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Mutually local fields from form factors

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Abstract

We compare two different methods of computing form factors. One is the well established procedure of solving the form factor consistency equations and the other is to represent the field content as well as the particle creation operators in terms of fermionic Fock operators. We compute the corresponding matrix elements for the complex free fermion and the Federbush model. The matrix elements only satisfy the form factor consistency equations involving anyonic factors of local commutativity when the corresponding operators are local. We carry out the ultraviolet limit, analyze the momentum space cluster properties and demonstrate how the Federbush model can be obtained from the $SU(3)_3$-homogeneous sine-Gordon model. We propose a new class of Lagrangians which constitute a generalization of the Federbush model in a Lie algebraic fashion. For these models we evaluate the associated scattering matrices from first principles, which can alternatively also be obtained in a certain limit of the homogeneous sine-Gordon models.

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1 Introduction

One of the most central concepts in relativistic quantum field theory, like Einstein causality and Poincaré covariance, are captured in local field equations and commutation relations. In fact this principle is widely considered as so pivotal that it constitutes the base of a whole subject, i.e. local quantum physics (algebraic quantum field theory) [1] which takes the collection of all operators localized in a particular region generating a von Neumann algebra, as its very starting point.

On the other hand, in the formulation of a quantum field theory, one may alternatively start from a particle picture and investigate the corresponding scattering theories. In particular for 1+1 dimensional integrable quantum field theories this latter approach has been proved to be impressively successful. As its most powerful tool one exploits here first the bootstrap principle [2, 3, 4], which allows to write down exact, i.e. non-perturbative, scattering matrices. Ignoring subtleties of non-asymptotic states, it is essentially possible to obtain the particle picture from the field formulation by means of the LSZ-reduction formalism [5]. However, the question of how to reconstruct the field content, or at least part of it, from the scattering theory is in general still an outstanding issue.

In the context of 1+1 dimensional integrable quantum field theories the identification of the operators is based on the assumption, dating back to the initial papers [6], that each solution to the form factor consistency equations [6, 7, 8, 9] corresponds to a particular local operator. Consequently one approach, as outlined in section 3.1., to construct the quantum field theory consists of solving systematically this set of equations and thereafter pin down the nature of the operator. To do this, numerous authors have used diverse arguments. For instance, most conventional, one may study the asymptotic behaviours or perform perturbation theory. More in the spirit of an exact formulation is to take symmetries into account and to formulate quantum equations of motion or conservation laws [8, 10, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. However, these computations can not really be regarded as a stringent identification, since they only relate particular solutions to each other and lack systematics. Even when taking them as a mere consistency check one should be cautious, since such equations also hold for matrix elements which do not satisfy the consistency equations, as argued in section 5.2 in more detail. An approach with somewhat more underlying systematic is to carry out the ultraviolet limit of the theory and appeal to the well understood classification scheme of conformal field theory [14, 13, 21, 16, 17, 18, 19]. Naming the operators in the massive model is then in one-to-one correspondence with the conformal field theory. So far it is still problematic here to unravel degeneracies [17]. Furthermore, one should be cautious when using this correspondence, since there might be operators, so-called “shadow operators”, in the massive model which do not possess a counterpart in the underlying conformal field theory [22].

This talk is also devoted to this question in the sense that we provide explicit expressions for operators $\mathcal{O}(x)$ located at x in terms of fermionic Fock fields. Particular emphasis is put on the question whether these operators are really local in the sense that they (anti)-commute for space-like separations with themselves,

$$[\mathcal{O}(x), \mathcal{O}(y)] = 0 \qquad \text{for} \qquad (x-y)^2 < 0 \tag{1}$$

and how this property is reflected in the form factor consistency equations. It will turn out that from possible matrix elements the form factor consistency equations select out those which correspond to mutually local operators. We argue that the presence of the factor of local commutativity in these equations is absolutely essential.

2 Prerequisites

The fundamental observation, on which all further analysis hinges, is that integrability, which means here the existence, one does not need to know its explicit form, of at least one non-trivial conserved charge, in 1+1 space-time dimensions implies the factorization of the n-particle scattering matrix into a product of two-particle scattering matrices

$$Z^\dagger_{\mu_n}(\theta_n)\ldots Z^\dagger_{\mu_1}(\theta_1)\,|0\rangle_{\text{out}} = \prod_{1\leq i<j\leq n} S_{\mu_i\mu_j}(\theta_{ij}) Z^\dagger_{\mu_1}(\theta_1)\ldots Z^\dagger_{\mu_n}(\theta_n)\,|0\rangle_{\text{in}}\ . \tag{2}$$

As common we parameterize the two-momentum $\vec{p}$ by the rapidity variable θ as $\vec{p} = m(\cosh\theta, \sinh\theta)$ and abbreviate $\theta_{ij} := \theta_i - \theta_j$. The $Z^\dagger_\mu(\theta)$ denote creation operators for stable particles of type μ with rapidity θ, which obey the Faddev-Zamolodchikov algebra [23, 24]

$$Z^\dagger_i(\theta_i)Z^\dagger_j(\theta_j) = S_{ij}(\theta_{ij})Z^\dagger_j(\theta_j)Z^\dagger_i(\theta_i) = \exp[2\pi i\delta_{ij}(\theta_{ij})]Z^\dagger_j(\theta_j)Z^\dagger_i(\theta_i)\,. \tag{3}$$

As indicated in equation (3), the two-particle scattering matrix $S_{ij}(\theta_{ij})$ can be expressed as a phase.

The basic assumption of the bootstrap program is now that every solution to the unitary-analyticity, crossing and fusing bootstrap equations*

$$S_{ij}(\theta) = S_{ji}(-\theta)^{-1} = S_{j\bar{\imath}}(i\pi-\theta), \qquad \prod_{l=i,j,k} S_{dl}(\theta + i\eta_l) = 1\ , \tag{4}$$

($\eta_l \in \mathbb{Q}$ are the fusing angles encoding the mass spectrum and the anti-particle of i is $\bar{\imath}$), which admits a consistent explanation of all poles inside the physical sheet (that is $0 < \text{Im}\,\theta < \pi$), leads to a local quantum field theory. There exists no

*For the purpose of this talk we suppose that there is no backscattering in the theory such that the Yang-Baxter equation constitutes no constraint.

rigorous proof for this assumption, however, it is supported by numerous explicitly constructed examples.

In order to pass from scattering theory to fields, we want to determine the form factors, i.e. the matrix element of a local operator $\mathcal{O}(x)$ located at the origin between a multi-particle in-state and the vacuum

$$F_n^{\mathcal{O}|\mu_1\ldots\mu_n}(\theta_1,\ldots,\theta_n) \equiv \left\langle \mathcal{O}(0)\, Z^\dagger_{\mu_1}(\theta_1),\ldots,Z^\dagger_{\mu_n}(\theta_n)\right\rangle_{\text{in}} . \tag{5}$$

We distinguish here between the mere matrix element $\tilde{F}_n^{\mathcal{O}}$ and the particular ones which also solve the consistency equations as stated in section 3.1. In that case we denote them as $F_n^{\mathcal{O}}$.

3 Determination of form factors

3.1 Solving the consistency equations

Various schemes have been suggested to compute the objects in equation (5). One of the original approaches is modeled in spirit closely on the set up for the determination of exact scattering matrices. It consists of solving a system of consistency equations which have to hold for the n-particle form factors based on some natural physical assumptions, like unitarity, crossing and bootstrap fusing properties [6, 7, 8, 9]

$$F_n^{\mathcal{O}|\ldots\mu_i\mu_j\ldots}(\ldots,\theta_i,\theta_j,\ldots) = F_n^{\mathcal{O}|\ldots\mu_j\mu_i,\ldots}(\ldots,\theta_j,\theta_i,\ldots)S_{\mu_i\mu_j}(\theta_{ij})\,, \tag{6}$$

$$F_n^{\mathcal{O}|\mu_1\ldots\mu_n}(\theta_1+2\pi i,\ldots,\theta_n) = \gamma^{\mathcal{O}}_{\mu_1}\, F_n^{\mathcal{O}|\mu_2\ldots\mu_n\mu_1}(\theta_2,\ldots,\theta_n,\theta_1)\;, \tag{7}$$

$$F_n^{\mathcal{O}|\mu_1\ldots\mu_n}(\theta_1+\lambda,\ldots,\theta_n+\lambda) = e^{s\lambda}F_n^{\mathcal{O}|\mu_1\ldots\mu_n}(\theta_1,\ldots,\theta_n)\;, \tag{8}$$

$$\operatorname*{Res}_{\bar\theta\to\theta_0} F_{n+2}^{\mathcal{O}|\bar\mu\mu\mu_1,\ldots,\mu_n}(\bar\theta+i\pi,\theta_0\quad,\theta_1\ldots\theta_n) = i(1-\gamma^{\mathcal{O}}_{\mu}\prod_{l=1}^{n}S_{\mu\mu_l}(\theta_{0l}))\times F_n^{\mathcal{O}|\mu_1\ldots\mu_n}(\theta_1,\ldots,\theta_n). \tag{9}$$

Here s is the Lorentz spin of the operator $\mathcal{O}$ and λ is an arbitrary real number. We omitted here the so-called bound state residue equation, which relates an $(n+1)$- to an n-particle form factor, since it will be of no importance to the explicit models we consider here. We stress the importance of the constant $\gamma^{\mathcal{O}}_{\mu}$, the factor of so-called local commutativity defined through the equal time exchange relation of the local operator $\mathcal{O}(x)$ and the field $\mathcal{O}_\mu(y)$ associated to the particle creation operators $Z^\dagger_\mu(\theta)$

$$\mathcal{O}_\mu(x)\mathcal{O}(y) = \gamma^{\mathcal{O}}_{\mu}\,\mathcal{O}(y)\,\mathcal{O}_\mu(x) \qquad \text{for} \quad x^1 > y^1\,, \tag{10}$$

with $x^\mu = (x^0,x^1)$. This factor $\gamma^{\mathcal{O}}_{\mu}$, which appeared for the first time in [8] is very often omitted in the analysis or simply taken to be one, but it can be seen that already in the Ising model it is needed to set up the equations consistently [8]. We also emphasize that this factor is not identical to the statistics factor, associated to an exchange of particles, which is sometimes extracted explicitly from the scattering matrix, see e.g. [9]. This factor carries properties of the operator and not just of the $Z^{\dagger}$'s. An immediate consequence of its presence is that a frequently made statement has to be revised, namely, that (6)-(9) constitute operator independent equations, which require as the only input the two-particle scattering matrix. Here we demonstrate that apart from ± 1, which already occur in the literature, this factor can be a non-trivial phase. Thus the form factor consistency equations contain also explicitly non-trivial properties of the operators.

To solve these equations at least for the lowest n-particle form factors is a fairly well established procedure, but it still remains a challenge to find closed analytic solutions for all n-particle form factors. We briefly recall the principle steps of the general solution procedure. For any local operator $\mathcal{O}$ one may anticipate the pole structure of the form factors and extract it explicitly in form of factorizing an ansatz. This might turn out to be a relatively involved matter due to the occurrence of higher order poles in some integrable theories, but nonetheless it is always possible. Thereafter the task of finding solutions may be reduced to the evaluation of the so-called minimal form factors and to solving a (or two if bound states may be formed in the model) recursive equation for a polynomial which results from (9) with the mentioned ansatz. The first task can be carried out relatively easily, especially if the related scattering matrix is given as a particular integral representation [6]. The second task is rather more complicated and the heart of the whole problem in this approach. Having a seed for the recursive equation, that is the lowest non-vanishing form factor one can in general compute from them several form factors which involve more particles. (This seed could be either a known form factor when the model reduces to some solved case or possibly the vacuum expectation value of the operator, which is not known in most cases.) Unfortunately, the equations become relatively involved after several steps. Aiming at the solution for all n-particle form factors, it is therefore highly desirable to unravel a more generic structure which enables one to formulate rigorous proofs. Several examples [10, 11, 16, 17, 18, 19] have shown that often the general solution may be cast into the form of determinants whose entries are elementary symmetric polynomials. Presuming such a structure which, at present, may be obtained by extrapolating from lower particle solutions to higher ones or by some inspired guess, one can rigorously formulate proofs such solutions. These determinant expressions allow directly to write down equivalent integral representations, see e.g. [16]. There exist also different types of universal expressions like for instance the integral representations presented in [7, 9]. However, these type of expressions are sometimes only of a very formal nature since to evaluate them concretely for higher n-particle form factors requires still a considerable amount of computational effort.

3.2 Direct computation of matrix elements

The most direct way to compute the matrix elements in (5) is to find explicit representations for the operators $Z^\dagger_\mu(\theta)$ and $\mathcal{O}(x)$. For instance in the context of lattice models this is a rather familiar situation and one knows how to compute matrix elements of the type (5) directly. The problem is then reduced to a purely computational task (albeit non-trivial), which may, for instance, be solved by well-known techniques of algebraic Bethe ansatz type, see e.g. [25]. In the context of field theory a similar way of attack to the problem has been followed by exploiting a free field representation for the operators $Z^\dagger_\mu(\theta)$ and $\mathcal{O}(x)$, in form of Heisenberg algebras or their q-deformed version. So far a successful computation of the n-particle form factors with this approach is limited to a rather restricted set of models and in particular for the sine-Gordon model, which is a model extensively studied by means of other approaches [7, 9, 15], only the free Fermion point can be treated successfully at present [26, 27, 28]. One of the purposes of this talk is to advocate another approach, namely the evaluation of the matrix elements (5) based on an expansion of the operators in the conventional fermionic Fock space. Recalling the well-known fact that in 1+1 space-time dimensions the notions of spin and statistics are not intrinsic, it is clear that both approaches are equally legitimate.

So, how do we represent the operators $Z^\dagger_\mu(\theta)$ and $\mathcal{O}(x)$? For the former this task is solved. A representation for these operators in the bosonic Fock space was first provided in [29]

$$Z_i^\dagger(\theta) = \exp\left[-i\int_\theta^\infty d\theta'\,\delta_{il}(\theta-\theta')a_l^\dagger(\theta')a_l(\theta')\right]a_i^\dagger(\theta)\,. \tag{11}$$

By replacing a constant phase with the rapidity dependent phase $\delta_{ij}(\theta)$ and turning the expression into a convolution with an additional sum over l, the expression (11) constitutes a generalization of formulae found in the late seventies [30], which interpolate between bosonic and fermionic Fock spaces for arbitrary spin. The latter construction may be viewed as a continuous version of a Jordan-Wigner transformation [31], albeit on the lattice the commutation relations are not purely bosonic or fermionic, since certain operators anti-commute at the same site but commute on different sites. Alternatively, one may also replace the bosonic a's in (11) by operators satisfying the usual fermionic anti-commutation relations

$$\{a_i(\theta), a_j(\theta')\} = 0 \qquad \text{and} \qquad \left\{a_i(\theta), a_j^\dagger(\theta')\right\} = 2\pi\delta_{ij}\delta(\theta-\theta') \tag{12}$$

and note that the exchange relations (3) are still satisfied [32]. In the following we want to work with this fermionic representation of the FZ-algebra. Having obtained a fairly simple realization for the Z-operators, we may now seek to represent the operator content of the theory in the same space. How to do this is not known in general and we have to resort to a study of explicit models at this stage.

4 Complex free Fermions

Let us consider N complex (Dirac) free Fermions described as usual by the Lagrangian density

$$\mathcal{L}_{\rm FF} = \sum\nolimits_{\alpha=1}^{N} \bar{\psi}_\alpha (i\gamma^\mu \partial_\mu - m_\alpha)\psi_\alpha \,. \tag{13}$$

We define a prototype auxiliary field

$$\begin{aligned}\chi_\kappa^\alpha(x) &= \int \frac{d\theta d\theta'}{4\pi^2} \Big[\kappa^\alpha(\theta,\theta') \left(a_\alpha^\dagger(\theta) a_{\bar\alpha}^\dagger(\theta') e^{i(p+p')\cdot x} + a_\alpha(\theta) a_{\bar\alpha}(\theta') e^{-i(p+p')\cdot x} \right) \\ &\quad + \kappa^\alpha(\theta,\theta'-i\pi) \left(a_{\bar\alpha}^\dagger(\theta) a_{\bar\alpha}(\theta') e^{i(p-p')\cdot x} - a_\alpha(\theta) a_\alpha^\dagger(\theta') e^{-i(p-p')\cdot x} \right) \Big] \end{aligned} \tag{14}$$

and intend to compute the matrix element of general operators composed out of these fields

$$\mathcal{O}^{\chi_\kappa^\alpha}(x) = :e^{\chi_\kappa^\alpha(x)}:, \;\; \hat{\mathcal{O}}^{\chi_\kappa^\alpha}(x) = : \int \frac{dp_\alpha^1}{2\pi p_\alpha^0} (a_\alpha(p) e^{-ip_\alpha\cdot x} + a_{\bar\alpha}^\dagger(p) e^{ip_\alpha\cdot x}) e^{\chi_\kappa^\alpha(x)}:. \tag{15}$$

Employing Wick's first theorem, we compute [20]

$$\tilde{F}_{2n}^{\mathcal{O}^{\chi_\kappa^\alpha}|n\times\bar\alpha\alpha}(\theta_1,\ldots,\theta_{2n}) = \int \frac{d\theta_1'\ldots d\theta_{2n}'}{n!} \prod_{i=1}^{n} \kappa^\alpha(\theta_{2i-1}',\theta_{2i}') \det \mathcal{D}^{2n} \;, \tag{16}$$

$$\tilde{F}_{2n+1}^{\hat{\mathcal{O}}^{\chi_\kappa^\alpha}|\alpha,n\times\bar\alpha\alpha}(\theta_1,\ldots,\theta_{2n+1}) = \int \frac{d\theta_1'\ldots d\theta_{2n+1}'}{n!} \prod_{i=1}^{n} \kappa^\alpha(\theta_{2i}',\theta_{2i+1}') \det \mathcal{D}^{2n+1}, \tag{17}$$

where $\mathcal{D}^\ell$ is a rank ℓ matrix whose entries are given by

$$\mathcal{D}_{ij}^\ell = \cos^2[(i-j)\pi/2]\delta(\theta_i'-\theta_j)\,, \qquad 1 \le i,j \le \ell\,. \tag{18}$$

Note that $\mathcal{O}^{\chi_\kappa^\alpha}(x)$ and $\hat{\mathcal{O}}^{\chi_\kappa^\alpha}(x)$ are in general non-local operators in the sense of (1). At the same time $\tilde{F}_n^{\mathcal{O}}$ is just the matrix element as defined on the r.h.s. of (5) and not yet a form factor of a local field, in the sense that it satisfies the consistency equations (6)-(9), which imply locality of $\mathcal{O}$. A rigorous proof of this latter implication to hold in generality is still an open issue. Let us now specify the function κ. The free fermionic theory possesses some very distinct fields, namely the disorder and order fields

$$\mu_\alpha(x) = :e^{\omega_\alpha(x)}: \qquad \text{and} \qquad \sigma_\alpha(x) = :\hat{\psi}_\alpha(x)\mu_\alpha(x):, \quad \alpha = 1,2, \tag{19}$$

respectively. We introduced here the fields

$$\omega_\alpha(x) = \chi_\kappa^\alpha(x), \quad \kappa^1(\theta,\theta') = -\kappa^2(-\theta,-\theta') = \frac{i}{2} \frac{e^{-\frac{1}{2}(\theta-\theta')}}{\cosh\frac{1}{2}(\theta-\theta')} \quad . \tag{20}$$

We compute [20] the integrals in (16) and (17) for this case and obtained a closed expression for the n-particle form factors of the disorder and order operators

$$\begin{aligned} F_{2n}^{\mu_1|n\times\bar{1}1}(\theta_1,\ldots,\theta_{2n}) &= (-1)^n F_{2n}^{\mu_2|n\times\bar{2}2}(-\theta_1,\ldots,-\theta_{2n}) \\ F_{2n}^{\mu_{\bar{1}}|n\times\bar{1}1}(-\theta_1,\ldots,-\theta_{2n}) &= (-1)^n F_{2n}^{\mu_{\bar{2}}|n\times\bar{2}2}(\theta_1,\ldots,\theta_{2n}) \\ &= i^n 2^{n-1}\sigma_n(\bar{x}_1,\bar{x}_3,\ldots,\bar{x}_{2n-1})\mathcal{B}_{n,n}\,, \end{aligned} \tag{21}$$

$$\begin{aligned} F_{2n+1}^{\sigma_1|1(n\times\bar{1}1)}(\theta_1,\ldots,\theta_{2n+1}) &= (-1)^n F_{2n+1}^{\sigma_2|2(n\times\bar{2}2)}(-\theta_1,\ldots,-\theta_{2n+1}) \\ F_{2n+1}^{\sigma_{\bar{1}}|1(n\times\bar{1}1)}(-\theta_1,\ldots,-\theta_{2n+1}) &= (-1)^n F_{2n+1}^{\sigma_{\bar{2}}|2(n\times\bar{2}2)}(\theta_1,\ldots,\theta_{2n+1}) \\ &= i^n 2^{n-1}\sigma_n(\bar{x}_1,\ldots,\bar{x}_{2n-1})\mathcal{B}_{n,n+1}, \end{aligned} \tag{22}$$

with

$$\mathcal{B}_{n,m} = \frac{\prod_{1\le i<j\le n}(\bar{x}_{2i-1}^2-\bar{x}_{2j-1}^2)\prod_{1\le i<j\le m}(x_{2i}^2-x_{2j}^2)}{\prod_{1\le i<j\le n+m}(u_i+u_j)}\,. \tag{23}$$

Associated with the particles and anti-particles we introduced here the quantities $x_i = \exp(\theta_i)$ and $\bar{x}_i = \exp(\bar{\theta}_i)$, respectively. The variable u_i can be either of them. We also employed the elementary symmetric polynomials $\sigma_k(x_1,\ldots,x_n)$. The remaining form factors are zero due to the U(1)-symmetry of the Lagrangian. One may easily verify that the expressions (21) and (22) indeed satisfy the consistency equations (6)-(9) with $\gamma_{\bar{\alpha}}^{\mu_\alpha} = -1$ and $\gamma_{\bar{\alpha}}^{\sigma_\alpha} = 1$ for $\alpha = 1,2$. We also compute [20] the form factors associated to the trace of the energy-momentum tensor

$$F_2^{T^\mu{}_\mu|\bar{\alpha}\alpha}(\theta,\tilde{\theta}) = F_2^{T^\mu{}_\mu|\alpha\bar{\alpha}}(\theta,\tilde{\theta}) = -2\pi i m_\alpha^2 \sinh\frac{\theta-\tilde{\theta}}{2}\,, \tag{24}$$

which plays a distinct role in the ultraviolet limit.

We want to conclude this section with a general comment on the comparison between the generic operators of the type (14), (15) with some general expressions for "local" operators which appear in the literature [41, 32, 38]. We carry out this argument in generality without restriction to a concrete model. Let us restore in equation (5) the space-time dependence, multiply the equation from the left with the bra-vector $\left\langle Z_{\mu_n}^\dagger(\theta_n)\ldots Z_{\mu_1}^\dagger(\theta_1)\right|$ and introduce the necessary amount of sums and integrals over the complete states such that one can identify the identity operator $\mathbb{I}$

$$\begin{aligned} &\sum_{\substack{n=1\ldots\infty\\ \mu_1\cdots\mu_n}}\int_{-\infty}^{\infty}\frac{d\theta_1\ldots d\theta_n}{n!(2\pi)^n}F_n^{\mathcal{O}|\mu_1\ldots\mu_n}(\theta_1\ldots\theta_n)\left\langle Z_{\mu_n}^\dagger(\theta_n)\ldots Z_{\mu_1}^\dagger(\theta_1)\right| e^{-i\sum_j p_j\cdot x} \\ &= \sum_{\substack{n=1\ldots\infty\\ \mu_1\cdots\mu_n}}\int_{-\infty}^{\infty}\frac{d\theta_1\ldots d\theta_n}{n!(2\pi)^n}\left\langle \mathcal{O}(x)\, Z_{\mu_1}^\dagger(\theta_1)\ldots Z_{\mu_n}^\dagger(\theta_n)\right\rangle\left\langle Z_{\mu_n}^\dagger(\theta_n)\ldots Z_{\mu_1}^\dagger(\theta_1)\right| \\ &= \langle\mathcal{O}(x)\ \mathbb{I}\,. \end{aligned}$$

Cancelling the vacuum in the first and last line, and noting that we can replace the product of operators, which is left over also by its normal ordered version, we obtain the expression defined originally in [41]

$$\tilde{\mathcal{O}}(x) = \sum_{\substack{n=1\ldots\infty \\ \mu_1\cdots\mu_n}} \int_{-\infty}^{\infty} \frac{d\theta_1 \ldots d\theta_n}{n!(2\pi)^n} F_n^{\mathcal{O}|\mu_1\cdots\mu_n} : Z^\dagger_{\mu_n}(\theta_n) \ldots Z^\dagger_{\mu_1}(\theta_1) : e^{-i\sum_j p_j \cdot x} . \tag{25}$$

Hence this field is simply an inversion of (5). From its very construction it is clear that $\tilde{\mathcal{O}}(x)$ is a meaningful field in the weak sense, that is acting on an in-state we will recover by construction the form factor related to $\mathcal{O}(x)$. In addition, one may also construct the well-known expression of the two-point correlation function expanded in terms of form factors, as stated in [41]. However, it is also clear that $\tilde{\mathcal{O}}(x) \neq \mathcal{O}(x)$, simply by comparing (25) and the explicit expressions for some local fields occurring in the free fermionic theory, e.g. (14), (15). The reason is that acting on an in-state with the latter expressions the form factors are generated in a non-trivial Wick contraction procedure, whereas when doing the same with (25) the Wick contractions will be trivial. Therefore general statements and conclusions drawn from an analysis based on the expression for $\tilde{\mathcal{O}}(x)$ should be taken with care. It is also needless to say that from a practical point of view the expression (25) is rather empty, since the expressions of the form factors $F_n^{\mathcal{O}|\mu_1\cdots\mu_n}(\theta_1 \ldots \theta_n)$ themselves are usually not known and their determination is in general a quite non-trivial task. In [41, 32, 38] the integration in the formula (25) is a rather artificial contour integration which takes care about analytic continuations of values of $i\pi$. This does not seem to be a fundamental feature, since it remains completely obscure how to incorporate bound states in this manner.

5 The Federbush Model

The Federbush model [33] was proposed forty years ago as a prototype for an exactly solvable quantum field theory which obeys the Wightman axioms [34]. It contains two different massive particles Ψ_1 and Ψ_2. A special feature of this model is that the related vector currents $J^\mu_\alpha = \bar{\Psi}_\alpha \gamma^\mu \Psi_\alpha$, $\alpha \in \{1, 2\}$, whose analogues occur squared in the massive Thirring model, enter the Lagrangian density of the Federbush model in a parity breaking manner

$$\mathcal{L}_{\mathrm{F}} = \sum\nolimits_{\alpha=1,2} \bar{\Psi}_\alpha (i\gamma^\mu \partial_\mu - m_\alpha) \Psi_\alpha - 2\pi\lambda \varepsilon_{\mu\nu} J^\mu_1 J^\nu_2 \tag{26}$$

due to the presence of the Levi-Civita pseudotensor ε. The scattering matrix was found to be [34, 37]

$$S^{\rm FB} = -\begin{pmatrix} 1 & 1 & e^{-2\pi i\lambda} & e^{2\pi i\lambda} \\ 1 & 1 & e^{2\pi i\lambda} & e^{-2\pi i\lambda} \\ e^{2\pi i\lambda} & e^{-2\pi i\lambda} & 1 & 1 \\ e^{-2\pi i\lambda} & e^{2\pi i\lambda} & 1 & 1 \end{pmatrix} . \tag{27}$$

For the rows and columns we adopt here the ordering $\{1, \bar{1}, 2, \bar{2}\}$. In close relation to the free fermionic theory one may also introduce the analogue fields to the disorder and order fields in the Federbush model

$$\Phi_\alpha^\lambda(x) = :\exp[\Omega_\alpha^\lambda(x)]: = \vdots\exp[-2\sqrt{\pi}i\lambda\phi_\alpha(x)]\vdots \tag{28}$$

$$\Sigma_\alpha^\lambda(x) = :\int \frac{dp_\alpha^1}{2\pi p_\alpha^0}(a_\alpha(p)e^{-ip_\alpha\cdot x} + a_{\bar\alpha}^\dagger(p)e^{ip_\alpha\cdot x})\,\Phi_\alpha^\lambda(x):, \tag{29}$$

where the κ-function related to Ω is

$$\hat\kappa^1(\theta,\theta') = -\hat\kappa^2(-\theta,-\theta') = \frac{i\sin(\pi\lambda)e^{-\lambda(\theta-\theta')}}{2\cosh\frac{1}{2}(\theta-\theta')} . \tag{30}$$

The last equality in (28) was found by Lehmann and Stehr [36], who showed the remarkable fact that the operator $\Phi_\alpha^\lambda(x)$ can be viewed in two equivalent ways. On one hand it can be defined through triple ordered free Bosons $\phi_\alpha(x)$, defined as $\vdots e^{\kappa\phi}\vdots = e^{\kappa\phi}/\left\langle e^{\kappa\phi}\right\rangle$ for κ being some constant, and on the other hand by means of a conventional fermionic Wick ordered expression. We compute [20] the following equal time exchange relations for $\alpha,\beta = 1,2$

$$\psi_\alpha(x)\Phi_\beta^\lambda(y) = \Phi_\beta^\lambda(y)\psi_\alpha(x)\, e^{2\pi i(-1)^\beta\lambda\delta_{\alpha\beta}\Theta(x^1-y^1)} , \tag{31}$$

$$-\psi_\alpha(x)\Sigma_\beta^\lambda(y) = \Sigma_\beta^\lambda(y)\psi_\alpha(x)\, e^{2\pi i(-1)^\beta\lambda\delta_{\alpha\beta}\Theta(x^1-y^1)} , \tag{32}$$

$$\Phi_\alpha^\lambda(x)\Phi_\beta^\lambda(y) = \Phi_\beta^\lambda(y)\Phi_\alpha^\lambda(x) \tag{33}$$

$$\Sigma_\alpha^\lambda(x)\Sigma_\beta^\lambda(y) = \Sigma_\beta^\lambda(y)\Sigma_\alpha^\lambda(x)\, e^{2\pi i(-1)^\beta\lambda\delta_{\alpha\beta}} . \tag{34}$$

where $\Theta(x)$ is the Heavyside step function. With the relevant exchange relations at our disposal, we can, according to (8), read off the factors of local commutativity for the operators under consideration

$$\gamma_\alpha^{\Phi_\beta^\lambda} = -\gamma_\alpha^{\Sigma_\beta^\lambda} = e^{2\pi i(-1)^\beta\lambda\delta_{\alpha\beta}} \quad \text{and} \quad \gamma_{\bar\alpha}^{\Phi_\beta^\lambda} = -\gamma_{\bar\alpha}^{\Sigma_\beta^\lambda} = e^{-2\pi i(-1)^\beta\lambda\delta_{\alpha\beta}} . \tag{35}$$

Proceeding again in the same way as in the previous section, we obtain as closed expressions for the n-particle form factors

$$F_{2n}^{\Phi_1^\lambda|n\times\bar{1}1}(\bar x_1, x_2\ldots\bar x_{2n-1}, x_{2n}) = (-1)^n F_{2n}^{\Phi_2^{-\lambda}|n\times\bar{2}2}(\bar x_1, x_2\ldots\bar x_{2n-1}, x_{2n}) =$$
$$F_{2n}^{\Phi_{\bar 1}^{-\lambda}|n\times\bar{1}1}(\bar x_1, x_2\ldots\bar x_{2n-1}, x_{2n}) = (-1)^n F_{2n}^{\Phi_{\bar 2}^{\lambda}|n\times\bar{2}2}(\bar x_1, x_2\ldots\bar x_{2n-1}, x_{2n}) =$$
$$i^n 2^{n-1}\sin^n(\pi\lambda)\sigma_n(\bar x_1\ldots\bar x_{2n-1})^{\lambda+\frac{1}{2}}\sigma_n(x_2\ldots x_{2n})^{\frac{1}{2}-\lambda}\mathcal{B}_{n,n}, \tag{36}$$

$$\tilde{F}_{2n+1}^{\Sigma_{\bar{1}}^{\lambda}|1(n\times\bar{1}1)}(\theta_1,\ldots,\theta_{2n+1}) = (-1)^n \tilde{F}_{2n+1}^{\Sigma_{2}^{-\lambda}|2(n\times\bar{2}2)}(\theta_1,\ldots,\theta_{2n+1}) =$$
$$\tilde{F}_{2n+1}^{\Sigma_{\bar{1}}^{-\lambda}|1(n\times\bar{1}1)}(\theta_1,\ldots,\theta_{2n+1}) = (-1)^n \tilde{F}_{2n+1}^{\Sigma_{\bar{2}}^{\lambda}|2(n\times\bar{2}2)}(\theta_1,\ldots,\theta_{2n+1}) = \frac{\sin^n(\pi\lambda)}{2}$$
$$\frac{(2i)^n\sigma_n(\bar{x}_2\ldots\bar{x}_{2n})^{\lambda+\frac{1}{2}}}{\sigma_n(x_1\ldots x_{2n+1})^{\lambda-\frac{1}{2}}}\prod_{1\leq i<j\leq n}(\bar{x}_{2i}-\bar{x}_{2j})\sum_k \frac{i^{k+1}\prod_{j<l;j,l\neq k}(x_j-x_l)}{(x_k)^{\frac{1}{2}-\lambda}\prod_{j\neq k}\prod_l (x_j+\bar{x}_l)} . \tag{37}$$

We may now convince ourselves, that the expressions for $F_{2n}^{\Phi_\alpha^\lambda|n\times\bar{\alpha}\alpha}$ indeed satisfy the consistency equations (6)-(9). However, the expressions of $\tilde{F}_{2n+1}^{\Sigma_\alpha^\lambda|\alpha(n\times\bar{\alpha}\alpha)}$ only satisfy the consistency equations (6)-(9) for $\lambda = 1/2$. This reflects the very important fact that $\Sigma_\alpha^\lambda(x)$ is only a mutually local operator for this value of λ, see equation (34), unlike $\Phi_\alpha^\lambda(x)$ which is mutually local for all value of λ. Thus, the equations (6)-(9) select out solutions corresponding to operators which are mutually local.

The form factors related to the trace of the energy-momentum tensor turn out to be the same as the ones for the complex free Fermion.

6 Momentum space cluster properties

Cluster properties in space, i.e. the observation that far separated operators do not interact, are quite familiar in quantum field theories [43] for a long time. In 1+1 dimensions a similar property has also been noted in momentum space. It states that whenever some of the rapidities, say κ, are shifted to plus or minus infinity, the n-particle form factor related to a local operator $\mathcal{O}$ factorizes into a κ and an $(n-\kappa)$-particle form factor which are possibly related to different types of operators $\mathcal{O}'$ and $\mathcal{O}''$. This type of behaviour has been analyzed explicitly for several specific models [10, 12, 13, 17]. The possibility of non-self-clustering, i.e. $\mathcal{O} \neq \mathcal{O}' \neq \mathcal{O}''$, was conjectured for the first time in [12] and the first explicit examples which confirm this were found in [17]. For self-clustering and a purely bosonic case this behaviour can be explained perturbatively by means of Weinberg's power counting theorem, see e.g. [35] for an explicit reasoning on this issue. Non-self-clustering still lacks an explanation at present. The cluster property serves not only a consistency check for possible solutions of (6)-(9), but also as a construction principle for new solutions, e.g. [17].

An interesting operator related property which the form factors satisfy is the momentum space cluster decomposition

$$\lim_{\Delta\to\infty} F_{k+l}^{\mathcal{O}}(\theta_1\ldots\theta_k,\theta_{k+1}+\Delta\ldots\theta_{k+l}+\Delta) = F_k^{\mathcal{O}'}(\theta_1\ldots\theta_k)F_l^{\mathcal{O}''}(\theta_{k+1}\ldots\theta_{k+l}) \ , \tag{38}$$

Writing instead of the matrix elements only the operators, we obtained [20] formally the following decomposition

$$\Phi_\alpha^\lambda \longrightarrow \Phi_\alpha^\lambda \times \Phi_\alpha^\lambda \qquad \sigma_\alpha \longrightarrow \begin{cases} \mu_\alpha \times \sigma_\alpha \\ \mu_{\bar\alpha} \times \sigma_\alpha \end{cases} \qquad \mu_\alpha \longrightarrow \begin{cases} \mu_\alpha \times \mu_\alpha \\ \sigma_\alpha \times \sigma_{\bar\alpha} \end{cases} \tag{39}$$

together with the equations for $\alpha \leftrightarrows \bar\alpha$. This means the stated operator content closes consistently under the action of the cluster decomposition operators. We also observe that non-self-clustering, i.e. $\mathcal{O} \neq \mathcal{O}' \neq \mathcal{O}''$, is possible. Unlike the self-clustering, which can be explained for the bosonic case with the help of Weinberg's power counting argument, this property is not yet understood from general principles.

7 Lie algebraically coupled Federbush models

The Federbush model as investigated in the previous section only contains two types of particles. In this section we propose a new Lagrangian, which admits a much larger particle content. The theories are not yet as complex as the homogeneous sine-Gordon (HSG) models, but they can also be obtained from them in a certain limit such that they will always constitute a benchmark for these class of theories.

Let us consider $\ell \times \tilde\ell$-real (Majorana) free Fermions $\psi_{a,j}(x)$, now labeled by two quantum numbers $1 \leq a \leq \ell$, $1 \leq j \leq \tilde\ell$ and described by the Dirac Lagrangian density $\mathcal{L}_{\mathrm{FF}}$. We perturb this system with a bilinear term in the vector currents $J^\mu_{a,j} = \bar\Psi_{a,j}\gamma^\mu\Psi_{a,j}$

$$\mathcal{L}_{\mathrm{CF}} = \sum_{a=1}^{\ell}\sum_{j=1}^{\tilde\ell} \bar\Psi_{a,j}(i\gamma^\mu\partial_\mu - m_{a,j})\Psi_{a,j} - \frac{1}{2}\pi\varepsilon_{\mu\nu}\sum_{a,b=1}^{\ell}\sum_{j,k=1}^{\tilde\ell} J^\mu_{a,j}J^\nu_{b,k}\Lambda^{jk}_{ab} \,, \tag{40}$$

and denote the new fields in $\mathcal{L}_{\mathrm{CF}}$ by $\Psi_{a,j}$. Furthermore, we introduced $\ell^2 \times \tilde\ell^2$ dimensional coupling constant dependent matrix Λ^{jk}_{ab}, whose further properties we leave unspecified at this stage. We computed [20] the related S-matrix to

$$S^{jk}_{ab} = -e^{i\pi\Lambda^{jk}_{ab}} \,. \tag{41}$$

where due to the crossing and unitarity relations we have the constraints

$$\Lambda^{jk}_{ab} = -\Lambda^{kj}_{ba} + 2\mathbb{Z} \qquad \text{and} \qquad \Lambda^{jk}_{ab} = \Lambda^{\bar k j}_{\bar b a} + 2\mathbb{Z} \tag{42}$$

on the constants Λ. Taking $\Lambda^{jk}_{ab} = 2\lambda_{ab}\varepsilon_{jk}\tilde I_{jk}K^{-1}_{a\bar b}$, with K, I being the Cartan and incidence matrix, respectively, provides the limit of the HSG-models.

8 The ultraviolet limit

The ultraviolet Virasoro central charge of the theory itself can be computed from the knowledge of the form factors of the trace of the energy-momentum tensor [42] by means of the expansion

$$c_{\rm uv} = \sum_{n=1}^{\infty} \sum_{\mu_1 \cdots \mu_n} \frac{9}{n!(2\pi)^n} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \frac{d\theta_1 \ldots d\theta_n \left| F_n^{T^{\mu}{}_{\mu}|\mu_1 \ldots \mu_n}(\theta_1, \ldots, \theta_n) \right|^2}{\left(\sum_{i=1}^{n} m_{\mu_i} \cosh \theta_i \right)^4} . \tag{43}$$

In a similar way one may compute the scaling dimension of the operator $\mathcal{O}$ from the knowledge of its n-particle form factors [21]

$$\begin{aligned} \Delta_{\rm uv}^{\mathcal{O}} &= -\frac{1}{2\langle \mathcal{O} \rangle} \sum_{n=1}^{\infty} \sum_{\mu_1 \cdots \mu_n} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \frac{d\theta_1 \ldots d\theta_n}{n!(2\pi)^n \left(\sum_{i=1}^{n} m_{\mu_i} \cosh \theta_i \right)^2} \\ & \quad \times F_n^{T^{\mu}{}_{\mu}|\mu_1 \ldots \mu_n}(\theta_1, \ldots, \theta_n) \left(F_n^{\mathcal{O}|\mu_1 \ldots \mu_n}(\theta_1, \ldots, \theta_n) \right)^* . \end{aligned} \tag{44}$$

In general the expressions (43) and (44) yield the difference between the corresponding infrared and ultraviolet values, but we assumed here already that the theory is purely massive such that the infrared contribution vanishes. Evaluating these formulae, we obtain

$$c_{\rm uv} = 2 \qquad \text{and} \qquad \Delta_{\rm uv}^{\mu_\alpha} = \Delta_{\rm uv}^{\mu_{\bar{\alpha}}} = \frac{1}{16} . \tag{45}$$

for the complex free Fermion and

$$c_{\rm uv} = 2 \qquad \text{and} \qquad \Delta_{\rm uv}^{\Phi_\alpha^\lambda} = \Delta_{\rm uv}^{\Phi_{\bar{\alpha}}^\lambda} = \frac{\lambda^2}{4} . \tag{46}$$

for the Federbush model Note, that $\Delta_{\rm uv}^{\Phi_\alpha^{1/2}} = \Delta_{\rm uv}^{\Phi_{\bar{\alpha}}^{1/2}} = 1/16$, which is the limit to the complex free Fermion. Yet more support for the relation between the $SU(3)_3$-HSG model and the Federbush model comes from the analysis of $\lambda = 2/3$, for which the $SU(3)_3$-HSG S-matrix is related to the one of the Federbush model. In that case we obtain from (46) the values $\Delta_{\rm uv}^{\Phi_\alpha^{2/3}} = \Delta_{\rm uv}^{\Phi_{\bar{\alpha}}^{2/3}} = 1/9$, which is a conformal dimension occurring in the $SU(3)_3$-HSG model. Thus precisely at the value of the coupling constant of the Federbush model at which the $SU(3)_3$-HSG S-matrix reduces to the $S^{\rm FB}$, the operator content of the two models overlaps.

9 Conclusions

We summarize our main results:

We computed explicitly closed formulae for the n-particle form factors of the complex free Fermion and the Federbush model related to various operators.

We carried out this computations in two alternative ways: On the one hand, we represent explicitly the field content (14) as well as the particle creation operators (11) in terms of fermionic Fock operators (12) and computed thereafter directly the corresponding matrix elements. On the other hand we verified that these expressions satisfy the form factor consistency equations only when the operators under consideration are mutually local, i.e. satisfying (1). It is crucial that the consistency equations contain the factor of local commutativity $\gamma_\mu^{\mathcal{O}}$ as defined in (10). Our analysis strongly suggest that *the form factor consistency equations select out operators, which are mutually local in the sense of (1).*

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Our analysis strongly suggest that *the form factor consistency equations select out operators, which are mutually local in the sense of (1).* To establish this in complete generality still remains an open issue. We have expressed our criticism on the analysis carried out in [41] in section 4. Further arguments which support this statement for specific situations can be found in [7, 44]. These type of analysis do not include the essential factor γ and the latter one does not allow higher order poles in the scattering matrix, which still excludes the majority of know diagonal theories.

Our solutions turned out to decompose consistently under the momentum space cluster property. This computations constitute next to the ones in [16, 17] the first concrete examples of non-self-clustering, i.e. $\mathcal{O} \to \mathcal{O}' \times \mathcal{O}''$ in the sense of (39).

Further support for the identification of the solutions of (4)-(7) with a specific operator was given by an analysis of the ultraviolet limit.

We demonstrated how the scattering matrix of the Federbush model can be obtained as a limit of the $SU(3)_3$-HSG scattering matrix. This "correspondence" also holds for the central charge, which equals 2 in both cases, and the scaling dimension of the disorder operator at a certain value of the coupling constant.

We proposed a Lie algebraic generalization of the Federbush models, by suggesting a new type of Lagrangian. We evaluate from first principles the related scattering matrices, which can also be obtained in a certain limit from the HSG-models.

We expect that the construction of form factors by means of free fermionic Fock fields can be extended to other models by characterizing further the function κ.

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