The Flip Side of Financial Synergies: Coinsurance versus Risk Contamination

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Abstract

This paper characterizes when joint financing of two projects through debt increases expected default costs, contrary to conventional wisdom. Separate financing dominates joint financing when risk-contamination losses (associated to the contagious default of a well-performing project that is dragged down by a poorly-performing project) outweigh standard coinsurance gains. Separate financing becomes more attractive than joint financing when the fraction of returns lost under default increases and when projects have lower mean returns, higher variability, more positive correlation, and more negative skewness. These predictions are broadly consistent with existing evidence on conglomerate mergers, spin-offs, project finance, and securitization.

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1 Introduction

Consider a firm that needs to finance two risky projects through a competitive credit market. The firm has the choice of financing the projects either separately with two independent loans or jointly with a single loan. With either financing regime, part of the returns are lost to default (or bankruptcy) costs when creditors do not obtain full repayment. When does joint financing lead to lower costs than separate financing? Answering this question allows us to shed light on the profitability of various corporate financial arrangements, such as:

- mergers that combine cash flows and the financing of otherwise separate corporations;
- holding companies, which protect the assets of individual subsidiaries from creditors' claims against other subsidiaries;
- spin-offs in which divisions are set up as independent corporations;
- project finance and securitization, in which projects or loans are financed through separate special-purpose vehicles.

At least since Lewellen (1971), conventional wisdom in corporate finance has largely settled on the view that default costs always generate positive financial synergies, so that joint financing is more profitable than separate financing in the absence of other frictions. According to this view, conglomeration brings about a reduction in the probability of default by allowing a firm to use the proceeds of a successful project to save an unsuccessful one, which would otherwise have failed. By aggregating imperfectly correlated cash flows, the argument goes, joint financing should reduce expected default costs and increase borrowing capacity. As aptly summarized by Brealey, Myers, and Allen's (2006, page 880) textbook, "merging decreases the probability of financial distress, other things equal. If it allows increased borrowing, and increased value from the interest tax shields, there can be a net gain to the merger."

This paper amends this conventional view by revisiting the *purely financial* effects of conglomeration. We argue that default costs alone create a non-trivial tradeoff for conglomeration, even ab-

stracting away from tax considerations and changes in borrowing capacity. While the literature has mostly focused on the *coinsurance* benefits of conglomeration, we show that the *risk-contamination* losses can turn the logic of the conventional argument on its head. Risk contamination losses arise when the failure of one project drags down another successful project that is financed jointly, thus increasing the probability of default and the expected default costs.

To illustrate the effects at work, consider the decision of a financial conglomerate, such as UBS, whether to spin off its investment banking division from the private banking operations. As acknowledged by the Financial Times, on the one hand a conglomerate can benefit from coinsurance gains ("its investment bank had access to such cheap funding [...] because UBS had a high credit rating, supported by its private banking business"). On the other hand, the conglomerate might also suffer from the effects of risk contamination, as a troubled investment-banking unit can drag down a highly profitable private-banking business ("the losses [in the investment banking unit] have prompted clients to withdraw cash from UBS's core wealth management business").[1]

To best understand the determinants of the tradeoff between coinsurance and risk contamination, we initially focus on a simple setting in which each of two projects has two possible realizations of returns, either low or high. We constrain financing to be obtained through standard debt. The low-return realization is insufficient to cover the initial investment outlay, thus generating the possibility of default. Separate financing involves two nonrecourse loans, so that, when the repayment obligation on one loan is not met, creditors do not have access to the returns of the other project. By contrast, joint financing aggregates the returns of the two projects, so that default costs are only incurred when the sum of the returns of the projects falls below the overall repayment obligation required by the creditors.

The repayment obligation is endogenously determined and depends on the financing regime, either separate or joint. In each regime, competition forces creditors to set the repayment obligation

[1] See "UBS does not have luxury of time before it splits up," Financial Times, March 17, 2008, and "Integration loses its attraction," Financial Times, August 13, 2008.

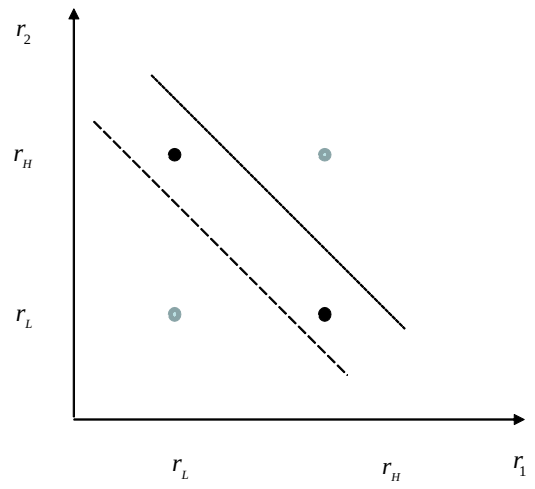

Figure 1: **Joint distribution of returns.** Each project $i = 1, 2$ yields an independent random return r^i with a binary distribution. The return is either low, $r^i = r_L > 0$, with probability $1 - p_i$, or high, $r^i = r_H > r_L$, with probability p_i.

at a level that allows the firm to obtain the projects' present value net of the expected default costs. If the projects are financed separately, each loan defaults when the corresponding project yields a low return. If, instead, the projects are financed jointly, default occurs if the *per-project* repayment obligation is higher than the average realized return of the two projects. Similar to the case of separate financing, default occurs if the returns of both projects are low (bottom-left realization of the joint distribution of returns in Figure 1) and does not occur if the returns of both projects are high (top-right realization). The key to the comparison with separate financing is whether or not the required repayment obligation can be met when one project yields a low return and the other project yields a high return, as illustrated by the top-left and bottom-right realizations in Figure 1.

There are two scenarios. First, suppose that the repayment obligation is *below* the average of the high and the low return, as illustrated by the dashed diagonal line in the figure. In this case, the probability of default and the expected default costs are reduced with joint financing. Ex post, a low-return project, which would have defaulted if it had been financed separately, is saved if the other project yields a high return. Ex ante, the two projects coinsure each other and there are positive financial synergies, equal to the reduction in expected default costs. In turn, a higher probability

of full repayment forces creditors to reduce the interest rate below the level required under separate financing. This coinsurance effect drives the classic logic of "good" conglomeration (positive financial synergies) stressed by Lewellen (1971).

This result is reversed if the per-project repayment obligation is above the average of the high and the low return, as illustrated by the dotted diagonal line in the figure. In this second scenario, the probability of default and the expected default costs are actually higher under joint financing. Ex post, a high-return project, which would have stayed afloat had it been financed separately, is now dragged into default when the other project has a low return. When projects risk-contaminate each other, there are ex ante financial dis-synergies (or negative synergies). If the default recovery rate is low, competing creditors are forced to increase the required interest rate above the level that results under separate financing because the loan will be repaid in full less often with joint financing. In this case, conglomeration is "bad" (financial synergies are negative) due to risk contamination.

The thrust of our analysis consists in characterizing the conditions on the model's primitives such that coinsurance prevails over risk contamination. To this end, we first solve for the equilibrium repayment obligations that result in the two financing regimes, and then determine the region of parameters for which the borrower is able to finance the projects jointly at a rate below the average of the high and the low return. In the context of the baseline model of two projects with independent binary returns, we derive a number of testable comparative statics predictions, such as the following:

- A reduction in the default recovery rate decreases the profitability of joint financing. Given that the amount available to creditors following default is lower when default costs are higher, the repayment obligation associated with joint financing increases with the level of default costs. It is then more difficult for the repayment obligation to be below the average of the high and the low return. Thus, the profitability of joint financing is reduced. Consistent with this theoretical prediction, Rossi and Volpin (2004) show that improvements in judicial efficiency and creditor rights significantly increase M&A activity, while Subramanian, Tung, and Wang

(2009) find that project finance is more prevalent than corporate finance in countries with less-efficient bankruptcy procedures and weaker creditor rights.

- For projects where good returns are more likely than bad ones, joint financing is also less profitable when the projects are riskier. This result is consistent with project finance being more widespread in riskier countries, as shown empirically by Kleimeier and Megginson (2000) among others.

- A mean-preserving increase in the negative skewness of the distribution of returns reduces the attractiveness of joint financing. This result is consistent with the finding that projects with negatively skewed returns, due, for example, to expropriation risk, are likely to be financed on a project basis (see Esty, 2003). Also, since debt returns are negatively skewed, this suggests a motive for the use of separate subsidiaries and securitization structures by banks and other lenders.

In the discussion so far we compared the profitability of separate and joint financing when both financing regimes are feasible. In the paper, we also characterize situations in which it is feasible to finance projects with positive net present value either only separately or only jointly. When the coinsurance effect prevails, joint financing increases the borrowing capacity, resulting in projects that can be financed jointly but cannot be financed separately. When risk contamination prevails, instead, joint financing decreases the borrowing capacity, so that there are projects that can be financed separately but not jointly.

We also show that a rule of thumb that prescribes adopting the financing regime associated with the lowest interest rate can be suboptimal. We illustrate situations in which it is more profitable for a firm to finance projects separately, even though joint financing at a *lower* interest rate is feasible. Indeed, when risk contamination prevails, joint financing can result in a lower interest rate despite being associated with a higher probability of default. When the recovery rate is sufficiently high (or, equivalently, the default costs are sufficiently low), at any given exogenous promised repayment rate,

creditors expect to obtain more with joint financing than with separate financing because default occurs more frequently. As a result, competition forces creditors to offer a lower rate to firms that finance projects jointly. This theoretical finding can explain the widespread use of project finance despite the fact that "project debt is often more expensive than corporate debt," solving one of the "apparently counterintuitive features [of project finance]" (Esty, 2003).

We then examine the impact of correlation between project returns. Intuitively, when returns are perfectly negatively correlated, the risk-contamination effect is absent and the coinsurance effect is so strong that it eliminates default altogether when projects are financed jointly. As the correlation increases, separate financing becomes optimal. In the limit case when returns are perfectly positively correlated separate financing and joint financing are clearly equivalent.

Having illustrated the simple logic of bad conglomeration for distributions with binary returns, we turn to the more general case with continuous returns. We show that the change in expected default costs of joint relative to separate financing can be analytically decomposed into coinsurance gains and risk contamination losses, which coexist with general continuous distributions. To initially abstract away from the advantage of the limited liability shelter, we begin by considering distributions of returns with a positive support, such as truncations of normal distributions. We then extend the results to distributions (such as the normal) that allow for negative returns and identify again the coinsurance and risk-contamination effects when limited liability considerations are also present.

Once we calibrate the model with realistic parameter values, we find that the risk-contamination effect dominates the coinsurance effect in a number of realistic scenarios. We verify the importance of risk contamination in a standard calibration of the stable distribution (McCulloch, 1997) that conveniently captures the skewness and fat tails of financial data. We also consider a calibration of a continuous bimodal distribution that has been recently used to explain features of the recent financial crisis (El-Erian and Spence, 2012). Confirming numerically the comparative statics predictions we obtained analytically for the baseline model with binary returns, we show that the risk-contamination effect dominates if the recovery rate is sufficiently small (or the financial distress costs are large), the

mean is low, the standard deviation is high, the correlation is high, and the skewness is negative.

By clarifying the conditions for the value of conglomeration in the presence of default costs, this paper contributes to a voluminous literature on the analysis of purely financial motives for mergers. In his discussion to Lewellen (1971), Higgins (1971) notes that joint financing also affects the riskiness of the lender's returns; hence, we abstract from risk concerns by assuming risk neutrality. Scott (1977) suggests that, by separating liabilities and selling secured debt, firms can increase the value of their equity by expropriating wealth from their existing unsecured creditors, such as suppliers and/or unsatisfied customers who are then unable to obtain compensation from the firm.[2] Similarly, Sarig (1985) shows that if cash flows can be negative, as "part of any production process (e.g., when customer or employee liabilities exceed future income)", a firm can exploit the limited liability shelter of the shareholders and creditors by financing projects through separate corporations, imposing again a loss on third-party holders of unsecured claims, such as customers, employees or government.

Our baseline model explicitly abstracts from these limited liability effects by assuming positive cash flows, so that creditors always break even and third parties are not affected. The financing regime affects the firm's payoffs because the creditors zero-profit condition creates an *endogenous* limited liability constraint.[3] The tradeoff in our model can be viewed as a borrowing firm's choice of replacing a single endogenously determined limited liability constraint by two separate constraints. As a result, in our model separate financing does not always dominate joint financing, contrary to the setting of Scott (1977) and Sarig (1985) with exogenous limited liability constraints.

[2] However, this "judgement proofness" effect is inconsistent with the notion of rationality on the part of customers and suppliers. Once the lower willingness to pay of customers and suppliers is taken into account, Smith and Warner (1979) argue that the firm's earnings should not be affected by the capital structure. See Section 4.2 for a related discussion and analysis.

[3] A number of papers (e.g., Higgins and Schall, 1975, and Kim and McConnell, 1977) have analyzed the effect of the current capital structure on merger incentives. These papers noted that, while mergers may increase total firm value, bondholders may gain at the expense of shareholders. We abstract from such a distributional conflict among (cashless) stakeholders, by considering the ex ante choice of corporate structure by shareholders and forcing bondholders to compete and therefore obtain no surplus.

In a precursor of this paper couched in the context of bank lending, Winton (1999) is the first to uncover the possibility of bad conglomeration. Our Proposition 4 develops Winton's (1999) third case of Proposition 3.1 in which a bank prefers to specialize even though the repayment rate for pooled projects is lower. Our systematic analysis of the tradeoff between coinsurance and risk contamination delivers a rich set of comparative statics predictions depending on the distributional characteristics of returns.[4]

Leland (2007) compares the profitability of separate and joint financing for a borrower who trades off default costs with tax shields by adjusting the mix of debt and equity. Instead, we consider fixed-investment projects that must be financed only with debt and thus we explicitly rule out the possibility of increasing leverage and re-optimizing the capital structure. As a result, unlike Leland (2007), our analysis uncovers situations in which separate financing is optimal even when the amount borrowed through debt does not depend on whether projects are financed jointly or separately. In addition, we obtain a comprehensive set of analytical predictions, including the effect of skewness and other features linked to nonsymmetric return distributions. See Section 4.2 for a detailed comparison.[5]

Banal-Estañol and Ottaviani's (2013) companion paper allows for financing through equity at a tax disadvantage, in addition to debt.[6] We show that if the tax advantage of debt is sufficiently low, joint financing is inconsequential because default is avoided altogether under either joint or separate financing. At the other extreme, if the tax advantage is sufficiently high, then no equity is used

[4] The literature on financial intermediation under costly state verification is also somewhat related, insofar as this focuses on how diversification across borrowers can reduce the verification costs of bank depositors when the bank defaults. Bond (2004) contrasts conglomerate financing with bank financing in the case of two independent projects. His work relies on the assumption that each project's scale requires large numbers of individual investors who cannot coordinate on costly state verification.

[5] Our results are also very different from those of Shaffer (1994), who studies the effect of joint financing on the probability of *joint* failure. Instead, we compare the firm's expected payoff when the interest rate is endogenously determined by competition among creditors.

[6] As we discuss in the next section, the costly state verification literature shows that debt is the optimal contractual arrangement if returns are privately observed by the borrower and can be verified by creditors only once default costs are incurred.

in either financing regime so that the choice between separate and joint financing is the same as in the debt-only model considered in the present paper. More interestingly, if the tax advantage is intermediate, joint financing becomes relatively more profitable than in the debt-only model, because equity financing makes it more likely to obtain debt repayment rates that avoid risk contamination. Debt capacity with joint financing, however, might need to be reduced substantially. At some point, the tax-disadvantage makes joint financing again unprofitable.[7] Contrary to the conventional wisdom, as shown in the quote of Brealey, Myers, and Allen (2006) reported above, conglomeration is then associated to less—rather than more—borrowing, with resulting losses in terms of tax shields.

John (1993), Hege and Ambrus-Lakatos (2002), and Inderst and Müller (2003) analyze the optimal corporate structure in models with agency costs due to debt overhang rather than default costs. For example, in Inderst and Müller's (2003) two-project version of Bolton and Scharfstein (1990), financing two projects within the same corporation can reduce the firm's ability to borrow when the firm is able to finance follow-up investments internally without returning to the external capital market.[8] Our predictions for the case with default costs are different (see, for example, the discussion following Prediction 2).

The paper proceeds as follows. Section 2 formulates the model. Focusing on a baseline version of the model with two projects with independent binary returns, Section 3 analyzes the conditions setting apart financial synergies from dis-synergies and performs comparative statics with respect to the default recovery rate and the distribution of returns, such as mean, variance, skewness, and correlation. Turning to the case of continuous distributions, Section 4 (i) provides an analytical decomposition of the net financial synergies in terms of coinsurance gains and risk contamination losses, (ii) shows through a number of numerical simulations that the risk contamination effect is empirically important and can outweigh the coinsurance effect in a number of realistic scenarios, and

[7] The preponderance of debt with separate financing is consistent with the many empirical studies that find that a large proportion of funding in project finance is in the form of debt (see, e.g., Kleimeier and Megginson, 2000).

[8] See also Faure-Grimaud and Inderst (2005), who focus on the trade-off between coinsurance and winner-picking incentives in this setting.

(iii) obtains comparative statics results that are fully consistent with those of the baseline model. Section 5 concludes with a summary of the main predictions of our theory and a discussion of avenues for future research. The Appendix collects the proofs.

2 Model

A risk-neutral firm has access to two ex-ante identical projects. Each project i requires at $t = 1$ an investment outlay normalized to $I = 1$ and yields at $t = 2$ a random payoff or return r_i with distribution function F. The projects have positive net present value, but the lowest return realization possible is insufficient to cover the initial investment outlay. Even though we focus for most of the analysis on the case with independently distributed returns, we also allow for correlated returns.

Before raising external finance, the firm chooses how to group the two projects into stand-alone corporations. This means that investors in each corporation have access to the returns of all projects in that corporation, but they do not have access to the returns of the projects in the other corporations set up by the firm. Financing each project in a separate corporation is equivalent to financing through separate nonrecourse loans, while joint financing of the two projects in a single conglomerate corporation is equivalent to financing through a large loan with recourse on the returns of both projects. Financing for each corporation (or loan) can be obtained in a competitive credit market. For notational simplicity, we stipulate that the firm seeks financing only when expecting to obtain a *strictly* positive expected payoff.

Creditors are risk neutral and lend money through standard debt contracts. Without loss of generality, we normalize the risk-free interest rate to $r_f = 0$. Therefore, creditors expect to make zero expected profits. This is equivalent to assuming that each corporation makes a take-it-or-leave-it repayment offer to a single creditor for each loan j, promising to repay r_j^* at $t = 2$ for each unit borrowed at $t = 1$.[9] Thus r_j^* denotes the promised repayment per project. According to our

[9] Thus, for the case in which each loan (or corporation) is financed by multiple creditors, we implicitly assume that there are no coordination failures across the creditors who syndicate the same loan.

accounting convention, this repayment rate comprises the amount borrowed as well as net interest.[10]

Creditors are repaid in full when the total realized return of the projects pledged is sufficient to cover the promised repayment. If instead the total realized return falls short of the repayment obligation, the corporation defaults and the ownership of the projects' realized returns is transferred to the creditor. Following default, the creditor is only able to recover a fraction $\gamma \in [0,1]$ of the realized returns r, so that the default costs following default are equal to $(1-\gamma)\, r$.[11] As we show in Banal-Estañol and Ottaviani (2013), our results hold robustly with a more general structure of default costs, provided that the economies or diseconomies of scale in default are not too extreme.

We restrict external financing to be obtained through debt. Note that debt is the optimal contractual arrangement if we assume that returns are privately observed by the borrower and can be verified by creditors only at a cost, as in the costly state verification model. As shown by Townsend (1979), Diamond (1984), and Gale and Hellwig (1985), the optimal financing arrangement is then the standard debt contract, under which returns are verified if and only if the borrower cannot repay the loan in full. Once default (or bankruptcy) costs are re-interpreted as CSV verification costs, the optimal contractual agreement between the entrepreneur and the creditor is thus a debt contract. That is, if two projects are available, the optimal contracting strategy is either two separate debt contracts, each of which is backed by the returns of one project, or one debt contract, which is backed by the returns of the two projects.

3 Binary Returns

To develop our main insight we initially analyze a baseline specification with two independently distributed projects with binary returns. Each project i yields either a low return r_L with probability $1-p$ or a high return $r_H > r_L$ with probability p and this return realization is independent of the

[10] The net interest rate i satisfies $1+i = r_j^*$ and therefore the repayment obligation can be interpreted as the gross interest rate. In our setting, given that projects require one unit of investment and they are fully financed with debt, the per-project market value of debt is always equal to one.

[11] For estimates of bankruptcy costs and other costs of financial distress across industries see, for example, Warner (1977), Weiss (1990), and Korteweg (2007).

return of the other project. Even though each project has a positive net present value, $(1-p)\,r_L + pr_H - 1 > 0$, the low return is insufficient to cover the initial investment outlay, $r_L < 1$.

In Section 3.1 we proceed to examine the conditions for when the borrower is able to finance the two projects separately and jointly. In Section 3.2 we compare the profitability of separate and joint financing, when they are both feasible. In Section 3.3 we characterize the effect of conglomeration on the firm's borrowing capacity. In Section 3.4 we derive a set of comparative statics predictions for the occurrence of joint and separate financing. In Section 3.5 we present a numerical illustration of the importance of financial dis-synergies. In Section 3.6 we show that the financing option with the lowest repayment rate *is not* necessarily optimal. In Section 3.7 we extend the model to allow for correlation across the returns of the two projects.

3.1 Financing Conditions

Consider first the possibility of financing the two projects through two separate nonrecourse loans or, equivalently, through two different limited liability corporations. Given that the two projects are ex ante identical, financing of each project, if possible, takes place at the same rate. In order for the creditor to break even, the rate r_i^* must satisfy $r_i^* > 1 > r_L$. Therefore, there is a positive probability that the loan is not repaid in full. To ensure that the borrower obtains strictly positive profits, the rate r_i^* must also satisfy $r_i^* < r_H$.

Given that the credit market is competitive, creditors must make zero expected profits. Thus the repayment requested by the creditor, r_i^*, is such that the gross expected proceeds, $pr_i^* + \gamma(1-p)r_L$, are equal to the initial investment outlay 1. As a result, each project can be financed through a separate loan if and only if

$$r_i^* := \frac{1-\gamma(1-p)r_L}{p} \leq r_H. \tag{1}$$

The repayment obligation, which is fully paid only in the case of a high return, is equal to the investment outlay, 1, less the expected proceeds from default, $\gamma(1-p)r_L$, divided by the probability of staying afloat, p. Intuitively, the creditor needs to recover the expected shortfall in the event of

default from the event in which the project yields a high return.

Next, consider joint financing of the two projects through a single loan or, equivalently, within the same corporation. Denote by r_m^* the equilibrium repayment obligation *per unit of investment*, so that $2r_m^*$ is the total repayment promised to the creditor in return for the initial financing of the two projects, $2I = 2$. Two cases need to be distinguished, depending on whether or not the required repayment rate induces default in the case when one project yields a high return while the other project yields a low return ("intermediate returns").

Suppose first that the equilibrium repayment rate r_m^* is such that $r_L \leq r_m^* \leq \frac{r_H + r_L}{2}$, so that there is no default with intermediate returns. As a result, the probability of default is reduced to $(1-p)^2$. Substituting again in the expected creditor profits, the borrower would only be able to obtain this rate in a competitive market if and only if

$$r_m^* := \frac{1 - \gamma (1-p)^2 r_L}{1 - (1-p)^2} \leq \frac{r_H + r_L}{2}. \tag{2}$$

Suppose now that the equilibrium rate r_m^{**} is such that $\frac{r_H + r_L}{2} \leq r_m^{**} \leq r_H$ and therefore the borrower defaults in the event of a high and a low return. Hence, default occurs with probability $1 - p^2$. In a competitive credit market, this rate can be obtained if and only if

$$r_m^{**} := \frac{1 - \gamma (1-p)(p r_H + r_L)}{p^2} \leq r_H. \tag{3}$$

Since the borrower's expected profits for a given distribution are decreasing in the equilibrium rate, if both conditions (2) and (3) are satisfied, the borrower prefers rate r_m^* to rate r_m^{**}.[12] Summarizing the results so far, we have the following proposition.

Proposition 1 (Financing conditions) *Two independent projects can be financed separately if and only if condition (1) is satisfied, in which case the equilibrium rate is r_i^*. Projects can be financed jointly if and only if conditions (2) or (3) are satisfied. If condition (2) is satisfied, the equilibrium rate is r_m^*, and if it is not satisfied, the rate is r_m^{**}.*

[12] It is straightforward to show that if $r_m^* > (r_H + r_L)/2$, then $r_m^{**} > (r_H + r_L)/2$. Therefore, if it is not possible to obtain r_m^*, then we can disregard the $r_m^{**} > (r_H + r_L)/2$ constraint.

3.2 Financial Synergies or Dis-synergies?

When both separate and joint financing are feasible, which regime is more profitable and thus optimal for the borrower? Obviously, in the absence of default costs (i.e., when $\gamma = 1$) the borrower is indifferent between financing the projects separately or jointly. The next proposition states the gains and losses when $\gamma < 1$.

Proposition 2 (Separate v. joint financing) *When the borrower can finance two independent projects separately as well as jointly:*
(a) If condition (2) is satisfied, it is optimal to finance the projects jointly, as the financial synergies are positive and equal to the coinsurance gains: $CI = p\,(1-p)\,(1-\gamma)r_L$.
(b) If condition (2) is not satisfied, it is optimal to finance the projects separately, as the financial synergies are negative and equal to the risk-contamination losses: $RC = p\,(1-p)\,(1-\gamma)r_H$.

Intuitively, when the borrower obtains a rate that avoids intermediate default, the probability of default under joint financing is lower than under separate financing. The low-return project is saved from default when the other project yields a high return, thereby reducing the inefficiency associated with default. Per-project expected savings when the projects are financed jointly rather than separately—the "coinsurance effect"—are equal to the probability that the first project yields a low return while the second project yields a high return, $p(1-p)$, multiplied by the default losses avoided, $(1-\gamma)r_L$.

If, instead, the borrower obtains a joint rate that does not avoid intermediate default, a project with low return drags down the other project, increasing the probability of default. Per-project expected losses when projects are financed jointly rather than separately—the "risk-contamination effect"—are equal to the probability that the first project yields a high return while the second project yields a low return, $p(1-p)$, multiplied by the additional default losses incurred, $(1-\gamma)r_H$.

The key is whether the equilibrium repayment rate for joint financing is below or above the crossing point, $(r_H + r_L)/2$. Notice that the crossing point is not necessarily at the mean. In

particular, if $p > 1/2$, so that the distribution is skewed to the left (i.e., returns are negatively skewed), the crossing point is below the mean. As a result, equilibrium rates above the crossing point are consistent with a probability of default below 50%. The resulting default probabilities are then $1 - p$ for separate financing and $1 - p^2$ for joint financing, which for a high enough p may be very low.

3.3 Borrowing Capacity

So far we have compared the profitability of separate and joint financing when both financing regimes are feasible. As we have seen in Section 3.1, there are situations in which it is feasible to finance projects with positive net present value either only separately or only jointly. Thus, conglomeration does not necessarily increase the firm's ability to finance projects.

Proposition 3 (Borrowing capacity) *Consider two independent projects:*
(a) If condition (2) is satisfied, there are projects that can be financed jointly but not separately.
(b) If condition (2) is not satisfied, any project that can be financed jointly can be financed separately and there are projects that can only be financed separately.

When the coinsurance effect prevails, there are projects that can be financed jointly but cannot be financed separately. In this first case, conglomeration increases the firm's borrowing capacity, as in Lewellen (1971). However, when risk contamination prevails, joint financing decreases the firm's borrowing capacity, so that there are projects that can be financed separately but not jointly.

3.4 Testable Predictions

We now derive comparative statics predictions with respect to changes in the characteristics of the projects: the recovery rates and the distribution of returns (mean, variability, and skewness). For each attribute, we study whether separate or joint financing is optimal for a larger range of the remaining parameters. At the same time, we contrast our predictions with those from existing theories and discuss how our predictions on joint and separate financing match existing empirical

evidence. Note that joint financing corresponds to mergers, especially conglomerate mergers, whereas separate financing corresponds to spin-offs of divisions. Also, as argued by Leland (2007) asset securitization and project finance are also methods for separately finance activities from originating or sponsoring organizations by placing them in bankruptcy-remote special-purpose vehicles (SPVs). From an analytical perspective, these entities have the key features of separate corporations.

Prediction 1 (Default costs) *For higher default costs (lower γ) then (a) both joint and separate financing can be obtained for a smaller region of parameters and (b) joint financing is optimal for a smaller region of the remaining parameters.*

Higher default costs decrease pledgeable returns, since the recovered returns in case of default are lower. Since default costs do not affect the crossing point, $(r_H + r_L)/2$, financing at a rate that avoids intermediate default is more difficult and thus joint finance is less likely. To the best of our knowledge, this prediction has not been formulated before.

Still, this prediction is consistent with empirical evidence indicating that merger activity is less likely and project finance is more likely in countries with weaker investor protection. Rossi and Volpin (2004) show that improvements in judicial efficiency and creditor rights significantly increase M&A activity. Comparing the incidence of bank loans for project finance with regular corporate loans for large investments, Subramanian, Tung, and Wang (2009) show that project financing is more frequent in countries with less efficient bankruptcy procedures and weaker creditor rights. Increases in these two measures of investor protection decrease the default costs and should favor, according to our model, joint financing (mergers or direct investment) over separate financing (project finance).

Prediction 2 (Mean) *For higher probability of a high return (higher p) then (a) both joint and separate financing can be obtained for a larger region of parameters and (b) joint financing is optimal for a larger region of the remaining parameters.*

If the probability of a high return increases, the expected return pledgeable to creditors also increases. It becomes easier to finance projects, and to finance them jointly at a rate that avoids

intermediate default.

This prediction contrasts with that of Inderst and Müller (2003). In their model, it is optimal to keep better projects separate to avoid self-financing and thus commit to return to the capital market. The existing empirical evidence on the productivity of conglomerate firms—one of the testable implications of this prediction—is mixed. While Maksimovic and Phillips (2002) find that conglomerate firms, for all but the smallest firms in their sample, are less productive than single-segment firms, Schoar (2002) finds that the productivity of plants in conglomerate firms is higher than in stand-alone firms.[13]

During booms, projects might have a higher expectation across-the-board. Our prediction would then be consistent with a large body of empirical evidence that shows that merger activity usually heats up during economic booms and slows down in recessions (see, for example, Maksimovic and Phillips, 2001). Similarly, Cantor and Demsetz (1993) show that off-balance sheet activity (separate financing) grows following a recession.

Prediction 3 (Mean-preserving spread) *Consider the effect of a mean-preserving spread in the project's return consisting of an increase in the high return r_H and a reduction in the low return r_L so as to maintain the mean return constant. Then, there exists $\overline{p} < 1/2$ such that the region of parameters for which joint financing is optimal decreases if and only if $p > \overline{p}$.*

That is, a mean preserving spread in the distribution of returns favors separate financing as long as the distribution of returns is not too positively skewed. If the distribution is symmetric ($p = 1/2$), a mean preserving spread increases r_H by as much as it reduces r_L. While the crossing point is unaffected, the joint financing rate that avoids intermediate default becomes more difficult to obtain

[13] Still, Schoar (2002) finds that conglomerates are less valued than focused firms (the so-called market diversification discount), and argues that the discrepancy can be attributed to conglomerates leaving more rents to workers. A number of papers have also argued that the diversification discount could also be spurious, because of measurement problems and selection biases. For example, Graham, Lemmon, and Wolf (2002) show that acquirers' excess values decline because the business units acquired are already discounted, thus explaining the diversification discount with a self-selection argument. See also Campa and Kedia (2002), Villalonga (2004), and Custodio (2009).

because the low return is even lower and the pledgeable returns before the crossing point are lower. If the distribution of returns is negatively skewed ($p > 1/2$), the crossing point is decreased and it becomes even more difficult to obtain joint financing below the crossing point.[14]

This prediction is consistent with a similar prediction obtained by Leland (2007). Empirical support can be found in the project finance literature. Kleimeier and Megginson (1999), for example, find that project finance loans are far more likely to be extended to borrowers in riskier countries, particularly countries with higher political and economic risks. They claim that: "As a whole, these geographic lending patterns are consistent with the widely held belief that project finance is a particularly appropriate method of funding projects in relatively risky (non-OECD) countries."

It is also worth noting that loans and other forms of debt typically have default rates well under 50%. Thus, according to our prediction, increases in loan risk should make it more likely that the loans are securitized. On the other hand, the relative risk of the loan originator and the loans will also play a role.

Prediction 4 (Skewness) *Consider the effect of a mean-preserving increase in negative skewness in the project's return consisting of a reduction in the low return level r_L and an increase in the probability of high return p so as to maintain the mean return constant. Then, it becomes optimal to finance the projects jointly for a smaller region of parameters if and only if the high return level r_H is sufficiently large.*

Increasing negative skewness has two conflicting effects. On the one hand, as r_L decreases, the crossing point is reduced and the returns in case of default are lower, so that joint financing at the rate that avoids intermediate default becomes more difficult. On the other hand, as p increases so as to keep the mean constant, the probability that both projects' returns are low is reduced, so that it becomes easier to finance the projects at the rate avoiding intermediate default. If r_H is sufficiently

[14] To maintain the mean constant, a given increase in r_H must be combined with a larger decrease in r_L, resulting in a reduction in the crossing point. Formally, from $r'_H = r_H + \varepsilon$ and $r'_L = r_L - \varepsilon p/(1-p)$, we have $(r'_H + r'_L)/2 = (r_H + r_L)/2 - \varepsilon(2p-1)/2(1-p)$.

high, the first effect dominates and separation becomes optimal for a larger set of parameters. Indeed, for a given increase in p, one needs a higher reduction in r_L to ensure a constant mean.

We can find support for this prediction in the literature on project finance. For example, Esty (2003) shows that project finance is widespread when it is possible to lose the entire value due to expropriation. This type of risk generates returns with large negative skewness, as opposed to more symmetric risks such those affecting exchange rates, prices, and quantities. Moreover, project finance is typically used for projects with high potential upside, satisfying the requirement that r_H be sufficiently high.

3.5 Illustration

We now provide an initial illustration of how joint financing can result in an increase in expected default costs for empirically plausible parameter values under the (admittedly strong) assumption that returns are binary. To this end, we perform a calibration of the four parameters (r_H, r_L, p, and γ) of the model of this section for the case with separate financing. As representative values, we set:
(i) the probability of default at 2.09% (parametrized by $1 - p^5 = 0.1$) by using Longstaff, Mittal, and Neis (2005) estimate of 10% for the default probability on bonds for BBB rated firms with a five-year horizon;
(ii) the mean return at 5% (so that $[pr_H + (1-p)\gamma r_L - 1]/1 = 0.05$), as in Parrino, Poteshman, and Weisbach (2005), who use a mean return of 10.63% given a risk-free rate of 5.22%;
(iii) the default recovery rate at $\gamma = 65\%$ (based on 35% liquidation losses as percentage of going concern value) from Alderson and Betker (1995); and
(iv) bankruptcy costs as a fraction of a firm's value at 11% (so that $(1-\gamma)r_L/[pr_H+(1-p)\gamma r_L] = 0.11$), at the mid point of Bris, Welch, and Zhu's (2006) range of estimates of 2% to 20%, at the low end of Altman's (1984) estimate of 11–17% for bankruptcy costs as a fraction of firm value up to three years before default and more conservative than Korteweg's (2010) estimate of 15–30% of firm value at the point of bankruptcy.

The calibrated values are then $r_H = 1.07$, $r_L = 0.33$, $p = 0.98$, and $\gamma = 0.65$. For these parameters, it is feasible to finance the projects separately, since $r_i^* = 1.02 < 1.07 = r_H$, as well as jointly, since $r_m^{**} = 1.02 < 1.07 = r_H$, but not at the rate below the crossing point, because $r_m^* = 1.01 > 0.70 = (r_H + r_L)/2$. Thus, separate financing is more profitable than joint financing. In this illustration, the risk-contamination effect identified in Proposition 2 is $p(1-p)(1-\gamma)r_H = 0.04$, 4% of the investment outlay $I = 1$, corresponding to 15% of the project's net present value. A key limitation of this initial numerical illustration is the restriction to binary returns. See Section 4 for more extensive and realistic calibrations for the model with continuous returns.

3.6 Managerial Implications

We now show that the financing regime with the lowest repayment rate does not necessarily entail the lowest likelihood of default and is thus not necessarily optimal. Borrowers would be misguided by choosing the financing regime with the lowest interest rate. The following proposition characterizes when it is more profitable to finance projects separately, even though joint financing is available at a lower rate.

Proposition 4 (Separate financing at higher rate) *Separate financing is optimal even though it results in a higher interest rate if and only if (i) condition (3) is satisfied but condition (2) is not satisfied and (ii)* $\gamma[pr_H + (1-p)r_L] > 1$.

To see what is going on, first suppose there were no default costs. Because the creditor's payoff is a concave function of firm cash flows, it is immediate that, for any fixed repayment rate r, the expected return to the creditor would be higher for joint financing than for separate financing, because joint financing has per unit returns that are less risky in the sense of second order stochastic dominance (see Rothschild and Stiglitz, 1970). As a result, the break-even rate for the creditor would be lower for joint financing than for separate financing—regardless of whether default occurred more often or not under joint financing. Nevertheless, the firm's expected cash flows would be the same under either financing method, so repayment rate is not a good indicator of which financing method to use.

Since there are in fact default costs, the break-even repayment rate must increase to offset the reduced cash flows in default states. If joint financing does not involve intermediate default (condition (2) holds), then expected default costs are lower under joint financing, the break-even rate is lower, and the firm prefers joint financing to separate financing. But if joint financing involves intermediate default (condition (2) does not hold but condition (3) holds), then expected default costs are higher under joint financing: default occurs more often, and costs once in default are at least as high as under separate financing. In this case, default costs make the repayment rate increase more under joint than under separate financing, and the firm's net expected cash flow is lower under joint financing. Still, since without default costs the repayment rate under conglomerate financing would definitely be lower than that for separate financing, the repayment rate with such costs may still be lower. Condition (ii) of the proposition guarantees that this is the case.

3.7 Correlated Returns

We now extend our baseline specification to add correlation in the distribution of joint returns. Suppose that the probability of two high returns is equal to $p\left[1-(1-p)(1-\rho)\right]$, the probability of two low returns is equal to $(1-p)\left[1-p(1-\rho)\right]$, and the probability that one of the projects yields a high return whereas the other yields a low one is equal to $p(1-p)(1-\rho)$. Thus ρ is the correlation coefficient between the two projects. For the joint probability distribution to be well defined, it is necessary to assume that $\rho \geq \max\langle -(1-p)/p, -p/(1-p)\rangle$. Clearly, if $\rho = 0$ we are back to the baseline scenario with independent returns.

Prediction 5 (Correlation) *If the correlation between the projects increases (ρ is larger), then separate financing is optimal for a larger set of parameters.*

This prediction is similar to the one obtained by Inderst and Müller (2003), but it is driven by a different logic. The probability of having two high returns and the probability of having two low returns increase simultaneously with ρ. As a result, the repayment rate when intermediate default is avoided is higher because the probability of two low returns is higher. When intermediate default

cannot be avoided, the repayment rate is lower because the probability of two high returns also increases. As a consequence, the financing conditions avoiding intermediate default are tighter and those not avoiding it looser.

The effects of correlation on the optimality conditions are also intuitive. In the extreme case with perfect negatively correlation (i.e., if $\rho = -1$ and $p = 1/2$), when one project has a high return the other necessarily has a low one, so that projects can always be jointly financed at a rate that avoids intermediate default.[15] Thus, it is clearly optimal to always finance projects jointly when the negative correlation is perfect. As correlation increases above $\rho = -1$, conglomeration is optimal for a smaller region of parameters. However, the probability of having intermediate returns decreases, so the difference in expected default costs between joint and separate financing shrinks. If projects have perfect positive correlation ($\rho = 1$), the conditions for joint and separate financing are identical and the firm is clearly indifferent between the two financing regimes.

4 Continuous Returns

Having illustrated the simple logic of bad conglomeration (and financial dis-synergies) for distributions with two possible return realizations, in the rest of the paper we extend the analysis to the more general case with continuous returns. We begin in Section 4.1 by considering only positive returns so as to abstract away from limited liability considerations. The net benefits of joint financing relative to separate financing are then equal to the reduction in expected default costs, which we analytically decompose into coinsurance gains and risk-contamination losses. Turning to a numerical calibration when the distribution of returns follows a truncation of the normal, we show that the risk-contamination effect can outweigh the coinsurance effect for continuous distributions. Consistent with the results from our baseline binary model, we verify that the risk-contamination losses dominate the coinsurance gains if the recovery rate is small (or the fraction of returns lost through

[15] This is not true for $p \neq 1/2$ because either the probability of two high realizations or the probability of two low realizations is greater than 0, even when the correlation is at the lowest possible level.

default is large), the mean is low, and the standard deviation is large.

In Section 4.2 we proceed to distributions with partly negative support. The net gains of conglomeration relative to separate financing are equal to the change in the limited liability shelter (which is negative and thus always favors separate financing) minus the change in expected default costs (which can be either positive or negative, as we show). Once we further decompose the reduction in expected default costs into coinsurance gains and risk-contamination losses, we characterize when conglomeration actually increases expected default costs. To quantify the occurrence of bad conglomeration, we calibrate the model with normally distributed returns. Conglomeration results in an increase (rather than a reduction) in expected default costs once the standard deviation is set at a sufficiently higher level than at the parameter specification considered by Leland (2007). Building on an argument originally put forward by Smith and Warner (1979), we initially net out the limited liability effect, so that the total gains of conglomeration are equal to the reduction in expected default costs, as in our baseline scenario. Again, we find that the risk-contamination effect dominates the coinsurance effect if the recovery rate is small, the mean is low, and the standard deviation is large. Similar results also hold when the limited liability effect is added to the tradeoff.

We conclude in Section 4.3 by extending the numerical analysis to allow for stable, bimodal, and correlated distributions.

4.1 Distributions with Positive Returns

While in our baseline model with binary returns the coinsurance and the risk-contamination effects are mutually exclusive, these two effects coexist when returns follow a continuous distribution. We begin by considering two identically and independently distributed projects with continuous density $f(r_i)$ and distribution $F(r_i)$ over a non-negative support. The mean of project i's returns at $t = 2$ satisfies $\mu > 1$ to ensure a positive net present value.

Decomposition of Reduction in Expected Default Costs. If the two projects are separately financed, each of them should be financed at the lowest possible rate r_i^*, if any, that ensures that the

creditors make zero expected profits, i.e.

$$r_i^* \left[1 - F(r_i^*)\right] + \gamma \int_0^{r_i^*} r_i f(r_i) dr_i = 1. \tag{4}$$

Substituting into the firm's profits, which are given by

$$\int_{r_i^*}^{\infty} r_i f(r_i) dr_i - r_i^* \left[1 - F(r_i^*)\right], \tag{5}$$

implies that the firm's profits are equal to the net expected returns minus the expected default costs,

$$\mu - 1 - \int_0^{r_i^*} (1 - \gamma) r_i f(r_i) dr_i. \tag{6}$$

If, instead, the projects are financed jointly, the zero-profit condition is given by the lowest r_m^* that satisfies

$$r_m^* \left[1 - H(r_m^*)\right] + \gamma \int_0^{r_m^*} r_m h(r_m) dr_m = 1 \tag{7}$$

where

$$h(r_m) := \int_0^{\infty} f(2r_m - r_i) 2 f(r_i) dr_i$$

is the density and H the distribution of the average of r_i and r_j, $r_m := (r_i + r_j)/2$. Per-project firm profits, which are given by

$$\int_{r_m^*}^{\infty} r_m h(r_m) dr_m - r_m^* \left[1 - H(r_m^*)\right], \tag{8}$$

are then equal to the net expected returns minus the expected default costs,

$$\mu - 1 - \int_0^{r_m^*} (1 - \gamma) r_m h(r_m) dr_m. \tag{9}$$

The net per-project gains of joint financing are then equal to the reduction in expected default costs. That is, subtracting (6) from (9), we obtain $\Delta\pi = -\Delta DC$, where

$$\Delta DC := \int_0^{r_m^*} (1 - \gamma) r_m h(r_m) dr_m - \int_0^{r_i^*} (1 - \gamma) r_i f(r_i) dr_i. \tag{10}$$

We now show that this expression can be rearranged to obtain an intuitive decomposition of the net gains of conglomeration.

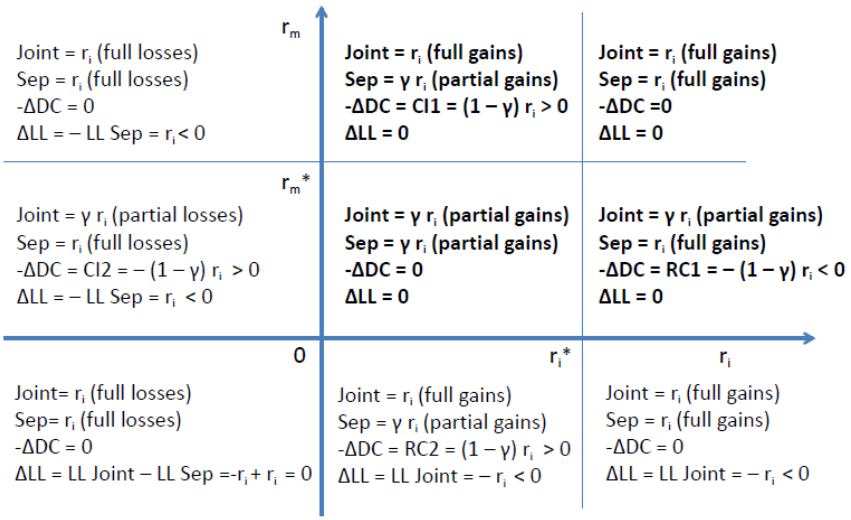

Figure 2: **Decomposition of reduction in expected default costs and limited liability shelter.** The horizontal axis reports the return of project i and the vertical axis the average return of project i and j, $r_m = (r_i + r_j)/2$. The entries report the composition of the reduction in expected default costs associated to project i in terms of risk-contamination and coinsurance effects, as well as the limited liability shelter.

Proposition 5 (Decomposition with positive returns) *The net financial synergies (and the reduction in expected default costs) of two independently distributed projects with continuous density f with positive support can be decomposed into the coinsurance gains and risk-contamination losses of conglomeration, i.e. $\Delta\pi = -\Delta DC = CI - RC$, where*

$$\begin{aligned} CI &: = \int_{r_m^*}^{\infty}\int_0^{r_i^*}(1-\gamma)r_i f(2r_m - r_i)2f(r_i)dr_i dr_m, \\ RC &: = \int_0^{r_m^*}\int_{r_i^*}^{\infty}(1-\gamma)r_i f(2r_m - r_i)2f(r_i)dr_i dr_m. \end{aligned} \tag{11}$$

The coinsurance gains (CI) are the expected savings obtained because a project i, which would have defaulted had it been financed separately (when $r_i < r_i^*$), is saved by the other project once the two projects are financed jointly (when $r_m = (r_i + r_j)/2 > r_m^*$). The risk-contamination losses (RC) are the expected losses suffered because a project i, which would have stayed afloat had it been financed separately (when $r_i > r_i^*$), is dragged down by the other project with which it is jointly financed (when $r_m = (r_i + r_j)/2 < r_m^*$).

The positive orthant of Figure 2 shows the returns of project i obtained by the firm and its creditors for each realization of the return, r_i (horizontal axis), and for each realization of the average return, r_m (vertical axis), depending on the joint or separate financing regime. The difference between the total returns in the two regimes, which is equal to the reduction in expected default costs, is assigned to coinsurance and risk contamination.

Numerical Analysis for Truncation of Normal Distribution. To quantify the effects we now turn to an example of a distribution with positive support that is obtained by truncating a normal distribution at zero and then assigning to the zero return a mass equal to the probability of the negative realizations of the original normal distribution. This construction leads to a mixed distribution with a probability mass at 0 and a normal distribution for the positive realizations. The decomposition derived in Proposition 5 can be easily extended to allow for a mixed distribution consisting in a probability mass q at 0 and a continuous function g in the positive support such that $\int_0^\infty r_i g(r_i) dr_i = 1 - q$.

Proposition 6 (Decomposition for mixed distributions) *The net financial synergies (and the reduction in expected default costs) of two independently distributed projects with a mixed distribution with a probability mass q at 0 and a continuous function g in the positive support can be decomposed into the coinsurance gains and risk-contamination losses of conglomeration, i.e. $\Delta\pi = -\Delta DC = CI - RC$, where*

$$\begin{aligned} CI &: = \int_{r_m^*}^\infty \int_0^{r_i^*} (1-\gamma) r_i g(2r_m - r_i) 2g(r_i) dr_i dr_m + \max\{q \int_{2r_m^*}^{r_i^*} (1-\gamma) r_i g(r_i) dr_i, 0\}, \\ RC &: = \int_0^{r_m^*} \int_{r_i^*}^\infty (1-\gamma) r_i g(2r_m - r_i) 2g(r_i) dr_i dr_m + \max\{q \int_{r_i^*}^{2r_m^*} (1-\gamma) r_i g(r_i) dr_i, 0\}. \end{aligned}$$

Note that there is an additional term with respect to Proposition 5, either in the coinsurance gains (if $r_i^* > 2r_m^*$) or in the risk-contamination losses (if $r_i^* < 2r_m^*$). If $r_i^* < 2r_m^*$, the new term corresponds to the additional risk-contamination losses generated by a project with return 0 and another with return r_i, such that $r_i > r_i^*$ and $r_i/2 < r_m^*$, because the positive-return project is

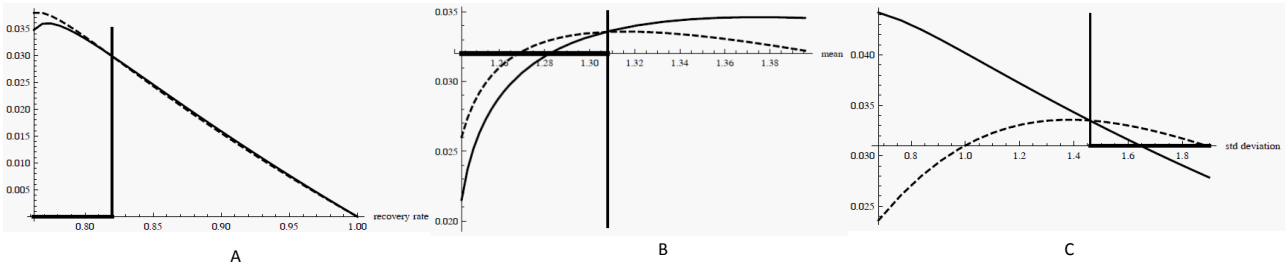

Figure 3: **Coinsurance versus risk contamination for the truncation of the normal distribution.** Panels A, B, and C plot the coinsurance gains CI (solid line) and risk-contamination losses RL (dashed line) with respect to the recovery rate, the mean, and the standard deviation, respectively.

saved with separate financing but defaults with joint financing. If, instead, $r_i^* > 2r_m^*$, the new term corresponds to the coinsurance gains generated because a project with positive return $r_i < r_i^*$ and $r_i/2 > r_m^*$ defaults with separate financing, but is saved with joint financing.

The three panels in Figure 3 perform comparative statics of the coinsurance gains and risk-contamination losses, with respect to the recovery rate, the mean, and the standard deviation. As a base case for this distribution**,** we take a recovery rate of $\gamma = 0.8$ (or financial distress costs of 20%, as estimated by Davydenko, Strebulaev, and Zhao, 2012) and a normal distribution with mean $\mu_o = 0.8$ and standard deviation $\sigma_o = 2.1$, resulting in a truncated mixed distribution with mean $\mu = 1.3$ and standard deviation $\sigma = 1.49$.[16] In each of the panels, we fix two of the parameters of the base case and perform comparative statics with respect to the third parameter.[17]

As can be seen from Figure 3, even with continuous distributions risk-contamination losses can outweigh coinsurance gains. For parameter values marked by a bold segment, risk contamination

[16] Keeping fixed the other parameters of the base case, projects can be financed both separately and jointly if $\gamma > 0.8$, $\mu > 1.29$ and for $\sigma < 3.2$. In the first plot, the lowest value of the recovery rate depicted corresponds to the lowest value such that the projects can be financed both separately and jointly. In the second and third plots, we have verified that the lines do not cross for higher or lower parameter values than those depicted.

[17] Note that a change in the standard deviation of an underlying normal distribution also affects the mean of the truncated distribution. To keep constant the mean of the truncated distribution, we thus also adjust the mean of the underlying normal distribution. Similarly, because changes in the mean of the underlying normal distribution affect the truncated distribution's standard deviation, we adjust the standard deviation of the original distribution so as to keep constant the standard deviation of the truncated distribution.

dominates coinsurance so that separate financing is optimal (i) for $\gamma < 0.85$ when $\mu = 1.3$ and $\sigma = 1.49$ in panel A (for $\gamma = 1$ the difference is 0), (ii) for $\mu < 1.31$ when $\sigma = 1.49$ and $\gamma = 0.8$ in panel B, and (iii) for $\sigma > 1.46$ when $\mu = 1.3$ and $\gamma = 0.8$ in panel C. Therefore, and consistent with the first three predictions of the baseline model, separation is optimal if the recovery rate is small (or the financial distress costs are large), the mean is low, and the standard deviation is high.

4.2 Distributions with Full Support

To better compare our results with Leland's (2007) numerical analysis for normally distributed returns, we turn to distributions with full support. The limited liability shelter allows both the creditor and the firm to walk out of negative returns through the bankruptcy process (Leland, 2007, top of page 770). Default occurs when returns are, instead, positive but insufficient to repay the creditors (page 771). Leland (2007) decomposes the difference in the firm's value from joint relative to separate financing into:

(i) the change in the limited liability shelter, which is always negative and thus favors separate financing,

(ii) plus the tax savings from optimal leveraging, which can favor separate or joint financing, and

(iii) minus the change in expected default costs, which is "negative in all examples considered" by Leland (2007, page 779).

By assuming that the projects need to be financed only with debt, we abstract away from the tax effect, (ii). We decompose further the change in the value of default costs, (iii), into a negative component (coinsurance effect) and a positive component (risk-contamination effect) for the case of distributions with partly negative support. We then show that risk contamination prevails so that the change in value of the default costs is actually positive when returns are normally distributed with variance sufficiently higher than in the calibration reported by Leland (2007).

We also note that the limited liability effect, (i), disappears when third parties who suffer the liability externality insist on obtaining proper compensation ex ante. The profitability of joint or separate financing is then determined exclusively by the change in expected default costs, on which we focus. More generally, we characterize the total effect of (i) and (iii).

Decomposition Revisited and Limited Liability Effect We now consider two identically and independently distributed projects with continuous density $f(r_i)$ and distribution $F(r_i)$ over the full support. Given that both the firm and the creditor can walk out of negative returns, the creditor profits are the same as in the case of positive support distributions, i.e. (4) and (7). Following the same procedure as before, the firm's profits, as defined in (5) and (8), are now equal to the net expected returns minus the expected default costs and plus the limited liability gains. That is, firm profits under separate financing are

$$\mu - 1 - \int_0^{r_i^*} (1-\gamma) r_i f(r_i) dr_i - \int_{-\infty}^0 r_i f(r_i) dr_i, \tag{12}$$

and under joint financing

$$\mu - 1 - \int_0^{r_m^*} (1-\gamma) r_m h(r_m) dr_m - \int_{-\infty}^0 r_m h(r_m) dr_m, \tag{13}$$

where

$$h(r_m) = \int_{-\infty}^{\infty} f(2r_m - r_i) 2 f(r_i) dr_i.$$

Therefore, the net gains of joint financing are given by $\Delta\pi = -\Delta DC + \Delta LL$, where ΔDC is defined as

$$\Delta DC := \int_0^{r_m^*} (1-\gamma) r_m h(r_m) dr_m - \int_0^{r_i^*} (1-\gamma) r_i f(r_i) dr_i. \tag{14}$$

and the limited liability effect is given by

$$\Delta LL := -\int_{-\infty}^0 r_m h(r_m) dr_m + \int_{-\infty}^0 r_i f(r_i) dr_i. \tag{15}$$

The following proposition summarizes this decomposition and further decomposes the changes in expected default costs.

Proposition 7 (Decomposition with possibly negative returns) *The net financial synergies of two independently distributed projects with continuous density f with full support can be decomposed into the limited liability effect and the reduction in expected default costs, $\Delta\pi = \Delta LL - \Delta DC$, and the reduction in expected default costs can be decomposed into the coinsurance gains and risk-contamination losses of conglomeration, i.e. $-\Delta DC = CI - RC$, where*

$$CI := \int_{r_m^*}^{\infty}\int_{0}^{r_i^*}(1-\gamma)r_i f(2r_m - r_i)2f(r_i)dr_i dr_m - \int_{0}^{r_m^*}\int_{-\infty}^{0}(1-\gamma)r_i f(2r_m - r_i)2f(r_i)dr_i dr_m,$$

$$RC := \int_{0}^{r_m^*}\int_{r_i^*}^{\infty}(1-\gamma)r_i f(2r_m - r_i)2f(r_i)dr_i dr_m - \int_{-\infty}^{0}\int_{0}^{r_i^*}(1-\gamma)r_i f(2r_m - r_i)2f(r_i)dr_i dr_m,$$

$$\Delta LL = -\int_{-\infty}^{0}\int_{-\infty}^{\infty} r_i f(2r_m - r_i)2f(r_i)dr_i dr_m + \int_{-\infty}^{0}\int_{-\infty}^{\infty} r_i f(2r_m - r_i)2f(r_i)dr_m dr_i.$$

To interpret the results, notice that the reduction in expected default costs is equal to the gains of conglomeration net of the limited liability effect, $-\Delta DC = \Delta\pi - \Delta LL$. In other words, the change in default costs equals the gain or loss from conglomeration in a setting with unlimited liability.

Using this observation, we can interpret the two terms in CI and RC that did not appear in the case of distributions with positive support. The second term in CI represents the additional gains that arise because, under separate financing, the returns of the project would have been negative $r_i < 0$, and therefore the firm would have been responsible for the full losses, whereas, under joint financing, the average returns are positive ($0 < (r_i + r_j)/2 < r_m^*$) and therefore part of the average returns (and thus part of the losses of the project i) are lost as financial distress costs. Similarly, the second term in RC represents the reduction in risk-contamination losses that arises because, under separate financing, the project would have had a positive return with financial distress losses ($0 < r_i < r_i^*$), but under joint financing the average returns are negative ($(r_i + r_j)/2 < 0$), and therefore the firm is fully liable for the average return losses, while also gaining all the returns of project i. Of course, in the case of positive support distributions, the new two terms are equal to 0. Figure 2 also includes these two terms (labelled $CI2$ and $RC2$), along with the limited liability effect, ΔLL.

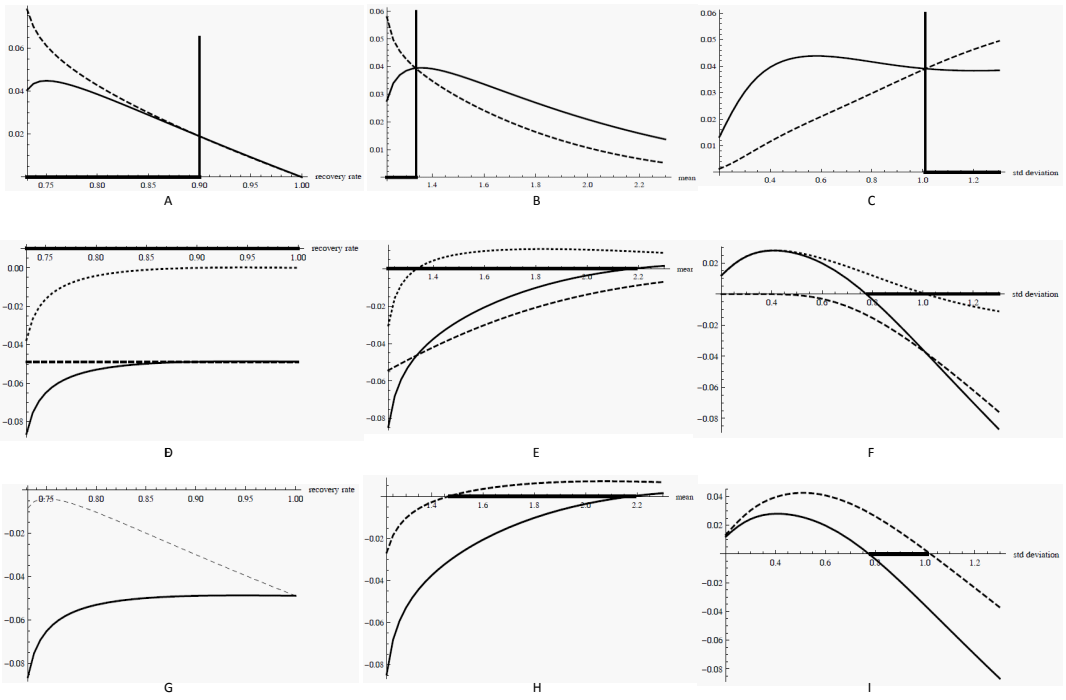

Figure 4: **Coinsurance, risk contamination, and limited liability for the normal distribution.** Panels A, B, and C in the first row plot the coinsurance gains CI (solid line) and risk-contamination losses RL (dashed line) against the recovery rate, the mean, and the standard deviation, respectively. Panels D, E, and F in the second row plot the total profit difference between joint and separate financing $\Delta\pi$ (solid line), the limited liability effect ΔLL (dashed line), and the reduction in expected default costs $-\Delta DC$ (dotted line). Panels G, H, and I in the third row plot the total profit difference between joint and separate financing $\Delta\pi$ (solid line), and the total difference net of the risk-contamination losses $\Delta\pi + RC$ $(= \Delta LL + CI)$ (dashed line).

Comparison with Leland (2007). Figure 4 displays when the reduction in expected default costs, $-\Delta DC = \Delta\pi - \Delta LL$, is negative, taking as base case a recovery rate of $\gamma = 0.8$ and a normal distribution with mean $\mu = 1.3$ and standard deviation $\sigma = 1.1$. Panels A, B, and C plot the coinsurance gains and risk-contamination losses with respect to the recovery rate, the mean, and the standard deviation, respectively. As before, the risk-contamination losses dominate the coinsurance gains if the recovery rate is small, the mean is low, and the standard deviation is high. The parameter regions for which joint financing increases expected default costs are marked by a bold segment on the horizontal axis of each plot.

Panels D, E, and F perform comparative statics of the incremental profits from joint (relative to separate financing), the limited liability effect, and the reduction in expected default costs. Once the limited liability effect is included, separate financing is optimal (i) for all values of γ when $\mu = 1.3$ and $\sigma = 1.1$ in panel D (for $\gamma = 1$ the profit difference is equal to the limited liability effect), (ii) for $\mu < 2.2$ when $\sigma = 1.1$ and $\gamma = 0.8$ in panel E, and (iii) for $\sigma > 0.8$ when $\mu = 1.3$ and $\gamma = 0.8$ in panel F.[18] These parameter regions correspond to the bold segments marked on the horizontal axes. For example, the dotted line in Panel F shows that the reduction in expected default costs is positive (or, equivalently, the change in default costs is negative) for the parameter specification used by Leland (2007), i.e. with mean $\mu = 1.3$ and standard deviation $\sigma = 0.5$. In addition, the reduction in expected default costs outweighs the limited liability effect (dashed line) and therefore joint financing is optimal (solid line). The same panel also shows that, with the same parameter specification but with a sufficiently higher standard deviation ($\sigma > 0.8$), the reduction in expected default costs becomes negative and thus separate financing is optimal. Comparing panels D, E, and F with panels A, B, and C, we confirm that taking into account the limited liability effect enlarges the set of parameters for which separate financing is optimal.

Panels G, H, and I compare the total incremental profit from joint financing, $\Delta\pi$ ($= \Delta LL - \Delta DC = \Delta LL + CI - RC$), with the incremental profits net of the risk-contamination effect, $\Delta LL + CI$ ($= \Delta\pi + RC$). The bold segments on the horizontal axes correspond to the parameter regions for which joint financing would have been chosen if risk-contamination losses were disregarded, even though separate financing is the preferred financing regime. Ignoring the risk-contamination effect would result in too much joint financing. Indeed, projects with $1.15 < \mu < 2.2$ for $\sigma = 1.1$ and $\gamma = 0.8$ as well as projects with $0.8 < \sigma < 1$ for $\mu = 1.3$ and $\gamma = 0.8$ would then be wrongly financed jointly rather than separately.

[18] In all panels, we show the range of parameters for which projects can be financed both jointly and separately. In panels A, D, and G the projects cannot be financed separately if $\gamma < 0.7$ and jointly if $\gamma < 0.75$. In panel B, E, and H projects cannot be financed if $\mu < 1.2$.

Exclusion of Limited Liability Effect. The limited liability effect favors separate financing because the firm's returns are assumed to be independent of its corporate structure. Suppose that the negative return realizations represent the payments that customers or suppliers filing legal claims against the firm are unable to receive because the firm enjoys limited liability. Since the amount lost by customers and suppliers is higher with separate financing, the returns of the firm under separate financing should be lowered by a corresponding amount.

A similar argument has been made by Smith and Warner (1979) in a reply to Scott's (1977) claim that, by issuing secured debt, a firm can increase the value of its securities by reducing the amount available to pay potential legal damages to customers and suppliers for defective products, should the firm go bankrupt. Smith and Warner (1979) point out that this is true only because Scott (1977) *unrealistically* assumes that the firm's net operating earnings are independent of its level of secured debt. They argue that a customer who buys the firm's product purchases both the services of the product and a "security" representing the right to sue the firm. If a firm increases its level of secured debt, it reduces the value of the above-mentioned security which customers receive. A similar argument applies to the externality imposed on suppliers. The earnings of the firm, which in part consist of the revenues it receives from the sale of these securities, should thus fall by an amount equal to the market value of the claim which an increase in secured debt has subtracted from customers or suppliers. Therefore, once this effect is taken into account, the value of the firm should be independent of the level of secured debt.

Following the logic of Smith and Warner's (1979) argument, the expected value of the firm should fall exactly by the increased amount that separate financing has taken away from customers and suppliers. As a result, the limited liability effect should be netted out from the gains of separate financing. An alternative way to make the same point is to view the security mentioned by Smith and Warner (1979) as an insurance warranty which the firm offers with the product traded. Suppose that the negative return realizations represent the losses incurred by customers or suppliers that trade with the firm in case the product is defective and causes damage. The firm offers a full

insurance security that promises a payment equal to the loss in each possible negative realization. The willingness to pay by customers or suppliers for this security is equal to the value of limited liability, $-E(r|r < 0)$. By financing the projects jointly, the value of this insurance warranty is reduced exactly by the limited liability effect, and therefore the gains of the firm is reduced exactly by this amount. However, the actuarially fair price that the firm has to pay for this insurance in a competitive insurance market will accordingly be reduced by the same amount.

If we use this argument we need to subtract the limited liability effect when computing the gains from joint financing. As a result, the profit difference is equal to the reduction in expected default costs, $\Delta\pi' = \Delta\pi - \Delta LL = -\Delta DC$. As shown in Figure 4, joint financing would then be more profitable because the limited liability effect favored separate financing. The financing decision would then be uniquely determined by the tradeoff between the coinsurance and risk-contamination effects, and the reduction in expected default costs. More generally, the firm could be forced to bear a fraction of the losses in case of a negative return. In terms of the graphs in Panels D, E, and F, the profit difference would move from the dotted line (full losses, i.e., without the limited liability effect) to the solid line (no loss, i.e., with limited liability effect) as we decrease the fraction of the losses borne by the firm.

4.3 Calibrated Specifications and Additional Comparative Statics

We continue our numerical investigations by displaying a number of additional realistic scenarios in which risk contamination dominates coinsurance so that financial dis-synergies prevail. We first consider a standard calibration of the stable distribution. Then, we consider a bimodal normal with the same mean and standard deviation as Leland (2007). We also perform comparative statics with respect to skewness, bimodality, and correlation.

Stable Distribution and Skewness. We consider first stable distributions (also known as α-Lévy stable distributions), which has been widely used by empiricists to model financial data. The class of stable distributions can be seen as a generalization of the normal distribution while allowing

for skewed (or asymmetric) returns and for tails of varying thickness, features that are frequently observed in financial data; see, for example, Mandelbrot (1963), Fama (1965), and Roll (1970). Stable distributions are the only distributions that retain their shape under addition. The sum (and average) of two stable distributions is stable, and, if any linear combination of two distributions follow the same distribution, then this distribution is stable.[19] The normal, the Cauchy, and the Lévy distributions have this property and thus belong to the stable class. In addition, by the Generalized Central Limit Theorem, the only possible non-trivial limit of normalized sums of independent identically distributed terms is a stable distribution (Nolan, 2005). Thus, stable distributions represent well observed data that result from the sum of a large number of small terms.

The class of stable distributions is described by four parameters $(\alpha, \beta, \eta, \delta)$. The parameter $\alpha \in (0, 2]$ is called the stability index. The normal distribution corresponds to the case $\alpha = 2$; if $\alpha < 2$ the distribution exhibits fat tails. The parameter $\beta \in [-1, 1]$ is called the skewness index: if $\beta = 0$, the distribution is symmetric, if $\beta > 0$ it is skewed towards the right, and if $\beta < 0$, it is skewed towards the left. The parameters α and β determine the shape. The parameter $\eta > 0$ is a scale parameter. The parameter δ is a location parameter that shifts the distribution to the right if $\delta > 0$, to the left if $\delta < 0$, and that is equal to the mean if $\alpha > 1$ (otherwise the mean is undefined). McCulloch (1997) calibrated the stable distribution with monthly stock market data from the Center for Research in Security Prices (CRSP). Using 40 years of the CRSP value-weighted stock index, including dividends and adjusted for inflation, his maximum likelihood estimates are $\alpha = 1.855$, $\beta = -0.558$, $\eta = 2.711$, and $\delta = 0.871$. These estimates indicate that the data exhibits fat tails ($\alpha < 2$) and negative skewness ($\beta < 0$).

Panel A of Figure 5 performs comparative static of the profit difference, the limited liability effect, and the reduction in expected default costs with respect to the skewness. We take as base case the parameters of the distribution estimated by McCulloch (1997) and, as before, Davydenko,

[19] Formally, let X_1 and X_2 be independent copies of a random variable X. Then X is said to be stable if for any constants $a > 0$ and $b > 0$ the random variable $aX_1 + bX_2$ has the same distribution as $cX + d$ for some constants $c > 0$ and d. The distribution is said to be strictly stable if this holds with $d = 0$. See Nolan (2005).

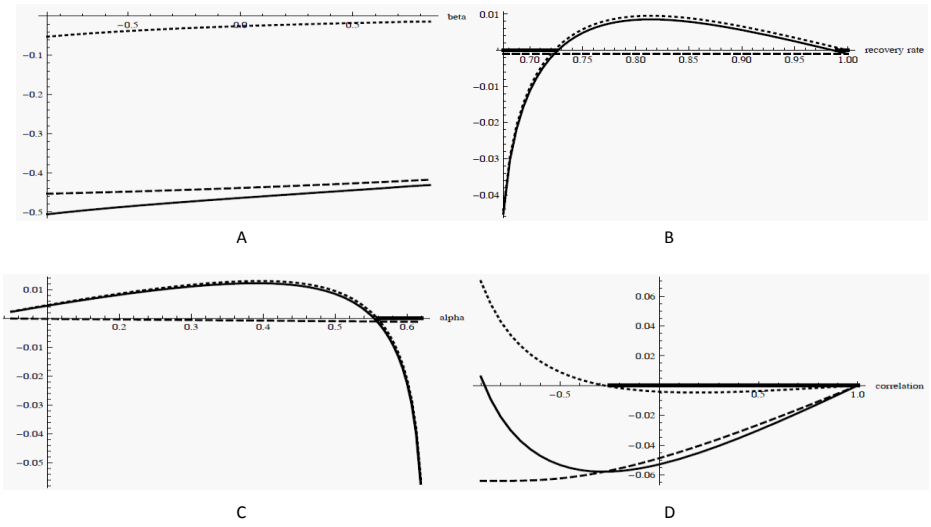

Figure 5: **Effects of skewness, bimodality, and correlation.** Total profit difference between joint and separate financing $\Delta\pi$ (solid line), the limited liability effect ΔLL (dashed line), and the reduction in expected default costs $-\Delta DC$ (dotted line) against the skewness parameter for the stable distribution in Panel A, the recovery rate and the mixture coefficient for the mixed normal model in Panels B and C, and the correlation coefficient for the correlated normal model in Panel D.

Strebulaev, and Zhao's (2012) estimate of the recovery rate.

The figure shows that the change in expected default costs is negative and the risk-contamination effect dominates the coinsurance effect for McCulloch's (1997) calibration of the stable distribution, which corresponds to $\beta = -0.558$. Importantly, and consistent with the results of our baseline model with binary returns, the risk-contamination effect is relatively more important as the skewness parameter (β) decreases. We have also verified that the risk-contamination effect dominates the coinsurance effect when $\alpha = 2$, corresponding to the normal distribution (β is irrelevant then), but it is even more important if the distribution exhibits fatter tails ($\alpha < 2$). In addition, the risk contamination effect is enhanced when the recovery rate decreases (or the financial distress costs increase) and when the mean decreases (i.e., if δ decreases).

Bimodal Distribution. We now turn to a bimodal distribution, which has been recently used to explain features of the financial crisis. El-Erian and Spence (2012) report the prevalence of subjective

bimodal distributions on the part of the investors. They claim in the current environment, there are "two or more scenarios, each quite different and each with its own distribution of outcomes, correlations, market functioning and returns. Investors are faced with the need to assess the relative likelihood of the scenarios, and then take a weighted average of usually two rather more normal looking distributions to end up with the bimodal one." Here, we show that the risk-contamination effect also dominates the coinsurance effect for a bimodal distribution with the same mean and standard deviation as Leland (2007).

Consider a mixture of two normal distributions, defined as one normal random variable with probability θ and another normal random variable with probability $1-\theta$, where $\theta \in (0,1)$ is the mixture coefficient. As a base case, take an equal probability ($\theta = 0.5$) of a normal with mean $\mu_1 = 0.9$ and standard deviation $\sigma_1 = 0.41$, and another with mean $\mu_2 = 1.7$ and standard deviation $\sigma_2 = 0.1$. The resulting distribution has mean $\mu = 1.3$ and standard deviation $\sigma = 0.5$, as in the base case of Leland (2007). As before, we take as base case a recovery rate of $\gamma = 0.8$.

Panels B and C of Figure 5 perform comparative statics of the profit difference, the limited liability effect, and the reduction in expected default costs with respect to the recovery rate and the mixture coefficient. Panel B shows that the risk-contamination effect dominates the coinsurance effect in a distribution with the same expectation and standard deviation as Leland (2007) as long as the recovery rate is small (i.e., if $\gamma < 0.73$), consistent with the baseline model. Panel C, in addition, shows that the effect is relatively more important when the bimodal distribution assigns more weight to the distribution with lower mean. We have also verified that the risk-contamination effect is relatively more important if the mean of any of the two normal distributions in the mixture is lower and/or their standard deviation is larger, as in our baseline binary model.

Correlation. Using the normal specification of Section 4.2 we now show that an increase in correlation favors separate financing.[20] Panel D in Figure 5 performs comparative statics of the profit

[20] The average distribution of two identical normal distributions with mean μ and standard deviation σ is a normal distribution with mean μ and standard deviation $\sqrt{(1+\rho)/2}\sigma$, where ρ is their correlation coefficient. Clearly, if $\rho = 0$

difference, the limited liability effect, and the reduction in expected default costs with respect to the correlation coefficient. The figure shows that the reduction in expected default costs from joint financing is negative if $\rho > -0.3$. When the limited liability effect is disregarded the increase in total profits is also negative, and also remains negative more generally because the limited liability effect is negative. As in the binary model, increasing correlation favors separate financing, while decreasing correlation favors joint financing.

5 Conclusion

This paper analyzes the simple economics of conglomeration with default costs. Our results qualify the long-standing claim that joint financing generates financial benefits by economizing on default costs. By turning on its head the classic logic that generates coinsurance savings from conglomeration, we characterize instances in which expected default costs increase because of risk contamination. For projects with binary returns we provide a complete characterization of the tradeoff between coinsurance and risk contamination. Broadly consistent with empirical evidence, the analysis predicts that:

- An increase in the fraction of returns lost due to default costs favors separate financing;
- An increase in average returns favors joint financing;
- An increase in the riskiness of returns favors separate financing;
- An increase in the negative skewness of returns favors separate financing;
- An increase in the correlation of returns favors separate financing.

In addition, we show that *separate* financing can be optimal even when joint financing involves paying a *lower* repayment rate or results in a *lower* probability of default.

The analysis in this paper restricts attention to two ex-ante identical projects that had to be financed with debt only and with default costs proportional to the value of the assets under default.

we are back to the baseline scenario with independent returns.

In a model with binary returns, Banal-Estañol and Ottaviani (2013) investigate the optimal structure of financial conglomeration with projects that have heterogeneous returns, a multiple number of projects, general specifications of default costs, and financing through tax-disadvantaged equity:

- Coinsurance and risk contamination effects may be present simultaneously in a setting with two projects with binary but heterogeneous returns, as in the case of identical projects with continuous return distributions.

- With more than two projects, sometimes it is optimal to partially conglomerate projects into subgroups of intermediate size. However, when the number of independent projects becomes arbitrarily large, the risk-contamination effect vanishes and it becomes optimal to finance all the projects jointly.

- Economies of scale in default costs (according to which per-project default costs are lower when projects are financed jointly) favor joint financing, while diseconomies of scale favor separate financing.

- Allowing for financing through tax-disadvantaged equity tends to favor joint financing, because equity financing sometimes makes it possible to obtain a repayment rate that avoids intermediate default when one project yields a high return and the other yields a low return.

In our setup, either investors in each of the two projects have recourse to the returns of the other project (with joint financing) or none of them have access to the returns of the other project (with separate financing). In reality, an asymmetric, intermediate situation could also arise whereby investors in one (recourse) project have access to the returns of the other (nonrecourse) project, but not conversely. In this case, one of the diagonal entries in Figure 1 would be akin to separate financing. That is, if the project without recourse yielded a low return while the project with recourse yielded a high return, the former project would go bankrupt while the latter project would stay afloat. In the other diagonal entry, however, both projects would stay afloat provided that the recourse project

is saved by the nonrecourse project. If this is the case, this intermediate solution would dominate separate financing, but the reverse would hold when the recourse project is dragged down by the nonrecourse project. A complete analysis for the resulting tradeoff is left to future research; see Nicodano and Luciano (2009) for an investigation in this direction in a setting with both default costs and taxes.

Saving an unsuccessful project might sometimes be optimal for reputational reasons, even if it has been financed with (nonrecourse) debt and the firm is under no legal obligation to save it. Gorton (2008), for example, points out that securitization issuers retain substantial *implicit* exposure even after mortgages are securitized. In the credit card asset-based securities (ABS) market, for example, Higgins and Mason (2004) document instances in which issuers of credit card ABS have taken back non-performing loans despite not being contractually required to do so. Similarly, Gorton and Souleles (2006) show that prices paid by investors in credit card ABS take into account issuers' ability to bail out their ABS. To capture this tradeoff, one could extend our static model to a dynamic framework. It is also natural to extend the model to allow for multiple (and possibly risk-averse) investors, as in Bond's (2004) analysis of conglomeration versus bank intermediation in the costly state verification model.

Finally, our model can also be extended to analyze the public policy problem of optimal conglomeration in the presence of systemic spillovers, a topic that has recently attracted attention (see, for example, Acharya, 2009, and Ibragimov, Jaffee, and Walden, 2011). In this case, bankruptcies create significant negative externalities and the borrower should minimize the probability of default instead of maximizing net returns. We leave the development of this extension to future research.

Appendix: Proofs

Proof of Proposition 1: The proof follows from the analysis reported in the text. Q.E.D.

Proof of Proposition 2: If projects can be financed separately, i.e. condition (1) is satisfied, the entrepreneur obtains a per-project return of $p(r_H - r_i^*)$, which is equal to the ex post net present value

$$pr_H + \gamma(1-p)r_L - 1. \tag{16}$$

Similarly, if condition (2) is satisfied, the entrepreneur obtains a per-project return of $p^2(r_H - r_m^*) + 2p(1-p)\left[(r_H + r_L)/2 - r_m^*\right]$, or

$$p^2 r_H + 2p(1-p)(r_H + r_L)/2 + \gamma\left(1-p\right)^2 r_L - 1, \tag{17}$$

and, if condition (3) but (2) is not satisfied, she obtains $p^2(r_H - r_m^{**})$, or

$$p^2 r_H + \gamma 2p(1-p)(r_H + r_L)/2 + \gamma\left(1-p\right)^2 r_L - 1. \tag{18}$$

Subtracting (17) from (16), we obtain $(1-\gamma)\,p(1-p)r_L$ and therefore joint financing is more profitable than separate financing. Instead, subtracting (16) from (17), we obtain $(1-\gamma)\,(1-p)pr_H$ and therefore separate financing is more profitable than joint financing. Q.E.D.

Proof of Prediction 1: The statements follow from the fact that the derivatives of the left-hand of (1), (2), and (3) with respect to γ are negative. Q.E.D.

Proof of Prediction 2: The statements follow from the fact that the derivatives of the left-hand of (1), (2), and (3) with respect to p are negative. Q.E.D.

Proof of Prediction 3: Letting ε be such that $\widehat{r}_H = r_H + \varepsilon$, we have that, in order to have a mean preserving spread, $\widehat{r}_L = r_L - \frac{p}{1-p}\varepsilon$. Substituting into condition (2), the derivative of the left-hand side less the derivative of the right-hand side is equal to

$$\frac{1-p}{2-p}\gamma + \frac{1}{2(1-p)} - 1,$$

which is positive if and only if $p > \overline{p}$, where $\overline{p} \equiv \left[1 + 4(1-\gamma) - \sqrt{1 + 8(1-\gamma)}\right] / 2(1-\gamma)$. Therefore, condition (2) is less likely to be satisfied following an increase in ε if and only if $p > \overline{p}$. It can be easily checked that $\overline{p} < 1/2$ for any γ. Q.E.D.

Proof of Prediction 4: Letting ε be such that $\widehat{r}_L = r_L - \varepsilon$, we have that, in order to have a mean preserving spread, $\widehat{p} = p - \frac{(1-p)\varepsilon}{r_H - r_L + \varepsilon}$. Following the same procedure as in the proof of the previous prediction, there exists $\overline{r}_H$, such that condition (2) is less likely to be satisfied following an increase in ε if and only if $r > \overline{r}_H$. Q.E.D.

Proof of Proposition 3: (i) Suppose that γ and r_L are arbitrarily close to 1, condition (2) is arbitrarily close to $\frac{r_H + r_L}{2} > 1$ whereas condition (1) simplifies to $r_H > 1$. Clearly there are situations in which condition (2) is satisfied, and therefore projects can be financed jointly, but condition (1) is not satisfied, and therefore projects cannot be financed separately.

(ii) If condition (2) is not satisfied, projects can only be financed jointly if condition (3) is satisfied. Condition (3) can be rewritten as

$$pr_H - p(1-p)r_H(1-\gamma) + (1-p)\gamma r_L > 1.$$

This implies that $pr_H + (1-p)\gamma r_L > 1$, which in turn implies that projects can be financed separately. Of course, the opposite is not true, if the parameters are such that $pr_H + (1-p)\gamma r_L$ is arbitrarily close to 1, then condition (3) is not satisfied. Q.E.D.

Proof of Proposition 4: Suppose first that a rate below the crossing point can be obtained. We have that

$$r_m^* = \frac{1 - (1-p)^2 \gamma r_L}{1 - (1-p)^2} < \frac{1 - (1-p)\gamma r_L}{p} = r_i^*,$$

because $1 > \gamma r_L$. Next, suppose that only a rate r_m^{**} above the crossing point can be obtained and therefore the probability of default is higher with joint financing. Nevertheless, the rate r_m^* associated

with joint financing is lower than r_i^* associated with separate financing whenever

$$r_m^{**} = \frac{1-(1-p)\,\gamma\,(pr_H + r_L)}{p^2} < \frac{1-(1-p)\,\gamma r_L}{p} = r_i^*,$$

or equivalently when

$$\gamma r_H > \frac{1-(1-p)\gamma r_L}{p} = r_i^*,$$

as claimed. Q.E.D.

Proof of Prediction 5: Clearly, separate financing is not affected by correlation. The joint financing repayment rates, r_m^* and r_m^{**} in Proposition 1, and the corresponding financing conditions, are now replaced by $r_{m,\rho}^*$ and $r_{m,\rho}^{**}$, respectively, where

$$r_{m,\rho}^* := \frac{1-(1-p)\,[1-p\,(1-\rho)]\,\gamma r_L}{1-(1-p)\,[1-p\,(1-\rho)]} < \frac{r_H + r_L}{2},$$

and

$$r_{m,\rho}^{**} := \frac{1-(1-p)\,\gamma r_L}{p\,[1-(1-p)\,(1-\rho)\,(1-\gamma)]} < r_H.$$

Note that $r_{m,\rho}^*$ and $r_{m,\rho}^{**}$ are respectively increasing and decreasing in ρ. Q.E.D.

Proof of Proposition 5: We show the split of the reduction of the expected default costs $-\Delta DC$, where ΔDC is defined in (*10*), into the two terms in the statement of the proposition. We first rewrite $\Delta DC/(1-\gamma)$ using that $h(r_m) = \int_0^\infty f(2r_m - r_i)2f(r_i)dr_i$ in the first term and introducing $\int_0^\infty f(2r_m - r_i)2dr_m (= 1 \text{ for all } r_i)$ in the second term, to obtain

$$\Delta DC/(1-\gamma) = \int_0^{r_m^*}\int_0^\infty r_m f(2r_m - r_i)2f(r_i)dr_i dr_m - \int_0^{r_i^*}\int_0^\infty r_i f(2r_m - r_i)2f(r_i)dr_m dr_i. \qquad (19)$$

We then decompose the first term in the right-hand side of (19) into two terms using $r_m = -(r_i - r_m) + r_i$, split the second integral that results, and also split the second term of (19) to obtain

$$\begin{aligned}\Delta DC/(1-\gamma) = &-\int_0^{r_m^*}\int_0^\infty (r_i - r_m)f(2r_m - r_i)2f(r_i)dr_i dr_m \\ &+\int_0^{r_m^*}\int_0^{r_i^*} r_i f(2r_m - r_i)2f(r_i)dr_i dr_m + \int_0^{r_m^*}\int_{r_i^*}^\infty r_i f(2r_m - r_i)2f(r_i)dr_i dr_m \\ &-\int_0^{r_i^*}\int_{r_m^*}^\infty r_i f(2r_m - r_i)2f(r_i)dr_m dr_i - \int_0^{r_i^*}\int_0^{r_m^*} r_i f(2r_m - r_i)2f(r_i)dr_m dr_i.\end{aligned}$$

Note that the first term is equal to 0. Using the law of iterating expectations to alter the order of the integrals of the second term, we the second and the fifth terms also cancel out. Applying again

the law of iterating expectations, and rearranging the remaining terms, we have that

$$-\Delta DC = \int_{r_m^*}^{\infty}\int_0^{r_i^*}(1-\gamma)r_i f(2r_m - r_i)2f(r_i)dr_i dr_m - \int_0^{r_m^*}\int_{r_i^*}^{\infty}(1-\gamma)r_i f(2r_m - r_i)2f(r_i)dr_i dr_m,$$

as we intended to show. Q.E.D.

Proof of Proposition 6: Define $j(r) := g(r)/(1-q)$ as the density of a distribution function defined on the strictly positive part, so that $\int_0^{\infty} j(r_i)dr_1 = 1$, and the mixed distribution function is given by

$$F(r) := q + (1-q)\int_0^r j(r_i)dr_i$$

whereas the distribution of the average is given by

$$H(r) := q^2 + 2q(1-q)\int_0^r j(2r_m)2dr_m + (1-q)^2\int_0^r\int_0^{\infty} j(r_i)j(2r_m - r_i)2dr_i dr_m.$$

Following the same procedure as before, the net per-project gains of joint financing are again equal to the reduction in the expected default costs, $\Delta\pi = -\Delta DC$, which are now given by

$$\begin{aligned}\Delta DC \quad : \quad &= (1-q)^2\int_0^{r_m^*}\int_0^{\infty}(1-\gamma)r_m j(r_i)j(2r_m - r_i)2dr_i dr_m + \\ &\quad 2q(1-q)\int_0^{r_m^*}(1-\gamma)r_m j(2r_m)2dr_m - (1-q)\int_0^{r_i^*}(1-\gamma)r_i j(r_i)dr_i\end{aligned}$$

Now, the expected default costs in joint financing are separated in two terms because, while the average return is lower than r_m^*, the return of one of them can be 0 or both of them are positive (if both are 0 there are no default costs).

We now provide the decomposition into coinsurance and risk-contamination effects. We rewrite $\Delta DC/[(1-\gamma)(1-q)]$ by decomposing the last term into two terms, using $1 = (1-q) + q$, and then introducing $\int_0^{\infty} j(2r_m - r_i)2dr_m (= 1 \text{ for all } r_i)$ in the first of those two terms to obtain

$$\begin{aligned}&\Delta DC/[(1-\gamma)(1-q)] = (1-q)\int_0^{r_m^*}\int_0^{\infty} r_m j(2r_m - r_i)2j(r_i)dr_i dr_m + 2q\int_0^{r_m^*} r_m j(2r_m)2dr_m \\ &-(1-q)\int_0^{r_i^*}\int_0^{\infty} r_i j(2r_m - r_i)2j(r_i)dr_m dr_i - q\int_0^{r_i^*} r_i j(r_i)dr_i.\end{aligned}$$

Now, from the proof of Proposition 5, the first and the third terms are equal to the first two terms below. The sum of the second and fourth terms, performing a change of variable, $2r_m = r_i$, in

the second one, can be written as the last term below

$$\Delta DC/[(1-\gamma)(1-q)] = (1-q)\int_0^{r_m^*}\int_{r_i^*}^{\infty} r_i j(2r_m - r_i)2j(r_i)dr_i dr_m$$

$$-(1-q)\int_0^{r_i^*}\int_{r_m^*}^{\infty} r_i j(2r_m - r_i)2j(r_i)dr_m dr_i + q\int_{r_i^*}^{2r_m^*} r_i j(r_i)dr_i.$$

Using the law of iterating expectations and rewriting, using $g(r) = (1-q)j(r)$, we have

$$\begin{aligned} -\Delta DC &= \int_{r_m^*}^{\infty}\int_0^{r_i^*}(1-\gamma)r_i g(2r_m - r_i)2g(r_i)dr_i dr_m \\ &\quad -\int_0^{r_m^*}\int_{r_i^*}^{\infty}(1-\gamma)r_i g(2r_m - r_i)2g(r_i)dr_i dr_m - q\int_{r_i^*}^{2r_m^*}(1-\gamma)r_i g(r_i)dr_i, \end{aligned}$$

as desired. Q.E.D.

Proof of Proposition 7: We show the split of the reduction in expected default costs $-\Delta DC$, defined in (*14*), into the four terms reported in the statement of the proposition. We first rewrite $\Delta DC/(1-\gamma)$ using that $h(r_m) = \int_{-\infty}^{\infty} f(2r_m - r_i)2f(r_i)dr_i$ and introducing $\int_{-\infty}^{\infty} f(2r_m - r_i)2dr_m$ (which is equal to 1 for all r_i) in the second term to obtain

$$\Delta DC/(1-\gamma) = \int_0^{r_m^*}\int_{-\infty}^{\infty} r_m f(2r_m - r_i)2f(r_i)dr_i dr_m - \int_0^{r_i^*}\int_{-\infty}^{\infty} r_i f(2r_m - r_i)2f(r_i)dr_m dr_i. \quad (20)$$

We then decompose the first term of the right-hand side of (20) into two terms using $r_m = -(r_i - r_m) + r_i$ and then split the second integral that results and the second term of (20) to obtain

$$\begin{aligned} \Delta DC/(1-\gamma) &= -\int_0^{r_m^*}\int_{-\infty}^{\infty}(r_i - r_m)f(2r_m - r_i)2f(r_i)dr_i dr_m \\ &+\int_0^{r_m^*}\int_{-\infty}^{r_i^*} r_i f(2r_m - r_i)2f(r_i)dr_i dr_m + \int_0^{r_m^*}\int_{r_i^*}^{\infty} r_i f(2r_m - r_i)2f(r_i)dr_i dr_m \\ &-\int_0^{r_i^*}\int_{r_m^*}^{\infty} r_i f(2r_m - r_i)2f(r_i)dr_m dr_i - \int_0^{r_i^*}\int_{-\infty}^{r_m^*} r_i f(2r_m - r_i)2f(r_i)dr_m dr_i. \end{aligned}$$

Note that the first term is equal to 0. Using the law of iterating expectations to alter the order of the integrals of the second term, part of the integral in the second and in the fifth terms cancel out. Applying again the law of iterating expectations, and rearranging the remaining terms, we obtain

$$\begin{aligned} -\Delta DC &= \int_{r_m^*}^{\infty}\int_0^{r_i^*}(1-\gamma)r_i f(2r_m - r_i)2f(r_i)dr_i dr_m - \int_0^{r_m^*}\int_{-\infty}^{0}(1-\gamma)r_i f(2r_m - r_i)2f(r_i)dr_i dr_m \\ &-\int_0^{r_m^*}\int_{r_i^*}^{\infty}(1-\gamma)r_i f(2r_m - r_i)2f(r_i)dr_i dr_m + \int_{-\infty}^{0}\int_0^{r_i^*}(1-\gamma)r_i f(2r_m - r_i)2f(r_i)dr_i dr_m, \end{aligned}$$

as we intended to show. Q.E.D.

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