PROCESS AND SYSTEMS BASED METHODOLOGIES RELATED TO CONTROL STRUCTURE SELECTION

BY

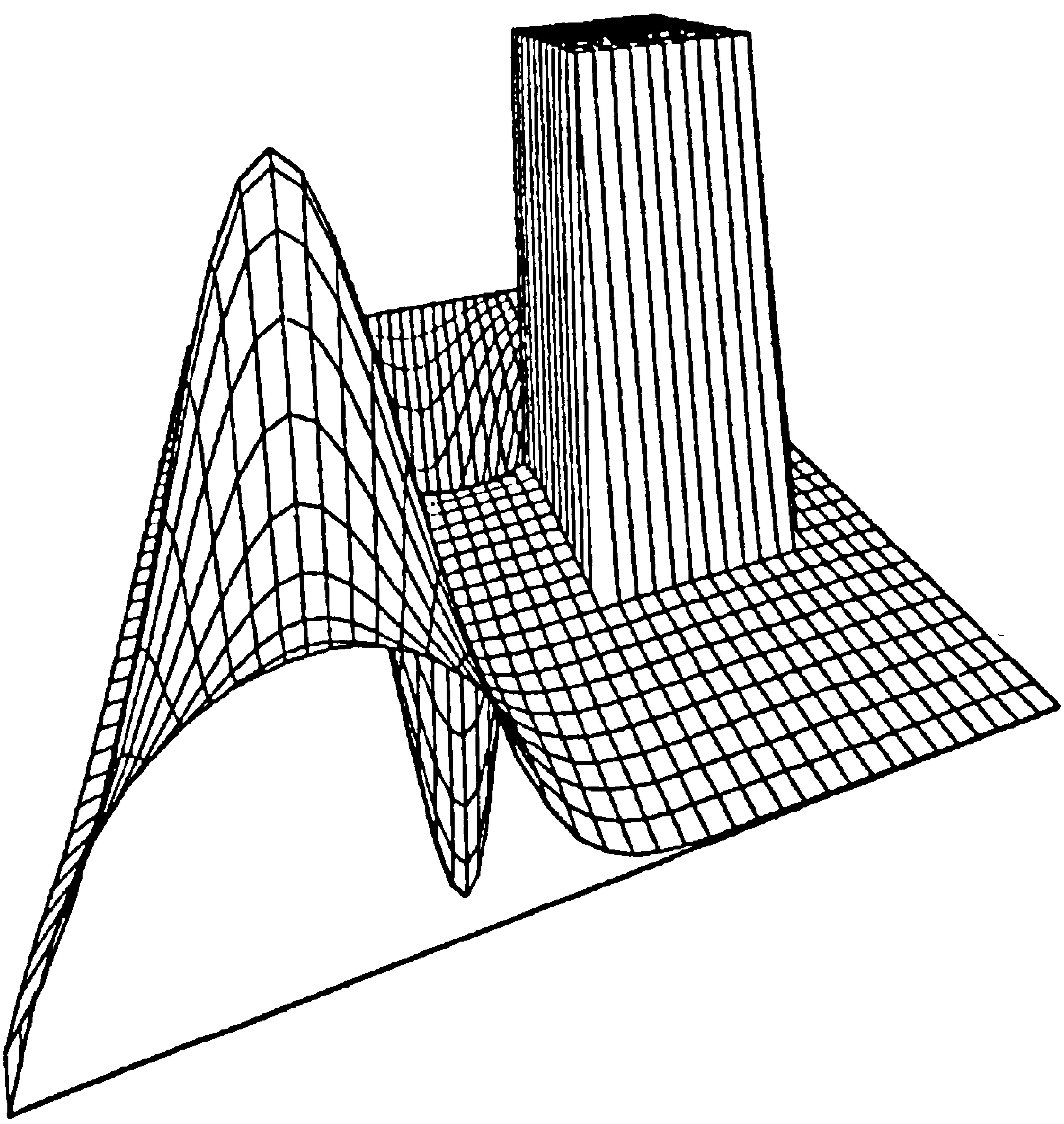
EMMANOUIL NISTAZAKIS

THESIS SUBMITTED FOR THE
AWARD OF THE DEGREE OF
DOCTOR OF PHILOSOPHY
IN
CONTROL ENGINEERING

CONTROL ENGINEERING CENTRE
DEPARTMENT OF ELECTRICAL, ELECTRONIC AND
INFORMATION ENGINEERING
CITY UNIVERSITY
LONDON EC1V 0HB

DECEMBER 1998

PROCESS AND SYSTEMS BASED METHODOLOGIES RELATED TO CONTROL STRUCTURE SELECTION



By wisdom a house is built,
and by understanding it is established;
by knowledge the rooms are filled
with all precious and pleasant riches.

Proverbs 24:3:4

Στους γονεὶς μου.

To my parents.

ACKNOWLEDGEMENTS

I can never adequately acknowledge the ever-encouranging generous and graceful spirit of my supervisor, Nicos Karcanias, whose many creative contributions and his invaluable criticism have been irreplaceable in the process of perfecting this thesis work. I am grateful for his intellectual guidance and support during the past years. His serious attitude towards research and his relentless insight to provide his students with the "whole" picture, provided me with the courage to continue in days of disillusionment, that inevitably accompany lost-lasting research. I know that I will succeed in whatever I do if I can match half of his intensity and dedication. But above all, I want to thank him for standing by me during some difficult periods of my life.

I had the privilege of working with two knowledgeable researchers, Dr. John Leventides and Dr. Stathis Milonidis. Without their contributions, the quality of this thesis would never have been the same. I look forward to many more fruitful interactions with them.

Despite my early expectations when I first arrived at City, I am leaving the place with many fond memories. Many friends have come along during the past eight years and their love and friendship have been of great importance to me. Even though my heart is overwhelmed with sadness for having to leave so much love and friendship behind, I am a better person for having known them. Names are unlisted here, but registered in my memory.

I would also like to thank my brother Costas Nistazakis for many conversations during my difficult and frustrating moments. And last, but certainly not least, I give special thanks to my parents. Without their love and support, I cannot imagine being where I stand now.

DECLARATION

ABSTRACT

This thesis is concerned with an important aspect of process control design, that is, the synthesis of the control structures. A review of the rapidly growing process methodologies' literature is presented and this leads to the identification of wider issues and new problems which are referred to as global instrumentation and forms the main subject of this thesis. The main objective has been the integration of existing process based tools and methodologies with a much more general approach of a systems and control theory character. The problem of Global Process Instrumentation concerns the selection of systems of measurement and actuation variables, found during the synthesis/design and operation of large-scale industrial processes/systems. The role of traditional instrumentation was considered but the emphasis has been on the systems aspects. In fact, instrumentation leads to the shaping of the final system and thus, is crucial in defining the control quality properties and operability characteristics of the final design. The development of these system aspects led to the emergence of an integrated framework for Global Instrumentation. An attempt was also made to abstract some results and formulate generic issues and problems, that would provide a wider scenario for activities in the future. Development of CAD to support the selection of control structures has been a major task undertaken here.

The system aspects of Global Instrumentation are demonstrated by studying two specific problems that involve the study of the structural properties of interconnected systems as a function of local selection of sensors and actuators and the problem of well-conditioning badly structured transfer functions. The role of selection of inputs and outputs, on the overall shaping of composite structure properties, at the subsystem level, was examined, and the significance of an assumption related to interconnections, referred to as the *completeness assumption,* was investigated. Specifically, the significance of the deviations from the completeness, was the subject of the investigation. Matrix Pencil Theory was used to examine the controllability, observability and zero structure related properties of composite systems under partial or total loss of inputs/outputs at the subsystem level. Selecting subsets of the original sets of inputs, outputs to guarantee full rank transfer function, was also an issue that was

examined. The above problems were presented as part of an integrated design philosophy that aims to explore the system structure.

An integrated approach to the overall problem of control structure selection was formulated and open issues and problems were identified. It was based on the assumption that there exists a progenitor model of the linear type for the process, which, however, may not be well defined. Structural analysis of the system theoretic framework, the interaction measures and the results for evaluation of alternative decentralisation schemes were then used, to specify a step by step approach to the control structure selection. The problem of handling alternative criteria was also considered and basic elements of a system procedure were given. There are many open issues, which were identified and are still open and thus the proposed structural approach should be considered as the first step to the development of an integrated methodology that involves the following major steps:

(a) Classification of system model variables and definition of well structured progenitor model.
(b) Definition of effective input, output structure based on operability, controllability criteria.
(c) Determining the structure of the control scheme by evaluation of alternative decentralised structures.

An important part of the integrated methodology for control structure selection is the – so called – interaction analysis. It consists of a number of diagnostics and structural tests that help to restrict the choice of the best scheme. Several of these tests/methodologies were reviewed and some of them were further expanded. The outcomes obtained by these methodologies provided promising results. These results gave the motivation for the construction of a complete CAD package, the "Interaction Analysis Toolbox", written in MATLAB®†. This Toolbox provides many tools and diagnostics that can be applied during the design stages, for the evaluation of the various alternative control structures.

† MATLAB® is a registered trademark of MathWorks Inc.

TABLE OF CONTENTS

NOTATION-ABBREVIATIONS

CPPM	: Centralised Pole Placement Map
DPPM	: Decentralised Pole Placement Map
DMP	: Decentralised Markov Parameters
DMPM	: Decentralised Markov Parameter Matrix
cmi	: column minimal indices
rmi	: row minimal indices
f.e.d.	: finite elementary divisors
i.e.d	: infinite elementary divisors
g.c.d	: greatest common divisor
i.z.p	: invariant zero polynomials
i.p.p	: invariant pole polynomials
l.c.m	: least common multiple
PFG	: Process Flowsheet Graph
MOP	: Model Orientation Problem

Chapter 1

INTRODUCTION

He who can at all times sacrifice Pleasure to Duty, approaches sublimity.

1. INTRODUCTION

Control Systems design is a topic that has received a lot of attention and it is well developed [Maciej., 1], [Marlin, 1]. The fundamental assumption in traditional design is that the system model is fixed, i.e. it has a given set of inputs and outputs. Furthermore, the structure of the controller is assumed to be given, as a certain way of coupling inputs and outputs, and usually, also, we assume the order of controller dynamics fixed. Thus, traditional control design is essentially a problem of tuning the parameters of the given structure and possibly specifying the dynamic complexity of the controller, to satisfy certain control design objectives. Seeing the problem of selection of inputs, outputs, as well as the structuring of the controller and the selection of its dynamic complexity as part of the overall control design, is what we refer to as Total Control Design (TCD) and it is a topic that has been addressed within the area of Process Control. So far, there has been no systematic methodology for tackling all issues involved. The general objectives of this thesis are to provide:

(a) A review of the methodologies from the Process Control area, which are relevant to the problem, and identification of the key issues, open for research.

(b) Provide CAD tools for Control Structure selection based on Process Control Methodologies and Diagnostics.

(c) Identify the Systems and Control Theoretic Issues involved in the Total Control Design problem and Introduce an integrated methodology based on System theoretic criteria and Process Control diagnostics and heuristics.

The problem of control structure selection is the main focus in this thesis. The issues, methodologies and concepts, that have emerged within the context of specific applications, are also reviewed. The approach is based on examining the particular, application based methodologies, first, and then extend them to generic ones that may be applied to problems, independent from application area nature. One of the most active areas, within which the control structure selection has been addressed, is that of process control and this area serves as a focus point. However, other application areas are also considered.

Articles and seminars evaluating the current status of process control and suggesting future research have been flourishing. A central point often raised is the unavailability of a systematic method for synthesising control structures for a complete plant. The problem is difficult because:

1. Chemical processes have non-linear, multiple couplings among variables.
2. The measurement and manipulation of process variables is limited to a relatively small number of variables.
3. The control objectives may not be clearly stated (or even known) at the beginning of the control system design.
4. Evaluation of the control system is based on a number of different objectives including: (a) safety, (b) reliability, (c) goodness of control (including stability), (d) range of control, (e) ease of start-up and shutdown, (f) cost of the control system, and (g) ease of operation of the system (including training).
5. The process structure may be changed to improve control.
6. There may be considerable uncertainty in the prediction of process behaviour.

Considering how many papers have been written on the control of a single unit operation like distillation, plant control has been discussed only a few times [Buckley 1], [Gov. & Pow., 1], [Umed., *et al.*, 1] because of its inherent complexity.

A control structure is composed of the following elements:

- a set of variables which are to be controlled to achieve a set of specified objectives.
- a set of variables which can be measured for control purposes.
- a set of manipulated variables, and
- a structure interconnecting measured and manipulated variables.

One of the key themes of process control is to develop a dynamic structure of measured and manipulated variables so that certain processing objectives are satisfied. Difficulties arise because, in certain cases, a variable will be both manipulated and controlled (e.g., ratio control of input streams). This implies that the various feasible sets of controlled, measured and manipulated variables and the interconnecting structure cannot be selected independently but should rather be considered simultaneously. To make matters more complicated, the optimal operating conditions change as a function of the external disturbances. [Maar. & Rij., 1] have demonstrated that the optimum operation of a plant switches discontinuously from one process constraint to another. Industrial experience also indicates that such operational policy is quite common and economically sound. It is clear though, that switching the operation of a plant from one given set of constraints to another implies a change in the plant's regulation structure.

While regulation is the principal control objective, the adopted regulatory control structure may not allow smooth, safe and reliable transition of the plant's operation to a new point for better economic performance. This conflict can be resolved by systematically formulating the regulatory structure and simultaneously optimizing the control structure. Further, if technically possible and economically justifiable, one will measure the controlled variables. Otherwise, secondary measurements will be chosen, in conjunction with estimation techniques, to infer the value of the unmeasured control objectives. The estimator will be part of the structure interconnecting the measurements and the manipulated variables. Apart from the macroscopic structuring difficulties, one faces a variety of local problems. Defining the lowest degree of model complexity, necessary to answer the posed questions, is an important initial task. Then, the development of preliminary control structures, which are feasible from an engineering and mathematical structural point of view, takes place, followed by an evaluation where more detailed static or dynamic models are required. The complexity of the encountered physicochemical systems makes checks for interaction and effects of nonlinearity necessary.

The selection of inputs, outputs and their coupling for control purposes is a complex problem that has to take into account a large, diverse set of requirements that arise within the area of process operations. Such problems are linked to quality issues, optimisation monitoring, fault diagnosis, overall assessment of the state of process operability, process flexibility etc. The process area is the subject of this review; other areas such as aerospace, also present similar problems. The main reason for giving emphasis on the process area is that, apart from its significance as an application area, it is characterised by strong interactions between alternative operational modes (when compared to discrete manufacturing) and this gives a more global significance to control structure selection issues. Furthermore, the process control area is quite active and many approaches and heuristics have emerged, which cannot be ignored in the development of an integrated methodology.

The structure of the thesis is the following: Chapter 2 is a review of background results and issues related to process based methodologies. The selection of control structures involves a number of fundamental issues. A brief, non-technical discussion of them is given, that includes the definition of the control objectives, the decomposition of the process and the selection of manipulated and controlled variables.

In chapter 3, some theoretical background material, from systems and process control is given. This is necessary, since most of the methods and results that appear in the subsequent chapters, require theory and theorems that are assumed to be known. The chapter provides a quick review of the theory of control systems and notions. By starting from the main definitions of a control system, it goes on to describe the McMillan form, with the relevant system properties and invariants, such as poles, zeros and matrix pencils, and it also outlines the main principles behind the Singular Value Decomposition. Finally, the notion of process controllability – in contrast to Kalman's definition (1960) of system controllability – is exploited.

Chapter 4 deals with the problem of global process instrumentation. The selection of systems of measurement and actuation variables is considered within the context of integrated design. It is now argued that, amongst the many different aspects of the problem, there are issues of Systems and Control Theory type which have not been considered before. The development of these system aspects and related methodologies are essential prerequisites for the emergence of an integrated framework for Global Instrumentation. An attempt was made to abstract the results of the review and to formulate generic issues and problems, that would provide a wider scenario for activities in the future.

Chapter 5 provides a study of the structural properties of interconnected systems, and examines the role of selection of inputs, outputs at subsystem level, on the overall shaping of composite structure properties. The importance of an assumption related to interconnections, which is referred to as the completeness assumption, and the significance of the deviation from that, is examined. The idea behind the work is an attempt to relate the structural aspects of the composite system in terms of the structural aspects of the subsystems and the nature of the interconnections. Of special interest is the investigation of the effect of changes in the structure of the composite system as the result of loss of inputs, outputs.

Chapter 6 deals with the development of integrated diagnostics and CAD tools based on interaction analysis and structural methodologies. The various methods are grouped into three categories: interaction matrices, interaction measures and control structure and system properties. The use of structural diagnostics, in the form of Markov matrices, for the selection/evaluation of alternative decentralisation schemes, is also exploited. Also, the use of graph theory and graph-theoretic methods for the examination of controllability of a given model is also presented. All these methods provide useful diagnostics that lead to the implementation of a CAD Toolbox, presented in chapter 7.

In chapter 8, an example demonstrating the developed software tools was given, that includes a detailed analysis of the application of the Toolbox.

Finally, in chapter 9, an attempt is made to formulate an integrated approach to the overall problem of control structure selection using a combination of systems theory and the previously considered diagnostics and to identify the main problems. Structural analysis of the system theoretic framework was used, and combined with some interaction measures, led to a step by step approach to the control structure selection. In this area we consider the problem of well structuring an oriented model that is non-degenerate and give a solution to this, as well as consider the general issues involved in the orientation of implicit models. These two problems are indicative of the significance of theoretical approaches to the problems of control structure selection, dominated so far by process based diagnostics and heuristics.

Chapter 10 provided the opportunity to summarise the main findings and point out the open issues.

Chapter 2

PROCESS BASED METHODOLOGIES: REVIEW OF BACKGROUND RESULTS AND RELATED ISSUES

Make sure there is a method to your madness.

2. PROCESS BASED METHODOLOGIES: REVIEW OF BACKGROUND RESULTS AND RELATED ISSUES

2.1 Introduction

In this chapter we provide a brief review of the problem areas within which the problem of control structure selection emerges. The main scope here is to introduce the main issues and provide a rather non-technical description of the fundamental issues. Certain aspects of the issues considered here will be examined in a greater depth later, if they have an impact on the general methodology, as well as the software development and CAD.

2.2 Definition of the Control Objectives

We must always start with a qualitative formulation of the control objectives for a given plant and any one determined by the specific nature of the process involved. In the first category of control objectives, we find those related to the operational feasibility. These objectives are always functions of process variables, which are to be kept within certain specified bounds, in spite of uncontrolled influences on the process. The origin of these requirements may be product quality specifications, safety considerations, operational requirements, environmental regulations etc. The second category of objectives is derived from economic considerations. These enter only if, after satisfying the first class of objectives, manipulated variables are left to adapt the operating conditions in order to stay at the most profitable point of operation. A feedforward adjustment of the manipulated variables in an optimal fashion is one such method, but it is relatively complex and unreliable. Along with classifying the objectives goes a classification of the control tasks, into regulatory and optimizing ones.

In almost every control system, disturbances are unavoidable, and hence, disturbance rejection is an issue that should be taken care of, during the early stages of design. Controllability measures for disturbances should always be a control objective. By the "controllability" or "dynamic resilience" of a plant, we mean the inherent control properties of a plant. That is, if a plant has poor controllability, then the responses for that plant will be poor, no matter what kind of controller we select to use. Some plants have better "built-in" disturbance rejection capabilities than others, that is, their dynamic resilience with respect to disturbance rejection is better. It should be noted however, that the general term "process controllability" is used in the process context in a much more general way than the classical control theoretic notion. In fact, it is used to denote the "potential" of a given plant to produce good performance under some suitable control design.

2.3 Decomposition of the Process

The study of large process systems always implies that the overall problem has to be divided, decomposed to subproblems. The decomposition of the process is not always dictated by computational considerations. Very frequently, it is part of the design strategy, very much in the same way as it was used for process flowsheet synthesis [Rudd & Wat., 1], optimization, optimal control etc. Process decomposition reveals the aggregates of unit operations and chemical reactors which must be centrally controlled. Note that the process decomposition can be directed towards developing the independently controlled groups of units, in terms of regulation or optimization. Both criteria can be applied to the same process simultaneously and nearly independently. Although this may sound contradictory, a process decomposition for regulatory purposes will be feasible within the bounds of the groups established from the process decomposition for optimizing control purposes.

In order to split a process into subprocesses, which are optimised separately, one must be able to decompose the overall objective function linearly, and one part of it must he associated with every subsystem [Fisher *et al.*, 1, 3]. The minimal size of a subsystem is usually dictated by that restriction. For optimization, the magnitude of the subproblems has to be balanced against the effort to coordinate solutions. In addition, the solution should not be too sensitive to the exact satisfaction of the interconnection of constraints. Otherwise, the required co-ordination algorithm has to be too involved.

2.4 Selection of Measurements

The first class of control objectives (product quality, safety regulations, etc.) dictates directly the measurements which should be made for monitoring the process. The second class (economic performance) can be translated into extra requirements under certain conditions to be described later, thus requiring additional measurements. These primary theoretically desirable measurements, are not always available. Often, they have to be substituted by secondary ones.

Measuring secondary variables allows us to estimate the primary ones on the basis of a process model. The choice of secondary measurements and the associated estimation problem can roughly be regarded to be independent of other decisions concerning the primary ones. (Loosely speaking we could invoke the separation principle of optimal control as a foundation of that statement). Selection criteria for secondary measurements and the development of a dynamic scheme for estimation and fault diagnosis is an active research area nowadays.

The complete set of measured variables for a feasible control structure must satisfy the extended conditions of structural observability. This includes the question of augmenting the set of measurements to obtain a structurally observable system. From the above discussion, it can be seen easily that alternative sets of measured variables will be developed during the synthesis of control structures. Which of these sets is the best is the central question in selecting the control structure.

2.5 Selection of the Manipulated and Controlled Variables

Selecting the manipulated variables will affect response capability to the external disturbances and the ability to keep the control objectives at the desired levels almost continuously. Hence, the question is whether or not there are an adequate number of manipulative variables for each of the process alternatives under consideration and/or the costs associated with ensuring that the alternatives can be made to be operable. Knowing that the process will be operable over the complete (reasonable) range of the disturbances will also mean that the problems will be well-defined at the starting of the construction of the dynamic models [Mor. *et al.*, 1]. Normally the controlled variables are the state variables that we desire to maintain at constant values. However, as disturbances enter the process, the optimum steady-state behaviour normally will change, and so we might want to change the set points for the controlled variables. Of course, if some of the state variables always remain constant at the optimum steady state conditions, when disturbances enter the process, then these can be chosen as controlled variables.

The more manipulated variables available, the better will be the control of the process. Structural aspects of the processing system and of the equations describing the processing units are of paramount importance in establishing feasible sets of manipulated variables. Certain manipulated variables will be more desirable than others, from an engineering point of view. [Gov & Pow., 1, 2] have listed a number of qualitative features that the selected manipulated variables should satisfy, those being the product of numerous discussions with practising engineers. Among these are reliability, ease of operation, start-up and shutdown, avoidance of the manipulation of "unpleasant" streams (solids, slurries) and of variables which influence a large number of other variables. However, such choices also have a control theoretic dimension, which has not taken much attention within the process control area.

2.6 Process Controllability Requirements

At the preliminary stages of a process design, most plants are difficult to control. That is, normally there are not enough manipulative variables in the flow sheet to be able to satisfy all of the process constraints and to optimise all of the operating variables as disturbances enter the plant. There are three ways that the controllability of the plant can be restored:

1. Modify the flow sheet to include more manipulative variables (e.g., add bypasses).
2. Modify the design so that some of the process constraints never become active over the complete (reasonable) range of disturbances that enter the process.
3. Neglect the least important optimisation variables.

In order to develop a systematic procedure for a controllability analysis, a series of simple controllability problems and diagnostics are considered, rather than attempting to evaluate the controllability of the complete flow sheet. The procedure that has emmerged from the process control area is the following:

1. Identification of the input streams and their classification as disturbances or manipulative variables.
2. Evaluation of the sensitivity to the disturbances.
3. Identification of the introduced process constraints.
4. Determination of the number of new design variables and equipment sizes that are specified, and calculation of the number of operating variables that can be optimised.
5. If the number of manipulative variables is equal to the number of constraints plus operating variables, as well as if the constraints and operating variables provide a well-posed problem, i.e., a non-singular Jacobian, then the process is controllable at that level. If the answer is no, several options exist, and we want to find the cheapest.

Such a methodology is largely based on rules and process based diagnostics and makes little use of the structural properties of the associated system model.

2.7 Control Structure Selection Relating the Measured and Manipulative Variables

Solutions to this problem will again be guided partly by engineering and cost, partly by control theoretical considerations [Mor. *et al.*, 1]. For example, if one insists on using single loop controllers only, the sets of manipulated variables must be chosen to result in the minimum possible interaction between the loops. If we allow for the possibility of multivariable control, be it decoupling, modal or optimal control, we gain more freedom.

Once we have selected a set of controlled variables and we are certain that there are an adequate number of manipulative variables, then we can start proposing control structures. The relative gain array (*RGA*) and singular value decomposition (*SVD*) can be used to eliminate proposed control structures that will have significant interactions in the control loops. However, a dynamic analysis is required to find the best control structure alternative. The existing structures range from single-input, single-output, non-interacting loops, to multivariable control schemes such as decoupling, modal, optimal or robust controllers.

2.8 The Concept of "Eigenstructure" in Process Control

Much of the work in the control structure selection of process control has been directed at finding control structures that minimise interaction among loops and decouple the system. However, what is really important in many chemical processes is a structure that does the best job in rejecting load disturbances. This problem is referred in the process literature as "eigenstructure" (choice of controlled and manipulated variables and their pairing); although the term is misleading in a traditional control setup [Luyb., 3]. Eigenstructure is that configuration which yields a system that is naturally self-regulating for load disturbances and self-optimising. It is claimed to be a unifying concept that links several previously published approaches to the process control problem. This general area addresses the problem of load disturbances rejection, regulation, rather than minimisation of interactions and it is thus the opposite to that of interaction analysis. The problem of load disturbances is intimately linked to the design of control structures for sections of the plant and then evaluating such designs in the overall plant setup. The term "eigenstructure", as used in the process area, is rather misleading and the overall problem has no clear definition in control theoretic terms. It is an issue that requires attention and a proper theoretical formulation. This area should be distinguished from the standard eigenstructure assignment design area of linear systems.

2.9 Inferential Control Schemes – Secondary Measurements

Control schemes utilising estimates of both unmeasured outputs and unmeasured input disturbances are termed Inferential Control Schemes. Significant progress has been accomplished in recent years towards the development of practical inferential control systems by Brosilow and his co-workers [Web. & Bros., 1], [Jos. & Bros., 1] and [Bros. & Tong, 1] and [Mor. & Steph., 1]. One of the parameters required in the construction of the estimator is the covariance of the input disturbance vector. Since accurate measurements of these statistics are rather difficult, the designer is often forced to estimate the input covariance matrix.

Since any linear model is valid only over a limited operating range, it is necessary to select secondary measurements that lead to a model that has a moderate or low condition number. Ideally, one seeks the smallest set of measurements that have both a low relative error and a low sensitivity to modelling errors. However, the relative error is generally a decreasing (strictly non-increasing) function of the number of measurements, while the condition number is generally an increasing function of the number of measurements. Thus, there is frequently a trade off between estimator accuracy, as measured by relative error and estimator sensitivity, as measured by condition number. The proper selection of secondary measurements is a task of paramount importance for the synthesis of control structures [Mor. & Steph., 1]. The measurements should be selected to minimize estimation error. The error can be caused by differences between the real system and the process model, that forms the basis for the design of the estimator, or by process and measurement noise.

If the measurement of a variable for quality regulation or optimizing control is desirable, it is technically and economically feasible to install a measurement device performing this duty. There are, however, many examples where a specific instrument is notoriously inaccurate, where the time lags associated with the sampling make the direct use of the result in a feedback loop impossible, or where a certain quantity (like catalyst activity) just cannot be measured on line. In those common instances, an estimation device is needed to infer the value of an unmeasurable variable from readily available measurements. Frequently, many measurements are available as inputs to the estimator. It is rarely technically feasible, desirable or necessary to use all of them. Intuitively it might appear that the quality of the estimate would improve uniformly with the number of measurements. However, as shown in [Jos. & Bros., 1], not even this is necessarily true. The question arises, then, which measurement should be used for the best estimate possible of the important, but unmeasurable, process variables. Although deterministic state reconstruction procedures for linear dynamic systems are available (e.g. Luenberger observer), the question of measurement selection can also be viewed in a stochastic environment. This problem has attracted the attention of many researchers [Johnson, 1], [Mül. & Web., 1], [Lück. & Mül., 1], [Mehra, 2]. Most of these approaches define the performance index of the estimator as the sum of measurement costs and the integral square estimation error.

In process control, state excitation noise is not only used to account for unmeasured process disturbances but also for the modelling error. Neglecting the state excitation noise would result in a design procedure of dubious value. A selection criterion based on this assumption will be of limited usefulness. Further, it is well known how difficult it is to choose *a priori* values for the weighting matrices in quadratic optimal control or for the covariance matrices for the design of a Kalman filter. Adding a measurement cost term, which has rarely any economic significance, increases the number of design parameters over which a trial and error search has to be performed. In addition, even for each selected cost parameter, the optimisation is an involved numerical procedure. Several criteria for the selection of secondary measurements were developed to minimise the mean square estimation error.

2.10 Process synthesis issues of chemical processes and system structure

The problem of synthesis of chemical processes is an issue that heavily depends on Chemical Engineering theory and practice. However, this area has structural implications on the overall system structure, as well as selection of control structures and thus we examine here briefly some of the dominant trends that affect the control structure selection. The techniques for the systematic synthesis of entire chemical processes, including reactors, separators, energy-transfer equipment, are classified into:

[Nish. *et al.*, 1] (1) approaches without an initial structure, and (2) structural parameter or integrated approaches and these are briefly considered below:

2.10.1 Approaches without an Initial Structure.

[Siirola *et al.*, 1], and [Powers, 1] developed a computer program called AIDES (Adaptive Initial Design Synthesizer), which utilises systematic heuristic procedures for process synthesis. AIDES performs the stream source/destination matching for the entire flowsheet in one step. It separately considers the flow of each species within the flowsheet, developing for each a scoring function which rates each possible source stream/destination stream match. The scoring attempts to account for potential separation costs, which might result, if the match is made. After scoring matches for all species, the entire stream matching is done in a single "parallel" step by solving a linear program to optimize the sum of match scores. [Mah. & Mot., 1] proposed a procedure for the synthesis of promising initial designs of chemical processing systems using the techniques employed for mechanical theorem proving. Underlying this method is the resolution principle [Robin., 1], where the designer attempts to derive conflicts among a set of facts (premises and axioms of chemical processing systems) and the desired goals (desired feasible flowsheet). The procedure begins with the consideration of production goals (desired product streams) one at a time, and ends with a process flowsheet which is feasible, in terms of mass and energy balances. Using a sequential depth-first procedure, the following structural rules are applied: (a) use the compositionally most similar source process streams to generate product streams, (b) give first preference to by-product streams already generated, and (c) reduce the mass load on separation sequences. The works of [Johns, 1] and [Johns & Rom., 1] were aimed at the early stages of process development, to select the optimal equipment configuration to transform given raw material streams into desired product streams using a mixture of dynamic programming and branch and bound arguments.

2.10.2 Integrated Approaches.

Since first proposed by [Ichik. *et al.*, 1], a number of papers using "structural parameters" have appeared. These methods can be divided into three categories: (a) the analytic and algorithmic methods, which employ the necessary condition for the optimal system and then develop a specific algorithm on the basis of necessary conditions, (b) the decomposition and/or transformation methods, which decompose or transform the synthesis problem into smaller problems so that the smaller problems are solved separately and their solution, co-ordinated in some way to assure the final solution of the individual problems, coincides with that of the overall problem, and (c) the direct application of optimization techniques of nonlinear programming. [Ichik. & Fan, 1] derived necessary conditions for the optimal system using the structural parameter approach. An evolutionary search for the optimal structure (ESOS) was developed, starting from a simple feasible structure.

Decomposition techniques may be one possible way to solve the structural parameter synthesis problem. To ease the difficulty of computations for structure optimization problems, several authors have proposed decomposition techniques. [Osak. & Fan, 1] used an infeasible two-level technique in conjunction with the structural parameter approach. Their method was applied to the synthesis problem of a simple reactor-separator synthesis problem. [Steph. *et al.*, 2], [Steph. *et al.*, 3] developed an infeasible two-level method, into which Hestene's method of multipliers was incorporated. A penalty term is used to guarantee the success of the method in the presence of functional non-convexities, often encountered in chemical process design [Steph. *et al.*, 1]. [Nish. & Powers, 1] proposed a feasible two-level method, which consists of the first-level and the second-level problem. Several authors have used non-linear optimisation techniques to solve various synthesis problems of chemical engineering interest. [Umed. *et al.*, 1] used a direct search technique namely Box's Complex method to synthesise a chemical process system consisting of two reactors, two distillation columns and several heat exchangers.

Process synthesis methodologies generate the process flowsheet and thus specify the first feasible set of control structure, based entirely on process synthesis criteria. In this sense, they provide the basis for the consideration of the overall control structure selection.

2.11 Fault diagnosis & fault tolerant design

The complex automatic systems, so widely employed in modern industry, can consist of hundreds of inter-dependent working parts, that are individually subject to malfunction or failure. Total failure of these systems can present unacceptable economic loss or even hazards to personnel. It is therefore necessary to provide the required operation of the entire system by (a) a plan of maintenance, which will replace worn parts before they malfunction or fail and (b) a scheme of monitoring, which detects a fault as it occurs, identifies the malfunction of a faulty component, and compensates for the fault of the component by substituting a configuration of redundant elements so that the system continues to operate satisfactorily [Mor. & Zaf., 1].

A plant, to be automated, can be considered to consist of three major types of subsystems, actuators, main structure (or process), and instrumentation or sensors. Early proposed schemes were concerned primarily with detecting sensor faults which, once detected, could usually be corrected by electronic switching techniques not requiring the reconfiguration of mechanical parts. The compensation of faults in actuators is usually more difficult than the re-direction of electrical signals. The compensation of malfunction in the main structure is even more difficult. Modern approaches for fault diagnosis and fault compensation tend to try to eliminate some or all of the redundant hardware. These new approaches to Instrument Fault Detection (IFD) are based upon the idea that three (or more) dissimilar sensors measuring different variables, and therefore producing entirely different signals, can be used in a comparison scheme. The rationale for this idea is that, even though the sensors are dissimilar, they are all driven by the same dynamic state of the system and are, therefore, functionally related. These newer schemes are called inherent, analytic redundancy or functional redundancy schemes, to distinguish them from physical or hardware redundancy [Le *et al.*, 1].

The functionally-redundant FDI schemes are basically signal processing techniques employing state estimation, parameter estimation, adaptive filtering, variable threshold logic, statistical decision theory, and various combinatorial and logical operations. Normally, both the input signals to the actuators and the sensor signals, i.e., the input and the output signals of the monitored plant, are available to the FDI Subsystem. The FDI schemes are therefore designed under an assumption that, either the dynamic nature of the system being monitored is known to a reasonable degree of precision, or, that it is possible to determine the values of certain physical parameters by on-line identification techniques, applied to the input and output signals of the monitored plant. FDI-related information can be extracted through the direct use of a parametric model. The parametric model is used as an estimator of a process variable using other process variables as inputs [Nwok. *et al.*, 1].

The robustness of an FDI scheme is the degree to which its performance is unaffected by conditions in the operating system that turn out to be different from what they were assumed to be in the design of the FDI scheme. However, in most operating plants, even those that are modelled accurately as linear and time invariant, some physical parameter values are known only approximately. Thus, the state estimators must be designed using only nominal values for the uncertain parameters or using some accommodating mechanism to compensate for the uncertainty. The result is that the state estimates are always in error, the severity of which, depends upon the deviations from normal operations of the monitored plant, in ways which are not easily determined.

Dynamic plants are always subjected to inputs other than those intended by the system designer. These inputs, called disturbances, are usually random functions originating in the environment, such as fluctuations in the wind. Furthermore, the sensors usually have electronic noise superimposed on their signals. The above lead to the robustness problem with respect to disturbances and noise. The robust fault detection problem becomes one of disturbance decoupling by design. If an observer or state estimator is used, modelling errors and dynamic uncertainty can be shown to act like a disturbance on a linear system. The special requirements of fault detection and compensation, either by design, or on-line, impose additional requirements on the selection of control structures. The problems of additional measurements for robust state estimation, of unmeasured quality variables, or identification of faults, and the problem of design of high operational integrity (robustness to faults), as well as control reconfiguration (correction of faults in an on-line mode), have special requirements on the overall control structure selection.

2.12 Economic Appraisal of Control Structure Selection

It has been recognised widely that the choice of measured and manipulated variables employed in a control system (the control structure) can have a strong effect on the performance of the process control system. Systematic methods to select the economically optimal control structure of a process, without designing the process controller, while maintaining good controllability characteristics, have been examined by a number of researchers. Examination of the effects of process dynamics on process economics, and how changes in the control structure alter these economics, is required. The scope of the problem so far has been limited to selecting economically optimal square regulatory feedback control structures for processes, whose operation is dominated by steady state aspects.

Many workers [Maar. & Rij., 1]; [Mor. & Steph., 1]; [Prett & Garc., 1]; [Marlin *et al.*, 1] have emphasised the role played by constraints limiting the steady state performance of the plant. The presence of disturbances causes plant personnel to choose operating points removed from the constraints determining optimum steady state operation in order to accommodate upsets within the feasible operating region. One benefit of control is to mitigate the effect of disturbances, so as to maintain feasibility and to allow operation closer to the key operating constraints. Different control structures possess differing abilities to modify the plant dynamics in this way, as well as having different capital and maintenance costs. It is the trade-off between instrumentation costs and operating benefits that accounts for the *a priori* assessment of the effect of disturbances on the economics of a given plant and control structure.

Typically, process design is performed by choosing a set of optimum steady state operating values, which minimise an appropriate process objective function, subject to a set of equality and inequality constraints. An optimal steady state process design will often result in plant operation on the operational constraints. It will not actually be possible to operate the plant on these constraints, as some process disturbances will cause the plant to violate the constraints. Thus, it is necessary to move the steady state operating point sufficiently far into the feasible region, to ensure that no constraint violation occurs during plant operation. From this, it can be seen that process control considering economic performance will try to minimise the process variability close to the active steady state constraints (the less the dynamic variability, the closer the plant can be operated to the constraint). An economic analysis of this type is detailed in [Narraw. *et al.*, 1]. It uses a linearised state space model of the system, which permits generation of first-order estimates of relevant quantities.

The analysis considers the variation in each of the variables and uses these values to estimate how large a step must be taken to ensure operability, and how this will affect the steady state economics. The economic analysis is carried out at the expected disturbance frequencies and amplitudes. One case of interest is to establish the effect of disturbances when no control action is taken (the open loop case). This effect is a property of the process and disturbances only. When controllers are implemented on the plant, the economic penalty will depend, not only on the process and disturbances, but also on the particular controller implemented. To avoid the need to design a controller, and to incorporate its effect into the plant dynamics, an estimate of the closed loop case is provided by assuming perfect control to the chosen control objectives. After calculating the open and closed loop economics over the disturbance frequency range, bounds on the cost of control may be found by selecting the worst economics for both the uncontrolled and perfectly controlled cases (i.e. the worst disturbance response for each case is used as one of the bounds). In addition to the consideration of economics, the use of the perfect control assumption, to analyse the closed loop case imposes a requirement to investigate the likelihood of approaching perfect control in practice. To do this, one may appeal to open loop indicators. An extension of the above method would be to use a generalised controller tuning technique like BLT tuning [Luyb. & Floud., 1] or H_∞ [Mor. & Zaf., 1], to generate an implementable control system. This control system can then be used to supply a practical estimate of the controlled economics.

A particularly convenient way to consider the economic effect of disturbances is to look at their impact on the variation of variables in the model measuring the violation of constraints active at the steady state optimum (the active constraint slack variables). If, under normal operation, the value of the slack variable of such a constraint takes a non-zero value, then that constraint will be violated. To remove the constraint violation, the operating point must be moved a sufficient distance into the operating region to ensure the constraint is no longer violated. A first-order sensitivity of the objective function, to such a move, is given by the Lagrange multiplier associated with the slack variable. If a measure of the dynamic slack variable amplitudes is available, then an estimate of economics of the operating point may be obtained, using these values and the Lagrange multipliers of the active constraints. This procedure is the reverse of that discussed by [Nishida *et al.*, 1,2], where control system synthesis is defined as starting with selection of the control objectives, and then determining appropriate manipulated and measured variables. This emphasises the point that it is preferable to determine the control objectives implicitly rather than explicitly, as an explicit set of control objectives are not guaranteed to be the optimal control objectives.

The area of economic appraisal is still in its early stages of development. Although the control structure plays an important role in the overall shaping of the design cost, the analysis should also take into account the overall process synthesis and optimisation. The fact that all aspects have to be considered together makes the problem of economic appraisal rather difficult.

2.13 Conclusions

Having defined the main control objectives, the problem of control structure selection, with all the various issues related to that, emerges. The "controllability" of a plant, in a broader term that includes operational, functional and economic considerations, emerges as an important issue for further study.

In this chapter, the main background issues have been presented in a rather non-technical form. Certain part of them will be, however, presented in a more formal and detailed form in the following chapters, and they will become part of a general methodology. The background results are also essential for the specification and development of a CAD Toolbox.

Chapter 3

BACKGROUND MATERIAL FROM SYSTEMS AND PROCESS CONTROL NOTIONS

Don't ignore known symptoms.

3. BACKGROUND MATERIAL FROM SYSTEMS AND PROCESS CONTROL NOTIONS

3.1 Introduction

In this chapter, we review the fundamental issues taken from linear systems and examine the key concept of process controllability, as it has been used in the process control area. We pay special attention to the latter concept, because it is fundamentally different from the classical linear systems notion. Usually, in the evaluation of process controllability, criteria are employed to guide the evolutionary development of the Process Design ; these are postulated on physical grounds and are used in an *ad hoc* manner. Only a few algorithmic synthesis procedures have been presented, which include some measure of controllability as part of the objective. The review has the intention to describe all the different schools of thought that have been formed during the last decade on the study of process controllability and extract from them the system notions which require special attention for further study.

3.2 Basics from linear control theory

The basic description of a linear multivariable system is usually taken to be a transfer-function matrix. This is simply a matrix $G(s)$ of transfer functions, in which all i,j elements $g_{ij}(s)$ are rational, and proper, scalar functions. For every such transfer function, one can find a state-space model [Kalman, 2]

$$\dot{x} = Ax + Bu, \quad y = Cx + Du \tag{3.1}$$

in which u is the vector of system inputs, y is the vector of outputs and x is a state vector. *A, B, C* and *D* are real matrices of appropriate dimensions, and are related to the transfer function $G(s)$ by

$$G(s) = C(sI - A)^{-1}B + D \tag{3.2}$$

The 4-tuple *(A, B, C, D)* is said to be a realisation of $G(s)$, and the expression *G(A, B, C, D)* is sometimes used to denote this.

The input/output behaviour, under zero initial conditions, of the system G in the frequency domain is described by

$$y(s) = G(s)u(s) \tag{3.3}$$

The short-hand notations

$$G \stackrel{s}{=} \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \quad and \quad (A,B,C,D) \tag{3.4}$$

are frequently used to describe a linear state-space model of a continuous system G given by (3.1)-(3.2).

Given a system G with state-space realisation (A,B,C,D) where A can be diagonalized (A has n linearly independent eigenvectors), then $G(s)$ can be written in the following partial fraction expansion

$$G(s) = \sum_{k=1}^{n} Cu_k \frac{1}{s - p_k} w_k^H B + D \tag{3.5}$$

In (3.12) w_k and u_k are left and right eigenvectors corresponding to the pole p_k, where w_k and u_k are scaled such that $w_k^H u_k = 1$.

Remark (3.1). Cu_k is a vector of dimension $\ell \times 1$, note that Cu_k indicates how much the k'th mode is observed in the inputs.

Remark (3.2). $w_k^H B$ is a vector of dimension $1 \times m$, and note similarly that $w_k^H B$ indicates how much the k'th mode is exited by the inputs. One problem with this view on controllability and observability, is that we are free to scale w_k and u_k arbitrarily, so the length of the vector $w_k^H B$ can be made as large as one wants by multiplying w_k with a non-zero constant c. However, then the length of the vector Cu_k becomes correspondingly small, since $w_k^H u_k = 1$ is required.

3.3 Zeros and zero directions in multivariable systems / Smith McMillan form and System Structure

Every rational transfer-function can be expressed as a polynomial matrix, divided by a common denominator polynomial. So, every polynomial matrix can be reduced to a canonical form known as the Smith form [Gant., 1].

Definition (3.1): A polynomial matrix $U(s)$ is called unimodular if it has an inverse which is also a polynomial matrix.

□

There are three elementary operations which can be performed on polynomial matrices:

- Interchange of any two rows, or columns,
- Multiplication of one row or column by a constant,
- Addition of a polynomial multiple of one row or column to another.

Each of these elementary operations can be represented by multiplying a polynomial matrix by a suitable matrix, called an *elementary matrix*. It is easy to show that all *elementary matrices* are unimodular.

Two (polynomial or rational) matrices $P(s)$ and $Q(s)$ are equivalent $(P(s) \sim Q(s))$ if there exist sequences of left and right elementary matrices $\{L_1(s),\ldots,L_l(s)\}$ and $\{R_1(s),\ldots,R_r(s)\}$ such that

$$P(s) = L_l(s)\ldots L_1(s)Q(s)R_1(s)\ldots R_r(s) \tag{3.6}$$

The next result states that every polynomial matrix is equivalent to a diagonal polynomial matrix known as the **Smith form** [Gant., 1].

Theorem(3.2): Let $P(s)$ be a polynomial matrix of normal rank r (i.e. of rank r for almost all s). Then $P(s)$ may be transformed by a sequence of elementary row and column operations into a pseudo-diagonal polynomial matrix $S(s)$ having the form

$$S(s) = diag\{\varepsilon_1(s), \varepsilon_2(s), \ldots, \varepsilon_r(s), 0, 0, \ldots, 0\} \tag{3.7}$$

in which each $\varepsilon_i(s)$ $(i = 1, \ldots, r)$ is a *monic polynomial* (i.e. has leading coefficient 1) satisfying the divisibility property

$$\varepsilon_i(s) | \varepsilon_{i+1}(s), \quad i = 1, \ldots, r-1 \tag{3.8}$$

(that is, $\varepsilon_i(s)$ divides $\varepsilon_{i+1}(s)$ without remainder). Moreover, if we define the determinantal divisors

$$D_0(s) = 1$$
$$D_1(s) = \textit{greatest common divisor of all } i \times i \textit{ minors of } P(s)$$

where each greatest common divisor is normalised to be a monic polynomial, then

$$\varepsilon_i(s) = \frac{D_i(s)}{D_{i-1}(s)}, \quad i = 1, \ldots, r \tag{3.9}$$

□

The matrix $S(s)$ is the Smith form of $P(s)$, and the $\varepsilon_i(s)$ are called the *invariant factors* of $P(s)$.

It is clear that the Smith form of a polynomial matrix is uniquely defined, and that two equivalent polynomial matrices have the same Smith form. The Smith form is thus a canonical form for a set of equivalent polynomial matrices. This can be extended to rational matrices:

Theorem(3.2): Let $G(s)$ be a rational matrix of normal rank r. Then $G(s)$ may be transformed by a series of elementary row and column operations into a pseudo-diagonal rational matrix $M(s)$ of the form

$$M(s) = diag\left\{\frac{\varepsilon_1(s)}{\psi_1(s)}, \frac{\varepsilon_2(s)}{\psi_2(s)}, \dots, \frac{\varepsilon_r(s)}{\psi_r(s)}, 0, \dots, 0\right\} \tag{3.10}$$

in which the monic polynomials $\{\varepsilon_i(s), \psi_i(s)\}$ are coprime for each i (i.e. they have no common factors) and satisfy the divisibility properties

$$\left.\begin{array}{l} \varepsilon_i(s) | \varepsilon_{i+1}(s) \\ \psi_{i+1}(s) | \psi_i(s) \end{array}\right\} \quad i = 1, \dots, r-1 \tag{3.11}$$

$M(s)$ is the Smith-McMillan form of $G(s)$.

Poles and Zeros of a transfer-function matrix

In SISO systems, the poles and zeros of the scalar transfer function

$$g(s) = \frac{n(s)}{d(s)} \tag{3.12}$$

(with $n(s)$ and $d(s)$ coprime polynomials) are given by the roots of $d(s)$ and $n(s)$, respectively. We now define the poles and zeros of a transfer-function matrix by means of the Smith-McMillan form [Rosen. 2].

Definition(3.2): Let $G(s)$ be a rational transfer-function matrix with Smith-McMillan form $M(s)$, and define the *pole polynomial* and *zero polynomial*

$$p(s) = \psi_1(s)\ldots\psi_r(s) \tag{3.13}$$

$$z(s) = \varepsilon_1(s)\ldots\varepsilon_r(s) \tag{3.14}$$

The roots of $p(s)$ and $z(s)$ are called the poles and zeros of $G(s)$, respectively. In other words, the poles of $G(s)$ are all the roots of the denominator polynomials $\psi_i(s)$ of the Smith-McMillan form of $G(s)$. If p_0 is a pole of $G(s)$, then $(s-p_0)^v$ $(v \geq 1)$ must be a factor of some $\psi_i(s)$. The number v is called the multiplicity of the pole, and if $v=1$ we say that p_0 is a simple pole. Zeros and their multiplicity are defined similarly, in terms of the numerator polynomials $\varepsilon_i(s)$ of the Smith-McMillan form.

Remark(3.3): If $G(s)$ is square, then $\det G(s) = c\dfrac{z(s)}{p(s)}$ for some constant c. In this case, although the pair of polynomials $\{\varepsilon_i(s), \psi_i(s)\}$ is coprime for each i, it is possible that there exist common factors between $p(s)$ and $z(s)$ which cancel out in forming $\det G(s)$.

Definition(3.3): The degree of the pole polynomial $p(s)$ is the McMillan degree of $G(s)$.

□

Zeros defined via the Smith-McMillan form are often called transmission zeros, in order to distinguish them from other kinds of zeros which have been defined. It is obvious that the rank of $G(s)$ drops below its normal value whenever $s = z_0$, if z_0 is a zero of $G(s)$. Hence, there exists a non-zero vector u_0 such that $G(z_0)u_0 = 0$. If the input-signal vector has transform

$$u(s) = \frac{u_0}{s - z_0} \tag{3.15}$$

then the output is given by

$$\begin{aligned} y(s) &= G(s)u(s) + \text{initial condition response} \\ &= \frac{G(z_0)u_0}{s-z_0} + \sum_i \frac{R_i u(p_i)}{s-p_i} + \text{initial condition response} \\ &= 0 \end{aligned} \quad (3.16)$$

if the initial conditions are chosen so as to cancel out the second term, in which p_i denotes a pole of $G(s)$, R_i denotes the residue of $G(s)$ at $s = p_i$ and all the poles are assumed to be simple. Hence, transmission zeros have a transmission-blocking property [McFar. & Karc., 1].

The significance of poles may be summarised very simply. Each pole p_i of a transfer-function matrix $G(s)$ must also appear as a pole of at least one of its elements. It is therefore possible to write $G(s)$ in 'partial fractions' as

$$G(s) = \sum_{i=1}^{\nu} \frac{G_i}{(s-p_i)^{k_i}} + G_0 \quad (3.17)$$

(assuming that $G(s)$ is proper), where G_i and G_0 are constant matrices, and k_i is some positive integer. Hence, the impulse-response matrix of the system, which is obtained as the inverse Laplace transform of $G(s)$, is

$$\overline{G}(t) = \sum_{i=1}^{\nu} G_i t^{k_i-1} e^{p_i t} + G_0 \delta(t) \quad (3.18)$$

The relation of pole locations to system stability is therefore the same as for SISO systems: a system is asymptotically stable if $\mathrm{Re}\{p_i\} < 0$ for each i, and it is stable if $\mathrm{Re}\{p_i\} \le 0$ and $k_i = 1$ whenever $\mathrm{Re}\{p_i\} = 0$.

Matrix-fraction description (MFD) of a transfer function

Throughout the thesis, $G(s)$ will denote an $m \times \ell$ proper, rational transfer-function matrix, namely one such that $G(\infty) = D$. Let $L(s)^{-1}$ and $R(s)^{-1}$ be the unimodular matrices that take $G(s)$ to its Smith-McMillan form $M(s)$ [Rosen. 2]

$$G(s) = L(s)M(s)R(s) = L(s)diag\left\{\frac{\varepsilon_1(s)}{\psi_1(s)}, \frac{\varepsilon_2(s)}{\psi_2(s)}, \ldots, \frac{\varepsilon_r(s)}{\psi_r(s)}, 0, \ldots, 0\right\}R(s) \quad (3.19)$$

The set of zeros of $\{\varepsilon_i(s),\ i = 1, \cdots, r\}$ are defined as the zeros of $G(s)$ and the zeros of $\{\psi_1(s), \cdots, \psi_r(s)\}$ are defined as the poles of $G(s)$ [Rosen. 2]

Assuming for simplicity $m \geq \ell$, one may write $M(s)$ as

$$M(s) = N'(s)D'(s)^{-1} \quad (3.20)$$

where $N'(s)$ and $D'(s)^{-1}$ are polynomial matrices defined as

$$N'(s) = \left[\begin{array}{ccc|c} \varepsilon_1(s) & & & \\ & \ddots & & 0 \\ & & \varepsilon_r(s) & \\ \hline & 0 & & 0 \end{array}\right] \quad (3.21)$$

$$D'(s) = diag\{\psi_1(s), \cdots, \psi_r(s), 1, \cdots, 1\} \quad (3.22)$$

and $D'(s)$ is a square matrix of dimension $\ell \times \ell$. Substitution of the two equations yields:

$$\begin{aligned} G(s) &= L(s)N'(s)D'(s)^{-1}R(s) \\ &= \left[L(s)N'(s)\right]\left[R(s)^{-1}D'(s)\right]^{-1} \\ &= N(s)D(s)^{-1} \end{aligned} \quad (3.23)$$

where

$$N(s) = L(s)N'(s) \quad and \quad D(s) = R(s)^{-1} D'(s) \tag{3.24}$$

The representation for $G(s)$ is called a right matrix-fraction description, and $N(s)$ and $D(s)$ are called the numerator matrix and the denominator matrix, respectively, of the MFD. Evidently, an MFD representation is not unique.

We are frequently interested in removing any unnecessary common factors, so:

Definition(3.4): Let $N(s)$ and $D(s)$ be polynomial matrices with the same number of columns. If there exist $N(s)$ and $D(s)$ such that

$$N(s) = \widetilde{N}(s)U(s) \quad and \quad D(s) = \widetilde{D}(s)U(s) \tag{3.25}$$

only for unimodular $U(s)$, then $N(s)$ and $D(s)$ are said to be right coprime.

□

An MFD $G(s) = N(s)D^{-1}(s)$ is said to be irreducible if $N(s)$ and $D(s)$ are right coprime; otherwise it is reducible.

The following theorem follows naturally [Kailath, 2]:

Theorem(3.3): If $G(s) = N(s)D^{-1}(s)$ and $N(s)$ and $D(s)$ are coprime, then

1. z is a (transmission) zero of $G(s)$ if and only if $N(s)$ loses rank at $s = z$
2. p is a pole of $G(s)$ if and only if $D(p)$ is singular.

□

The results for left MFDs i.e. factorisations of the type

$$G(s) = \overline{D}^{-1}(s)\overline{N}(s) \tag{3.26}$$

where $\overline{D}(s)$ is $m \times m$, and $\overline{N}(s)$ is $m \times \ell$ polynomial matrices, follow by duality (right MFD's on the transposed transfer function $G^T(s)$).

Corollary (3.1): If $G(s) = N(s)D^{-1}(s) = \overline{D}^{-1}(s)\overline{N}(s)$ are right, left coprime MFDs, then

(1) The zeros of $G(s)$ are defined as the zeros of the Smith form of $N(s), \overline{N}(s)$

(2) The poles of $G(s)$ are defined as the zeros of the Smith form of $D(s), \overline{D}(s)$.

□

3.4 System properties and invariants

Controllability is a property of the coupling between the input and the state, and thus involves the matrices A and B.

Definition 3.5: [Wonham, 1] A linear system is said to be *controllable* at t_0 if it is possible to find some input function $u(t)$, defined over $t \in \Im$, which will transfer the initial state $x(t_0)$ to the origin at some finite time $t_1 \in \Im$, $t_1 > t_0$. That is, there exists some input $u_{[t_0,t_1]}$, which gives $x(t_1) = 0$ at a finite $t_1 \in \Im$. If this is true for all initial times t_0 and all initial states $x(t_0)$, the system is *completely controllable.*

The definition given above is referred to as *state controllability,* and it is the most common definition. *Complete controllability* is obviously a very important property. If a system is not completely controllable, then for some initial states no input exists which can drive the system to the zero state.

Observability is a property of the coupling between the state and the output and thus involves the matrices A and C.

Definition 3.2: [Wonham,1] A linear system is said to be *observable* at t_0 if $x(t_0)$ can be determined from the output function $y_{[t_0,t_1]}$ for $t_0 \in \Im$ and $t_0 \le t_1$, where t_1 is some finite time belonging to $\Im$. If this is true for all t_0 and $x(t_0)$, the system is said to be *completely observable.*

Both controllability and observability are defined in terms of the state of the system. For a given physical system, there are many ways of selecting state variables. It is therefore possible that a given physical system will have one state model which is controllable but not observable and another state model which is observable but not controllable. These properties are characteristics of the model $\{A, B, C, D\}$ rather than the physical system *per se*. However, if one nth-order state variable model is both controllable and observable, then all possible state variable models of order n will have these properties. If either property is lacking in a given nth-order state variable model, then every state variable model of that order will fail to have either one or the other property.

Polynomial matrices play an important role in system theory and a special type that is often used, is called *matrix pencil. Matrix Pencils* are polynomial matrices of degree one, i.e. they have the form $sF-G$ where F,G are real matrices. The role of matrix pencils in systems theory is important, since they are directly related to first order differential systems [Karc., 8] [Karc. & Hayt., 1]. The notion of *strict equivalence* for matrix pencils is defined below [Gant., 1]:

Definition 3.7: Two matrix pencils sF_1-G_1 and sF_2-G_2 are called *strictly equivalent* if there exist constant non-singular matrices P,Q such that

$$sF_2-G_2=P(sF_1-G_1)Q \tag{3.27}$$

If the pencil $sF-G$ is square and $\det\{sF-G\}\neq 0$ then the pencil is called *regular*, otherwise it is called *singular*. If $sF-\hat{s}G$ and $f_i(s,\hat{s}), i=1,\cdots,r$, $r=rank\{sF-G\}$, are the *homogeneous invariant polynomials* (obtained by reduction to *Smith form*), then elementary divisors of the type $\hat{s}^q$ are referred to as *infinite elementary divisors* and those of the type $(s-a\hat{s})^p$ as *finite elementary divisors*. If the pencil is singular, at least one of the following equations has a solution for polynomial vectors $x(s)$, $y^T(s)$

$$(sF-G)x(s)=0 \quad \text{and / or} \quad y^{T}(s)(sF-G)=0^{T} \tag{3.28}$$

If $\left[x_1(s),\cdots,x_\mu(s)\right]$ and $\left[y_1^T(s),\cdots,y_v^T\right]^T$ are minimal polynomial bases for the right and left null space of $sF-G$ respectively and $\varepsilon_i, i=1,\cdots,\mu$, $\eta_j, j=1,\cdots,v$ denote the corresponding degrees, then ε_i are known as *column minimal indices* and η_j as *row minimal indices* of the pencil. The sets of *elementary divisors* and *minimal indices* uniquely characterises the strict equivalence class of $sF-G$ and there exists a canonical form obtained by some appropriate transform pair (P,Q) and defined by $P(sF-G)Q=sF_k-G_k$ where

$$sF_k-G_k=\left[\begin{array}{c|cccc} 0_{q,h} & & 0 & & \\ \hline & L_\eta(s) & & 0 & \\ 0 & & L_\varepsilon(s) & & \\ & 0 & & sH-I & \\ & & & & sI-J \end{array}\right] \tag{3.29}$$

where $0_{g,h}$ is a zero block defined by the g *row minimal indices*, h *zero column minimal indices*, $L_\varepsilon(s)$, $L_\eta(s)$ are blocks associated with *nonzero column minimal indices* and *row minimal indices* respectively, $sH-I$ a block associated with the *infinite elementary divisors* and $sI-J$ a block associated with the *finite elementary divisors*. The structure of these blocks is defined below:

$$L_\varepsilon(s)=block-diag[\cdots,L_{\varepsilon_i}(s),\cdots], \quad L_{\varepsilon_i}(s)=s\left[I_{\varepsilon_i},\underline{0}\right]-\left[\underline{0},I_{\varepsilon_i}\right]$$

$$L_\eta(s)=block-diag[\cdots,L_{\eta_i}(s),\cdots], \quad L_{\eta_i}(s)=s\left[\begin{array}{c} I_{\eta_i} \\ \hline \underline{0}^T \end{array}\right]-\left[\begin{array}{c} \underline{0}^T \\ \hline I_{\eta_i} \end{array}\right]$$

$$sH-I=block-diag\left[\cdots,sH_{q_i}-I_{q_i},\cdots\right], \quad H_{q_i}=\left[\begin{array}{c|c} 0 & I_{q_i}-1 \\ \hline 0 & 0 \end{array}\right]$$

$$sI - J = block - diag[\cdots, sI_{p_i} - J_{p_i}(\alpha), \cdots], \quad J_{p_i}(\alpha) = \alpha I_{p_i} - H_{p_i}$$

The above canonical form is called *Kronecker canonical form* of $sF - G$. In the case where the pencil is regular, it is characterised only by the infinite and finite elementary divisors and the canonical form has only the blocks $sI - J$ and $sH - I$. In this case the canonical form is called *Weierstrass canonical form.*

Controllability and Observability

The pencil $[sI - A, -B]$ is known as input-state, or controllability pencil [Rosen., 1] [Karc., 5] and the invariants of $[sI - A, -B]$ are very closely associated with the controllability properties of the system. A system is uncontrollable iff there exist finite elementary divisors in $[sI - A, -B]$. We have the following definition [Rosen, 2]:

Definition (3.8): A point x_c in the state-space is called controllable, if there exists control input $u(t)$ such that if $x(0) = x_c$, the state may be driven to the origin in finite time and the trajectory $x(t)$ is continuously differentiable.

□

The pencil $[sI - A, -C]$ is known as state-output, or observability pencil. The system is unobservable iff there exists finite elementary divisors of the pencil $[sI - A', -C']'$ [Rosen., 2] which is referred to as the state-output pencil [Karc., 5].

Definition (3.9): A system is called *observable* if there exists a $t_1 > 0$ such that given u and y on an interval $[0, t]$ it is possible to deduce $x(0)$.

□

Transmission Blocking and Zeros

The concept of a zero is strongly connected with the physical problem of a system $S(A, B, C, D)$ whose output response remains identically zero even though the system input and states are themselves non-zero. This situation may be represented diagrammatically as [McFar. & Karc., 1]

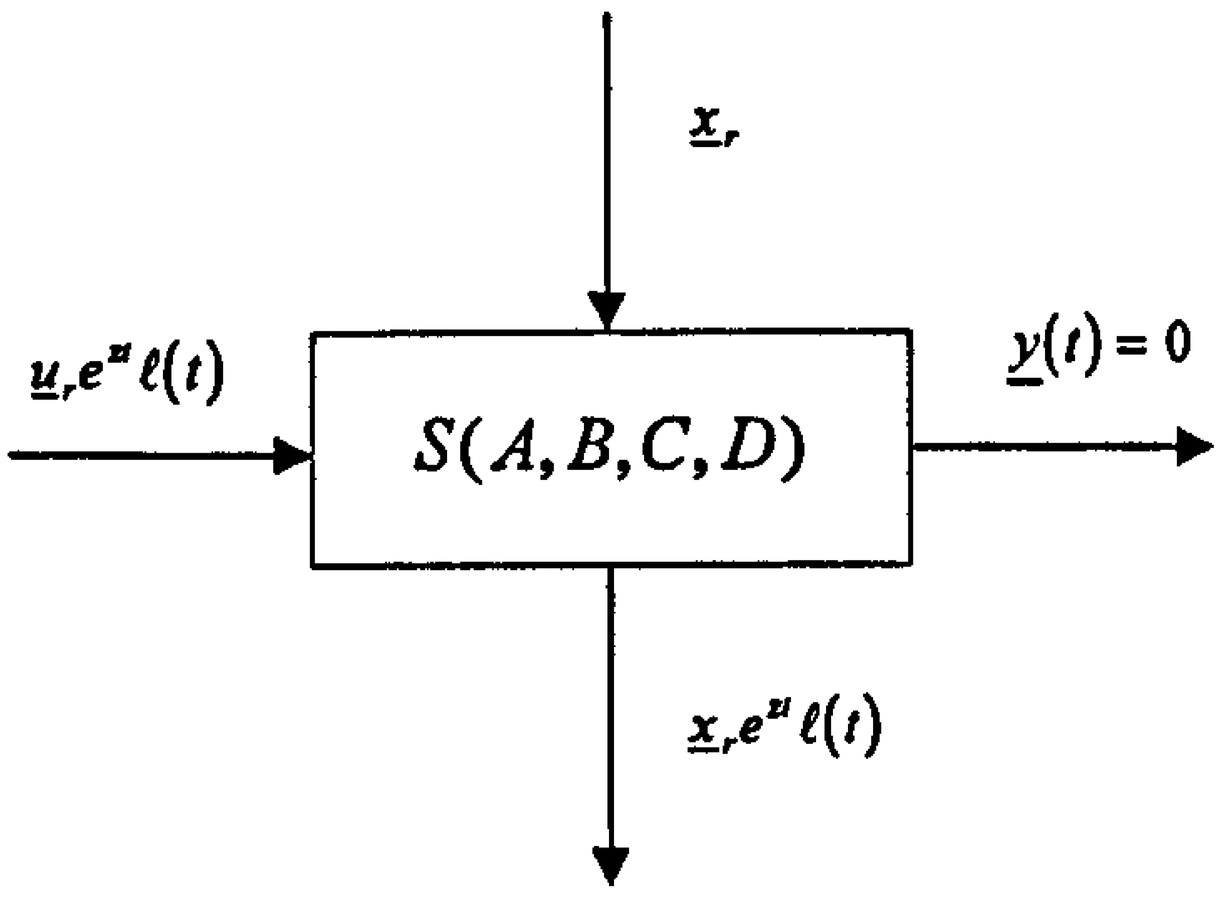


Figure (3.1): Output zeroing Problem

The conditions for the solution of this problem are expressed in the following theorem:

<u>Theorem (3.4)</u>: [McFar. & Karc., 1] For a proper system $S(A,B,C,D)$ with full rank transfer function for which the number of inputs ℓ is less than or equal to the number of outputs m, a necessary and sufficient condition for an input

$$\underline{u}(t)=\underline{u}_r e^{zt}, \quad t \geq 0 \tag{3.30}$$

to yield a rectilinear motion in the state space of the form

$$\underline{x}(t)=\underline{x}_r e^{zt}, \quad t \geq 0 \tag{3.31}$$

and to be such that

$$\underline{y}(t) \equiv 0 \quad for\ t>0 \tag{3.32}$$

is that

$$P(z)\begin{bmatrix}\underline{x}_r \\ \hline \underline{u}_r\end{bmatrix}=0, \quad P(z)=\begin{bmatrix} zI-A & -B \\ -C & -D\end{bmatrix} \tag{3.33}$$

From (3.8) we have that the solutions in z give the values of the complex variable s for which $P(s)$ loses column rank. This is only possible for values of s which coincide with the finite elementary divisors of $P(s)$. The frequencies z define the set of finite invariant zeros. The vector solutions $\underline{x}_r, \underline{u}_r$ that correspond to the finite invariant zeros are called the *state* and *input zero directions* [McFar. & Karc., 1]. The matrix $P(s)$ is known as the Rosenbrock's system matrix and its invariant structure defines the zero structure of the system [Karc. & Kouv., 1].

3.5 Singular Value Decomposition (SVD)

Principal gains (singular values)

In a SISO system the performance of a feedback loop is determined by the variation of the loop gain with frequency; disturbance rejection, noise transmission and differential sensitivity to parameter variations all depend only on the gain (assuming that stability is achieved). If the open-loop transfer function (return ratio) has no right half-plane zeros, then stability margins and the closed loop transient response are also determined by the open-loop gain characteristic.

In attempting to extend this correlation to multivariable feedback, the main problem is that a matrix does not have a unique gain: the norm $\|G(s)u(s)\|$ depends on the direction of the vector $u(s)$. However, we can bound the ratios

$$\frac{\|G(s)u(s)\|}{\|u(s)\|} \quad and \quad \frac{\|G^{-1}(s)y(s)\|}{\|y(s)\|} \tag{3.34}$$

using matrix norms. ($G(s)$ is usually assumed to be square and invertible, although neither of these assumptions is necessary). Thus, the idea of a simple gain is replaced by the notion of a range of gains, this range being bounded below and above.

If $\|x\|$ denotes any vector norm, then an *induced* (or subordinate) *matrix norm* is defined by

$$\|G\| = \sup_{x \neq 0} \frac{\|Gx\|}{\|x\|} \tag{3.35}$$

In particular, using the *Euclidean vector norm* (for complex vectors)

$$\|x\| = \sqrt{\left(x^H x\right)} \tag{3.36}$$

then the *induced matrix norm* is the *Hilbert* or *spectral norm*:

$$\|G\|_s = \bar{\sigma} \tag{3.37}$$

where $\bar{\sigma}^2$ is the *maximum eigenvalue* of $G^H G$ (or of GG^H). Here x^H denotes $\bar{x}^T$, and similarly for G^H. Now, if G has m rows and l columns, and $m \geq l$, then the positive square roots of the eigenvalues of $G^H G$ are called the *singular values* of G.

If instead of G we have $G(s)$, and set $s = j\omega$ $(0 \leq \omega < \infty)$, then the *singular values* of $G(s)$ are functions of ω, and they are then called the *principal gains* of $G(s)$. They are denoted by $\{\sigma_i(\omega)\}$ to emphasize their dependence on frequency, or by $\{\sigma_i(G)\}$ to distinguish the *principal gains* of G from those of some other system. The ordering $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_m$ is adopted, when necessary, and σ_1, σ_m are denoted by $\bar{\sigma}$ and $\underline{\sigma}$ respectively, when the use of the largest or smallest principal gain needs to be emphasized. It should be noted that

$$\bar{\sigma}(G(j\omega)) = \|G(j\omega)\|_s \tag{3.38}$$

which is a norm on the matrix $G(j\omega)$, which changes with ω. $\|G\|_2$ and $\|G\|_\infty$ are norms of the transfer function G that are independent of frequency.

The singular-value decomposition

Let $\sum = diag\{\sigma_1, \sigma_2, \ldots, \sigma_m\}$, and let G be a complex matrix. Then, G can always be written as

$$G = Y \sum U^H \tag{3.39}$$

where

if $m \geq \ell$:	*if* $m \leq \ell$:
$Y \in C^{m \times \ell}$	$Y \in C^{m \times m}$
$\Sigma \in R^{\ell \times \ell}$	$\Sigma \in R^{m \times m}$
$U^H \in C^{\ell \times \ell}$	$U^H \in C^{m \times \ell}$
$Y^H Y = I_\ell$	$Y^H Y = YY^H = I_m$
$U^H U = UU^H = I_\ell$	$U^H U = I_m$

and $C^{m \times \ell}$ denotes the set of complex matrices with m rows and ℓ columns, and $R^{m \times \ell}$ denotes the set of real matrices with these dimensions. This is known as the *singular-value decomposition.* This decomposition is not, however, unique: $G = Y' \Sigma U'^H$, where $Y' = Ye^{j\theta}$ and $U' = Ue^{-j\theta}$, for any θ, also gives a *singular-value decomposition.* However, the σ_i are unique.

Consider $H = U\Sigma^{-1}Y^H$

Then

$$\begin{aligned} HGH &= \left(U\Sigma^{-1}Y^H\right)\left(Y\Sigma U^H\right)\left(U\Sigma^{-1}Y^H\right) \\ &= H \end{aligned} \tag{3.40}$$

and

$$\begin{aligned} GHG &= \left(Y\Sigma U^H\right)\left(U\Sigma^{-1}Y^H\right)\left(Y\Sigma U\ \right) \\ &= G \end{aligned} \tag{3.41}$$

which shows that H is a pseudo-inverse of G. Thus we have

$$G^\dagger = U\Sigma^{-1}Y^H \tag{3.42}$$

where $G^{\dagger}$ denotes the pseudo-inverse of G. It is assumed that $\text{rank}(G) = \min(\ell, m)$. If $r < \min(\ell, m)$ then instead of Σ^{-1} we take

$$\begin{bmatrix} \Sigma_r^{-1} & 0 \\ 0 & 0 \end{bmatrix} \tag{3.43}$$

where $\Sigma_r = diag\{\sigma_1, \ldots, \sigma_r\}$.

If G is square and non-singular, then

$$G^{-1} = U\Sigma^{-1}Y^H \tag{3.44}$$

from which

$$\left\| G^{-1}(j\omega) \right\|_s = \frac{1}{\underline{\sigma}(\omega)} \tag{3.45}$$

We can also easily prove that

$$\underline{\sigma}(\omega) \le \frac{\left\| G(j\omega)u(j\omega) \right\|}{\left\| u(j\omega) \right\|} \le \overline{\sigma}(\omega) \tag{3.46}$$

which shows that the gain of a multivariable system is sandwiched between the smallest and largest principal gains.

With each principal gain, we can associate a pair of principal directions, as follows: Assume that $m \ge \ell$, and let the rows of Y^H be $y_1^H, y_2^H, \cdots, y_\ell^H$, and the columns of U be $u_1, u_2, \cdots, u_\ell$. Then, we can write

$$\begin{aligned} y &= Gu \\ &= \sum_{k=1}^{\ell} \sigma_k y_k u_k^H u \end{aligned} \tag{3.47}$$

Since $\|u_k\| = 1$, we have

$$\left| u_k^H u \right| \le \|u_k\| \, \|u\| = \|u\| \tag{3.48}$$

the equality holding only if $u = \alpha u_k$ for some scalar α. Suppose, then, that $u = \alpha u_i$, with $|\alpha| = 1$, so that $\|u\| = 1$. Then $u_k^H u = 0$ for $k \neq i$, and hence, the resulting output is

$$y = \sigma_i y_i \alpha$$

so that

$$\|y\| = \sigma_i$$

This shows that the gain of the system is precisely σ_i if the input signal is in the direction of u_i. The set $\{u_1, u_2, \cdots, u_\ell\}$ is called the set of *input principal directions* of G. In particular, the greatest possible gain $\bar{\sigma} = \sigma_1$ occurs if the input signal is in the direction of u_1, and the smallest possible gain $\underline{\sigma} = \sigma_\ell$ occurs if it is in the direction of u_ℓ. Note that the principal directions are *orthogonal* to each other (that is $u_j^H u_j = 0$ if $i \neq j$), since $U^H U = I$.

If the input vector is in the direction u_i, then the output vector is in the direction y_i. The set $\{y_1, y_2, \cdots, y_\ell\}$ is called the set of *output principal directions*. Again, these are orthogonal to each other.

A useful characteristic of the system is its *condition number*, which is defined as

$$\text{cond}(G) = \frac{\bar{\sigma}}{\underline{\sigma}} \tag{3.49}$$

and which depends, of course, on frequency. In numerical analysis the condition number measures the difficulty of inverting a matrix. It also has a control-theoretic significance, in that it measures the inherent difficulty of controlling a given plant.

We can also define two further functions, that are widely used for assessing performance, the *sensitivity* $S(s)$ and the *complementary sensitivity* $T(s)$:

$$S(s) = \left[I + G(s)K(s)\right]^{-1} \tag{3.50}$$

$$T(s) = S(s)G(s)K(s) \tag{3.51}$$

To assess the disturbance-rejection (sensitivity) properties of a loop, one should examine the principal gains of S (see figure 3.2). If the region lying between $\overline{\sigma}(S)$ and $\underline{\sigma}(S)$ is very narrow, then we are almost back with the SISO single gain, and we can describe the loop's sensitivity properties very accurately.

Usually the region will not be narrow – in this case, since we are always interested in keeping sensitivity as small as possible, it is the upper boundary of the region (that is, $\overline{\sigma}$) which will be important.

To assess the propagation of measurement noise we can look at the principal gains of T. Again, the upper boundary is important, since we wish to minimise the propagation. Note that there are now two loop bandwidths: ω_b, the bandwidth at the output, and ω_t, the bandwidth at the input.

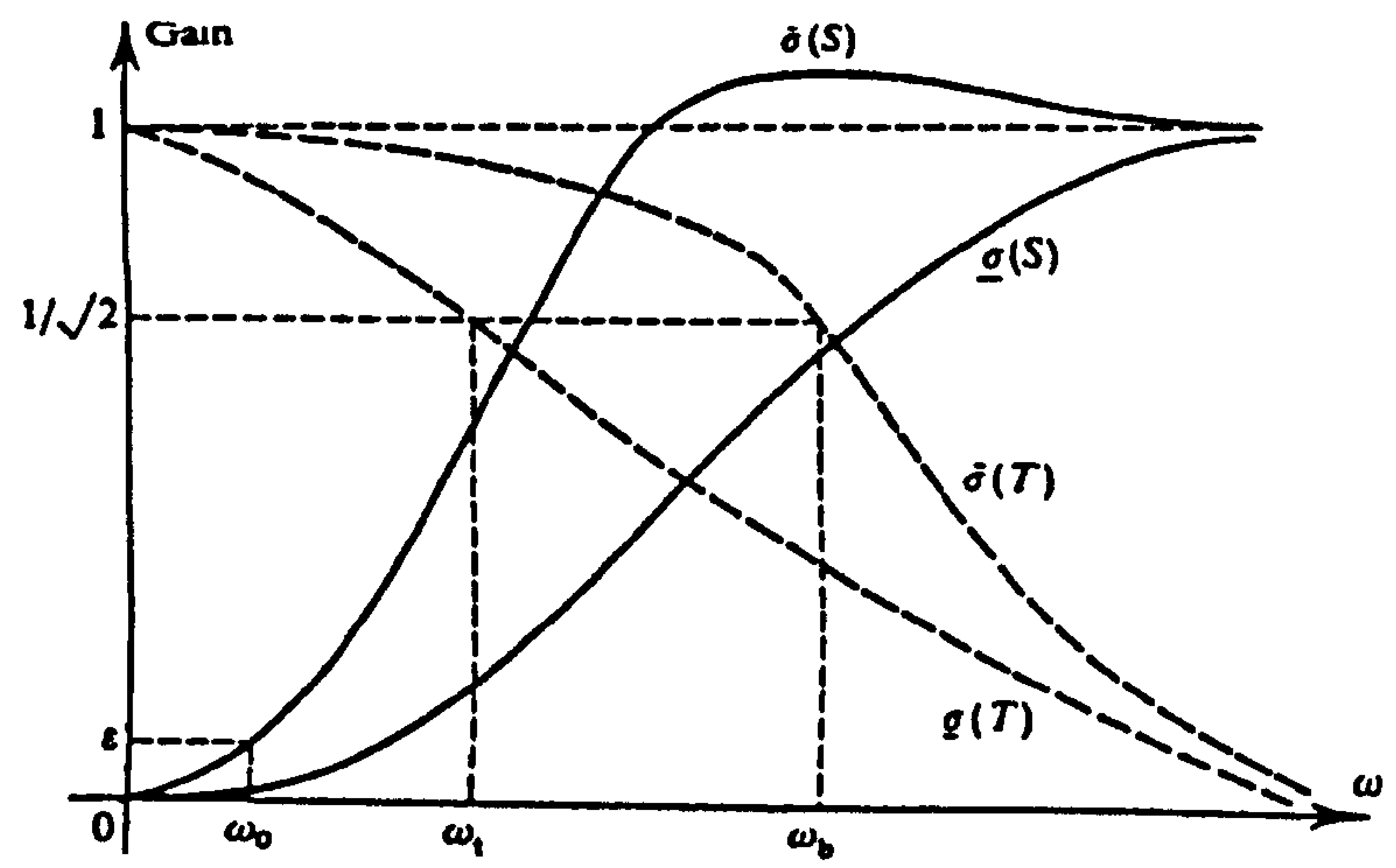


Figure (3.2) Smallest and largest prinsipal gains of multivariable sensitivity function S (solid curves) and complementary sensitivity function T (dashed curves).

Matrix norms and the operator norms $\|G\|_2$ and $\|G\|_\infty$

The 'gain' of a transfer function is usually considered as a function of frequency (i.e. measured at discrete points). But it is possible, and useful, to have a cruder measure of the 'gain'. Two such measures can be defined, known as norms, denoted by $\|G\|_2$ and $\|G\|_\infty$.

<u>**Definition (3.10):**</u> Let $G(s)$ be a proper transfer function with no poles on the imaginary axis. Then

$$\|G\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{tr}\left[G(j\omega)G^T(-j\omega)\right]\mathrm{d}\omega} \tag{3.52}$$

and

$$\|G\|_\infty = \sup_\omega \bar{\sigma}\left(G(j\omega)\right) \tag{3.53}$$

It can be shown that $\|G\|_2$ and $\|G\|_\infty$ both satisfy the usual properties of norms. These norms are often referred to as *operator norms*, as the system represented by a transfer function is an operator which maps functions – input signals – into other functions – output signals, and these norms measure the amplification (or at least the greatest possible amplification) of this mapping.

3.6 The notion of Process Controllability

3.6.1 Introduction

The concept of controllability as introduced by Kalman (1960), states that a linear system described by the state differential equation:

$$\underline{\dot{x}}(t) = A\underline{x}(t) + B\underline{u}(t) \tag{3.54}$$

is completely controllable, if the state of the system can be transferred from the zero state at any initial time, to any terminal state $x(t_1) = x_1$ within a finite time $t_1 - t_0$, through the use of a piecewise continuous control input $u(t)$. In the area of process control, the same term has been used with a much wider meaning. In this section, we review these alternative notions which dominate chemical engineering practices.

The notion of controllability of a process structure, has a much more general meaning and plays a key role, if we wish to integrate effectively the design of a process and the control system design problem. Broadly speaking, the controllability involves two aspects. The first is the *performance* of a process under a transient condition, where the operating mode of the process, or, equivalently the set point of its control system, is shifted from one to another; obviously, a change in the set point, usually induces a long-term disturbance. The second, which is probably more important than the first, is the *stability* under a given operating mode, where the normal values of the input and output variables of the process are fixed, but under the constant influence of various disturbances; any of such disturbances is usually regarded as a short-term disturbance. How well the synthesised process responds to these disturbances and how effective its control functions are, determine the process' dynamic characteristics, and consequently, its controllability. One aspect of the controllability of a process assessment is by examining the disturbance propagation in a process structure.

3.6.2 Process Controllability and Steady State Analysis

A series of papers by Douglas and co-workers [Fisher *et al.*, 2,3] describe a systematic procedure for assessing process controllability at the preliminary stages of a process design, so that some of the economic penalties, associated with control, can be used as an additional criterion for screening process alternatives. For improving controllability, they consider (1) modifying the flowsheet to add more manipulative variables, (2) over-designing certain pieces of equipment so that the process constraints never become active for the complete range of process disturbances, or (3) ignoring the optimisation of the least important operating variables. The goal of their controllability analysis is to determine which of these alternatives have the smallest cost penalty. In another related case study, they demonstrate that for a distillation column over-design in terms of the number of trays is important to account for tray efficiencies, but it does not significantly increase the range of operability of a column.

The controllability problem for heat exchanger networks was defined first in [Mars. *et. al.*, 1], where a procedure was developed to yield designs which handle stream flow rate and temperature fluctuations with maximum energy efficiency. The problem as studied in [Linnh. & Kotj., 1] addressed the trade-offs between controllability and economics more effectively through a combination of "downstream paths" and "sensitivity tables". Recently, Georgiou and Floudas [Georg. & Floud., 3] have proposed an automated synthesis procedure based on a superstructure approach, which will, either generate heat recovery networks with low capital costs and total disturbance rejection, or, if this is not possible, it will minimize disturbance propagation. Shinnar [Shinnar, 1] introduces a new definition of the concept of controllability which takes into account model uncertainty and the fact that many important process variables may not be measurable in practice.

3.6.3 Controllability evaluation based on linear dynamic models

Given a linear dynamic model of the process, some performance measures related to the desired closed-loop performance, and some restrictions on the controller structure (PI, decentralised, etc.), the objective is to find the optimal controller parameters and to compare the achieved optimal performance, a measure of controllability, among different designs. Several difficulties are associated with this approach. The traditional objective function in optimal control was the integral square error (ISE). It was used to distinguish alternate designs, for example, by Lenhoff and Morari [Lenh. & Mor., 1]. By itself the ISE is hardly a measure which is of direct interest in practice. [Lee *et al.*, 1] used multiple performance criteria: integral square error, maximum deviation of exit response, maximum magnitude of control variable, and saturation magnitude. Any such assessment however, would become prohibitively complex in the multi-variable context. A further difficulty noted in the constrained-structure optimal control approach is that the search for the control parameters is notorious for its multiple local optima nature.

Both previously mentioned drawbacks are alleviated to a large extent by adopting robust performance as a control objective [Mor. & Zaf., 1] and by removing any restriction on the controller structure. In the H_∞ context, the objective function is also scalar but it can be quite easily formulated to express frequency-dependent constraints on various outputs and manipulated variables, as well as the effect of model uncertainty. Usually, the structured singular value μ is used as a normalised indicator, with values less than one signifying satisfaction of the performance specifications in the presence of uncertainty. Even better methods for the computation of μ-optimal controllers (without complexity constraints) become available. Nowadays, restricting control complexity is hardly an issue in any new installation. [Skog. *et al.*, 4] used μ-optimal controllers to distinguish various control structures for high-purity distillation columns. Jacobsen et al. employed μ-optimal PID controllers to evaluate various designs of a homogeneous azeotropic distillation column. Despite these successes, it must be stated that formulating a meaningful robust control problem and determining the optimal robust controller is far from trivial. Therefore, the search for alternate, simple criteria which would allow a rank ordering of design alternatives, roughly consistent with that obtained from a μ analysis, has continuing importance. Criteria such as zero and time-delay analysis, singular values, condition number, *RGA* etc. are important in this context.

A number of process characteristics, which limit the achievable control-performance independently of controller design, have already been identified [Morari, 6]: non-minimum-phase behaviour (i.e., time delays and right-half-plane zeros), actuator limitations and more uncertainty. Occasionally, measurement noise may have a dominant effect. Various researchers have proceeded to analyse the effect of time delays, right-half-plane zeros and model uncertainty individually on closed-loop performance. Multivariable delays were analysed in [Holt & Mor., 1], [Perk. & Wong, 1]. Exact bounds on achievable performance, with and without decoupling constraints, were identified through a mixed integer linear programming procedure in [Psarris & Floud., 3]. The properties of multivariable right-half-plane transmission zeros were investigated in [Holt & Mor., 1], [Mor. & Zaf., 1]. Conditions under which decoupling is integral-square-error optimal were identified. This work was followed by a more complete treatment by Psarris and Floudas [Psar. & Floud., 1, 2] who dealt with the case of an infinite number of zeros which usually arise in the presence of multivariable time delays. The drawback of these analysis techniques is that, in any but the simplest situations, it is essentially impossible to rank the order of alternatives.

In [Psar. & Floud., 1], [Jacob. & Skog., 1], the condition number of the plant transfer matrix as a function of frequency was suggested as an indicator of closed-loop sensitivity to model error. This criterion was first applied in 1982 to a system of two CSTRs with heat integration [Mor. *et al.,* 4]. In the last ten years, we have gained some understanding of the role of the condition number [Skog. & Mor., 2], but the basis for its use for controllability assessment is still somewhat tentative. The main problem is that all the conditions relating the condition number and closed-loop stability and performance [Mor. & Zaf.,] are sufficient but not necessary. While we can say with certainty that the closed loop performance of low-condition-number plants tends to be insensitive to model error, we cannot reject high-condition-number plants with certainty, though there are many indications that the performance for these types of plants will be bad. Over the years, there has also been some discussion on the scalings of the plant inputs and outputs and their effect on the magnitude of the condition number. Our experience, supported by theory [Skog. & Mor., 2], indicates that minimising the condition number by input and output scaling tends to distort any conclusions on model error sensitivity.

Skogestad et al. [Skog. *et al.,* 1] make a strong case for using a combination of the frequency-dependent relative gain array (*RGA*) and the closed-loop disturbance gain (*CLDG*) to judge the relative controllability of alternate designs. These tools have been employed in many recent case studies. The conclusions tend to correlate well with the closed-loop performance obtained with μ-optimal control systems. The steady-state *RGA* was introduced by [Bristol, 1] (1966) and continues to be widely used in industry as a controllability indicator. While the steady-state *RGA* is indicative of the fault tolerance of multivariable control systems [Grosd. *et al.,* 1], it can be very misleading as a controllability indicator [Skog. *et al.,* 2]. The *CLDG* was introduced by Hovd and Skogestad [Hovd & Skog., 2] based on the relative disturbance gain defined by McAvoy and co-workers [Stanl. *et al.,* 1]. Hovd and Skogestad found that the *CLDG* enters nicely into the relation between control error and disturbances, while the *RGA* enters in a similar way into the relation between control error and set point changes.

In the last few years, many case studies have appeared, where these concepts were applied. An ethyl-benzene production facility and a two-column separator system in styrene manufacturing is analysed at steady state in [Mizsey & Fonyo, 1]. The controllability of ordinary distillation columns is investigated in [Jacob. & Lar., 1]. A fluid catalytic cracker is studied in [Hovd & Skog., 1], heat exchanger networks in [Wolf *et al.,* 1], [Mass. *et al.,* 1], homogeneous azeotropic distillation columns in [Jacob. & Lar., 1], and floatation circuits in [Lear *et al.,* 1]. It is often possible to interpret the behaviour of the frequency-dependent *RGA* and *CLDG* on physical grounds. This and their theoretical basis make the *RGA* and the *CLDG* highly appealing and easily applicable tools for controllability assessment. Finally, it should be mentioned that several researchers [Denn & Lavie, 1], [Rinard, 2], [Fisher *et al.,* 3] have studied the effect of recycle on the dynamic behaviour in linear models. Silverstein and Shinnar [Silv. & Shin., 1] use essentially linear techniques to analyse the nonlinear model of a reactor with feed-effluent heat exchanger.

3.6.4 Controllability evaluation based on nonlinear dynamic models

In most cases, a controllability evaluation based on linear models, as described above, suffices even when the system is strongly nonlinear and when a linear control system is inadequate. Usually, it is quite easy to design simple static nonlinear compensators which remove most of the process nonlinearity. The compensated system can then be analysed with the proposed linear techniques. As a typical example, high purity distillation columns can behave in quite a nonlinear fashion. However, when relative composition deviations are controlled or alternatively the logarithm of the compositions, the system is linearized sufficiently so that adequate performance is obtained with linear control systems [Skog. & Mor., 3].

In rare but very important instances, the system can exhibit nonlinear behaviour which is not easily correctable with simple nonlinear transformations. For example, a reactor may display high parametric sensitivity and the temperature may rise to excessively high levels when small perturbations occur [Shinnar *et al.,* 1]. Also, the system may exhibit multiple steady states, limit cycles, or even chaotic behaviour. It was suggested recently that nonlinear characteristics should be examined at the design stage [Seider *et al.,* 1], [Seider *et al.,* 2] and that nonlinear analysis techniques, like bifurcation analysis and singularity theory, should be used more routinely in process design. Indeed, this has been done in the area of reactor design for decades. The question is what to do with this type of analysis. [Seider *et al.,* 2] suggest that modern nonlinear control algorithms allow us to deal with almost any difficult control situation, and consequently regions of unusual dynamic behaviour should not be avoided in process design. Carried to the extreme, one could conclude that such a nonlinear analysis is not needed at all at the design stage because any complex nonlinear behaviour can be fixed later on by the control algorithm. It is possible that future developments may lead us somewhat in this direction (just like for linear systems, where the issue of control complexity became less and less important in the last decade), but we should not forget that nonlinear control theory is in its infancy. Even if the applicability of a particular algorithm (for example, nonlinear model predictive control) is established in principle, the control effort required may be enormous [Lenh. & Mor., 1] and totally uneconomical.

There is no doubt that increased quality standards, stricter environmental regulations, and economic pressures will push designs into regions which were previously avoided, and where unusual nonlinear behaviour occurs. Therefore nonlinear analysis techniques will undoubtedly be needed increasingly at the design stage. For chemical reactors, unusual dynamic behaviour is almost expected nowadays, but recently similar phenomena were discovered for other systems as well. The results obtained by Seader and co-workers [Chavez *et al.*, 1], [Lin *et al.*, 1] suggest that multiplicities may be one reason why the so-called Petlyuk distillation configuration is not used in industry despite established energy advantages. It is known that heterogeneous azeotropic distillation columns can exhibit multiple steady states [Magnus. *et al.*, 1], [Roval. & Doh., 1]. Rovaglio et al. [Roval. *et al.*, 1] have studied the control problems associated with these multiplicities and the parametric sensitivity also found in these columns. Recently, Laroche et al. [Laroche *et al.*, 1, 2] discovered multiple steady states in homogeneous azeotropic distillation columns where such phenomena were believed not to exist. Fortunately, continuously improving software (e g. [Doedel, 1]) allows today's designer to carry out bifurcation analyses on large systems (e g., of the order of hundred differential equations in the case of the homogeneous azeotropic distillation columns), which was essentially infeasible a decade ago. Often, the bifurcation diagrams can be used to redesign the process in order to be more attractive from an economic and environmental point of view, while at the same time avoiding complex dynamics [Ray, 1], [Teym. & Ray, 1]. At typical operating conditions emulsion polymerisation of vinyl acetate carried out in a CSTR exhibits limit cycle behaviour. The bifurcation analysis shows that limit cycles can be avoided either by increasing the solvent fraction (a traditional technique which leads to costly recycle problems) or by decreasing the solvent volume fraction and increasing the initiator feed concentration. Another example is the novel feed system for a CSTR for continuous emulsion polymerisation, which was suggested by Penlidis [Penl. *et al.*, 1], and was shown, in experiments, to remove the highly undesirable oscillatory (limit cycle) behaviour of conventional CSTRs. More examples are discussed in the review paper by Seider et al. [Seider *et al.*, 1].

Though a particular system may not exhibit multiple steady states, limit cycles and other exotic dynamic behaviour, it may be extremely sensitive to disturbances and small changes in its operating parameters. This has been observed in catalytic reactors, where runaway has been the subject of numerous studies, starting with [Wilson, 1] and [Lenh & Mor., 1] up to the recent work by [Balak. & Luss., 1], where some simple design rules to avoid runaway were derived.

3.6.5 Algorithmic Synthesis involving Process Controllability Criteria

Very little has been published on algorithmic synthesis techniques for processes which are both economical and controllable or where economics and controllability are traded off automatically in some intelligent manner. Apart from the early work by Ichikawa [Nish. & Ichik., 1], [Nish. *et al.*, 1], there is the more recent work by Floudas [Lin. & Kotj., 1], [Georg. & Floud., 1], where the power of mixed integer nonlinear programming techniques is exploited for the synthesis of heat exchanger networks which exhibit minimal disturbance propagation. The same problem was studied in [Huang & Fan, 1], where a knowledge engineering approach is proposed and mass exchanger networks are also considered. In [Luyb. & Floud., 1] a multiobjective optimization technique is used to study the trade-off between various measures of steady state controllability and economics in the design of binary distillation columns. The experience, so far, has not clarified how useful the automatic synthesis techniques will be in the near future. Usually, they require a scalar performance index to be specified and they cannot exercise judgement on something like the behaviour of the frequency-dependent *RCA* and *CLDG*. The multiobjective approach introduced in [Luyb. & Floud., 1] looks very promising, though the controllability assessment is done at steady state and any implications for the dynamic behaviour are tenuous at best.

3.7 Conclusions

The basis of systems and control have been examined in this chapter and a review of the notion of Process Controllability has been performed. The results here serve as a background to the following chapters.

Chapter 4

THE PROBLEM OF GLOBAL PROCESS INSTRUMENTATION: EMERGING ISSUES

Happy is the man who seeks understanding.

4. THE PROBLEM OF GLOBAL PROCESS INSTRUMENTATION: EMERGING ISSUES

4.1 Introduction

The synthesis/design and operation of large scale industrial processes/systems has, as an integral part, the selection of systems of measurement and actuation variables. Although the way we measure individual physical variables and act upon them is governed by the traditional instrumentation rules, the selection of systems of measurement and actuation variables has a significant effect on the shaping of the final system and thus, it is crucial in defining the control quality properties (process controllability) and operability characteristics of the final design. We have reviewed so far the different aspects of control structure as they have emerged in the process control literature and use them to identify some fundamental issues related to the selection of systems of measurement and actuation variables; the review clearly suggests that this problem should be considered within the context of integrated design. It is now argued that, amongst the many different aspects of the problem, there are issues of Systems and Control Theory type which, have not been considered before. The development of these system aspects and related methodologies are essential prerequisites for the emergence of an integrated framework for Global Instrumentation [Karc., 8], [Karc. & Mil., 1], where traditional instrumentation, signal and communication aspects, artificial intelligence tools, process heuristics, and overall control performance characteristics are considered simultaneously, in an interactive manner, and not as independent issues. In this chapter, we try to abstract the results of the review so far and formulate generic issues and problems, which provide a wider scenario for activities in the future.

4.2 Model, Event Identification and Input-Output Selection

The selection of input test signals and output measurements is an integral part of the setting up of model identification experiments, as well as statistical experimental design. In fact, the identified model is always a function of the way the system is excited and its behaviour is subsequently observed, that is the way the system is embedded in its experimental environment. Most of the work so far has concentrated on SISO identification techniques and on the effect of test signal characteristics on the identification aspects of the model. The study of effect of location of the group of excitation signals and corresponding group of extracted measurements on the identification problems has not been properly examined so far. Issues, such as how and whether additional excitation signals and extracted measurements may enhance the scope and accuracy of identifiable models, are important topics for research. Within this context, the selection of inputs, outputs in a system is closely related to the overall modelling exercise and frequently becomes indistinguishable from it.

A similar class of problems to model identification is the family of problems, where the selection of measurements aims at providing information for identifying the occurrence of events, such as faults, or drifting of the process model from a nominal operating condition. The spatial distribution of sensors is of crucial importance in the fault diagnosis, as well as experimental design methodology. A dual problem to the fault diagnosis and location of measurements is the problem of adequate actuation variables for control rescheduling in the event of operational faults. In both problems, the effect of input-output structure shaping on the potential of the resulting system model to accept solutions either of the estimation, or the control type, is fundamental.

4.3 Global Information Processing

The role of sensors and actuators and their location may be viewed within the wider area of information machines [Finkel. & Grat., 1], [SESDIP 1]. In fact, instruments are information machines, which sense a power or material flow from an object under measurement at the input, assign to it a symbol and carry out operations on the symbol, providing at the output, either a display symbol to a human operator, or finally, effectuate the information by operating actuators, or similar machines. In this context, an instrumentation system is an information machine, which has as its function, the acquisition, processing, outputting and effectuation of information from the real world. In information machines, information is carried by the magnitude or attributes of the time variations of physical variables. Viewing sensors and actuators as information machines allows the linking of instrumentation to alternative aspects of process operations and the use of information type models of variable degree of complexity and abstraction.

Information-machine systems, such as the instrumentation systems [Finkel. & Grat., 1], are formed by joining blocks with suitable connections, the structure of which is referred to as architecture. A physical system may have many components, or parts; each part is considered as block with its own input-output relationship. The blocks representing the various elements are connected in a specific way and they produce composite forms, architectures, realising composite functions. General modelling and tools from computer science, as well as general information theory, together with communication and data management issues, emerge as the building blocks of this alternative view of an instrumentation system, which may have as role, the monitoring, diagnosis etc. functions. The selection of input-output structures on simple, or composite information machines has a similar impact on the resulting model as that studied on dynamic models. The importance of the information viewpoint is greater when we consider higher level issues of the process operations hierarchy, or lower level problems of real time control. The information approach is a separate issue worth pursuing further than the current stage of the problem.

4.4 Integrated Process and Control Design

The problem of control structure selection is one of the important issues in process control area and although a variety of criteria and heuristics have been developed, there is up to now no systematic methodology for synthesising control structures for the whole plant. This area has been previously examined in detail and has served as a focus point for this work; in fact, the control structure selection problem within which the input-output scheme selection problem is embedded, has many different challenging aspects due to the close integration of higher and lower operational stages especially for continuous processes. The overall problem for continuous process presents a number of challenges due to that:

(i) The processes have non linear multiple couplings among the variables.

(ii) The measurement and manipulation of process variables is limited to a relatively small number of variables.

(iii) The control objectives may not be clearly stated (or even known) at the beginning of control design.

(iv) Evaluation of the control system is based on a variety of objectives such as: (a) safety, (b) reliability, (c) quality of control (including stability), (d) flexibility of process operations, i.e. range of control, (e) ease of start-up and shutdown, (f) cost of the control system, and (g) ease of operation of the system (including training).

(v) The process structure may be changed to improve control.

(vi) There may be considerable uncertainty in the prediction of process behaviour.

(vii) Control structure selection issues may have to be addressed at early stages of design with rough models and limited information on the exact objectives.

(viii) For processes with no buffers between layers of the operational hierarchy, such as the continuous processes, control structure selection has either to take into account the translated higher level requirements, or become itself an aspect of the solution of the higher level problem (such as optimisation).

(ix) Very frequently, the main features and choices in control structure are determined during process synthesis and, thus, the range of possible solutions is significantly affected by modelling considerations.

The problem we consider here is in a sense part of the extended control design problem which has the following main parts:

(a) Selection of process inputs and outputs.

(b) Selection of coupling of controlled (outputs) and manipulated (inputs) variables, as well as specification of controller dynamics.

(c) Design of the control scheme specified in (b) with a variety of control performance objectives and criteria.

So far, Control Design has addressed predominantly the area (c) and has assumed that both the input-output structure and the coupling of variables have been previously decided. In the process control area, the issues in (b) have been previously considered, but there is no systematic methodology emerging yet that covers all different aspects. Area (b) has also been addressed within Control theory and Design, but not as a design of decentralisation schemes. In fact, the study of solvability of types of decentralised problems always assumes that the decentralisation scheme is given; however, the derived results may be used to parametrise the schemes which allow solvability of certain problems and thus, traditional interaction analysis is complementary to the parametrisation of decentralisation schemes. [SESDIP 1].

The requirements of integrated design imply that issues related to design of input-output structures for processes have to be seen together with the selection of control structure, as well as control design. The evolution of properties, as we go through the successive design stages, has to be properly understood and - if possible - to direct such an evolutionary process. An additional issue, arising due to integration, is that the overall problem has to be addressed within the context of wider operational requirements, well beyond those of traditional control design. Issues, such as safety, have implications on aspects such as design with high integrity, fault diagnosability and ability to provide redesign of the control structure. The flexibility of operations requirements implies that operational requirements, such as optimisation, are essential ingredients of the solution and that one solution for each of the problems (a), (b), (c) may not be adequate for a wide range of operational conditions. Important problems that arise within this context are:

(P.1) Classification of the operating regimes (which include start-up, shut-down, emergencies and smooth transition from one operating condition to another) for which we require alternative solutions for (a), (b), (c) areas of problems.

(P.2) Simultaneous, Robust design of either of the (a), (b), (c) areas, when common solutions are feasible for groupings of operating points.

(P.3) When more than one grouping of operating regimes emerges, which implies switching, there is a need for an appropriate supervisory strategy for running effectively and safely such schemes.

(P.4) Taking into account operational criteria (i.e. optimisation on plant level, total quality) in the design or redesign of sections of the process and evaluating the impact of local designs on the general performance indices referred to sections, or whole plant.

4.5 Process Modelling and Input-Output Selection

The selection and classification of process variables is an integral part of the overall exercise and it is influenced by (a) the purpose which the model is to serve and (b) the boundaries of the system to be modelled. The purpose of the model clearly influences the choice of relevant variables to be included in the model, the detail and accuracy desired of the model and the procedures necessary to derive it. Given the purpose of the model, the next step is to specify the boundaries of the system, which is to be modelled. For example, we may be concerned with developing a mathematical model of an entire corporation, of a refinery or an integrated plant, of a processing system, of a unit process, such as an individual heat exchanger, or we may desire a model of the flow pattern in the elbow joint of a pipe; each of these is an appropriate subject for modelling. The location of boundaries determines the particular variables which must be taken into consideration, as well as their status as independent and dependent quantities in the model. The above two factors are instrumental in the overall classification of variables and are considered as external modelling factors in the classification process.

From the point of view of control, the typical process can be looked upon as a multivariable system with a number of input and output variables. The inputs are the independent variables of the process; they may be considered as casual factors, in the sense that the dependent variables, the outputs may be considered as effects, or responses to inputs. A diagram summarising the classification of variables is shown in Figure (4.1), where the independent and dependent variables are classified further into controlled-uncontrolled and performance-intermediate respectively. Such a classification is intimately related to the purpose and boundaries of the modelling exercise.

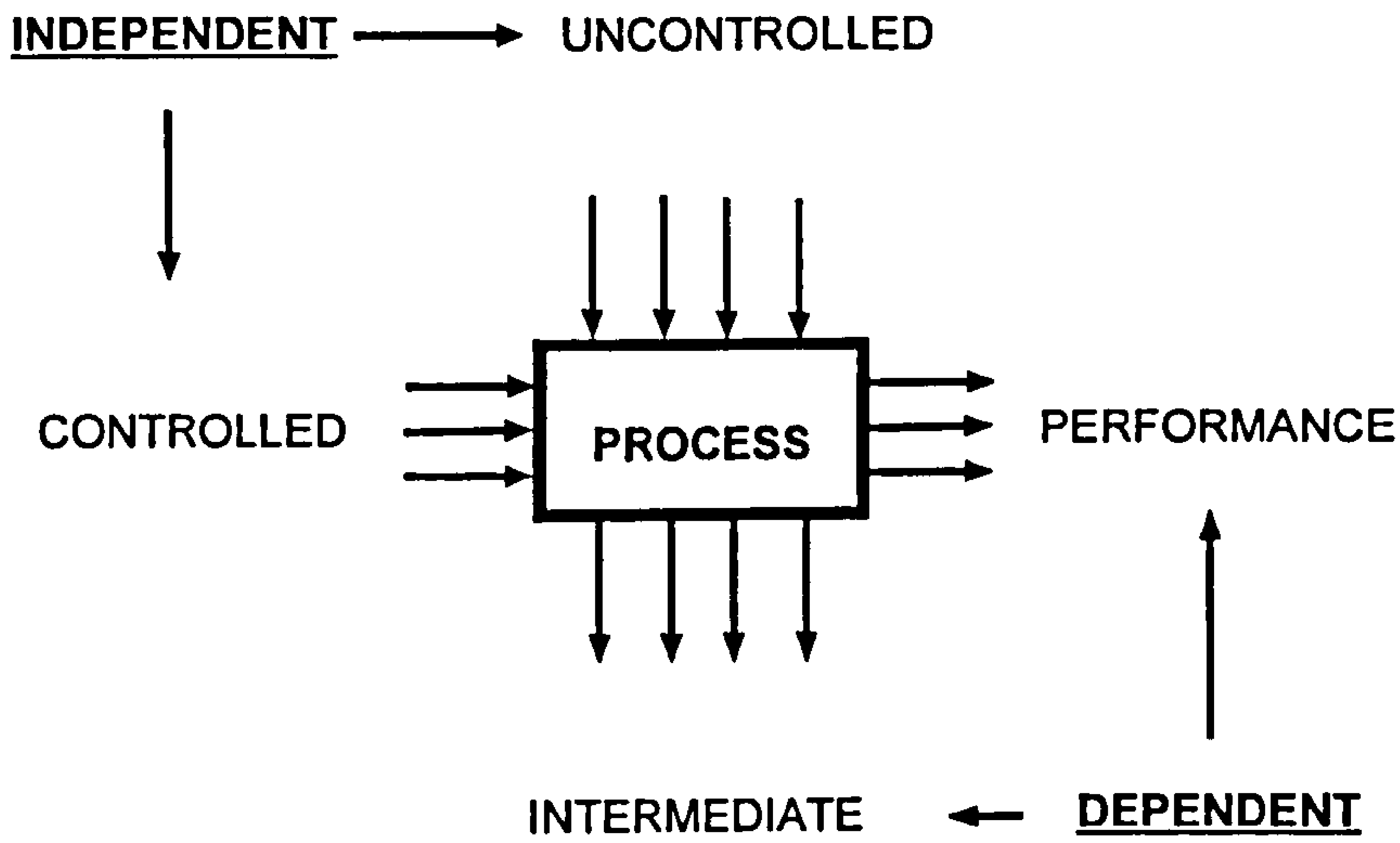


Figure (4.1): Classification of model variables

(a) Uncontrolled Variables: An uncontrolled variable, also called a disturbance, is a quantity which affects the process operation, but over which the operator has no direct control; its value is often determined by some known, or unknown agency external to the process boundary. Uncontrolled variables may be classified into five categories, those with (1) raw materials, (2) ambient conditions, (3) equipment condition, (4) economic factors, and (5) loading effects.

(b) Controlled Variables: These are variables over which the operator can exercise control. Such variables may be classified to basic control variables and transformed control variables. The first are primary variables, which the operator can handle set. Usually, in analysing the process variables and formulating a model for control purposes, it is more convenient to think, not in terms of the basic control variables, but rather in terms of a set of transformed control variables linked to some fundamental properties of the process. It is clear thàt the transformed independent controlled variables do not form a unique set, but depend on the preferences, approach of the designer. However, each model must be internally consistent and the number of transformed independent variables must be equal to the number of basic control variable. Given the past and present values of the independent variables, the dependent quantities are completely determined. Dependent variables enter the model for two reasons; either they are directly related to process performance or they arise as intermediate variables, which indirectly affect the operation of the process. Thus, we distinguish:

(c) **Performance Variables**: Performance variables are those which serve to evaluate directly the performance, or condition of the process. In practise, these are the variables which the operator should constantly bear in mind while running the process. We may classify these variables into:
(1) Economic variables, (2) Constrained variables. The first family include those which provide a direct measure of the economic performance of the process. According to the nature of the process and the management policy, a number of such variables are specified. The second category, the constrained variables, include quantities, which are restricted, or limited to a certain range of values. Constrained variables are further classified to physical and managerial types. Physical constraints are imposed principally by capacity, safety etc., considerations, whereas managerial constraints relate to policy decisions. In the latter family, we distinguish those related to product quality and size of production. In general, these are many quality and quantity constraints on process and their nature is limited to the particular physical and operational characteristics of them.

(d) **Intermediate Variables**: Intermediate dependent variables constitute the remainder of the pertinent process variables. They are not of direct, immediate or explicit use in evaluating the performance and conditions of the process, in the sense that they do not have direct economic impact, nor are they explicitly constrained. Their role however may be significant in the overall control of the process, as well as the development of advanced schemes for evaluating key quality variables, which cannot be directly measured.

4.6 The Global Instrumentation within the Field of Integrated Design

The specific role of the selection of measurement and actuation variables in the context of overall process design is examined in this section. This also serves to illustrate the general philosophical approach on integrated design previously stated.

The main Design Stages and the need for Integration

The selection of systems of actuation, sensor variables, referred to here as Global Instrumentation, is part of three main engineering stages represented in Figure (4.2). The general features of the technological stages are briefly considered first, before we focus on the significance of Global Instrumentation.

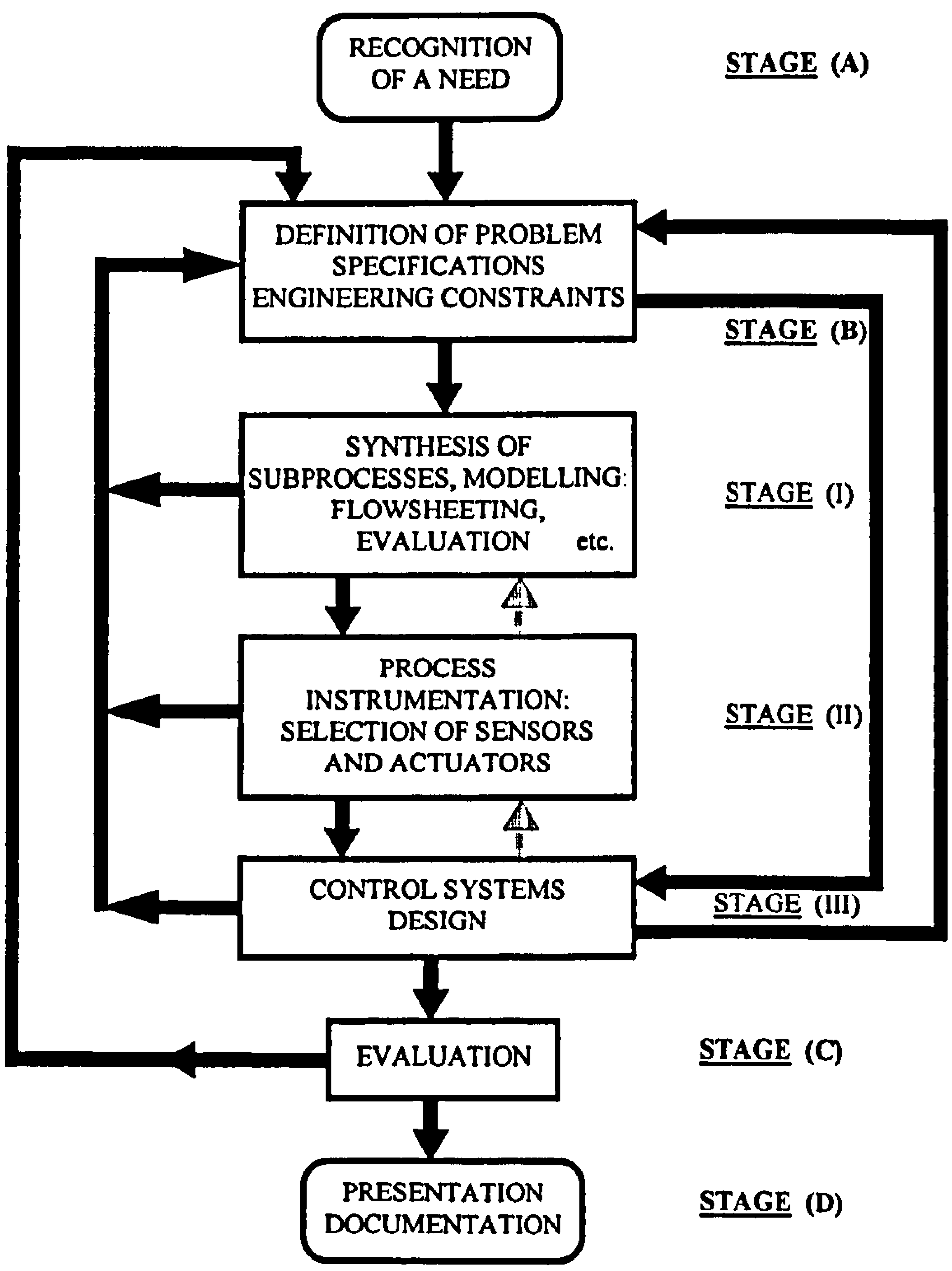


Figure (4.2): Simplified form of Engineering Design Process

The exchange of information, illustrated in the above diagram, between the different design stages has a short prediction horizon, as far as the impact on the subsequent design stages are concerned, and it is of a local character. This local character is dominated by the specialised skills, theory and techniques needed for a given engineering task. The ability to translate local decisions as actions assigning certain structure to the stage model is currently missing. The common engineering practice is dominated by simulations, trial and error and finally the overall design is tested on a pilot plant. Accelerating this design process, with the obvious advantages on financial costs, performance, operability, safety etc., is an important task; this may be helped by developing a Global Co-ordination Theory for the above design process. Such a development may also help in the effort to modify existing designs by moderate redesign of them.

Our attention is focused on the purely technological nature stages of design, that is :

STAGE (I) : Process Synthesis

STAGE (II) : Process Instrumentation

STAGE (III) : Process Control.

Each of the above stages operates under a set of engineering specifications and constraints, which together with the economic constraints, define the boundaries of the local decision making. Experience from building similar processes provides rules and guidelines of what to do and what to avoid. This body of knowledge is indispensable, but not sufficient for the fulfilment of the original task, that is, deriving final designs, which have desirable performance characteristics, with the minimal effort and economic cost. The problem of control structure selection is within the overall problem of global process instrumentation. In fact, we may view this empirical knowledge and rules as an intermediate stage co-ordination layer with a rather short prediction horizon. GCT aims at enlarging the knowledge required for an improved process synthesis by introducing system and control based criteria, rules and techniques.

In summary, the general aims and objectives of Global Co-ordination are to provide the theory, tools, rules and criteria for:

(i) Understanding and where possible, control the model evolution process through the different design stages

(ii) Shaping sets of feasible and compatible specifications

4.7 Generic Issues and Problems Associated with Global Instrumentation

The traditional role of instrumentation is well developed and deals with the problem of measurements, or implementation of action upon given physical variables [Ekh., 1]. This is closely related to the physics of the particular problem and issues related to signal processing, are also crucial. The focus point in traditional instrumentation is the particular variable, whereas the effect and significance of such a selection on the shaping of the overall process model is not considered. It has been noted [McFar. & Kouv., 1], [Kouv. & McFar., 1], [Rosen. & Power, 1] that the selection of sensors and actuators (their location, as well as the way we measure, implement and act) plays a decisive role in the formation of the characteristics of the final design. These aspects are those which have been examined here and they refer to the selection of locations and the properties of the set of actuation and sensing variables on the resulting model. Aspects, such as communications, information theory etc., although important, they do require separate consideration, and are beyond the scope of this thesis. The current dominant theme is the model shaping role of global instrumentation. The internal process characteristics and dynamics (the result of the process synthesis design stage) are essential, since they determine the progenitor basic characteristics of the final design; however, the manner we observe and try to act upon the variables of the progenitor system, determines the final characteristics of the system. The final system model is the product of interaction of the internal dynamics and its environment; the role of instrumentation lies in the building of bridges between the internal mechanism of the process and its environment.

The main tasks involved in the development of concepts, methodology and tools for Global Instrumentation are:

(i) Translating a variety of requirements for different operational modes and for technical specifications on the process model, as well as interpreting technical features of the design in the different operational indices.

(ii) Characterising the desirable and undesirable performance characteristics of the overall system, and the limits of the best results that can be achieved under compensation.

(iii) Relate the best achievable performance, or undesirable performance characteristics to the system model structural type characteristics and their values.

(iv) Establishing the functional relations between model parameters and structural characteristics.

(v) Formulating and solving suitable structure formation problems.

The overall problem, which is considered here, is an attempt to shape the final characteristics of the process model that emerges from the process synthesis and process instrumentation stage. This attempt would make the final control design problem as simple as possible, with natural consequences on costs, operability, safety etc. of the final process. The formation of the structural characteristics of the overall process is reminiscent of an evolution process. In fact, each design stage starts with a model (parent gene) and decisions, taken there, contribute to the gradual shaping of the final structural characteristics, but only within a range of possible options; thus, structural properties and characteristics evolve, but not in a simple manner. The main objective is to drive the model evolution along certain paths, so as to avoid the formation of undesirable structural characteristics and – where possible – to assign desirable ones. Most of the approaches deal with diagnostics, rather than trying to define a synthesis methodology based on both aspects of structure, that is, graph and parameter dependent invariants. The area is extremely important and involves the translation of requirements and specifications from the higher levels of the operational hierarchy to the lower level technical requirements, which shape the specific problem; inversely, translating lower level issues, technical achievement in terms of variables affecting higher level indices is a similar problem area.

The long term objective of the current work is to establish a conceptual framework and appropriate tools which will allow the eventual integration of global process instrumentation with control operability criteria. This is seen as the backbone of a Coordination Methodology that is required for directing the process synthesis, global instrumentation and control design. The desirable features of such a methodology are:

Desirable Features of Global Coordination Theory

The general desirable features of co-ordination theory, described in the previous section, take the following more specific form:

(a) The general common language of the co-ordinator has to be that of the signals, information and system model and thus, systems theory is predominant. Specifications may then be interpreted as constraints, and alternative designs as possible models.

(b) Each element of the family of local possible models has a different potential, as far as the subsequent design at the following stages is predominant. This potential is expressed by the structural characteristics of the model and the values, shape of design indicators associated with it. The interpretation of the effect of alternative local designs on the shaping of the structure and of the possible values of design indicators, is an essential task of GCT.

(c) The range of possible alternative structural characteristics and possible values of design indicators may be communicated between different design stages. Each one of them has a different depth prediction horizon, as far as the possible further shaping of process model at subsequent design stages is concerned. The depth of the prediction horizon of a structural characteristic, design indicator, depends on its nature and functional dependence to model parameters. It is this predictive property of structural characteristics and indicators that will allow the enrichment of previous stages design decisions with successively broader and, finally, global criteria.

(d) Deriving global criteria, as far as the desirability, of the types and values of structural characteristics is concerned, is one of the tasks, but not the final of GCT. Assigning desirable structural features, or shapes to property indicators and more generally avoiding the formation of undesirable properties is of paramount importance. These features heavily rely on the understanding of dependence between system structure, property indicators and model parameters.

(e) The ability to translate global criteria to the local design level and to evaluate the impact of local optimality to the global one, is also important.

4.8 Conclusions

The general area of Global Process Instrumentation has been examined and the needs for a general integrating methodology of the system type have been specified. The overall area is clearly multidisciplinary and will require considerable effort for many years to come, as well as the synergy of many specialisations and disciplines. The requirements of this very important area were pointed out, as well as the specifications of a Systems and Control Methodology framework. The Control Theory and Design framework stems from the general requirements considered here, as well as the problems and issues of the application areas previously examined.

Chapter 5

THE ROLE OF INPUT, OUTPUT SELECTION IN THE FORMATION OF COMPOSITE SYSTEMS STRUCTURE

The shell must break before the bird can fly.

5. The role of Input, Output Selection in the formation of Composite Systems Structure.

5.1 Introduction

This chapter provides a study of the structural properties of interconnected systems and examines the role of selection of inputs, outputs at subsystem level on the overall shaping of composite structure properties. In particular, the significance of an assumption related to interconnections, which is referred to as the completeness assumption, is examined. The significance of the deviations from the completeness is a subject of investigation in this chapter. This initial study is restricted on the examination of controllability, observability and zero structure related properties and the adopted approach is based on Matrix Pencil Theory. The controllability (observability) properties of composite systems under partial, or total lose of inputs (outputs) at the subsystem level takes special attention.

The theory of structural properties of composite systems has attracted a lot of attention in recent years [Rosen., 1] [Cal. & Dos., 1] [Pugh & Kafai, 1] [Kailath, 1] etc. The main idea behind the work here is to try to relate the structural aspects of the composite system in terms of the structural aspects of the subsystems and the nature of the interconnections. Of special interest is the investigation of the effect of changes in the structure of the composite system as the result of loss of inputs, outputs. The present approach relies on the use of the restricted pencils [Karc. & McBean, 1], [Karc. & Kouv, 1] for the composite system; this analysis leads to the computation of the restricted pencils of the composite systems which are expressed in a simple way in terms of the restriction pencils of the subsystems. Some basic assumptions in dealing with composite system are that the transfer function of each subsystem provides a representation for the subsystem, that is, each subsystem is both controllable and observable.

The general case of input-state restricted pencil of composite and aggregate systems with full inputs and outputs is considered first and then the approach is extended to the cases where one or more inputs or outputs are lost at the subsystem level. It will be shown that, controllability, observability and zero structure properties of composite system under full input, output structure are simply given as aggregates (direct sum) of corresponding properties of subsystems if the interconnection scheme uses all available subsystem inputs, outputs. The use of all available inputs, outputs for interconnections at the subsystem level is what we refer to here as the *completeness assumption* of the composite scheme. In this case, it emerges that the interconnection graph is of little importance for such properties.

In the case of systems where the idealistic assumption of completeness does not hold true, i.e. where there is a loss of inputs, outputs at the subsystem level, the dependency of the results on the underlying graph is very explicit; most of the ideas presented here are illustrated in terms of examples. Three examples are used to illustrate the above ideas, that is the effects of loss of inputs, outputs on the structural properties of the composite system. We shall concentrate on one of the examples with 4 subsystems where the first input is lost. As a result, it will be shown that total loss of input (output) channels for any of the 4 subsystems results in structural uncontrollability (unobservability). It will also be shown that the observability structure of the subsystem where we have lost its inputs, enters into the zero structure of the aggregate system. General results are derived for cases of total loss of inputs, outputs at subsystem level, whereas for cases outside completeness and total subsystem loss we have to use graph analysis. We summarise first some results on the role of pencils in the characterisation of the system properties.

5.2 Matrix pencils and structural properties

In this section, the basic definitions and properties of matrix pencils related to system fundamental properties will be reviewed. These properties will later be used to describe the structural properties of composite systems.

5.2.1 Input-state pencil

The pencil $[sI-A,-B]$ is known as input-state, or controllability pencil [Rosen., 1] [Karc., 5] and the invariants of $[sI-A,-B]$ are very closely associated with the controllability properties of the system. A system is uncontrollable iff there exist finite elementary divisors in $[sI-A,-B]$. This implies the existence of a non-zero constant vector $\underline{v}'$ and a frequency s_0 such that

$$\underline{v}'[s_0 I-A,-B]=0 \tag{5.1}$$

Let N be a left annihilator of B (a basis matrix for $N_\ell(B)$) and $B^\dagger$ be a left inverse of B, i.e.

$$NB=0, \qquad B^\dagger B=I_\ell \tag{5.2}$$

we may now write $\underline{v}'=[\underline{v}_1',\underline{v}_2']\left[\dfrac{N}{B^\dagger}\right]$ where $\left[\dfrac{N}{B^\dagger}\right]$ is a full rank matrix and thus it may be readily shown that (5.1) is equivalent to

$$\underline{v}_1'(s_0 N-NA)=0 \tag{5.3}$$

The last equation implies that there exist finite elementary divisors of the pencil $sN-NA$ iff the system $S(A,B)$ is uncontrollable. The pencil $sN-NA$ is known as the restricted input-state pencil [Karc., 5] [Karc., 6]. It can be proved that a controllable system yields an input-state pencil characterised only by a set of column minimal indices $\{\varepsilon_i+1=\sigma_{\ell-i+1}, i\in\ell\}$, where σ_k denotes the controllability indices of the pair (A,B) and ε_i are the *c.m.i.* of the restricted input-state pencil $sN-NA$. For uncontrollable systems the canonical form of $sN-NA$ contains additional blocks to those corresponding to the *column minimal indices* [Karc. & McBean, 1]; these new blocks correspond to finite elementary divisors, which in turn define the input decoupling zeros of the system [Rosen., 1]. The pencils $[sI-A,B]$ and $sN-NA$ have the same *f.e.d.*, but their *c.m.i.* are related by the "*plus one*" property described above.

It was shown that if T is the co-ordinate transformation bringing the pair (A,B) in the Luenberger controllable companion form [Karc., 6], then a mere multiplication of $sN-NA$ on the right by T^{-1} brings the pencil in the Kronecker canonical form. The transformation T^{-1} belongs to the class of strict equivalent transformations [Gant., 1] and, as such, does not affect the Kronecker invariants. Another important set of transformations on the pair (A,B), is the set of state feedback transformations; the input-state pencil that corresponds to a closed loop pair $(A-BL,B)$ is $sN-N(A-BL)=sN-NA$, since $NB=0$ and thus we are led to the following theorem:

<u>Theorem (5.1)</u>: [Karc., 6] The input-state pencil $sN-NA$ corresponding to the pair (A,B) and thus also its Kronecker canonical form, are invariant under state feedback.

5.2.2 State output pencil

In the previous section we have considered a pencil with reduced dimensions than those of $[sI-A,-B]$, which characterised the equivalence class of the systems $S(A,B)$ under state feedback. In this section we shall repeat the analysis for the $S(A,C)$ pair using the concepts of observability and/or unobservability, instead of those of controllability. Note that a system is unobservable iff there exists finite elementary divisors of the pencil $[sI-A',-C']'$ [Rosen., 2] which is referred to as the state-output pencil [Karc., 5]. This implies the existence of a non-zero vector $\underline{u}$ and a frequency $s_0 \in C$ such that

$$\begin{bmatrix} s_0 I - A \\ -C \end{bmatrix} \underline{u} = 0 \tag{5.4}$$

Let M be a right annihilator of C (i.e. a basis matrix for $N_r(C)$) and $C^\dagger$ be a right inverse of C, i.e.

$$CM = 0, \qquad CC^\dagger = I_m \tag{5.5}$$

we may always write

$$\underline{u} = \left[M \mid C^\dagger\right]\begin{bmatrix}\underline{u}_1 \\ \underline{u}_2\end{bmatrix} \tag{5.6}$$

then the system (5.4) may be expressed as

$$\begin{bmatrix}s_0 I_n - A \\ -C\end{bmatrix}\left[M \mid C^\dagger\right]\begin{bmatrix}\underline{u}_1 \\ \underline{u}_2\end{bmatrix} = 0$$

which leads to $\underline{u}_2 = 0$ and thus we have the equivalent condition

$$(s_0 M - AM)\underline{u}_1 = 0 \tag{5.7}$$

Condition (5.7) implies that there exist finite elementary divisors of the pencil $sM - AM$ iff the system $S(A,C)$ is unobservable. The pencil $sM - AM$ is known as the restricted state-output pencil [Karc. & McBean, 1] [Karc., 5].

It can be proved [Karc. & McBean, 1] that observable systems yield a state-output pencil characterised only by a set of row minimal indices $\{\eta_i + 1 = \rho_{m-i+1}, i \in m\}$, where ρ_k denotes the controllability indices of the pair (A,C) and η_i are the *r.m.i.* of $sM - AM$. For unobservable systems the canonical form of $sM - AM$ contains additional blocks to those corresponding to the row minimal indices; these new blocks correspond to finite elementary divisors, which in turn define the output decoupling zeros of the system. Once more the state-output and restricted state-output pencils have the same *f.e.d.* and their *r.m.i.* are characterised by the "*plus one*" property described above. It was shown [Karc. & McBean, 1] that if V is the coordinate transformation bringing the pair (A,C) to the Luenberger observable form, then a mere multiplication of $sM-AM$ on the right by V^{-1} brings the pencil into Kronecker form. Note that V^{-1} belongs to the class of strict equivalence transformations and, as such, preserves the Kronecker canonical form.

5.3 The zero pencil

The finite zeros and zero directions are related to the finite elementary divisors on the system matrix pencil $P(s)$. In the study of the properties of zeros and zero directions a simpler form than $P(s)$ has also being used [Karc. & Kouv., 1]. A new pencil is derived of reduced dimensions, which simplifies the study of the zero behaviour, since it is restricted only to the properties of state; this pencil is known as the zero pencil and may be defined from the conditions characterising the output zeroing problem for a strictly proper system as shown next. We should first note that condition (3.8) for strictly proper systems

$$P(z)\begin{bmatrix} \underline{x}_r \\ \hline \underline{u}_r \end{bmatrix} = 0, \quad P(z) = \begin{bmatrix} zI-A & -B \\ -C & -D \end{bmatrix} \tag{5.8}$$

implies:

$$(zI-A)\underline{x}_r = B\underline{u}_r \tag{5.9}$$

$$C\underline{x}_r = 0 \tag{5.10}$$

The last equation implies that $\underline{x}_r \in \ker C$, so that

$$\underline{x}_r = M\underline{v}_r \tag{5.11}$$

where M is a basis matrix representation of $\ker C$ and $\underline{v}_r$ is an appropriate constant vector. Substitution of equation (5.11) into (5.8) and premultiplication by the full rank transformation $\begin{bmatrix} N \\ \hline B^{\dagger} \end{bmatrix}$, where N is a left annihilator of B and $B^{\dagger}$ is a left inverse of B gives

$$(zNM - NAM)\underline{v}_r = 0 \tag{5.12}$$

$$\underline{u}_r = B^{\dagger}(zI - A)\underline{x}_r \tag{5.13}$$

since equations (5.11), (5.12) and (5.13) are equivalent to (5.8). These conditions lead to the definition of frequencies z and vectors $\underline{u}_r$ and $\underline{x}_r$, which are the zeros and the zero directions of the system. The matrix pencil $sNM - NAM$ is known as the *zero pencil* [Karc. & Kouv., 1] and its structure characterises the zero structure of the system, which is also the structure that remains invariant under the general set of state space transformations. These transformations are those of the *Kronecker group* which involves state feedback, output injection, and state, input, output coordinate transformations [Morse, 1]. Under these transformations the system $S(A,B,C,D)$ may be reduced to a canonical form, $S(A_k,B_k,C_k,D_k)$ known as the *Kronecker canonical form* [Morse, 1] [Thorp, 1] [Karc. & McBean, 1]. The relationship between the *Kronecker canonical form* $S(A_k,B_k,C_k,D_k)$ and the Kronecker form of the zero pencil is established by the following result [Karc. & McBean, 1]:

Theorem (5.2): Let $S(A,B,C)$ be a strictly proper linear system with the following set of invariants, defined by the system matrix pencil $P(s)$.

i) $(s-s_i)^{\tau_i}, i=1,\ldots,r$ finite elementary divisors

ii) $\hat{s}^{q_i}, i=1,\ldots,\mu,\ \ 0<q_1\leq\cdots\leq q_\mu$ infinite elementary divisors

iii) $0\leq\varepsilon_1\leq\varepsilon_2\leq\cdots\leq\varepsilon_p$ column minimal indices

iv) $0\leq\eta_1\leq\eta_2\leq\cdots\leq\eta_t$ row minimal indices.

Then,

(a) If B,C have full rank and $G(s)$ has full normal rank, then $\varepsilon_1 > 0$, $\eta_1 > 0$ and $q_1 \geq 2$.

(b) Let $S_k(A_k, B_k, C_k)$ be the *Kronecker canonical form* [Karc. & McBean, 1] of the system $S(A,B,C)$. The zero pencil $Z_k(s)$ computed on the system $S_k(\hat{A}_k, \hat{B}_k, \hat{C}_k)$ is in *Kronecker canonical form* with blocks corresponding to the *infinite elementary divisors* and the *row minimal indices* rearranged. The invariants of $Z_k(s)$ are related to the invariants of $S(A,B,C)$ in the following manner:

1) The *finite elementary divisors* of $Z_k(s)$ are equal to the *finite elementary divisors* of $S(A,B,C)$.
2) The infinite elementary divisors of $z_k(s)$, $\hat{s}^{\tilde{q}_i}$ are defined by $\tilde{q}_i = q_i - 2, i = 1,\ldots,\mu$
3) The column and row minimal indices of $z_k(s)$ are defined by

$$\tilde{\varepsilon}_i = \varepsilon_i - 1, i = 1,2,\ldots,p$$
$$\tilde{\eta}_j = \eta_j - 1, j = 1,2,\ldots,t$$

The results of this section are used in the following section to establish links between the structural properties of composite systems and those of the subsystems.

5.4 Composite System: The Equivalent Feedback Configuration

5.4.1 Basic Interconnection Schemes

A process is always synthesised by connecting subprocesses (subsystems). The aim here is to investigate the links between the structural aspects of the composite system, the structural aspects of the subsystems and the interconnection graph. This problem is of immense importance, especially in the early stages of designing systems by interconnecting subprocesses, since it has important implications on the synthesis of composite structures with desirable control structure characteristics.

Some basic assumptions in dealing with composite systems, represented by their transfer function matrices, or by their *minimal state space* descriptions are summarised below:

(i) These is no loading effect in any connection of two subsystems; that is, the transfer function of each subsystem remains unchanged after the connection [Chen, 1].

(ii) A system is represented by its transfer function matrix (that is, it is controllable and observable), or more generally, the system is stabilisable and detectable [Wonham, 1]. It will be also assumed that the transfer functions are rational and proper.

Note that the assumption that the subsystems are completely characterised by their transfer functions, does not imply that the composite systems are completely characterised by their transfer functions.

We consider proper systems $S_i(A_i, B_i, C_i, D_i)$ with transfer function matrices $G_i(s) = C_i(sI - A_i)^{-1}B_i + D_i$, $i = 1,2,\ldots$. An interconnected system consisting of a number of subsystems S_i will be denoted by $\sum_c$. The composite system will be called *well formed*, if all closed-loop transfer functions are well-defined and *well posed* if all closed-loop transfer functions are well defined and proper [Cal. & Des., 1]. The basic interconnection schemes for two systems are shown below:

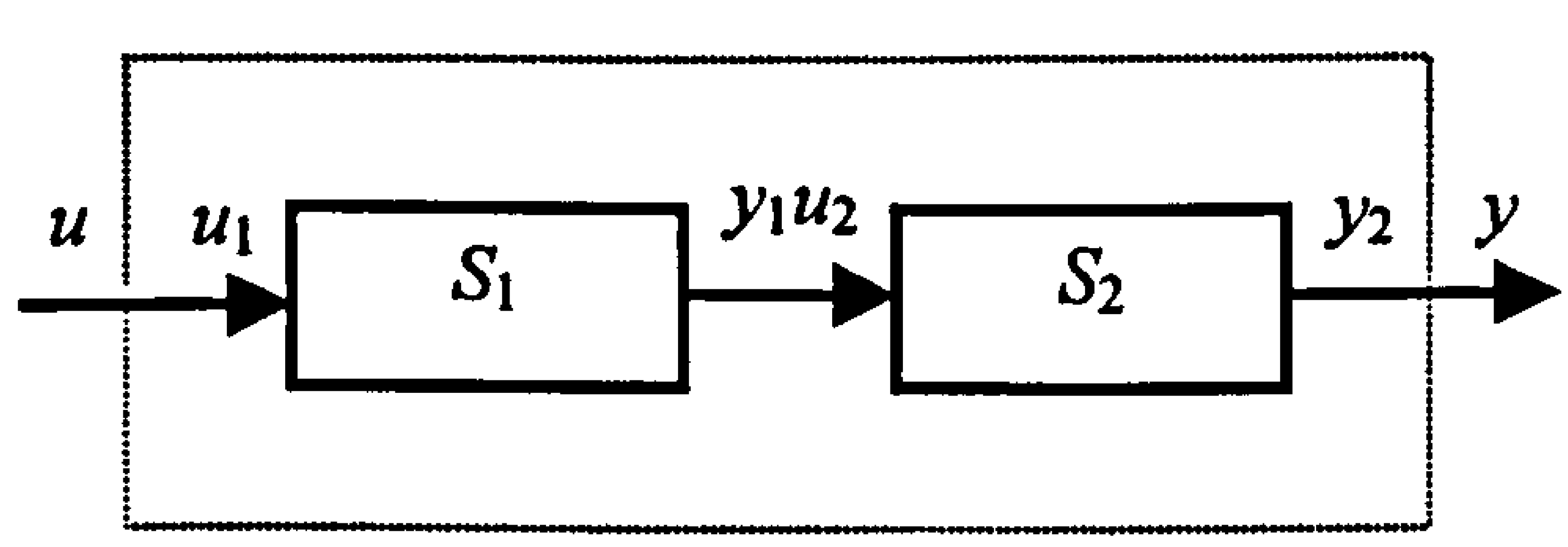


Figure (5.1). Cascade or Tandem Connection

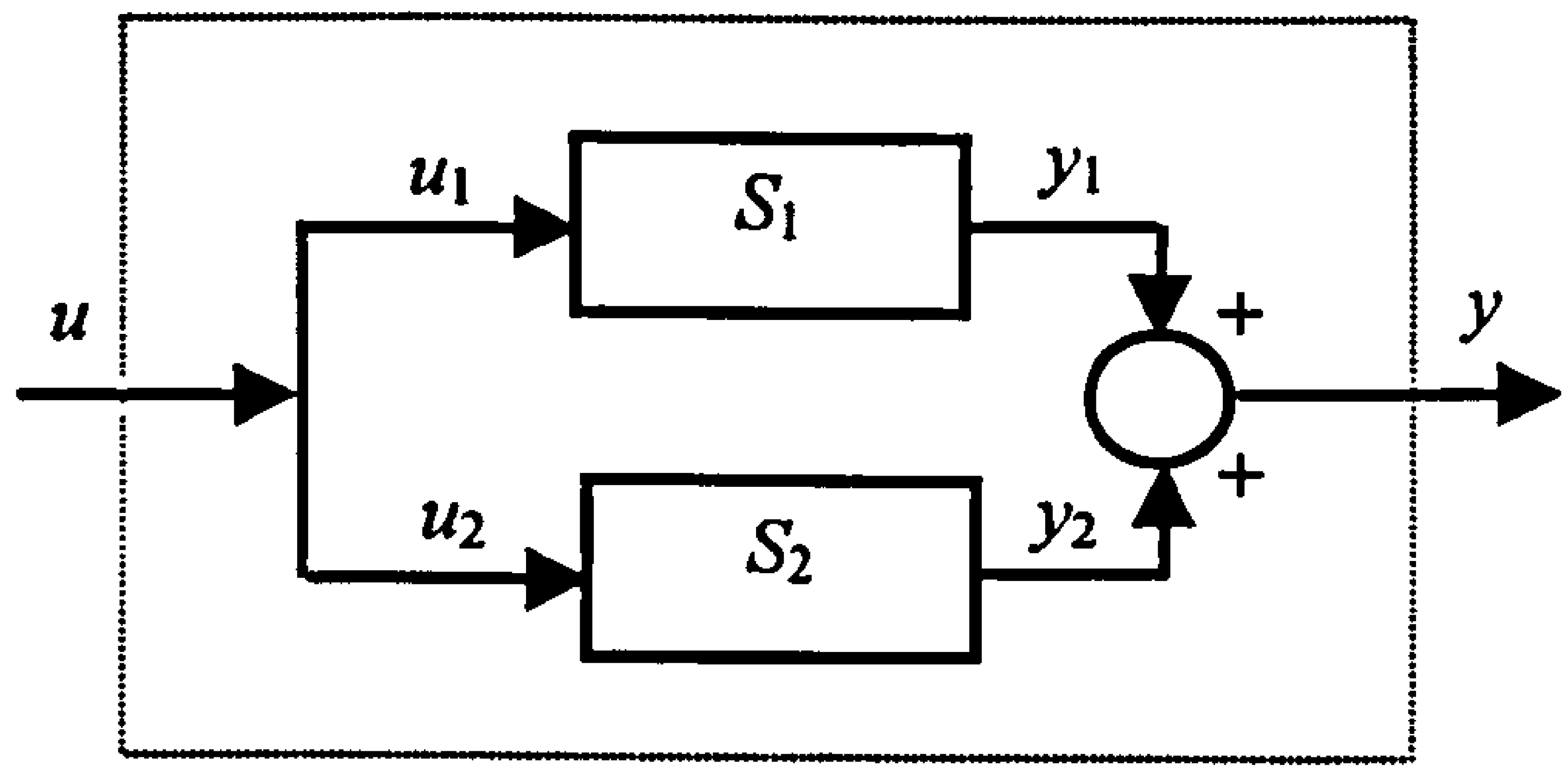


Figure (5.2). Parallel Connection

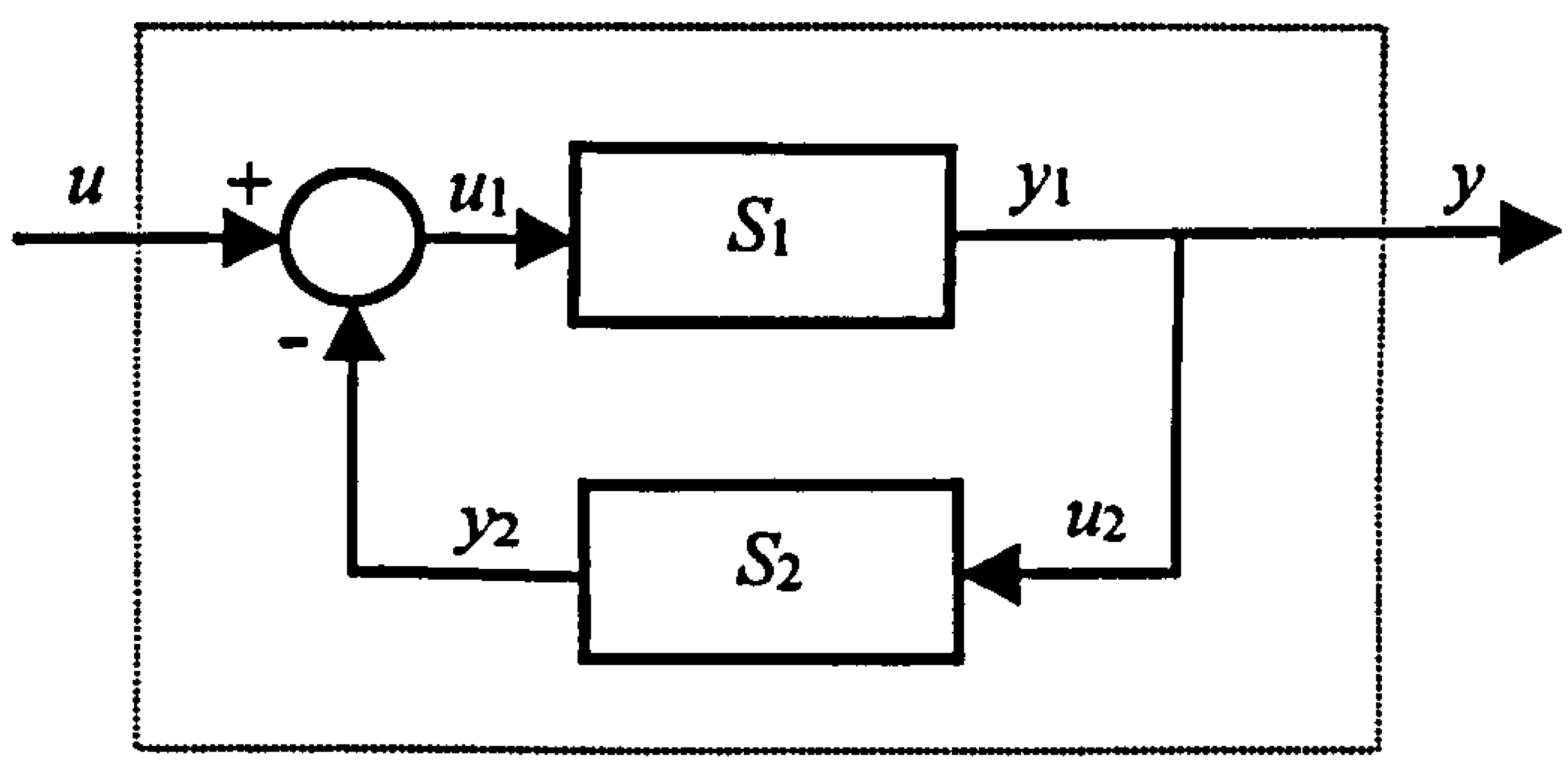


Figure (5.3). Feedback Connection

The composite systems described above are defined by the composite state space descriptions, and whether the composite transfer functions describe these systems depends on the relationships between poles and zeros of the subsystems [Rosen, 2], [Chen, 1] etc. Note that the above connections are well posed under the following conditions:

(a) Tandem connection: Always

(b) Parallel connection: If $G_1(s) \neq -G_2(s)$

(c) Feedback connection: If $|I + G_1(\infty)G_2(\infty)| = |I + D_1 D_2| \neq 0$

For two systems S_1, S_2 which are completely characterised by their proper transfer function matrices $G_1(s), G_2(s)$, any composite well posed connection of S_1 and S_2 is completely characterised by its composite transfer matrix $G_{12}(s)$, if and only if [Chen, 1]

$$\delta_m(G_{12}(s)) = \delta_m(G_1(s)) + \delta_m(G_2(s)) \tag{5.14}$$

For the different types of connections described above, the latter condition for the representation of the composite system by its composite transfer function matrix may become more explicit as conditions for coprimeness of the polynomial matrices defined by the R[s]-*irreducible* MFDs of $G_1(s)$ and $G_2(s)$ (see [Chen, 1], [Kailath, 1] etc.). For the simple case of single-input, single-output (SISO) systems S_i which are completely characterised by their proper rational functions $g_i(s), i=1,2$ we have the following:

(a) The <u>tandem connection</u> of S_1 and S_2 is completely characterised by $g_{12}(s)=g_2(s)g_1(s)$, if and only if there is no pole-zero cancellation between $g_1(s)$ and $g_2(s)$.

(b) The <u>parallel connection</u> of S_1 and S_2 is completely characterised by $g_{12}(s)=g_1(s)+g_2(s)$, if and only if $g_1(s)$ and $g_2(s)$ do not have any pole in common.

(c) The <u>feedback connection</u> of S_1 and S_2 is completely characterised by $g_{12}(s)=(1+g_1(s)g_2(s))^{-1}g_1(s)$, if and only if there is no pole of $g_2(s)$ cancelled by any zero of $g_1(s)$.

The problem of representation of composite systems by the composite transfer function is always related to controllability and observability of the composite system. The feedback configuration of figures (5.1 – 5.3) does not always have these two properties. Controllability and observability of a system always depend on the selection of the inputs and outputs. An enlarged feedback configuration, denoted in figure (5.4) always has the property of controllability and observability for the composite input vector $[r_1', r_2']'$ and output vector $[y_1', y_2']'$ and will be called the <u>*complete feedback configuration*</u>. Such configuration will be used again in the discussion of the general control design problem and it is *well-posed* if $|I+G_1(\infty)G_2(\infty)| \neq 0$. For such a configuration we may define:

$$\begin{bmatrix} \underline{u}_1(s) \\ \underline{u}_2(s) \end{bmatrix} = H(s)\begin{bmatrix} \underline{r}_1(s) \\ \underline{r}_2(s) \end{bmatrix}, \quad where\ H(s) = \begin{bmatrix} I_1 & G_2(s) \\ -G_1(s) & I_2 \end{bmatrix}^{-1} \tag{5.15}$$

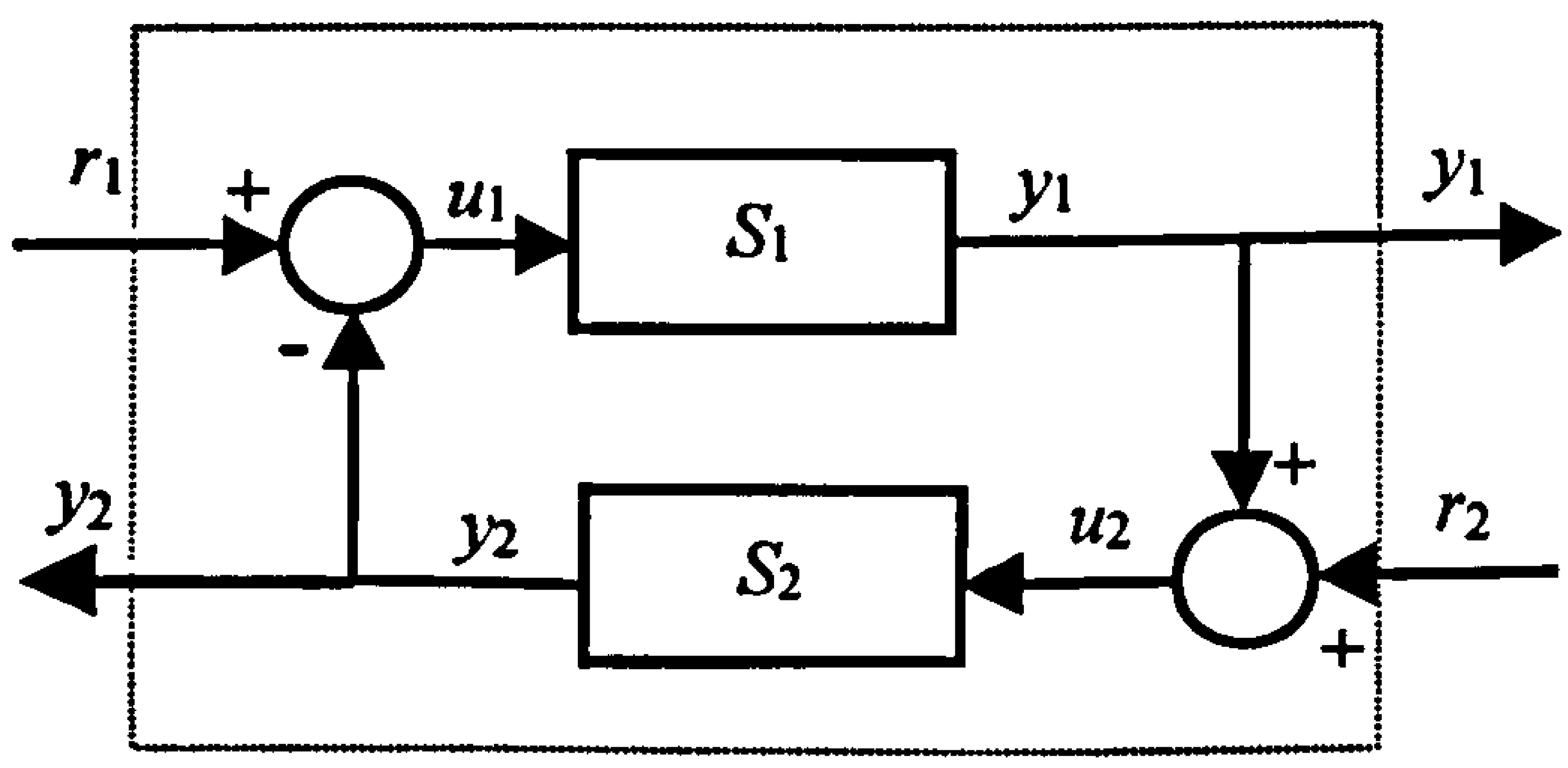


Figure (5.4) $\sum$: Complete feedback configuration

and $H(s)$ exists under the *well posedness* assumption and it is known as *error transfer function* (other transfer functions may also be defined). If $\sum_{12}^{CF}$ denotes the composite state space equations and assume that $G_i(s)$ are complete representations of S_i, then $H(s)$ completely describes the $\sum_{12}^{CF}$ composite system [Chen, 1], [Vidyas., 1].

5.4.2 The General Configuration of Composite Systems

The feedback configuration above is a natural representation of general interconnected systems [Cal. & Des., 1]. Thus assume that the interconnected system $\sum$ is obtained by coupling p subsystems, S_k, each one of them described completely by their proper transfer functions $G_i(s)$, i.e. $G_k(s) \in R_{pr}(s)^{m_k \times \ell_k}$. For example, consider the interconnection shown in figure (5.5). (In the following, we work in the s-domain (Laplace transforms) and thus we omit (s)). The following assumptions are made [Cal. & Des., 1] as far as the nature of the interconnections is concerned:

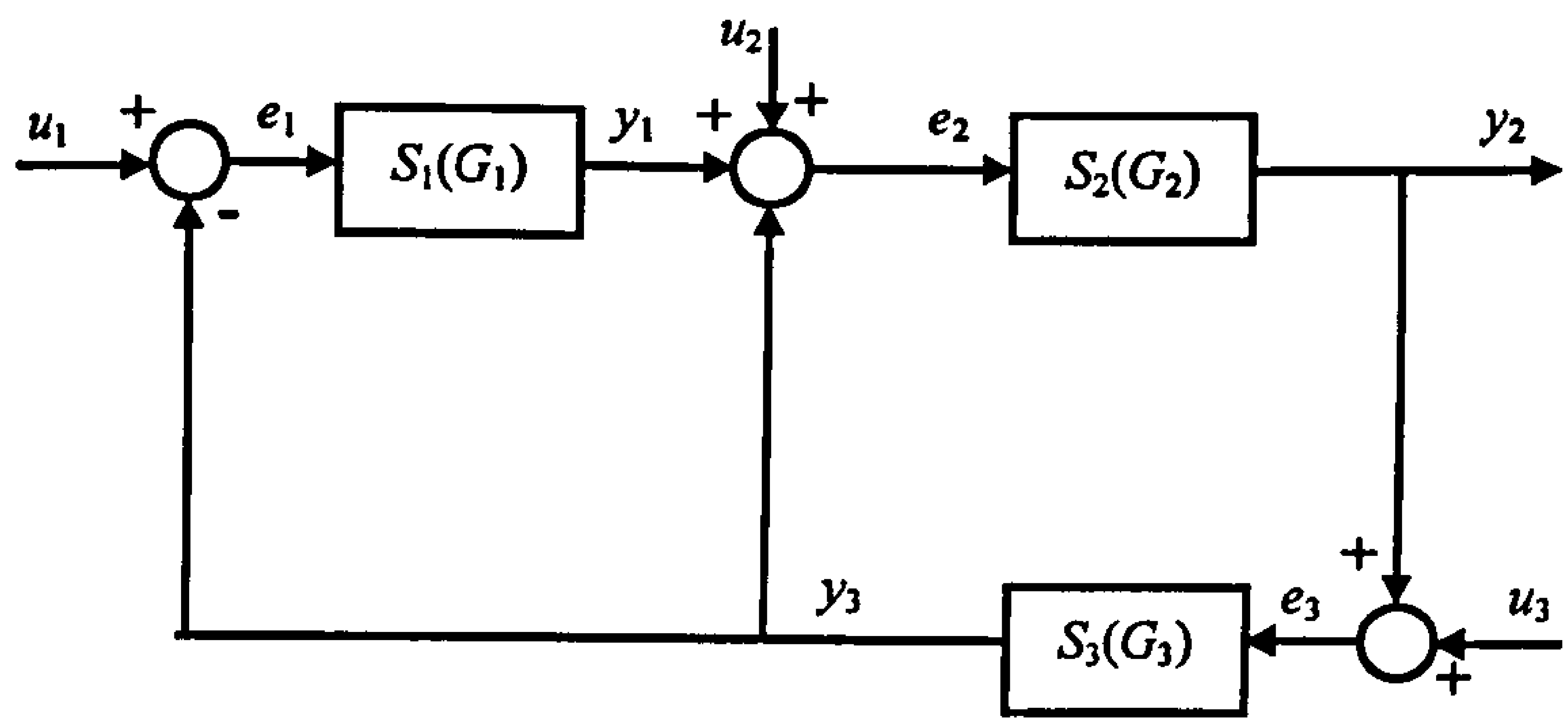


Figure (5.5). Interconnected system

<u>**Assumptions**</u>: For each subsystem $S_k\left(G_k \in R_{pr}(s)^{m_k \times \ell_k}\right)$ $k=1,\ldots p$ we have the interconnection structure shown in *figure (5.7)* which is characterised by the properties: to <u>each subsystem</u> S_k with <u>input</u> e_k and <u>output</u> y_k we associate a <u>summing node</u> with the following characteristics:

<u>**a: Well Structuring**</u> [Cal. & Des., 1]

i) Its outputs are defined by the subsystem outputs $\underline{y}_k$, i.e. $\underline{y}=\left[\underline{y}_1',\ldots,\underline{y}_p'\right]'$, or a subset of them.

ii) Its inputs at the subsystem level are:

a) an <u>exogenous input</u> $\underline{u}_k$ (arbitrarily assignable, or disturbance signal)

b) other inputs which are feedbacks of the form $F_{kj}\underline{y}_j$, $j=1,2,\ldots,p$ where $F_{kj} \in R_{pr}(s)^{\ell_k \times m_k}$ denotes a proper dynamic matrix from $\underline{y}_j$ to the k[th] summing node (very frequency F_{kj} may be real and some of them may be zero).

<u>**b: Completeness**</u> [Karc., 10]

c) $\underline{u}_k$ has as many independent coordinates as those needed to define a basis for $col.sp\left\{\left[F_{k1};\cdots;F_{k\mu}\right]\right\}$ and has subsystems outputs $\underline{y}_k=\underline{z}_k$, where $\underline{z}_k$ contains all subsystems $\sum_k$ variables which feed to the other subsystems.

□

An interconnected system satisfying (i), (ii)a, (ii)b will be called *well structured* and if also (ii)c holds true, then it will be called a *complete composite system* and shall be denoted by $\sum_c(T,F)$. In the present thesis, we restrict ourselves to the case where all subsystems are represented by proper transfer functions; note that the definition of completeness is also valid in the case where we have non-proper transfer functions, or singular subsystems.

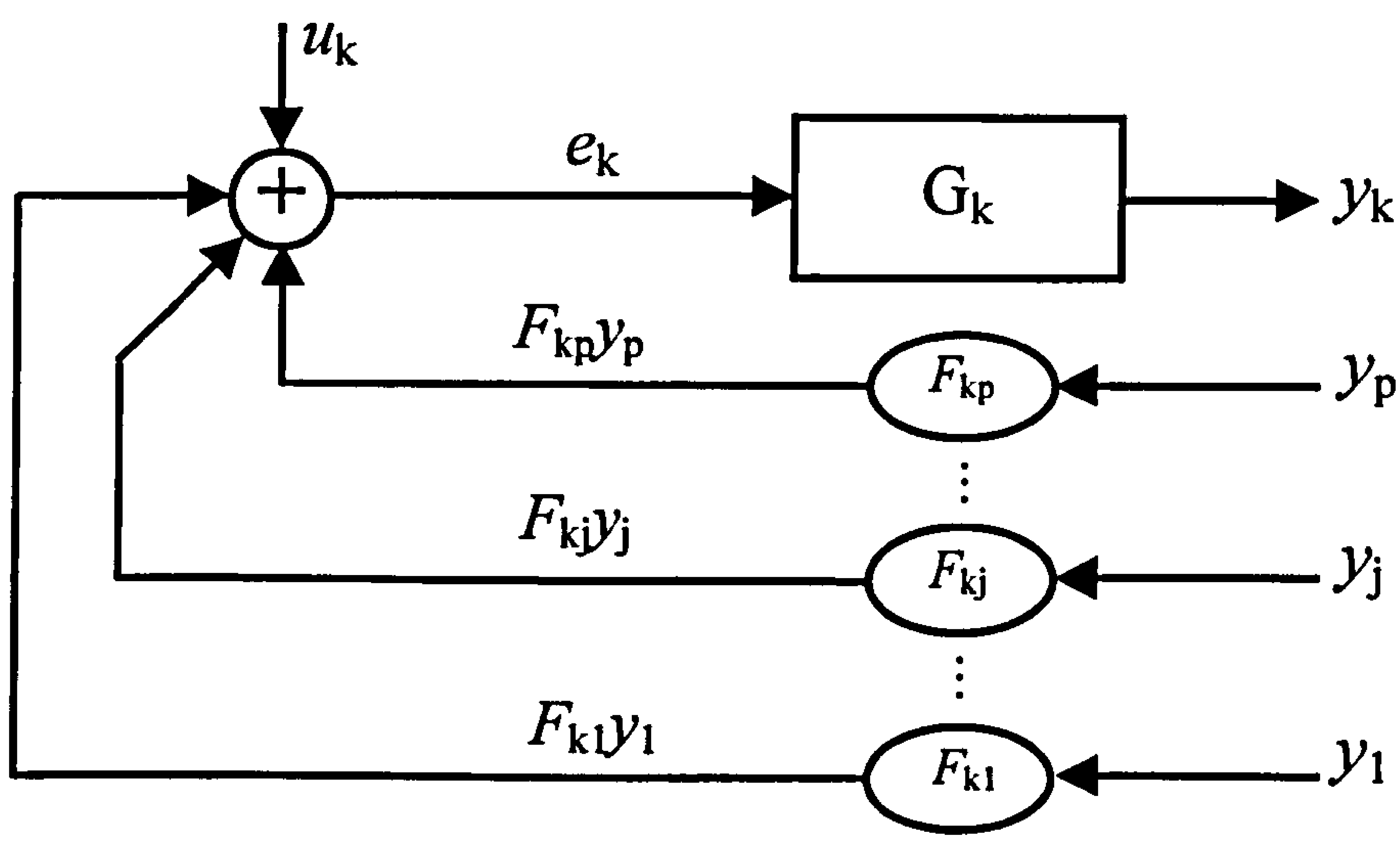


Figure (5.6).

In the following, we shall assume that the interconnection matrices are constant, since, in the general case, they can always be treated by considering the interconnection matrices as subsystem themselves. The implications of the above assumptions are that the subsystems $S_k, k=1,\dots,p$ are interconnected according to the equations

$$\underline{e}_k = \underline{u}_k + \sum_{j=1}^{p} F_{kj}\underline{y}_j,\ \underline{y}_k = G_k\underline{e}_k \tag{5.16}$$

Hence, by aggregation, i.e. by defining global quantities

$$
\begin{aligned}
& m=\sum_{k=1}^{p} m_k,\ L=\sum_{k=1}^{p} L_k \\
& \underline{u}=\left[\underline{u}_1^t,\ldots,\underline{u}_p^t\right]^t \in R^{\ell} \\
& \underline{y}=\left[\underline{y}_1^t,\ldots,\underline{y}_p^t\right]^t \in R^{m} \\
& \underline{e}=\left[\underline{e}_1^t,\ldots,\underline{e}_p^t\right]^t \in R^{\ell} \\
& F=\left[F_{kj}\right]_{k,j\in<p>} \in R_{pr}^{\ell\times m}(s) \\
& G=block-diag\{G_k\}_{k=1}^{p} \in R_{pr}^{m\times\ell}(s)
\end{aligned}
\tag{5.17}
$$

then (5.16) may be equivalently written as

$$\underline{e}=\underline{u}+F\underline{y},\quad \underline{y}=G\underline{e} \tag{5.18}$$

which describes the feedback system shown in figure (5.6).

For the example of figure (5.5), we have

$$
G=\begin{bmatrix} G_1 & 0 & 0 \\ 0 & G_2 & 0 \\ 0 & 0 & G_3 \end{bmatrix}\begin{matrix} m_1 \\ m_2 \\ m_3 \end{matrix} \quad (\text{columns } \ell_1\ \ell_2\ \ell_3)
\qquad
F=\begin{bmatrix} 0 & 0 & -I \\ I & 0 & -I \\ 0 & I & 0 \end{bmatrix}\begin{matrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{matrix} \quad (\text{columns } m_1\ m_2\ m_3)
$$

Because of the above configuration, we shall refer to $\underline{u},\underline{e},\underline{y}$ as the <u>input</u>, <u>error</u> and <u>output</u> respectively of $\sum$. The matrix F will be called the gain, or interconnection matrix of $\sum$ and G the *open-loop*, or *aggregate transfer function* of Σ.

The above representation of composite systems (as a feedback configuration) has important implications for the present work:

(i) It provides a systematic method for representing composite systems (with implications on the transition from process configurations to process transfer functions).

(ii) It allows the formulation of the process synthesis problem (interconnection of subprocesses) as a feedback design problem.

Equation (5.18) shows that the interconnected system $\sum$ has the transfer function matrices

$$H_{eu}:\underline{u}\rightarrow\underline{e} \quad and \quad H_{yu}:\underline{u}\rightarrow\underline{y} \tag{5.19}$$

which are defined as long as $|I-FG|\neq 0$ (*well-formedness* assumption) and are called the *input-error* and *input-output transfer functions* respectively. These transfer functions are defined by

$$H_{eu}=(I-FG)^{-1}\in R(s)^{\ell\times\ell},\quad H_{yu}=G(I-FG)^{-1}\in R(s)^{m\times\ell} \tag{5.20}$$

where the two transfer functions are related by:

$$H_{eu}=I+FH_{yu} \tag{5.21}$$

Note that under the well-formedness assumption, all closed-loop transfer functions of $\sum$, i.e. from $\underline{u}\rightarrow\underline{e},\underline{y},F\underline{y}$ respectively, exist and they are given by H_{eu}, H_{yu}, $FH_{yu}=H_{eu}-I$.

Remark (5.1): [Vidyas., 1] If F,G are proper transfer functions, then the complete feedback configuration is well-posed iff

$$|I+F(\infty)G(\infty)|=|I+G(\infty)F(\infty)|\neq 0$$

For a complete system, the interconnections are equivalent to output feedback and thus we have the following representation of the action of system composition:

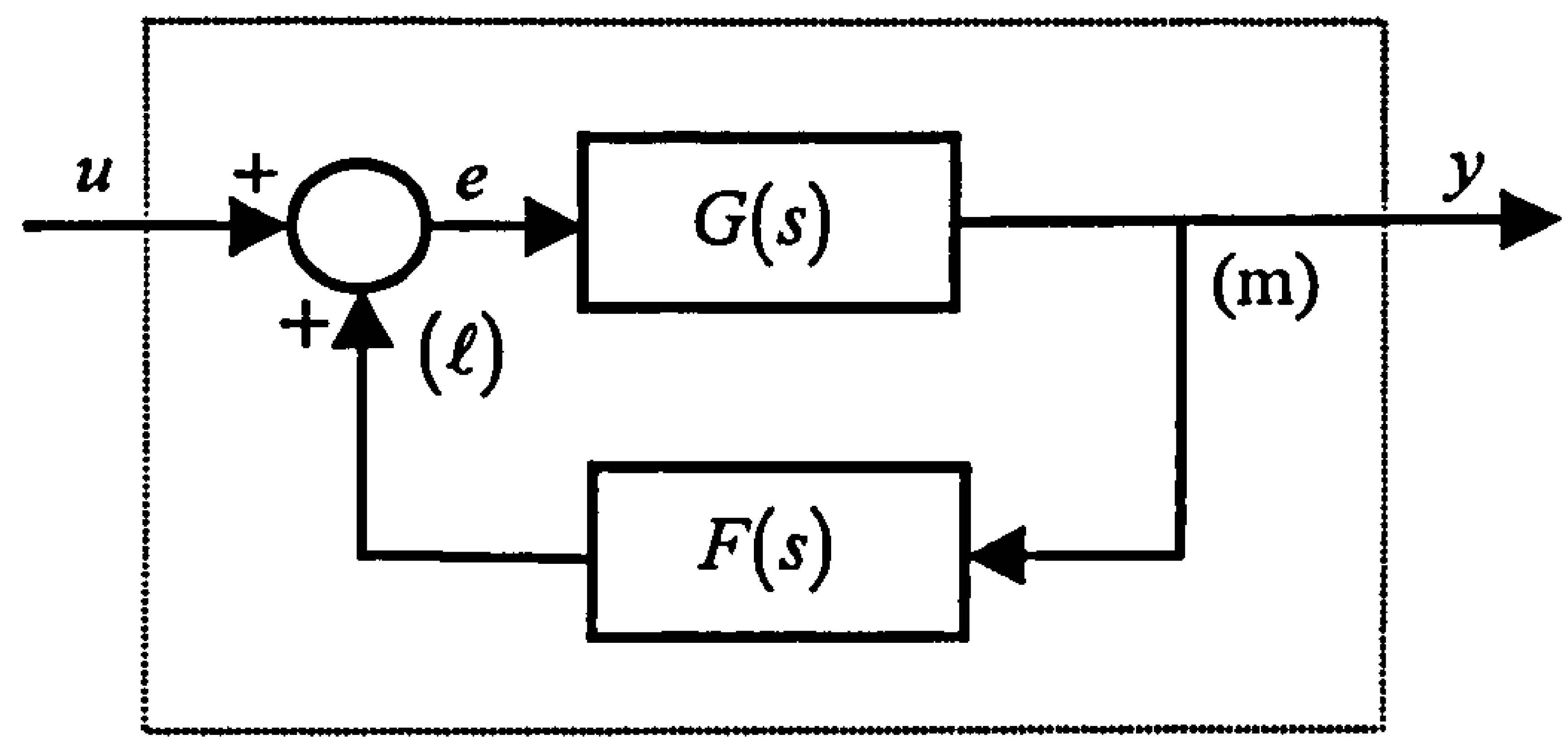


Figure (5.7). Interconnected System $\sum$ under the Completeness assumption.

Lack of completeness implies that the numbers of free variables in $\underline{u}$ are fewer than those in $\underline{e}$ and/or that we do not measure all interconnection variables.

5.5 General State Space Description of Well-posed Complete Composite Systems

The complete composite system may be represented as a feedback structure as shown in figure (5.7). Under the assumption of *well-posedness*, all transfer functions H_{yu}, H_{eu} are proper. The state space form description of the composite system is considered next. Note that if the system equations are defined by

$$S_i: \begin{cases} \dot{\underline{x}}_i = \overline{A}_i \underline{x}_i + \overline{B}_i \underline{e}_i \\ \underline{y}_i = \overline{C}_i \underline{x}_i + \overline{D}_i \underline{e}_i \end{cases} \quad i = 1,2,\ldots,k \tag{5.22}$$

where

$$\underline{e} = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_k \end{bmatrix}, \quad \underline{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_k \end{bmatrix}, \quad \underline{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix}, \quad \underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix} \tag{5.23}$$

then the aggregate system is described by

$$\underbrace{\begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \\ \vdots \\ \dot{\underline{x}}_k \end{bmatrix}}_{=\dot{\underline{x}}} = \underbrace{\begin{bmatrix} A_1 & & & 0 \\ & A_2 & & \\ & & \ddots & \\ 0 & & & A_k \end{bmatrix}}_{=\overline{A}} \underbrace{\begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \vdots \\ \underline{x}_k \end{bmatrix}}_{=\underline{x}} + \underbrace{\begin{bmatrix} B_1 & & & 0 \\ & B_2 & & \\ & & \ddots & \\ 0 & & & B_k \end{bmatrix}}_{=\overline{B}} \underbrace{\begin{bmatrix} \underline{e}_1 \\ \underline{e}_2 \\ \vdots \\ \underline{e}_k \end{bmatrix}}_{=\underline{e}} \tag{5.24}$$

or

$$\dot{\underline{x}} = \overline{A}\underline{x} + \overline{B}\underline{e} \tag{5.25}$$

where

$$\underline{e} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_k \end{bmatrix} + F \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix} \tag{5.26}$$

where F is a matrix expressing interconnections. Furthermore

$$\underbrace{\begin{bmatrix} \dot{\underline{y}}_1 \\ \dot{\underline{y}}_2 \\ \vdots \\ \dot{\underline{y}}_k \end{bmatrix}}_{=\dot{\underline{y}}} = \underbrace{\begin{bmatrix} C_1 & & & 0 \\ & C_2 & & \\ & & \ddots & \\ 0 & & & C_k \end{bmatrix}}_{=\overline{C}} \underbrace{\begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \vdots \\ \underline{x}_k \end{bmatrix}}_{=\underline{x}} + \underbrace{\begin{bmatrix} D_1 & & & 0 \\ & D_2 & & \\ & & \ddots & \\ 0 & & & D_k \end{bmatrix}}_{=\overline{D}} \underbrace{\begin{bmatrix} \underline{e}_1 \\ \underline{e}_2 \\ \vdots \\ \underline{e}_k \end{bmatrix}}_{=\underline{e}} \tag{5.27}$$

or

$$\underline{y} = \overline{C}\underline{x} + \overline{D}\underline{e} \tag{5.28}$$

Thus, the composite system equations become

$$\begin{cases} \dot{\underline{x}} = \overline{A}\underline{x} + \overline{B}\underline{e} \\ \underline{y} = \overline{C}\underline{x} + \overline{D}\underline{e} \end{cases} \tag{5.29}$$

and

$$\underline{e} = \underline{u} + F\underline{y} \tag{5.30}$$

where F is a matrix expressing interconnections. From the above (i.e. (5.29) and (5.30)) we have $\underline{e} = \underline{u} + F\underline{y} = \underline{u} + F(\overline{C}\underline{x} + \overline{D}\underline{e})$ or

$$\underline{e} = \underline{u} + F\overline{C}\underline{x} + F\overline{D}\underline{e}$$
$$(I - F\overline{D})\underline{e} = \underline{u} + F\overline{C}\underline{x} \tag{5.31}$$

Assume that the feedback configuration is well formed i.e. $|I - F\overline{D}| \neq 0$ then we may define

$$\Delta = (I - F\overline{D})^{-1} \tag{5.32}$$

From (5.31), (5.32) we have

$$\underline{e} = \Delta\underline{u} + \Delta F\overline{C}\underline{x} \tag{5.33}$$

and thus $\underline{\dot{x}} = \overline{A}\underline{x} + \overline{B}\underline{e} = \overline{A}\underline{x} + \overline{B}\Delta\underline{u} + \overline{B}\Delta F\overline{C}\underline{x}$ or

$$\underline{\dot{x}} = (\overline{A} + \overline{B}\Delta F\overline{C})\underline{x} + \overline{B}\Delta\underline{u} \tag{5.34}$$

and $\underline{y} = \overline{C}\underline{x} + \overline{D}\Delta\underline{e} = \overline{C}\underline{x} + \overline{D}\Delta\underline{u} + \overline{D}\Delta F\overline{C}\underline{x}$ or

$$\underline{y} = (I + \overline{D}\Delta F)\overline{C}\underline{x} + \overline{D}\Delta\underline{u} \tag{5.35}$$

We may summarise the above analysis as follows:

Proposition (5.1): The composite system state equations of the well-posed system are given by:

$$\underline{\dot{x}} = \overbrace{\left(\overline{A} + \overline{B}\Delta F\overline{C}\right)}^{\tilde{A}}\underline{x} + \overline{B}\Delta\underline{u}$$
$$\underline{y} = \left(I + \overline{D}\Delta F\right)\overline{C}\underline{x} + \overline{D}\Delta\underline{u} \tag{5.36}$$

where $\Delta = \left(I - F\overline{D}\right)^{-1}$, $\left|I - F\overline{D}\right| \neq 0$ and $\overline{A}, \overline{B}, \overline{C}$ and $\overline{D}$ are the state space parameters describing the aggregate model.

We assume, as before, that the component subsystems $S_i(A_i, B_i, C_i, D_i)$ are both *controllable* and *observable* (or stabilisable and detectable) for all $i = 1,2,\ldots,k$. The problem we consider next is the investigation of the controllability properties of the composite system under the partial, or total loss of inputs at the subsystem level. The results for observability are of similar character.

5.6 Input-state Restriction Pencil of Complete Composite and Aggregate Systems

Consider the composite system described by equation (5.36) and let $\overline{N}$ be a left annihilator of $\overline{B}$, where

$$\overline{N} = \begin{bmatrix} N_1 & & & 0 \\ & N_2 & & \\ & & \ddots & \\ 0 & & & N_k \end{bmatrix} \tag{5.37}$$

and N_i are left annihilators for B_i. The input state Restriction Pencil of the composite system is $s\overline{N} - \overline{N}\tilde{A}$ and may be expressed as shown below. We first note that the pencil

$$s\overline{N}-\overline{NA}=\begin{bmatrix} sN_1-N_1A_1 & & 0 \\ & sN_2-N_2A_2 & \\ & & \ddots & \\ 0 & & & sN_k-N_kA_k \end{bmatrix} \tag{5.38}$$

is the Input-state Restriction pencil of the aggregate system without the interconnection. Given that $\overline{NB}\Delta=0$, then the restriction pencil of the composite system is:

$$s\overline{N}-\overline{N}\tilde{A}=s\overline{N}-\overline{N}\left(\overline{A}+\overline{B}\Delta F\overline{C}\right)=\begin{bmatrix} sN_1-N_1A_1 & & 0 \\ & sN_2-N_2A_2 & \\ & & \ddots & \\ 0 & & & sN_k-N_kA_k \end{bmatrix} \tag{5.39}$$

and this leads to the following result:

<u>**Theorem (5.4):**</u> If $S_i(A_i,B_i)$, $i=1,\ldots,k$ are controllable and the composite system is well formed, then the composite system with full inputs, that is, the complete composite system, is also controllable.

Proof: Since the input-state restriction pencil of the composite system is the direct sum of the input-state restriction pencils of the subsystems, controllability properties are expressed as the aggregates of the corresponding properties defined on the subsystems.

□

<u>**Corollary (5.1):**</u> If the composite system is complete, then uncontrollability of a subsystem results in uncontrollability of the composite system. Furthermore, the dimension of the controllable space of the composite system is the sum of the dimensions of the controllable subspaces of subsystems.

□

Corollary (5.2): The set of controllability indices of the composite system is given as the union of the controllability indices of the subsystems. Furthermore, the set of input decoupling zeros of the composite system is given as the union of the set of input decoupling zeros of the subsystems.

□

The above Corollaries follow in a straightforward manner from condition (5.39) which is a direct consequence of the completeness assumption.

Now we investigate the controllability properties of the composite system under total loss of subsystem inputs.

5.7 Input-state Restriction Pencils under Total Loss of Subsystem Inputs

Consider a complete composite system and assume that all the i-th subsystem external inputs are not used (i.e. this occurs when interconnection elements are dynamic and no assignable input is available for them). In this case, the corresponding subsystem has as inputs those coming from the interconnections only and it does not possess any more the completeness property. The composite system description with total loss of subsystem inputs is described by:

$$\begin{cases} \dot{\underline{x}} = \left(\overline{A} + \overline{B}\Delta\overline{C}\right)\underline{x} + \overline{B}\Delta\underline{u} \\ \underline{y} = \left(I + \overline{D}\Delta F\right)\overline{C}\underline{x} + \overline{D}\Delta\underline{u} \end{cases}$$

where for the case of strictly proper systems $(\Delta = I)$, we have

$$\underline{x} = \left(\overline{A} + \overline{B}\Delta\overline{C}\right)\underline{x} + \overline{B}\underline{u}'_i, \quad \underline{u}'_i = \begin{bmatrix} \underline{u}_l \\ \vdots \\ \underline{u}_{i-1} \\ 0 \\ \underline{u}_{i+1} \\ \vdots \\ \underline{u}_k \end{bmatrix} \tag{5.40}$$

and

$$\underline{\dot{x}} = \left(\overline{A} + \overline{B}\Delta\overline{C}\right)\underline{x} + \widetilde{B}_i\underline{\widetilde{u}}_i \tag{5.41}$$

where

$$\widetilde{B}_i = \begin{bmatrix} B_1 & & & & & 0 \\ & \ddots & & & & \\ & & B_{i-1} & & & \\ & & & & & \\ & & & B_{i+1} & & \\ & & & & \ddots & \\ 0 & & & & & B_k \end{bmatrix}, \quad \underline{\widetilde{u}}_i = \begin{bmatrix} \underline{u}_1 \\ \vdots \\ \underline{u}_{i-1} \\ \underline{u}_{i+1} \\ \vdots \\ \underline{u}_k \end{bmatrix} \tag{5.42}$$

i.e. the block containing B_i has been deleted. We may define the left annihilator of $\widetilde{B}_i, \widetilde{N}_i$ by solving the equation $\widetilde{N}_i\widetilde{B}_i = 0$. It is obvious that

$$\underbrace{\begin{bmatrix} N_1 & & & & & 0 \\ & \ddots & & & & \\ & & N_{i-1} & & & \\ & & & I_{n_i} & & \\ & & & & N_{i+1} & \\ & & & & & \ddots \\ 0 & & & & & N_k \end{bmatrix}}_{=\widetilde{N}_i} \underbrace{\begin{bmatrix} B_1 & & & & & 0 \\ & \ddots & & & & \\ & & B_{i-1} & & & \\ & & & & & \\ & & & B_{i+1} & & \\ & & & & \ddots & \\ 0 & & & & & B_k \end{bmatrix}}_{=\widetilde{B}_i} = 0 \tag{5.43}$$

and thus the corresponding restriction pencil is

$$R_i(s) = s\widetilde{N}_i - \widetilde{N}_i\left(\overline{A} + \overline{B}F\overline{C}\right) = s\widetilde{N}_i - \widetilde{N}_i\overline{A} - \widetilde{N}_i\overline{B}F\overline{C} \tag{5.44}$$

The controllability properties are investigated by examining the above pencil, where

$$s\tilde{N}_i - \tilde{N}_i\overline{A} = \begin{bmatrix} sN_1 - N_1A_1 & & & & & 0 \\ & \ddots & & & & \\ & & sN_{i-1} - N_{i-1}A_{i-1} & & & \\ & & & sI - A_i & & \\ & & & & \ddots & \\ 0 & & & & & sN_k - N_kA_k \end{bmatrix} \tag{5.45}$$

The problem of computing $R_i(s)$ is thus reduced to the computation of $\tilde{N}_i\overline{B}F\overline{C}$. Note that

$$\tilde{N}_i\overline{B} = \begin{bmatrix} & & & 0 & & & \\ & & & \vdots & & & \\ & & & 0 & & & \\ 0 & \cdots & 0 & B_i & 0 & \cdots & 0 \\ & & & 0 & & & \\ & & & \vdots & & & \\ & & & 0 & & & \end{bmatrix}, \quad F = \begin{bmatrix} F_{11} & F_{12} & \cdots & F_{1i} & \cdots & F_{1k} \\ \vdots & \vdots & & \vdots & & \vdots \\ F_{i1} & F_{i2} & \cdots & F_{ii} & \cdots & F_{ik} \\ \vdots & \vdots & & \vdots & & \vdots \\ F_{k1} & F_{k2} & \cdots & F_{ki} & \cdots & F_{kk} \end{bmatrix} \tag{5.46}$$

and thus

$$\tilde{N}_i\overline{B}F\overline{C} = \begin{bmatrix} & & 0 & & \\ & & \vdots & & \\ & & 0 & & \\ B_iF_{i1} & \cdots & B_iF_{ii} & \cdots & B_iF_{ik} \\ & & 0 & & \\ & & \vdots & & \\ & & 0 & & \end{bmatrix}\overline{C} \tag{5.47}$$

$$\tilde{N}_i \bar{B} F \bar{C} = \begin{bmatrix} & 0 & \\ & \vdots & \\ & 0 & \\ B_i F_{i1} C_1 \quad \cdots & B_i F_{ii} C_i & \cdots \quad B_i F_{ik} C_k \\ & 0 & \\ & \vdots & \\ & 0 & \end{bmatrix} \tag{5.48}$$

From (5.45) and (5.48) we have that the input-state restricted pencil $R_i(s)$ is given by

$$R_i(s) = s\tilde{N}_i - \tilde{N}_i \bar{A} - \tilde{N}_i \bar{B} F \bar{C} =$$

$$\begin{bmatrix} sN_1 - N_1 A_1 & & & & & 0 \\ 0 & \ddots & & & & \\ -B_i F_{i1} C_1 & \cdots & -B_i F_{i,i-1} C_{i-1} & sI - A_i - B_i F_{ii} C_i & \cdots & -B_i F_{ik} C_k \\ & & & & \ddots & 0 \\ 0 & & & & & sN_k - N_k A_k \end{bmatrix} \tag{5.49}$$

The above expression may be generalised for every i and extended to any combination of indices i, j, etc. The expression for $R_i(s)$ may be used for:

(i) Studying the effect of the structure F on the loss of controllability, when total loss of subsystem inputs occurs, as well as the location of the formed input decoupling zeros.

(ii) Distinguish the phenomena depending on the parameters of subsystems $S_i(A_i, B_i, C_i)$ and those depending only on F structure.

It should now be pointed that loss of external inputs results in a pencil $R_i(s)$, whose Kronecker structure is no longer expressed as a direct sum of the Kronecker structure of the subsystems. The role of the matrix F expressing the interconnections is now crucial in determining the composite system properties.

5.8 State-Output Restriction Pencil of Composite and Aggregate Systems with Full Outputs

Consider the composite system described by equation (5.36) and let $\overline{M}$ be a right annihilator of $\overline{C}$, where

$$\overline{M} = \begin{bmatrix} M_1 & & & 0 \\ & M_2 & & \\ & & \ddots & \\ 0 & & & M_k \end{bmatrix} \tag{5.50}$$

where M_i are right annihilators for C_i. The state-output restriction pencil is $s\overline{M} - \tilde{A}\overline{M}$ and may be expressed as shown below. We first note that the pencil

$$s\overline{M} - \tilde{A}\overline{M} = \begin{bmatrix} sM_1 - A_1M_1 & & & 0 \\ & sM_2 - A_2M_2 & & \\ & & \ddots & \\ 0 & & & sM_k - A_kM_k \end{bmatrix} \tag{5.51}$$

is the state-output restriction pencil of the aggregate system without the interconnection. Given that $\Delta\overline{C}\overline{M} = 0$, then the restriction pencil of the composite system is:

$$\begin{aligned} s\overline{M} - \tilde{A}\overline{M} &= s\overline{M} - \left(\overline{A} + \overline{B}\Delta F\overline{C}\right)\overline{M} \\ &= \begin{bmatrix} sM_1 - A_1M_1 & & & 0 \\ & sM_2 - A_2M_2 & & \\ & & \ddots & \\ 0 & & & sM_k - A_kM_k \end{bmatrix} \end{aligned} \tag{5.52}$$

The above leads to the following result:

Theorem (5.4): If $S_i(A_i, C_i), i = 1, \ldots, k$ are observable and composite system is well formed, then the composite system with full outputs or equivalently, the complete composite system, is also observable.

Proof: Since the state-output restriction pencil of the composite system is the direct sum of the state-output restriction pencils of the subsystems, observability properties are expressed as the aggregates of the corresponding properties defined on the subsystems.

Corollary (5.3): Unobservability of a subsystem results in a unobservable composite system. Furthermore, the dimension of the unobservable space of the composite system is the sum of the dimensions of the unobservable subspaces of subsystems.

□

Corollary (5.4): The set of observability indices of the composite system is given by the union of the observability indices of the subsystems. Furthermore, the set of output decoupling zeros of the composite system is given as the union of the set of output decoupling zeros of the subsystems.

□

Now we investigate the observability properties of the composite system under total loss of subsystem outputs.

5.9 State-Output Restriction Pencils under Total Loss of Subsystem Outputs.

We consider here the composite system and shall examine the case where all the i-th subsystem outputs are not measured. The composite system description is described by:

$$\begin{cases} \underline{\dot{x}} = \left(\overline{A} + \overline{B}\Delta F\overline{C}\right)\underline{x} + \overline{B}\Delta\underline{u} \\ \underline{y} = \left(I + \overline{D}\Delta F\right)\overline{C}\,\underline{x} + \overline{D}\Delta\underline{u} \end{cases}$$

where for the case of strictly proper systems $(\Delta = I)$

$$\begin{cases} \dot{\underline{x}} = (\overline{A} + \overline{B}\overline{C})\underline{x} + \overline{B}\Delta\underline{u} \\ \underline{y} = \overline{C}\underline{x} \end{cases} \tag{5.53}$$

Under the assumption that the output associated with the i-th subsystem is not measured, then the new output is written as $y = \widetilde{C}_i \underline{\widetilde{x}}_i$ where

$$\widetilde{C}_i = \begin{bmatrix} C_1 & & & & & & 0 \\ & \ddots & & & & & \\ & & C_{i-1} & 0 & & & \\ & & 0 & C_{i+1} & & & \\ & & & & \ddots & & \\ 0 & & & & & & C_k \end{bmatrix} \tag{5.54}$$

i.e. the block containing C_i has been deleted. We may define the right annihilator of $\widetilde{C}_i$, $\widetilde{M}_i$ by solving the equation $\widetilde{C}_i \widetilde{M}_i = 0$. It is obvious that

$$\underbrace{\begin{bmatrix} C_1 & & & & & 0 \\ & \ddots & & & & \\ & & C_{i-1} & 0 & & \\ & & 0 & C_{i+1} & & \\ & & & & \ddots & \\ 0 & & & & & C_k \end{bmatrix}}_{=\widetilde{C}_i} \underbrace{\begin{bmatrix} M_1 & & & & & 0 \\ & \ddots & & & & \\ & & M_{i-1} & & & \\ & & & I_{m_i} & & \\ & & & & M_{i+1} & \\ & & & & \ddots & \\ 0 & & & & & M_k \end{bmatrix}}_{=\widetilde{M}_i} = 0 \tag{5.55}$$

and thus the corresponding restriction pencil is

$$T_i(s) = s\widetilde{M}_i - A\widetilde{M}_i = s\widetilde{M}_i - (\overline{A} + \overline{B}F\overline{C})\overline{M}_i = s\widetilde{M}_i - \overline{A}\widetilde{M}_i - \overline{B}F\overline{C}\widetilde{M}_i \tag{5.56}$$

The observability properties are investigated by examining the above pencil, where

$$s\widetilde{M}_i - A\widetilde{M}_i = \begin{bmatrix} sM_i - A_1M_1 & & & & & \\ & \ddots & & & & \\ & & sM_{i-1} - A_{i-1}M_{i-1} & & & \\ & & & sI - A_i & & \\ & & & & \ddots & \\ & & & & & sM_k - A_kM_k \end{bmatrix} \tag{5.57}$$

The problem of computing $T_i(s)$ is thus reduced to the computation of $\overline{B}F\overline{C}\widetilde{M}_i$. Note that

$$\overline{C}\widetilde{M}_i = \begin{bmatrix} & & & 0 & & & \\ & & & \vdots & & & \\ & & & 0 & & & \\ 0 & \cdots & 0 & C_i & 0 & \cdots & 0 \\ & & & 0 & & & \\ & & & \vdots & & & \\ & & & 0 & & & \end{bmatrix}, \quad F = \begin{bmatrix} F_{11} & F_{12} & \cdots & F_{1i} & \cdots & F_{1k} \\ \vdots & \vdots & & \vdots & & \vdots \\ F_{i1} & F_{i2} & \cdots & F_{ii} & \cdots & F_{ik} \\ \vdots & \vdots & & \vdots & & \vdots \\ F_{k1} & F_{k2} & \cdots & F_{ki} & \cdots & F_{kk} \end{bmatrix} \tag{5.58}$$

and thus

$$\overline{B}F\overline{C}\widetilde{M}_i = \overline{B}\begin{bmatrix} & F_{1i}C_i & \\ & \vdots & \\ & F_{i-1,i}C_i & \\ 0 & F_{ii}C_i & 0 \\ & 0 & \\ & & \\ & F_{ki}C_i & \end{bmatrix} \tag{5.59}$$

$$\overline{B}F\overline{C}\widetilde{M}_i = \begin{bmatrix} & B_1F_{1i}C_i & \\ & \vdots & \\ 0 & B_iF_{ii}C_i & 0 \\ & \vdots & \\ & B_kk_{ki}C_i & \end{bmatrix} \tag{5.60}$$

From (5.57) and (5.60) we have that the state-output restricted pencil $T_i(s)$ is given by

$$T_i(s) = s\widetilde{M}_i - \overline{A}\widetilde{M}_i - \overline{B}F\overline{C}\widetilde{M}_i$$

$$= \begin{bmatrix} sM_1 - A_1M_1 & -B_1F_{1i}C_i & 0 \\ \ddots & \vdots & \\ & -B_{i-1}F_{i-1,i}C_i & \\ & sI - A_i - B_iF_{ii}C_i & \\ & & \ddots \\ 0 & -B_kF_{ki}C_i & sM_k - A_kM_k \end{bmatrix} \tag{5.61}$$

The above expression can be generalised for every i and extended to any combination of indices i, j, etc. The expression for $T_i(s)$ may be used for:

(i) Studying the effect of the structure F on the loss of observability, when total loss of subsystem outputs occurs, as well as the location of the formed output decoupling zeros.

(ii) Distinguishing the phenomena depending on the parameters of subsystems $S_i(A_i, B_i, C_i)$ and those depending only on F structure.

It should now be pointed that loss of outputs results in a pencil $T_i(s)$, whose Kronecker structure is no longer expressed as a direct sum of the Kronecker structure of the subsystems. The role of the matrix F expressing the interconnections is now crucial in determining the composite system properties.

5.10 The Zero Pencil of Composite and Aggregate Systems with Full Inputs and Outputs

Consider the composite system described by equation (5.36) and let $\overline{N}, \overline{M}$ be left and right annihilators of $\overline{B}, \overline{C}$ respectively, where $\overline{N}, \overline{M}$ are as in (5.37) and (5.50).

The zero pencil is $s\overline{N}\overline{M}-\overline{N}\tilde{A}\overline{M}$ and may be expressed as shown below. We first note that the pencil

$$s\overline{N}\overline{M}-\overline{N}\overline{A}\overline{M}=\begin{bmatrix} sN_1-N_1A_1 & & & & 0 \\ & \ddots & & & \\ & & sN_i-N_iA_i & & \\ & & & \ddots & \\ 0 & & & & sN_k-N_kA_k \end{bmatrix}\begin{bmatrix} M_1 & & & & 0 \\ & \ddots & & & \\ & & M_i & & \\ & & & \ddots & \\ 0 & & & & M_k \end{bmatrix}$$

$$=\begin{bmatrix} sN_1M_1-N_1A_1M_1 & & & & 0 \\ & \ddots & & & \\ & & sN_iM_i-N_iA_iM_i & & \\ & & & \ddots & \\ 0 & & & & sN_kM_k-N_kA_kM_k \end{bmatrix} \quad (5.62)$$

is the zero pencil of the aggregate system without the interconnection. Given that $\overline{N}\overline{B}\Delta=0$, $\Delta\overline{C}\overline{M}=0$, then the zero pencil of the composite system is

$$s\overline{N}\overline{M}-\overline{N}\tilde{A}\overline{M}=s\overline{N}\overline{M}-\overline{N}\left(\overline{A}+\overline{B}\Delta F\overline{C}\right)\overline{M}=$$

$$=\begin{bmatrix} sN_1-N_1A_1 & & & & 0 \\ & \ddots & & & \\ & & sN_i-N_iA_i & & \\ & & & \ddots & \\ 0 & & & & sN_k-N_kA_k \end{bmatrix}\begin{bmatrix} M_1 & & & & 0 \\ & \ddots & & & \\ & & M_i & & \\ & & & \ddots & \\ 0 & & & & M_k \end{bmatrix}$$

$$=\begin{bmatrix} sN_1M_1-N_1A_1M_1 & & & & 0 \\ & \ddots & & & \\ & & sN_iM_i-N_iA_iM_i & & \\ & & & \ddots & \\ 0 & & & & sN_kM_k-N_kA_kM_k \end{bmatrix} \quad (5.63)$$

Theorem (5.5): The zero pencil of the composite system is the direct sum of the zero pencils of the subsystems, and thus the zero properties are expressed as the aggregate of the corresponding properties defined on the subsystems.

□

The study of the zero pencil expressed in the form of equation (5.63) provides all the necessary insight for the analysis of the zero structure of the composite system. We may now investigate the zero properties of the composite system under total loss of subsystem inputs and/or outputs.

5.11 The Zero Pencil under Total Loss of Subsystem Inputs

Consider the composite system and assume that all the i-th subsystem external inputs are not used (i.e. this occurs when interconnection elements are dynamic and no assignable input is available for them). In this case, the corresponding subsystem has as inputs those coming from the interconnection only. The zero pencil of the composite system is described by:

$$s\tilde{N}_i\overline{M} - \tilde{N}_i\overline{A}\overline{M} = \begin{bmatrix} sN_1 - N_1A_1 & & & & 0 \\ \vdots & \ddots & & & \vdots \\ 0 & & & & 0 \\ -B_iF_{i1}C_1 & \cdots & sI - A_i - B_iF_{ii}C_i & \cdots & -B_iF_{ik}C_k \\ 0 & & & & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & & & & sN_1 - N_1A_1 \end{bmatrix} \begin{bmatrix} M_1 & & & & 0 \\ & \ddots & & & \\ & & M_i & & \\ & & & \ddots & \\ 0 & & & & M_k \end{bmatrix}$$

$$= \begin{bmatrix} sN_1M_1 - N_1A_1M_1 & & & & 0 \\ & \ddots & & & \\ & & sM_i - A_iM_i & & \\ & & & \ddots & \\ 0 & & & & sN_kM_k - N_kA_kM_k \end{bmatrix} \quad (5.64)$$

where $\tilde{N}_i$ has been defined by equation (5.43).

<u>**Remark (5.2):**</u> The above pencil may be generalised for every i and extended to any combination of indices i, j etc.

□

The above matrix may be used to study the location of the zeros when total loss of subsystem inputs occurs. It should now be pointed out that loss of external input results in the zero pencil, whose Kronecker structure is again expressed as a direct sum of the Kronecker structures originating from the subsystems. The role of the matrix F in expressing the interconnection is not crucial in determining the zero properties of the composite system. In connection with the above statement, we have the following theorem.

Theorem (5.6): The observability structure of the subsystem where we lost its inputs, as defined by the corresponding state output pencil, enters into the zero structure of the composite system, where by observability structure we mean the structure of output decoupling zeros and *r.m.i.* of the state-output pencil.

□

Now we investigate the zero properties of the composite system under the total loss of subsystem outputs.

5.12 The Zero Pencil under Total Loss of Subsystem Outputs

Consider the composite system and assume that all the i-th subsystem outputs are not measured. The zero pencil of the composite system with total loss of subsystem outputs is described by:

$$s\tilde{N}_i\tilde{M}_i - N_i\bar{A}\tilde{M}_i = \begin{bmatrix} N_1 & & & & 0 \\ & \ddots & & & \\ & & N_i & & \\ & & & \ddots & \\ 0 & & & & N_k \end{bmatrix} \begin{bmatrix} sM_1 - M_1A_1 & & B_1F_{1i}C_i & & 0 \\ & \ddots & \vdots & & \\ & & & & \\ & & \vdots & \ddots & \\ 0 & & B_kF_{ki}C_i & & sM_k - A_KM_k \end{bmatrix}$$

$$= \begin{bmatrix} sN_1M_1 - N_1A_1M_1 & & & & 0 \\ & \ddots & & & \\ & & sN_i - N_iA_i & & \\ & & & \ddots & \\ 0 & & & & sN_kM_k - N_kA_kM_k \end{bmatrix} \quad (5.65)$$

Remark (5.3): The above pencil may be generalised for every i and extended to any combination of indices i, j etc.

□

The above matrix may be used to study the zero system structure when total loss of subsystem outputs occurs. In previous sections the role of matrix F in expressing the interconnection was highlighted. A similar argument can be used when total loss of subsystem outputs occurs, to characterise the resulting zero structure.

Theorem (5.7): The controllability structure of the subsystem where we lost its outputs, enters into the zero structure of the composite system, where by controllability structure we mean the structure defined by the input decoupling zeros and $c.m.i.$ of the input-state pencil.

□

We illustrate the above statements by means of the following examples:

Example (1): COMPOSITE STRUCTURE (I)

The following figure shows the block diagram of a composite system with two subsystems.

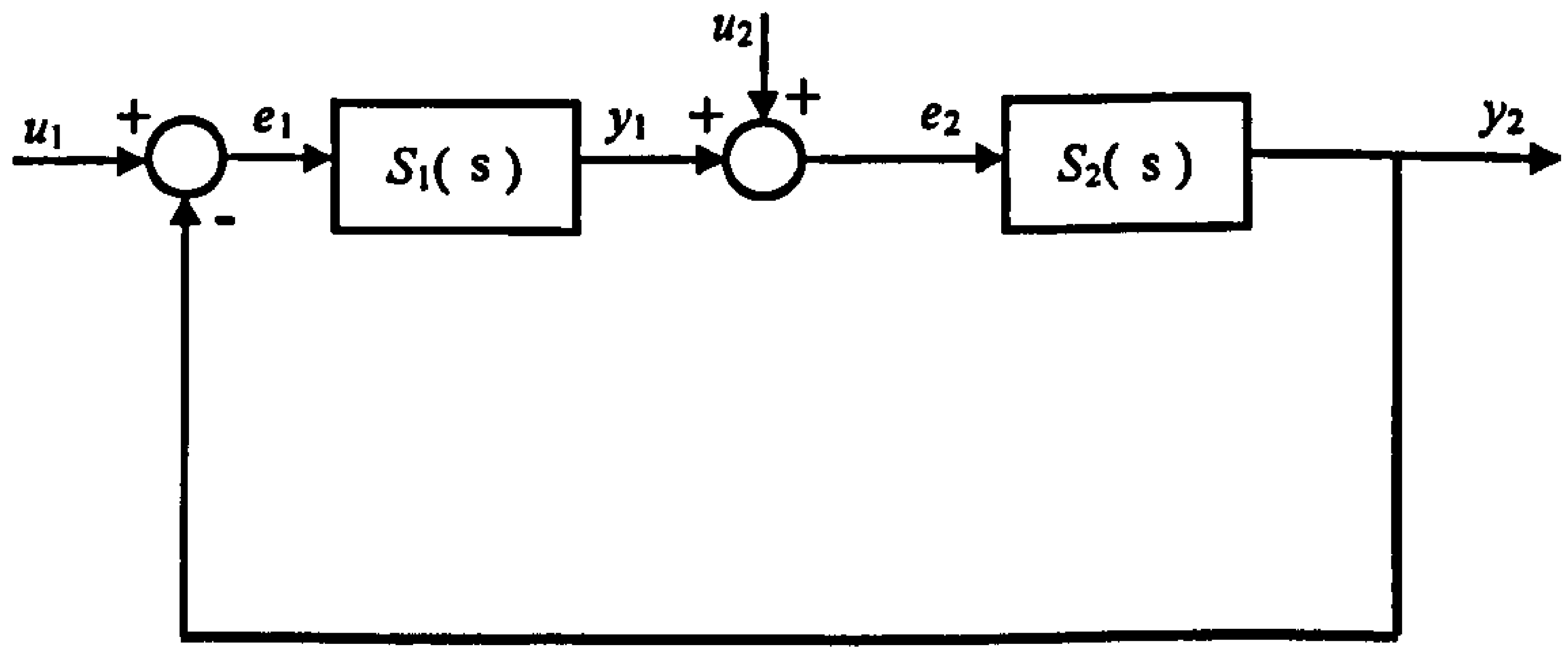


Figure (5.8): Composite Structure (I)

The system equations derived from the above figure are:

$$\begin{cases} \underline{e}_1 = \underline{u}_1 - \underline{y}_2 \\ \underline{e}_2 = \underline{u}_2 - \underline{y}_1 \end{cases} \Rightarrow \begin{bmatrix} \underline{e}_1 \\ \underline{e}_2 \end{bmatrix} = \begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \end{bmatrix} \underbrace{\begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix}}_{=F} \begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \end{bmatrix} \tag{5.66}$$

$$\begin{cases} \begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}}_{=\overline{A}} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}}_{\overline{B}} \begin{bmatrix} \underline{e}_1 \\ \underline{e}_2 \end{bmatrix} \\ \begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}}_{=\overline{C}} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} \end{cases} \tag{5.67}$$

Thus, the composite state matrix equations are

$$\begin{aligned} \overline{A} + \overline{B}F\overline{C} &= \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \\ &= \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} + \begin{bmatrix} 0 & -B_1 \\ B_2 & 0 \end{bmatrix} \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \\ &= \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} + \begin{bmatrix} 0 & -B_1C_2 \\ B_2C_1 & 0 \end{bmatrix} \end{aligned}$$

or

$$\overline{A} + \overline{B}F\overline{C} = \begin{bmatrix} A_1 & -B_1C_2 \\ B_2C_1 & A_2 \end{bmatrix} \tag{5.68}$$

Therefore, the composite system description can be written as:

$$\begin{cases} \begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & -B_1C_2 \\ B_2C_1 & A_2 \end{bmatrix}}_{=\overline{A}} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}}_{\overline{B}} \begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \end{bmatrix} \\ \begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}}_{=\overline{C}} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} \end{cases} \tag{5.69}$$

Consider next the restriction pencils of the composite system. In this example, three input-output cases are considered. First, the full input, full output case is given.

We first note that given the system, we associate the input state restriction pencil, state output restriction pencil and zero pencil as shown below:

$$\begin{Bmatrix} S_1(A_1,B_1,C_1):\rightarrow sN_1 - N_1A_1, sM_1 - A_1M_1, sN_1M_1 - N_1A_1M_1 \\ S_2(A_2,B_2,C_2):\rightarrow sN_2 - N_2A_2, sM_2 - A_2M_2, sN_2M_2 - N_2A_2M_2 \end{Bmatrix} \quad (5.70)$$

Consider the composite system described above and let N and M be left and right annihilators of B and C such that

$$NB = 0 \quad where\ N = \begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix}, \quad CM = 0 \quad where\ M = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \quad (5.71)$$

To study the controllability properties of the system with full input, full output, the input-state restriction pencil is derived as follows:

$$\begin{aligned} sN - NA &= \begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix} \begin{bmatrix} sI - A_1 & B_1C_2 \\ -B_2C_1 & sI - A_2 \end{bmatrix} \\ &= \begin{bmatrix} sN_1 - N_1A_1 & 0 \\ 0 & sN_2 - N_2A_2 \end{bmatrix} \end{aligned} \quad (5.72)$$

For an insight into the observability properties, the state output restriction pencil is given by:

$$\begin{aligned} sN - AM &= \begin{bmatrix} sI - A_1 & B_1C_2 \\ -B_2C_1 & sI - A_2 \end{bmatrix} \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \\ &= \begin{bmatrix} sM_1 - A_1M_1 & 0 \\ 0 & sM_2 - A_2M_2 \end{bmatrix} \end{aligned} \quad (5.73)$$

To investigate the zero properties of the system, from either of the above pencils the zero pencil can be derived:

$$sNM - NAM = \begin{bmatrix} sN_1M_1 - N_1A_1M_1 & 0 \\ 0 & sN_2M_2 - N_2A_2M_2 \end{bmatrix} \tag{5.74}$$

we have therefore verify the results previously stated and summarised as:

Remark (5.4): The controllability, observability, zero structure properties of the composite system (I) under full input, output structure are simply given as aggregates (direct sum) of corresponding properties of two subsystems.

□

We consider next the case where the total loss of subsystem input structure has occurred. Assume that $\underline{u}_2 = 0$ without loss of generality. This leads to the following reduced composite system description:

$$\begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \end{bmatrix} = \begin{bmatrix} A_1 & -B_1C_2 \\ B_2C_1 & A_2 \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} \underline{u}_1 \\ 0 \end{bmatrix} \tag{5.75}$$

or

$$\dot{\underline{x}} = \begin{bmatrix} A_1 & -B_1C_2 \\ B_2C_1 & A_2 \end{bmatrix} \underline{x} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \underline{u}_1, \quad \underline{y} = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \underline{x} \tag{5.76}$$

Let

$$\tilde{N}_1 = \begin{bmatrix} N_1 & 0 \\ 0 & I \end{bmatrix} \tag{5.77}$$

To study the controllability properties of the system when there is a loss of input, the input state restriction pencil is derived as follows:

$$\begin{aligned} s\tilde{N}_1 - \tilde{N}_1 A &= \begin{bmatrix} N_1 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} sI - A_1 & B_1C_2 \\ -B_2C_1 & sI - A_2 \end{bmatrix} \\ &= \begin{bmatrix} sN_1 - N_1A_1 & 0 \\ -B_2C_1 & sI - A_2 \end{bmatrix} \end{aligned} \tag{5.78}$$

Equation (5.78) shows that the loss of inputs of one of the subsystems may result in the presence of finite elementary divisors in the input-state restriction pencil. Therefore, the system may become uncontrollable; however the later property depends on $-B_2C_1$ matrix.

Remark (5.5): Total loss of input channels for any of the two subsystems may result in structural uncontrollability for the resulting system.

□

To investigate the zero properties of the system when there is a loss of input, we derive the zero pencil as follows:

$$\begin{aligned} s\tilde{N}_1M-\tilde{N}_1AM &= \begin{bmatrix} sN_1-N_1A_1 & 0 \\ -B_2C_1 & sI-A_2 \end{bmatrix}\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \\ &= \begin{bmatrix} sN_1M_1-N_1A_1M_1 & 0 \\ 0 & sM_2-A_2M_2 \end{bmatrix} \end{aligned} \tag{5.79}$$

which verifies the general result previously derived.

Last, let us assume that only the first output is measured. This leads to the total loss of subsystem output, and the reduced composite system description is given by:

$$\begin{cases} \dot{\underline{x}} = \begin{bmatrix} A_1 & -B_1C2 \\ B_2C_1 & A_2 \end{bmatrix}\underline{x} + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}\underline{u} \\ \underline{y} = \underbrace{[C_1, 0]}_{=\tilde{C}_1}\underline{x} \end{cases} \tag{5.80}$$

Let

$$\tilde{M}_1 = \begin{bmatrix} M_1 & 0 \\ 0 & I \end{bmatrix} \tag{5.81}$$

To investigate the zero structure properties, when there is a loss of output, we may define the state-output restriction pencil as follows:

$$
\begin{aligned}
s\widetilde{M}_1 - A\widetilde{M}_1 &= \begin{bmatrix} sI - A_1 & B_1C_2 \\ -B_2C_1 & sI - A_2 \end{bmatrix} \begin{bmatrix} M_1 & 0 \\ 0 & I \end{bmatrix} \\
&= \begin{bmatrix} sM_1 - A_1M_1 & 0 \\ 0 & sI - A_2 \end{bmatrix}
\end{aligned} \tag{5.82}
$$

Equation (5.82) shows that the loss of output of one of the subsystems may result in the presence of finite elementary divisors in the state-output restriction pencil. Therefore, the system may become unobservable; however, the later property depends on the B_1C_2 matrix.

Remark (5.6): Loss of outputs of one of the subsystems may result in system unobservability.

To obtain the zero properties of the system when there is a loss of output, we defined the zero pencil as follows:

$$
\begin{aligned}
sN\widetilde{M}_1 - NA\widetilde{M}_1 &= \begin{bmatrix} N_1 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} sM_1 - A_1M_1 & B_1C_2 \\ 0 & sI - A_2 \end{bmatrix} \\
&= \begin{bmatrix} sN_1M_1 - N_1A_1M_1 & 0 \\ 0 & sN_2 - N_2A_2 \end{bmatrix}
\end{aligned} \tag{5.83}
$$

The above verifies the previously worked out result.

Example (2): COMPOSITE STRUCTURE (II)

The following figure shows the block diagram of a complete composite system with three subsystems.

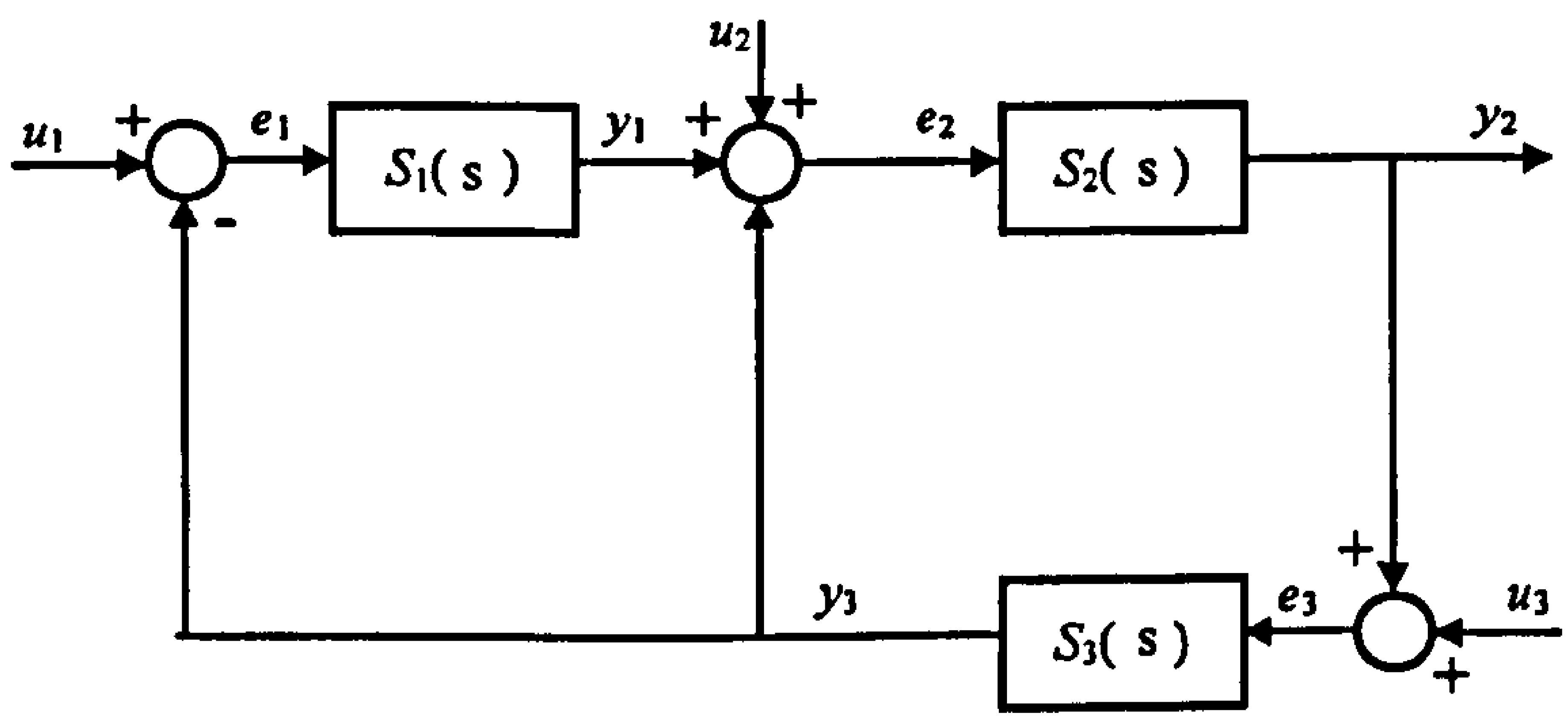


Figure (5.9): Example of composite system (II)

The system equations as derived from the above figure are:

$$\left\{\begin{array}{l}\underline{e}_1=\underline{u}_1-\underline{y}_3\\ \underline{e}_2=\underline{u}_2+\underline{y}_1-\underline{y}_3\\ \underline{e}_1=\underline{u}_3-\underline{y}_2\end{array}\right] \Rightarrow \begin{bmatrix}\underline{e}_1\\ \underline{e}_2\\ \underline{e}_3\end{bmatrix}=\begin{bmatrix}\underline{u}_1\\ \underline{u}_2\\ \underline{u}_3\end{bmatrix}+\begin{bmatrix}0&0&-I\\ I&0&I\\ 0&I&0\end{bmatrix}\begin{bmatrix}\underline{y}_1\\ \underline{y}_2\\ \underline{y}_3\end{bmatrix} \tag{5.84}$$

This leads to the aggregate system equations

$$\left\{\begin{array}{l}\begin{bmatrix}\dot{\underline{x}}_1\\ \dot{\underline{x}}_2\\ \dot{\underline{x}}_3\end{bmatrix}=\underbrace{\begin{bmatrix}A_1&0&0\\ 0&A_2&0\\ 0&0&A_3\end{bmatrix}}_{=\bar{A}}\begin{bmatrix}\underline{x}_1\\ \underline{x}_2\\ \underline{x}_3\end{bmatrix}+\underbrace{\begin{bmatrix}B_1&0&0\\ 0&B_2&0\\ 0&0&B_3\end{bmatrix}}_{=\bar{B}}\begin{bmatrix}\underline{e}_1\\ \underline{e}_2\\ \underline{e}_3\end{bmatrix}\\ \begin{bmatrix}\underline{y}_1\\ \underline{y}_2\\ \underline{y}_3\end{bmatrix}=\underbrace{\begin{bmatrix}C_1&0&0\\ 0&C_2&0\\ 0&0&C_3\end{bmatrix}}_{=\bar{C}}\begin{bmatrix}\underline{x}_1\\ \underline{x}_2\\ \underline{x}_3\end{bmatrix}\end{array}\right. \tag{5.85}$$

Thus, the composite state matrix equations are:

$$\bar{A}+\bar{B}F\bar{C}=\begin{bmatrix}A_1&0&0\\ 0&A_2&0\\ 0&0&A_3\end{bmatrix}+\begin{bmatrix}B_1&0&0\\ 0&B_2&0\\ 0&0&B_3\end{bmatrix}\begin{bmatrix}0&0&-I\\ I&0&-I\\ 0&I&0\end{bmatrix}\begin{bmatrix}C_1&0&0\\ 0&C_2&0\\ 0&0&C_3\end{bmatrix}$$

or

$$\bar{A}+\bar{B}F\bar{C}=\begin{bmatrix} A_1 & 0 & -B_1C_3 \\ B_2C_1 & A_2 & -B_2C_3 \\ 0 & B_3C_2 & A_3 \end{bmatrix} \tag{5.86}$$

Therefore the composite system description can be written as:

$$\begin{cases} \begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \\ \dot{\underline{x}}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & 0 & -B_1C_3 \\ B_2C_1 & A_2 & -B_2C_3 \\ 0 & B_3C_2 & A_3 \end{bmatrix}}_{=\bar{A}} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \end{bmatrix} + \underbrace{\begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix}}_{=\bar{B}} \begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \\ \underline{u}_3 \end{bmatrix} \\ \begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \underline{y}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{bmatrix}}_{=\bar{C}} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \end{bmatrix} \end{cases} \tag{5.87}$$

Consider next the restriction pencil of the composite system. In this example, three different cases are considered. First, the full input-full output case is given. We first note that given the system, we associate the input-state restriction pencil, state-output restriction pencil, zero pencil as shown below for each subsystem:

$$\begin{aligned} S_1(A_1,B_1,C_1): &\quad \rightarrow \quad sN_1-N_1A_1, \quad sM_1-A_1M_1, \quad sN_1M_1-N_1A_1M_1 \\ S_2(A_2,B_2,C_2): &\quad \rightarrow \quad sN_2-N_2A_2, \quad sM_2-A_2M_2, \quad sN_2M_2-N_2A_2M_2 \\ S_3(A_3,B_3,C_3): &\quad \rightarrow \quad sN_3-N_3A_3, \quad sM_3-A_3M_3, \quad sN_3M_3-N_3A_3M_3 \end{aligned}$$

For the composite system described by the above figure (5.9), let N and M be left and right annihilators of B and C respectively such that

$$NB=0, \quad where\ N=\begin{bmatrix} N_1 & 0 & 0 \\ 0 & N_2 & 0 \\ 0 & 0 & N_3 \end{bmatrix}, \quad CM=0 \quad where\ M=\begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix}$$

To study the controllability properties of the system with full input, full output, the input-state restriction pencil is derived as follows:

$$
\begin{aligned}
sN - NA &= \begin{bmatrix} N_1 & 0 & 0 \\ 0 & N_2 & 0 \\ 0 & 0 & N_3 \end{bmatrix} \begin{bmatrix} sI - A_1 & 0 & -B_1C_3 \\ B_2C_1 & sI - A_2 & -B_2C_3 \\ 0 & B_3C_2 & sI - A_3 \end{bmatrix} \\
&= \begin{bmatrix} sN_1 - N_1A_1 & 0 & 0 \\ 0 & sN_2 - N_2A_2 & 0 \\ 0 & 0 & sN_3 - N_3A_3 \end{bmatrix}
\end{aligned} \tag{5.88}
$$

To investigate the observability properties of the system, the state-output restriction pencil is given by:

$$
\begin{aligned}
sN - AM &= \begin{bmatrix} sI - A_1 & 0 & -B_1C_3 \\ B_2C_1 & sI - A_2 & -B_2C_3 \\ 0 & B_3C_2 & sI - A_3 \end{bmatrix} \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix} \\
&= \begin{bmatrix} sM_1 - A_1M_1 & 0 & 0 \\ 0 & sM_2 - A_2M_2 & 0 \\ 0 & 0 & sM_3 - A_3M_3 \end{bmatrix}
\end{aligned} \tag{5.89}
$$

To obtain the zero properties of the system, with full input, full output, the zero pencil is defined by:

$$
sNM - NAM = \begin{bmatrix} sN_1M_1 - N_1A_1M_1 & 0 & 0 \\ 0 & sN_2M_2 - N_2A_2M_2 & 0 \\ 0 & 0 & sN_3M_3 - N_3A_3M_3 \end{bmatrix} \tag{5.90}
$$

The above verifies the previously given general results. Consider next the case where total loss of subsystem input structure has occurred. Assume that $\underline{u}_1 = 0$ (without loss of generality). This leads to the following reduced composite system description:

$$
\begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \\ \dot{\underline{x}}_3 \end{bmatrix} = \begin{bmatrix} A_1 & 0 & -B_1C_3 \\ B_2C_1 & A_2 & -B_2C_3 \\ 0 & B_3C_2 & A_3 \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \end{bmatrix} + \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} \begin{bmatrix} 0 \\ \underline{u}_2 \\ \underline{u}_3 \end{bmatrix} \tag{5.91}
$$

or

$$\dot{\underline{x}} = \begin{bmatrix} A_1 & 0 & -B_1C_3 \\ B_2C_1 & A_2 & -B_2C_3 \\ 0 & B_3C_2 & A_3 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 & 0 \\ B_2 & 0 \\ 0 & B_3 \end{bmatrix} \begin{bmatrix} \underline{u_2} \\ \underline{u_3} \end{bmatrix} \tag{5.92}$$

Then,

$$\tilde{N}_1 = \begin{bmatrix} I & 0 & 0 \\ 0 & N_2 & 0 \\ 0 & 0 & N_3 \end{bmatrix} \tag{5.93}$$

To investigate the controllability properties when there is a total loss of subsystem input, we may define the input-state restriction pencil as follows:

$$\begin{aligned} s\tilde{N}_1 - \tilde{N}_1 A &= \begin{bmatrix} I & 0 & 0 \\ 0 & N_2 & 0 \\ 0 & 0 & N_3 \end{bmatrix} \begin{bmatrix} sI - A_1 & 0 & -B_1C_3 \\ B_2C_1 & sI - A_2 & -B_2C_3 \\ 0 & B_3C_2 & sI - A_3 \end{bmatrix} \\ &= \begin{bmatrix} sI - A_1 & 0 & -B_1C_3 \\ 0 & sN_2 - N_2A_2 & 0 \\ 0 & 0 & sN_3 - N_3A_3 \end{bmatrix} \end{aligned} \tag{5.94}$$

Equation (5.94) shows that the loss of inputs of one of the subsystems may result in the presence of *finite elementary divisors* in the input-state restriction pencil. Therefore, the system may become uncontrollable; however the later property depends on $-B_1C_3$ matrix.

To demonstrate that now the properties of controllability depend on the interconnection graph, we observe the following:

(a) The pencil $R(s) = s\tilde{N}_1 - \tilde{N}_1 A$ in (5.94) is strict equivalent by permutation of blocks to

$$R'(s) = \left[\begin{array}{c|cc} sN_2 - N_2A_2 & 0 & \\ \hline 0 & sI - A_1 & -B_1C_3 \\ & 0 & sN_3 - N_3A_3 \end{array}\right] = \left[\begin{array}{c|c} sN_2 - N_2A_2 & 0 \\ \hline 0 & \overline{R}(s) \end{array}\right]$$

The above suggests that part of the controllability structure, that is, that connected with 2nd subsystem, is part of the resulting controllability structure of the resulting system. The rest of the properties depend on the structure of the reduced pencil $\overline{R}(s)$.

(b) If (Q,T) is a pair that reduces $sN_3 - N_3A_3$ to its standard form $Q(sN_3 - N_3A_3)T = s[I,0]-[P_1,P_2]$ and $-B_1C_3T = -[E_1,E_2]$ (compatible partitioning with that of $Q(sN_3 - N_3A_3)T$), then $\overline{R}(s)$ is strict equivalent to

$$\overline{R}(s)=\begin{bmatrix} sI-A_1 & -E_1 & -E_2 \\ 0 & sI-P_1 & -P_2 \end{bmatrix}$$

The controllability properties of the remaining subsystem are those of the pair

$$A'=\begin{bmatrix} A_1 & E_1 \\ 0 & P_1 \end{bmatrix}, \qquad B'=\begin{bmatrix} E_2 \\ P_2 \end{bmatrix}$$

and thus clearly affected by the graph of the system. We summarise as:

Remark (5.7): Total loss of input channels for any of the three subsystems may result in structural uncontrollability. This however, is a property entirely dependent on the system graph and the location of the deviation from completeness.

□

To investigate the zero properties of the system, when there is a loss of input, we derive the zero pencil as follows:

$$\begin{aligned} s\widetilde{N}_1M_1 - \widetilde{N}_1AM_1 &= \begin{bmatrix} sI-A_1 & 0 & -B_1C_3 \\ 0 & sN_2-N_2A_2 & 0 \\ 0 & 0 & sN_3-N_3A_3 \end{bmatrix}\begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix} \\ &= \begin{bmatrix} sM_1-A_1M_1 & 0 & 0 \\ 0 & sN_2M_2-N_2A_2M_2 & 0 \\ 0 & 0 & sN_3M_3-N_3A_3M_3 \end{bmatrix} \end{aligned} \quad (5.95)$$

which once more verifies the previously stated general result.

Last, let us assume that the first and second outputs are measured (again without loss of generality). This leads to the total loss of subsystem output, and the reduced composite system description is given by:

$$\begin{cases} \dot{\underline{x}} = \begin{bmatrix} A_1 & 0 & -B_1C_3 \\ B_2C_1 & A_2 & -B_2C_3 \\ 0 & B_3C_2 & A_3 \end{bmatrix} \underline{x} + \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} \underline{u} \\ \underline{y} = \underbrace{\begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \end{bmatrix}}_{=\tilde{C}} \underline{x} \end{cases} \tag{5.96}$$

and a right annihilator of $\tilde{C}_1$ is given by

$$\tilde{M}_1 = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & I \end{bmatrix} \tag{5.97}$$

To investigate the observability properties, when there is a loss of output, we may define the state-output restriction pencil as follows:

$$\begin{aligned} s\tilde{M}_1 - A\tilde{M}_1 &= \begin{bmatrix} sI - A_1 & 0 & B_1C_3 \\ -B_2C_1 & sI - A_2 & B_2C_3 \\ 0 & -B_3C_2 & sI - A_3 \end{bmatrix} \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & I \end{bmatrix} \\ &= \begin{bmatrix} sM_1 - A_1M_1 & 0 & B_1C_3 \\ 0 & sM_2 - A_2M_2 & B_2C_3 \\ 0 & 0 & sI - A_3 \end{bmatrix} \end{aligned} \tag{5.98}$$

Equation (5.98) shows that the loss of output of one of the subsystems may result in the presence of finite elementary divisors in the state-output restriction pencil. Therefore, the system may become unobservable; however the later property depends on B_1C_3, B_2C_3 matrices.

Remark (5.8): Loss of outputs of one of the subsystems may result in system unobservability.

□

To obtain the zero properties of the system when there is a loss of output, we define the zero pencil as

$$
\begin{aligned}
s\tilde{M}_1 - A\tilde{M}_1 &= \begin{bmatrix} sI - A_1 & 0 & B_1C_3 \\ -B_2C_1 & sI - A_2 & B_2C_3 \\ 0 & -B_3C_2 & sI - A_3 \end{bmatrix} \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & I \end{bmatrix} \\
&= \begin{bmatrix} sM_1 - A_1M_1 & 0 & B_1C_3 \\ 0 & sM_2 - A_2M_2 & B_2C_3 \\ 0 & 0 & sI - A_3 \end{bmatrix}
\end{aligned} \tag{5.99}
$$

The above verifies the previously derived result.

Let us now consider the following example which extends the composite system by a further subsystem.

Example (3): COMPOSITE STRUCTURE (III)

The following figure shows the block diagram of a complete composite system with four subsystems:

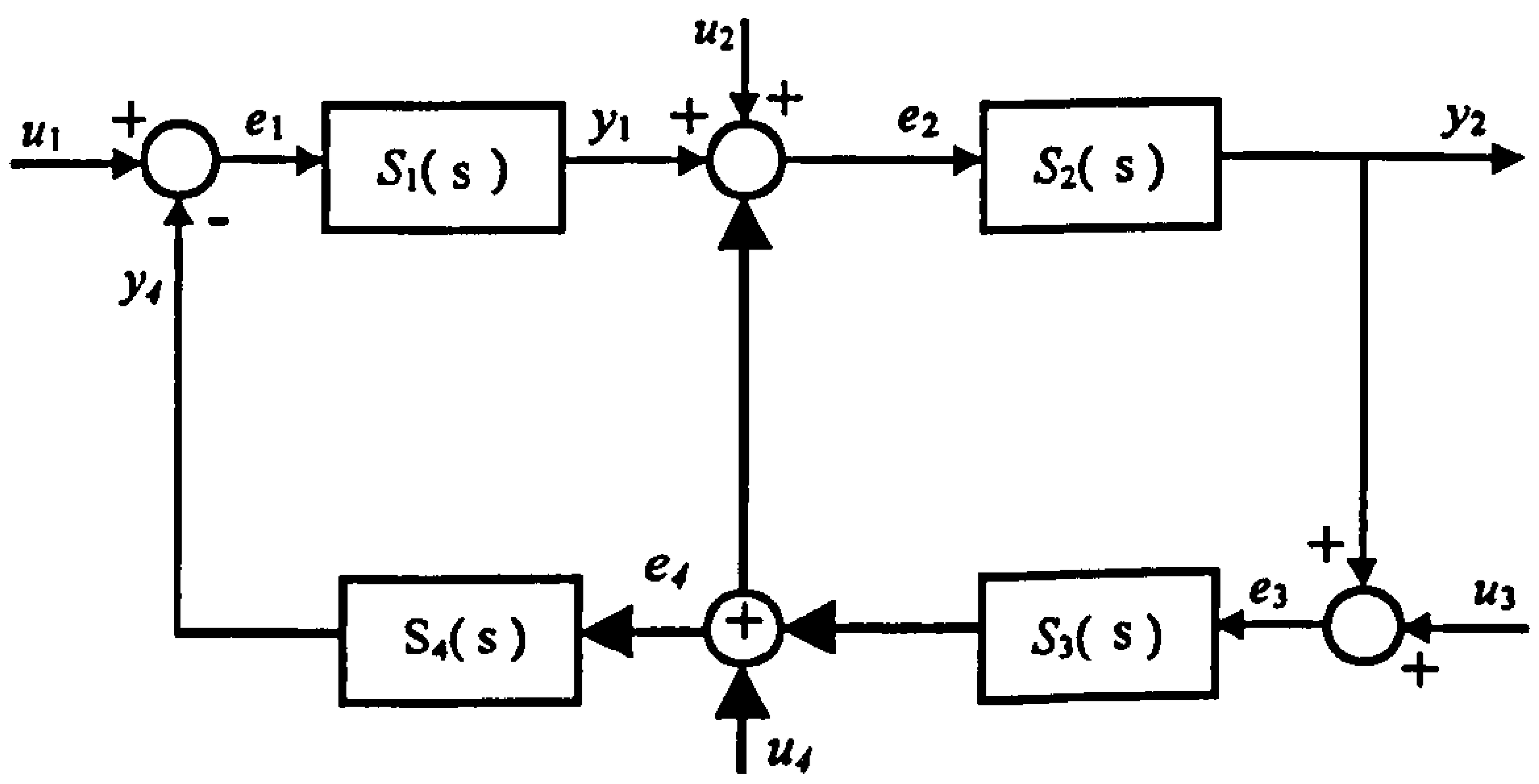


Figure (5.10): Example of composite system (III)

The system equations, as derived from the above figure, are:

$$\begin{cases} \underline{e}_1 = \underline{u}_1 - \underline{y}_4 \\ \underline{e}_2 = \underline{u}_2 + \underline{y}_1 - \underline{y}_3 \\ \underline{e}_3 = \underline{u}_3 + \underline{y}_2 \\ \underline{e}_4 = \underline{u}_4 + \underline{y}_3 \end{cases} \Rightarrow \begin{bmatrix} \underline{e}_1 \\ \underline{e}_2 \\ \underline{e}_3 \\ \underline{e}_4 \end{bmatrix} = \begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \\ \underline{u}_3 \\ \underline{u}_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & -I \\ I & 0 & -I & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \underline{y}_3 \\ \underline{y}_4 \end{bmatrix} \tag{5.100}$$

This leads to the following aggregate system equation:

$$\begin{cases} \begin{bmatrix} \underline{\dot{x}}_1 \\ \underline{\dot{x}}_2 \\ \underline{\dot{x}}_3 \\ \underline{\dot{x}}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & 0 & 0 & 0 \\ 0 & A_2 & 0 & 0 \\ 0 & 0 & A_3 & 0 \\ 0 & 0 & 0 & A_4 \end{bmatrix}}_{=\overline{A}} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \\ \underline{x}_4 \end{bmatrix} + \underbrace{\begin{bmatrix} B_1 & 0 & 0 & 0 \\ 0 & B_2 & 0 & 0 \\ 0 & 0 & B_3 & 0 \\ 0 & 0 & 0 & B_4 \end{bmatrix}}_{=\overline{B}} \begin{bmatrix} \underline{e}_1 \\ \underline{e}_2 \\ \underline{e}_3 \\ \underline{e}_4 \end{bmatrix} \\ \begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \underline{y}_3 \\ \underline{y}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} C_1 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 \\ 0 & 0 & C_3 & 0 \\ 0 & 0 & 0 & C_4 \end{bmatrix}}_{=\overline{C}} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \\ \underline{x}_4 \end{bmatrix} \end{cases} \tag{5.101}$$

Thus, the composite state matrix equation is:

$$\overline{A} + \overline{B}F\overline{C} = \begin{bmatrix} A_1 & 0 & 0 & 0 \\ 0 & A_2 & 0 & 0 \\ 0 & 0 & A_3 & 0 \\ 0 & 0 & 0 & A_4 \end{bmatrix} + \begin{bmatrix} B_1 & 0 & 0 & 0 \\ 0 & B_2 & 0 & 0 \\ 0 & 0 & B_3 & 0 \\ 0 & 0 & 0 & B_4 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & -I \\ I & 0 & -I & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} C_1 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 \\ 0 & 0 & C_3 & 0 \\ 0 & 0 & 0 & C_4 \end{bmatrix}$$

or

$$\overline{A} + \overline{B}F\overline{C} = \begin{bmatrix} A_1 & 0 & 0 & -B_1C_4 \\ B_2C_1 & A_2 & B_2C_3 & 0 \\ 0 & B_3C_2 & A_3 & 0 \\ 0 & 0 & B_4C_3 & A_4 \end{bmatrix} \tag{5.102}$$

Therefore, the composite system description can be written as:

$$\begin{cases}\begin{bmatrix}\dot{\underline{x}}_1\\ \dot{\underline{x}}_2\\ \dot{\underline{x}}_3\\ \dot{\underline{x}}_4\end{bmatrix}=\underbrace{\begin{bmatrix}A_1 & 0 & 0 & -B_1C_4\\ B_2C_1 & A_2 & B_2C_3 & 0\\ 0 & B_3C_2 & A_3 & 0\\ 0 & 0 & B_4C_3 & A_4\end{bmatrix}}_{=A}\begin{bmatrix}\underline{x}_1\\ \underline{x}_2\\ \underline{x}_3\\ \underline{x}_4\end{bmatrix}+\underbrace{\begin{bmatrix}B_1 & 0 & 0 & 0\\ 0 & B_2 & 0 & 0\\ 0 & 0 & B_3 & 0\\ 0 & 0 & 0 & B_4\end{bmatrix}}_{=B}\begin{bmatrix}\underline{u}_1\\ \underline{u}_2\\ \underline{u}_3\\ \underline{u}_4\end{bmatrix}\\ \begin{bmatrix}\underline{y}_1\\ \underline{y}_2\\ \underline{y}_3\\ \underline{y}_4\end{bmatrix}=\underbrace{\begin{bmatrix}C_1 & 0 & 0 & 0\\ 0 & C_2 & 0 & 0\\ 0 & 0 & C_3 & 0\\ 0 & 0 & 0 & C_4\end{bmatrix}}_{=C}\begin{bmatrix}\underline{x}_1\\ \underline{x}_2\\ \underline{x}_3\\ \underline{x}_4\end{bmatrix}\end{cases} \quad (5.103)$$

Consider next the restriction pencils of the composite system. In this example, two input-output cases are considered. We first note that given the system we associate input-state restriction pencil, state-output restriction pencil and zero pencil as shown below:

$$S_i(A_i,B_i,C_i): \quad \rightarrow \quad sN_i-N_iA_i, \quad sM_i-A_iM_i, \quad sN_iM_i-N_iA_iM_i, \quad i=1,2,3,4$$

The total loss of subsystem input structure is occurred now. Assume that $u_1=0$, without loss of generality. This leads to the following reduced composite system description:

$$\begin{bmatrix}\dot{\underline{x}}_1\\ \dot{\underline{x}}_2\\ \dot{\underline{x}}_3\\ \dot{\underline{x}}_4\end{bmatrix}=\begin{bmatrix}A_1 & 0 & 0 & -B_1C_4\\ B_2C_1 & A_2 & B_2C_3 & 0\\ 0 & B_3C_2 & A_3 & 0\\ 0 & 0 & B_4C_3 & A_4\end{bmatrix}\begin{bmatrix}\underline{x}_1\\ \underline{x}_2\\ \underline{x}_3\\ \underline{x}_4\end{bmatrix}+\begin{bmatrix}B_1 & 0 & 0 & 0\\ 0 & B_2 & 0 & 0\\ 0 & 0 & B_3 & 0\\ 0 & 0 & 0 & B_4\end{bmatrix}\begin{bmatrix}0\\ \underline{u}_2\\ \underline{u}_3\\ \underline{u}_4\end{bmatrix} \quad (5.104)$$

or

$$\dot{\underline{x}}=\begin{bmatrix}A_1 & 0 & 0 & -B_1C_4\\ B_2C_1 & A_2 & B_2C_3 & 0\\ 0 & B_3C_2 & A_3 & 0\\ 0 & 0 & B_4C_3 & A_4\end{bmatrix}\underline{x}+\begin{bmatrix}0 & 0 & 0\\ B_2 & 0 & 0\\ 0 & B_3 & 0\\ 0 & 0 & B_4\end{bmatrix}\begin{bmatrix}\underline{u}_2\\ \underline{u}_3\\ \underline{u}_4\end{bmatrix} \quad (5.105)$$

It is clear that a left annihilator is defined by

$$\widetilde{N}_1 = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & N_2 & 0 & 0 \\ 0 & 0 & N_3 & 0 \\ 0 & 0 & 0 & N_4 \end{bmatrix} \tag{5.106}$$

To investigate the controllability properties of the system, when there is a loss of input, we may define the input-state restriction pencil as follows:

$$s\widetilde{N}_1 - \widetilde{N}_1 A = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & N_2 & 0 & 0 \\ 0 & 0 & N_3 & 0 \\ 0 & 0 & 0 & N_4 \end{bmatrix} \begin{bmatrix} sI - A_1 & 0 & 0 & -B_1C_4 \\ B_2C_1 & sI - A_2 & B_2C_3 & 0 \\ 0 & B_3C_2 & sI - A_3 & 0 \\ 0 & 0 & 0 & sI - A_4 \end{bmatrix}$$

$$= \begin{bmatrix} sI - A_1 & 0 & 0 & -B_1C_4 \\ B_2C_1 & sN_2 - N_2A_2 & B_2C_3 & 0 \\ 0 & B_3C_2 & sN_3 - N_3A_3 & 0 \\ 0 & 0 & 0 & sN_4 - N_4A_4 \end{bmatrix} \tag{5.107}$$

The above matrix can be rearranged in order to get

$$\left[\begin{array}{cc|cc} sN_2 - N_2A_2 & 0 & 0 & 0 \\ 0 & sN_3 - N_3A_3 & 0 & 0 \\ \hline 0 & 0 & sI - A_1 & -B_1C_4 \\ 0 & 0 & 0 & sN_4 - N_4A_4 \end{array}\right]$$

This shows the rearranged matrix partitioned into four blocks. It is obvious that the canonical form of $s\widetilde{N}_1 - \widetilde{N}_1 A$ contains an additional block to those corresponding to the column minimal indices. This block has to be further investigated to find out about the existence of uncontrollable modes. This can be achieved through a rank test. Note that a system is uncontrollable, iff there exists finite elementary divisors in

$$\begin{bmatrix} sI - A_1 & -B_1C_4 \\ 0 & sN_4 - N_4A_4 \end{bmatrix}$$

This implies the existence of a nonzero constant vector $\underline{v}'$ and an eigenvalue λ such that

$$\begin{bmatrix} \underline{v}_1' & \underline{v}_2' \end{bmatrix} \begin{bmatrix} \lambda I - A_1 & -B_1C_4 \\ 0 & \lambda N_4 - N_4A_4 \end{bmatrix} = 0 \tag{5.108}$$

or

$$\underline{v}_1'(\lambda I - A_1) = 0 \tag{5.109}$$

$$-\underline{v}_1'B_1C_4 + \underline{v}_2'(\lambda N_4 - N_4A_4) = 0 \tag{5.110}$$

We have to test (5.110) for the left eigenvalues and eigenvectors of A_1.

Consider next the case when there is a total loss of subsystem input and output. Assume that $\underline{u}_1 = 0$ and first output is not measured. To investigate the zero properties of the system, from matrix (5.107), the zero pencil can be derived:

$$s\tilde{N}_1\tilde{M}_1 - \tilde{N}_1A\tilde{M}_1 =$$
$$= \begin{bmatrix} sI - A_1 & 0 & 0 & -B_1C_4 \\ 0 & sN_2 - N_2A_2 & 0 & 0 \\ 0 & 0 & sN_3 - N_3A_3 & 0 \\ 0 & 0 & 0 & sN_4 - N_4A_4 \end{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 \\ 0 & 0 & M_3 & 0 \\ 0 & 0 & 0 & M_4 \end{bmatrix} \tag{5.111}$$
$$= \begin{bmatrix} sI - A_1 & 0 & 0 & 0 \\ 0 & sN_2M_2 - N_2A_2M_2 & 0 & 0 \\ 0 & 0 & sN_3M_3 - N_3A_3M_3 & 0 \\ 0 & 0 & 0 & sN_4M_4 - N_4A_4M_4 \end{bmatrix}$$

<u>**Theorem (5.8):**</u> Internal dynamics, as defined by the eigenvalues of corresponding subsystem, become part of the zero structure of the composite system under the total loss of input and output of corresponding subsystem.

□

There are some cases where two or more inputs are lost, in which case the procedure for solving the problem is more complex. The above three examples demonstrate that when there is partial loss of inputs, or outputs, then the interconnection structure plays a crucial role in defining the controllability, observability properties of the resulted system. The characterisation of the resulting structural properties then depends on the properties of the interconnection graph manifested in the structure of the matrix F.

Although we have considered cases of total loss of inputs, outputs at subsystem level, the approach may be extended to partial losses, i.e. more generic forms of deviations from completeness. Such cases may be treated as cases of squaring down at subsystem inputs, and/or outputs. Such analysis becomes much more complicated.

5.13 Conclusions

The different types of Restriction pencils nave been used for studying the effect of the structure F on the loss of controllability, observability and changes in zero structure, when total loss of subsystem input, output, occurs. It was shown that controllability, observability, zero structure properties of composite system under full input, output structure or under the completeness assumption are simply given as aggregates (direct sum) of corresponding properties of subsystems. It was also shown that total loss of input (output) channels for any of the subsystems may result in structural uncontrollability, unobservability. A number of examples have demonstrated that there is a need for further work in obtaining general criteria for system controllability, observability under partial loss of input, output channels at the subsystem level, which corresponds to forms of input and/or output squaring down. Exploring further the role of system graph in determining the properties of systems deviating from the completeness assumption is an issue that requires further work.

Chapter 6

INTERACTION ANALYSIS AND STRUCTURAL METHODOLOGIES

Solve mystery with contradiction and elimination.

6. Interaction Analysis and Structural Methodologies

6.1 Introduction

An important part of the integrated methodology for control structure selection is the so-called interaction analysis which is made up of a number of diagnostics and structural tests that help to restrict the choice of the best scheme: Here we provide a brief review that underpins the software development following in the successive chapter.

The design of decentralised controllers involves the control structure selection problem as an integral part. It has to be decided, which set of measurements will affect which set of inputs and, thus, the problem of interaction between different control loops has to be analysed. Interaction analysis methods can be divided into three groups:

- Interaction matrices
- Interaction measures
- Control structure and system properties

The first group of methods, referred to as interaction matrices, help the control system designer to find possible control structure candidates. The number of possible control structures rapidly increases as the number of inputs and outputs increases. If a process with p inputs and m outputs is to be controlled using m SISO controllers, then the number of possible control structures is $m!$. It is clear that control structure selection for large multivariable systems is a tedious task, if all possible structures have to be tested. A faster method than the exhaustive search of all possible structures is needed. Interaction matrices are a group of methods that offer a solution to this problem. If we are considering a process with p inputs and m outputs, then an interaction matrix is an $m \times p$ matrix in which each element describes interaction between the corresponding input and output. The control structure should be selected so that the inputs and outputs that strongly interact with each other are connected in the feedback controller. Control structures including weakly interacting input-output pairs do not deserve any further study. In this way the number of control structures to be tested can be reduced.

The best-known interaction matrix is the *Relative Gain Array* proposed by Bristol (1966). It is easy to compute, and only the steady state gain matrix of the process under study is needed. Because the *Relative Gain Array* reveals only static interactions in a process, extensions to it have been proposed, which include dynamic interaction effects too, for example in [Wit. & McAvoy, 1] and [Tung & Edgar, 1]. [Lau *et al.*, 2] use a different approach in defining an interaction matrix. Their method is based on the singular value decomposition (*SVD*) of the process model. The problem with this *SVD* based method is that the results are dependent upon the scaling of the process model. The *Scaled Gain Matrix* proposed by [Liesl., 1] is based on the scaling of input and output variables.

The second group of methods, interaction measures, help the designer to analyse and compare the quality of the selected control structure candidates. They measure the performance loss caused by different control structures compared to the system with a full controller matrix. A unified treatment of different interaction measures is given in [Grosd. & Mor., 1]. Manousiouthakis has also produced a method for selection of decentralisation, using a similar approach. The developed tools are very general and easy to implement. The technique can be effectively used in making comparative assessment of different designs.

Another important aspect in decentralised control design is fault tolerance. It is desirable to select a control structure so that separate controllers can be detuned or taken out of service, while maintaining the stability of the overall system. A system with these properties is said to be *Decentralised Integral Controllable.* A set of simple, but only necessary, conditions for DIC is given in [Mor. & Zaf., 1]. A more complex DIC screening method, that is based on a sufficient condition, is proposed by [Le *et al.*, 1], and [Nwok. et al., 1].

Finally, last but not least, (Lev. & Karc. 3) present a method that extends the results previously derived for the properties of the centralised pole placement map under complex and real output feedback (Lev. & Karc, 2) to the case of decentralised constant output feedback and investigate some general properties related to measuring the size of the set of polynomials for a given system that can be assigned. Instrumental in the above study is the differential of the decentralised pole placement map (DPPM), which provides a measure of the size of the set at a generic decentralised feedback K_{dec}. The differential of DPPM is computed and this provides a link with the decentralised Plucker matrix of the problem [Karc. *et al.* 1] and a set of state space based parameters which incorporate the decentralisation structure and are referred to as decentralised Markov Parameters (DMP). The study of pole assignability under decentralised control schemes provides criteria that link the control structure selection to the underlying invariant system structure. In fact, the Markov matrix arising as the representation of the differential of the pole placement map characterises both pole assignability as well as absence of fixed modes for the system. An additional advantage of the Markov Parameter framework, centralised or decentralised, is that due to its direct link to the state space description, it provides the means for modifying the selection of the C, or B, matrices, such that the centralised or decentralised Markov matrix has full rank and thus achieve the very important linear assignment property which excludes the presence of fixed and almost fixed modes and preconditions well the system to accept a certain type of control solution.

The above fundamental methodologies provide the basics of a CAD package for Control Structure Selection, which has been developed in this thesis. A summary of these techniques, which have been implemented, are given here.

6.2 Interaction Matrices

6.2.1 Relative Gain Array

The relative gain array (*RGA*) proposed by Bristol [Bristol, 1] is probably the best known and most widely used interaction matrix. The element λ_{ij} in the matrix Λ is a measure of the relative gain between y_i and u_j. The relative gain is the ratio of the transfer function between the two variables, with all other outputs uncontrolled, and the transfer function between the same variables, when all other outputs are perfectly controlled. The only information needed for the calculation of the *Relative Gain array* is the steady state gain matrix u_j.

The basic strategy of the static *RGA* is to choose a control loop in which the manipulated variable u_j and the controlled variable y_i are most sensitive to each other and hence less sensitive to other input-output pairs. Because of interaction effects, one must consider the so-called open loop sensitivity:

$$\left(\frac{\partial y_i}{\partial u_j}\right)_{uk,k\neq j} \tag{6.1}$$

as well as the closed loop sensitivity:

$$\left(\frac{\partial y_i}{\partial u_j}\right)_{yk,k\neq i} \tag{6.2}$$

where $u_{k,k\neq j}$ indicates all controllers except u_j are held constant. A measure of relative sensitivity is given by the relative gain array, whose elements are defined as follows:

$$\lambda_{ij} = \frac{\left(\frac{\partial y_i}{\partial u_j}\right)_{uk,k\neq j}}{\left(\frac{\partial y_i}{\partial u_j}\right)_{yk,k\neq i}} \tag{6.3}$$

Bristol has shown that the proper input-output pair for single loop control is the one having the nearest to one λ_{ij} value.

For the interpretation of the *RGA*, Bristol gives the following two pairing rules:

1. Pair together inputs and outputs indicated by positive *RGA* elements that are closest to unity.

2. Avoid pairing together inputs and outputs indicated by negative *RGA* elements, because such pairings result in, either an unstable system, or an inverse responding system.

Always pair on positive relative gains that are closest to unity and check the resulting pairings for stability using Niederlinski's theorem; if the pairings are unstable, choose other possible pairings with values closest to unity, avoiding negative pairings if possible.

Theorem (6.1) (Niederlinski Stability Theorem):

The closed loop system under investigation is unstable if :

$$\frac{|A|}{\prod_{i=1}^{n} a_{ii}} < 0 \tag{6.4}$$

where $|A|$ is the determinant of A.

This theorem is particularly powerful. All that it requires for its use is steady-state gain information and the assumption that perfect steady-state control is achieved in all loops. Niederlinski's theorem is actually equivalent to pairing on a negative *RGA* element.

Interactions in a multivariable control system have implications beyond merely propagation of disturbances between loops. Consider for example a two input-output system where the controllers have been tuned independently. Even though the individual performance of the two controllers, when tuned, may be quite satisfactory, the overall system will sometimes go unstable, when both loops are operated together. The interacting controllers, therefore, create conditions which can destabilise an otherwise stable system. Thus, it appears that the *RGA* can be used not just as a measure of variable interactions but also as a measure of system stability. Since in the derivation of the RGA it was assumed that the output variables were subject to perfect steady-state control, it seems natural to seek a relationship between the *RGA* and systems under integral control and it is an issue considered in section 6.4.2.

6.2.2 Dynamic Relative Gain Array

The *relative gain array* is a steady-state analysis method and does not explicitly include dynamic effects. Several extensions, which include dynamic effects, have been proposed. The dynamic relative gain array proposed by [Tung & Edgar, 2] is one such extension. Several investigators have proposed different definitions for a dynamic *RGA*. In some cases, these definitions require that the feedback controller be designed. Since the *RGA* is most valuable in screening alternative control system designs, the requirement that the controller must be designed, limits the utility of these definitions. The approach often used, does not require that the controllers must be specified. Starting with zero initial conditions, it is desired to bring the process output y to the new set point r_o. The basic idea behind this method is to divide the change in the output y into parts, caused by the different elements of the input vector, going from 0 to u_{i_o}:

Let a linear dynamic process be described by:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{6.5}$$

The system is assumed to be controllable and observable. Consider a change in set point from zero to y_o, and let the required control change necessary to bring about this set point change be $u = u_o$. At steady state,

$$0 = Ax_o + Bu_o \tag{6.6}$$

Assuming the system is stable, then the above equation can be solved for x_o,

$$x_o = (-A)^{-1} Bu_o \tag{6.7}$$

but

$$y_o = Cx_o = C(-A)^{-1} Bu_o \tag{6.8}$$

therefore, for y and u of the same dimension,

$$u_o = \left[C(-A)^{-1}B\right]^{-1} y_o \tag{6.9}$$

With zero initial conditions, the output response is, in the s-domain,

$$\begin{aligned} y(s) &= Cx(s) \\ &= C(sI-A)^{-1}Bu_o \cdot 1/s \\ &= C(sI-A)^{-1}B \circ \left[C(-A)^{-1}B\right]^{-1} y_o \cdot 1/s \\ &= G(s) \circ \left[G(0)\right]^{-1} y_o \cdot 1/s \end{aligned} \tag{6.10}$$

where $G(s)$ is the process transfer function matrix and $\circ$ denotes the element by element multiplication. The last equation can be written in more detail:

$$\begin{bmatrix} y_1(s) \\ y_2(s) \\ \vdots \\ y_m(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) & \cdots & G_{1m}(s) \\ G_{21}(s) & G_{22}(s) & \cdots & G_{2m}(s) \\ \vdots & \vdots & & \vdots \\ G_{m1}(s) & G_{m2}(s) & \cdots & G_{mm}(s) \end{bmatrix} \circ \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \cdots & \Gamma_{1m} \\ \Gamma_{21} & \Gamma_{22} & \cdots & \Gamma_{2m} \\ \vdots & \vdots & & \vdots \\ \Gamma_{m1} & \Gamma_{m2} & \cdots & \Gamma_{mm} \end{bmatrix} \cdot \begin{bmatrix} y_{1_o} \\ y_{2_o} \\ \vdots \\ y_{m_o} \end{bmatrix} \cdot \frac{1}{s} \tag{6.11}$$

that is,

$$y_i(s) = \sum_{j=1}^{m}\left(\sum_{k=1}^{m} G_{i,k}(s)\Gamma_{kj}\right)\frac{y_{j_o}}{s}, \quad i = 1,2,\cdots,m \tag{6.12}$$

where $G_{i,j}(s)$ and $\Gamma_{k,j}$ are elements of the $C(sI-A)^{-1}B$ matrix and the $\left[C(-A)^{-1}B\right]^{-1}$ matrix $G^{-1}(0)$, respectively.

Now consider a step change in y_{l_o} only. The response of y_l is simply:

$$y_i(s) = \left(\sum_{k=1}^{m} G_{i,k}(s)\Gamma_{kj}\right)\frac{y_{l_o}}{s} \tag{6.13}$$

Note that the kth term in the summation results from the kth controller. The above equation indicates that if y_i is to be controlled by controller u_r, the term $G_{i,r}\Gamma_{r,i}/s$ should be the dominant term. This equation can lead to the formation of a dynamic RGA matrix:

$$\begin{array}{c|cccc} & u_1 & u_2 & \cdots & u_m \\ \hline y_1 & a_{11}(s) & a_{12}(s) & \cdots & a_{1m}(s) \\ y_2 & a_{21}(s) & a_{22}(s) & \cdots & a_{2m}(s) \\ \vdots & \vdots & \vdots & & \vdots \\ y_m & a_{m1}(s) & a_{m2}(s) & \cdots & a_{mm}(s) \end{array} \tag{6.14}$$

where

$$\alpha_{ij}(s) = G_{ij}(s)\Gamma_{ji}\cdot 1/s, \quad \begin{cases} i = 1,2,\cdots,m \\ j = 1,2,\cdots,m \end{cases} \tag{6.15}$$

Having presented a frequency domain version of the dynamic RGA, a time domain interpretation can easily be defined. Such a procedure will finally lead us to the following result:

$$\begin{array}{c|ccc} & u_1 & \cdots & u_m \\ \hline y_1 & \left(\left.\frac{\partial y_1}{\partial u_1}\right|_u\right) \Big/ \left(\left.\frac{\partial y_1}{\partial u_1}\right|_y\right) & \cdots & \left(\left.\frac{\partial y_1}{\partial u_m}\right|_u\right) \Big/ \left(\left.\frac{\partial y_1}{\partial u_m}\right|_y\right) \\ \vdots & \vdots & & \vdots \\ y_m & \left(\left.\frac{\partial y_m}{\partial u_1}\right|_u\right) \Big/ \left(\left.\frac{\partial y_m}{\partial u_1}\right|_y\right) & \cdots & \left(\left.\frac{\partial y_m}{\partial u_m}\right|_u\right) \Big/ \left(\left.\frac{\partial y_m}{\partial u_m}\right|_y\right) \end{array}$$

Therefore, the notion of the static RGA can then be derived from a detailed dynamic analysis.

Tung and Edgar propose that a proper control structure can be selected by finding the dominant terms in the dynamic relative gain array. The elements of the dynamic relative gain array having large absolute values indicate the recommended feedback loops. It is possible that different loop pairings are recommended at low and high frequencies. The use of a multivariable controller could be beneficial in such cases. Because this interpretation is based on gains only, one should analyse the effect of delays separately.

6.2.3 Performance - Relative Gain Matrix (P-RGA)

The notion of the *D-RGA* has already been presented and its use as a screening tool for alternative control structures has already been established. However, the *RGA* was never considered to be a panacea. It can be very handy in numerous cases but it has its limitations. Lets look at it in more detail:

The *RGA* matrix, as already defined, has some interesting algebraic properties (e.g. [Grosd. *et al.*, 1]):

(a) It is scaling independent. Mathematically, $\Lambda(D_1GD_2)=\Lambda G$ where y_i and D_2 are diagonal matrices.
(b) All row and column sums equal one.
(c) Any permutation of rows or columns in G results in the same permutation in the RGA.
(d) If $G(s)$ is triangular (and hence also if it is diagonal), $\Lambda(G)=I$.
(e) Relative perturbations in elements of G and in its inverse are related by $d[G^{-1}]_{ji}/[G^{-1}]_{ji}=-\lambda_{ij}dg_{ij}/g_{ij}$.

One inadequacy of the *RGA* is that it, because of property *d*, may indicate that interaction is no problem, but significant one-way coupling may exist. To overcome this problem, the performance relative gain array (*P-RGA*) can be introduced [Hovd. & Skog., 2]. The *PRGA*-matrix is defined as

$$P(s)=\tilde{G}(s)G(s)^{-1}$$

where $\tilde{G}(s)$ is the matrix consisting of only the diagonal elements of $G(s)$, i.e. $\tilde{G} = diag\{g_{ii}\}$. The matrix P was originally introduced at steady-state by [Grosd., 1] in order to understand the effect of directions under decentralised control. The elements of P are given by

$$p_{ij} = g_{ii}(s)\left[G^{-1}\right]_{ij} = \frac{g_{ii}(s)}{g_{ji}(s)}\lambda_{ji}(s)$$

Note that the diagonal elements of *RGA* and *PRGA* are identical, but otherwise *PRGA* does not have all the algebraic properties of the *RGA*. *PRGA* must be recomputed whenever G is rearranged, whereas *RGA* only needs to be rearranged in the same way as G. *PRGA* is independent of *input* scaling, that is $P(GD_2) = P(G)$, but it depends on output scaling. This is reasonable since performance is defined in terms of the magnitude of the outputs.

The measures above may be extended to non-square systems by introducing the pseudoinverse. However, the usefulness of the measures, at least for analysing decentralised control, then seems limited.

6.2.4 Scaled Gain Matrix (SGM)

This method was first proposed by [Liesl., 1]. The method aims to provide a control system designer useful information on interactions in a form that is easy to interpret. It is based on the scaling of input and output variables. Although a large gain between an input and an output indicates strong interaction, the process gain matrix can not be directly used for interaction analysis, because it depends on the scaling of input and output variables. In this method the input and output variables are rescaled, so that in the new gain matrix, corresponding to the rescaled variables, the elements are directly comparable with each other.

Consider an $m \times n$ process transfer function matrix $G(s)$. The basic idea behind the method is to scale input and output variables in such a way that the average gain in each row and column of the process model is one at a given frequency. This is achieved using the following iterative procedure:

Step 1. Calculate the gain matrix at the desired frequency ω^*, $|G(j\omega^*)|$. This is the first estimate of the scaled gain matrix Ψ, i.e. for $k=1$ set

$$\psi_{ij}^k = |g_{ij}(j\omega^*)|$$

Step 2. Scale the rows of Ψ^k in such a way that in each row the average value of the elements is equal to one.

$$\psi_{ij}^{k+1} = \frac{n \cdot \psi_{ij}^k}{\sum_{j=1}^{n} \psi_{ij}^k}$$

Step 3. Scale the columns of Ψ^{k+1} in such a way that in each column the average value of the elements is equal to one.

$$\psi_{ij}^{k+2} = \frac{m \cdot \psi_{ij}^{k+1}}{\sum_{j=1}^{m} \psi_{ij}^{k+1}}$$

Step 4. Stop if the changes between Ψ^k and Ψ^{k+2} are sufficiently small. Otherwise set $k \leftarrow k+2$, and go to step 2.

The procedure converges towards the *scaled gain matrix* (*SGM*) that is unique for each matrix.

In the *Scaled Gain Matrix*, the average value of the elements in each row and column is one. The interpretation of this interaction matrix is simple: values larger than one indicate strong interaction and values smaller than one indicate weak interaction. The largest elements in Ψ then indicate the inputs and outputs, which should be connected in the feedback controller. The *SGM*, unlike the *RGA*, can be used even when the number of inputs and outputs is unequal. The *SGM* is also applicable to the analysis of dynamic effects without any special extensions.

The *SGM* is a method for finding possible control structure candidates. Stability and performance constraints imposed by these control structures have to be analysed using alternative measures.

6.2.5 Block Relative Gain (BRG)

Because of its many useful properties, the *RGA* has found wide applications among engineers. However, at the same time, its original development as a scalar and its presentation in a single array, unnecessarily limited its applicability exclusively to SISO control loops. By formulating and extending the relative gain concept and its properties from a scalar to a matrix, a more powerful synthesis framework is formed, that can address a broader class of control problems, such as the synthesis of decentralised control structures that are not restricted to SISO control loops. This new concept is referred to as *Block Relative Gain.*

Control system synthesis starts with a given set of measurements, y, and manipulated variables, u. The input-output model $y(s) = G(s)u(s)$ is usually assumed to be the one to describe the plant dynamics, with the transfer function matrix $G(s)$ considered to be square. In decentralised plant control, different subsets of outputs are assigned to different subsets of inputs and each such assignment forms a subsystem G_{ii}. In classical feedback terms this implies that output measurements of an individual subsystem will affect the manipulated inputs of that subsystem only via its own control law. Alternative subsystems and, thus, decentralised control structures can be systematically generated by partitioning $G(s)$ into blocks of different dimensions and also due to alternative ways of assigning inputs and outputs to the blocks, figure (6.1).

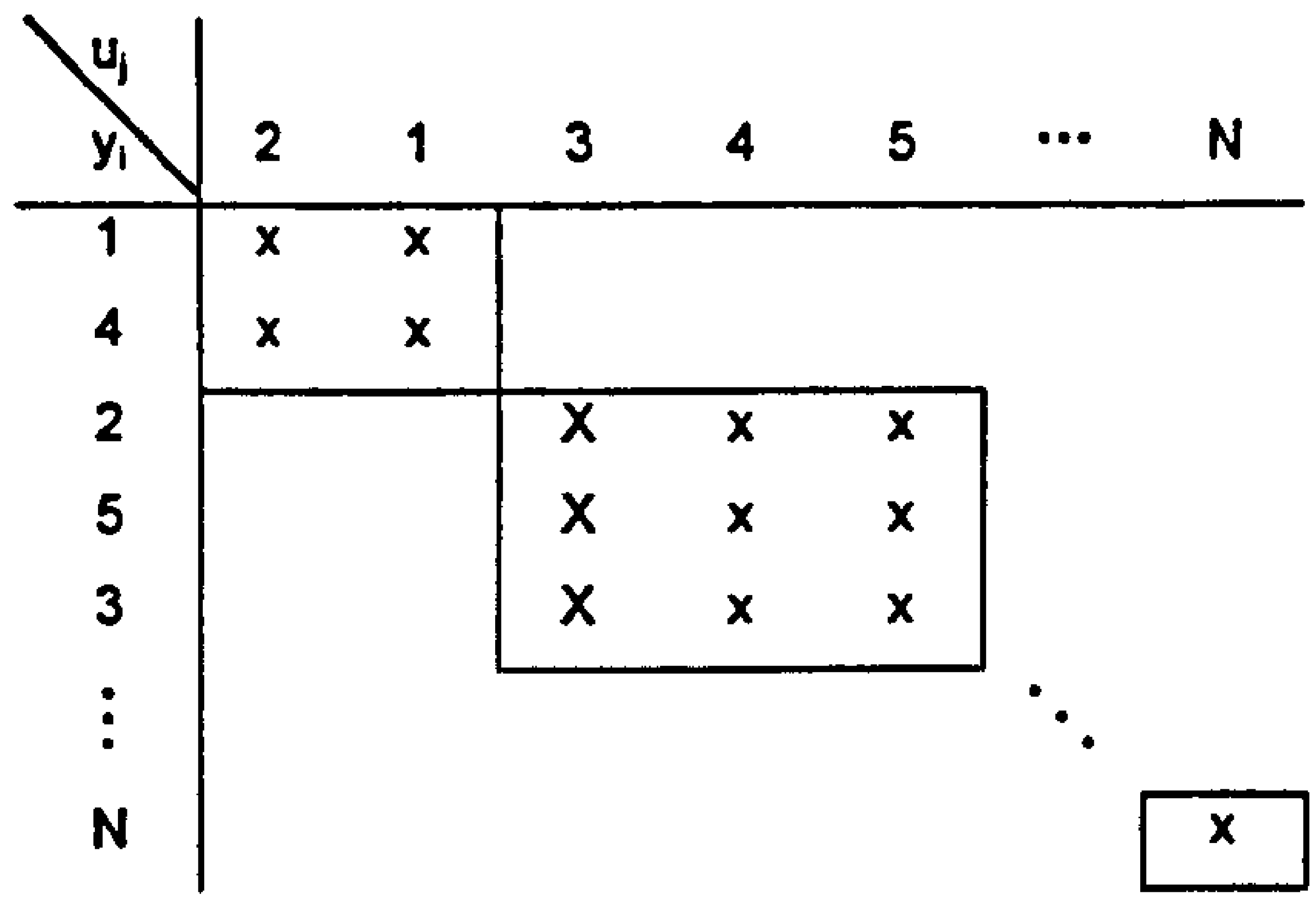


Figure 6.1. Partitioning of $G(s)$ into blocks of different dimensions.

Note that in this type of partitioning, subsystems are viewed as aggregates of control loops and not as groups of process units. Thus, block partitioning of $G(s)$ may not necessarily correspond to a particular process decomposition and the resulting decentralised control system does not have to be compatible with any arrangement of subsystems of process unit operations. However, this does not preclude the possibility of specifying the process decomposition first and then structuring the decentralised control systems within the boundaries of the individual process subsystems. In some cases, this may eliminate the synthesis of undesirable decentralised control structures right from the beginning and reduce the potential combinatorial problems encountered in the block partitioning procedure.

To better understand the concept of BRG, one has to consider a square ($n \times n$) transfer function matrix $G(s)$, partitioned as follows (the s is dropped for convenience):

$$G = \begin{array}{c} \begin{array}{cc} m & n-m \end{array} \\ \left[\begin{array}{c|c} G_{11} & G_{12} \\ \hline G_{21} & G_{22} \end{array}\right] \end{array} \begin{array}{l} m \\ n-m \end{array} \qquad \text{with} \quad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = G \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{6.16}$$

The plant is to be controlled by a decentralised control structure in which the first m outputs y_1 are interconnected with the first m inputs u_1 and the last $n-m$ outputs y_2 are interconnected with the last $n-m$ inputs u_2. The corresponding feedback configuration is shown in figure 6.2. The controller K and the filter F are given by:

$$K = \begin{array}{c} \begin{array}{cc} m & n-m \end{array} \\ \left[\begin{array}{c|c} K_1 & 0 \\ \hline 0 & K_2 \end{array}\right] \end{array} \begin{array}{c} m \\ n-m \end{array} \quad , \quad F = \begin{array}{c} \begin{array}{cc} m & n-m \end{array} \\ \left[\begin{array}{c|c} F_1 & 0 \\ \hline 0 & F_2 \end{array}\right] \end{array} \begin{array}{c} m \\ n-m \end{array} \tag{6.17}$$

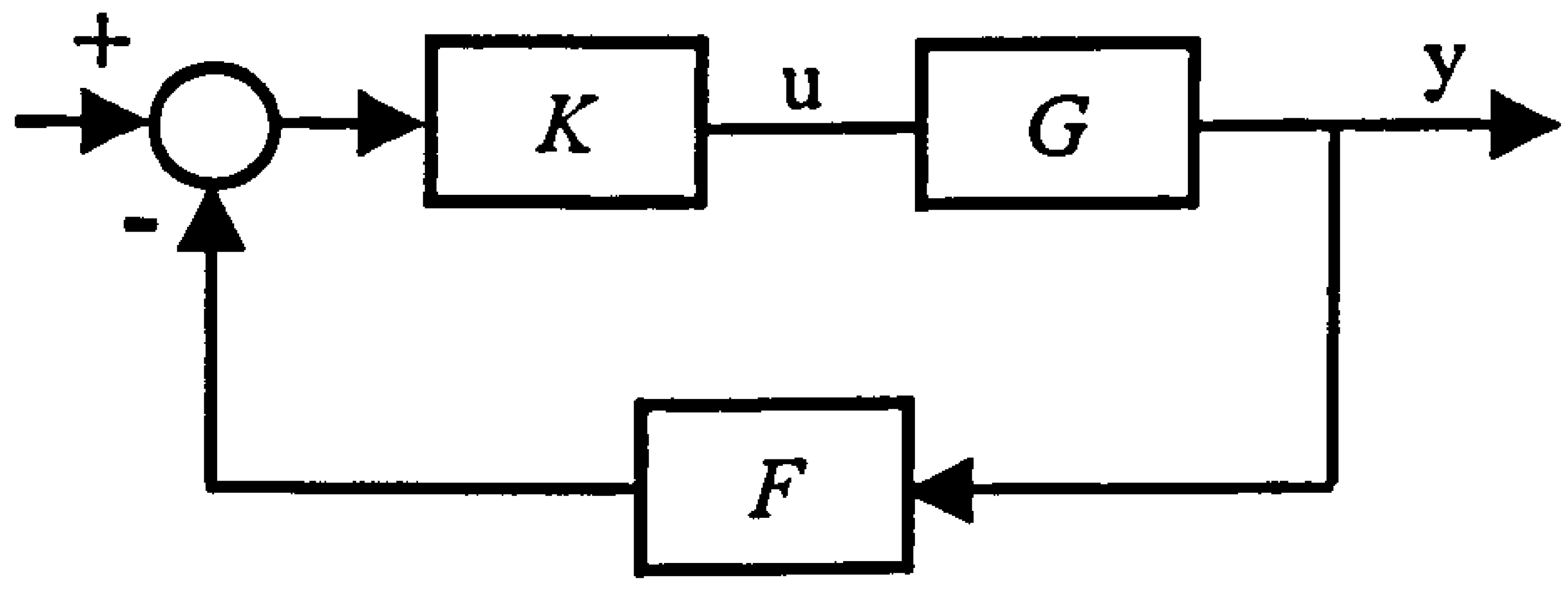


Figure 6.2. Decentralised feedback system.

The following relations hold:

$$y = Gu \tag{6.18}$$

$$u = G^{-1}y \qquad \text{(assuming } G^{-1} \text{ exists)} \tag{6.19}$$

Then

$$\left.\frac{\partial y_1}{\partial u_1}\right|_{\substack{u_2=0 \\ F=0}} = G_{11} \tag{6.20}$$

$$\left.\frac{\partial y_1}{\partial u_1}\right|_{\substack{u_2=0 \\ F_1=0 \\ F_2=I}} = \left(\left[G^{-1}\right]_{11}\right)^{-1} = G_{11} - G_{12}G_{22}^{-1}G_{21} \quad \text{(if } G_{22} \text{ is nonsingular)} \tag{6.21}$$

where $\left[G^{-1}\right]_{11}$ is the first $m \times m$ block of G^{-1}:

$$G^{-1} = \begin{bmatrix} \left[G^{-1}\right]_{11} & \left[G^{-1}\right]_{12} \\ \left[G^{-1}\right]_{21} & \left[G^{-1}\right]_{22} \end{bmatrix} \tag{6.22}$$

According to eq. (6.20), G_{11} denotes the block gain between y_1 and u_1 when all the loops are open, i.e. $F=0$. Similarly, $\left(\left[G^{-1}\right]_{11}\right)^{-1}$ is the block gain between y_1 and u_1 when the first m loops are open, i.e. $F_1=0$, and the last $n-m$ loops are closed, i.e., $F_2=I$, and under perfect control, i.e., $y_2=0$.

The m-dimensional *Block Relative Gain* (left and right) can then be defined as:

$$BRG_\ell = \left[\left.\frac{\partial y_1}{\partial u_1}\right|_{\substack{u_2=0\\ F=0}}\right] \cdot \left[\left.\frac{\partial y_1}{\partial u_1}\right|_{\substack{u_2=0\\ F_1=0\\ F_2=I}}\right]^{-1} = G_{11}\cdot\left[G^{-1}\right]_{11} \tag{6.23}$$

$$BRG_r = \left[\left.\frac{\partial y_1}{\partial u_1}\right|_{\substack{u_2=0\\ F_1=0\\ F_2=I}}\right] \cdot \left[\left.\frac{\partial y_1}{\partial u_1}\right|_{\substack{u_2=0\\ F=0}}\right] = \left[G^{-1}\right]_{11}\cdot G_{11} \tag{6.24}$$

Note that in the case of one-dimensional *BRG*, left and right *BRG*'s become identical (since G_{11} is scalar) and reduce to the classical Bristol's *RGA*. In the case of n-dimensional *BRG*, $BRG_\ell = BRG_r = I$.

The significance of the *BRG* in relation to the closed-loop performance can be derived from a study of the following three cases:

<u>Case 1</u>: $F_1=0, F_2=0$ (no feedback).
<u>Case 2</u>: $F_1=0, F_2=I$ (feedback of the last $n-m$ outputs to the last $n-m$ inputs).
<u>Case 3</u>: $F_1=0, F_2=0$ (feedback of the first m $[\text{last } n-m]$ outputs to the first $m\,[\text{last } n-m]$ inputs, respectively).

From the results (the extensive calculations can be found in [Manous. *et al*, 2], one can find the answer to the question: What is the significance of the relative gain for the performance of the closed-loop system? The answer is: The closed-loop performance of the $m\times m$ block under consideration, when the other $n-m$ outputs are under perfect control, is a continuous function of BRG_ℓ. When $BRG_\ell = I$, which implies $BRG_r = I$, the closed-loop performance of the $m \times m$ block is as if this block was isolated from the rest of the plant and operating under the influence only of its own control law. This makes it clear what kind of information one should expect from *BRG* and in what sense it can be considered as a measure of interaction.

6.2.6 Dynamic Block Relative Gain (D-BRG)

When defining the *block relative gain* and deriving its relation to the closed-loop performance, the usual assumption of perfect control for the plant outputs has been made. This assumption always holds at zero frequency (i.e., at steady state) by the use of integral control action. However, it may not hold for all the frequencies especially when non-minimum phase and/or strictly proper blocks are present. For such cases, the assumption of perfect control over the whole frequency range can be relaxed, [Manous. *et al.,* 1]. If one wished to investigate interactions over the whole frequency range, *BRG* could be extended to a *Dynamic-BRG* and become a frequency-dependent interaction measure, that need not be modified, if there are no right half-plane transmission zeros in the complementary subsystem, which is supposed to work under perfect control. On the other hand, when RHP zeros exist, one either evaluates the equations at steady state only or, if interested in all the frequencies, one can use an appropriately modified *D-BRG* as given in [Manous. *et al.,* 1].

BRG, as it was previously defined, is related to the first m outputs and m inputs of the plant. As a result, it will depend on how the n outputs and n inputs are ordered in $G(s)$. Since the number of all possible rearrangements of n objects is $n!$, n outputs and n inputs can be ordered in $(n!)\cdot(n!)$ possible combinations. Calculating an m-dimensional *BRG* for each such combination would result to a total of $(n!)^2$ *BRG* computations, which would be an enormous task for large n's. In order to resolve this combinatorial problem, certain theorems were presented [Manous. *et al.,* 2]. The results were the following:

- BRG_ℓ (BRG_r) is not affected at all by the ordering of the last $n-m$ inputs and $n-m$ outputs and the ordering of the first m inputs (outputs). Furthermore, for all

G's that contain the same first m inputs and m outputs but different arrangements, corresponding BRG's turn out to be trivial rearrangements of each other. Consequently, they can be considered equivalent. Thus for an m-dimensional subsystem containing a unique group of inputs and outputs, only one of the equivalent BRG's needs to be examined. This means that for an n-dimensional system, the number of calculations for an n-dimensional BRG drops from $(n!)^2$ to $\binom{n}{m}\cdot\binom{n}{m}$, which is a significant reduction for large systems.

- BRG_ℓ (BRG_r) does not depend on the scaling of the last $n-m$ inputs and outputs and the first m inputs (outputs), but it does depend on the scaling of the first m outputs (inputs). However, if the first m outputs (inputs) are all scaled in the same way, then BRG_ℓ (BRG_r) is not affected at all.

- The diagonal elements of BRG's are well-defined, i.e. they remain on the diagonal but not necessarily at the same locations when G is trivially rearranged. This implies that, for all the BRGs corresponding to a particular group of m inputs and outputs, the designer needs to examine only m diagonal terms.

- The well-known property of the RGA, that elements of each row and each column add to 1, also applies to BRGs. This is a direct consequence of the previous result and also of the fact that the n-dimensional BRG is the identity matrix.

The aim of the $DBRG$ is to provide an acceptable block partitioning of the plant matrix $G(s)$. Such a task is considered to be accomplished if all the BRG's of different dimensions corresponding to the diagonal blocks of different dimensions $G_{ii}(s)$'s, are close to an identity matrix. To quantify this closeness and define the set of viable BRG's, the following procedure is necessary:

Let $B(1,\varepsilon)$ denote a neighbourhood in the complex plane with centre at $(1,0)$ and radius $\varepsilon=\varepsilon(\omega)$. Then we say that a BRG is viable (i.e. close to identity) if its diagonal elements and eigenvalues belong to neighbourhoods $B(1,\varepsilon_1)$ and $B(1,\varepsilon_2)$, respectively, for all frequencies ω. The selection process is the following:

First consider the highest degree of decentralisation – i.e. 1×1 block partitioning of G – that would yield a total of N SISO assignments (or pairings). For this, all the one-dimensional *BRG*s are first evaluated at $s=0$. Among the viable ones, those which establish a $1-1$ correspondence between the plant's inputs and outputs are selected. If such alternatives do not exist, then there is no acceptable partitioning using $1-1$ blocks only. In that case, assignment is not complete and one proceeds with two-dimensional *BRG*s. In case there exists an acceptable $1-1$ block partitioning for $s=0$ but viability and/or acceptance are violated at frequencies other than $\omega=0$, the study of two-dimensional *BRG*s is again necessary.

The next step in the process is the study of two-dimensional *BRG*s. We first study only BRG_ℓs. The BRG_ℓs, whose diagonal elements are not close to 1, are screened out first. Among the remaining BRG_ℓs, only those with eigenvalues close to 1 are retained since these are the BRG_ℓs that are close to the identity matrix. Since the eigenvalues of BRG_r and BRG_ℓ are the same, a detailed study of BRG_rs is deemed unnecessary, in case the eigenvalues of BRG_ℓ are close to 1. If this is not the case, the diagonal elements of BRG_rs should be calculated from the *RGA* and their closeness to one should be examined as a final screening criteria.

The diagonal terms of all the two-dimensional BRG_ℓs are the elements of the column vectors that result from every possible addition of two columns of the *RGA*. Thus if one of these column vectors has $p>2$ elements within $B(1,\varepsilon_1)$ this implies that there exist only $p!/[2!(p-2)!]$ two-dimensional BRG_ℓs that should be further considered. Among these BRG_ℓs, those with eigenvalues outside $B(1,\varepsilon_2)$ are rejected. The remaining BRG_ℓs are the two-dimensional viable BRG_ℓs for $s=0$ and for one of the column vectors discussed above. The screening process is repeated for all the possible column vectors and for all frequencies other than $\omega=0$ and ultimately gives all viable two-dimensional BRG_ℓs.

Searching for an acceptable partitioning over the sets of both two- and one-dimensional viable BRG_ℓs is the next step. If one is found, the procedure concludes; otherwise it continues with the study of BRG_ℓs of higher dimension, in the same manner, until a solution is achieved. The process is guaranteed to conclude since, in the worst case, it will lead to a centralised full control structure that corresponds to an n-dimensional BRG. It should be mentioned that $\varepsilon_1(\omega)$, $\varepsilon_2(\omega)$ are free parameters through which the designer can affect the screening process and establish what an acceptable degree of interaction is.

Having presented the procedure, one can easily understand the advantages of the *DBRG*. Different block partitioning of input and output sets leads to alternative decentralised control structures, among which the best are selected by the systematic screening procedure that utilises various important properties of *BRG*. These properties effectively reduce the combinatorial problems and make the analysis of large-scale systems feasible.

6.2.7 Gershgorin Analysis

Multivariable frequency response techniques are some of the most promising modern control techniques for analysing interaction and designing controllers. Rosenbrock [Rosen., 2] developed and used both the Direct Nyquist Array (*DNA*) and the Inverse Nyquist Array (INA). Since these two techniques are similar, only the *INA* is discussed here.

To use the *INA*, the G matrix is arranged so that a diagonal pairing of loops results. Next, the inverse of the matrix of process transfer functions, G^{-1}, is calculated as a function of frequency. The elements of G^{-1} will be denoted as $\hat{G}_{ij}$. A complex plane plot of the diagonal elements of G^{-1}, $\hat{G}_{ii}$, is made as a function of frequency, ω. Next, the radii of circles, called Gershgorin bands, are calculated as a function of ω as:

$$r_i = \sum_{\substack{i=1 \\ j \neq i}}^{n} \left|\hat{G}_{ij}\right|$$

These radii measure the importance of off-diagonal (interacting) elements relative to the diagonal elements. Circles with these radii are then superimposed on the previous curves.

The goal of the INA is to achieve a system that is diagonally dominant. This is similar to a decoupled system but less restrictive. We first start with the definition for a *diagonally row dominant* matrix:

Definition 6.1: A rational $k \times k$ matrix $Z(s)$ is said to be *diagonally row dominant* on the contour D if $z_{ii}(s)$ has no pole on D, $i = 1,2,\cdots,k$, and

$$\left|z_{ii}(s)\right| - \sum_{\substack{j=1 \\ j \neq i}}^{k} \left|z_{ij}(s)\right| > 0 \qquad \text{for } i = 1,2,\cdots,k \text{ and all } s \text{ on } D$$

Definition 6.2: A rational $k \times k$ matrix $Z(s)$ is said to be *diagonally column dominant* on the contour D if $z_{ii}(s)$ has no pole on D, $i = 1,2,\cdots,k$, and

$$\left|z_{ii}(s)\right| - \sum_{\substack{j=1 \\ j \neq i}}^{k} \left|z_{ji}(s)\right| > 0 \qquad \text{for } i = 1,2,\cdots,k \text{ and all } s \text{ on } D$$

Diagonal dominance on D is defined as follows:

$$\textit{For each } s \textit{ on } D \begin{cases} \textit{either} \; \left|z_{ii}(s)\right| - \sum_{\substack{j=1 \\ j \neq i}}^{k} \left|z_{ij}(s)\right| > 0 & i = 1,2,\cdots,k \\ \\ \textit{or} \quad \left|z_{ii}(s)\right| - \sum_{\substack{j=1 \\ j \neq i}}^{k} \left|z_{ji}(s)\right| > 0 & i = 1,2,\cdots,k \end{cases}$$

In a diagonally dominant system, the off-diagonal elements of G taken together are less important than the diagonal elements. If none of the Gershgorin bands encircles the origin, the system is diagonally dominant. Before the analysis can proceed further, it is necessary to make the system dominant. The first step is to try different pairings. Next, one can try using combinations of manipulative and/or controlled variables. For example, one might try a steady-state decoupler. It is important to note that the INA does not require perfect decoupling. The INA only requires that the system be diagonally dominant. Once a dominant system is achieved, the rigorous stability theorems can be used to design feedback controllers. In essence, once diagonal dominance is achieved, one can treat the design of each controller as a single-loop problem and this suggests possible pairings on the original system. If a system is not dominant, one should be able to try various changes of manipulated and/or controlled variables in a simple, straightforward manner. A methodology for using dominance ideas in the control structure selection has been recently developed in [McAvoy, 1], which may allow alternative systematic procedures. A drawback of this setup is the need to achieve dominance before we apply the selection procedure, which may not always be feasible.

6.2.8 Process Instrumentation Matrix (PIM)

Advanced process design leads to more complicated flowsheets, hence it requires an early assessment of process operability & controllability. Many tools can be used for selecting control structures and for designing control systems. To visualise these structures, the **PIM** (Process Instrumentation Matrix) table can be used. This constitutes one of the main viewpoint in the EPIC (Early Process design Integrated with Control) user interface package [Epic, 1]. In its simplest form, it has columns for correcting variables (locations for actuators) and rows for candidate controlled variables (associated with sensors), from which the controlled variables have to be selected. The cell can display control functions, process and control data, and other information pertaining to the relationship. Usually, control power (the static sensitivity of the controlled variable to a change in the correcting variable) and control speed (the bandwidth concept in servomechanism design, corresponds to the resonance frequency of the control loop for a well-tuned PID-controller) are included in a PIM.

The latter two measures of control quality refer directly to the frequency domain. Although this is not a limitation, straight forward computation can be made, that yield graphs of control errors as a function of frequency for the various inputs. Transformations to the time domain (by inverse Fourier transform) can be planned., so as to find the maximum deviation in case of "hard" constraints. For critical control tasks, better performance allows a more favourable compromise between economy and reliability of process operation, as the margin between set point and critical constraint can be decreased.

6.3 Interaction Measures

Modelling uncertainties and constantly changing operating conditions make it very difficult to develop reliable dynamic models for chemical processes. Often, only steady-state gain information is available. In multi-input-multi-output (MIMO) systems, these data may be represented as a matrix of steady-state gains $G(0)$. Since this matrix $G(0)$ is often the only information available on the system, any method, that will allow the extraction of useful feedback properties from it, is clearly of great practical importance. Very important closed-loop properties can easily be extracted from the steady-state gain matrix. Such techniques include closed-loop stability, sensor and actuator failure tolerance, feasibility of decentralised control structures and robustness with respect to modelling errors. Instead of using matrices, scalar indicators expressing the cumulative effect of different performance aspects may be used to express interaction properties in selecting the appropriate pairings. Alternatively, pairing may be decided on the basis of guaranteeing specific system properties.

6.3.1 Singular Value Analysis

Singular Value Decomposition (*SVD*) is a promising tool in the structural analysis of multivariable systems [Lau *et al.*, 2]. The method can provide a powerful and computationally efficient tool for analysing matrix systems [Fors., 2] and it is the basis for many diagnostics for control system design.

A systematic approach to the synthesis of regulatory process control structure can be formulated. The analysis can be performed over the frequency range that is of practical importance for the particular process, so that both static and dynamic aspects can be considered. An additional important feature of the *SVD* strategy is its ability to identify modelling aspects, such as model mismatch, which affect the performance of the resulting process control structure. Also, the strategy can show whether or not a structural decoupler will be effective in minimising interactions between loops. A compensator can be designed for the range of frequencies most likely to affect the process. Since we are primarily interested in the control structure, rather than in the actual controller design, the analysis is based on the open loop transfer function. The approach provides insights into important closed loop system properties: stability (Post. *et al.*, 1), sensitivity [Weber & Bros., 1], and invertibility [Morari, 3].

An option for *SVD Analysis* was implemented in the CAD Toolbox. It is based on the singular values of the open-loop transfer function. The magnitude of the singular values measures the sensitivity of MIMO systems in the same manner as the amplitude ratio is employed in SISO systems. In order to encompass both static and dynamic features, the analysis is carried out over a range of frequencies of practical significance for a given process. The theory behind the method is presented in the following lines.

The application of the *SVD* to the $m \times n$ transfer function matrix $G(s)$ leads to the equation

$$G(s) = Z(s)\Lambda(s)V^{\mathrm{T}}(s)$$

where

$$\Lambda(s) = \begin{array}{c} \left[\begin{array}{c|c} \Delta(s) & 0 \\ \hline 0 & 0 \end{array}\right] \begin{array}{l} p \\ m-p \end{array} \\ \quad p \qquad n-p \end{array}$$

$$p = rank\ G(s) \le \min(m,n)$$

and $(\cdot)^{\mathrm{T}}$ denotes transposition. $\Delta(s)$ is a diagonal matrix whose entries are the singular values of $G(s)$. This decomposition implies that $(m-p)$ measurements and $(n-p)$ manipulated variables can be deleted without altering the input-output accessibility and manipulability of the system. However, a preliminary system analysis should include all the input and output variables.

Suppose $\sigma_1(s), \sigma_2(s), \cdots, \sigma_p(s)$ are distinct singular values of $G(s)$, then $Z(s)$ and $V(s)$ can be partitioned as $Z(s) = [z_1(s) \vdots z_2(s) \vdots \cdots \vdots z_p(s) \vdots z_{p+1}(s), \cdots, z_m(s)]$, $V(s) = [v_1(s) \vdots v_2(s) \vdots \cdots \vdots v_p(s) \vdots v_{p+1}(s), \cdots, v_n(s)]$ where $z_i(s)$ and $v_i(s)$ $(i = 1, \cdots, p)$ are the singular decomposition vectors which correspond to the i-th singular value and $z_j(s)$ and $v_j(s)$ $(j = p+1, \cdots, n)$ are the remaining decomposition vectors which correspond to the zero singular values. An alternative expression would be to write $G(s)$ as a sum of dyads:

$$\begin{aligned} G(s) &= \sum_1^p \sigma_i(s) z_i(s) v_i^\dagger(s) \\ &= \sum_1^p \sigma_i(s) W_i(s) \end{aligned}$$

The above equations do not contain a $p+1$ term because it is multiplied by a zero singular value. Thus, a singular value decomposition of the matrix $G(s)$ defines an input space spanned by a set of orthonormal basis vectors $\{v_i(s)\}_1^p$, and a gain space defined by the set of singular values $\{\sigma_i(s)\}_1^p$. Furthermore, a one-to-one correspondence is established between these spaces as it is illustrated in figure (6.3).

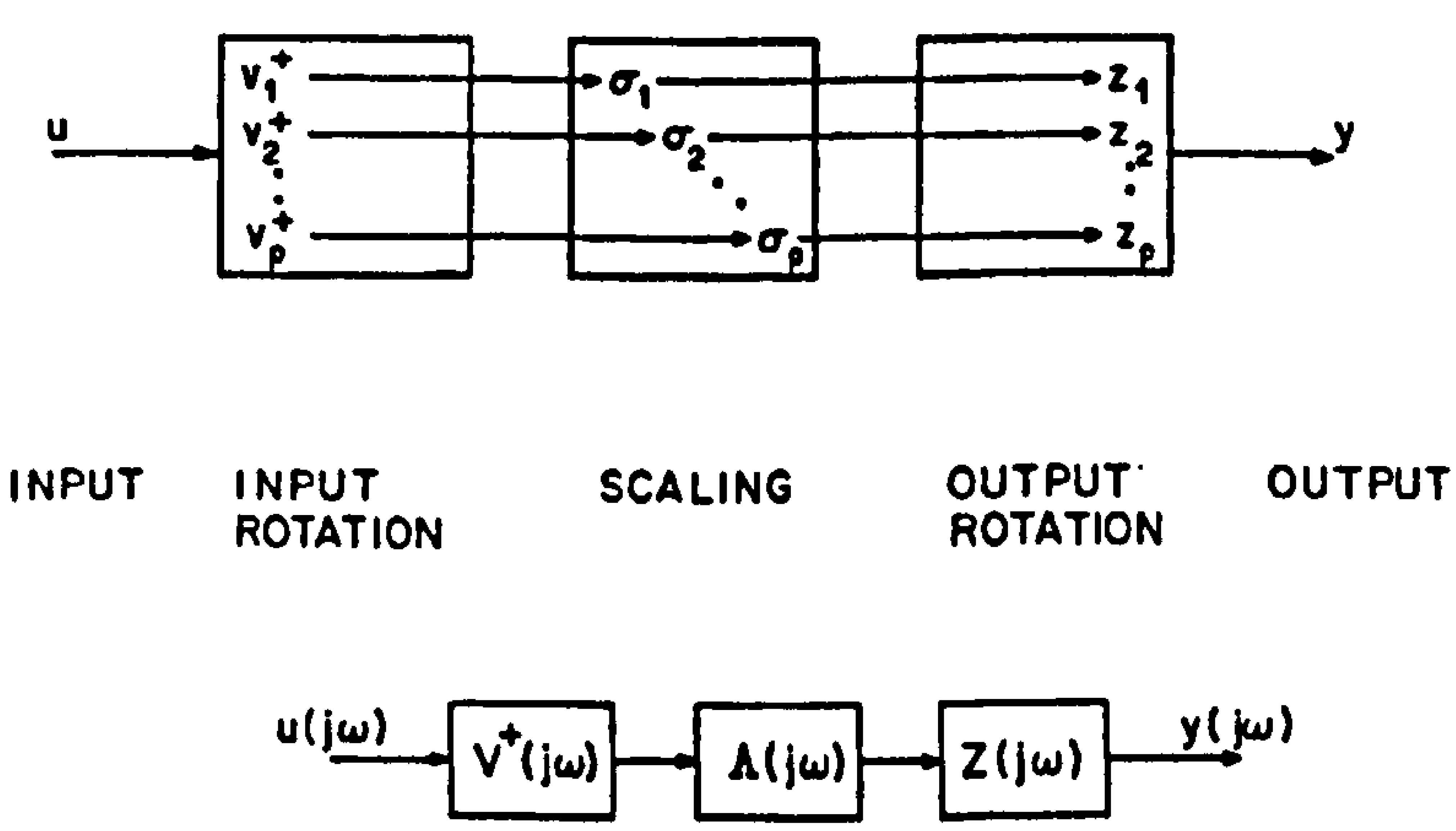


Figure (6.3) Geometric interpretation of the SVD. [Lau *et al.*, 2]

It is now possible to interpret the transfer function matrix geometrically. An input vector in the direction of $v_i^T(s)$ propagates through the input space, is scaled by the gain $\sigma_i(s)$, and reappears in the output direction $z_i(s)$.

From the above, it is easy to express $G(s)$ in terms of the singular values:

$$y(s) = \left[\sum_{1}^{p} \sigma_i(s) z_i(s) v_i^{\dagger}(s)\right] u(s)$$

By expanding $y(s)$ and $u(s)$ in the standard basis vectors $\{e_k^m\}_{k=1}^{m}$ and $\{e_j^n\}_{j=1}^{n}$, where the superscripts refer to the vector dimension, we finally obtain:

$$y_k(s) = \sum_{i=1}^{p} \sigma_i(s) \sum_{j=1}^{n} u_j(s) \langle W_i(s), E_{kj} \rangle$$

where $\langle W_i(s), E_{kj} \rangle \equiv \left(e_k^{m^T} z_i(s)\right)\left(v_i^T(s) e_j^n\right)$. The product $\langle W_i(s), E_{kj} \rangle$ may be interpreted geometrically as a measure of the alignment of the singular decomposition vectors $z_i(s)$ and $v_i^T(s)$ to the standard basis vectors in the appropriate space, as is illustrated in figure (6.4):

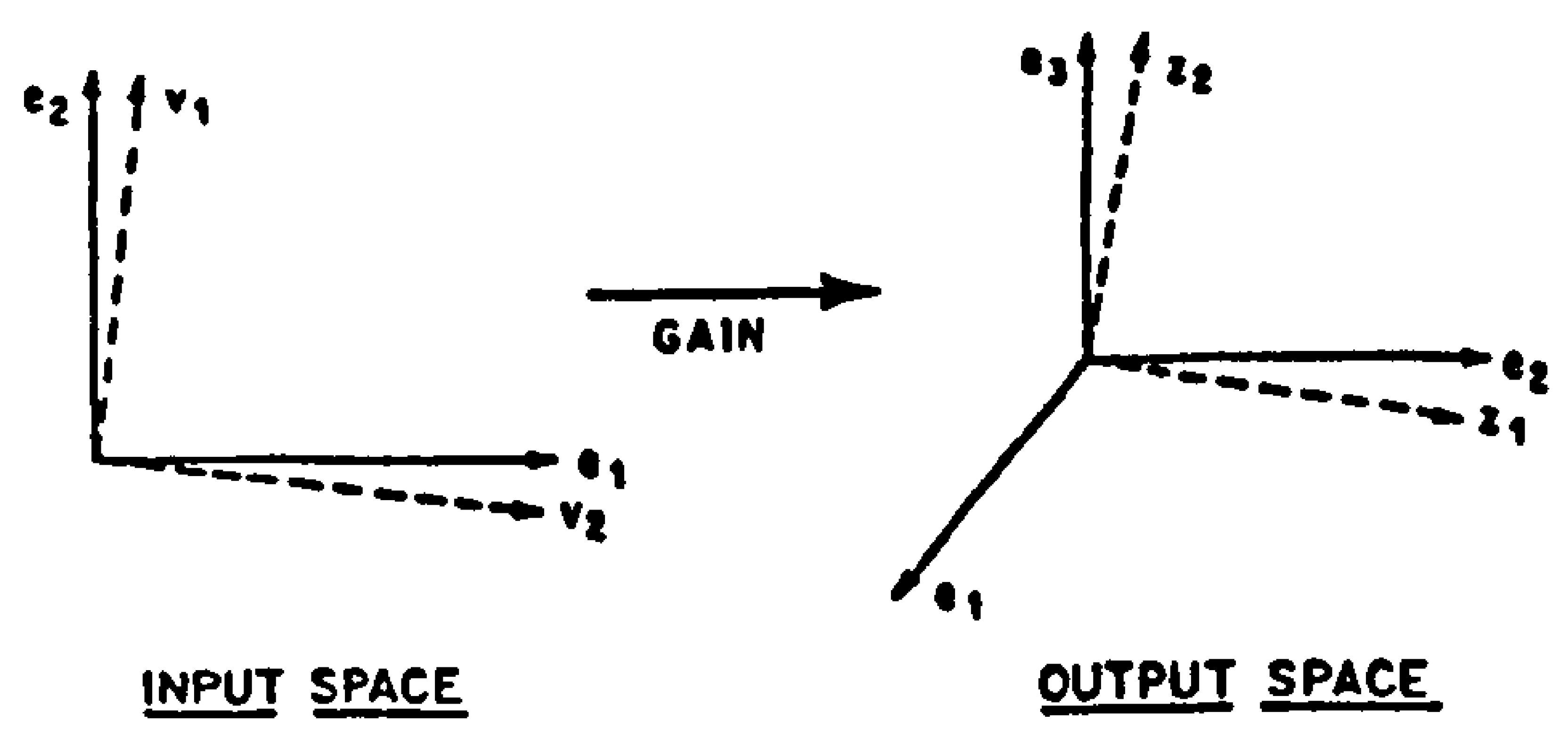


Figure (6.4) Pictorial representation of alignments between singular value vectors and standard basis vectors for a 3-input X 2-input system.

Suppose that for some i_o and for $s = j\omega$

$$\left|\left\langle W_{i_o}(j\omega), E_{k\ell}\right\rangle\right| \cong 1$$

This implies that the i_o dyad is aligned closely with the basis dyad defined by the kth output and the ℓth input, or alternatively defined the (k,ℓ)th loop. Since the basis vectors $\{v_i(j\omega)\}_1^p$ and $\{z_i(j\omega)\}_1^p$ are orthonormal sets, we have

$$\left|\left\langle W_{i_o}(j\omega), E_{k\ell}\right\rangle\right| \cong 0 \qquad \forall\, j \neq i_o$$

$$\left|\left\langle W_{i_o}(j\omega), E_{sp}\right\rangle\right| \cong 0 \quad p \neq \ell,\ s \neq k$$

thus, we can conclude that except when the system is poorly conditioned, the (k,ℓ)th loop interacts minimally with other loops which we may select to control the system. Therefore, the (k,ℓ)th loop is called the natural loop of the system.

The *SVD* can also be used for directional analysis. This can be formalised by starting with the following definition:

$$\theta_{i_o} = \arccos\left|\left\langle W_{i_o}, E_{k\ell}\right\rangle\right|$$

where θ_{i_o} is the angle between the dyad, W_{i_o}, and the basis dyad $E_{k\ell}$. When θ_{i_o} is less than 15°, over 95% in magnitude of the i_o dyad comes from the (k,ℓ)th term and consequently the (k,ℓ)th loop is defined to have good directional property. Also θ_{i_o} is restricted between 0° and 90° by definition. The derivation of the interaction measure can be found in [Lau *et al.*, 2].

To obtain a measure of how different the transfer function matrix G is from that of a completely decoupled system, the following **total interaction measure** can be used:

$$\theta = \arccos \sqrt{\left(\frac{\sum_{1}^{p} \sigma_i^2 \cos^2 \theta_i}{\sum_{1}^{p} \sigma_i^2} \right)}$$

where the individual angles θ_i are defined by $\theta_i = \arccos|\langle W_i, E_{kl} \rangle|$.

The physical interpretation of this total interaction measure is the ratio between the sums of the squares of the alignment of each dyad with a certain loop, weighted by the appropriate squared singular values and the perfect alignment ($\cos\theta$=1), weighted by the squared singular values. It can also be viewed as a geometric average of the contributions to the interaction by each node. If the maximum singular value is much greater than any other singular value and the corresponding alignment angle θ^* is of the same order of magnitude as the other alignment angles, then $\theta \cong \theta^*$.

By using singular value analysis and the angles defined above, many system properties can be characterised by plots similar to Bode diagrams (see chapter 8 for graphs/results). The singular values and the directional angles can be plotted as functions of frequency. The same applies to the condition number and the total interaction measure. Note that the condition number can be visualised by taking the distance between the maximum and minimum singular value in the first set of plots. The two sets are complementary because the former indicates the sensitivity and directionality of components in the system, whereas the latter depicts total system properties.

Singular values of the transfer function matrix may be used to evaluate stability margins for multi-input-multi-output (MIMO) systems in the same manner as the amplitude ratio is used in single-input/single-output (SISO) systems [Doyle & Stein, 1]. [Smith *et al.*, 1] have adapted *SVD* to the loop selection in a steady-state system. However, no measure of interaction or systematic search procedure is considered. [Morari, 2] used the *SVD* to quantify the control performance attainable in a process and interpreted implementability and sensitivity of the plant, concepts, which quantify the resiliency of the plant, in terms of the norms of the transfer function operator. It is interesting that two problems, at opposite ends of the hierarchy in process design and control structure synthesis, emerge closely related after the appropriate analysis. In addition, the *SVD* strategy can be used to identify modelling aspects, such as model mismatch, which affect the performance of the resulting control structure.

As a first attempt to design a control configuration, one could choose to synthesise a multi-loop control system. This approach is attractive because it is relatively easy to implement. However, it is well-known that interactions between loops may lead to tuning and stability problems. The key decision in this simple multi-loop control is the proper pairing of measured and manipulated variables. In some instance, there may exist natural interactions within the system, which should be exploited by the proper combination of variables [Foss, 1]; [Mor. & Stef., 1]. Moreover, the performance of the control system should be satisfactory over the frequency band of characteristic disturbances. Thus, the strategy to synthesise a control configuration should include a frequency domain analysis, particularly emphasising the frequency spectrum of disturbances, which affect the system. Since we are primarily interested in the control structure, rather than in the actual controller design, the analysis is based on the open loop transfer function. The singular value analysis provides a variety of indicators for selection of control structures, but it is far from being a complete methodology.

6.3.2 Robustness and the Relative Gain Array

Controllers are designed on the basis of inaccurate models and must be tuned such that stability is preserved, even when the system changes due to changes in operating conditions, for example. The ability of a closed-loop system to remain stable in the presence of model/plant mismatch is referred to as robustness. Though apparently the RGA cannot be used as a tool to determine when a closed-loop system will become unstable if the plant and model do not agree, it might provide some information on when a system is particularly "sensitive"; "ill conditioned" matrices display this sensitivity.

The search for a relationship between the condition number and the *RGA* of a process transfer matrix is spurred by the following observations. (1) The condition number is rigorously related to system sensitivity and robustness but is scale dependent. (2) A relationship between the *RGA* and sensitivity has only been demonstrated empirically, but the *RGA* has the advantage of being scale independent. (3) The elements of the *RGA* and the condition number display a striking mathematical resemblance. Indeed, the condition number can be related to the *RGA* (Grosdidier & Morari, 1985). This shows that the *RGA* is itself a measure of error sensitivity, a result which has been argued for in the past [Bristol, 1]; [Shinskey, 3]; [McAvoy, 1]. A difficulty, in attempting to link the *RGA* to the condition number, is the fact that, whereas the *RGA* is scale independent, the condition number is not. The latter is therefore a function of the units of the transfer matrix G. This problem can be circumvented by scaling the transfer matrix G with diagonal matrices, in such a way that a minimum or "optimal" condition number is obtained. Optimal scaling simply ensures that the least conservative value of the condition number is obtained and it should not be given a physical interpretation.

6.4 Control Structure and System Properties

6.4.1 Integral Controllers and Integral Stabilizability.

The term integral controller is used to designate PI and PID controllers or any multivariable feedback controller, which includes integral action. All such controllers can be decomposed into a matrix of integrators $\frac{k}{s} * I$ and a compensator matrix $C(s)$. Such controllers are widely used, due to their ability to guarantee zero error for step tracking and disturbance rejection.

Conditionally stable systems are clearly undesirable from a practical point of view. Not only is it difficult to determine the range of gains for which the closed loop system is stable, but this range is likely to change with evolving process operating conditions. The concepts of integral controllability and fault tolerance are based on this idea [Morari, 6]. An open-loop stable system H(s) is called integral controllable, if there exists a $k^{*} > 0$ such that the closed-loop system is stable for all values of k satisfying $0 < k < k^{*}$ and has zero tracking error, for asymptotically constant disturbances.

In this definition, the emphasis is placed on the existence of a range of positive gains starting from zero rather than any exact value. A practical consequence of this definition is that integral controllable systems can be tuned on-line, starting with a very low gain for which stability is guaranteed, and then increasing the gain, until acceptable performance is achieved.

Integral controllable systems are very desirable in practice. The control loops can be tuned starting from very small gains, and unstable closed-loop systems can easily be stabilised by decreasing the gain. On the other hand, for systems which are only integral stabilizable, increasing the gain might be necessary for stability and stability might only be maintained for a narrow range of gains. Using the newly introduced idea of integral controllability, it is also possible to strengthen the concept of failure sensitivity. The behaviour of such schemes under failure conditions is an important indicator, in deciding the nature of control structure.

The concepts of sensor and actuator failure designate actual hardware failure, as well as the saturation of a manipulated variable. Both sensor and actuator failure can have adverse consequences on a control system. The failure of a sensor, for example, means that an erratic electrical signal is sent to the controller. When this happens the integral controller will take action, with the aim of eliminating the offset between the received signal and the setpoint. Because the action of the controller is based on an erroneous input signal, it may be totally inappropriate for the system and ultimately lead to instabilities. Actuator failure, on the other hand, brings an end to all control action - an equally dangerous situation.

The control problems, created by the failure of a sensor or an actuator can be remedied by placing the controller in the failure loop in the off-line mode. In such a situation, it is desirable that without readjustments to the other parts of the control system, system stability be preserved.

A system is *j-sensor failure sensitive* (*j*-SFS) [Morari, 6], if the complete system is integral stabilizable but the reduced system, with the *j*th sensor removed ($k_j = 0$), is not. To make this definition meaningful, we assume that the failure has been recognised and that the loop, with the faulty sensor, has been taken out of service; i.e., k_j has been set to zero. The practical implications of this definition are straightforward. If the complete system is integral stabilizable, there exists a $k>0$, such that the closed loop system is stable. If the system is *j*-SFS, then the system will become unstable as soon as the *j*th sensor is removed ($k_j = 0$), regardless of controller tuning, i.e. regardless of how $k>0$ was chosen. More care has to be used in the definition of actuator failure sensitivity. If only (n-1) actuators are operating, only (n-1) variables can be controlled in an offset-free manner. Thus, any actuator failure requires that one controlled variable be left uncontrolled. The selection of control structure affects the above property, but it is not yet clear how to systematically explore it, without going through all possible alternatives.

6.4.2 Decentralised Integral Controllability

If it is desirable to select a control structure, so that separate controllers can be de-tuned or taken out of service, while maintaining the stability of the overall system, then the system has the property of *Decentralised Integral Controllability* (DIC). This topic was considered by [Mor. & Zaf., 1] and subsequently by [Nwok. *et al.*, 1], who have given some useful tests for *Decentralised Integral Controllability.*

Such tests are based on properties of $G(0)$ and, in particular, the positivity of such elements, as well as the existence of a diagonal controller of the form $K(s)D^{*}s^{-1}$ with integral action satisfying $k_i(0) > 0$, $d_i^{*} > 0$, $i = 1, 2, \ldots, n$ such that:

1. the zeros of $\det\left(I + G(s)K(s)D^{*}s^{-1}\right) = 0$ are in the left half of the complex plane;

2. these same zeros continue to remain in the left half plane, for all real positive diagonal, $D = diag(d_1, d_2, \cdots, d_n)$ satisfying $0 \le d_i \le d_i^{*}$.

The assumption, that the diagonal elements of $G(0)$ are all positive, does not restrict the generality of the test, because it is possible to multiply each column with $+1$ or -1, so that all diagonal elements become positive.

The overall philosophy in the study of this problem is to derive necessary conditions and if possible, also some sufficient conditions based on the steady state value $G(0)$ of the open loop transfer function, as well as relative error matrix and relative gain array, such that for the scheme of fig.(6.5) there exists a diagonal controller K(s) which satisfies the condition of Decentralised Integral Controllability.

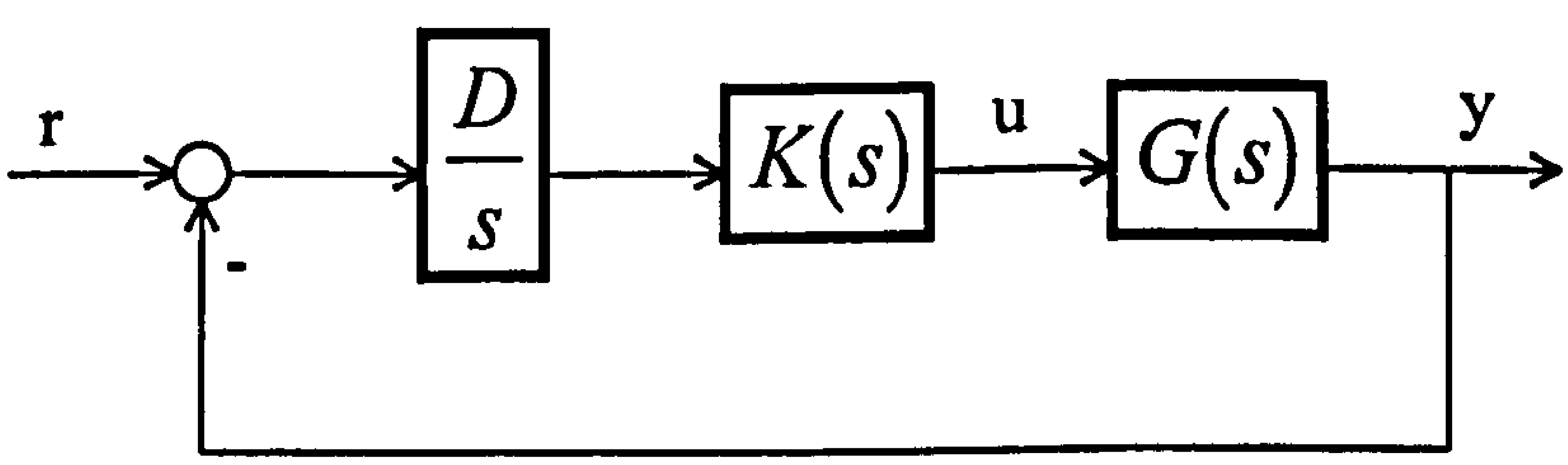


Figure (6.5)

6.4.3 Selection of Control Structure for Load Disturbance Attenuation.

Many researchers have concentrated on choosing the structure, such that interaction among the control loops is minimised. The use of decouplers assumes coupling is undesirable. The much-used *Relative Gain Array* (*RGA*) method and the *Inverse Nyquist Array* (INA) method are based on the assumption that control loop interaction is bad. This may be true in systems, where set-point changes are the principal disturbance. But set-point disturbances are usually much less frequent in chemical process control, than load disturbances. Most industrial applications require a control system that can hold the process at desired values of performance (composition, yield, etc.) in the face of load disturbances, such as variations in feed composition and throughput. In fact, as [Nied., 2] pointed out over two decades ago, designing control systems, such that they are noninteracting, can degrade the performance of the system in rejecting load disturbances.

Each process has an intrinsically self-regulating control structure, which makes the system as insensitive as possible to load disturbances and is self-optimizing. An alternative control system design philosophy has been proposed, that contain as its first step the use of the notion of eigenstructure, which deals with control structure selection, based on load disturbance rejection properties, rather than noninteraction. Several researchers have dealt with this problem, such as [Buckley, 1], dealing with this overall plant control strategy, [Luyben, 2] examining the impact of steady-state energy consumption on control system structure, [Douglas, 1], [Fisher *et al.*, 2], dealing with steady-state plant-wide control issues, [Stanley *et al.*, 1] considering the *Relative Disturbance Gain*, [Tyreus, 1], with his integration of steady-state optimization into the regulatory control structure; [Georgk., 1], with his reaction rate or extensive variable control.

[Buckley, 1] approached the problem of plant-wide control by splitting the problem horizontally, i.e., considering the slow material-balance control structure of the entire plant, first, and then, later, establishing the faster composition control structure of each individual unit. This two-level approach is in contrast to the vertical splitting of the plant-wide control problem, that most academics attempt to employ, i.e., slicing the plant up into many little sub-units in series. Buckley's material-balance control structure (slow liquid level and gas pressure loops) produced a plant, that intrinsically handled disturbances well. The effects of load changes were attenuated, as they worked their way through the process. This slow material-balance control is a component of what it is referred to as eigenstructure.

[Luyben, 2] pointed out that the optimum control structure, that minimised energy consumption in distillation columns, required controlling product compositions at both ends of the column (dual composition control). However, he suggested that a more simple, single-end control structure, that used very little additional energy, could often be found. Steady-state rating programs were used to calculate how the manipulated variables (vapour boil-up reflux, and reflux ratio) had to change at steady state as feed composition changed, over the expected range of variability, to keep product compositions constant at their specified values. If one of these manipulated variables was found to vary only slightly, a simple control structure was recommended to hold this variable constant at its maximum value and manipulate the other variable to control the composition of only one product. Feed rate disturbances could be effectively handled by ratio schemes in a feed-forward sense. This is one of the first examples of finding an eigenstructure that yields a self-optimizing and more stable control system. This same basic notion was extended to an entire chemical plant by Douglas. The paper by [Fisher *et al.*, 2] proposed the use of steady-state rating programs to find the optimum operating conditions of the overall plant for different disturbances. Then simple relationships were found between controlled and manipulated variables, such that the system was inherently held at or near the optimum point.

Load disturbances are considered in the search for an eigenstructure. [Tyreus, 1] has recently proposed a design procedure that combines the concepts of Buckley, Douglas, and Luyben. [Georgk., 1] has proposed the control of calculated "extensive" variables, such as reaction rate, total energy content, or total light component content, instead of the traditional intensive variables (temperature, pressure, etc.). Manipulated variables can also be chosen to be sums, differences, or ratios of flow rates. The resulting eigenstructure handles disturbances more effectively.

6.4.4 Nyquist arrays and Gershgorin Bands – Diagonal Dominance.

The Nyquist array of $G(s)$ is an array of graphs, the (i,j)[th] graph being the Nyquist locus of $g_{ij}(s)$. The inverse Nyquist array (defined only when $G(j\omega)$ is square) is the array of graphs of Nyquist loci of the elements of $G^{-1}(s)$.

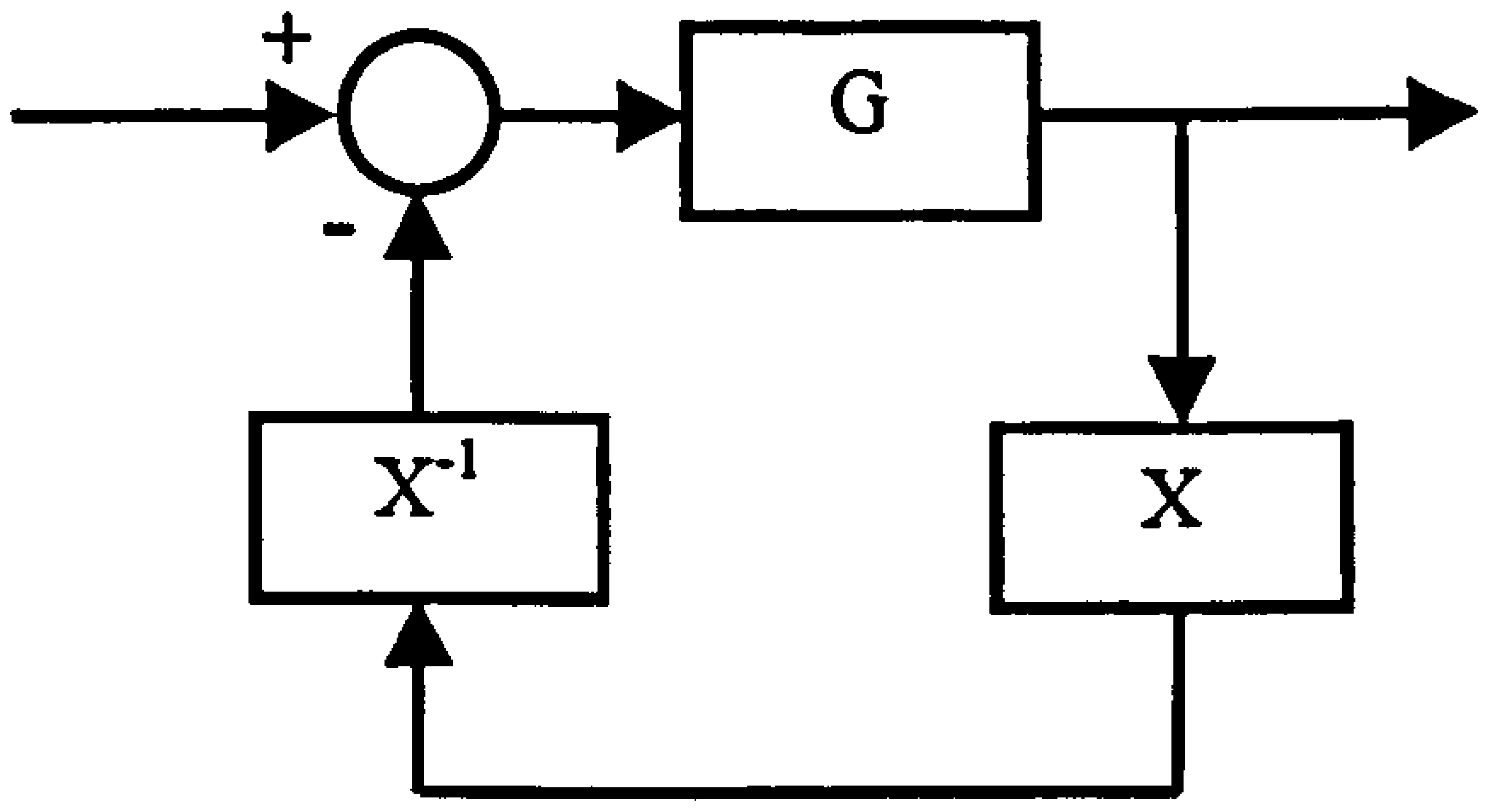
If the Gershgorin bands of $G(s)$ exclude the origin, then $G(s)$ is said to be diagonally dominant (row dominant or column dominant, if applicable). Note that for stability $[I+G(s)]$ must be diagonally dominant.

The greater the degree of dominance (of $G(s)$ or of $I+G(s)$) – that is, the narrower the Gershgorin bands – the more closely does $G(s)$ resemble m non-interacting SISO transfer functions.

These Nyquist-array-based tests check sufficient, but not necessary, conditions for stability (or instability). If any Gershgorin band does overlap the point –1, in other words if $I+G(or\ I+G^{-1})$ is not diagonally dominant, then we cannot infer whether the system is stable or not. But suppose that we replace G by

$$\tilde{G} = XGX^{-1}$$

This is a similarity transformation, so the characteristic loci of $\tilde{G}$ must be the same as those of G, and hence we can check stability by displaying Gershgorin bands of $\tilde{G}$ (or $\tilde{G}^{-1}$) just as well as we can with those of G. Physically, the last equation corresponds to inserting a system X at the output of G, and X^{-1} at the input of G, as shown in the following figure:



Figure(6.6): Feedback loop containing the system defined by system equation.

The point of doing this is that $\tilde{G}$ may be diagonally dominant, even if G is not, for some choices of X.

A method for finding a suitable X [Mees, 1], is the following: For any matrix M with elements $m_{i,j}$, we define

$$\text{abs } M = \left[\left|m_{i,j}\right|\right]$$

that is, the elements of abs M are $\left|m_{i,j}\right|$. Such a matrix is called positive. If M is square, let the eigenvalues of abs M be $\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$, and let them be ordered so that $|\lambda_1| \geq |\lambda_2| \geq \cdots$. M is called primitive if (abs $M)^r$ has positive entries only for some integer *r*. For primitive positive matrices λ_1 is real and $\lambda_1 > |\lambda_i|$ for $i \neq 1$; λ_1 is called the Perron-Frobenius eigenvalue of M, and we shall denote it by $\lambda_p(M)$. The corresponding left and right eigenvectors of abs M are called the Perron-Frobenius eigenvectors of M (and are also real and positive).

Now, let

$$M_{diag} = diag\{m_{11}, m_{22}, \ldots, m_{nm}\}$$

[Mees, 1] proves the following theorem:

<u>**Theorem (6.1):**</u> If G is square and primitive, then there exists a diagonal matrix X such that

$$\tilde{G} = XGX^{-1}$$

is diagonally dominant, if and only if

$$\lambda_p\left(GG_{diag}^{-1}\right) < 2$$

If the last condition is satisfied, and the Perron-Frobenious left eigenvector of GG_{diag}^{-1} is $(x_1, x_2, \ldots, x_n)^T$, then an X which achieves diagonal dominance is

$$X = diag\{x_1, x_2, \ldots, x_n\}$$

Note that the X found by this theorem can be interpreted as a scaling of the outputs of G, and X^{-1} as a scaling of the inputs of G, since X is diagonal and real. A different X is required at each frequency, of course; this is no problem if we are using X only for analysis, since X does not correspond to a system that has to be built. This theory can also be used for design, and in that case, we shall have to build systems whose gain behaviour approximates that of X as frequency varies.

The Perron-Frobenious theory allows us to check whether a plant can be made diagonally dominant by input and output scaling. This can clearly be applied to design, but with a proviso: in general, we must be careful about using output scaling as part of the strategy for achieving diagonal dominance. On the face of it, output scaling corresponds to inserting a post-compensator (that is, inserting a compensator between the outputs and the variables being controlled). This is physically impossible, since the meaningful plant outputs (which are variables such as velocity, or thickness of steel strip) cannot be affected by mathematical operations. It is certainly possible to change the scaling of the measurements of the output variables, however, by placing the post-compensator in the feedback path.

One can use the scalings S^{-1}, at the plant input, and S, in the feedback path, to obtain a diagonally dominant return ratio. But we must be wary of falling into the trap of believing that this return ratio tells us anything about interaction at the plant output. The output variables may be interacting with each other to a considerable extend, and this interaction may be being hidden by the measurement scaling S. Thus the choice of measurement scaling is really part of the specification of the feedback design problem, and changing this scaling is not usually admissible as a step in a design technique. The output scaling is, in effect, built into the definition of the plant. If the specification is rather loose, however (for example, that closed-loop stability and good long-term following of set points are to be achieved, with no specification on interaction), then output scaling may be considered. Fortunately, the Perron-Frobenius theory gives useful results, even if only pre-compensation (input scaling) is allowed.

6.4.5 Structural Diagnostics

The selection of control structure is an important problem within the area of integrated systems and control design [Karc., 3] that has not taken much attention in the mainstream control literature so far. This problem involves two important subproblems: the first deals with the selection of input output structure of the system model and the second with the structuring of the control scheme, i.e. deciding on centralisation versus decentralisation and if decentralised solutions are sought, then defining the nature of decentralisation. The study of pole assignability under decentralised control schemes provides criteria that link the control structure selection to the underlying invariant system structure. In fact, the Markov matrix arising as the representation of the differential of the pole placement map, characterises both pole assignability as well as absence of fixed modes for the system. As for different decentralisation schemes, we get different submatrices of the same full Markov Matrix, we have thus, the means for a simple evaluation of the alternative decentralisation schemes by choosing that scheme corresponding to the best conditioned submatrix. This selection can be based on criteria that guarantee avoidance of fixed modes and well conditioning for arbitrary pole assignability.

[Lev. & Karc., 1] present a method that extends the results previously derived for the properties of the centralised pole placement map under complex and real output feedback [Lev. & Karc., 3] to the case of decentralised constant output feedback and investigate some general properties related to measuring the size of the set of polynomials for a given system that can be assigned. Instrumental in the above study is the differential of the decentralised pole placement map (DPPM), which provides a measure of the size of the set at a generic decentralised feedback K_{dec}. The differential of DPPM is computed and this provides a link with the decentralised Plucker matrix of the problem [Karc., *et al.*, 1] and a set of state space based parameters which incorporate the decentralisation structure and are referred to as decentralised Markov Parameters (DMP). In fact, it is shown that the matrix formed by the DMPs is a submatrix in the decentralised Plucker matrix. By ensuring that, the matrix formed by the DMPs and referred to as the decentralised Markov Parameter Matrix (DMPM), has full rank n (n is the number of states), we ensure the absence of fixed modes, as well as the good conditioning for the Plucker matrix to have full rank; the latter property guarantees that we also avoid the formation of almost fixed modes and satisfy the necessary condition for complete assignment by decentralised output feedback [Karc., *et al.*, 1]. The link of DMPM to the decentralised Plucker matrix enables the tackling of two important problems:

(i) The screening of all possible decentralisation alternatives and

(ii) The introduction of a natural set up for design, redesign of C, B matrices to guarantee certain important properties for the system.

In fact, the computation of DMPMs for the different decentralisation schemes is straightforward, avoids exterior algebra computations and requires only knowledge of the system Markov Parameters. The link of Markov parameters to the Plucker matrices provides a mechanism for affecting the shaping of the properties of Plucker matrices by intervening appropriately in the shaping of the properties of the Markov parameters. The results which were originally presented for the decentralised constant output feedback were then extended to the case of decentralised PI compensation.

The pole placement map under the decentralisation assumption [Lev. & Karc., 3]

The centralised pole placement map assigns, to every feedback compensator, the closed loop poles or coefficient vector of the closed loop polynomial (modulo scaling) or any other quantity corresponding uniquely to the close loop poles. Therefore, the centralised pole placement map (excluding infinite poles) X^c is a map of the form $X^c\colon F^{mp} \rightarrow F^n$. A central role in the study of assignability properties of a system, as well as the determination of fixed modes plays the differential of the pole placement map. This can be calculated by first rewriting the closed loop polynomial as:

$$p(s) = \det\big(D(s)(I + KG(s))\big) \tag{6.25}$$

and then expanding the above as:

$$p(s) = f(s)\left(1 + \sum_{i,j}(-1)^{i+j} k_{ij} g_{ij}(s) + higher \ order \ terms\right) \tag{6.25}$$

or equivalently:

$$p(s) = f(s) + \sum_{i,j}(-1)^{i+j} k_{ij} f_{ji}(s) + h.o.t \tag{6.26}$$

where $f(s) = \det D(s)$. The highest order coefficient of $p(s)$ is equal to:

$$p_n = 1 + \sum_{i,j}(-1)^{i+j} k_{ij} f_{jin} + h.o.t = 1 + k^t \cdot f + h.o.t \tag{6.27}$$

where $f_{ji}(s) = f(s) g_{ji}(s)$, $g_{ji}(s)$ are the elements of $G(s)$ and f_{jin} is the n-th order coefficient of $f_{ji}(s)$ (Coefficient of s^n). This analysis leads to:

Proposition (6.1): The differential of X^d at K=0 is equal to

$$D(X^c)_{K=0} = \mathcal{D} - \underline{f} \cdot \underline{f}_n^T \tag{6.28}$$

where $\mathcal{D}$ is the coefficient matrix of the polynomial vector $\{f_{ji}(s)\}_{ij}$, $\underline{f}_n^T$ is the coefficient vector of s^n of $\{f_{ji}(s)\}_{ij}$ and $\underline{f}$ is the coefficient vector of $f(s)$.

A state-space representation of this differential is given by:

Proposition (6.2): A matrix representation of $(D_{X^c})_{K=0}$, denoted by $R(X^c)_K$, with respect to the basis $(-1)^{i+j}\left(\frac{\partial}{\partial Kij}\right)$ of $T(F^{mp})_K(a \in \Omega)$, is a $n \times mp$ matrix given by:

$$Q^T\left[\text{col}CB, \text{col}CAB, \cdots, \text{col}CA^{n-1}B\right]^T \tag{6.29}$$

where 'col' maps an $m \times p$ matrix to the $mp \times 1$ matrix formed by superimposing its columns, and Q is given by:

$$Q = \begin{bmatrix} 1 & f_1 & \cdots & f_{n-1} \\ 0 & 1 & & f_{n-2} \\ \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \tag{6.30}$$

where the f_i's are the coefficients of the open loop pole polynomial $f(s)$.

The above analysis for the centralised case has been extended in [Lev. & Karc., 2] to the decentralised case and this in turn provides systematic procedures for screening alternative decentralisation structures.

□

Any structured pole placement map X^s with characteristic χ^s can be viewed as a restriction of the centralised pole placement map X^s on $F^{|\Omega_s|}$ and can be factored as

$$X^s: \; F^{|\Omega_s|} \xrightarrow{E} F^{m\times p} \xrightarrow{X^s} F^n \tag{6.31}$$

where E is the natural injection of the set of structured gains into the set of all gains. Therefore, the differential X^s at K is given by:

$$D(X^s)_K = D(X^c)_{E(K)} \circ D(E)_K \tag{6.31}$$

The differential $D(E)_K$ has a very simple structure induced by the natural injection $F^{|\Omega|} \to F^{m\times p}$, i.e.

$$D(E)_K\left(\sum_{\omega\in\Omega_s} x_\omega e_\omega\right) = \sum_{\omega\in\Omega_s} x_\omega e_\omega + \sum_{\omega\notin\Omega_s} 0 e_\omega \tag{6.32}$$

where $\{e_\omega\}_{\omega\in\Omega_s} \subseteq \{e_\omega\}_{\omega\in\Omega}$ are the bases for the tangent spaces $T_K\left(F^{|\Omega_s|}\right) \subseteq T_{E(K)}F^{m\times p}$ respectively. In other words, $D(X^s)_K$ can be represented as a submatrix of the representation of $D(X^c)_{E(K)}$ formed by those columns corresponding to $\omega \in \Omega_s$. The above analysis may be summarised as follows:

<u>**Theorem (6.2)**</u>: A representation of the differential at K=0 of the structured pole placement map X^s corresponding to Ω_s is given by:

$$Q^T\left[\mathrm{col}\hat{C}B, \mathrm{col}\hat{CA}B, \cdots, \mathrm{col}\hat{CA^{n-1}}B\right]^T \tag{6.33}$$

where $\mathrm{col}\hat{CA^i}B$ denotes the reduced column obtained from $\mathrm{col}CA^iB$ after eliminating all the entries that do not correspond to the set of indices Ω_s.

Remark (6.1): The Markov parameter corresponding to constant term D, does not enter the definition of the Markov matrix and thus does not affect its properties.

Corollary (6.1): The rank of the differential of the pole placement map $D(X^c)_{K=0}$ is equal to:

$$rank\left[\mathrm{col}\hat{C}B, \mathrm{col}\hat{C}\hat{A}B, \cdots, \mathrm{col}\hat{C}\hat{A}^{n-1}B\right] \quad (6.34)$$

The matrix $M_d = \left[\mathrm{col}\hat{C}B, \mathrm{col}\hat{C}\hat{A}B, \cdots, \mathrm{col}\hat{C}\hat{A}^{n-1}B\right] = \left[\hat{m}_1; \hat{m}_2; \cdots; \hat{m}_n\right]$ is referred to as *Structured or Decentralised Markov Matrix.* The rank of the structured (decentralised) Markov matrix may be used as an estimate of the size of the set of assignable polynomials via static (structured) decentralised output feedback. The definition of the *Decentralised Markov Parameters* can provide alternative simple tests for avoiding fixed modes and for satisfying the linear assignment condition for decentralised constant output feedback. The simple method for constructing the DMP from the centralised and the structure of the decentralised feedback suggests that the screening of all possible decentralisation schemes may be achieved in a very efficient way from the set of standard Markov parameters. Thus, the alternative exterior algebra tests, based on the rank of the Decentralised Plucker matrix may be avoided in the first instance. The Decentralised Plucker matrix requires exterior algebra computations that can be carried out with the use of the corresponding toolbox [Mitr. & Karc., 1] and its full rank provides a necessary and sufficient condition for avoiding the presence of fixed modes; this matrix can be used for further testing fixed modes, if all DMPs are rank deficient. The testing for fixed modes depends on the decentralisation structure of the compensator and it is independent of its dynamics; however the rank of the corresponding dynamic decentralised Plucker matrix is a function of the compensator dynamics. The main steps of the procedure are summarised below:

(1) Define the physical decentralisation set and from this all feasible sets.

(2) Starting from the set of Markov parameters $\left[D, CB, CAB, \cdots, CA^{n-1}B\right]$ form the Markov matrix: $M = \left[\mathrm{col}CB, \mathrm{col}CAB, \cdots, CA^{n-1}B\right]$.

(3) Calculate the matrix: $M_1 = MQ[e_n(s_1),\cdots,e_n(s_k)]$ where $e_n(s)=[s^n,\cdots,1]'$ and $s_1,\cdots,s_k$ are the unstable eigenvalues of A.

(4) Calculate the minimum column norm of the submatrices of M_1 corresponding to all feasible sets whose cardinality is equal to n. The column norms indicate how far from zero these are and as each column correspond to an unstable pole, these norms provide a measure for the absence of fixed modes [Karc. *et al,* 1]. For every submatrix, the minimum of the norms of its columns corresponds to that unstable pole who is closer to being fixed. Taking the maximum of these minimum numbers over all selections will provide us with that selection which is the "least worse", as far as the existence of a fixed unstable pole is concerned.

The procedure for selection of decentralisation, based on Markov parameters uses the numerical values of the model parameters. Generic solvability conditions such as those developed in [Lev. & Karc., 1] may be used to provide a first listing of possible decentralisation schemes; such a combination of results may limit the large number of possible combinations to be tested. An additional advantage of the Markov Parameter framework, centralised, or decentralised, is that due to its direct link to the state space description, it provides the means for modifying the selection of the B, or C, matrices, such that the centralised or decentralised Markov matrix has full rank and thus achieve the very important linear assignment property which excludes the presence of fixed and almost fixed modes and preconditions well the system to accept a certain type of control solution. The current results establish a framework and provide the tools for affecting the shaping of properties of centralised, or decentralised Plucker matrices at early design stages.

6.5. GRAPH TYPE METHODOLOGIES

6.5.1 Graphs and Process Flowsheets

Process systems are composite structures made from different types of units and connected in a certain way, usually referred to as process flowsheet. According to the degree of modelling, the composite structure takes different forms, but the general interconnection rule evolves within this new set-up. Graph methodologies describe the main features of this evolving composite structure and play a significant role in determining properties of decomposition and control structure selection. In this section, we review the methodologies of the graph type, as these have appeared in the process literature.

Synthesis of configuration, using the static input-output characteristics of the process, has been explored by [Bristol, 1] and [Weber & Bros., 1], the latter including also the effects of measurement errors. However, an approach that does not include the effects of process dynamics is not sufficient for synthesising process control systems. [Nied. 1] proposed that control links could be selected from a multivariable generalisation of the simple loop gain-bandwidth criterion. [Gov. & Pow., 4], have addressed a basic problem of converting a steady-state process flowsheet into a piping and instrument diagram. A flowsheet defines the equipment in the process and the streams which interconnect them. Also, the control objectives must be stated for the process. A large variety of methods have emerged, which are based on the concept of graphs and they are examined here.

The "graph theoretic approach" is an attempt to model a large scale system by a suitably chosen graph representation. Based on this representation, important properties such as decomposability, structural controllability, and observability can be checked. The problems of pole placement, disturbance rejection, and decoupling by static state feedback can be investigated from the graph theoretic point of view. The graph approach is based on a mapping of state space equations into a directed graph, represented as Boolean matrices in the computer. Many authors have presented various methods related to control structure synthesis. Some of them are being dealt within the following paragraphs, with the main points of their respective methodologies in perspective.

6.5.2. Piping and Instrumentation Diagrams

[Gov. & Pow., 4], have proposed a systematic procedure to generate alternative control structures, based on the cause-and-effect representation of a process. The final product is a set of control schemes from which the final system may be selected or evolved. The work is significant in that it is the first attempt to apply non-numerical problem-solving techniques to the problem of synthesising process control structures. The approach is based on three main ideas:

1. The models used in the synthesis of control systems, must be simple.
2. The propagation of control constraints through the process flow diagram will generate the candidate control structures.
3. Evaluation of alternate control structures will depend primarily on: (a) the feasibility and simplicity of measuring and manipulating candidate control structure variables; and (b) the steady-state interactions which occur between control substructures within the process. Detailed dynamic performance needs to be considered only when the alternate control structures have strong dynamic interactions or the dynamic achievements of the control objective are questionable.

Hence, this approach has a strong component of steady-state control [Buckley, 1]; [Shinskey, 1], integrated with dynamic considerations, when it is indicated. The alternate structures are evaluated using heuristics derived from both theoretical and practical considerations. The final product is a set of control schemes (say 5 to 10) from which the final system may be selected or evolved. The control objectives must be stated for the process. The safety parameters, operational constraints, production specifications, and environmental regulations can be specified relatively easily. However, the variables that govern the economic performance are not so easily specified as control objectives. In fact, the statement of the economic objectives is tantamount to writing the objective function (both steady-state and dynamic) for the complete system.

These control objectives place constraints on the variables within the process. If properly stated, objectives can be the starting points for the synthesis of control structures. The control objective is firstly translated into the process variables that might be controlled. In cases where the control objective directly translates into variables which can be sensed, the problem is to find the appropriate set of variables, which must be sensed or manipulated. In other cases where it may not be feasible or economical to measure the control objective variable, secondary variables related to the objective variables have to be measured. The important point is to know the structural and dynamic relationships between the control objective and the secondary variables. The sensed variable selection algorithm generates ways in which the constraint variable can be computed or estimated from other process variable measurements. This results in a set of derived constraints, each of which have to be satisfied in order to satisfy the primary constraint. Algorithms for the synthesis of feedback, feedforward and cascade control structures are then executed for the derived constraint to obtain structures for controlling the primary constraint. Finally, an evaluation function is used to screen the possibilities to obtain a small set of potential control structures for the process.

Graph Modelling

The chemical process is represented by a directed graph called the *Process Flowsheet Graph* (PFG). The *Process Flowsheet Graph* represents the material flow in the process, wherein the nodes designate the stream flow-rates and the directed edges, the units in the process. A directed edge exists from node *i* to node *j*, if material flows from the stream represented by node *i* to the stream designated by node *j*. Figure 6.7 illustrates a simple liquid–liquid heat exchange process, and its process flowsheet graph is shown in Figure 6.8.

The structure of the *Process Flowsheet Graph* (PFG) directly reflects the structure of the process. A recycle loop in the process is represented by a feedback loop in the PFG and a bypass, in the process, results in a feedforward loop in the graph.

The unit operations in the chemical process are represented at two levels: 1. equation level, and 2. cause-and-effect level. At the equation level, each unit operation is represented by its transfer function, that relates each output variable to the input variables. The transfer function, which is in the Laplace domain, designates the steady-state gain and dynamics, associated with the cause-and-effects between the input and output variables. In general, a unit operator can be represented by the following algebraic equations:

$$O_i(s)=\sum_{j=1}^{m} G_{ij}(s) I_j(s) \quad i \in n$$

where

$O_i(s)$ = ith output variable in the Laplace domain,
$G_{ij}(s)$ = transfer function between the input variable $I_j(s)$ and output variable $O_i(s)$,
$I_j(s)$ = jth input variable in the Laplace domain,
m = number of input variables,
n = number of output variables.

The structure of the equations is represented by a table called the "structural array" (Rudd & Watson, 1]. The columns in the array correspond to all the variables and the rows correspond to all the equations. An X is placed whenever a variable appears in an equation. The direction of the edges in the cause-and-effect graph indicates the causality of the process and are derived from the process transfer function equations. Heuristically, the notion of causality is intended as the property that the present value of the output (effect) of a physical system is not affected by future values of the input (cause). Based on this notion, the input variables on the right hand side of a dynamic equation in the Laplace domain affect the output variable on the left-hand side.

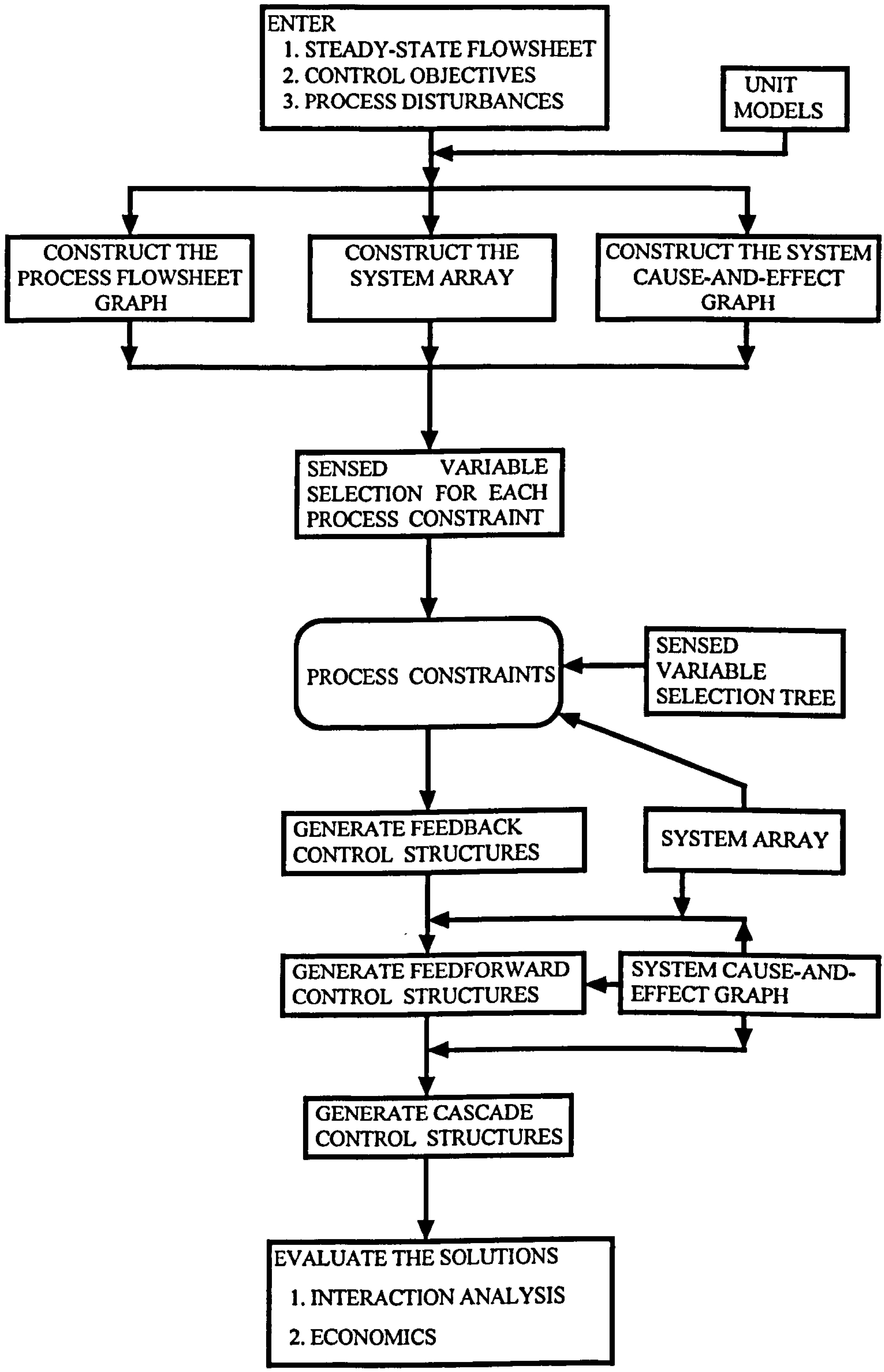


Figure 6.6. Algorithm to transform a steady-state flowsheet into a piping & instrument diagram [Gov. & Pow., 4].

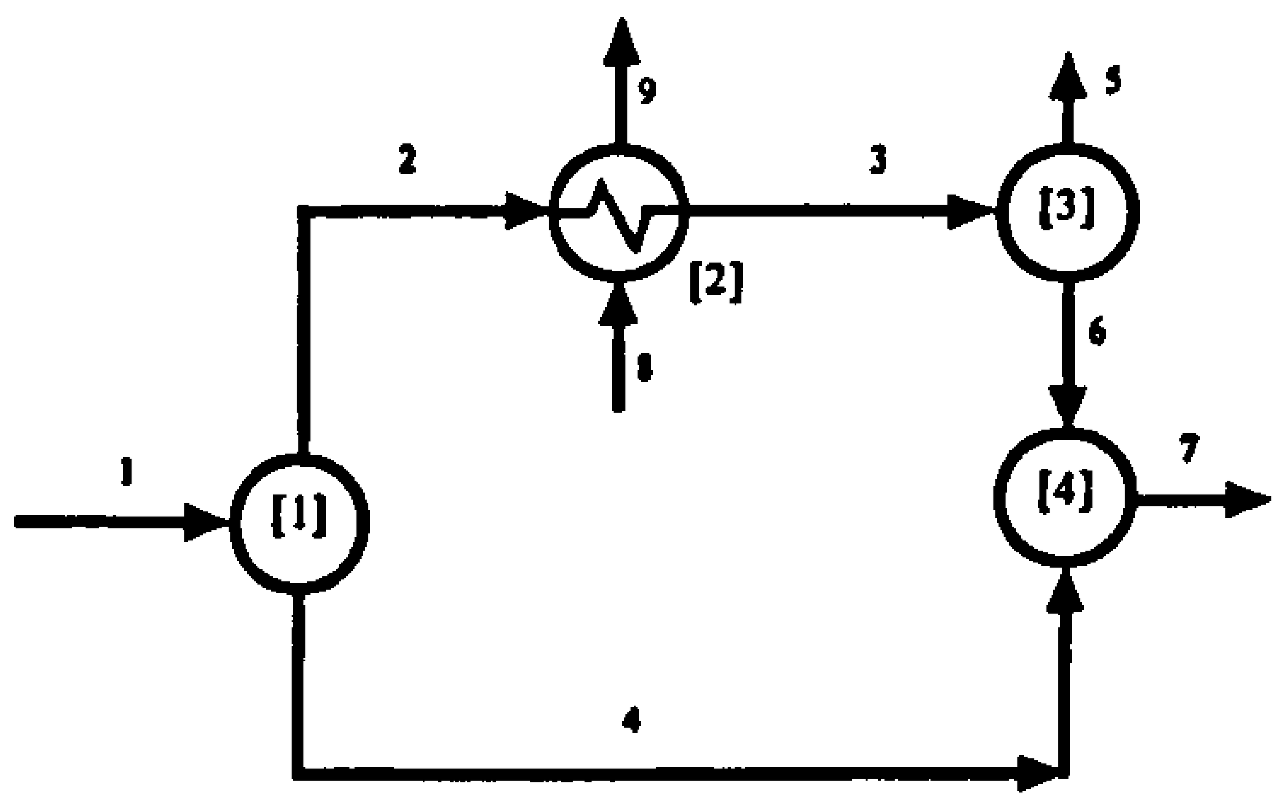


Figure 6.7. Heat exchanger process [Gov. & Pow., 4].

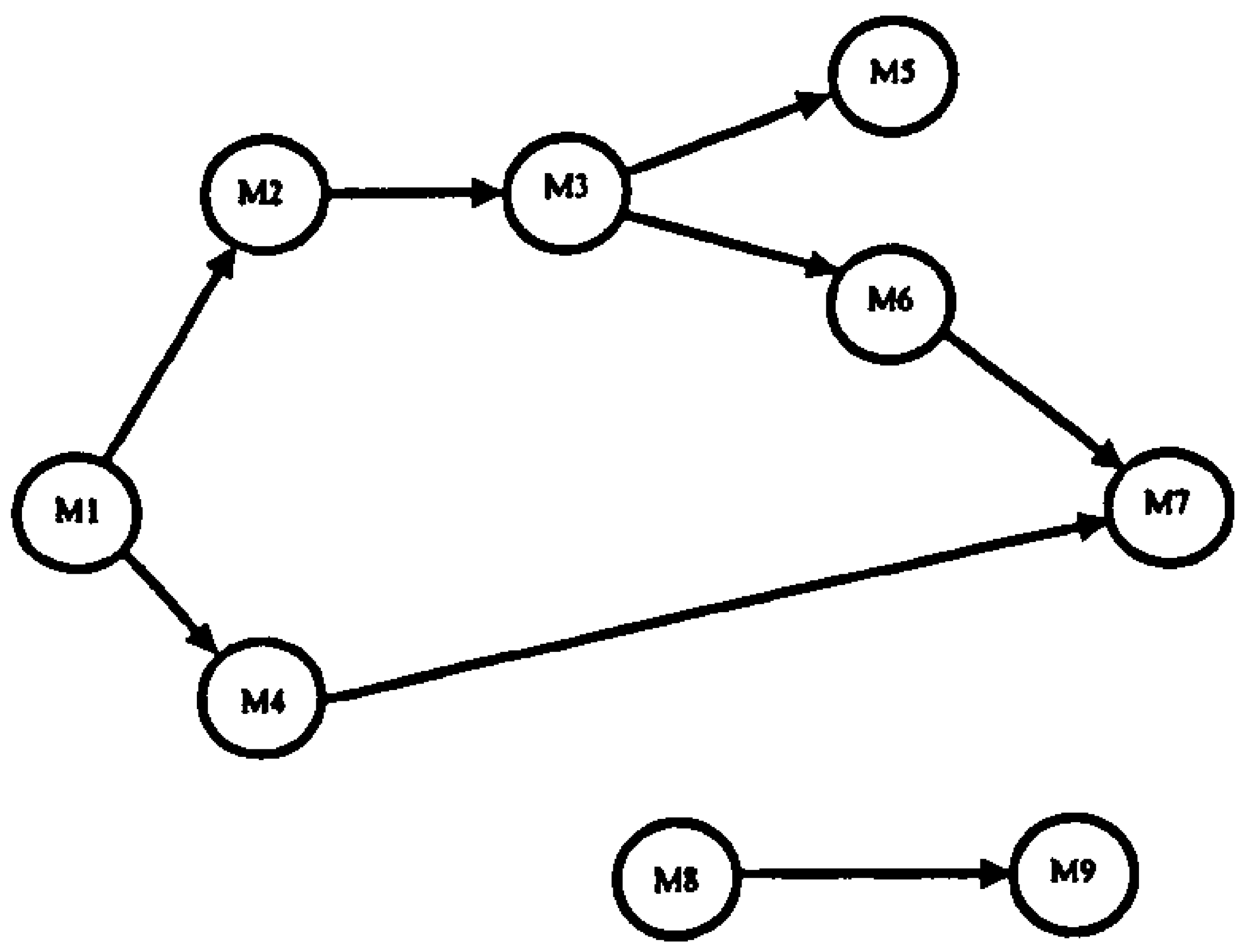


Figure 6.8. Process flowsheet graph [Gov. & Pow., 4].

The cause-and-effect graph is derived directly from the process dynamic equations in the Laplace domain. The direction of the edges, corresponding to steady-state equations, are drawn from a physical understanding of the unit operations. It is important to note that a cause-and-effect graph is different from a signal flow graph, since an edge on the signal flow graph does not represent causality. It simply indicates that the output variable can be computed from the input variables. The cause-and-effect graph allows easy detection of variable interactions that would normally be difficult to derive from the process equations. The structure of the graph also has a very important bearing on the controllability and stability of the system [Lin, 1]; [Gov. & Pow., 2]. An edge on the cause-and-effect graph is associated with a transfer function that relates the output variable to the input variable. This transfer function gives the steady-state gain, time constants and dead time associated with the cause-and-effect. These parameters can also be identified experimentally, if the process already exists. Simple methods for defining the effective lag and time constants of the process have been presented in the literature [Gilib. and Lees, 1].

Given the steady-state flowsheet for a process, it is possible to combine the structural arrays for individual units in the process to obtain the system array for the complete process. The cause-and-effect graph for the individual units in the process are also interconnected to obtain the system cause-and-effect graph for the process. This system graph gives all known interactions between the process variables. The system array and the system cause-and-effect graph represent different aspects of the process and can be used in the synthesis of process control systems. Following the generation of the system array and cause-and-effect graph, the control objectives may be specified. These objectives define the variables that need to be maintained within a certain error bound around the steady-state value.

The problem of sensed variable selection involves (1) finding the correct variables to measure; and (2) measuring or calculating a true representation of the variable. The sensed variable possibilities are expressed by the *Sensed Variable Tree* consisting of AND-OR gates. A set of variables are ANDed with respect to a constraint variable on the tree if all the variables have to be measured in order to obtain the value of the constraint variable. If a single variable alone is sufficient to obtain the constraint, then the variable is ORed on the *Sensed Variable Tree.* The sensed variable possibilities vary in their degree of inference. No variable is ever directly measured. There is always an inference mode.

In addition to the degree of inference, the measurement of the constraint variable is limited by several factors that affect the overall accuracy. These factors are:

(a) Transducer characteristics - nonlinearity, irreproducibility, hysteresis, deadband response time, environmental effects,

(b) Installation characteristics - physical location of the sensor and noise.

The overall accuracy of the measurement is considered with respect to its cost, i.e., cost of the sensor. If the cost to attain a desired accuracy is high, other sensed variable possibilities may have to be considered. Inferential measurements are only used in instances when a "direct" measurement is not available or prohibitively expensive.

An important aspect of control system design - synthesis of the control structure is addressed. Process cause-and-effects are assumed to be deterministic and in order for control to exist, there must be a causal path between the controlling and the controlled variables. The synthesis problem is subdivided into three sub-tasks:

1. Find possible ways of measuring the process constraint variable
2. Find possible ways of manipulating the constraint variable
3. Combine solutions of subtasks 1 and 2 to generate feedback, feedforward, and cascade control structures.

6.5.3 Structural Design - Decomposition of Process Subsystems

Synthesis of control structures, based on aspects of structural controllability and observability to yield a controllable and observable system, have been presented [Mor. & Steph., 1]. The structural design of alternative regulatory control schemes has also been dealt with. Within the framework of hierarchical control, criteria are developed for the further decomposition of the process subsystems, reducing the combinatorial problem, while not eliminating feasible control structures. Structural models are used to describe the interactions among the units of a plant and the physicochemical phenomena occurring in the various units. The relevance of controllability and observability in the synthesis of control structures is discussed, and modified versions are used to develop all the alternative feasible regulatory structures in an algorithmic fashion. During the structure of controllers, the following problems are addressed:

1. Development of a suitable type of system representation (model), requiring a minimal amount of information.
2. Formulation of mathematical criteria to be satisfied by every feasible control structure.
3. Development of guidelines for decomposing problem into manageable subproblems.
4. Algorithmic procedure to develop alternative control structures.

An approach can be based on the structural characteristics describing (a) the interactions among the units of a chemical process and (b) the logical dependence (of the Boolean type) among the variables used to model the dynamic behaviour of the various units. Thus, detailed dynamic modelling at an early stage is avoided. The mathematical feasibility criteria for the generated alternative control structures are based on the concepts of controllability and observability. Systems are represented as structured matrices and extended versions of the conditions for structural controllability and observability can be used as feasibility criteria. Mainly feedback control structures are addressed, and feedforward compensation is developed as a logical extension. This way, a theory-based method for developing alternative control structures - excluding the possibility of singularities, overspecifications and undetectable local instabilities is accomplished. Engineering heuristic criteria can enter at any stage of the synthesis procedure. Also, the discontinuity between the employed, completely heuristic, structuring procedures and the available sophisticated, detailed design techniques for multivariable control loops, is reduced.

As a first step, the variables that have to be measured and controlled to guarantee smooth plant operation, have to be determined. Before the actual control algorithms can be designed, the alternative sets of manipulated variables, which can be used in a feedback arrangement, must be developed. Establishing suitable process models offers great difficulties. The underlying philosophy is to make initial decisions based on a crude model and refine the model appropriately after each design step. The use of a simple model at the start can be adopted. The most primitive model for control purposes is one which displays the structural dependencies of the variables only, showing if the time derivative of one variable depends on another or not. The most efficient way to determine feasible sets of measured and manipulated variables, and to keep the model as simple as possible, is to use criteria of structural controllability and observability [Lin, 1]. These properties, however, are neither necessary nor sufficient for a control system to work in practice. Extended concepts of output structural controllability and observability have been formulated to remedy these deficiencies [Mor. & Steph., 1].

A graph can be associated with the system matrix, which shows the mutual influence of the variables. The system variables form the state-nodes of the graph. There is a directed edge from node j to node i, if the structural system matrix has a nonzero entry in the i^{th} row and the j^{th} column. Each manipulated variable and disturbance can be represented by a node, and its influence on the state variables shown graphically. The structural representation of a staged system gives rise to large matrices with repeated common structural elements. The generic rank ρ_g of a structural matrix is defined to be the maximal rank, a matrix achieves, as a function of its free parameters.

The synthesis problem is very complex. A computer program, which checks for an entire plant the feasibility of the different possible sets of manipulated and controlled variables, according to the structural rank and accessibility criteria, can be created. The number of resulting solutions would however, be enormous, and screening them would represent an almost insurmountable task. [Gov. & Pow., 3] use essentially this approach employing heuristics to screen, in addition to known control-theoretical considerations like speed of response, time lags and structural criteria. Instead of eliminating the majority of the alternatives after they have been synthesised, it appears preferable not to generate them at all. This can be achieved by decomposing the process, synthesising the regulators only within the subsystem's boundaries, and finally combining the subsystems appropriately. Guiding principles for the decomposition must be established. Decomposition has to be performed along arguments involving system dynamics. It is not reasonable to generate control structures which suggest manipulating a variable at the first stage of a process, in order to influence a variable in the last stage. This can be easily avoided if decomposition precedes the synthesis, and if the control objectives of a subsystem are met - as much as possible - through the manipulation of variables located in the same subsystem. The following operations can be applied to the integrated plant:

1. Precedence order and grouping of the units (e.g. [Sarg. & Wester., 1]. Through that algorithm, a chain of groups in sequential order is obtained.
2. Determine the minimum number of "torn streams" in the irreducible groups [Bark. & Mot., 1], [Pho & Lap., 1] and break up the irreducible groups into chains of functional units.
3. Generate the control structures for each of the sequentially arranged functional units separately.

At this point, all the primary regulatory control objectives and all the secondary regulatory control objectives arising from the development of feedback optimising control structures have been defined. Synthesising the alternative regulatory schemes can be done at the level of the units resulting from the decomposition. For each unit *i* proceed to synthesise the feasible control structures. Finally, a test for accessibility is performed, and, if it is not satisfied, the set of manipulated variables selected is not feasible and it is rejected. If it is satisfied, then the set of manipulated variables selected is retained for further screening.

6.5.4 Graphs and Control Theoretic Methods

This section summarises the fundamentals of graph theory tools that allow the structural analysis of control systems to be undertaken. This analysis can be carried out on the so-called *linear state-space* model representation of systems. This model represents the system as a set of linear first order differential equations. The model consists of a set of inputs, outputs, states and, optionally, disturbances. The matrix-vector equations making up this model are as follows,

$$\dot{x}(t) = Ax(t) + Bu(t) + Dv(t)$$
$$y(t) = Cx(t)$$

Structural analysis of the model considers only the structure of the A, B, C and D matrices. By *structure* in this case we mean the matrices formed by assigning an arbitrary symbol to elements of these matrices known to be non-zero and keeping zero those known to be zero due to physical constraints. A numerical realisation of a structure matrix is defined as any matrix with numerical values replacing the non-zero elements. This approach is particularly useful, considering the fact that in most practical situations some elements of these matrices will have a level of uncertainty as to their actual values, while others will be precisely zero due to the physical properties of the system. Hence a *structural* or *structure* matrix can be precisely known for the system while the usual case for a numerical matrix is that it will have some degree of uncertainty associated with it.

The structural matrices are used as a basis for the graph representation of the model. A *graph* or more precisely a *digraph* consists of a set of vertices and a set of directed edges connecting the vertices together. The graph model of the system will consist of a separate vertex to represent each of the states, inputs, outputs and disturbances. The vertices are connected by edges; a separate edge representing each of the non-zero elements of the matrices. Each matrix will contribute the following edges to the graph:

1) The A matrix an edge from state vertex *j* to state vertex *i* for each non-zero a_{ij}.
2) The B matrix an edge from input vertex *j* to state vertex *I* for each non-zero b_{ij}.
3) The C matrix an edge from a state vertex *j* to output vertex *i* for each non-zero c_{ij}.
4) The D matrix an edge from a disturbance vertex *j* to a state vertex *i* for each non-zero d_{ij}

Figure 6.9 gives an example of a state-space model and its equivalent graph representation.

$$A=\begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & 0 & a_{23} \\ 0 & a_{32} & 0 \end{bmatrix}, B=\begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \\ 0 & 0 \end{bmatrix}, C=\begin{bmatrix} c_{11} & 0 & 0 \\ 0 & c_{22} & 0 \end{bmatrix}, D=\begin{bmatrix} 0 \\ 0 \\ d_{13} \end{bmatrix}$$

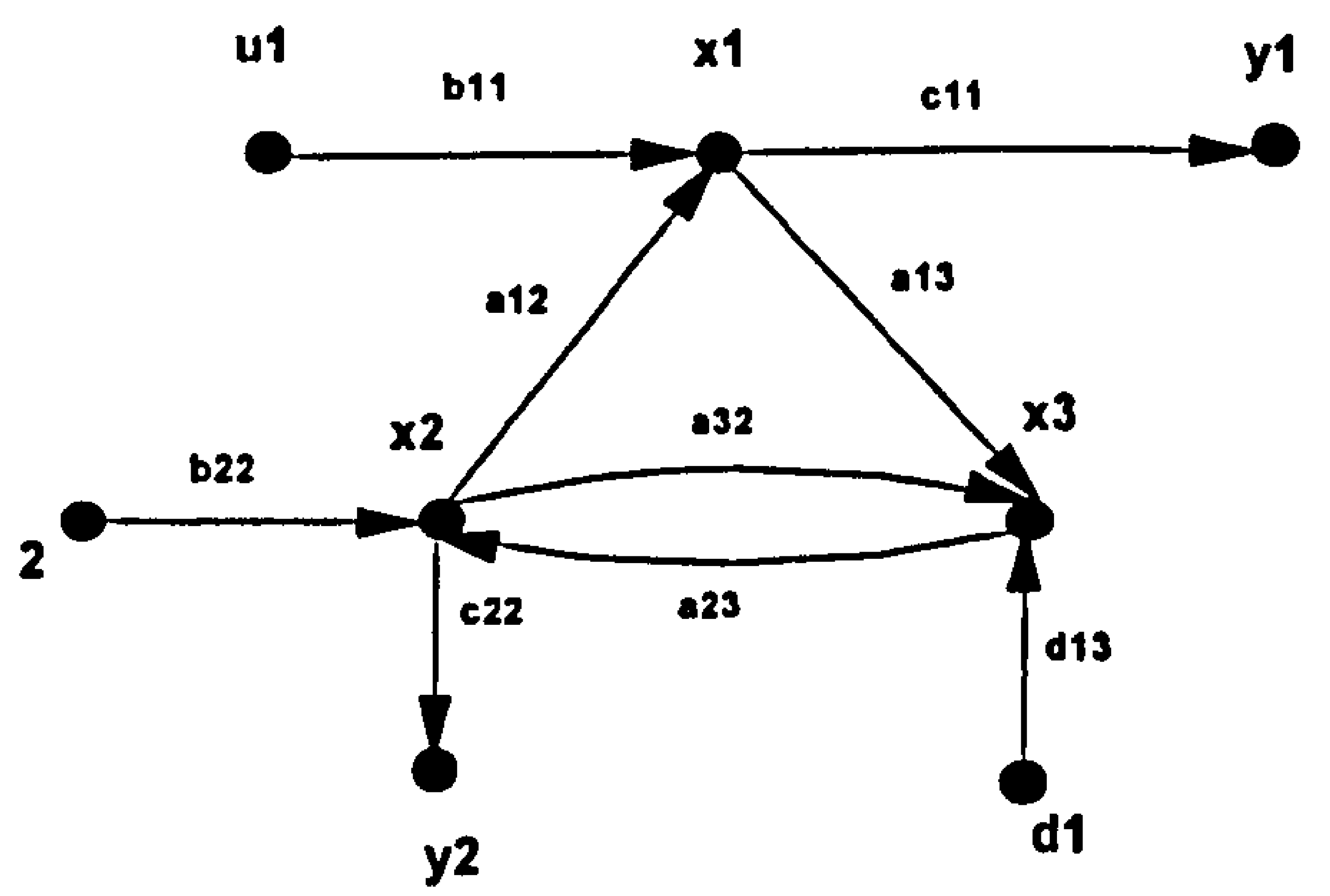


Figure 6.9 - Example of a state-space model and its equivalent graph representation

It should be noted that the graph representation of the model implicitly ignores non-zero elements in the matrices. Using the graph representation of the state-space model the very rich mathematics of graph theory can be applied to give a very different approach to the analysis than the classical, heavily numeric, matrix based approach.

A *graph*, as defined in [Harary, 1], is an abstract structure that consists of a set of vertices, denoted by *V*, and a set of edges, denoted by *E*. An edge is a line or arc whose two endpoints are vertices. *G(V,E)* is the symbol used for a graph consisting of a vertex set *V* and an edge set *E*. If we have a graph that the edges have a direction associated with them then this graph is called directed graph or *digraph*. In this text, we will only consider graphs with directed edges and may frequently use the term *graph* instead of the more precise term *digraph*. The following figure gives an example of a digraph.

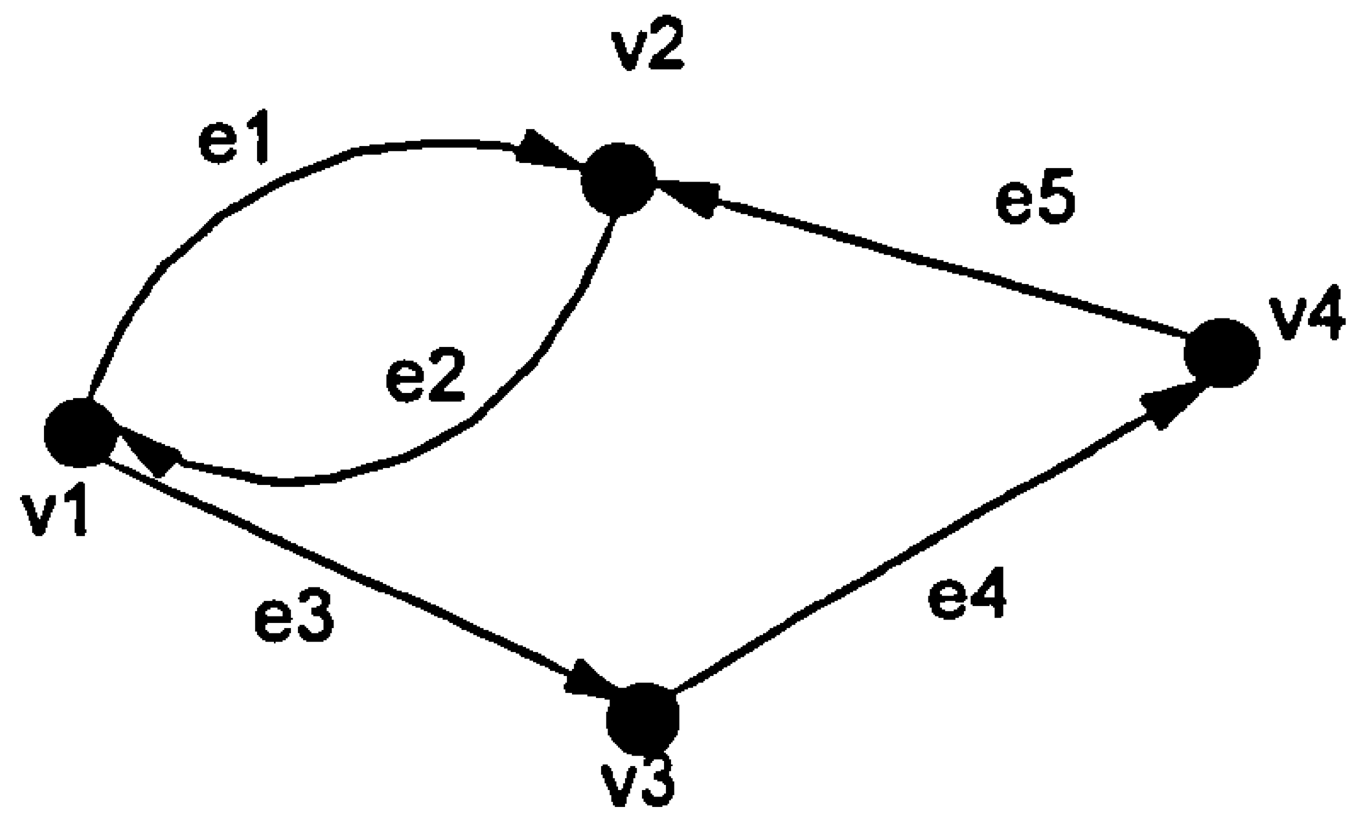


Figure 6.11. Example of a digraph

An edge may be specified by the vertices at each end. The vertex an edge is directed to is denoted the *final vertex* and the vertex an edge is directed from the *initial vertex*. Vertices at either end of an edge are said to be adjacent to or incident with one another. A vertex is said to have an *in-degree n* if it has n incoming edges and *out-degree n* if it has n outgoing edges. Hence, in figure 6.11, vertices v_3 and v_4 have in-degree 1 and out-degree 1, v_1 has in-degree 1 and out-degree 2 and v_2 has in-degree 2 and out-degree 1. Sometimes it is appropriate to assign numbers or *weights* to each of the edges of the digraph. In this case the graph is called a *weighted digraph*.

Paths, Cycles, Cutsets and Trees

A *path* is a sequence of edges such that the final vertex of the preceding edge is the initial vertex of the succeeding edge [Christof., 1]. Some authors allow the edges of the path not to be necessarily distinct; however, in this text we use the definition of the sometimes called *simple path* which does not use the same edge more than once. If the initial vertex of a path is the same as the final vertex then the path is called a *cycle* (or circuit). Figure 6.12 shows an example of a *path* and a *cycle* .

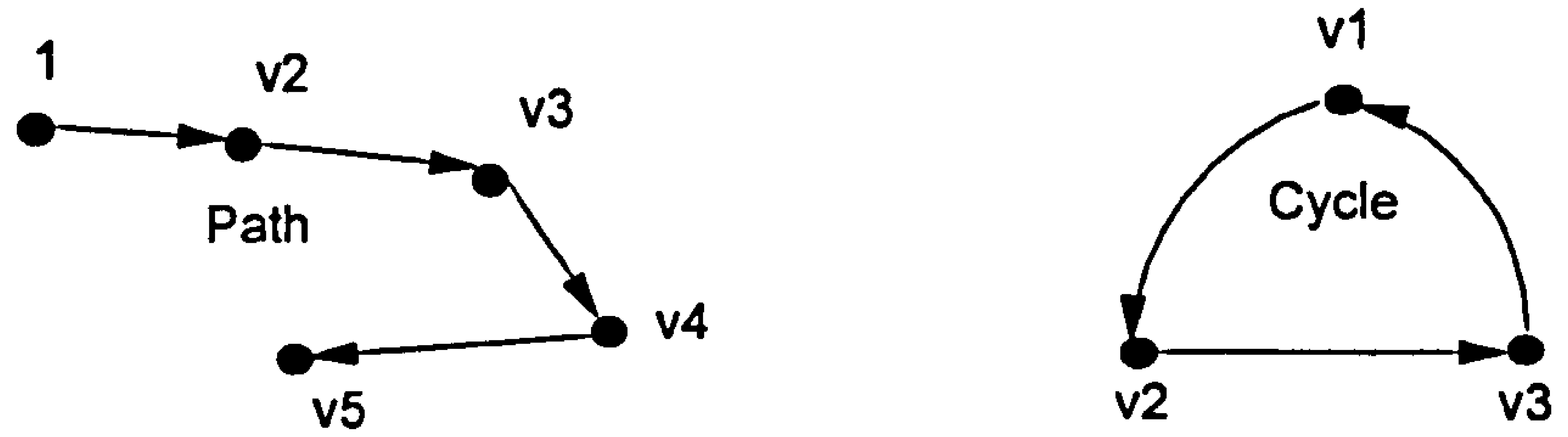


Figure 6.12 Example of a path and a cycle

If in a digraph there is a path from vertex v_1 to vertex v_2, then vertices v_1 and v_2 are said to be *connected*. In addition, two vertices, v_1 and v_2 are said to be *strongly connected* if there is a directed path from v_1 to v_2 *and* a directed path from v_2 to v_1. If there exists only one of these two paths then v_1 and v_2 are said to be *weakly connected*. A group of vertices are said to be *strongly connected* if every pair of vertices are *strongly connected*.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12} & 0\\ 0 & 0 & 0\\ a_{31} & a_{32} & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}b_{11} & 0\\ 0 & b_{22}\\ 0 & 0\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix}=\begin{bmatrix}c_{11} & 0 & 0\\ 0 & 0 & c_{23}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

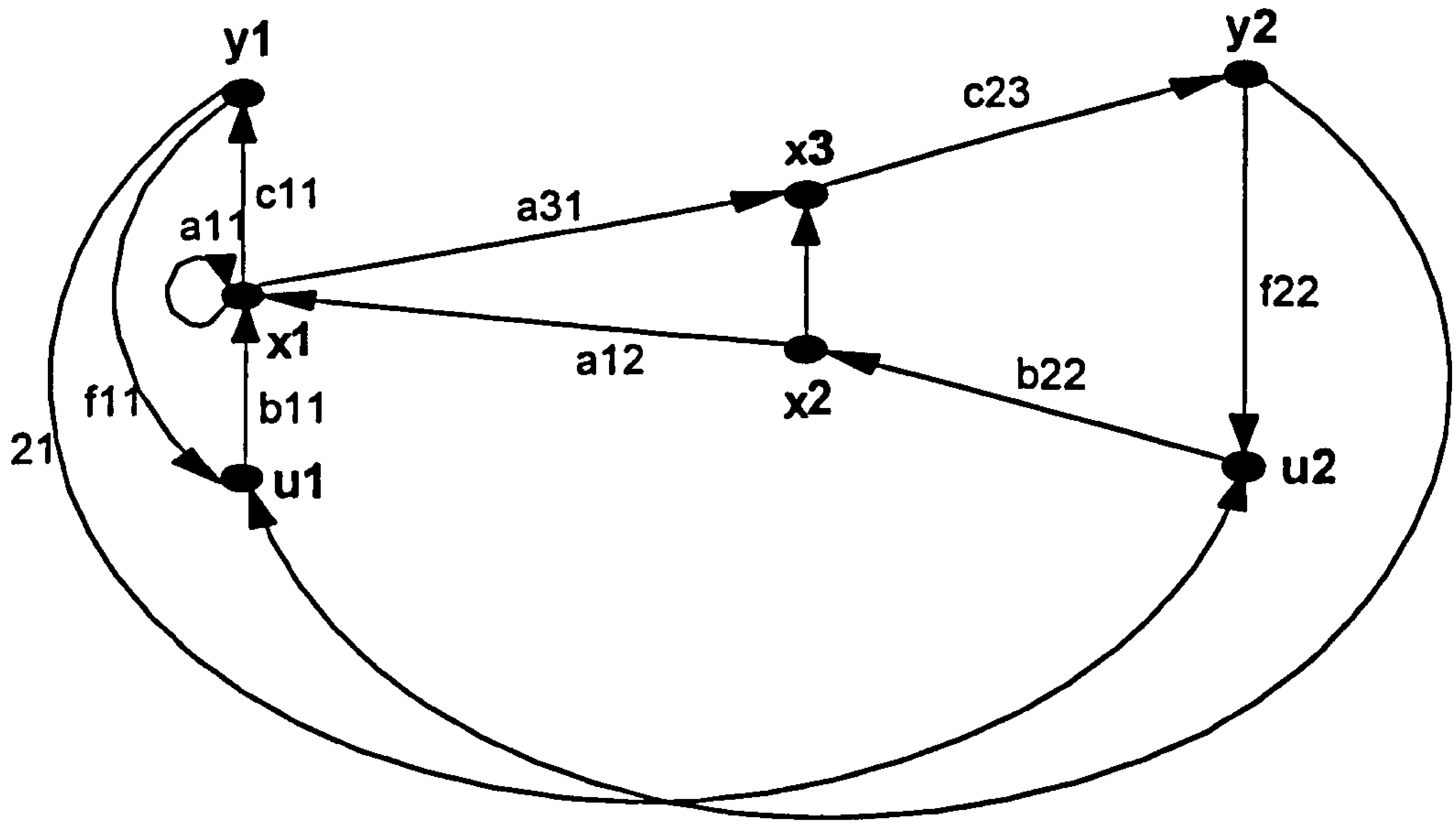


Figure 6.13. Example of a state-space model and its Q_4 digraph representation

Another property is the easy extraction of the original model equations from the digraph representation. For example, if we consider the graph in figure 6.13 we can see that vertex y_1 has one input edge (an in-degree of 1) with weight c_{11} from state vertex x_1. In an equation form this becomes

$$y_1 = x_1 \, c_{11}$$

State vertex x_1 has three inputs one from vertex x_2, one from input u_1 and another from itself. This means

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + b_{11}u_1$$

In this way we can built up the original matrix equations of the model. A significant feature of the graph representation of the model is the implicit exclusion of the zeros from the system matrices. This makes for a more compact model and in some cases a clearer representation of the input, state and output dependencies would be obtained from the matrix notation. Basically, the structural representation of the system is equivalent to the unweighted digraph representations of the system described above.

Structural Analysis and Structure Matrix

When modelling a system using the state space approach, the entries of the A, B, C and D matrices will usually fall into two classes.

i) Those whose values are strictly zero due to physical constraints.

ii) Those whose values are non-zero but have some (maybe small) degree of uncertainty as to their actual value.

Properties of state space models can easily be sensitive to small numerical perturbations in these matrix entries. *Structural Analysis* allows all entries with any degree of uncertainty to vary independently of one another. The elements of a *structure matrix* [Q] are either fixed zero or indeterminate value, usually denoted by L, which are assumed to be independent of one another. A numerically given matrix Q is called an *admissible numerical realisation* (with respect to [Q]) if it can be obtained by fixing all indeterminate entries of [Q] at some particular values. Two matrices Q′ and Q″ are called *structurally equivalent* if both Q′ and Q″ are admissible numerical realisations of the same structure matrix [Q]. A property holds structurally within a class of structurally equivalent systems if the property under investigation holds numerically on 'almost all' admissible numerical realisations [Rein., 2].

An example of a property of matrices that is important in control theory is what is known as *rank*. The *rank* of a matrix is defined as the dimension of the largest minor having a non-zero determinant. The *structural rank* (s-rank, term rank or generic rank) of a structure matrix is defined as the maximal number of elements contained in at least one set of independent entries. As an example consider the following matrix A and its equivalent structure matrix [A] [Jantzen, 1].

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 4 \\ 0 & 5 & 0 \end{bmatrix} \qquad [A] = \begin{bmatrix} \underline{L} & 0 & L \\ L & 0 & \underline{L} \\ 0 & \underline{L} & 0 \end{bmatrix}$$

The determinant of the matrix A is given by $|A| = 2\times2\times5 - 5\times4\times1 = 0$. Thus, it is obvious that matrix A is rank *deficient*, i.e. its rank is less than 3. This is because the numbers are such that some summands cancel each other; it is not because of its structure. The rank of A actually increases to 3 if say A(1,1) increases by ten percent. In the structural domain the rank of A is 3, since there are three entries (marked with an underscore) that exist in different rows and columns.

It is now important to summarise some useful results that structural analysis give an engineer [Rein., 2]:

i) If a property does not hold structurally then no numerical realisation can have that property.

ii) If a property holds structurally and the varying entries of the structure matrix are independent then this property will hold for *almost all* numerical realisations.

iii) If a property holds structurally but the varying entries are dependent then the space of all numerical realisations having this property is undetermined. It is even possible that no numerical realisation has this property.

Structural Controllability and Observability

Controllability of a system means that the manipulation of the inputs should be able to cause the states to behave in any way desired. From this definition it can be seen that in the time-invariant linear state space model of a system only the A and B matrices are needed to determine controllability. The numerical rank condition for controllability (and observability) depends on the fortuitous selection of parameter values and for well behaved physical systems, it fails at isolated points only. Thus it does not provide any useful, *global* information about the behaviour of a controlled system. It is this last aspect which dictates that any meaningful information should depend on the invariant structural aspects of a dynamic systems. Indeed, *structural controllability* concerns only the structure matrix pair ([A],[B]).

An obvious precondition of controllability is that the system inputs are able to influence all state variables. Said in graph theoretic terms, there must exist paths from input vertices to all state vertices; the definition of input connectability follows [Rein., 2]

Definition 6.3:

"A class of systems is said to be *input connectable* (or input reachable) if in the digraph of $G(Q_1)$ there is, for each state vertex, a path from at least one of the input vertices to the chosen vertex state."

The concept of structural controllability was first introduced for the single input case ([Lin, 1]); many authors extended this work to the multi-input case ([Shields & Pear., 1], [Dav., 2] and many others).

Observability and controllability are known as dual concepts. Controllability concerns the interaction of inputs to states whilst observability concerns the equivalent interaction between states and outputs. Therefore, theorems concerning controllability can be changed to their dual theorems concerning observability by simply replacing input with output and the B matrix with the C matrix.

The duality of observability and controllability can also be interpreted by stating that the structural pair (C,A) is observable, if, and only if, the structural pair (A^T, C^T) is controllable.

Finally, the important concept of structural completeness should be introduced [Rein., 2].

Definition 6.4:

"A class of systems being *both* s-controllable and s-observable is said to be *structurally complete*."

Disturbance Rejection by means of State Feedback

A more realistic model of a dynamical system would include disturbances as additional inputs to the system. Disturbance inputs are unwanted effects, and typically reflect such physical phenomena as electrical noise and external vibrations. Graph theory can be used as a useful tool in the problem of rejection of these disturbances. In the graph theoretic approach, the disturbance rejection problem can be approached from what is, in effect, a simple set theoretic approach related to the digraph.

[Rein., 2] discusses two types of analysis for disturbance rejection, namely, the *rejection of the full variety of disturbances* and the *rejection of actual disturbances.* The first analysis finds a subset of state vertices that definitely cannot have disturbances attached and assumes disturbances are attached to all other vertices. It then sees if simultaneous rejection of all these disturbances can be achieved. If it is possible then the analysis defines a set of state vertices at which disturbances can be attached and a state feedback matrix capable of their rejection. If, however, it is not possible then this does not mean all disturbance rejection is impossible.

A second analysis can be done where *actual disturbances* are attached to the system, a disturbance output cutset is found, and then it is assessed whether the simultaneous compensation of this cutset is feasible. If it is not then another cutset can be looked for and rejection using this new cutset attempted. Only when all cutsets have been tried and failed can we say that rejection of these disturbances is impossible.

Vertex Decomposition

A very common feature of nearly all large-scale systems is the high degree of sparsity in their system matrices. Sparsity means the number of zero entries is greater than the number of non-zero entries. This will also be reflected in the adjacency matrices of any graph representation. *Decomposition* of graph into subgraphs should always be considered when investigating a system; it involves the sorting of the vertices into individual subgraphs, or vertex groups, based on the connectivity of the graph. In a decomposable system the eigenvalues of the components are also the eigenvalues of the whole system. Thus, a controller design can sometimes be performed locally on the components rather than taking the whole system into consideration.

Not all systems are decomposable. If the system is one big cyclic (strongly connected) component, it is not decomposable. A matrix model with many small uncertain elements may not be decomposable. These 'almost zero elements' could result from rounding errors during matrix inversion in a computer. However, large scale systems are often decomposable.

Generally, decomposition is related to matrix *reduction* [Jantzen, 1]. That is, after decomposition the original matrix is reordered in such a way that it becomes a *lower* (or upper) *block triangular* matrix, with square blocks along the diagonal and zeros above (or below) the block diagonal. The blocks on the diagonal are themselves irreducible; they correspond to the cyclic components of the graph.

Fixed Modes under Decentralised Control

In large scale systems, e.g. flexible space structures, electric power networks, and chemical processes, the requirement for decentralised control structure is common, due to physical constraints and/or economic factors. Sometimes, a constraint on information exchange between various control agents is imposed due to geographical separation, transport delay, etc. However, in most control applications, decentralisation arises because of the cost of communication links and the reliability of maintaining a centralised control system. The decentralised control of such large-scale systems, whose essential characteristics are a very large number of variables and a spatial distribution, generally requires some restriction on the output-input pairs which the controller may connect. With such a structurally constrained information flow, the problems of stabilisation and pole placement are to be considered outside the classical framework. For example, the output feedback matrix F for a plant with 100 inputs and 100 outputs consists of 10000 elements for a centralised control system, whereas the corresponding matrix for a decentralised system may have only 100 elements or less. In this case, the selection of a feasible decentralised structure is critical to the design and operation of the closed-loop system.

The fundamental concept in the study of this problem is that of *fixed modes* which was introduced by Wang and Davison [Wang & Davis., 1]. The existence of a solution for the stabilisation or the pole placement problem using a structurally constrained controller depends critically on the properties of this set. The algebraic characterisation of fixed modes using exterior algebra tools [Karc. *et al.*, 1], provides the means to relate their formation to the structure system invariants, as discussed previously in the section with decentralised Markov parameters [Lev. & Karc., 2]. When considering time-invariant regulators, the presence of unstable fixed modes indicates that stabilisation is impossible, while the absence of any sort of fixed modes rules out arbitrary pole placement. Consequently, the concept of fixed modes appears as an extension of the concept of uncontrollable and unobservable modes which exists in classic control problems. In the classical framework, it was shown that relevant results can be obtained by carrying out a purely structural study [Lin, 1], which is particularly appropriate when dealing with large scale systems. The concept of structurally controllable and observable modes was then introduced. The same approach was developed by Sezer and Siljak [Siljak *et al.*, 2] for structurally constrained control problems giving rise to the concept of *structurally fixed modes* whose existence depends only on the structure of the system. Structurally fixed modes are thus generic property of the system with respect to a specified feedback structure. Therefore, structurally fixed modes have an essential part in those control problems concerned with large-scale systems because parameter values are generally subject to uncertainty; an obvious advantage offered by the concept of structurally fixed modes is numerical. Since existence of such modes is a qualitative property of the system, it can be characterised in terms of *directed graphs* leading to computationally attractive tests involving binary computations only.

The concept of Vulnerability [Pichai, 1]

A great deal of system theory is concerned with problems of uncertainty, with emphasis on solutions that ensure a satisfactory performance despite significant changes in the system structure. Examples of this trend are the studies of stability and optimality under structural perturbations whereby a number of variables, or a part of the system, is disconnected and again connected during operation. It has been possible to identify structures that are basic to the design of reliable control, subject to plant and controller failures. A missing ingredient in these studies, however, has been the consideration of the effects of disconnections of variables on controllability and observability properties of the system, which are crucial in reliability design of control and estimation schemes for complex dynamic processes.

A graph is considered vulnerable at a line or a point if a removal of that line or point destroys connectivity of the graph [Harary, 2]. When a graph is associated with a linear dynamic system, the role of connectivity is taken by input reachability or structural controllability. As we have already mentioned, a system is input reachable if each state variable can be influenced by an input variable either directly or via other state variables. In terms of the corresponding digraph this means that to each state point there is a path from at least one input point. A question of vulnerability arises: Does a removal of a line or a point from the digraph destroys input reachability ? It is obvious that the essential part of controllability (observability) is input (output) reachability, and if by a line or point removal the system becomes input (output) unreachable, it becomes uncontrollable (unobservable) independently of the size of the existing interconnections among the system variables. By using the concept of structural controllability instead of input reachability, the vulnerability analysis of controllability can be refined at the price of a more extensive analysis; we require to find the minimal inputs of a given system which are sufficient to preserve the property of s-controllability.

A brute force approach to the vulnerability problem would be to determine input reachability for each possible structural perturbation of the corresponding digraph *G(V,E)*. This simplistic approach becomes inefficient when the size of the system increases. To assess vulnerability of input reachable systems in a more efficient way, a different approach is required [Pichai *et al.*, 1].

Structural Controllability and Observability

A possible refinement of the concept of vulnerability of dynamic systems, is to consider structural controllability instead of input reachability. As we have already mentioned: "A class of systems characterised by the n×(n+m) structure matrix [A,B] is s-controllable if and only if the digraph *G([Q1])*, meets the following conditions.

(a) For each state vertex in *G([Q1])* there is at least one path from one of the m input vertices to the chosen state vertex.

(b) There is at least one *cycle family* of width *n* in *G([Q1])*."

Therefore an obvious way of finding the minimal inputs and outputs in order to preserve the properties of controllability and observability can be outlined as follows :

i) Find the *input line basis* for the given system i.e. the set of the minimal inputs/outputs required to preserve input reachability.

ii) Check among these inputs/outputs to find those that satisfy the second criterion for structural controllability/observability

This procedure was implemented in software by mainly using the *cutsets* routine [Econ., 1]. That is, it tries to find a *minimal* set of input edges whose removal from the graph will disconnect all paths from the set of input vertices to the set of state vertices. By *minimal set of edges* we mean a set of edges such that their removal would be the minimal sufficient conditions to disconnect all the required paths from the input vertices to the state vertices.

6.6 Discussion

The *relative gain array* (*RGA*) is currently the principal tool for interaction analysis. The *RGA* has proven to be a useful method, although it has its shortcomings. Its interpretation is unambiguous, only in the case of a two input, two output process. The *SGM* of a large system with several inputs and outputs is, in general, easier to interpret than the corresponding *RGA*.

The input-output pairings that seem to be promising, based on the analysis of interaction matrices, should be further analysed using interaction measures. However, the definition of interaction measures given in *Section 6.3* has some limitations and, therefore, the results should be interpreted with caution. The reason is that the interaction measures are based on the block diagonal closed-loop system that might or might not be indicative of the actual full closed-loop system. The definition guarantees the system to be stable, but the performance can be poor, even if the interaction measures indicate small interactions.

The application of the tests for decentralised integral controllability (DIC) can sometimes be problematic in actual practice. The conditions involved are easy to check, but they are only necessary. A systematic procedure that goes through the various steps and criteria and, for each case employs the most efficient test, is currently missing. Furthermore, most of the tests have a heuristic basis and lack a systematic theoretic foundation.

The Singular Value Decomposition can provide a powerful and computationally efficient tool for analysing matrix systems and it is the basis for many diagnostics for control system design. The analysis can be performed over the frequency range that is of practical importance for the particular process, so that both static and dynamic aspects can be considered. An additional important feature of the SVD strategy is its ability to identify modelling aspects, such as model mismatch, which affect the performance of the resulting process control structure. Also, the strategy can show whether or not a structural decoupler will be effective in minimising interactions between loops. A compensator can be designed for the range of frequencies most likely to affect the process. The approach can provide insights into important closed loop system properties such as stability, sensitivity and invertibility [Morari, 3].

The procedure for selection of decentralisation, based on Markov parameters, uses the numerical values of the model parameters. Generic solvability conditions may be used to provide a first listing of possible decentralisation schemes; such a combination of results may limit the large number of possible combinations to be tested. An additional advantage of the Markov Parameter framework, centralised, or decentralised, is that due to its direct link to the state space description, it provides the means for modifying the selection of the B, or C, matrices, such that the centralised or decentralised Markov matrix has full rank and thus achieve the very important linear assignment property which excludes the presence of fixed and almost fixed modes and preconditions well the system to accept a certain type of control solution. The current results establish a framework and provide the tools for affecting the shaping of properties of centralised, or decentralised Plucker matrices at early design stages.

The 'graph theoretic approach' is another useful tool. The control system under investigation is modelled by a suitably chosen graph representation. Therefore, the investigator obtains a better visual insight and feeling for the system and its properties. Provided he succeeds in showing that a desired property holds *generically* i.e. independently of numerical parameter values, the indeterminate parameters may be considered to be degrees of freedom during further steps of design or optimisation. If the graph theoretic approach shows that a specific property does not hold, the investigator is able to suggest system modifications with the aid of which the desired property could be fulfilled.

Chapter 7

SOFTWARE DEVELOPMENT

Every noble work is at first impossible.

7. SOFTWARE DEVELOPMENT

7.1 Introduction : Interaction Analysis Toolbox.

In this chapter, a number of control theories are brought together to develop systematic methods for control structure selection. They form a CAD Toolbox that can be used to provide inside information of a control structure and draw some useful results that can lead to improvements and probably, to a complete restructuring of the model.

Many different methodologies exist, that deal with the interaction analysis and the process controllability notion of a system. However, most of them rely on heuristics, and some others, are case dependent. An attempt was made to unify most of them under a single CAD package, so that the complementary nature of these methodologies can be fully exploited, and, hence, provide some useful indicators/measures of the likely performance of the control structure under observation. No single method is suitable for every control problem, nor every method can be applied to every problem.

The software developed here addresses the interaction indicators, a list of which is given below:

1. RGA
2. D-RGA
3. PRGA
4. D-PRGA
5. SGM
6. D-SGM
7. BRG
8. D-BRG
9. SVD

A detailed description of the above methods has been given in the previous chapter. Here we summarise those aspects linked to software development. The testing of the software is considered in terms of examples, in the following chapter.

7.2 RGA

Purpose: Evaluate coupling of inputs, outputs based on single loop controllers.

Description: The Interaction Analysis Toolbox provides an option for (static) *RGA* analysis. This option uses the steady-state gain matrix of a process. The relative gain is the ratio of the transfer function between two variables, with all other outputs uncontrolled, and the transfer function between the same variables, when all other outputs are perfectly controlled. The only information needed for the calculation of the *Relative Gain array* is the steady state gain matrix $G(0)$.

For the interpretation of the *RGA*, the following two pairing rules are used [Bristol, 1]:

1. Pair together inputs and outputs indicated by positive *RGA* elements that are closest to unity.

2. Avoid pairing together inputs and outputs indicated by negative *RGA* elements, because such pairings result in, either an unstable system, or an inverse responding system.

The resulting pairings are checked for stability using Niederlinski's theorem; if the pairings are unstable, other possible pairings with values closest to unity should be used; negative pairings should be avoided, if possible.

Algorithm: This option uses the **frga** function of the Matlab MFD Toolbox. It also uses the **inv** to find the inverse of the steady-state gain matrix G. They are all included in the **emrga**, where provision was made to embody the routines necessary for the checking of the Niederlinski condition.

Diagnostics: The function provides diagnostics for a square system. It computes the interactions among system's input-output pairs. Also, is uses the Niederlinski theorem. This theorem is particularly powerful. All that it requires for its use is steady-state gain information and the assumption that perfect steady-state control is achieved in all loops. Niederlinski's theorem is actually equivalent to pairing on a negative *RGA* element.

The use of the gain matrix means that the *RGA* is based on local linearisation around steady state. The *RGA* can be applied to non-linear systems but one has to make sure that accurate process gains are calculated.

References: [Bristol, 1], [Grosd. *et al.*, 1], [Liang, 1].

7.3 Dynamic-RGA (D-RGA)

Purpose: Evaluate frequency-dependent coupling of inputs, outputs based on single loop controllers.

Description: *D-RGA* is an extension of the traditional *RGA* so that dynamic effects are included. Several investigators have proposed many different definitions for a dynamic *RGA*. In some cases, these definitions require that the feedback controller be designed. Since the *RGA* is most valuable in screening alternative control system designs, the requirement that the controller must be designed, limits the utility of these definitions. The approach used here does not require that the controllers must be specified. Starting with zero initial conditions, it is desired to bring the process output to a new set point. For the interpretation of the *D-RGA*, the pairing rules used are similar to the ones used for the static-*RGA* (see *RGA*).

Algorithm: This option uses the **frga** function of the Matlab MFD Toolbox. It also uses the **inv** to find the inverse of the gain matrix G. The **emdrga** function is used to encompass all the various procedures. Note that if g_i is the i-th component matrix of G then the relative gain array at frequency $w(i)$ is defined as $g_i \circ \left[(g_i)^{-1}\right]^{\dagger}$.

Diagnostics: The function provides diagnostics for a square system. The usual limitations due to linearisation – for non-linear systems – also apply (see also *RGA*).

Much empirical evidence suggests that feedback loops associated with large elements of the relative gain array are inherently difficult to control. Hence, examination of the array can aid the decision on the pairing of sensors and actuators in a decentralised control scheme.

References: [Bristol, 1], [Tung & Edg., 1], [Skog. *et al.*, 3], [Hagg., 1], [Cao & Biss, 1].

7.4 Performance Relative Gain Array (PRGA)

Purpose: Indicate (one-way) couplings of inputs to output.

Description: *P-RGA* is a slightly different definition of the traditional *DRGA*. It tries to overcome a problem sometimes encountered with *DRGA*, the wrong indication of no severe interaction, while, at the same time, significant one-way couplings may exist. It is defined in a quite similar way to the *DRGA*, i.e. $P(s)=\tilde{G}(s)G(s)^{-1}$, but in this case, $\tilde{G}(s)$ is a matrix consisting of only the diagonal elements of $G(s)$, i.e. $\tilde{G}=diag\{g_{ii}\}$.

Algorithm: This option uses the **frga** function of the Matlab MFD Toolbox. It also uses the **inv** to find the inverse of the gain matrix G and the **diag** to extract the necessary diagonal elements of G. The **emprga** function is used to encompass all the various procedures. The elements of P (the *PRGA* matrix) are given by: $p_{ij}=g_{ii}(s)\left[G^{-1}\right]_{ij}=\dfrac{g_{ii}(s)}{g_{ji}(s)}\lambda_{ji}(s)$.

Diagnostics: The function provides diagnostics for a square system. It should be noted that although the diagonal elements of the *RGA* and the *PRGA* are identical, the *PRGA* does not have all the algebraic properties of the *RGA*. *PRGA* must be recomputed whenever G is rearranged, whereas *RGA* only needs to be rearranged in the same way as G. *PRGA* is independent of input scaling, but it depends on output scaling. This is reasonable since performance in defined in terms of the magnitude of the outputs. The measures above may also be extended to non-square systems by introducing the pseudoinverse.

References: [Hovd & Skog., 3].

7.5 Scaled Gain Matrix (SGM).

Purpose: Scaling of inputs, outputs provides direct comparison with each other.

Description: The method aims to provide useful information on interactions, in a form that is easy to interpret. It is based on the scaling of input and output variables. A large gain between an input and an output can indicate strong interaction. However, this can not be directly used for interaction analysis, because the process gain matrix depends on the scaling of input and output variables. In this method the input and output variables are rescaled, so that in the new gain matrix, corresponding to the rescaled variables, the elements are directly comparable with each other.

The iterative procedure used is the following:

Step 1. Calculate the gain matrix. This is the first estimate of the scaled gain matrix Ψ, i.e. for $k=1$ set

$$\psi_{ij}^{k} = |g_{ij}|$$

Step 2. Scale the rows of Ψ^{k} in such a way that in each row the average value of the elements is equal to one.

$$\psi_{ij}^{k+1} = \frac{n \cdot \psi_{ij}^{k}}{\sum_{j=1}^{n} \psi_{ij}^{k}}$$

Step 3. Scale the columns of Ψ^{k+1} in such a way that in each column the average value of the elements is equal to one.

$$\psi_{ij}^{k+2} = \frac{m \cdot \psi_{ij}^{k+1}}{\sum_{j=1}^{m} \psi_{ij}^{k+1}}$$

Step 4. Stop if the changes between Ψ^k and Ψ^{k+2} are sufficiently small. Otherwise set $k \leftarrow k+2$, and go to step 2.

The procedure converges towards the *scaled gain matrix* (*SGM*) that is unique for each matrix.

Algorithm: This option uses matrix manipulation functions. It produces the results with the use of **emsgm** function.

Diagnostics: In the *Scaled Gain Matrix* the average value of the elements in each row and column is one. The interpretation of this interaction matrix is simple: values larger than one indicate strong interaction and values smaller than one indicate weak interaction. The largest elements in Ψ then indicate the inputs and outputs, which should be connected in the feedback controller.

References:

[Liesl., 1].

7.6 Dynamic Scaled Gain Matrix (D-SGM).

Purpose: Scaling of inputs, outputs provides direct comparison with each other, over the desired frequency range.

Description: This method (an extension of the previously presented static-*SGM*), aims to provide information on interactions in a form that is easy to interpret. It is based on the scaling of input and output variables. In this method the input and output variables are rescaled, so that in the new gain matrix, corresponding to the rescaled variables, the elements are directly comparable with each other.

Consider an $m \times n$ process transfer function matrix $G(s)$. The basic idea behind the method is to scale input and output variables in such a way that the average gain in each row and column of the process model is one at a given frequency. This is achieved using the same iterative procedure that was used for the static-*SGM*, but in this case, the elements change with frequency, i.e. $\psi_{ij}^{k} = \left|g_{ij}(j\omega^{*})\right|$, where ω^{*} is the desired frequency. The procedure again, converges towards the *scaled gain matrix* (*SGM*) that is unique for each matrix.

Algorithm: This option utilises many different functions. Extensive matrix manipulation is used. The function **emdsgm** presents the resulting *SGM* in the form of a MVFR matrix, i.e. the values of the matrix, alongside the corresponding frequency vector. The results are also presented in a graph, with the use of the **plot** command.

Diagnostics: In the *Scaled Gain Matrix* the average value of the elements in each row and column is one. So, values larger than one indicate strong interaction and values smaller than one indicate weak interaction. The largest elements in Ψ then indicate the inputs and outputs, which should be connected in the feedback controller. The *SGM*, unlike the *RGA*, can be used even when the number of inputs and outputs is unequal.

References: [Liesl., 1].

7.7 Block Relative Gain (BRG)

Purpose: Evaluate SISO or small order MIMO couplings of inputs, outputs.

Description: By formulating and extending the *RGA* concept and its properties from a scalar to a matrix, a more powerful synthesis framework is formed, that can address a broader class of control problems, such as the synthesis of decentralised control structures that are not restricted to SISO control loops. This concept is referred to as *Block Relative Gain*. According to this method, decentralised control structures can be systematically generated by partitioning $G(s)$ into blocks of different dimensions. To extend this method further, alternatives can also be generated due to the alternative ways of assigning inputs and outputs to the blocks. It should be noted that in this type of partitioning, subsystems are viewed as aggregates of control loops and not as groups of process units. Thus, block partitioning of $G(s)$ may not necessarily correspond to a particular process decomposition and the resulting decentralised control system does not have to be compatible with any arrangement of subsystems of process unit operations. However, this does not preclude the possibility of specifying the process decomposition first and then structuring the decentralised control systems within the boundaries of the individual process subsystems. In some cases, this may eliminate the synthesis of undesirable decentralised control structures right from the beginning and reduce the potential combinatorial problems encountered in the block partitioning procedure. The main steps of the procedure are the following:

First we consider the highest degree of decentralisation – i.e. 1×1 block partitioning of G – that would yield a total of N SISO assignments (or pairings). For this, all the one-dimensional *BRG*s are evaluated. Among the viable ones, those which establish a $1-1$ correspondence between the plant's inputs and outputs are selected. If such alternatives do not exist, then there is no acceptable partitioning using $1-1$ blocks only. In that case, assignment is not complete and one proceeds with two-dimensional *BRG*s. The next step in the process is the study of two-dimensional *BRG*s. This ultimately gives all viable two-dimensional *BRG*s. Searching for an acceptable partitioning over the sets of both two- and one-dimensional viable *BRG*s is the next step. If one is found, the procedure concludes; otherwise it continues with the study of *BRG*s of higher dimension, in the same manner, until a solution is achieved. The process is guaranteed to conclude since, in the worst case, it will lead to a centralised full control structure that corresponds to an n-dimensional *BRG*.

<u>Algorithm:</u> This option uses the **fbrga** built-in function of the MFD toolbox and also the purpose-built **embrga**, to formulate the recursive procedure.

Diagnostics: The function provides diagnostics for a square system. Also, the results are not difficult to interpret. The significance of the *BRG* for the performance of the closed-loop system is the following: The closed-loop performance of an $m \times m$ block, when the other $n-m$ outputs are under perfect control, is a continuous function of *BRG*. The closed-loop performance of the $m \times m$ block is as if this block was isolated from the rest of the plant and operating under the influence only of its own control law. This makes it clear what kind of information one should expect from *BRG* and in what sense it can be considered as a measure of interaction.

References:

[Manous. *et al*, 2].

7.8 Dynamic Block Relative Gain (D-BRG)

Purpose: Evaluate frequency-dependent SISO or small order MIMO couplings of inputs, outputs.

Description: When defining the *block relative gain* and deriving its relation to the closed-loop performance, the usual assumption of perfect control for the plant outputs has been made. This assumption always holds at zero frequency (i.e., at steady state) by the use of integral control action. However, it may not hold for all the frequencies. To investigate interactions over the whole frequency range, *BRG* could be extended to a *Dynamic-BRG* and become a frequency-dependent interaction measure.

The aim of the *DBRG* is to provide an acceptable block partitioning of the plant matrix $G(s)$. Such a task is considered to be accomplished if all the *BRG*'s of different dimensions corresponding to the diagonal blocks of different dimensions $G_{ii}(s)$'s, are close to an identity matrix. To quantify this closeness and define the set of viable *BRG*'s, the following procedure is necessary:

First consider the highest degree of decentralisation – i.e. 1×1 block partitioning of G – that would yield a total of N SISO assignments (or pairings). For this, all the one-dimensional *BRG*s are first evaluated at $s=0$. Among the viable ones, those which establish a $1-1$ correspondence between the plant's inputs and outputs are selected. If such alternatives do not exist, then there is no acceptable partitioning using $1-1$ blocks only. In that case, assignment is not complete and one proceeds with two-dimensional *BRG*s. In case there exists an acceptable $1-1$ block partitioning for $s=0$ but viability and/or acceptance are violated at frequencies other than $\omega=0$, the study of two-dimensional *BRG*s is again necessary. Otherwise, the procedure can conclude at this point with the resulting $1-1$ block partitioning and the corresponding control structures.

The next step in the process is the study of two-dimensional *BRG*s. The *BRG*s, whose diagonal elements are not close to 1, are screened out first. The screening process is repeated for all the possible column vectors and for all frequencies other than $\omega=0$ and ultimately gives all viable two-dimensional *BRG*s.

Searching for an acceptable partitioning over the sets of both two- and one-dimensional viable *BRG*s is the next step. If one is found, the procedure concludes; otherwise it continues with the study of *BRG*s of higher dimension, in the same manner, until a solution is achieved. The process is guaranteed to conclude since, in the worst case, it will lead to a centralised full control structure that corresponds to an n-dimensional *BRG*. It should be mentioned that $\varepsilon_1(\omega)$, $\varepsilon_2(\omega)$ are two free parameters that are used in the procedure, through which the designer can affect the screening process and establish what an acceptable degree of interaction is.

Algorithm: This option uses the **fbrga** built-in function of the MFD toolbox and also the purpose-built **emdbrga**, to formulate the recursive procedure.

Diagnostics: The function provides diagnostics for a square system. The elements of each row and each column of a *DBRG* add to 1. Having presented the procedure, one can easily understand the advantages of the *DBRG*. Different block partitioning of input and output sets leads to alternative decentralised control structures, among which the best are selected by the systematic screening procedure that utilises various important properties of *BRG*. These properties effectively reduce the combinatorial problems and make the analysis of large-scale systems feasible.

References:

[Manous. *et al*, 1], [Manous. *et al*, 2], [Hagg., 1], [Reev. & Ark., 1].

7.9 Singular Value Decomposition (SVD)

Purpose: To provide insights into important closed loop properties: stability, sensitivity, invertibility. Model mismatch can also be indicated.

Description: The Interaction Analysis Toolbox provides an option for *SVD* analysis. This option actually, activates a second window (figure 7.1) that provides the user with four choices. There are three tests that can be executed, and there is also a fourth option, for the simultaneous depiction of all the results on the same figure-window.

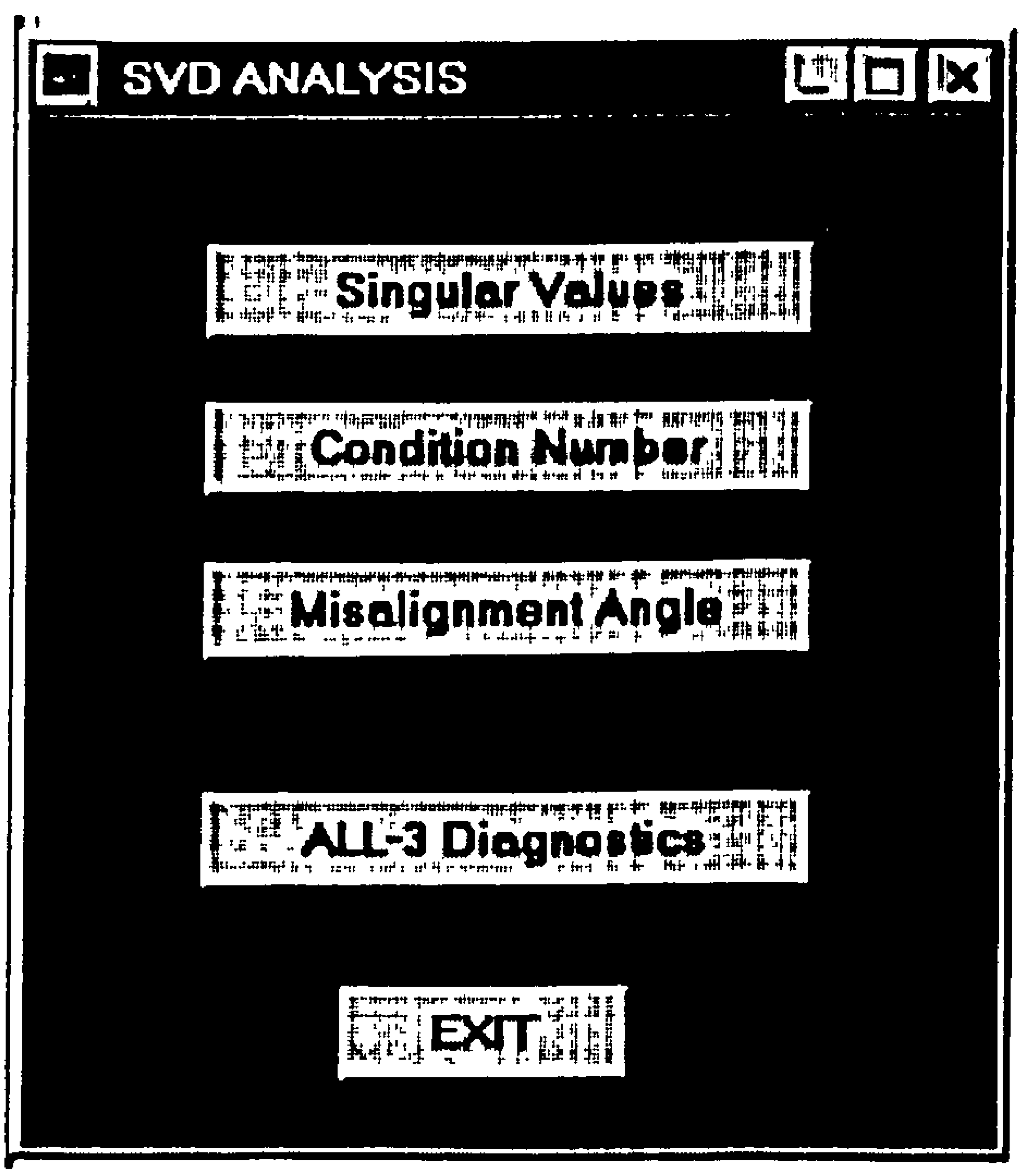


Figure (7.1). SVD Analysis options.

The first one presents the singular values, as a function of frequency. The magnitude of the minimum singular value is a measure of the minimum distance to the nearest singular matrix and hence also a measure of the invertibility of the system. Thus, $\underline{\sigma}$ discloses potential difficulties when implementing feedback control. The best performance can be achieved when s is large.

The second one, is the condition number, i.e. the ratio of the largest and the smallest singular value. This number can be used to quantify the sensitivity of the system.

The third one calculates the misalignment angle (see 6.3.1).

Based on the theoretical analysis of the system, one can develop strategies for the control structure. Four different cases can be identified:

(1) <u>Case 1.</u> Good condition and good directional property.

$$\gamma(j\omega) < 10$$
$$\theta_i(j\omega) < 15^{\circ}$$

where $\omega \in [\omega_1, \omega_n]$ and $G\ (s)$. In this situation, modelling uncertainties can be tolerated and a natural loop structure exists. The good condition number of the system implies that the system will be well-behaved with the selected control structure for moderate modelling inaccuracies.

(2) <u>Case 2.</u> Good condition and poor directional property.

$$\gamma(j\omega) < 10$$
$$\theta_i(j\omega) > 15^{\circ}$$

A natural loop structure does not exist. However, a structural compensator can be used to improve the directional property. The system's condition number is good, so that compensation can actually be beneficial.

(3) Case 3. Poor condition and good directional property.

$$\gamma(j\omega) > 10$$
$$\theta_i(j\omega) < 15^\circ$$

The system exhibits a good direction property but its condition number is poor. As a consequence, it is conceivable that small misalignments between the input and output spaces could be amplified because of the large differences in the magnitudes of the singular values. However, a gain compensator can easily be designed, since each singular value of $G(j\omega)$ can be changed by the appropriate gain without altering the other singular values. One should bear in mind, though, that the compensator's performance may be strongly affected by system perturbations. Also, the magnitude of the gains that can be implemented on the system may be limited by physical restrictions such as control valve saturations.

(4) Case 4. Poor condition and poor directional property.

$$\gamma(j\omega) > 10$$
$$\theta_i(j\omega) > 15^\circ$$

Theoretically, a combination of a structural and a gain compensator can be used, so as to develop a control interconnection structure. However, both compensators are model-dependent and one should proceed with extreme caution.

Algorithm: This option uses many different functions. Some were built-in functions of the MFD toolbox (fsvd, fmisalg) and some others were purpose-built (fcond, emsvd, emsvdsv).

Diagnostics: The function provides diagnostics for a square system. It also uses a Quasi-Newton optimisation algorithm (due to fssv). From svd, if the limit of 75 QR step iterations is exhausted while seeking a singular value, the following message will appear:

Solution will not converge

It should be noted that the function automatically optimises the scaling of the axis, so that the best screen output can be obtained and the results can easily be compared.

In summary, this approach provides a diagnosis of the system. For the cases with good condition number, the strategy selected is less dependent on model accuracies. For the cases with poor condition, the control synthesis is more difficult. This function demonstrates how *SVD* analysis can yield structural information of the open-loop system and enables one to design a structural compensator to minimise loop interactions without changing the system sensitivity. It decouples the interaction and sensitivity analysis so that they can be handled independently. Consequently, the feedback control system can easily be designed once the analysis and the proper compensations have been performed.

References: [Lee & Mor., 1], [Lau et al., 2], [Lee et al., 2], [Hauv. & Skog., 1], [Hovd & Skog., 3].

7.10 Conclusions

The various methodologies for interaction analysis have been presented. They are all based on interaction matrices (with the exception of *SVD*). Although they all share a common task, i.e. the analysis of interactions in the system, the way to do it and the various diagnostics all serve to form a toolbox that is as complete as possible. Not all methods can be applied to any problem, nor can all of them have the same weighting.

The *RGA* (and *D-RGA*) are well known, the *P-RGA* can be used when independence of input scaling is important and *SGM* is very useful when input and output scaling is a problem and when we deal with non-square systems (unequal number of inputs and outputs). The *BRG* and *D-BRG* provide an interesting alternative to the

SISO model, and can be applied when a low-order MIMO controller is not a problem. Finally, *SVD* is a very handy tool when one deals with multivariable systems. It can indicate sensitivity or model mismatch problems and can be applied over the frequency range that is of practical importance.

To conclude, no single method can be seen as a panacea, but they all contribute towards the formation of an – almost complete – Interaction Analysis Toolbox. The use of the toolbox will be presented with the use of an example in the following chapter.

Chapter 8

EXAMPLES

Step back and view the forest.

8. EXAMPLES

8.1 Introduction

The various methodologies presented in chapter 6, led to the creation of an accompanying Matlab† Toolbox. The CAD Toolbox that was developed, deals with the interactions that are present between the various control loops. It supports both a transfer function and state-space models as an input. The analysis can be performed either for 0 frequency (static) or for a range of frequencies (dynamic), since each methodology provides options for both.

8.2 The Toolbox

The Toolbox was implemented in Matlab† v4.2 for Windows‡. It can be installed as an additional toolbox, under the directory `C:\matlab\toolbox\interact` . When first run (by typing "mmenu"), the user is presented with a welcome screen (figure 8.1). The screen that follows, includes the main options of the toolbox, i.e. it prompts for the insertion of the system data and then the various methods can be applied to it (figure 8.2).

† MATLAB® is a registered trademark of MathWorks Inc.

‡ Windows is a registered trademark of Microsoft Corporation

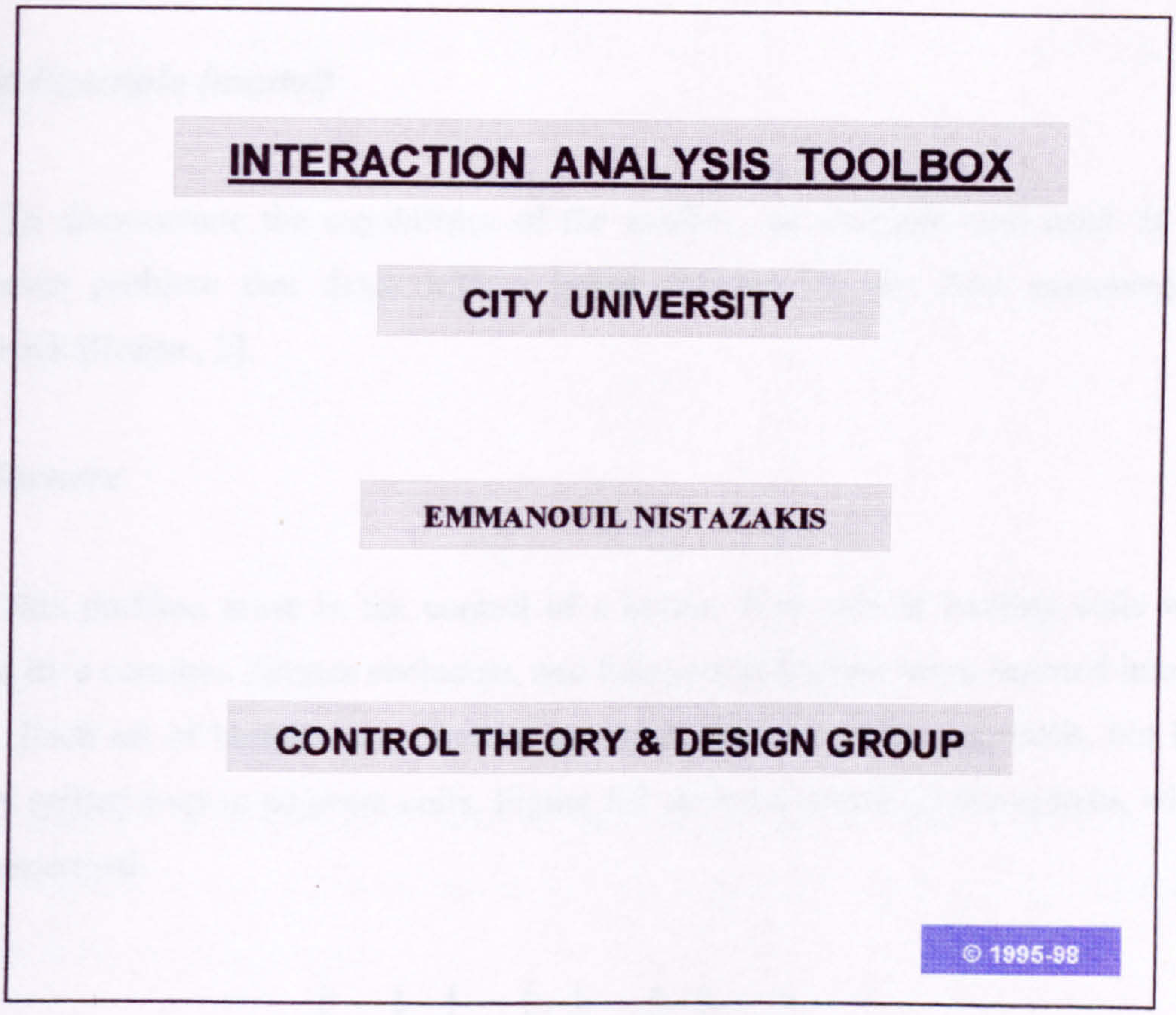


Figure 8.1. Introductory screen.

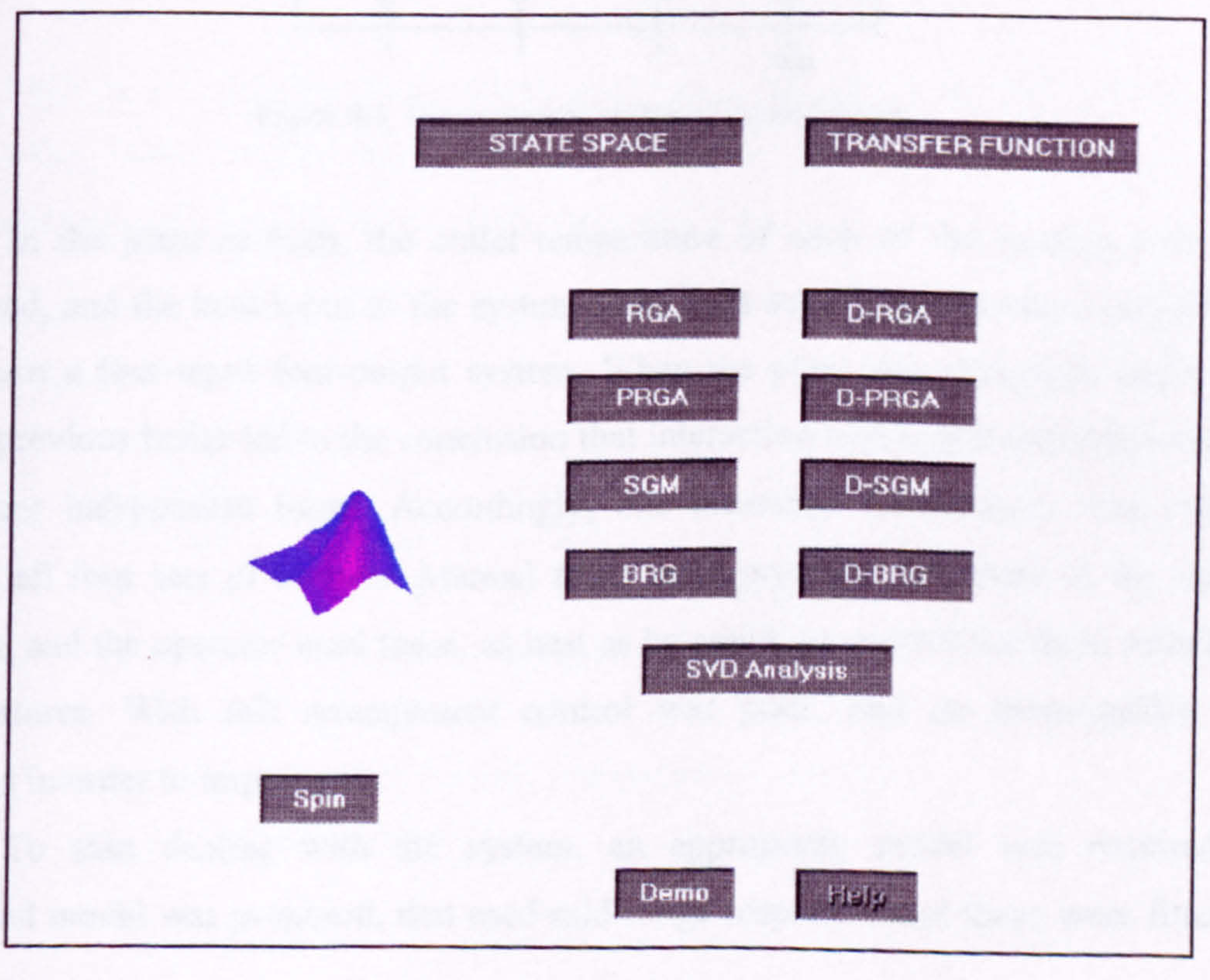


Figure 8.2. Interaction Analysis Toolbox – Main Menu.

8.3 The Example (model)

To demonstrate the capabilities of the toolbox, an example was used. It is a well-known problem that deals with a boiler furnace. It was first presented by Rosenbrock [Rosen., 2]..

Boiler furnace

This problem arose in the control of a boiler. Four sets of heating coils were enclosed in a common furnace enclosure, and four sets of burners were inserted into the furnace. Each set of burners was directed at one of the sets of heating coils, but heat naturally spilled over to adjacent coils. Figure 8.3 shows a sketch of the system, which was symmetrical.

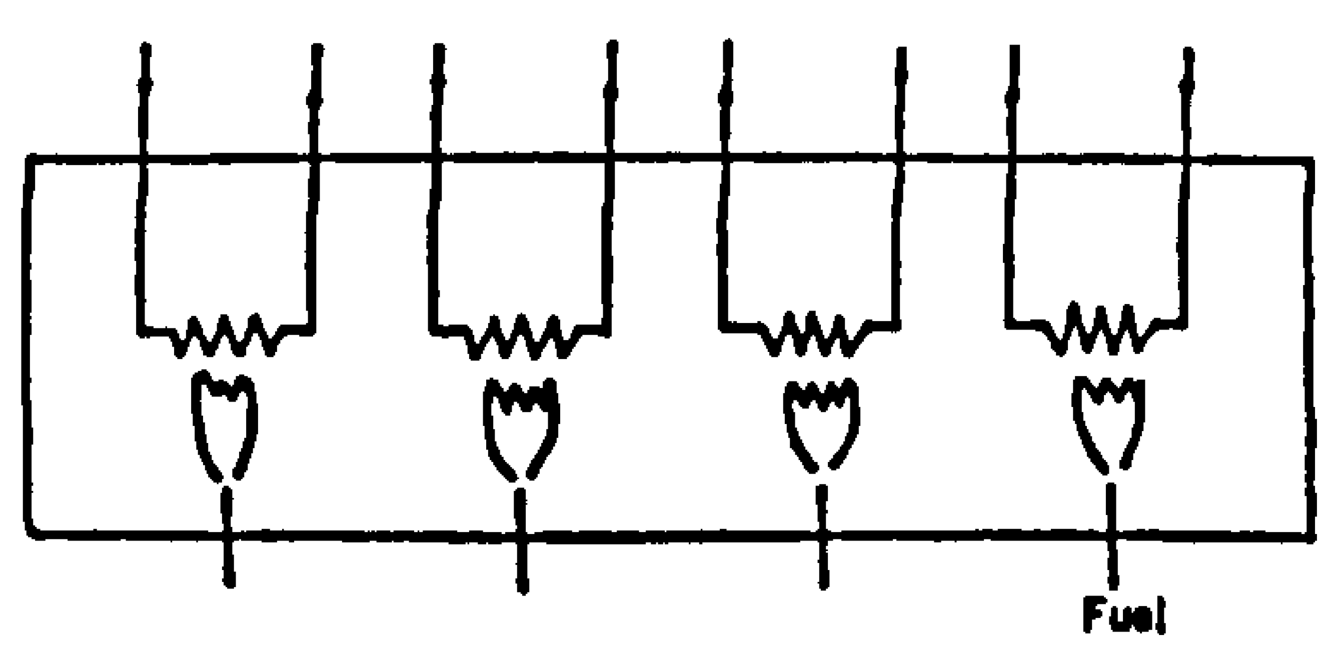


Figure 8.3. Diagrammatic sketch of boiler furnace.

In the plant as built, the outlet temperature of each of the heating coils was measured, and the heat input to the system from each set of burners was manipulated. This gave a four-input four-output system. When the plant was designed, experience with a previous boiler led to the conclusion that interaction would prevent stable control with four independent loops. Accordingly, one measured temperature was used to control all four sets of burners. Manual trims were provided for three of the sets of burners, and the operator used these, as best as he could, to control the three remaining temperatures. With this arrangement control was poor, and an investigation was required in order to improve it.

To start dealing with the system, an appropriate model was required. A linearised model was proposed, that used mid-range responses and these were fitted by first-order transfer functions. In this way, the elements of $G(s)$ were found, and the result was:

$$G(s)=\begin{bmatrix} \frac{1.0}{1+4s} & \frac{0.7}{1+4s} & \frac{0.3}{1+4s} & \frac{0.2}{1+4s} \\ \frac{0.6}{1+4s} & \frac{1.0}{1+5s} & \frac{0.4}{1+5s} & \frac{0.35}{1+5s} \\ \frac{0.35}{1+5s} & \frac{0.4}{1+4s} & \frac{1.0}{1+5s} & \frac{0.6}{1+5s} \\ \frac{0.2}{1+5s} & \frac{0.3}{1+5s} & \frac{0.7}{1+5s} & \frac{1.0}{1+4s} \end{bmatrix} \tag{8.1}$$

No great accuracy was justified in view of the nonlinearity, and the values shown are rounded. The model was inserted in the Interaction Analysis Toolbox and the results are presented in the following pages.

8.4 RGA

RGA is probably the most widely used interaction measure. The relative gains between the various inputs and outputs are represented in this array. For the above mentioned example, the following results were obtained:

```
RGA =

    1.6402   -0.7959    0.2871   -0.1314
   -0.6025    2.1757   -1.0711    0.4979
   -0.0644   -0.2678    2.1757   -0.8435
    0.0268   -0.1120   -0.3917    1.4770

 According to Niederlinski stability theorem,
 the system is stable.
```

Figure 8.4. RGA results

One may say that the results are somewhat expected. Indeed, the highlighted diagonal elements are the most appropriate ones for pairing. They are positive and dominant, compared with the rest. However, their values are far from the ideal one of "unity". The Niederlinski stability theorem was also applied to the system (for details see 6.2.1). Although this is normally used to reject a solution – as unstable – in this case, the system appears to be stable. The outcome is that strong interactions require further study of the system.

8.5 D-RGA

The D-RGA, an extension of the RGA which includes dynamics, can be used to track the changes of the interactions with frequency. In real life, the appropriate frequency range (bandwidth) would be known beforehand. For our example, the range is taken to be 0.01 to 10 rad/sec. The outcome, in this case, is the following:

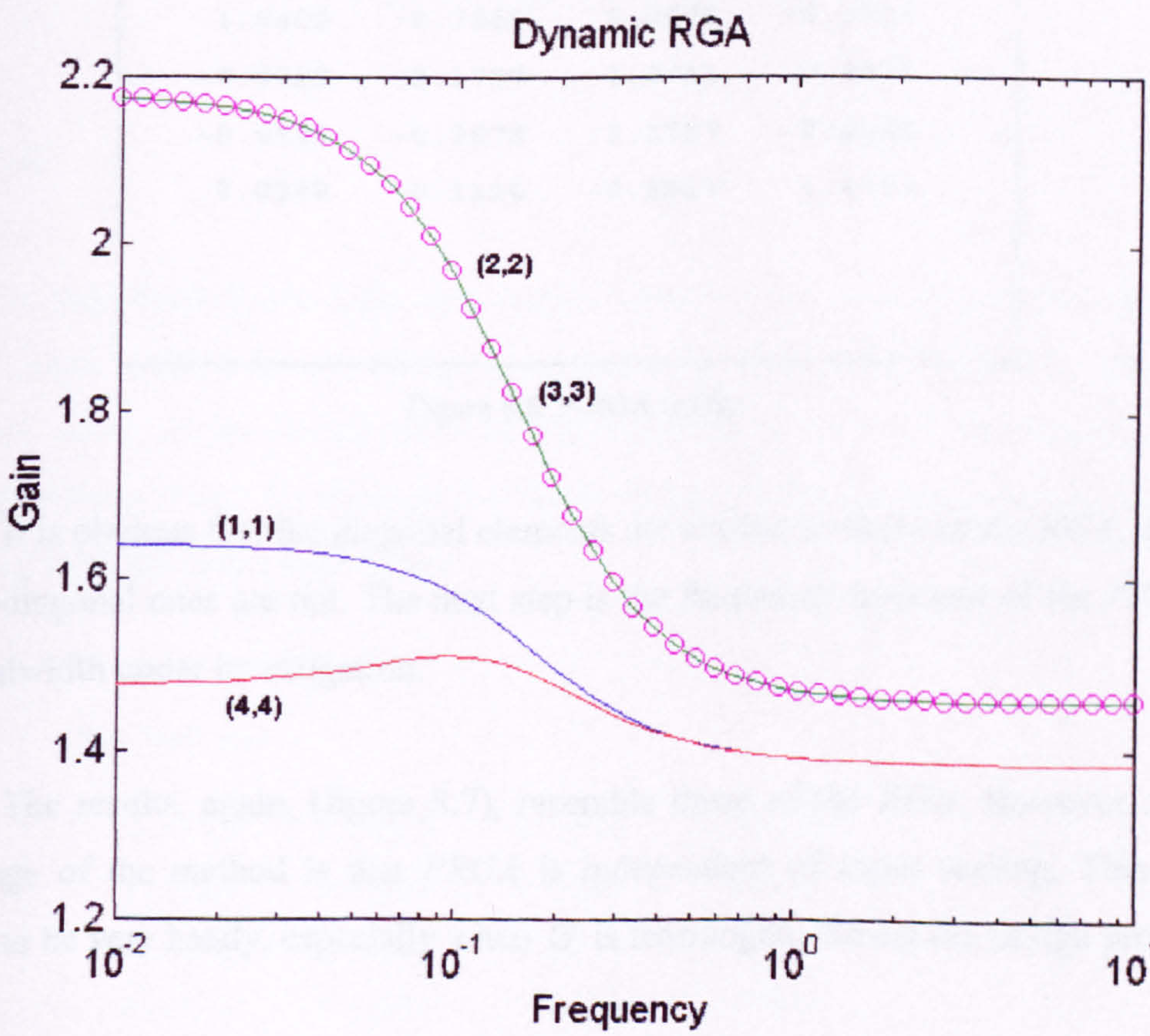


Figure 8.5. Dynamic-RGA results

The figure discloses the behaviour of the four proposed loops. The interactions seem to be quite high for low frequencies, although they settle down, as frequency goes up.

Note that since the analysis is based on gains only, the effect of any possible delays should be analysed separately.

8.6 Performance RGA (PRGA)

The notion of the *RGA* has already been presented and its use as a screening tool for alternative control structures has already been established. However, due to the limitations already presented in 6.2.4, a screening with the *PRGA* tool seems necessary.

```
ans =

    1.6402   -0.7959    0.2871   -0.1314
   -0.6025    2.1757   -1.0711    0.4979
   -0.0644   -0.2678    2.1757   -0.8435
    0.0268   -0.1120   -0.3917    1.4770
```

Figure 8.6. P-RGA results

It is obvious that the diagonal elements are similar to those of the *RGA*, although the off-diagonal ones are not. The next step is the frequency response of the *PRGA*, for the bandwidth under investigation.

The results, again, (figure 8.7), resemble those of the *RGA*. However, the real advantage of the method is that *PRGA* is independent of *input* scaling. This can be proved to be very handy, especially when G is rearranged, during the design process.

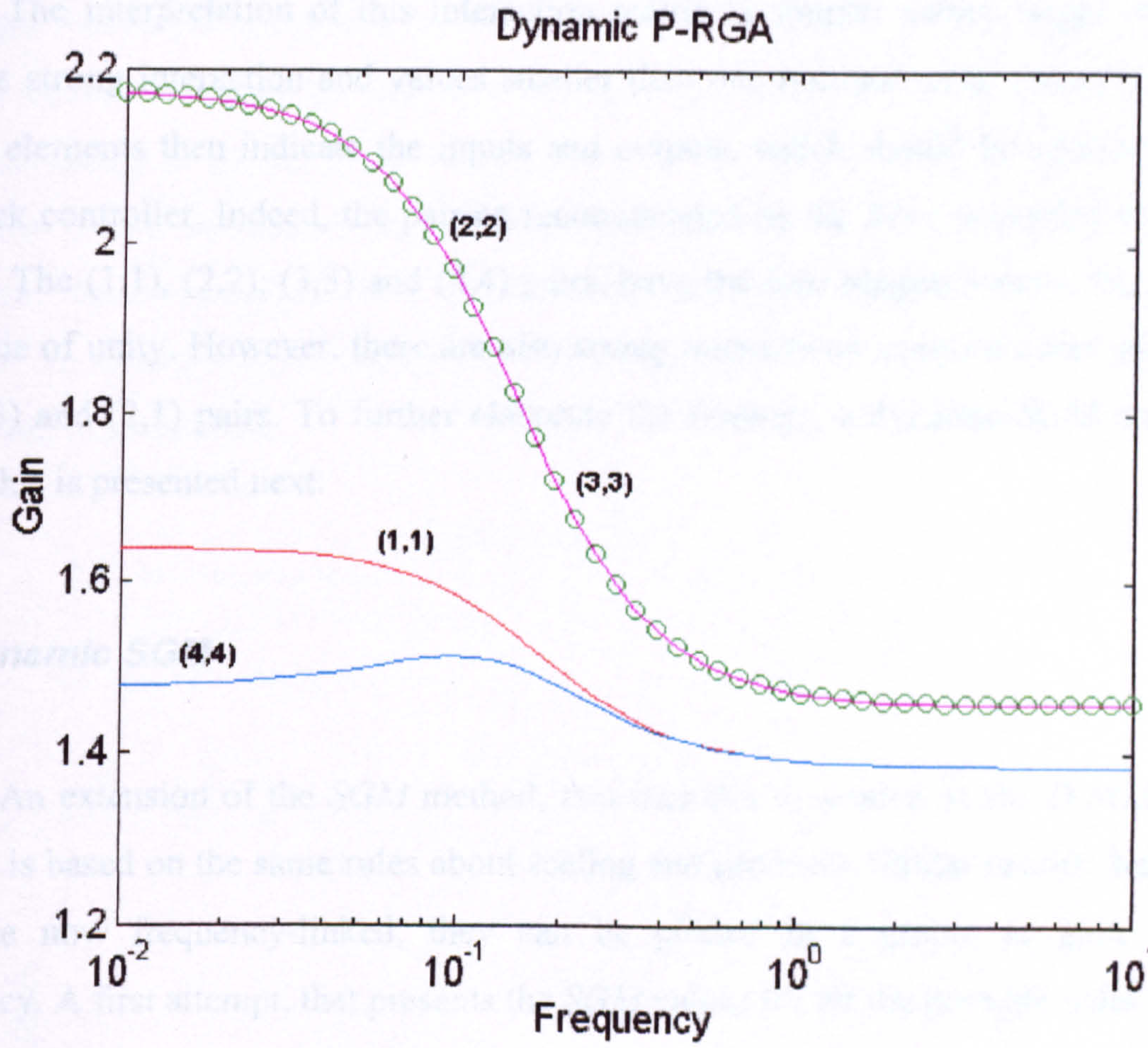


Figure 8.7. Dynamic P-RGA results

8.7 SGM

In this method, the input and output variables are rescaled, so that in the new matrix, corresponding to the rescaled variables, the elements are directly comparable with each other. In the *Scaled Gain Matrix*, the average value of the elements in each row and column is one. By utilising the Matlab-based *SGM* routine (`emsgm.m`), we are presented with the following results:

```
The SGM is:

1.9723    1.0390    0.5730    0.4149
1.1384    1.4278    0.7350    0.6985
0.5181    1.1141    1.4337    0.9343
0.3712    0.4191    1.2583    1.9523
```

The interpretation of this interaction matrix is simple: values larger than one indicate strong interaction and values smaller than one indicate weak interaction. The largest elements then indicate the inputs and outputs, which should be connected in a feedback controller. Indeed, the pairing recommended by the *RGA* is confirmed by the results. The (1,1), (2,2), (3,3) and (4,4) pairs, have the four biggest values, well above the value of unity. However, there are also strong interactions amongst other pairs, e.g. the (4,3) and (2,1) pairs. To further elaborate the findings, a dynamic-*SGM* routine is used. This is presented next.

8.8 Dynamic SGM

An extension of the *SGM* method, that includes dynamics, is the *D-SGM*. This method is based on the same rules about scaling and produces similar results, but, since they are now frequency-linked, they can be plotted in a graph, as gain against frequency. A first attempt, that presents the *SGM* values for all the possible pairs (16), is the following:

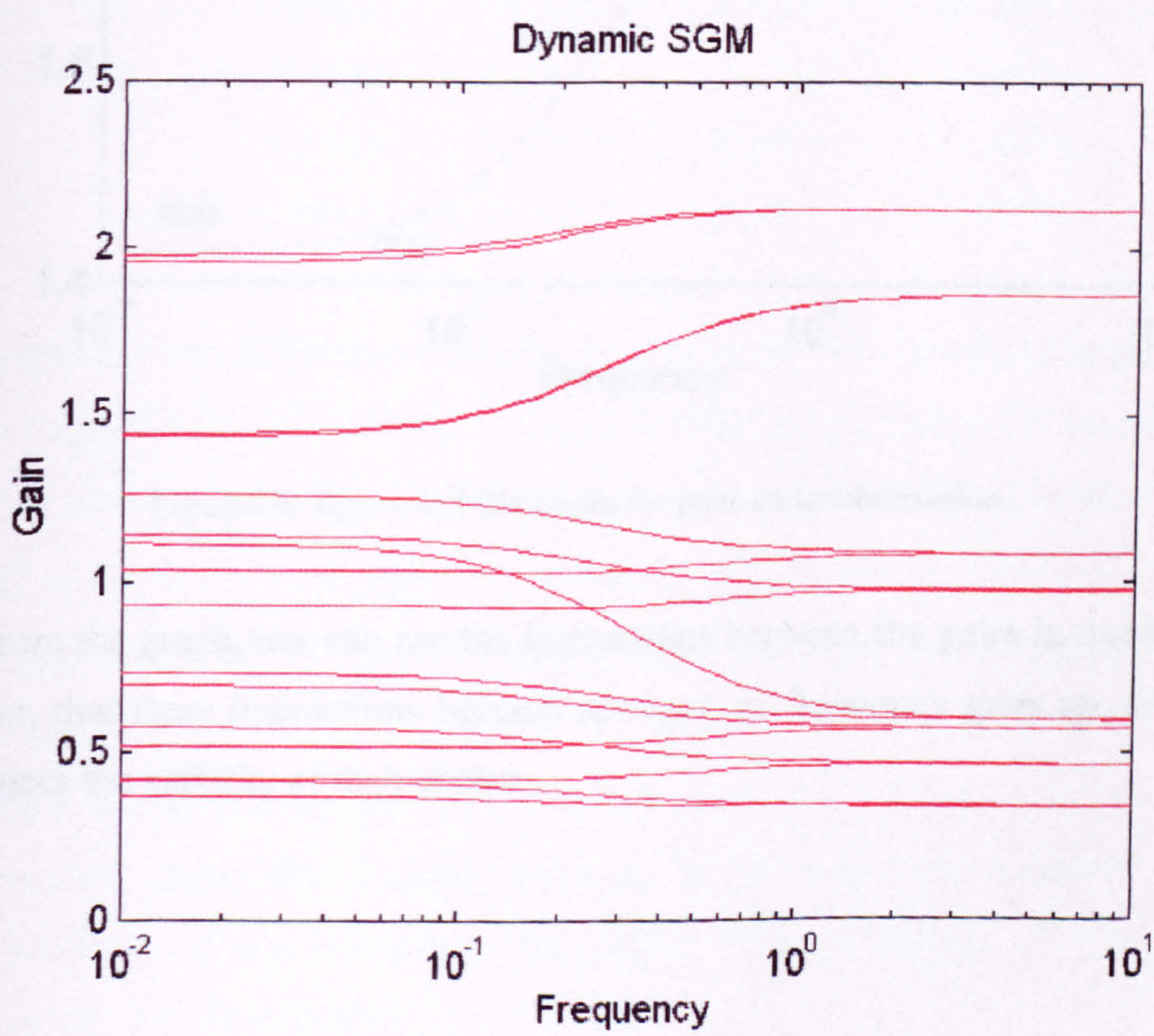


Figure 8.8. Dynamic-SGM results for all 16 pairs.

Although this might seem as a bit complicated, it is a good starting point, since the various trends can easily be seen. For example, the off-diagonal pairs (4,3) and (2,1) that – according to (static) *SGM* – appeared to have strong interactions, do not pose a big threat. These interactions seem to die out with frequency. On the other hand, the (1,2) pair, that seemed O.K. for 0 frequency, seems to play a greater role, as frequency goes up. To further analyse the system, the same graph, but focusing only on the pairs under test, was obtained:

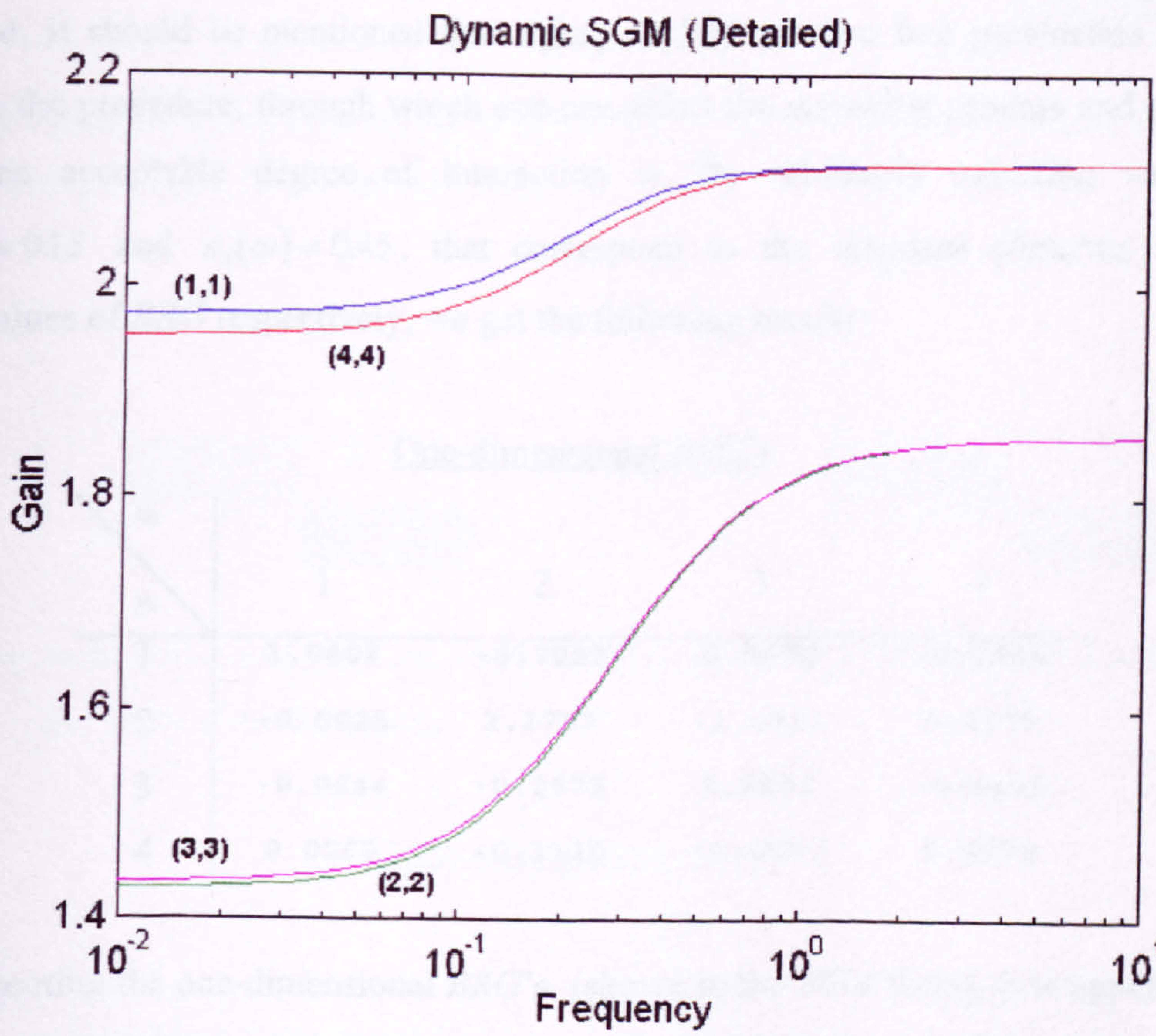


Figure 8.9. Dynamic-SGM results for pairs under observation.

From the graph, one can see the interactions between the pairs in question. It is quite clear, that these interactions become stronger, as frequency goes up, and hence, this enhances the viability of their choice.

8.9 BRG

One method that resembles *RGA* but extends its concept even further, is the *BRG*. In this method, first, we consider the highest degree of decentralisation – i.e. 1×1 block partitioning of G – that would yield a total of N SISO assignments (or pairings). For this, all the one-dimensional *BRG*s are evaluated. Among the viable ones, those which establish a $1-1$ correspondence between the plant's inputs and outputs are selected. It should be mentioned that $\varepsilon_1(\omega)$, $\varepsilon_2(\omega)$ are two free parameters that are used in the procedure, through which one can affect the screening process and establish what an acceptable degree of interaction is. By arbitrarily selecting values of $\varepsilon_1(\omega)=0.15$ and $\varepsilon_2(\omega)=0.45$, that correspond to the diagonal elements and the eigenvalues of *BRG* respectively, we get the following results:

One-dimensional *BRG*'s

y_i \ u_j	1	2	3	4
1	**1.6402**	-0.7959	0.2871	-0.1314
2	-0.6025	**2.1757**	-1.0711	0.4979
3	-0.0644	-0.2678	**2.1757**	-0.8435
4	0.0268	-0.1120	-0.3917	**1.4770**

By inspecting the one-dimensional *BRG*'s, (shown in the *RGA* form), it is apparent that no viable ones exist for our choice of $\varepsilon_1, \varepsilon_2$. It is interesting to note that the highlighted ones are the closest ones to the unity. However, due to the restriction of interaction with the use of $\varepsilon_1, \varepsilon_2$, they are rejected. If one is happy with a greater degree of interaction (e.g. for $\varepsilon_1(\omega)=0.4770$ or $\varepsilon_1(\omega)=0.6402$), then the (4,4) or (1,1) pairs would be accepted, respectively.

Since no one-dimensional *BRG*'s are selected, the procedure should move to two-dimensional ones. The diagonal elements in this case are available when all possible additions of two columns of the above *RGA* are performed. The resulting column vectors are shown below:

Two-dimensional *BRG*'s

y_i \ u_j	1+2	1+3	1+4	2+3	2+4	3+4
1	0.844	1.927	1.509	-0.509	-0.927	0.156
2	1.573	-1.674	-0.105	1.105	2.674	-0.573
3	-0.332	2.111	-0.908	1.908	-1.111	1.332
4	-0.085	-0.365	1.504	-0.504	1.365	1.085

The only promising elements are those marked in columns 1+2, 3+4. The corresponding two-dimensional *BRG*'s are the following:

y_i \ u_j	1	2
1	0.844	0.519
2	-0.153	1.573

y_i \ u_j	3	4
3	1.332	0.327
4	0.117	1.085

Working in the same way, one can verify that no viable three-dimensional BRG's exist. The resulting table is the following:

Three-dimensional *BRG*'s

y_i \ u_j	1	2	3
1	1.131	-0.285	0.281
2	0.230	0.502	0.492
3	0.394	-0.854	1.844

y_i \ u_j	2	3	4
2	1.603	0.111	-0.081
3	0.352	1.064	-0.047
4	0.201	0.037	0.973

The highlighted pairs are the most promising ones, although they do require a relaxation of the restrictions, i.e. an increase of $\varepsilon_1, \varepsilon_2$ to 0.6402 (see *RGA*).

Having found the detailed one, two and three-dimensional *BRG*'s, it is now easy to understand the overall process of *BRG*. This process can be further automated, and this was actually done by the `embrg.m` procedure. This tries to brake the system to the simplest possible sub-systems, given the $\varepsilon_1, \varepsilon_2$, i.e. the acceptable degree of interaction. For the example under investigation, this procedure would give the following results:

```
Please input desired value for error step : 0.0001
w = 0
error = 1.0000e-004
system =  4
--------------------------------------------------
error = 0.5733
system =  2    2
--------------------------------------------------
error = 0.6402
system =  1    3
--------------------------------------------------
error = 0.9080
system =  1    2    1
--------------------------------------------------
error =  1.1757
system =  1    1    1    1
```

The interpretation of these results is the following:

At the beginning, the program prompts for the desirable *error step* value. This represents the step increase of the value of ε_1, (i.e. $\varepsilon_1 = 0$, $\varepsilon_1' = 0.0001$, $\varepsilon_1'' = 0.0002$). Then, the procedure tries to find the simplest mix of sub-systems, for the given error. For error=0, the result is "system = 4", which means we have a 4×4 MIMO system. If the condition is relaxed, then when it reaches the point of error=0.5733, (i.e. $\varepsilon_1 = 0.5733$), the system can be split in two 2×2 subsystems:

y_i \ u_j	1	2	3	4
1	x	x	.	.
2	x	x	.	.
3	.	.	x	x
4	.	.	x	x

As the procedure continues and the error condition reaches the point of 0.6402, then the system can be split in one SISO and one 3×3 MIMO, a result that agrees with the previous findings:

y_i \ u_j	1	2	3	4
1	x	.	.	.
2	.	x	x	x
3	.	x	x	x
4	.	x	x	x

If one is happy with an even greater degree of interaction, but wants a smaller-degree system, then for $\varepsilon_1 = 0.9080$ the system can be split as "system = **1** **2** **1**", i.e. two 1×1 and one 2×2 systems. Attention should be paid to the ordering of the inputs and the outputs:

y_i \ u_j	1	2	3	4
1	X	.	.	.
2	.	x	x	.
3	.	x	x	.
4	.	.	.	x

The procedure concludes when the most decentralised system is found (i.e. n SISO systems). In this case though, $\varepsilon_1 = 1.1757$, a result that corresponds with the 2.1757 element of our *BRG*. The proposed system is the following:

y_i \ u_j	1	2	3	4
1	x	.	.	.
2	.	x	.	.
3	.	.	x	.
4	.	.	.	x

This procedure has produced some useful results. However, to fully explit the method, one should proceed with the *D-BRG*, so that the frequency response of the system is also evaluated.

8.10 Dynamic BRG

Having found the 0-frequency pairs, it is important to establish their behaviour at high frequencies. The `emdbrga` procedure provides this kind of information. The same pairs that were found from the *BRG* are treated for frequency response. The results are the following (figure 8.10).

It is interesting to note that the results are the expected ones and also that – due to symmetry – the pairs terminate to the same final values.

Next, the eigenvalues are examined. From figure 8.11, one can easily see that they belong to $(1 \pm \varepsilon_2)$ space, and thus they are viable. Note again, that because of the physical symmetry, the eigenvalues of the two *BRG*s are identical. The first set is printed with lines and the second is superimposed in the form of small circles.

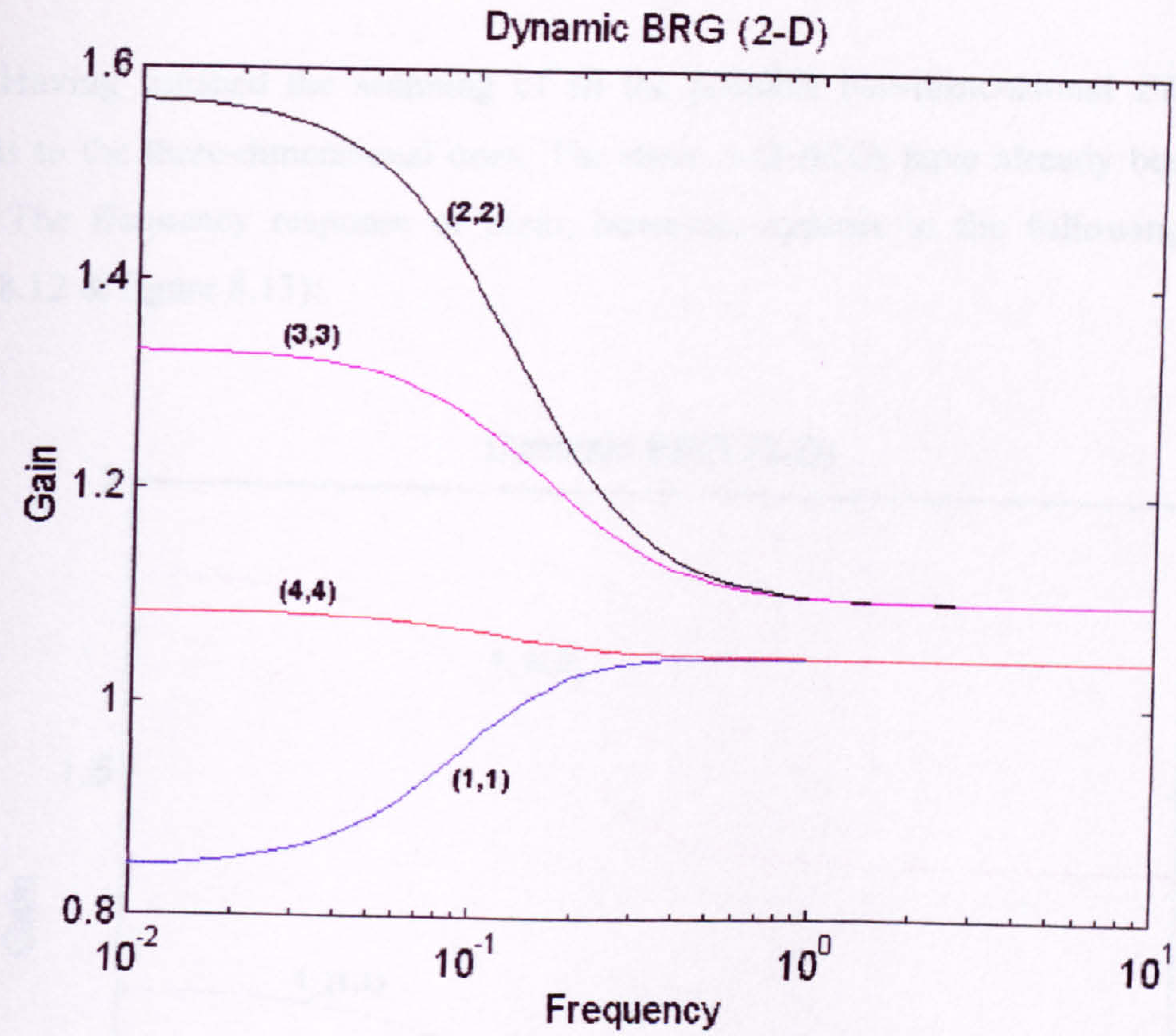


Figure 8.10. Dynamic-*BRG* results for two-dimensional *BRG*s.

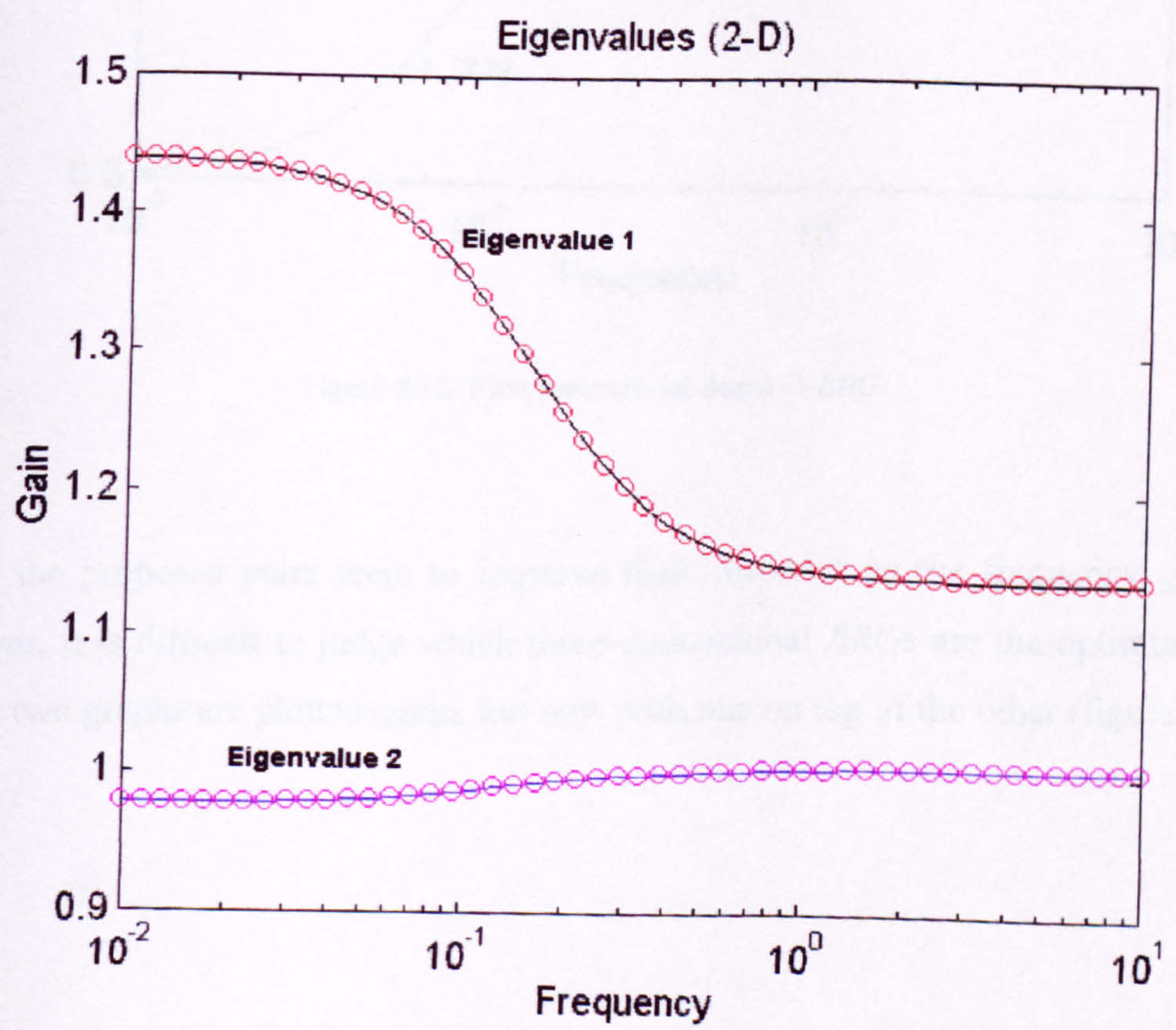


Figure 8.11. Eigenvalue results for two-dimensional *BRG*s.

Having finished the scanning of all the possible two-dimensional *BRG*s, one proceeds to the three-dimensional ones. The static 3-D *BRG*s have already been found in 8.8. The frequency response of them, however, appears in the following graphs (figure 8.12 & figure 8.13):

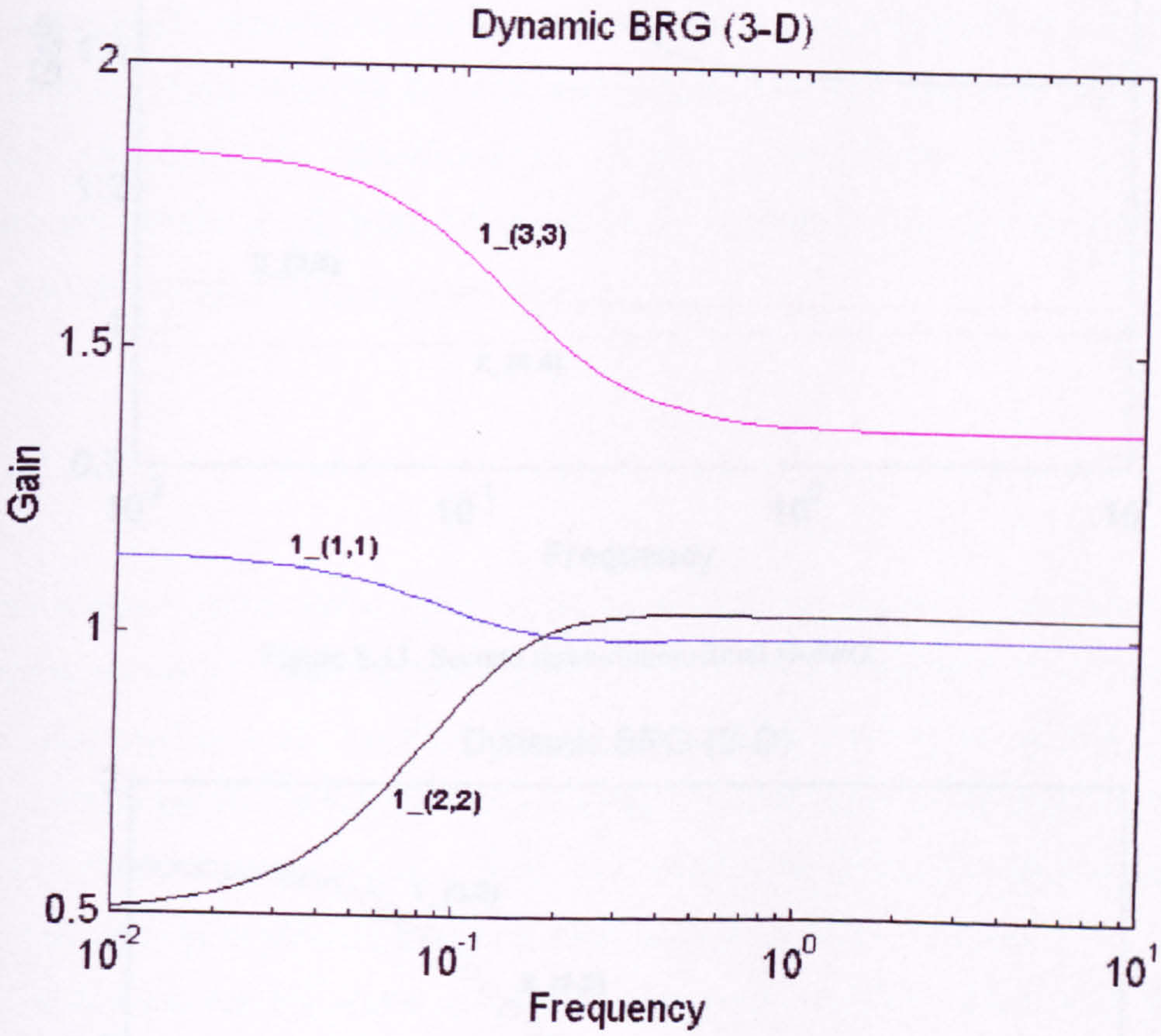


Figure 8.12. First three-dimensional *D-BRG*.

Again, the proposed pairs seem to improve their response as the frequency goes up. However, it is difficult to judge which three-dimensional *BRG*s are the optimum ones. So, the two graphs are plotted again, but now with one on top of the other (figure 8.14):

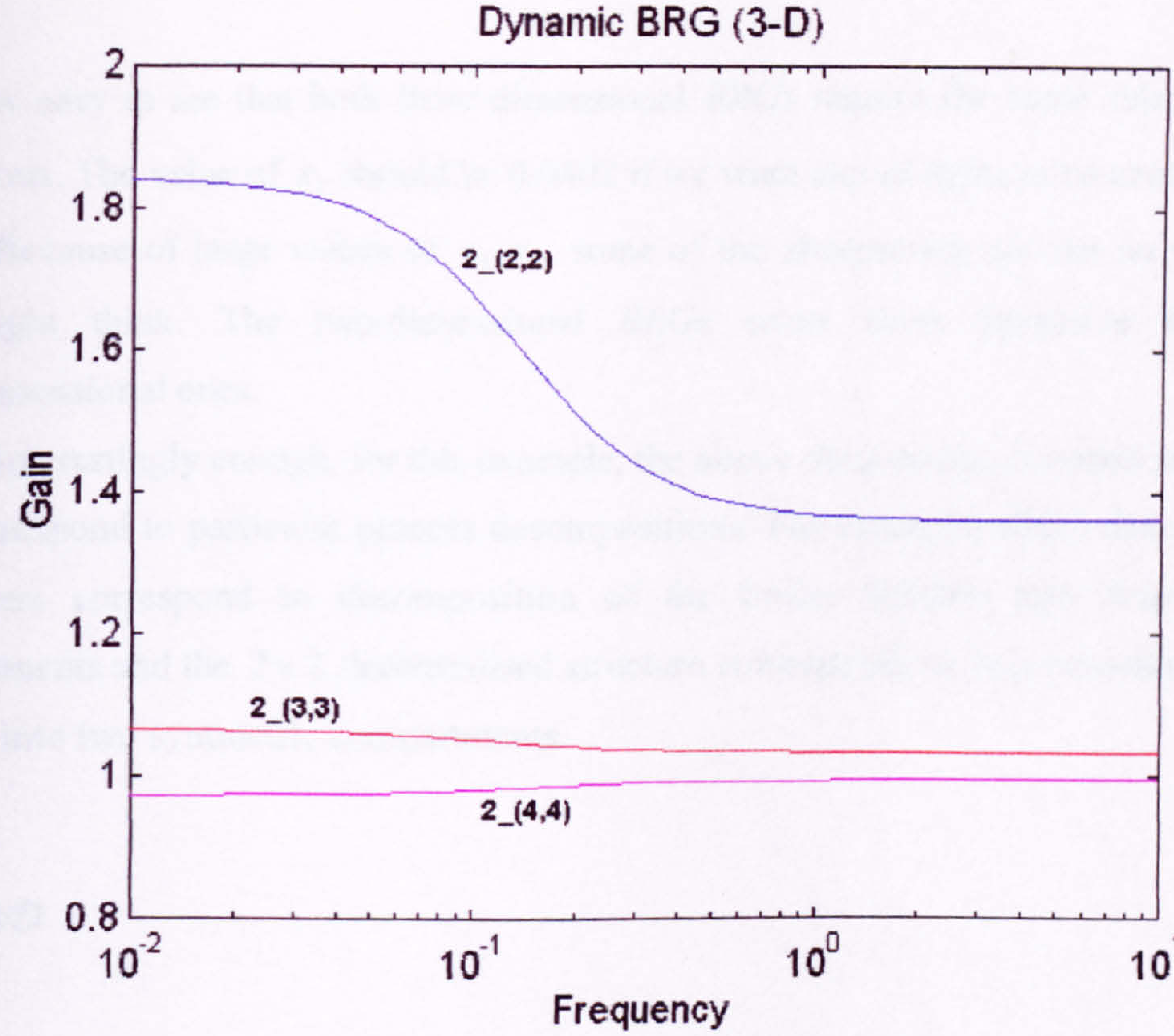


Figure 8.13. Second three-dimensional *D-BRG*.

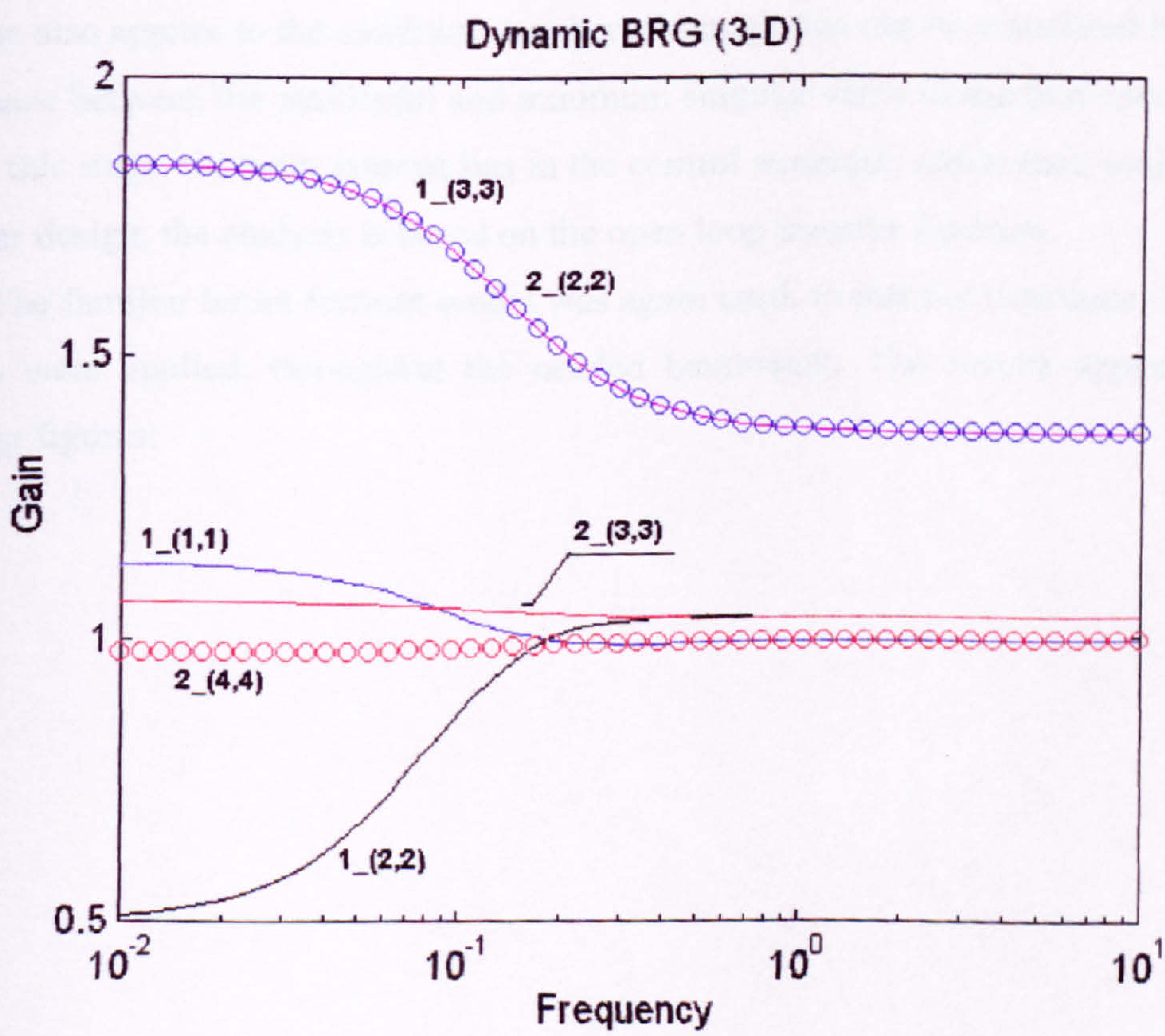


Figure 8.14. Dynamic-*BRG* results for three-dimensional *BRG*s.

It is now easy to see that both three-dimensional *BRG*s require the same relaxation of restrictions. The value of ε_1 should be 0.6402 if we want any of them to be accepted.

Because of large values of $\varepsilon_1, \varepsilon_2$, some of the alternatives are not so viable as one might think. The two-dimensional *BRG*s seem more attractive than the three-dimensional ones.

Interestingly enough, for this example, the above decentralised control structures also correspond to particular process decompositions. For example, SISO decentralised controllers correspond to decomposition of the boiler furnace into four similar compartments and the 2×2 decentralised structure corresponds to decomposition of the furnace into two symmetric compartments.

8.11 SVD

By using singular value analysis, many system properties can be characterised. The singular values and the directional angles can be plotted as functions of frequency. The same also applies to the condition number, although this can be visualised by taking the distance between the maximum and minimum singular value in the first set of plots. Since at this stage, the main interest lies in the control structure, rather than in the actual controller design, the analysis is based on the open loop transfer function.

The familiar boiler furnace model was again used, to test the functions. All three methods were applied, throughout the needed bandwidth. The results appear in the following figures:

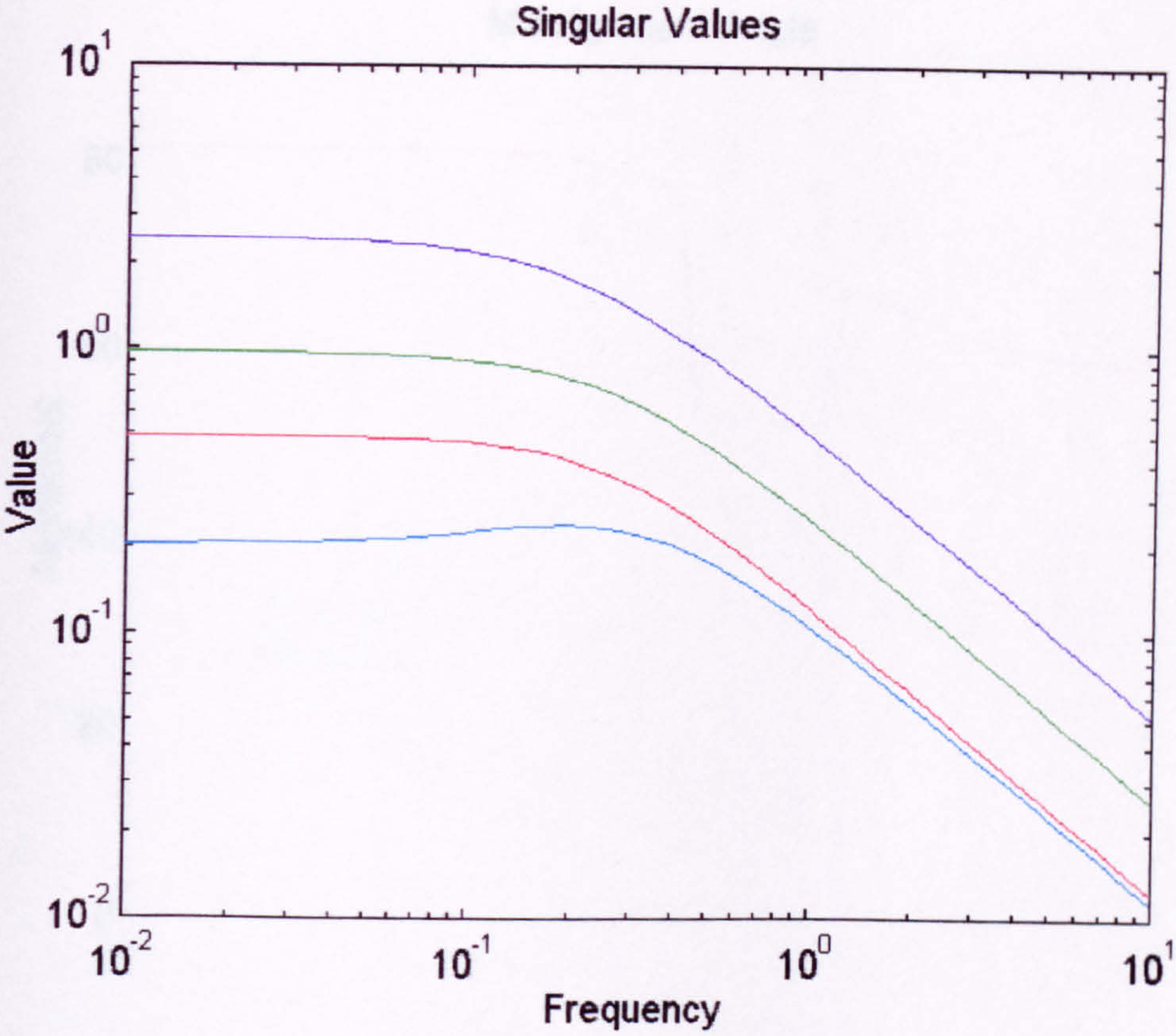


Figure 8.15. Singular Values of the system.

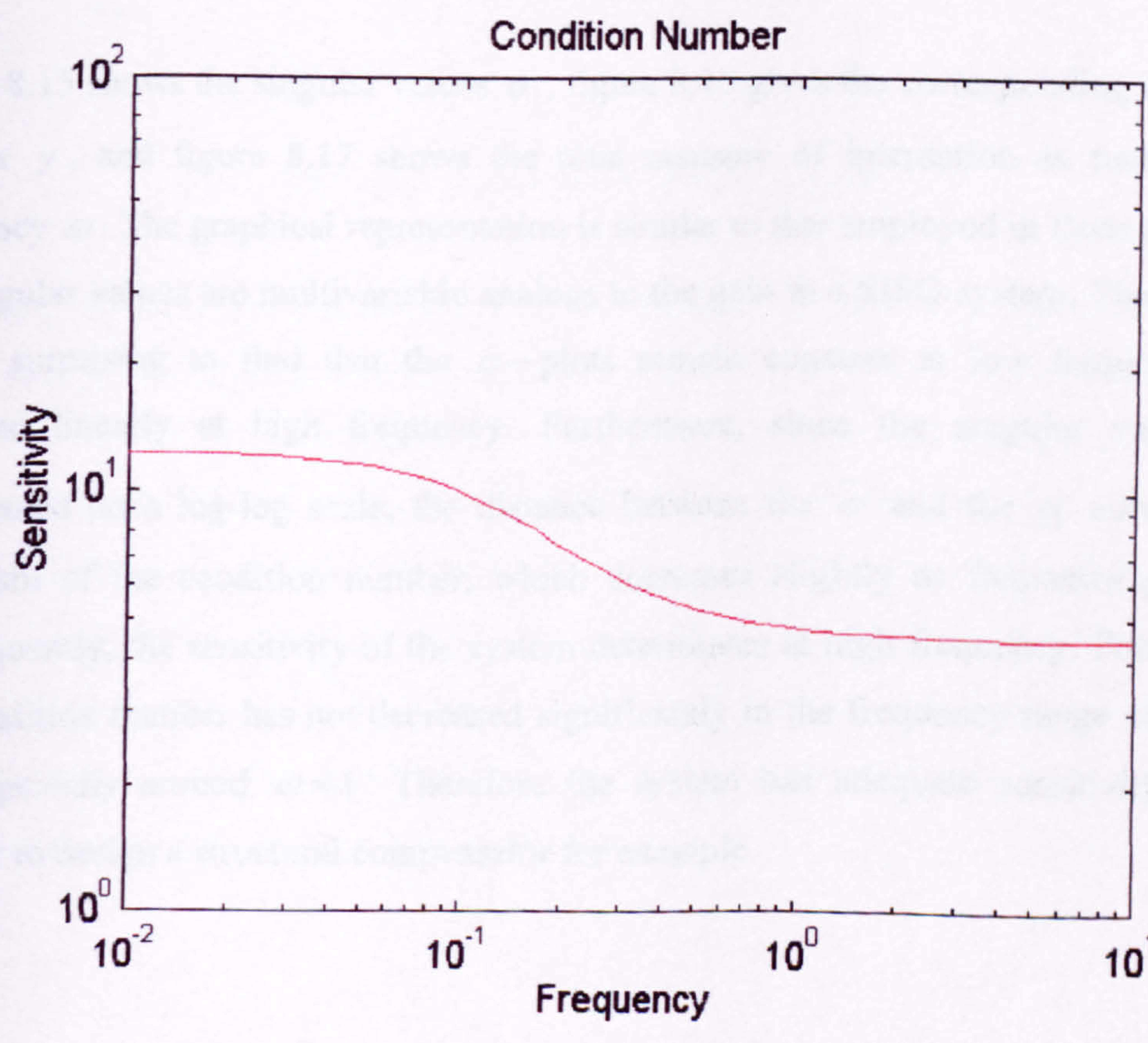


Figure 8.16. Condition number.

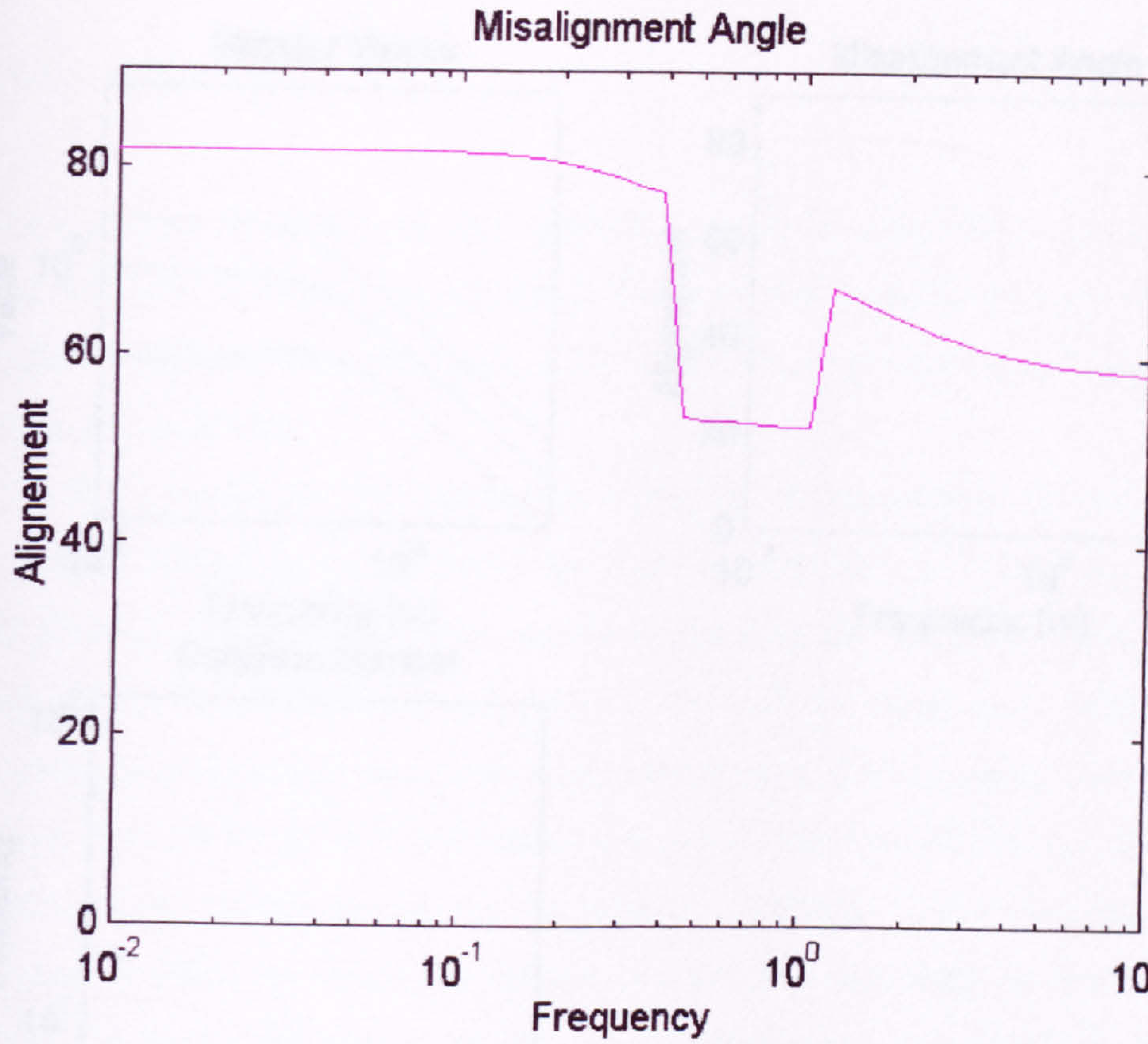


Figure 8.17. Misalignment Angle.

Figure 8.15 shows the singular values σ_i, figure 8.16 gives the corresponding condition number γ, and figure 8.17 shows the total measure of interaction as functions of frequency ω. The graphical representation is similar to that employed in Bode plots and the singular values are multivariable analogs to the gain in a SISO system. Therefore, it is not surprising to find that the σ − plots remain constant at low frequency and decrease linearly at high frequency. Furthermore, since the singular values are represented on a log-log scale, the distance between the $\overline{\sigma}$ and the $\underline{\sigma}$ curve is the logarithm of the condition number, which decreases slightly as frequency goes up. Consequently, the sensitivity of the system deteriorates at high frequency. Fortunately, the condition number has not decreased significantly in the frequency range of interest and especially around $\omega = 1$. Therefore the system has adequate sensitivity for an attempt to design a structural compensator for example.

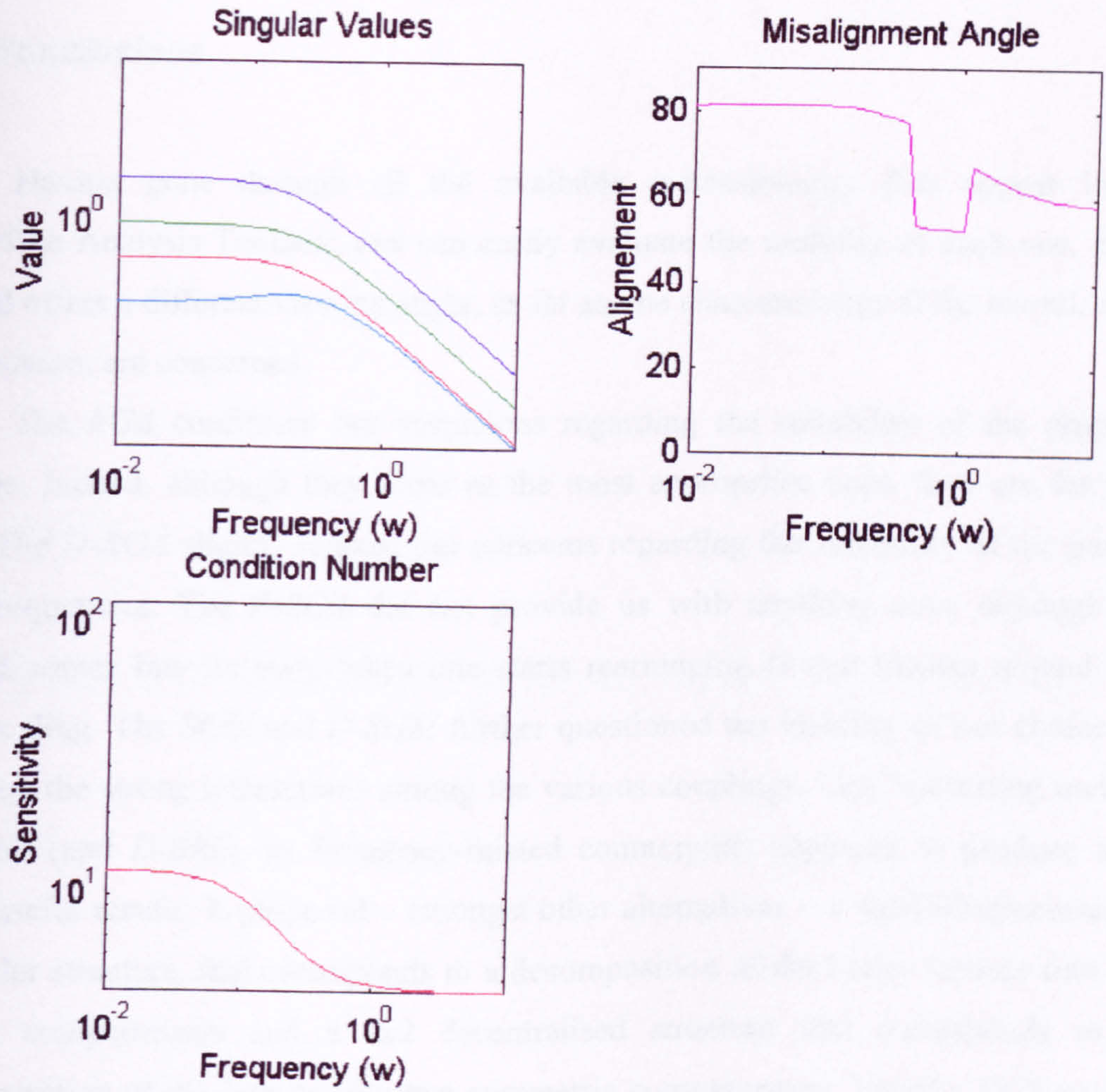


Figure 8.18. The collection of all three results.

The angle of alignment (figure 8.17) by far exceeds 15° for all frequencies, so the system has no natural loop structure and compensators must be introduced to reduce the interactions within the system (see 6.3.1 and 7.10). Figure 8.18 represents a collection of all three diagnostics, so that results can easily be drawn for the system.

This example demonstrates how *SVD* analysis yields structural information on the open-loop system. The current approach also has the advantage of decoupling the interaction and sensitivity analysis, so that they can be handled independently. Consequently, a feedback control system can easily be designed once the analysis and the proper compensations have been performed.

8.12 Conclusions

Having gone through all the available methodologies that appear in the Interaction Analysis Toolbox, one can easily evaluate the usability of each one. Every method offers a different viewing angle, as far as, the characteristics of the model, under examination, are concerned.

The *RGA* confirmed our suspicions regarding the suitability of the proposed pairings. Indeed, although they seem as the most appropriate ones, they are far from ideal. The *D-RGA* slightly relaxed our concerns regarding the suitability of the pairs at high frequencies. The *P-RGA* did not provide us with anything new, although this method comes into its own, when one starts rearranging *G* and messes around with input scaling. The *SGM* and *D-SGM* further questioned the viability of our choice and indicated the strong interactions among the various couplings. One interesting method, the *BRG* (and *D-BRG*, its frequency-related counterpart) appeared to produce some really useful results. It proposed – amongst other alternatives – a 4xSISO decentralised controller structure, that corresponds to a decomposition of the boiler furnace into four similar compartments and a 2x2 decentralised structure that corresponds to the decomposition of the furnace into two symmetric compartments. Finally, *SVD* analysis was unable to produce results related to the sensitivity of the system and provide some insights for the possible use of compensators.

It is well-known that no single method can deal with every control structure selection related problem. The solutions also tend to be very case dependent. However, the constructed Interaction Analysis Toolbox uses a complete set of diagnostics, so that the designer can pick and match from all the available results and indicators.

Our intention was not to automate the control structure selection process – an almost impossible mission – but rather to present a tool that can be applied in almost every case and provide the richest possible set of diagnostics.

Chapter 9

INTEGRATED STRUCTURAL METHODOLOGY

Use inference to narrow suspects.

9. INTEGRATION METHODOLOGIES FOR CONTROL STRUCTURE SELECTION AND SYSTEM CONSIDERATIONS

9.1 Introduction

The problem of control structure selection may be seen as involving three major steps: (a) Classification of variables and definition of system progenitor models; (b) Definition of effective sets of inputs, outputs and; (c) Structuring of the feedback coupling of the control scheme. The overall structural methodology, that has been adopted in SESDIP project [SESDIP, 1] and used here, suggests a natural procedure for the study of the above three problems and poses a number of concrete problems for each of the three areas. The ordering of subproblems, we address in each of the above families, is based on the generality of the issues and the progression from simple models to more detailed dynamic models. The current approach is based on linear models only. Apart from specifying tests, that may help answering questions which are part of the future developments in the area, we address a few representative problems here to demonstrate the issues. However, the proper development of the field is a topic of longer-term research.

The classification of variables is a problem that is not always solved using physical modelling arguments. Very frequently, it may lead to progenitor models, which are not well defined. The specific issues involved in the selection of a well-defined progenitor model and the procedure, that can be used to define a well-behaved model, are considered in section 9.2. The structuring of an effective input, output structure is considered in section 9.3, where a procedure progressing from generic properties on unstructured models, to graph properties, parameter dependent invariants and performance indicators is suggested, which reflects the overall structural philosophy we have introduced. Having decided the required input, output structure of the feedback scheme, the issue that has to be decided is that of structure of the feedback scheme i.e. centralised versus decentralised, and if decentralised, then the exact nature of decentralisation. The latter involves the partitioning and the pairing, as well as order of dynamics, for the particular channels. The methodology and diagnostics are based on the use of simple models first that progressively move to more detailed models and more detailed structural criteria. The current emphasis in the approach is the screening of the bad choices. Then, the final selection is left to performance dependent criteria and multi-objective optimisation. A procedure for sorting out various criteria is based on specifying first the structure, then use optimisation for the fine-tuning of parameters.

9.2 Classification of Variables and Definition of Well-Structured Progenitor Models

We assume that we are given a linearised model with a large number of exogenous variables, potential measurements and controlled variables and a given number of states. Very frequently, we may start with a matrix pencil, or auto-regressive model.

General problem: Define an oriented effective model that has the "best" possible properties and a control structure that allows the solvability of a number of important control problems that may be posed.

□

The model that is given is partially non-oriented, since the exogenous variables are not classified to control variables and disturbances. Approaches, that may be followed to tackle the above mentioned problem, are of the following type:

(i) Physical approach.

(ii) Structure Assignment on implicit models approach.

(iii) Hybrid approach.

Here we consider the fundamental issues of the first two, whereas the third is based on the composition of the first two.

9.2.1 Conditioning of an Oriented Model Derived by Engineering Considerations

The starting point of our investigation may be an implicit model of the matrix pencil type [Karc. & Hay., 1]:

$$Fp\underline{\xi} = G\underline{\xi}, \qquad F, G \in \Re^{l \times k} \tag{9.1}$$

where $\xi \in \Re^k$ is a mixed variable vector, or more generally auto-regressive forms of the behavioural type [Wil., 1]

$$T(p)\underline{\zeta} = 0, \quad T(p) \in \Re^{v \times \mu}[p], \ \underline{\zeta} \in \Re^{\mu} \tag{9.2}$$

The classification of variables may be based on engineering arguments and it is summarised below:

Problem (1): Using knowledge of the physics, chemistry of the problem, as well as assuming a given system boundary (see report on Global Instrumentation [SESDIP 1]), we can provide a classification of the exogenous variables into: (a) potential control variables and (b) disturbances.

□

Remark (9.1): Resolving the issues involved above we use physical modelling arguments (knowledge of process) and knowledge, specification of the system boundaries (design scope, assumption).

□

The result of this step is an oriented model (separation of control, disturbance variables and outputs), but not necessarily well defined. In fact, the inputs, outputs may not be independent and the input, output transfer function may not be of full rank. We, thus, consider the system descriptions:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}, \quad A \in \Re^{n \times n}, \quad B \in \Re^{n \times r} \tag{9.3}$$

$$\underline{y} = C\underline{x} + D\underline{u}, \quad C \in \Re^{q \times n}, \quad D \in \Re^{q \times r} \tag{9.4}$$

with corresponding transfer function $H(s) = C(sI - A)^{-1}B + D \quad \in \Re^{q \times r}(s)$.

Problem (2): Devise a methodology for the selection of a well structured, progenitor model (or a family of well structured models) such that the matrices B, C, or the corresponding input, output matrices for proper systems have full rank and the transfer function $H(s)$ has full rank.

□

This procedure implies the need to compute the normal rank of $H(s)$. For a number of distinct frequencies (randomly) selected, use *SVD* to compute rank of $H(j\omega)$. This leads to the definition of the normal rank ρ of the transfer function.

Remark (9.2): ρ defines the maximal number of output variables that may be independently controlled (output function controllability criterion). Furthermore, ρ defines the minimal number of independent inputs required for control of ρ outputs (if it is less than ρ, then we control fewer variables).

□

The number ρ emerges as one of the most basic structural characteristics that determine fundamental problems of the final model. Some important structural properties related to degeneracy follow from the study of the Kronecker structure of system matrices [Karc. & McBean, 1] and some of the properties are summarised below:

Remark (9.3): If $H(s) \in \Re^{q \times r}(s)$ and $rank_{\Re(s)}\{H(s)\} = \rho < \min(q,r)$, then

$$N_\ell\{H(s)\} \neq \{0\} \text{ and } N_r\{H(s)\} \neq \{0\}$$

a) If $N_r\{H(s)\} \neq \{0\}$ and there exist constant vectors in it, then

$$\text{rank}\left\{\begin{bmatrix} B \\ D \end{bmatrix}\right\} < r \text{ and } \text{rank}(B) < r \tag{9.5}$$

Furthermore, if

$$n_r\left\{\begin{bmatrix} B \\ D \end{bmatrix}\right\} = r - \rho \tag{9.6}$$

then all right indices of $H(s)$ are zero, then the system is called totally input degenerate. Similarly:

b) If $N_l\{H(s)\} \neq \{0\}$ and there exist constant vectors in it, then

$$\text{rank}\{[C \quad D]\} < q \text{ and } \text{rank}(C) < q \tag{9.7}$$

Furthermore, if

$$n_\ell\{[C \quad D]\} = q - \rho \tag{9.8}$$

then all the left indices of $H(s)$ are zero and the system is called totally output degenerate.

For a general system $H(s) \in \Re^{q \times r}(s)$ we define the numbers of 0-right and 0-left (nulling) indices (0-cmi, 0-rmi of the system matrix) as

$$t_r = r - \text{rank}\left\{\left[B' \quad D'\right]'\right\} \leq r - \rho \tag{9.9}$$

$$t_\ell = q - \text{rank}\{[C \quad D]\} \leq q - \rho \tag{9.10}$$

and we refer to them as the input-output redundancy indices of the $H(s)$ model; with this notation, total input (output) degeneracy $t_r = r - \rho$, ($t_\ell = q - \rho$), implies that all right (left) indices of the system are of the 0-type. Alternatively, these conditions imply that degeneracy of the transfer function is entirely due to redundancy in the input or output scheme.

The above discussion suggests that an important problem that has to be solved at this stage is the following:

Full Rank Conditioning Problem (FRCP): Consider a system with rank deficient transfer function and possibly having nonzero redundancy indices. Define maximal subsets of the existing input, output variables, such that the resulting system is non-degenerate and has full rank input and output structure.

□

Note that implicit in the above problem formulation is the assumption that the original model inputs and outputs are physical variables and thus we want to select a maximal subset of them rather than carrying out general transformations of the input, output sets. This additional condition influences considerably the study of the problem. The rest of this section deals with the study of solutions to the above problem.

For the sake of simplicity, we shall consider the case of systems with $q \geq r$ and shall assume that $\rho = \text{rank}_{\Re}(s)\{H(s)\} < r$. The analysis for the $q < r$ case follows by transposed duality arguments. We start the analysis by considering the case first of 0-right (*0-ri*), 0-left indices (*0-li*).

9.2.1a Zero Indices case

If $P(s)$ is the Rosenbrock's system matrix

$$P(s) = \begin{bmatrix} sI - A & -B \\ -C & -D \end{bmatrix} \in \Re^{(n+q)\times(n+r)}[s] \tag{9.11}$$

then we have:

(a) There exists a 0-right index (*0-ri*) if there exists a vector $[\underline{0}', \underline{u}']'$ such that

$$\begin{bmatrix} sI - A & -B \\ -C & -D \end{bmatrix} \begin{bmatrix} 0 \\ \underline{u} \end{bmatrix} = 0 \quad \rightarrow \quad \begin{bmatrix} B \\ D \end{bmatrix} \underline{u} = 0 \tag{9.12}$$

(b) There exists a 0-left index (*0-li*), if there exists a vector $\left[\underline{0}', \underline{y}'\right]$ such that

$$\left[\underline{0}', \underline{y}'\right]\begin{bmatrix} sI-A & -B \\ -C & -D \end{bmatrix} = 0 \quad \rightarrow \quad \underline{y}'\begin{bmatrix} C & D \end{bmatrix} = 0 \tag{9.13}$$

Remark (9.4): If $q \geq r$, then the presence of *0-ri* implies that $H(s)$ is degenerate and is equivalent to redundancy in the actuator scheme. The presence of *0-li* is equivalent to redundancy in the actuator scheme, but does not necessarily imply system degeneracy, unless $q = r$. The reverse holds for the $q \leq r$ case.

□

Let us now denote by

$$F = \begin{bmatrix} B \\ D \end{bmatrix} = \left[\underline{f}_1, \ldots, \underline{f}_r\right] \in \Re^{(n+q)\times r}, \quad \text{rank}(F) = \tau_0 \tag{9.14}$$

$$H = \begin{bmatrix} C & D \end{bmatrix} = \begin{bmatrix} \underline{h}_1' \\ \vdots \\ \underline{h}_q' \end{bmatrix} \in \Re^{q\times(n+r)}, \quad \text{rank}(H) = \sigma_0 \tag{9.15}$$

The selection of a maximal τ_0-cardinality subset of $\left\{\underline{f}_1, \cdots, \underline{f}_r\right\}$, or σ_0-cardinality subset of $\left\{\underline{h}_1, \cdots, \underline{h}_r\right\}$ to guarantee ranks τ_0, σ_0 respectively does not have a unique solution. Selection of the most orthogonal τ_0 (σ_0) cardinality subset of $\left\{\underline{f}_i\right\}$ $\left(\left\{\underline{h}_j\right\}\right)$ is a problem that can be solved by using the "Best Uncorrupted Basis Algorithm" [Mitr. & Karc., 2]. A description of this Algorithm will be given at the end of this section. The result of applying this algorithm is that we obtain a smaller dimension model

$$H'(s) \in \Re^{q' \times r'}(s), \quad \begin{cases} q' \leq q \\ r' \leq r \end{cases} \tag{9.16}$$

where $\text{rank}\{[C' \quad D']\}$ and $\text{rank}\{[B'' \quad C'']'\}$ are full and thus, there are no zero minimal indices in $N_l\{H'(s)\}$ and $N_r\{H'(s)\}$. However, we might have

$$\text{rank}\{H'(s)\} = \rho' < \min\{q', r'\} \tag{9.17}$$

9.2.1b Nonzero Indices Case

We consider now the case where $q \geq r$, $\rho < r$ and assume that the system has a right index with value k. Then, there exists a pair of polynomial vectors $\underline{x}(s)$, $\underline{u}(s)$ such that

$$\underline{x}(s) = \underline{x}_0 + s\ \underline{x}_1 + \ldots + s^{k-1}\underline{x}_{k-1} \tag{9.18a}$$

$$\underline{u}(s) = \underline{u}_0 + s\ \underline{u}_1 + \ldots + s^{k-1}\underline{u}_{k-1} \tag{9.18b}$$

$$(sI - A)\underline{x}(s) = B\underline{u}(s) \tag{9.19}$$

$$C\underline{x}(s) + D\underline{u}(s) = 0 \tag{9.20}$$

The above lead to the following result

<u>**Proposition (9.1):**</u> The system $S(A,B,C,D)$ with $q \geq r$ and $\rho < r$ has a right index with value k, if and only if there exists a set of vectors $\{\underline{u}_0, \underline{u}_1, \cdots, \underline{u}_k\}$ such that the following conditions are satisfied:

$$\begin{bmatrix} A^kB & A^{k-1}B & A^{k-2}B & \cdots & A^2B & AB & B \\ CA^{k-1}B & CA^{k-2}B & CA^{k-3}B & \cdots & CAB & CB & D \\ CA^{k-2}B & CA^{k-3}B & CA^{k-4}B & \cdots & CB & D & 0 \\ \vdots & \vdots & \vdots & & & & \vdots \\ CAB & CB & D & \cdots & 0 & 0 & 0 \\ CB & D & 0 & \cdots & 0 & 0 & 0 \\ D & 0 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{u}_k \\ \underline{u}_{k-1} \\ \underline{u}_{k-2} \\ \vdots \\ \underline{u}_2 \\ \underline{u}_1 \\ \underline{u}_0 \end{bmatrix} = \underline{0} \tag{9.21}$$

Proof:

Substituting the expressions of $\underline{x}(s)$, $\underline{u}(s)$ from (9.18a), (9.18b) into (9.19), (9.20) we have

$$(sI-A)\left(\underline{x}_0 + s\underline{x}_1 + \cdots + s^{k-1}\underline{x}_{k-1}\right) = B\left(\underline{u}_0 + s\underline{u}_1 + \cdots + s^k\underline{u}_k\right)$$

$$C\left(\underline{x}_0 + s\underline{x}_1 + \cdots + s^{k-1}\underline{x}_{k-1}\right) + D\left(\underline{u}_0 + s\underline{u}_1 + \cdots + s^k\underline{u}_k\right) = 0$$

By equating coefficients of equal powers, it follows that

$$\begin{aligned} \underline{x}_{k-1} &= B\underline{u}_k \\ \underline{x}_{k-2} &= AB\underline{u}_k + B\underline{u}_{k-1} \\ &\vdots \\ \underline{x}_0 &= A^{k-1}B\underline{u}_k + A^{k-2}B\underline{u}_{k-1} + \cdots + AB\underline{u}_2 + B\underline{u}_1 \end{aligned} \tag{9.22a}$$

$$0 = A^kB\underline{u}_k + A^{k-1}B\underline{u}_{k-1} + \cdots + A^2B\underline{u}_2 + AB\underline{u}_1 + B\underline{u}_0 \tag{9.22b}$$

and

$$\begin{aligned} C\underline{x}_0 + D\underline{u}_0 &= 0 = CA^{k-1}B\underline{u}_k + CA^{k-2}B\underline{u}_{k-1} + \cdots + CAB\underline{u}_2 + CB\underline{u}_1 + D\underline{u}_0 \\ C\underline{x}_1 + D\underline{u}_1 &= 0 = CA^{k-2}B\underline{u}_k + CA^{k-3}B\underline{u}_{k-1} + \cdots + CAB\underline{u}_3 + CB\underline{u}_2 + D\underline{u}_1 \\ &\vdots \\ C\underline{x}_{k-1} + D\underline{u}_{k-1} &= 0 = CB\underline{u}_k + D\underline{u}_{k-1} \\ D\underline{u}_k &= 0 \end{aligned} \tag{9.23}$$

By combining (9.23) and (9.22b), condition (9.21) follows.

The above condition may now be used to derive conditions for non-degeneracy of transfer functions and thus also procedures for redesign of the system to guarantee non-degeneracy. For the given system, we define the following set of matrices:

$$M_o = \begin{bmatrix} B \\ D \end{bmatrix},\ M_1 = \begin{bmatrix} AB & B \\ CB & D \\ D & 0 \end{bmatrix},\ M_2 = \begin{bmatrix} A^2B & AB & B \\ CAB & CB & D \\ CB & D & 0 \\ D & 0 & 0 \end{bmatrix}, \cdots$$

$$\cdots, M_k = \begin{bmatrix} A^kB & A^{k-1}B & \cdots & AB & B \\ CA^{k-1}B & CA^{k-2}B & \cdots & CB & D \\ CA^{k-2}B & CA^{k-3}B & \cdots & D & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ CAB & CB & \cdots & 0 & 0 \\ CB & D & \cdots & 0 & 0 \\ D & 0 & \cdots & 0 & 0 \end{bmatrix} = \left[\begin{array}{ccc|c} A^kB & \cdots & AB & B \\ \hline & & & D \\ & N_k & & \vdots \\ & & & 0 \end{array}\right] \tag{9.24}$$

In terms of the above matrices, we may state the following result:

Theorem (9.1): For the system $S(A,B,C,D)$ with $q \geq r$, the following properties hold true:

(i) If M_0 has full rank, then the system has no right indices of any value and it is thus non-degenerate.

(ii) If τ is the smallest integer for which M_τ has full rank, then the maximal possible value of a right index is $\tau - 1$. The existence of such indices is determined by the solutions of the following equation

$$\left[\begin{array}{cccc|c} A^{\tau-1}B & A^{\tau-2}B & \ldots & AB & B \\ \hline & & & & \mathrm{D} \\ & & N_{\tau-1} & & \vdots \\ & & & & 0 \end{array}\right] \begin{bmatrix} \underline{u}_{\tau-1} \\ \vdots \\ \underline{u}_1 \\ \underline{u}_0 \end{bmatrix} = 0 \tag{9.25}$$

Proof:

(i) From Proposition (9.1), it follows that if $\text{rank}(D) = r$, then from the last of (9.21) we have that $D\underline{u}_k = 0$. Clearly, this implies $\underline{u}_k = 0$ and this in turn (from (9.21)) yields $D\underline{u}_{k-1} = 0$; again we have $\underline{u}_{k-1} = 0$ and by obvious induction, $\underline{u}_k = 0$ for all $k = 0,1,2,\cdots$. It is now clear that since there is no $u(s)$ and thus no $x(s)$ satisfying (9.19) and (9.20), the system is non-degenerate.

(ii) Once more, if τ is the smallest integer for which M_τ is full rank, then for any $k > \tau$ we have from (9.21) that

$$M_\tau \begin{bmatrix} \underline{u}_k \\ \vdots \\ \underline{u}_{k-\tau-1} \end{bmatrix} = 0$$

from which $\underline{u}_n = \cdots = \underline{u}_{k-\tau-1} = 0$ and thus, if indices exist, then their values cannot be larger than τ.

□

For the case of strictly proper systems, we may define the matrices:

$$\widetilde{M}_o = [B],\ \widetilde{M}_1 = \begin{bmatrix} AB & B \\ CB & 0 \end{bmatrix},\ \widetilde{M}_2 = \begin{bmatrix} A^2B & AB & B \\ CAB & CB & 0 \\ CB & 0 & 0 \end{bmatrix} \cdots \widetilde{M}_k = \begin{bmatrix} A^kB & \cdots & A^2B & AB & B \\ CA^{k-1}B & \cdots & CAB & CB & 0 \\ CA^{k-2}B & \cdots & CB & 0 & 0 \\ \vdots & & \vdots & \vdots & \vdots \\ CAB & \cdots & 0 & 0 & 0 \\ CB & \cdots & 0 & 0 & 0 \end{bmatrix} \quad (9.26)$$

and Theorem (9.1) leads to the following corollary:

Corollary (9.1): For the system $S(A,B,C,D)$ with $q \geq r$, the following properties hold true:

(i) If $\widetilde{M}_1$ is full rank, then the system has no right indices and the system is non-degenerate.

(ii) If τ is the smallest integer for which $\widetilde{M}_\tau$ has full rank, then the maximal possible value of a right index is $\tau-1$. The existence of such indices is determined by the solutions of

$$\left[\begin{array}{cccc|c} A^{\tau-1}B & A^{\tau-2}B & \cdots & AB & B \\ \hline & & & & 0 \\ & N_{\tau-1} & & & \vdots \\ & & & & 0 \end{array}\right] \tag{9.27}$$

□

Proof:

(i) From (9.21) we have that there exists a 0-right index if the matrix $[B',0]'$ or equivalently B looses rank. However, if $\text{rank}(CB)=r$, then it is necessary that $\text{rank}(B)=r$, because, otherwise $\exists \underline{v} : \underline{v} \neq \underline{0}$ and $B\underline{v}=0 \rightarrow CB\underline{v}=0$ and this leads to a contradiction. Thus, there is no 0-right index. Following similar arguments to those in the proof of the Theorem, it follows also that there is no other right index of any value k.

(ii) Part (ii) follows along similar lines.

□

The above results provide for formulating redesign procedures for $H(s)$, which can lead to transfer functions with non-degeneracy and non-redundancy in the input, output structure. Redesign that may lead to the minimal possible reduction of the numbers q,r has to be based on the investigation of conditions of the type (9.25), (9.27). In fact, what we are aiming at is the reduction of the number of inputs such that the above conditions are not satisfied. Such investigations are possible, but quite complicated. An alternative simpler approach for redesign is to rely on sufficient conditions. This is summarised below:

Remark (9.5): If the system $S(A,B,C,D)$ with $q \geq r$ is degenerate, a redesign procedure leading to $S'(A,B',C,D')$ with D' full rank guarantees the creation of a system which is non-degenerate and has full rank input structure. Some redesign of the output structure may be required if $\begin{bmatrix} C & D' \end{bmatrix}$ is rank deficient.

□

Remark (9.6): If the system $S(A,B,C)$ with $q \geq r$ is degenerate, a redesign procedure leading to $S'(A,B',C')$ with $C'B'$ full rank guarantees the creation of a system which is non-degenerate and has full rank input structure. Some redesign of the output structure may be required if C' is rank deficient.

□

The meaning of redesign of D, or CB is that we aim to define a maximal subset of the columns of D, or CB that guarantee the maximal full rank. This procedure is clearly sufficient, but not necessary and leads to a system of smaller dimensions, as far as input, output structure is concerned. The procedure is described below:

Redesign procedure: Let $T \in \Re^{q \times r}$, $q \geq r$ be a matrix (which may be D, CB, or nay other) with $rank(T) = \rho_0$, and let

$$T = [\underline{t}_1, \underline{t}_2, \cdots, \underline{t}_r] \tag{9.28}$$

If $\{i_1, i_2, \cdots, i_{\rho_0}\}$ is the subset of column indices that corresponds to the "best uncorrupted base" selection of the set $\{\underline{t}_1, \cdots, \underline{t}_r\}$, than $\{i_1, i_2, \cdots, i_{\rho_0}\}$ defines the required selection that leads to a matrix

$$T' = \left[\underline{t}_{i_1}, \underline{t}_{i_2}, \cdots, \underline{t}_{i_{\rho_0}}\right] \in \Re^{q \times \rho_0} \tag{9.29}$$

□

If the rank ρ_0 is too small, then the sufficient procedure above has to be avoided and the full conditions have to be used. Clearly, all the above mentioned results apply also to the $q \le r$ case by use of transposed duality. We close this section by summarising the procedure for selection of the "*best uncorrupted basis*" of a set of vectors.

9.2.1c Selection of Best Uncorrupted Basis

If $\{\underline{x}_i, i \in \underline{m}\}$ is a set of vectors of $\Re^n$, $X = sp\{\underline{x}_i, i \in \underline{m}\}$ and $\dim X < m$, then the selection of a basis for X is a problem [Mit. & Karc., 1] that may be handled by *the Gram-Schmidt orthogonalisation procedure*, or use of the *Singular Value Decomposition.* Such procedures yield orthogonal bases, but transform the original data. In many applications, such as the problems considered above and in problems of "non-generic computations" ([Karc. & Mit., 1]) it is essential to select a subset of the existing set, without transforming the original set. This may be done according to some rule, and in this case we may consider the "*degrees of orthogonality*" of the orthogonality. This problem may be referred to as selection of the "*best uncorrupted basis*". The approach presented here for the selection of the best uncorrupted basis is based on the properties of the Gram matrix and uses tools from the theory of compound matrices (Mar. & Minc, 1]. Some useful definitions and tools are considered first.

<u>**Definition (9.1)**</u>: [Gant., 1]: Let $\underline{x}_1, \underline{x}_2, \cdots, \underline{x}_m$ be vectors $\in \Re^n$. The matrix defined by

$$G = \begin{bmatrix} (\underline{x}_1 \cdot \underline{x}_1) & (\underline{x}_1 \cdot \underline{x}_2) & \cdots & (\underline{x}_1 \cdot \underline{x}_m) \\ (\underline{x}_2 \cdot \underline{x}_1) & (\underline{x}_2 \cdot \underline{x}_2) & \cdots & (\underline{x}_2 \cdot \underline{x}_m) \\ \vdots & \vdots & & \vdots \\ (\underline{x}_m \cdot \underline{x}_1) & (\underline{x}_m \cdot \underline{x}_2) & \cdots & (\underline{x}_m \cdot \underline{x}_m) \end{bmatrix} \tag{9.30}$$

is called the Gram matrix of the vectors $\underline{x}_1, \underline{x}_2, \cdots, \underline{x}_m$ and the determinant $G_m = G(\underline{x}_1, \underline{x}_2, \cdots, \underline{x}_m) = |G|$ is called their <u>Gramian</u>.

□

Note [Gant., 1] that the vectors $\underline{x}_1, \underline{x}_2, \cdots, \underline{x}_m$ are linearly independent, if and only if their Gramian is nonzero; in general we have that $|G| \ge 0$ and we have the following property that holds true (Hadamard's inequality):

$$G(\underline{x}_1,\underline{x}_2,\cdots,\underline{x}_m)\le G(\underline{x}_1)\cdot G(\underline{x}_2)\cdots G(\underline{x}_m) \tag{9.31}$$

Note that $G(\underline{x}_i)=\|\underline{x}_i\|_2$ and if the vectors are of unit length (i.e. $\|\underline{x}_i\|_2=1$, $i=1,2,\cdots,m$), then

$$0\le G(\underline{x}_1,\underline{x}_2,\cdots,\underline{x}_m)\le 1 \tag{9.32}$$

Remark (9.7): An alternative test for closeness to normality of a normalised selected set can be based on the condition number of the corresponding matrix. In fact, the deviation from unity of the condition number is a measure of proximity to orthogonality.

□

If $A=[\underline{r}_1,\underline{r}_2,\cdots,\underline{r}_m]'\in\Re^{m\times n}$, then the normalisation of A is a matrix $A_N=[\underline{v}_1,\underline{v}_2,\cdots,\underline{v}_m]'\in\Re^{m\times n}$ with the property: $\underline{v}_i=\underline{r}_i/\|\underline{r}_i\|_2$, $i=1,2,\cdots,m$; it is obvious that $\underline{v}_i\in\Re^{n\times 1}$, $i=1,2,\cdots,m$ are unit length vectors $(\|\underline{v}_i\|_2=1)$. The test on closeness to normality defined by condition (9.32) provides a solution to the problem of selection of an uncorrupted basis as shown below:

Proposition (9.2): Let $A=[\underline{r}_1,\underline{r}_2,\cdots,\underline{r}_m]'\in\Re^{m\times n}$, $\rho(A)=r\le\min\{m,n\}$, $A_N=[\underline{v}_1,\underline{v}_2,\cdots,\underline{v}_m]'\in\Re^{m\times n}$ the normalisation of A. Suppose $G=G(\underline{v}_1,\underline{v}_2,\cdots,\underline{v}_m)\in\Re^{m\times m}$ the Gram matrix of the vectors $\{\underline{v}_1,\underline{v}_2,\cdots,\underline{v}_m\}$ and $C_r(G)=[c_{i,j}]\in\Re^{\binom{m}{r}\times\binom{m}{r}}$ the r-th compound matrix of G. If $c_{ii}=\det\{G[a/a]\}$, $a=(i_1,i_2,\cdots,i_r)\in Q_{r,m}$ is the maximum diagonal element of $C_r(G)$, then a most orthogonal uncorrupted base for the row space of A, consists from the vectors $\{\underline{r}_{i_1},\underline{r}_{i_2},\cdots,\underline{r}_{i_r}\}$.

□

The proof of the above mentioned result readily follows from the relationship between $C_r(G)$ and the Gramian. This result provides a procedure for selection of the most orthogonal set of rows, which is rather simple. An alternative procedure based on the condition number is more complicated since it involves the computation of condition number on all possible combinations of r-sets of vectors.

9.2.2 Model Orientation Problems for Implicit Systems

Implicit equations of the type (9.1), (9.2) occur naturally as system models in a number of practical situations, such as [Aple, 1], [Karc.& Hay, 1]:

- Modelling of composite systems using linear subsystem models.
- Linear system identification.
- Solution of operating points (the 'load-flow' problem in electric power systems analysis or the 'D.C. analysis' problem for non-linear circuits).
- Simulation of possibly large and sparse non-linear systems.
- Design of controllers for multivariable systems using algebraic or frequency domain performance specifications.
- Study of dynamics of linear systems, in the context of geometric theory.

System descriptions of the (9.1), (9.2) type are referred to as matrix pencil and polynomial model implicit descriptions respectively. The characteristic of both (9.1), (9.2) descriptions is that the vector of system behaviour $\xi(t)$, $\zeta(t)$, referred to as implicit vector contains all the variables of importance to the study of the system, without making a distinction between control, observation and internal dynamic variables and without making any assumption on the independence of them. Descriptions with the above properties are called non-oriented [Aple, 1], [Karc., 3]. For a number of processes, the classification of the variables in the implicit vectors, referred to as implicit variables is not known *a priori* [Wil., 1]. Although the study of dynamics may be carried out on implicit non-oriented forms [Wil., 1], when it comes to observing (measuring), controlling, or trying to connect the process as part of a composite structure, the classification of the implicit co-ordinates into inputs, outputs and internal variables arises naturally. The problem of classifying the implicit variables into inputs, outputs and internal variables is called Model Orientation Problem (MOP) and it is considered here as a problem of algebraic assignment or structure assignment. A summary of the objectives of such a problem (considered within the ESPRIT project SESDIP [SESDIP, 1]) is given below. The proper study of this problem has been outside the scope of this thesis.

9.2.2a Issues and Problems in Model Orientation

For most of the applications, the nature of the problem defines part of the partitioning exercise, but there are degrees of freedom in the overall problem and these have to be explored. We consider two types of MOP, the realistic version, where part of the classification is defined by the nature of the problem and referred to as *restricted MOP* and the idealistic case where all implicit variables are unconstrained, as far as orientation is concerned, and it is called the *free MOP.* The issues involved in the study of the *free MOP* are also present in the restricted version of the problem. Thus, in the following, we examine the free version. The free MOP versions are defined as:

Definition (9.2): (i) Given the matrix pencil implicit model of equation (9.1), define a transformation $Q{:}\xi = Q\hat{\xi}$, $Q \in \Re^{k\times k}$, $|Q| \neq 0$, such that it is equivalent to

$$\begin{bmatrix} p\hat{E}-\hat{A} & -B \\ -\hat{C} & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} 0 \\ -y(t) \end{bmatrix} \tag{9.33}$$

and where $x \in \Re^{n}$, $u \in \Re^{\ell}$ and $y \in \Re^{m}$. The system $S\left(\hat{E},\hat{A},\hat{B},\hat{C}\right)$ is called an orientation of the $(pF-G)$ and $\sum(F,G)$ denotes the family of all such systems.

(ii) Given the polynomial implicit model of equation (9.2), define a transformation $R(p){:}\xi = R(p)\hat{\xi}$, $R(p) \in \Re^{\mu\times\mu}[p]$, $|R(p)| \neq 0$ such as (9.2) is equivalent to

$$\begin{bmatrix} T(p) & U(p) \\ -V(p) & W(p) \end{bmatrix}\begin{bmatrix} w(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} 0 \\ -y(t) \end{bmatrix} \tag{9.34}$$

and $\hat{\zeta}=\left[w',u',y'\right]'$, where $w\in\Re^{v}$, $u\in\Re^{\ell}$, $y\in\Re^{m}$ and $\mu=v+\ell+m$. The system described by the system matrix [Ros, 1] in equation (9.34), is called (v,ℓ,m)-*Rosenbrock orientation* and will be denoted by $L(T,U,V,W)$ and the family of such models will be denoted by $\sum(H)$.

□

The families $\sum(F,G)$ or $\sum(H)$ contain more than one solution. Such solutions may be classified according to the invariant structural characteristics of the corresponding orientation, as well as the input, output type properties of the resulting oriented model. Furthermore, we might have a variety of solutions due to the variability of the number of inputs, outputs we specify, as well as the selection of alternative sets. For the case of polynomial implicit descriptions, the current definition of orientation is based on equivalence that preserves only the smooth space of solutions of the original and oriented model. Alternative orientation problems may also be defined, which preserve also impulsive behaviour, if specialised transformations of the type described in [Pug. *et. al.*, 1] are used. An important issue in selecting oriented models is the issue of model minimality [kni & Sch., 1] [Bont & Mal., 1] which is equivalent to selecting a minimal number of internal variables. Issues of minimality, as well as assignment of desirable structural characteristics are important criteria which have to be used in the parametrisation of the $\sum(F,G)$, $\sum(H)$ families. A problem that is similar in nature to MOP, is that of invariant realisation presented in [Karc, 8]; the results there provide a useful methodology for the study of MOP.

9.3 Definition of Effective Input-Output Structure on Well structured Models

We consider a well-structured Progenitor Model represented by the transfer function matrix $H(s) \in \Re^{q \times r}(s)$. Such a model may be of excessively large dimensions and the problem, which is considered here, is the definition of a smaller dimension model,

$$\widetilde{H}(s) \in \Re^{m \times p}(s), m \leq q, p \leq r \qquad (9.35)$$

which has adequate input, output structure for the control and measurement requirements of the problem. The selection of the effective input, output structure is based on criteria using system properties on models which are progressively more detailed. Such a framework involves the following steps:

<u>**Problem (3)**</u>: Determine the minimal required cardinality of the input, output structure, which is required to guarantee certain control and measurement properties.

If m, p are the effective numbers of outputs, inputs respectively, then assuming that n is the McMillan degree of the $\widetilde{H}(s)$ progenitor model, we can use the results on the generic solvability of control problems derived in Control Theory and summarised in report [SESDIP, SDCU046], as well as any structural information, such as Segré Index to define desirable values for m, p. These results may be used as theoretical background information, which can be used to well-condition the model. A brief summary of some of these results is given below to indicate the nature of these criteria.

9.3.1 Criteria for selection of numbers of inputs, outputs based on Generic Solvability of Control Problems.

In this section, we will review the generic solvability conditions for exact synthesis problems in terms of discrete, as well as continuous system invariants, with the aim to characterise the desirable properties of invariants form the viewpoint of characterising the potential of systems for accepting certain types of control solutions.

These results provide means to exclude bad choices in the initial phase of the design of the system and aim to well-condition it. Here, we provide a representation sample of criteria, which may become part of a theory library for control structure selection.

Results: Segré Index and Frequency Assignment [Karc., 11].

I) Let $\kappa(A)$ be the maximum of the geometric multiplicities of the eigenvalues of A, known as Segré Index, then if $\kappa(A) > p$, then the pair (A, B) is uncontrollable, for any choice of the parameters in A, B that preserve the above assumption. Furthermore,

II) For a given A, if $\kappa(A) \leq p$, then for a generic B, (A, B) is controllable.

As far as the pole shifting property via static state feedback, we have:

Necessary condition for eigenvalue placement:

Let κ_i be the geometric multiplicity of the i-th eigenvalue of A, then if $\kappa_i > p$ then this eigenvalue cannot be shifted via static state feedback.

The following table summarises the previous results including the dual problem of observer design:

Problem	Compensation Scheme	Generic Solvability Condition	Desirable Values for Structural Characteristics	Structural Characteristics to be avoided
Pole Assignment	Static State Feedback	$\kappa(A) \leq p$	$\kappa(A) = 1$	Large eigenvalue multiplicities
Observer Design	Static Output Injection	$\kappa(A) \leq p$	$\kappa(A) = 1$	Large eigenvalue multiplicities

Results: Static Output Feedback Pole-Assignment

If $N(s)D(s)^{-1}$ is a right MFD of the transfer function $C(sI-A)^{-1}B$ then the equation:

$$\det\left\{\left[I_p, K\right]\begin{bmatrix} D(s) \\ N(s) \end{bmatrix}\right\} = p(s)$$

has to be solved with respect to K.

A necessary condition for the solvability of the above problem for every polynomial $p(s)$ of degree n is that [Lev. & Kar., 1]

Necessary condition:	$mp \geq n$ and $rank(P) = n+1$

where P is the so called Plücker matrix, which is the coefficient matrix of the compound $C_p\left(\left[D(s)', N(s)'\right]'\right)$. It was also proven that the following is a sufficient condition for generic pole placement:

Generic Sufficient condition:	$mp > n$

The special structure of the matrix A, as defined by the Segré Index has the following implications [Kar. 10]:

> Let $\kappa(A)$ be the maximum of the geometric multiplicities of the eigenvalues of A then if $\kappa(A) > \min(p,m)$ then the Plücker matrix of the system has rank less than $n+1$ and therefore the system is not arbitrarily assignable by static output feedback.

The following table summarises the above results:

Problem	Compensation Scheme	Generic Solvability Condition	Desirable Values for Structural Characteristics	Structural Characteristics to be avoided
Pole Assignment	Static Output Feedback	$mp > n$ $rank(P) = n+1$	Small "n" $\kappa(A) \le \min(m,p)$	Large $\kappa(A)$

Results: Dynamic Output Feedback Pole Assignment

Given a system of n-states represented by a $m \times p$ transfer function $G(s) = N(s)D(s)^{-1}$, find a feedback controller $K(s) = D_1(s)^{-1}N_1(s)$ of degree n_1 such that the closed loop characteristic polynomial is equal to a given one $p(s)$. In other words, the equation:

$$\det\left\{\left[D_1(s)\; N_1(s)\right]\begin{bmatrix} D(s) \\ N(s) \end{bmatrix}\right\} = p(s) \tag{9.36}$$

has to be solved with respect to $\left[D_1(s)\; N_1(s)\right]$. The solvability conditions of the above problem depend again on the rank of a generalised Plücker matrix related to the problem. We define the n_1 *Toeplitz-Plucker matrix* [Lev. & Kar., 2] [Lev. & Kar., 4]

$$T = \overbrace{\begin{bmatrix} \underline{p}_n & \underline{p}_{n-1} & \cdots & \underline{p}_0 & 0 & \cdots & 0 \\ 0 & \underline{p}_n & \underline{p}_{n-1} & \cdots & \underline{p}_0 & \cdots & 0 \\ \vdots & 0 & \underline{p}_n & \ddots & & & \vdots \\ \vdots & & & & \ddots & \underline{p}_0 & \\ 0 & \cdots & 0 & \underline{p}_n & \underline{p}_{n-1} & \cdots & \underline{p}_0 \end{bmatrix}}^{n+n_1+1} \tag{9.37}$$

where the p_i's satisfy

$$C_p\left(\begin{bmatrix} D(s) \\ N(s) \end{bmatrix}\right) = s^n \underline{p}_n + s^{n-1}\underline{p}_{n-1} + \cdots + \underline{p}_0 \tag{9.38}$$

(in other words the p_i's are the columns of the Plücker matrix P). Based on this matrix we have the following two necessary conditions for the solvability of the arbitrary pole placement via feedback controllers of degree n_1:

Necessary conditions:

1. $rank(T) = n + n_1 = 1$
2. $mp + n_1(m+p) \geq n + n_1$

Remark (9.8)

(a) According to the above, given a system of p-inputs, m-outputs and n-states, one has to choose a controller of degree at least $(n-mp)/(m+p-1)$. If the rank of Plücker matrix P is $n+1$, then the rank of T is generally $n+n_1+1$ and thus condition 1. is satisfied. In the other case $(rank(P) < n+1)$ for T to have full rank n_1 must be greater than or equal to $(n+1-rank(P))/(rank(P)-1)$. To summarise:

$$n_1 \geq \max\left\{\frac{n-mp}{m+p-1}, \frac{n+1-rank(P)}{rank(P)-1}\right\} \tag{9.39}$$

is a necessary condition for achieving arbitrary pole assignability using controllers of degree n_1.

(b) The above conditions do not always ensure arbitrary pole assignability for a generic system and in some cases higher degree controllers have to be selected. In fact arbitrary pole assignability holds true (for a generic system satisfying $m \geq p$) if n_1 is greater than or equal to the smallest multiple of p exceeding $(n-mp)/(m+p-1)$ (for $p \geq m$ we have a dual result). The selection of such a degree guarantees that T has full rank (at least for a generic system); there are however nongeneric cases of plants that T may be rank deficient.

Problem	Compensation Scheme	Generic Solvability Condition	Desirable Values for Structural Characteristics	Structural Characteristics to be avoided
Pole Assignment	Output Feedback of degree n_1	$\eta \geq \max\left\{\frac{n-mp}{m+p-1}, \frac{n+1-\operatorname{rank}(P)}{\operatorname{rank}(P)-1}\right\}$	Small "n" $\kappa(A) \leq \min(p,m)$	Large n, Large $\kappa(A)$

Results: Simultaneous Pole Assignment, Stabilisation

The issue of simultaneous stabilisation or pole assignment arises when a plant is subject to a discrete change. In this case the problem is to find a feedback controller $k(s)$ that simultaneously assigns poles and stabilises all plants $G_i(s)$ in a given family. For the simultaneous pole assignment we have:

Simultaneous pole placement via static controllers:

Consider k generic systems G_i of p-inputs, m-outputs and n_i-states respectively then if:

$$m \geq k + \sum_{i=1}^{k} c_i \tag{9.40}$$

where c_i is the smallest controllability index of the i-th system, then their poles can be shifted arbitrarily via the same static controller.

The work so far has been representative of the type of results that may be used, but by no means exhaustive. An important emerging future task is:

Task (I): Develop a library of structural conditions and a procedure for working out the optimal values of m, p given the control and measurement requirements.

□

The integral part of the above analysis is the solution of the following problem:

Problem (4): Identify robustly the basic structural characteristics, such as McMillan degree, orders of infinite zeros, Segré characteristic, etc. on early models which may be characterised by uncertainty in dynamics and parameters.

□

The problem of structural identification, defined in [Kar. & Mil., 1], is related to the above; however, the area is still in its early stages of development. It is worth noting that in this step we require the least possible information from the progenitor model to decide on the required number of inputs and outputs.

9.3.2 Use of Structural Graph Analysis.

The type of analysis used above makes no assumption on the progenitor model, apart from the basic structural features considered above (numbers of inputs, outputs, states, Segré index, etc.). Assuming a bit more about the underlying model leads to the following problem:

Problem (5): Define all possible pairs of subsets of the input, output structure which are needed to guarantee basic structural properties, such as structural controllability, observability, system vulnerability, etc.

□

In this step, we exploit the fundamental underlying graph structure of the progenitor model, which requires some more detailed information. We use graph theory for such an evaluation and some of the first results in this area are presented in [SDCU058, SESDIP] report. The aim of this investigation is to produce more well structured alternatives than those specified by the investigation previously, which then have to be further investigated with criteria which are more detailed than those of the graph type structure. An important issue here is the use of existing and development of new graph type diagnostics for the selection of input, output structures of interconnected system for which there is an explicit knowledge of the underlying interconnection graph. This topic has been considered in a previous chapter and a systematic approach has been presented in [SESDIP, SDCU058].

9.3.3 Parameter dependent invariants and diagnostics

Progressing from models described from dimensions and graph structural information on the interconnections to models, having fixed numerical parameters, we consider problems of invariant structure assignment, which may be described as:

Problem (6): Evaluate the pairs of input, output structures produced by the previous step and specify new alternatives using parameter depended structural invariants such as zeros, specific values, controllability, observability indices, properties of Plücker matrices, Forney orders, etc.

At this stage, we use linear models and rely on problems of the general structural family of model projection [Karc., 10] to assign desirable structural characteristics, or avoid the formation of undesirable ones. The area of assignment of invariants has not been properly studied with the exception of the problem of zero assignment by squaring down [Kar. & Gia., 1] the fundamentals of which are described below.

9.3.3a Squaring down and zero Assignment

For a non-square plant whose number of measured output variables is greater than the number of control inputs ($m > p$), the problem of combining all outputs together into a new set of outputs, whose number is equal to the number of control inputs has been called the "squaring down" problem [Kar. & Gia.,1]. It is evident that the control of the general squaring down problem has significant consequences on the zero structure of the corresponding loop transmission transfer function matrix and therefore, it vitally affects the final control design process.

We consider a system S whose input-output behaviour is described by the transfer function $G(s) = N(s)D(s)^{-1}$ where $m > p$. Under the coprimeness assumption, the zeros of the system S are given by the zeros of the numerator $N(s)$. Squaring down at the plant outputs makes sense as a post-compensation with dynamics representing those of the sensors used, or constant.

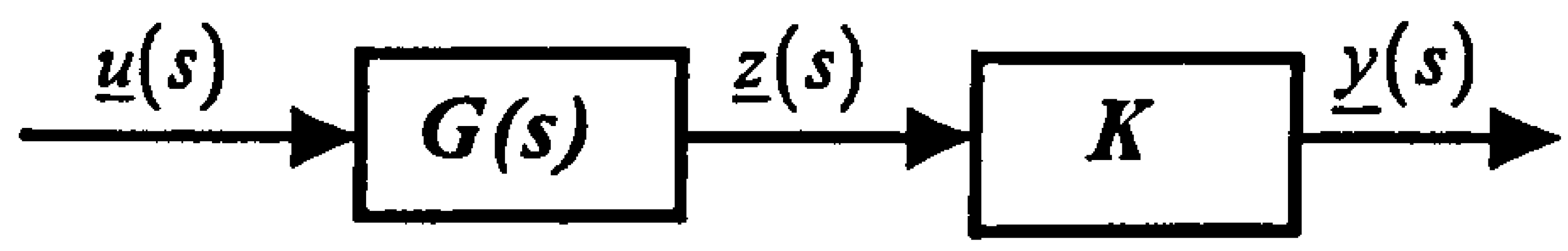


Figure 9.1.

Then the zeros of the overall system are given by the zeros of $KN(s)$ and its invariant zero polynomial is given by:

$$z_k(s) = \det(KN(s)) = \det(K\overline{N}(s))\det(Z(s)) \tag{9.41}$$

where $Z(s)$ is a greatest right divisor of $N(s)$ and where $\overline{N}(s)$ is the least degree polynomial matrix of the rational vector space $colsp\{G(s)\}$. It is clear that $\det(Z(s))$ is a fixed divisor of $z(s)$ for all K (invariance of existing zeros under squaring down). The newly introduced zeros, are the zeros of the polynomial:

$$f(s) = \det(K\overline{N}(s)) \tag{9.42}$$

where $f(s)$ is a polynomial with degree equal to the Forney's dynamical order δ [For., 1] of the previous rational vector space. The problem of zero assignment by squaring down, can be defined as defining a full rank K that assigns the zeros of (9.42) arbitrarily.

For this problem we have the following generic result [Karc. & Gia., 1]:

Generic result:

If $p(m-p) > \delta$, then a generic system, the zeros can be placed to arbitrary positions.

Summary:

Problem	Compensation Scheme	Generic Solvability Condition	Desirable Values for Structural Characteristics	Structural Characteristics to be avoided
Zero Assignment	Static postcompensator	$p(m-p)>\delta$	Small n	Right half plane zeros

Algorithmic procedures for solving this problem are of similar nature to those developed in [Lev. & Karc., 2].

The overall area of structure assignment using Model Projection Problems is in its early stages of development. The generic solvability results may become part of the structural library for design. The results of such an investigation lead to smaller sets of input, output structure alternatives, which probably have to be further evaluated with some additional criteria. In fact, the definition of certain structure assignment problems may lead to a parametrisation of the possible solutions from this set. The alternative means required, can be provided through the following alternative step:

9.3.4 Input, output selection and Performance Indicators

Structure assignment is one way of affecting the potential of a system model to have an easy control problem. An alternative is to use the freedom available in the Model Projection Problems to Shape System Performance indicators. In general, Performance Indicators are affected by Control Design, but here we consider this alternative design. We may summarise as:

Problem (7): Specify the free parameters, or use the free variables in the parameters form of solution of structure assignment, or structure avoidance of the previous step by exploiting criteria based on the values of performance indicators, such as energy transfer, or requirements, degree of controllability, observability, robustness of properties under system uncertainty, etc.

□

At this stage, we deal with a well-structured linear model, or a family with free parameters, which satisfy certain structural conditions. The problem we face is to retain the achieved structural features and achieve some additional properties for the input, output structure by tuning parameters. We may use a great variety of performance tests and criteria, such as energy requirements for control and observation, condition number of $G(s)$, as well as other properties such as maximising the degree of controllability, observability, reduction of sensitivity to parameter uncertainty, etc. The current stage of development of this area is dominated by the effort to define meaningful tests and criteria. The next stage has to do with the formulation of appropriate optimisation problems for achieving the best possible tuning.

It should be noticed that the analysis, so far, is based on structural characteristics first, which determine the potential for control performance and progressively move to performance indicators shaping, after having specified the basic structure. The overall philosophy, which underlines this approach, is to sort out first the structure formation by solving well defined synthesis problems, define families of such solutions and then use multi-objective optimisation for selection of free parameters in the available alternatives. The result of this procedure is a well-structured model $\widetilde{H}(s) \in \Re^{m \times p}(s)$, on which the control design problem has to be addressed. The important subproblem of this major activity is the definition of the structure of the control scheme, i.e., sorting out issues on decentralisation, versus centralisation.

9.4 Structuring Of Control Scheme: Evaluation Of Decentralised Options

The problem, we now address, is the selection of the structure of the compensation scheme that involves answering questions on whether we have to use centralised, or decentralised schemes and if decentralisation is needed, then to decide on the partitioning of the input, output channels, as well as the way we have to couple them in a feedback, or precompensation configuration. An integral part of this design stage is also the specification of the required order of dynamics. The problem of selection of the coupling (interaction analysis, structure analysis, etc.) has been considered in the previous chapters. We may summarise the overall approach by specifying a number of important problems, steps.

Problem (8): Use knowledge on the process, geographical location of process units and operational requirements to define a first appraisal of options as far as centralisation versus decentralisation.

□

This step aims to take into account the particulars of the applications area and nature of the problem. This knowledge is indispensable and it is part of the overall problem specification. What is expected at this stage is the development of the first structuring of the schemes in terms of superblocks, which themselves may require some further structuring subsequently. It is worth mentioning that the requirements of the overall problem decomposition, based either on performance optimisation (operational), or on sub-problem design need to be taken into account here. This area is dominated by the process dependent specifics, heuristics, but there is also need for work which has to be based on the systematic study of the problem decomposition (operational and design aspects). This area of work may be considered as a part of the control structure selection on a whole plant.

Problem (9): Use results on the generic solvability of decentralised control problems to produce a first parameterisation of alternatives.

□

The study of decentralised control problems has produced some results characterising generic solvability of control problems, which lead to parameterisation of possible partitions of input, output channels, which permit solvability of control problems. These results depend on structural characteristics such as the McMillan degree and the numbers of inputs, outputs. A review of this methodology and available results are given [Karc. & Mil., 2]. This analysis is the first of the analytical steps in the evaluation of the alternative schemes.

Problem (10): Use of graph analysis methodology and the concept of structural fixed modes for evaluation of alternatives defined by the previous step.

□

For systems, which have an explicit graph structure, a procedure for evaluating alternatives based on the exclusion of structural fixed modes may be used as a first structural methodology that uses the most basic structural aspect, the system graph. It is clear that the results have exploited deeper structural characteristics based on the graph rather than those of the previous step.

Problem (11): Use of interaction analysis diagnostics based on steady state models, or simple dynamic models to evaluate the alternatives produced at the previous stage.

□

Progressing from graph models to steady state, or simple dynamic models, we may use the large number of diagnostics of the *RGA*, *BRGA* type to evaluate further the options specified in the last step. The previous chapters contain a variety of tests for interaction analysis. After this stage we progress to the further evaluation described below.

Problem (12): Advanced structure selection diagnostics based on linear dynamic models and parameter dependent structural characteristics.

□

At this stage, we proceed with the evaluation of the available options using linear models and parameter dependent properties such as fixed modes (non structural), almost fixed modes under various dynamic modes, properties of the rank of decentralised Plücker matrices, strong instability and minimum phase phenomena, etc. A set of exterior algebra based diagnostics is described in [SESDIP, SDCU 53]. Within this family, the Decentralised Markov parameters are first used, since the computations involved are relatively simple, and then we proceed to the more complex algebra tests. In all these studies, we use as a test the avoidance of formation of undesirable characteristics (fixed, almost fixed modes, loss of rank of Plücker matrices) or preconditioning of properties (full rank of Plücker matrices). In fact, the decentralised Markov parameter test also provides the means to modify the centralised input, output structure in order to guarantee certain properties.

Problem (13): On a full dynamics linear model, use diagnostics based on performance indicators to evaluate the alternative decentralisation schemes, which have been specified by the previous stage.

□

Having exhausted all structural methodologies and tests to reduce the set of options (necessary conditions have been mostly used) we now use computationally intensive methodologies such as singular value analysis, structural singular values, properties of cost balanced realisations, energy requirements for coupling, etc. This area of diagnostics is quite rich, but there is still need for improvement, as well as sorting out alternative criteria.

9.5 Multiobjective Criteria and Control Structure Selection

According to the previous sections, selection of inputs and outputs, as well as the control structure selection depends on a variety of criteria, which can be accordingly imposed on the non-oriented or oriented Progenitor model. The majority of the tests are rank tests on matrices related to the model data and the specific input-output or control structure selection. As the dimension of the matrices for these types of problems is large and the rank is not a well-defined numerical quantity, the condition number may be used instead. For a given criterion, we can calculate a vector consisting of condition numbers each one related to a specific input-output or control structure selection. The condition number is a measure of the extent to which the property exists for a specific input-output selection. These vectors therefore may be used for the derivation of a total index, which will classify the input-output or the control structure selection. This can be done in two steps:

1. Use factor analysis or singular value decomposition to produce a small number of vectors (related to properties) which they sufficiently represent the others.
2. Use an appropriately weighted linear combination of the last vectors to produce a single one.

Remark (9.9): The first step depends on how accurately we would like to represent the properties or how many dominant factors we would like to have. The second depends on the importance that is assigned to every property.

□

Having calculated a single vector, we then may select the input-output group that corresponds to the minimum entry of the vector. This vector provides a classification of all input-output groups of interest, as to each of these, a single index is assigned.

9.6 CONCLUSIONS

An attempt has been made, for the first time, to provide an integrated methodology for selection, classification of process variables, shaping of the input, output structure and evaluation of alternative decentralisation schemes. The overall approach has been based on exploiting the different aspects of the underlying system structure going progressively from unstructured model diagnostics, to graph structure based results, to model parameter dependent invariants and finally, performance indicators. This structural methodology reflects the overall structural philosophy and it is quite logical for the overall problem. In fact, starting with a large number of options, we first use simple theory and criteria and progressively by reducing the set of options we start using more detailed and meaningful criteria, which however are associated with more computationally intensive procedures. What we have provided so far is an overall methodology and in the various steps, new, as well as known results are used. There are many areas which need development if we are to move to an integrated and substantial structure selection diagnostics framework. Generating the different alternatives in a systematic, and not in an *ad hoc* manner, sorting out the multiobjective decision problem of alternative criteria and finally, moving from evaluation to design, are open challenges for the future. So far, we have relied on the structural approach which is quite meaningful at early stages and for sorting out many options. At the later stages, there is a need to develop optimisation methodologies for tuning parameters within a given selected structure. This is also an important area for future research, where tools from the H_∞ optimisation methodology may be combined with the structural approaches to provide powerful hybrid methodologies.

Chapter 10

CONCLUSIONS

Blessed is he who takes nothing for granted.

10. CONCLUSIONS – (Ideas for future work)

A thorough investigation of the methodologies, heuristics and approaches for the selection of control structures which originate from the process area has been performed. This has revealed a number of important diagnostic tools for selection of control structures, as well as a large number of open issues. We have produced software for the different type of diagnostics, which in a sense provides some way towards integration of them. However, more work is needed to classify them, link them, and provide a mechanism for weighting their significance. Some optimisation technique incorporating these diagnostics and heuristics is needed and it is an important direction for research. The extended review and presentation of the process based diagnostics given here serves as an assisting tool to the use of the software toolbox.

An attempt to divert from the traditional process based methodologies was made by defining an overall systems based framework for Global Instrumentation, by considering a few typical problems within that and then providing the elements of an integrated philosophy that combines systems and process based tools. The problem of conditioning degenerate transfer functions has been solved and the general model orientation has been formulated. However, its solution, especially of the restricted type, is still open and subject for future work.

The role of selection of inputs, outputs at subsystem level, has been examined with respect to the completeness assumption. Further work is needed here to relate the general structure of the underlined system graph and the selection of the required set of inputs, outputs. Integral part in such an investigation is making the analysis independent of the state-space setup, i.e. moving to transfer function subsystem descriptions.

An alternative philosophy in the overall design has been presented here, in terms of use of theoretical properties parametrising families of systems. This together with the graph methodology offer a new insight in to how we structure systems with desirable properties at the initial stages of design. Integrating heuristics, theoretical properties and analytic diagnostics remains a challenge.

The area of Global Process Instrumentation, as we describe the cluster of problems related to the classification of process variables, the definition of inputs, outputs and the selection of coupling of input, output variables (i.e. the structuring of the control scheme) are issues which have been very prominent in areas such as process and control design of chemical processes and aerospace areas, such as flexible space structures. This cluster of problems, has not however been seen as one problem and there is very little interaction between the different issues. The whole area is dominated by partial results, borrowed from the Control Design area and by heuristics. It is not clear what is the type of applications, or systems where the heuristics are valid and what are the more general implications of the Control Design indicators, since it is difficult to relate them to procedures linked to process or control redesign. The current dominant practice of listing all possible alternatives and then trying on them the various heuristics and partial results in an unguided manner is time consuming and not satisfactory as far as linking the failure to the modifications needed. The current practice is to link the control structure selection to the lower level of controllability criteria; it becomes, however, evident nowadays that wider criteria from the overall process operations area (higher layers of the process operations hierarchy) have also to be taken into consideration. This links the local process we are currently addressing to the global problem of Control Structure Selection and Control Design for the overall plant. Issues related to the economic appraisal of resulting structure selection, have been identified as important, but they are still in their early developments. It emerges clearly from the systematic reviewing of many issues so far, that there is a strong need for further work in areas such as:

(i) Development of a unifying system based methodology for control structure selection that allows the integration of diagnostics and their linking to design, or redesign issues.

(ii) Unification of existing criteria and heuristics, classification of systems where particular heuristics are valid and development of methodology tuned for specific applications.

(iii) Embedding of the control structure selection problem in the wider problem of total control design of a plant and evaluation of technical alternatives in financial terms.

There is a need to develop a structural methodology that may act as a unifying basis for structure evaluation diagnostics, linking with issues of design, or redesign and provide the means for integrating alternative non-structural diagnostics and heuristics.

The underlying philosophy of this structural approach, is motivated by the early efforts in the process area for predicting properties of full designs at early stages. Given that only structure information of process models has such a predictive capability, motivates the suggestion that a structural framework for the study of the problem is necessary. The initial work has to be based on simple and rich, as far as results are concerned, theory and thus restriction mainly to linear models, at the beginning, is essential.

In the second area, the emphasis is on the unification of the different diagnostics for control structure selection as well as their enrichment with alternative new ones, coming from the control theory area. This will lead to the development of a systematic procedure for using such a diagnostics framework and then tackle issues related to difficult real life processes, where dedicated tools, such as neural networks may become useful. The overall appraisal of heuristics and their link to the other diagnostics is an important issue which has to be examined. The third area is also important, but it is considered as a much longer horizon area, which requires the development of the first two. In the first instance, there is a need for considerable effort in the following main directions:

(i) Unification and Classification of existing results and heuristics.
(ii) Development of the Systems and Control based framework and methodology.
(iii) Development of methodology and tools to address the specifics of application areas.
(iv) Expansion of the Systems based theoretical framework into a general framework for Global Process Instrumentation.
(v) Exploring the impact of emerging technologies on the shaping of strategies, methodology and tools for GPI.

REFERENCES

[Albr. *et al.*, 1] Albrecht, R. F., C. R. Reeves, N. C. Steele "The application of neural sensors to fermentation." *Proceedings of the international conference of Artificial Neural Nets and Genetic Algorithms*, pp.292-299, Berlin, Germany, 14-16 April 1993.

[And. & Clem., 1] Anderson, B. D., D. J. Clements "Algebraic characterisation of fixed modes in decentralised control", *Automatica*, Vol. **17**, pp.703-712, 1981.

[Anders., 1] Anderson, J. S. "A practical problem in dynamic heat transfer." *Chem. Eng.*, pp. CE97-CE103, May 1966.

[Apl., 1] Aplevich, J. D. "Implicit Linear Systems." *Lecture Notes in Control and Information Sciences*, Vol. **152**, Springer, Berlin, 1991.

[Arkun, 1] Arkun, Y. "Dynamic Block Relative Gain and its connection with the performance and stability of decentralized control structures", *Int. J. Control.* Vol. **46**, No. **4**, pp.1187-1193, 1987.

[Ark. *et al.*, 1] Arkun, Y., B. Manousiouthakis, and A. Palazoglu "Robustness Analysis of Process Control Systems. A Case Study of Decoupling Control In Distillation." *I & EC Process Design and Development*, October 1982.

[Astrom, 1] Astrom, K. J. *Introduction to Stochastic Control Theory*, Academic Press Inc, New York 1970.

[Athans, 1] Athans, M. "On the Determination of Optimal Costly Measurement Strategies for Linear Stochastic Systems." *Automatica*, Vol. **8**, page 397, 1972.

[Balak. & Luss, 1] Balakotaiah, V. and D. Luss "Explicit runaway criterion for catalytic reactors with transport limitations." *AIChE Journal*, Vol. **37**, pp.1780-1788, 1991.

[Barkel., 1] Barkelew, C. H. "Stability of Adiabatic Reactors", *ACS Symposium Series, American Chemical Society*, Vol. **237**, page 337, 1984.

[Bark. & Mot., 1] Barkley. R. W. and R. L. Motard, "Decomposition of Nets" *Chem. Eng. J.*, Vol. **3**, page 266, 1972.

[Bart. *et al.*, 1] Barton, G. W., M. A. Padley and J. D. Perkins "Incorporating Operability Measures into the Process Synthesis Stage of Design",

[Bauel, 1] Bauel, F. L. "Optimally scaled matrices." *Numer. Math.* Vol. **5**, page 73, 1963.

[Beale, 1] Beale, E. M. L. *Integer programming. In The State of the Art in Numerical Analysis*, Academic Press, 1977.

[Bonilla, 1] Bonilla, M. and M. Malabre "External reachability (reachability with pole assignment by P.D. feedback) for implicit descriptions." *Kybernetica*, Vol. **29**, No. **5**, pp.499-510, 1993.

[Bristol, 1] Bristol, E. H. "On a new measure of interactions for multivariable process control." *IEEE Trans. Automat. Control*, **AC-11**, 1966.

[Bristol, 2] Bristol, E. H. "General (n x n) Proof of the Negative Value RGA Property," Personal communication.

[Bros. & Tong, 1] Brosilow, C. B., and M. Tong "Inferential Control Applied to Multicomponent Distillation," *paper presented at AIChE 79th National Meeting in Houston*, Tex., Nov. 1974.

[Buckley, 1] Buckley, P. S. *Techniques of Process Control*, John Wiley and Sons, Inc., New York, 1964.

[Calan. & Steph., 1] Calandranis, J. and G. Stephanopoulos "Structural operability analysis of heat exchanger networks." *Chem. Eng. Res. Des.* Vol. **64**, pp.347-364, 1986.

[Cal. & Dos., 1] Callier, F. M. and C. A. Dosoer *Multivariable Feedback Systems*, Springer-Verlag, New York, 1982.

[Cao & Biss, 1] Cao, Y. and D. Biss, "New screening techniques for choosing manipulated variables." *IFAC World Congress*, San Francisco, USA, pp.103-108, 1996.

[Chavez, *et al.*, 1] Chavez, C. R., J. D. Seader, and T. L. Wayburn "Multiple steady-state solutions for interlinked separation systems." *Ind. Enq. Chem. Fundam.* Vol. **5**, pp.566-576, 1986.

[Chen, 1] Chen, C. T. *Linear Systems Theory and Design*, Holt-Rinehart and Winston, New York, 1984.

[Chen & Mars, 1] Chen, J. R. and P. Mars *Artificial neural networks and nonlinear system identification.* School of Engineering Report, Univ. of Durham, England, 1990.

[Chen & Bil., 1] Chen, S. and S. A. Billings "Neural networks for nonlinear dynamic system modelling and identification." *Int. Journal of Control,* Vol. **56**, No. **2**, pp. 319-346, 1992.

[Chiang *et al.*, 1] Chiang, Hsiao-Dong, Jin-Cheng Wang, James S. Thorp and J. Lu "Multi-objective Controller Placements in Large-Scale Linear Systems", 1993.

[Christof., 1] Christofides N. *Graph Theory - An Algorithmic Approach*, Academic Press, 1975.

[Cough. & Kop., 1] Coughanowr, D. and L. Koppel *Process Systems Analysis and Control.* New York, McGraw-Hill, 1965.

[Cut. & Ram., 1] Cutler, C. R. and B. L. Ramaker "Dynamic Matrix Control-A Computer Control Algorithm." *AIChE 86th National Meeting, April 1979*; also in *Joint Automatic Control Conference Proceedings*, San Francisco, 1980.

[Davison, 1] Davison, E. J., (Ed) "Benchmark problems for control system design", *IFAC report of IFAC Theory Committee*, 2-3, (Scholossplatz 12, A-2361 Luxembourg, Austria), 1990.

[Davison, 2] Davison, E. J. "Connectability and structural controllability of composite systems", *Automatica*, Vol. **13**, pp. 109-127, 1977.

[Denn & Lavie, 1] Denn, M. M. and R. Lavie "Dynamics of plants with recycle." *Chem. Eng. Journal*, Vol. **24**, pp.55-59, 1982.

[Doedel, 1] Doedel, E. J. "AUTO: Software for Continuation and Bifurcation Problems in Ordinary Differential Equations." *Concordia University*: Montreal, Canada, 1986.

[Doug. & Ath. 1] Douglas, J. and M. Athans *The Control Handbook*, 30ed., CRC Press, 1995.

[Douglas, 1] Douglas, J. "A Preliminary Design Procedure for Control Systems for Complete Chemical Plants", *Paper 5a, AIChE 73rd Ann. Meet.*, Chicago, Nov., 1980.

[Doyle, 1] Doyle, J. C. and G. Stein, "Multivariable Feedback Design: Concepts for a Classical/Modern Synthesis." *IEEE Trans. Aut. Contr.*, **AC-26** (1), 4, 1981.

[EPIC, 1] Early Process design Integrated with Control (EPIC). "Procedure Specification of Process Instrumentation Matrix (PIM)." *Paper No. P1.2.2B*, March 1990.

[Economou, 1] Economou, C. "Graph theory tools for the analysis and synthesis of multivariable control systems", *BEng Thesis, City University*, 1996.

[Evans *et al.*, 1] Evans F. J., C. Schizas and J. Chan J "Control system designs using graphical decomposition techniques", *IEE Proc. part D*, Vol. **128**, pp. 77-84, 1981.

[Ekh., 1] Ekhoff, P. "Identification in Measurement and Instrumentation." *In Concise Encyclopedia of Measurement and Instrumentation*, Edit. L. Finklestein, K.T.V. Grattan, Pergamon Press, pp.137-142, 1994.

[Fewings, 1] Fewings, J. H., P. J. Slaughter, E. Manlapig, and A. J. Lynch. "Experiences with process control of the chalcopylite flotation circuit at Mount Isa Mines Limited." *In 13th Int. Min. Proc. Gongress*, Warsaw, page 1540, 1979.

[Finkel., 1] Finkelstein L. and K. T. V. Grattan, *Concise Encyclopedia of Measurement and Instrumentation*, Pergamon Press. 1994.

[Fisher *et al.*, 1] Fisher, W. R., M. F. Doherty and J. M. Douglas, *Ind. Eng. Chem. Fund*, Vol. **28**, pp.60-73, 1988.

[Fisher *et al.*, 2] Fisher, W. R., M. F. Doherty, J. M. Douglas, "Synthesis of Steady-State Control Structures For Complete Chemical Plants, Part III: Control Structure Synthesis Strategies." *Paper 82b, presented at the AIChE Meeting*, San Francisco, Nov. 1984.

[Fisher *et al.*, 3] Fisher, W.R., M. F. Doherty and J. M. Douglas, "The interface between design and control. 1. process controllability." *Ind. Enq. Chem. Res.*, Vol. **27**, pp.597-605, 1988.

[Fisher *et al.*, 4] Fisher, W.R., M. F. Doherty and J. M. Douglas, "The interface between design and control. 2. Process operability." *Ind. Eng. Chem. Res.*, Vol. **27**, pp.606-611, 1988.

[Fisher *et al.*, 5] Fisher, W. R., M. F. Doherty, and J. M. Douglas, "Effect of overdesign on the operability of distillation columns." *Ind. Eng. Chem. Process Des. Dev.* Vol. **24**, pp.593-598, 1985.

[Fors., 1] Forsythe, G. E., C. B. Moler, *Computer Solution of Linear Algebraic Systems*, Pentice-Hall, Inc.: Englewood Cliffs, NJ, pp.20-26, 1967.

[Fors., 2] Forsythe. G. E., M. A. Malcolm, and C. B. Moler, *Computer Methods for Mathematical Computations*, Prentice Hall, Englewood Cliffs, NJ, 1977.

[Foss, 1] Foss, A. S. "Critique of Chemical Process Control Theory", *AIChE J.*, Vol. **19**, No. **2**, page 209, 1973.

[Gag. & Seb.,1] Gagnepain, J. P., and D. E. Seborg, "Analysis of Process Interactions with Applications to Multi-loop Control System Design." *Ind. Eng. Chem. Proc. Des. Dev.*, Vol. **21**, No. **5**, 1982.

[Gant., 1] Gantmacher, F. R. *The Theory of Matrices*, Chelsea, New York, vol. **1**,**2**, 1959.

[Gar. & Mor.,1] Garcia, C. E., and M. Morari, "Internal Model Control-A Unifying Review and Some New Results." *IEC Proc. Des. Dev.* Vol. **21**, page 308, 1982.

[Gawron., 1] Gawronski, W. *Balanced Control of Flexible Structures*, Lecture Notes in Control & Inform. Sc., Vol. **211**, Springer Verlag, 1996.

[Georgk., 1] Georgakis, C. *Chem. Eng. Sci.* Vol. **41**, page 1471, 1986.

[Georg. & Floud., 1] Georgiou, A. and C. A. Floudas, "Structural properties of large scale systems." *Int. J. Control*, Vol. **51**, pp. 169-187,1990.

[Georg. & Floud., 2] Georgiou, A. and C. A. Floudas, "Structural Analysis and Synthesis of Feasible Control Systems - Theory and Applications." *Chem. Eng. Res. Des.* Vol. **67**, page 600, 1989.

[Georg. & Floud., 3] Georgiou, C. A. and C. A. Floudas, "Simultaneous process synthesis and control: Minimization of disturbance propagation in heat recovery systems." In J.J. Siirola, I.E. Grossman, and G. Stephanopoulos, editors, *Proc. of 3rd Conf. On Foundation of Computer Aided Process Design*, Snowmass, Colorado, pp.435-450, 1989.

[Gibbons, 1] Gibbons A. '*Algorithmic Graph Theory*', Cambridge University Press, 1989.

[Gilib. & Lees, 1] Giliboro, L. G. and F. P. Lees, "The Reduction of Complex Transfer Function Models to Simple Models Using the Method of Moments." *Chem. Eng. Sci.* Vol. **24**, page 85, 1969.

[Gong & Ald., 1] Gong, Z. and M. Aldeen, "On the characterisation of fixed modes in decentralised control", *IEEE Trans.Autom.Control*, Vol. **37**, No.7, pp. 1046-1049, 1992.

[Gov. & Pow., 1] Govind, R. "Causal Modelling of Large Scale Complex Systems", *Proceedings of the Ninth Annual Pittsburgh Conference on Modelling and Simulation*, Vol. **9**, page 689, 1978a.

[Gov. & Pow., 2] Govind, R. *Control System Synthesis Strategies*. Ph.D. Thesis, Carnegie-Mellon University, 1978b.

[Gov. & Pow., 3] Govind, R. and G. J. Powers, "Control System Synthesis Strategies", *AIChE 82nd National Meeting, Atlantic City,* 1976.

[Gov. & Pow., 4] Govind, R. and G. J. Powers, "Control System Synthesis Strategies", *AIChE J.*, Vol. **28**, page 60, 1982.

[Grosd. & Mor., 1] Grosdidier, P. and M. Morari, "Interaction measures for systems under decentralised control." *Automatica*, Vol. **22**, pp.309-319, 1986.

[Grosd. *et al.*, 1] Grosdidier, P., M. Morari, and B. R. Holt, "Closed loop properties from steady state gain information." *Ind. Eng. Chem. Fundam.* Vol. **24**, pp.221-235, 1985.

[Gross. & Floud., 1] Grossman, I. E. and C. A. Floudas, "Active constraint strategy for flexibility analysis in chemical processes." *Comp. Chem. Eng.* Vol. **11**, 1987.

[Gross. & Mor., 1] Grossmann, I. E. and M. Morari, "Operability, resiliency and flexibility, in process design-objectives for a changing world." In A.W. Westerberg and H.H. Chien, editors, *Proc. of 2nd Conf. on Foundations of Computer Aided Process Design*, Snowmass, Colorado, pp.931-1010, 1984.

[Guha, 1] Guha, A. "Neural network based inferential sensing and instrumentation." *Proceedings of the 1992 Summer Workshop on "Neural Network Computing for the Electric Power Industry"(*Editor D. J. Sobajic), Stanford. CA, August 17-19, 1992.

[Guo & Uhrig, 1] Guo, Z. and R. E. Uhrig, "Use of artificial neural networks to analyse nuclear power plant performance." *Nuclear Technology*, Vol. **99**, pp.36-42, July 1992.

[Gupta *et al.*, 1] Gupta, P. K., A. W. Westerberg, J. E. Hendry, and R. R. Hughes, *AIChE*, Vol. **20**, page 397, 1974.

[Hägg., 1] Häggblom, K. E. "Control structure selection via relative gain analysis of partially controlled systems." *European Control Conference*, Brussels, 1-4 July 1997.

[Haggin *et al.*, 1] Haggin, J. *AIChE J*, Vol. **28**, page 60, 1984.

[Harary, 1] Harary, F. *Graph Theory*, Addison Wesley, 1969.

[Harary, 2] Harary, F., R. Z. Norman and D. Cartwright, *Structural Models: An Introduction to the Theory of Directed Graphs*, John Wiley, New York, 1965.

[Hav. & Skog., 1] Havre, K. and S. Skogestad, "Input/ output selection and partial control." *IFAC World Congress*, San Francisco, USA, pp.181-186, 1996.

[Herget, 1] Herget, J. Charles and Ching-Fang Lin, "Advanced Control Systems Design, an Overview", *IEEE*, page 935, 1994.

[Her. & Mel., 1] Herring, K. D. and J. L. Melsa, "Optimum Measurements for Estimation." *IEEE Trans. Autom. Contr.*, **AC-19**, page 264, 1974.

[Hertz, 1] Hertz, J. A., R. G. Palmer and A. S. Krogh, *Introduction to the theory of neural computation.* Addison Wesley, 1991.

[Holt & Mor., 1] Holt, B. R. and M. Morari, "Design of resilient processing plants-VI. the effect of right-half- plane zeros on dynamic resilience." *Chem. Eng. Sci.*, Vol. **40**, pp.59-74, 1985.

[Holt & Mor., 2] Holt, B. R. and M. Morari. "Design of resilient processing plants-V. the effect of deadtime on dynamic resilience." *Chem. Eng. Sci.* Vol. **40**, pp.1229-1237, 1985.

[Hovd & Skog., 1] Hovd, M. and S. Skogestad, "Controllability analysis for the fluid catalytic cracking process." In *AIChE Annual Meeting*, Los Angeles, California, November 1991.

[Hovd & Skog., 2] Hovd, M. and S. Skogestad. "Use of frequency-dependent RGA for control system analysis, structure selection and design." *Automatica*, 1991.

[Hovd & Skog., 3] Hovd, M. and S. Skogestad, "Simple Frequency-dependent Tools for Control System Analysis, Structure Selection and Design." *Automatica*, Vol. **28**, No. **5**, pp.989-996, 1992.

[Huang & Fan, 1] Huang, Y. L. and L. T. Fant, "Distributed Strategy for Integration of Process Design and Control: A Knowledge Engineering Approach to the Incorporation of Controllability into Exchanger Network Synthesis", *Computers Chem. Eng.* Vol. **16**, pp.497-522, 1992.

[Hunt, 1] Hunt, K. J., D. Sbarbaro, R. Zbikowski and P. J. Gawthrop, "Neural networks for control systems - a survey." *Automatica*, Vol. **28**, No. **6**, November 1992.

[Hutch.,1] Hutchison, W. J. "Decoupling of Multivariable Systems by Function Optimization," *Int. J. Contr.*, Vol. **30**, **No.1**, page 001, 1979.

[Jacob. & Skog., 1] Jacobsen, E. W. and S. Skogestad, "Design modifications for improved controllability of distillation columns." *In COPE-91*, Barcelona, Spain, October 1991.

[Jacob. *et al.*, 1] Jacobsen, E.W., L. Laroche, M. Morari, S. Skogestad, and H.W. Anderson. "Robust control of homogeneous azeotropic distillation columns". *AIChE Journal*, 37:1810-1824, 1991.

[Jaf. & McAvoy, 1] Jafarey, A. and T. J. McAvoy, "Degeneracy of Decoupling in Distillation Columns." *I&EC Process Design Develop.* Vol. **17**, pp.485-490, 1978.

[Jantzen, 1] Jantzen J. *Digraph Approach to Multivariable Control*, Technical University of Denmark: Electric Power Eng. Dept, publication No. 9408 (Lecture Notes), 1994.

[Jan. & Mehrd., 1] Jantzen J. and B. Mehrdad, *Digraph Programs for Multivariable Control*, ESPRIT project No. 8924, SESDIP.

[Jensen *et al.*, 1] Jensen, N., D. G. Fisher and S. L. Shah, "Interaction Analysis in Multivariable Control System Design." *AIChE J.*, 1981.

[Johnson, 1] Johnson, C. D. "Optimisation of a Certain Quality of Complete Controllability and Observability for Linear Dynamical Systems." *ASME J. Basic Eng.* Vol. **91**, Ser. D, page 228, 1969.

[Jos. & Bros., 1] Joseph. B. and C. Brosilow, "Inferential Control of Processes. I: Steady State Analysis and Design," *AIChE J.* Vol. **24**, page 485, 1978.

[Kailath, 1] Kailath, T. *Linear Systems*, Prentice Hall, Englewood Cliffs, NJ, 1980.

[Kalman, 1] Kalman R. E. "Mathematical description of linear dynamical systems", *SIAM J. Control*, Vol. **1**, No. **2**, page 152, 1963.

[Kalman, 2] Kalman R. E. "On minimal realisations of a linear input-output map", in *Topics in Mathematical System Theory*, McGraw-Hill, New York, 1969.

[Kalman, 3] Kalman, R. E. "Kronecker invariants and feedback", in *Ordinary differential equations*, L. Weiss (ed.), Academic Press, New York, 1972.

[Kalman, 4] Kalman, R. E. "Canonical structure of linear dynamical systems." *Proc. Nat. Acd. Sci.*, Vol. **48**, page 596, 1962.

[Karc. & Kouv., 1] Karcanias, N. and B. Kouvaritakis, "The output zeroing problems and its relationship to the invariant zero structure: a matrix pencil approach", *International Journal of Control*, Vol. **30**, pp.395-415, 1979.

[Karc. & Gian., 1] Karcanias, N. and C. Giannakopoulos, "Necessary and Sufficient conditions for zero assignment by constant squaring down." *Linear Algebra and its Applications*, Vol. **122-124**, pp.415-446, 1989.

[Karc. & Vaf., 2] Karcanias, N. and D. Vafiadis, "Model Orientation Problems: An algebraic approach." *Control Engineering Centre research report*, City University, 1995.

[Karc. & Vaf., 1] Karcanias, N. and D. Vafiadis, "On the Cover Problems of Geometric Theory." *Kybernetica*, Vol. **29**, No. **6**, pp.547-562, 1993.

[Karc. & Mil., 1] Karcanias, N. and E. Milonidis, "Global Process Instrumentation: Systems and Control Theory Issues Concepts and Tools for model structure shaping." *Control Engineering Centre research report*, City University, 1995.

[Karc. & Mil., 2] Karcanias, N. and E. Milonidis, " Structural Identification in Large Scale Systems: Generic McMillan Degree Computation." *IFAC Conference on Large Scale Systems, LSS'98*, Univ. of Patras, Greece, 1998.

[Karc. & Hayt., 1] Karcanias, N. and G. E. Hayton, "Generalised autonomous dynamical systems, algebraic duality and geometric theory." *Proc of 8th IFAC World Congress, Kyoto, Japan*, pp.289-294, Pergamon Press, 1981.

[Karc. & Kalog., 1] Karcanias, N. and G. Kalogeropoulos, "Geometric Theory and Feedback Invariants of Generalised Linear Systems: A Matrix Pencil Approach." *Circuits Systems and Signal Process*, Vol. **8**, No. **3**, pp.395-375, 1989.

[Karc. & Levent., 1] Karcanias, N. and J. Leventides, "Two parameter Squaring down and zero assignment." *Control Engineering Centre, Research Report*, City University, London, 1995.

[Karc. & McBean, 1] Karcanias, N. and P. MacBean, "Structural invariants and canonical forms of linear multivariable systems", *Third IMA Conference on Control Theory*, Academic Press, pp.257-282, 1981.

[Karc. & Shan., 1] Karcanias, N. and X. Y. Shan, "The problem of Structural McMillan degree identification." *Control Engineering Centre Research Report*, City University, London, 1995.

[Karc., 1] Karcanias, N. "An overall control theory framework for the model projection problems in the global instrumentation of a process", *Control Engineering Centre, Research Report*, City University, 1993.

[Karc., 2] Karcanias, N. "General Methodology and philosophy of EPIC." *ESPRIT II, Project EPIC, Report* EPCK **0044**, Jan. 1992.

[Karc., 3] Karcanias, N. "Global Process Instrumentation: Issues and Problems of a System and Control Theory Framework." *Measurement*, Vol. **14**, pp.103-113, 1994.

[Karc., 4] Karcanias, N. "Linear system: Kronecker canonical forms and invariants", *Systems and Control Encyclopedia, Section: Linear Systems: General Aspects*, Ed. M. G. Singh, Pergamon Press, pp. 2866-2871, 1987.

[Karc., 5] Karcanias, N. "Matrix pencil approach to geometric system theory", *Proc. IEE*, Vol. **126**, pp.585-590, 1979.

[Karc., 6] Karcanias, N. "On the basis matrix characterization of controllability subspaces." *Int. J. Control*, Vol. **29**, pp.767-786, 1979.

[Karc., 7] Karcanias, N. "On the characteristic, Weyr sequences, the Kronecker invariants etc.", *Proc. 10th IFAC World Congress*, Munich, 1987.

[Karc., 8] Karcanias, N. "Proper invariant realisations of singular systems problems." *IEEE Trans. Autom. Control*, Vol. **AC-35**, No. **5**, pp. 230-233, 1990.

[Karc., 9] Karcanias, N. "The problem of early process design: An overall framework from the control design viewpoint", *ESPRIT II, Project EPIC, Report* EPCK **0005**, City University, Control Engineering Centre, May 1989.

[Karc., 10] Karcanias, N. "Control Problems in Global Process Instrumentation: A Structural Approach", *Computers in Chem. Eng.* Vol. **20**, page 1101, 1966.

[Karc. *et al.*, 1] Karcanias, N., B. Laios and C. Giannacopoulos, "Decentralised Determinantal assignment problems: fixed and almost fixed modes and zeros." *Int. J. Control,* Vol. **23**, pp.149-181, 1988.

[Karc. *et al.*, 2] Karcanias, N., J. Leventides and E. Milonidis, "An integrated Framework for Input, output and Control Structure Selection: Advanced Control Diagnostics", *Proc. Of 5th IEEE Med. Conference*, Cyprus, 1997.

[Karc. *et al.*, 3] Karcanias, N., H. N. Koivo, E. Milonidis and E. Nistazakis, "The problem of global process instrumentation: general issues and control design implications." *WAC'96*, France, May 1996.

[Kest. *et al.*, 1] Kestenbaum, A., R. Shinnar and F. E. Thau, "Design Consepts for Process Control", *I&EC Process Design and Development*, Vol. **15**, page 2, 1976.

[Koiv. *et al.*, 1] Koivisto, H., V. Ruoppila and H. N. Koivo, "Properties of the neural network internal model control." *Preprints of IFAC/IFIP/IMACS International Symposium on Artificial Intelligence in Real-Time Control AIRTC* 92), Delft, The Netherlands, pp. 221-226, June 16-18, 1992.

[Koppel, 1] Koppel, L. B. "Conditions imposed by process statistics on multivariable process dynamics." *AIChE J.* Vol. **31**, page 70, 1985.

[Koppel, 2] Koppel, L. B. "Input-Output Pairing in Multivariable Control", *AIChE J.* January 1982.

[Kouv. & McFar., 1] Kouvaritakis, B. and A. G. J. MacFarlane, "Geometric approach to analysis and synthesis of system zeros; Part II: non-square systems", *Int. J. Control.* Vol. **23**, pp.167-181, 1976.

[Kuijp. & Schum., 1] Kuijper, M. and J. M. Schumacher, "Minimality of descriptor representations under external equivalence." *Automatica*, Vol. **27**, pp.985-995, 1991.

[Kumar & Sein., 1] Kumar, S. and J. H. Seinfeld, "Optimal location of measurements for distributed patameter estimation." *IEEE Transactions on Automatic Control*, **AC-23**, pp.690-698, 1978.

[Kumar & Sein., 2] Kumar, S. and J. H. Seinfeld. "Optimal location of measurements in tubular reactors." *Chem. Eng. Sci.*, Vol. **33**, pp.1507-1516, 1978.

[Lamp. *Et al.*, 1] Lampinen, Jouko, Taipale and Ossi, "Optimization and simulation of quality properties in paper machine with neural networks." *Proceedings of IEEE International Conference on Neural Networks*, Orlando, Florida, 2729, June, 1994.

[Laroche *et al.*, 1] Laroche, L., H. W. Anderson, and M. Morari, "Homogeneous azeotropic distillation: Separability and flowsheet synthesis." *Ind. Eng. Chem. Research*, 1992.

[Laroche *et al.*, 2] Laroche, L., N. Bekiaris, H. W. Anderson, and M. Morari. "The curious behaviour of homogeneous azeotropic distillation-implications for entrainer selection." *AIChE Journal*, 1992.

[Lau, 1] Lau, H. *Studies on Control of Integrated Chemical Processes*, Ph.D. Dissertation, Univ. Minnesota, 1982.

[Lau *et al.*, 1] Lau, H., J. Alvarez, and K. F. Jensen, "Design of Noninteracting Control Structures by Singular Value Analysis." *Presented at 1982 Annual AIChE Meeting*, Los Angeles, November 14-19, 1982.

[Lau *et al.*, 2] Lau, H., J. Alvarez and K. F. Jensen, "Synthesis of Control Structures by Singular Value Analysis: Dynamic Measures of Sensitivity and Interaction." *AIChE J.* Vol. **31**, No. **3**, page 427, 1985.

[Le *et al.*, 1] Le, D. K., O. D. Nwokah and A. E. Frazho, "Multivariable decentralised integral controllability." *Int. J. Control*, Vol. **54**, pp.481-496, 1991.

[Lear *et al.*, 1] Lear, J. B., G. W. Barton, and J. D. Perkins, "Optimising control as an economic design tool." *In PSE '91 4th International Symposium on Process Systems Engineering*, Montebello, Quebec, Canada, pp. I.13.1-I.13.15, 1991.

[Lee *et. al.*, 1] Lee, H. H., L. B. Koppel, and H. C. Lim, "Integrated approach to design and control of a class of countercurrent processes." *Ind. Eng. Chem. Process Des. Dev.*, Vol. **11**, pp.376-382, 1972.

[Lee *et. al.*, 2] Lee, J. H., R. D. Braatz, M. Morari and A. Packard, "Screening Tools for Control Structure Selection." *Automatica*, Vol. **31**, No. **2**, pp. 229-235, 1995.

[Lee & Mor., 1] Lee, J. H. and M. Morari, "Robust control structure selection and control system design methods applied to distillation column control." *In Proc. of IEEE Conf. On Decision and Control*, pp. 2041-2046, 1990.

[Lee & Mor., 2] Lee, J. H. and M. Morari, "Robust measurement selection". *Automatica*, Vol. **27**, pp.519-527, 1991.

[Lee & Week.,1] Lee, W., and V. W. Weekman, Jr. "Advanced Control Practice in the Chemical Process Industry: A View from Industry." *AIChE J.* Vol. **22**, page 27, 1976.

[[Lenh. & Mor., 1] Lenhoff, A. M. and M. Morari, "Design of resilient processing plants. I. process design under consideration of dynamic aspects." *Chem. Eng. Sci.* Vol. **37**, pp.245-258, 1982.

[Leon. *et al.*, 1] Leonard, J. A., M. A. Kramer, L. H. Ungar, "Using radial basis functions to approximate a function and its error bounds." *IEEE transactions on neural networks*, Vol. **3**, No. **4**, July 1992.

[Lev. & Karc., 1] Leventides, J. and N. Karcanias, "Sufficient conditions for arbitrary Pole Assignment by Constant Decentralised Output Feedback." *Math. Control Signals Systems,* Vol. **8**, pp.222-240, 1995.

[Lev. & Karc., 2] Leventides, J. and N. Karcanias, "The decentralized Markov parameters and the selection of control structures." *Int. J. Control*, Vol. **70**, pp.815-830, 1998.

[Lev. & Karc., 3] Leventides, J. and N. Karcanias, "The Pole Placement Map, its Properties and Relationships to System invariants." *IEEE Trans. on Aut. Control*, **AC-38**, pp.1266-1270, 1993.

[Lev. & Karc., 4] Leventides, J. and N. Karcanias, "Dynamic Pole Assignment using Global, Blow up Linearisation: Low Complexity Solutions." *Journal of Optimisation Theory and Applications*, Vol. **96**, pp.57-86, 1998.

[Lewis, 1] Lewis, F. L. "A survey of linear singular systems. Circuits, Systems and Signal Processes." Vol. **8**, pp.375-397, 1989.

[Liang, 1] Liang Q. "Is the Relative Gain Array a Sensitivity Measure?", *IFAC Workshop*, pp.133-138, September 1992.

[Liesl., 1] Lieslehto, J. *An Expert System for Multivariable Controller Design*. Dr.Tech Thesis, Tampere Univ. of Tech., Finland, 1993.

[Limeb., 1] Limebeer, D. J. N. "The application of generalized diagonal dominance to linear system stability theory." *Int. J. Control.* Vol. **36**, pp.185-212, 1982.

[Lin, 1] Lin, Ching-Tai, "Structural Controllability", *IEEE Transactions on Automatic Control*, **AC-19**, No. **3**, page 201, 1976.

[Lin *et al.*, 1] Lin, W. J., J. D. Seader, and T. L. Wayburn, "Computing multiple solutions to systems of interlinked separation columns." *AIChE Journal*, Vol. **33**, page 886, 1987.

[Lin. & Kotj., 1] Linnhoff, B., and E. Kotjabasakis, "Downstream paths for operable process design." *Chem. Eng. Progress*, Vol. **82**, pp.23-28, 1986.

[Loiseau, 1] Loiseau, J. J. "Sur la modification de la structure a l'infi par un retour d'etat statique." *SIAM J. Control and Optimization*, Vol. **26**, pp.251-273. London, Intern. Stud. Edit. 1988.

[Lück. & Mül., 1] Lückel, J., and P. C. Müller, Analyse von Steuerbarkeits-Beobachtbarkeits- und Störbarkeitsstrukturen linearer zeitinvarianter Systeme, Regelungstechnik, Heft **5**, page 163, 1975.

[Luyb. & Floud., 1] Luyben, M. L., and C. A. Floudas, "A multiobjective optimisation approach for analysing the interaction of design and control, part 1: Theoretical framework." *In IFAC Workshop on Interactions between Process Design and Process Control*, London, England, September 1992.

[Luyben, 1] Luyben, W. L. "Distillation Decoupling," *AIChE J.*, Vol. **16**, page 198, 1970.

[Luyben, 2] Luyben, W. L. *Ind. Eng. Chem. Fundam.* Vol.**14**, page 321, 1975.

[Luyben, 3] Luyben, W. L. *Ind. Eng. Chem. Res.* Vol. **27**, page 206, 1988.

[Maar. & Rij., 1] Maarleveld, A. and J. E. Rijnsdorp, "Constraint control on distillation columns." *Automatica* Vol. **6**, page 51, 1970.

[McFar. & Karc., 1] MacFarlane, A. G. J. and N. Karcanias, "Poles and zeros of linear multivariable systems: A survey of the Algebraic Geometric and Complex Variable Theory." *Int. J. Control*, Vol. **24**, pp.33-74, 1976.

[McFar. & Kouv.,1] MacFarlane, A. G. J., and B. Kouvaritakis, "A Design Technique for Linear Multivariable Feedback Systems," *Int. J. Contr.*, Vol. **25**, page 837, 1977.

[Maciej., 1] Maciejowski, J. M. *Multivariable Feedback Design,* Addision-Wesley, Wokingham, England, 1989.

[Magnus. *et al.*, 1] Magnussen, T., M. L. Michelsen, and A. Fredenslund. "Azeotropic distillation using unifac". In *Third International Symposium on Distillation,* I. Chem. E. Symposium Series No. **56**, pp.1-19, The Institution of Chemical Engineers, 1979.

[Manous., 1] Manousiouthakis, B. *Design of Control Systems for Uncertain Chemical Plants.* Ph.D Thesis, Rensselaer Polytech. Inst., Troy, NY, 1985.

[Manous. *et al.*, 1] Manousiouthakis, B., and Y. Arkun, "Robust Controller Design. A Hybrid Approach." *Am. Control Conf. Proc. San Diego*, page 1277, 1984.

[Manous. *et al.*, 2] Manousiouthakis, V., R. Savage and Y. Arkun, "Synthesis of Decentralised Process Control Structures Using the Concept of Block Relative Gain." *AIChE J.*, Vol. **32**, No **6**, page 991, June 1986.

[Marlin *et al.*, 1] Marlin, T. E., J. D. Perkins, G W. Barton, and M. L. Brisk, "Benefits from process control results of a joint industry-university study." *J. Process Control*, Vol. **1**, page 68, 1991.

[Mars. *et al.*, 1] Marselle, D. F., M. Morari, and D. F. Rudd, "Design of resilient processing plants. II. Design and control of energy management systems." *Chem. Eng. Sci.*, Vol. **37**, pp.259-270, 1982.

[Mass. *et al.*, 1] Massimo, C. D., G. A. Montague, M. J. Willis, M. T. Tham, and A. J. Morris, "Towards improved penicillin fermentation via artificial neural netoworks." *Computers Chemical Engineering*, Vol. **16**, No. **4**, pp. 283-291, 1992.

[Mathisen, 1] Mathisen, K. W., S. Skogestad, and E. A. Wolff, "Controllability of heat exchanger networks." *In AIChE Annual Meeting*, Los Angeles, California, November 1991.

[McAvoy, 1] McAvoy, T. J. *Interaction Analysis.* Instrument Soc. America, Research Triangle Park. Monograph Series, 1983.

[McAvoy, 2] McAvoy, T. J. "Physical Example of Control Systems." *Modular Instructional Series of AIChE, Series A: Process Control*, No. **A6.2**, *AIChE Corp.* 1981.

[McAvoy, 3] McAvoy, T. J. "Some Results on Dynamic Interaction Analysis of Complex Control Systems." *I&EC Process Design Develop.* March 1982.

[Meert, 1] Meert, Kurt "Estimation of the average chain length of polymers with neural classifiers." *Proceedings of IEEE International Conference on Neural Networks*, Orlando, Florida, 27-29 June 1994.

[Mees, 1] Mees, A. I. "Achieving diagonal dominance." *Systems and Control Letters*, Vol. **1**, pp.155-158, 1981.

[Mehra, 2] Mehra, R. K. "Optimization of Measurement Schedules and Sensor Design for Linear Dynamic Systems." *IEEE Trans. Autom. Contr.*, **AC-21**, page 55, 1976.

[Mehra *et al.*, 1] Mehra, R. K., R. Rouhani, J. Eterno, J. Richalet, and J. Rault, "Model Algorithmic Control: Review and Recent Developments," *Eng. Foundation Conf. on Chemical Process Control II*, Sea Island, Georgia, pp.287-309, 1981.

[Mell. & Sarg., 1] Mellefont, D. J., and R. W. H. Sargent, "Optimal Measurement Policies for Control Purposes." *Int. J. Contr*, Vol. **26**. Page 595, 1977.

[Mell. & Sarg., 2] Mellefont, D. J., and R. W. Sargent, "Selection of Measurements for Optimal Feedback Control." *Ind. Eng. Chem. Proc. Des. Dev.*, Vol. **17**, page 349, 1978.

[Mij. *et al.*, 1] Mijares, G., J. D. Cole, N. W. Naugle, H. A. Preisig and C. D. Holland, "A new criterion for the pairing of control and manipulated variables." *AIChE J*, Vol. **32**, pp.1439-1449, 1986.

[Mitr. & Karc., 1] Mitrouli, M. and N. Karcanias, "Algebraic, Exterior algebra Computations and Control Analysis Diagnostics." *ESPRIT project SESDIP 8924, Research report* **SDCU054**, Control Eng. Centre, City University, 1996.

[Mizsey & Fonyo, 1] Mizsey, P., and Z. Fonyo, "Assessing plant operability during process design." *ID COPE-91*, Barcelona, Spain, October 1991.

[Mor. & Steph., 1] Morari M. and G. Stephanopoulos, "(a) Studies in the synthesis of control structures for chemical processes : Part II: Structural aspects and the synthesis of alternative feasible control schemes." *AIChE Journal*, Vol. **26**, pp.232-246, 1980, "(b) Part III Optimal selection of secondary measurements." *AIChE Journal*, Vol. **26**, pp.247-260, 1980.

[Morari, 4] Morari, M. "Design of resilient processing plants-III. a general framework for the assessment of dynamic resilience." *Chem. Eng. Sci.* Vol. **38**, pp.1881-1891, 1983.

[Mor. & Zaf., 1] Morari, M. and E. Zafiriou, *Robust Process Control.* Prentice-Hall, Englewood Cliffs, 1989.

[Mor. & Steph., 2] Morari, M. and G. Stephanopoulos, "Minimizing unobservability in inferential control schemes." *Int. J. Control.* Vol. **31**, page 367, 1980.

[Mor. & Lee, 1] Morari, M. and J. H. Lee, "Control structure selection: Issues and a new methodology." In *Signal Processing, Part II. Control Theory and Applications*, edited by F. A. Grunbaum, J. W. Helton, and P. Khargonekar, Springer Veriag, pp.195-219, 1990.

[Morari, 5] Morari, M. "Operability measures for process design." In *Understanding Process Integration, I. Chem. E. Symposium Series* No.74, pp.131-140, The Institution of Chemical Engineers, 1982.

[Morari, 6] Morari, M. "Design of resilient processing plants-III. a general framework for the assessment of dynamic resilience." *Chem. Eng. Sci.*, Vol. **38**, pp.1881-1891, 1983.

[Morari, 1] Morari, M. "Effect of Design on the Controllability of Chemical Plants." *IFAC Workshop on Interactions Between Process Design and Process Control*, London, 1992.

[Morari, 2] Morari, M. "Flexibility and Resiliency of Process Systems," *Proc. Int. Symp. Proc. Sys. Eng.*, Kyoto, Japan, 1982.

[Morari, 3] Morari, M. "Integrated Plant Control: A Solution at Hand or a Research Topic for the Next Decade." *Proc. Chem. Proc. Control II*, edited by Seborg and Edgar, 1981.

[Mor. & Zaf., 1] Morari, M. and E. Zafiriou *Robust Process Control*, Prentice Hall, Englewood Cliffs, 1989.

[Mor. & Lee, 1] Morari, M. and J. H. Lee, "Control structure selection: issues and a new methodology." In *Signal Processing, Part II: Control Theory and Applications*, edited by F.A. Grunbaum, J.W. Helton, and P. Khargonekar, pp.195-219, Springer Verlag, 1990.

[Mor. *et al.*, 5] Morari, M., E. Zafiriou, and B. R. Holt, "Design of resilient processing plants-X. characterisation of the effect of rhp zeros." *Chem. Eng. Sci.* Vol. **42**, pp.2425-2428, 1987.

[Mor. *et al.*, 2] Morari, M. *et al.* "Design of Resilient Processing Plants. Design of Energy Management System for Unstable Reactors-New Insights." *Chem. Eng. Sci.* Vol. **40**, page 187, 1985.

[Mor. *et al.*, 4] Morari, M., W. Grimm, M. J. Ogelsby, and I. D. Prosser, "Design of resilient processing plants-VII. design of energy management system for unstable reactors-new insights." *Chem. Eng. Sci.* Vol. **40**, pp.187-198, 1985.

[Mor. *et al.*, 3] Morari, M., Y. Arkun, and G. Stephanopoulos, "An Integrated Approach to the Synthesis of Process Control Structures." *JACC Proc.* Philadelphia, 1978.

[Mor. *et al.*, 1] Morari, M., Y. Arkun and G. Stephanopoulos, "Studies in the Synthesis of Control Structures for Chemical Process." *AIChE J.* Vol. **26**, page 220, 1980.

[Morse, 1] Morse, A. S. "Structural invariants of linear multivariable systems." *SIAM J. Control*, Vol. **11**, pp.446-465, 1973.

[Mül. & Web., 1] Müller, P. C., and H. I. Weber, "Analysis and Optimization of Certain Qualities of Controllability and Observability for Linear Dynamical Systems." *Automatica*, Vol. **8**, page 237, 1972.

[Murt. & Saund., 1] Murtagh, B. A. and M. A. Saunders, "MINOS 5.0 users guide." *Technical Report SOL*, 83-20 Stanford University, 1983.

[Naren. *et al.*, 1] Narendra, S. Kumpati, and Parthasarathy Kannan, "Identification and control of dynamical systems using neural networks." *IEEE Transactions on Neural Networks*, Vol. **1**, No. **1**, 1990.

[Narraw. *et al.*, 1] Narraway, L. T., J. D. Perkins, and G. W. Barton, "Interaction between process design and process control: economic analysis of process dynamics." *J. Proc. Cont.* Vol. **1**, page 243, 1992.

[Narraw., 2] Narraway, L. T. *Selection of process control structure based on economics.* PhD thesis, Dept. Chem. Eng. Imperial College, University of London, 1992.

[Nayl.,1] Naylor, A. W., and G. R. Sell, *Linear Operator Theory in Engineering and Science*, Springer-Verlag, New York, 1982.

[Nemh. & Wols., 1] Nemhauser G. L. and L. A. Wolsey, *Integer and combinatorial optimization.* John Wiley and sons, New York, 1988.

[Nett & Manous., 1] Nett, C. N. and V. Manousiouthakis, "Euclidian Condition and Block Relative Gain: Connections, Conjectures and Clarifications." *IEEE Trans. Aut. Control*, Vol. **AC-32**, No. **5**, May 1997.

[Newell & Fish., 1] Newell, R. and D. Grant Fisher, "Model Development, Reduction, and Experimental Evaluation for an Evaporator." *Ind. Eng. Chem. Process Des. Develop.* Vol. **11**, page 213, 1972.

[Nied., 1] Niederlinski, A. "A heuristic approach to the design of linear multivariable control systems." *Automatica*, Vol. **7**, pp.691-701, 1971.

[Nied., 2] Niederlinski, A. "Two-variable Distillation Control: Decouple or not decouple." *AIChE J.* Vol. **17**, page 1261, 1971.

[Nish. & Ichik., 1] Nishida, N., and A. Ichikawa *Ind. Eng. Chem. Proc. Des. Dev.* Vol. **14**, page 263, 1975.

[Nish. *et al.*, 1] Nishida, N., G. Stephanopoulos, and A. W. Westerberg, "A Review of Process Synthesis." *AIChE J.* Vol. **27**, No. **3**, page 321, 1981.

[Nish. *et al.*, 2] Nishida, N., G. Stephanopoulos, and A. W. Westerberg, "A review of process synthesis." *AIChE J.* Vol. **27**, No. **4**, page 423, 1981.

[Nish. *et al.*, 3] Nishida, N., G. Stephanopoulos, and A. W. Westerberg, "A review of process synthesis." *AIChE Journal*, Vol. **27**, pp.321-351, 1981.

[Nish. *et al.*, 1] Nishida, N., Y. A Liu, and A. Ichikawa, *AIChE Journal,* Vol. **22**, page 539, 1976.

[Nistazak. *et al.*, 1] Nistazakis, E., E. Milonidis, and A. Nissinen, "Process control methodologies for control structure selection: a review of issues and tools." Presented in *World Automation Congress '96*, Montpellier, France, 27-30 May.

[Nob. & Dan.,1] Noble, B., and J. W. Daniel, *Applied Linear Algebra*, Prentice-Hall, Englewood Cliffs, N J, 1977.

[Nwok. *et al.*, 1] Nwokah, O. D., A. E. Frazho and D. K. Lee, "A note on decentralised integral controllability." *Int. J. Control*, Vol. **57**, pp.485-494, 1993.

[Ogun. & Ray.,1] Ogunnaike, B. A., and W. H. Ray, "Multivariable Controller Design for Linear Systems Having Multiple Time Delays." *AIChE J.* Vol. **25**, No. **1**, page 43, 1979.

[Oldaker, 1] Oldaker R. *The MATLAB Graph Theory Toolbox and its Application to Structural Analysis and Design*, PhD Thesis, City University, 1992.

[Pars. & Sull., 1] Parsaei, H. R. and W. G. Sullivan, *Concurrent Engineering: Contemporary Issues and Modern Design Tools*, Chapman & Hall, London, 1993.

[Patton *et al.*, 1] Patton, R., P. Frank and R. Clark, *Fault Diagnosis in Dynamic Systems: Theory and Application.* Prentice Hall, Hemel Hempstead, UK, 1989.

[Penl. *et al.*, 1] Penlidis, A., J. F. MacGregor, and A. E. Hamielec, "Continuous emulsion polymerization: Design and control of CSTR trains", *Chem. Eng. Sci.* Vol. **44**, pp.273-281, 1989.

[Perk. & Wong, 1] Perkins and M. P. F. Wong, "Assessing controllability of chemical plants." *Chem. Eng. Res. Des.* Vol. **63**, pp.358-362, 1985.

[Pichai *et al.*, 1] Pichai V., M. E. Sezer, and D. D. Siljak, "Vulnerability of dynamic systems." *Int.J.Control,* Vol. **34**, No. **6**, pp. 1049-1060, 1981.

[Post. *et al.*, 1] Postethwaite, L., J. M. Edmunds, and A. G. J. MacFarlane, "Principal Gains and Principal Phases in the Analysis of Linear Multivariable Feedback Systems." *IEEE Trans. Auto. Contr.* Vol. **AC-26**, No. **1**, page 32, 1981.

[Prett & Gil., 1] Prett, D. M. and R. Gillette, "Optimization and Constrained Multivariable Control of a Catalytic Cracking Unit." *AIChE 86th National Meeting*, April 1979; also in *Joint Automatic Control Conference Proceedings*, San Francisco, 1980.

[Prett & Garc., 1] Prett. D. M. and C. E. Garcia, *Fundamental process control.* Butterworths, Boston, 1988.

[Prosys, 1] Prosys Technology, *SPEEDUP case book.* Prosys Technology, Cambridge, 1989.

[Psar. & Floud., 1] Psarris, P. and C. A. Floudas, "Dynamic Operability of Mimo Systems with Time Delays and Transmission Zeroes - I. Assessment." *Chem. Eng. Sc.* Vol. **46**, pp.2691-2707, 1991.

[Psar. & Floud., 2] Psarris, P. and C. A. Floudas, "Dynamic Operability of MIMO Systems with Time Delays and Transmission Zeroes- II. Enhancement." *Chem. Eng. Sc.* Vol. **46**, pp.2709-2728, 1991.

[Psar. & Floud., 3] Psarris, P. and C. A. Floudas, "Improving Dynamic Operability in Mimo Systems with Time Delays." *Chem. Eng. Sc.* Vol. **45**, page 3505, 1990.

[Pugh & Kafai, 1] Pugh, A. C. and A. Kafai, "Simple condition for composite system properness." *Int. J. Control,* Vol. **45**, 1987.

[Ray, 1] Ray, W. H. "Multivariable Process Control - A Survey." *Proc. Int. Symp. Proc. Syst. Eng.* Kyoto, Japan, 1992.

[Ray, 2] Ray, W. H. *Advanced Process Control,* McGraw Hill, New York, 1981.

[Ray, 1] Ray, W. H. "Progress in pollution prevention through precision process modeling, design, and operation." In *Proceedings of EFChE Conference on Precision Process Technology - Perspectives for Pollution-Prevention,* Delft, Netherlands, October 1992.

[Reev. & Ark., 1] Reeves, D. E. and Y. Arkun, "Interaction Measures for Nonsquare Decentralised Control Structures." *AIChE Journal,* Vol. **35**, No. **4**, pp. 603-613, April 1989.

[Reinschke, 1] Reinschke K. J. "Graph Theoretic characterisation of fixed modes in centralised and decentralised control." *Int.J.Control,* Vol. **39**, pp. 715-729, 1984.

[Reinschke, 2] Reinschke K. J. *Multivariable Control - A graph theoretic approach.* Lecture Notes in Control and Information Sciences, Vol. **108**, Springer-Verlag, Berlin, 1988.

[Rinard, 1] Rinard, I. H. "Process control of highly integrated processes." In *National Science Foundation Conference on Innovative Design Techniques for Energy Efficient Processes,* Northwestern University, Evanston, Illinois, 1975.

[Rinard, 2] Rinard, I. H., and B. W. Benjamin, "Control of recycle systems. part 1. Continuous control." In *American Control Conference,* Los Angeles, California, 1982.

[Roat, 1] Roat, S. D., J. J. Downs, E. F. Vogel, and J. E. Doss, "The integration of rigorous dynamic modelling and control system synthesis for distillation columns: An industrial approach." In *Proceedings of the Third Int. Conf. on Chemical Process Control - CPCIII*, edited by M. Morari and T. J. McAvoy, Asilomar, California, CACHE-Elsevier, 1986.

[Rosen., 1] Rosenbrock, H. H. *State Space and Multivariable Theory*, Nelson London, 1970.

[Rosen., 2] Rosenbrock, H. H. *Computer-Aided Control System Design*, Academic Press, New York, 1974.

[Rosen., 3] Rosenbrock, H. H. "Computer-Aided Design of Multivariable Control Systems." *Multivariable Control, Proceedings of Second Annual Control Conf.* Purdue University, West Lafayette, Ind. 1975.

[Rosen., 4] Rosenbrock, H. H. and A. C. Pugh, "Contributions to a hierarchical theory of systems." *Int. J. Control*, Vol. **19**, pp.845-867, 1974.

[Rosen., 5] Rosenbrock, H. H. "Design of Multivariable Control Systems Using the Inverse Nyquist Array." *Proc. IEEE*, Vol. **116**, page 1929, 1969.

[Rosen., & Power, 1] Rosenbrock, H. H. and B. A. Power, "Allocation of poles and zeros." *Proc. IEE*, Vol. **117**, No. 9, pp.1079 – 1082, 1970.

[Roval. & Doh., 1] Rovaglio, M., and M. F. Doherty, "Dynamics of heterogeneous azeotropic distillation columns." *AIChE Journal*, Vol. **36**, pp.39-52, 1990.

[Roval. *et al.*, 1] Rovaglio, M., T. Faravelli, and M. F. Doherty, "Operability and control of azeotropic heterogeneous distillation sequences." *In PSE '91 4th International Symposium on Process Systems Engineering Montebello*, Quebec, Canada, pp.II.17.1-II.17.15, 1991.

[Rudd & Watson, 1] Rudd, D. F. and C. C. Watson, *Strategy of Process Engineering*, John Wiley & Sons, New York, 1968.

[Russ. & Perk., 1] Russell, L. W. and J. D. Perkins, "Towards a method for diagnosis of controllability and operability problems in chemical plants." *Chem. Eng. Res. and Design*, Vol. **65**, pp.453-461, 1987.

[Ryskamp,1] Ryskamp, C. J. "Explicit vs. Implicit Decoupling in Distillation Control." *Proc. Chem. Proc. Control II*, edited by Seborg and Edgar, 1981.

[Saeks & DeCarlo, 1] Saeks, R. and R. A. DeCarlo, *Interconnected Dynamical Systems*, Marcel Dekker, New York, 1981.

[Safanov, 1] Safanov, M. G., A. I. Laub, and G. L. Hartmann, "Feedback Properties of Multivariable Systems: The Role and Use of Return Difference Matrix." *IEEE Trans. Auto. Contr.* Vol. **AC-26**, No. **1**, page 47, 1981.

[Sarg. & Wester., 1] Sargent, R. W. H. and A. W. Westerberg, "SPEED-UP in Chemical Engineering Design." *Trans. Inst. Chem. Eng.* Vol. **42**, page 190, 1964.

[Savage, 1] Savage, B. *Block Relative Gain and its Application to the Synthesis of Control Structures for Chemical Planls.* M.S. Thesis, Rensselaer Polytech. Inst., Troy. NY, 1981.

[Savas, 1] Savas, E. S. *Computer Control of Industrial Processes*, McGraw-Hill, 1965.

[Scheuring, 1] Scheuring R. *Control System Structure Design.* Dept. of Engineering, University of Cambridge, England, 1989.

[Schizas, 1] Schizas C. *A Graph Theoretic Approach for the Analysis of Large Scale systems.* PhD Thesis, University of London, Queen Mary College, Dept. of EE Eng. 1981.

[Seider *et al.*, 1] Seider, W. D., D. D. Brengel, A. M. Provost, and S. Widagdo, "Nonlinear analysis in process design. Why overdesign to avoid complex nonlinearities? " *Ind. Eng. Chem. Res.* Vol. **29**, pp.805-818, 1990.

[Sheider *et al.*, 2] Seider, W. D., D. D. Brengel, and S. Widagdo, "Nonlinear analysis in process design." *AIChE Journal*, Vol. **37**, pp.1-38, 1991.

[Shah, 1] Shah. N. *Efficient scheduling, planning and design of multipurpose batch plants.* PhD thesis, Dept. Chem. Eng., Imperial College, University of London, 1992.

[Shields & Pear., 1] Shields R. W., Pearson J. B. "Structural Controllability of multi-input linear systems", *IEEE Trans.* Vol. **AC-21**, pp.203-212, 1976.

[Shigley, 1] Shigley, J. E. *Mechanical Engineering Design.* Third edition, McGraw-Hill, New York, 1977.

[Shinnar, 1] Shinnar, R. "Chemical reactor modelling for purposes of controller design." *Chem. Eng. Com.* Vol. **9**, pp.73-99, 1981.

[Shinnar *et al.*, 1] Shinnar, R., F. J. Doyle, H. M. Budman, and M. Morari, "Design considerations for tubular reactors with highly exothermic reactions." *AIChE Journal*, 1992.

[Shigley, 2] Shinskey, F. G. *Process Control Systems*, McGraw Hill Co. 1967.

[Shinskey, 1] Shinskey, F. G. "Uncontrollable processes and what to do about them." *Hydrocarbon Process*, page 62, 1983.

[Shigley, 3] Shinskey, F. G. *Distillation Control for Productivity and Energy Conservation.* New York: McGraw-Hill, pp.301-302, 1979.

[Shigley, 4] Shinskey, F. G. *Process Control Systems*, Second edition, New York: McGraw-Hill, pp.210-212, 1979.

[Siljak *et al.*, 1] Siljak D. D, V. Pichai and M. E. Sezer, "A Graph-Theoretic characterisation of structurally fixed modes" *Automatica*, Vol. **20**, No. **2**, pp.247-250, 1984.

[Siljak *et al.*, 2] Siljak D. D, V. Pichai and M. E. Sezer, "Graph-theoretic analysis of dynamic systems", *Report DE-AC 037 ET 29138-35*, University of Santa Clara, California, 1982.

[Siljak, 1] Siljak D. D. *Decentralised Control of Complex Systems.* Academic Press inc., 1991.

[Silv. & Shin., 1] Silverstein, J. L., and R. Shinnar, "Effect of design on the stability and control of fixed bed catalytic reactors with heat feedback. 1 concepts." *Ind. Eng. Chem. Process Des. Dev.* Page 21, 1982.

[Skog. & Mor., 3] Skogestad, S., and M. Morari, "LV-control of a high-purity distillation column." *Chem. Eng. Sci.* Vol. **43**, pp.33-48, 1988.

[Skog. & Mor., 1] Skogestad, S., and M. Morari, "Control configurations for distillation columns." *AIChE Journal,* Vol. **33**, pp.1620-1635, 1987.

[Skog. & Mor., 2] Skogestad, S., and M. Morari, "Design of resilient processing systems-IX. effect of model uncertainty on dynamic resilience." *Chem. Eng. Sci.* Vol. **42**, pp.1765-1780, 1987.

[Skog. *et al.*, 2] Skogestad, S., E. W. Jacobsen, and M. Morari, "Inadequacy of steady-state analysis for feedback control: Distillate bottom control of distillation columns." *Ind. Eng. Chem. Research*, Vol. **29**, pp.2339-2346, 1990.

[Skog. *et al.*, 1] Skogestad, S., M. Hovd, and P. Lundström, "Towards integrating design and control: Use of frequency-dependent tools for controllability analysis." In *PSE '91 4th International Symposium on Process Systems Engineering*, Montebello, Quebec, Canada, pp. III.3.1-III.3.15, 1991.

[Skog. *et al.*, 4] Skogestad, S., M. Morari, and J. C. Doyle, "Robust control of ill-conditioned plants: High purity distillation." *IEEE Trans. Autom. Control*, Vol. **33**, pp.1092-1105, 1988.

[Skog. *et al.*, 3] Skogestad, S., P. Lundström, and E. W. Jacobsen, "Selecting the best distillation control configuration." *AIChE Journal*, Vol. **36**, page 753, 1990.

[Smith, 1] Smith, C. R. *Multivariable Process Control Using Singular Value Decomposition.* Ph.D. Thesis Univ. Tennessee, 1981.

[Smith *et al.*, 1] Smith, C. R., C. F. Moore, and D. D. Bruns, "A Structural Framework for Multivariable Control Applications." *JACC*, 1981a.

[Sokol. & Redh., 1] Sokolnikoff, I. S. and R. M. Redheffer, *Mathematics of Physics and Modern Engineering.* New York, McGraw-Hill, 1958.

[Stanley *et al.*, 1] Stanley, G., M. Marino-Galarraga, and T. J. McAvoy, "Shortcut operability analysis. 1. the relative disturbance gain." *Ind. Eng. Chem. Process Des. Dev.* Vol. **24**, pp.1181-1188, 1985.

[Stanl. & Mar., 1] Stanley, G., M. Marino-Galarraga, T. J. McAvoy, *Ind. Eng. Chem. Process Des. Dev.* Vol. **24**, page 1181, 1985.

[Stef., 1] Stephanopoulos, G. "Synthesis of Control Systems for Chemical Plants - A Challenge for Creativity." *Proc. Int. Symp. Proc. Syst. Eng.* Kyoto, Japan, 1982.

[Takamatsu *et al.*, 1] Takamatsu T., I. Hashimoto, and Y. Nakai, "A Geometric Approach to Multivariable Control System Design of a Distillation Column", *Automatica*, Vol. **15**, pp. 387-403, 1979.

[Talou & Yah., 1] Talou, T., G. Yahiaoui, "An intelligent gas odor sensor: application to quality control for food industry." *Montpellier '94, 3rd International Conference. Interface to real and virtual worlds. Proceedings*, pp.369-77, 1994.

[Teym. & Ray, 1] Teymour, F. and W. H. Ray, "The dynamic behavior of continuous solution polymerization reactors - IV. dynamic stability and bifurcation analysis of an experimental reactor". *Chem. Eng. Sci.* Vol. **44**, pp.1967-1982, 1989.

[Thorp, 1] Thorp, J. P. "The singular pencil of linear dynamical systems." *Int. J. Control*, Vol. **18**, pp.577-596, 1973.

[Tung & Edg., 1] Tung, L. S. and T. F. Edgar, "Analysis of Control-Output Interactions in Dynamic Systems." *AIChE J.* Vol. **27**, page 690, 1981.

[Tung & Edgar, 1] Tung, L. S. and T. F. Edgar, "Dynamic Interaction Index and Its Application to Distillation Control." *Proc. IEEE Conference on Decision and Control*, Vol. **1**, page 107, New Orleans, December 1977.

[Tung & Edgar, 2] Tung, L. S. and T. F. Edgar, "Analysis of control-output interactions in dynamic systems." *AIChE J.*, Vol. **27**, pp.690-693, 1981.

[Tyreus, 1] Tyreus, B. D. "Optimisation and Multivariable Control of Distillation Columns". *Advances In Instrumentation* Vol. **42**, Part 1, *Proceedings of ISA 87 International Conference*, 1987.

[Tyreus, 2] Tyreus, B. *Industrial Applications of Multivariable Control.* Presented at Short Course on Distillation Dynamics and Control, Lehigh University, Bethlehem, Penn., May 1982.

[Tyreus, 3] Tyreus, B. "Multivariable Control System Design for an Industrial Distillation Column." *I&EC Process Design Develop.* page 18, 1979.

[Tyreus, 4] Tyreus, B. *Variable Pairin, in Multi-loop Control Systems.* Presented at Short Course on Distillation Dynamics and Control, Lehigh University, Bethlehem, Penn., May 1982.

[[Uhrig, 1] Uhrig, R. E., L. H. Tsoukalas, and A. Ikonomopoulos, "Application of neural networks and fuzzy systems to power plants." *Proceedings of IEEE International Conference on Neural Networks*, Orlando, Florida, 27-29 June, 1994.

[Umed., *et al.*, 1] Umeda, T., T. Kuriyama, and A. Ichikawa, "A logical structure for process control system synthesis." *In Proc. IFAC Congress,* Vol. 1, Helsinki, 1978.

[Uppal *et al.*, 1] Uppal, A., W. H. Ray, and A. B. Poore, "On the dynamic behavior of continuous stirred tanks." *Chem. Eng. Sci.* Vol. **29**, 1974.

[Vidyas., 2] Vidyasagar, M. *Input-Output Analysis of Large Scale interconnected Systems: Decomposition, Well-Posedness and Stability.* Lecture Notes in Control and Inform. Sciences. Vol. **29**, Springer-Verlag, Berlin, 1981.

[Vidyas., 1] Vidyasagar, M. *Control System Synthesis: A Factorization Approach,* MIT Press, Cambridge, Mass, 1985.

[Waller *et al.*, 1] Waller, K. V., D. H. Finnerman, P. M. Sandelin, K. E. Haggblom, "On the Difference Between Distillation Column Control Structures." *Report 86-2, 1986, Process Control Laboratory,* Abo Akademi, Abo, Finland.

[Wang & Davis., 1] Wang S. H., E. J. Davison, "A note in the simple modes of *multivariable control", IEEE Trans. Autom. Control,* Vol. **18**, pp. 473.

[Web. & Bros., 1] Weber, R., and C. Brosilow, "The Use of Secondary Measurements to Improve Control" *AIChE J.* Vol. **18**, page 614, 1972.

[Weis. & McAv.,1] Weischedel, K., and T. J. McAvoy, "Feasibility of Decoupling in Conventionally Controlled Distillation Columns." *Ind. Eng. Chem. Fund.* Vol. **19**, page 379, 1980.

[Willems, 1] Willems, J. C. "Model for Dynamics." *Dynamics Report 2*, pp. 171-269, 1989.

[Wilson, 1] Wilson, K. B. "Calculation and analysis of longitudinal temperature gradients in tubular reactors." *Trans. Inst. Chem. Engrs.* Vol. **24**, 1946.

[Winter, 1] Winter, P. "Computer-aided process engineering: The evolution continues." *Chem. Eng. Progress*, Vol. **88**, pp.76-83, February 1992.

[Wit. & McAvoy, 1] Witcher, M. F., and T. J. McAvoy, "Interacting Control Systems: Steady State and Dynamic Measurement of Interaction." *ISA Trans.* Vol. **16**, No. **3**, page 35, 1977.

[Wolf *et al.*, 1] Wolf, E. A., K. W. Mathisen, and S. Skogestad, "Dynamics and controllability of heat exchanger networks." In *COPE-91*, Barcelona, Spain, October 1991.

[Wonham, 1] Wonham, W. M. *Linear Multivariable Control: A Geometric Approach.* Springer-Verlag, New York, Second Edition, 1979.

[Wood & Ber., 1] Wood, R. K., and M. W. Berry, "Terminal Composition Control of a Binary Distillation Column." *Chem. Eng. Sci.* Vol. **28**, No. **1**, page 707, 1973.

[Woolv., 1] Woolverton, P. "How to Use Relative Gain Analysis with Integrating Variables." *Intech. 27*, pp.63-65, September 1980.

[Zhou *et al.*, 1] Zhou, K., J. C. Doyle and K. Glover, *Robust and Optimal Control,* Prentice-Hall, Upper Saddle River, 1996.

[Ziegl. & Nich., 1] Ziegler, J. and N. Nichols, "Optimum Settings for Automatic Controllers." *ASME Trans.* Vol. **64**, page 759, 1942.

[Ziegler & Nich., 2] Ziegler, J. G., and N. B. Nichols, "Process lags in automatic-control circuits." *Transactions of the A.S.M.E.* Vol. **65**, pp.433-444, 1943.

[Grosd., 1] Grosdidier, P. "Analysis of interaction with the singular value decomposition." *Comp. Chem. Eng.*, Vol. **14**, pp.687-689, 1990.

[SESDIP 1] *Global Process Instrumentation: Issues and Problems of a Systems and Control Theory/design Framework*, SDCU038, Project Report, ESPRIT III project SESDIP (8924), City University, Sept. 1995.

[SESDIP 2] *Review of Process Methodologies for Global Instrumentation,* SDCU037, Project Report, ESPRIT III project SESDIP (8924), City University, Nov. 1995.

PAGE/PAGES EXCLUDED UNDER INSTRUCTION FROM UNIVERSITY