BN(t) = \int_e^t R(s) * (1+i)^{t-s} ds \qquad \textit{for all } t \leq r$$

From Definition 3.3.4.1.1 and Definition 3.3.4.2.1, if SC(t) = R(t) for all t,

$$P(t) = 0 \qquad \textit{for all } t.$$

When this equation holds, the size of the individual pot is equal to the accumulation of the past standard contributions. There will be no gains /losses on the member's exit.

Example 3.3.4.2.1.1

The Current Unit method satisfies this condition. (This is discussed later.)

Example 3.3.4.2.1.2

Money purchase schemes also satisfy the condition if the investment performance is the same as the valuation rate of interest assumed.

Now, the standard contributions paid into the individual pot are separated into two parts.

Definition 3.3.4.2.2

$$X(t) = SC(t) - R(t)$$

This is the X function in connection with contribution incomes. Owing to the strict assumptions made in section 3.3.3, this definition covers a narrower meaning than the description in section 3.2.

Proposition 3.3.4.2.3

$$P(t) = \int_e^t X(s)(1+i)^{t-s} ds$$

The proof is straightforward from the definitions of P and X.

From Theorem 3.3.4.1.2, P(t) cannot be positive for all t. Similarly, the following Corollary holds.

Corollary 3.3.4.2.4

There exists t ($e < t < r$) such that $X(t) \leq 0$, and

there exists s ($e < s < r$) such that $X(s) \geq 0$.

Proof

Only the first half is shown here.

If $X(t) > 0$ for all t, from Proposition 3.3.4.2.3, $P(t) > 0$ for all t. This is contradictory to Theorem 3.3.4.1.2.

QED

3.3.5 Description of the individual model

Sections 3.3.3 and 3.3.4 gave all the assumptions and definitions which are needed for the individual model. Here, a prose explanation of the model is given.

Although the scheme has only one member, the fund is separated into the individual pot and the rest. The individual pot is represented by BN(x). The rest is represented by P(x).

The contributions paid into the scheme are divided between the portion going to the individual pot, R(x), and the balance, X(x). If the growth in BN(x) is faster than SC(x), the shortfall is met by a negative X(x).

The individual pot also receives investment proceeds at the assumed rate. (see Definition 3.3.4.2.1, where investment incomes are included in the definition of R.)

So, the amount of assets allocated to the individual pot is always kept equal to BN(x).

The rest of the fund is represented by P, which is the accumulation of X at the assumed rate of return. P can be positive, 0, or negative. On the exit of the member, P is released. (see section 3.3.7)

3.3.6 Standard fund and P, X

This section and section 3.3.7 develop a mathematical discussion regarding the surpluses/losses recognised under the individual model.

Direct expressions of the Standard Fund depend on the funding method chosen. Here, from Conditions 3.3.2.2.3 and 3.3.2.2.4, the Standard Fund for the member aged t at time t can be expressed as follows.

Proposition 3.3.6.1

$$SF(t) = \int_t^r \mu_s \frac{l_s}{l_t} BN(s) v^{s-t} ds + \frac{l_r}{l_t} BN(r) v^{r-t} - \int_t^r SC(s) \frac{l_s}{l_t} v^{s-t} ds$$

The proof is directly given by applying Conditions 3.3.2.2.3 and 3.3.2.2.4.

Here, the future Standard contributions, which are derived from the application of a particular funding method, are used to express the Standard Fund.

Now, PVP is defined as the present value of P on expected exits.

Definition 3.3.6.2

$$PVP(t) = \int_t^r \mu_s \frac{l_s}{l_t} P(s) v^{s-t} ds + \frac{l_r}{l_t} P(r) v^{r-t}$$

The following relationship holds.

Theorem 3.3.6.3

$$SF(t) = BN(t) + P(t) - PVP(t) \qquad \text{for all } t$$

Proof

Applying the definition of P(t) to the definition of PVP(t) above,

$$
\begin{aligned}
PVP(t) &= \int_t^r \frac{l_s}{l_t}\mu_s v^{s-t}\left[\int_0^s SC(u)v^{u-s}du - BN(s)\right]ds + \frac{l_r}{l_t}v^{r-t}\left[\int_0^r SC(u)v^{u-r}du - BN(r)\right] \\
&= \int_t^r \frac{l_s}{l_t}\mu_s \int_0^s SC(u)v^{u-t}duds + \frac{l_r}{l_t}\int_0^r SC(u)v^{u-t}du - \int_t^r \frac{l_s}{l_t}\mu_s v^{s-t}BN(s)ds - \frac{l_r}{l_t}v^{r-t}BN(r) \\
&= \left[-\frac{l_s}{l_t}\int_0^s SC(u)v^{u-t}du\right]_{s=t}^r - \int_t^r -\frac{l_s}{l_t}SC(s)v^{s-t}ds + \frac{l_r}{l_t}\int_0^r SC(u)v^{u-t}du \\
&\qquad - \int_t^r \frac{l_s}{l_t}\mu_s v^{s-t}BN(s)ds - \frac{l_r}{l_t}v^{r-t}BN(r) \\
&= \int_0^t SC(u)v^{u-t}du + \int_t^r \frac{l_s}{l_t}SC(s)v^{s-t}ds - \int_t^r \frac{l_s}{l_t}\mu_s v^{s-t}BN(s)ds - \frac{l_r}{l_t}v^{r-t}BN(r)
\end{aligned}
$$

From Proposition 3.3.6.1,

$$PVP(t) = \int_0^t SC(u)v^{u-t}du - SF(t)$$

Applying Definition of P(x) (see 3.3.4.1.1),

$$PVP(t) = BN(t) + P(t) - SF(t)$$

QED

From Theorem 3.3.6.3, the following are derived:

Corollary 3.3.6.4

The following conditions i) and ii) are equivalent:

i) $SF(t) = BN(t)$

ii) $P(t) = PVP(t)$

The proof is obvious from Theorem 3.3.6.3.

Theorem 3.3.6.5

The following conditions I) and II) are equivalent:

I) $SF(t) = BN(t)$ *for all t*

II) $P(t) = 0$ *for all t*

Proof

From Corollary 3.3.6.4, the condition I) is equivalent to:

I1) $P(t) = PVP(t)$ *for all t*

Applying Definition 3.3.6.2, the definition of PVP(t), this is equivalent to:

I2) $$P(t) = \int_t^r \mu_s \frac{l_s}{l_t} P(s) v^{s-t} ds + \frac{l_r}{l_t} P(r) v^{r-t}$$ *for all t*

Now, let us perform partial integration on the first element of the right hand side of I2).

$$\begin{aligned}
\int_t^r \mu_s \frac{l_s}{l_t} P(s) v^{s-t} ds &= \frac{1}{l_t v^t} \int_t^r (P(s) v^s)(l_s \mu_s) ds \\
&= -\frac{1}{l_t v^t} \int_t^r (P(s) v^s) \frac{d}{ds} l_s ds \\
&= -\frac{1}{l_t v^t} [P(s) v^s l_s]_{s=t}^r + \frac{1}{l_t v^t} \int_t^r l_s \frac{d}{ds}(P(s) v^s) ds \\
&= -P(r) \frac{l_r}{l_t} v^{r-t} + P(t) + \frac{1}{l_t v^t} \int_t^r l_s \frac{d}{ds}(P(s) v^s) ds
\end{aligned}$$

Substituting the first element of the right hand side of I2) with this and multiplying by $l_t v^t$, I2) is shown to be equivalent to:

I3) $$\int_t^r l_s \frac{d}{ds}(P(s) v^s) ds = 0$$ *for all t*

Because I3) holds for all t, the differential of the left hand side of I3) by t must be 0. And if the differential is 0 for all t, I3) should hold. That is, I3) is equivalent to:

I4) $$\frac{d}{dt} \int_t^r l_s \frac{d}{ds}(P(s) v^s) ds = 0$$ *for all t*

This is equivalent to

$$l_t \frac{d}{ds}(P(s) v^s)\big|_{s=t} = 0$$ *for all t*

That is:

I5) $$P(t) v^t = k$$ *for all t*

where k : constant.

At entry to the scheme,

$$P(e) = 0$$

Hence, k must be 0. That is:

II) $$P(t) = 0$$ *for all t*

QED

3.3.7 Surplus

Now, we shall discuss the surplus. There can be various definitions of surplus. Here, two different approaches for defining the surplus are given, and the relationship between these is discussed.

For each approach to the definition of surplus, separate consideration in respect of the members who exit the scheme and those who remain, and separate definitions of surplus are given. i.e. two sets of two definitions of surplus are given below.

The first approach is based on the idea that the value of each member's liability is represented by the standard fund for the member. If the member leaves the scheme, the difference between the standard fund and the value of actual benefit given is taken as a surplus/deficit. If the member remains in the scheme, the difference between the increase in the standard fund and the standard contributions plus investment proceeds on the standard fund is taken as a surplus/deficit. This approach does not need the concepts of individual pots nor X-functions, and may be regarded as a conventional approach.

Definition 3.3.7.1

$$SUR(t) = SF(t) - BN(t) \qquad \textit{for an exit at } t$$

$$SUR'(t) = \delta SF(t) + SC(t) - \frac{d}{dt}SF(t) \qquad \textit{for a person surviving as a member to time } t$$

An alternative definition of the surplus from a different point of view as follows may be possible in line with the model which is being discussed in this chapter.

Definition 3.3.7.2

$$SURa(t) = P(t) \qquad \textit{for an exit at } t$$

$$SURa'(t) = 0 \qquad \textit{for a person surviving as a member to time } t$$

The second view is based on the cash accumulation for a member. From the definition of P(t), P(t) + BN(t) represents the accumulated contributions for the member. So, SURa(t) can be interpreted as equivalent to [P(t) + BN(t)] - BN(t) That is, the excess of the accumulated contributions over the value of the benefit granted is recognised as the surplus on an exit. The surplus is not recognised until the exits actually occur.

SF(t) - BN(t) was shown by Theorem 3.3.6.3 to be equal to P(t) - PVP(t). So,

$$SURa(t) - SUR(t) = PVP(t)$$

From the definition of PVP(t), this may be interpreted as the present value of the anticipated surpluses (on a cash accumulation basis) from expected future exits the realisation of which are prevented by the exit now.

That is, the first view anticipates future SURa, and allows them to be recognised in advance. An exit means a loss in the possibilities for such future gains (SURa), and the present value of such future losses (PVP) is deducted from the arising surplus (SURa at the time of the exit).

Now, we shall examine the surplus arising from survivals from the first viewpoint.

Theorem 3.3.7.3 ---

$$SUR'(t) = \frac{d}{dt}PVP(t) - \delta PVP(t)$$

To prove the theorem, the following two lemmas are needed:

Lemma 3.3.7.4 ---

$$\frac{d}{dt}BN(t) = \delta BN(t) + R(t)$$

Proof of Lemma 3.3.7.4

From Definition 3.3.4.2.1, the definition of R(t),

$$\begin{aligned}
\frac{d}{dt}BN(t) &= \frac{d}{dt}\int_0^t R(s) * (1+i)^{t-s}ds \\
&= \frac{d}{dt}[(1+i)^t * \int_0^t R(s)v^s ds] \\
&= \frac{d}{dt}(1+i)^t * \int_0^t R(s)v^s ds + (1+i)^t * \frac{d}{dt}\int_0^t R(s)v^s ds \\
&= \delta(1+i)^t \int_0^t R(s)v^s ds + (1+i)^t R(t)v^t \\
&= \delta BN(t) + R(t)
\end{aligned}$$

QED

Lemma 3.3.7.5 ---

$$\frac{d}{dt}P(t) = \delta P(t) + X(t)$$

Proof of Lemma 3.3.7.5

Similar to the proof of Lemma 3.3.7.4 above, the proof is easily given by differentiating the both sides of Proposition 3.3.4.2.3. QED

Proof of Theorem 3.3.7.3

By applying Theorem 3.3.4.3 and Definition 3.3.4.2.2, the definition of X(t), to Definition 3.3.7.1, the definition of $SUR'(t)$,

$$SUR'(t) = \delta[BN(t)+P(t)-PVP(t)] + [X(t)+R(t)] - \frac{d}{dt}SF(t)$$

$$= [\delta BN(t)+R(t)] + [\delta P(t)+X(t)] - \delta PVP(t) - \frac{d}{dt}SF(t)$$

Applying Lemma 3.3.7.4 and Lemma 3.3.7.5 to the above,

$$SUR'(t) = \frac{d}{dt}BN(t) + \frac{d}{dt}P(t) - \frac{d}{dt}SF(t) - \delta PVP(t)$$

$$= \frac{d}{dt}PVP(t) - \delta PVP(t)$$

QED

The right hand side of the Theorem 3.3.6.3 is the change in PVP excluding interest on PVP. This supports the comment on the difference in the two viewpoints on the surplus above.

By applying the definition of PVP, the surplus is shown to be equal to

$$-\mu_t[P(t) - PVP(t)]$$

From Theorem 3.3.6.3, this is equal to,

$$-\mu_t[SF(t) - BN(t)]$$

This is equal to the surplus anticipated to arise by exits, which did not occur.

The second view is simpler, and fits well with the model with P, X functions. Particularly, this is useful for treating the influences of withdrawals separately.

3.3.8 Interpretation of typical funding methods

Now, typical funding methods are defined in a continuous mode, and P and X are calculated for an illustrative scheme.

Let us assume the following benefit formula;

$$BN(t) = k(t-e)S(t)v^{r-t}\bar{a}_r$$

Here, no revaluation of annuities during the deferred period is assumed. (A generalisation, which allows for revaluations, can be easily made by using a different interest rate during the deferment, but without changes in the formulae.)

By differentiating BN(t) and deducting the element of interests on BN(t), (Lemma 3.3.7.4)

$$R(t) = kS(t)v^{r-t}\bar{a}_r + \frac{d}{dt}S(t) * BN(t)$$

3.3.8.1 Current Unit Method

This method is represented by

$$SF(t) = BN(t)$$

$$SC(t) = kS(t)v^{r-t}\bar{a}_r + \frac{d}{dt}S(t) * BN(t)$$

From Theorem 3.3.4.3,

$$P(t) = 0 \qquad \textit{for all } t$$

And from the definition of X,

$$X(t) = 0 \qquad \textit{for all } t$$

3.3.8.2 Projected Unit Method

This method has

$$SF(t) = \int_t^r k(t-e)S(s)v^{r-t}\bar{a}_r\frac{\mu_s l_s}{l_t}ds$$

$$=k(t-e)v^{r-t}\bar{a}_r\int_t^r S(s)\frac{\mu_s l_s}{l_t}ds$$

and

$$SC(t) = kv^{r-t}\bar{a}_r\int_t^r S(s)\frac{\mu_s l_s}{l_t}ds$$

Here,

$$SC(t) = \frac{SF(t)}{t-e}$$

Applying these to the definition of P,

$$\begin{aligned}
P(t) &= \int_e^t SC(s)(1+i)^{t-s}ds - BN(t) \\
&= \int_e^t kv^{r-s}\bar{a}_r\int_s^r S(u)\frac{\mu_u l_u}{l_s}du * (1+i)^{t-s}ds - BN(t) \\
&= \int_e^t kv^{r-t}\bar{a}_r\frac{1}{l_s}\int_s^r S(u)\mu_u l_u du\ ds - BN(t) \\
&= kv^{r-t}\bar{a}_r\int_e^t\frac{1}{l_s}\int_s^r S(u)\mu_u l_u du\ ds - k(t-e)S(t)v^{r-t}\bar{a}_r \\
&= kv^{r-t}\bar{a}_r\left[\int_e^t\frac{1}{l_s}\int_s^r S(u)\mu_u l_u du\ ds - (t-e)S(t)\right]
\end{aligned}$$

From Definition 3.3.4.2.2,

$$\begin{aligned}
X(t) &= SC(t) - R(t) \\
&= kv^{r-t}\bar{a}_r\int_t^r S(s)\frac{\mu_s l_s}{l_t}ds - kS(t)v^{r-t}\bar{a}_r - \frac{d}{dt}S(t) * BN(t) \\
&= kv^{r-t}\bar{a}_r\left\{\int_t^r S(s)\frac{\mu_s l_s}{i_t}ds - S(t)\right\} - \frac{d}{dt}S(t) * BN(t)
\end{aligned}$$

Now, let us define adjusted P, or AdjP.

Definition 3.3.8.2.1 ---

$$AdjP(t) = P(t)\frac{v^{t-r}}{\bar{a}_r}$$

Proposition 3.3.8.2.2 ---

AdjP for Projected Unit Method is independent of the interest rate assumption in the basis.

The proof is obvious from the above.

Hence, P can be decomposed into the interest related element and the interest non-related element, AdjP.

3.3.8.3 Entry Age Method

Here, the individual entry age method is examined.

In this method, the standard contribution rate is set at the entry as a level percentage of the salary throughout the working life.

$$SC(t) = \frac{\int_e^r BN(s)\frac{\mu_s l_s}{l_e} v^{s-e} ds}{\int_e^r S(s)\frac{l_s}{l_e} v^{s-e} ds} * S(t)$$

$$= \frac{\int_e^r BN(s)\mu_s l_s v^{s-e} ds}{\int_e^r S(s) l_s v^{s-e} ds} * S(t)$$

$$= \frac{\int_e^r k(s-e)S(s)v^{r-s}\bar{a}_r \mu_s l_s v^{s-e} ds}{\int_e^r S(s) l_s v^{s-e} ds} * S(t)$$

$$= \frac{\int_e^r kv^{r-e}\bar{a}_r (s-e)S(s)\mu_s l_s ds}{\int_e^r S(s) l_s v^{s-e} ds} * S(t)$$

$$= kv^r \bar{a}_r \frac{\int_e^r (s-e)S(s)\mu_s l_s ds}{\int_e^r S(s) l_s v^s ds} * S(t)$$

$$SF(t) = \int_t^r BN(s)\frac{\mu_s l_s}{l_t} v^{s-t} ds - \int_t^r SC(s)\frac{l_s}{l_t} v^{s-t} ds$$

$$= \int_t^r k(s-e)S(s)v^{r-s}\bar{a}_r \frac{\mu_s l_s}{l_t} v^{s-t} ds - \int_t^r SC(s)\frac{l_s}{l_t} v^{s-t} ds$$

$$= \frac{kv^{r-t}\bar{a}_r}{l_t}\int_t^r (s-e)S(s)\mu_s l_s ds - \int_t^r SC(s)\frac{l_s}{l_t} v^{s-t} ds$$

From the definition of P,

$$P(t) = \int_e^t SC(s)(1+i)^{t-s} ds - BN(t)$$

$$= kv^r \bar{a}_r \frac{\int_e^r (u-e)S(u)\mu_u l_u du}{\int_e^r S(u) l_u v^u du} \int_e^t S(t)(1+i)^{t-s} ds - BN(t)$$

From the definition of X,

$$X(t) = kv^{r}\bar{a}_r S(t) \left[\frac{\int_e^r (s-e)S(s)\mu_s l_s ds}{\int_e^r S(s) l_s v^s ds} - (1+i)^t \right] - \frac{d}{dt}S(t) * BN(t)$$

In the Projected Unit method and the Entry Age method, X function is designed to be positive while the member's past service is short, and negative as the service becomes longer. On the contrary, in the Current Unit method, X function is designed to be 0 all the time.

Usually, the Entry Age method uses only one entry age for the whole scheme. The resulting standard contribution rate may be interpreted as the average of individual rates on the Individual Entry Age method.

3.3.8.4 Attained Age Method

This method does not satisfy Condition 3.3.2.2.4. Further analysis may be carried out on this model as a modified version of the Entry Age method or the Projected Unit method. i.e. the same P and X as the Entry Age method or the Projected Unit method may be used.

For the analysis of the Aggregate method, the same is applied.

3.3.8.5 Initial Funding

This has

$$SF(t) = \int_t^r BN(s) \frac{\mu_s l_s}{l_t} ds = \frac{kv^{r-t}\bar{a}_r}{l_t} \int_t^r (s-e)S(s)\mu_s l_s ds$$

and

$$SC(t) = \begin{cases} 0 & \text{if } t > e \\ SF(e) & \text{if } t = e \end{cases}$$

From the definition of P,

$$P(t) = SF(e)(1+i)^{t-e} - BN(t)$$

From the definition of X,

$$X(t) = \begin{cases} -\frac{d}{dt}BN(t) & \text{if } t > e \\ SF(e) & \text{if } t = e \end{cases}$$

3.3.8.6 Terminal Funding

Under the assumptions of the individual model, this method is equivalent to the pay-as-you-go method. The contributions are demanded whenever the benefits need to be given. So, it is not possible to have the Standard contribution rate, and this method does not satisfy the assumption in section 3.3.3.

However, if no withdrawal benefits are given, it would be possible to treat the Terminal Funding as a funding method satisfying the requirements in this section.

The contributions received are immediately used to pay the benefits. Hence,

$$SF(t) = 0$$

There are no need to pay benefits before the normal retirement age. So,

$$SC(t) = \begin{cases} 0 & \text{if } t < r \\ BN(r) & \text{if } t = r \end{cases}$$

Obviously,

$$P(t) = X(t) = 0$$

3.4 Individual Model and Global Model

We now have two models; the global model defined in section 3.2, and the very restrictive individual model in section 3.3. The relationship between them is discussed in this section.

The two models share various basic concepts; the individual pot, SC, X and R. Also, there are no inconsistencies between the two. So, the individual model may be regarded as a special case of the global model.

The rest of the fund in the individual model corresponds to the common pool in the global model. The Standard Fund less the individual pot in the individual model is equivalent to the standard common pool in the global model.

The differences are that only one member exists currently, in the past and in the future, that the experience exactly follows the assumptions except withdrawals, and that the valuation basis cannot be changed in the individual model.

It would be possible to expand the individual model as follows. The individual model deals with only one active member. However, if several individual models are added together, the model can be applied to multiple membership schemes. Here, on exit of the members, P is retained in the scheme, and accumulated by investment returns. The total of P for existing and past members corresponds to the common pool in the global model.

Although the expanded individual model can deal with different withdrawal experience from the assumptions, it is also necessary to generalise the model to allow for a different experience from the assumptions in other factors and for a change in the valuation basis.

For example, we shall look at the X function in both models. In the individual model, the X function only appeared in conjunction with contributions. This is because the contribution incomes, investment incomes and benefit payments are made exactly in line with the assumed basis in Section 3.3. When they are different from the basis, the X function in the global model is required to keep each individual pot equal to the value of the leaving service benefit from time to time. In this sense, the X function in the global model may be regarded as a generalisation of the X function in the individual model.

Although the global model may have a wider application, the individual model is also useful because of its rigid mathematical definitions. For example,

-it is useful to clarify each member's contribution to the common pool.

-as seen in section 3.3.8, the individual model may be powerful in analysing the principles of funding methods.

3.5 Application

Now we shall apply the model in the analysis of pension fund finances. In the following, various financial events are given explanations using the model.

3.5.1 Withdrawals

Take a particular period which is short enough to be able to ignore the impacts of investment proceeds and accruals of benefits.

If a withdrawal or a normal retirement occurs during the period without any enhancements in the benefit, the amount of the individual pot for the exiting member is equal to the benefit. So, no X is required to increase/decrease the size of individual pot, and the common pool is not affected by such withdrawals.

By the exit, the standard common pool will be reduced by the exiting member's portion, P. Consequently, a surplus of P (a loss if P is negative) in the first sense is accruing.

At the same time, regarding a remaining member, the individual pot remains the same during the period. However, if some withdrawals are assumed in the calculation of the Standard Fund, the member's portion of the Standard Fund would increase (or decrease if the Standard Fund is negative), the member's portion of the standard common pool also increases (decreases if the Standard Fund is negative). Because the individual pot is unaffected, these changes do not require X functions.

If anticipated withdrawals occur during the period, the total changes in the standard common pool balances to 0. However, surplus/loss (the sum of P for the exits) in the first sense is recognised. As mentioned in section 3.3.7, such surpluses/losses are anticipated by the funding method which assumes some withdrawals.

Accrued surpluses are invisible because the standard common pool is not changed. However, the standard common pool will be used to meet the shortages in individual pots for the long term survivors in the future through negative X functions. So, indirectly, the surpluses accrued on the withdrawing members are redistributed to the remaining members. Such redistributions may be observed in the Entry Age method and the Project Unit method.

To be able to meet the demands for future negative X's, the common pool needs to be sufficiently large. The larger the negative X is expected to be, the larger the standard common pool should be.

So, it is seen that the funding method and basis which has a large standard common pool and P is affected more by the difference between the actual and expected withdrawal experience than the method and basis which has a smaller (or 0) standard common pool and P. In such cases of large standard common pools, the withdrawal assumption is relatively important. On the other hand, the Current Unit method has nil standard common pool, and the withdrawal assumption is irrelevant for the finances of the pension fund.

3.5.2 Salary increases

In the case of a salary related scheme, increases in salary expand the individual pots. Such expansions are met by SC and negative X.

Firstly, the situation where the actual salary increases follow the assumptions is discussed. If the Current Unit method is used, all the requirements are designed to be met by SC because SC is determined to meet this requirement, and no X may be required.

If the funding method is the Projected Unit method or the Entry Age method, future salary rises are anticipated, and SC is made more stable than R. So, initially, SC includes allowances for such rises and positive Xs would appear. Later, when SC becomes less than R, the shortfall is filled by negative X.

The degree of dependence on negative X function depends on the rate of salary increases assumed and the average length of period during which the salary increases are assumed. The larger the two factors are, the more the degree of dependence would be.

A high dependence in X functions means a high level of the standard common pool. So, this affects the impact from the withdrawal experience.

Secondly, the situation where the actual salary increases are different from the assumptions is discussed. If the actual salary increase is higher than anticipated, the individual pots increase in line with the actual salary increases rather than the anticipated salary increase. So, because of a gearing effect, the difference between the actual required contribution, R', and the anticipated, R, grows more than proportionate to the gap between the actual and anticipated salary increases.

At the same time, SC increases in line with the actual salary increases because SC is determined by the Standard Contribution rate (which is unaffected) times the actual salary.

That is,

$$R' / R \quad > \quad SC' / SC,$$

where

SC: Standard contribution anticipated, and

SC': the actual Standard contribution.

If X (= SC - R) is positive, R is smaller than SC, and the higher rate of the increase in R may be cancelled to some extent by the increase in SC. The increase in SC might well be larger than the increase in R. Then, X may be reduced modestly or increased.

If X is negative, the increase in R is larger than the increase in SC, and extra negative Xs are required. The increase in the negative X is, because of a gearing effect, higher than the increase in R, which is higher than the increase in SC.

Thus, the total effect to X depends on the membership structure.

At the same time, the Standard Fund becomes higher than that under the anticipated salary increases by the ratio of

'Average actual salary increases' / 'Anticipated salary increases'.

Because of the increase in individual pots, the standard common pool is also increased by the same ratio as the Standard Fund.

Although the common pool receives the balance of the contributions which are increased in line with the increase in the salaries, the existing assets corresponding to the common pool stay the same. Consequently, the growth in the common pool is less than the standard common pool, and a deficit arises.

3.5.3 Investment returns

Differences from the assumption on investment income affect the individual pot and require X to maintain the size of individual pots. Combined with the investment returns on the common pool, the relative size of the common pool and the standard common pool change. This appears as surpluses/deficits.

3.5.4 Extra benefits

They increase individual pots for the members who are given extra benefits. These are financed by negative X or extra contributions for these particular members. Some can be explained as insurance within the fund (see 3.2.3) as long as such extra benefits are anticipated in the basis. Otherwise, the extra benefits bring losses to the fund because the common fund is reduced by the negative X unless it is met by specific contributions.

3.5.5 Asset value

Asset values assessed on the basis may not be the same as the market value. Such differences give the fund surpluses/deficits from the positive/negative cash flows. In the following, the impacts of the differences in the market values and the assessed values are discussed separately for each item.

3.5.5.1 Contribution

Contribution rates may be calculated on the basis, but adjustments to market values may not be made. Consequently, the assessed value of contributions received may be more/less than required. In the model, SC is not affected, while the value of the total contributions is affected by the difference. Consequently, the value of total contributions less the sum of SC, which is directly paid into the common pool, is affected.

3.5.5.2 Investment proceeds

The investment proceeds, which consist of investment incomes and any revaluation of assets, are recognised on the valuation basis. So, the differences between the market values and the assessed values affect the investment return, which is used as the basis for crediting investment proceeds to individual pots and the common pool. So, the impact is the same as section 3.5.3 above.

3.5.5.3 Benefits

Benefits have two aspects; the difference between the market value of assets and the assessed value and the value of benefits on the valuation basis and that on the market basis. The latter arises when the benefits are in the form of the annuities, but paid as lump sums referring to the market interest rates.

Any balance of the individual pots after the benefit payment is met by the X function, and may change the relative sizes of the common pool and the standard common pool.

3.5.6 Basis changes

Changes in the valuation basis have the following influences;

- Changes in the sizes of individual pots
- Change in the size of the Standard Fund
- Change in the assessed value of total assets in the fund

Changes in the sizes of individual pots occur because the values of leaving service benefits may have changed, and will be filled by the X functions. The change in the sizes of the Standard Fund occurs because the values of the benefits and contributions may have changed, and combined with the changes in individual pots, the standard common pool will change its size.

Because of the change in the assessed value of the total assets and the change in the standard common pool, the size of the common pool will change. Alternatively, this may be explained as the change in the assessed value of the assets corresponding to the common fund and X functions required by the changes in the individual pots.

Chapter 4 Cross-Subsidies

4.1 Introduction

The 'cross-subsidy' problem is often talked as a disadvantage of a defined benefit pension scheme. There are several papers referring to the 'cross-subsidy' problem (see Kaye(1985) and Wilkie(1985a)), without offering a precise definition of the problem. The definition should be the starting point for the discussion. This chapter aims to identify the 'cross-subsidy' problem and to analyse it. Also, this chapter presents possible ways to deal with it.

First, we shall consider how the pension benefits are evalued in sections 4.2 to 4.5. This is the starting point to consider the 'cross-subsidy' problem. In section 4.6, the 'cross-subsidy' problem is identified. In sections 4.7 and 4.8, the causes of the problem are analysed., and in section 4.9, possible solutions to the problem are presented. As a part of one of the solutions, the application of the model developed in Chapter 3 is proposed in section 4.10. Finally, section 4.11 concludes the discussion.

4.2 Benefits from a defined benefit scheme

The benefits provided by a defined benefit pension scheme are basically determined by the formulae or the scheme rules. The forms and the amounts of the benefits can vary from a member to another.

The form of the benefits can either be a lump sum, an annuity or a combination of the two. The form may be specified by the scheme rules according to the circumstances under which the benefits are payable. The members may be given an option to choose one form from several alternatives.

The amount of the benefits may be determined by the scheme rules referring to several factors; such as final salary, past service, year to Normal Retirement Age, the reason of exit, marital status, the member's contributions already made, and so on. A discretionary element can be added to the benefits provided if the rules permit.

4.3 Comparison of the benefits

To appreciate the benefits given under a pension scheme, it would be natural to check the form and the amount of the actual benefits paid or to be potentially paid.

Such approach may be suitable to examine the appropriateness of the design of the scheme benefits. Various benefits given to different members may be compared based on the actual form and amount of the benefits.

The perceived value of an identical benefit could be different for each member. Through this approach, such subjective elements might be able to be taken into account, although this would make the appraisal of the benefits subjective.

The main difficulty in this approach is the need to consider every possible situation where a different benefit is granted. The withdrawals with preserved benefits, the withdrawals with transfer values, the normal retirement, the death in service, and so on, must be separately appraised, and then compared with each other.

The comparison can be made by giving a monetary value of each benefit. This method allows us to compare the annuities and the lump sums directly.

However, this does not solve the difficulty above. Also, further problems arise as follows;

First, the value depends on the basis of calculation chosen. Because we cannot specify the definitely correct basis in the real world, the value of the benefits cannot be absolutely correct.

Secondly, the timing of the benefit payments vary for different members. Even if all the members receive a fixed lump sum on retirement, inflation reduces the real value. There can be some measures of inflation; e.g. Retail Price Index in the UK. However, the measure cannot be absolute. For example, Retail Price Index can include or exclude mortgage payments, and it is difficult to tell which should be used. National Average Earnings figures might be more appropriate.

As a conclusion, it is essential to check each benefit directly in order to appraise the benefit design. However, this approach is not straightforward for assessing the level of benefits granted.

4.4 Accrual method

As stated above, the direct appraisal of the pension fund benefits is difficult. As an alternative, the appraisal may be done by looking at the value of the accrued benefits. The values reflect all the future possible benefits, and the individually calculated values could represent the average value of the benefits the member is expected to receive in the future. This approach effectively abandons the direct comparison of the benefits, and the timing of the appraisal is antedated to the dates of the benefit accruals rather than the dates of the benefit payments.

The concept of benefit accrual during active service is widespread. The Projected Unit funding method, which is widely used in the UK, is based on this concept. Also, the British accounting practice is to recognise the pension costs as they accrue, although the costs are only recognised collectively.

Also, this approach allows all the possible benefits to be summarised in a single figure.

The shortfalls of this approach may be;

-This is a theoretical approach, and may not be easily understood by the general public.

-The value depends on the basis. The sensitivity of the value to the changes in the basis may be much larger than that of the benefits paid. Also, there is no absolutely correct basis.
-This does not indicate the sizes of benefits given for different circumstances. So, the appropriateness of the benefit design cannot be judged from the values calculated.

4.5 Contribution method

As an alternative to the accrual method, the value of the benefits may be assessed by the contributions actually paid in.
The contributions are more likely to be paid in advance because of a widespread practice of pre-funding rather than pay-as-you-go funding. Projected Unit funding is common in the UK, and this method may be a proxy for the accrual method. Hence, this method inherits some of the characteristics of the accrual method; namely, antedating of the assessment of the benefits, expression in a single figure, and unsuitability for judging the benefit design.

Because the contribution payments are visible, this method may be easier to use in practice. When the employees make contributions to the scheme, they may naturally think that their benefits accrue as the contributions are paid. So, this method fits well with their perspective.

This approach is, also, in harmony with the idea of the pension as deferred pay. (Refer to section 18.3 of Fenton et al(1993).)

Furthermore, this approach can give a basis for the comparison with defined contribution schemes.

However, this method is misleading. In particular, this can give a false impression of objectivity. Although the amounts of the contributions made are objectively recognised, the contributions rates are calculated on some set of assumptions, i.e. a basis. There is no absolutely correct basis, and the contributions calculated may not appropriately represent the value of the benefits.

Also, the contribution rates may be decided using various funding methods, and the resulting contribution rates can be very different from one funding method to another.

Moreover, the contribution rates can be adjusted within a certain band to take into account the surplus/deficit. In an extreme case, the contribution payments may be halted until the surplus is removed. Hence, adjustments to the contribution figures are required for this method.

Such misleading features were one of the reasons to adopt the accrual method in the accounts.

4.6 Cross-subsidy

Now, we shall identify the 'cross-subsidy' problem.

Cross-subsidies can arise with the accrual method. However, practically, the contribution method poses more problems owing to its visible nature. Hence, from now on, discussion is made only on the contribution method. The main arguments below may also hold with the accrual method.

Even when the value of the benefits is recognised by the contribution method, people may wish to check the value of the actual benefits paid. This is natural because the benefits paid are the essential concern for the defined benefit scheme design. However the two perspectives are different.

The cross-subsidy problem arises when people think there are some gaps between the value recognised under the contribution method and the value of the benefits actually given.

Under the contribution method, each individual's portion of the contributions is identified during the active service. Here the amount of the individual's portion contains all the future possibilities; possibilities for a high and low salary growth, possibilities for early leaving, a death in service, retirement, etc. So, the contribution method may be used to assess the benefits when the individual's portion is allocated, but it is not suitable for the assessment at the time of benefit payment by rolling up the allocated portions.

If this rolling up is performed, the difference in the rolled-up individual's portions and the value of the benefit actually given needs to be explained. This difference is explained as the internal re-allocation of the resources, i.e. the cross-subsidies.

This cross-subsidy problem would be more prominent if the contributions could be allocated to each member in some convincing way. For example, if the contributions need to be taxed as the income of the members, the contributions would need to be allocated for the taxation purpose. The benefits may be still determined by the rules, and the contributions made may not be directly relevant to the benefits. However, in such a case, it would be difficult to ignore the cross-subsidy problem. So, in this case, the best way might be to adopt a defined contribution pension scheme. We shall not discuss such cases any more in the following sections, and assume there are no such convincing methods of allocating the contributions.

4.7 Misuses of the contribution method

From the arguments in the previous section, the cross-subsidy problem arises from the misuse of the contribution method. The contribution method can only be applied when the contributions are made. Rolling-up assumes a link between the individual's contributions and the benefits. This may stem from an analogy with a defined contribution scheme. However, in a defined benefit scheme, there are no such links.

Also, the allocation of the contributions may not be appropriate. When a uniform contribution rate is applied to the total of the salaries, people might take an individual's portion as the uniform rate times the individual's salary.

The shortfalls in the contribution method discussed above also affect this argument. If the allocated contributions are inappropriate for the assessment of the benefits, the rolled-up figure is unlikely to be a appropriate measure as well.

When the cross-subsidies are talked of as a problem, it may be often true that there are problems in the benefit design. In such cases, it should be possible to discuss the benefit design directly. However, people may still prefer the perspective of the contribution method. If the contributions allocated to the individuals are satisfactory, or as in the previous paragraph, if the result of inappropriate allocation is satisfactory, the blame of unsatisfactory benefits falls onto the cross-subsidy.

Also, the basis for the valuation can change. Then the values placed on the accrued benefits become totally different, and there will be a discontinuity in the contribution rates. This makes the roll-up of the contributions meaningless.

4.8 Equity

The cross-subsidy problem may be often discussed with the concept of equity among the members.

The equity of the benefits may not necessarily be the aim of a pension scheme. The pension benefits provided may be regarded as a part of the employment terms. Also, the employees may not necessarily seek equal benefits. They may also want rewards for the successful careers.

However, the operation of the pension fund needs to appear fair, and the cross-subsidy may be a hazard to this.

If the members understand the benefits provided by the scheme in full, there may not be such a cross-subsidy problem. Any problems on the benefit design may be discussed directly.

In practice, this may not be the case. People concerned may stick on to the contribution method. The method itself is controversial, but the use of the method may also be wrong. For example, the members may have an impression of equity in the benefits from the uniformly expressed contribution rates. Such illusions may not be corrected until the benefits are actually paid. Then, the members who were given relatively small benefits may feel that they have been deceived. e.g. early leavers, members with low salary growths, and so on.

4.9 Solutions

Now, possible actions to be taken to remove the illusory problem of the cross-subsidies are proposed.

Firstly, the cross-subsidy problem is often a restatement of the problems in the benefit design. Discussions on the contribution basis cannot give the solutions to the problem. For the solutions, a review of the benefit structure may be required. If the difference in the benefits between the early leavers and the retired members, or between the high salary growth members and the low salary growth members in a final salary scheme, a move to a defined contribution scheme, a career average scheme, or a fixed benefit scheme may be found suitable. However, the optimum benefit design may vary among the interested parties, and there cannot be any theoretically correct design for everyone.

Secondly, the cross-subsidy problem tends to be based on several misunderstandings about the workings of a defined benefit pension fund. There is no doubt that the contributions made are the sources of the benefit payments. However, the link between the two is not just a simple roll-up. A proper understanding of the operation of the fund can explain the nature of the cross-subsidy, and solve the problem. For this, the model discussed in the previous chapter, the X-function model, may be useful.

Thirdly, all the pension benefits may be explained as an insurance rather than an investment. It may be a proper way to explain the functions of the pension scheme benefits. Should such arguments be accepted by the members, the question of the cross-subsidies would never arise. However, this cannot answer the problem of the cross-subsidies directly.

4.10 Use of X-function model

The model developed in chapter 3 may be applied to assist the second solution in section 4.9. In the following, an explanation of the cross-subsidy using the X-function model is given. The notation from chapter 3 is used here.

Let

(A) = {Contributions based on the uniform contribution rate} - {individual contribution, SC}

(B) = SC - R ; i.e. the X function when contributions are paid

(C) : X function when BN increases

When the uniform contribution rate is taken as the basis for individual portion of the contributions, the cross-subsidy is interpreted as (A) + (B) + (C).

If the individual contribution rate, which is the standard contribution rate for the individual on a particular funding method, is taken as the basis for individual portion of the contributions, the cross-subsidy is interpreted as (B) + (C) above, the accumulated total of which is the P function in the model.

In either case, if the cross-subsidy figure is positive after all the payments of the benefits, the member may be identified as the member who loses out because of the cross-subsidies. If it is negative, the member may be identified as a beneficiary from the cross-subsidies.

The individual pot is limited to the value of the leaving service benefit in the model. This works to separate the contributions into the part corresponding to the benefits actually accrued and the balance. The latter, if it is positive, is directed to the common pool. If the former is more than the contributions, SC, the balance is provided from the common pool.

These movements may be interpreted in two ways.

Firstly, the funding method may make the increases in the contribution rates as the age goes up less than the increase in the required contribution, R. This results in the accumulation of the excessive contributions when the member is young, and it is used later to supplement the shortfall of R - SC.

Secondly, the contributions directed to the common pool contain the contributions corresponding to the contingent extra benefits over the leaving service benefits. The flow from the common pool to the individual pots may be taken as a realisation of the contingent extra benefits. This function was described in the previous chapter as 'insurance within the fund'. In a sense, this part of the explanation resembles the third solution in section 4.9.

Also, the common pool works as the vessel for the contributions and the investment incomes not attributable to the individual pots. This feature is useful for dealing with the difference between the total of the theoretical individual standard contributions and the actual contributions.

The changes of the basis are dealt with by the changes in the sizes of the individual pots and the consequent X-functions.

4.11 Conclusion

The cross-subsidies are the reallocation of the resources among the individually earmarked contributions. Such earmarking is purely notional for a defined benefit scheme. The cross-subsidy may thus be just a notional problem.

However, as discussed in section 4.9, real problems may lie in the benefit design, and this can only be solved by re-examining the benefit design.

Otherwise, the identification of potential cross-subsidies may be a result of a lack of understanding of the workings of the pension fund. For this problem, as explained in section 4.10, the X-function model discussed in the previous chapter may be useful.

Firstly, faults in earmarking the contributions may be addressed using the X-function model. The model distinguishes contributions needed for the growth of individuals' vested benefits (expressed as "R") from any contributions which are notionally assigned to individuals. This feature would be useful in explaining that the rolling up of notional individual contributions which are inappropriately allocated may not represent the value of benefits to which the members are entitled.

Also, the model is useful in explaining the principles of pre-funding, and thus in explaining the difference between R and the contributions, SC.

Secondly, the model may be effective in explaining the pooled nature of funding. The model separates individual pots, which are the minimum entitlements to the members, from the balance, and any higher benefits are considered to be financed from the common pool. This feature would emphasise any additional benefits/benefit entitlements, which are contingent on certain events; for example, deaths and higher than anticipated salary rises. Consequently, the mechanism for providing such contingent additional benefits from the common pool may be easier to explain than the case without any subdivision between individual pots and a common pool.

It may be possible to explain in full the cross-subsidy problem applying the X-function model. However, time constraints have meant that this possibility has not been realised, and is left for future research.

PART III

Chapter 5 Valuation and Matching

5.1. Introduction

In this chapter, valuation methods of pension funds with their related areas of topics are discussed.

The major part of the research in the areas of funding methods and valuation bases is described in other chapters (Chapter 3 for funding methods, Chapters 6-8 for valuation basis). This involves mathematical developments of the theory and large scale calculations.

In contrast, this chapter mainly consists of a general discussion on the theory and practice, rather than mathematical developments of formulae or numerical calculations although some numerical examples are included. This chapter combined with other parts of this thesis completes the discussion on various aspects of the valuation of pension funds. Another aim of this chapter is to give theoretical explanations or evaluations to the actual practices related to valuations.

A valuation is principally an evaluation of future cash flows, and it is a direct application of cash flow projection techniques. So, this chapter starts with a discussion on cash flow projections in section 5.2. Cash flow projections are widely used in various areas, e.g. investment strategy and matching, and this section also provides a foundation for the discussion in the following sections. A general discussion on projections is followed by a discussion of two distinctive approaches; a deterministic approach and a stochastic approach.

Valuations deal with assets as well as liabilities. For the valuation of assets, it is necessary to consider investment strategies because they influence the prospects of future cash flows from the assets held. In particular, a matched position is often mentioned in practice as a benchmark. Section 5.3, hence, discusses investment strategies and the matching of assets and liabilities as a background to valuation techniques. There has been considerable work carried out and published by actuaries, economists and other reseachers in this area. Some of the most important among these works will be discussed and given interpretations using a theoretical framework developed by the author.

Following the basic consideration of cash flow projections and investment strategies, the purposes of valuations are discussed in section 5.4. Then, valuation methods which are in practical use are discussed in detail in section 5.5. In section 5.6, the use of the concept of matching in the valuation process is discussed. In section 5.5 and 5.6, discussions are limited to deterministic valuations.

In the last section of this chapter, possibilities for the use of a stochastic approach to valuations are explored.

5.2. Projection

5.2.1 Procedure for projections

To begin with, let us explore the basic techniques used for matching and valuations. To think about matching or valuation, it is necessary to project future cash flows in some way. The reasons why projections are necessary for matching or valuation will be discussed later. Here, the discussion is focused on how the cash flows are projected. There are two principal methods: deterministic and stochastic approaches.

Before looking into each approach in sections 5.2.2 and 5.2.3, the meaning of a cash flow projection is clarified, and each step of a projection is discussed in general terms. The steps are summarised in the following diagram.

```
 Objectives of entity
        |
        |
        v
 Purpose of projection
        |            |
        |            |
        v            |
      Model          |
        |            |
        |            |
        v            |
   Assumption        |
        |            |
        |            |
        v            v
 Criteria for evaluation
```

5.2.1.1 Cash flows

If someone possesses an asset, the asset is expected to produce a series of incomes or a lump sum income in the future. The incomes may be obtained by just holding the asset for a certain period, and/or by selling it. The pattern of future incomes depends on the amount of the assets held, the nature of the asset, the market at the time and in the future and the holder's intention to exercise options (this includes when to sell the asset). On the other hand, if someone has a liability, a series of payments or a lump sum payment must be made in the future to discharge the liability. The pattern of the payments also depends on the size and nature of the liability, the economic environment at the time and in the future and the possibility of exercising options by the creditors. The patterns of both the income from the assets and the outgo in respect of the liabilities may be known or may not. This depends on the factors mentioned above.

Also, new capital or liabilities may be added in the future. They will produce additional cash flows. When new capital is introduced, it is recognised as a cash in-flow. Alternatively, if it is invested in some assets, it can be recognised as future income. When a new liability is added, it will create future outgo.

5.2.1.2 Cash flow projection

When we deal with problems of a financial entity, it is essential to make estimate of the possible future cash flows in terms of the timing and the amount. This process of estimate is called a 'cash flow projection'.

5.2.1.3 Purpose of the projection

In order to carry out the projection, it is necessary to clarify the questions to be answered. In other words, the purposes of the projection must be identified. The whole process of projection should be consistent and in line with the purposes. Because projections are merely tools to solve the problem, the projections should be designed and performed appropriately according to the purpose. (For examples, see sections 5.2.1.4 and 5.2.1.5.)

It is not unusual that the financial entity does not have clear objectives, or has mutually conflicting objectives. In such cases, it may be difficult to identify clearly the problem, or what answers are required. However, this does not mean that the financial entity does not have any problems to be solved. It is more likely that the entity still requires to make various decisions on its financial management. Then, projections may be made assuming particular purposes. Alternatively, several projections may be made under different purposes. The financial entity still has to identify the problems or the objectives of the entity, but the projection results may assist the discussion.

In an extreme case, a projection might be carried out to show the most likely cash flows in the future without any particular problems to be solved. In this case, the purpose of projection may be merely to find possible future problems. To solve the problem found, further projections may required.

5.2.1.4 Modelling

Making a model is the next step in the projection. The model can be regarded as a framework in which the projections are to be performed. Because the real situation is usually very complex, it is necessary to abstract the essential features related to the future cash flows. Here, two sub-steps are involved. The first is to choose the items to be looked at; the second is to define any relationships among the items chosen. For example, item A may be defined to keep a relationship with item B, which could be $A + B = \text{constant}$. Alternatively, A may be assumed independent from B.

The first obvious point to note is that the model must be reasonable. The working of the model should be justifiable. Also, the results compared with the assumptions should look reasonable. For example, if realistic assumptions are chosen, the results should lie in a realistic range.

The model must be constructed in line with the purposes of the projection. Selection of essential features can be different for different purposes; an essential feature in answering one question may not be important for another question. The degree of simplicity in the model construction and operation also depends on the purposes.

For example, to forecast the short term profits of a company, it may be necessary to include details of the company's activities and its environment (economic, social, geographic, and so on) in the model. On the other hand, a simple and less detailed model may be appropriate for a long term projection.

Also, it is important that the model is sufficiently practicable to be run efficiently. This aspect depends on the means available for projections, particularly, on the capacity and speed of the computer systems allocated to this task.

5.2.1.5 Assumptions

The parameters, or assumptions, for the projection need to be set before the actual calculations can be performed. The word 'assumption' can be taken to include both the design of the model and the parameters; however, here, I use 'assumption' only to represent parameters.

The assumptions should be made according to the purposes of the projections.

For example, if the purpose of the projection is to show the workings of a funding method, it would be appropriate to use the assumptions which are exactly the same as the assumptions for funding.

On the other hand, if the purpose is to assess the degree of matching of assets and liabilities, a series of projections (possibly stochastic projections) under various assumptions. In such a case, it would be appropriate to choose each assumption from a realistic range, and the assumptions for the projections may not be the same as the funding assumptions (or, they could be significantly different from the funding assumptions if large margins are included in the funding assumptions).

5.2.1.6 Criteria for evaluation

After running a projection of future cash flows, some evaluation process must be followed to draw a conclusion on the matter. This is necessary even when the cash flows are completely known without making any assumptions. To evaluate a particular pattern of cash flows, the criteria for judgement must be identified. The criteria should be expressed in mathematical formulae. The evaluation process then assesses how closely a particular cash flow pattern satisfies the criteria.

The criteria must be chosen to reflect the purposes closely. Where the purposes of the projection are clearly defined, the criteria may be the same or a direct consequence of these purposes. If the purposes are not definite enough, selecting appropriate criteria can be a difficult job. For example, if the purpose is to maximise the value of assets in 10 years, the criterion to be used may be the same, and the projections can be evaluated by the market value in 10 years time. However, if the purpose is just to maximise the value of assets, a criterion specifying the timing, e.g. 10 years' time, may need to be set. In this case, the appropriateness of this particular choice of the timing, 10 years, may become contentious.

Another important point is that the evaluation method, or the application of criteria to the projected cash flows, should be practicable. If the method is too complicated, it may be impossible to derive any conclusions owing to, for example, insoluble equations. Or, the process may take too much time and resources. These could be overcome, in due course, by the progress of theory and techniques. However, if a conclusion is required now, practical alternatives need to be sought.

Furthermore, simplicity of the method helps understanding of the evaluation process. This can be very important in a practical context to persuade the various parties involved and to obtain support from them on the conclusion drawn from the investigation.

For example, let us consider the situation where someone needs to buy one of two assets, A and B, which have the same current market price. The first step to take may be to estimate the cash flows for A and B. Then, the cash flows must be compared and the asset which produces more suitable cash flow should be chosen. For this assessment, the internal rate of return of the cash flow may be used, and the asset with a larger internal rate of return may be preferred if the objective of the selection of the asset is to maximise the value of the assets held in the future. However, if the entity's objective is to avoid the risk of default of the assets, the method of evaluation must be different, and the conclusion regarding which asset to choose may be different. It may be wrong to use the internal rate of return to assess the risk of default.

5.2.1.7 Summary

In summary, the projection process can be subdivided into the following:

-Identify the objectives of the financial entity
-Identify the purposes of the projection
-Modelling
-Set assumptions
-Create cash flows
-Set criteria of evaluation
-Evaluate the result

The effectiveness of the projection for answering a particular question a financial entity has may be judged from the answers to the following questions:

-Regarding the purpose

Are the purposes well defined?

-Regarding the model

Is the model reasonable?

Is the model appropriate for the purposes?

Is the model practicable to run?

-Regarding the assumptions

Are the assumptions appropriate for the purposes?

-Regarding the evaluation

Are the criteria for evaluation fit with the purposes?

Is the method of evaluation practicable?

5.2.2 Deterministic approach

We shall now discuss two approaches for projections. In this section, the advantages and disadvantages of the deterministic approach is considered; in section 5.2.3, the stochastic approach is discussed in comparison with the deterministic approach.

The deterministic approach takes only one set of assumptions for the cash flow projection. The assumptions can be set in various ways according to the purposes of the projection. They may be taken as best estimates, to obtain the most probable cash flow pattern. Or, they may be taken as the best possible or worst possible cases to see the extent of the range of the results.

Under a deterministic approach, a relatively limited volume of calculations are required because only a single projection is performed. Hence, the model can be relatively complex without increasing the burden of calculation too much.

Because the result is a single pattern of cash flows, it is possible to analyse the full details of the results. If the model is close enough to the real situation, it may be possible to analyse the result in a similar way to the analysis of the past financial experience. This means that a variety of evaluation methods are practically available for evaluating the results.

Furthermore, because the volume of the output from the projection is limited, the results can be presented in various ways. In other words, the style of presentation may not be restricted by the volume of the output.

Another advantage of the deterministic approach is that it allows projections under any assumptions, which can be just notional and might not be realistic. This feature is very useful in practice. It may be possible to create some simple numerical example by setting simplified assumptions. Furthermore, regulations for the financial affairs of the entity could be set in reference to the deterministic projections under a fixed set of assumptions. An example is the definition of the upper limit of tax exempt funding for the UK approved funded scheme.

However, the deterministic approach does have shortfalls. Firstly, the result obtained is derived from only one set of assumptions, and the information the projection can give is limited. Also, it is not possible to indicate the probability that the projection result would be realised. On the other hand, because only one projection is given as the result, there is a danger that the result might be taken as a fact, or a prediction. Furthermore, it may be difficult to choose one particular set of assumptions with enough justification, and ensure that these are self-consistent.

To reduce the impact of these shortfalls, the discussion on the setting of the assumptions and model may be attached when the projection results are presented. Furthermore, as supplementary to the main projection, several projections under slightly modified assumptions may be made to assess the sensitivity of the results with respect to each item of the assumptions. This will increase the amount of information, and reduce the risk of the results being taken as a precise prediction.

To avoid mutually conflicting assumptions, the assumptions may be set within the constraints of a model view of world. For example, the salary inflation rate may be set assuming a certain correlation with the price inflation rate.

5.2.3 *Stochastic approach*

The stochastic approach as opposed to the deterministic approach allows the variations of the parameter values in the cash flow projection. Different values are chosen according to a probabilistic mechanism, e.g. investment returns are assumed to follow a lognormal distribution.

It might be possible to calculate required figures, e.g. the variance of the rate of return, directly using mathematical formulae without carrying out cash flow projections if the model is simple enough. However, the model may not be always simple enough, and cash flow projections may be required numerically for every possible set of figures of the variables used in the model. In practice, the number of possibilities for a particular variable is often very large or infinite, and sampling may be necessary. In any case, a large number of parallel projections are created at one time.

This feature of parallel projections may be said to be shared with the deterministic approach supplemented by the sensitivity analysis, although the number of the projections needs to be much larger under the stochastic approach in order that meaningful statistical indices or probability estimates can be calculated or to cover the range of possible values fully. Also, a deterministic projection with sensitivity checks cannot be produced in accordance with a probabilistic mechanism as is seen in stochastic projections.

Because of the large volume of the output, the methods of presentation and evaluation of the results are very important for the stochastic approach. In some cases, it may be appropriate to choose some particular item of the output or a ratio of two items, and its mean, variance, the distribution, or any other statistical values associated with the item chosen can be shown as the result. Alternatively, a probability of a particular event to happen can be used as the index for the result. e.g. probability of ruin. There can be many different ways for expressing the results. The choice depends on the situation. While the index must appropriately express the essential feature of the problem, a simple index is preferred to complex ones because of the ease of calculation and explanation. In the case of the stochastic approach, the volume of information obtained from a series of projections is very large, and it may be difficult to utilise the information fully and effectively.

What really distinguishes the stochastic approach from the deterministic approach is the feature that the approach needs/allows a model to be constructed for the assumptions themselves. In the case of the stochastic approach, a large number of sets of parameters have to be generated according to certain rules. Firstly, the movement of the variables is described as a statistical distribution or process. Secondly, items of the assumption can be set to hold some particular relationship concurrently. Finally, the assumption values at time t may be made dependent on the values at previous times or on previous states. Such rules constitute a model separate from the main model regarding the mechanism of the process under investigation.

On the other hand, in the case of the deterministic approach, all the parameters are given as a single set of parameters. Although the determination of each parameter may be based on a particular structure or model, such structures or models are not an integral part of the deterministic projections.

Because of these features, it may be more appropriate to set the projection on realistic assumptions rather than using notional assumptions isolated from reality. It is natural to fix realistically the distribution of an item from the set of assumptions. However, this does not preclude the possibility of setting the assumptions pessimistically or optimistically by adjusting the probability distribution which the experience suggests.

However, it would be pointless to use unrealistic probability distributions or mechanisms because they may not improve the accuracy of the projections while they increase the volume of calculation required. If a distribution or a mechanism is not known, a simple assumption with supporting discussion or evidence from experience and experiments might be more appropriate. On the other hand, a set of assumptions made up of notional figures may be acceptable for a deterministic projection.

This argument on realistic assumptions also leads us to the choice between explicit and implicit assumptions. Some items of the assumption set which can affect the projections may be omitted or modified to an unrealistic extent under the implicit assumption approach. The projection as a whole can be, however, acceptable by claiming that the omission or modification of some assumptions are implicitly compensated for by other items of the set of assumptions. Such an approach may be possible under a deterministic approach, but not under a stochastic approach. Stochastic projections need explicit assumptions in that implicit assumptions cannot be realistic.

Benjamin(1976) mentioned that the use of realistic assumptions may be required for a stochastic projection, but pointed out practical difficulties in setting such explicit margins. He argued that, although explicit margins seemed natural and right, people may not necessarily know what figures made the assumptions realistic. So, he argued that a crude approach using implicit margins may be more practical, and the explicit approach may be impossible or would give a spurious impression of accuracy.

Therefore, a danger regarding the stochastic approach may be noted here. The results are totally dependent on the model assumed and assumptions made. However, the results could be perceived as a product of actual experiments because of closeness to the reality. So, it should not be forgotten to note that the results obtained from a stochastic model are the consequence of the model and assumptions.

Such a possible misinterpretation of the results in a stochastic approach may be shared with the deterministic approach. However, the causes of the misinterpretation may be slightly different for the two approaches. In the case of the deterministic approach, this misinterpretation could originate from a solid-looking result. Presenting only one future might give a false impression of confidence in the projection result. In the case of the stochastic approach, however, the misinterpretation may be the consequence of the complexity of the model and its closeness to reality. Furthermore, the recipient of the results might not be accustomed to the stochastic approach, and could be just overwhelmed by the volume of output produced.

Another danger is the choice of the index for the presentation of the results. Once an index is chosen and accepted widely, people may start thinking that the ultimate goal is to achieve a certain level of the index. This point can be also made in respect of the deterministic approach, but to a lesser extent. This is because a stochastic model produces a large volume of results and the users of the results tend to rely more on a set of specific indices for summary and comparison purposes.

As an addition to the main features, the stochastic approach is usually performed forward only. The projection starts from a fixed situation, such as "now", and is projected toward the future. Projecting backward from a particular situation at a particular time in the future involves difficulties in respect of the underlying probability theory. Further, the results projected back to the present may not coincide with the real situation now, and such a retrospective approach to projection may not give useful or meaningful information.

Here, it should be noted that projecting backward does not include stochastic discounting because stochastic discounting is applied to the projection results carried out in a forwards direction. Stochastic discounting is discussed further in section 5.7.1.

5.3. Investment strategy

5.3.1 Investment strategy for pension fund valuation

Investment strategies taken by the pension fund now and in the future affect future cash flows and may change the value placed on the assets. So, investment strategies need to be discussed when we deal with the valuation of pension funds. Particularly, a matching of assets and liabilities is an important investment strategy. In fact, it is not unusual in the UK to use a so-called 'notionally matched portfolio' for the valuation of the assets.

In the past, various works have been published in this area. Haynes & Kirton(1952) introduced the idea of 'absolute matching' for actuarial applications.

As a variant to matching, Redington(1952) introduced the concept of 'immunization' for fixed liabilities. (The concept of duration which is essential to immunization theory was known to Macaulay in 1930s. (see Shiu(1988))) Immunization is an arrangement of the distribution of the term of the assets which reduces the possibility of loss arising from a change in interest rates, under certain assumptions. Redington(1952) noted that immunisation is important for a financial entity to remain solvent. Redington(1952) also showed mathematically that,

assuming a uniform interest rate for all fixed interest securities regardless of the term to redemption, no transaction costs and taxes, availability of immediate trading without restrictions in the volume, and fixed liabilities,

-immunization is achieved by making the mean term of the value of the asset-proceeds equal to the mean term of the value of the liability-outgoes, and

-if the spread of the value of the asset-proceeds about the mean term is greater than the spread of the value of the liability-outgoes, after a small change in the interest rate, a profit can be made by switching to a new immunized position under the new interest rate.

However, it must be remembered that immunisation only deals with fixed liabilities and fixed interest assets, while the majority of assets and liabilities of pension funds (particularly in the U.K.) tend to produce cashflows which are more or less linked to inflation.

Further works (eg. Boyle(1978) and Milgrom(1986)) have been carried out to generalise the idea of immunization under various yield curve models.

Wise(1984a,b) constructed an asset liability model which does not allow future switchings of the assets, and gave an interpretation of the meaning of 'matching' by looking at the mean of the residual assets after meeting all the liability-outgoes and the mean of the square of the residual assets. The model developed is stochastic.

Wilkie(1985b) generalised this, and has shown how Wise's model can be interpreted in the framework of

Modern Portfolio Theory, which uses mean and standard deviation as the measure. He also introduced the initial amount of assets as an independent factor for the portfolio selection problem against a specific set of liabilities, and created the P-E-V space model, which deals with the portfolio selection problem by looking at the initial amount of the assets, P, the expected value of the residual assets, E, and their standard deviations, V.

Wise(1987a,b) developed the P-E-V space model further. In particular, Wise(1987a) identified the valuation of given liabilities by two steps in the P-E-V theory:

-the actuary chooses the level of expected residual assets, E, after meeting all the expected liability outgoes and the level of its standard deviation, V,

-then, obtains the amount of the initial asset, P, required to achieve the specified E and V through an efficient portfolio.

Sherris(1992) developed a framework based on the idea of maximising the utility of residual assets, and analysed the choice of portfolio. In the paper, the P-E-V model above has been explained as a special case of his framework with the utility function $U(S) = -\exp\{-S/r\}$, with S: the residual assets and r:the risk tolerance of the fund. A further extention of the framework to allow rebalancing of the portfolio has been briefly discussed. However, he argued that the amount of the initial asset is in practice more likely to be determined so as to limit the risk of insolvency within a specified level, and expressed some reservation on the P-E-V model.

Booth et al(1993) applied the utility function introduced by Sherris(1992) to further investigations on the asset allocation problem for investors with and without specific liabilities, and the results were extended to the case of a with-profit life assurance company.

The idea of using a stochastic model and the mean and the standard deviation of the residual assets as the indicators was introduced by Markowitz(1952), which started the Modern Portfolio Theory, although no liabilities were taken into account. Markowitz(1959 and 1987) and many others have further developed mean-variance analysis applied to portfolio selection (for a summary of the development of the Modern Portfolio Theory, see Moore(1972 and 1983) and Cummins(1990)).

Arthur & Randall(1990) discussed the valuation of assets. If the assets are valued by a discounted cash flow technique, the value of assets depends on the projected future incomes from the assets. So, the value of the assets is affected by the assumptions in the investment policies taken. They argue that the valuation should be independent from the actual investment decisions, and the use of the notional matched position is recommended, which would avoid the valuation results being affected by the investment policies actually taken.

The phrase 'matching of assets and liabilities' has been widely used in academic and practical contexts with various meanings, and the precise meaning may not be clear. So, the main aim of this section is to clarify the meaning of 'matching'. To achieve this aim, it is felt necessary to construct a more general framework for investments and place 'matching' and other investment strategies in this framework.

To deal with a wide range of arguments in a single framework, the definitions given may be found non-specific. However, this, I believe, gives generality to the discussion, and more specific arguments are used as examples.

The discussion starts in section 5.3.2 with an exploration of the meanings of 'matching' as used in various contexts. This section also contains the arguments on the appropriateness of a deterministic or stochastic approach for the discussion of matching. Sections 5.3.3 and 5.3.4 give a theoretical framework of investments in general. Then, this is applied in section 5.3.5 and 5.3.6 to identify a type of matching, 'V-matching'. Another type of matching, 'S-matching', is described in section 5.3.7. Then, possible meanings for 'matching' to a pension fund are considered in section 5.3.8, and the 'mismatching reserve' will be discussed briefly in the last section.

5.3.2 Matching

The phrase 'matching of assets and liabilities' can have at least two distinctive meanings for a financial institution which accepts liabilities and holds assets to pay these liabilities.

Firstly, it can mean equalising the future incomes from the existing assets and outgoes from the existing liabilities at any time in the future. This guarantees the payment of liabilities as they fall due without the need for extra resources or any future reinvestments. In other words, the liabilities cancel out the assets, and the financial institution only has to hold the assets and apply the investment incomes to the payments of the liabilities. Such matching is sometimes referred as 'absolute matching', the concept of which comes from Haynes & Kirton(1952).

The phrase 'matching by duration' is also often used. This matching emphasizes the matching of the timings of the cash flows, although the amounts of outgoes may not be the same as those of incomes. In this sense, the 'matching by duration' may be regarded as a weakened form of 'absolute matching'.

Secondly, it can mean equalising the degree of increases/decreases of the future income from the assets and the future outgo from the liabilities. This is sometimes referred as 'matching by type'. This does not guarantee full repayment of liabilities. Rather, the phrase can be used even if there is an apparent shortage in assets compared with the liabilities. Also, the liabilities in consideration may not be limited to the existing ones. Future cash injections, e.g. contributions, could also be anticipated. The value of liabilities held by a financial institution may increase or decrease because of the changes in economic environment. If the matching in this meaning is achieved, the value of assets will increase or decrease according to the changes in the value of liabilities to the same extent. Then, the ratio of the value of assets to the value of liabilities will be unchanged by such changes in the economy. Thus, a financial institution can reduce the volatility in the difference between liabilities and assets, and stabilise its finance.

The phrase 'matching of assets and liabilities' can have much wider applications in practice although this chapter only deals with the above two.

Now, we shall discuss the suitability of the deterministic and stochastic approaches for the general discussion of investment strategies and matching.

When we consider the concept of matching, the liabilities under consideration must be specified. It is usual that the liabilities are given, although the amounts and the timings of the cash flows of these items will vary under different possible future course of events and will need to be projected. Usually, any new liabilities accruing in the future are not taken into account. However, it may be possible to extend the calculation to allow the future accruals of liabilities and to project the cash flows caused by them.

The current market value of the assets held is usually known. Future new monies may be either given or set to be 0. Where the future new monies are given, the amounts and the timings may be fixed or subject to stochastic variation.

The task is to find an asset portfolio which matches the given liabilities. However, in some circumstances, there may not be an answer owing to too many constraints. For example, if there are not enough assets and no new monies are assumed, the matching of future cash flows in the first sense may be impossible.

Let us consider the steps to be taken to obtain a matched portfolio in the first sense, i.e. to find a portfolio of assets which makes the future incomes from them plus the new monies assumed equal to the outgoes from the liabilities at any time in the future. Here, the amount of the assets cannot be specified as discussed in the previous paragraph. The first step is to project the cash flows from the liabilities less the new monies. Then, projections of cash in-flows from various asset portfolios under the same assumptions as the liabilities may be tested until the same cash flow pattern is obtained. Alternatively, cash flows from a typical asset in each type of assets may be projected, and a combination of these types may be sought to match the liabilities' cash flows.

This can be done by a deterministic approach. Under a set of deterministic assumptions, the cash flows from the assets are made equal to those from the liabilities. However, matching is only certain if the outgoes and incomes are fixed in terms of the amounts and timings. If the liabilities or incomes from the assets are variable, the matched portfolio obtained by the deterministic approach may not give matching in practice. Also, once re-investments and dis-investments are required, the projection results will contain uncertain factors because of changeable re-investment and dis-investment conditions. Sensitivity checks may provide further information but, essentially, a stochastic approach is required to cope with such uncertainty.

Under a stochastic approach, stochastic projections of the outgoes from the liabilities and the incomes from a typical asset in each asset type may be made. Then, the combination of the types of assets which gives the closest match of cash in-flows to the liability outgoes throughout the range of the projections may be chosen as the matched portfolio. Here, it needs to be defined how the closeness between particular cash in-flows and the corresponding cash out-flows can be measured. Possible ways to measure this will be discussed later. In a stochastic model, it is unlikely that the outgoes and incomes can be always kept equal unless the cash flows are fixed amounts or matching is theoretically guaranteed. Also, the result depends on the model and assumptions.

When the phrase 'matching by duration' is used, the timings of the cash flows from the assets and the liabilities need to be matched, but the amounts need not be matched as long as the ratios of the liability outgoes and the investment proceeds are kept the same. Hence, for the case of matching by duration, the amount of the initial assets does not need to be a certain amount. The only concern here is the composition of the initial assets. The composition must have the same proportion of each asset as that of the absolutely matched portfolio. Similarly to absolute matching, matching by duration is only possible when the liabilities are fixed. If the liabilities are subject to a stochastic variability, matching by duration can only be discussed in relative terms.

Now, the stochastic approach can also give a matched portfolio in the second sense, i.e. equalising the degree of increases/decreases of the future incomes from the assets and the outgoes from the liabilities. To judge if particular liabilities and assets are matched in this sense or how close they are to the matched position, it is necessary to examine how the cash flows can vary as the economic environment and demographic factors change. Under a stochastic approach, cash flow projections are carried out on a range of economic environments and demographic factors according to the statistical distributions assumed, e.g. market interest rate, price inflation, salary increases, mortality, and so on. Hence, a stochastic approach should be able to give information on the degree of changes in liabilities and assets related to the changes in the environment. All we need to do is to choose sensible indicators. The standard deviation of the residual amount of assets after paying all the liabilities may be a possible candidate.

Matching in the second sense cannot be examined by a deterministic approach because this approach allows only one particular economic and demographic scenario. Although sensitivity checks may work as a rough guide, these checks do not take into account the probability of each economic and demographic incidence, and we may not be able to measure the closeness to the matched position.

As a conclusion, for a theoretical discussion of matching, a stochastic approach should be used. A deterministic approach supplemented by sensitivity checks should be regarded as only giving an approximation to this more theoretically sound approach.

5.3.3 Stochastic approach to asset allocation

Now, a general discussion of the asset allocation problem using the stochastic approach is made to construct a general framework for the discussion of investments in this and the next section. Matching will be dealt with later in sections 5.3.5, 5.3.6 and 5.3.7 as a special case. As we dicussed in the previous section, the framework needs to be built using a stochastic approach, not a deterministic approach. In this section, the details of the stochastic projections, on which the framework is to be constructed, are discussed. The framework itself is described in the next section.

Let us consider a process to decide asset allocation using stochastic approach. Here, we assume that the market value of the assets held is given. Also, the future cash flows caused by the liabilities and the new monies are estimated by making assumptions if necessary.

Firstly, assets need to be categorised into a manageable number of groups. The assets within each group are regarded as identical. This is important because, in the real world, there are so many different assets that projecting cash flows from all the different assets may be impossible.

The second step is to model the performances of assets by setting a series of formulae. For example, the market value of gilts may be set as the discounted value of future coupons and redemption payments at the 'market gilts interest rate' at the time.

This step also includes the modelling of relationships between various economic and other factors and the assumptions regarding the probability distribution of these factors. For example, the relationship between the average dividend yield on all equities in the market and the real yield on index-linked gilts. It is possible to set some relationships between the values of the factors before a particular time and the probability distributions of some factors at that particular time, for example, the relationship between the probability distribution of the market gilts yields and the yields in recent past.

The two steps described above define the investment environment model. Then, the probability distributions of the independent variables in the model are set. The model formulated by Wilkie(1986) is a well-known example of such a model.

Then, a model of the liabilities is created. The amounts of the future liability outgoes are expressed in formulae, which may simplify the situations. For example, all the members at the same age may be treated identically, i.e. the same salary scale and the same decrement rates may be applied. The members might be grouped in 5 year age bands for the projection. Some modes of decrements may be ignored or combined with other decrements. e.g. ill-health retirements might be ignored.

If the liabilities and new monies are also subject to the stochastic approach to projection, the logic for these calculations is also included in the model.

For the projection of liability outgoes, the assumptions required for the projection have to be set. They can be set to maintain certain relationship with the investment variables. The assumptions including decrements can be allowed to vary stochastically, or for simplicity, the items subject to stochastic variations may be limited only to economic factors, e.g. the salary inflation rate, but not the number of future new entrants.

The next step is to set an investment strategy for the stochastic projections. The investment strategy consists of the initial asset allocation to each category of assets, the allocations of future net cash in-flows, and rules for switching the assets. An example of such a rule may be that a half of the equities are switched to conventional gilts if the total assets becomes less than a certain amount.

Cash flow projections can now be carried out on a set of specified liabilities, a specific amount of current assets, any future additions by way of contributions and so on. The future contributions could be given, or calculated according to a formula as a part of the projection.

Then, a set of projections under varying assumptions according to the specified asset distribution will be made. Here, there are a number of combinations of assumption parameters; they may be infinite. So, projections may be carried out only for a selection of combinations.

To decide what investment strategy should be taken, such projections need to be repeated for several different investment strategies, and the results must be compared. Theoretically, the number of strategies to be tried can be infinite, but this is impossible to implement practically. So, only a few likely candidates for the investment strategy may be tested.

After running a projection, an indicator, or indicators, needs to be chosen for the evaluation of the results. There can be various choices, but for the following discussion, the indicator chosen is the difference between the market value and the liabilities, i.e. surplus, at some specific time T, say.

T may be chosen as the time by which all the liabilities have been paid out for all parallel projections. In this case, the surplus is the same as the residual amount. However, T could be very large or infinite. Alternatively, T could be set as a fixed value, and at time T, there still could exist some liabilities to be paid. This approach has the advantage of limiting the length of the projection, and consequently reduces the volume of calculations. However, in this case, the surplus needs to be defined by valuing the liabilities outstanding at time T deterministically; if this is done stochastically, the advantage of reducing the volume of calculation is lost. So, if T were chosen in this way, the method of determining the value of liabilities will become important. For the following discussion, T is assumed to be the former unless stated otherwise. However, it may be worth mentioning that much of the following discussion is general enough to allow both choices for T.

Wise & Annable(1990) have summarised the practical points regarding asset-liability modelling, such as the objectives, modelling techniques, models for assets and liabilities, definition of risk, and so on. This provides us with a review of asset-liability modelling techniques practically in use in the U.K. when the paper was written.

An example of a calculation using a stochastic asset-liability model can be seen in Daykin et al(1993). In the paper, a stochastic asset-liability model was developed for a closed pension fund, and the means and variances of various portfolios were calculated to examine which portfolio can provide the least variance.

5.3.4 Theoretical framework

In this section, a system of evaluating investment strategies is developed using a stochastic approach. This will give us the framework for discussions of investment strategies. The framework is, in a sense, a generalisation of Wise-Wilkie's P-E-V model (Wise(1984a,b and 1987a,b) and Wilkie(1985b)). The main differences are
-the criteria for the risk is left open, and
-switching of the assets can be allowed.

Sherris(1992) developed a generalised framework on investments based on the idea of maximising utility, and the P-E-V model has been identified as a special case within his framework. (see section 5.3.1) In his paper, the risk of insolvency was treated as an additional constraint, while the risk of volatile residual assets can be dealt with as a part of maximising utility. The framework developed in this section is different from this by treating these risks equally and separately.

From the discussion on projections in section 5.2, we need to specify the objectives of the finantial entity, purposes of the projection, model, assumptions, and criteria for evaluation.

The financial entities which invest their monies are varied, and the objectives can be very different for each entity. A financial entity might have several objectives which are mutually conflicting. The priority, or the balance, among them must be clearly sorted out. Otherwise, the finantial entity cannot choose one particular investment strategy out of possible candidates. However, as investors, they may share the desire to maximise the return and to minimise the risks taken. So, here, these two are considered as objectives.

Now, the precise meaning of the risk needs to be identified. The risk that people talk about may involve various factors, some of which can be based on human feelings rather than measurable facts. Also, risk depends on the situations where each financial entity is placed. Hence, there are wide possibilities for the meaning of the term "risk", which is discussed later in this section.

The purpose of the projection is to evaluate each investment strategy in the light of the objectives. The model and assumptions have already been discussed in the previous section.

Now, the criteria for evaluation are discussed.

From the discussion above, we have chosen two objectives. Also, in the last part of the previous section, we decided to use as an indicator the amount of residual assets after paying all the liability outgoes. That is, this residual amount is identified as the financial result referred to earlier.

Hence, the criterion corresponding to the first objective may be expressed as to maximise the expected market value of assets at time T, where T is defined as as the minimum value by which time all the liability outgoes have already paid out in all branches of the projections. The mathematical expression of this may be to maximise the mean of the market value at time T.

The second objective is more difficult to be interpreted mathematically. If the risk is identified as the volatility of the financial outcomes, possible mathematical expressions may be to minimise the standard deviation or mean deviation of the residual assets (positive or negative). If the risk is identified as the possibility of insolvency, minimising the probability of ruin may be the mathematical criterion. In either case, some form of utility function could be included in the criterion of risk to reflect each financial entity's special concerns or circumstances.

Now, let us review the risks identified by previous authors.

Markowitz(1952) identified risk as the variability of the financial outcome, and adopted the standard deviation of the residual assets as the measurement of the risk.

Markowitz(1959) gave a more generalised discussion on the risk, and listed alternative measurements; semi-variance, expected value of loss, expected absolute deviation, probability of loss and the maximum loss. Also, he further developed the discussion using utility functions. However, Modern Portfolio Theory has been developed using the standard deviation of the residual assets (Moore(1972 and 1983) and Cummins(1990)), and other measurements of risk have not been discussed so fully. Markowitz(1987) argued that the use of mean-variance is justifiable

-as a quadratic approximation to utility functions,

-because of the ease of performing the numerical and theoretical calculations, and

-because of its objectivity (There is no need to set the utility functions individually).

Wise(1987a,b) and Wilkie(1985b) also used the standard deviation of the residue as the measurement of the risk. Another criterion for the risk, the probability of insolvency, was included in their model as a boundary condition in the selection of a portfolio.

However, the risk of insolvency is also regarded as important. Daykin et al(1987) took the probability of ruin as a measurement of risk for a general insurance company. Clarkson(1989 and 1990) identified the risk as the expected value of W(x-L), where

W:	severity of loss
L:	minimum achievement required to make W = 0
x:	the amount of the residual asset.

Smink(1991) used two measurements of risk; the standard deviation and the below target standard deviation.

Sherris(1992 and 1993) interpreted the problem of the trade-off between the volatility and the expectation of the returns as the maximisation of the utility in a generalised framework. Constraints on the solvency level have been added as boundary conditions.

It would be possible to develop general discussions on investments using a generalised utility function without specifying the exact mathematical expression of the utility function. This approach may be able to cover a wide range of discussions, and may have some theoretical advantages. However, the discussion might become too generalised for the purpose of the identification of the 'matching'. An example of such an approach is seen in section 2 of Sherris(1992).

On the other hand, the objectives and the risks of investments may be expressed in exact formulae including the use of particular utility functions. Then, mathematical analyses might be applied, and the discussion may become clearer. However, the results of the discussion might be specific to the particular utility, risk or other functions. Also, it would not be easy to justify the use of particular functions to express the investors' objectives and risks. Examples of such an approach are section 3 of Sherris(1992), Booth et al(1993), Wise(1984a,b and 1987a,b), Wilkie(1985b), Clarkson(1989 and 1990) and Smink(1991), which have already been mentioned above.

Here, an approach between the two is adopted. When 'matching' is considered, there are at least two types of risks; the volatility of the residual assets and the risk of insolvency. The two risks are different in character, but both are important for institutional funds including pension funds. So, separate discussions are made for the two risks.

At the same time, the exact expression of the risks may be different from one fund to another. So, if particular mathematical expressions are chosen for the investors' risks, the discussion might become too specific. Also, this chapter only aims to develop a broad framework of investment, and detailed mathematical arguments are not required. Thus, while considering two different types of risks, it is decided that each risk is discussed in general terms as far as possible. This approach allows the discussion in this chapter to be applied to any definitions of the risks as long as they are measurable and they satisfy certain specified conditions.

The use of utility functions may allow the objectives of maximising the return and minimising the risk of variable results to be achieved simultaneously. However, if risk is identified as the risk of insolvency, it would be difficult to deal with two objectives altogether. Although Sherris(1992) states that the risk measure given in Clarkson(1989 and 1990) can be re-expressed using a utility function, such a utility function only measures the degree of damage caused by the loss, but would not give any positive value for good investment returns. So, another measure for investment returns is required, and investment strategies need to be judged by two different measures; one is a utility function as a measure of risk, another a measure of investment returns. Alternatively, as in Sherris(1992), a utility function can be used to measure the achievement towards the objective of maximising investment returns with an additional condition for solvency requirement. This requirement of two different measures is an inherent difficulty in utility theory when risk is identified as the risk of insolvency. To recognise that higher returns are better, it is necessary to use a utility function which is monotonously increasing as the value of assets increases. However, this means that a possibility of loss can be compensated by possibilities of high returns, and the risk of insolvency cannot be expressed within such a framework. So, the risk of insolvency needs to be added as an extra constraint. Should the risk of insolvency be expressed using a utility function, to avoid such a trade-off between the possibilities of gains and losses, the utility for solvent results needs to be uniformly 0. Then, the maximisation of returns cannot be expressed by this utility function.

Hence, in this framework, the use of utility functions is avoided, although some parts of the discussion might be re-expressed using certain utility functions.

We shall now develop a framework in order to analyse the meaning of matching.

First of all, it should be made clear that the framework assumes a particular stochastic investment and demographic model. (see section 5.3.3.) Therefore, the analysis will depend on the investment and demographic model chosen. In the following discussion, the setting of an investment and gemographic model is not discussed further, and the discussion is applicable to any investment and demographic model. However, it is also very important to consider the appropriateness of the investment and demographic model chosen.

Before starting the discussions on each type of risk, first, a general framework is developed as follows. The evaluation process for investment strategies applying the criteria above is described.

Let us think about the comparison between two investment strategies A and B under a particular investment and demographic model. (Regarding the meaning of an "investment strategy", refer to section 5.3.3.)

Let

m(x,t): mean surplus of investment strategy x at time t

s(x,t): measurement of risk (e.g. standard deviation) of investment strategy x at time t,

where x can be either A or B. 5.3.4.1

The definition of s(x,t) above has various possibilities, but here, the following are imposed as the conditions to be satisfied when setting the definition:

-the minimum possible value of s(x,t) is 0, and

-the risk increases as the value of s(x,t) increases. 5.3.4.2

Now, let t = T, where T is as defined in section 5.3.3. If

$$m(A,T) > m(B,T)$$

and

$$s(A,T) < s(B,T),$$

Strategy A may be judged to be unambiguously superior to B.

However, if

$$m(A,T) > m(B,T)$$

but

$$s(A,T) > s(B,T),$$

neither of them can be said to be superior.

By trying every possible investment strategy and eliminating strategies which are inferior to other strategies, an efficient range of strategies can be identified. Here, the term 'efficient strategy' means that there are no superior strategies to this.

Let us summarise the above in the following definitions.

Definition 5.3.4.3...

Assume Investment strategy A and B have the same market value at time 0. Investment strategy A is <u>superior</u> in terms of time T to Investment strategy B if

$$m(A,T) > m(B,T)$$

and

$s(A,T) < s(B,T)$.

At the same time, Investment strategy B is said to be <u>inferior</u> to Investment strategy A.

....

Definition 5.3.4.4...

An investment strategy is said to be <u>efficient</u> in terms of time t if there are no investment strategies superior to it.

....

The task for the investor is to clarify the priority and balance between the two criteria; maximising the expected return and minimising the risk, and to choose an appropriate investment strategy out of the efficient strategies.

5.3.5 V-Matching

As an application of the theoretical framework described in the previous section, we shall now discuss matching. In this section, the definition of 'V-matching', a type of matching, is given within this framework. This terminology is used to avoid possible confusions caused by various types of matching in discussion. The relationship between 'V-matching' in this section and the 'matching' in section 5.3.2 will be discussed in section 5.3.6.

In the previous section, the definition of risk was left open for generality. As is discussed in section 5.3.4, the risk to be considered can vary significantly. Here, two typical risks, the volatility of the residual amount (sections 5.3.5 and 5.3.6) and the risk of insolvency (section 5.3.7), will be considered.

Thus, in this section, risk is identified as the volatility of the residual amount. For the measurement of volatility, various mathematical values, such as standard deviation, mean deviation, and so on., may be used. In this discussion, we do not specify the measurement of the volatility. The only condition that needs to be imposed on the measurement, $s(x,T)$, is that

$s(x,T) = 0$ means that the residual amount will be equal to $m(x,T)$ with probability 1.

.....5.3.5.1

This condition is imposed on top of the general conditions 5.3.4.3. Standard deviation and mean deviation obviously satisfy these conditions.

The interpretation of 'matching' in the framework is given by the following definitions.

Definition 5.3.5.2...

An <u>investment position</u> is a combination of a particular amount of initial assets and an investment strategy.

....

Definition 5.3.5.3 ....

The V-matched position is a investment position with the expected surplus of 0 and 0 volatility in terms of time T.

....

The V-matched position consists of two elements: a V-matched amount and a V-matched strategy.

Definition 5.3.5.4 ....

A V-matched amount is the initial amount of assets which can create a V-matched position together with some investment strategy.

....

Definition 5.3.5.5 ....

A V-matched strategy is the investment strategy which can create a V-matched position together with some initial amount of assets.

....

There can be several V-matched positions for a particular liability. Taking a V-matched amount as the initial amount of assets, the V-matched strategies may not necessarily belong to the efficient range. As an extreme case, if there is more than enough money, V-matching can be achieved by investing only a part of the fund, and the rest can be thrown away. Instead of throwing away the surplus, it could be invested in a riskless asset should this exist, i.e. investing in the asset with known return and zero standard deviation if the risk is measured by the standard deviation. This is more efficient than the V-matched strategy. Thus, a V-matched strategy is not necessarily an efficient strategy. However, there can exist a V-matched and efficient strategy.

Theorem 5.3.5.6..

Take a particular liability.

If there is a V-matched strategy regarding this liability, and
if a fixed amount of sum at time T can be exchanged for a fixed sum at time 0,

there exists a V-matched and efficient strategy.

....

Proof

From the assumption, there is at least one V-matched strategy. So, the set of the V-matched strategies is not an empty set. Also, from the definition of the V-matched strategy, there exists a V-matched amount for each V-matched strategy. Let us take a V-matched strategy, A, which has the minimum V-matched amount. This A is a V-matched and efficient strategy.

Now, suppose A is not efficient, and a contradiction will be shown to prove the theorem.

If A is not efficient, there exists an investment strategy, B, with the same amount of initial assets as the V-matched amount of A and a higher mean, $m(B,T) > 0$. Then, $s(B,T)$ must be greater than 0. Otherwise, $s(B,T) = 0$. This means that at time T, the amount of surplus is $m(B,T)$ with probability 1.

From the second assumption, it is possible to make $m(B,T) = 0$ by reducing the initial amount of assets. This will create a V-matched strategy with a lower market value. However, this contradicts against the assumption that A is the V-matched strategy with minimum market value.

QED

V-matched positions with higher V-matched amounts than A are not efficient. The investor can take the V-matched position A, and the excess can be used to buy a secure zero volatility sum at time T. The existence of such an investment is guaranteed by the second assumption. This will create a superior strategy.

The second assumption (in the statement of the theorem) may not hold in practice. This depends on the investment model assumed. When a fixed sum at T is needed to be bought at time 0, there may not be a zero coupon bond maturing at time T. Thus, the V-matched positions with higher V-matched amounts could be efficient. On the other hand, when a fixed sum at time T needs to be removed, it may not be allowed in the model to give up a fixed sum at time T by selling such zero coupon bonds and throw the proceed away. If this is not allowed, the reduction needs to be done by reducing certain types of assets currently held, but this would make the investment position no longer V-matched. Hence, it is possible that the investment position A is not efficient.

Further, in practice, there may not exist any V-matched position for particular liabilities and new monies because of the unavailability of particular types of assets required for V-matching. So, positions close to the V-matched position may be chosen instead of a V-matched position which is unattainable. Such investment position close to a matched position may be called as 'quasi-V-matched position'.. Such a position can be defined by weakening the conditions for a V-matched position. There can be various ways of weakening, and the choice of definition may depend on the situation. An example of the definition of quasi-V-matched position may be that

a quasi-V-matched position is an investment position which satisfies the following conditions:
(a) the expected surplus is 0, and
(b) the volatility is minimum among the investment strategies which satisfy (a) 5.3.5.7

Wise's 'unbiased match' (see Wise(1984a)) is, as interpreted by Wilkie(1985b), equivalent to this quasi-V-matched position without any switching of assets. Wilkie(1985a) also pointed out that an 'unbiased match' is not in general efficient. (Refer to Theorem 5.3.5.6.)

5.3.6 Interpretation of V-matched position

In this section, the relationship between the V-matched position defined in section 5.3.5 and the 'matching' discussed in section 5.3.2 is examined.

Firstly, we will examine the relationship with the matching in the second sense in section 5.3.2., i.e. the matching by type.

A V-matched position is required to have zero mean and zero volatility. From section 5.3.5, this means that the residual is 0 with probability 1.

Now, let us take an investment position with a V-matched strategy but with the initial amount of x times the V-matched amount. Here, x is a positive value, and can be less than, equal to or more than 1..I assume, here, that the investment returns are not affected by the size of the fund. In such a case, the investment position can provide returns just enough but not in excess to pay x times the liability outgoes in any possible circumstances. The value of the assets and liabilities may be measured by the future cash flows. (Detailed discussions on the valuation will be made later in this chapter.) Thus, the value of the assets can be said to be 100x percent of the value of liabilities at time 0.

This situation is the same as the description of the matching in the second sense of section 5.3.2. That is, the ratio of the value of assets to the value of liabilities will be unchanged by the changes of the environment which affect the liability outgoes. Hence, matching in the second sense ('matching by type') is identified in the framework as a V-matched strategy.

Then, what is the relationship between the matching in the first sense, i.e. the absolute matching or the matching by duration, and the V-matched position?

Absolute matching may be re-expressed as follows:

Definition 5.3.6.1..

An investment position is in an absolutely matched position with liabilities if,

(a) The asset income is always equal to the liability outgo, and

(b) After all the liabilities are paid out, there are not any residual assets or any shortfalls.

....

From (a), there is no need for re-investment in the future. Also, if switching between different assets is allowed, the terms of switching depend on the market conditions, which may change stochastically. Hence, if switching of assets is allowed, the condition (b) in Definition 5.3.6.1 may not hold. Therefore, the following holds;

Proposition 5.3.6.2...

If an investment position is in an absolutely matched position with liabilities,

and unless the investment and demographic model assumed allows fixed price ratios between different assets,

no asset switching is allowed except for switching between identical assets.

....

From 5.3.6.1(a), there is no need for re-investment in the future. Also from Proposition 5.3.6.2, the investment position is defined only by the initial amount and initial asset allocation except for special cases.

From 5.3.6.1(b), it is clear that the following proposition holds;

Proposition 5.3.6.3..

If an investment position is an absolutely matched position, the investment position is a V-matched position.

....

Here, the absolutely matched position may not necessarily be an efficient position. Throwing away the monies at some point in the future is excluded by condition (a), and throwing away monies at inception may not be a practical idea. However, the following example shows a non-trivial existence of an absolutely matched, but not efficient position if the investment environment assumptions allow this.

Example 5.3.6.4..

The only liability is a payment of £1,000 after 10 years.

Assume there are two zero-coupon bonds with no risk of default. Both have 10 years to redemption, and the redemption amount is £1000. One is Bond A currently priced at £400. Another is Bond B priced at £300 owing to its limited marketability.

Buying either A or B will secure absolute matching, but buying Bond A is not efficient.

....

We shall now consider if a V-matched position is always absolutely matched.

The answer is "no". We consider the immunisation concept in Redington(1952) under an investment model which allows the guaranteed existence of appropriate gilts, no transaction charges, uniform market yield, and the ability to transact quickly enough to make 'surpluses' at all transactions. If the 'surpluses' arising are thrown away, this will create a V-matched position. However, this is not an absolutely matched position because the cash inflows from this investment position are not equal to the liability outgoes, and this is against the condition 5.3.6.1(a).

Now, we shall consider the conditions to be added to make a V-matched position absolutely matched.

From the definition of a V-matched position, it is obvious that the condition 5.3.6.1(b) is satisfied.

If throwing away the monies is allowed, a V-matched position may not be absolutely matched as shown in the example above. Hence, throwing away the monies needs to be excluded from consideration.

Now, the conditions required to make a V-matched position an absolutely matched would depend on the model which specifies the investment environment and the demographic movements. For a trivial example, a stochastic model with probability 1 on one particular outcome may be considered. Because this model does not allow varying investment returns and other effects, this is also a deterministic model. Then, it is obvious that a V-matched position will provide an absolute matching under this model, and if the V-matched position assumes switching of the assets in the future, this can be an example of the case of absolute matching allowing future switchings.

Let us consider a more conventional stochastic model. i.e. the investment environment and the factors affecting liabilities are subject to stochastic variation and the elements which affect the cash flows in the projection are not totally dependent on each other. Such a conventional model satisfies the second condition of Proposition 5.3.6.2, and for absolute matching, any switchings or any future new investments are not allowed.

Under such a conventional model, it is not possible to achieve a V-matched position if the assets and liabilities are independently affected by such stochastic movements. This is because the varying liability outgoes may not be cancelled by the changes in investment returns which are not totally dependent on the liabilities.

However, if both the liabilities and assets are fixed, or the two are totally dependent (e.g. fully index linked annuities and the assets fully linked to the same index) by nature rather than by the restrictions imposed by the projection model, it might be possible to achieve V-matching under such a conventional stochastic asset-liability model.

In such cases, if future investments are required, V-matching will not be achieved because the future investments needs to be done in uncertain conditions. For the same reason, switching of the assets is excluded. Hence, the cash inflows from the initial assets must be exactly equal to the liability outgoes. That is, a V-matched position will always satisfies the condition 5.3.6.1(a).

In summary, a V-matched position in a conventional stochastic asset-liability model is also an absolute matched position if throwing away the monies is not allowed.

5.3.7 S-Matching

In the previous two sections 5.3.5 and 5.3.6, we have developed an interpretation of matching by confining the meaning of risk as the volatility of the residual amount. Now, we shall discuss another type of matching by redefining the meaning of risk as the insolvency of the fund. Here, this type of matching is called the 'S-matching' to distinguish it from the V-matching discussed in the previous sections.

Again, the measurements of risk will not be specified as long as they represent the risk of insolvency, i.e. a shortage in the fund to pay off all the liabilities. The only condition needs to be imposed on the measurement of risk, $s(x,T)$, is that

Definition 5.3.7.1 ---

$s(x,T) = 0$ means the fund remains solvent until time T with probability 1, where T is the time when the last liability outgo is due.

This measurement s(x,T) counts all the shortages in the fund at any time until time T. The shortage here means a negative cash balance, but not a state of the existence of a positive assessed value of liabilities in excess of the value of assets. An example of such measurements is the probability of ruin.

It should be noted that this definition of the risk is different from the following alternative definition 5.3.7.2.

Definition 5.3.7.2 --

s(x,T) = 0 means the fund is not negative at time T with probability 1, where T is the time when the last liability outgo is due.

Under Definition 5.3.7.2, any shortfalls in cash balance during the period between time 0 and T are not counted as risk. This s(x,T) only depends on the cash balance at time T. Definition 5.3.7.2 includes the risk represented by the loss function defined in Clarkson(1989 and 1990) and the use of the semi-variance of the residual amount by which only negative residual amounts will be taken into account.

Both types of measurements have been in use, and they are mathematically different. If s(x,T)=0 in the sense of 5.3.7.1, s(x,T) is also 0 in the sense of 5.3.7.2. So, definition 5.3.7.1 is stricter than definition 5.3.7.2.

However,

-once the fund becomes negative before time T, it would be unusual to come back to positive unless new monies are injected,

-where the probability of ruin is high, s(x,T) in the sense of 5.3.7.2 tends to be high because the probability distribution of the residual assets tend to be continuous, and the expected severity of the losses tend to increase as the probability of ruin increases.

So, the difference between the two types of definitions only matters materially regarding the exact timings of cash flows, which may be regarded as subsidiary in this discussion. Hence, the following discussions are based on definition 5.3.7.1, but the similar discussion can be developed under definition 5.3.7.2, as well.

'S-matching' is defined as follows.

Definition 5.3.7.3 ....

The S-matched position is an investment position with the risk of insolvency 0 in terms of time T.

....

From the definition, the mean of the expected residual amount must be at least 0.

The S-matched position consists of two elements: an S-matched amount and an S-matched strategy.

Definition 5.3.7.4 ....

A S-matched amount is the initial amount of assets which can create an S-matched position together with some investment strategy.

....

Definition 5.3.7.5 ....

A S-matched strategy is the investment strategy which can create an S-matched position together with some initial amount of assets.

....

Similar to V-matching, quasi-S-matching is defined by relaxing the condition for S-matching in Definition 5.3.7.3.

Definition 5.3.7.6 ....

A quasi-S-matched position is an investment position with the risk of insolvency equal to or less than z in terms of time T. Here, z is a given positive value.

....

Similar to above, quasi-S-matched strategies and quasi-S-matched-amounts can be defined.

The value "z" in Definition 5.3.7.6 represents the acceptable level of the risk of insolvency to a particular financial entity. Any S-matched position is obviously a quasi-S-matched position in terms of any level of risk, z. So, for the sake of generality, the following discussion is made in respect of quasi-S-matching. If $z = 0$, all the arguments apply to the case of S-matching.

There can be several quasi-S-matched positions for a particular liability. Here, a quasi-S-matched position may not necessarily be efficient. An inefficient quasi-S-matched position can be easily created from a quasi-S-matched position by adding an initial asset which is expected to perform badly, or to become value 0. However, there can exist a quasi-S-matched and efficient strategy.

Proposition 5.3.7.7 ....

Take a particular set of liabilities.

Let

Q1: the set of all the quasi-S-matched positions with the level of risk z for the given liabilities.

Q2: the set of all the investment positions which belong to Q1 and have the minimum quasi-S-matched amount. Q2 is a subset of Q1.

Q3: the set of all the investment positions which belong to Q2 and have the minimum $s(x,T)$. Q3 is a subset of Q2.

Assume

(a) there is at least one quasi-S-matched position in terms of the level of risk, z,

(b) there exists a minimum quasi-S-matched amount, A, within Q1,

(c) there exists a minimum s(x,T) = z', within Q2, and

(d) there exists a maximum m(x,T) within Q3,

where z' is a certain real number.

Then, there exists a quasi-S-matched and efficient position with the minimum amount of assets.

....

Proof

From (a), Q1 is not an empty set.

Also, from (b), there exists the minimum quasi-S-matched amount, A, within Q1. Thus, Q2 is not an empty set. The quasi-S-matched positions in Q2 can have s(x,T) which is between 0 and z inclusive.

From (c), there exists the minimum s(x,T), z', within Q2, and Q3 is not an empty set.

From (d), there exists the maximum m(x,T) within Q3. So, take the position, M, with the highest m(x,T) within Q3. This position M is efficient.

The efficiency of M is shown as follows.
With the initial amount A, M has risk z', which is the lowest possible value. If M is not efficient, there must be a position with

a higher or equal m(x,T), and

a lower or equal s(x,T).

However, there cannot be such positions from the definition of M.

QED

The conditions (a) - (d) in Proposition 5.3.7.7 may look very restrictive, but they are necessary to exclude extreme counter-examples.

Condition (a) may not hold for a particular z. For example, if z is very low (e.g. 0) and the investment model assumes very high volatilities in the investment return, it is possible that no investment positions are S-matched. However, if the initial amount is sufficiently large, s(x,T) can be below z. Also, if z is chosen at a high level, the chance of the existence of quasi-S-matched positions will be larger. Hence, Condition (a) is likely to be satisfied in practice.

Condition (b) is required to exclude the case where there are infinite elements in Q1, and there is not a minimum quasi-S-matched amount. For example, there can be an amount A such that all the quasi-S-matched amount exceed A, but for any a > 0, A+a can be a quasi-S-matched amount. In this case, A is the lower boundary, but not the minimum.

Conditions (c) and (d) are required for a similar reason. Theoretically, such possibilities are important. However, if we consider a practical situation where small differences in the initial amount, s(x,T) or m(x,T) can be negligible, Conditions (b)-(d) may be regarded to be satisfied all the time. For example, the initial amount and m(x,T) may be rounded to the nearest £1000, and s(x,T) may be rounded to the nearest .01.

An efficent quasi-S-matched position may have a higher volatility than inefficient positions.

Example 5.3.7.8 ....

Take a liability of £100 due in 1 years time. Suppose the initial asset is a cash of £100.

There are two investment strategies as follows:

Strategy A is to keep the asset in a safe place for a year.

Strategy B is to deposit the money in a variable rate bank account which offers capital guarantee but the interest rate can vary daily.

Then, both A and B are S-matched because the payment of the liability is certainly secured. However, B is superior to A because the expected return is higher. If there are no superior strategies to B with risk equal to 0, then B is efficient. However, the volatility of B is higher than A.

....

Proposition 5.3.7.9 ....

Take a particular set of liabilities. Also, take an investment position, P, which is not quasi-S-matched at the level of risk z in terms of the given liabilities.

Now, assume

(a) it is possible to create a quasi-S-matched position at the level of risk z, R, by adding an investment position, Q, to P, and

(b) there exists a quasi-S-matched and efficient position with the minimum amount of assets.

Then, the initial amount of R, B, cannot be less than the minimum initial amount of efficient quasi-S-matched positions.

....

Proof

From the assumptions (a) and (b), this is obvious.

QED

The assumption (b) can be substituted by the conditions (a)-(d) in Proposition 5.3.7.7 because Proposition 5.3.7.7 guarantees the existence of a quasi-S-matched and efficient position with the minimum amount of assets.

From the discussion on Conditions (a)-(d) in Proposition 5.3.7.7, these conditions can be supposed to be fulfilled all the time in practice. Hence, Proposition 5.3.7.9 means that, to make the risk of insolvency equal to or less than a certain level, the minimum initial amount of assets is required when some quasi-S-matched position is taken. Q in Proposition 5.3.7.9 can be taken as an investment reserve, or a solvency margin, and a non-quasi-S-matched position plus an extra fund needs more initial assets.

We shall now give an interpretation of Redington's immunisation theory using the concept of S-matching. (For the brief description of Redington's immunisation, see section 5.3.1.) Here, we shall take the risk as the probability of ruin. The immunised position in the sense of Redington achieves the condition that the probability of ruin equals 0. So, *immunisation in the sense of* Redington is an S-matched position from Definition 5.3.7.3.

In Redington's immunisation, the initial amount of assets is equal to the value of liabilities, which are fixed in terms of amounts and timings of payments. It would be possible to match absolutely, i.e. the mean and the volatility of the residue are equal to 0. However, by taking an immunised position, profits are expected from the switching of assets when the market interest rate changes, while there are no risks of insolvency.

This can be explained using the concept of S-matching.
Let us take the fixed amount of initial assets which is equal to the initial amount of an S-matched (immunised) position.

V-matched positions with the same amount of initial assets may exist. If so, the V-matched positions have zero volatility, and they satisfy the condition of 5.3.7.3 as well. Hence, they are also S-matched.

However, the S-matched (immunised) position allows the mean to be more than 0. It may be possible to choose the S-matched position with the highest mean, i.e. to choose an efficient position in terms of risk measured by the probability of ruin. However, such an efficient S-matched position has a higher volatility. This is another example of 5.3.7.8.

5.3.8 Matching for a defined benefit pension scheme

A difficulty in a pension fund is that there can be several different meanings of the word 'matching', and often various 'matchings' are required at one time.

Matching can be defined in various ways in a pension fund because there are many ways to recognise the liabilities. The choices of the liabilities may be:

(1) leaving service liabilities for the existing members
(2) liabilities on winding up for the existing members (This can be different from (1).)
(3) accrued liabilities with allowance for future salary increases for the existing members
(4) total service liabilities less future contributions for the existing members

(5) (4) plus future new entrants

(4) and (5) can have variations according to the definition of the contributions. They can be defined as a fixed rate, the Standard Contribution rate under some funding method, recommended contribution rate, and so on. (see Chapter 2 for the descriptions of typical funding methods.)

Also, matching can be defined in various ways according to the risk considered.

When a fund is concerned with the solvency of liabilities (1) - (3), the risk is the possibility of insolvency. So, the investment strategy needs to be examined in the light of the risk of insolvency, and the type of matching to be considered will be the S-matching in section 5.3.7.

It would also be possible to extend the arguments to the solvency for an on-going scheme. Assuming the number, age and salary of future new entrants for a certain period, and assuming a particular system of setting the contribution rates, a set of stochastic projections may be carried out. The assumptions on the new entrants may reflect each scheme's realistic prospect, e.g. if a company is expanding rapidly, a high level of new entrants may be assumed for the first, say, 5 years or so. Then, the probability of ruin may be measured as the risk. If the period when new entrants are allowed is reasonably long, this may be used as a proxy for the risk of insolvency for an on-going scheme.

The probability of ruin will be affected by not only the investment policy, but also the method of contribution calculations, which includes the choice of the funding method, the choice of the amortisation method and the choice of the valuation basis.

On the other hand, when a fund is concerned with the volatility of the future contributions to be paid in, the liabilities in consideration are likely to be (4), (5), or the liabilities accruing in the future. The risk may be taken as the volatility of future contribution rates which may be determined by a specified formula. Alternatively, the risk may be expressed indirectly by taking the volatility of the amount of the residual assets assuming a particular level of future contribution rates. In this case, the matching to be considered will be the V-matching in sections 5.3.5 and 5.3.6.

However, when we discuss the stability of the contribution rate, it is usual that the scheme is assumed to continue indefinitely, and it is not possible to apply the discussion in sections 5.3.5 and 5.3.6 directly, where 'matching' was discussed for a limited time span.

One approach to assess the stability of contribution rates might be to treat the fund separately for the existing members and future new entrants. For each sub-fund, the concept of V-matching may be applied. Regarding the existing members, all of the assets currently held and future contributions in respect of the existing members are taken into account. For the future new entrants, only future contributions are relevant. Hence, only the calculations for the existing members are required to identify the V-matched position as at the valuation date. However, this approach cannot deal with, for example, a case where a scheme is expanding rapidly and all the payouts are expected to be met out of each year's contributions for a while. In such a case, taking into account the future developments, an extremely long investment position may be close to 'matching'.

Hence, for the purpose of stability, the use of the residual assets may need to be abandoned, and similarly to the above, new entrants may be added to the stochastic structure. The risk in this case may be the volatility in the contribution rate. Again, the result may be heavily affected by the choice of the method and the bases for future valuations.

5.3.9 Mismatching reserve

Now, briefly, the idea of a 'mismatching reserve' is discussed in connection with the discussion of 'matching'.

Mismatching reserves and solvency margins, which are often discussed in life assurance and general insurance, are a set of extra assets to reduce the risk created by an investment strategy which is not 'matched' to the liabilities. The idea is that these extra funds can reduce the 'risk'. If the risk is the risk of insolvency, as we have discussed earlier, such extra funds can reduce the risk. From Proposition 5.3.7.9, it is shown that such a fund with mismatching reserve cannot be smaller than an efficient S-matched amount.

If the risk under consideration is the volatility in contribution rates or the volatility in the residual assets, such extra funds themselves cannot reduce the risk unless the extra funds are invested in such a way that the total fund is matched to the liabilities.

The extra funds might be used to reduce the volatility in the contribution rate without threatening the solvency of the fund. However, this approach may not reduce the volatility in the level of funding because the extra funds themselves are part of the fund. Volatilities in the experience of the cash flows cannot be absorbed without causing some volatility in the finances of the fund.

A reduction in volatility of the funding level without changing the contribution rates may be achieved if a mismatching reserve, which is held separately from the main fund, is allowed to change its size. The volatility in the funding level would be reduced if the mismatching reserve is excluded from the calculation of the funding level unless the mismatching reserve exhausts. However, this approach does not make the total funding level (the main fund plus the mismatching reserve) remain stable.

Such an approach based on explicit extra reserves is not adopted in the U.K. pension funds. Possibilities for achieving similar effects to this may be

-to amortise the surpluses over a long period, which would increase the fund temporarily, and would reduce the volatility in the contribution rate without causing a solvency problem, or

-to modify the valuation basis from time to time to keep the contribution rate stable unless there are solvency problems or the possibility of over funding. However, this may result in hiding surpluses/deficits temporarily, and the delay in remedial actions may worsen the financial situation of the fund.

5.4. Purposes of the valuation

Valuations are essential for the management of defined benefit pension schemes. This is because the ultimate costs of the defined benefit schemes are not known in advance, but some estimates for the costs are required for the financial planning. So, one can say that the purpose of valuations is to estimate the costs of providing the defined benefits. However, this description does not cover all the aspects of the valuation. In this section, further consideration is given to the purposes of the valuation.

The discussion in this section is general, and not dependent on the valuation methods used.

5.4.1 Funding

The first question to be solved by the valuation is the size of the contributions required. A wide variety of contribution rates may be justified if there are no other constraints. As long as all the benefits promised are paid in full, this year's contributions can be any amount. However, to make sure that the future contributions cover the liabilities in full, theoretical and systematic ways of calculating the contribution rates, that is, the funding methods, have been developed. There are various funding methods, each of which has been developed on the basis of a different logical framework. (For the details of each funding method, see Chapter 2.)

Usually, a funding method has a standard contribution rate and a standard fund. The standard contribution rate will be quoted as a recommended contribution rate if the value of assets held is equal to the standard fund. If there are differences, the elimination of the difference is planned in a specified way by adjusting the contribution rate. The method of elimination may be considered as a part of funding method. Hence, setting the contribution rate can be subdivided into the following steps;

-calculate the standard contribution rate

-identify the difference between the standard fund and the value of assets

-adjust the contribution rate to remove this difference.

The second step may be considered as a separate purpose of valuation; that is to check the progress of funding in the light of the funding method.

The method of valuation then depends on the funding method chosen.

5.4.2 Security

There is another purpose of the valuation; to check the security of the future benefit payments the rights of which have already accrued.

This can be done under various contexts. Firstly, the accrued benefits may be taken as the benefits in reference to the past service and projected salary in the case of final salary schemes. Secondly, the accrued benefits can be taken as the benefits payable if all the members leave the scheme voluntarily at the valuation date. Finally, the accrued benefits may be taken as the benefits guaranteed by the rules if the scheme discontinues at the valuation date. In the last case, the benefits could be enhanced to more than the leaving service benefits.

The second purpose in section 5.4.1 (i.e. to check the progress of funding in the light of the funding method) may be the same as the first one listed above if the funding method is the Projected Unit method, or as the second one listed above if the Current Unit method is used. However, the perspective is slightly different, and it is possible that different bases can be used for identifying the surplus/deficit and checking the security at the same valuation.

Different types of benefit security can be measured under different bases reflecting the precise purpose. For example, if security of the discontinuance benefits is to be checked, the basis should be chosen to reflect the realistic situation on winding up at the date of the valuation. So, one approach to such a valuation may be to assume the disposal of all the assets and the purchase of deferred annuity policies from life assurance offices. Then, the value of the assets may be taken as their market value, and the liabilities may be taken as single premium rates for these policies.

5.5. Deterministic valuation

Before discussing matching and valuation further, this section summarizes the deterministic valuation procedure in practice.

Valuations are carried out deterministically in current practice, possibly with some supplementary stochastic calculations. Under a deterministic approach, single figures for the values of the assets, liabilities and future salaries are given under a particular set of valuation assumptions. Then, they are input into a set of formulae to obtain the valuation results. The formulae and precise definition of the figures required vary according to the purpose and funding method, but the consistent treatment of all the items is needed to justify a particular valuation.

The details of the valuation methods are not discussed here. (See Chapter 2) Only a broad outline of the deterministic approach is described.

5.5.1 Liabilities

The extent of the liabilities included in the calculation depends on the purpose of the valuation and the funding method. For example, if the purpose of the valuation is to check the solvency of past service benefits, the liabilities to be valued are the past service liabilities with fully projected salaries.

If the purpose is to set the contribution rate and the Entry Age funding method is chosen, the liabilities to be valued are

-the future service liability for a new entrant at the assumed single entry age, and

-the past and future service liabilities for existing members.

The normal approach is to project the liability outgoes and to discount back these cash flows to the valuation date.

Sometimes, the value of the liabilities might be measured by using the single premiums of life policies to cover these liabilities. This method values the liabilities using other actuaries' measurement methods and assumptions. So, the results would depend on the prevailing environment in the life assurance market. Also, there may not be any policies to cover the liabilities exactly. So, the use of this method is limited to particular circumstances, e.g. the liability on winding up.

5.5.2 Assets

There are two major methods to value the assets; market value method and discounted cash flow method.

The market value method measures the value of a particular asset in accordance with the market prices of the asset. This method does not involve projecting future cash flows. This method can give the value of the assets objectively, and the values are realistic. So, this method may be suitable for the checking of the solvency position assuming an immediate winding up.

However, the objectivity is subject to qualifications because there are various ways to define market values. Firstly, the market might have more than one price and some adjustments to the market price may be introduced. For example, quoted equities are priced differently by different parties and have bid and offer prices. Secondly, the prices fluctuate continuously. An example of the definition might be the past one month average of the middle of the highest bid price and the lowest offer price of each day. Finally, for those assets which do not have active markets, some methods of estimation are required.

The discounted cash flow approach involves a cash flow projection in respect of the assets. For the projection, some assumptions need to be made, e.g. future dividend growth rate. Under this method, consistency between the values of the assets and the liabilities can be more easily maintained than the market value method. Shortfalls may be a possible presentational difficulty because of the difference between the value of assets and the market value. Also, to obtain the value of assets, more calculations may be required than the use of market values.

There are some other methods, e.g. book value method. However, values placed by such other methods may be less realistic than market values, and less consistent with the value of liabilities than discounted cash flow values.

5.5.3 Future contributions/future salaries

Usually, projections of salaries are required. The technique for these projections is the same as those of cash flow projections.

In the first instance, the value of future contributions might be taken into account for the valuation under some funding methods at a known rate, and a projection of future salaries is required if the contribution rate is expressed as a percentage of salary. For example, the standard fund under the Entry Age method is defined as the present value of total benefits less the present value of future contributions at the standard contribution rate which needs to be set before this calculation.

Also, to spread the cost over a certain period, the value of future salaries are required. To obtain a contribution rate as a percentage of salaries which matches the cost, the value of the cost will be divided by the present value of the salaries during that period. For example, if a deficit is spread over the working life time of the membership, the adjustment to the contribution rate is calculated as the amount of deficit divided by the present value of all the future salaries of existing members. Also, the standard contribution rate of the Projected Unit method is calculated as the present value of the benefits accruing in the next year divided by the present value of the next year's salaries. Thus, the cost of the next year's accrued benefits is spread over the year.

5.6. Valuation and matching

In the U.K., as a technique to value the assets, a notional switching to a matched position is often used. Hence, it is important to discuss the use of the concept of matching for the valuation.

In this section, firstly, we will consider the meaning of matching in the context of valuations, and then try to interpret practical valuation methods in the light of these considerations.

5.6.1 General consideration

In section 5.3, we discussed various types of matchings and their possible applications to pension funds. Also, in section 5.4, we discussed the purposes of the valuation. Now, based on these, we shall consider matching in the context of the valuation.

In this section 5.6.1, the discussion is rather theoretical, and uses the stochastic framework discussed in section 5.3. So, the discussion here may not be directly applicable to the practical situation. Possibilities of the use of a stochastic approach to the valuations will be discussed in section 5.7.

In section 5.3.8, we discussed the meaning of matching for a pension fund. Now, the discussion on matching is applied to the valuation.

5.6.1.1 Solvency

One of the purposes of the valuation is to identify the minimum level of funding required to secure particular liabilities, and to identify the current funding level. In such a case, the extent of the liabilities under consideration is usually known, e.g. leaving service liabilities, accrued liabilities taking into account future salary increases, or the total service liabilities for the existing members. Then, the cash flows will cease at some time in the future, and the discussion on matching in section 5.3 can be applied.

When we consider the solvency of the fund, the risk we are concerned about is the risk of insolvency. Hence, the discussion on S-matching in section 5.3.7 is most relevant. As discussed in section 5.3.8, solvency in an on-going scheme might be checked by using a (quasi-)S-matching which explicitly recognises the probability of ruin.

It may also be possible to discuss the solvency using V-matching because when the expected residue is large enough and its standard deviation is low enough, the probability of having a negative residue can be made lower than a certain specified level. However, the approach using V-matching deals with solvency only indirectly, and V-matching approach is not discussed further for the checking of solvency.

There can be several possibilities for carrying out the valuation for this purpose;

(a) The actual investment position is taken, i.e. the actual current asset allocation and future likely investment strategies are assumed. Calculations are then carried out so as to identify an extra amount of initial assets to bring the risk s to 0, or a certain value which represents the acceptable level of risk. In the U.K., in practice, such extra funds may not be identified explicitly. Instead, a cautious basis, i.e. implicit assumptions, might be used to allow such extra funds implicitly.

(b) A (quasi-)S-matched position is sought by 'trial and error' or some other method. There may be a range of (quasi-)S-matched amounts. The minimum, i.e. an efficient-(quasi-)S-matched position, may be taken as the amount required for 100 percent funding. Then, the percentage covered by the current assets is identified.

From Proposition 5.3.7.9, the result from (a) requires the fund which has at least the equal market value to the fund derived from (b). In other words, the required amount of assets for particular liabilities is always the minimum for a specified level of risk of insolvency when the assets are assumed to be invested in a (quasi-)S-matched position.

5.6.1.2 Stability

Another purpose of the valuation is to set the contribution rate. Here, the rates may be set to achieve stability in the future contribution rates, stability in the future funding level, or any other criteria. The second criterion is more relevant to the previous section (5.6.1.1) though the solvency in the future needs to be checked as well as the solvency at the valuation.

When we consider stability in the future contribution rates, the risk may be identified as the volatility in the future contribution rate.

The stability may be affected by the investment returns and changes in the liabilities. The stability in the relationship between the assets and the liabilities would contribute to reduce the volatility in the contribution rate. So, the discussion on V-matching in sections 5.3.5 and 5.3.7 is relevant.

Also, the stability is affected by the process of setting the contribution rate, i.e. the valuation method, i.e. the funding method, the method of amortisation and the mechanism of choosing the basis.

It could be the case that a particular investment strategy may reduce the volatility in the contribution rate. However, in the valuation, the investment strategy assumed may not be relevant to the actual strategy. Only the value placed on the assets can affect the contribution rate through the valuation process, and the choice of portfolio for the valuation purposes cannot directly influence the stability of the contribution rate.

As discussed in section 5.3.9, an additional fund may not reduce the volatility. Hence, a single valuation result itself may not affect the stability. That is, the choice of the portfolio for the valuation does not have a direct consequence on the stability of the contributions, although the actual portfolio selection affects the stability. So, stability may not be a reason for selecting a particular investment strategy for the valuation of the assets.

Stability may be more affected by the entire valuation method, i.e. the funding method, the method of amortisation and the mechanism of choosing the basis. In this sense, the valuation method of the assets indirectly affects the stability through a series of valuations. Stability might be achieved in a more satisfactory way with methods based on the dynamic control of the fund. (see Chapters 6-8)

5.6.2 Interpretation of valuation in practice

Now, we shall apply the discussion in the previous section 5.6.1 to interpret various valuation methods in practice. This section only covers deterministic valuations.

The method of valuation is basically a combination of a cash flow projection and discounting, at least for the valuation of liabilities, which have already been described in section 5.5. In that section, two distinctive methods of valuing the assets, i.e. the market value method and the discounted cash flow method, were mentioned. In order to maintain the consistency between the values of the assets and the liabilities, the choice of the basis for the valuation of the liabilities is affected by the method of the asset valuation. So, the two cases are discussed separately.

Where the assets are valued by a discounted cash flow method, there is a further choice concerning the portfolio to be valued. The notional fund approach and the approach based on the actual portfolio are common in the U.K, and they are discussed separately.

5.6.2.1 Valuation basis

Firstly, we shall consider the discounted cash flow approach for the asset valuation. Here, a common valuation basis is set for the assets and the liabilities. The discount rate may be based on the long term expected rate of return, or could be any value. Other assumptions need to be consistent with the choice of discount rate. This is a widely used valuation method in the UK for the purpose of setting contribution rates.

For the valuation of the assets, cash inflows are projected and discounted. To carry out this projection, the initial portfolio and the investment strategy in the future need to be identified. Although it would be possible to include changes in the investment policy during the projection, such approaches are not used in practice because of the resulting complexity of the calculations. Hence, the main approaches are
-to assume a 'matched' position throughout the projection, and
-to assume that the current investment strategy is continued.

In either case, the calculation may be approximate.

If the projected incomes from the assets exceed the projected liability outgoes at a certain future time, it would be logical to assume that the excess incomes are re-invested, and to project the further incomes from the re-invested assets. On the other hand, if the projected incomes from the assets are less, a part of the assets may be assumed to be sold.

However, the impact of re-investments and dis-investments depends on future market conditions, and it is difficult to make explicit assumptions about future market conditions. Also, such explicit allowance would add further complexity to the calculations.

Hence, in practice, an implicit allowance may be made by adjusting the discount rate to take account of risks from unfavourable changes in the market condition in the future. The discount rate may be lowered if re-investments are projected to be more significant than dis-investments. This is because a lower discount rate means lower returns from future reinvestments, and the valuation result becomes more conservative, i.e. the value of liabilities less the value of assets increases. If future dis-investments are likely to be more significant than re-investments, to make the valuation result more conservative, a higher discount rate may be chosen.

5.6.2.1.1 Notionally matched portfolio

This involves a notional redistribution of the assets at market value. The cash flows on the notionally redistributed portfolio are projected, and then discounted to give the value of the assets.

It would be technically possible to use any portfolio as the notional portfolio. However, the notional portfolio is often taken as a 'matched portfolio', or an approximation to it. Here, the word 'matched' is used ambiguously. The word 'matching' can have various meanings as is discussed in section 5.3. Also, the extent of matching can be varied as shown in section 5.3.8.

However, as discussed in section 5.6.1, only S-matching is considered here. Also, from the discussion in sections 5.3.8 and 5.6.1, it is possible to include new entrants in the future to some extent.

As discussed in section 5.6.1.1, to identify the efficient (quasi-)S-matched position to be used, a stochastic approach may be applied. However, the precise identification of the efficient (quasi-)S-matched position may be difficult in practice, and merely an approximation to it may be used. The approximation may be made without specific calculations by referring to broad cash flow patterns for the liabilities and particular categories of assets and their likely changes under changes in the underlying economic assumptions.

From the discussion in section 5.6.1.1, the use of the efficient (quasi-)S-matched position as a notional investment position for the valuation makes the assets required for a particular set of liabilities minimum among the various investment positions for the valuation which keep the risk of insolvency within a specified level.

Now, let us consider the meaning of the use of an efficient (quasi-)S-matched strategy in a deterministic valuation.

In a deterministic valuation, the stochastic financial projections which assess the risk of insolvency through the probability distribution of the projection results are not carried out. So, the risk of insolvency for a particular investment position may not be evaluated. Consequently, the amount of assets which is needed to keep the risk of insolvency within a particular limit may not be identified explicitly.

In practice, an efficient and S-matched/quasi-S-matched position may be used as a benchmark. By notional switching to this benchmark position, the amount of assets which are required to cover a particular set of liabilities is assessed.

This amount identified is, in theory, the minimum amount for the given level of risk allowable. If a different investment strategy is adopted, a higher amount of assets is required to keep the level of risk within the limit. However, the extra amount required may not be explicitly identified because of the lack of information on the degree of the risk of insolvency.

There are two methods to deal with the difference between the notional position and the actual investment position.

The first is to ignore the difference. The valuation is carried out assuming the future investments are made in line with the minimum risk strategy, which is equal to the notionally (quasi-)S-matched strategy. From the discussion in section 5.6.1.1, this gives a more optimistic view than the valuation using actual assets because of the lower required asset amount for the same liabilities.

If the actual investment strategy is riskier than an (quasi-)S-matched strategy, the higher risk may be realised as a deficit or a surplus in the future valuations. This approach maintains the neutrality of the valuation from the investment strategy, and Arthur & Randall(1990) have recommended this.

The second approach is to increase the value of the liabilities or to reduce the value of the assets by adjusting the valuation basis. Because a deterministic valuation cannot identify the extra amount of assets required for reducing the extra risk, this can be done only approximately, and implicitly. This approach takes into account the effect of the actual investment strategy, and the risk of insolvency can be incorporated into the valuation result. However, this approach has a difficulty in justifying a particular margin included in the valuation basis because the underlying risk of insolvency is not measured explicitly.

Now, there can be different (quasi-)S-matched positions to be used for the purpose of notional matching.

The matched position may correspond only to the liabilities which are covered by the Standard Fund under a particular funding method in use. For example, the past service liabilities are considered when the Projected Unit method is used. Then, the difference between the value of the actual assets and that required for 100% funding is recognised as a surplus/deficit. The amortisation of the surplus/deficit is then considered separately.

On the other hand, if a matched position is determined taking into account future contributions realistically, it may be theoretically more satisfactory. It may be difficult to obtain such a matched position, but crude approximations to this are seen in practice. For example, for an underfunded scheme, expecting high contribution incomes, the matched position for valuation can be set to make the duration of the assets very long. For an overfunded scheme, matching at a shorter duration may be assumed.

There can be various ways of applying notional (quasi-)S-matching in actual valuations. We will now examine each of these.

5.6.2.1.1.1 Absolute matching

This method assumes that there exists a portfolio where the income precisely matches the liability outgoes. In practice, it may be impossible to find such a portfolio. So, this method is mentioned here only for theoretical interest, and practical use of this method may be limited.

The valuation technique is based on the fact that if the cash flows from two portfolios are exactly the same, the value of the cash flows must be the same under any discount rate.

That is,

Value of matched assets = Value of liabilities,

where the discount rate can take any value.

Now, let us go back to a particular valuation. Consider the two situations where the discount rate is the market discount rate and where it is the valuation discount rate. The liabilities are valued at the valuation discount rate. Hence, the assets need to be valued at the same valuation discount rate as well. If the funding level is 100 percent, the market value of assets must be equal to the value of liabilities at the market discount rate, and the value of the assets for the valuation purposes must be equal to the value of liabilities at the valuation discount rate. By notionally switching to the portfolio which has the same proportion of each investments as the perfectly matched portfolio, the level of funding on the market value basis should be equal to that on the valuation basis. Hence, the value of assets can be calculated as

Market value of assets
* Value of liabilities at valuation discount rate
/ Value of liabilities at market discount rate.

So, the assets are valued without the requirement to project investment incomes although liabilities need to be valued twice at the valuation discount rate and the market discount rate.

The difficulties with this method are:
-The market discount rate may not be easily identified.
-It is customary to use economic assumptions consistent with each other. This means that if the discount rate is different, other assumptions may need to be different, and the cash flow patterns may be different for different discount rates. This does not fit well with the discussion above, where unchanged cash flows for all discount rates are assumed.

The second point is important. But, if we only think about temporary fluctuations in interest rate in the market without any fundamental changes in the economic and demographic environment, the unchanged cash flows may be justifiable.

5.6.2.1.1.2 Approximate matching

The portfolio used for the valuation of assets is determined referring to the broad nature of the liabilities, but the portfolio may not be absolutely matched. The portfolio may consist of only a few types of assets; for example, the assets which represent the average of all equities, the average of all fixed interest securities, and the average of other categories of assets. Assumptions on the future investment incomes may be simplified; e.g. uniform dividend growth assumption. Such a simplification would reduce the volume of calculations required.

This approach may be interpreted as an approximation of 5.6.2.1.1.1. Then, the assumptions on future investment incomes need to reflect the shape of the liability outgoes as far as possible. For example, if the pension scheme membership is very young and not much outgo is expected in the near future, a 100% equity portfolio which has a high dividend growth rate and a low current running yield may be assumed to match the liability outgoes. On the other hand, if the scheme is being retracted, and large outgoes are expected in the next 10 years or so, a 100% equity portfolio which has a low dividend growth rate or a portfolio which includes a high proportion of high yielding fixed interest securities might be assumed in order to provide a high level of immediate income for matching.

Similarly, the notionally matched position for the discontinuance solvency may be different from that for the on-going solvency. The discontinuance liabilities are usually subject to lower growth rates in amount than the on-going liabilities and no future entrants are taken into account. Hence, the cash out-flows to pay the discontinuance liabilities have a shorter duration than those of the on-going liabilities. Consequently, for (quasi-)S-matching with the discontinuance liabilities, assets of shorter duration may be required, and the portfolio to be assumed may contain more fixed securities and/or equities with low expected dividend growths than equities with high expected dividend growths.

Where a portfolio which consists entirely of equities with low dividend growths, i.e. the high yielding equities, is assumed as a notional portfolio, the value of the assets may be given by

	Market value of assets
*	Market dividend yield of the high yielding equities
/	Dividend yield assumption

Here, the dividend yield assumption is set at a high level to make the valuation of assets consistent. Hence, this may not necessarily mean a lower value of the assets compared with the average equities because the market dividend yield of the high yielding equities is higher than the dividend yield of average equities, and the relative values of

Market dividend yield / Dividend yield assumption

for the high yielding equities and the average equities may not be significantly different as long as the dividend yield assumption is reasonable.

Such detailed notional matchings are complex, and may be difficult to implement in practice. For example, in the case of the high yielding equities above, there are difficulties in
-identifying high yielding equities in the market, and
-setting an appropriate dividend yield assumption for them.

Hence, instead of taking into account the details of the liabilities, we may just make standard assumptions on future investment income regardless of the nature of the liabilities and the current market conditions. This approach can be interpreted as a further approximation to the (quasi-)S-matched position. Here, only the proportion of each category of assets will matter.

5.6.2.1.2 Actual portfolio

This approach tries to project the expected cash flows from the current portfolio or the portfolio under a likely future investment strategy. The cash flows from individual assets might be projected, but this is too complex in practice because of the need for making a large number of assumptions. Hence, the projection is usually done for each broad category of the assets.

The current investment policy may be different from a (quasi-)S-matched strategy, and the risk of insolvency may be higher than (quasi-)S-matched strategy with the same market value of assets. However, the value of assets calculated assuming the actual portfolio may not reflect the degree of the risk of insolvency as discussed in section 5.6.1.1. It might be higher or lower than that under the notionally matched strategy. Hence, some margins may be added in the valuation basis to adjust the result taking into account the risk of insolvency.

Alternatively, no adjustments may be made. In such a case, the risk of insolvency has to be assessed independently from the valuation result itself because the valuation result does not include any information on the risk.

In order to assess the degree of risk of insolvency, some forms of stochastic projections would be required using the actual or likely future portfolio rather than the (quasi-)S-matched portfolio. (see section 5.7) However, the efficient (quasi-)S-matched position is still very important as a benchmark to indicate the reduction in the risk by changing the strategy from the current strategy to the efficient (quasi-)S-matched strategy.

5.6.2.2 Market value basis

This approach uses the market value as the value of the assets, and attempts to value the liabilities and future contributions on the basis consistent with the market value of assets. Hence, for the valuation of the liabilities, the rate of interest, salary increase, inflation for liabilities, and so on, need to be set consistently with the market value of assets. This may mean that these need to be what the market expects at the time of the valuation. It may also be possible to set them as varying according to the length of the period between the valuation and the time of the expected occurence of cash flows.

However, the task of setting the valuation basis is difficult and very important due to the following reasons:

Firstly, these rates of interest, salary increase and inflation may not be easily obtained. For example, there may not be any indicators in the market directly related to the salary increases.

Secondly, the choice of the discount rate may be affected by the future investment policy. If the entire fund is going to be invested in equities, the realistic rate may be the expected yield on equities. On the other hand, if the entire fund is going to be invested in gilts, the realistic rate may be the current gilts yield.

Finally, if any margins are to be allowed, they should be included in the assumptions for the liabilities. Although there are some variations in the way of defining the market values as discussed in section 5.5, the portfolio to be valued is always the actual portfolio, and there may not be much room for margins in the valuation of the assets.

Because the market value is readily available, or can be assessed independently from the valuation process, there are no choices for the portfolio to be valued. However, the valuation can be carried out taking into account the future investment strategy. Here, two different approaches are possible through the setting of the valuation basis.

The first approach is to assume the actual or likely future investment strategy, which might be different from a (quasi-)S-matched strategy. The risk stemming from any mismatching can only be taken into account implicitly by modifying the valuation basis for the liabilities although it is difficult in practice as discussed above.

The second approach is not to make any such adjustments. This would imply that the valuation is carried out assuming the assets and the liabilities are (quasi-)S-matched. That is, a (quasi-)S-matched investment strategy is assumed. Then, similar arguments to the use of discounted cash flow approach with a notionally matched portfolio would apply, but any discussion on implicit margins is not applicable here. (see section 5.6.2.1.1.)

Again, to assess the level of risk of insolvency, some forms of stochastic valuations may be required. (see section 5.7) To carry out such stochastic valuations, future income from the fund needs to be projected, and a future investment strategy has to be assumed.

5.7. Stochastic valuation

As we have seen, deterministic valuations can be interpreted within the wider framework of a stochastic asset-liability model. Through these interpretations, the meaning of the valuation may become clearer, and the setting of the valuation basis may be carried out more appropriately.

However, a deterministic approach only presents one set of results although sensitivity checks might accompany the main result. This hides the real meaning of the valuation from the possible users of the valuation results. In particular, the degree of risk of insolvency or the volatility of the results is unlikely to be shown explicitly. The only hint to the degree of risk would be obtained by assessing the degree of margin included in the assumptions. However, such a guesswork is inevitably subjective, and may be difficult for non-specialists.

Information regarding risk can be communicated if the valuation result is presented stochastically. That is, the probability distributions of future contribution rates, future funding levels or other items of the valuation results may be shown. Then, we can learn what level of risk has been taken according to the criteria for the funding. Of course, to present the results stochastically, the valuation itself needs to be carried out applying a stochastic approach. As discussed in section 5.2.3, a stochastic approach should be based on a realistic, i.e. market value, basis.

We shall explore the possibilities of a stochastic approach in the valuation of a pension fund in this section. In the case of deterministic valuations, projected future cash flows are discounted to obtain the present values of liability outgoes and incomes from the assets. Therefore, firstly, the possible use of discounting in a stochastic approach is briefly considered in section 5.7.1. Then, in sections 5.7.2 and 5.7.3, applications of a stochastic approach to valuations are explored, and section 5.7.4 concludes the discussion.

5.7.1 Stochastic Discounting

In order to discount future cash flows and obtain present values, a discount rate (or discount rates) has to be determined.

It may be possible to set a fixed discount rate. For example, the discount rate could be the expected rate of interest at the valuation date. This may be a practical solution. However, if a particular branch of the projection is considered, the discount rate is not related to the rate of returns assumed in that branch of the projection, and the discounted values regarding the branch itself may not be theoretically explicable. So, a theoretical justification for applying the expected rate of interest for all the branches of the projection is required.

It might also be possible to use a notional rate of return required by the investors if the stochastic approach is used for the valuation of company shares. However, in the case of pension funds, the payments of the benefits are the prime objective, and the absolute rate of return is not important. Therefore, the discount rate should be the rate of investment return achievable.

Therefore, for theoretical soundness, different discount rates needs to be applied for each branch of stochastic projections by reference to the investment returns achievable in that branch. (See Buhlmann(1992) for a mathematical discussion on the subject of stochastic discounting.) Then, the average and the standard deviation of the present value of the discounted cash flows may be calculated. Such an approach is referred to as 'stochastic discounting'. Loades(1988) has shown an example of this approach.

By comparing the market value of the assets held and the probability distribution of the present value of the liabilities, the degree of risk may be assessed. Where a different investment strategy is assumed, this may lead to a different set of discount rates because they are determined referring to the investment returns achievable in each branch of projections. Thus, the probability distribution under the different investment strategy may be different. In this way, the risk and the investment strategy can be linked.

Also, the use of discounted values can make the projection results to be presented compactly. Furthermore, the concept of stochastic discounting may be useful for purely mathematical investigations.

However, such discounting calculations might require a very large computer capacity because the discounted value needs to be calculated for each branch of the projection. Hence, it might be practically difficult to apply this approach to the real pension fund valuation in full scale.

As another disadvantage of discounting, if future cash flows are discounted, information on the timing of the cash flow is not utilised. For example, it would be possible that, in a particular branch of the projection, the fund becomes temporarily insolvent but recovers by future contributions. Discounted figures may not reveal such situations. Also, the projected liability outgoes would have been useful to set an investment strategy, but such information is not included in the discounted figures. Indeed, the discounting approach can show the impact of the investment strategy only indirectly. Furthermore, investigations on the level of and the fluctuation in future contribution rates may be better dealt with by applying non-discounted information.

In conclusion, although stochastic discounting may be a useful method, other approaches should also be considered to explore the potential of a stochastic approach fully.

In the following sections, no further discussion on discounting is made, and, in the following, we shall discuss non-discounting approaches only. Although some of the following discussion might apply to discounting approach as well as non-discounting approaches, a full investigation in the application of discounting is left to future reserches.

5.7.2 Checking of funding level / security

In this section, I will discuss the possible use of a stochastic approach to check the funding level or security.

Daykin et al(1987) developed such a stochastic asset-liability model to check the solvency of a general insurance company. Several different investment strategies now and in the future were investigated. The results were evaluated by taking the probability of ruin and the distribution of the remaining assets. The model was also extended to allow for the inflows of new business for up to two years.

Coutts & Clark(1991) developed a stochastic asset-liability model for a general insurance company, where the risk was taken as the volatility of residual assets. Daykin et al(1993) have developed a stochastic asset-liability model for a closed pension fund, and an investment strategy to minimise the risk of insolvency was sought. Here, the risk was assessed by the probability distribution of estimated future surpluses after 15 years.

Here, the possibilities for the application to a pension fund are examined further. I assume that the portfolio of assets to be valued is already specified. It is then possible to perform cash flow projections of the assets and the liabilities stochastically. The liabilities under consideration are limited to these which have already accrued, and only existing members are involved. Future salary increases may be allowed for either stochastically or deterministically. To maintain the consistency between the salary growth assumption and other economic assumptions, a stochastic salary growth assumption which is linked to other assumptions is preferred. The projection can be carried out until the last liability outgo is made.

Liabilities and assets are projected together to measure either the residual assets after all the liabilities are paid out, or the probability of ruin. The asset allocation can either be the actual portfolio or a notional one according to the investment strategies to be taken. Here, the purpose of the projection is to check if the fund is solvent. So, the measurement of the risk should be related to the probability of insolvency, e.g. the probability of ruin.

A possible method of valuation may be to try projections with various values of current assets and establish the relationship between the amount of current assets and the expected residue or the probability of ruin. The minimum amount of assets which satisfies a specified criterion, e.g. probability of ruin to be less than .5%, may be taken as the "100% funding level". Alternatively, the measurement of the risk may be related to the distribution of the residual assets, and the "100% funding level" may be identified according to a criterion expressed by the measurement of the risk.

The "100% funding level" can be calculated either assuming a specific investment strategy or without specifying the strategy.

In the first case, the "100% funding level" depends on the investment strategy specified. The investment strategy specified can be simply the current investment strategy, and the "100% funding level" is the level at which the risk of insolvency is below a certain level under the current investment strategy.

In the second case, in theory, an efficient (quasi-)S-matched position has to be identified after trying every possible investment strategy. Then, the "100% funding level" is the lowest possible value of assets which can satisfy the specified criterion. So, the "100% funding level" in the second sense is less than or equal to that in the first sense.

The argument on which approach to the investment strategy should be taken is essentially the same as for the deterministic approach discussed in section 5.6.1. However, under a stochastic approach, margins are explicitly and quantitatively shown.

Such a "100% funding level" may be identified as the Standard Fund of the Current Unit or the Projected Unit Methods, although the fund is assumed to be closed and the appropriate on-going level of the Standard Funds may be different from these.

Then, the actual funding level may be obtained by dividing the market value of assets held by the market value of assets required for the "100% funding level".

The liabilities may be extended to the future liabilities of the existing members and the liabilities for new entrants who enter the scheme within a limited time period from the valuation date. In this case, it is necessary to assume future contribution rates, which may be just assumed at a certain rate, or a method of calculation may be specified. Such a approach may be, for example, applied to obtain the Standard Fund of the Entry Age Method stochastically after specifying the Standard Contribution Rate which can be determined either stochastically or deterministically.

However, this approach treats the existing fund in isolation, and ignores possible interactions between the existing fund and the contributions and liabilities accruing in the future. Thus, one may criticise that the results obtained from this approach may be distorted. To answer this criticism, an integrated aproach would be required. (see section 5.7.3.2)

5.7.3 Setting the contribution rate

It is possible to follow various paths for this purpose. We now consider two distinct methods, and the treatment of future new entrants.

5.7.3.1 Stepped method

This method follows the steps taken by the traditional deterministic approach;

1. The Standard Fund and the Standard Contribution Rate are calculated according to a specific funding method.
2. The surpluses/deficits are identified by comparing the Standard Fund and the value of assets.
3. The surpluses/deficits are amortised by adjusting the contribution rate.

In step 1, the order of the calculations of the Standard Fund and the Standard Contribution Rate can be different for each funding method.

The calculation of the Standard Fund and identifying the current funding level against the Standard Fund may be carried out using the method in Section 5.7.2.

At the same time, the Standard Contribution Rate may be set by performing stochastic projections for particular future liabilities allowing for various rates of contributions. The future liabilities are defined according to the funding method. For example, under the Projected Unit Method, they are the liabilities accruing in the next year; under the Entry Age Method, they are the total future liabilities for a new entrant at the Entry Age. Then, a series of projections with different contribution rates will reveal the relationship between the contribution rate and the risk. The minimum contribution rate which provides an acceptable level of risk may be chosen as the Standard Contribution rate.

The amortisation of the surpluses/deficits is dealt with separately. It can be carried out deterministically by regarding the surplus/deficit to be a fixed amount and amortised systematically. Alternatively, a stochastic model may be applied on the initial surpluses/deficits and the adjustment of contribution rate separately from the main model which involves the stochastic projections of the scheme liabilities, the standard fund and the standard contributions. In such a model, the initial surplus/deficit plus (or minus if the contribution is negative) a particular level of contributions is rolled up at the assumed interest rates which are stochastic. A set of stochastic projections will give the probability distribution of the residual amount at the end of year x, when the amortisation is expected to be completed. The same sets of projections are performed assuming different contribution rates, and the contribution rate which gives the best probability distribution of the residue may be chosen as the adjustment of contribution rate for amortisation. Here, the best distribution depends on the criteria imposed; for example, in the case of the amortisation of a deficit, 'the probability of still having a deficit at the end of year x should be at most 5%'. Also, the contribution rate should be the minimum among those which satisfy the criteria above.

This method does not fully model the working of a pension fund, but carries out each step separately. So, there can be inconsistencies among these steps when an S-matched portfolio is sought and used.

For example, consider the Projected Unit Method. The Standard Fund and the Standard Contribution Rate are identified separately under a closed fund situation. They are calculated as the minimum levels which are required to satisfy certain solvency constraints. Under an on-going situation, the Standard Fund and the Standard Contribution Rate which are calculated separately should be sufficient to meet all the liabilities. However, there can be a synergy by combining the existing fund and the funds created from future Standard Contributions. So, under an on-going situation, lower levels of the standard Fund and the Standard Contribution Rate might be suggested for the same solvency requirements.

The stepped method is in line with traditional deterministic approaches, and is relatively easy to implement as long as no new entrants are assumed. It is possible to use a stochastic approach to only one or two of the steps and use a deterministic approach for the rest. Also, all the existing funding methods are valid under this process.

However, a main drawback of this approach would be that the results obtained by combining together all the separate steps might not be appropriate to achieve certain funding objectives, although each step may be carried out in reference to such stated objectives.

5.7.3.2 Integrated method

Another possibility is to use a full model of pension funds. This approach starts from projecting future cash flows from the assets and liabilities on existing members. Different levels and patterns of future contributions are added to the model, and the projection results are judged by the expected value and the volatility of the value of residual assets or by the probability of ruin. It may also be possible to use the volatility of contribution rates as another criterion.

The pattern of contributions can be linked to some variables in the projection. This includes all the traditional funding methods, but also this could allow for a wider range of contribution patterns than the traditional funding methods would suggest. The level and pattern of contribution rate which satisfy certain conditions in terms of risk may be chosen as the recommended contribution rates.

Such projections can be carried out under various investment strategies, and the combination of investment strategy and contribution rate which fits the criteria best will be chosen in practice. The strategy can be a matched one, or a more aggressive strategy. Instead, a particular investment strategy is assumed, and the appropriate contribution rates for the investment strategy may be determined.

Future new entrants may be included. If future entrants are not included, the calculation becomes simpler, but may not represent realisically the on-going situation.

If future entrants are included perpetually, the evaluation of projections using the residual assets will become impossible.

In such cases, the assets and the liabilities might be valued after a certain period using some deterministic formula, and the solvency criteria, which should be satisfied by the resulting contribution patterns, could be expressed in terms of the valuation results.

Alternatively, the probability of ruin for a certain period of time might be used to express the solvency criteria. If the period during which ruins are counted is sufficiently long, the result may be used as an on-going recommended contribution rate because the rates are expected to be re-calculated at the next valuation.

Another way would be to allow new entrants only for a certain period, and then assume the scheme to be closed to new entrants. If the period is long enough, the result may be suitable for actual use.

The integrated method is satisfactory theoretically because it treats the whole process consistently. Also, this method allows flexible contribution patterns and assists the setting of the investment strategy.

This method is also useful to check if a given stream of contributions satisfies a certain set of solvency and other criteria, or for measuring the degree of risk associated with the given stream of contributions.

However, there are some shortfalls. Firstly, a large volume of calculations is required.

Secondly, the contribution pattern may lack the theoretical backing which is seen under traditional funding methods.

Thirdly, it is not possible to identify the ongoing funding level as a percentage of a Standard Fund, which is not required under this approach.

Of course, the check on the discontinuance funding level at the valuation date may be done separately at the same time. However, it may be nearly impossible to check the projected future funding level. To do this, the checking of future funding levels needs to be performed at all the times in future in all possible projection branches. This means that the criteria in terms of future discontinuance funding level cannot be set for the determination of contribution rates unless discontinuance funding levels are checked using a deterministic formula.

Another disadvantage is the absence of a Standard Contribution Rate, which may be useful for accounting purposes.

5.7.4 Conclusions

As discussed in section 5.6.2, the assessment of risk of insolvency of a particular investment position cannot be made by deterministic valuations. Some stochastic projections, or stochastic valuations, would be necessary for this task.

As discussed in section 5.7.2, this task can be carried out using existing techniques of stochastic projections. However, treating the existing fund separately from future contributions and future liabilities may cause a distortion to the results.

Alternatively, an integrated approach may be taken. Through this approach, not only the assessment of risk but also other uses, such as the setting of a more flexible funding plan, may become possible. However, because of the difficulty in dealing with new entrants, currently it would be difficult to take this approach in a theoretically satisfactory manner. Further technical developments in this area are awaited.

To manage any risk, first of all, the degree of risk needs to be appropriately assessed. Therefore, the further development of valuation technique using stochastic projections are very important for an improved management of a pension fund.

PART IV

Chapter 6 Control of Pension Fund through Valuation

6.1 Introduction

Chapters 6, 7 and 8 together with Appendices 1, 2 and 3 form Part IV of the thesis. This part deals with the issues related to the control of a pension fund. In particular, the focus is put on the choice of a valuation basis under a changing environment. A general discussion of the ideas is given in this chapter. Chapter 7 is devoted to explaining a series of cash flow projections that have been performed. The main results are presented in Chapter 8, with some details left to Appendices.

A pension fund may be managed so as to achieve certain financial objectives, such as;
-maintaining on-going solvency,
-maintaining discontinuance solvency, and
-stabilising the contribution rates.

To achieve these and other objectives, the pension fund needs to be controlled by some means. To manage a pension fund, the valuation is very important for collecting information on the finances of the fund and for influencing the future by changing the contribution rate and/or by some other means. Consequently, valuation has a major role in controlling the pension fund. Hence, the thesis focuses on control through the valuation, although other forms of control may be possible.

To ensure the financial objectives are achieved, it is desirable that such controls can be made systematically. Goford(1985) proposed such a systematical control in the life assurance. Once such a system is created, it is important to examine how the system works to judge if the system is useful for achieving the objectives.

In the area of engineering, control theory has been developed to control the output from some processes. This theory may give some insight to the problem of controlling a pension fund.

In section 6.2, a brief review of control theory based on Dorf(1992) will be made. For further details, Dorf(1992) or other text books on control theory may be consulted.

In section 6.3, ways to apply control theory to a pension fund will be explored. Possible control systems for a pension fund will be discussed in sections 6.4 and 6.5. Section 6.5 concentrates on control through the valuation bases.

6.2 Control theory

The concepts of control system and feedback have been developed in the field of engineering as an attempt to obtain desired responses from a system. For example, the quantity of the output from a production line may be stabilised by adjusting the speed of the process according to the quantity of the output actually made.

The process to be controlled together with the mechanism of control may be complex. To analyse a control system, it may need to be modelled. The model should contain all the essential features of the system. At the same time, the simpler the model is, the easier the analysis would be. In particular, for a complete mathematical analysis, the model needs to be very simple.

6.2.1 Types of control systems

A process to be controlled consists of, in principle, three elements; the input, the main process and the output.

```
                          ________________
                         |                |
INPUT   ----->           |  MAIN PROCESS  |  ---->   OUTPUT
                         |________________|
```

Diagram 6.2.1.1 Process

There are two different control systems; an open-loop control system and a closed-loop feedback control system.

An open-loop control system controls the process directly. The actual output will not affect the controller.

```
                          ________________
                         |                |
INPUT   ----->           |  MAIN PROCESS  |  ---->   OUTPUT
                         |________________|
                                 ^
                                 |
                                 |
                          _______|________
DESIRED                  |                |
 OUTPUT  ----->          |   CONTROLLER   |
                         |________________|
```

Diagram 6.2.1.2 Open-loop Control System

A closed-loop feedback system measures the output, and this is compared with the desired output. The system is controlled by referring to this comparison. So, the future process may be affected by the actual output produced up to the current time.

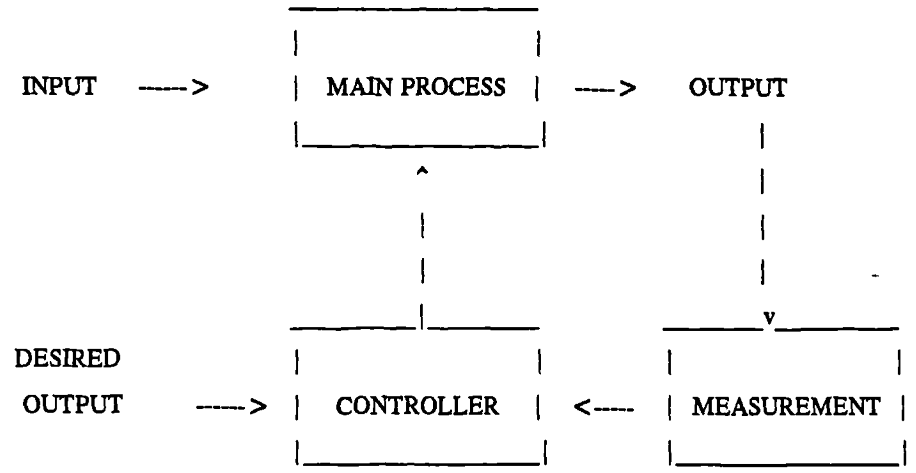


Diagram 6.2.1.3 Closed-loop Feedback System

Comparing open-loop systems and closed-loop feedback systems, the advantages of the latter would be as follows;

-Feedback systems can reduce importance of the main process.

In the case of an open-loop system, there are no means of adjusting the main process to achieve the desired output. So, reaching the desired output totally depends on the main process. A feedback system, on the other hand, can adjust the process according to the previous output. Hence, even if the main process is not particularly suitable to produce the desired output, the system may eventually achieve the desired output.

-Feedback systems can allow the responses to be time related.

The adjustments to the process as a result of the current output can be delayed, or an average of the output for a period might be used to control the process. This feature gives a wider possibility to feedback systems.

-The effect of disturbance signals as inputs may be controlled and partially eliminated.

Undesired effects on the output as a result of unexpected changes in the input may be reduced or removed by a feedback system. For example, if an increase in the amount of the input makes the amount of the output larger, and if a stable output is desired, a feedback system to delay the process when the amount of the output becomes large may be used. Then, if an increased level of input is given for a period, the impact to the output may be mitigated. Initially, the output will increase due to the increased level of the input. However, the increased output triggers a reduction in the speed of the process, and the output may be reduced. After the input comes back to the normal level, the output may gradually come down to the normal level. Thus, the output may be more stable than a system without feedback.

-Feedback systems may reduce the 'steady-state errors'.

Here, the 'steady-state errors' mean the difference between the desired and actual output in a stationary situation. Feedback systems can adjust the process to reduce such errors. This is not possible under open-loop systems.

On the other hand, there are some disadvantages to feedback systems.

-Feedback systems are more complex than open-loop systems.

The reactions to various inputs are not straightforward, and it may make the analysis of the system difficult. So, it could be difficult to judge if a particular system helps to achieve any of the desired objectives.

-The responses from a feedback system may be slow.

The effects of feedback may make the system slow to respond to a new situations.

-Feedback systems might introduce instability.

It was listed as an advantage that feedback systems might reduce the impact of erratic inputs. On the other hand, feedback systems might increase the impact of such erratic inputs, for example, by an over reaction to the output.

6.2.2 Analysis of control system

A control system may be evaluated by investigating its reactions to various inputs. A good system may be able to achieve the desired output target under various patterns of input. The patterns of input may be infinite in number, but the effect on the output from a particular pattern of the input may be a sum of the effects from a set of basic input signals. Hence, in order to analyse a control system, these basic input signals may be used as the test inputs, and the responses from the system may be studied. Even if the impact of a particular input is not the sum of the impacts from the basic signals, the summation may give an approximation. Also, after studying the responses from such basic signals, compound effects might be investigated as the next stage.

Typical test signals to be used may be;

Unit impulse:	a temporary change in input,
Step:	a constant input which starts at a particular time and continues indefinitely,
Ramp:	a straight line increase in the amount of the input,
Parabolic:	a quadratic increase in the amount of the input, and
Sinusoidal:	cyclical changes in the input.

Usually, the model is expressed in a mathematical formula, and the responses to these test signals are examined mathematically.

6.3 Application of control theory

We shall now discuss the possibility of applying the control theory to the analyses of a pension fund.

Balzer & Benjamin(1980) and Balzer(1982) applied the idea of the control system to the context of general insurance using a simple model. The model takes the premium rate and the claim experiences as the inputs and the profit as the output. A feedback module is built into the model, and the premium rate can be changed according to the profits/losses. The control system consists of the inputs, the output, the black box to transform inputs to the output, and the feedback module. The control system aims to achieve stability of the profits, ie. the output. The papers examined the responses to test signals, and tried to identify the optimum feedback system mathematically. Cyclical trends of insurance profits from year to year [i.e. the underwriting cycle: see Berger(1988)] have been explained as resulting from the non-optimal features of the feedback system.

Benjamin(1984 and 1989) tried to apply this idea to the pension fund. Using a simplified model, the stability of the Recommended Contribution Rate is sought under varying investment returns. The impacts of the test signals were examined mathematically. Cash flow projections were also used to visualise the effects.

Loades(1992) investigated the impact of a cyclical change in real interest rates (i.e., in excess of salary inflation) on a pension fund. The system which allows the valuation interest rate to be changed according to the past investment results was examined using cash flow projections. The system's trend towards a stable contribution rate was appraised using the standard deviation of the contribution rate.

6.3.1 Nature of pension fund and method of investigation

The pension fund as a financial process is very different from a general insurance fund, which was modelled in Balzer & Benjamin(1980) and Balzer(1982).

Firstly, a pension fund has several different items to be stabilised, while it would be appropriate to put an emphasis on the stability of the profit in the general insurance model. The operators of a pension fund may wish to stabilise the contribution rates and the funding level simultaneously, and they may be found mutually conflicting under ever changing circumstances. Some systems might be more effective for a particular objective, while other systems might be more effective for different objectives. So, it would be difficult to judge whether a particular system is superior to another.

Secondly, there are many items affecting the financial outcomes; investment return, salary growth, inflations, withdrawal rates, mortality, and so on. The impact from each item is unique, and the combined effects may not be a mere sum of the effects from individual items because of the interdependence among these items.

Thirdly, a long timespan needs to be considered. From the entry of a member, his/her retirement, his/her death after the retirement, to the eventual death of his/her dependant, it may well take 100 years.

Fourthly, every year, new entrants may be admitted to the scheme, and the pension fund may contain generations of members. So, the modelling of the pension fund may be more complex than that of a general insurance fund.

Consequently, mathematical approaches may be difficult to apply. Mathematical approaches require all the details of the model to be expressed in relatively simple formulae. If this approach is adopted, some of the important factors of the pension fund may need to be modified or ignored for the sake of simplicity. Instead, numerical approaches, which involves cash flow projections on asset-liability models, may be found more useful.

If we change the contribution rates or exercise some other controls according to the actual experience with regards to a particular member after the member finally exits, the control may not be effective because a long time may have passed and many generations of new members may have been admitted to the scheme before the control is exercised, and surpluses/deficiencies may have accumulated to a very high level, and because the experiences of the exiting members may be out of date by the time they finally exit the scheme.

Also, for controls after a member finally exits, it may be necessary to subdivide the fund for each generation of members to measure the outcomes in respect of those members who belong to a particular generation, but such subdivision of the fund may not be easy.

So, some form of control before the final results, i.e. the eventual cessation of the benefit payouts, is required. Such control may be found more difficult to design and analyse than controls directly based upon the final outcomes.

Also, when the responses from a control system are checked, long term accumulation effects need to be examined.

6.3.2 Items to be controlled

In order to control the process, firstly, the output to be controlled need to be identified according to the objectives of the system. Then, in the case of an open-loop system, a controller should be designed so as to obtain the desired output. In the case of a feedback system, a system of measuring the output and modifying the main process according to the result of the measurements needs to be designed.

As discussed in section 6.3.1, the control system cannot wait until the final outcomes. Hence, the output to be controlled need to be some items which currently exist.

6.3.2.1 On-going solvency

Because the pension fund aims to provide promised benefits to the members, it is important to maintain the fund at a sufficient level. This is referred to as on-going solvency. However, the sufficiency of the fund cannot be definitively judged until all the contributions have been received and all the benefits have been paid.

A valuation is a means of measuring the sufficiency, but the valuation result itself may not be the output to be controlled unless the stabilisation of the valuation result is desired because of possible practical constraints. Assume that the sole objective of managing a pension fund is to obtain the valuation results which show that the fund is sufficient. Then, the control systems to achieve this objective may involve the changes of the valuation basis because the valuation result is dependent on the choice of the valuation basis. However, the modifications in the valuation result do not actually affect the real cost of providing the benefits, and such control systems may not be suitable for ensuring the on-going solvency.

So, as the output to be controlled for the objective of ensuring the on-going solvency, a rather notional concept of "the current financial state of the fund" may be chosen. The current financial state is created by the assets held, the extent of liabilities to be covered by the assets held, and the likely future events. The current financial state represents the likely sufficiency of the assets held to cover the specified range of liabilities. The current financial state, thus, does not depend on the choice of the valuation basis. The valuation is identified as the measurement of this current financial state of the fund for the purpose of feedback.

In the valuation, the value of the liabilities may be identified as the Standard Fund, or the Actuarial Liabilities, under the funding method in use. The value of the liabilities may be assessed making assumptions about the future likely values of the investment returns, rate of salary increases, mortality rates, withdrawal rates, and so on. These demographic and economic factors cannot be predicted with complete accuracy, and for the valuation, estimates are required. Thus, inevitably, the valuation results might not represent the realistic financial state because of our inability to predict exactly the future. Also, deliberate departures from the realistic estimates may be attempted in the choice of assumptions for the valuation.

The financial state is measured by the valuation, and some actions may be taken to achieve a sound financial state. This will be discussed further in section 6.4.

6.3.2.2 Discontinuance solvency

At the same time, the sufficiency of the fund if the scheme winds up now is also important. This is referred to as discontinuance solvency.

Similar arguments to those given in the on-going solvency case apply here. The amounts of fund to secure the benefits on a winding up may be assessed by estimating the future events and their impact. This may be the case if the money to be allocated to each member is defined as the value of the benefits accrued.

It is possible, however, that the money allocated is not calculated using these estimates. For example, the money allocated might be assessed assuming that the benefits are secured by means of insurance policies. Then, the discontinuance financial position depends on the assets and liabilities of the pension scheme and the current insurance premium rates, but not the future likely values of the demographic and economic factors mentioned earlier. It would be necessary to make estimates in respect of these factors in order to set the premium rates, but this would be carried out by the life assurance companies. To the pension scheme, the premium rates may be regarded simply as a part of the current financial environment.

In this case, the long term nature and the compound effect mentioned in section 6.3.1 may not be relevant, and the modelling might be simpler than otherwise.

Although discontinuance solvency is important for the protection of the members' rights, on-going solvency may be more essential for the management of the pension fund which is expected to continue. Also, the nature of the discontinuance problem may be similar to or simpler than the on-going solvency case, and the discussion on the on-going solvency may be directly applicable. So, no further discussion will be made of discontinuance solvency in this chapter.

6.3.2.3 Stability of contribution rate

The third candidate for the item to be controlled may be the Recommended Contribution Rate because its stability may be regarded as an important objective of the fund.

We shall now consider the question whether it is possible to create a control system which directly aims to stabilise the contribution rate.

There are defined contribution pension schemes, the benefits under which depend on the investment returns on the contributions paid in and the annuity rates at the time of retirement. The stability of the contribution rate is fully achieved by such schemes at the cost of instability in the level of benefits.

In the case of defined benefit schemes, the real cost of providing the predetermined benefits depends on the actual experience in terms of the investment performance, salary growth and the demographic factors. If the fund built up by a constant contribution rate proves to be insufficient to provide the promised benefits, extra contributions will be demanded. On the other hand, if the experiences have been so favourable that there are surpluses after paying all the benefits, the surpluses may be returned as a negative contribution. Thus, it would not be possible to achieve a constant contribution rate except by coincidence.

There may be a system under which the contribution rate is likely to be more stable than others. For example, the contribution rate under the Projected Unit funding method may be more stable than that under the Current Unit Method. Also, the amortisation of the surpluses/deficits over a long period may well work to stabilise the contribution rates.

However, in these examples, the changes in the contribution rates do not affect the process of deciding the contribution rate. In other words, the contribution rate as an output is not fedback to achieve the stability. So, these examples may be interpreted as open-loop control systems with the contribution rate as an output, but not closed-loop feedback systems. (see section 6.3.2.4 and Benjamin(1984 and 1989))

If the contribution rate is taken as the output, and measured by a feedback routine, the system does not work.

Assume such a control system exists, and suppose the contribution rates are calculated on a particular valuation method and basis. When the contribution rate calculated on this method and basis increases, the system will work to reduce the contribution rate at next valuation by modifying the valuation method and basis. There are a range of valuation methods and bases which can be used to assess the appropriate contribution rate. So, it is possible that both the initial and the modified valuation methods and bases are appropriate compared with the benefits to be paid in the future.

However, an increase in the contribution rate under a specific valuation method and basis tends to be the result of unfavourable differences between the expected and the actual experience in demographic and economic factors. Then, if the valuation method and basis are modified to reduce the contribution rate at the next valuation, the difference would widen, and the financial state of the fund would deteriorate. Unless the future experiences of the fund turn out to be favourable in the future, the fund would not be able to pay off all the benefits. So, subsequent rises in the contribution rate would be inevitable.

Another possible reason for rises in the contribution rate is the increase automatically built into the valuation method and basis. For example, the Projected Unit Method would give increasing Standard Contribution Rates for a closed membership. In such cases, rises in the contribution rate are expected and are necessary to finance the future benefit payments. If the increases in the contribution rates are cut down, a shortage in the fund in the future would follow even if the actual experience exactly follows the assumptions made for the calculation of the contribution rates.

In either case, stabilising the contribution rate may not be realised without causing shortages in the fund.

The stability of the contribution rates can be a reason to build up a pension fund rather than paying the benefits from each year's revenue. So, it is important to check the stability of the contribution rate under a particular system of funding. However, for a defined benefit pension fund, this cannot be the primary objective as discussed above. Particularly, it is impossible to design a closed-loop feedback system with the contribution rate as the output.

6.3.2.4 Conclusion to section 6.3.2

As a conclusion, the financial state of the fund should be taken as the output from the process. The feedback routine measures the financial state of the fund, and the system tries to stabilise it. In other words, all the control systems should be designed primarily to ensure the payment of the benefits promised.

As another important objective, stabilising the contribution rate may be attempted through a control system. However, the control systems which have been discussed above treat this objective as secondary. The systems has been designed to achieve a sound financial state, and the stability of the contributions is tested as a desirable but secondary feature.

Benjamin(1984 and 1989) identified the Recommended Contribution Rate as the output. However, changes in the Recommended Contribution Rate do not trigger any changes in the process of calculating the Recommended Contribution Rate in Benjamin(1984 and 1989). So, from the definition of open and closed loop system in section 6.2.1, the control system in this case is regarded as an open-loop system. The objective of securing on-going solvency, which is treated as the main objective in this section, was designed to be satisfied automatically in the main process in the case of Benjamin(1984 and 1989). Such an approach represents a different view from this thesis, and would be useful for the analyses of particular features of a pension fund. However, it was felt that, by taking the financial state of the fund as the output, wider issues can be discussed, and the approach taken by Benjamin(1984 and 1989) has not been followed in this thesis.

6.4 Examples of control

The sponsoring employer, the trustees, or any other people who are responsible for managing a pension fund need to make judgements on the benefits to be paid, the investment strategy and the pace of funding.

Control of the pension fund can be exercised through each of these. Under each particular circumstance, the actions to be taken may be chosen out of a set of limited possible courses of actions. In this sense, the judgements may be classified to some extent in practice. So, we might be able to identify control systems in use in practice, although these control systems might not be clearly formulated.

In this section, possible methods of controlling a pension fund are explored.

6.4.1 Benefits

The benefits can be increased or reduced by discretion or through changes in the rules. Such changes may be made aiming to meet the objectives of the fund.

If the objective is to pay the benefits as prescribed in the rules, such changes in benefits may not be appropriate. However, if changes in the benefits are acceptable, the changes may be made to keep the financial state sound.

If the fund is in surplus, the surplus can be used to increase the benefits. For example, discretionary increases in the amount of the pensions in payment may be made.

On the other hand, if the fund is in financial difficulties, the level of benefits might be cut back. For example, if the current level of benefit accrual cannot be sustained, the future benefit accruals may be reduced. Also, certain payments, e.g. transfer values, may be temporarily reduced, or their payment may be delayed.

Terms offered in exchange for a transfer payment into the fund are also controllable by the operators.

The changes in the benefits have direct effects on the solvency of the benefits and the stability of the contributions. However, reductions in the benefits may, in particular, be restricted to emergency measures.

```
                        ____________
[INPUT]                 |          |          [OUTPUT]
Experience  ---->       |  PROCESS |  ---->   Financial State
                        |__________|          of the Fund
                              ^                    |
                              | Benefit            |
                              | Increase/Cut       |
                        ______|_____         ______v_________
[Desired Output]        |          |         | [MEASUREMENT] |
Funding level  --->     | CONTROLLER |  <----  |   Valuation   |
     = 100%             |____________|       |_______________|
```

Diagram 6.4.1.1   Benefit Change

6.4.2   Investment strategy

The investment of the fund can be made taking into account the nature of the liabilities. A so-called 'matched' strategy may be taken to reduce the risk of insolvency, or to reduce the volatility in the financial state of the fund. A higher level of risk or volatility might be adopted if the operators so wish. (see Chapter 5 for further discussion on investment strategies).

Switches between the matched position and unmatched positions may be made systematically according to the level of funding. For example, if the funding level is low, the matched position may be taken. If the funding level becomes high enough, the investments may be shifted to an unmatched position gradually.

A control mechanism may be illustrated as follows.

```
                        ____________
[INPUT]                 |          |          [OUTPUT]
Experience  ---->       |  PROCESS |  ---->   Financial State
                        |__________|          of the Fund
                              ^ Change in          |
                              | Investment         |
                              | Strategy           |
                        ______|_____         ______v_________
[Desired Output]        |          |         |[MEASUREMENT] |
Funding level  ---->    | CONTROLLER |  <----  |   Valuation   |
     = 100%             |____________|       |_______________|
```

Diagram 6.4.2.1   Change in Investment Strategy

By taking a particular investment decision, the operator of the fund can only reduce the probabilities of certain consequences while increasing others. Owing to the uncertainty in the financial markets, the control through investments may only have indirect effects on the solvency of the fund and the stability of the contribution rates.

The impacts of changes in the investment strategies may be fed back through valuations gradually as investment returns are achieved. Then, the process of feedback may take a very long time.

Alternatively, the changes in strategy may be reflected in the valuation basis. For example, the valuations of the assets may be carried out using the actual portfolio, or a notional portfolio in line with the new strategy. Also, margins included in the valuation assumptions may be increased or decreased reflecting the changes in the level of risk taken. In these cases, the process of feedback is quicker than the cases using notional portfolios, but the valuation results may be heavily dependent on subjective judgements.

6.4.3 Pace of funding

The pace of funding can be changed to meet various requirements. That is, the contribution rates can be changed without changing the benefits. Under the current practice of the use of deterministic funding methods, five items can be separately considered.

Firstly, the funding method can be changed. There are various funding methods, and the choice of a particular funding method affects the current and future expected contribution rates. For example, the Projected Unit Method may provide a lower current contribution rate than the Attained Age Method at a cost of higher contribution rates expected in the future. For the descriptions of typical funding methods, see Chapter 2.

The use of a particular funding method may be interpreted as an example of an open-loop system. The funding method aims to achieve the full payment of the benefits without any surpluses or deficits. However, the funding method itself cannot cope with existing surpluses/deficits.

```
                           _____________
[INPUT]                   |             |           [OUTPUT]
Experience     ---->      |   PROCESS   |   ----->  Financial State
                          |_____________|           of the Fund
                                 ^
                                 | Give Formula
                                 | to Calculate Contribution Rates
                          _______|_______
[Desired Output]         | [CONTROLLER]  |
Funding level   ---->    | Funding Method|
     = 100%              |_______________|
```

Diagram 6.4.3.1 Funding Method

It might be argued that the choice of the funding method can be triggered by changes in the financial state, and thus a closed-loop system is formed. For example, if the Attained Age Method is used and the current funding level becomes high, it might be possible to switch to the Projected Unit Method in order to avoid further accumulation of surpluses.

A change in the funding method means a change in planning for the pace of funding. If such changes are to be made frequently, it would be said that there is not a plan for funding for the scheme, and the merit of using a particular funding method would be lost. Hence, the scope for changing the funding method may be very limited.

Secondly, the method of removing the surpluses/deficits can be considered as a control system, which ensures the 100% funding level at some stage in the future.

```
                          ____________
[INPUT]                  |            |          [OUTPUT]
Experience ----->        |  PROCESS   |  ----->  Financial State
                         |____________|          of the Fund
                               ^                       |
                               | Amortisation of       |
                               | Surplus/Deficit       |
                         ______|______           ______v_________
[Desired Output]         |            |          | [MEASUREMENT] |
Funding level   ----->   | CONTROLLER |  <----   |   Valuation   |
    = 100%               |____________|          |_______________|
```

Diagram 6.4.3.2 Amortisation of Surplus/Deficit

The control system can be flexible in terms of the pace of amortisation. To increase the security of the benefits, deficits need to be removed quickly. On the other hand, for the stability of the contributions, the surpluses/deficits may be removed slowly.

A so-called "contribution holiday" is an example of the above, where contributions are suspended for a period to reduce the funding level. Refunds are the extreme means to remove surpluses.

Dufresne (1986 and 1988) and Haberman (1990 and 1991) have investigated the relationship between the pace of amortisation and the volatility in the contributions in a model where the investment return varies stochastically.

In the case of the Aggregate Method, no surpluses/deficits are recognised, and the pace of amortisation is not controllable. To make the pace of amortisation flexible, the Attained Age Method or the Entry Age Method may be used instead of the Aggregate Method, which can be regarded as the Attained Age Method or the Entry Age Method with a fixed pace of amortisation; i.e. the amortisation can be regarded as a uniform percentage reduction/increase in the contribution rate over the remaining period of membership of the existing members.

Thirdly, the modelling of the assets and liabilities, or the method of valuation, can be changed. For the valuation of the assets and the liabilities, simplifying the calculations through the use of a model is necessary.

Also, in order to value the assets, various methods of valuation are possible; e.g. discounted cash flow approach with notional switching to the matched position, the market related value approach, etc. The nature of the current portfolio and investment policy may need to be considered when valuing the assets. On the other hand, investment decisions may be affected by the nature of the liabilities. Their inter-relationship is important. This has been discussed in detail in Chapter 5. Also, see section 6.4.2.

Fourthly, the assumptions used in the valuation, the valuation basis, can be changed. This will be discussed in detail in section 6.5.

Finally, the frequency of the valuations can be changed. This has been investigated by Loades(1992) using a simulation approach and by Haberman(1993) using an analytical approach.

As discussed in Section 5.7, a stochastic approach might enable us to deal with each of the five components mentioned above simultaneously.

6.5 Control through valuation basis

The valuation basis is chosen out of a wide range of possibilities. The contribution rate depends on the choice of the valuation basis, and the financial state will be affected by this change. So, the output, the financial state of the fund, can be controlled by the choice of the valuation basis.

However, controls through the choice of basis have different characters from other means of control which have been discussed in section 6.4. So, it is separately discussed in this section.

This section aims to identify the mechanism of the control through the choice of the valuation basis. Such discussion is a starting point of the analysis of the effects of the choice of the valuation basis.

Firstly, the valuation basis is described. Then, it is discussed whether the setting of the basis is an open-loop system, or a feedback system. There is a separate discussion on two different parts of the valuation basis. Finally, the possibility of setting up and using of a systematic approach to the setting of the basis is considered.

6.5.1 Valuation basis

The exact items required in the basis depend on the model used for the valuation. The basis may include the following;

-interest rate (A different rate might be used after retirement.)
-salary inflation rate
-promotional salary scale
-mortality (for active members and after retirement separately)
-withdrawal rate
-retirement rate (for ill-health and normal health early retirements separately)
-rate of increase in pension (for deferred pensions and pensions in payment)
-assumptions on the dependants (percentage married, age difference with spouse, dependants' mortality, and so on).

Many works have been written on the subject of the choice of the basis. Colbran(1982) has given a brief discussion on how economic and demographic assumptions may be set in practice with reference to the current and past experience. Lee(1986) has discussed the principles of the setting of the basis. Parsons(1990) discussed how the best estimate of each economic and demographic assumption may be set for the calculation of pension costs under SSAP24, a U.K. accounting standard. Thornton & Wilson(1992a) reviewed economic and demographic assumptions in the light of past experiences, and argued that the prevailing practice then tended to include unnecessary margins in the valuation basis. All the papers referred here have included suggestions of possible ranges of values of economic assumptions referring to the data available then. The possible assumptions on demographic factors have also been suggested in Colbran(1982), Lee(1986) and Thornton & Wilson(1992a).

However, such discussion on suggesting the valuation basis referring to the past experience and the current environment is not a part of the thesis, and no further discussion on this subject will be made here.

6.5.2 Interpretation as a control system

The mechanism of the controls over the fund through the selection of the valuation basis may be analysed by identifying the process as a control system, which may either be an open-loop system, or a feedback system. The possible interpretations are discussed in this section.

If the output, the financial state of the fund, is measured, and the results of the measurement influence the choice of the valuation basis, this is a feedback system. On the other hand, if there are no links between the output and the setting of the basis, this is an open-loop system.

It might be the case that the existence of surpluses and deficits might trigger a change, or a reconsideration, of the basis. Through the analysis of surplus, an appropriate valuation basis might be suggested. (Although such a system of setting the basis may not be practical, this is discussed here for theoretical completeness.)

When the financial state is poor, an increase in the contribution rate through a change in the basis may improve the real financial state in the future, although the apparent funding level would fall instantly. On the other hand, when the assets are recognised to be excessive in a realistic sense, reductions in the contribution rate can be sought through a change in the basis, although this would lead to an immediate improvement in the apparent funding level.

In these sense, it can be interpreted as a feedback system.

However, the current state of the fund is measured through the valuation process, which is affected by the choice of the basis. So, if it is a feedback system, the system may need to include the mechanism of changing the measurements. Such a system might be very complex.

Also, the choice of the basis may not necessarily be a direct consequence of the valuation result. The valuation basis may be chosen referring to the past experience and the future view. This information may be obtained without carrying out the valuation,or measuring the output.

So, the choice of valuation basis can be interpreted as an open-loop system.

Now, we shall consider the valuation basis further.

The valuation basis may be set referring to the most likely future course of events, although these might be difficult to identify.

However, the best estimates may not be realised in practice, and the departure of the future experiences from the valuation assumptions may cause financial difficulties to the fund. So, in order to reduce the risks of failing to meet the objectives, it is common to include some margins implicitly in the basis. (see Benjamin(1976) for a discussion of the concepts of implicit and explicit margins).

So, the basis may be regarded as consisting of two parts: the best estimates of future events and the margins. Each part may have its own characteristics when we consider controls of the fund through them. In the following, further discussions are made on each part of the basis separately.

6.5.2.1 Best estimate

In order to fund in advance for a pension scheme, contribution rates to be made need to be assessed by setting assumptions on the future course of events. To avoid the contribution rate becoming excessive or too low, it is important to assess the most likely future course of events. Otherwise, the pattern of the contribution rates could become erratic unwantedly.

If the contribution rates are calculated on unrealistic assumptions, the contribution rate may prove unrealistic. Then, the difference between the assumptions and the actual experiences may need to be met by large extra positive or negative contributions, which may distort the smooth pattern of the contribution payments.

So, best estimates of the future course of events need to be made and reflected in the valuation basis.

The changes in the best estimate part of the basis should follow the changes in the likely future events. If the likely future events are unchanged, it would be difficult to change the best estimate part of the valuation basis. The control may be more likely to be exercised through other means, e.g. the amortisation of surpluses/deficits. On the other hand, if the likely future events change, changes in the best estimate part of the basis may need to be considered.

So, the likely future events need to be assessed from time to time.

Such processes of setting the best estimate part of the valuation basis may be interpreted as a closed-loop control system as illustrated below.

```
                               ____________
[INPUT]                        |          |              [OUTPUT]
Experience    ---->            | PROCESS  |    ---->     Financial State
                               |__________|              of the Fund
                                    ^                         |
                                    | Change                  |
                                    | Valuation Basis         |
                               _____|_____               _____v_______
[Desired Output]               |          |              | [MEASUREMENT] |
Real Funding State  --->       |CONTROLLER|   <---       | Assessment of future |
is satisfactory                |__________|              |_______________|
```

Diagram 6.5.2.1.1 Choice of Valuation Basis (1)

This interpretation takes the view that the financial state of the fund includes the likely future events, and that they can be measured, or assessed. (See section 6.3.2.1.)

However, although the likely future events form a part of the current financial state, the financial state itself is not measured in this case. In this sense, the control over the financial state may not be direct.

Also, although the likely future events can be regarded as a part of the output, they form part of the environment surrounding the fund, and may be regarded as inputs, rather than output.

So, an interpretation as an open-loop system may also be possible as follows.

```
[INPUT]                                  |              |               [OUTPUT]
Experience            ---->   | PROCESS    |   ----->   Financial State
Current Environment           |____________|             of the Fund
                                            ^
                                            | Give Basis
                                            | to Calculate Contribution Rates
                                     _______|_______
[Desired Output]              | [CONTROLLER] |
Satisfactory       --->       |  Set the Basis  |
Financial State               |______________|
```

Diagram 6.5.2.1.2 Choice of Valuation Basis (2)

The controller sets the basis referring to the past experience and the current environment which might both affect the view on the likely future events.

To conclude as to whether choosing the best estimate is a feedback control or an open-loop control, the actual procedures used would be consulted.

Because the future cannot be predicted precisely, the basis needs to be set referring to the limited information available. The possible sources may be the past economic and demographic experience, the current economic environment, any theories or reasonings which might assist in forming a view on the future, the sponsoring employer's intentions towards salary rises, recruitments, etc. The nature of the liabilities, current and future investment policy and the current funding level may also influence the decision.

Apart from the funding level, no elements are derived from the measurement of the output. Some might argue that the past experiences may be derived from the analysis of surplus, and thus they are from the output. However, the past experiences themselves should be obtainable directly from the accumulated record of inputs.

So, as a conclusion, setting the best estimate may not be a part of a closed-loop feedback system designed to stabilize the financial state of the fund.

6.5.2.2 Margins in the basis

Now, we shall discuss the second part of the valuation basis.

The valuation basis may be deliberately different from the best estimates of the likely future events. Here, the differences are separately treated as the margins.

It may be possible to change the margins included in the basis as a result of the financial state of the fund.

For example, if *the fund is assessed to be deficient in assets compared with the liabilities*, the margins might be increased in order to make the contribution rates higher and hence to increase the size of the fund.

The margins are a part of the funding strategy, and may be considered together with the funding method. So, the same discussion as for the funding method in section 6.4.3 would be applicable, although changes of the margins may be more practicable than changes of the funding method.

As a conclusion, the choice of the level of margins may be regarded as a type of open-loop control.

6.5.3 Systematic approach

In section 6.5.2, we discussed the framework which might be employed to describe the process of setting the valuation basis and of influencing the financial state of the fund.

Now, we shall consider how we can set an optimum valuation basis in the light of various objectives of managing a pension fund.

A valuation basis is set not only referring to the past experience and the current environment, but also making various subjective judgements. The past experience and the current environment may be expressed numerically and treated objectively. However, the impacts of the past experience and the current environment on the future course of events may be so complex that a simplification would be required. Furthermore, the future course of events is affected by random factors which cannot precisely be predicted from the past and current experience. Therefore, the best estimates of the future course of events may not be set in a satisfactory manner only from the objective data, but some judgements regarding the simplification and the randomness are required. Such judgements may be based on well-established economic theories, but adopting particular theories itself is a subjective judgement. Also, the margins may reflect various considerations, and the process of setting this part of the basis may include substantial subjectivities. Therefore, subjective elements are a very important part of setting a valuation basis in practice.

Such subjectivity may not be eliminated even when stochastic approaches are adopted to assess the degree of risk explicitly. Firstly, the level of the risk acceptable needs to be specified; this is a subjective judgement. Also, a stochastic approach is based on the past and current data, and the data are applied to esssess the likely future according to statistical theories and any associated assumptions. Here, the decision to follow a particular statistical theory and to accept any associated assumptions is a subjective judgement, too.

When we explore the optimum way of setting the basis, one approach of investigation would be to formulate various ways of setting the basis, and the consequences of each way of setting the basis can be tested on a model through cash flow projections or mathematical analyses.

As discussed above, in setting the basis, subjective elements have important roles. Although this subjectivity may not necessarily deny the value of systematic methods for setting the basis, it would be difficult to formulate the actual process in use. So, this approach may be difficult to apply in practice.

As an alternative, various simplified systematic methods may be considered and tested. Each simplified method may not represent fully the actual process of setting the basis; particularly subjective elements may be omitted or restricted to a minimum in number. The knowledge from the tests on such systematic methods may assist in the setting of the valuation basis in practice. Firstly, if a systematic approach seems to give good results in the light of the objectives of the fund, the systematic approach itself might be adopted. Secondly, a basis obtained from such a systematic method could be a starting point for setting the basis, and the actual basis may be obtained by modifying this reflecting subjective elements. Finally, comparisons between the possible impacts of various methods of setting the basis may give a guide to the possible consequences of certain modifications in the valuation basis, and these would be useful in deciding the type and degree of modification in the valuation basis.

The discussion in section 6.5.2 would be very useful for formulating such simplified systematic methods.

An obvious system to be adopted as a model is the averaging of the past experience over a certain period. The averaging may be made in various ways: for example, using different weightings on each year's experience, geometric or arithmetic averages. Loades(1992) uses exponential smoothing which is a modified arithmetic average. This allows higher weights to be placed on the more recent experience.

There may be more sophisticated systems taking into account the current market conditions, or systems which delay the timing of the change of the basis.

As a conclusion, the creation of systematic and objective approaches for setting the valuation basis and investigating the impact of the basis would be useful when considering the control of a pension fund.

Chapters 7 and 8 will present an attempt to find a reasonable and objective process for setting the valuation basis and analysing the impact of different methods of setting the valuation basis on the finances of a pension fund.

Chapter 7 Overview of Projection Model

7.1 Introduction

We have discussed various aspects of controlling a pension fund in Chapter 6. In particular, the setting of the valuation basis was noted as an important means of control.

There are many works on the choice of the valuation basis. [see Colbran(1982), Lee(1986), Parsons(1990), Thornton & Wilson(1992a) and also section 6.5.1]

Benjamin(1990) and Loades(1992) have investigated the setting of a basis where only the real investment returns change from year to year. However, it was felt that there is a scope for basic research in this field, and an investigation which aims to obtain information on the choice of the basis and the impact to the scheme finance has been planned.

The investigation looks into the choice of the basis under changing environments. The outline of the investigation is given in this chapter. The summary of the results will be presented in Chapter 8. Some details are left to Appendices.

7.2 Purpose and method of research

7.2.1 Purpose of research

From the discussions in section 6.5, we have learned that control using the valuation basis can be interpreted as an open-loop control system. However, the actual process of setting the basis may involve various subjective judgements, and the process is difficult to be expressed in a set of formulae. This stems from the fact that the future cannot be predicted precisely. The data available consist only of the past experience and the current economic outlook. These data by themselves do not lead to the prediction of the future course of events. So, the best estimate of future events can only be made by taking a view.

Also, owing to this unpredictability of the future, some margins may be included in the basis. However, the extent of the margins may only be decided subjectively. (see section 6.5.3)

If the best estimate has changed since the last valuation, the setting of the basis involves the decision as to what extent this change should be incorporated. To obtain a stable contribution rate, the extent may be limited, while a larger change might be suggested from the objective of maintaining solvency. Deciding the balance between them is a subjective matter.

Some mechanical methods for the setting of the basis may be created and the possible consequences of adopting such methods may be investigated in order to assist the setting of the basis. A valuation basis created by such a mechanical method may be used as a starting point for the setting of the basis. This basis may be adjusted reflecting various additional factors, some of which might be subjective. These adjustments may be made referring to the knowledge from investigations on the possible impact of the adjustments.

An example of such a mechanical method would be the use of the averages of the last n years' experiences as an assumption for each item of the valuation basis. Here n might be 5, 10, or any other figures. It is also possible to include margins automatically; for example, the investment return assumption may be set as the five year average less 1%.

Here, it must be noted that, although such formulae may create a basis without subjective judgements, the process of setting the formulae can be subjective. In the above examples, the choice of the length of averaging period and the extent of the margin might be determined in an arbitrary way. Also, the choice of a particular mechanical method from all the possible methods could be a matter of subjective judgement.

However, projections on a model would be carried out to investigate the possible consequences of the use of such a method of setting the assumptions. Based on the results of the investigation, an actuary might be able to judge if such an approach were appropriate, or to choose the most suitable approach from various candidates in the light of the financial objectives of the fund.

Hence, it may be worthwhile to consider some possible mechanical methods for setting the basis and the behaviour of each as a control system. These are the final aims of this investigation.

7.2.2 Approach of research

For an investigation of a control system, it may be more common to specify a control system, and check the responses under various test signals. However, this approach may not fulfil the first aim above; to create an objective system. Hence, a backward approach has been chosen. The research does not assume a particular control system. Under various test signals, various responses are tried, and the response which an optimum control system ought to give is identified for each signal. This approach was taken because this may lead us to some unknown, but effective system for setting the basis.

Based on these projections, optimum systems may be suggested as a conclusion to this investigation.

The discussion in section 6.3.1 suggests that the investigations of the work of pension funds may be better done by numerical cash flow projections rather than mathematical approaches. So, an asset liability model has been created, and long term cash flow projections have been carried out under various situations. The term of projection has been set as 100 years to allow long term effects to be observed.

Also, the projections are deterministic. It would be possible to take a stochastic approach. However, to achieve the abovementioned aims, it has been considered more important to understand the interaction between the changes in the external environment and the system for changing the basis. For this purpose, a deterministic approach is more suitable. Also, should a stochastic approach be adopted, to limit the volume of computation, the model may need to be simplified further, or the range of the investigation may need to be narrowed. In the future, as an extention to this research, stochastic approaches might be applied.

7.2.3 Appraisal of results

According to the discussion in section 6.3.2, the output from the pension fund control system should be the current financial state of the fund, and the stabilisation of the contribution rates may not be the prime purpose of the system. This argument should be applicable to open-loop systems as well as closed-loop systems.

However, the financial state of the fund may not be expressed in a satisfactory way. The financial state could be expressed as the valuation results, but there is a question of the selection of the basis. Because we are investigating how to set the basis, the valuation basis used in the projections may not be appropriate to represent the likely future course of events. Further, the future is of course unknown, and there are no appropriate measures of the financial state which are dependent on the future experience.

So, the projection results are mainly expressed in terms of the Recommended Contribution Rates. The on-going solvency is indirectly assessed by the long term changes in the contribution rate. For example, if the contribution rate continuously increases without any signs of convergence, it may be interpreted that the fund is insolvent.

Although the Recommended Contribution Rates are used to express the projection results, this does not mean that control systems with the recommended contribution rate as the output are in consideration. It should be noted that the movements in the Recommended Contribution Rates are only treated as a proxy to measure the progress of the financial state of the fund.

7.2.4 Coverage

The finances of a pension scheme are affected by various factors: investment returns, salary growth, demographic experience, and so on. In an actual situation, these factors may move concurrently. If the investigation needs to deal with such concurrent changes, the complexity of the pension fund means that the model would be too complex to analyse and derive useful results.

So, as a first step, movements in only one factor are examined at a time; movements in the investment returns in excess of the rate of salary growth and those in excess of the rate of pension increase, movements in the market dividend yield of equities, and movements in the rates of withdrawals are each investigated separately.

These three are chosen because they may be the most significant factors influencing the finances of a pension fund, and they are expected to have very different characteristics from each other.

The investment returns in excess of the rate of salary growth and those in excess of the rate of pension increases significantly affect the eventual cost of providing the benefits. The assumptions on these elements are dominant factors for the value of the liabilities and the contribution rates.

The market dividend yield and the assumption on the dividend yield affect the value of assets which are held as, or deemed to be held as, equities under a discounted cash flow approach. In the UK, it is not unusual to use a discounted cash flow approach with a significant part, sometimes 100%, of the assets valued as equities. So, the impact of the dividend yield on the finances of a pension fund may be significant.

Withdrawals affect the fund in several different ways. Firstly, the value of the benefits is usually smaller than the value of the projected accrued benefits for the withdrawing members, and the fund may gain from the withdrawals. Secondly, the withdrawals may take out transfer values, and the cash flow of the fund would be affected. Thirdly, the withdrawals will change the membership structure, and the contribution rates might be affected. Although, in the UK, the significance of the first point has been reduced by recent statutory improvements in the benefits paid to withdrawing members (see Fenton, Ham & Sabel(1993)), the combined effect of withdrawals on the finances of the fund cannot be overlooked, and is distinctive from the effect of the investment returns or the equity market dividend yield. Also, the impact of other decrements may share some common features, and the study of withdrawals may give some insights here.

Further details of the investigations in these areas will be described in section 7.5.1.

There are other factors affecting the finances of the fund; for example, pre- and post-retirement mortality rates, early retirement rates, the level of new entrants and the proportions married. Their impacts may be investigated as a supplement. Also, the effects may not be simply cumulative because of the possible interaction and dependence between the factors. So, the combined effects may need to be investigated as the next step in future researches.

7.3 Notation

In order to describe the model and results precisely, the following notation is used throughout Part IV, i.e. Chapters 7 and 8 and Appendices 1,2 and 3. (In Chapter 6, no mathematical formulae were used, and this notation is not relevant.)

The time is measured by the number of years passed from the origin. At the beginning of a scheme year, the time is expressed by an integer. In order to describe the timing of events accurately, T-0, T and T+0 are used to express different timings.

T-0: The end of year T-1, but before any end year cash flows; benefit payments and dividend receipts.

T: The end of year T-1, when the valuation takes place.

T+0: The beginning of year T after receiving contributions.

D(t): Amount of the dividend received at time t

M(t): Market value of the assets held at time t

C(t): Contributions received at time t.

B(t): Benefits paid at time t.

I(t): Rate of return achieved during the period (t-1,t) on the market value basis.

MV(T): Market value (ex-dividend) of a specific asset at time T excluding dividends at time T

Div(T): Dividend receivable at time T

i"(t): Market interest rate at time t. The long term interest rate in the market

g"(t): Market expectation of dividend growth at time t

d"(t): Market dividend yield at time t. For the precise definition, see section A1.3.3.

p"(t): Market expectation of long term future price inflation at time t

g'(t): Actual dividend growth from t-1 to t

p'(t): Actual price inflation rate from t-1 to t

e'(t): Increase in general level of salary from t-1 to t

ss'(t): Promotional salary increase experienced between t-1 and t

w'(t): Rate of withdrawals experienced between t-1 and t

VM(t): Assessed value placed on M(t)

VD(t): Assessed value placed on D(t)

VC(t): Assessed value placed on C(t)

c(t): Conversion factor of the market value to the assessed value: i.e,

$$VM(t) = c(t) * M(t)$$
$$VD(t) = c(t) * D(t)$$
$$VC(t) = c(t) * C(t)$$

I'(t): Rate of return achieved during the period (t-1,t) on the valuation basis.

i(t): Interest rate assumption for the valuation

g(t): Dividend growth assumption for the valuation

d(t): Dividend yield assumption for the valuation

p(t): pension increase rate assumption in payment and deferment for the valuation

e(t): salary inflation rate assumption for the valuation

ss(t): promotionary salary increase assumption for the valuation

w(t): withdrawal rate assumption for the valuation

VM^(t): assessed value on an alternative basis

g^(t): the alternative dividend growth rate assumption

d^(t): the alternative dividend yield assumption

VSp(t): the valuation surplus arising from the change of the basis from the original one to the alternative one.

f(y): Rate of reduction in the retirement pension for a member who withdrawed at age y l(y): Survival function at age y

dw(y): Number of withdrawals at age y

V(y): Value of liabilities at age y exact

Where there is no fear of confusion, (t) may be omitted.

7.4 Model

The investigation was carried out on the asset liability model described below.

7.4.1 Benefits

The benefits provided under the model scheme are;

- On retirement at age 65, retirement pensions of 1/60 per year of service times final salary are paid.
- On withdrawals from the scheme, deferred pensions based on the same scale above are given.
- Pension are paid annually in advance, and the pensions in payment and in deferment are increased in line with price inflation.
- No other benefits are paid. As stated in section 7.4.2 below, no other decrements are modelled.

In fact, on withdrawals or retirement, all the pension rights are assumed to be transferred out. The transfer values are calculated on the basis of the prevailing market interest rate, the market expectation for inflation and with PMA80B35 (see Continuous Mortality Investigation Bureau(1990)) for the mortality rates after age 65. Zero mortality before age 65 is assumed.

7.4.2 Membership

Initially, a stationary population is assumed under the following basis;

-There are fixed numbers of new entrants at age 25. The initial salary is increased at 6.5% per annum

-The salaries are increased at 6.5% per annum as the salary inflation, plus 1% per annum as the promotional element

-The withdrawal rate is 5% uniform at all ages.

-No other decrements occur.

-At age 65, all the remaining members retire.

The salary inflation rate may vary, but the promotional element remains the same.

The withdrawal rate may vary, but a uniform rate is always applied to all ages.

So, if the withdrawal rates stay the same, the population structure will become stationary, except that certain variables will be increasing in line with salary inflation.

7.4.3 Investments

Initially, the funding level is 100% of the projected accrued liabilities under the standard basis. (see section 7.4.5)

A model portfolio of 100% equities is assumed, and the assets held are assumed to behave exactly in the same way as the whole market.

7.4.4 Economic environment model

The whole equity market consists of only one type of shares, and all the shares are identical. The dividends are paid at the end of each year (just before the valuation).

Regarding the economic environment, 5 variables are given as inputs for each year;

-market interest rate at the end of each year,	$i''(T)$
-market expectation for price inflation at the end of each year,	$p''(T)$
-market expectation for the average dividend growth at the end of each year,	$g''(T)$
-actual price inflation rate during each year,	$p'(T)$
-actual dividend growth during each year,	$g'(T)$

The market dividend yield, $d''(T)$, is derived by the following formula (See A1.3.3.4 and A1.3.4.1.);

$$d''(T) = (1 + i''(T)) / (1 + g''(T)) - 1$$

A single market interest rate and a single expected dividend growth rate are used.. Also, a single expected inflation rate is assumed. These market related variables may not be readily recognisable in the real world. However, for theoretical simplicity, the model assumes that these variables exist and are measurable.

The model is potentially capable of using different figures for the inflation experience during a particular year and the expected inflation rate at the end of the year. However, for this investigation, the use of the model is restricted to reduce the complexity, and these factors are assumed to be the same all the time. That is, the market expectation for future inflation is exactly the same as the inflation just experienced during the last 12 months. The same applies to the dividend growths. That is,

$$p''(T) = p'(T)$$
$$g''(T) = g'(T).$$

The market price of a specific asset (equity) at the end of each year, T, is determined by the formula;

$$MV(T-0) = Div(T) * (1 + i''(T)) / (i''(T) - g''(T)) \qquad .....7.4.4.1$$

where

MV(T-0):	the market price of a specific asset at the end year T including Div(T)
Div(T):	the dividend just received on the specific asset(equity). (refer to Corollary A1.3.4.2.)

The actual investment return during each year is calculated from the factors above.

7.4.5 Valuation

A valuation takes place every year using a specified basis.

The funding method is the Projected Unit Method. The surpluses/deficits are spread over *the remaining* working lifetime of the existing members. At each valuation, the surpluses/deficits are respread. The assets are valued on a discounted cash flow approach assuming 100% of the fund is invested in equities.

The valuation basis consists of the following;

-	Rate of interest,	i(T)
-	Dividend yield,	d(T)
-	Dividend growth,	g(T)
-	Pension increase rate in payment and deferment,	p(T)
-	Salary inflation,	e(T)
-	Promotional salary increase,	ss(T)
-	Withdrawal rate,	w(T)

The pension increase rate is equal to the rate of price inflation.

Here, it is assumed that the following holds;

$$1 + i(T) = (1 + g(T)) * (1 + d(T))$$

Therefore, if two of the three assumptions, i(T), g(T) and d(T), are set, the other assumption is automatically obtained from this formula.

For the exact timing of each cash flow, see Appendix 2.

There is a standard basis;

i	=	9%
g	=	5%
p	=	4.5%
e	=	6.5%
ss	=	1%
w	=	5%.

The initial amount of assets is calculated so as to achieve 100% funding level under this standard basis.

The basis used in the projections may depart from the standard basis.

7.4.6 Input

As each years' experience,

-market interest rate at the end of each year,	$i''(T)$
-market expectation for price inflation at the end of each year,	$p''(T)$
-market expectation for the average dividend growth at the end of each year,	$g''(T)$
-actual inflation rate during each year,	$p'(T) = p''(T)$
-actual dividend growth during each year,	$g'(T) = g''(T)$
-actual salary inflation during each year,	$e'(T)$
-actual withdrawal rate during each year (a uniform rate is assumed over all the ages),	$w'(T)$
-actual promotional salary increase,	$ss'(T)$

As each year's valuation basis,

-	Rate of interest,	$i(T)$
-	Dividend yield,	$d(T)$
-	Dividend growth,	$g(T)$
-	Rate of price inflation,	$p(T)$
-	Salary inflation,	$e(T)$
-	Promotional salary increase,	$ss(T)$
-	Withdrawal rate,	$w(T)$

7.4.7 Output

Full details of the cash flows and valuation results on the current and initial valuation basis are available. However, as discussed in section 7.2, our attention has been principally directed to the Recommended Contribution Rates.

7.5 Range of investigation

The model described in section 7.4 simplifies the real operations of a pension fund. However, the model is still complex, and a thorough investigation of the model may not be practicable because of the limitation in the time and resources available. So, the investigation was restricted to a certain area. The following summarizes the range covered in this investigation.

7.5.1 Area of investigation

The model has many elements which can be variable inputs. As mentioned in section 7.2, the investigation was limited to the areas as follows;

- the movements in the real investment returns,

  that is the investment returns in excess of the rate of salary growth and those in excess of the rate of pension increase,
- the movements in the dividend yield, and
- the movements in the rates of withdrawals.

Each factor was investigated separately, and no investigations on the interactions between the factors were made.

7.5.1.1 Movement in real investment returns

In this set of projections, various patterns of changes in the actual investment returns in excess of the rate of salary growth and those in excess of the rate of pension increase are assumed. Under each pattern of changes in the experience, the effects of a range of ways of modifying the valuation basis are examined.

There are several ways to investigate the effect of the movements in the real investment returns;

- investment returns can be varied,
- rates of salary growth and pension increase can be varied, or
- the combination of the two.

The last option may represent the real situation more closely than the others, but the added complexities may not be accompanied with any enhancement in the information obtained from the projections.

The first option may be a natural approach. However, the model does not express the investment returns in a simple way. The actual investment returns are obtained from the interactions between the dividend growth during a specified period and the market dividend yield at the beginning and the end of the period. Also, the assessed value of the assets may be different from the market value. So, in order to change the investment returns, various elements may need to be changed, and any subsidiary effects need to be checked.

Also, the transfer values paid to the withdrawing members are assumed to be based on the market interest rate. So, if the movements in the investment returns is accompanied by changes in the market interest rate, there would be impacts on the transfer values, and a further level of complexity may be added.

Hence, to avoid these complexities, the second option has been adopted. Changes in the investment returns in excess of the rates of salary growth and those in excess of pension increase are realised by changing the rates of salary growth and pension increase while the rate of actual dividend growth and the market dividend yield remain unchanged.

The dividend growth rate may have a strong correlation with the rate of salary growth or the rate of price inflation (which is equal to the rate of pension increase in this model). The effects of the changes in the rates of salary growth and pension increase and accompanying changes in the dividend growth rate may be discussed by combining the results of the analyses of each factor. The investigations on the changes in the dividend growth rate will be discussed in the next section (7.5.1.2).

To make the situation simpler, the relationship between the actual salary growth rate and the actual price inflation rate was kept fixed. In particular, it is assumed that $(1+e')/(1+p')$ is constant at $1.065/1.045 = 1.0191387$. So, if e' is increased, p' is also increased so as to keep this ratio constant. Also, the promotional salary increases were kept at 1% per annum.

For simplicity of the investigation, the valuation basis is also assumed to maintain the ratio $(1+e)/(1+p)$ fixed at the same value 1.0191387, and s is kept fixed at 1% per annum.

So, the only independent variables in this part of the investigation are the actual salary growth rate, e', and the salary growth rate assumption, e.

7.5.1.2 Movement of dividend yield

This part of investigation deals with the effects of the changes in the equity market level. Different patterns of movements in the market dividend yield are assumed, and the effects of various patterns of changing the valuation basis are examined.

There are no impacts on the amounts and the timings of the benefit payments.

The model requires each year's market interest rate, i", and market dividend growth expectation, g", as inputs. The market dividend yield, d", is derived from the market interest rate, i", and the market dividend growth expectation, g" by the formula (See Corollary A1.3.4.1.);

$$d''(T) = (1 + i''(T)) / (1 + g''(T)) - 1$$

The impact of the changes in the real investment returns have been dealt with by the set of projections discussed in section 7.5.1.1. The projections in this section have been designed to be as independent as possible from the earlier ones, and the market interest rate has been kept constant. A rise in the market dividend yield is, thus, accompanied by a fall in the market dividend growth expectation.

Here, from the calculations in Appendix 1 (see A1.4.1.4.), the rate of investment return on the market value basis from time T to T+1, I(T+1), is expressed as follows;

$$1 + I(T+1) = (1 + g'(T+1))(1+d''(T+1))\, d''(T) / d''(T+1)$$

Because $g''(T+1) = g'(T+1)$ (see section 7.4.4),

$$1 + I(T+1) \quad = \quad (1 + i''(T+1))\, d''(T) \,/\, d''(T+1)$$

That is, the real investment return is affected by the move in the market dividend yield. [For further discussion, see Appendix 1.]

Now, if the dividend yield assumption for the valuation stays constant, the rate of return on the valuation basis, I', will be expressed as follows (See A1.5.1.2.);

$$1 + I'(T+1) \quad = \quad (1 + g'(T+1))(1 + d''(T+1))$$

$$= \quad 1 + i''(T+1)$$

That is, the change in the market price level does not affect the return on the valuation basis. However, the market dividend yield, or the market interest rate, affects the return as shown in the formula above. Also, cash flows affect the fund, and this part of the projections deals with such effects, and possibilities for changing the dividend yield assumption. Other assumptions have not been changed. So, the value of the liabilities is not affected at all. Also, the Standard Contribution Rates are not affected, and remain at the same constant level. In the projections, only the value of the assets, and consequently the adjustments of contribution rates for amortisation are affected.

7.5.1.3 Movement of withdrawal rate

This part of the projections deals with movements in the withdrawal rates and changes in the withdrawal assumptions.

The model assumes that all the withdrawals immediately take transfer values. So, there are no deferred pensioners' liabilities left in the fund. Also, the transfer value is the cash equivalent of the preserved pension benefit based on the market interest rate, which is equal to the valuation interest rate and is constant in this part of the projections. Thus, the interactions between the movements in the withdrawal rates and the market interest rate levels have not been investigated here.

The main focus of these projections is on the level of surpluses arising from the withdrawals and the consequent changes in the population structure.

7.5.2 Patterns of movements

The total of 123 projections (41 for each area of investigation) are described in Appendix 3; however, further projections have been carried out, but have not been reported on in detail.

These projections are designed to examine various combinations of patterns of experiences and choices of valuation bases in each area. The same set of combinations is applied to all three areas of investigation to enable comparisons between the results to be made.

The combinations used are described in the following sections (7.5.2.1 and 7.5.2.2.)

7.5.2.1 Pattern of experience

As discussed in section 7.2, the projections are carried out for various input signals. The signals used are;

-No Change

The experience is always equal to the standard experience, which is the same as the standard valuation basis.

-Step (One permanent change)

The experience in year one is different from the standard experience, and the same is experienced throughout the projections.

-Ramp (Gradual straight line change)

The experience in year one is different from the standard experience. From year two onwards, the divergence escalates at a constant pace throughout the projections.

-Cyclical (Sine curve)

The experience changes cyclically around the standard experience. The period of one cycle is 20 years. Here, the changes in the experiences do not exactly follow sine curves; instead, the cycles consist of piecewise linear elements, which are a 5 year increasing element, a 10 year decreasing element and another 5 year increasing element, as an approximation to the sine curve.

Originally, 10 year cycles were also examined. However, the preliminary results showed no distinctive differences from 20 year cycles, and the results for 10 year cycles are not presented in the thesis.

In section 6.2.2, unit impulses and parabolic changes were also mentioned as typical test signals.

A unit impulse may be expressed in the model as a temporary departure of the experience from the standard one in a particular year. Possible reactions to be taken would be either to keep the valuation basis unchanged or to take an average of past experiences as the basis. A series of projections might be able to reveal which option is more preferable in the light of particular funding objectives.

However, such temporary fluctuations may not affect the long term expectations of the future experiences, and a change in the valuation basis may not be considered to be a practical action. Such fluctuations may be better dealt with by the amortisation of surpluses/deficits. So, the inclusion of unit impulses was not thought essential. To reduce the volume of calculations which are already very large, unit impulses have been excluded from the scope of this research.

Parabolic changes may be important if the impacts were to be investigated mathematically. However, through cash flow projections, the differences between the impacts of ramps (gradual straight line changes) and those of parabolic changes may not be significant in terms of quality although quantitative impacts may be different. So, parabolic changes have also been excluded.

Here, the direction of the movements in the experiences is limited to one. For example, a gradual reduction in the salary growth is investigated, but not gradual increases. The results from the other direction are expected to be the reverse, and to save the amount of computation, such projections have been omitted.

7.5.2.2 Pattern of valuation basis

To each input test signal, various responses as changes in the valuation basis are tested. As stated in section 7.2, one of the aims of this investigation is to find out optimum responses to particular changes in experiences. So, it is desired that as many patterns of changes in the valuation basis as possible should be investigated. However, the volume of calculations has to be limited, and only a small number of patterns can be tested. To make the investigation effective, the patterns should be chosen from a wide range of possibilities.

As an answer to this requirement, the use of typical test signals is considered (as in section 7.5.2.1 for the actual experience). The choice of a valuation basis may be regarded as an input signal to the pension fund. So, the patterns of changes in the valuation basis may be represented by typical test signals. So, the projections are carried out for each combination of typical test signals for the actual experience and the valuation basis.

However, some combinations have been excluded because they are obviously unsuitable combinations. For example, the combination of an unchanged experience and a sine curve change in the valuation basis.

On the other hand, there can be minor variations in some of the combinations. For example, we consider the combination of step changes in both the experience and the valuation basis. The change in the valuation basis can be immediate or can be delayed. The magnitude of the change in the valuation basis can be the same as or different from the change in the experience. So, a series of variations have been prepared for each combination.

Also, setting the basis as an average of past experiences is an important option, but may not be covered in the approach above. So, averaging has been added as a pattern of reactions where appropriate. (Taking an average of past investment returns as the valuation interest rate has been investigated in Benjamin(1984 and 1989) and Loades(1992).)

Here, changing the frequency of the valuations and the method of amortising the surpluses/deficits might have been added to this investigation. However, the volume of computation which is already large has meant that these aspects have been left for future researches.

Further investigations may be needed to examine the effects of any other reasonable reactions not covered in this research.

In the following, an outline of the combinations and variations investigated are described. A full description is made in Appendix 3 with brief comments on each result. A corresponding set of the projections have been carried out in each area described in section 7.5.1, namely the movements in the rate of real investment returns, the dividend yield and the withdrawal rates.

7.5.2.2.1 Unchanged experience

Only constant valuation bases are examined. The standard basis is applied to show the cash flow projection of a stationary state, and as a basis for comparisons with other projections. In order to examine the cumulative effects of the differences between the experiences and the valuation basis, the constant valuation bases which have higher or lower salary growth rate/dividend growth rate/withdrawal rate assumptions than the experience are also tested.

7.5.2.2.2 Step (One permanent change in experience)

The patterns of change in the valuation basis tested are: Constant, Step and Ramp.

As a constant valuation basis, only the standard valuation basis is examined.

For step changes in the valuation basis, variations in the timing and magnitude are examined.

For ramp changes in the valuation basis, variations in the magnitude and the frequency of the changes are examined.

The averaging case shows a gradual change of the valuation basis from the standard basis to the new experience level after the step change in the experience has occurred. So, the averaging case is a special case of ramp changes. The difference from the genuine step changes in the valuation basis is that the valuation basis will remain stable after reaching the new experience level in the averaging case. The investigation is mainly carried out for the averaging case because the genuine step changes may not be practically important. The difference in the length of averaging period corresponds to the magnitude of each year's change. For completeness, a projection under which the basis continue to change even after reaching the experience level has been added.

7.5.2.2.3 Ramp (Gradual change in experience)

The patterns in the change in the valuation basis tested are: Constant and Ramp.

As a constant valuation basis, only the standard valuation basis is examined.

For ramp changes in the valuation basis, variations in the timing, magnitude and the frequency of the changes are examined.

The averaging case is a special case of ramp changes. Because the experience is changing monotonically throughout the projections, in the averaging case, the magnitude of each year's change in the valuation basis is the same as the changes in the experience after an initial period. During the initial period, the paces of the changes in the valuation basis and the experience are different because the experience before the commencement of the projections is assumed to be constant at the standard level. The longer the length of the averaging period is, the longer the initial period would be. Hence, the averaging case and genuine ramp changes are different during the initial period. To investigate the differences, some averaging cases are examined in addition to genuine ramp changes in the basis,

7.5.2.2.4 Cyclical change in experience

The patterns in the change in the valuation basis tested are: Constant and Cyclical.

As a constant valuation basis, three cases are examined: the standard valuation basis, the highest experience as the basis and the lowest.

For cyclical changes in the valuation basis, variations in the magnitude and the frequency of the changes are examined.

The averaging case is a special case of cyclical changes. Some projections are made for three different averaging periods.

Chapter 8 Summary of Projection Results

8.1 Introduction

Here, the main results from the series of cash flow projections are summarised. For more details of the projection results, the readers are referred to Appendix 3, which provides the notes on all the projections carried out with the graphs of contribution rate movements. The details of the assumptions made and the calculations are described in Chapter 7 and Appendices 1 and 2.

Three different areas; the real investment return changes, the dividend growth changes and the withdrawal rate changes are discussed separately in sections 8.2, 8.3 and 8.4. Each section includes the general analysis of impact and a summary of the results from the projections.

Section 8.5 provides some concluding comments on the whole set of projections. In particular, common features observed in all the areas are summarised and discussed.

In this chapter, the notation defined in section 7.3 is used.

8.2 Real investment return changes

8.2.1 Basic impact

8.2.1.1 Ultimate cost

The real investment return changes are achieved through the changes in the salary growth rates and the price inflation rates while the nominal investment returns are unchanged (as described in section 7.5.1.1).

Although the total salaries for each age are affected by the changes in the salary growth rates, the relative sizes between different ages are not affected because the same salary growth rate is applied uniformly to all the members. Also, the new entrants' salary level follows the same growth rate. So, the relative membership structure is not affected although the absolute size is affected.

Where the salary growth is low permanently, the eventual final salaries would be low, and the amounts of the benefits paid in the future, which are related to the final salary, would be reduced. So, a low salary growth experience leads to a low cost to the pension fund. The ultimate cost for a particular member is dependent on the total salary growth experienced by the member from joining the scheme to the cessation of his/her active membership. Short term fluctuations are less important.

Lower price inflation reduces the transfer values to withdrawing members and the costs of buying out pensions for retiring members because they are based on the market expectation for inflation at the times of exit in this model. So, the reduction in the market inflation rate would reduce the cost of providing benefits promptly.

There is no impact on the assets held except the impact through the changes in the level of benefit payments and the changes in the contribution rates.

In summary, the reductions in the salary growth and price inflation rates changes the relative sizes of assets and liabilities; the former is not affected while the latter is reduced. Thus, the ultimate cost of providing benefits is reduced.

8.2.1.2 Gap between experience and assumption

If the salary growth experience during a particular period is lower than the assumption, surpluses arise. This is because

-the value of the liabilities at the end of the period will become less than anticipated, and

-the benefits to be paid to the retiring and withdrawing members during the period are less than anticipated.

Here, a low salary growth may not have to be permanent to produce surpluses. Only one year's low salary growth can cause surpluses.

If the market price inflation expectations at the time of the withdrawals and retirements of the members are lower than the assumption, surpluses arise. Because there are no pensions in payment and in deferment in this model, the reduction in the price inflation rate has no impact on existing members' liabilities.

Where the relationship between the experience and the assumptions is reversed, deficits arise. The surpluses/deficits arising are proportionate to the width of the gap between the experience and the assumptions.

In the model, the ratio of salary growth and price inflation is kept constant for both experience and assumptions. So, the impact from the changes in salary growth and that from the changes in price inflation described above is observed simultaneously.

8.2.1.3 Valuation surplus/deficit

If the salary growth assumption is reduced, the value of the liabilities is reduced. The value of the assets is not affected by the change, and the reduction in the value of liabilities appears as a valuation surplus. The same applies to a reduction in the price inflation assumption.

Also, the standard contribution rate is reduced reflecting a lower expected ultimate cost of providing benefits.

The surpluses from the changes in the valuation basis are more significant than those from the gap between the experience and the basis. If the salary growth experience in a particular year is 1 percent lower than the assumption, the liabilities at the end of the year is based on the 1 percent lower salaries than anticipated, and the reduction in the value of the liabitities compared with the anticipated value would be only about 1 percent of the total liabilities.

However, if the salary growth assumption is reduced by 1 percent, the liabilities are assessed assuming that the future salary growth until the final exit from active membership would be lower by 1 percent per annum. So, the impact is cumulative during the remaining service life time. The average remaining service life time may well be more than 10 years, and the surpluses arising may be 10 times or more larger than the surpluses above.

The same cumulative effects are seen for the price inflation assumptions during the period of deferment and payment.

Combined with the reductions in the standard contribution rate, which are also affected cumulatively, the impact of the changes in the valuation basis would be significantly larger than those arising from a gap between the experience and the assumptions.

However, if the gap were very large, the impact of the gap can exceed valuation surpluses/deficits. For example, if the experience gradually but endlessly changes in one direction while the valuation basis is unchanged, the impact of the gap may be negligible initially, but would grow very large and dominate the situation later.

8.2.2 Unchanged experience

Refer to section A3.1.1 and graphs A3.1.1.a-c in Appendix 3.1.

The experience remains stable and is equal to the standard experience. Three different bases which are constant throughout the projections have been applied. One basis is equal to the standard basis, and the experience exactly follow the basis. Another basis has a larger real interest rate than the experience, and the other has a smaller real interest rate.

Because the experience is stable, and the membership structure is stationary, the use of any constant basis leads to a stationary fund after the impact of the initial surpluses/deficits are settled down.

The higher the real interest rate assumption is, the lower the stationary funding level will be. (A higher real interest rate assumption will result in perpetual accrual of deficits from the gap between the assumption and the experience. At a stationary state, the accruing deficits are met by additional contributions in excess of the Standard Contribution Rate. This implies a lower funding level than 100%.) Where the valuation basis is different from the experience, the level of the stationary fund also depends on the pace of amortisation of surpluses/deficits. It is obvious from a simple calculation that the slower the pace is, the larger the accumulated surpluses/deficits will be. Also, the quicker the amortisation is, the quicker a stationary state is achieved.

In terms of the objective to achieve stable contribution rates, all the valuation bases satisfy this.

However, the funding level under the standard valuation basis might be preferred. The use of the standard valuation basis, which is equal to the experience, eliminates surpluses/deficits, and the fund is equal to the actuarial liabilities under the funding method employed. So, this is most in line with the funding method chosen. Should the constant experience continue, the actuarial liabilities under the standard valuation basis would represent the realistic cost of providing the promised benefits. So, if the fund is equal to this amount, the financial state of the fund may be said to be satisfactory.

On the other hand, the use of a lower real interest rate assumption may be acceptable in order to provide deliberately extra security of benefit payments through a higher level of fund. However, it is not easy to identify the optimum level of margin because overfunding might cause practical difficulties.

8.2.3 One permanent change in experience

The actual real investment return rises at the beginning of the projections, and remains at the higher level thereafter.

Here, the realistic value of the benefits may be the actuarial liabilities calculated under the basis with the real interest rate assumption which is equal to the actual real investment return after the change. We shall call this level of fund the optimum level.

A constant valuation basis, one permanent change in the basis, and gradual changes in the basis have been examined.

8.2.3.1 Constant basis

Refer to section A3.1.2.1 and graph A3.1.2.a in Appendix 3.1.

The standard basis is applied to see the impact of the change in the experience without any moves in the basis. The projection is done as a basis for comparison with other projections which use changing bases.

If the real interest rate in the valuation basis is not increased, the funding level will eventually settle down to a higher level than the funding level achieved by the 'follow the experience' case in section 8.2.3.2, which may be regarded as the optimum level.

8.2.3.2 One permanent change in basis

Refer to section A3.1.2.2 and graphs A3.1.2.b-g in Appendix 3.1.

Once and permanent increases in the real interest rate assumption have been examined. Various combinations of the timing and the magnitude of the changes are investigated. The different cases are now discussed in turn.

-Follow the experience
If the basis is changed immediately after the change in the experience and to the same degree, initially, a surplus arises owing to the change in the valuation basis. After amortising the initial surplus, the fund will eventually achieve the stationary state at the optimum level.

-Magnitude of change
If the magnitude of the change in the valuation basis is different, the optimum level of the fund cannot be achieved.

However, when the magnitude is less than the changes in the experience, the initial surplus produced by the change in the basis is reduced, and smoother contribution rate changes may be achieved.

-Delayed change
A short delay in the change of the valuation basis may not cause significant impact on the long term contribution and funding level patterns.

While the change in the valuation basis is delayed, the gap between the experience and the valuation basis produces surpluses, which are accumulated. So, the level of the surplus at the change of the basis would be higher. So, in the short run, the delays may make the state of funding more volatile.

8.2.3.3 Gradual change in basis

Refer to section A3.1.2.3 and graphs A3.1.2.h-n in Appendix 3.1.

The averaging of past experience gives a gradual change in the valuation basis. Various combinations of the magnitude of the changes and the frequency of the changes in the basis are examined. The different cases are now discussed in turn.

-Gradual change
Where a small increase in the real interest rate assumption is made every year starting immediately after the change in the experience, the emergence of the surpluses is spread over years.

The surpluses arising each year consist of two elements; the surpluses because of the change in the valuation basis and the surpluses arising from the gap between the experience and the basis. The latter is initially large, but reduces as the basis approaches the experience level.

The gradual changes in the standard contribution rate also contribute to reduce the volatility in the recommended contribution rate.

Eventually, the fund achieves the optimum stationary state.

The changes in the basis are slower than the immediate changes, and it takes a longer time to achieve this optimum stationary state. So, the funding level stays higher for a longer time. There is a trade off between the stability of the contribution rate and the rapid achievement and maintenance of the optimum funding level.

-Different speed in change
If a longer averaging period is used, each year's change in the valuation basis is smaller, and this will reduce the volatility in the recommended contribution rate.

On the other hand, a slower change in the basis results in a slower amortisation of surpluses, and the funding level stays higher for a longer time. So, the trade off between the stability of the contribution rate and quickly achieving the optimum funding level (mentioned above) is observed again.

-Frequency of change
Projections where the change in the basis is only made triennially although the valuation takes place every year have been carried out.

The general tendency of the pattern of the contribution rates and the funding levels in the long run is not affected by the less frequent basis changes.

However, in the short run, the recommended contribution rates are more volatile from year to year because of the different treatments in the years of basis changes and other years.

-Further change

Here, a projection where the change in the basis does not stop after reaching the level of experience has been carried out.

Until the basis reaches the experience level, the movements are the same as averaging.

However, owing to the continuing weakening of the basis, valuation surpluses continue to be released. Also, the standard contribution rate continues to fall. These make the funding level lower than optimum. So, deficits arise from the difference between the experience and the assumptions. The deficits, which are increasing, becomes more dominant gradually, and the recommended contribution rate increases.

The funding level becomes low and the pace of the increase in the contribution rate accelerates as the projection continues.

8.2.3.4 Summary of 8.2.3

To achieve the optimum funding level, it is essential to move to the valuation basis which is equal to the experience.

Short term fluctuations in the contribution rate and the funding level may be reduced by gradual changes in the basis, where the reductions in the standard contribution rate and the emergence of valuation surpluses are made gradually. However, there is a trade off between this reduction in the fluctuations and the speed of achieving the optimum level of the fund.

A few years' delay in the timing of the changes in the valuation basis may not alter the situation so much.

8.2.4 Gradual change in experience

Here, the real investment returns achieved are gradually increasing. A constant valuation basis and gradual changes in the bases are applied.

Because the real investment returns are improving, the realistic value of the liabilities might be decided taking future improvements into account. Such a realistic value of the liabilities might be referred to as the optimum funding level. However, this involves a generation approach to the real investment return assumptions, and this is beyond the capacity of the projection system.

Because the change in the experience progresses continuously, the optimum level changes as time passes. This changing nature is considered more important than the absolute level of the optimum funding level, which may be difficult to identify and calculate.

So, here, the changing patterns of the valuation bases are appraised by the ability to cope smoothly with the changes.

8.2.4.1 Constant basis

Refer to section A3.1.3.1 and graph A3.1.3.a in Appendix 3.1.

If the valuation basis is not changed at all, the gap between the actual and assumed real investment returns widens gradually, and surpluses, which are increasing, arise.

The surpluses accumulate, and the funding level increases, where the optimum funding level should fall as the experience improves. The recommended contribution rate falls accordingly.

The unchanged valuation basis may not be appropriate.

8.2.4.2 Gradual change in basis

Refer to section A3.1.3.2 and graphs A3.1.3.b-l in Appendix 3.1.

In this section, the valuation bases are changed gradually as straight lines. Various combinations of the magnitude of the changes, the timings of the start of changes and the frequency of the changes in the basis have been examined.

If the valuation basis is set as an average of past experience, the pace of the changes in the basis will be the same as that in the experience after the year when the period from the start of the projection exceeds the averaging period. However, initially, the pace of the basis changes is different from straight line changes which are seen at later periods of the projections. This is because the experience before the start of the projections is assumed to be stationary. To see the initial differences, some projections using the bases which are the averages of the past experience are also investigated.

The different cases are now discussed in turn.

-Follow the experience

Where the changes in the valuation basis start immediately with exactly the same magnitude as the experience, there are no gaps between the experience and the assumptions. So, only valuation surpluses, which are steady over the years, arise. Consequently, the amortisation element of the contribution rate is kept small and stable.

At the same time, the standard contribution rate is gradually reduced reflecting the changes in the experience.

So, the size of the fund is smoothly reduced reflecting the changes in the experience.

Also, owing to the relatively small accumulated surpluses, the results may not be affected much by the choice of the amortisation method.

-More moderate change

More moderate changes than the changes in experience have been examined.

A more moderate change in the basis compared with the changes in the experience makes the changes in the standard contribution rate and the emergence of valuation surpluses smaller. So, the recommended contribution rate is more stable initially. However, if the changes in the experience develop further, surpluses arising from the gaps between the experience and the assumptions increase steadily, and a large surplus is accumulated. The reduction in the size of the fund becomes very slow, or the size might be increased initially. This would lead to a rapid fall in the contribution rates later, and they would be unstable in the long run.

Also,the contribution rate changes are heavily dependent on the amortisation method.

-Delayed change

Delays in the start of the changes in the bases have been examined.

The effect of delay is a higher level of accumulated surplus because of the gap between the experience and the basis which is created by the delay and never removed.

However, two years' delay does not lead to significant effects on the basic shape of the contribution rate changes in the long run.

-Less frequent changes in basis

Some projections have been carried out, where the valuations take place every year, but the changes in the valuation basis are made only every three years or ten years.

Again, the basic pattern of the projection results in the long run is not affected. However, fluctuations in the contribution rates from year to year are higher.

-Averaging

Here, the valuation basis is set as the average of the past experience. For the averaging period, 5 years and 10 years are examined.

Averaging has a similar effect as delaying the change of the basis. Because the pace of the change in the valuation basis is the same as that of the experience, the reduction in the size of the fund is smoothly carried out. There are no large surpluses accumulated, but small and constant surpluses.

8.2.4.3 Summary of 8.2.4

Against a gradual change in the real investment returns, it is important to keep the changes in the valuation assumption roughly at the same pace. Otherwise, surpluses/deficits arising from the gaps between the experience and the assumptions would increase, while the reduction in the standard contribution rate is not sufficient. Then, the funding level may be driven in unwanted directions, and the contribution rates may become more volatile.

Different frequencies and the delays in the basis changes do not have significant impact.

Averaging has a similar effect as delaying, and keeps the pace of changes in the basis the same as that in the experience.

8.2.5 Cyclical change in experience

Here, the actual real investment returns are assumed to follow a twenty year cycle.

Constant valuation bases and cyclical changes in the bases are applied.

Although the experience keeps changing, long term experience level may not change at all. So, the realistic value of the liabilities may not be affected by the cyclical changes in the experience, and may be calculated on the long term average basis.

8.2.5.1 Constant basis

Refer to section A3.1.4.1 and graphs A3.1.4.a-c in Appendix 3.1.

Three different constant bases are tried; the highest, middle and the lowest real investment return in the experience cycle.

The average funding level and the contribution rate show similar tendencies as for the projections with unchanged experience. The use of a strong basis results in a permanent surplus.

The fluctuations in the recommended contribution rates are relatively moderate because of the lack of changes in the standard contribution rates and the valuation surpluses/deficits.

Reflecting the cyclical changes in the experience, the funding level also shows the same length of cycle. The timings of peaks and troughs are different between the experience and the funding level. This is because the latter represents the accumulated effects of experience and the progression in the amortisation.

8.2.5.2 Cyclical change in basis

Refer to section A3.1.4.2 and graphs A3.1.4.d-l in Appendix 3.1.

In this section, the valuation bases are changed cyclically with the same length of cycles, i.e. 20 year cycles. Various combinations of the magnitude and the frequency of the changes in the basis have been examined.

Also, the effects of averaging are analysed by applying three different averaging periods.

The different cases are now discussed in turn.

-Follow the experience

Where the changes in the valuation basis are made exactly the same as the changes in the experience without delays, the contribution rate and the funding level become more volatile than for a constant basis. This is because of the fact that the combined effects of the changes in the standard contribution rate and valuation surpluses/deficits are more significant than the impact of the gaps between the experience and the basis as discussed in section 8.2.1.3.

-More moderate changes

More moderate changes in the valuation basis have been examined.

The result is somewhere between the case with the unchanged basis and the case where the basis exactly follows the experience. Thus, the changes in the contribution rate are smaller than the case where the basis exactly follows the experience.

-Less frequent changes

Although valuations are made annually, the basis is changed only every 3 or 5 years.

In the short run, the fluctuations in the funding level and the contribution rate are more volatile. However, the long term general patterns of the contribution rates and the funding level are not greatly affected by less frequent changes in the valuation bases.

-Averaging

The valuation basis is set as the average of past experience. For the averaging period, 3, 5 and 10 years are investigated.

The effects of averaging on the setting of the bases are the moderation of the changes in the basis and the delay in the changes. The first effect makes the range of the valuation basis applied in the projections narrower than that of the experience, and the changes in the contribution rates are smaller than the case where the basis exactly follows the experience (similar to the case of more moderate changes above). The second effect appears as gaps between the phases of the experience cycle and the basis cycle.

The effect of the changes in the basis is dominant on the pattern of the changes in the contribution rate. The main effects of using a longer averaging period are a reduction in the volatility of the contribution rates because of the reduced range of the changes in the valuation basis, and the delay in the phase of the cycle.

8.2.5.3 Summary of 8.2.5

When the real investment return experience is moving cyclically, constant valuation bases make the funding level most stable. The optimum funding level may be achieved by the use of the middle rate of real investment return as the assumption.

Any movements in the valuation bases would increase the fluctuations in the funding level and the contribution rates owing to the changes in the standard contribution rates and the valuation surpluses/deficits.

Averaging produces some fluctuations in the recommended contribution rate, but the degree is less than the basis exactly following the experience. Also, averaging makes the phase of the cycle of the changes in the valuation basis different from that of the experience cycle.

8.2.6 Conclusions to real investment return changes

Where the real investment returns are changing, the impact of the changes in the standard contribution rate is important as well as the effects of surpluses/deficits arising.

With an unchanged experience, an unchanged valuation basis, which is equal to the experience, may be the best.

With a once and permanent change in the experience, in order to avoid permanent surpluses/deficits, it is important to use the basis which eventually becomes equal to the experience. To make the changes in the contribution rates and the funding level change smoothly, gradual changes, or averaging, are effective. However, this would delay the achievement of a new stationary state.

With a gradual change in the experience, it is essential to keep the pace of the changes in the basis roughly the same as the experience. Short delays in the response may not significantly affect the broad pattern of changes in the contribution rates. Averaging may give desirable results because it maintains the pace of changes though the response is delayed a little.

With a cyclical change in the experience, the use of the basis which represents the middle experience is effective to keep the contribution rate and the funding level stable.

Delays in the changes and less frequent changes in the valuation bases are not important for the long term effect on the fund, although they would increase fluctuations in the short run.

In reality, no one knows the future, and it is not possible to identify the pattern of the changes in the experience in advance. For example, even if there has been a history of gradual changes, it may only be a first part of a very long cyle, or the experience may become stable at some early stage.

So, the realistic approach would be to find a way which suits all the situations well. In this sense, a long term averaging may be recommended because it suits all the situations examined above reasonably well though it might not be the best in individual cases. The changes in the basis may be further delayed to clarify the situation.

However, averaging and the use of a delay may not give suitable results in the short run, where more control may be exercised through amortisation of the surpluses/deficits.

8.3 Dividend growth changes

8.3.1 Basic impact

Changes in the experience and the valuation basis only affect the market and actuarial values of the assets. They do not affect the membership structure and the liabilities at all.

8.3.1.1 Ultimate cost

Consider permanent changes in the dividend growth experience. Rises in the dividend growth experience increase dividend income from the same unit of equities.

If the market dividend growth expectations and the market interest rate are unchanged (although this is not the case in the projection model of this thesis), the market dividend yield would be unaffected. So, the market values of the existing assets grow in line with the dividend growth experienced which is at a high level. The returns achieved on the existing assets would be enhanced because of the increased dividend income and a higher pace of growth in capital value.

New monies can be invested at the unchanged dividend yield level. The unchanged dividend yield means that the market prices are based on a lower level of dividend growth expectations. However, the actual dividend growth rates would be higher than that. So, the same arguments as for the existing assets apply, and the returns on the new monies would be enhanced to the same degree.

The same would apply if both the market dividend growth expectations and the market interest rate are increased by the same degree simultaneously. If the new market dividend growth expectations are in line with the new level of dividend growth experience, the new market interest rate would be the same as the new rate of return achieved on the existing assets and new monies.

Such rises in the dividend growth experience may occur when the inflation rate rises. If the liabilities are simultaneously increased to the same degree, there may not be any changes in real terms, and no further discussion of this case is needed here.

It is also possible that the change may not be accompanied with a rise in the inflation rate. Then, this would result in an increase in the real interest rate. However, this case has already been investigated in the previous part of the projections, and is not discussed here.

We shall now consider the projection model which is being discussed. In this part of the investigation, the situations where the market dividend growth expectations are increased in line with the experience and the market interest rate stays the same are examined.

If such changes occur, the equity prices go up reflecting the enhanced expectations for future dividend growth. The model assumes the market prices are obtained from a discounted cash flow approach, and the market prices are affected by all the future dividend incomes expected. The dividend growth expectation is applied cumulatively to calculate the amount of the future expected dividends. So, the increase in the market price is more than proportionate to the increase in the expected dividend growth rate.

So, the growth in the market values would be higher than the case above, and higher capital gains may be obtained if the existing assets are sold. Thus, the investment returns on the existing assets are improved more than the case above.

On the other hand, new monies can only be invested in equities at a higher price level, which reflects the new dividend growth experience. The expected returns on the new monies, which are equal to the market interest rate, would not change.

This applies to the projections carried out in this section, and improved returns are obtained from the existing assets while the returns from the new monies are not affected. There are initial funds, which are not trivial, and the impact of improved returns may not be negligible. On the other hand, the amounts of benefits to be paid are not affected by the changes. So, as a whole, a reduction in the ultimate cost is expected.

8.3.1.2 Gap between experience and assumption

Now, the impact of the gap between the actual and assumed dividend growth rate is analysed. The analysis is relevant to Appendix 1, the results of which are referred to in this section.

The calculations below are made assuming the same set of assumptions as for the projections (see section 7.4). For the notation, refer to section 7.3.

If the market expectation for the dividend growth is higher than the assumption, the actuarial value of the equities is less than the market value.

Consider the end of a year, T, when the valuation takes place.

Firstly, we shall examine the impact on the value of assets held excluding any impact from cash flows. Therefore, we consider no benefit payouts and contribution income in this part. That is, we consider the impact on the value of the fund at time T-0, at which time benefit payouts and contribution income have not yet taken place. If the actual dividend growth for the last 12 months, $g'(T)$, had been the same as the assumed dividend growth rate, $g(T)$, the dividend received would have been

$$D(T) * (1 + g(T)) / (1 + g'(T)) \qquad .....8.3.1.2.1$$

At the same time, the market dividend yield would have been

$$d(T) \qquad .....8.3.1.2.2$$

Now, the following relationship is known from Corollary A1.3.4.2;

$$M(T\text{-}0) = D(T) * (1 + d''(T)) / d''(T) \qquad \text{.....8.3.1.2.3}$$

Also, applying Corollary A1.3.4.3 to 8.3.1.2.3,

$$\begin{aligned} VM(T\text{-}0) &= M(T\text{-}0) * c(T) \\ &= D(T) * (1 + d''(T)) / d''(T) * d''(T) / d(T) \\ &= D(T) * (1 + d''(T)) / d(T) \end{aligned}$$

.....8.3.1.2.4

If the actual experience were as 8.3.1.2.1 and 8.3.1.2.2 above, by replacing D(T) and d"(T) in formula 8.3.1.2.4 with 8.3.1.2.1 and 8.3.1.2.2 respectively, the actual value of assets at T-0 would have been

$$D(T) * (1 + g(T)) / (1 + g'(T)) * (1 + d(T)) / d(T)$$

.....8.3.1.2.5

Therefore, the surplus arising from this source is

Actual value of assets (8.3.1.2.4) - Expected value of assets (8.3.1.2.5)

$$\begin{aligned} &= D(T) * (1 + d''(T)) / d(T) \\ &\qquad - D(T) * (1 + g(T)) / (1 + g'(T)) * (1 + d(T)) / d(T) \\ &= D(T) / d(T) * \{1 + d''(T) - (1 + g(T)) * (1 + d(T)) / (1 + g'(T))\} \end{aligned}$$

.....8.3.1.2.6

Here, from section 7.4.5,

$$(1 + g(T)) * (1 + d(T)) = 1 + i(T)$$

Now, in this model,

$$g''(T) = g'(T)$$

Therefore,

$$\begin{aligned} (1 + g'(T)) * (1 + d''(T)) &= (1 + g''(T)) * (1 + d''(T)) \\ &= 1 + i''(T) \end{aligned}$$

Because it is assumed that $i(T) = i''(T)$ in this part of the projections,

$$(1 + g(T)) * (1 + d(T)) / (1 + g'(T))$$

$$= \quad (1 + i(T)) / (1 + g'(T))$$

$$= \quad (1 + i''(T)) / (1 + g''(T))$$

$$= \quad 1 + d''(T)$$

.....8.3.1.2.7

Applying 8.3.1.2.7 to 8.3.1.2.6, 8.3.1.2.6 is equal to 0.

.....8.3.1.2.8

The value of M(T-0)-D(T) grows from the value of M(T-1+0) by $1 + g'(T)$. So, if $g'(T) > g(T)$, the actual value exceeds the expected value. Furthermore, the market value of the dividend income, D(T), is higher than expected.

However, because the market expectation for the future dividend growth rate, $g''(T)$, is assumed to be equal to the last 12 months' experience, $g'(T)$, $d''(T)$ becomes lower than $d(T)$. Consequently, the assessed value of the assets is devalued by $d''(T) / d(T)$.

These two effects are shown in 8.3.1.2.4 to cancel each other out.

Next, we shall consider the impact on the cashflow. If the actuarial value is less than the corresponding market value, the contribution incomes are given a lower valuation than the face value, and this is a source of a deficit. If the actuarial value is higher, the contribution incomes are valued higher than the face value. In other words, the surpluses/deficits from the subsequent investment differences are already included in the capital element above.

In the model, the contributions are paid at the beginning of each year. So, the contribution income during year T is expressed as C(T-1). The surplus, which is recognised as at T, from the contribution paid in at time T-1 is

$$C(T-1) * (1 + i(T)) * \{ d''(T) / d(T) - 1 \}$$

$$= \quad C(T-1) * (1 + i(T)) * (d''(T)) - d(T)) / d(T)$$

Because $d''(T) < d(T)$, this is a deficit.

On the other hand, the face values of benefit payments are not affected by the changes in the dividend growths, and the costs recognised on the valuation basis are less than the face value. So, a surplus arises. The surplus from this source is

$$-B(T) * (d''(T)) - d(T)) / d(T)$$

These two are added to be

$$\{C(T-1) * (1 + i(T)) - B(T)\} * (d''(T)) - d(T)) / d(T)$$

This depends on the amount of contributions with interest less the benefit payments. If it is positive, the cash flow provides a deficit. If it is negative, this becomes a surplus. The gap affects the size of surpluses/deficits.

Where the fund is in deficit, the contribution rate is relatively high because of amortisation. So, the cash flow is more likely to be positive, and a deficit may arise. On the other hand, where the fund is in surplus, the cash flow is more likely to be negative, and a surplus may arise.

The cash flow is also influenced by the size of benefit outgoes. For example, where the fund is relatively new, and there are few benefit payments, the cash flow is more likely to be positive, and a deficit may arise.

In the projections which are shown in Appendix 3.2, the fund is already mature when the projections start, and the fund is relatively close to a stationary state. A stationary fund is expected to grow at the rate of salary inflation, e, each year, while investment proceeds are earned at a higher rate, i. Therefore, part of the investment proceeds is paid out. That is, the net cashflow, contributions less benefits (or more precisely, $C(T-1) * (1+i(T)) - B(T)$) is negative if the fund is stationary. Because the actual fund is relatively close to a stationary state, the actual cashflow is more likely to be negative than positive.

Therefore, a positive gap of $g' - g$ is likely to produce surpluses except for the situations with very low funding levels.

Where there is a surplus, the cashflow tends to be negative owing to the reduction in contribution rates for amortisation. This will lead to further accruals of surpluses. On the other hand, if the funding level is very low and the cashflow is positive, further deficits will arise. Hence, a positive gap might cause further accumulation of surpluses/deficits, and the funding level can be potentially unstable. However, as shown in the following numerical examples, the level of surpluses/deficits accruing is relatively small, and this potential instability may not be recognised in the projection results.

If g'' is smaller than g, the opposite would occur.

Here, if there is a surplus and cashflow is negative, deficits will arise. On the other hand, if the fund has a large deficit and the cashflow is positive, surpluses will arise. Hence, unlike a positive gap, a negative $g' - g$ gap is expected to reduce the surpluses/deficits automatically, and the funding level is expected to be relatively stable. However, the level of surpluses/deficits accruing is relatively small, and this stabilising effect may not be recognised in the projection results.

We shall now examine a numerical example. The stationary fund under the standard experience and the standard valuation basis in the projection produces the cash flows as follows;

Contribution incomes at the start of year 1:	269
Dividend incomes at the end of year 1:	203
Benefit payments at the end of year 1:	414
Capital growth in year 1:	254
Value of the fund at the end of year 1:	5,120

The contribution rate is 10.8578%.

Although, in this case, there are no differences in g" and g, the relative size of the two elements might be appreciated by an approximate calculation.

Here,

$$C(T-1) * (1+i) - B(T) \quad = \quad 269 * 1.09 - 414 \quad = \quad -121$$

So, if g"(T) = 5.5% and g(T) = 5%,

$$\text{The surplus arising} \quad = \quad -121 * (.03318-.03810)/.03810 \quad = \quad 15.6$$

This is only 0.3% of the fund.

If the fund was only a half, i.e. 2,560, the situation might be as follows.

The benefit payments are not affected at 414.
The contribution rate would be increased reflecting the lower funding level. Here, the standard fund is 5,120. Assuming the spread factor is, say, 15, the increase in the contributions would be

$$(5{,}120 - 2{,}560) / 15 = 171$$

So,

$$\text{The surplus} \quad = \quad \{(269+171) * 1.09 - 414\} * (.03318-.03810)/.03810 \quad = \quad -8.5$$

i.e. a deficit of 8.5 arises. This shows that a deficit can arise if the funding level is very low.

On the other hand, if there is a surplus of 2,560, i.e. the size of the fund is 7,680, the reduction in the contribution might be 171.

So,

$$\text{The surplus} \quad = \quad \{(269-171) * 1.09 - 414\} * (.03318-.03810)/.03810 \quad = \quad 39.7$$

Therefore, a larger surplus would arise if the fund is larger because of the effect of amortisation. However, the size of the surplus arising in relation to the size of the fund is small. Therefore, it may be said that surpluses/deficits from the gap are unlikely to accumulate to a large sum unless the gap is very wide.

8.3.1.3 Valuation surplus/deficit

The changes in the dividend growth and dividend yield assumptions do not affect the value of liabilities as long as the interest rate assumption is the same. Only the value of the assets is affected.

If the dividend growth assumption is increased, the value of assets are increased. The increase in the value of assets appears as a valuation surplus. From Corollary A1.3.4.3 in Appendix 1,

$$VM(T) = M(T) * d''(T) / d(T)$$

So, if we define

$VM\hat{}(T)$:	actuarial value of assets on the new basis
$g\hat{}(T)$:	new dividend growth assumption
$d\hat{}(T)$:	new dividend yield assumption,
$VSp(T)$:	valuation surplus

the following equations hold;

$$1 + i(T) = (1 + g\hat{}(T)) * (1 + d\hat{}(T)) \qquad .....8.3.1.3.1$$

$$VM\hat{}(T) = M(T) * d''(T) / d\hat{}(T) \qquad .....8.3.1.3.2$$

Then,

$$VSp(T) = VM\hat{}(T) - VM(T)$$

$$= M(T) * d''(T) * \{1/d\hat{}(T) - 1/d(T)\} \qquad .....8.3.1.3.3$$

For example, if

$$i(T) = i''(T) = 9\%$$
$$g(T) = g''(T) = 5\%$$
$$g\hat{}(T) = 5.5\%,$$

then

$$d(T) = d''(T) = 3.810\%$$
$$d\hat{}(T) = 3.318\%$$

So, from 8.3.1.3.3,

$$VSp(T) = M * .148$$

So, in this case, a change in the dividend growth assumption by .5% results in a 14.8% change in the value of assets. Similar to the real interest rate assumption, the surplus from the change in the assumption is usually much larger than the surplus from the gap between the experience and the assumption of the same degree.

This is because the surpluses arising from the gap is only related to the cash flow, which is usually much smaller than the fund as a whole.

However, if the gap between the experience and the assumptions is very large, the impact of the gap can exceed the valuation surpluses/deficits. For example, if the experience gradually but endlessly changes in one direction while the valuation basis is unchanged, the impact of the gap may be negligible initially, but would grow very large and dominate the situation later.

Also, if the scheme is growing rapidly, reducing its size rapidly, or with a large deficit, the effect of the gap may become significant because the relative size of the positive/negative cash flow to the size of the fund can be large.

Because the standard contribution rate is not affected by the changes in the dividend growth assumption, the impact of the changes in the valuation basis is relatively moderate compared with the changes in the real interest rate assumptions.

8.3.2 Unchanged experience

Refer to section A3.2.1 and graphs A3.2.1.a-c in Appendix 3.2.

The experience is the same as the standard experience and remains stable throughout the projections. Three different bases, which are also constant throughout the projection, are used, and the long term effects of the choice of the basis are examined. One basis is equal to the standard basis, another has a higher dividend growth assumption, and the other has a lower one.

The accumulations of the surpluses/deficits are similar to section 8.2. However, the standard contribution rates are unchanged for all projections.

To achieve the optimum level of funding, the use of the standard basis, which is equal to the experience, is necessary.

8.3.3 One permanent change in experience

The dividend growth experience is increased from year 1, and the dividend growth market expectation, which is the same as the experience in the previous year, from the end of year 1. They both remain constant at the increased level throughout the projections thereafter. The market interest rate stays the same, and the market dividend yield is reduced at the end of year 1, and remains so thereafter.

There are improved dividend incomes from existing assets, while new monies can only be invested on the assets with lower dividend yields and higher dividend growth expectations. However, the overall expected return on the new money stays the same at 9%.

A constant valuation basis, one permanent change in the basis, and gradual changes in the basis are investigated.

8.3.3.1 Constant basis

Refer to section A3.2.2.1 and graph A3.2.2.a in Appendix 3.2.

The standard basis is applied to see the impact of the change in the experience without any moves in the basis. The projection is done as a basis for comparison with other projections which use changing bases.

This case is similar to section 8.2.3.1. The funding level is initially 100%, but gradually increases because of the surpluses arising which are gradually increasing as the funding level goes up and the recommended contribution rate falls. However, the amortisation catches up with the accumulation of surpluses eventually, and the relative level of surplus becomes stable. This stable funding level is higher than the eventual funding level achieved in the case where the valuation basis exactly follows the experience, which may be regarded as optimum.

8.3.3.2 One permanent change in basis

Refer to section A3.2.2.2 and graphs A3.2.2.b-g in Appendix 3.2.

The valuation bases are changed only once, here. Various combinations of the timing and the magnitude of the changes are examined. The different cases are now discussed in turn.

The standard contribution rate stays the same after the changes in the valuation bases. However, the pattern of the emergence and accumulation of the surpluses/deficits is similar to section 8.2.

-Follow the experience
Initially, a surplus arises owing to an increase in the value of assets. From year 2 onwards, no surpluses/deficits arise, and the fund becomes stationary at the optimum level after amortising the initial surplus.

-Magnitude of change
Under different bases, although the fund eventually achieves a stability, the level of funding is not the optimum level.

Where the change in the basis is less than the experience, a cancellation effect between the reduced contribution rates and the newly accruing surpluses shown is observed, and the contribution rate changes are more moderate.

On the other hand, when the change in the valuation basis is larger, the contribution rate shows wider movements.

-Delayed change
A short delay in the change of the valuation basis may not cause significant impact on the long term contribution and funding level patterns.

While the change in the valuation basis is delayed, the gap between the experience and the valuation basis produces surpluses, which are accumulated. So, the level of the surplus at the change of the basis would be slightly higher. So, in the short run, the delays may make the state of funding more volatile.

8.3.3.3 Gradual change in basis

Refer to section A3.2.2.3 and graphs A3.2.2.h-n in Appendix 3.2.

The averaging of past experience gives a gradual change in the valuation basis. Various combinations of the magnitude of the changes and the frequency of the changes in the basis are also examined. Now, the different cases are discussed in turn.

-Gradual change

The changes in the contribution rate are more moderate than one large change in the basis.

Eventually, the fund achieves the optimum stationary state. However, the changes in the basis are slower than immediate changes, and it takes a longer time to achieve this optimum stationary state than the cases with one permanent changes in the basis. So, the funding level stays higher for a longer time than the cases with one permanent changes in the basis. Thus, there is a trade off between the stability of the contribution rate and achieving quickly the optimum funding level.

-Different speed in change

If a longer averaging period is used, each year's change in the valuation basis is smaller, and this will reduce the volatility in the recommended contribution rate.

On the other hand, a slower change in the basis results in a slower amortisation of surpluses, and the funding level stays higher for a longer time. So, the trade off between the stability of the contribution rate and obtaining quickly the optimum funding level mentioned above is observed again.

-Frequency of change

Projections where the change in the basis is only made triennially although the valuation takes place every year have been carried out.

The general tendency of the pattern of the contribution rates and the funding levels in the long run is not affected by the less frequent basis changes, although the fluctuations in the short run are larger.

-Further change

Here, a projection where the change in the basis does not stop after reaching the level of experience has been carried out.

Until the basis reaches the experience level, the movements are the same as averaging.

However, owing to the continuing weakening of the basis, valuation surpluses continue to be released. This makes the funding level lower than optimum. So, deficits arise from the difference between the experience and the assumptions. The deficits, which are increasing, becomes more dominant gradually, and the recommended contribution rate increases.

The funding level continues to reduce and the increase in the contribution rate is accelerated towards the end of the projection period. Hence, should the projection be carried out for a longer period, we might see that the fund would become nil while liabilities still remains.

8.3.3.4 Summary of 8.3.3

To achieve the optimum funding level, it is essential to move to a valuation basis which is equal to the experience.

In general, owing to the lack of the changes in the standard contribution rate and different patterns of surpluses accruing (see section 8.3.1.), the magnitude of the changes in the contribution rates is smaller than the case with a 1% fall in the salary growth rate (see section 8.2.3).

Short term fluctuations in the contribution rate and the funding level may be reduced by gradual changes in the basis. However, there is a trade off between this reduction in the fluctuations and the speed of achieving the optimum level of fund.

A few years' delay in the change may not alter the situation so much.

8.3.4 Gradual change in experience

Here, the actual dividend growth rates are gradually increasing. A constant valuation basis and gradual changes in the bases are applied.

Similar to the arguments in section 8.2.4, the ability to cope with the changing situations is considered essential.

Because the value of the liabilities is not changing, here, the changing patterns of the valuation bases are appraised by the ability to maintain a constant funding level under changing dividend growth assumptions.

8.3.4.1 Constant basis

Refer to section A3.2.3.1 and graph A3.2.3.a in Appendix 3.2.

If the valuation basis is not changed at all, the gap between the actual and assumed real investment returns widens gradually, and increasing surpluses arise.

The surpluses accumulate, and the funding level increases, which had been hoped to be stable. The recommended contribution rate falls accordingly.

Thus the unchanged valuation basis may not be appropriate.

8.3.4.2 Gradual change in basis

Refer to section A3.2.3.2 and graphs A3.2.3.b-l in Appendix 3.2.

The valuation bases are changed gradually as straight lines. Various combinations of the magnitude of the changes, the timings of the start of changes and the frequency of the changes in the basis have been examined. Similarly to section 8.2.4.2, some projections using the bases which are the averages of the past experience have also been carried out. The different cases are now discussed in turn.

-Follow the experience

Where the changes in the valuation basis start immediately with exactly the same magnitude as the experience, there are no gaps between the experience and the assumptions. So, only valuation surpluses, which are steady over the years, arise. Consequently, the amortisation element of the contribution rate is kept small and stable.

Also, the standard contribution rate remains stable.

So, the size of the fund is smoothly reduced reflecting the changes in the experience.

Also, because of the relatively small accumulated surpluses, the results may not be affected much by the choice of the amortisation method.

-More moderate change

More moderate changes than the changes in experience have been examined.

A moderate change in the basis compared with the changes in the experience makes the contribution rate more stable initially. However, if the changes in the experience develop more, a large surplus is accumulated, and the reduction in the size of the fund becomes very slow, or the size might be increased initially. This would lead to rapid falls in the contribution rates later, and they would be unstable in the long run.

Also,the contribution rate changes are heavily dependent on the amortisation method.

-Delayed change

Delays in the start of the changes in the bases have been examined.

The effect of delay is a higher level of accumulated surplus because of the gap between the experience and the basis which is created by the delay and never removed.

However, two years' delay does not give significant effects on the basic shape of the contribution rate changes in the long run.

-Less frequent changes in basis

Projections have been carried out, where the valuations take place every year, but the changes in the valuation basis are made only every three years or ten years.

Again, the basic pattern of the projection results in the long run is not affected. However, the fluctuations in the contribution rates from year to year are higher.

-Averaging

Here, the valuation basis is set as the average of the past experience. As the averaging period, 5 years and 10 years are examined.

Averaging has a similar effect as delaying the change of the basis. Because the pace of the change in the valuation basis is the same as that of the experience, the reduction in the size of the fund is smoothly carried out. There are no large surpluses accumulated. The accumulated surplus remains small throughout the projections.

8.3.4.3 Summary of 8.3.4

Against a gradual change in the dividend growths, it is important to keep the changes in the valuation assumption roughly at the same pace. Otherwise, the surpluses/deficits arising would increase, and the funding level may be driven in unwanted directions.

Unlike section 8.2, it is possible to maintain the funding level and the contribution rate relatively constant because the standard contribution rate is not affected by the changes in the dividend growth assumption.

Different frequencies and delays in the basis changes do not have a significant impact.

Averaging has a similar effect to delaying the changes in the basis, and keeps the pace of changes the same as that of the experience.

8.3.5 Cyclical change in experience

Here, the dividend growth experience are assumed to follow a twenty year cycle. The market dividend growth expectations are also following the same cycle.

Constant valuation bases and cyclical changes in the bases have been examined.

Although the experience keeps changing, the long term experience level may not change at all. So, the realistic value of the liabilities may not be affected by the cyclic changes in the experience, and may be calculated on the long term average basis.

8.3.5.1 Constant basis

Refer to section A3.2.4.1 and graphs A3.2.4.a-c in Appendix 3.2.

Three different constant bases have been applied; the highest, middle and the lowest dividend growth rates in the experience cycle.

The average funding level and the contribution rate show similar tendencies as the projections with unchanged experience. The use of a strong basis results in a permanent surplus.

Reflecting the cyclical changes in the experience, the funding level also shows a cycle with the same length. The peaks and troughs of the actual experience differ from those of the funding level. This is because the latter represents the accumulated effects of experience and the progression in the amortisation.

8.3.5.2 Cyclical change in basis

Refer to section A3.2.4.2 and graphs A3.2.4.d-l in Appendix 3.2.

In this section, the valuation bases are changed cyclically with the same length of cycles, i.e. 20 year cycles. Various combinations of the magnitude and the frequency of the changes in the basis are investigated.

Also, the effects of averaging are analysed by considering three different averaging periods. Now, the different cases are discussed in turn.

-Follow the experience
Where the changes in the valuation basis are made exactly the same as the changes in the experience without delays, the contribution rate and the funding level become more volatile than the constant basis although the standard contribution rates are not affected in this case. This is because of the fact that the effects of the changes in the valuation basis are more significant than the impact of the gap between the experience and the basis as discussed in section 8.3.1.3.

-More moderate changes
More moderate changes in the valuation basis have also been examined.

The result lies somewhere between that for the unchanged basis and that for the basis which exactly follows the experience.

-Less frequent changes
Although valuations are made annually, the basis is changed only every 3 or 5 years.

In the short run, the fluctuations in the funding level and the contribution rate are more volatile. However, the long term general patterns of the contribution rates and the funding level are only marginally affected by less frequent changes in the valuation bases.

-Averaging
The valuation basis is set as the average of past experience. As the averaging period, 3, 5 and 10 years are used.

The effects of averaging in setting the assumptions for the basis are the moderation of the changes in the basis and the delay in the changes. The first effect is observed from the narrower range of the valuation bases used than when the basis exactly follows the experience (see earlier). The second effect appears as gaps between the phases of the experience cycle and the basis cycle.

The effect of the changes in the basis is dominant on the pattern of the changes in the contribution rate although the standard contribution rate is unchanged. The main effects of using a longer averaging period are a reduction in the volatility of the contribution rates and a delay in the phase of the cycle.

8.3.5.3 Summary of 8.3.5

When the dividend growth experience is moving cyclically, constant valuation bases make the funding level most stable. The optimum funding level may be achieved by the use of the middle dividend growth rate experienced as the assumption.

Any movements in the valuation basis would increase the fluctuations in the funding level and the contribution rates.

Averaging leads to some fluctuations, but the degree is less than for the basis which exactly follows the experience. Also, averaging makes the phase of the cycle of the valuation basis changes different from that of the experience cycle.

8.3.6 Conclusions to dividend growth changes

A .5 percent change in the dividend growth rate affects the finances of the fund (and consequently the contribution rate) less than a 1 percent change in the real investment returns. The reasons may be as follows.

Firstly, the standard contribution rates are not affected at all. So, the impact on the recommended contribution rate comes only from the amortisation of surpluses/deficits.

Secondly, the valuation surpluses arising are smaller. A .5 percent change in the dividend growth rate assumption at the beginning of the projection would release a surplus of about 15 percent of the fund, while a 1 percent change in the real investment returns would release around 25 percent.

Finally, the surpluses arising from the gaps between the experience and the assumptions are smaller. A salary growth 1 percent lower than the assumption would produce a surplus of around 1 percent of the fund at the beginning of the projections where the fund is equal to the value of the actuarial liabilities. However, in the case of a .5 percent gap in the dividend growth assumption and experience, the surplus only arise from the cash flow, and is more likely to be less than 1 percent of the fund. (See section 8.3.1.2.)

Consequently, there are differences in the magnitude and the patterns of the changes in contribution rates and the funding level between those in section 8.2 and those in section 8.3. However, as we have gone through the results, the conclusions are similar to section 8.2.6.

For an unchanged experience, an unchanged basis may be the best response.

In response to a single permanent change in the experience, it is important to use eventually the basis equal to the experience. To make the changes in the contribution rates and the funding level, gradual changes, or averaging of the past experience, are both effective. However, these would delay the achievement of a new stationary state.

In response to a gradual change in the experience, it is essential to keep the pace of the changes in the basis roughly the same as the experience. Averaging would qualify for this criterion.

In response to a cyclical change in the experience, the use of the basis which represents the middle experience is effective to maintain the stability.

Delays in the changes and less frequent changes in the valuation bases are not important for the long term effect on the fund, but they would increase short time fluctuations.

For the similar reasons as section 8.2.6, a long term averaging period and possibly a delay in the changes in the basis may be recommended.

8.4 Withdrawal rate changes

8.4.1 Basic impact

Changes in the withdrawal experience affect the cash flow and the membership structure.

In this model, all the benefits are assumed to be paid as lump sums at the time of exit. So, an increase in the withdrawal rates would increase immediate cash outgoes. Also, a higher level of withdrawals than before would reduce the total salaries, and future contribution incomes are reduced unless the contribution rate is increased.

The model assumes constant in-flows of new entrants at age 25, but does not allow replacements at any other ages. So, once a higher withdrawal rate occurs, the existing membership shrinks, and the average age of the membership reduces. However, the movements of the average age thereafter depend on the actual membership structure, and the average age could either increase or decrease. If the high withdrawal experience is a one off event, the original stationary population is restored after 40 years when all the members affected by the one off high withdrawal rates cease to be active members.

The exact impact on the population structure depends on the pattern of the changes in the withdrawal rates and the initial population structure. So, the details are discussed separately in each of the following sections.

Owing to the changes in the population structure, the average age moves. So, even if the valuation basis is unchanged, the standard contribution rate rises when the average age becomes older, and the standard contribution rate falls when the average age becomes younger.

Also, the spread of surpluses/deficits needs to be done over a smaller population if a large number of withdrawals occur. This may increase the degree of the adjustment to contribution rates from amortisation compared with the absolute size of the surpluses/deficits.

8.4.1.1 Ultimate cost

The benefits given to withdrawing members are deferred annuities which are increased in line with increases in the price inflation between the date of withdrawal and the normal retirement age. If these members had stayed until the normal retirement age, their benefits would have been increased in line with the salary increases up to the normal retirement age, which are assumed to be higher than the increases in price inflation.

Although the pension increase rate is the same after the normal retirement age, because of the lower growth in the benefits before the normal retirement age, the ultimate cost would be lower if members withdraw from the scheme.

Now, the cost of providing a unit pension to a younger member is lower than that for an older member because there is more time until the eventual pension payments and more investment proceeds can be expected during that period.

We shall consider a lump sum of 1 at age 65, the normal retirement age. If no withdrawals occur and all other standard experience are followed, the equivalent cash amount to the cost of this benefit at different ages would be;

1 / {1.09^(65-x)} 8.4.1.1.1

For some ages, the figures would be as follows;

Age	Equivalent Amount
25	3.18%
30	4.90
35	7.54
40	11.60
45	17.84
50	27.45
55	42.24
60	64.99

So, it is known that the cost of providing the benefits per unit is higher for older members. Once a member withdraws, the need to provide benefits to the member later when the member becomes older disappears. In this model, the member is not replaced by a new member at the same age, but new entrants are always aged 25. So, a withdrawal reduces the average cost to provide a unit pension.

From the discussion above, it is clear that the ultimate cost is reduced by a higher actual withdrawal rate in two senses; the reduction in the cost of growth in the accrued benefits and the reduction in the average cost to provide benefits because of a reduction in the average age.

Further, as mentioned earlier, the cash flow pattern is changed by the changes in the withdrawal rate. This may affect the investment returns achieved by the fund. Also, there may be external factors which lead to changes in withdrawal rates, salary inflation rates and investment returns. Hence, there may be a scope for investigation into the combined effects of withdrawal rate changes, salary inflation rate changes and investment return changes. However, such an investigation is beyond the scope of this research.

We shall now consider the impact of age at withdrawal further.

In the projections, a price inflation rate of 4.5%, a salary growth rate of 6.5% and a promotional salary increase of 1% are assumed to occur. So, the proportionate saving in the ultimate cost per a lump sum of 1 at age 65, the rate of cost reduction, is

1 - {1.045/(1.065*1.01)}^(65-x) 8.4.1.1.2

where x is the age nearest to withdrawal. Because the withdrawals are assumed to occur just before the birthday just after the salary increases, a withdrawal at age x means a withdrawal at age x-0 after salary increases. The rates of cost reduction for some ages are shown below;

Age	Rate of Cost Reduction
25	68.54%
30	63.64
35	57.99
40	51.46
45	43.91
50	35.18
55	25.11
60	13.46

So, the earlier they withdraw, the less the benefits cost.

However, the cost of providing benefits reduces as the age becomes younger. The actual proportionate cost reduction by a withdrawal compared with the case where the member stays until age 65 is;

$$[1 - \{1.045/(1.065*1.01)\}^{\wedge}(65\text{-}x)] * 1 / \{1.09^{\wedge}(65\text{-}x)\} \qquad .....8.4.1.1.3$$

For some ages, the actual proportionate reduction would be as follows;

Age	Actual Reduction
25	2.18%
30	3.12
35	4.37
40	5.97
45	7.83
50	9.66
55	10.61
60	8.75

So, the saving in cost as the equivalent amount at each age tends to be higher for older members, but the impact does not necessarily increase simply as the age increases.

This shows that the impact of withdrawals is dependent on the membership structure. If the average age is young, the impact tends to be small.

8.4.1.2 Gap between experience and assumption

If the actual withdrawals in a particular year are higher than assumed, more transfer values than expected will be paid out, and the assets at the end of year will be less than expected. Because the contributions are paid before the withdrawals take place in the model, there is no impact on the contributions received in the year.

At the end of year, the value of the liabilities will be reduced because there will be no need to fund for the members who have withdrawn. Should the members have stayed in the scheme, the value of liabilities would have taken into account future salary increases. However, the transfer values only include allowances for future price inflation, which is assumed to be lower than salary increases. So, the reduction in the value of the liabilities is higher than the reduction in the value of assets, which is the excess transfer payments over the assumed amount. Hence, a surplus arises.

On the same menbership structure and on the same valuation basis, the amount of surpluses arising is proportionate to the width of the gap.

In section 8.4.1.1, the age related impact on the ultimate costs was discussed by comparing with the costs without any further withdrawals in the future. Although the valuation basis may include some allowance for withdrawals, then similar arguments would hold. That is, the impact of withdrawals is dependent on the membership structure. If the average age is young, the surplus arising from a higher withdrawal rate than assumed tends to be small.

So, even if the gap is constant, the surpluses arising may change from year to year according to the change in the average age.

We shall now consider the impact of the valuation basis.

If the withdrawal rate assumption is 0%, the surplus arising will be in line with the reduction in cost calculated in section 8.4.1.1.

If a non-zero withdrawal rate is assumed, the anticipated cost, i.e. the value of liabilities, is calculated taking into account future withdrawals. So, the value of liabilities will be lower, while the transfer values paid will not be affected by the withdrawal rate assumption.

This means that the financial difference to the fund between the case where a particular member withdraws and the case where he/she remains active is smaller if the assumed withdrawal rate is higher.

Consider 100 members at the same age. If no withdrawals are assumed, but 1 member withdraws, a surplus arises.

If the withdrawal rate of 5% is assumed, surpluses from 5 withdrawals are anticipated, and the value of the liabilities at the beginning of the year is lower than that with the withdrawal rate of 0%. If, actually, 5 members withdraw, there will be no surpluses/deficits arising.

However, if 6 members withdraw, the financial difference between the experience and the assumption will be the value of liabilities for one active member at the end of year less the amount of transfer value for one withdrawal. Because the value of liabilities is less than that with the withdrawal rate of 0%, the difference, i.e. the surplus arising, will be smaller.

Thus, the surplus arising from the same gap between the experience and the assumption would be less if the withdrawal rate assumed is higher.

As a consequence, the surplus arising from actual withdrawals at 5% with a 0% valuation assumption would be less than the deficit arising from nil actual withdrawals with a 5% assumption on the same membership structure at the beginning of the year.

8.4.1.3 Valuation surplus/deficit

If a higher withdrawal rate is assumed, the anticipated cost of providing benefits would decrease because withdrawals mean reductions in the costs as discussed in section 8.4.1.1. So, when the withdrawal rate assumption is increased, a valuation surplus arises.

The impact would depends on the membership structure and the original level of withdrawal assumption.

We shall calculate the value of liabilities for age x on the assumption that the withdrawal rate is W at all ages. Here, age x means the age x exact just after all the exits are made.

Similar to 8.4.1.1.2, the ratio of withdrawal benefits to the full retirement benefits for a withdrawing member aged y-0 is;

$$f(y) = \{1.045/(1.065*1.01)\}\hat{}(65-y) \qquad .....8.4.1.3.1$$

Now, let

l(y): survival function, where l(x) = 1. That is, the probability of a member aged x will remain as a member at age y.

dw(y): expected number of withdrawals at age y-0 (with l(x) = 1)

V(x): value of liabilities at age x exact

From 8.4.1.1.1, the value at age x of a benefit 1 paid at age 65 is 1/1.09^(65-x). So,

$$V(x) = \{dw(x+1)*f(x+1) + dw(x+2)*f(x+2) + ... + dw(64)*f(64) + l(65)*1\} / \{1.09\hat{}(65-x)\} \qquad .....8.4.1.3.2$$

Here,

$$l(y+1) = l(y) - dw(y+1)$$

and

$$dw(y+1) \quad = \quad l(y) * W \qquad \text{for } y < 64$$

So,

$$l(x+n) \quad = \quad l(x) * (1 - W) \wedge n \quad = \quad (1 - W) \wedge n \qquad \text{for } x+n < 65$$

and

$$dw(x+n) \quad = \quad l(x+n-1) * W \quad = \quad W * (1 - W) \wedge (n-1) \qquad \text{for } x+n < 64$$

where

$$l(65) \quad = \quad l(64)$$

and

$$dw(64) \quad = \quad 0 \qquad .....8.4.1.3.3$$

Applying 8.4.1.3.1. and 8.4.1.3.3 to 8.4.1.3.2,

$$\begin{aligned} V(x) \quad = \quad & \{\ W \quad * \{1.045/(1.065*1.01)\}\text{^}(65\text{-}x\text{-}1) \\ & + W * (1\text{-}W) \quad * \{1.045/(1.065*1.01)\}\text{^}(65\text{-}x\text{-}2) \\ & + W * (1\text{-}W)\text{^}2 \quad * \{1.045/(1.065*1.01)\}\text{^}(65\text{-}x\text{-}3) \\ & + \quad .... \\ & + W * (1\text{-}W)\text{^}(64\text{-}x\text{-}1) * \{1.045/(1.065*1.01)\} \\ & + (1\text{-}W)\text{^}\ (65\text{-}x\text{-}1)\ \} \quad / \quad \{1.09\text{^}(65\text{-}x)\} \end{aligned}$$

$$\begin{aligned} = \quad & W\ /\ 1.09\ *\ \{1.045/(1.065*1.01*1.09)\}\text{^}(64\text{-}x) \\ & \quad *\ [1 - \{(1\text{-}W)*1.065*1.01/1.045\}\text{^}(64\text{-}x)] \\ & \quad /\ [1 - \{(1\text{-}W)*1.065*1.01/1.045\}] \\ + \quad & (1\text{-}W)\text{^}(64\text{-}x)/\{1.09\text{^}(65\text{-}x)\} \end{aligned}$$

$$(\text{if } W <> 1\text{-}1.045/(1.065*1.01) = .02849)$$

$$\begin{aligned} = \quad & (64\text{-}x)\ /\ 1.09 * \{1\text{-}1.045/(1.065*1.01)\} \\ & \quad * \{1.045/(1.065*1.01*1.09)\}\text{^}(65\text{-}x\text{-}1) \\ + \quad & \{1.045/(1.065*1.01)\}\text{^}(64\text{-}x)/\{1.09\text{^}(65\text{-}x)\} \end{aligned}$$

$$(\text{if } W = 1\text{-}1.045/(1.065*1.01) = .02849)$$

.....8.4.1.3.4

V(x) figures for various ages and withdrawal rates are shown in Table 8.4.1.3.5.

Table 8.4.1.3.5

(a) V(x) (Valuation Factor per Normal Retirement Benefit of 1 Due at Age 65) (%)

Withdrawal rate assumption(w)	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Age(x)														
25	3.18	2.74	2.40	2.14	1.94	1.79	1.67	1.57	1.49	1.43	1.38	1.34	1.31	1.28
30	4.90	4.35	3.91	3.56	3.28	3.06	2.88	2.74	2.62	2.52	2.44	2.37	2.32	2.27
35	7.54	6.88	6.34	5.89	5.52	5.22	4.96	4.75	4.58	4.43	4.30	4.19	4.10	4.02
40	11.60	10.85	10.22	9.68	9.22	8.83	8.49	8.20	7.95	7.73	7.54	7.38	7.23	7.11
45	17.84	17.07	16.39	15.79	15.26	14.79	14.38	14.01	13.68	13.39	13.13	12.90	12.69	12.51
50	27.45	26.76	26.12	25.54	25.01	24.52	24.07	23.66	23.29	22.95	22.63	22.34	22.08	21.84
55	42.24	41.75	41.29	40.85	40.44	40.04	39.67	39.32	38.99	38.67	38.37	38.09	37.82	37.57
60	64.99	64.81	64.64	64.47	64.30	64.14	63.98	63.82	63.67	63.51	63.37	63.22	63.08	62.94

(b) Difference in V(x) by a 1% increase in w

w(old) (%)	0	1	2	3	4	5	6	7	8	9	10	11	12
w(new) (%)	1	2	3	4	5	6	7	8	9	10	11	12	13
x													
25	-0.44	-0.34	-0.26	-0.20	-0.16	-0.12	-0.10	-0.08	-0.06	-0.05	-0.04	-0.03	-0.03
30	-0.55	-0.44	-0.35	-0.28	-0.22	-0.18	-0.15	-0.12	-0.10	-0.08	-0.07	-0.06	-0.05
35	-0.66	-0.54	-0.45	-0.37	-0.30	-0.25	-0.21	-0.18	-0.15	-0.13	-0.11	-0.09	-0.08
40	-0.74	-0.63	-0.54	-0.46	-0.39	-0.34	-0.29	-0.25	-0.22	-0.19	-0.16	-0.14	-0.13
45	-0.77	-0.68	-0.60	-0.53	-0.47	-0.42	-0.37	-0.33	-0.29	-0.26	-0.23	-0.21	-0.19
50	-0.70	-0.64	-0.58	-0.53	-0.49	-0.45	-0.41	-0.37	-0.34	-0.31	-0.29	-0.27	-0.24
55	-0.49	-0.46	-0.44	-0.41	-0.39	-0.37	-0.35	-0.33	-0.32	-0.30	-0.28	-0.27	-0.25
60	-0.18	-0.17	-0.17	-0.17	-0.16	-0.16	-0.16	-0.15	-0.15	-0.15	-0.14	-0.14	-0.14

(c) Percentage Change in V(x) by a 1% increase in w

w(old) (%)	0	1	2	3	4	5	6	7	8	9	10	11	12
w(new) (%)	1	2	3	4	5	6	7	8	9	10	11	12	13
x													
25	-13.94	-12.35	-10.80	-9.34	-8.00	-6.80	-5.76	-4.86	-4.11	-3.47	-2.95	-2.51	-2.15
30	-11.28	-10.06	-8.88	-7.78	-6.76	-5.85	-5.04	-4.33	-3.72	-3.20	-2.75	-2.38	-2.06
35	-8.75	-7.87	-7.03	-6.24	-5.52	-4.85	-4.26	-3.73	-3.26	-2.85	-2.50	-2.19	-1.92
40	-6.41	-5.83	-5.28	-4.76	-4.28	-3.83	-3.43	-3.06	-2.73	-2.43	-2.17	-1.94	-1.73
45	-4.31	-3.98	-3.66	-3.36	-3.07	-2.81	-2.56	-2.34	-2.13	-1.94	-1.76	-1.60	-1.46
50	-2.54	-2.38	-2.23	-2.09	-1.95	-1.82	-1.70	-1.58	-1.47	-1.37	-1.28	-1.19	-1.10
55	-1.16	-1.11	-1.06	-1.02	-0.97	-0.93	-0.89	-0.85	-0.81	-0.77	-0.74	-0.70	-0.67
60	-0.27	-0.27	-0.26	-0.26	-0.26	-0.25	-0.25	-0.24	-0.24	-0.23	-0.23	-0.22	-0.22

From Table 8.4.1.3.5, the following may be observed;

-By following the figures in Table 8.4.1.3.5 (b) and (c) horizontally, the degree of the change in V(x) arising from a 1 % increase in the withdrawal rate assumption is observed to reduce as the level of withdrawal rate assumption increases.

-By following the figures in Table 8.4.1.3.5 (b) and (c) vertically, the changes in V(x) arising from a 1 % change in the withdrawal rate assumption are observed to be higher if the age is younger. That is, the impact of the change in the basis depends on the membership structure.

Although a mathematical analysis of V(x) may reveal more details, this has not been investigated further in this research.

Now, the valuation surpluses arising from a 1% increase in the withdrawal rate assumption are larger than the surpluses arising from the actual experience being higher than the valuation assumption by 1%. This is because the former changes expectations for future withdrawals, while the latter is only related to one year's difference (see section 8.4.1.2).

Further, the spread of surpluses/deficits is made according to the valuation basis. So, when we compare the use of two different withdrawal rate assumptions on an identical experience, the pace of amortisation becomes faster for the case with the higher withdrawal rate assumption. This is because the use of a higher withdrawal rate assumption leads to a shorter average expected remaining membership period.

8.4.2 Unchanged experience

Refer to section A3.3.1 and graphs A3.3.1.a-c in Appendix 3.3.

The experience is the same as the standard basis and remains stable throughout the projections. So, there are no changes in the membership structure.

Three different bases, which are also constant throughout the projection, are used, and the long term effects of the choice of the basis are examined. The first basis is equal to the standard basis, the second basis has a higher withdrawal rate assumption than the standard basis, and the third has a lower one.

The pattern of the accumulations of the surpluses/deficits are similar as section 8.2, although the details are different. (see section 8.4.1)

To achieve the optimum level of funding, the use of the standard basis, which is equal to the experience, is necessary.

8.4.3 One permanent change in experience

The withdrawal rate experience is increased from year 1. It remains constant at the increased level throughout the projections thereafter.

Because of this change, the existing members reduce their weights within the membership structure compared with new entrants, the in-flow of which is unchanged. So, the average age gradually reduces. The pace of reduction is relatively quick initially, but becomes moderate as the change proceeds.

After 39 years, when the last initial members retire, the membership structure becomes stable again, but with a lower average age.

A constant valuation basis, one permanent change in the basis, and gradual changes in the basis are investigated.

8.4.3.1 Constant basis

Refer to section A3.3.2.1 and graph A3.3.2.a in Appendix 3.3.

The standard basis is applied to see the impact of the change in the experience without any moves in the basis. The projection is done as a basis for comparison with other projections which use changing bases.

Similarly to section 8.2.3.1, the funding level is initially 100%, but gradually increases because of the surpluses arising every year. The surpluses arising are almost constant, but gradually reduce as the average age goes down. The amortisation catches up with the accumulation of surpluses eventually, and the relative level of surplus becomes stable. This funding level is higher than the funding level achieved for the case where the valuation basis exactly follows the experience, which may be regarded as optimum.

8.4.3.2 One permanent change in basis

Refer to section A3.3.2.2 and graphs A3.3.2.b-g in Appendix 3.3.

In this case, the valuation basis is changed only once. Various combinations of the timing and the magnitude of the changes are examined. The different cases are now discussed in turn.

The standard contribution rate moves as the average age reduces, but stays the same after the population becomes stable again after year 40. However, the broad patterns of the emergence and accumulation of the surpluses/deficits are similar to the cases with a fall in the salary inflation rate (see section 8.2.3.2).

-Follow the experience

Initially, a surplus arises because of a reduction in the value of liabilities. From year 2 onwards, no surpluses/deficits arise, and the fund becomes stationary at the optimum level after the population becomes static and after amortising the initial surplus.

In this particular case, the pace of fall in the standard contribution rate and that in the reduction in the contribution rate arising from the amortisation show a mutually cancelling effect, and the recommended contribution rate has been kept relatively constant after year 1. However, this does not mitigate the large fall in the contribution rate at the end of year 1.

-Magnitude of change
Under different bases, although the fund eventually achieves stability, the level of funding is not at the optimum level.

Where the change in the basis is less than the experience, a cancellation effect between the reduced contribution rates and the newly accruing surpluses is observed, and the contribution rate changes are more moderate. On the other hand, the change in the valuation basis is larger, and the contribution rate shows wider fluctuations.

-Delayed change
A short delay in the change of the valuation basis may not cause a significant impact on the long term contribution and funding level patterns.

While the change in the valuation basis is delayed, the gap between the experience and the valuation basis produces surpluses, which are accumulated. So, the level of the surplus at the time of changing the basis would be slightly higher. So, in the short run, the delays may make the state of funding more volatile.

8.4.3.3 Gradual change in basis

Refer to section A3.3.2.3 and graphs A3.3.2.h-n in Appendix 3.3.

The averaging of past experience gives a gradual change in the valuation basis. Various combinations of the magnitude of the changes and the frequency of the changes in the basis are also examined. Now, the different cases are discussed in turn.

-Gradual change
The changes in the contribution rate are more moderate than when there is one large change in the basis.

Surpluses arise because of the change in the valuation basis and the gap between the experience and the assumption. Both factors are reducing as time passes.

Eventually, the surpluses are amortised, and the fund achieves the optimum stationary state. However, the changes in the basis are slower than the case with immediate changes, and it takes a longer time to achieve this optimum stationary state. So, the funding level stays higher for a longer time. There is a trade off between the stability of the contribution rate and quickly achieving the optimum funding level.

-Different speed in change

If a longer averaging period is used, each year's change in the valuation basis is smaller, and this will reduce the volatility in the recommended contribution rate.

On the other hand, a slower change in the basis results in a slower amortisation of surpluses, and the funding level stays higher for a longer time. So, the trade off between the stability of the contribution rate and quickly reaching the optimum funding level mentioned above is observed again.

-Frequency of change

Projections where the change in the basis is only made triennially although the valuation takes place every year have been carried out.

The general tendency of the long run pattern of the contribution rates and the funding levels is not affected by the less frequent basis changes, although the fluctuations in the short run are larger.

-Further change

Here, a projection where the change in the basis does not stop after reaching the level of experience has been carried out.

Until the basis reaches the experience level, the movements in the standard and recommended contribution rates are the same as averaging.

However, owing to the continuing weakening of the basis, the standard contribution rate increases. Also, valuation surpluses continue to be released, although they are reducing as the withdrawal rate assumption becomes higher. This makes the funding level lower than the optimum. So, deficits arise from the difference between the experience and the assumptions. The deficits, which are increasing, gradually become more dominant, and the recommended contribution rate increases together with the increases in the standard contribution rate.

After the valuation assumption for withdrawal rates becomes higher than the actual experience, the funding level continues to reduce and the degree of increases in the contribution rate rises.

8.4.3.4 Summary of 8.4.3

To acieve the optimum funding level, it is essential to move to the valuation basis which is equal to the experience.

Because of the change in the population structure and the different patterns of the accrual of surpluses (See section 8.4.1.), the impact are slightly different from section 8.2.3.

Short term fluctuations in the contribution rate and the funding level may be reduced by gradual changes in the basis. However, there is a trade off between this reduction in the fluctuations and the speed of achieving the optimum level of fund.

A few years' delay in the change of the valuation basis may not greatly alter the movements of the funding level and the contribution rates.

8.4.4 Gradual change in experience

Here, the actual withdrawal rate is gradually increasing.

Similarly to a single permanent change, this change makes the weighting for the existing members within the membership structure reduced compared with new entrants, the in-flow of which is unchanged. So, the average age gradually reduces.

Because the change each year is small, the reduction in the average age is small. However, because the changes continue, the average age continue to reduce throughout the projection.

In the case of the one permanent change in the withdrawal experience, the reduction in the average age continues for 39 years after the change. If we consider the case of a gradual change as the accumulation of single permanent changes for the last 39 years, it is clear that the pace of the reduction in the average age escalates initially.

However, because of the changing shape of the membership structure, the degree of the reduction in the average age becomes smaller as the withdrawal rate increases. For example, the effect of a change in the withdrawal rate from 10% to 10.1% is smaller than that from 5% to 5.1%.

A constant valuation basis and gradual changes in the bases are investigated.

Using arguments similar to section 8.2.4, the changing patterns of the valuation bases are appraised by the ability to cope smoothly with the changes.

8.4.4.1 Constant basis

Refer to section A3.3.3.1 and graph A3.3.3.a in Appendix 3.3.

If the valuation basis is not changed at all, the gap between the actual and assumed real withdrawal rates widens gradually, and surpluses which are increasing, arise.

The surpluses accumulate, and the funding level increases, which had been hoped to be stable. The recommended contribution rate falls accordingly.

The unchanged valuation basis may not be appropriate.

8.4.4.2 Gradual change in basis

Refer to section A3.3.3.2 and graphs A3.3.3.b-l in Appendix 3.3.

The valuation bases are changed gradually as straight lines. Various combinations of the magnitude of the changes, the timings of the start of changes and the frequency of the changes in the basis have been examined. Similar to section 8.2.4.2, additionally, some projections using the bases which are the averages of the past experience have also been carried out. The different cases are now discussed in turn.

-Follow the experience
Where the changes in the valuation basis starts immediately with exactly the same magnitude as the experience, there are no gaps between the experience and the assumptions. So, only valuation surpluses, which are steady over the years, arise. Consequently, the amortisation element of the contribution rate is kept small and stable.

The standard contribution rate is gradually reduced as the valuation basis is weakened and the average age becomes younger.

So, the size of the fund is smoothly reduced reflecting the changes in the experience.

Also, owing to the relatively small accumulated surpluses, the results may not be affected much by the choice of the amortisation method.

-More moderate change
More moderate changes than the changes in experience have been examined.

A moderate change in the basis compared with the changes in the experience makes the contribution rate more stable initially because of smaller changes in the standard contribution rate and a smaller level of valuation surpluses accruing.

However, as the changes in the experience develop more, more surpluses arise from the gap between the experience and the assumption, and a large surplus is accumulated. The reduction in the size of the fund becomes very slow, or the size might be increased initially. This would lead to rapid falls in the contribution rates later, and they would be unstable in the long run.

Also,the contribution rate changes are heavily dependent on the amortisation method.

-Delayed change
Delays in the start of the changes in the bases have been examined.

The effect of a delay is a higher level of accumulated surplus because of the gap between the experience and the basis which is created by the delay and never removed.

However, two years' delay does not have a significant effect on the basic shape of the contribution rate changes in the long run.

-Less frequent changes in basis

Projections have been carried out, where the valuations take place every year, but the changes in the valuation basis are made only every three years or ten years.

Again, the basic pattern of the projection results in the long run is not affected. However, the fluctuations in the contribution rates from year to year are higher than the case of annual changes in the valuation basis.

-Averaging

Here, the valuation basis is set as the average of the past experience. As the averaging period, 5 years and 10 years are examined.

Averaging has a similar effect as delaying the change of the basis. Because the pace of the change in the valuation basis is the same as that of the experience, the reduction in the size of the fund is smoothly carried out. There are no large surpluses accumulated, but instead small and constant surpluses emerge.

8.4.4.3 Summary of 8.4.4

With a gradual change in the withdrawal rates experienced, it is important to keep the changes in the valuation assumption roughly at the same pace. Otherwise, surpluses/deficits arising would increase, and the funding level may be driven in unwanted directions.

Different frequencies of and delays in the changes to the basis do not have a significant impact on the long term pattern of changes in the funding level and the contribution rates.

Averaging is similar in the pattern of changes in the valuation basis to delays in the changes, and keeps the pace of changes the same as that of experience.

8.4.5 Cyclical change in experience

Here, the withdrawal rate experience is assumed to follow a twenty year cycle.

Similar to gradual changes, the impact on the membership structure may be considered to be the accumulation of the effects from the previous 39 years' changes in the withdrawal rate.

For the first five years, owing to the increasing withdrawal rates, the average age of the membership reduces. Although, after year 6, the withdrawal rate falls, the average age continue to fall owing to the effects of earlier increases in the rate. However, the effects of these falls in the rates accumulate and appear later as increases in the average age.

Increases and falls in the withdrawal rates are repeated cyclically. After 40 years, the initial members disappear, and the membership structure shows a cyclical pattern. One cycle consists of 20 years. Owing to the above cumulative effects, the phase of the cycle of the membership structure is different from that of the experience in the withdrawal rate.

Now, after 40 years, the cumulative withdrawal rate for any consecutive 20 years is constant. Namely, the cohort who have been members for 20 years or more should have experienced for the last 20 years the cumulative withdrawal rate of

1 - (1-.05)*(1-.06)*(1-.07)*(1-.08)*(1-.09)*(1-.10)*(1-.09)*(1-.08)*(1-.07)*(1-.06)*(1-.05)*(1-.04)*(1-.03)*(1-.02)*(1-.01)*(1-.0)*(1-.01)*(1-.02)*(1-.03)*(1-.04) = 1 - .355 = .645,

or 5.045% per year on average regardless of the current phase of the cycle.

.....8.4.5.0.1

Therefore, the number of members aged y (45 =< y < 65) is always 35.5% of the number aged y-20. In particular, the number of members at age 45 should remain the same because the number of new entrants remains the same. However, because the withdrawal rate is always assumed to be 0 between age 64 and 65, the number of retiring members fluctuates according to the moves in the withdrawal rate.

The shape of the membership structure may look like a pyramid with a slight wave pattern, which shows a 20 year cycle. Now, this is clarified using formulae.

If the withdrawal rate has been constant at 5.045% per year, the membership structure is expressed as a reducing geometric sequence. Namely, on the basis of l(25) = 1,

l(y) = .94955 ^ (y-25) for y < 65.

.....8.4.5.0.2

This forms the basic pyramid shape. Here, it should be noted that l(45) is always expressed by the formula 8.4.5.0.2.

For ages between 26 and 44, l(y) depends on the cumulative withdrawal rate experienced since joining the pension scheme. If the avarage withdrawal rate experienced is lower than 5.045%, l(y) becomes larger than 8.4.5.0.2. On the other hand, if the average is higher, l(y) becomes smaller. Because the withdrawal rate experience changes gradually in a cyclical form, it can be observed that the changes in the average withdrawal rate experiences from one age to the next age are gradual.

This variance from the basic pyramid shape is a half of the wave pattern, which has a relatively small range of oscillation. From 8.4.5.0.1, the same pattern as the ages between 25 and 44 is repeated between ages 45 and 64. Therefore, this wave pattern has a 20 year cycle.

Now, in response to the cyclical change in the withdrawal rate experience, constant valuation bases and cyclical changes in the bases have been examined.

Although the experience keeps changing, the long term experience level may not change at all. So, the realistic value of the liabilities may not be affected by the cyclical changes in the experience, and may be calculated on the long term average basis.

8.4.5.1 Constant basis

Refer to section A3.3.4.1 and graphs A3.3.4.a-c in Appendix 3.3.

Three different constant bases have been considered; the highest, middle and the lowest withdrawal rates in the experience cycle.

The average funding level and the contribution rate show similar tendencies as the projections with unchanged experience. The use of a strong basis results in a permanent surplus.

Reflecting the cyclical changes in the experience, the funding level also shows a cycle with the same length. The peaks and troughs in the experience cycle are different from those in the funding level cycle. This is because the latter represents the accumulated effects of experience and the progression in the amortisation.

8.4.5.2 Cyclical change in basis

Refer to section A3.3.4.2 and graphs A3.3.4.d-l in Appendix 3.3.

In this section, the valuation bases are changed cyclically with the same length of cycles, i.e. 20 year cycles. Various combinations of the magnitude and the frequency of the changes in the basis are investigated.

Also, the effects of averaging are analysed by applying three different averaging periods. Now, the different cases are discussed in turn.

-Follow the experience
Where the changes in the valuation basis are made exactly the same as the changes in the experience without delays, the standard contribution rate changes and valuation surpluses/deficits arise each year. Consequently, the contribution rate and the funding level become more volatile than for the constant basis. As discussed in section 8.4.1.3, the effects of the changes in the valuation basis are more significant than the impact of the gap between the experience and the basis.

-More moderate changes
More moderate changes in the valuation basis have been examined.

The result lies somewhere between the unchanged basis and the exactly changing basis.

-Less frequent changes
Although valuations are made annually, the basis is changed only every 3 or 5 years.

In the short run, the fluctuations in the funding level and the contribution rate are more volatile. However, the long term general patterns of the contribution rates and the funding level are not affected much by less frequent changes in the valuation bases.

-Averaging

The valuation basis is set as the average of past experience. As the averaging period, 3, 5 and 10 years are used.

The effects of averaging on the setting of the bases are the moderation of the changes in the basis and the delay in the changes. The first effect is observed from the narrower range of the valuation basis than the basis exactly following the experience. The second effect appears as gaps between the phases of the experience cycle and the basis cycle.

The effect of the changes in the basis is dominant on the pattern of the changes in the contribution rate. The main effects of using a longer averaging period are a reduction in the volatility of the contribution rates and an increase in the delay in the phase of the cycle.

8.4.5.3 Summary of 8.4.5

When the withdrawal experience is moving cyclically, constant valuation bases make the funding level most stable. The optimum funding level may be achieved by the use of the central withdrawal rate experienced as the assumption.

Any movements in the valuation bases would increase the fluctuations in the funding level and the contribution rates.

Averaging leads to some fluctuations, but the degree is less than the basis exactly following the experience. Also, averaging makes the phase of the cycle of the valuation basis changes different from that of the experience cycle.

8.4.6 Conclusions to withdrawal rate changes

The impact of the changes in the withdrawal rates depends on the membership structure, which is itself affected by the changes in the experience. So, the changes in the contribution rates and the funding level are complex although the assets are not affected.

The difference between the value of benefits paid to withdrawing members is increased in line with price inflation before attaining age 65, while the rights to the benefits are increased in line with the increases in the salary if they stay in the scheme as active members.

The difference between these two values leads to surpluses when larger than expected withdrawals occur. However, because the difference is not large, up to a 5 percent increase in the withdrawal rate experience, up to a 5 percent increase in the withdrawal rate assumption and up to a 5 percent difference in the withdrawal rate experience and assumption all produce relatively small changes in the funding level and the contribution rates compared from the cases discussed in sections 8.2 and 8.3.

Although the details are different, the main conclusions are similar to section 8.2.6.

In response to an unchanged experience, an unchanged basis may be the best.

In response to a single permanent change in the experience, it is important to use eventually the basis equal to the experience. To make the changes in the contribution rates and the funding level, gradual changes, or averaging, are both effective. However, this would delay the achievement of a new stationary state.

In response to a gradual change in the experience, it is essential to keep the pace of the changes in the basis roughly the same as the experience. Averaging would qualify for this criterion.

In response to a cyclical change in the experience, the use of the basis which represents the middle experience is effective for maintaining stability.

Delays in the changes and less frequent changes in the valuation bases are not important for the long term effect to the fund, but they would increase short time fluctuations.

For the similar reasons as section 8.2.6, a long term averaging period and possibly a delay in the changes in the basis may be recommended.

8.5 Conclusions

As we have examined in previous sections, the three areas share many common properties although the mechanism of the changes in the contribution rates and the funding level are different for each area. In fact, the main conclusions have been the same for all the areas. For the differences, sections 8.2.1, 8.3.1 and 8.4.1 may be referred.

Here, the conclusions are given based on the main common observations.

8.5.1 Optimum funding level

From the discussion in section 6.3.2, the prime aim of controlling a pension fund has been identified as the maintenance of an adequate funding level. (see also section 7.2.3.)

Here, the term "an adequate funding level" does not mean that

the value of assets / the value of liabilities

under the valuation basis in use will be within a certain range of figures (e.g. 90% to 110%), but means that the assets are not excessive or too little compared with the liabilities in an absolute sense. The term "the optimum funding level" is the best level among all adequate funding levels.

Here, the difficulty is how to interpret the phrase "an absolute sense". In fact, "the optimum funding level" should depend on the acceptable level of risk of insolvency. If the risk is to be kept at a very low level, the optimum funding level would be higher than otherwise. Such investigations are left for future research, which would involve a stochastic approach so as to deal with the risk of insolvency.

In the cases of unchanged experience and one permanent change in experience, "the optimum funding level" was taken as the level where the value of the assets is equal to the value of actuarial liabilities calculated on a realistic valuation basis which is equal to the new stable experience.

If we accept the definition of "the optimum funding level" above, it may be concluded from the analysis of projections that, in order to achieve the optimum level of funding, it is essential to use the valuation basis which is the same as the level of experience. Deliberate margins in the basis will lead to a permanent surplus/deficit over the optimum level.

However, it must be noted that, if a higher criterion for the risk of insolvency is applied, the optimum funding level would be higher, and margins over the realistic basis may be regarded as necessary.

In the case of gradual changes in experience, it was found difficult to define cleary the optimum funding level. In the case of cyclical changes in experience, "the optimum funding level" was considered to be the level where the value of the assets is equal to the value of actuarial liabilities calculated on a realistic valuation basis which is equal to the long term average of experience, which is the mid value of the cyclical range. This definition may be contentious.

In summary,
-"the optimum funding level" must be investigated further, and
-if the optimum funding level is identified as the level where the value of the assets is equal to the value of actuarial liabilities calculated on a realistic valuation basis, the valuation basis to be used should not include deliberate margins.

8.5.2 Long term implications

The analysis of the projections shows that the level of accumulated surpluses/deficits should be kept within a specified range so as to maintain the stability in the funding level in the long run. Once the surpluses/deficits start accumulating rapidly, the consequence may be very volatile contribution rates and funding levels.

The pattern of the accumulation of surpluses/deficits depends on the pattern of the accrual of surpluses/deficits. If the surpluses/deficits accruing are increasing, it would be very difficult to stop their accumulation. This was particularly so in the projections carried out where the method of amortisation is fixed and the amortisation progresses slowly.

In this sense, in the case of gradual changes in the experience, it is essential to keep the pace of the changes in the valuation basis broadly matched to the pace of the changes in the experience in the long run, while the long term situation may not be significantly affected by short term departures.

Also, it has been observed that delayed changes and less frequent changes in the valuation basis may not cause any large long term effects. However, in the short run, these may increase volatility in the contribution rates.

In summary,
-the choice of the valuation basis is very important for the long term stability of the fund,
-in order to maintain the stability, the pace of the changes in the valuation basis should be kept broadly matched to the pace of the changes in the experience in the long run, and
-delayed changes and less frequent changes in the valuation basis may not cause any large long term effects, despite short term fluctuations in the contribution rates.

8.5.3 Short term implications

It has been observed that the impact of the changes in the valuation basis tends to be much larger than those of the gap between the experience and the assumption. So, for a smooth transition in the state of funding, gradual changes in the basis would be effective. Where the experience is cyclical, the best result can be obtained by the use of an unchanged valuation basis.

However, by smoothing the changes in the valuation basis, the achievement of the new optimum funding level may be delayed. There is a trade off between the smoothness of the transitions and the speed to achieve the optimum funding level.

Here, it should also be noted that the patterns of the changes in the contribution rates and the funding level are affected by the choice of the amortisation method. In particular, the choice of the amortisation method becomes very important if the surpluses/deficits accumulate to a high level. For example, where a large initial valuation surplus is released, the moves in the contribution rates and the funding level are dominated by this choice, at least initially.

8.5.4 Practical points

Finally, it must be pointed out that the future course of the events are unknown, and it would be practically important to choose the option which reasonably fits in with all the possible situations.

For example, it would be best to follow the experience exactly if the experience is changing gradually in one direction. However, the gradual change might stop suddenly, or the changes may start to move in the opposite direction. Then, there may have been better courses of changes in the valuation basis in order to keep the contribution rate more smooth.

In this sense, long term averaging is useful because, although it might not be the optimum approach in individual cases, it does fit reasonably well with the various situations considered here. Averaging also helps with smoothing the contribution rates by changing the valuation basis gradually.

Also, delays in the changes in the valuation basis may be useful for identifying more clearly the trend in the actual experience.

Averaging and delays would also be found more useful in practice where the experience fluctuates over a short period.

PART V

Chapter 9 Conclusions

In this closing chapter to the thesis, we summarise the main findings of the investigations. Various aspects of the mechanism of a pension fund, particularly those related to valuation, have been investigated. We shall briefly review each main part. For the full results, each part itself should be revisited.

In Part II, the mechanism of a pension fund has been analysed using a model which notionally separates the fund into individual pots and a common pool. An individual pot represents the value of the leaving service benefit for each member. The balance of the fund forms the common pool. Contributions are also separated into each individual's portion, which is the standard contribution individually calculated according to the funding method used, the member's age, sex and class, and the balance. The individual portion of the contributions are regarded to be paid into the individual's pot, and the balance to the common pool. Investment incomes are apportioned between the individual pots and the common pool. Benefits are paid by applying the individual's pot. Any excess/shortfall in the individual pot is met by a transfer of fund between the individual pot and the common pool whenever it is necessary. The amount of this transfer is defined as X-function, which plays a key role in the model.

This model seems to be particularly effective in explaining the effect of withdrawals and the cross-subsidy problem. Also, the model may be used to explain the operation of different funding methods.

In Part III, the matching of liabilities and assets has been discussed. The main findings are as follows.

-The precise meaning of matching depends on the risk under consideration. The risk may be expressed by a particular utility function appropriate to the pension fund.
-The risks are related to the objectives of the pension fund.
-The major risks may be the instability in the contribution rate and insolvency, and two distinctive types of matching, V-matching and S-matching respectively, can be defined according to these risks.

The discussion above has been applied to interpret the meaning of the use of a notionally matched fund combined with the discounted cash flow approach for the valuation of assets. The conclusions are as follows.

-The matching for this sense should be the S-matching, which is defined according to the risk of insolvency.
-The use of a notionally matched portfolio would give the minimum amount of assets required to realise the risk of insolvency within a specified level.
-However, the use of a notional matched portfolio ignores a possible higher level of risk of insolvency

resulting from the difference between the notionally matched portfolio and the actual portfolio.
-The use of any portfolio for valuation purposes cannot allow for the degree of risk of insolvency under a deterministic valuation.
-The only way to take into account the risk of insolvency inherent in the actual investment policy would be to add an extra solvency reserve either explicitly or implicitly.
-A stochastic approach to the actual portfolio may allow us to assess the degree of risk.

Indeed, the assessment of risk is a first step to the management of the risk, and for this reason, the development of such a stochastic valuation is awaited. The possibility of the use of a stochastic technique in valuation has been discussed at the end of Chapter 5.

In Part IV, control of a pension fund through valuation has been investigated. Chapter 6 is central to the theoretical discussion, while Chapters 7 and 8 provide a large numerical experiment.

In Chapter 6, from the general discussion on the control of a pension fund, the following has been argued.
-The financial objectives of managing a pension fund need to be identified before considering a control system.

-The prime objective to be achieved should be maintaining an adequate level of fund.
-Stabilising the contribution rate is another possible objective, but this ranks second to the objective above.
-Hence, a control system which aims to maintaining an adequate level of fund should be investigated further.

According to control theory, there are two types of control systems; open-loop control systems and closed-loop feedback system. Various means of control for a pension fund are examined and classified as one of the types of control systems. In particular, control through the setting of the valuation basis has been analysed in detail, and the following conclusion has been obtained.
-A control system which uses the choice of the valuation basis as the means of control can be interpreted as an open-loop control system.

Also, it has been remarked that
-The best way to control a particular pension fund will depend on the particular circumstances.
-The more means to control there are, the easier to find the best way to control the fund.

To exercise controls, valuations are essential to monitor the situation of the fund. Also, the setting of contribution rates through the valuation process is one of the most important ways of controlling the fund.

Chapters 7 and 8 have shown a series of projections carried out to investigate optimal ways of selecting the valuation basis based on the discussion of controls in Chapter 6. The research covered three independent areas separately; changes in real investment returns, changes in dividend growth rates, and changes in withdrawal rates.

The investigation has been carried out only to a limited extent. However, the conclusions from different areas have been similar although the mechanism of the impacts of these changes are totally different. In particular, it has been noticed that the impact of changes in the valuation basis is in general far larger than that from the difference between the actual and expected experience by the same degree as the changes in the valuation basis. Therefore, it has been found that large changes in the valuation basis tend to cause fluctuations in the contribution rates and the funding level. However, it has also been found that, in the long run, the valuation basis should be changed in line with the changes in the actual experience in order to maintain an adequate level of funding. As a result, the use of the averaging of past experience for the setting of valuation assumptions and the delaying of changes in the valuation basis have been recommended in all the areas.

I would like to end this thesis by pointing out possible future extensions to this research.

Firstly, each part of this thesis can be deepened further. For example, as an extention to Part III, a stochastic valuation might be applied on a model. The projections in Part IV can be extended further. Also, in Part IV, during the course of the analysis of projections, some difficulty was noted in identifying the optimum funding level when the experience is changing gradually and continuously. If the optimum funding level can be defined in a theoretically satisfactory way (in terms of maximising or minimising a well-defined objective function, for example), the corresponding part of the analysis may be improved.

Each aspect of valuation of a pension fund has been investigated separately in this thesis. However, these aspects are in fact closely related each other. For example, the understanding of the mechanisms of a pension fund discussed in Part II may help the consideration of the control approach. Although such combined discussions have not been made in this thesis, they might give interesting results.

Another possible development from this thesis may be researches of the subjects in this thesis in conjunction with other sets of scientific theories. For example, the choice of the valuation basis discussed in Part IV might be approached from the basis of decision theory (or game theory).

Appendix 1 Mathematical Relationsip of Dividend Yield, Dividend Growth and Investment Return

A1.1 Introduction

Mathematical relationships among the market dividend yields, the market dividend growth expectation and the actual rates of investment return on the market value of assets are developed first in this appendix. Then, the discussion is extended to the rates of investment return on the assessed value of assets, which are determined by a discounted cash flow approach on the valuation basis .

The assumptions made here are almost identical to those for the projection model except for some conditions which are more generalised in this appendix. So, the discussion below is used to construct the projection model.

Finally, a short discussion on the setting of the valuation assumptions is developed in section A1.6.

A1.2 Notation

The notation defined in section 7.3 is used in this appendix. It should be noted that between i(t), d(t) and g(t), the following relationship is assumed;

$$1 + i(t) = (1 + g(t)) * (1 + d(t)) \qquad \text{.....A1.2.1}$$

A1.3 Assumptions

The following is the list of assumptions made for the discussion in this appendix.

A1.3.1 Pension fund

A1.3.1.1 100% of the assets are invested in the equities.

A1.3.1.2 Valuations take place at the end of each year.

A1.3.1.3 Valuation of the assets is done using the discounted cash flow method.

A1.3.1.4 All the benefit payments are made once a year just before the valuation.

A1.3.1.5 Contributions are received once a year just after the valuation; i.e. at the beginning of each year.

A1.3.1.6 The cash available is immediately invested into equities.

A1.3.2 Financial market

A1.3.2.1 All the equities in the market are identical.

A1.3.2.2 Dividends are paid once a year just before the valuation.

A1.3.2.3 The market at any time t has an expectation of a constant dividend growth rate, g"(t), over the future.

A1.3.2.4 The market at any time t has an market interest rate, i"(t).

A1.3.2.5 The market prices are set by the discounted cash flow method using g"(t) and i"(t).

A1.3.3 Timing

For the definitions of T-0, T and T+0, section 7.3 should be referred to.

Benefit payments and dividend incomes occur between T-0 and T. However, B(T) and D(T) are used to represent these cash flows because there are no fears of notational confusion.

It is assumed that the market conditions, which is represented by i"(t) and g"(t), stay the same during the period T-0 and T+0. .....A1.3.3.1

M(T-0) is the market value of assets just before the dividend payouts, and includes the value of dividends due at time T. i.e, M(T-0) is a cum-dividend value. On the other hand, M(T) is the market value of assets after benefit payments, B(T), and dividend receipts, D(T). The receipt of the dividend has two effects. Firstly, it reduces the market value of existing assets. Secondly, the dividends are received by the pension fund, and thus the market value is increased. Here, the amounts of increase and reduction in the market value are the same. Hence, no changes are made in the market value of assets by the dividend payments. Therefore,

$$M(T) = M(T-0) - B(T) \qquad \text{.....A1.3.3.2}$$

Here, it should be noted that, under the assumptions in this appendix, the exact order of the timings of the benefit payout and the receipt of dividends does not affect the investment performance of the pension fund because M(T-0) is a cum-dividend value. If the benefits are paid before the receipt of dividends, the equities are sold with the rights to the imminent dividends. However, the sale price includes the value of the dividends. Therefore, there are no financial disadvantages to the pension fund by selling the equities compared with the case where the benefits are paid after receiving the dividends.

Now, M(T+0) and M(T) has the relationship;

$$M(T+0) = M(T) + C(T) \qquad \text{.....A1.3.3.3}$$

At time T, the market dividend yield is defined as follows;

$$d''(T) = D(T) \ / \ \{M(T-0) - D(T)\} \qquad \text{.....A1.3.3.4}$$

Where t is not an integer, d"(t) is not defined.

A1.3.4 Basic formulae

Corollary A1.3.4.1

$$1 + i''(T) = (1 + d''(T)) * (1 + g''(T))$$

Proof

D(T) is paid on the equities which have the market value of M(T-0). M(T-0) - D(T) represents the market value of the equities after the dividend payment but before the reinvestment of the dividends.

From A1.3.2.5, {M(T-0) - D(T)} is a discounted value of all expected future dividends except for the dividends due at time T. The dividend income at time T is D(T). So, the next expected dividends are D(T) * (1 + g"(T)) in one year time, and then, annual dividends increasing by 1 + g"(T) per annum compound thereafter. The projected incomes are discounted at the interest rate i"(T). Therefore,

$$\begin{aligned} M(T-0) - D(T) &= (1 + g''(T)) / (1 + i''(T)) * D(T) + (1 + g''(T))^2 / (1 + i''(T))^2 * D(T) \\ &\quad + (1 + g''(T))^3 / (1 + i''(T))^3 * D(T) + \ldots\ldots \\ &= D(T) * (1 + g''(T)) / (1 + i''(T)) / \{1 - (1 + g''(T)) / (1 + i''(T))\} \\ &= D(T) * (1 + g''(T)) / \{i''(T) - g''(T)\} \qquad \ldots\ldots A1.3.4.1.1 \end{aligned}$$

So, from the definition of d"(T) (A1.3.3.4),

$$d''(T) = \{i''(T) - g''(T)\} / \{1 + g''(T)\} \qquad \ldots\ldots A1.3.4.1.2$$

Adding 1 to both side,

$$\begin{aligned} 1 + d''(T) &= \{i''(T) - g''(T)\} / \{1 + g''(T)\} + 1 \\ &= \{1 + i''(T)\} / \{1 + g''(T)\} \end{aligned}$$

QED

Corollary A1.3.4.2

$$M(T-0) = D(T) * (1 + d''(T)) / d''(T)$$

Proof

From the definition of d"(T) (A1.3.3.4),

$$M(T-0) - D(T) = D(T) / d''(T) \qquad \ldots\ldots A1.3.4.2.1$$

Therefore,

$$M(T-0) = D(T) + D(T) / d''(T)$$
$$= D(T) * (1 + d''(T)) / d''(T)$$

QED

N.B. A1.3.4.2.1 can also be derived from A1.3.4.1.1 and A1.3.4.1.2.

Corollary A1.3.4.3 ---

$$c(T) = d''(T) / d(T)$$

Proof

From assumptions A1.3.1.1 and A1.3.2.1, the entire assets consist of only one type of assets, a particular type of equities, and from assumption A1.3.1.3, the assets are valued according to a discounted cashflow approach. Therefore, the ratio of VM(T) / M(T) does not depend on the size of M(T). i.e. there is a unique c(T), which is independent of the size of the assets which are to be valued. Therefore, it is sufficient to show Corollary A1.3.4.3 for a particular size of fund.

Here, we shall take a fund, the market value at T of which is equal to M(T-0) - D(T). This is the ex-dividend value of the fund excluding the dividend income at time T. From the definition of d"(T) (A1.3.3.4),

$$M(T-0) - D(T) = D(T) / d''(T)$$

At the time of valuation, T, the next dividend payment is expected at time T+1. At time T, a dividend of D(T) is produced from this fund, M(T-0) - D(T), and the dividend income is expected to grow at the rate of g(T) on the valuation basis. Hence, following a similar calculation as in A1.3.4.1.1,

$$\text{Value of } \{M(T-0) - D(T)\} = D(T) * (1 + g(T)) / \{i(T) - g(T)\}$$

Applying A1.2.1,

$$\text{Value of } \{M(T-0) - D(T)\} = D(T) / d(T)$$

Therefore,

$$c(T) = \text{Value of } \{M(T-0) - D(T)\} \ / \ \{M(T-0) - D(T)\}$$

$$= \{D(T) / d(T)\} \ / \ \{D(T) / d''(T)\}$$

$$= d''(T) / d(T)$$

QED

A1.4 Investment return on the market basis

A1.4.1 Investment return

We shall now calculate the investment returns on the market value.

At time T+0, the total market value of the fund is M(T+0). There are no cash flows during the period T+0 and T+1-0.

At time T+1, dividend of (1 + g'(T+1)) * D(T) would be paid if the capital held were M(T-0) - D(T) at time T. Hence, the dividend income at time T+1, D(T+1), will be

$$D(T+1) = (1+g'(T+1)) * D(T) * M(T+0) / \{M(T-0) - D(T)\}$$

$$= (1+g'(T+1)) * d''(T) * M(T+0) \qquad \text{.....A1.4.1.1}$$

Here, from A1.3.3.4, the definition of d",

$$M(T+1-0) - D(T+1) = D(T+1) / d''(T+1)$$

$$= (1+g'(T+1)) * d''(T) * M(T+0) / d''(T+1) \qquad \text{.....A1.4.1.2}$$

So,

$$M(T+1-0) = (1+g'(T+1))\, d''(T)/d''(T+1)\, M(T+0) + (1+g'(T+1))\, d''(T)\, M(T+0)$$

$$= (1+g'(T+1))\, d''(T)/d''(T+1)\, (1+d''(T+1))\, M(T+0) \qquad \text{.....A1.4.1.3}$$

Because there are no cash flows between T+0 and T+1-0, the rate of return can be expressed as the rate of increase in the market value. Hence, the investment return during year T is,

$$1 + I(T+1) = M(T+1-0) / M(T+0)$$

$$= (1+g'(T+1))\, (1+d''(T+1))\, d''(T) / d''(T+1) \qquad \text{.........A1.4.1.4}$$

Here, d"(T) / d"(T+1) may be interpreted to represent the market price level changes. This is affected by the market interest rate and the market dividend growth expectations.

A1.4.2 Comparison with expected interest rate

Now, the actual return, I(T+1), and the market expectation, i"(T), are compared.

From the definition of i"(T),

$$(1 + i''(T)) = (1 + g''(T)) * (1 + d''(T))$$

So, from A1.4.1.4,

$$(1 + I(T+1)) / (1 + i''(T)) = (1 + g'(T+1)) / (1 + g''(T))$$

$$* \quad (1 + d''(T+1)) / (1 + d''(T)) * d''(T) / d''(T+1) \qquad .....A1.4.2.1$$

If d"(T) = d"(T+1), i.e, if the dividend yield is unchanged, A1.4.2.1 becomes;

$$(1 + I(T+1)) / (1 + i''(T)) = (1 + g'(T+1)) / (1 + g''(T)) \qquad .....A1.4.2.2$$

The market price of one share will increase by the actual dividend growth rate, g'(T+1). (See assumption A1.2.2.4.) Also, the dividend income will grow at the same rate. Hence the first element of A1.4.2.1, namely the right hand side of A1.4.2.2, may be interpreted as the "capital element" which represents the difference in the capital growth excluding the market level changes.

On the other hand, if the dividend growth were the same as expected, A1.4.2.1 becomes;

$$(1 + I(T+1)) / (1 + i''(T))$$

$$= (1 + d''(T+1)) / (1 + d''(T)) * d''(T) / d''(T+1) \qquad .....A1.4.2.3$$

Because the dividend growth was equal to what had been expected, the dividend income at time T+1 should be equal to the expectation. So, the difference expressed as A1.4.2.3 may be interpreted as representing the "market element". This expression is, unlike c(T) in Corollary A1.3.4.2, not the simple ratio of two dividend yields. This is because the investment returns include dividends received at time T+1.

Hence, the difference in the expected investment return, i"(T), and the actual, I(T+1), can be split into two elements; namely,

$(1 + g'(T+1)) / (1 + g''(T))$: the factor representing the difference in the dividend growth

$\{(1 + d''(T+1)) / (1 + d''(T))\} * d''(T) / d''(T+1)$: the factor representing the difference in the market price level.

If the dividend growth were as expected and the market dividend yield is the same as one year before, the return achieved is the same as the market interest rate one year before.

Now, if $g''(T+1) = g'(T+1)$ as assumed in section 7.4.4, A1.4.2.1 becomes

$$(1 + I(T+1)) / (1 + i''(T)) = (1 + g''(T+1)) / (1 + g''(T))$$

$$* \quad (1 + d''(T+1)) / (1 + d''(T)) * d''(T) / d''(T+1)$$

$$= (1 + i''(T+1)) / (1 + i''(T)) * d''(T) / d''(T+1) \qquad \text{.....A1.4.2.4}$$

Further, if $i''(T) = i''(T+1)$ as assumed in section 7.5.1.2, A1.4.2.4 becomes

$$(1 + I(T+1)) / (1 + i''(T)) = d''(T) / d''(T+1) \qquad \text{.....A1.4.2.5}$$

That is, in this case, the difference between the market interest rate at T and the actual investment return is only the difference in the market price level, or the market dividend yield.

A1.5 Investment return on the valuation basis

Now, the investment return on the valuation basis is analysed. Thornton & Wilson(1992b) have carried out a similar calculation of the investment return on the valuation basis without considering any changes in the valuation assumptions. In this section, the changes in the valuation assumptions are allowed for.

A1.5.1 Investment return

The assessed value of the fund at time T after receiving the dividends before the payment of benefits is VM(T+1-0). Hence,

$$\begin{aligned} 1 + I'(T+1) &= VM(T+1-0) \,/\, VM(T+0) \\ &= \{ c(T+1)\,(M(T+1-0) \} \;\; / \;\; \{ c(T)\, M(T+0) \} \\ &= c(T+1) \,/\, c(T) \;\; * \;\; (1 + I(T+1)) \end{aligned}$$

Applying A1.4.1.4,

$$1 + I'(T+1) = c(T+1) \,/\, c(T) \; * \; (1 + g'(T+1)) \; * \; (1 + d''(T+1)) \; * \; d''(T) \,/\, d''(T+1) \qquad \text{.....A1.5.1.1}$$

Applying Corollary A1.3.4.2,

$$\begin{aligned} 1 + I'(T+1) &= d''(T+1) \,/\, d(T+1) \\ &\quad * \; d(T) \,/\, d''(T) \\ &\quad * \; (1 + g'(T+1)) \; * \; (1 + d''(T+1)) \; * \; d''(T) \,/\, d''(T+1) \\ &= d(T) \,/\, d(T+1) \; * \; (1 + g'(T+1)) \; * \; (1 + d''(T+1)) \end{aligned} \qquad \text{.....A1.5.1.2}$$

A value on the valuation basis at time T is translated into the corresponding value on the basis at time T+1 by multiplying the first part of A1.5.1.2, d(T) / d(T+1). In fact, this translation factor has a similar form as c(T). (see Corollary A1.3.4.3) Therefore, this part would represent the effect of the changes in the valuation basis.

As we can see from A1.5.1.2, the rate of return I'(T+1) is dependent on d"(T+1), but not dependent on d"(T). In order to explain this, we shall now take a unit of equity which has the market value of MV(T) at time T. Here, for the sake of clarity, separate notation, MV and Div, is used instead of M and D respectively.

(see section 7.4.4) $MV(T+1)$ is the market value of the same asset at time $T+1$ excluding dividend income at time $T+1$, $Div(T+1)$. If the valuation basis is not changed, the value of the capital, $MV(T+1)$, will grow in line with the actual growth of the dividend. i.e, the value of the capital will grow by $(1 + g'(T+1))$. (This is a consequence of assumption A1.3.2.5.)

At the same time, the value of the assets will be increased by the dividend income at time $T+1$, $Div(T+1)$. Because $c(T+1)$ is independent of the size of the fund, the value of the assets will be increased proportionately to the increase in the market value of the fund. i.e. the value of the assets will be increased by $(1 + d''(T+1))$.

A1.5.2 Comparison with expected return

Dividing the both sides of A1.5.1.1 by A1.2.1, we obtain

$$(1 + I'(T+1)) / (1 + i(T)) = d(T) / d(T+1) * (1 + g'(T+1)) / (1 + g(T)) * (1 + d''(T+1)) / (1 + d(T)) \quad .....A1.5.2.1$$

So, the difference in the expected and the actual returns can be separated into three parts; namely,

$d(T) / d(T+1)$: the effect of the changes in the valuation basis (as already explained in section A1.5.1.)

$(1 + g'(T+1)) / (1 + g(T))$: difference in the dividend growth

$(1 + d''(T+1)) / (1 + d(T))$: difference in the dividend income

Here, if $g''(T+1) = g'(T+1)$ as assumed in section 7.4.4, A1.5.2.1 becomes

$$(1 + I'(T+1)) / (1 + i(T)) = d(T) / d(T+1) * (1 + g''(T+1)) / (1 + g(T)) * (1 + d''(T+1)) / (1 + d(T))$$
$$= d(T) / d(T+1) * (1 + i''(T+1)) / (1 + i(T)) \quad .....A1.5.2.2$$

Similar to A1.4.2.5, if $i''(T+1) = i(T)$ is also assumed as section 7.5.1.2, this becomes

$$(1 + I'(T+1)) / (1 + i(T)) = d(T) / d(T+1) \quad .....A1.5.2.3$$

A1.6 Setting the valuation basis

In this section, a brief discussion on the approach to setting the valuation basis is made in the light of the formulae developed in previous sections. So, this discussion is also restricted by the assumptions made in section A1.3. Particularly, this discussion would only be valid for the pension scheme which invests its entire funds in equities.

When we wish to value the pension fund realistically, we would use an assumption which represents a long term average expectation of each element; a long term likely average investment return, a long term likely average salary growth rate, and so on.

From A1.5.2.1, the differences in the actual and expected investment returns on the valuation basis would arise from the difference between $g'(T+1)$ and $g(T)$ and the difference between $d''(T+1)$ and $d(T)$.

So, when the dividend yield assumption needs to be set, the history of the market dividend yields may be investigated.

Similarly, it would be logical to investigate the historical dividend growth rates when we set the dividend growth assumption. The market dividend growth expectations, which are difficult to assess, are irrelevant.

Consequently, the market interest rates are not directly relevant to the investment return assumption because the market interest rate is related to the market dividend growth expectations rather than the actual dividend growths.

The market dividend growth expectations and the actual dividend growths can be different, and can remain different for ever at least in theory. As discussed earlier, the investment returns depend on the actual dividend growth and the market dividend yield. So, even if the assessment of the market interest rate for the equities is appropriate, the investment return assumption might prove to be far from the actual experience. This way of setting the investment return assumption can only be appropriate when the market expectations on the dividend growths are on average right in the long run.

Hence, the investment return assumption may be decided from the dividend growth assumption and the dividend yield assumption. Alternatively, investigations into the historical investment returns, which might be adjusted to remove market fluctuations, may be carried out.

Appendix 2 Details of the Projection Calculations

This appendix describes the details of the projection calculations. Section 7.3 should also be referred to.

The notation in Appendix 1 is followed here, but some new notation is added in the text as necessary.

A2.1 Construction of initial membership

The projections start from a stationary population under the standard experience. From the assumptions made in section 7.3,

- the new entrants are aged 25,
- the normal retirement age is 65, and
- all the benefits are paid as lump sums at the time of exits.

So, the stationary population was constructed from the state of no members by running the main routine for 40 years. (See section A2.3 below)

A2.2 Initial valuation

At the beginning of the projection, a valuation of the liabilities is carried out on the standard valuation basis. The initial value of assets is set to be equal to the value of the liabilities. The market value of the assets are calculated as;

$$M(0) \quad = \quad VM(0) \ * \ d(0) \ / \ d''(0)$$

Because the initial market conditions are always equal to the standard experience, here, $d(0) = d''(0) = .03810$

The recommended contribution rate is set as the standard contribution rate derived from the valuation.

A2.3 Main routine

Let T be the time at the start of a particular scheme year.

The main routine starts from just after the valuation at the end of the previous year. The market value of assets held is M(T).

At the beginning of the year, i.e. time T, contributions are paid in as the recommended contribution rate calculated in the valuation at time T multiplied by the total members' salaries. The contributions received are immediately invested in equities. So, the market value of assets increase by the amount of the contributions received. i.e.

$$M(T+0) \quad = \quad M(T) + C(T)$$

During the year, no cash flows and member movements occur. That is, no dividends are paid during the year.

At the end of year, the new market interest rate, $i''(T+1)$, the new market dividend growth expectation, $g''(T+1)$, the new market dividend yield, $d''(T+1)$, and the new market price inflation expectation, $f''(T+1)$, are applied.

The market value of assets before any cashflows at the end of year T (i.e, the market value excluding dividend payments at time T+1, and before benefit payouts and contribution incomes) is calculated as; (Refer to A1.4.1.3.)

$$M(T+1-0) - D(T+1) \quad = \quad M(T+0) * \{1 + g'(T+1)\} * d''(T) / d''(T+1)$$

For theoretical discussions, Appendix 1 should be consulted.

Events then occur in the following order.

Firstly, dividends are received. According to A1.4.1.1, the amount of dividend received is expressed as follows;

$$D(T+1) \quad = \quad M(T+0) * d''(T) * \{1 + g'(T+1)\}$$

Secondly, salaries are increased to all the members according to the salary growth, $e(T)$, and promotional salary increase. All the members have their birthdays on the same date, which is the same date as the end of the scheme year, and the salaries are increased on their birthday.

Let $p(t)$ be the rate of promotional salary increase between time t-1 and t. This rate is applied to all ages uniformly. (see section 7.3.2.)

Then, the new salary at time T is the old salary multiplied by $(1 + e(T)) * (1 + p(T))$.

Thirdly, withdrawals and retirements occur immediately after their birthday, and their salary rises. The withdrawal and retirement benefits are paid out as lump sums.

Let

$L(t,x)$: the number of active members aged x at time t

$R(t,x)$: the number of retiring members aged x at time t

$W(t,x)$: the number of withdrawing members aged x at time t.

Here, all retirements and withdrawals are assumed to occur at the end of each scheme year. (see section 7.3) Therefore,

$R(t,x) \;=\; W(t,x) \;=\; 0 \quad$ if t is not an integer.

Retirements occur only when members reach age 65. Hence,

$$R(T+1,65) \quad = \quad L(T+0,64)$$

$$R(T+1,x) \quad = \quad 0 \qquad \text{if } x < 65$$

Here, no withdrawals have been assumed for the members aged 64.

The number of withdrawing members at age x, W(T+1,x), is calculated from the active members aged x at the beginning of the year, L(T+0,x-1), and the withdrawal rate for year T, w(T). Here, all the members who are aged 64 at the beginning of year are excluded because they have already been treated as retired.

$$W(T+1,x) \quad = \quad L(T+0,x-1) \ * \ w(T) \qquad \text{for } x < 65$$

So, the remaining members aged x becomes

$$L(T+1+0,x) \quad = \quad L(T+0,x-1) \ - \ W(T+1,x)$$

It should be noted that the benefits are calculated after salary increases. The total amount paid, B(t+1), is calculated in the benefit calculation routine below.

Fourthly, new entrants are admitted to the scheme. They are aged exactly 25, with the same birthdays as existing members. The number of the new entrants is kept constant. So,

$$L(T+1+0,25) \quad = \quad \text{a constant number given.}$$

The joining salary is set artificially at the beginning of the creation of the initial population (See section A2.1), and is increased every year by the experienced salary growth during the year. Here, promotional increases are excluded.

Finally, a valuation takes place, and the new standard and recommended contribution rates are calculated. (See section A2.4) At the time of the valuation, the market value of assets, M(T+1), is

$$M(T+1) \;=\; M(T+1-0) \ - \ B(T+1)$$

This routine is repeated until the projection comes to the end.

A2.4 Valuation routine

The valuation at time T is carried out using the membership and assets at time T and the valuation assumptions for time T, which are inputs to the system.

The valuation follows the Projected Unit funding method, and the surpluses/deficits are spread over the remaining service life time of existing members as a level percentage of salaries. At each valuation, the total surpluses/deficits outstanding are respread.

Detailed assumptions, for example, the timing of pension payments, and so on, are made exactly in the same way as the actual model.

A2.5 Benefit calculation routine

The benefits are defined in the form of pensions from age 65. However, alternatively, the cash equivalents to these defined benefits are paid out in respect of withdrawing members and retiring members. The calculation of the cash equivalents is based on the market interest rate, $i''(t)$, and the market price inflation rate, $p''(t)$, at the time of the exit. Here, the market interest rate and the market inflation rate represent the rates which are expected in the market at the time, and can be input independently from the actual past experience.

Appendix 3 Full Projection Results

In Chapter 7, the objectives and the setting of a series of cash flow projections, which are 123 projections in total, have been described. In this appendix, the results of the projections are shown as the graph of the Standard Contribution Rate, Adjustment in Contribution Rate and the Recommended Contribution Rate throughout the 100 years' projection period for each projection. Also, the significant features of each projection are described in a note form.

The 123 projections are grouped into three; the projections regarding real investment return changes, those regarding the dividend growth rate changes and those regarding the withdrawal rate changes. Each group is dealt with in Appendices 3.1, 3.2 and 3.3 in turn.

Notation used in this Appendix is defined in section 7.3.

Appendix 3.1 Real Investment Return Changes

A3.1.1 Unchanged experience

The experience is the same as the standard experience and remains stable throughout the projections. Three different bases, which are also constant throughout the projection, are used, and the long term effects of the choice of the basis are examined.

List of Projections

A3.1.1.a	e=6.5%, p=4.5% [Standard basis]
A3.1.1.b	e=5.5%, p=3.5188%
A3.1.1.c	e=7.5%, p=5.4912%

The bases are stable from the beginning of year 1. However, the fund at the beginning of year 1 is equal to the actuarial liabilities calculated on the standard basis, and the initial fund is the same for all 3 projections.

A3.1.1.a:
The experience and the valuation basis are equal and static throughout the projection. This shows a stationary fund which grows in line with the salary growth rate, i.e. 6.5% p.a. The Recommended Contribution Rate is always the same as the Standard Contribution Rate, which is constant at 10.8578%. The funding level on the standard basis is static at 100%.

A3.1.1.b:
In this projection, the liabilities are optimistically valued, and the Standard Contribution Rate is lower than for A3.1.1.a. The population structure is stable, and the Standard Contribution Rate is stable at 8.2279%.

Initially, there is a surplus. For the amortisation of this surplus, the Recommended Contribution Rate is lower than the Standard Contribution Rate. The initial Recommended Contribution Rate is 5.3616%.

The actual real investment returns are lower than the assumption, and deficits arise every year from these differences, and they accumulate. So, gradually, the funding level becomes lower, and the Recommended Contribution Rate rises.

The amount of newly accruing deficit is proportionate to the size of the fund which grows in line with the actual salary growth. On the other hand, the deficits accumulate, and the total amount of the deficit grows quicker than the amount of deficits accruing each year. The additional contribution rate in excess of the Standard Contribution Rate is a constant percentage of the total deficit, and the growth rate is higher than the growth rate of the newly accuing deficits. Initially, the newly accruing deficit is larger than the additional contributions, but gradually, the two become closer. Consequently, the fund approaches a stationary state with a higher contribution rate, about 11.7%, and a lower funding level, about 75% on this valuation basis or about

61% on the standard valuation basis, than for A3.1.1.a.

Because the salary growth assumption is lower than the actual experience, the value of future salaries is understated. So, adjustment of the contribution rates for amortisation is larger than that on a realistic basis, and the speed of amortisation is quicker than planned. That is, the stationary state seems to be reached relatively quickly.

A3.1.1.c:

This is an oposite to A3.1.1.b. The liabilities are pessimistically valued, and the Standard Contribution Rate is higher, stable at 14.5094%, than for A3.1.1.a.

Initially, there is a deficiency. For the amortisation of this deficiency, the Recommended Contribution Rate is higher than the Standard Contribution Rate. The initial Recommended Contribution Rate is 17.6668%.

In contrast to A3.1.1.b, surpluses arise every year, and gradually, the funding level becomes higher, and the Recommended Contribution Rate falls.

Eventually, the fund approaches a stationary state with a lower contribution rate, about 9.2%, and a higher funding level, about 129% on this valuation basis or about 161% on the standard valuation basis, than for A3.1.1.a.

The speed of amortisation is slower than planned because of the opposite reason from A3.1.1.b. So, the ultimate stationary state seems to be reached more slowly than for A3.1.1.b.

Summary

Where the experience is constant, the membership structure remains stationary, and a range of stable valuation bases may achieve a stationary fund eventually. The use of an optimistic basis may not give sufficient security of benefit payments. On the other hand, a pessimistic basis may cause over-funding.

The initial movements in the Recommended Contribution Rates are avoided by the use of the standard basis. However, the situation can be totally different if the initial asset is different.

A basis with a wider gap between the interest rate and the salary growth gives optimistic values of the liabilities, but at the same time, the adjustments for amortisations are set larger than otherwise. This leads to a quicker amortisation than planned, and the convergence to a stationary state is achieved more quickly. A quick convergence means a more stable contribution rate after a shorter period, but a higher volatility in the contribution rate initially. There is a trade off between these two effects. A similar observation may be made on the effects of the choice of the pace of amortisation.

A3.1.2 One permanent change in experience

In this section, the salary growth experience, which is 6.5% before the projections start, is reduced to 5.5% from year 1, and remains so throughout the projections. The rate of pension increases, which is equal to the rate of price inflation, is simultaneously reduced from 4.5% to 3.5188%.

A constant valuation basis, a single permanent change in the basis, and gradual changes in the basis are applied.

A3.1.2.1 Constant basis

The standard basis is tried to see the impacts of the change in the experience without any moves in the basis. The projection is done as a basis for comparison with other projections which use changing bases.

List of Projections

A3.1.2.a Standard basis

A3.1.2.a:

Initially, the funding level is 100%, and the Recommended Contribution Rate is equal to the Standard Contribution Rate.

The change in the salary growth rate is applied to all the active members uniformly, and the age weighting of the total salaries is not affected. So, the Standard Contribution Rate stays the same at 10.8578%.

The experienced salary growth rate and the pension increase rate are lower than the assumptions, and surpluses arise every year. They are accumulated, and the Recommended Contribution Rate is reduced gradually to amortise the surpluses.

The fund reaches an equilibrium where the Recommended Contribution Rate is about 6.0% and the funding level is about 130%.

Except for a nil impact at the beginning, the projection is similar to A3.1.1.c.

A3.1.2.2 One permanent change in basis

In this section, the valuation bases are changed only once. Various combinations of the timing and the magnitude of the changes are tried.

-Immediate change

The bases are changed at the end of year 1. So, there are no delays in the reaction.

List of Projections

A3.1.2.b	Immediate, e=5.5% from year 1 [Same as experience]
A3.1.2.c	Immediate, e=5.0% from year 1
A3.1.2.d	Immediate, e=6.0% from year 1

The inflation rate is also changed to maintain the ratio of 1+e:1+p

A3.1.2.b:

At the end of year 1, a surplus arises because of a reduction in the value of liabilities. Also, the Standard Contribution Rate is reduced from 10.8578% at the beginning of year 1 to 8.2279%. The Recommended Contribution Rate is even lower than this at 4.6107% to amortise the surplus. So, the initial change in the contribution rate is very large.

Similar to A3.1.2.a, the age weighting does not change, and the Standard Contribution Rate stays the same thereafter.

From year 2 onwards, the experience and the valuation basis are equal, and no surpluses/deficits arise.

The initial surplus is gradually amortised to nil, and the fund becomes stationary with the Standard Contribution Rate equal to the Recommended Contribution Rate and the funding level of 100%.

A3.1.2.c:

The valuation basis is more optimistic than for A3.1.2.b, and the initial surplus is larger. Also, the fall in the Standard Contribution Rate is larger, and the rate becomes 7.1970%. The Recommended Contribution Rate is 1.9974% reflecting the larger surplus and the lower Standard Contribution Rate than for A3.1.2.b.

The Standard Contribution Rate remains stable.

The salary growths and the pension increases are higher than the assumptions, and deficits arise every year. The deficits reduce the initial surpluses and, then, accumulate gradually. Consequently, the Recommended Contribution Rate increases, and becomes higher than the Standard Contribution Rate.

Eventually, the fund approaches a stationary state where the Recommended Contribution Rate is about 9.0%. The contribution rate is higher than for A3.1.2.b. This suggests that the security is lower.

A3.1.2.c starts from a lower contribution rate and ends in a higher contribution rate than for A3.1.2.b. So, the contribution rate is less stable than for A3.1.2.b.

A3.1.2.d:

Similar, but to a lesser extent, to A3.1.2.b, there is an initial surplus. Also, the Standard Contribution Rate falls to 9.4368%. The Recommended Contribution Rate at the end of year 1 is 7.4647%.

The Standard Contribution Rate stays the same thereafter.

The actual rates of salary growth and pension increase are lower than the assumptions, and surpluses arise every year. By a coincidence, the amount of surplus arising is almost equal to the reduction in the contributions for the amortisation of the initial surplus, and they almost cancel each other. As a consequence, the level of funding remains similar to that at the end of year 1.

Hence, the contribution rate is stable, and the funding level is higher than for A3.1.2.b and is stable.

Summary

The stability shown in A3.1.2.d is created by an accident, but the effect of the cancellation between the reduced contribution rates and the newly accruing surpluses may be observed to some extent if the valuation basis is between A3.1.2.a and A3.1.2.b. So, the contribution rates may be more stable than for A3.1.2.a or A3.1.2.b.

On the other hand, valuation bases more optimistic than for A3.1.2.b would produce more volatile contribution rates and lower funding levels.

-Delayed change

The changes in the valuation bases take place at the end of year 3. So, there is two years' delay in the change.

List of Projections

A3.1.2.e	Delayed, e=5.5% from year 3 [Same as experience]
A3.1.2.f	Delayed, e=5.0% from year 3
A3.1.2.g	Delayed, e=6.0% from year 3

A3.1.2.e:

Basically, the pattern of changes in the cntribution rates is similar to A3.1.2.b. The Standard Contribution Rate after the basis change and the ultimate level of the contribution rate are the same.

The effect of the delay in the change of the basis appears as a higher level of additional surpluses which were caused by lower salary growth rates and price inflation rates than expected before the change of the basis.

From year 4 onwards, the experience and the valuation basis are equal, and no surpluses/deficits arise. So, the surpluses at the end of year 3 are gradually amortised to nil. Because the surplus is larger than for A3.1.2.b, the initial reduction in the contribution rate is larger, and it takes longer to amortise the surplus.

A3.1.2.f:

The effect of the delay appears as a larger initial surplus than for A3.1.2.c, and it takes longer to reach an equilibrium.

However, the basic pattern of changes in contribution rates are the same as A3.1.2.c. The Standard Contribution Rate after the change in the basis is the same as that of A3.1.2.c, and the ultimate contribution rate is almost the same.

A3.1.2.g:

The Standard Contribution Rate after the change in the basis is the same as that of A3.1.2.d, and the ultimate contribution rate is almost the same.

The effect of the delay appears as a larger initial surplus than for A3.1.2.d, and the difference between the initial Recommended Contribution Rate and the ultimate rate is wider than for A3.1.2.d.

Similar to A3.1.2.d, the contribution rate is relatively stable after the change in the basis. A slight difference is observed in the pattern of contribution rate to approach a stationary state. While A3.1.2.d shows a gradual reduction in contribution rate, this projection shows a gradual increase in the contribution rate.

This may be explained as the effect of the balance between the amortisation of the initial surplus and newly accruing surpluses. In A3.1.2.c, the reduction in the contribution for the amortisation of the initial surplus is slightly smaller than newly accruing surpluses each year, and the surpluses gradually accumulate until an equilibrium is reached. In A3.1.2.g, the initial surplus is larger, and the reduction in the contribution for the amortisation of the initial surplus is slightly larger than newly accruing surpluses each year. Consequently, gradually the level of surpluses goes down, i.e. the Recommended Contribution Rate gradually increases, until an equilibrium is reached.

Summary

The delay in the change of basis appears as a higher level of surplus when the basis is changed. This results in a lower initial contribution rate, and, in A3.1.2.e and A3.1.2.f, a slower convergence to an equilibrium. So, a delay in the change tends to increase the volatility of the Recommended Contribution Rate after the change of the basis.

A3.1.2.3 Gradual change in basis

In this section, the valuation bases are changed gradually as straight lines. Various combinations of the magnitude of the changes and the frequency of the changes in the basis are tried.

They are interpreted as the averaging of the past experience. The magnitude of the changes corresponds to the length of averaging period. There are three levels of magnitude tested; Quick (.04% p.a.), Medium (.02% p.a.) and Slow (.008% p.a.). 'Quick' corresponds to 25 year averaging, 'Medium' corresponds to 50 year averaging, and 'Slow' corresponds to 125 year averaging.

Because the valuation bases are the averages of the past experience, the changes in the basis will cease when the basis arrives at the experience level.

For completeness, one additional projection has been made where the change in the basis does not stop when the basis is equal to the experience.

-Annual change

The change in the basis starts from the end of year 1, and the chages are made every year thereafter.

List of Projections

A3.1.2.h	Averaging, Annual change, Quick(.04%pa)
A3.1.2.i	Averaging, Annual change, Medium(.02%pa)
A3.1.2.j	Averaging, Annual change, Slow(.008%pa)

A3.1.2.h:

Until year 25, the Standard Contribution Rate gradually falls, reflecting the change in the basis.

Until year 25, owing to the continuing weakening of the basis, valuation surpluses arise. At the same time, the salary growth and price inflation experience are lower than the valuation basis, and surpluses arise. The level of surpluses arising from the latter source reduces gradually as the gap between the experiennce and the basis reduces. The total effect is dominated by the second factor, and the surpluses arising are gradually reduced. These surpluses accumulate until year 25, and the Recommended Contribution Rate gradually falls.

After year 26, no surpluses/deficits are accruing because the experience exactly matches the assumptions. The accumulated surpluses are amortised in due course. Eventually, the surpluses are reduced to nil.

A3.1.2.i:

Until year 50, the Standard Contribution Rate gradually falls, reflecting the change in the basis.

Until year 50, similar to A3.1.2.h, surpluses arise, and the level of surpluses arising reduces gradually as the gap between the experience and the basis reduces.

These surpluses accumulate until year 26, and the Recommended Contribution Rate gradually fails. However, after year 27, the accruing surpluses are smaller than the reduction in the contrubution for the amortisation of accumulated surpluses, and the surpluses gradually fall.

Combined with the fall in the Standard Contribution Rate until year 50, the Recommended Contribution Rate falls until year 37, but it gradually increases after year 38.

After year 51, no surpluses/deficits are accruing because the experience exactly match the assumptions. So, the fall in the accumulated surpluses is accelerated, and eventually, the surpluses are eliminated.

The maximum level of accumulated surpluses is slightly higher than for A3.1.2.h, but owing to a smoother change in the Standard Contribution Rate, the variation in the Recommended Contribution Rate is smaller than for A3.1.2.h.

A3.1.2.j:
The change in the valuation basis is so slow that the basis cannot reach the experience level at the end of 100 years.

So, the Standard Contribution Rate continuously falls, and surpluses arise every year during the projection. The level of surpluses arising reduces gradually as the gap between the experience and the basis reduces.

The surpluses accumulate until year 37. After year 38, the accruing surpluses become smaller than the reduction in the contribution for amortisation, and the surpluses are gradually reduced.

Combined with the fall in the Standard Contribution Rate, the Recommended Contribution Rate falls until year 49, but it gradually increases after year 50.

The maximum level of accumulated surpluses is slightly higher than for A3.1.2.i, but owing to a smoother change in the Standard Contribution Rate, the variation in the Recommended Contribution Rate is the smallest among the three.

Summary
A slower averaging gives a higher level of accumulated surpluses. This effect is similar to the delay in the change of basis.

However, unlike the delayed changes, the gradual changes in the Standard Contribution Rate will reduce the volatility in the Recommended Contribution Rate.

On the other hand, a slower change in the basis results in a slower amortisation of surpluses, and the funding level stays higher for a longer time. There is a trade off between the stability of the contribution rate and maintaining an adequate funding level. (Here, a surplus, similar to a deficit, may be considered undesirable.)

The characteristics described above may be emphasised if these projections are compared with A3.1.2.b, where the basis may be interpreted as one year average of the experience.

Even if the change is very slow, e.g. 125 year average, the surpluses are expected to be eliminated eventually. This feature is totally different from A3.1.2.a, where the basis is not changed at all and an equilibrium with a surplus is achieved.

-Triennial change

Although the valuation takes place every year, the change in the basis is only made triennially. The first change of the basis is made at the end of year 1. After three years, the change in basis is made by three times the annual rate of change. i.e. a change by .02% pa means a change by .06% every three years.

List of Projections

A3.1.2.k	Averaging, Triennial change, Quick(.04%pa)
A3.1.2.l	Averaging, Triennial change, Medium(.02%pa)
A3.1.2.m	Averaging, Triennial change, Slow(.008%pa)

The general tendency is the same as the projections with annual changes.

During the three year period of unchanged bases, the Standard Contribution Rate stays the same. Also, no valuation surpluses arise. However, the surpluses arising from the gap between the experience and the assumptions are larger because of a wider gap particularly in the second and third year after the change in the basis.

The maximum accumulated surpluses are slightly larger than the projections with annual basis changes.

Consequently, at the change of the basis, the impact of the accumulated surpluses is large. Also, there is a release of valuation surpluses which would have arisen every year in the case of the projections with annual changes. Combined with the large change in the Standard Contribution Rate, the change in the Recommended Contribution Rate tends to be larger than the projections with annual basis changes.

So, the Recommended Contribution Rates are more volatile from year to year.

-Further change

The change in the valuation basis starts similar to A3.1.2.h, but the change goes on at the same pace even after the valuation basis reaches at the experience level.

List of Projections

A3.1.2.n	Change continue, Anuual change, Quick(.04%pa)

A3.1.2.n:
Up to year 25, this is the same as A3.1.2.h.

After year 26, owing to the continuing weakening of the basis, deficits arise from the difference between the experience and the assumptions. Although valuation surpluses continue to arise, the deficits gradually become more dominant, and the level of deficits increases as time goes on.

The accumulated surpluses are amortised by the reductions in the contribution rate and these newly accruing deficits. So, the amortisation is done very quickly. After that, deficits accumulate, and this requires an additional contribution on top of the standard contributions.

Although the Standard Contribution Rate continuously falls, the impact of the accumulating deficits is more significant, and the Recommended Contribution Rate increases.

At the end of year 100, there are still no signs of convergence, and the contribution rates may increase further while the funding level deteriorates.

A3.1.3 Gradual change in experience

Here, the salary growth rate experience is assumed to reduce from 6.5% by .02% per annum. So, in year 1, the salary growth rate is 6.48%, and in year 100, it is 4.5%. The pension increases, which are equal to the price inflation rates, are simultaneously reduced keeping the ratio 1+e':1+p' constant.

A constant valuation basis and gradual changes in the bases are applied.

A3.1.3.1 Constant basis

The standard basis is tried to see the impact of the change in the experience without any moves in the basis.

List of Projections

A3.1.3.a Standard basis

A3.1.3.a:

The Standard Contribution Rate remains stable under an unchanged valuation basis.

The experience in the salary growth rates and the price inflation rates gradually decrease, and surpluses arise. The amount of surpluses arising is initially very small, but gradually increases.

The surpluses accumulate, and the Recommended Contribution Rate falls accordingly.

Because the amount of surpluses arising is increasing, it always exceeds the reduction in contributions for amortisation which are based on the previous amount of accumulated surpluses. The two do not balance at any stage of the projection. So, there are no signs of arriving at an equilibrium.

Towards the end of the projection, a huge surplus has been accumulated, and the contribution rate has not become stable.

A3.1.3.2 Gradual change in basis

In this section, the valuation bases are changed gradually as straight lines. Various combinations of the magnitude of the changes, the timings of the start of changes and the frequency of the changes in the basis have been tried.

The magnitude of the changes are expressed as annual rates of reductions in the salary growth assumption. The price inflation assumptions are also reduced simultaneously keeping the ratio 1+e:1+p constant.

If the valuation basis is set as an average of past experience, the pace of the changes in the basis will be the same as that in the experience after the year when the period from the start of the projection exceeds the averaging period. However, the initial pace of the basis changes is different from straight line changes in the basis. This is because the experience before the projections start are assumed to be stationary. To see the initial differences, some projections using the bases which are the averages of the past experience are added.

-Immediate change

The changes in the valuation basis starts at the end of year 1. So, there are no delays. Three different magnitudes of changes are examined; one follows exactly the experience, others show lower changes in the bases than the experience. Projections with higher paces of the changes in the valuation bases have been omitted because they are unlikely to be suitable.

List of Projections

A3.1.3.b	Immediate, Annual change, .02%pa[Same as experience]
A3.1.3.c	Immediate, Annual change, .008%pa
A3.1.3.d	Immediate, Annual change, .004%pa

A3.1.3.b:

The Standard Contribution Rate gradually falls owing to the changes in the valuation basis.

Owing to the continuous weakening of the basis, a valuation surplus arises every year. Because the changes in the valuation basis is matched by the changes in the experience, there are no surpluses/deficits arising from the gap between the experience and the assumptions. The level of surpluses arising every year is fairly constant because the degree of changes in the basis is constant.

Initially, the reduction in the contributions for the amortisation of the surpluses is less than the surpluses accruing, and the surpluses accumulate. However, as the surpluses accumulate, the reduction in the contributions increases, and they eventually balance each other. Thus, after year 30, the level of outstanding surplus becomes relatively stable. In fact, after year 30, the reduction in the contribution rate arising from amortisation remains at the level of around 1%, and only decreases at most .01% from one year to the next year.

Although the Recommended Contribution Rate falls without convergence, the funding position may be said to be stable and sound because the level of surplus is stable and small.

A3.1.3.c:

The Standard Contribution Rate falls as the basis is weakened, but to a lesser degree than for A3.1.3.b.

Every year, owing to the weakening of the basis, a valuation surplus arises. This surplus is relatively small and stable from year to year. At the same time, the difference in the experience and the assumptions causes further surpluses. Because the pace of the changes in the valuation basis is less than that of the experience, the difference increases as the time passes. So, the surplus arising from this source gradually increases.

Because the surpluses arising are increasing, the reduction in the contribution for amortisation of already accumulated surpluses cannot match them. So, the surpluses accumulate continuously, and further reductions in the contribution rates are required.

Combined with the fall in the Standard Contribution Rate, the Recommended Contribution Rate falls without convergence.

The Recommended Contribution Rate is more stable than for A3.1.3.b initially, but changes more rapidly afterwards.

A3.1.3.d:
Similar results as A3.1.3.c are seen, but all the features emerge more significantly.

Summary
A3.1.3.b keeps the amortisation element of the contribution rate small and stable. So, the results may not be affected much by the choice of the amortisation method.

A moderate change in the basis compared with the changes in the experience makes the contribution rate more stable initially. However, if the changes in the experience develop further, a large surplus/deficit may be accumulated, and the contribution rates would be unstable in the long run. Also,the contribution rate changes are heavily dependent on the amortisation method.

-Delayed change

The changes in the valuation basis start at the end of year 3. So, there is two years' delay compared with the projections with immediate changes. Two different magnitudes of the changes in the bases are examined.

The initial changes are made only for one year's portion of the changes. There are no extra changes included in the bases reflecting the existence of the delay. So, even in A3.1.3.e, the valuation basis is always two years behind the experience.

List of Projections

A3.1.3.e	From year 3, Annual change, .02%pa
A3.1.3.f	From year 3, Annual change, .008%pa

A3.1.3.e:
The Standard Contribution Rate falls gradually after year 3 owing to the change in the valuation basis.

Owing to the delay in the start of the change in the valuation basis, there exists a gap between the experience and the assumptions. This is a source of surpluses every year. The gap is constant because the pace of the change in the valuation basis is equal to that of the experience. So, the surpluses accruing from this source are constant throughout the projection except for the first three years, where the gap gradually widens.

Also, owing to the change in the valuation basis, a valuation surplus arises every year. Because the change of the basis is steady, the surplus arising from this source is also constant each year.

So, the surpluses arising each year is fairly constant, but because of the first element of the surplus, the level of surpluses arising is higher than for A3.1.3.b.

Consequently, the eventual level of accumulated surpluses, where each year's accruing surpluses and the reduction in the contribution for amortisation cancel each other to maintain stability, is higher.

The total contribution rate continues to fall owing to the fall in the Standard Contribution Rate, but the level of surplus accumulated is stable and relatively low although it is higher than for A3.1.3.b.

A3.1.3.f:

The Standard Contribution Rate falls as the valuation basis is weakened. The degree of fall is less than A3.1.3.e.

Surpluses arise every year owing to the gap between the experience and the assumptions and the weakening of the valuation basis. The former increases because the changes in the assumptions are less than the changes in the experience. The amount of this part of surplus is larger than that in A3.1.3.c because of the delay in the basis change. The valuation surpluses arise constantly.

Because the surpluses arising are increasing, the surpluses accumulate without any signs of convergence, and more reductions in contribution rates for amortisation are required.

Combined with the fall in the Standard Contribution Rate, the Recommended Contribution Rate falls. The Recommended Contribution Rates are initially more stable than for A3.1.3.e, but later, they move more quickly.

The level of accumulated surplus is higher than for A3.1.3.c because of the delay, and the contribution level is lower.

Summary

The effect of delay is a higher level of accumulated surplus.

However, two years' delay may not be significant. The basic shape of the contribution rate changes is not affected. So, it may be justifiable to delay the basis changes until the tendency in the experience becomes clearer.

-Triennial changes

The valuations take place every year, but the changes in the valuation basis are made only every three years. The first change is made at the end of year 1.

The degree of change each time is three years' portion of the changes. So, if the reduction in the salary growth assumption is .02% p.a., .06% is deducted every three years.

List of Projections

A3.1.3.g		Immediate, Triennial change, .02%pa

A3.1.3.h		Immediate, Triennial change, .008%pa

A3.1.3.g:

The general shape of the contribution rate changes are similar to A3.1.3.b.

At the basis change, a large valuation surplus is released, while there are no such surpluses between the basis changes.

The surpluses arising from the gap between the experience and the assumptions are increased between the basis changes because the basis is the same while the experience changes.

Owing to these factors, the fluctuations in the Recommended Contribution Rates are higher than for A3.1.3.b.

A3.1.3.h

The general shape of the contribution rates changes are similar to A3.1.3.c.

Similar comments as A3.1.3.g apply, and the fluctuations in the contribution rate are higher than in A3.1.3.c.

Summary

Triennial changes in the basis do not affect the basic pattern of the projection results. Only fluctuations in the contribution rates from year to year are higher.

-Decennial changes

Similar to triennial changes, the valuations take place every year, but the changes in the valuation basis are made only every ten years. The first change is made at the end of year 1.

The degree of change each time is ten years' portion of the changes. So, if the reduction in the salary growth assumption is .02% p.a., .2% is deducted every ten years.

List of Projections

A3.1.3.i		Immediate, Decennial change, .02%pa

A3.1.3.j		Immediate, Decennial change, .008%pa

The long term pattern of the contribution rates is similar to the cases of annual changes and triennial changes.

In A3.1.3.i, the level of amortisation seems to progress between the basis changes. During the period, no valuation surpluses arise. Only the gap between the experience and the assumptions creates surpluses. The latter factor of surplus is larger than for A3.1.3.b, especially for years close to the next basis change. However, the lack of valuation surpluses helps the amortisation to progress.

At the change of the basis, a large valuation surplus is released, and the temporary effect of the progress in the amortisation disappears. The accumulated surpluses then return to the similar level as at the previous basis change.

Such a temporary progression in the amortisation is not clearly seen in A3.1.3.j. This is because the valuation surpluses are not significant when the changes in the basis are slower than experience.

-Averaging

The valuation basis is set as the average of the past experience. As the averaging period, 5 years and 10 years are examined.

List of Projections

A3.1.3.k	Immediate, Annual change, 5 year average
A3.1.3.l	Immediate, Annual change, 10 year average

A3.1.3.k:

Except for the valuations performed in the first 4 year period, the valuation bases used are exactly the same as those under A3.1.3.e, 2 years' delay.

The differences in the initial period are not large, and they do not give any significant impacts.

The contribution rate changes is almost identical to A3.1.3.e. The differences in the Recommended Contribution Rates are within .01% throughout the projections.

A3.1.3.l:

After 10 years, the pace of the changes in the valuation basis is the same as that of the experience. But the basis never catches up with the experience.

The Standard Contribution Rate continuously falls as the basis is weakened.

Similar to A3.1.3.e, there are two sources of surpluses arising every year; valuation surpluses and the gap between the experience and the assumptions.

The first element depends on the magnitude of the changes in the basis, and the level of surpluses are the same as A3.1.3.b or A3.1.3.e. This part of the surplus arising is fairly constant over years.

10 years' averaging is equivalent to 4.5 years' delay in the start of the change in the basis except for the initial

10 years. So, the gap between the experience and the assumptions is wider than for A3.1.3.e, and the surplus is larger. However, because the pace of the changes in the valuation basis becomes the same as that of the experience, the gap remains constant, and this part of the surplus arising is also fairly constant.

Consequently, the level of accumulated surpluses reaches an equilibrium where the accruing surpluses and the reduction in contribution rates balance each other.

Although the level of surplus is higher than for A3.1.3.b, the basic pattern of the results is the same as for A3.1.3.b.

Summary

Averaging has similar effects to delaying the change of the basis, and averaging may not produce significantly different impacts on the finances of the fund compared with the use of a non-averaged basis.

A3.1.4 Cyclical change in experience

Here, the salary growth experience, e', are assumed to follow a twenty year cycle. Namely, e' is reduced at .2% per annum until e' reaches 5.5% in the fifth year. Then, e' is increased by .2% per annum for ten years to reach 7.5%. After that, e' is again reduced at .2% per annum to return to 6.5% in the twentieth year. During the 100 year projection, thus, five cycles are made. The pension increases, which are equal to the price inflation, are simultaneously reduced keeping the ratio 1+e':1+p' constant.

Constant valuation bases and cyclical changes in the bases are applied.

A3.1.4.1 Constant basis

Three different constant bases are tried.

List of Projections

A3.1.4.a	e=6.5% [Standard basis]
A3.1.4.b	e=5.5% [Lowest value]
A3.1.4.c	e=7.5% [Highest value]

A3.1.4.a:

The Standard Contribution Rate is constant because the valuation basis has not changed at all.

According to the cyclical changes in the experience, surpluses arise when the actual salary growth is below 6.5%, and deficits arise when it is above. The surpluses/deficits arising are cyclical, and the same level of surpluses/deficits (relative to the total salary) arise in 20 years' time again.

At the beginning of year 1, there are no accumulated surpluses/deficits.

For first five years, surpluses arise, and the level of these accruing surpluses increases. The reduction in the contribution rate for amortisation is less than the accruing surpluses, and the surpluses accumulate. Consequently, the contribution rate reduces.

For the second five years, surpluses continue to arise, but the amount arising falls from year to year. The reduction in the contribution rate for amortisation is still less than the accruing surpluses initially, and the surpluses accumulate until year 8. So, the contribution rate continues to fall until year 8. However, the reduction in the contributions for amortisation exceeds the accrual of the surpluses eventually because the former increases while the latter decreases. In year 9 and 10, the contribution rate is increased reflecting the progress of the amortisation of surpluses. At the end of year 10, there is a surplus outstanding.

For the next five years, deficits arise, the amount of which gradually increases. The deficits are used to remove the outstanding surpluses together with the reduction in the contributions for amortisation. By the end of year 15, the amortisation of the surpluses is completed.

For the last quarter of the cycle, deficits arise, and the amount reduces gradually to 0. Until year 19, the deficits accumulate, and the contribution rate is increased. However, in year 20, the accrual of the deficit is exceeded by the additional contributions for amortisation, and the accumulated deficit is slightly reduced.

At the end of the first cycle, thus, there is a deficit carried forward.

The accrual pattern of the surpluses and deficits for each consecutive cycle is the same as the first one. However, because of the accumulated deficit at the beginning of the cycle, the pattern of the accumulation of the surpluses and deficits becomes different.

Because of the initial deficit, the accumulation of surpluses in the first half of the second cycle is smaller. So, the size of the reduction in the contribution for amortisation is relatively smaller than for the first cycle, while the accruing surpluses are the same. Consequently, the timing of balancing the two is delayed until the 9th year of the cycle.

So, the second half of the second cycle starts with a smaller accumulated surplus, and the level of accumulation of the deficits becomes higher than for the first cycle.

At the end of the second cycle, there is a higher level of accumulated deficits.

As the cycles go on, the level of the deficit at the beginning of each cycle becomes higher, but the degree of the increase is reduced each time. So, the pattern of the contribution rate changes, which represents the pattern of the funding level, seems to become cyclical and stable.

In the fifth cycle, the highest contribution rate is 11.5448%, and the lowest is 10.2055%. The difference is 1.3393%. The differences from the Standard Contribution Rate, 10.8578%, are similar to the both directions.

A3.1.4.b:
The Standard Contribution Rate is lower than for A3.1.4.a, and stable.

The real interest rate assumed in the valuation basis is always higher than or equal to the experience. So, except in year 5,25,45,..., where they are equal, deficits arise owing to this gap every year. According to the cyclical pattern of the experience, the level of deficits arising changes cyclically. The level of the deficit arising starts from a non-zero level, and reduced to 0 by year 5. Then, the level increases steadily until year 15. From year 16 to year 20, the level reduces to the initial level.

Initially, there is a valuation surplus because the initial amount of the assets is equal to the actuarial liability under the standard basis. So, the contribution rates are reduced to amortise this initial surplus. The pace of amortisation is quick helped by the accruing deficit every year. By the end of year 13, the surpluses disappear. During the rest of the first cycle, the deficits accumulate, and the contribution rate increases.

The second cycle starts with a deficit. The deficits continue to accumulate and the addition to the contribution from amortisation increases. However, the level of accruing deficits is reducing, and in year 22, the accrued deficit becomes less than the addition to the contribution, and the accumulated deficit reduces. The reduction in the accumulated deficits continues while the accumulated deficit level is low until year 27. Then, the gradually increased deficit accruing exceeds the addition in the contribution, and the accumulation of the deficits starts again. The accumulation continues until the end of the second cycle. The second cycle ends with higher deficits than the beginning of the cycle.

A similar pattern is repeated for each subsequent cycle. The timings of the peak and the trough of the accumulated deficits are slightly affected by the increased level of the initial deficit, and the peak is in the 19th year and the trough in the 9th year.

The levels of the deficit at the beginning of the fourth and the fifth cycle are slightly increased. The deficit at the beginning of each cycle seems to settle down to a stable level after the fifth cycle.

In the fifth cycle, the contribution rates are between 11.1709% and 12.2687%. The contribution level is, thus, higher than for A3.1.4.a. and the funding level is lower. The difference in the contribution rates is 1.0978%, which is narrower than for A3.1.4.a.

A3.1.4.c:
This is a reverse of A3.1.4.b. The Standard Contribution Rate is constant and higher than for A3.1.4.a.

The real interest rate assumed in the valuation basis is always lower than or equal to the experience. So, except in year 15,35,55,..., where they are equal, surpluses arise owing to this gap every year. According to the cyclical pattern of the experience, the level of surpluses arising changes cyclically. The level of the surplus arising starts from a non-zero level, and increases to the maximum level in year 5. Then, the level reduces steadily to 0 until year 15. From year 16 to year 20, the level increases back to the initial level.

There is an initial valuation deficit, and the Recommended Contribution Rate is higher than the Standard Contribution Rate.

After amortising the initial deficits, surpluses are accumulated. After several cycles, the contribution rate pattern seems to become cyclical and stable.

The contribution rates in the fifth cycle are between 8.4352% (year 89) and 10.0991% (year 99). The level is lower than for A3.1.4.a, and the funding level is higher. The difference in the highest and the lowest contribution rates is 1.6639%. The gap is the widest of the three projections.

Summary
The average funding level and the contribution rate show similar tendencies as the projections with unchanged experience. The use of a strong basis results in a permanent surplus.

Reflecting the cyclical changes in the experience, the funding level also shows a cycle with the same length.

The peaks and troughs are different between the experience and the funding level. This is because the latter represents the accumulated effects of experience and the progression in the amortisation.

The fluctuation in the contribution rate is the largest in A3.1.4.c, and the smallest in A3.1.4.b. This may be a result of the difference in the size of the fund; The funding level of A3.1.4.c is the largest, and the level of surpluses/deficits compared with the total salary is relatively large.

A3.1.4.2 Cyclical change in basis

In this section, the valuation bases are changed cyclically with the same length of cycles, i.e. 20 year cycles. Various combinations of the magnitude and the frequency of the changes in the basis are tried.

The magnitude of the changes are expressed as annual rates of changes in the salary growth rate assumption. The price inflation assumptions are also reduced simultaneously keeping the ratio 1+e:1+p constant.

Also, the effects of averaging are analysed by applying three different averaging periods.

-Immediate changes
The changes in the valuation basis starts from the standard basis at the beginning of the projections, to the direction as that in experience.

List of Projections

A3.1.4.d Immediate, Annual change, .2%pa[Same as experience]

A3.1.4.e Immediate, Annual change, .1%pa

A3.1.4.d:
In this projection, the valuation basis is exactly the same as the experience in the last 12 months.

The Standard Contribution Rate moves according to the changes in the valuation basis. During the first five years of a cycle, the rate reduces as the real investment return assumption becomes larger. In the next ten years, the rate increases. At the end of the tenth year, the Standard Contribution Rate return to the original level, 10.8578%. After reaching a peak at the end of the fifteenth year, the rate reduces, and comes back again to the original level at the end of the twentieth year. This twenty year cycle is repeated.

The projection starts with no surpluses/deficiencies.

Although there is no gap between the experience and the valuation basis, every year, a valuation surplus/deficit arises owing to the changes in the valuation basis.

In the first five years, the valuation basis continues to be weakened, and valuation surpluses arise. The level of each year's accrual is proportionate to the value of the actuarial liabilities and the degree of the change in the valuation basis. Here, the value of the liabitities is relatively stable, although it moves cyclically, and the degree of the change is constant. Hence, the surpluses accruing each year are also relatively constant. The effect of the amortisation is relatively small compared with the accruing surpluses. So, the surpluses accumulate, and the degree of the reduction in the contribution rate increases. Combined with the fall in the Standard Contribution Rate, the Recommended Contribution Rate shows a rapid fall.

In the next ten years, valuation deficits arise consistently throughout.

First, the accumulated surpluses are amortised quickly, and then deficits accumulate. The accumulated deficit is at its peak in year 15. This peak is slightly larger than the peak in the accumulated surplus in year 5. The Recommended Contribution Rate rises combined with the rise in the Standard Contribution Rate.

After year 16, valuation surpluses constantly accrue at the same level as the first five years, and the accumulated dificits are quickly amortised by the end of year 20. At the end of year 20, there is a small surplus outstanding. The Recommended Contribution Rate falls combined with the fall in the Standard Contribution Rate.

Because the surplus at the beginning of the second cycle is small, a similar pattern is repeated. However, the funding level is in general slightly higher.

This gradual rise in the funding level continues, but seems to become stable and cyclical. In the fifth cycle, the lowest Recommended Contribution Rate is 4.4105% in year 85, and the highest is 18.2225% in year 95. The difference is 13.812%. This is much wider than projections with constant bases.

Consequently, the funding level is more volatile than for A3.1.4.a, although the average level is similar.

A3.1.4.e:
In this projection, the changes in the valuation basis are less than the changes in experience.

The Standard Contribution Rate moves according to the changes in the valuation basis. The pattern of the moves is the same as A3.1.4.d, but the movements are smaller.

The projection starts with no surpluses/deficiencies.

In the first five years, the valuation basis continues to be weakened, and valuation surpluses arise. The level of each year's accrual is relatively constant for similar reasons as in A3.1.4.d. The level is less than for A3.1.4.d reflecting the smaller changes in the basis. At the same time, surpluses arise from the gap between the experience and the valuation basis. This part of the surplus increases towards year 5. The total accrual of surpluses is less on average than for A3.1.4.d. The surpluses accumulate, and the degree of the reduction in the contribution rate increases. Combined with the fall in the Standard Contribution Rate, the Recommended Contribution Rate shows a fall. The degree of fall is less than for A3.1.4.d.

In the next five years, valuation deficits arise constantly, while the gap between the experience and the basis produces surpluses. The surpluses arising gradually reduce to 0 towards year 10 reflecting the narrowing gap. The surpluses accruing from the gap seem to be less significant than the valuation deficits, and the surpluses are amortised quickly. Combined with the rise in the Standard Contribution Rate, the Recommended Contribution Rate rises.

In the third quarter of the cycle, both valuation deficits and deficits from the gap between the experience and the basis arise at the same time. The latter increases gradually. However, the impact of the latter seems to be small, and the Recommended Contribution Rate increases at a similar pace as during the second quarter. Towards the end of year 15, deficits are accumulated to reach the highest level.

After year 16, valuation surpluses constantly accrue at the same level as the first five years, while the gap between the experience and the basis creates deficits, which are reducing. Again, the latter is insignificant, and the accruing surpluses help with amortising the deficits. At the end of the first cycle, there is a small accumulated surplus. Combined with the reducing Standard Contribution Rate, the Recommended Contribution Rate goes down.

Because the surplus at the beginning of the second cycle is small, a similar pattern is repeated. However, the funding level is in general slightly higher.

This gradual rise in the funding level continues, but seems to become stable and cyclical. In the fifth cycle, the lowest Recommended Contribution Rate is 7.4440% in year 85, and the highest is 14.5482% in year 95. The difference is 7.1042%. This is much less than for A3.1.4.d, but much wider than the projections with constant bases.

Consequently, the funding level is less volatile than for A3.1.4.d, but more than for A3.1.4.a, although the average level is similar.

Summary

The changes in the basis according to the cyclical change in the experience make the contribution rate more volatile.

In these projections, the effects of the changes in the valuation basis are more significant than the impact of the gap between the experience and the basis. A 1% difference in the experience and the basis only produces a surplus/deficit of 1% of the total liabilities, while a 1% change in the valuation basis change is accumulated from the valuation date to the final payments of the benefits, and the surpluses/deficits could be more than 10 times larger than those from the 1% gap. So, by using a fixed basis as A3.1.4.a, the first factor is eliminated, and the volatility in the contribution rate is reduced.

-Triennial changes

Although valuations are made annually, the basis is changed only every three years. The experience cycle is 20 years and the basis change is every 3 years. So, in the long run, all the combinations of experience and the basis evenly appear.

List of Projections

A3.1.4.f Immediate, Triennial change, .2%pa

A3.1.4.g Immediate, Triennial change, .1%pa

On each change of the basis, the moves in the contribution rate and the funding level are very large. In other years, the moves are moderate. So, the year to year movements in the Recommended Contribution Rate and the funding level are less smooth than for A3.1.4.d or A3.1.4.e.

However, the general pattern, the average and the difference between the highest and the lowest of the contribution rates are similar to A3.1.4.d and A3.1.4.e respectively.

-Quinquennial changes

Here, the changes in the basis take place every five years. This means that the same valuation basis is used for each quarter of the cycle; the basis close to the standard basis during the first and the third quarter, a weak basis for the second quarter, and a strong basis for the fourth quarter.

List of Projections

A3.1.4.h Immediate, Quinquennial change, .2%pa

A3.1.4.i Immediate, Quinquennial change, .1%pa

The year to year variations in the Recommended Contribution Rate is less smooth than triennial changes. Also, owing to the use of the same valuation bases for every cycle, the contribution rate pattern is more clearly cyclical.

However, the general pattern, the average and the difference between the highest and the lowest of the contribution rates are similar to A3.1.4.d and A3.1.4.e respectively.

So, the frequency of the basis changes may not be significant as long as the changes take place within a relatively short period after the last change.

-Averaging

The valuation basis is set as the average of past experience. As the averaging period, 3, 5 and 10 years are used. A3.1.4.d can be regarded as the projection with one year's averaging period.

The effects of averaging on the setting of the bases are the moderation of the changes in the basis and the delay in the changes. The impact from the first effect, that is the narrower range of the valuation basis than in A3.1.4.d, is similar to the impact observed in 3.1.4.e. The second effect appears as gaps between the phases of the experience cycle and the basis cycle.

List of Projections

A3.1.4.j	Immediate, Annual change, 3 year average
A3.1.4.k	Immediate, Annual change, 5 year average
A3.1.4.l	Immediate, Annual change, 10 year average

A3.1.4.j:

The range of the salary growth assumption is 5.6333% and 7.3667%. This is slightly narrower than that of the experience, 5.5% to 7.5%. The lowest salary growth assumption is used in year 6, and the highest in year 16. So, there is one year's delay in the phase of the cycle.

As mentioned in A3.1.4.d and A3.1.4.e, the effects of valuation changes are dominant, and the effects of the gap between the experience and the basis are less significant. So, the changes in the Recommended Contribution Rates follow the pattern of A3.1.4.d with one year's delay with smaller movements.

A3.1.4.k:

The range of the salary growth assumption is 5.74% and 7.26%. This is slightly narrower than that of A3.1.4.j. The lowest salary growth assumption is used in year 7, and the highest in year 17. So, there is two years' delay in the phase of cycle compared with the experience cycle. Similar comments to A3.1.4.j apply.

A3.1.4.l:

The range of the salary growth assumption is 6.0% and 7.0%. This is slightly narrower than that of A3.1.4.k. This range is the same as A3.1.4.e. The lowest salary growth assumption is used in year 9 and 10, and the highest in year 19 and 20. So, there is 4.5 years' delay in the phase of cycle compared with the experience cycle.

Again, similar comments to A3.1.4.j apply.

Although the valuation basis's cycle is almost a quarter behind that of the experience, only the effects of valuation surpluses/deficits are significant, and the pattern of the widening and narrowing of the gap between the experience and the basis seems less important.

The range of the valuation bases used is the same as A3.1.4.e, but the distribution within the range is different. The distribution of the salary growth assumptions used during the fifth cycle are as follows;

	A3.1.4.e	A3.1.4.l
less than 6.125	4	6
6.125 - 6.275	2	2
6.275 - 6.425	4	2
6.425 - 6.575	2	0
6.575 - 6.725	4	2
6.725 - 6.875	2	2
more than 6.875	4	6

A3.1.4.e covers the range evenly, while A3.1.4.l use more from the boundaries than the centre of the range.

Consequently, the variations in the contribution rates are more significant in A3.1.4.l than for A3.1.4.e.

Summary

The effect of the changes in the basis is dominant on the pattern of the changes in the contribution rate. The main effects of using a longer averaging period are a reduction in the volatility of the contribution rates and the delay in the phase of the cycle.

Note

Although the following combinations have also been examined, a full description is omitted from the results in Chapter 8 and this Appendix because these results are judged to add little to the main discussion.

-Gradual changes in experience

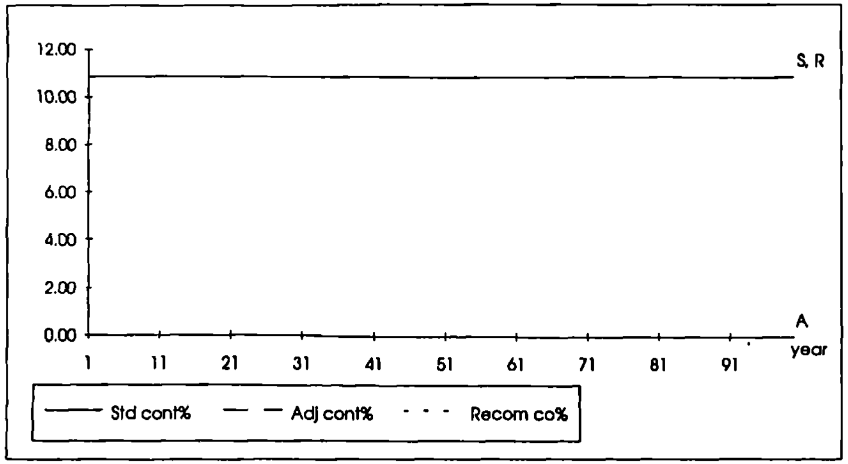
A3.1.3.m	From year 3, Annual change, .004%pa
A3.1.3.n	Immediate, Triennial change, .004%pa
A3.1.3.o	Immediate, Decennial change, .004%pa
A3.1.3.p	Immediate, Triennial change, 5 year average
A3.1.3.q	Immediate, Triennial change, 10 year average
A3.1.3.r	Immediate, Decennial change, 5 year average
A3.1.3.s	Immediate, Decennial change, 10 year average

-Cyclical changes in experience

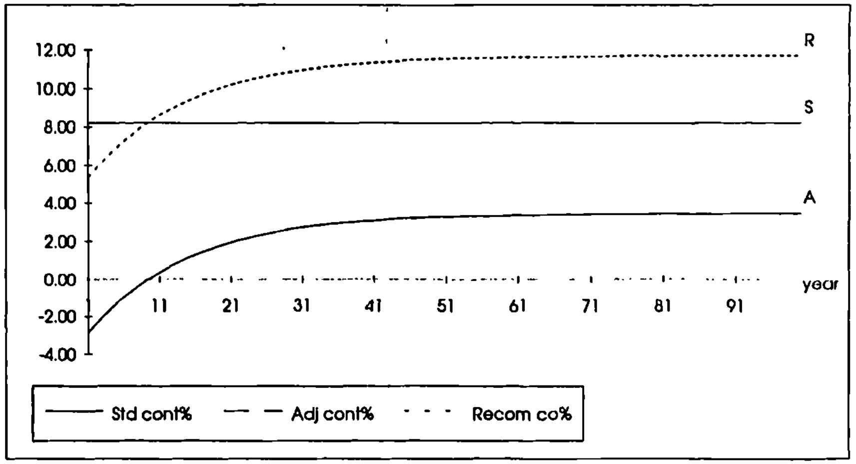
A3.1.4.m	From year 2, Triennial change, .2%pa
A3.1.4.n	From year 3, Triennial change, .2%pa
A3.1.4.o	Immediate, Triennial change, 3 year average
A3.1.4.p	Immediate, Triennial change, 5 year average
A3.1.4.q	Immediate, Triennial change, 10 year average
A3.1.4.r	Immediate, Quinquennial change, 3 year average
A3.1.4.s	Immediate, Quinquennial change, 5 year average
A3.1.4.t	Immediate, Quinquennial change, 10 year average

Also, all the corresponding combinations with a 10 year cycle.

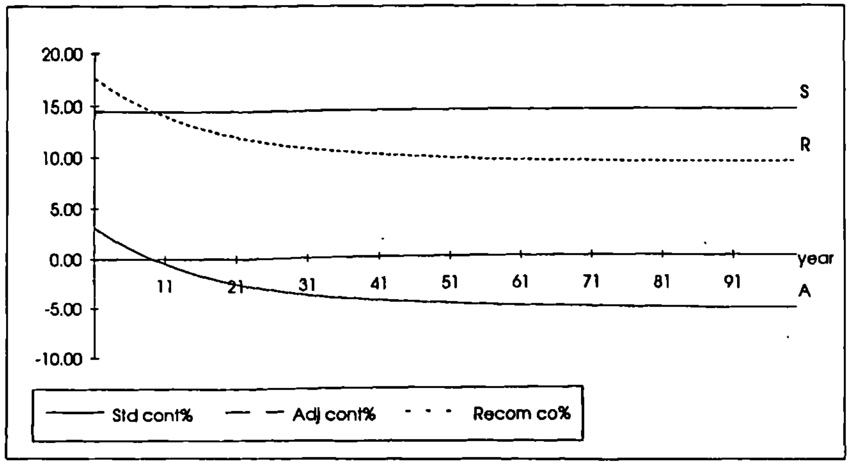
Graph A3.1.1.a



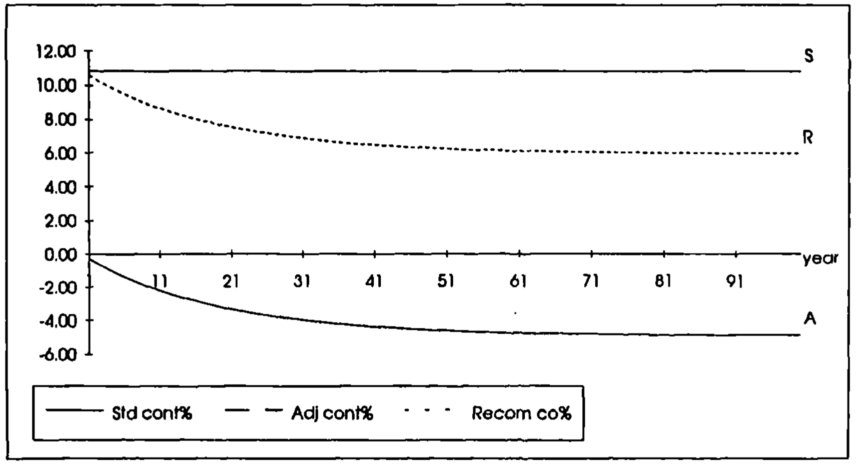
Graph A3.1.1.b



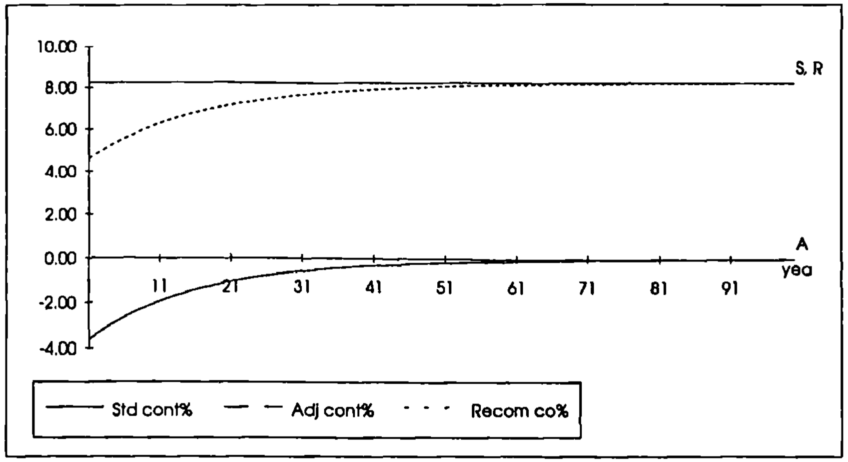
Graph A3.1.1.c



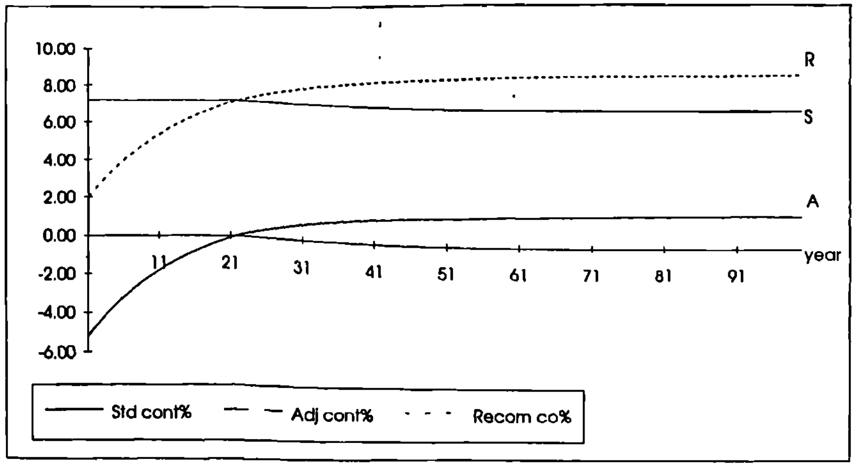
Graph A3.1.2.a



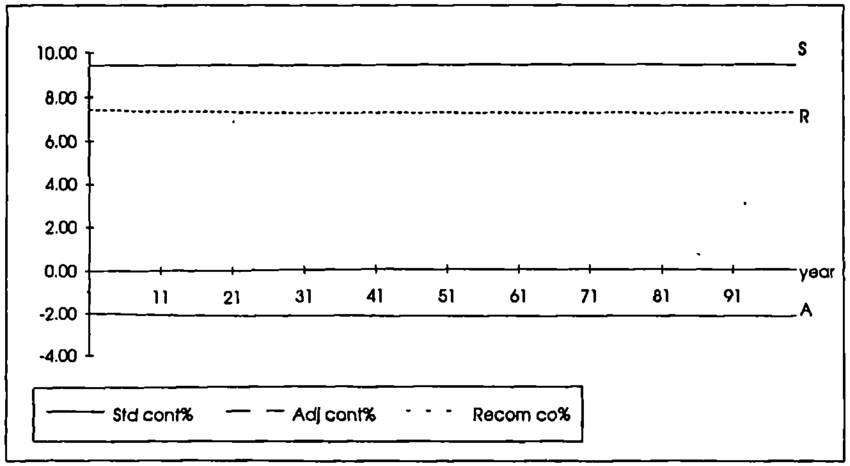
Graph A3.1.2.b



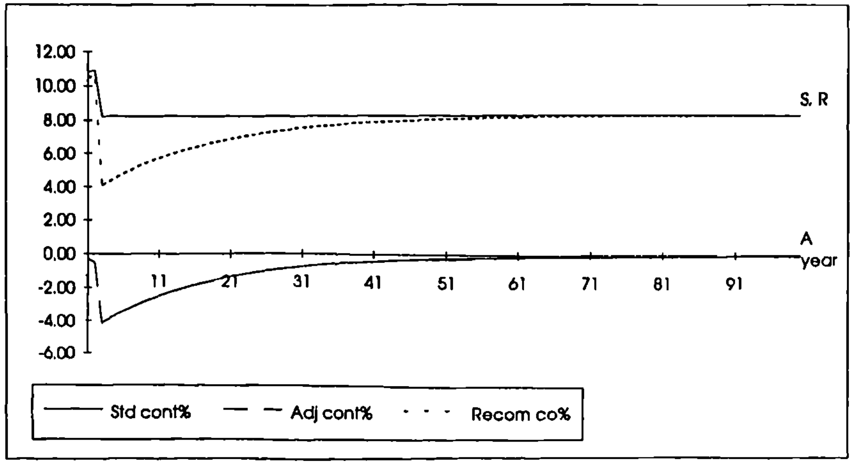
Graph A3.1.2.c



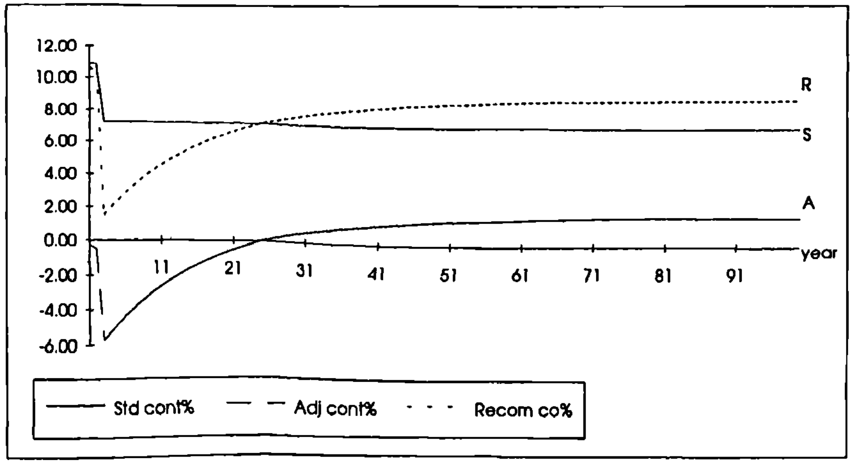
Graph A3.1.2.d



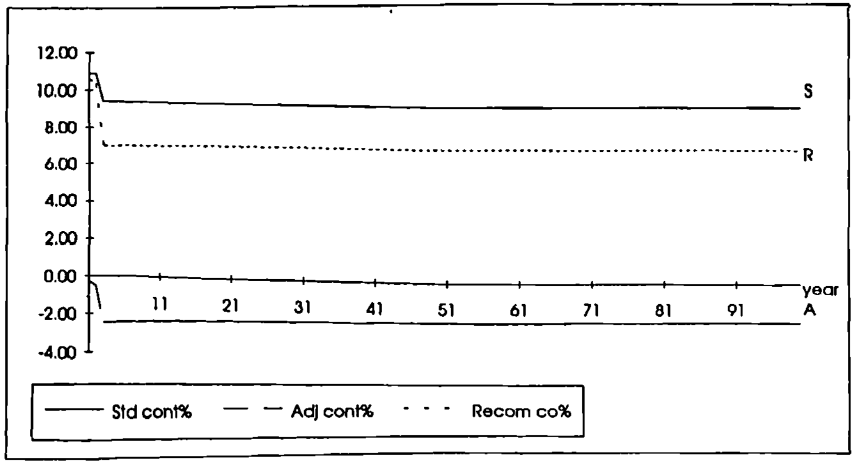
Graph A3.1.2.e



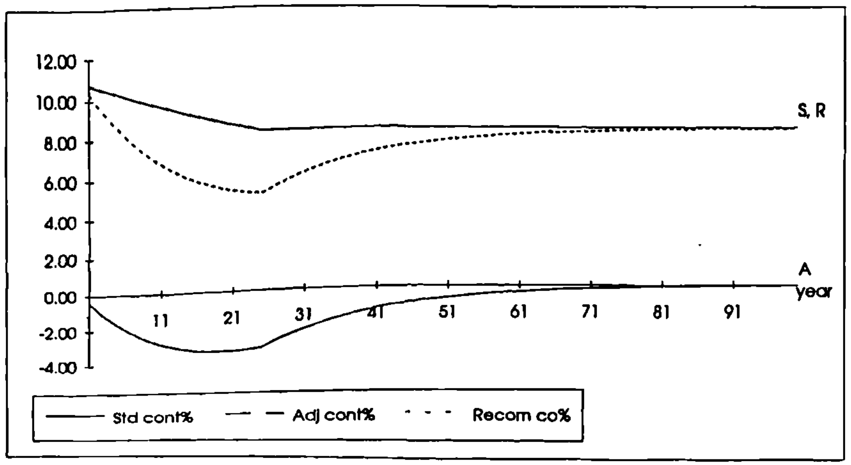
Graph A3.1.2.f



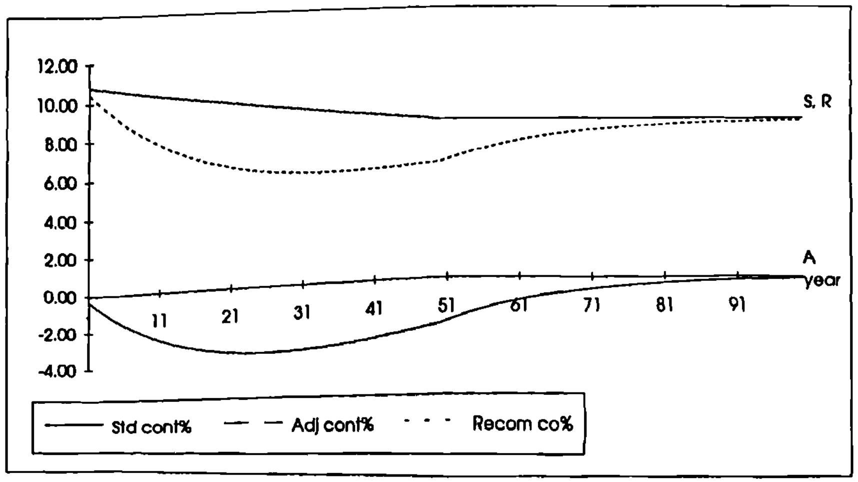
Graph A3.1.2.g



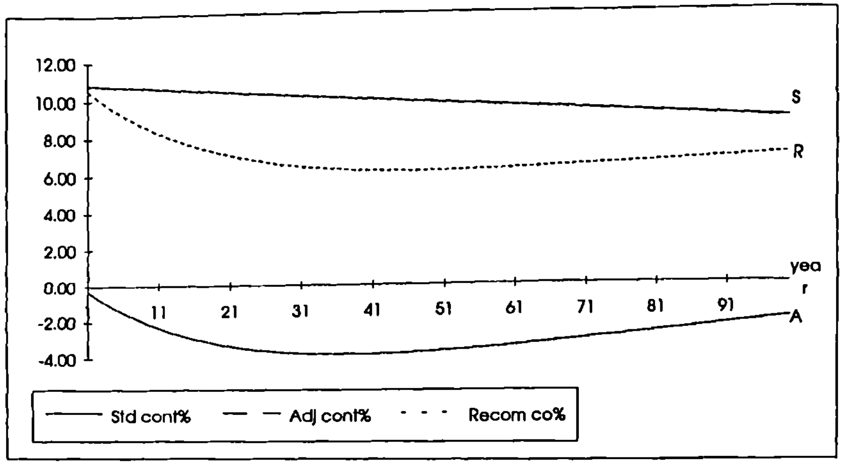
Graph A3.1.2.h



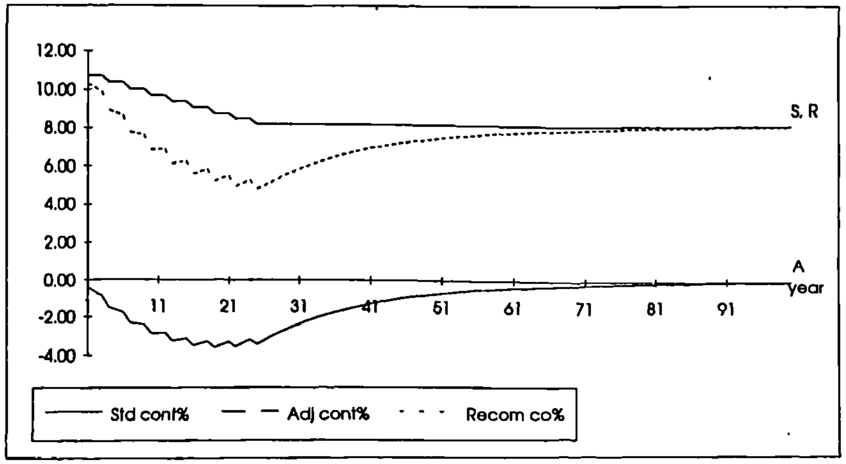
Graph A3.1.2.I



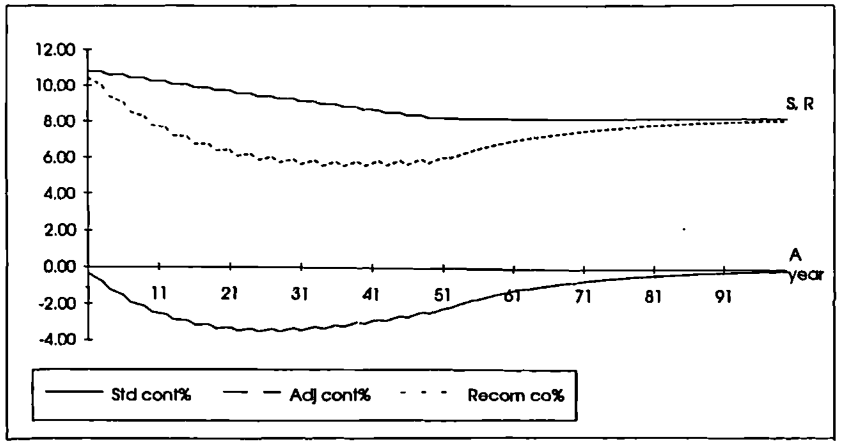
Graph A3.1.2.j



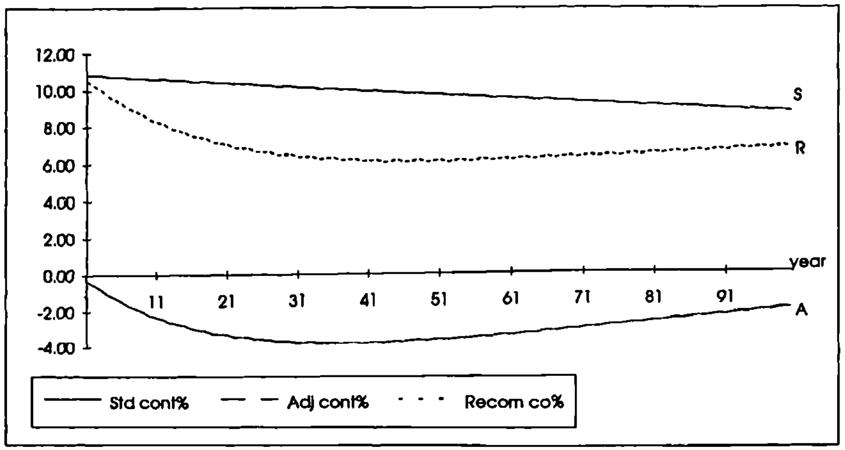
Graph A3.1.2.k



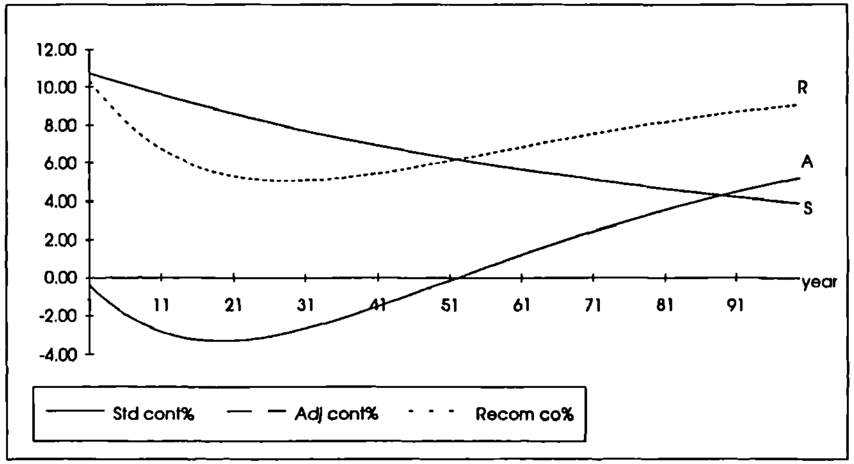
Graph A3.1.2.l



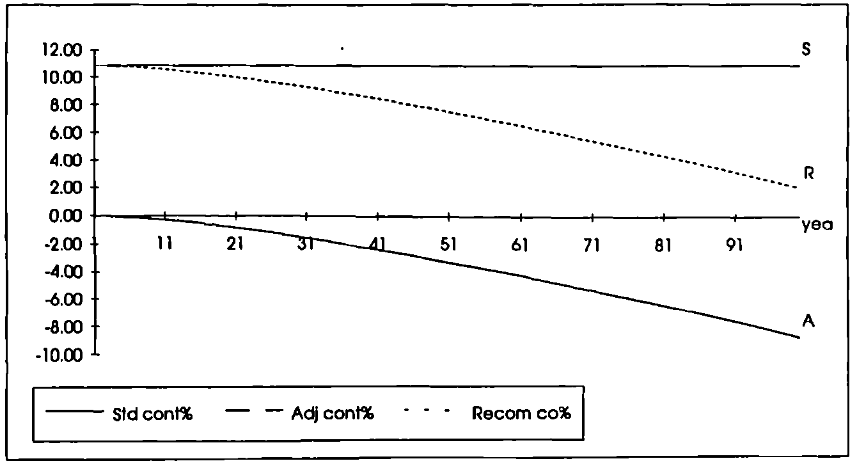
Graph A3.1.2.m



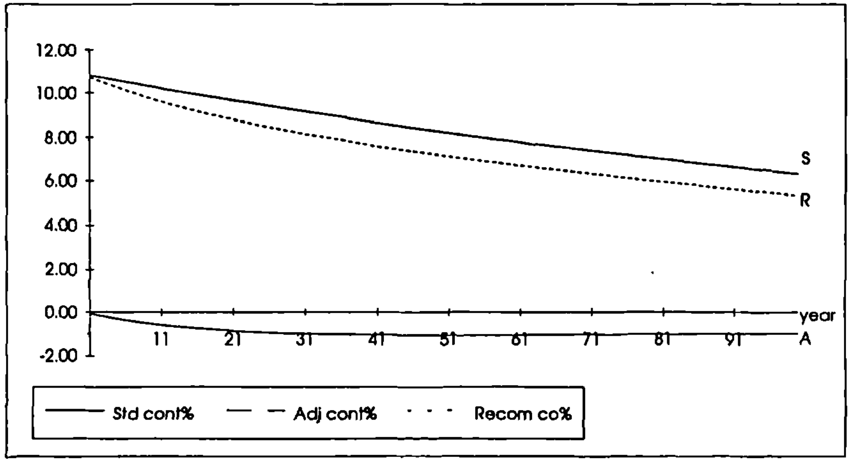
Graph A3.1.2.n



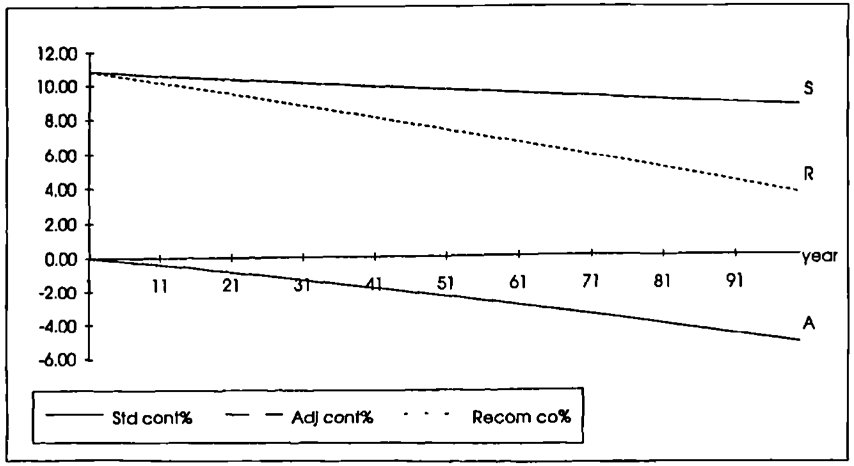
Graph A3.1.3.a



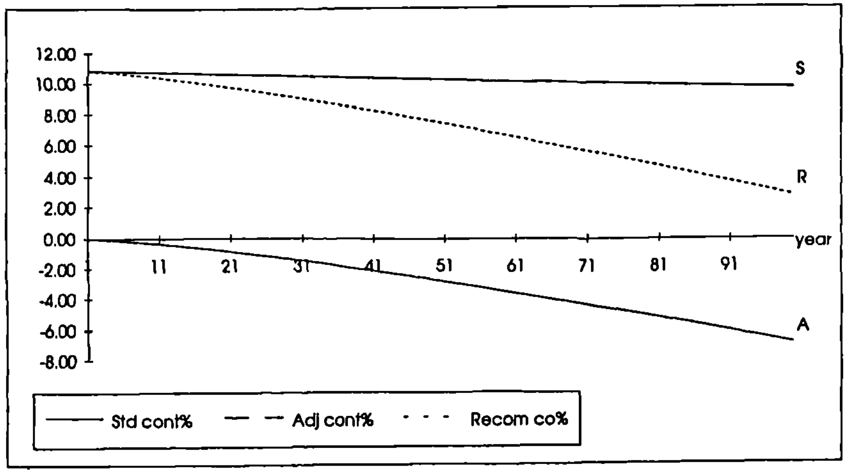
Graph A3.1.3.b



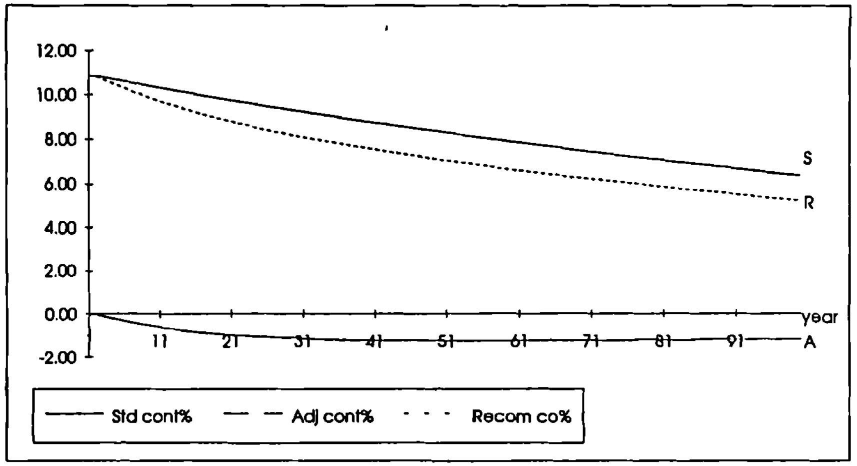
Graph A3.1.3.c



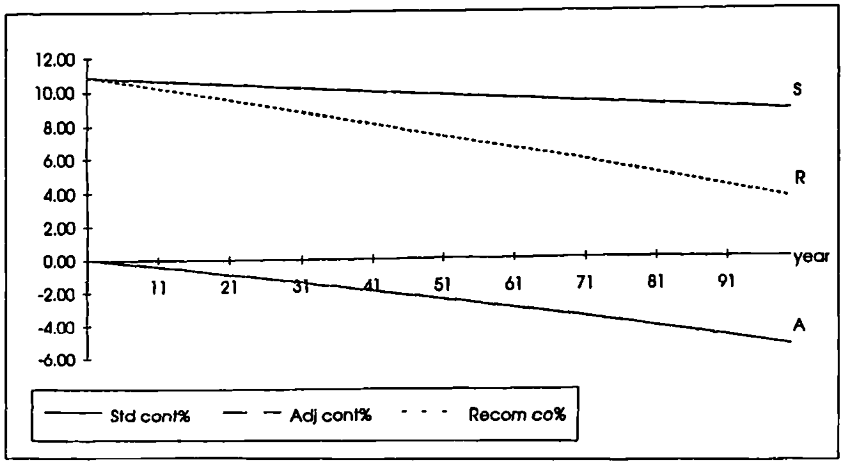
Graph A3.1.3.d



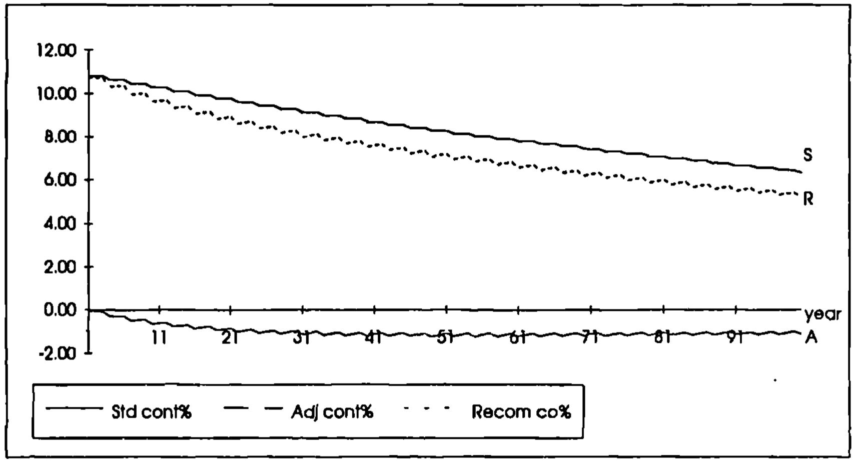
Graph A3.1.3.e



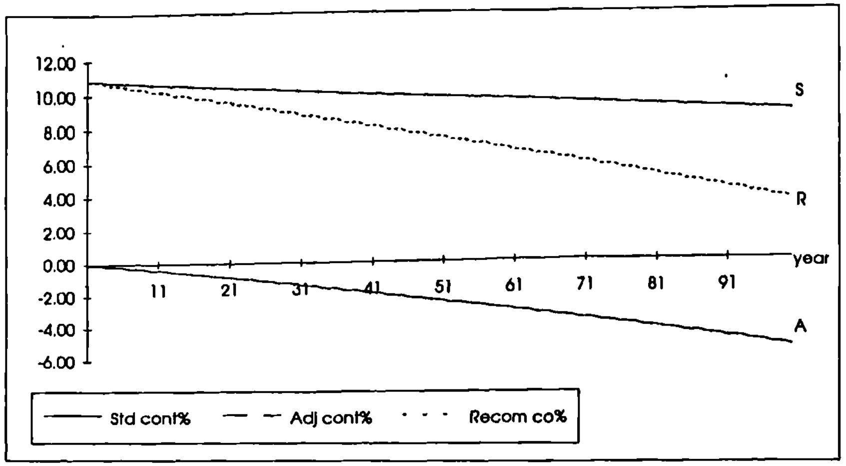
Graph A3.1.3.f



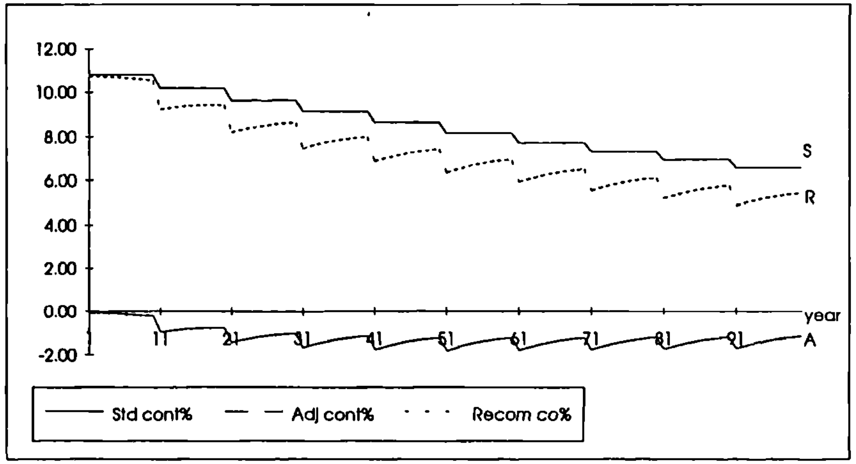
Graph A3.1.3.g



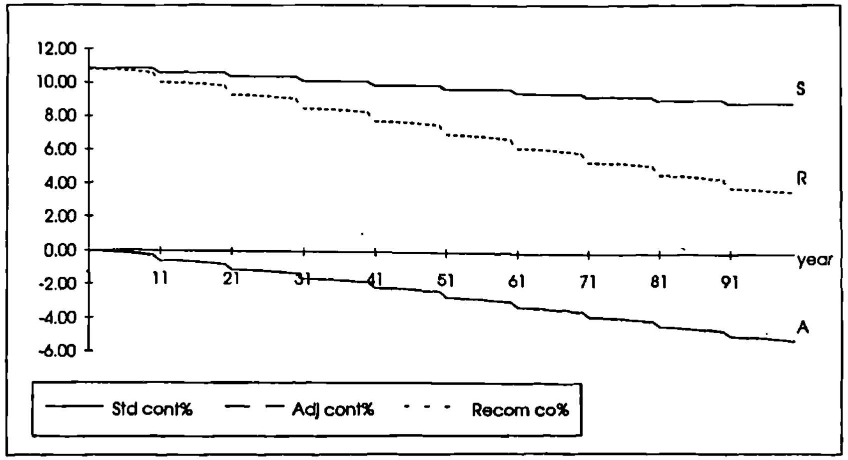
Graph A3.1.3.h



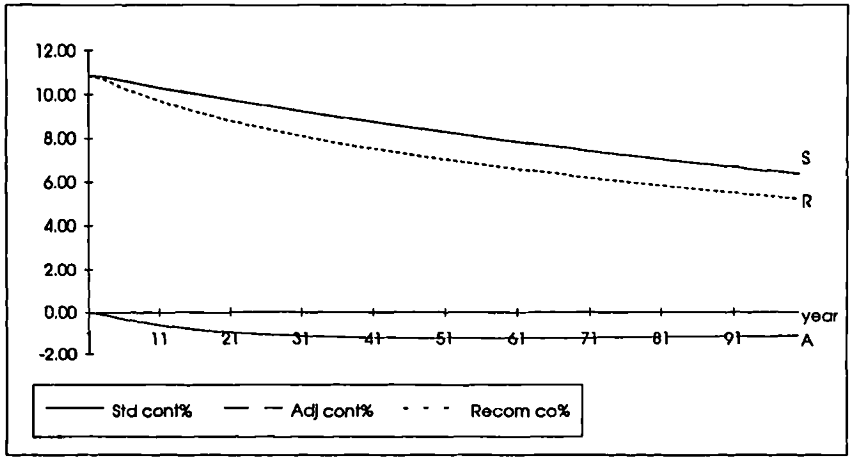
Graph A3.1.3.i



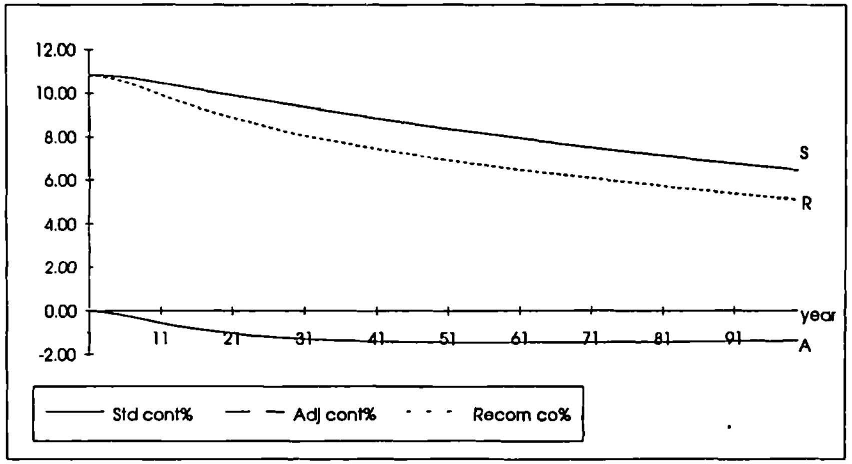
Graph A3.1.3.j



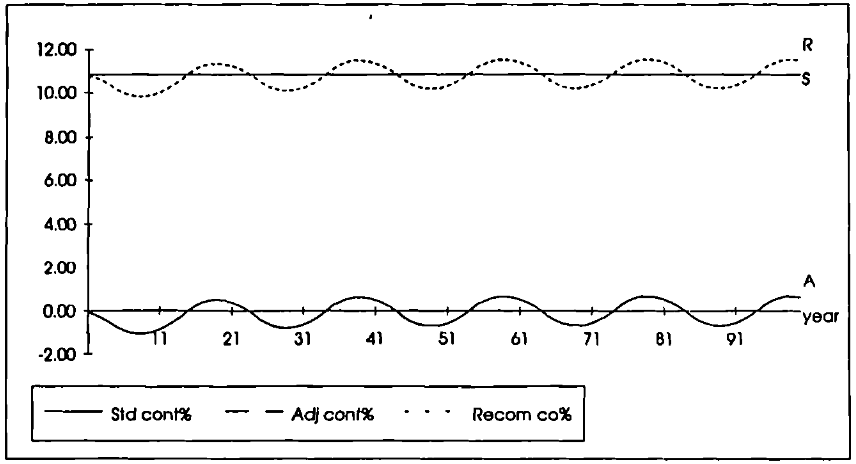
Graph A3.1.3.k



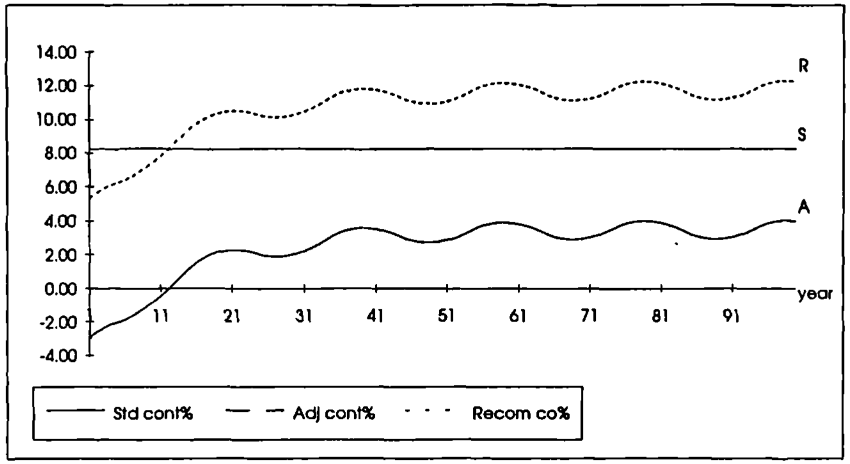
Graph A3.1.3.I



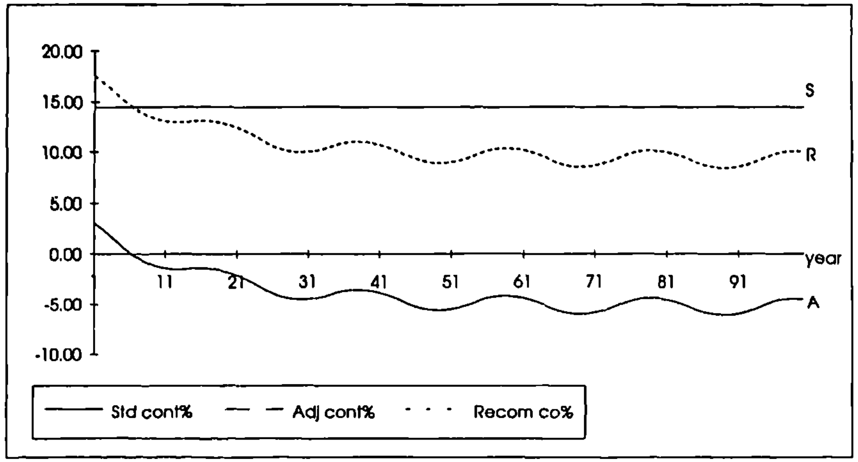
Graph A3.1.4.a



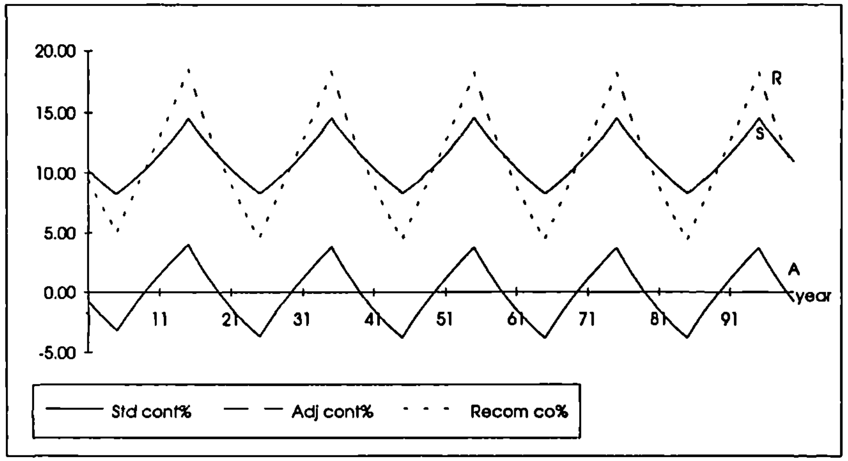
Graph A3.1.4.b



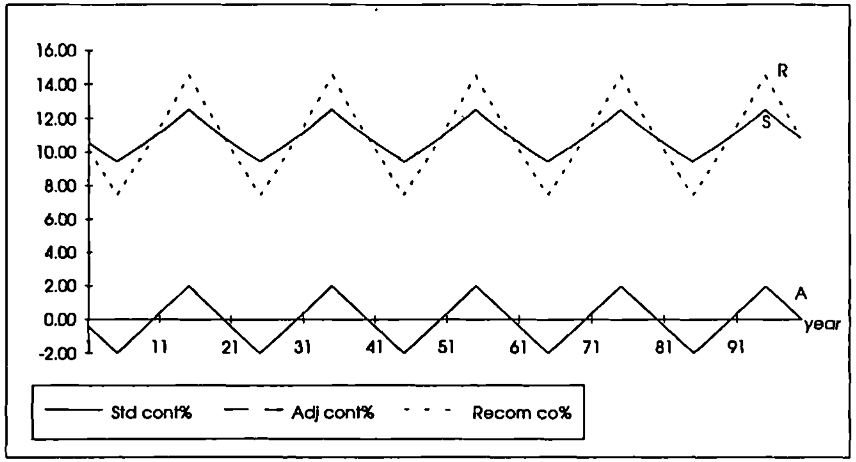
Graph A3.1.4.c



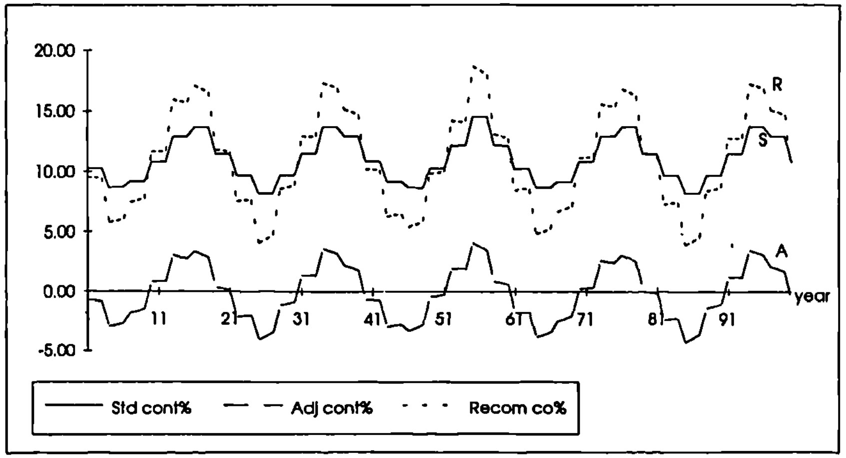
Graph A3.1.4.d



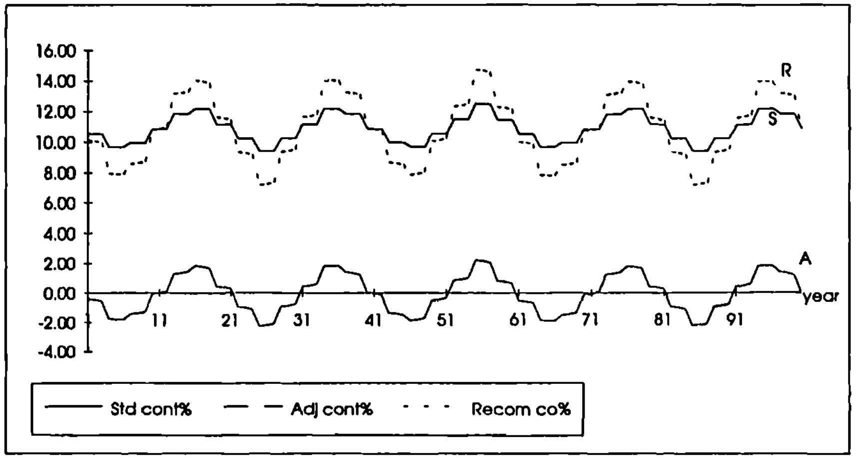
Graph A3.1.4.e



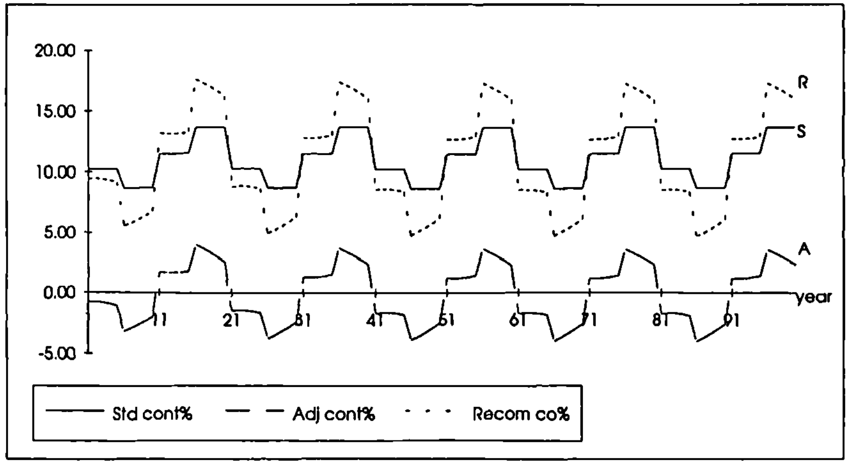
Graph A3.1.4.f



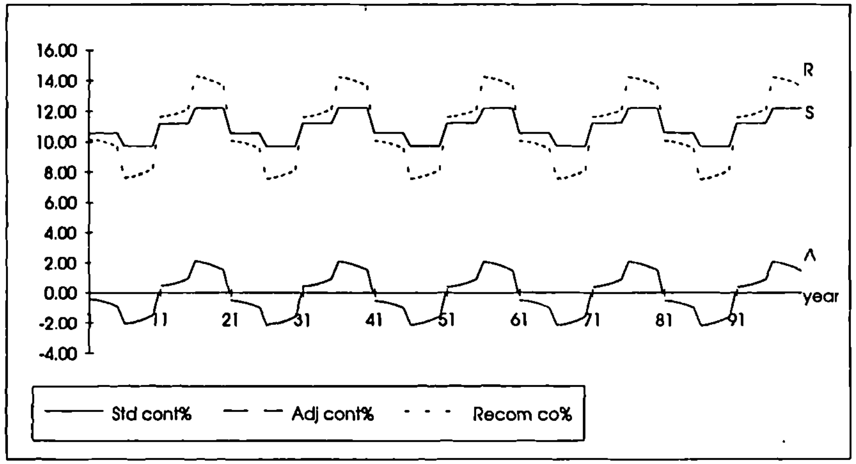
Graph A3.1.4.g



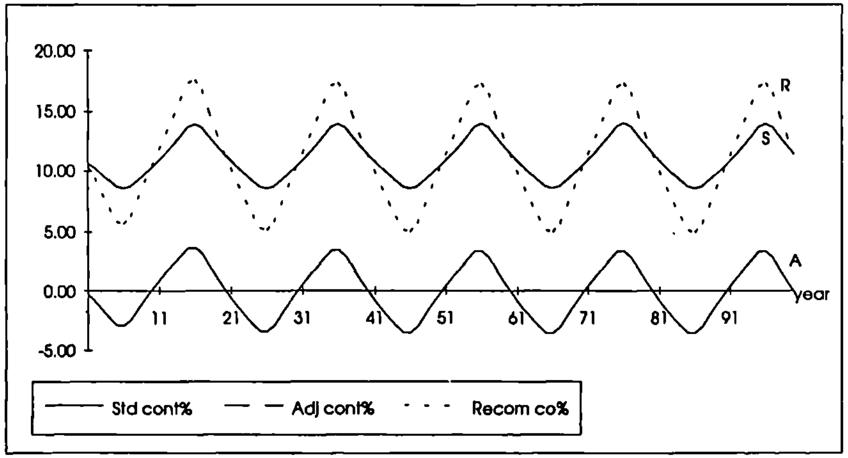
Graph A3.1.4.h



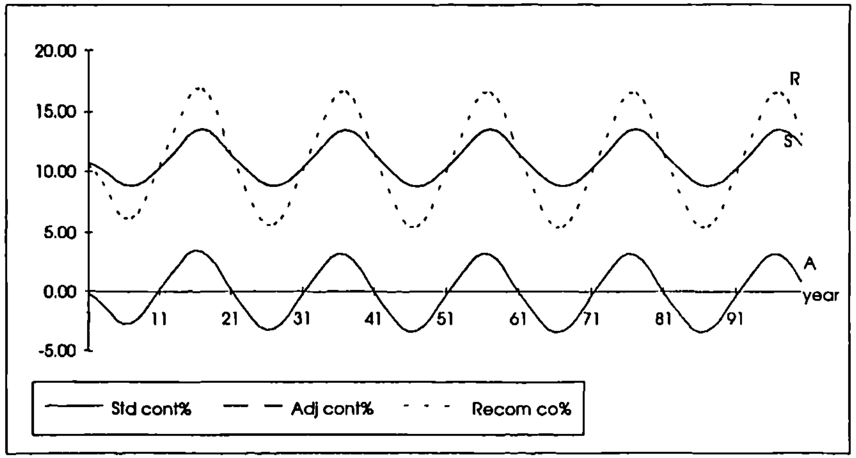
Graph A.3.1.4.l



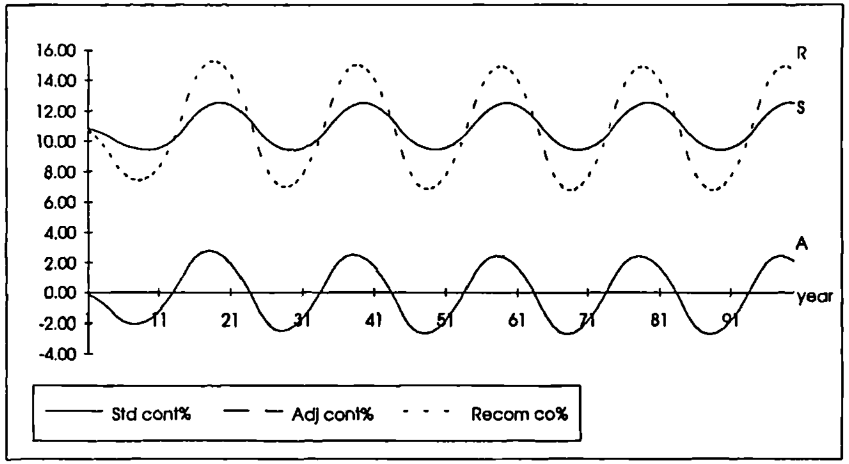
Graph A3.1.4.j



Graph A3.1.4.k



Graph A3.1.4.I


16.00
14.00
12.00
10.00
8.00
6.00
4.00
2.00
0.00
-2.00
-4.00
11
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41
51
61
71
81
91
year
R
S
A
Std cont%
Adj cont%
Recom co%

Appendix 3.2 Dividend Growth Changes

As commented in section 7.5.1.2, the value of the liabilities and the Standard Contribution Rates are not affected. The Standard Contribution Rate remains stable at 10.8578% for all the projections. So, the following does not particularly comment on the Standard Contribution Rates.

A3.2.1 Unchanged experience

The experience is the same as the standard experience and remains stable throughout the projections. Under the standard experience, the market interest rate is 9.0% and the market dividend growth expectation is 5.0%. So, the market dividend yield stays at 3.8095%.

Three different bases, which are also constant throughout the projection, are used, and the long term effects of the choice of the basis are examined.

List of Projections

A3.2.1.a	g=5.0%, d=3.8095% [Standard basis]
A3.2.1.b	g=5.5%, d=3.3175%
A3.2.1.c	g=4.5%, d=4.3062%

The bases are stable from the beginning of year 1. However, the fund at the beginning of year 1 is equal to the actuarial liabilities calculated on the standard basis, and the *initial fund is the same for all three* projections.

A3.2.1.a:
The experience and the valuation basis are equal and static throughout the projection. This shows a stationary fund which grows in line with the salary growth, i.e. 6.5% p.a. The Recommended Contribution Rate is always the same as the Standard Contribution Rate, which is constant. The funding level on the standard basis is static at 100%.

A3.2.1.b:
In this projection, the assets are valued higher than in A3.2.1.a by 14.8%. (because the basis is "weaker" - see section 8.3.1.3)

Initially, there is a surplus. For the amortisation of this surplus, the contribution rate is reduced to 8.7039% initially.

Because of the differences between the experience and the assumption, deficits arise from negative cashflows (see section 8.3.1.2). Because of the progress of amortisation and the accumulation of the deficits, the contribution rate increases. This reduces the negative cashflow. So, from section 8.3.1.2, the deficits arising gradually decrease.

On the other hand, as the initial surplus is amortised, and then deficits accumulate, the contribution rate increases. Then, at some stage, the additional contribution for amortisation and the deficit arising each year balance each other, and an equilibrium is achieved at a lower funding level than for A3.2.1.a.

A3.2.1.c:

This case is opposite to A3.2.1.b. The assets are valued lower than in A3.2.1.a by 11.5%, and initial deficits arise.

For reasons opposite to those for A3.2.1.b, surpluses arise from the negative cashflow every year, which accelerate to amortise the initial deficits. Further, surpluses accumulate, and the contribution rate falls.

The level of surplus arising every year increases gradually as the recommended contribution rate reduces and consequently the negative cashflow increases. However, the pace of reduction in contributions for amortisation exceeds the pace of increase in the surpluses arising every year. So, the difference between the two approaches to 0, and an equilibrium state with a higher funding level than for A3.2.1.a is achieved.

Summary

Where the experience is constant, the membership structure remains stationary, and a range of stable valuation bases may achieve stationary fund eventually. The use of an optimistic basis may not give a sufficient security of benefit payments. On the other hand, a pessimistic basis may cause over funding.

The initial movements in the Recommended Contribution Rates are avoided by the use of the standard basis. However, the situation can be totally different if the amount of initial assets is different from the level assumed in these projections.

A3.2.2 One permanent change in experience

In this section, the dividend growth experience, which is 5.0% before the projections start, is increased to 5.5% from year 1, and dividend growth market expectation, which is also 5.0% before the projections start, is increased to 5.5% from the end of year 1. They remain at 5.5% throughout the projections thereafter. The market interest rate stays at 9.0%, and the market dividend yield is reduced from 3.8095% to 3.3175% at the end of year 1, and remains so thereafter.

There are improved dividend incomes from existing assets, while new monies can only be invested on the assets with low dividend yield and high dividend growth expectation. However, the overall expected return on the new money stays the same at 9%.

A constant valuation basis, a one permanent change in the basis, and gradual changes in the basis are investigated.

A3.2.2.1 Constant basis

The standard basis is tried to see the impacts of the change in the experience without any moves in the basis. The projection is done as a basis for comparison with other projections which use changing bases.

List of Projections

A3.2.2.a Standard basis

A3.2.2.a:

By the change in the experences, the market value of existing assets is increased by 14.8% (=.038095/.033175 - 1. See A1.4.2.5 in Appendix 1.) at the end of year 1, but the actuarial value is not affected. So, initially, the funding level is 100%, and the Recommended Contribution Rate is equal to the Standard Contribution Rate.

After the change in the experience, there is always a gap between the actual dividend growth and the assumption. So, similar to A3.2.1.c, surpluses emerge every year. The surpluses are gradually increasing as the contribution rate falls. However, the amortisation catches up with this, and an equilibrium state which is in surplus is attained.

Unlike A3.2.1.c, there are no initial deficits, and the process of achieving an equilibrium is quicker.

A3.2.2.2 One permanent change in basis

In this section, the valuation bases are changed only once. Various combinations of the timing and the magnitude of the changes are tried.

-Immediate change

The bases are changed at the end of year 1. So, there are no delays in the reactions.

List of Projections

A3.2.2.b	Immediate, g=5.5% from year 1 [Same as experience]
A3.2.2.c	Immediate, g=5.75% from year 1
A3.2.2.d	Immediate, g=5.25% from year 1

A3.2.2.b:

At the end of year 1, a surplus arises owing to an increase in the value of assets. The Recommended Contribution Rate is 2.58% less than the Standard Contribution Rate to amortise the surplus.

From year 2 onwards, the experience and the valuation basis are equal, and no surpluses/deficits arise.

The initial surplus is gradually amortised to nil, and the fund becomes stationary with the Standard Contribution Rate as the Recommended Contribution Rate and the funding level equal to 100%.

A3.2.2.c:

The valuation basis is more optimistic than for A3.2.2.b, and the initial surplus is larger. The Recommended Contribution Rate is 4.06% less than the Standard Contribution Rate reflecting a surplus which is larger than for A3.2.2.b.

The actual dividend growths are lower than the assumptions, and similar to A3.2.1.b, deficits arise because of the negative cashflow every year. The deficits reduce the initial surpluses and, then, accumulate gradually. Consequently, the Recommended Contribution Rate increases.

Eventually, the emerging deficits, which are decreasing, and the additional contributions for amortisation, which are increasing as the deficits accumulate, balance each other, and the fund approaches a stationary state where the funding level is lower than for A3.2.2.b. The contribution rate is higher than for A3.2.2.b. This suggests that the security is lower.

A3.2.2.c starts from a lower contribution rate and ends in a higher contribution rate than for A3.2.2.b. So, the contribution rate is less stable than for A3.2.2.b.

A3.2.2.d:

Similar to A3.2.2.b, there is an initial surplus, but the amount is smaller than for A3.2.2.b. The Recommended Contribution Rate at the end of year 1 is 1.29% lower than the Standard Contribution Rate.

The actual dividend growths are higher than the assumptions, and surpluses arise because of the negative cashflow every year. The surpluses are falling gradually as the contribution rate rises.

Initially, the reduction in the contributions for amortisation is larger than the surpluses arising. However, as the amortisation progresses, the degree of the reduction in the contributions falls. Eventually, the surplus arising and the reduction in the contributions are balanced to achieve an equilibrium at a higher funding level than for A3.2.2.b.

From the outset, the surpluses arising are mitigating the effect of the reduction in the contributions for the amortisation of the initial surplus. So, the contribution rate is relatively stable.

Summary

The effect of the cancellation between the reduced contribution rates and the newly accruing surpluses shown in A3.2.2.d may be observed if the valuation basis is between A3.2.2.a and A3.2.2.b, although the degree of the effect can be more or less significant than for A3.2.2.d. So, the conribution rates may be more stable than for A3.2.2.a or A3.2.2.b if such a basis is chosen.

On the other hand, valuation bases more optimistic than for A3.2.2.b (e.g.A3.2.2.c) would produce more volatile contribution rates and lower funding levels.

-Delayed change

The changes in the valuation basis take place at the end of year 3. So, there is two years' delay in the change.

List of Projections

A3.2.2.e	Delayed, g=5.5% from year 3 [Same as experience]
A3.2.2.f	Delayed, g=5.75% from year 3
A3.2.2.g	Delayed, g=5.25% from year 3

A3.2.2.e:

Basically, the pattern of changes in the contribution rates is similar to A3.2.2.b. The ultimate level of the contribution rate is the same as the Standard Contribution Rate.

The effect of the delay in the change of the basis appears as a higher level of additional surpluses which are caused by higher dividend growths than expected for the first three years, rather than only for one year as in the case of A3.2.2.b.

From year 4 onwards, the experience and the valuation basis are equal, and no surpluses/deficits arise. So, the surpluses at the end of year 3 are gradually amortised to nil. Because the surplus is larger than for A3.2.2.b, the initial reduction in the contribution rate is larger, and it takes longer to amortise the surplus.

A3.2.2.f:

The effect of the delay appears as a larger initial surplus than for A3.2.2.c, and it takes longer to reach an equilibrium.

However, the basic pattern of changes in contribution rates are the same as A3.2.2.c. The ultimate contribution rate is almost the same as that of A3.2.2.c.

A3.2.2.g:

The effect of the delay appears as a slightly larger initial surplus than for A3.2.2.d, and it takes a little longer to come to an equilibrium. The ultimate contribution rate is almost the same as that of A3.2.2.d.

Similar to A3.2.2.d, the contribution rate is relatively stable after the change in the basis.

Summary

The delay in the change of basis appears as a higher level of surplus when the basis is changed. This results in a lower initial contribution rate, and a slower convergence to the equilibrium state. So, a delay in the change tends to increase the volatility of the Recommended Contribution Rate after the change of the basis.

A3.2.2.3 Gradual change in basis

In this section, the valuation bases are changed gradually as straight lines. Various combinations of the magnitude of the changes and the frequency of the changes in the basis are tried.

They are interpreted as the averaging of the past experience. The magnitude of the changes corresponds to the length of averaging period. There are three levels of magnitude tested; Quick (.02% p.a.), Medium (.01% p.a.) and Slow (.004% p.a.). 'Quick' corresponds to 25 year averaging, 'Medium' corresponds to 50 year averaging, and 'Slow' corresponds to 125 year averaging.

Because the valuation bases are the averages of the past experience, the changes in the basis will cease when the basis arrives at the experience level.

For the completeness, one additional projection has been made where the change in the basis does not stop when the basis is equal to the experience.

-Annual change

The change in the basis starts from the end of year 1, and the changes are made every year thereafter.

List of Projections

A3.2.2.h	Averaging, Annual change, Quick(.02%pa)
A3.2.2.i	Averaging, Annual change, Medium(.01%pa)
A3.2.2.j	Averaging, Annual change, Slow(.004%pa)

A3.2.2.h:

Until year 25, owing to the continuing weakening of the basis, the assets are revalued, and valuation surpluses arise. At the same time, the dividend growth experience is higher than the valuation basis, and surpluses arise because of the negative cashflows. The level of surpluses arising from the latter source reduces gradually as the gap between the experience and the basis reduces. These surpluses accumulate until year 25, and the Recommended Contribution Rate gradually falls.

After year 26, no surpluses/deficits are accruing because the experience exactly matches the assumptions. The accumulated surpluses are then amortised in due course. Eventually, the surpluses are reduced to nil.

A3.2.2.i:

Until year 50, similar to A3.2.2.h, surpluses arise, and the level of surpluses arising reduces gradually as the gap between the experience and the basis reduces.

These surpluses accumulate until year 32, and the Recommended Contribution Rate gradually falls. However, after year 33, the accruing surpluses are smaller than the reduction in the contrubution for the amortisation of accumulated surpluses, and the surpluses gradually fall.

After year 51, no surpluses/deficits are accruing because the experience exactly matches the assumptions. So, the fall in the accumulated surpluses is accelerated, and eventually, the surpluses are eliminated.

The maximum level of accumulated surpluses is lower than for A3.2.2.h, and the variation in the Recommended Contribution Rate is smaller than for A3.2.2.h.

A3.2.2.j:

The change in the valuation basis is so slow that the basis cannot reach the experience level at the end of 100 years.

So, surpluses arise every year during the projection. The level of surpluses arising reduces gradually as the gap between the experience and the basis reduces.

The surpluses accumulate until year 41. After year 42, the accruing surpluses become smaller than the reduction in the contribution for amortisation, and the surpluses are gradually reduced.

The maximum level of accumulated surpluses is slightly lower than for A3.2.2.i, and the variation in the Recommended Contribution Rate is the smallest among the three.

Summary

A longer averaging gives a lower level of accumulated surpluses. This effect is opposite to the delay in the change of basis. This is because the surpluses arising are relatively small, and the amortisation progresses at the same time.

A slower change in the basis results in a delay in the completion of the amortisation of surpluses, and the funding level stays higher for a longer time. There is a trade off between the stability of the contribution rate and the maintaining the adequate funding level. (Here, a surplus, similar to a deficit, may be considered undesirable.)

The characteristics described above may be emphasised if these projections are compared with A3.2.2.b, where the basis may be interpreted as one year average of the experience.

Even if the change is very slow, e.g. 125 year average, the surpluses are expected to be eliminated eventually. This feature is totally different from A3.2.2.a, where the basis is not changed at all and an equilibrium with a surplus is achieved.

-Triennial change

Although the valuation takes place every year, the change in the basis is only made triennially. The first change of the basis is made at the end of year 1. After three years, the change in basis is made by three times the annual rate of change. i.e. a change by .02% pa means a change by .06% every three years.

List of Projections

A3.2.2.k	Averaging, Triennial change, Quick(.02%pa)
A3.2.2.l	Averaging, Triennial change, Medium(.01%pa)
A3.2.2.m	Averaging, Triennial change, Slow(.004%pa)

The general tendency is the same as for the projections with annual changes.

During the three year period of unchanged bases, the gap between the experience and the assumptions may be wider, and a larger surplus may arise from this. However, there are no valuation surpluses arising, and as a total, the surplus arising is lower than the annual changes, and amortisation tends to progress.

Consequently, at the change of the basis, the impact of the accumulated surpluses is large. Also, there is a release of valuation surpluses which would have arisen every year in the case of the projections with annual changes. So, the change in the Recommended Contribution Rate tends to be larger than the projections with annual basis changes.

-Further change

The change in the valuation basis starts similar to A3.2.2.h, but the change goes on at the same pace even after the valuation basis reaches at the experience level.

List of Projections

A3.2.2.n	Change continue, Anuual change, Quick(.02%pa)

A3.2.2.n:

Up to year 25, this is the same as A3.2.2.h.

After year 26, owing to the continuing weakening of the basis, deficits from the gap between the experience and the assumptions arise because of the negative cashflows. Although valuation surpluses continue to arise, the deficits seem to be larger, and the level of deficits increases as the gap is gradually widened.

The accumulated surpluses are amortised by the reductions in the contribution rate and these newly accruing deficits. After that, deficits accumulate, and this requires additional contributions on top of the standard contributions.

At the end of year 100, the contribution rate is still increasing. The future course of the changes depends on the pattern of future accruals of deficits/surpluses, and the future course of events beyond 100 years may not be easily predicted.

Valuation surpluses would continue to accrue constantly. However, the deficits from the gap between the experience and the assumption would increase less than proportionately to the gap. This is because the deficits arising are also proportionate to the level of the negative cashflow, which is decreasing. The cashflow might even become positive if a sufficiently large deficit is accumulated. Should it be the case, surpluses might arise from the cashflow. [See section 8.3.1.2]

A3.2.3 Gradual change in experience

Here, the dividend growth experience is assumed to increase from 5.0% by .01% per annum. So, in year 1, the dividend growth rate is 5.01%, and in year 100, it is 6.0%.

A constant valuation basis and gradual changes in the bases are investigated.

A3.2.3.1 Constant basis

The standard basis is applied to see the impacts of the change in the experience without any moves in the basis.

List of Projections

A3.2.3.a Standard basis

A3.2.3.a:

The dividend growth rates experienced gradually increase, and surpluses arise because of the negative cashflows. The amount of surpluses arising is initially very small, but gradually increases as the level of negative cashflow increases and the gap becomes wider.

The surpluses accumulate, and the Recommended Contribution Rate falls accordingly.

Because the amount of surpluses arising is increasing, it always exceeds the reduction in contributions for amortisation which are based on the previous amount of accumulated surpluses. The two do not balance at any stage of the projection. So, there are no signs of arriving at an equilibrium.

Towards the end of the projection, a huge surplus has been accumulated. The contribution rate has not been stable.

A3.2.3.2 Gradual change in basis

In this section, the valuation bases are changed gradually as straight lines. Various combinations of the magnitude of the changes, the timings of the start of changes and the frequency of the changes in the basis are tried.

The magnitude of the changes are expressed as annual rates of increases in the dividend growth assumption.

If the valuation basis is set as an average of past experience, the pace of the changes in the basis will be the same as that in the experience after the year when the period from the start of the projection exceeds the averaging period. However, the initial pace of the basis changes is different from straight line changes in the basis. This is because the experience before the projections start is assumed to have been stationary. To see the initial differences, some projections using the bases which are the averages of the past experience are added.

-Immediate change

The changes in the valuation basis starts at the end of year 1. So, there are no delays. Three different magnitudes of changes are examined; one follows exactly the experience, others show less changes in the bases than the experience. Projections with higher paces of the changes in the valuation bases have been omitted because they are unlikely to be suitable.

List of Projections

A3.2.3.b	Immediate, Annual change, .01%pa[Same as experience]
A3.2.3.c	Immediate, Annual change, .004%pa
A3.2.3.d	Immediate, Annual change, .002%pa

A3.2.3.b:

Owing to the continuous weakening of the basis, a valuation surplus arises every year. Because the changes in the valuation basis is matched by the changes in the experience, there are no gaps, and this factor is not a source of surpluses/deficits. So, the level of surpluses arising every year is fairly constant because the changes are constant.

Initially, the reduction in the contributions for the amortisation of the surpluses is less than the surpluses accruing, and the surpluses accumulate. However, as the surpluses accumulate, the reduction in the contributions increases, and the pace of accumulation slows down.

Although the contribution rate continues to fall, the funding position may be said to be stable and sound because the reduction in the contribution rate is very small and constant.

A3.2.3.c:

Every year, owing to the weakening of the basis, a valuation surplus arises. The level of this surplus accruing is relatively small and stable from year to year. At the same time, the difference in the experience and the assumptions causes further surpluses. Because the pace of the changes in the valuation basis is less than that of the experience, the difference increases as the time passes. So, the surplus arising from this source gradually increases.

Because the surpluses arising are increasing, the reduction in the contribution for amortisation of already accumulated surpluses cannot match them. So, the surpluses accumulate continuously, and more reductions in the contribution rates are required.

The Recommended Contribution Rate is more stable than for A3.2.3.b initially, but changes more rapidly afterwards.

A3.2.3.d:

Similar results as A3.2.3.c are seen, but all the features emerge more significantly. So, the result is between A3.2.3.c and A3.2.3.a.

Summary

A3.2.3.b keeps the amortisation element of the contribution rate small and stable. So, the results may not be affected much by the choice of the amortisation method.

A moderate change in the basis compared with the changes in the experience makes the contribution rate more stable initially. However, if the changes in the experience persist, a large surplus/deficit may be accumulated, and the contribution rates would be unstable in the long run. In the long run, the changes in the basis would need to be large to achieve stability.

Also,the contribution rate changes are heavily dependent on the amortisation method.

-Delayed change

The changes in the valuation basis start at the end of year 3. So, there are two years' delays compared with the projections with immediate changes. Three different magnitudes of the changes in the bases are examined.

The initial changes are made only for one year's portion of the changes. There are no extra changes in the bases reflecting the delay. So, even in A3.2.3.e, the valuation basis is always two years behind the experience.

List of Projections

A3.2.3.e	From year 3, Annual change, .01%pa
A3.2.3.f	From year 3, Annual change, .004%pa

A3.2.3.e:

The Standard Contribution Rate falls gradually after year 3 because of the change in the valuation basis.

Owing to the delay in the start of the change in the valuation basis, there exists a gap between the experience and the assumptions. This is a source of surpluses every year. The gap is constant because the pace of the change in the valuation basis is equal to that of the experience. The surpluses accruing from this source become relatively stable after the accumulation of surpluses slows down.

Also, owing to the change in the valuation basis, a valuation surplus arises every year. Because the change of the basis is steady, the surplus arising from this source is constant over the years.

So, the surpluses arising each year is fairly constant, but because of the wider gap between the experience and the assumptions, the level of surpluses arising is higher than for A3.2.3.b.

Consequently, the eventual level of accumulated surpluses is higher, with each year's accruing surpluses and the reduction in the contribution for amortisation cancelling each other out to maintain stability.

A3.2.3.f:

Surpluses arise every year owing to the gap between the experience and the assumptions and the weakening of the valuation basis. The former increases because the changes in the assumptions are less than the changes in the experience. The amount of this part of surplus is larger than that in A3.2.3.c because of the delay in the basis change. The valuation surpluses arise constantly.

Because the surpluses arising are increasing, the surpluses accumulate without any signs of convergence, and more reductions in contribution rates for amortisation are required. This increases the level of negative cashflows, and accelerates the accrual of surpluses arising from the gap between the experience and the assumptions.

The level of accumulated surplus is higher than for A3.2.3.c owing to the delay, and the contribution level is lower.

Summary

The effect of delay is a higher level of accumulated surplus.

However, two years' delay may not be significant. The basic shape of the contribution rate changes is not affected. So, it may be justifiable to delay the basis changes until the tendency in the experience becomes clearer.

-Triennial changes

The valuations take place every year, but the changes in the valuation basis are made only every three years. The first change is made at the end of year 1.

The degree of change each time is three years' portion of the changes. So, if the increase in the dividend growth assumption is .01% p.a., .03% is added every three years.

List of Projections

A3.2.3.g	Immediate, Triennial change, .01%pa
A3.2.3.h	Immediate, Triennial change, .004%pa

A3.2.3.g:

The general shape of the contribution rates changes are similar to A3.2.3.b.

As the basis changes, a large valuation surplus is released, while there are no such surpluses emerging during the interval between the times when the basis changes.

The surpluses arising from the gap between the experience and the assumptions are increased during the period between the times when the basis changes because the basis is the same while the experience changes and the gap is widened.

Because of these factors, the fluctuations in the Recommended Contribution Rates are higher than for A3.2.3.b.

A3.2.3.h

The general shape of the contribution rate changes is similar to A3.2.3.c.

Similar comments as A3.2.3.g apply, and the fluctuations in the contribution rate are higher than for A3.2.3.c.

Summary

Triennial changes in the basis do not affect the basic pattern of the projection results. However, the fluctuations in the contribution rates from year to year are higher.

-Decennial changes

Similar to triennial changes, the valuations take place every year, but the changes in the valuation basis are made only every ten years. The first change is made at the end of year 1.

The degree of change each time is ten years' portion of the changes. So, if the increase in the dividend growth assumption is .01% p.a., .1% is added *every ten years.*

List of Projections

A3.2.3.i	Immediate, Decennial change, .01%pa
A3.2.3.j	Immediate, Decennial change, .004%pa

The long term pattern of the contribution rates is similar to the cases of annual changes and triennial changes.

In A3.2.3.i, amortisations seem to progress during the period between the times when the basis changes. During this period, no valuation surpluses arise. Only the gap between the experience and the assumptions creates surpluses. This latter element of surplus is larger than for A3.2.3.b, especially closer to the next basis change. However, the lack of valuation surpluses helps the amortisation to progress.

At the change of the basis, a large valuation surplus is released at a time, and the temporary effect of the progress in the amortisation disappears. The accumulated surpluses then come back to a similar level as at the previous basis change.

Such a temporary progression of the amortisation is not clearly seen in A3.2.3.j. This is because the valuation surpluses are not significant and the surpluses arising from the gap between the experience and the assumption are larger than for A3.2.3.i. This is a result of slower changes in the basis than for the actual experience.

-Averaging

The valuation basis is set as the average of the past experience. As the averaging period, 5 years and 10 years are examined.

List of Projections

A3.2.3.k	Immediate, Annual change, 5 year average
A3.2.3.l	Immediate, Annual change, 10 year average

A3.2.3.k:

Except for the first 4 valuations, the valuation bases used are exactly the same as those under A3.2.3.e, the case with 2 years' delay.

The differences in the initial period are not large, and they do not give any significant impacts.

The contribution rate changes are almost identical to A3.2.3.e. The differences in the Recommended Contribution Rates are within .01% throughout the projections.

A3.2.3.l:

After 10 years, the pace of the changes in the valuation basis is the same as that of the experience. But the basis never catches up with the experience.

Similarly to A3.2.3.e, there are two sources of surpluses arising every year: the valuation surpluses and the gap between the experience and the assumptions.

The first element depends on the magnitude of the changes in the basis, and the level of this part of the surpluses is the same as for A3.2.3.b or A3.2.3.e. This part of the surplus arising is fairly constant over the period of the projection.

10 years' averaging is equivalent to 4.5 years' delay in the start of the change in the basis except for the initial 10 years. So, the gap between the experience and the assumptions is wider than for A3.2.3.e, and the surplus is larger. However, because the pace of the changes in the valuation basis becomes the same as that of the experience, the gap remains constant. This part of the surplus arising slightly increases as the recommended contribution rate falls (and the fund increases), but is fairly constant because the funding level rises only very slowly.

Consequently, the level of accumulated surpluses reaches an equilibrium where the accruing surpluses and the reduction in contribution rates balance each other.

Although the level of surplus is higher than for A3.2.3.b, the basic pattern of the results are the same as A3.2.3.b.

Summary

Averaging has a similar effect as delaying the change of the basis, and averaging may not have a significant impact on the finances of the pension fund compared with the use of a non-averaged basis.

A3.2.4 Cyclical change in experience

Here, the dividend growth experience, g', is assumed to follow a twenty year cycle. Namely, g' is increased at .1% per annum until g' reaches 5.5% in the fifth year. Then, g' is reduced by .1% per annum for ten years to reach 4.5%. After that, g' is again increased at .1% per annum to return to 5.0% in the twentieth year. During the 100 year projection, thus, five cycles are made. Constant valuation bases and cyclical changes in the bases are applied.

A3.2.4.1 Constant basis

Three different constant bases are tried.

List of Projections

A3.2.4.a	g=5.0% [Standard basis]
A3.2.4.b	g=5.5% [Highest value]
A3.2.4.c	g=4.5% [Lowest value]

A3.2.4.a:

According to the cyclical changes in the experience, surpluses arise when the actual dividend growth is above 5.0%, and deficits arise when it is below because the cashflow is negative. The surpluses/deficits arising are almost cyclical, but they are also affected by the level of negative cashflow.

At the beginning of year 1, there are no accumulated surpluses/deficits.

For the first five years, surpluses arise from the negative cashflows, and the level of these accruing surpluses increases. The reduction in the contribution rate for amortisation is less than the accruing surpluses, and the surpluses accumulate. Consequently, the contribution rate reduces.

For the second five years, still surpluses arise, but the amount arising falls from year to year. The reduction in the contribution rate for amortisation is still less than the accruing surpluses initially, and the surpluses still accumulate in year 6. So, the contribution rate continues to fall until year 6. However, the reduction of the contributions for amortisation exceeds the accrual of the surpluses eventually because the former increases while the latter decreases. From year 7, the contribution rate is increased reflecting the progress of the amortisation of surpluses. At the end of year 10, there is a surplus outstanding.

For the next five years, deficits arise, the amount of which gradually increases. The deficits are used to remove the outstanding surpluses together with the reduction in the contributions for amortisation. By the end of year 14, the amortisation of the surpluses is completed. At the end of year 15, there is a deficit.

For the last quarter of the cycle, deficits arise, the amount of which reduces gradually to 0. During year 16, the deficits still accumulate, and the contribution rate is increased. However, from year 17 onwards, the accrual of the deficit is exceeded by the additional contributions for amortisation, and the accumulated deficit is slightly reduced.

At the end of the first cycle, thus, there is a deficit carried forward.

The accrual pattern of the surpluses and deficits is the same for every consecutive cycle as the first one. However, because of the accumulated deficit at the beginning of the cycle, the pattern of the accumulation of the surpluses and deficits becomes different.

Owing to the initial deficit, the accumulation of surpluses in the first half of the second cycle is smaller. So, the size of the reduction in the contribution for amortisation is relatively smaller than for the first cycle, while the accruing surpluses are almost the same.

So, the second half of the second cycle starts with a smaller accumulated surplus, and the level of accumulation of the deficits becomes higher than for the first cycle.

At the end of the second cycle, there is a higher level of accumulated deficits.

As the cycles go on, the level of the deficit at the beginning of each cycle becomes higher, but the degree of the increase is reduced each time. So, the pattern of the contribution rate changes, which represents the pattern of the funding level, seems to become cyclical and stable.

In the fifth cycle, the highest contribution rate is 11.0229% (year 96), and the lowest is 10.6739% (year 86). The difference is 0.3490%. The differences from the Standard Contribution Rate, 10.8578%, are similar in both directions.

A3.2.4.b:

The dividend growth assumed in the valuation basis is always higher than or equal to the experience. So, except in years 5,25,45,..., where they are equal, deficits arise because of this gap every year. According to the cyclical pattern of the experience, the level of deficits arising changes almost cyclically although they are also affected by the level of cashflow, and consequently, by the size of the fund. The level of the deficit arising starts from a non-zero level, and reduces to 0 by year 5. Then, the level increases steadily until year 15. From year 16 to year 20, the level reduces to the initial level.

Initially, there is a valuation surplus because the initial amount of the assets is equal to the actuarial liability under the standard basis. So, the contribution rates are reduced to amortise this initial surplus. The pace of amortisation is quick helped by the accruing deficit every year. By the end of year 17, the surpluses disappear. During the rest of the first cycle, the deficits accumulate, and the contribution rate increases.

The second cycle starts with a deficit. The deficits continue to accumulate and the addition to the contribution rate for amortisation increases. However, the level of accruing deficits are reducing, and in year 22, the accrued deficit becomes less than the addition in the contribution, and the accumulated deficit reduces. The reduction in the accumulated deficits continues while the accumulated deficit level is low until year 25. Then, the gradually increased deficit accruing exceeds the addition in the contribution, and the accumulation of the deficits starts again. The accumulation continues until the end of the second cycle. The second cycle ends with a higher deficit than at the beginning of the cycle.

A similar pattern is repeated for each coming cycle. The timings of the peak and the trough of the accumulated deficits are slightly affected by the increased level of the initial deficit, and the peak is in the 15th year and the trough in the 5th year.

The levels of the deficit at the beginning of the fourth and the fifth cycle are slightly increased. The deficit at the beginning of each cycle seems to settle down to a stable level after the fifth cycle.

In the fifth cycle, the contribution rates are between 11.4028% (year 85) and 11.8010% (year 95). The contribution level is, thus, higher than for A3.2.4.a. and the funding level is lower. The difference in the contribution rates is 0.3982%, which is slightly wider than for A3.2.4.a.

A3.2.4.c:

This is a reverse of A3.2.4.b.

The dividend assumed in the valuation basis is always lower than or equal to the experience. So, except in year 15,35,55,..., where they are equal, surpluses arise each year because of this gap. According to the cyclical pattern of the experience, the level of surpluses arising changes almost cyclically although this is also affected by the level of cashflow, which reflects the funding level. The level of the surplus arising starts from a non-zero level, and increases to the maximum level in year 5. Then, the level reduces steadily to 0 in year 15. From year 16 to year 20, the level increases back to the initial level.

There is an initial valuation deficit, and the Recommended Contribution Rate is higher than the Standard Contribution Rate.

After amortising the initial deficits, surpluses are accumulated. After several cycles, the contribution rate pattern seems to become cyclical and stable.

The contribution rates in the fifth cycle are between 9.8688% (year 87) and 10.1850% (year 97). The general level is lower than for A3.2.4.a, and the funding level is higher. The difference in the highest and the lowest contribution rates is 0.3162%. The gap is the narrowest of the three projections, although they are very close.

Summary

The average funding level and the contribution rate show similar tendencies as the projections with unchanged experience. The use of a strong basis results in a permanent surplus.

Reflecting the cyclical changes in the experience, the funding level also shows a cycle with the same length. The peaks and troughs of the actual experience differ slightly from those of the funding level. This is because the latter represents the accumulated effects of experience and the progression in the amortisation.

A3.2.4.2 Cyclical change in basis

In this section, the valuation bases are changed cyclically with the same length of cycles, i.e. 20 year cycles. Various combinations of the magnitude and the frequency of the changes in the basis are tried.

The magnitude of the changes are expressed as annual rates of changes in the dividend growth assumption.

Also, the effects of averaging are analysed by applying three different averaging periods.

-Immediate changes

The changes in the valuation basis starts from the standard basis at the beginning of the projections, and follows the same direction as in the experience.

List of Projections

A3.2.4.d Immediate, Annual change, .1%pa[Same as experience]

A3.2.4.e Immediate, Annual change, .05%pa

A3.2.4.d:

In this projection, the valuation basis is exactly the same as the experience in last 12 months.

The projection starts with no surpluses/deficiencies.

Although there is no gap between the experience and the valuation basis, every year, a valuation surplus/deficit arises owing to the changes in the valuation basis.

In the first five years, the valuation basis continues to be weakened, and valuation surpluses arise. The level of each year's accrual is relatively constant because the changes in the valuation basis are constant, but increases as the funding level increases. The effect of the amortisation is relatively small compared with the accruing surpluses. So, the surpluses accumulate, and the contribution rate falls rapidly.

In each of the next ten years, valuation deficits arise.

First, the accumulated surpluses are amortised quickly, and then deficits accumulate. The accumulated deficit is at its peak in year 15. This peak is slightly smaller than the peak in the accumulated surplus in year 5. The Recommended Contribution Rate rises.

After year 16, valuation surpluses constantly accrue at a similar level as in the first five years, and the accumulated deficits are quickly amortised by the end of year 20. At the end of year 20, there is a small surplus outstanding. The Recommended Contribution Rate falls.

Because the surplus at the beginning of the second cycle is small, a similar pattern is repeated, but the funding level is in general slightly higher.

This gradual rise in the funding level continues, but seems to become stable and cyclical. In the fifth cycle, the lowest Recommended Contribution Rate is 8.2794% in year 85, and the highest is 12.8592% in year 95. The difference is 4.5798%. This is much wider than projections with constant bases.

Consequently, the funding level is more volatile than for A3.2.4.a, although the average level is similar. The impact of a 1% change in the dividend growth assumption on the funding level and the contribution rates is much larger than the impact of a 1% gap between the experience and the assumptions. This may be the reason for this wider variation in the contribution rate.

Also, the surpluses/deficits arisen from the previous changes in the basis is cancelled when the basis comes back to the level before the previous changes in the basis. So, the changes in the contribution rate match without delay the changes in the experience.

A3.2.4.e:

In this projection, the changes in the valuation basis are less than the changes in experience. Consequently, the dividend growth assumed in the valuation basis is lower than the experience in the first half of each cycle, and is higher than the experience in the second half of each cycle.

The projection starts with no surpluses/deficiencies.

In the first five years, the valuation basis continue to be weakened, and valuation surpluses arise. The level of each year's accrual is relatively constant because the changes in the valuation basis are constant, but increases as the funding level increases. The level is less than for A3.2.4.d reflecting the smaller changes in the basis. At the same time, surpluses due to negative cashflows arise from the gap between the experience and the valuation basis. This part of the surplus increases towards year 5. The total accrual of surpluses is less than for A3.2.4.d on average. The surpluses accumulate, and the degree of the reduction in the contribution rate increases. The Recommended Contribution Rate shows a fall. The degree of fall is less than for A3.2.4.d.

In each of the next five years, valuation deficits arise, while the gap between the experience and the basis produces surpluses due to negative cashflows. The surpluses arising gradually reduce to 0 towards year 10 reflecting the narrowing gap. The surpluses accruing seem to be less significant than the valuation deficits, and the surpluses are amortised quickly. The Recommended Contribution Rate rises.

In the third quarter of the cycle, both valuation deficits and deficits from the gap between the experience and the basis arise at the same time. The latter increases gradually. However, the impact of the latter seems to be small, and the Recommended Contribution Rate increases at a similar pace as during the second quarter. Towards the end of year 15, deficits are accumulated to the highest level.

After year 16, valuation surpluses constantly accrue at the same level as the first five years, while the gap between the experience and the basis creates deficits, which are reducing.

Again, the latter is insignificant, and the accruing surpluses help with the amortising of the deficits. At the end of the first cycle, there is a small accumulated surplus. The Recommended Contribution Rate goes down.

Because the surplus at the beginning of the second cycle is small, a similar pattern is repeated. However, the funding level is in general slightly higher.

This gradual rise in the funding level continues, but seems to become stable and cyclical. In the fifth cycle, the lowest Recommended Contribution Rate is 9.5282% in year 85, and the highest is 11.9655% in year 95. The difference is 2.4373%. This is much less than for A3.2.4.d, but much wider than the projections with constant bases.

Consequently, the funding level is less volatile than for A3.2.4.d, but more than for A3.2.4.a, although the average level is similar.

Summary

The changes in the basis according to the cyclical change in the experience make the contribution rate more volatile.

In these projections, the effects of the changes in the valuation basis are more significant than the impact of the gap between the experience and the basis. So, by using a fixed basis as for A3.2.4.a, the first factor is eliminated, and the volatility in the contribution rate is reduced.

-Triennial changes

Although valuations are made annually, the basis is changed only every three years. The experience cycle is 20 years and the basis change is every 3 years. So, in the long run, all the combinations of experience and the basis evenly appear.

List of Projections

A3.2.4.f	Immediate, Triennial change, .1%pa
A3.2.4.g	Immediate, Triennial change, .05%pa

On each change of the basis, the moves in the contribution rate and the funding level are very large. In other years, the moves are more moderate. So, the year to year movements in the Recommended Contribution Rate and the funding level are less smooth than for A3.2.4.d or A3.2.4.e.

However, the general pattern, the average and the difference between the highest and the lowest of the contribution rates are similar to A3.2.4.d and A3.2.4.e respectively.

-Quinquennial changes

Here, the changes in the basis take place every five years. This means that the same valuation basis is used for each quarter of the cycle; a basis close to the standard basis during the first and the third quarter, a weak basis for the second quarter, and a strong basis for the fourth quarter.

List of Projections

A3.2.4.h	Immediate, Quinquennial change, .1%pa
A3.2.4.i	Immediate, Quinquennial change, .05%pa

The year to year variations in the Recommended Contribution Rate is less smooth than for the case of triennial changes. Also, owing to the use of the same valuation basis for every cycle, the contribution rate pattern is more clearly cyclical.

However, the general pattern, the average and the difference between the highest and the lowest of the contribution rates are similar to A3.2.4.d and A3.2.4.e.

So, the frequency of the basis changes may not be significant as long as the changes take place within a relatively short period after the last basis change.

-Averaging

The valuation basis is set as the average of past experience. As the averaging period, 3, 5 and 10 years are used. A3.2.4.d can then be regarded as a projection with one year's averaging period.

The effects of averaging on the setting of the valuation basis are the moderation of the changes in the basis and the delay in the implementation of these changes. The first effect is observed from the narrower range of the valuation basis compared to A3.2.4.d. The second effect appears as gaps between the phases of the experience cycle and the basis cycle.

List of Projections

A3.2.4.j	Immediate, Annual change, 3 year average
A3.2.4.k	Immediate, Annual change, 5 year average
A3.2.4.l	Immediate, Annual change, 10 year average

A3.2.4.j:

The range of the dividend growth assumption is 4.5667% to 5.4333%. This is slightly narrower than that of the experience, 4.5% to 5.5%. The lowest dividend growth assumption is used in year 16, and the highest in year 6. So, there is one year's delay in the phase of cycle.

As mentioned for projections A3.2.4.d and A3.2.4.e, the effects of the valuation changes are dominant, and the effects of the gap between the experience and the basis are less significant. So, the changes in the Recommended Contribution Rates follow the similar pattern as for A3.2.4.d with one year's delay and reduced movement.

A3.2.4.k:

The range of the dividend growth assumption is 4.62% to 5.38%. This is slightly narrower than that of A3.2.4.j. The lowest dividend growth assumption is used in year 17, and the highest in year 7. So, there is two years' delay in the phase of cycle compared with the experience cycle.

Similar comments to A3.2.4.j apply.

A3.2.4.l:

The range of the dividend growth assumption is 4.75% to 5.25%. This is slightly narrower than that of A3.2.4.k. This range is the same as A3.2.4.e. The lowest dividend growth assumption is used in year 19 and 20, and the highest in year 9 and 10. So, there is 4.5 years' delay in the phase of cycle compared with the experience cycle.

Again, similar comments as for A3.2.4.j apply.

Although the valuation basis's cycle is almost a quarter behind that of the experience, only the effects of the valuation surpluses/deficits are significant, and the pattern of the widening and narrowing of the gap between the experience and the valuation basis seems less important.

The range of the dividend growth rate assumption used is the same as for A3.2.4.e. In the case of A3.2.4.e, the pace of the change in the assumption is constant, and the values in the range are evenly used as the dividend growth rate assumption. However, in A3.2.4.l, the dividend growth rate assumption changes more quickly when it is in the centre of the range and it changes more slowly when it is near the boundary of the range. Thus, in A3.2.4.l, more values near the boundary are used for the assumption.

Consequently, the variations in the contribution rates are more significant in A3.2.4.l than for A3.2.4.e.

Summary

The effect of the changes in the basis is dominant on the pattern of the changes in the contribution rate. The main effects of using a longer averaging period are a reduction in the volatility of the contribution rates and a delay in the phase of the Recommended Contribution Rate cycle compared with the experience cycle.

Note

Although the following combinations have also been examined, they are omitted from the results because they are considered to be of lesser importance for the main discussion.

-Gradual changes in experience

A3.2.3.m	From year 3, Annual change, .002%pa
A3.2.3.n	Immediate, Triennial change, .002%pa
A3.2.3.o	Immediate, Decennial change, .002%pa

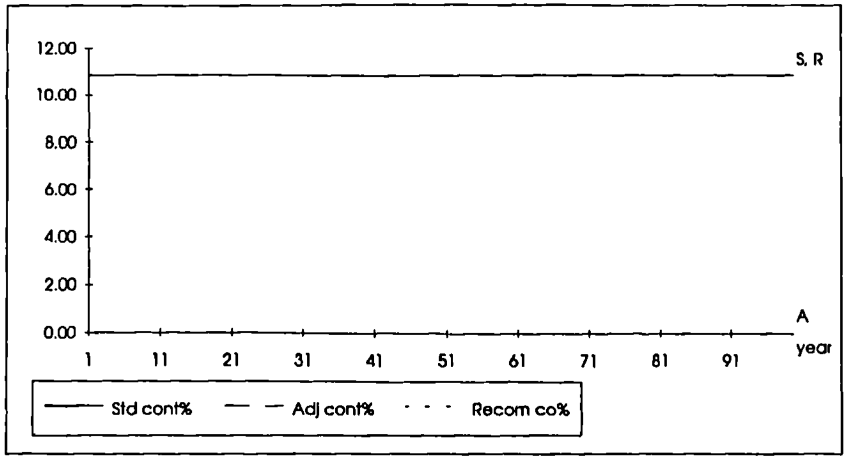
A3.2.3.p	Immediate, Triennial change, 5 year average
A3.2.3.q	Immediate, Triennial change, 10 year average
A3.2.3.r	Immediate, Decennial change, 5 year average
A3.2.3.s	Immediate, Decennial change, 10 year average

-Cyclical changes in experience

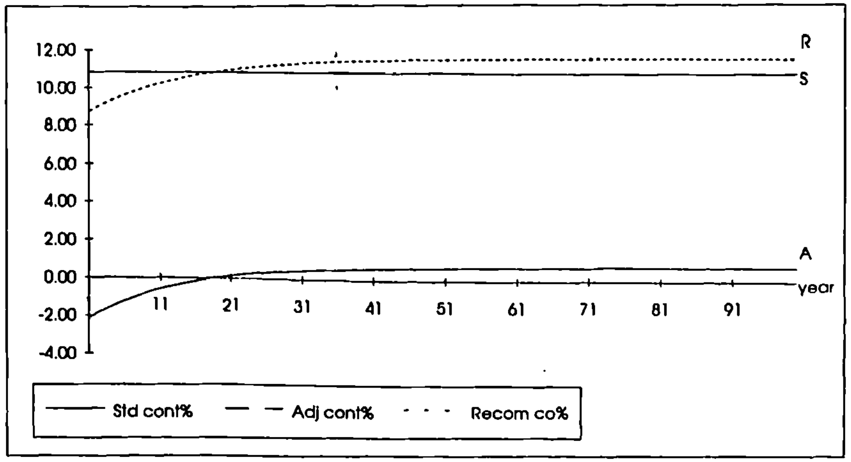
A3.2.4.m	From year 2, Triennial change, .1%pa
A3.2.4.n	From year 3, Triennial change, .1%pa
A3.2.4.o	Immediate, Triennial change, 3 year average
A3.2.4.p	Immediate, Triennial change, 5 year average
A3.2.4.q	Immediate, Triennial change, 10 year average
A3.2.4.r	Immediate, Quinquennial change, 3 year average
A3.2.4.s	Immediate, Quinquennial change, 5 year average
A3.2.4.t	Immediate, Quinquennial change, 10 year average

Also, all the corresponding combinations with a 10 year cycle.

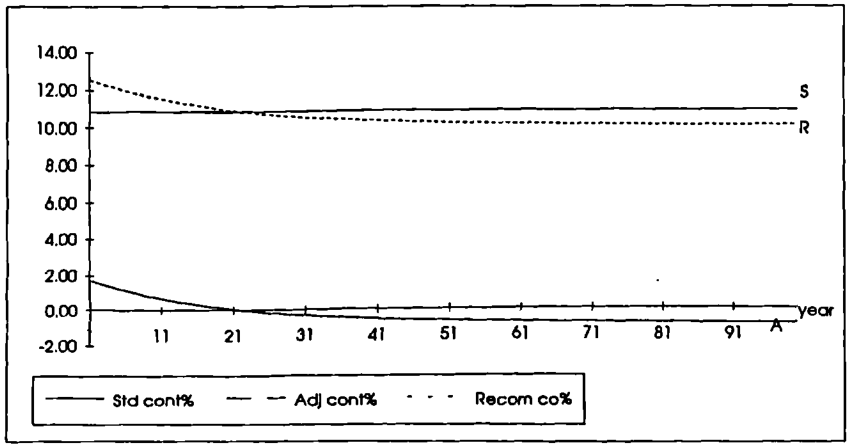
Graph A3.2.1.a



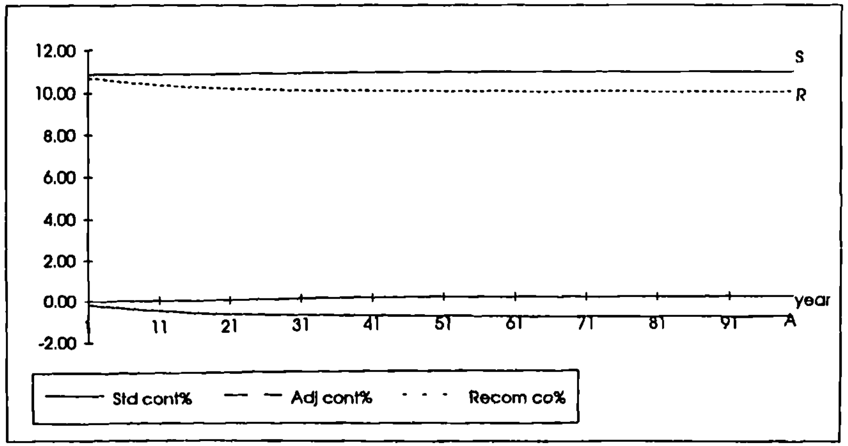
Graph A3.2.1.b



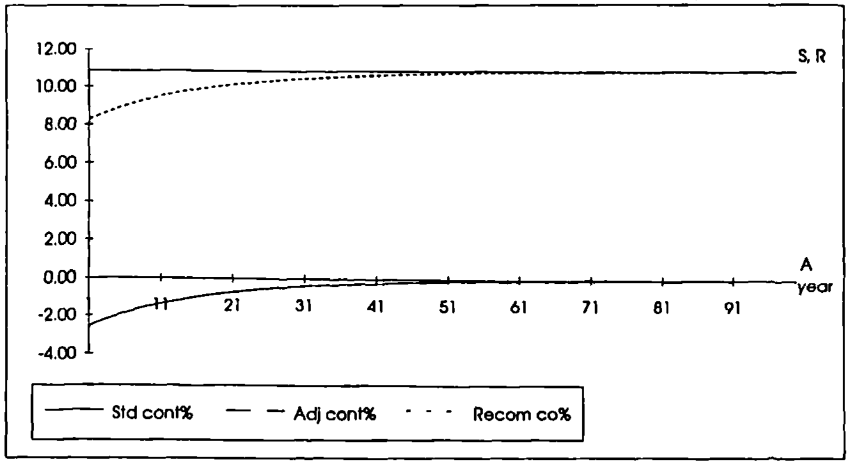
Graph A3.2.1.c



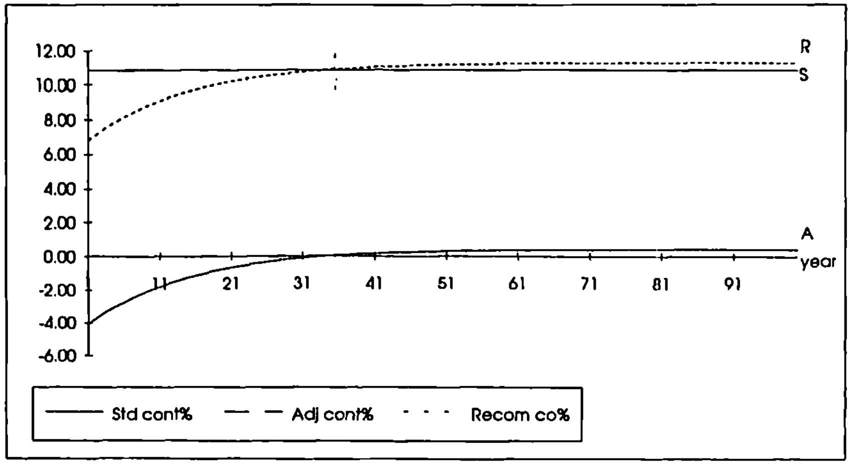
Graph A3.2.2.a



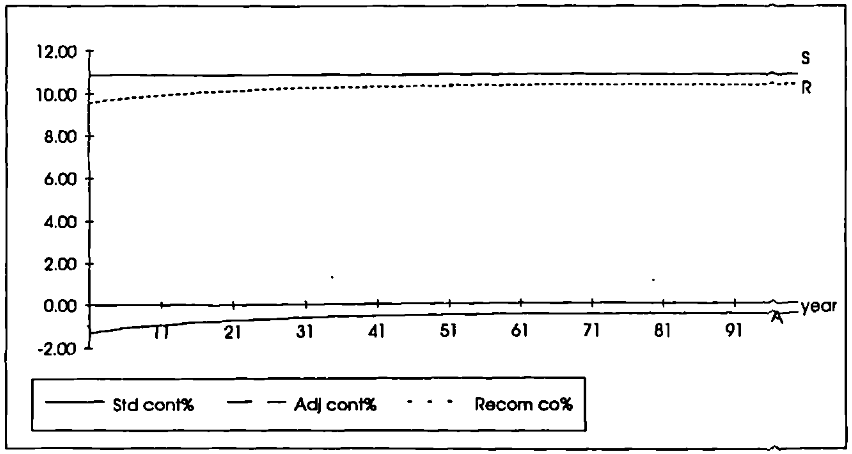
Graph A3.2.2.b



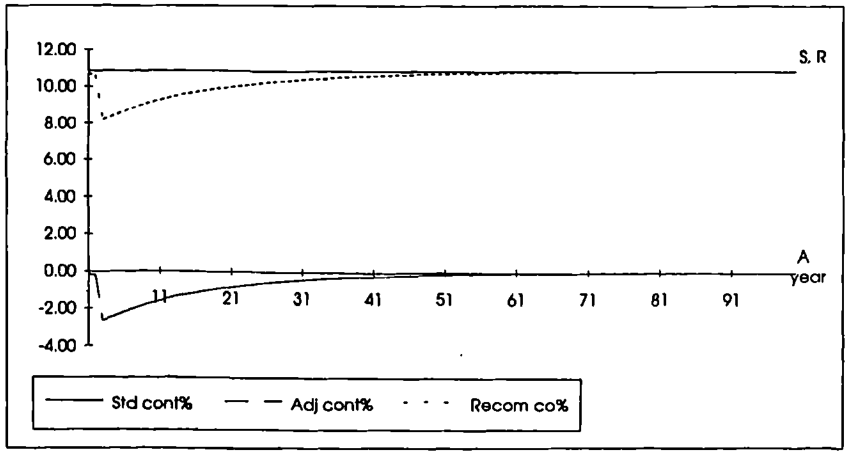
Graph A3.2.2.c



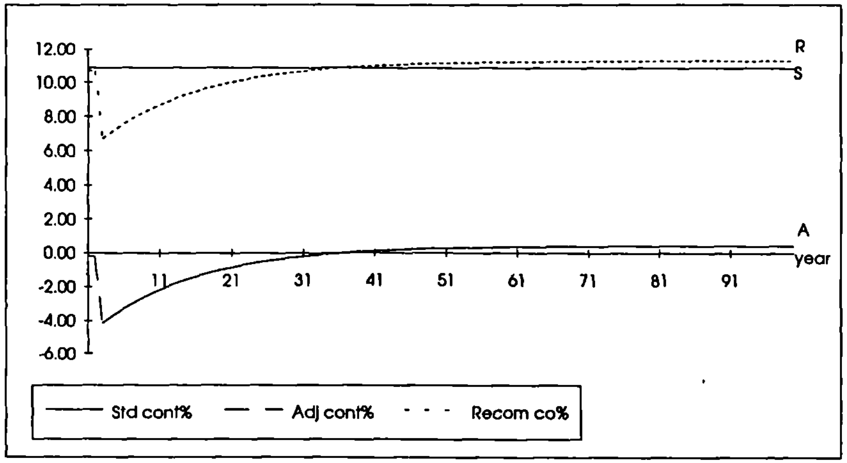
Graph A3.2.2.d



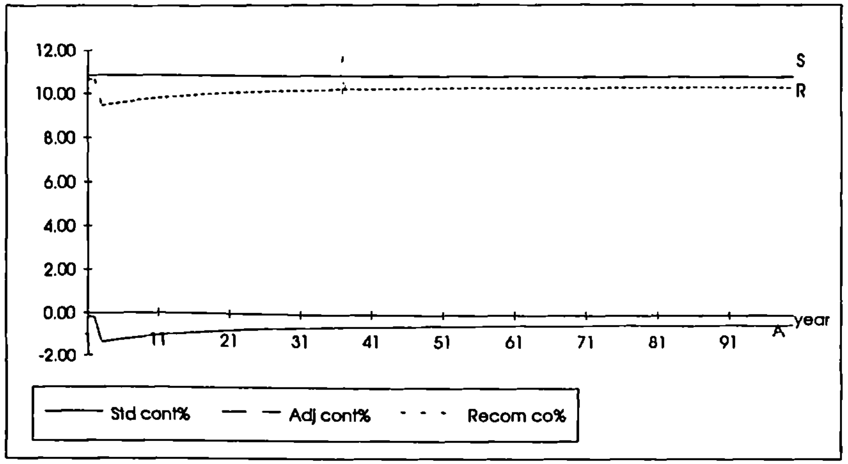
Graph A3.2.2.e



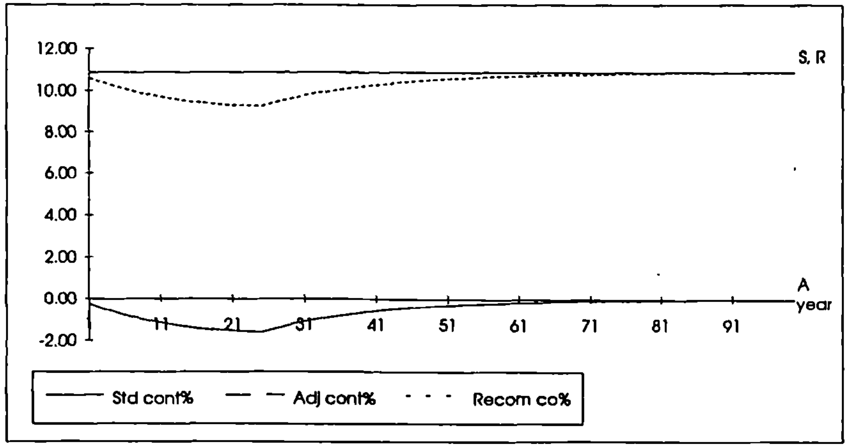
Graph A3.2.2.f



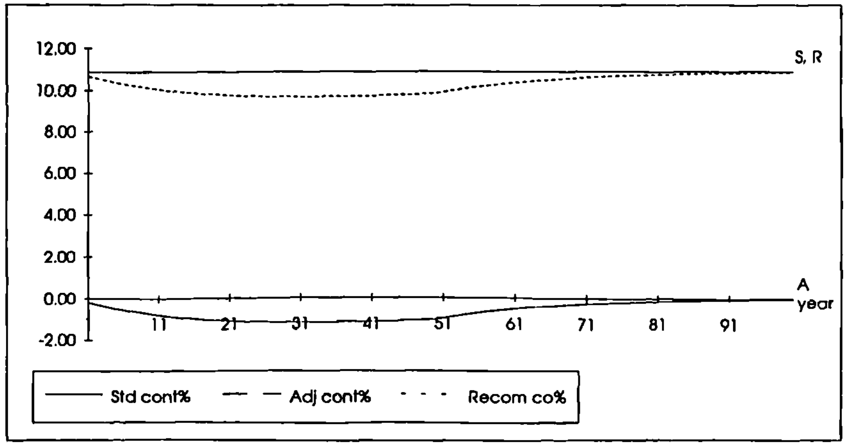
Graph A3.2.2.g



Graph A3.2.2.h



Graph A3.2.2.i



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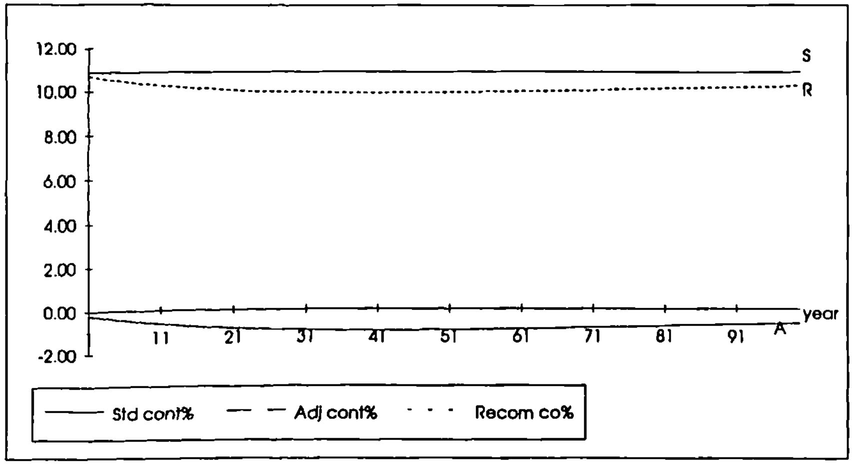

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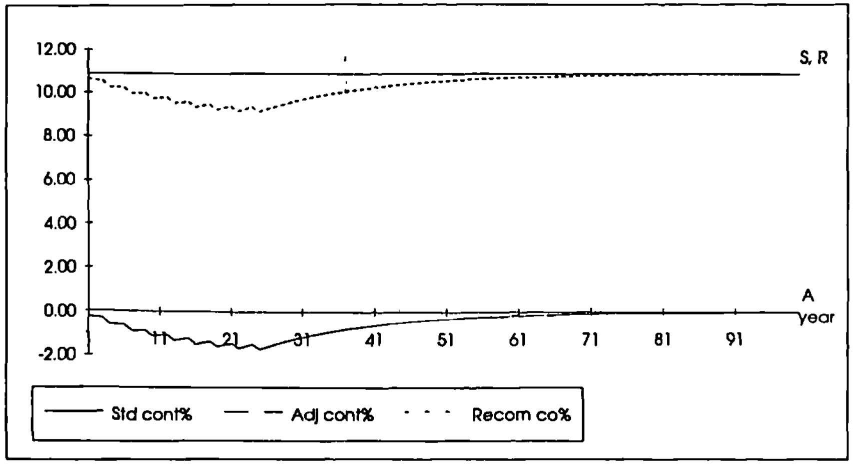

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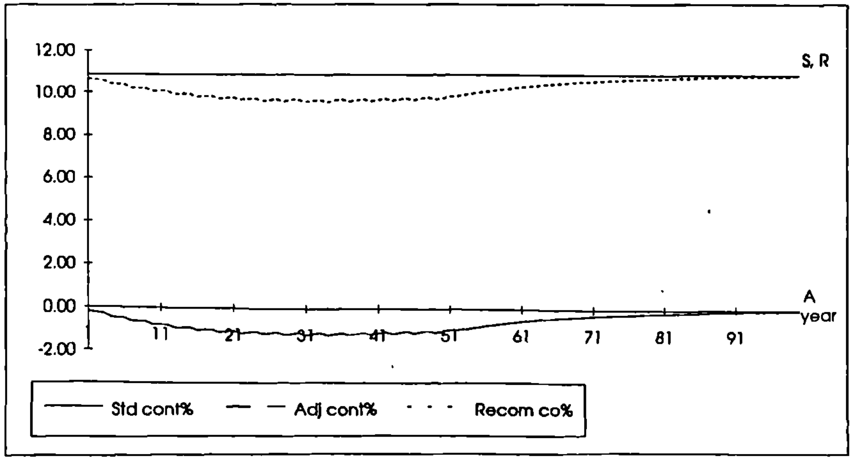

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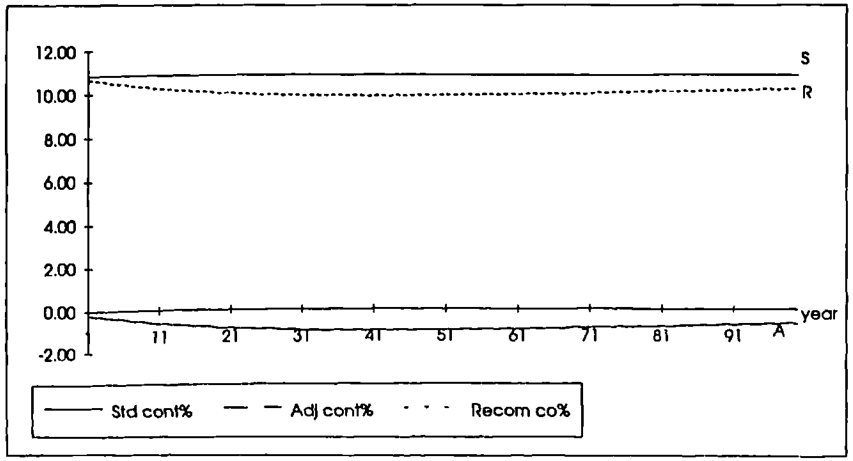
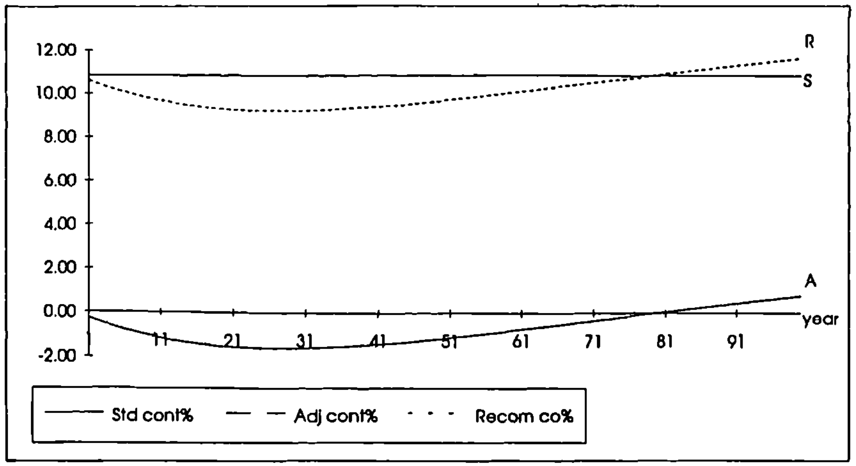
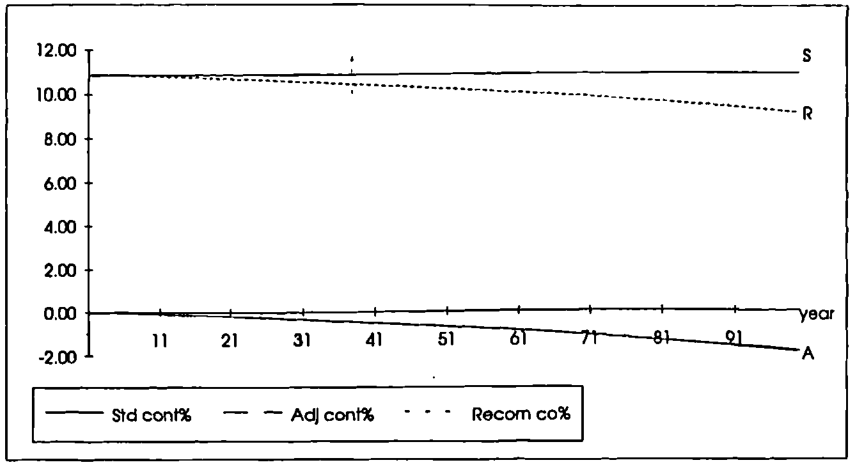


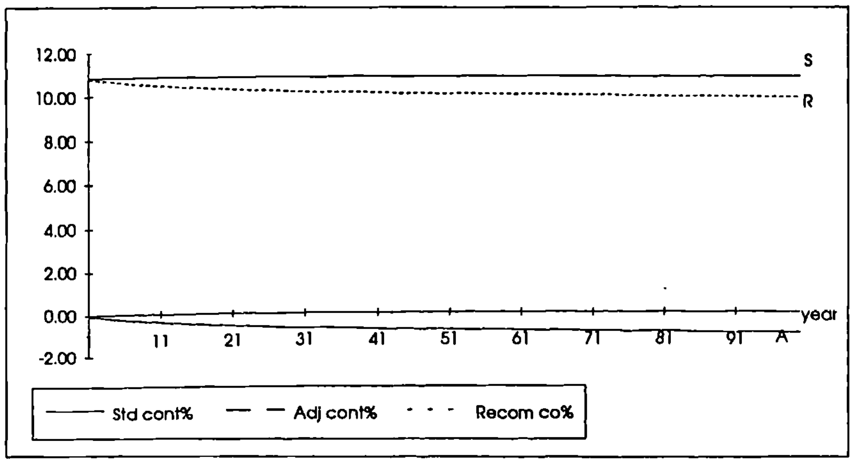
Graph A3.2.2.n



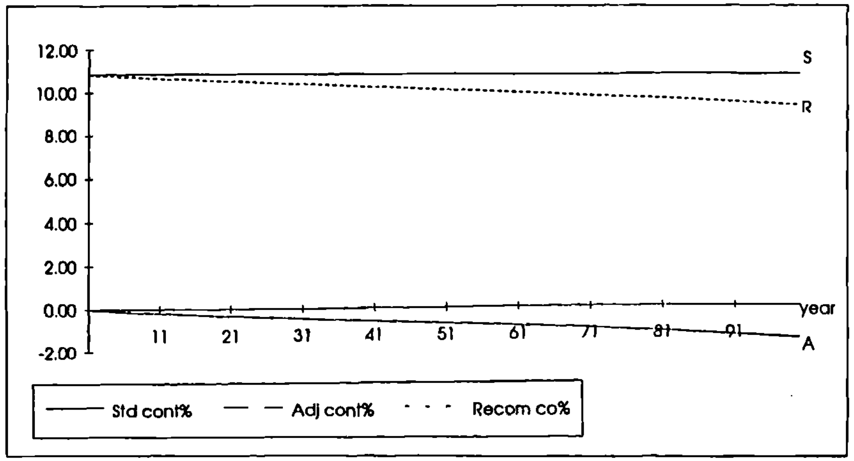
Graph A3.2.3.a



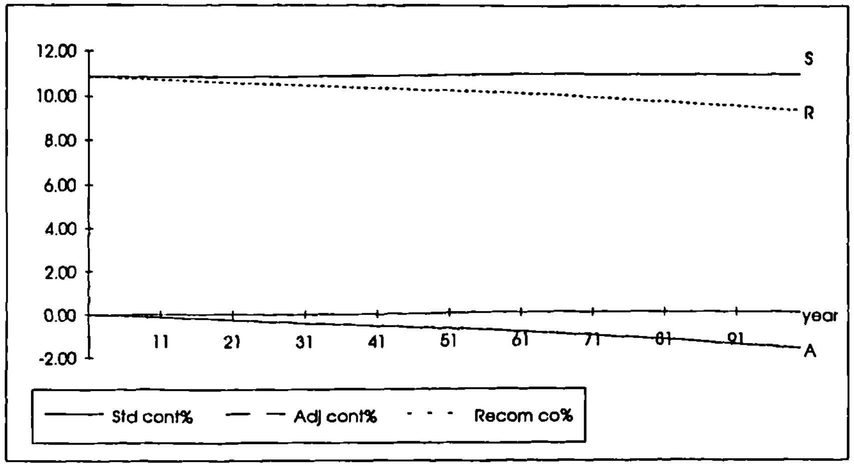
Graph A3.2.3.b



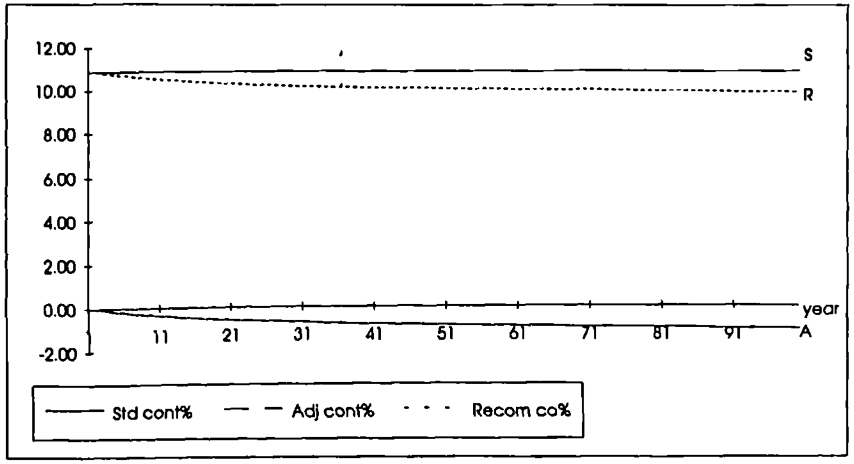
Graph A3.2.3.c



Graph A3.2.3.d



Graph A3.2.3.e



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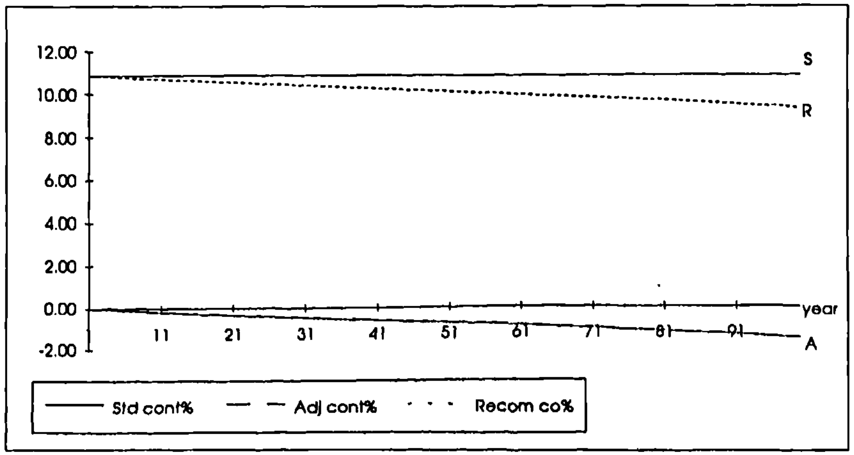
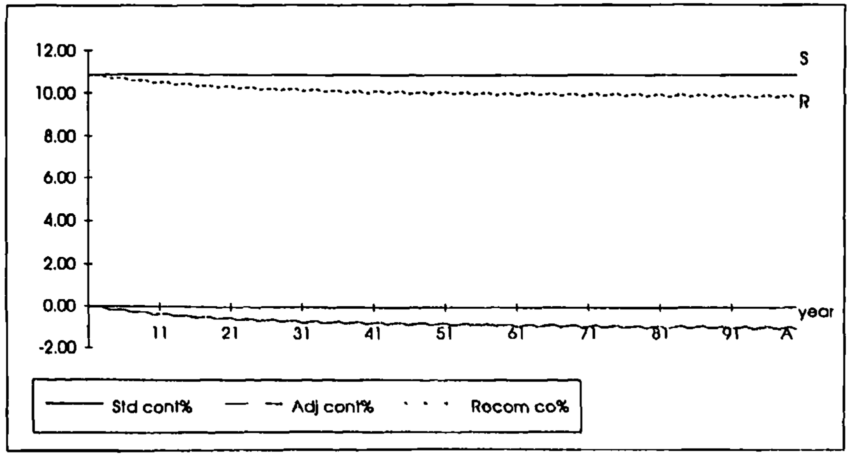
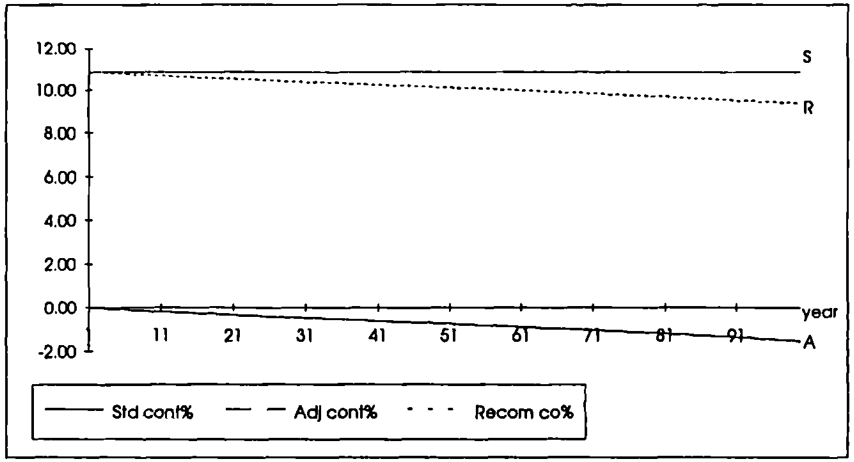


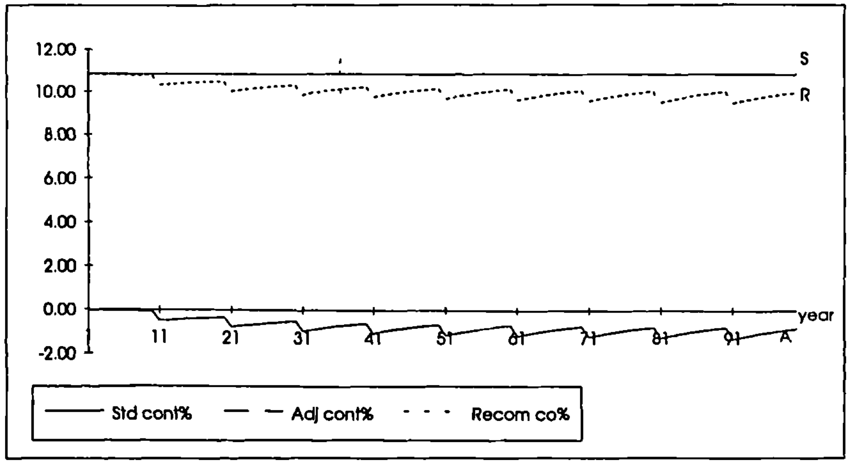
Graph A3.2.3.g



Graph A3.2.3.h



Graph A3.2.3.i



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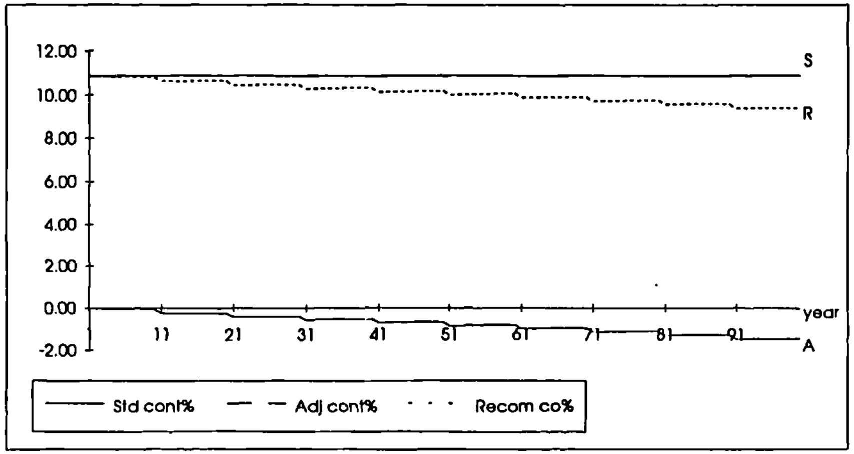

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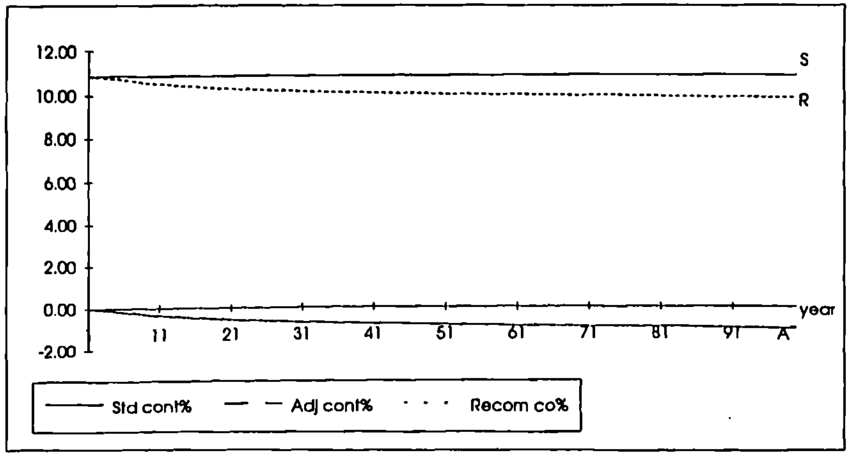
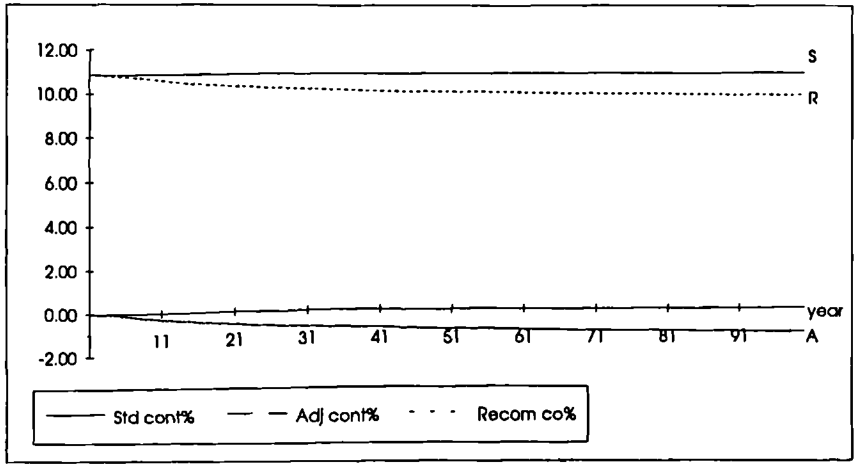
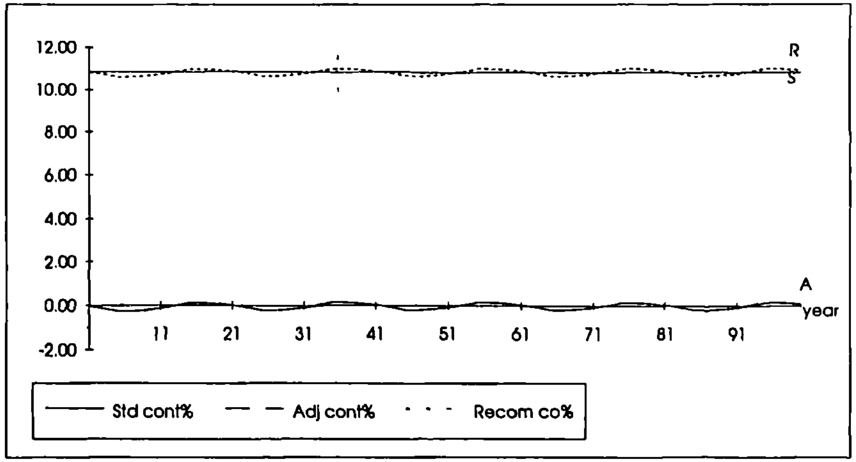


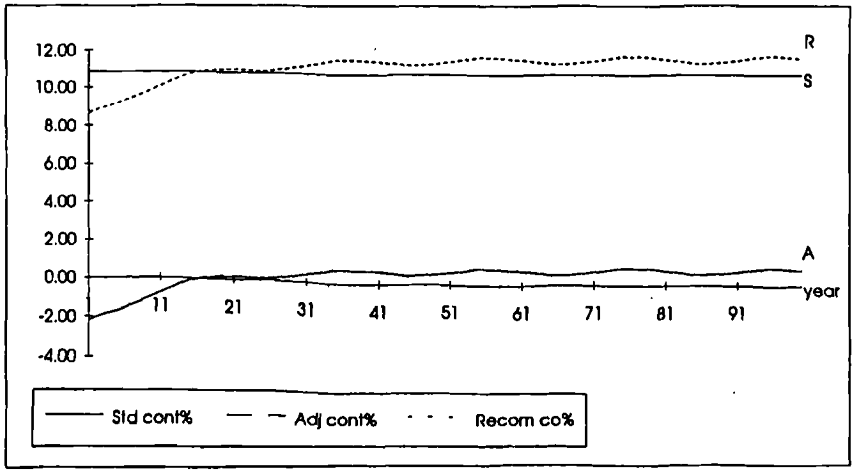
Graph A3.2.3.I



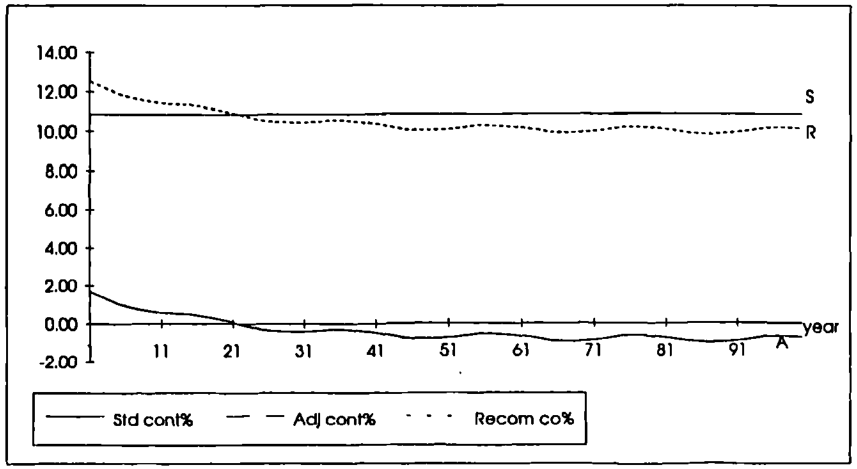
Graph A3.2.4.a



Graph A3.2.4.b



Graph A3.2.4.c



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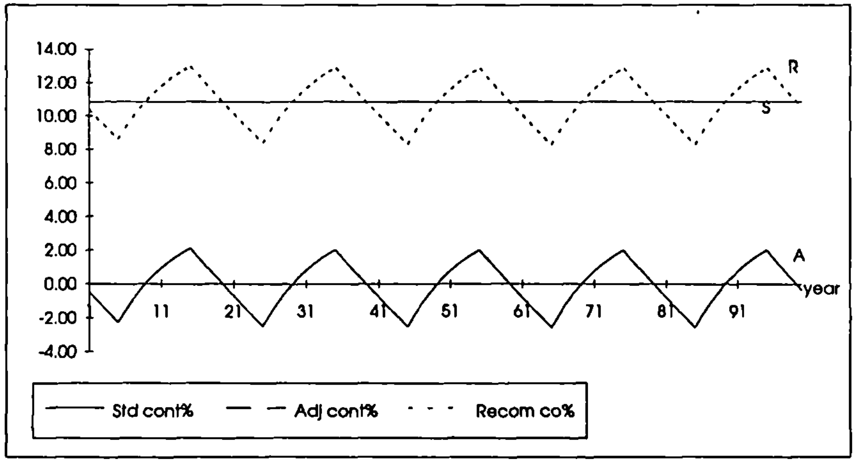
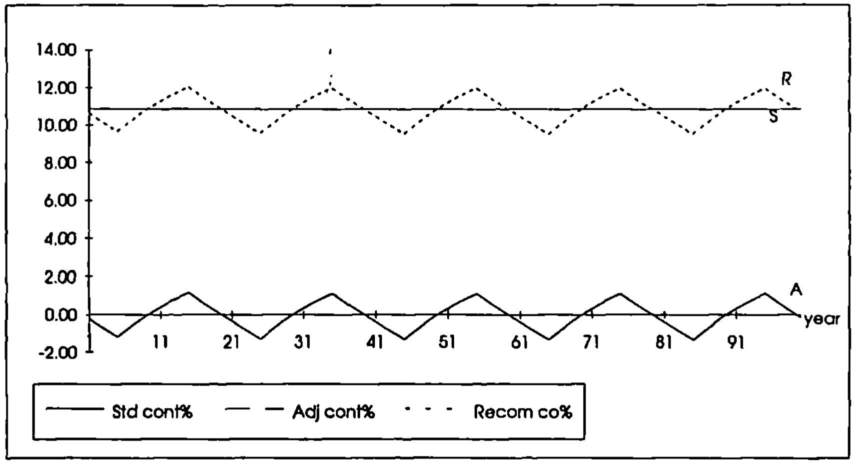
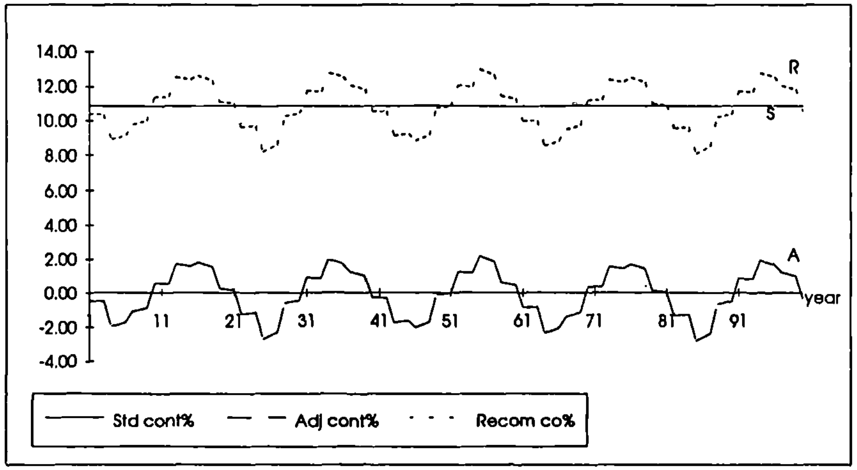


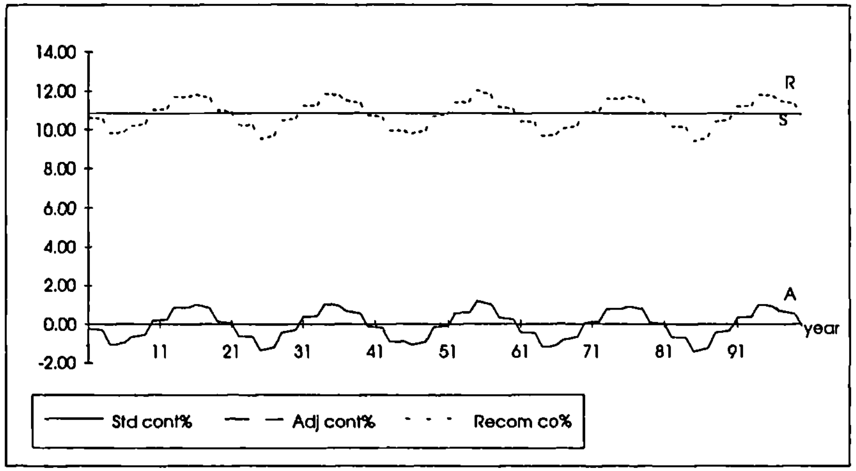
Graph A3.2.4.e



Graph A3.2.4.f



Graph A3.2.4.g



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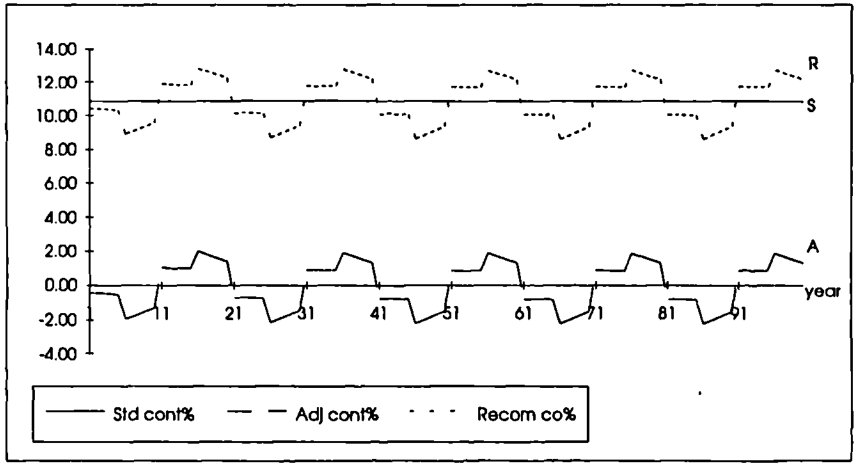
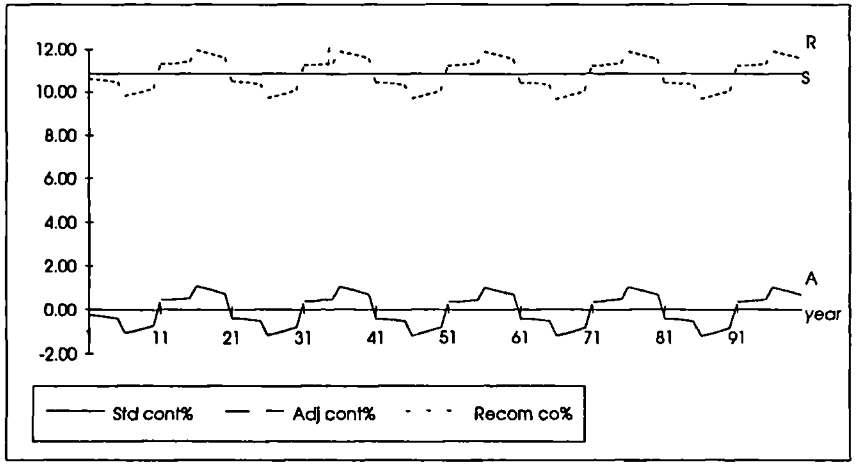


Graph A3.2.4.i



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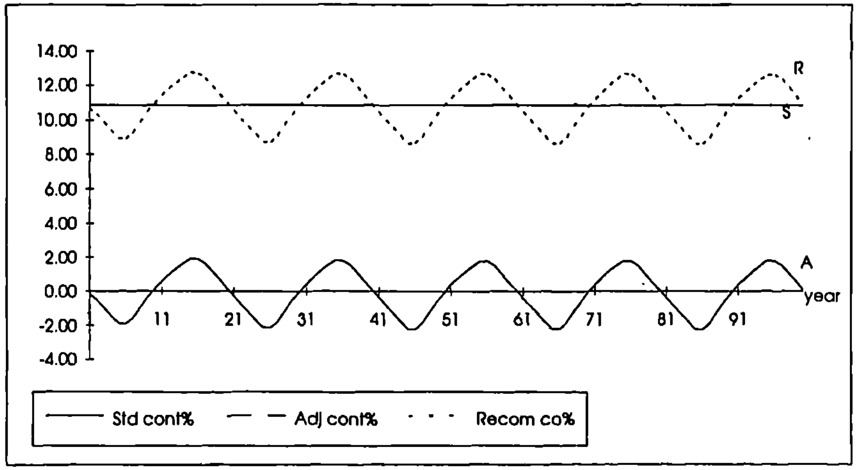
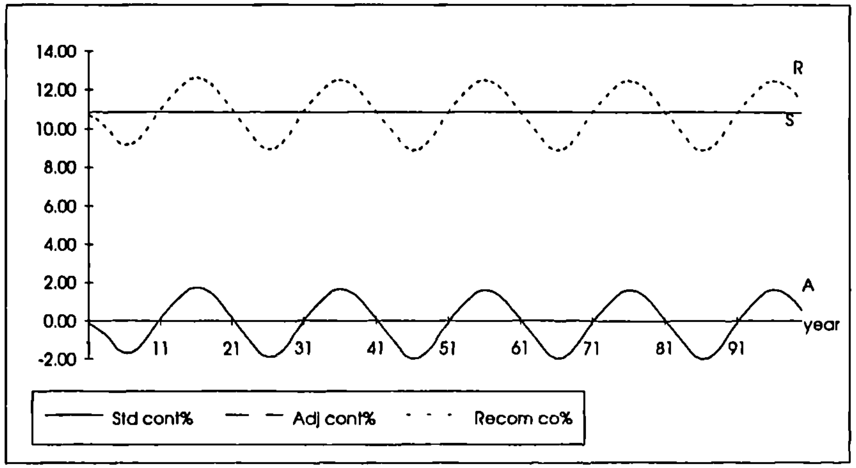
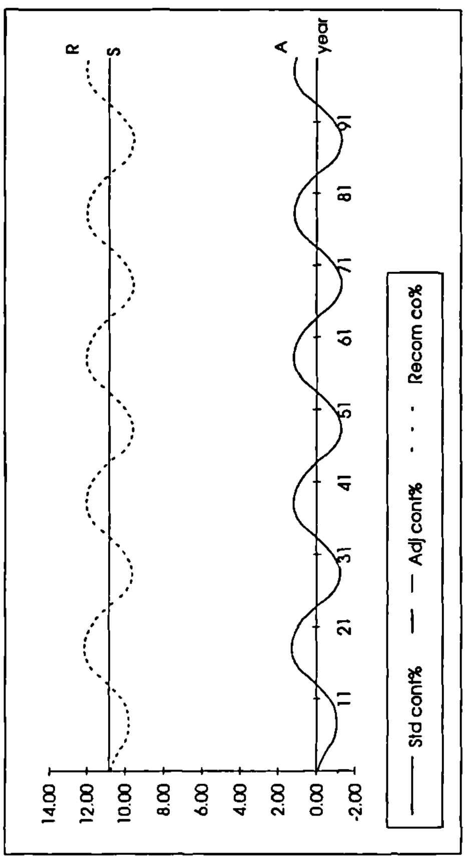


Graph A3.2.4.k



Graph A3.2.4.1



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Recom co%

Appendix 3.3 Withdrawal Rate Changes

A3.3.1 Unchanged experience

The experience is the same as the standard experience and remains stable throughout the projections. Three different bases, which are also constant throughout the projection, are used, and the long term effects of the choice of the basis are examined.

List of Projections

A3.3.1.a	w=5% [Standard basis]
A3.3.1.b	w=10%
A3.3.1.c	w=0%

The bases are stable from the beginning of year 1. However, the fund at the beginning of year 1 is equal to the actuarial liabilities calculated on the standard basis, and the initial fund is the same for all 3 projections.

A3.3.1.a:
The experience and the valuation basis are equal and static throughout the projection. This shows a stationary fund which grows in line with the salary growth, i.e. 6.5% p.a. The Recommended Contribution Rate is always the same as the Standard Contribution Rate, which is constant at 10.8578%. The funding level on the standard basis is static at 100%.

A3.3.1.b:
In this projection, the liabilities are optimistically valued, and the Standard Contribution Rate is lower than for A3.3.1.a. The population structure is stable, and the Standard Contribution Rate is stable at 9.5228%.

Initially, there is a surplus. For the amortisation of this surplus, the Recommended Contribution Rate is lower than the Standard Contribution Rate. The initial Recommended Contribution Rate is 8.0345%.

The actual withdrawal rates are lower than the assumption, and deficits arise every year from these differences, and they accumulate. So, gradually, the funding level becomes lower, and the Recommended Contribution Rate rises.

The size of the newly accruing deficit compared with the size of the fund is constant. The deficits accumulate, but at the same time, the additional contributions for amortisation over the Standard Contribution Rate grow at the same pace as the growth in the accumulated deficits.

Eventually, the accrual of deficits balances with the additional contributions, and the fund approaches a stationary state with a higher contribution rate, about 11.6%, and a lower funding level than for A3.3.1.a, about 94% on this valuation basis or about 84% on the standard valuation basis.

Because the withdrawal assumption is lower than the actual experience, the value of the future service lifetime is understated. So, the degree of the adjustment of contribution rates for amortisation is larger than that on a realistic basis, and the speed of amortisation is quicker than planned. That is, the stationary state seems to be reached relatively quickly.

A3.3.1.c:

This is the opposite case to A3.3.1.b. The liabilities are pessimistically valued, and the Standard Contribution Rate is higher, stable at 14.1952%, than for A3.3.1.a.

Initially, there is a deficiency. For the amortisation of this deficiency, the Recommended Contribution Rate is higher than the Standard Contribution Rate. The initial Recommended Contribution Rate is 15.4079%.

In contrast to A3.3.1.b, surpluses arise every year, and gradually, the funding level becomes higher, and the Recommended Contribution Rate falls.

Eventually, the fund approaches a stationary state with a lower contribution rate, about 7.0%, and a higher funding level than for A3.3.1.a, about 168% on this valuation basis or about 195% on the atandard valuation basis.

The speed of amortisation is slower than planned for the opposite reason from A3.3.1.b. So, the stationary state seems to be reached marginally more slowly than A3.3.1.b.

The impacts seen in this projection are much larger than for A3.3.1.b. The difference in the values of liabilities using w=5% and w=0% seems to be larger than that between w=10% and w=5%.

Summary

Where the experience is constant, the membership structure remains stationary, and a range of stable valuation bases may lead to a stationary fund eventually. The use of an optimistic basis may not give a sufficient security level for the benefit payments. On the other hand, a pessimistic basis may cause over funding.

The initial movements in the Recommended Contribution Rates are avoided by the use of the standard basis. However, the situation can be totally different if the initial asset is different.

A basis with a higher withdrawal rate assumption gives optimistic values of the liabilities, but at the same time, the adjustments for amortisations are set larger than otherwise. This leads to a quicker amortisation than planned, and the convergence to a stationary state is achieved more quickly. A quick convergence means a more stable contribution rate after a shorter period, but a higher volatility in the contribution rate initially. There is thus a trade off between these two effects. A similar observation may be made on the effects of the choice of the pace of amortisation.

A3.3.2 One permanent change in experience

In this section, the withdrawal experience, which is 5% for all ages before the projections start, is increased to 10% from year 1, and remains so throughout the projections.

The change in the withdrawal rate is applied to all the active members uniformly, and the age weighting of the membership is not affected initially. However, the new entrants are joining at the same pace as before, and the average age gradually becomes younger.

A constant valuation basis, one permanent changes in the basis, and gradual changes in the basis are considered.

A3.3.2.1 Constant basis

The standard basis is used to see the impact of the change in the experience without any moves in the basis. The projection is done as a basis for comparison with other projections which use changing bases.

List of Projections

A3.3.2.a Standard basis

A3.3.2.a:

Initially, the funding level is 100%, and the Recommended Contribution Rate is equal to the Standard Contribution Rate.

Because of the change in the membership structure, the Standard Contribution Rate moves from the original 10.8578% to 9.1550% at the end of year 39. After year 40, a new stationary population is maintained, and the Standard Contribution Rate stays the same.

The experienced withdrawal rates are higher than the assumptions, and surpluses arise every year. They are accumulated, and the Recommended Contribution Rate is reduced gradually to amortise the surpluses.

The fund reaches an equilibrium state where the Recommended Contribution Rate is about 6.6% and the funding level is about 132%.

The difference between the experience and the assumed withdrawal rates is 5%, which is the same as for A3.3.1.c. However, the level of surpluses accruing is lower for A3.3.2.a than for A3.3.1.c (see the last paragraph of the commens on A3.3.1.c in section A3.3.1.), and an equilibrium state is reached at an earlier stage for A3.3.2.a than for A3.3.1.c.

A3.3.2.2 One permanent change in basis

In this section, the valuation basis is changed once. Various combinations of the timing and the magnitude of the change are tried.

-Immediate change

The basis is changed at the end of year 1. So, there are no delays in the reactions.

List of Projections

A3.3.2.b	Immediate, w=10% from year 1 [Same as experience]
A3.3.2.c	Immediate, w=12.5% from year 1
A3.3.2.d	Immediate, w=7.5% from year 1

A3.3.2.b:

At the end of year 1, a surplus arises because of a reduction in the value of liabilities. Also, the Standard Contribution Rate is reduced from 10.8578% at the beginning of year 1 to 9.5104%. The Recommended Contribution Rate is even lower than this at 7.2749% to amortise the surplus. So, the initial change in the contribution rate is very large.

Because of the gradual reduction in the average age, the Standard Contribution Rate reduces to 7.6777% in year 39. Then, the population structure becomes stable again, and the Standard Contribution Rate stays at 7.6777% thereafter.

From year 2 onwards, the experience and the valuation basis are equal, and no surpluses/deficits arise.

Usually, when the average age changes, surpluses/deficits would be expected during the intervaluation periods because of the difference between the average age used for the calculation of the Standard Contribution Rate and the actual average age during the period. However, here, the model assumes that

-a valuation is carried out every year, and

-the contributions are received in full at the beginning of the year before any changes in the membership occur. Therefore, no surpluses/deficits arise from this source in all the projections.

The initial surplus is gradually amortised to nil, and the fund becomes stationary with the Standard Contribution Rate as the Recommended Contribution Rate and a funding level of 100%.

During the first 39 years, the gradual fall in the Standard Contribution Rate and the gradual fall in the reduction of contribution rate arising from the amortisation component are to some extent cancelling each other, and the Recommended Contribution Rate is relatively stable during this period after the initial fall at the end of year 1. After year 40, the Standard Contribution Rate becomes stable, and the amortisation is almost completed. So, a similar contribution rate as for the first 39 years is maintained throughout the projection.

A3.3.2.c:

The valuation basis is more optimistic than for A3.3.2.b, and the initial surplus is larger. Also, the Standard

Contribution Rate falls more to 9.1427%. The Recommended Contribution Rate is 5.7974% reflecting the larger surplus and the lower Standard Contribution Rate than for A3.3.2.b.

Because of the change in the membership structure, the Standard Contribution Rate falls to 7.2981% until year 39, then remains stable.

The actual withdrawal rates are lower than the assumptions, and deficits arise every year. The size of the deficit is relatively constant. The deficits reduce the initial surpluses and, then, accumulate gradually. Consequently, the Recommended Contribution Rate increases, and becomes higher than the Standard Contribution Rate. Eventually, the accrual of deficits balances with the additional contributions for amortisation, and the level of the accumulated deficits becomes stable at about 4% of the actuarial liabilities.

After the Standard Contribution Rate and the additional contribution rate become stable, the fund becomes stationary, where the Recommended Contribution Rate is about 7.8%. The contribution rate is higher than for A3.3.2.b. This suggests that the security is lower.

Similar to A3.3.2.b, the cancelling effects between the fall in the Standard Contribution Rate and the increase in the additional contributions (or the decrease in the reduction in the contribution) arising from the amortisation are seen during the first 39 years. However, A3.3.2.c starts from a lower contribution rate and ends in a higher contribution rate than for A3.3.2.b. So, the contribution rate is less stable than for A3.3.2.b.

A3.3.2.d:

Similarly to A3.3.2.b, there is an initial valuation surplus, but the amount of surplus is smaller than A3.3.2.b. Also, the Standard Contribution Rate falls to 10.0426%. The Recommended Contribution Rate at the end of year 1 is 8.8583%.

Because of the change in the membership structure, the Standard Contribution Rate falls to 8.2505% until year 39, then remains stable.

The actual withdrawal rates are higher than the assumptions, and surpluses arise every year. By a coincidence, the amount of surplus arising is almost equal to the reduction in the contributions for the amortisation of the initial surplus, and they almost cancel each other. As a consequence, the level of funding remains similar to that at the end of year 1.

Because the adjustment of contribution rates for amortisation is almost constant, the cancelling effect with the fall in Standard Contribution Rate does not appear. So, during the first 39 years, the Recommended Contribution Rate gradually falls. So, the moves in the contribution rate throughout the projection are greater than for A3.3.2.b.

The eventual funding level is higher than for A3.3.2.b and is stable at about 109% of the actuarial liabilities.

Summary

The cancelling effects between the fall in the Standard Contribution Rates and the increase in the additional contributions (the decrease in the reduction in contributions) arising from the amortisation have been observed in A3.3.2.b and A3.3.2.c. So, in terms of the stability of the Recommended Contribution Rate, A3.3.2.b seems to be the most satisfactory.

Also, only A3.3.2.b can achieve a funding level of 100%.

However, in terms of the stability of the funding level during the projections, A3.3.2.d shows a relatively constant funding level. In the case of A3.3.2.b, the contribution rate is relatively stable at the cost of a higher funding level temporarily.

-Delayed change

The changes in the valuation basis take place at the end of year 3. So, there is two years' delay in the change.

List of Projections

A3.3.2.e	Delayed, w=10% from year 3 [Same as experience]
A3.3.2.f	Delayed, w=12.5% from year 3
A3.3.2.g	Delayed, w=7.5% from year 3

A3.3.2.e:

The pattern of changes in the cntribution rates is similar to A3.3.2.b. The Standard Contribution Rate falls at the change of basis, and continues to fall gradually until the average age becomes stable. An initial valuation surplus arises, but no further surpluses accrue. So, the surpluses are gradually amortised to nil.

Similar cancelling effects with the decreases in the reduction of contributions arising from the amortisation are seen, and the Recommended Contribution Rate is relatively constant.

The effect of the delay in the change of the basis appears as a higher accumulated surplus at the change of the basis. This was caused by the higher actual withdrawals than expected before the change of the basis.

Because the initial level of surpluses is larger than for A3.3.2.b, the initial reduction in the contribution rate is larger, and it takes longer to amortise the surplus. Because of this effect, the contribution rate is less stable for the initial few years.

A3.3.2.f:

The effect of the delay appears as a larger initial surplus than for A3.3.2.c, and it takes longer to reach an equilibrium state, and the contribution rate is less stable, initially in particular.

However, the basic pattern of changes in contribution rates are the same as for A3.3.2.c. The ultimate Standard Contribution Rate is the same as that of A3.3.2.c, and the ultimate contribution rate is almost the same.

A3.3.2.g:

The basic patterns of the changes are the same as for A3.3.2.d. The ultimate levels of the contribution rate and the funding level are almost the same.

The effects of the delay appear as a higher initial surplus and a larger fall in the Standard Contribution Rate at the end of year 3. Because of the higher level of surplus, the cancellation effect which is not seen in A3.3.2.d appears to an extent. So, the changes in the Recommended Contribution Rate are slightly less than for A3.3.2.d.

However, the ultimate state of the fund is little changed compared with A3.3.2.d, where the change in the basis takes place immediately after the change in the experience level.

Summary

The effects of the delay tend to appear as more volatile contribution rates and funding levels initially. However, the delay may not significantly affect the ultimate state of the fund.

A3.3.2.3 Gradual change in basis

In this section, the valuation bases are changed gradually as straight lines from the original experience level of 5% to the new experience level of 10%. Various combinations of the magnitude of the changes and the frequency of the changes in the basis are investigated.

Gradual changes in the basis are interpreted as the averaging of the past experience. The magnitude of the changes corresponds to the length of averaging period. There are three levels of magnitude in the change of the withdrawal assumption tested; Quick (.2% p.a.), Medium (.1% p.a.) and Slow (.04% p.a.). 'Quick' corresponds to 25 year averaging, 'Medium' corresponds to 50 year averaging, and 'Slow' corresponds to 125 year averaging.

Because the valuation bases are the averages of the past experience, the changes in the basis will cease when the basis arrives at the experience level.

For completeness, one additional projection has been made where the change in the basis does not stop when the basis becomes equal to the new experience level of 10%.

-Annual change

The change in the basis starts from the end of year 1, and the chages are made every year thereafter.

List of Projections

A3.3.2.h	Averaging, Annual change, Quick(.2%pa)
A3.3.2.i	Averaging, Annual change, Medium(.1%pa)
A3.3.2.j	Averaging, Annual change, Slow(.04%pa)

A3.3.2.h:

Until year 25, the Standard Contribution Rate gradually falls reflecting the change in the basis and the fall in the average age. Between year 26 and year 39, the Standard Contribution Rate still falls because of the fall in the average age but to a lesser degree.

Until year 25, owing to the continuing weakening of the basis, valuation surpluses arise. At the same time, the withdrawal experience is higher than the valuation basis, and surpluses arise. The level of surpluses arising from the latter source reduces gradually as the gap between the experience and the basis reduces. The total effect is dominated by the second factor, and the surpluses arising are gradually reduced. These surpluses accumulate until year 15, but are slowly amortised after year 16.

After year 26, no surpluses/deficits are accruing because the assumptions exactly match the experience. The accumulated surpluses are amortised in due course. Eventually, the surpluses are reduced to nil.

By the combination of the changes in the Standard Contribution Rate and the effect of the amortisation, the Recommended Contribution Rate falls until year 23, rapidly at first and more slowly later on. After year 24, the progress of amortisation becomes more significant, and the contribution rate gradually increases to reach stability.

A3.3.2.i:

Until year 50, the Standard Contribution Rate gradually falls reflecting the change in the basis. Until year 39, the fall in the average age also contributes to the reduction in the Standard Contribution Rate.

Until year 50, similar to A3.3.2.h, surpluses arise, and the level of surpluses arising reduces gradually as the gap between the experience and the basis reduces.

These surpluses accumulate until year 18, and the Recommended Contribution Rate gradually falls. However, after year 19, the accruing surpluses are smaller than the reduction in the contribution arising from the amortisation of accumulated surpluses, and the surpluses gradually fall.

Combined with the fall in the Standard Contribution Rate until year 50, the Recommended Contribution Rate falls until year 28, but it gradually increases after year 29.

After year 51, no surpluses/deficits are accruing because the assumptions exactly match the experience. So, the fall in the accumulated surpluses is accelerated, and eventually, the surpluses are eliminated.

The maximum level of accumulated surpluses is slightly higher than for A3.3.2.h, but because of a smoother change in the Standard Contribution Rate, the variation in the Recommended Contribution Rate is smaller than for A3.3.2.h.

A3.3.2.j:

The change in the valuation basis is so slow that the basis cannot reach the experience level at the end of 100 years.

So, the Standard Contribution Rate continuously falls, and surpluses arise every year during the projection. The level of surpluses arising reduces gradually as the gap between the experience and the basis reduces.

The surpluses accumulate until year 22. After year 23, the accruing surpluses become smaller than the reduction in the contribution for amortisation, and the surpluses are gradually reduced.

Combined with the fall in the Standard Contribution Rate, the Recommended Contribution Rate falls until year 33, but it gradually increases after year 34.

The maximum level of accumulated surpluses is slightly higher than for A3.3.2.i, but because of a smoother change in the Standard Contribution Rate, the variation in the Recommended Contribution Rate is the smallest among the three projections considered.

Summary

All these projections seem to converge to the same ultimate funding level as for A3.3.2.b, where surpluses are nil.

A slower averaging gives a higher level of accumulated surpluses. This effect is similar to the delay in the change of basis.

However, unlike the delayed changes, the gradual changes in the Standard Contribution Rate will reduce the volatility in the Recommended Contribution Rate.

On the other hand, a slower change in the basis results in a slower amortisation of surpluses, and the funding level stays higher for a longer time. There is a trade off between the stability of the contribution rate and maintaining an adequate funding level. (Here, a surplus, similar to a deficit, may be considered undesirable.)

In A3.3.2.b, the Recommended Contribution Rate is stable after the change of the basis, but there is a large drop at the end of year 1. The gradual changes in the valuation basis enables the contribution rates to move from the original level to the ultimate level smoothly.

Even if the change is very slow, e.g. 125 year average, the surpluses are expected to be eliminated eventually. This feature is totally different from A3.3.2.a, where the basis is not changed at all and an equilibrium state with a surplus is achieved.

-Triennial change

Although the valuation takes place every year, the change in the basis is only made triennially. The first change of the basis is made at the end of year 1. After three years, the change in basis is made by three times the annual rate of change. i.e. a change by .2% pa means a change by .6% every three years.

List of Projections

A3.3.2.k	Averaging, Triennial change, Quick(.2%pa)
A3.3.2.1	Averaging, Triennial change, Medium(.1%pa)
A3.3.2.m	Averaging, Triennial change, Slow(.04%pa)

The general tendency is the same as for the projections with annual changes.

During the three year period of unchanged bases, the Standard Contribution Rate changes only by the fall in the average age. Also, no valuation surpluses arise. However, the surpluses arising from the gap between the experience and the assumptions are larger than the cases with annual changes in the basis because of a wider gap particularly in the second and third year after the change in the basis.

The maximum accumulated surpluses are slightly larger than for the projections with annual changes in the basis.

Consequently, at the time of changing of the basis, the impact of the accumulated surpluses is large. Also, there is a release of valuation surpluses which would have arisen every year in the case of the projections with annual changes. Combined with the large change in the Standard Contribution Rate, the change in the Recommended Contribution Rate tends to be larger than for the projections with annual changes in the basis.

So, the Recommended Contribution Rates are more volatile from year to year.

-Further change

The change in the valuation basis starts in a similar way to A3.3.2.h, but the change continues at the same pace even after the valuation basis reaches the experience level.

List of Projections

A3.3.2.n	Change continue, Anuual change, Quick(.2%pa)

A3.3.2.n:
Up to year 25, this is the same as for A3.3.2.h.

After year 26, owing to the continuing weakening of the basis, deficits arise from the difference between the experience and the assumptions. Although valuation surpluses continue to arise, the deficits gradually become more dominant, and the level of deficits increases as time goes on.

The accumulated surpluses are amortised by the reductions in the contribution rate and these newly accruing deficits. So, the amortisation is done very quickly. After that, deficits accumulate, and this requires additional contributions on top of the standard contributions.

Although the Standard Contribution Rate continuously falls, the impact of the accumulating deficits is more significant, and the Recommended Contribution Rate gradually increases. At the same time, the funding level is gradually deteriorating.

At the end of year 100, the contribution rate is still increasing. However, the fall in the Standard Contribution Rate and the rise in the additional contribution rate are more moderate than for the case of real interest rate changes, and the contribution rates seem to be more stable than for A3.1.2.n.

A3.3.3 Gradual change in experience

Here, the withdrawal experience is assumed to increase from 5% by .1% per annum for all ages. So, in year 1, the withdrawal rate is 5.1%, and in year 100, it is 15%.

The withdrawal rate is continuously increasing, while the same level of new entrants are admitted at age 25. So, the population structure is continuously changing, and the average age falls. Unlike section A3.3.2, the change of withdrawal experience each year is very small, and the change in the membership structure is, therefore, small.

A constant valuation basis and gradual changes in the bases are investigated.

A3.3.3.1 Constant basis

The standard basis is tried to see the impact of the change in the experience without any moves in the basis.

List of Projections

A3.3.3.a Standard basis

A3.3.3.a:

The Standard Contribution Rate continuously reduces as the average age falls.

The withdrawal rate experienced gradually increases, and surpluses arise. The amount of surpluses arising is initially very small, but gradually increases as the difference from the assumption widens.

The surpluses accumulate, and the Recommended Contribution Rate falls accordingly.

Because the amount of surpluses arising is increasing, it always exceeds the reduction in contributions for amortisation which are based on the previous amount of accumulated surpluses. The two do not balance at any stage of the projection. So, there are no signs of arriving at an equilibrium state.

Towards the end of the projection, a huge surplus has been accumulated. The contribution rate has not been stable.

A3.3.3.2 Gradual change in basis

In this section, the valuation bases are changed gradually as straight lines. Various combinations of the magnitude of the changes, the timings of the start of changes and the frequency of the changes in the basis are considered.

The magnitude of the changes are expressed as annual rates of increases in the withdrawal rate assumption.

If the valuation basis is set as an average of past experience, the pace of the changes in the basis will be the same as that in the experience after the year when the period from the start of the projection exceeds the averaging period. However, the initial pace of the basis changes is different from straight line changes in the basis. This is because the experience before the projections start is assumed to be stationary. To see the initial differences, some projections using the bases which are the averages of the past experience are added.

-Immediate change

The changes in the valuation basis starts at the end of year 1. So, there are no delays. Three different magnitudes of changes are examined; one follows exactly the experience, others show smaller changes in the bases than the experience. Projections with higher paces of the changes in the valuation bases have been omitted because they are unlikely to be suitable.

List of Projections

A3.3.3.b	Immediate, Annual change, .1%pa[Same as experience]
A3.3.3.c	Immediate, Annual change, .04%pa
A3.3.3.d	Immediate, Annual change, .02%pa

A3.3.3.b:

The Standard Contribution Rate gradually falls because of the changes in the valuation basis and in the average age.

Owing to the continuous weakening of the basis, a valuation surplus arises every year. Because the changes in the valuation basis are matched by the changes in the experience, there are no surpluses/deficits from the gap between the experience and the valuation basis. The level of surpluses arising every year is fairly constant although the growth slows down a little as the level of withdrawal assumed increases. (see section 8.4.1.3)

Initially, the reduction in the contributions for the amortisation of the surpluses is less than the surpluses accruing, and the surpluses accumulate. However, as the surpluses accumulate, the reduction in the contributions increases, and they eventually balance each other. After year 19, the reduction in the contribution rate arising from amortisation remains relatively stable at the level around 1%, and only changes by at most .01% per annum. Hence, the level of surplus is relatively stable.

Although the contribution rate falls without convergence, the funding position may be said to be stable and sound because the level of surplus is stable and small.

A3.3.3.c:

The Standard Contribution Rate falls as the basis is weakened and the average age is reduced, but the degree of the fall in the Standard Contribution Rate is smaller than for A3.3.3.b.

Each year, owing to the weakening of the basis, a valuation surplus arises. This surplus is relatively small and stable from year to year. At the same time, the difference between the experience and the assumptions causes further surpluses. Because the pace of the changes in the valuation basis is less than that of the experience, the difference increases as time passes. So, the surplus arising from this source gradually increases at least initially.

Eventually, the reduction in the contribution arising from the amortisation of already accumulated surpluses eventually becomes large enough to match with the surplus arising each year. So, the funding level seems to become stable at a higher level than for A3.3.3.c.

The Recommended Contribution Rate is more stable than for A3.3.3.b initially, but changes more rapidly afterwards.

A3.3.3.d:
The direction of changes in the contribution rates is the same as for A3.3.3.c, but the degree of changes are greater than for A3.3.3.c.

Summary
A3.3.3.b keeps the amortisation element of the contribution rate small and stable. So, the results may not be affected much by the choice of the amortisation method.

More modest changes in the basis than in the experience make the contribution rate smooth initially at the cost of a higher level of surplus/deficit. If reversal changes occur after this, the use of more modest changes in the basis may mitigate the impact. But if the next change is in the same direction, it results in an accumulated surplus/deficit. In the projections above, such accumulation of surpluses are observed, and they caused more volatile contributions and funding levels.

-Delayed change

The changes in the valuation basis start at the end of year 3. So, there are two years' delays compared with the projections with immediate changes. Three different magnitudes of the changes in the bases are examined.

The initial changes are made only for one year's portion of the changes. There are no extra changes in the bases reflecting the delay. So, even in A3.3.3.e, the valuation basis is always two years behind the experience.

List of Projections

A3.3.3.e	From year 3, Annual change, .1%pa
A3.3.3.f	From year 3, Annual change, .04%pa

A3.3.3.e:
The Standard Contribution Rate falls gradually owing to the fall in the average age and the change in the valuation basis after year 3.

Because of the delay in the start of the change in the valuation basis, there exists a gap between the experience and the assumptions. This is a source of surpluses every year. The gap is constant because the pace of the change in the valuation basis is equal to that of the experience. So, the surpluses accruing from this source are constant throughout the projection except for the first three years, where the gap gradually widens.

Also, owing to the change in the valuation basis, a small valuation surplus arises every year. Because the change of the basis is steady, the surplus arising from this source is also constant over time.

So, the surpluses arising each year are fairly constant, but because of the first element of the surplus, the level of surpluses arising is higher than for A3.3.3.b.

Consequently, the eventual level of accumulated surpluses is slightly higher, with each year's accruing surpluses and the reduction in the contribution arising from amortisation cancelling each other out to maintain stability.

The total contribution rate continues to fall because of the fall in the Standard Contribution Rate, but the level of surplus accumulated is stable and relatively low although it is higher than for A3.3.3.b.

A3.3.3.f:

The Standard Contribution Rate falls as the valuation basis is weakened and the average age falls. The degree of fall is less than for A3.3.3.e.

Surpluses arise each year owing to the gap between the experience and the assumptions and the weakening of the valuation basis. The former increases at least initially, while the valuation surpluses arise constantly.

The patern of the contribution rates and the funding level changes are similar to A3.3.3.c. Owing to the delay, the level of accumulated surplus is slightly higher than for A3.3.3.c, and the contribution level is lower.

Summary

The effect of delay is a higher level of accumulated surplus.

However, two years' delay may not be significant. The basic shape of the contribution rate changes is not affected by the delay. So, it may be justifiable to delay the basis changes until the tendency in the experience becomes clearer.

-Triennial changes

The valuations take place every year, but the changes in the valuation basis are made only every three years. The first change is made at the end of year 1.

The degree of change each time is three years' portion of the changes. So, if the increase in the withdrawal rate assumption is .1% p.a., .3% is added every three years.

List of Projections

A3.3.3.g Immediate, Triennial change, .1%pa

A3.3.3.h Immediate, Triennial change, .04%pa

A3.3.3.g:

The general shape of the contribution rates changes is similar to A3.3.3.b.

At the time of the change in the basis, a large valuation surplus is released, while there are no such surpluses between the times of basis changes.

The surpluses arising from the gap between the experience and the assumptions are increased between the basis changes because the basis is the same while the experience change.

Because of these factors, fluctuations in the Recommended Contribution Rates are higher than for A3.3.3.b.

A3.3.3.h:

The general shape of the contribution rates changes are similar to A3.3.3.c.

Similar comments as A3.3.3.g apply, and the fluctuations in the contribution rate are higher than for A3.3.3.c.

Summary

Triennial changes in the basis do not affect the basic pattern of the projection results. However, the fluctuations in the contribution rates from year to year are higher than the cases with annual changes.

-Decennial changes

Similar to triennial changes, the valuations take place every year, but the changes in the valuation basis are made only every ten years. The first change is made at the end of year 1.

The degree of change each time is ten years' portion of the changes. So, if the increase in the withdrawal rate assumption is .1% p.a., 1% is added every ten years.

List of Projections

A3.3.3.i Immediate, Decennial change, .1%pa

A3.3.3.j Immediate, Decennial change, .04%pa

The broad pattern of the contribution rates in the long run is similar to the cases of annual changes and triennial changes. More short term fluctuations than for triennial changes are seen.

-Averaging

The valuation basis is set as the average of the past experience. As the averaging period, 5 years and 10 years are examined.

List of Projections

A3.3.3.k	Immediate, Annual change, 5 year average
A3.3.3.l	Immediate, Annual change, 10 year average

A3.3.3.k:

Except for the initial 4 valuations, the valuation bases used are exactly the same as those under A3.3.3.e, with 2 years' delay.

The differences in the initial period are not large, and they do not lead to any significant impact.

The contribution rate changes are almost identical to A3.3.3.e.

A3.3.3.l:

The Standard Contribution Rate continuously falls as the basis is weakened and the average age reduces.

10 years' averaging is equivalent to 4.5 years' delay in the start of the change in the basis except for the initial 10 years. So, the gap between the experience and the assumptions is wider than for A3.3.3.e, and the surplus is larger. However, because the pace of the changes in the valuation basis becomes the same as that of the experience, the gap remains constant.

The differences in the results with A3.3.3.k or A3.3.3.e are not large.

Summary

Averaging has a similar effect as delaying the change of the basis, and averaging may not have a significant impact on the finances of the fund compared with the use of a non-averaged basis.

A3.3.4 Cyclical change in experience

Here, the withdrawal experience, w', is assumed to follow a twenty year cycle. Namely, w' is increased at 1% per annum until w' reaches 10% in the fifth year. Then, w' is reduced by 1% per annum for ten years to reach 0%. After that, w' is again increased at 1% per annum to return to 5% in the twentieth year. During the 100 year projection, thus, five cycles are made.

Initially, owing to the increase in the withdrawal rate, the average age reduces. Although the withdrawal rate starts to reduce in year 6, the reduction in the average age continues until year 13. After year 14, the average age slightly increases because of the low withdrawal level. The increase continues until year 24 although the withdrawal rate starts rising again after year 16.

After 40 years when all the initial members have exited the scheme, a cyclical pattern of the membership structure changes is established. The phase of the cycle is different from the withdrawal rate cycle; the highest average age is achieved at the end of the 4th year of the cycle, and the lowest at the end of 13th year.

Constant valuation bases and cyclical changes in the bases are investigated.

A3.3.4.1 Constant basis

Three different constant bases are tried.

List of Projections

A3.3.4.a	w=5% [Standard basis]
A3.3.4.b	w=10% [Highest value]
A3.3.4.c	w=0% [Lowest value]

A3.3.4.a:

The Standard Contribution Rate moves according to the changes in the average age. So, eventually, it shows a cycle with a different phase from the experience cycle. The highest Standard Contribution Rate is achieved at the end of the 4th year of the cycle, and the lowest at the end of 13th year.

According to the cyclical changes in the experience, surpluses/deficits arise from the gap between the experience and the assumptions.

At the beginning of year 1, there are no accumulated surpluses/deficits.

For the first five years, surpluses arise, and the level of these accruing surpluses increases. The reduction in the contribution rate arising from amortisation is less than the accruing surpluses, and the surpluses accumulate. Combined with the fall in the Standard Contribution Rate, the Recommended Contribution Rate reduces.

For the second five years, still surpluses arise, but the amount arising falls from year to year. The reduction in the contribution rate arising from amortisation is still less than the accruing surpluses initially, and the surpluses accumulate until year 8. However, the reduction of the contributions for amortisation exceeds the accrual of the surpluses eventually because the former increases while the latter decreases. In year 9 and 10, the contribution rate is increased reflecting the progress of the amortisation of surpluses. At the end of year 10, there is a surplus outstanding. Combined with the continued fall in the Standard Contribution Rate, the Recommended Contribution Rate falls until year 9, but slightly increases in year 10.

For the next five years, deficits arise, the amount of which gradually increase. The deficits are used to remove the outstanding surpluses together with the reduction in the contributions for amortisation. By the end of year 15, the surpluses become very small. Although the Standard Contribution Rate continues to fall until year 13, the decrease in the reduction in the contribution rate for amortisation is larger, and the Recommended Contribution Rate increases. After year 14, the Standard Contribution Rate also increases, and the Recommended Contribution Rate increases further.

For the last quarter of the cycle, deficits, the amount of which gradually reduces to 0, arise. Until year 19, the deficits accumulate. Combined with the increase in the Standard Contribution Rate, the Recommended Contribution Rate increases. However, in year 20, the accrual of the deficit is exceeded by the additional contributions for amortisation, and the accumulated deficit is slightly reduced.

At the end of the first cycle, thus, there is a deficit carried forward.

The accrual pattern of the surpluses and deficits is the same for every consecutive cycle as the first one. However, the pattern of the membership structure changes are different in the subsequent cycles. Also, because of the accumulated deficit at the beginning of the cycle, the pattern of the accumulation of the surpluses and deficits becomes different.

Because of the initial deficit, the accumulation of surpluses in the first half of the second cycle is smaller. So, the size of the reduction in the contribution for amortisation is relatively smaller than for the first cycle, while the accruing surpluses are the same. Consequently, the time when the two effects balance is delayed to the 8th year.

So, the second half of the second cycle starts with a smaller accumulated surplus, and the level of accumulation of the deficits becomes higher than for the first cycle.

At the end of the second cycle, there is a higher level of accumulated deficits.

After year 40, the changes in the membership structure becomes cyclical, and the Standard Contribution Rate follows an established pattern of a 20 year cycle. The highest Standard Contribution Rate is achieved at the end of the 4th year of the cycle, and the lowest at the end of 13th year.

As the cycles go on, the level of the deficit at the beginning of each cycle becomes higher, but the degree of the increase is reduced each time. So, the pattern of the contribution rate changes, which represents the pattern of the funding level, seems to become cyclical and stable.

Combining the cycle of the Standard Contribution Rate and the cycle of the adjustments of contribution rate arising from amortisation, the Recommended Contribution Rate seems to follow a cyclical pattern.

In the fifth cycle, the highest contribution rate is 11.3395% in year 100, and the lowest is 10.1861% in year 89. The difference is 1.1534%.

A3.3.4.b:

The general level of the Standard Contribution Rate is lower than for A3.3.4.a. The Standard Contribution Rate shows a cyclical pattern because of the changes in the average age. Eventually, it seems to follow a cycle with peaks in the 4th year and troughs in the 13th year.

The withdrawal rate assumed in the valuation basis is always higher than or equal to the experience. So, except in year 5,25,45,..., where they are equal, deficits arise because of this gap each year. According to the cyclical pattern of the experience, the level of deficits arising changes cyclically. The level of the deficit arising starts from a non-zero level, and reduces to 0 by year 5. Then, the level increases steadily until year 15. From year 16 to year 20, the level reduces to the initial level.

Initially, there is a valuation surplus because the initial amount of the assets is equal to the actuarial liability under the standard basis. So, the contribution rates are reduced to amortise this initial surplus. The amortisation is quickly helped by the accruing deficit every year. By the end of year 11, the surpluses disappear. During the rest of the first cycle, the deficits accumulate, and the addition to the contribution rate arising from amortisation increases.

The second cycle starts with a deficit. The deficits continue to accumulate and the addition in the contribution arising from amortisation increases. However, the level of accruing deficits are reducing, and in year 22, the accrued deficit becomes less than the addition to the contribution, and the accumulated deficit reduces. The reduction in the accumulated deficits continues while the accumulated deficit level is low until year 28. Then, the gradually increased deficit accruing exceeds the addition in the contribution, and the accumulation of the deficits starts again. The accumulation continues until the end of the second cycle. The second cycle ends with higher deficits than the beginning of the cycle.

A similar pattern is repeated for each coming cycle. The timings of the peak and the trough of the accumulated deficits are slightly affected by the increased level of the initial deficit, and the peak is in the 19th year and the trough is in the 9th year.

The levels of the deficit at the beginning of the fourth and the fifth cycle are slightly increased. The deficit at the beginning of each cycle seems to settle down to a stable level after the fifth cycle.

Combining the cycles of the Standard Contribution Rate and the adjustments of contribution rates for amortisation, the Recommended Contribution Rate seems to follow a 20 year cycle ultimately.

In the fifth cycle, the contribution rates are between 11.1700% in year 90 and 11.8592% in year 81. The contribution level is, thus, higher than for A3.3.4.a. and the funding level is lower. The difference in the contribution rates is 0.6892%%, which is narrower than for A3.3.4.a.

A3.3.4.c:
This is a reverse of A3.3.4.b. The Standard Contribution Rate is higher than for A3.3.4.a. The Standard Contribution Rate shows a cyclical pattern because of the changes in the average age. Eventually, it seems to follow a cycle with the peak in the 4th year and the trough in the 13th year.

The withdrawal rate assumed in the valuation basis is always lower than or equal to the experience. So, except in year 15,35,55,..., where they are equal, surpluses arise because of this gap each year. According to the cyclical pattern of the experience, the level of surpluses arising changes cyclically. The level of the surplus arising starts from a non-zero level, and increases to the maximum level in year 5. Then, the level reduces steadily to 0 until year 15. From year 16 to year 20, the level increases back to the initial level.

There is an initial valuation deficit, and the Recommended Contribution Rate is higher than the Standard Contribution Rate.

In A3.3.4.c, a lower withdrawal rate than for the other two cases (A3.3.4.a and A3.3.4.b) is assumed for the calculation of the remaining working life time, and the surplus/deficit is spread over a longer period than the other cases. Hence, the adjustments in the contribution rate arising from amortisation becomes smaller, and consequently the pace of amortisation becomes slower than for A3.3.4.a or A3.3.4.b.

After amortising the initial deficits, surpluses are accumulated. After several cycles, the pattern of accumulation of surpluses/deficits and the pattern of Standard Contribution Rate become cyclical. Combining the two, the Recommended Contribution Rate pattern also seems to become cyclical.

The contribution rates in the fifth cycle are between 5.7125% in year 89 and 8.2977 in year 99. The level is lower than for A3.3.4.a, and the funding level is higher.

Summary
The average funding level and the contribution rate show similar tendencies as the projections with unchanged experience. The use of a strong basis results in a permanent surplus.

Reflecting the cyclical changes in the experience, the funding level also shows a cycle with the same length. The timings of peaks and troughs in the experience cycle are different from those in the funding level cycle. This is because the latter represents the accumulated effects of experience and the progression in the amortisation.

A3.3.4.2 Cyclical change in basis

In this section, the valuation bases are changed cyclically with the same length of cycles, i.e. 20 year cycles. Various combinations of the magnitude and the frequency of the changes in the basis are investigated.

The magnitude of the changes are expressed as annual rates of changes in the withdrawal assumption.

Also, the effects of averaging are analysed by applying three different averaging periods.

-Immediate changes
The changes in the valuation basis starts from the standard basis at the beginning of the projections, and moves in the direction as the experience.

List of Projections

A3.3.4.d	Immediate, Annual change, 1%pa[Same as experience]
A3.3.4.e	Immediate, Annual change, .5%pa

A3.3.4.d:
In this projection, the valuation basis is exactly the same as the experience in last 12 months.

The Standard Contribution Rate moves according to the changes in the average age and the valuation basis. After 40 years, the former becomes cyclical, and the Standard Contribution Rate moves cyclically between 9.6359% in the 5th year and 14.1162% in the 15th year. Here, the impact of the changes in the basis is dominant, and the cycle follows the cycle of valuation basis changes.

The projection starts with no surpluses/deficiencies.

Although there are no gaps between the experience and the valuation basis, each year a valuation surplus/deficit arises because of the changes in the valuation basis.

In the first five years, the valuation basis continues to be weakened, and valuation surpluses arise. (see section 8.4.1 for a detailed analysis.) The level of each year's accrual is proportionate to the value of actuarial liabilities and the degree of the change in the valuation basis. The effect of the amortisation is relatively small compared with the accruing surpluses. So, the surpluses accumulate, and the degree of the reduction in the contribution rate increases. Combined with the fall in the Standard Contribution Rate, the Recommended Contribution Rate shows a rapid fall during this period.

In the next ten years, valuation deficits arise. First, the accumulated surpluses are amortised quickly, and then deficits accumulate. The accumulated deficit is at its peak in year 15. This peak is slightly smaller than the peak in the accumulated surplus in year 5. The Recommended Contribution Rate rises combined with the rise in the Standard Contribution Rate during this ten year period.

After year 16, valuation surpluses constantly accrue at a similar level as the first five years, and the accumulated deficits are quickly amortised by the end of year 19. At the end of year 20, there is a small surplus outstanding. The Recommended Contribution Rate falls because of the fall in the Standard Contribution Rate and the reduction in the level of deficit during this period.

Because the surplus at the beginning of the second cycle is small, a similar pattern is repeated. However, the funding level is in general slightly higher. Also, the average age is different from the first cycle, and the Standard Contribution Rate is slightly different.

This gradual rise in the funding level continues, but seems to become stable and cyclical. Combined with the establishment of the cyclical pattern of the Standard Contribution Rate after year 40, the Recommended Contribution Rate seems to follow a cyclical pattern.

In the fifth cycle, the lowest Recommended Contribution Rate is 6.7752% in year 85, and the highest is 15.4480% in year 95. The difference is 8.6728%. This is much wider than for the projections with constant bases.

Consequently, the funding level is more volatile than for A3.3.4.a, although the average level is similar.

A3.3.4.e:
In this projection, the changes in the valuation basis are less than the changes in experience.

The Standard Contribution Rate moves according to the changes in the valuation basis and the average age. The pattern of the moves is the same as for A3.3.4.d, but the movements are smaller.

The projection starts with no surpluses/deficiencies.

In the first five years, the valuation basis continue to be weakened, and valuation surpluses arise. The level of each year's accrual is relatively constant because of similar reasons as for A3.3.4.d. The level is less than for A3.3.4.d reflecting the smaller changes in the basis. At the same time, surpluses arise from the gap between the experience and the valuation basis. This part of the surplus increases towards year 5. The total accrual of surpluses is less on average than for A3.3.4.d. The surpluses accumulate, and the degree of the reduction in the contribution rate increases. Combined with the fall in the Standard Contribution Rate, the Recommended Contribution Rate shows a fall. The degree of the fall is less than for A3.3.4.d.

In the next five years, valuation deficits arise constantly, while the gap between the experience and the basis produces surpluses. The surpluses arising gradually reduce to 0 towards year 10 reflecting the narrowing gap. The surpluses accruing from the gap seem to be less significant than the valuation deficits, and the surpluses are amortised quickly. Combined with the rise in the Standard Contribution Rate, the Recommended Contribution Rate rises.

In the third quarter of the cycle, both valuation deficits and deficits from the gap between the experience and the basis arise at the same time. The latter increases gradually. However, the impact of the latter seems to be small, and the Recommended Contribution Rate increases at a similar pace as during the second quarter. Towards the end of year 15, deficits are accumulated to the highest level.

After year 16, valuation surpluses constantly accrue at the same level as the first five years, while the gap between the experience and the basis creates deficits, which are reducing. Again, the latter is insignificant, and the accruing surpluses help with the amortising of the deficits. At the end of the first cycle, there is a small accumulated surplus. Combined with the reducing Standard Contribution Rate, the Recommended Contribution Rate goes down.

Because the surplus at the beginning of the second cycle is small, a similar pattern is repeated. However, the funding level is in general slightly higher.

This gradual rise in the funding level continues, but seems to become stable and cyclical. Combined with the cyclical pattern of the Standard Contribution Rate after year 40, the Recommended Contribution Rate seem to follow a cyclical pattern.

In the fifth cycle, the lowest Recommended Contribution Rate is 8.7943% in year 85, and the highest is 12.8469% in year 95. The difference is 4.0526%. This is much less than for A3.3.4.d, but much wider than for the projections with constant bases.

Consequently, the funding level is less volatile than for A3.3.4.d, but more than for A3.3.4.a, although the average level is similar.

Summary

The changes in the basis according to the cyclical change in the experience make the contribution rate more volatile.

In these projections, the effects of the changes in the valuation basis are more significant than the impact of the gap between the experience and the basis. So, by using a fixed basis as in A3.3.4.a, the first factor is eliminated, and the volatility in the contribution rate is reduced.

-Triennial changes

Although valuations are made annually, the basis is changed only every three years. The length of the experience cycle is 20 years, while the basis change is every 3 years. The numbers 20 and 3 do not have any common divisors. Therefore, if we take any consecutive 20 basis changes, each basis change takes place at a different phase of the experience cycle. That is, in the long run, all the combinations of experience and the basis evenly appear.

List of Projections

A3.3.4.f	Immediate, Triennial change, 1%pa
A3.3.4.g	Immediate, Triennial change, .5%pa

On each change of the basis, the moves in the contribution rate and the funding level are large. In other years, the moves are moderate. So, the year to year movements in the Recommended Contribution Rate and the funding level are less smooth than for either A3.3.4.d or A3.3.4.e.

However, the general pattern in A3.3.4.f and A3.3.4.g, the average Recommended Contribution Rate and the difference between the highest and the lowest of the Recommended Contribution Rates are similar to those for A3.3.4.d and A3.3.4.e respectively.

-Quinquennial changes

Here, the changes in the basis take place every five years. This means that the same valuation basis is used for each quarter of the cycle; the bases close to the standard basis during the first and the third quarter, a weak basis for the second quarter, and a strong basis for the fourth quarter.

List of Projections

A3.3.4.h	Immediate, Quinquennial change, 1%pa
A3.3.4.i	Immediate, Quinquennial change, .5%pa

The year to year variations in the Recommended Contribution Rate are less smooth than triennial changes. Also, because of the use of the same valuation bases for every cycle, the contribution rate pattern is more clearly cyclical.

However, the general pattern in these two projections, the average Recommended Contribution Rate and the difference between the highest and the lowest of the Recommended Contribution Rates are similar to those for A3.3.4.d and A3.3.4.e.

So, the frequency of the basis changes may not be significant as long as the changes take place within relatively short periods.

-Averaging

The valuation basis is set as the average of past experience. As the averaging period, 3, 5 and 10 years are used. A3.3.4.d can be regarded as a projection with one year's averaging period.

The effects of averaging on the setting of the bases are the moderation of the changes in the basis and the delay in these changes. The first effect is observed from the fact that, in the case of averaging, the withdrawal rate assumptions used fall in a narrower range than for A3.3.4.d. The second effect appears from the gaps between the phases of the experience cycle and the basis cycle.

List of Projections

A3.3.4.j	Immediate, Annual change, 3 year average
A3.3.4.k	Immediate, Annual change, 5 year average
A3.3.4.l	Immediate, Annual change, 10 year average

A3.3.4.j:

The range of the withdrawal rate assumption is between 0.6667% and 9.3333%. This is slightly narrower than that of the experience, 0% to 10%. The lowest withdrawal rate assumption is used in year 16, and the highest in year 6. So, there is one year's delay in the phase of cycle.

As mentioned in A3.3.4.d and A3.3.4.e, the effects of the valuation changes are dominant, and the effects of the gap between the experience and the basis are less significant. So, the changes in the Recommended Contribution Rates follow the pattern of A3.3.4.d with one year's delay and the degree of the changes is smaller than for A3.3.4.d.

A3.3.4.k:

The range of the withdrawal rate assumption is between 1.2% and 8.8%. This is slightly narrower than for A3.3.4.j. The lowest withdrawal rate assumption is used in year 17, and the highest in year 7. So, there is two years' delay in the phase of cycle compared with the experience cycle.

Similar comments as A3.3.4.j apply.

A3.3.4.l:

The range of the withdrawal rate assumption is between 2.5% and 7.5%. This is slightly narrower than for A3.3.4.k. This range is the same as A3.3.4.e. The lowest withdrawal rate assumption is used in year 19 and 20, and the highest in year 9 and 10. So, there is 4.5 years' delay in the phase of cycle compared with the experience cycle.

Again, basically, similar comments as A3.3.4.j apply.

Although the cycle for the valuation basis is almost a quarter of a cycle behind that of the experience, only the effects of valuation surpluses/deficits are significant, and the pattern of the widening and narrowing of the gap between the experience and the basis seems less important.

The range of the withdrawal rate assumption used is the same as for A3.3.4.e, but the way of the changes from year to year is different. In A3.3.4.e, the degree of changes in the withdrawal rate assumption is constant. However, in A3.3.4.l, the rate tends to stay longer near the upper and lower boundary of the range, and the rate changes quickly when the rate is in the centre of the range.

Consequently, the variations in the contribution rates are more significant in A3.3.4.l than for A3.3.4.e.

Summary

The effect of the changes in the basis is dominant on the pattern of the changes in the contribution rate. The main effects of using a longer averaging period are a reduction in the volatility of the contribution rates and an increase in the delay in the phase of the cycle.

Note

Although the following combinations have also been examined, they are omitted from the results because they are judged to be unimportant for the main discussion.

-Gradual changes in experience

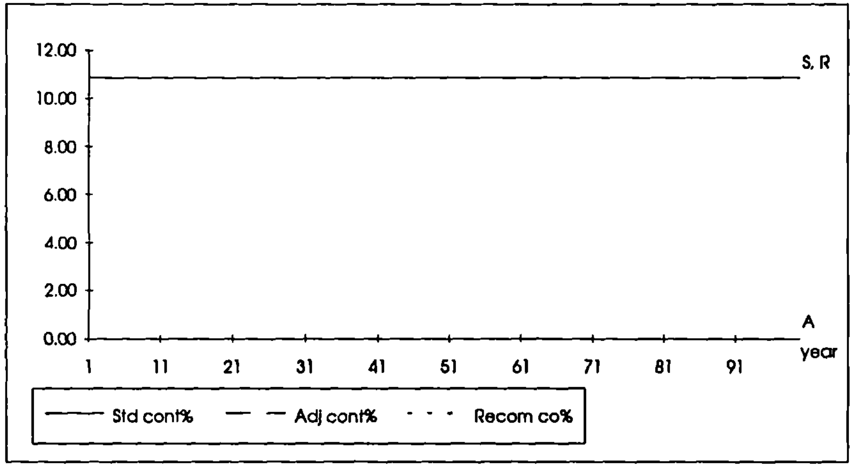
A3.3.3.m	From year 3, Annual change, .02%pa
A3.3.3.n	Immediate, Triennial change, .02%pa
A3.3.3.o	Immediate, Decennial change, .02%pa
A3.3.3.p	Immediate, Triennial change, 5 year average
A3.3.3.q	Immediate, Triennial change, 10 year average
A3.3.3.r	Immediate, Decennial change, 5 year average
A3.3.3.s	Immediate, Decennial change, 10 year average

-Cyclical changes in experience

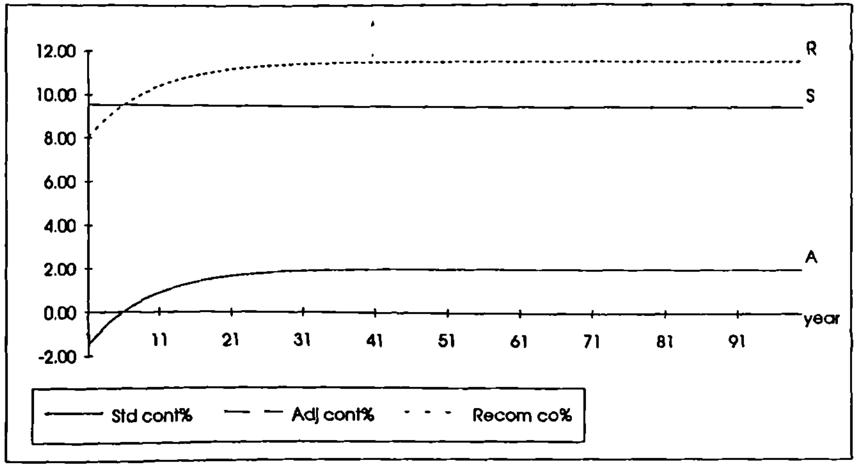
A3.3.4.m	From year 2, Triennial change, 1%pa
A3.3.4.n	From year 3, Triennial change, 1%pa
A3.3.4.o	Immediate, Triennial change, 3 year average
A3.3.4.p	Immediate, Triennial change, 5 year average
A3.3.4.q	Immediate, Triennial change, 10 year average
A3.3.4.r	Immediate, Quinquennial change, 3 year average
A3.3.4.s	Immediate, Quinquennial change, 5 year average
A3.3.4.t	Immediate, Quinquennial change, 10 year average

Also, all the corresponding combinations with a 10 year cycle.

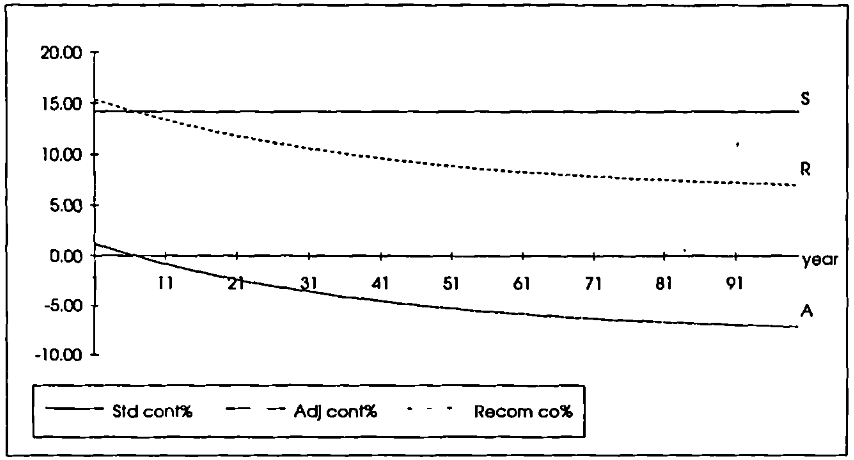
Grapf A3.3.1.a



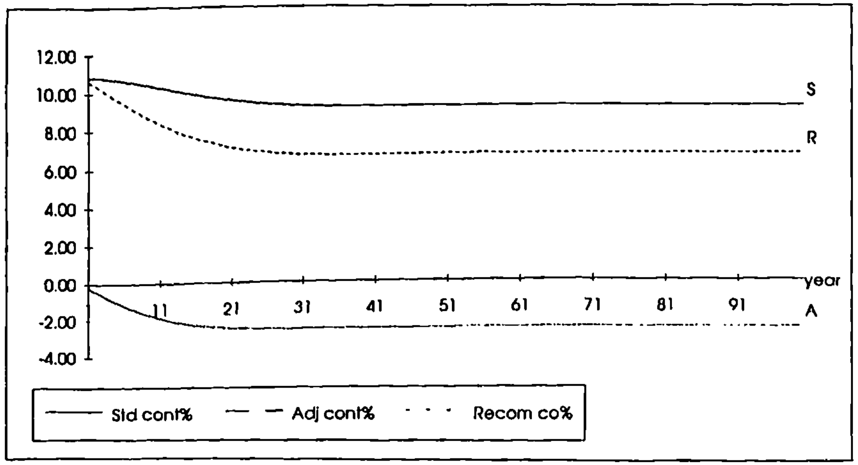
Graph A3.3.1.b



Graph A3.3.1.c



Graph A3.3.2.a



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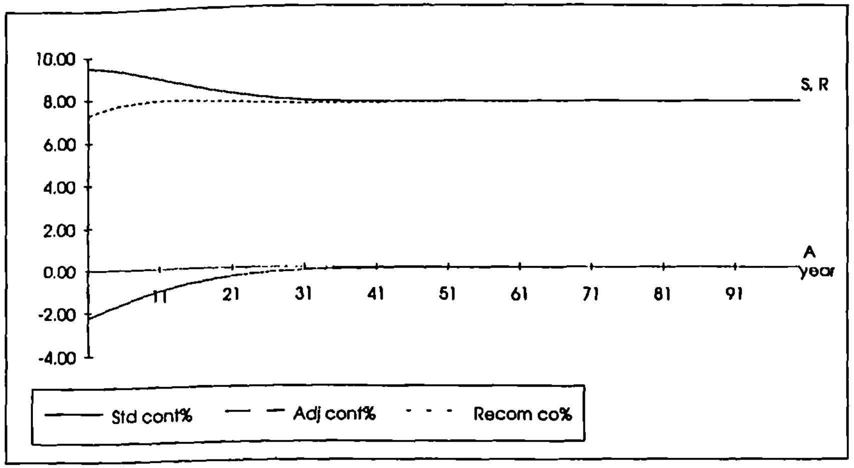

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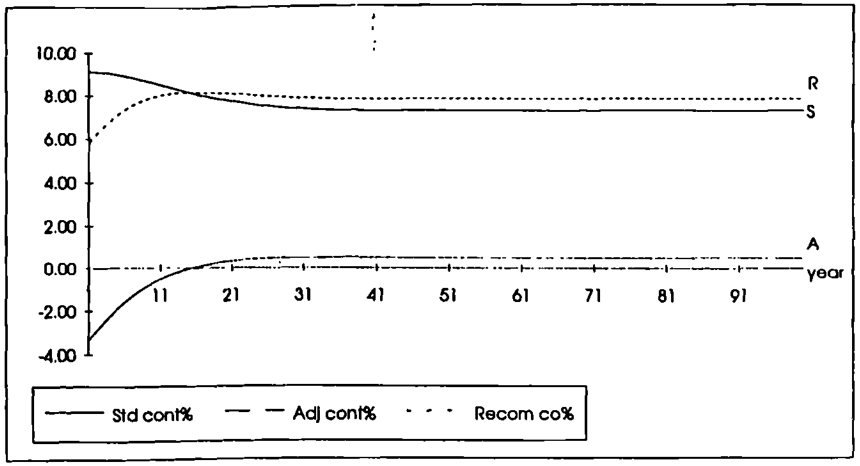
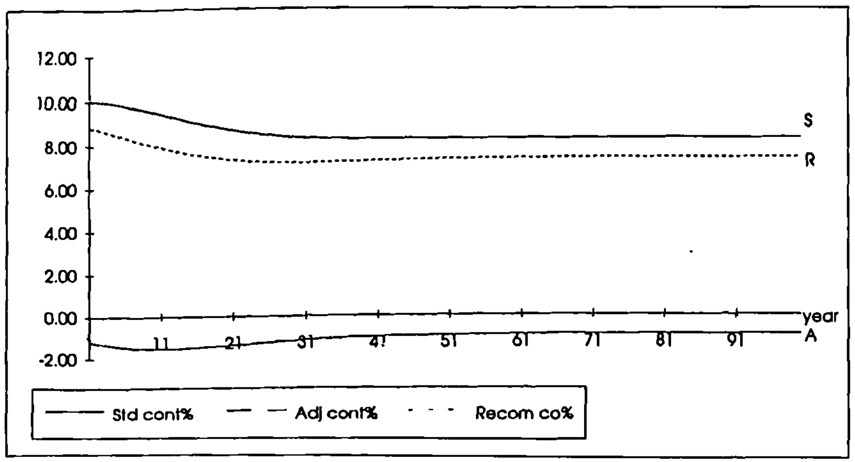
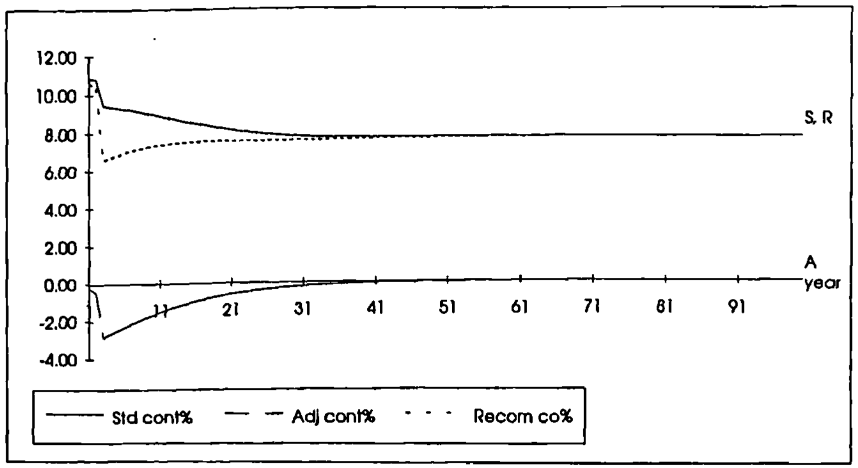


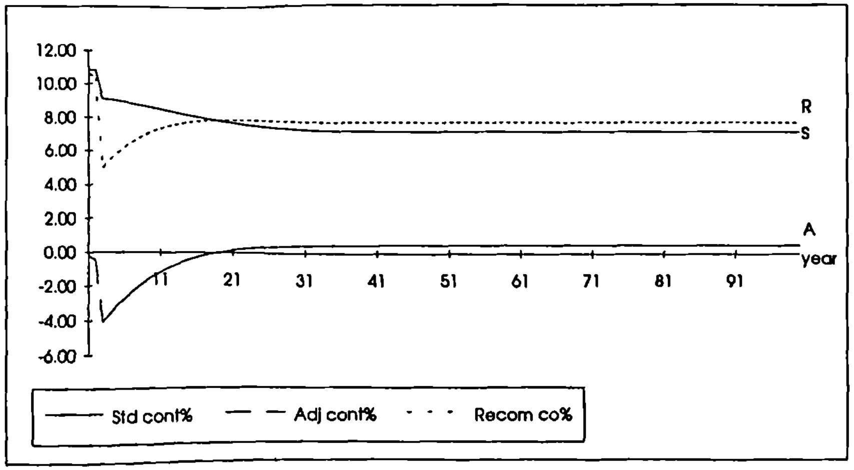
Graph A3.3.2.d



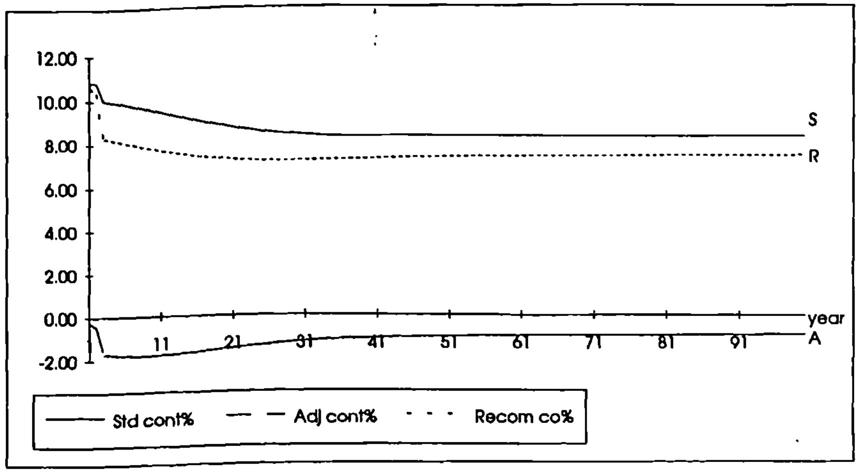
Graph A3.3.2.e



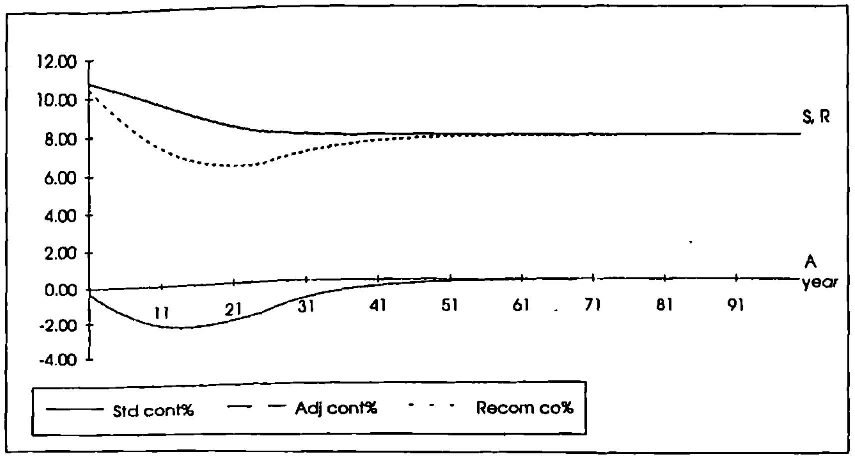
Graph A3.3.2.f



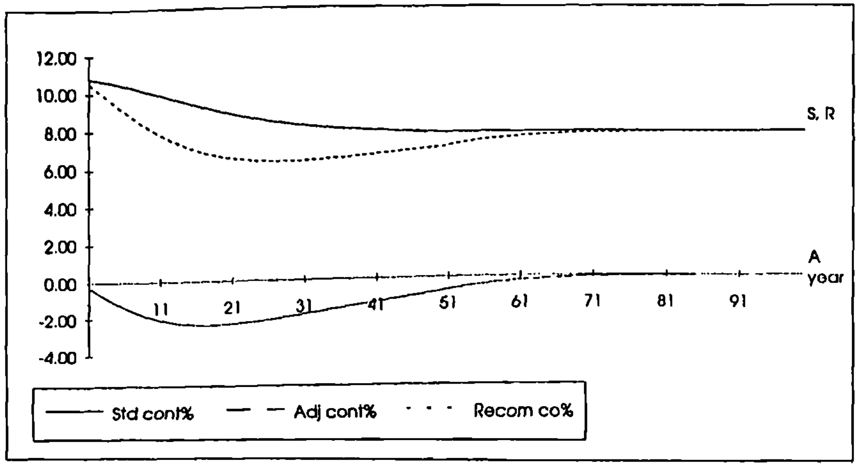
Graph A3.3.2.g



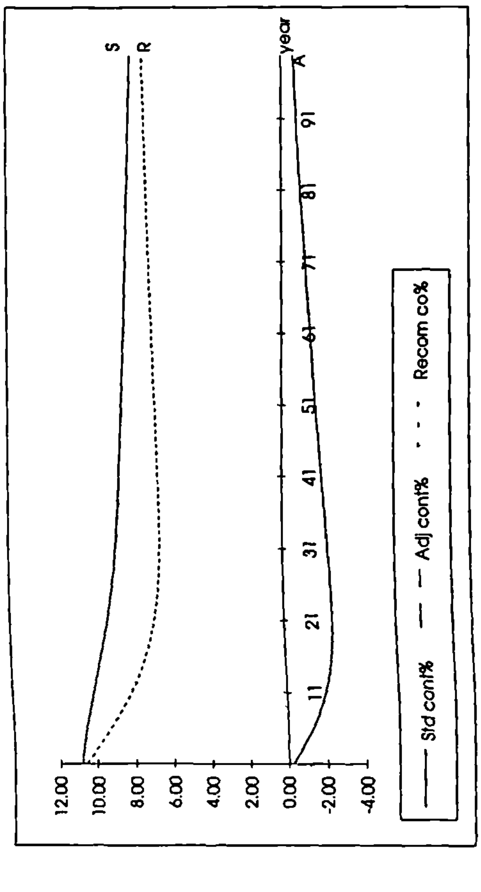
Graph A3.3.2.h



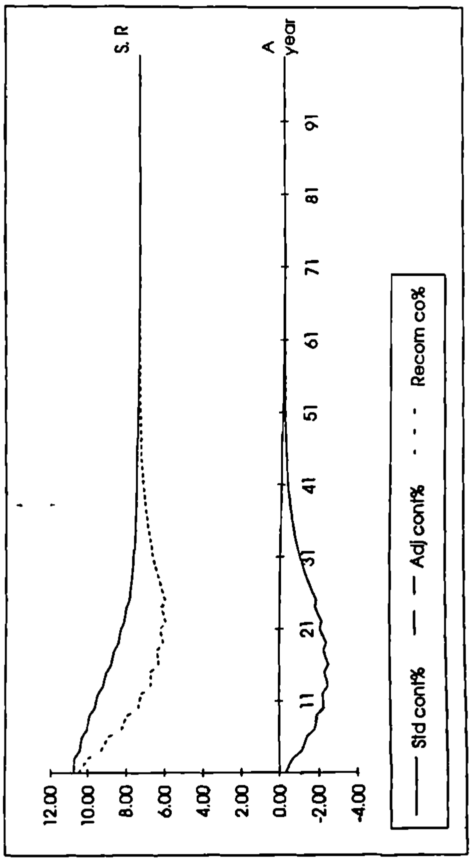
Graph A3.3.2.i



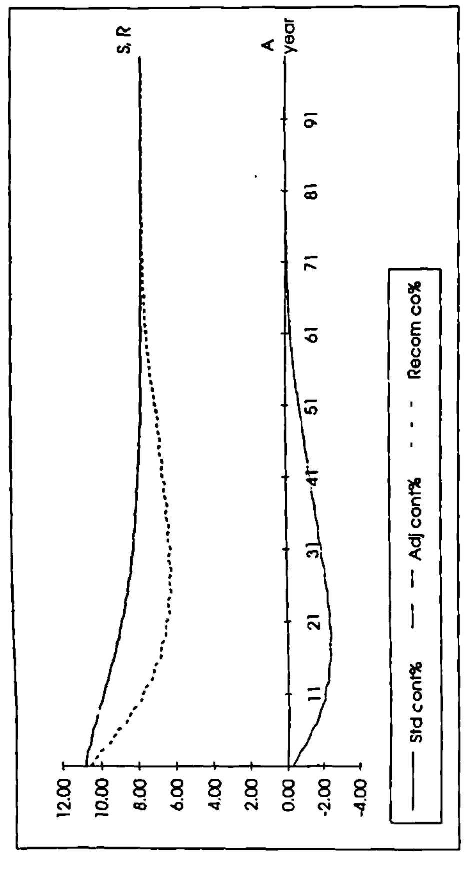
Graph A3.3.2.j



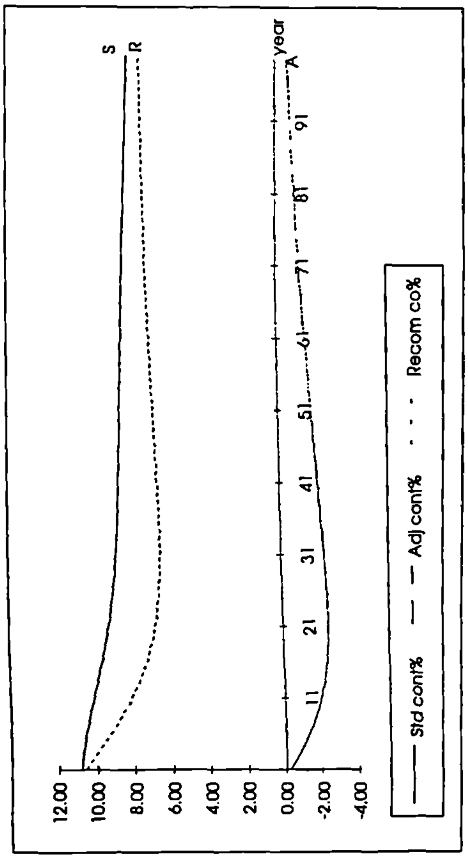
Graph A3.3.2.k



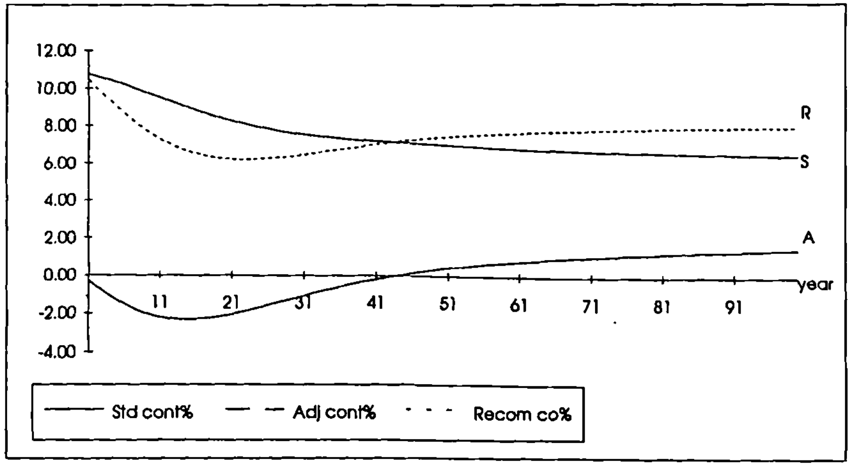
Graph A3.3.2.l



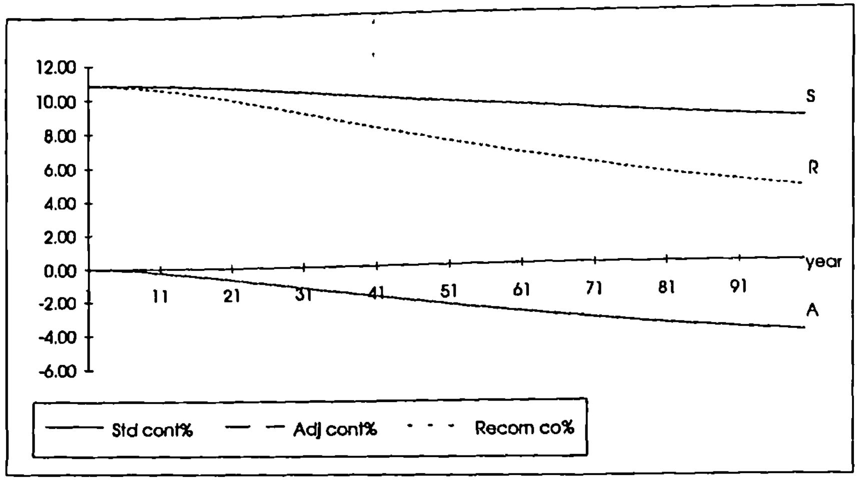
Graph A3.3.2.m



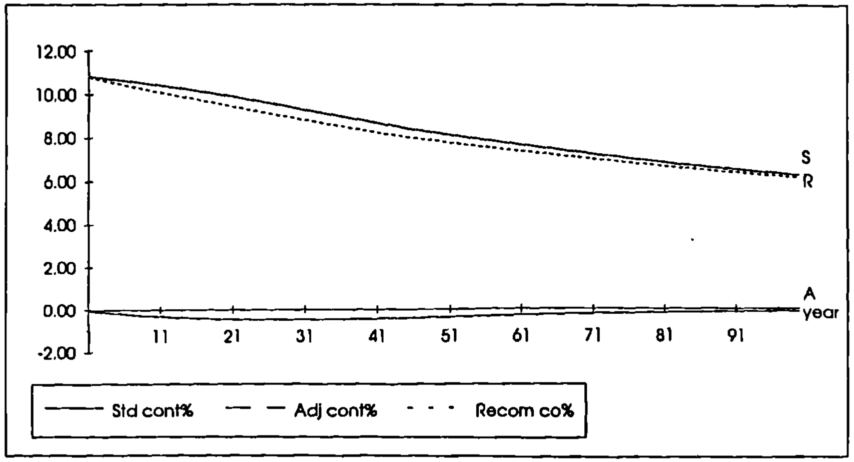
Graph A3.3.2.n



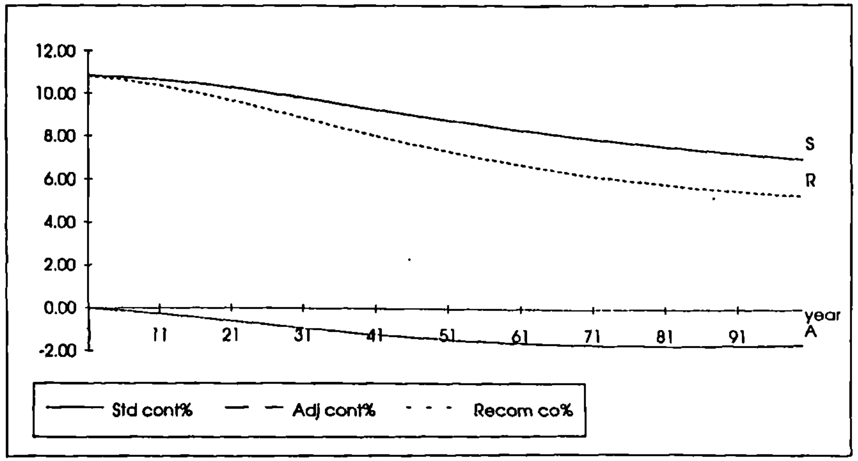
Graph A3.3.3.a



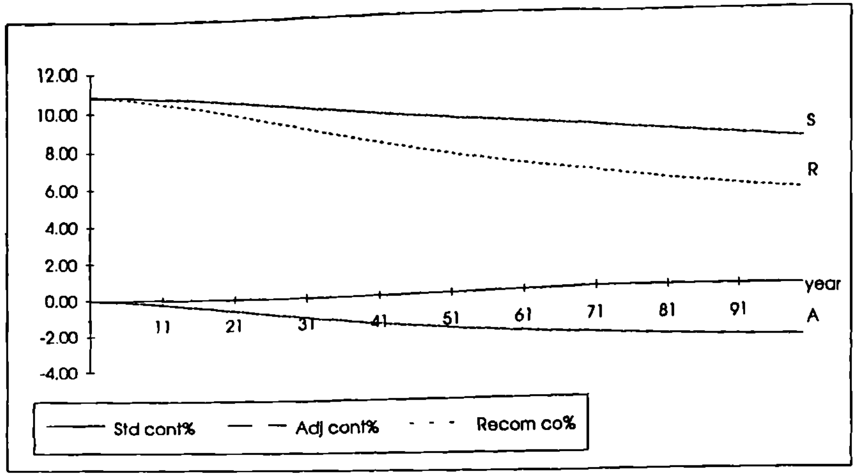
Graph A3.3.3.b



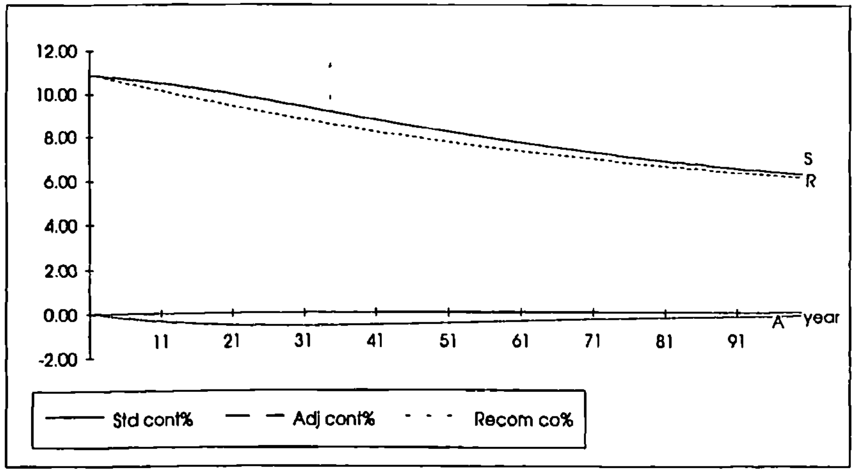
Graph A3.3.3.c



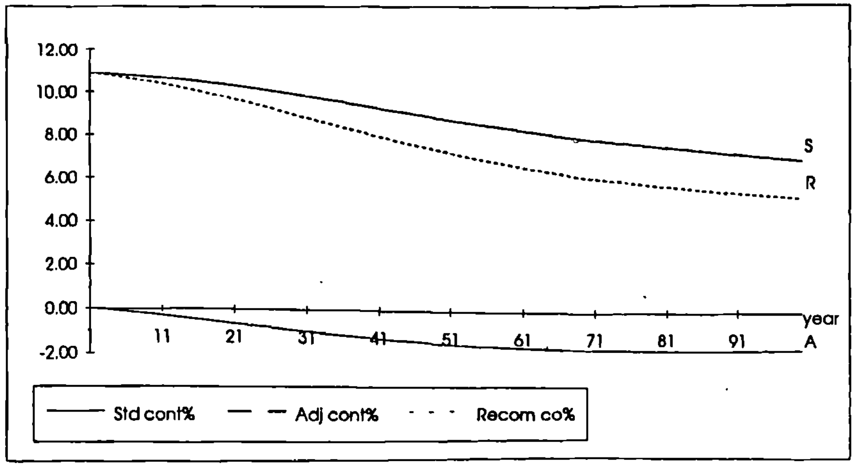
Graph A3.3.3.d



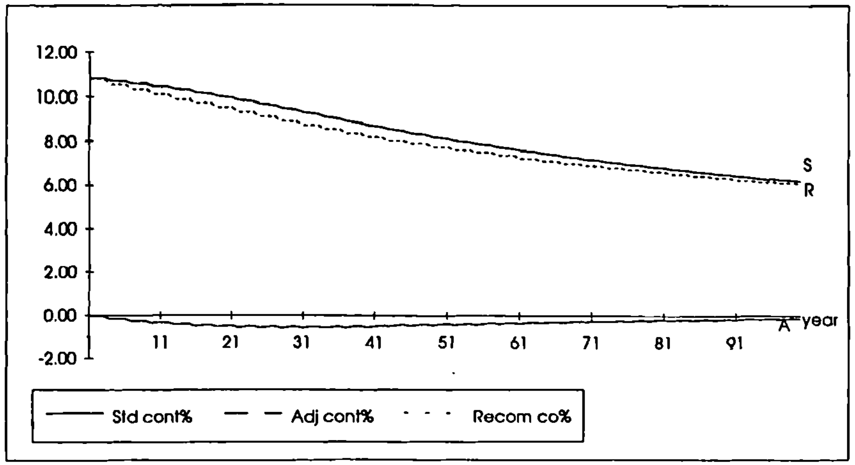
Graph A3.3.3.e



Graph A3.3.3.f



Graph A3.3.3.g



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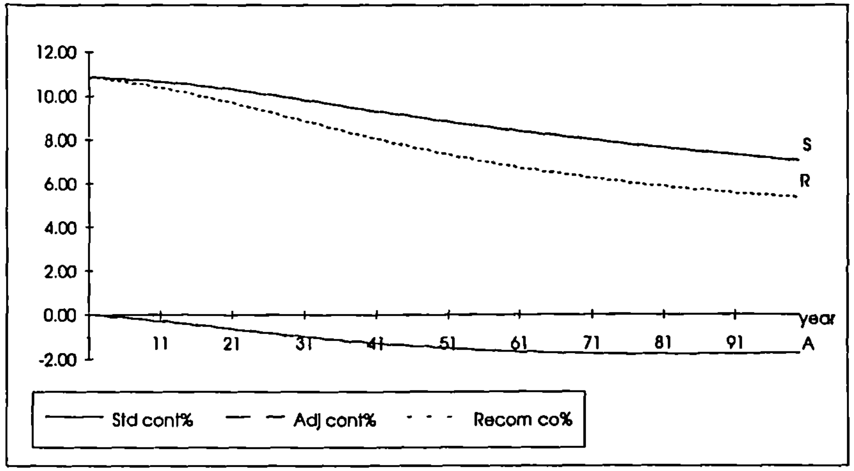

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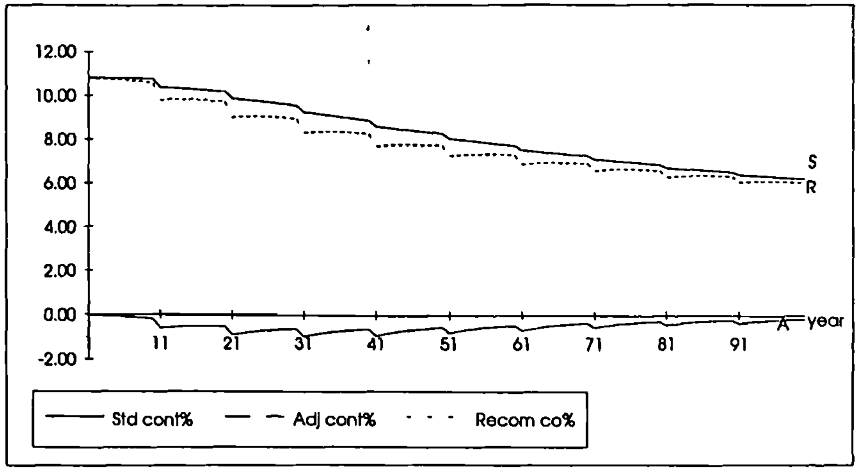

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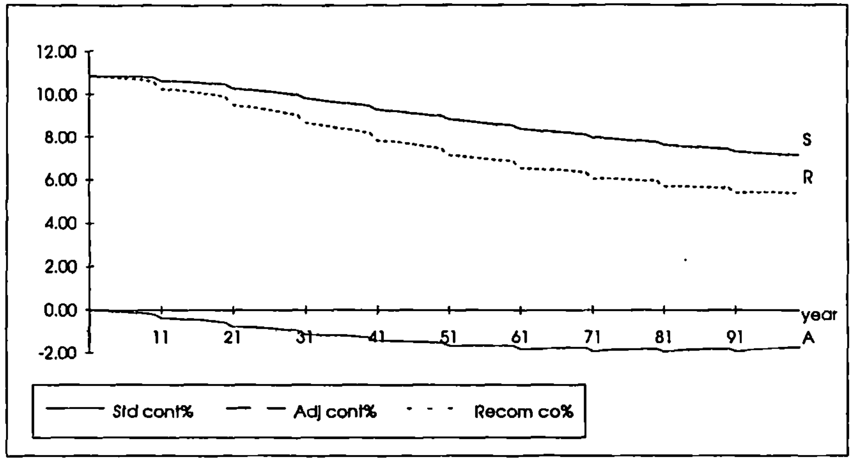
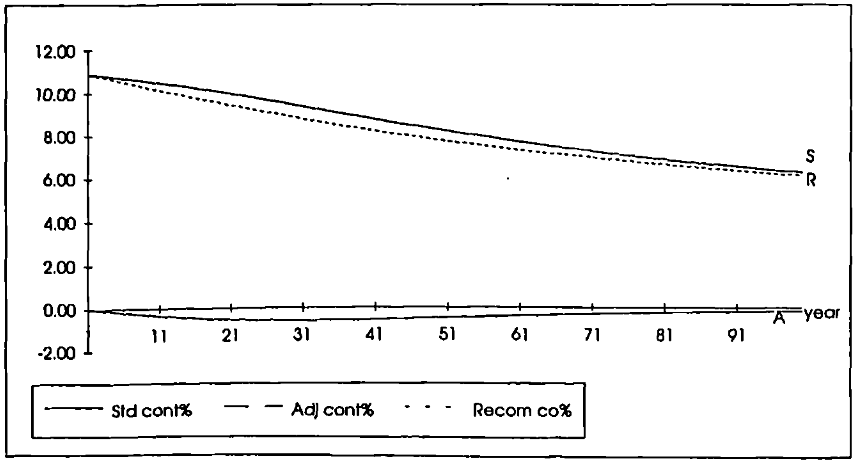
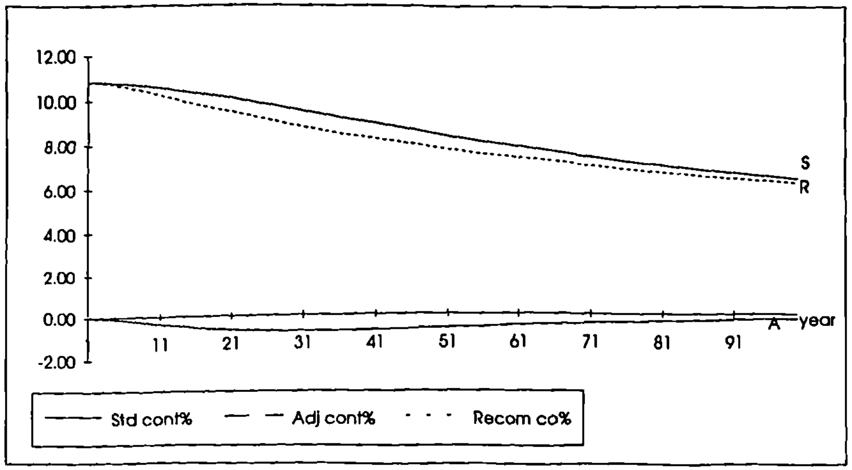


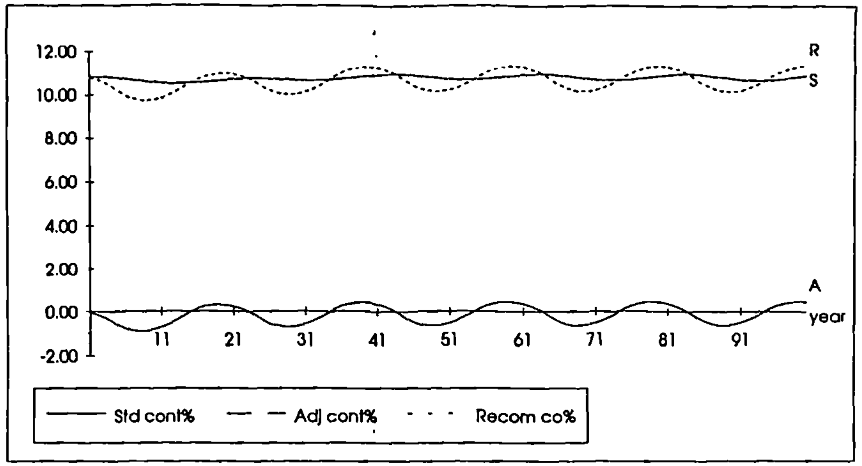
Graph A3.3.3.k



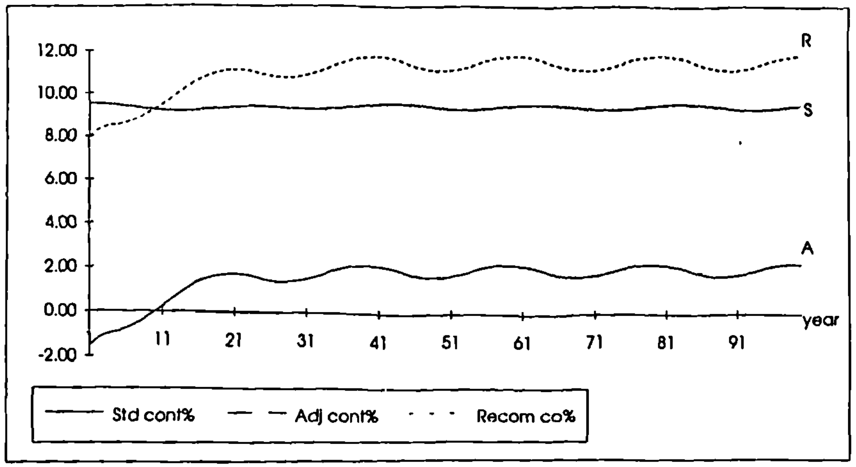
Graph A3.3.3.l



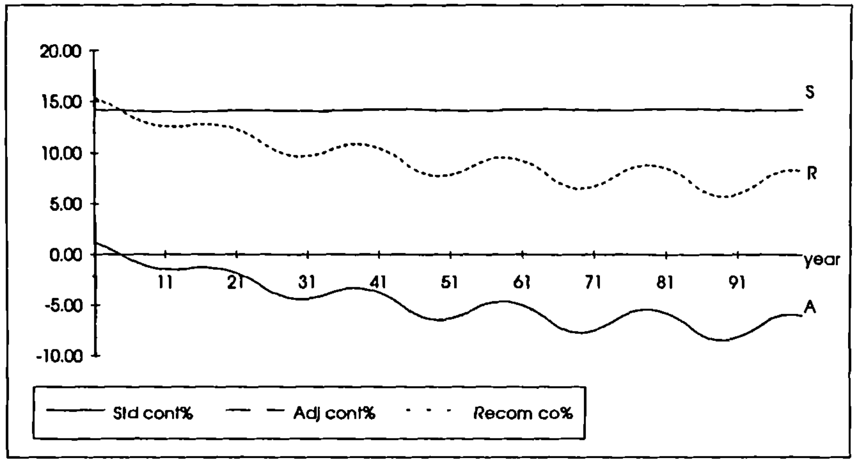
Graph A3.3.4.a



Graph A3.3.4.b



Graph A3.3.4.c



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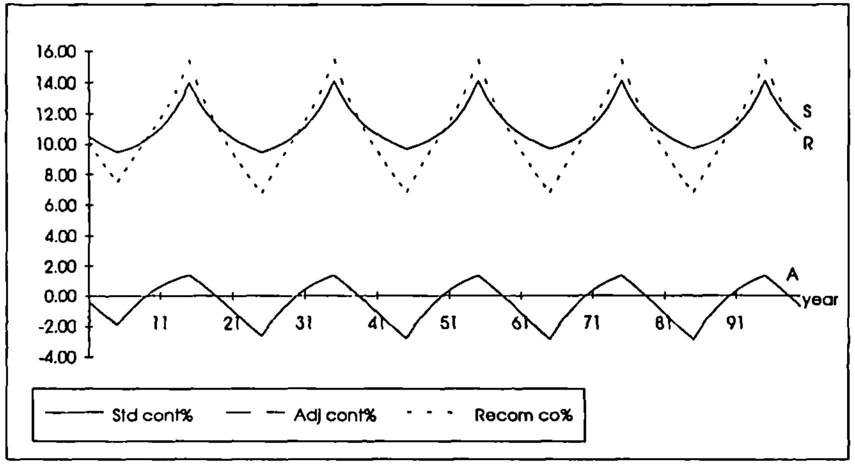

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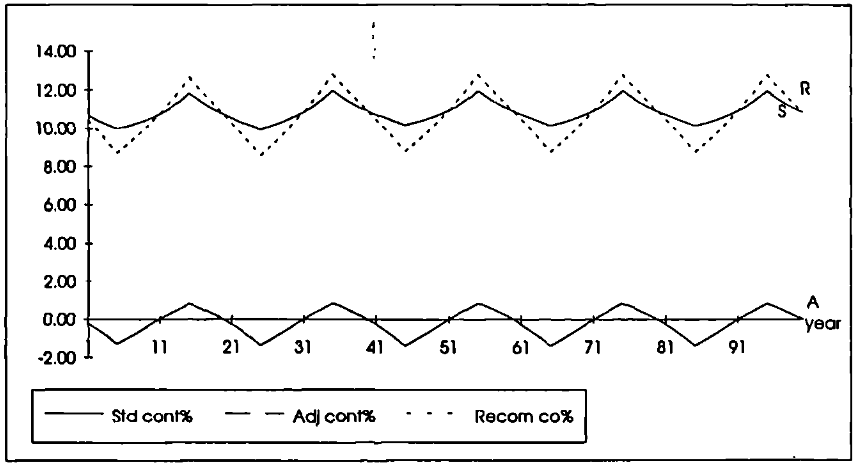
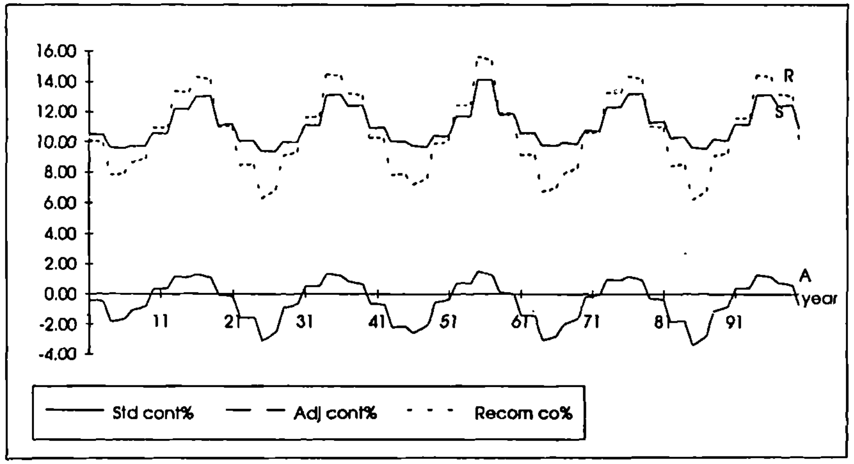
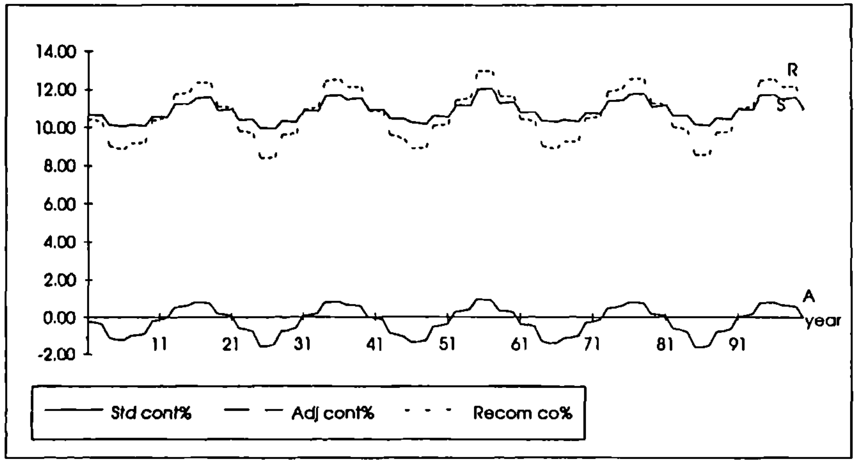


Graph A3.3.4.f



Graph A3.3.4.g



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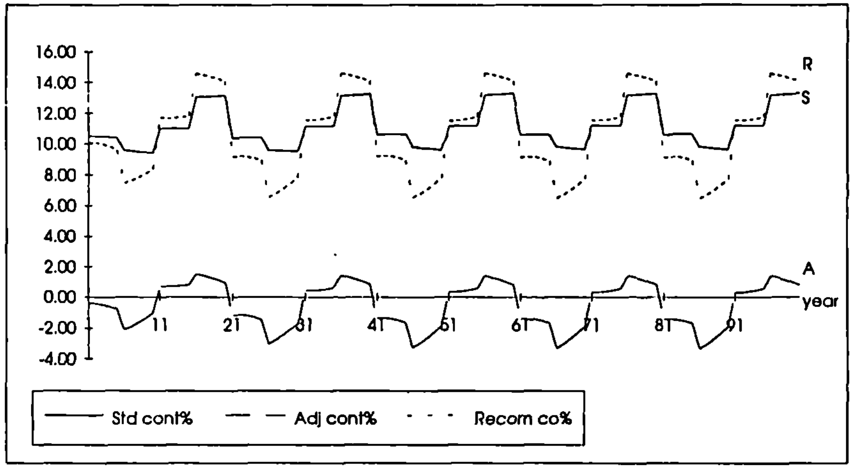

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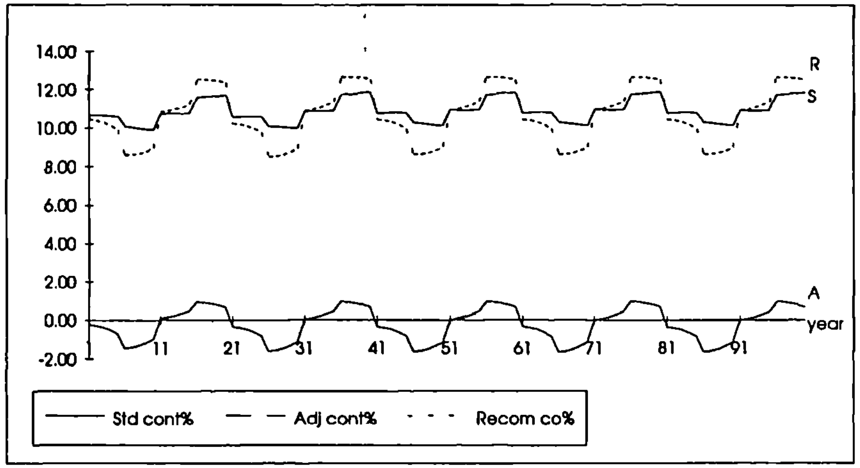
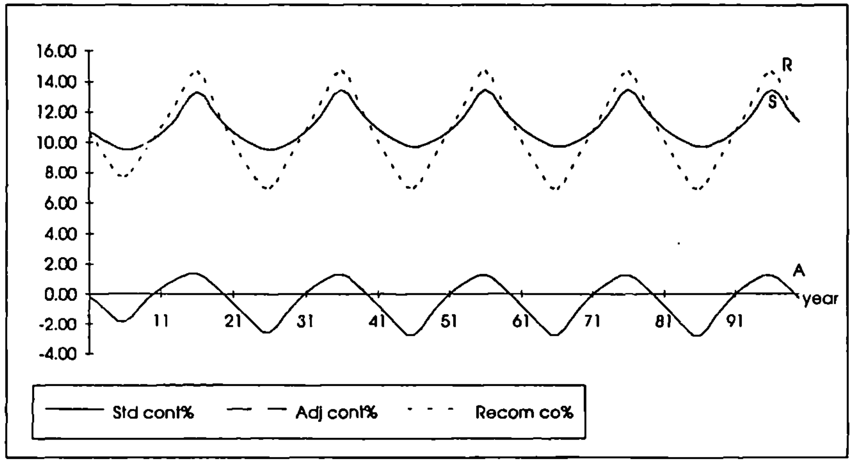


Graph A3.3.4.J



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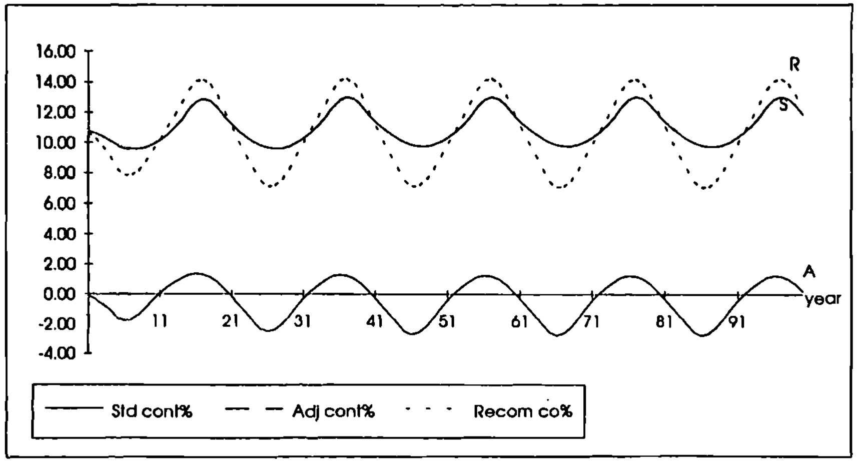

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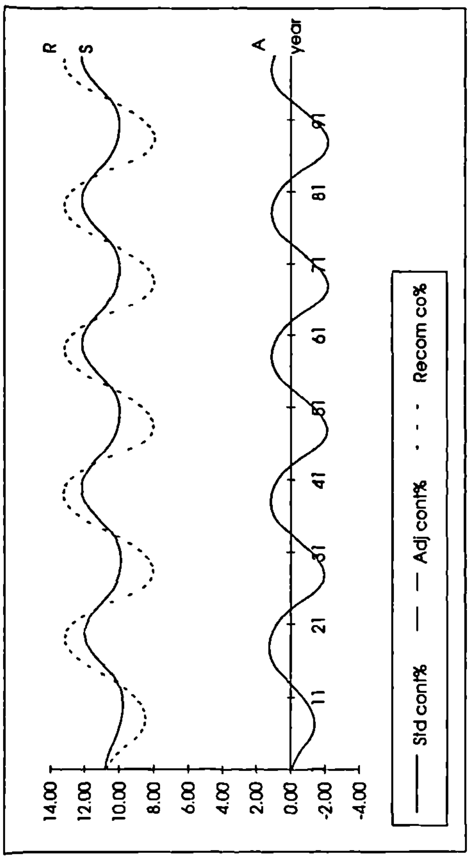

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