Price Expectations in Goods and Financial Markets

New Developments in Theory and Empirical Research

Edited by
François Gardes

Professor of Economics at the University of Paris I-Panthéon-Sorbonne, member of the research group TEAM, France

Georges Prat

Director of Research at the National Center of Scientific Research (CNRS), member of the research group MODEM, University of Paris-X Nanterre, France

Preface by
M. Hashem Pesaran

Edward Elgar
Cheltenham, UK • Northampton, MA, USA

Published by
Edward Elgar Publishing Limited
Glensanda House
Montpellier Parade
Cheltenham
Glos GL50 1UA
UK

Edward Elgar Publishing, Inc.
136 West Street
Suite 202
Northampton
Massachusetts 01060
USA

A catalogue record for this book
is available from the British Library

Library of Congress Cataloguing in Publication Data

Price expectations in goods and financial markets : new developments in theory and empirical research / edited by François Gardes, Georges Prat.
Includes bibliographical references and index.
1. Prices—Econometric models. 2. Rational expectations (Economic theory)
I. Gardes, François, 1950– II. Prat, Georges, docteur d'Etat en sciences économiques.

HB221 .P69 2000
338.5′2—dc21 99–087208

ISBN 1 84064 322 6

Printed and bound in Great Britain by MPG Books Ltd, Bodmin, Cornwall

8. Variance rationality: evidence from inflation expectations

Roy Batchelor and Firoozeh Zarkesh

INTRODUCTION

In a much-cited paper, Muth (1961) defines expectations as rational if:

> expectations of firms (or more generally the subjective probability distribution of outcomes) tend to be distributed for the same information set, about the prediction of the theory (or the 'objective' probability distribution of outcomes).

More formally, suppose that at time t forecaster i has information consisting of variables x_{it}, and makes a forecast for the value p_{t+k} of some variable k periods in the future in the form of a probability distribution $p^e_{it,k}$ with mean $\mu_{it,k}$ and variance $\sigma^2_{it,k}$. Then the subjective probability distribution is rational in the sense of Muth if it is identical to the objective conditional probability distribution for p_{t+k}, with conditioning done on the information set x_{it}.

In this paper we further define $p^e_{it,k}$ as 'mean-rational' if $\mu_{it,k}=E\{p_{t+k}|x_{it}\}$, and as 'variance-rational' if $\sigma_{it,k}=E\{|p_{t+k}-\mu_{it,k}|x_{it}\}$. Muth-rationality is sufficient for a forecast to be both mean and variance rational. Mean-rationality and variance-rationality are necessary but not sufficient for a forecast to be Muth-rational. Mean-rationality and variance-rationality respectively imply that in:

$$p_{t+k}-\mu_{it,k}=\epsilon_{it,k} \tag{8.1}$$

$$|\epsilon_{it,k}|-\sigma_{it,k}=\eta_{it,k} \tag{8.2}$$

the errors $\epsilon_{it,k}$ and $\eta_{it,k}$ in subjective assessments of the mean and standard deviation of the forecast variable must satisfy the unbiasedness and orthogonality conditions:

$$E(\epsilon_{it,k})=0,\ \mathrm{Cov}(\epsilon_{it,k}, x_{it,k})=0 \tag{8.3}$$

$$E(\eta_{it,k})=0,\ \mathrm{Cov}(\eta_{it,k}, x_{it,k})=0 \tag{8.4}$$

Many empirical tests of the mean-rationality conditions (8.3) have been conducted, using published forecasts as estimates of the means of individual forecasters' probability distributions for future values of economic variables, including McNees (1978); Figlewski and Wachtel (1981); Pearce (1984); Urich and Wachtel (1984); Zarnowitz (1985); and Swidler and Ketcher (1990). Most recently, Keane and Runkle (1990) have shown that errors in one-quarter of ahead price level forecasts made by the professional economists contributing to the ASA–NBER surveys pass stringent orthogonality tests.

In contrast, few tests of variance-rationality have been conducted. This is not because the question of whether individuals are variance-rational is unimportant. Economic theory suggests that, in general, consumers and firms will change their behaviour in response not only to changing mean expectations but also to changes in uncertainty, as measured by the variance surrounding these means (see, for example, the literature reviewed in Hey, 1979). Econometric practice is to maintain the rationality assumption in order to obtain empirical proxies for uncertainty. Cukierman and Wachtel (1982), for example, use (8.2) and (8.4) to argue that the variance of expected inflation can be proxied by the mean squared error in the inflation forecasts made by contributors to the Livingston survey of economic forecasters. Engle (1982, 1983) assumes a particular information set x_{it}, and argues that inflation uncertainty can be proxied by a time-varying conditional variance of a model relating inflation to these variables, the ARCH model. However, it has been difficult to obtain measures of subjective variances on which to test the validity of these procedures. Batchelor and Jonung (1988, 1989) do conduct tests for variance rationality, but their variance measure is simply a qualitative score, measuring the 'degree of confidence' of lay individuals about their inflation forecasts.

In this paper, we use quantitative estimates of the mean and variance of the inflation forecasts of professional forecasters to test the bias and orthogonality conditions (8.4) for variance-rationality, as well as the mean-rationality conditions (8.3). The variance estimates come from responses to the ASA–NBER surveys, and are described in section 8.1. Because the data consist of pooled cross-section and time series information on forecasts for various forecast horizons, and because they contain measurement error, care is necessary in making inferences about rationality. Section 8.2 describes the econometric techniques used. Section 8.3 analyses the test results. Mean rationality is rejected for long-horizon but not short-horizon forecasts. Variance rationality is strongly rejected at all horizons. There is no significant correlation between a forecaster's estimate of the subjective variance surrounding an inflation forecast and the accuracy of his/her mean forecast. Implications of this finding are discussed in the concluding section of the paper.

8.1 ESTIMATING THE VARIANCE OF INFLATION FORECASTS

Every quarter since 1968Q4, the American Statistical Association (ASA) and the National Bureau of Economic Research (NBER) have conducted a survey of the outlook for various US macroeconomic variables, by means of a mail questionnaire among a panel of economic forecasters. The survey is described in Zarnowitz (1969), and the accuracy and rationality of aggregate and individual price level forecasts are analysed in Zarnowitz (1985).

A unique feature of the ASA–NBER survey is that forecasters are invited not only to provide point predictions of price levels for the current and five following quarters, but also a summary probability distribution for future annual inflation rates. The precise question asked about the probability distribution of inflation has changed a little over time. Initially, forecasters were asked:

> Please indicate what probabilities you would attach to the following percentage changes from 1968 to 1969 in . . . the implicit price deflator (annual figures):
>
> 10 per cent or more/9 to 9.9 per cent/8 to 8.9 per cent/ . . . 0 to 0.9 per cent/−1 to −0.1 per cent/ . . . /below −3 per cent.

As mean expected inflation rose in the 1970s, these ranges were all raised by 2 per cent in 1973Q2 and again by 4 per cent in 1974Q4. In 1981Q3 the number of ranges was halved, their size doubled, and levels reduced by 2 per cent, so that possible responses became:

> 12 per cent or more/10 to 11.9 per cent/8 to 9.9 per cent/6 to 7.9 per cent/4 to 5.9 per cent/rise less than 4 per cent.

The ranges were further reduced by 2 per cent in 1985Q2.

To estimate the mean and variance of inflation forecasts from responses to the survey questions, assumptions must be made about the limits of the highest and lowest response ranges, and about the form of the subjective probability distributions. We follow Zarnowitz and Lambros (1987) in assuming that extreme ranges have a width of 2 per cent, and that probability is uniform within each range. Lahiri and Teigland (1987) compute means, variances and higher moments using slightly different assumptions,

but note that their results are not sensitive to alternative assumptions about the little-used extreme ranges. We additionally apply Sheppard corrections to all variance estimates to compensate for the effect of the change in the width of the central ranges after 1981Q3.

Suppose that the upper and lower bounds of the jth range in the survey conducted in quarter t are written U_{jt} and L_{jt}; that the width of the central ranges is W_t; and that the probability provided by forecaster i at time (quarter) t for the inflation rate in the year ending quarter $t+k$ lying in the jth range is $P_{ijt,k}$. Our estimates of the mean and variance of forecaster i's subjective probability distribution for inflation are:

$$\mathrm{MEAN}_{it,k} = \sum_{j=1}^{r} P_{ijt,k} \cdot \frac{(U_{jt} - L_{jt})}{2} \tag{8.5}$$

$$\mathrm{VAR}_{it,k} = \sum_{j=1}^{r} P_{ijt,k} \cdot \frac{(U_{jt}^3 - L_{jt}^3)}{3(U_{jt} - L_{jt})} - \mathrm{MEAN}_{it,k}^2 - W_t^2/12 \tag{8.6}$$

where $r=12$ or 6, and $W_t=2$ or 4, depending on whether data are from surveys before or after 1981Q3. The subjective standard deviation of inflation is estimated by the square root of $\mathrm{VAR}_{it,k}$, and is denoted $\mathrm{SDEV}_{it,k}$. $\mathrm{MEAN}_{it,k}$ and $\mathrm{SDEV}_{it,k}$ are estimates of the expected value and standard deviation $\mu_{it,k}$ and $\sigma_{it,k}$ of the subjective probability distribution for future inflation.

Because they depend on possibly incorrect assumptions about the form of the probability distribution, these estimates may contain measurement error. In addition, because incentives on forecasters to supply accurate figures to the ASA–NBER survey are weak, the estimates may not represent the best that forecasters can do. This is unlikely to be a problem for the mean forecasts, since respondents typically also publish their mean inflation forecasts in a commercial environment, so their accuracy can be easily monitored. On the other hand, the variance estimates are not subject to any similar market test, and so may contain more noise.

In the surveys conducted in the first three quarters of each year up to 1981, the surveys asked for forecasts of the average rate of inflation between the previous year and the current year. In the fourth quarter forecasts switched to inflation between the current year and the following year. This means that the maximum forecast horizon k shrinks from four quarters (forecast made in Q4 for the following year) to one quarter (forecast made in Q3 for the current year). From 1981Q3 onwards, forecasts are sought for both the current and the following year, so that the forecasting horizon shrinks from seven quarters to zero quarters. In this paper, we consider only one- to four-quarter horizons, for which data are available throughout the history of the survey.

The number of forecasters providing probability distribution information has fallen steadily: from over 60 in the early years of the survey to less than 20 in recent years. The ASA–NBER survey was discontinued in the first quarter of 1990 and responsibility for the survey passed to the Federal Reserve Bank of Philadelphia. Details of the survey are given at their website http:///www.phil.frb.org. Our data end with the 1989Q4 forecasts of inflation between 1988 and 1989. As a result, our data consist of 3361 mean and variance estimates, covering the 21 target years 1969–89. Of these, 815 are one-quarter ahead forecasts, 833 are two-quarter ahead, 790 three-quarter ahead and 923 four-quarter ahead. Because the composition of the panel changes over time and not all panellists provide probability distribution data, it is not possible to compile complete time series on individual forecasters.

Questionnaires are mailed out towards the end of the first month of each quarter, and are typically returned towards the end of the second month. This means that when they complete the survey in the first quarter of each year, respondents know the preliminary estimate of inflation in the previous year, released in mid-February. We assume that forecasters are judged on their ability to predict this first available estimate of inflation in the GNP deflator rather than any revised estimate. Inflation between year ending with quarter $t+k-4$ and target year $t+k$ is denoted INFL_{t+k}.

8.2 ECONOMETRIC ISSUES IN RATIONALITY TESTING

Our rationality tests are based on the properties of errors in the mean and standard deviations of subjective probability distributions. We write the error in the mean forecast made by individual i at time t for horizon k as $\text{MERR}_{it,k} = \text{INFL}_{t+k} - \text{MEAN}_{it,k}$ and the error in the subjective standard deviation as $\text{SDERR}_{it,k} = |\text{MERR}_{it,k}| - \text{SDMEAN}_{it,k}$. Bias and orthogonality in these errors is tested by conducting three regressions with $\text{MERR}_{it,k}$ as dependent variable, and three parallel regressions with $\text{SDERR}_{it,k}$ as dependent variable. The regressions are:

Mean rationality:

$$\text{MERR}_{it,k} = a + e_{it,k} \tag{8.7}$$

$$\text{MERR}_{it,k} = a + b\text{MEAN}_{it,k} + c\text{SDEV}_{it,k} + e_{it,k} \tag{8.8}$$

$$\text{MERR}_{it,3} = a + b\text{MERR}_{it-2,1} + c\text{SDERR}_{it-2,1} + e_{it,k} \tag{8.9}$$

Variance rationality:

$$\text{SDERR}_{it,k} = a + h_{it,k} \tag{8.7$'$}$$

$$\text{SDERR}_{it,k} = a + b\text{MEAN}_{it,k} + c\text{SDEV}_{it,k} + h_{it,k} \tag{8.8$'$}$$

$$\text{SDERR}_{it,3} = a + b\,|\text{MERR}_{it-2,1}| + c\text{SDERR}_{it-2,1} + h_{it,k} \tag{8,9$'$}$$

Regressions (8.7) and (8.7′) test the unbiasedness conditions $\text{E}(\epsilon_{it,k}) = 0$ and $\text{E}(\eta_{it,k}) = 0$. Under rationality, we expect $a = 0$ in (8.7) and (8.7′). Note that if expectations are rational, the regression residuals $e_{it,k}$ and $h_{it,k}$ are estimates of the forecast errors $\epsilon_{it,k}$ and $\eta_{it,k}$ in the subjective mean and standard deviation of expections.

Regressions (8.8) and (8.8′) test the orthogonality of errors in subjective expectations of the mean and standard deviation of future inflation with respect to two variables which are certainly in forecaster i's information vector x_{it} at t – namely, the levels of the subjective mean and standard deviation themselves. Under the rationality conditions $\text{cov}\{\epsilon_{it}, (\mu_{it,k}, \sigma_{it,k})\} = 0$ and $\text{cov}\{\eta_{it,k}, (\mu_{it,k}, \sigma_{it,k})\} = 0$ we expect $a = b = c = 0$ in (8.8) and (8.8′) respectively. Regression (8.8) can be regarded as a generalization of the conventional test for mean-rationality in which the actual level of inflation is regressed on the mean expectation, as:

$$\text{INFL}_{it,k} = a + (1+b)\text{MEAN}_{it,k} + \text{e}_{it,k} \tag{8.10}$$

and the coefficents tested for $a = b = 0$. Although often termed an 'unbiasedness test' (for example, Brown and Maital, 1981), (8.10) actually tests for orthogonality between the forecast error in the mean and the level of the subjective mean, as can be seen if it is rearranged as:

$$\text{MERR}_{it,k} = a + b\text{MEAN}_{it,k} + \text{e}_{it,k} \tag{8.11}$$

Our regression (8.8) adds another known parameter of the subjective probability distribution, the subjective standard deviation, to the right-hand side of (8.11).

Regressions (8.9) and (8.9′) test the orthogonality of errors in the subjective mean and standard deviation with respect to past expectations errors. Since past errors made in forecasting are first revealed when the preliminary inflation figures are published in the first quarter of each year, (8.9) and (8.9′) regress errors in the mean and standard deviations of forecasts made in the first quarter of each year, $\text{MERR}_{it,3}$ and $\text{SDERR}_{it,3}$, on known errors in the most recent forecasts made for inflation in the previous year, the one-quarter ahead forecasts made in the third quarter of the previous year, $\mathrm{MERR}_{it-2,1}$ and $\mathrm{SDERR}_{it-2,1}$. Since the subjective standard deviation is likely to be influenced by the size of any past errors, irrespective of their sign, the absolute value of $\mathrm{MERR}_{it-2,1}$ is used as a regressor in (8.9′). Since not all forecasters provide probability distribution information in all quarters, (8.9) and (8.9′) are estimated on a sub-sample of 437 observations of forecasters who provided such data in both the first quarter of one year and the third quarter of the previous year. Under the rationality conditions $\mathrm{cov}\{\epsilon_{it,3}, (\epsilon_{it-2,1}|, \eta_{it-2,1})\} = 0$ and $\mathrm{cov}\{\eta_{it,3}, (|\epsilon_{it-2,1}|, \eta_{it-2,1})\} = 0$, we expect $a = b = c = 0$ in (8.9) and (8.9′) respectively.

Inferences about the coefficients of regressions (8.7)–(8.9) and (8.7′)–(8.9′) are complicated by two main factors: non-independence and heteroscedasticity of the residuals, and measurement error in the regressors.

As we have seen, under rationality the regression residuals are forecast errors in the mean and standard deviation of expectations. These forecast errors are likely to be correlated across forecasts made for the same target date but at different horizons, and across forecasts made by different forecasters for a given target date. Under these conditions, the conventional OLS estimate of the variance–covariance matrix of the regression coefficients is biased and inconsistent. Because some years may be easier to forecst than others, forecast errors for different target dates may also be heteroscedastic.

We have eliminated the problem of overlapping forecast horisons by estimating (8.7) and (8.8), (8.7′) and (8.8′) separately on data for different forecast horizons. In general, this is undesirable since it reduces the efficiency of estimates of the regression coefficients. However, in our particular data set, the hypothesis that coefficients are equal at all horizons is rejected, so there is no justification for pooling.

We have dealt with the problem of correlations in errors across forecasters and with the possibility of heteroscedasticity by estimating the coefficients of all regressions by OLS, but estimating the variance–covariance matrix of the coefficients by a special generalized method of moments (GMM) estimator which Hansen (1982) has shown to be autocorrelation- and heteroscedasticity-consistent. This approach has been used in several earlier rationality tests, including Brown and Maital (1981) and Keane and Runkle (1990). The estimator has the general form:

$$\mathrm{V} = (\mathrm{X}'\mathrm{X})^{-1}\mathrm{X}'\mathrm{C}\mathrm{X}(\mathrm{X}'\mathrm{X})^{-1} \tag{8.12}$$

where X is the matrix of observations on regressors. For our problem, C is a block-diagonal weighting matrix of the form:

$$\mathrm{C} = \begin{bmatrix} \mathrm{C}_1 & 0 & \dots & 0 \\ 0 & \mathrm{C}_2 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & \mathrm{C}_m \end{bmatrix} \tag{8.13}$$

and the submatrices $\mathrm{C}_1, \mathrm{C}_2, \dots, \mathrm{C}_m$ contain estimates of the variances and covariances of forecast errors made by forecasters for the m (=21) target years covered by this study.

These variances and covariances are estimated using the OLS residuals $_{eit,k}$ from (8.7)–(8.9), or $_{hit,k}$ from (8.7′)–(8.9′), as follows. We assume that the error in an individual's k-quarter forecast made at t for the year ending in quarter $t+k$ consists of independent individual-specific and common components, as $e_{it,k} = f_{it,k} + g_{it,k}$, with $\mathrm{var}(f_{it,k}) = \pi^2_{it,k}$, $\mathrm{var}(g_{t,k}) = \tau^2_{t,k}$ and $\mathrm{cov}(f_{it,k}, g_{t,k}) = 0$. That is, the error variances of individual k-horizon forecast errors made at t are all equal to $\pi^2_{t,k} + \tau^2_{t,k}$, but may vary with the forecast horizon and with the target year. Similarly, the covariances between all pairs of k-horizon forecasts made at t are all equal to $\tau^2_{t,k}$, but may vary with forecast horizon and target year. Estimates of the variances and covariances of errors in the forecasts based on the OLS residuals are then:

$$\hat{\pi}^2_{t,k} + \hat{\tau}^2_{t,k} = \sum_{i=1}^{n_{t,k}} (\hat{e}_{it,k})^2 / n_{t,k} \tag{8.14}$$

$$\hat{\tau}^2_{t,k} = \sum_{i=1}^{n_{t,k}} \sum_{j \neq i} (\hat{e}_{it,k})(\hat{e}_{jt,k}) / (n^2_{t,k} - n_{t,k}) \tag{8.15}$$

where $n_{t,k}$ is the number of forecasters producing probability distributions for inflation in the year ending $t+k$. Suppose the target year ending in quarter $t+k$ is indexed as T. Then the corresponding $(n_{t,k} \times n_{t,k})$ submatrix C_T of estimated forecast error variances and covariances in (8.13) is:

$$\mathrm{C}_\mathrm{T} = \begin{Bmatrix} \hat{\pi}^2_{t,k} + \hat{\tau}^2_{t,k} & \hat{\tau}^2_{t,k} & \dots & \hat{\tau}^2_{t,k} \\ \hat{\tau}^2_{t,k} & \hat{\pi}^2_{t,k} + \hat{\tau}_{t,k} & \ddots & \hat{\tau}^2_{t,k} \\ \hat{\tau}^2_{t,k} & \hat{\tau}^2_{t,k} & \dots & \hat{\pi}^2_{t,k} + \hat{\tau}^2_{t,k} \end{Bmatrix} \tag{8.16}$$

This generalizes the GMM covariance matrix estimator suggested by Keane and Runkle (1990) for pooled cross-section and time series observations, to the case where forecast errors are heteroscedastic.

If the coefficient vectors in (8.7)–(8.9), or (8.7′)–(8.9′), are written as a, then the rationality restrictions $a = 0$ can be tested using the result that, under this null, the statistic $a'\mathrm{V}a$ is distributed as χ^2_ℓ where ℓ is the number of coefficients.

As noted earlier, measurement error is liable to be present in our estimates of the mean and standard deviation of inflation forecasts, and hence of inflation forecast errors. OLS estimates of coefficients in regressions like (8.8), (8.9), (8.8′) and (8.9′) are therefore liable to be biased and inconsistent. Apart from the discussions in Lahiri (1981) and Pesaran (1987, ch 8.5), little account has been taken of the effects of measurement error in earlier tests of rationality. The general problem of obtaining consistent coefficient estimates in regressions such as (8.8) and (8.9), and (8.8′) and (8.9′), where two regressors are measured with error, is, however, treated in Maddala (1988, ch. 11.3). If measurement errors are independent, serially uncorrelated, and uncorrelated with the true values of the regressors, consistent coefficient estimates b^* and c^* are related to their OLS estimates $\hat{b}$ and $\hat{c}$ as:

$$(1-\rho^2-\lambda_b)b^*+\rho\lambda_c c^*=(1-\rho^2)\hat{b} \tag{8.17}$$

$$(1-\rho^2-\lambda_c)c^*+\rho\lambda_b b^*=(1-\rho^2)\hat{c} \tag{8.18}$$

where λ_b and λ_c are the ratios of the variance of measurement error to total variance of the regressors to which the coefficients b and c are attached, and ρ is the correlation between the two regressors.

Fortunately, an estimate of the size of measurement errors in our probability-distribution-based estimates of the mean inflation forecast can be obtained by comparing them with the point inflation estimates implied by the price level forecasts which are also provided by the ASA–NBER respondents. As noted by Zarnowitz and Lambros (1987), differences between the point inflation forecasts and the means of subjective probability distributions are small. Assuming that the price level forecasts represent the true means of forecasters' subjective probability distributions, the standard errors in the means of the probability distributions in our data are 0.38, 0.47, 0.45 and 0.53 per cent for the one-, two-, three- and four-quarter horizons respectively. These compare with standard deviations of probability-distribution-based mean forecasts of 2.27, 2.10, 2.26 and 2.25 per cent, giving estimates of λ_b of 0.02, 0.05, 0.04 and 0.05. For simplicity, we take $\lambda_b=0.05$. In view of our earlier argument that forecasters have fewer incentives to report their variance estimates accurately, we assume $\lambda_c=4\lambda_b=0.20$. The correlations ρ between regressors are -0.05, 0.10, 0.24 and -0.02 for the one-, two-, three- and four-quarter ahead versions of (8.8) and (8.8′), and -0.02 and 0.83 for (8.9) and (8.9′).

Finally, it is worth noting that the regressors in all our test equations are sub-sets of the whole information set x_{it} of forecaster i at t. The omission of some possible determinants of forecast errors will therefore bias coefficient estimates and standard errors. However, Abel and Mishkin (1983) show that the bias will be towards non-rejection of rationality. Hence if rationality is rejected for one of the sub-sets we consider, it will certainly be rejected for the whole information set.

8.3 RESULTS OF RATIONALITY TESTS

Table 8.1 contains the results of tests for mean rationality based on (8.7) and (8.8). The column headed 'Bias' sets out estimated values of the bias coefficient a in regression (8.7). Beneath each estimated coefficient is an estimated standard error based on the GMM covariance matrix V of (8.12) above. None of the estimated bias coefficients is significantly non-zero, so the hypothesis that mean forecasts are unbiased at all horizons cannot be rejected.

Table 8.1: Tests for mean rationality

Dependent variable	n	Bias	Coefficient on: Constant	$\mathrm{MEAN}_{it,k}$	$\mathrm{SDEV}_{it,k}$	χ^2_3
$\mathrm{MERR}_{it,1}$	815	−0.0497	0.8164	−0.1212	−0.2047	5.87
				−0.1303	−0.2564	
		(0.17)	(0.48)	(0.08)	(0.13)	
$\mathrm{MERR}_{it,2}$	833	−0.0165	0.6559	−0.0262	−0.5947*	15.12*
				−0.0117	−0.7452	
		(0.19)	(0.53)	(0.09)	(0.17)	
$\mathrm{MERR}_{it,3}$	790	0.2038	1.6600*	−0.1400	−0.7502*	21.51*
				−0.0967	−0.9507	
		(0.31)	(0.50)	(0.10)	(0.22)	
$\mathrm{MERR}_{it,4}$	923	0.1699	2.1763*	−0.2705*	−0.5988*	17.43*
				−0.2879	−0.7489	
		(0.33)	(0.58)	(0.09)	(0.17)	

Notes:

1 Results of tests for forecast bias, and for orthogonality of errors in mean forecasts with respect to subjective mean and standard deviation of forecast, based on text equations (8.7) and (8.8). Two sets of coefficient estimates are shown for (8.8), the first based on OLS, the second on OLS corrected for measurement error in the regressors, as in text equations (8.17) and (8.18).

2 Figures in parentheses beneath estimated coefficients are GMM estimates of coefficient standard errors.

3 The χ^2_3 statistic tests all coefficients in (8.8) against zero, and has a 5 per cent critical value of 7.81.

4 Statistics rejecting zero restrictions at the 5 per cent level are marked with an asterisk.

The rest of Table 8.1 sets out the estimated coefficients *a*, *b* and *c* in regression (8.7) of the mean forecast error on the mean and standard deviation of the subjective probability distribution. The first row shows OLS coefficient estimates. Beneath these are shown measurement-error-consistent coefficient estimates, based on (8.17) and (8.18) above. In parentheses beneath the coefficient estimates are estimated standard errors, based on the GMM covariance matrix.

The χ^2_3 statistics test the restriction that the OLS coefficients are jointly zero. They show that this restriction cannot be rejected for the one-quarter ahead horizon, but that it is rejected for longer forecast horizons. The main reason for rejection is that mean forecast errors are negatively correlated with forecaster's subjective standard deviations; that is, more confident forecasters tend to underestimate inflation, and vice versa. In addition, at the longest four-quarter horizon, there is negative correlation between mean forecast errors and the mean expected inflation rate. This constitutes 'bias' in the conventional sense of equation (8.10), with forecasters tending to underestimate inflation when inflation is expected to be high, and vice versa.

Because measurement error is small and collinearity between regressors low, our consistent estimates are close to the OLS estimates. More important, in all cases they suggest that measurement error has if anything biased the significant coefficient estimates towards zero. The presence of significant deviations of the OLS coefficients from the zero values expected under rationality cannot therefore be ascribed to measurement error.

These findings are less favourable to the mean rationality hypothesis than, but not inconsistent with, the results of previous work on the ASA–NBER forecasters. Keane and Runkle (1990) find that errors in short-term, one-quarter ahead, price level forecasts are unbiased, uncorrelated with the level of the mean forecast itself, and with known past errors and a range of published economic indicators. We also find short-term forecasts to be mean-rational. But at longer horizons, we find that errors in mean forecasts are not uncorrelated with an element of the forecasters' information set not considered by Keane and Runkle, namely the level of the standard deviation surrounding the mean forecast.

Table 8.2 contains results of the parallel regressions (8.7′) and (8.8′) testing for variance rationality. The first column shows the estimated bias in the subjective standard deviation of inflation forecasts, the difference between the mean absolute forecast error and the mean subjective standard deviation. For one- and two-quarter horizons, the bias is negative and very small, and not significantly non-zero. For the three-quarter horizon the bias is positive, but not significant. For the four-quarter horizon, the bias

Table 8.2: Tests for variance rationality

Dependent variable	n	Bias	Coefficient on: Constant	$\text{MEAN}_{it,k}$	$\text{SDEV}_{it,k}$	χ^2_3
$\text{SDERR}_{it,1}$	815	−0.0842	0.6696*	−0.0317	−0.6835*	113.86*
				−0.0424	−0.8550	
		(0.07)	(0.18)	(0.03)	(0.08)	
$\text{SDERR}_{it,2}$	833	−0.0201	0.5944*	0.0206	−0.8348*	139.69*
				0.0440	−1.0464	
		(0.09)	(0.23)	(0.05)	(0.08)	
$\text{SDERR}_{it,3}$	790	0.2086	0.8024*	0.0535	−0.9980*	186.71*
				0.1247	−1.2689	
		(0.19)	(0.28)	(0.86)	(0.11)	
$\text{SDERR}_{it,4}$	923	0.3714*	1.0190*	0.0056	−0.7587*	69.45*
				0.0019	−0.9485	
		(0.18)	(0.30)	(0.10)	(0.10)	

Notes:
1 Results of tests for forecast bias, and for orthogonality of errors in standard deviation forecasts, with respect to subjective mean and standard deviation of forecast, based on text equations (8.7′) and (8.8′). Two sets of coefficient estimates are shown for (8.8), the first based on OLS, the second on OLS corrected for measurement error in the regressors, as in text equations (8.17) and (8.18).
2 Figures in parentheses beneath estimated coefficients are GMM estimates of coefficient standard errors.
3 The χ^2_3 statistic tests all coefficients in (8.8) against zero, and has a 5 per cent critical value of 7.81.
4 Statistics rejecting zero restrictions at the 5 per cent level are marked with an asterisk.

is more positive still, and significant. The unbiasedness condition for variance rationality is therefore rejected at the longest forecast horizon.

The bias estimates show that for four-quarter ahead forecasts, forecasters tend to be overconfident about the likely size of their errors. In fact, not only are forecasters too confident relative to the actual errors they make in their four-quarter ahead forecasts, they are also on average just as confident about their four-quarter ahead forecasts as about their shorter-term forecasts. The mean subjective standard deviations for four-, three-, two- and one-quarter horizons in our sample are 0.89, 0.90, 0.88 and 0.84 per cent. However, the mean absolute errors in the mean forecast decrease uniformly as the forecast horizon decreases from four- to one-quarter, from 1.28 to 1.10, 0.86 and 0.75 per cent.

The rest of Table 8.2 sets out estimates of the coefficients in regression (8.8′), and tests them against zero. The χ^2_3 statistics show that this condition for variance rationality is very strongly rejected at all horizons. The coefficients b are not significantly non-zero, so there is no suggestion that forecasters who expect a high inflation rate are over- or underconfident about the accuracy of their forecasts. However, the coefficients c are all significantly negative. That is, forecasters who make a high estimate of the standard deviation surrounding their mean forecast typically overestimate the size of the absolute error in their mean forecast, and vice versa. In the cases of two-, three- and four-quarter ahead forecasts the hypothesis $c = -1$ cannot be rejected, suggesting that there is no relation whatever between forecasters' subjective standard deviations and the size of the errors in their mean forecasts.

In all the regressions reported in Table 8.2, measurement error can again be seen to bias the significant OLS coefficients towards zero. Our finding of deviations from variance rationality cannot therefore be ascribed to measurement error.

Table 8.3 sets out the results of tests for the orthogonality of errors in both subjective mean and variance estimates with respect to known past errors, based on regressions (8.9) and (8.9′). Although the estimated coefficient c in (8.9) is significantly non-zero, the χ_3^2 statistic does not reject the hypothesis that all coefficients are jointly zero at the 5 per cent significance level. The mean rationality hypothesis cannot therefore be

Table 8.3: Tests for error orthogonality with respect to past errors

Dependent variable	n	Coefficient on: Constant	$MERR_{it-2,1}$	$\lvert MERR_{it-2,1}\rvert$	$SDERR_{it-2,1}$	χ_3^2
$MERR_{it,3}$	437	0.1253	0.0875		0.2909*	7.43
			0.0936		0.3638	
		(0.34)	(0.18)		(0.15)	
$SDERR_{it,3}$	437	0.7128*		−0.5762*	0.7826*	35.74*
				−2.7275	3.2103	
		(0.29)		(0.16)	(0.18)	

Notes:
1 Results of tests for orthogonality of forecast errors with respect to known past forecast errors, based on text equations (8.9) and (8.9′). Two sets of coefficient estimates are shown, the first based on OLS, the second on OLS corrected for measurement error in the regressors, as in text equations (8.17) and (8.18).
2 Figures in parentheses beneath estimated coefficients are GMM estimates of coefficient standard errors.
3 The χ_3^2 statistic tests all coefficients in (8.8) against zero, and has a 5 per cent critical value of 7.81.
4 Statistics rejecting zero restrictions at the 5 per cent level are marked with an asterisk.

rejected. However, both coefficients in (8.9′) are significant, and the χ_3^2 statistic strongly rejects variance rationality. Although the effects of measurement error on the regression (8.9′) are large, they again imply that use of OLS biases the tests against rejecting rationality.

Our estimates of (8.9′) show that forecasters are prey to two systematic errors in revising their subjective standard deviations for future inflation. The negative b coefficient means that a forecaster's estimate of the standard deviation surrounding the mean forecast for the current year is likely to be too high if the absolute error in the mean forecast made in the previous year was large. Forecaster confidence is excessively sensitive to recent forecast performance. The positive c coefficient means that the standard deviation for the current year is also likely to be too high if the forecaster's subjective standard deviation for the previous year was too high. Forecasters persist in being overly optimistic or pessimistic about the accuracy of their mean forecasts.

8.4 CONCLUSIONS

In this paper, we have introduced the concept of variance rationality, and tested the rationality of the mean and variance of the inflation forecasts made by a group of US forecasters. We have found that the forecasts are mean-rational at short forecast horizons, but not at long horizons. However, forecasts are not variance-rational. Forecasters are typically overconfident at long forecast horizons. Their subjective standard deviations bear little relation to the accuracy of their mean forecasts; and they fail to adjust their standard deviation estimates appropriately in the light of past performance, giving too much weight to the size of recent errors in mean forecasts, and too little to their long-term accuracy record.

Our findings imply that the practices of proxying inflation uncertainty by mean squared forecast errors (Cukierman and Wachtel, 1982) or by variances from regressions of inflation on some specific information set x_{it} (Engle, 1982, 1983) are questionable, since both rely on the maintained assumption of variance rationality. Similar techniques are in common use in finance, where the riskiness of investments is regularly proxied by the variance (and covariances) of returns around mean values predicted by some information set (for an ARCH implementation, see Bollerslev *et al.*, 1988). If our finding of irrationality in estimates of the variance of price inflation in goods markets is due to the innate difficulty of the problem, then it is also unlikely that variance rationality will characterize forecasts of asset market prices, since price fundamentals are much less clear. However, if our finding of irrationality in inflation forecasts simply reflects the lack of incentives to produce accurate risk measures for goods prices, then it is less likely to carry over into asset markets, where financial incentives to produce variance-rational forecasts are greater.

REFERENCES

Abel, Andrew B. and Frederic S. Mishkin (1983), 'An integrated view of tests of rationality, market efficiency and the short-run neutrality of monetary policy', *Journal of Monetary Economics*, 11, 3–24.

Batchelor, Roy and Lars Jonung (1988), 'Confidence about inflation forecasts: evidence from surveys of Swedish consumers', in K. H. Oppenlander and G. Poser (eds), *Contributions of Business Cycle Surveys to Empirical Economics*, Aldershot: Gower Press, 211–40.

Batchelor, Roy and Lars Jonung (1989), 'Cross-sectional evidence on the rationality of the mean and variance of inflation expectations', in K. Grunert and F. Olander (eds), *Frontiers in Economic Psychology*, Dordrecht: Reidel, 93–106.

Bollerslev, Tim, Robert F. Engle and Jeffrey M. Wooldridge (1988), 'A capital asset pricing model with time-varying covariances', *Journal of Political Economy*, 96, 116–31.

Brown, Bryan W. and Shlomo Maital (1981), 'What do economists know? An empirical study of experts' expectations', *Econometrica*, 49, 491–504.

Cukierman, Alex and Paul Wachtel (1982), 'Inflationary expectations and further thoughts on inflation uncertainty', *American Economic Review*, 72, 508–12.

Engle, Robert F. (1982), 'Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation', *Econometrica*, 50, 987–1007.

Engle, Robert F. (1983), 'Estimates of the variance of US inflation based upon the ARCH model', *Journal of Money, Credit and Banking*, 15, 286–301.

Figlewski, S. and P. Wachtel (1981), 'The formation of inflationary expectations', *Review of Economics and Statistics*, 63, 1–10.

Hansen, Lars-Peter (1982), 'Large sample properties of generalized method of moments estimators', *Econometrica*, 50, 1029–54.

Hey, John D. (1979), *Uncertainty in Microeconomics*, Oxford: Martin Robertson.

Keane, Michael P. and David E. Runkle (1990), 'Testing the rationality of price forecasts: new evidence from panel data', *American Economic Review*, 80, 714–35.

Lahiri, Kajal (1981), *The Econometrics of Inflationary Expectations*, Amsterdam: North-Holland.

Lahiri, Kajal and Christie Teigland (1987), 'On the normality of probability distributions of inflation and GNP forecasts', *International Journal of Forecasting*, 3, 269–79.

Maddala, G.S. (1988), *Introduction to Econometrics*, New York: Macmillan.

McNees, Stephen K. (1978), 'The rationality of economic forecasts', *American Economic Review*, 68, 301–5.

Muth, John F. (1961), 'Rational expectations and the theory of price movements', *Econometrica*, 29, 315–35.

Pearce, Douglas K. (1984), 'An empirical analysis of expected stock price movements', *Journal of Money, Credit and Banking*, 16, 317–27.

Pesaran, M. Hashem (1987), *The Limits to Rational Expectations*, Oxford: Basil Blackwell.

Swidler, Steve and David Ketcher (1990), 'Economic forecasts, rationality, and the processing of new information over time', *Journal of Money, Credit and Banking*, 22, 65–76.

Urich, Thomas and Paul Wachtel (1984), 'The structure of expectations of the weekly money supply announcements', *Journal of Monetary Economics*, 13, 183–94.

Zarnowitz, Victor (1969), 'The new ASA–NBER survey of forecasts by economic statisticians', *American Statistician*, 23, 12–16.

Zarnowitz, Victor (1979), 'An analysis of annual and multiperiod quarterly forecasts of aggregate income, output and the price level', *Journal of Business*, 52, 1–33.

Zarnowitz, Victor (1985), 'Rational expectations and macroeconomic forecasts', *Journal of Business and Economic Statistics*, 3, 293–311.

Zarnowitz, Victor and Louis A. Lambros (1987), 'Consensus and uncertainty in economic prediction', *Journal of Political Economy*, 95, 591–621.