A STUDY OF THE FUNDAMENTALS OF ACTUARIAL ECONOMIC MODELS

by

Paul Philip Huber

A thesis submitted for the degree of Doctor of Philosophy

City University, London
Department of Actuarial Science and Statistics
December 1996

Contents

List of Tables

List of Figures

Acknowledgements

Chapter 8 draws on "A review of Wilkie's stochastic investment model" (Huber 1995a,b). Section 4.3 is drawn from "A note on the jump-equilibrium model" (Huber 1996). These papers have been revised and submitted to the *British Actuarial Journal.*

I am indebted to my supervisors Professor Steven Haberman and Dr Richard Verrall for their support during these past three years and for their helpful comments on earlier drafts of this thesis. Mr Philip Booth was also closely involved with this thesis and I am grateful to him for many useful discussions.

Thanks are due to the many others who have contributed to this research in some way, including: my colleagues Mike Boskov, Peter Hatzopoulos, and Alen Ong; Professor David Wilkie who offered valuable comments and supplied the data used in Chapter 8; Andrew Smith who made insightful comments on Chapter 4; and Dr Russell Gerrard who provided the ideas behind the proof in Appendix 4B.1.

I am very grateful to Meghan for her tireless encouragement, inspiration, and patience. My extended family, especially my Mother, also provided much support and encouragement, for which I am grateful.

I wish to acknowledge the Department of Actuarial Science and Statistics at City University for funding this research.

Declaration

I grant powers of discretion to the University Librarian to allow this thesis to be copied in whole or in part without further reference to me. This permission covers only single copies made for study purposes, subject to normal conditions of acknowledgement.

Abstract

This thesis examines the methods that have been used by actuaries to describe and model the economic variables required for actuarial calculations. Traditionally actuaries have only used average future values to describe these variables and they have tended to use relatively informal methods for determining these averages. These informal methods are potentially subject to numerous biases. The likelihood of these biases occurring could be reduced by using the more formal methods of financial economics and stochastic modelling. Stochastic models also provide additional information that is essential for some applications.

The main UK stochastic asset models that are considered include: Wilkie's (1995b) model, Dyson and Exley's (1995) expectations model, and Smith's (1996) jump-equilibrium model. Wilkie's model was developed primarily from data considerations, whilst the other two models were developed from theoretical considerations. Wilkie's model is shown to be inconsistent with the rational expectations hypothesis, the efficient market hypothesis, and aspects of portfolio theory. Dyson and Exley's model is shown to be inconsistent with portfolio theory. The jump-equilibrium model is consistent with most financial theories, but it is shown to produce returns with moments that are inconsistent with historical data.

The importance of these limitations is then examined from a methodological perspective. This review emphasises the importance and difficulty of empirical testing. It also suggests that economic predictive success is always likely to be limited. As a result, it is argued that a model's pragmatic qualities are relatively more important than they would otherwise be, that a theoretical framework is invaluable for motivating economic models and for directing research activities, and that actuaries should aim to develop models with shorter time horizons. The empirical adequacy of financial economic theories is then examined and many persistent problems are reported. Despite these problems it is suggested that financial economics provides a useful theoretical framework. Lastly, the empirical adequacy of Wilkie's model is considered using the criteria of Hendry's (1995) general-to-specific approach. This review identifies some apparent weaknesses. In particular, the out-of-sample residuals from Wilkie's (1986a) model do not seem to be independent.

Table of Symbols

General

SSAP	statement of standard accounting practice
CAPM	capital asset pricing model
CCAPM	consumption based capital asset pricing model
ICAPM	intertemporal capital asset pricing model
APT	arbitrage pricing theory
ARCH	autoregressive conditional heteroskedastic
VAR	vector autoregressive
BVAR	Bayesian vector autoregressive

Asset classes

s	stocks, shares, or equity securities
p	property
bn	n-year fixed-interest securities, $n \in [0, \infty)$
b	general bonds or long-term fixed-interest securities
m	money market or short-term fixed-interest securities
rn	n-year index-linked fixed-interest securities, $n \in [0, \infty)$
r	long-term index-linked fixed-interest securities
q	retail or consumer prices
w	wages or earnings
x	sterling
xj	exchange rate between pound sterling and currency j
g	gross domestic product
f	risk-free asset
Ω	market portfolio of assets

Asset variables

$R_j(t)$	total return index for asset j at time t
$r_j(t) = \Delta \log_e R_j(t)$	force of return for asset j in year t

$\rho_j(t) = R_j(t)/R_j(t-1) - 1$	rate of return for asset *j* in year *t*
$v_j(t) = 1/(1+\rho_j(t))$	discount factor for asset *j* in year *t*
$P_j(t)$	price index for asset *j* at time *t*
$p_j(t) = \Delta \log_e P_j(t)$	force of price growth for asset *j* in year *t*
$D_j(t)$	cash flow for asset *j* in year *t*
$d_j(t) = \Delta \log_e D_j(t)$	force of cash flow growth for asset *j* in year *t*
$Y_j(t) = D_j(t)/P_j(t)$	yield (running yield) on asset *j* at time *t*
$y_j(t) = \log_e Y_j(t)$	logarithm of the yield on asset *j* at time *t*
$X_j(t)$	notional currency capital value for asset *j* at time *t*
$RX_j(t)$	notional currency total return index for asset *j* at time *t*
$i_j(t,\tau)$	par redemption yield on a τ-year asset *j* security at time *t*
$\delta_j(t,\tau)$	spot force of interest on a τ-year asset *j* security at time *t*
$\mathrm{B}_j(t,\tau)$	price of a τ-year asset *j* zero-coupon bond at time *t*

Notes:

1 The convention that variables with time subscripts *t* are always known at time *t* has been used throughout. Therefore, $D_j(t)$ is used to denote dividends received in year *t* rather than $D_j(t-1)$, which was used by Campbell and Shiller (1987, 1988).

2 Dividends are assumed to be paid at the end of each interval.

3 For all variables, the subscript *j* is only used in formulae that refer to a specific asset and the subscript *t* is only used for variables that are assumed to vary over time.

4 Retail prices and currency exchange rates are assumed to be assets with no cash flows. $R_q(t)$ represents the retail price index at time *t*. $R_{xj}(t)$ represents the pound sterling versus currency *j* exchange rate at time *t*.

Company variables (Section 2.3.3)

b	debt to equity ratio
g	company growth rate
$Q(t)$	equity capital employed at time *t*

Utility theory variables

$u_t(\cdot)$	von Neumann-Morgenstern utility function at time *t*

$C(t)$	real aggregate consumption at time t
$W(t)$	real aggregate wealth at time t
$\kappa_j(t)$	risk premium over asset j at time t
$\beta_{j,k}$	covariance between security j and k divided by the variance of security k
ϕ_k	kth factor of a K factor model

Other, statistical variables

AR(p)	autoregressive model of order p
MA(q)	moving average model of order q
ARIMA(p, d, q)	autoregressive integrated moving average model of order p, d, and q
VAR(p)	vector autoregressive model of order p
μ_x	mean of a random variable x
σ_x	variance of a random variable x
γ_x	skewness of a random variable x
$\varepsilon(t)$	general error term at time t
$z_j(t)$	standard normal random variable at time t
$\eta_j(t)$	shifted gamma random variable at time t
$N(\mu, \sigma^2)$	normal distribution with mean μ and standard deviation σ
$\Gamma(\alpha, \lambda)$	gamma distribution with parameters α and λ
$G(\cdot)$	compound Poisson process
L	lag operator: $L \cdot x(t) = x(t-1)$
Δ	first difference operator: $\Delta x(t) = x(t) - x(t-1)$
$P(x1 \mid x2)$	probability of $x1$ given $x2$
$D_x[\cdot]$	joint density function of X
$E_t[\cdot]$	expectations operator conditional on all information at time t
$H(t)$	history, information available at time t
$\mathrm{cov}(x1, x2)$	covariance between the random variables $x1$ and $x2$
$h'(x)$	derivative of a function h at x
$\lvert\cdot\rvert$	absolute value
$\exp[\cdot]$	exponential function

Chapter 1

INTRODUCTION

1.1 Problem Statement

The core domain of actuarial responsibility is related to the provision and management of contracts of life assurance, pensions and annuities, health insurance, and general insurance. These contracts are characterised by the advance payment of a premium, or a series of premiums, in return for payments that are contingent on pre-specified but unpredictable future events. For example, the basic contingent event in life assurance contracts is the death of the life assured. As the premiums for these contracts are paid in advance, and often well in advance, of the benefits received, they require the policyholder to place substantial trust in the organisation offering the contracts. The gravity of this requirement is usually intensified by the financial vulnerability of the policy beneficiary when a contingent event occurs. Consequently, there is considerable public interest in the financial soundness of these organisations. This interest is represented in the numerous Acts of Parliament of Great Britain, which have been designed to protect the policyholders (see Institute of Actuaries 1996: C4). These Acts have entrusted much of this responsibility to the actuarial profession and the actuarial profession binds its members to provide: "the best possible service and advice" (Institute of Actuaries 1996: A1.1).

Actuaries usually prepare this service and advice with the aid of mathematical models. These models generally represent simplified descriptions of the relevant future environment and they are used to estimate, amongst other things, the funds currently required to provide for the contingent payments of insurance contracts. Important elements of the actuarial environment are economic variables such as interest rates and inflation rates. This thesis critically reviews the methods that have been used by UK actuaries to describe and model these variables in relation to the obligations and objectives of the actuarial profession. The other relevant elements of the future environment, such as mortality and morbidity rates, are not considered.

Actuaries have traditionally used average future values, or deterministic assumptions, to describe the relevant economic variables. These assumptions tend to be based on relatively informal justifications and they do not provide any information on the likelihood of particular events occurring. As a result of these limitations, actuaries have developed stochastic models. These models provide more complete descriptions of the behaviour of the relevant variables. Stochastic models are also generally supported by formal statistical and theoretical justifications. They are seen by many actuaries as becoming an important new actuarial technique. A recent study on the future of the actuarial profession recommended that it should: "Increase emphasis on stochastic and other methods over deterministic approaches" (Nowell *et al.* 1995: 5). This study also identified stochastic modelling as a particular skill that is likely to be required by all actuaries. However, Thomson *et al.* (1995) reported that stochastic techniques were only seldom used by life offices: less than 10% of life offices surveyed were actively using stochastic methods and half were not using any stochastic methods. Hence, Nowell *et al.* (1995: 15) reported that a concern was: "The low levels of involvement of actuaries in probabilistic techniques and realistic evaluation."

An important reason why actuaries have been reluctant to embrace stochastic methods is the lack of a clearly adequate stochastic model. The economy is an extremely complex system that has proved difficult to model. Economic predictions and econometric models have tended to be unreliable, especially over the long time horizons of some insurance contracts. Until an adequate stochastic model is discovered, relatively little confidence can be placed in the results of stochastic investigations. This thesis examines how actuarial stochastic models could be justified and whether the available models are sufficiently adequate. Furthermore, it considers how economists have succeeded in studying the economy and the limitations of economics. This examination offers some insight into how future actuarial economic models could be developed and justified.

1.2 Overview

Chapter 2 provides an overview of the methods used by UK actuaries to determine the economic assumptions. The professional duties of actuaries in relation to economic bases are initially considered. The traditional methods of setting these assumptions are then reviewed. These methods tend to be relatively informal and attempt to incorporate a variety of information. The alternative, more formal, approaches of selecting the economic assumptions suggested by financial economics and stochastic methods are then briefly described and discussed. These approaches appear to offer methods for improving traditional actuarial techniques, but their worth is not proven. The remainder of this thesis attempts to examine whether financial economic and stochastic methods can significantly enhance actuarial investigations.

The available stochastic models have tended to be developed primarily from either the probabilistic structure of the historical data or from theoretical considerations. Chapter 3 describes the main data based comprehensive stochastic asset models, especially Wilkie's (1986a, 1995b) model. Wilkie's model is examined in detail as it has been the most influential UK stochastic asset model. The theoretical and statistical properties of Wilkie's model are discussed.

Chapter 4 describes the main theoretical stochastic asset models, including Dyson and Exley's (1995) expectations model and Smith's (1996) jump-equilibrium model. The theoretical and statistical properties of these models are compared with those of Wilkie's model.

All the models surveyed in Chapters 3 and 4 are inadequate in some respect and none has acquired universal support. Chapter 5 attempts to lay the groundwork that is required for assessing these models. It initially reviews some of the relevant issues from the philosophy of science. This literature examines past scientific achievements and it considers, amongst other things, how scientific investigations should be conducted and appraised. The more specific philosophical issues from economic methodology are then considered. This literature emphasises the limitations of knowledge, the importance and difficulty of empirical testing, and the role of a theoretical framework. It also stresses the particular difficulties associated with economic investigations. Hausman's (1992) interpretation of economic methodology is then used to justify the theoretical approach to developing actuarial stochastic models.

Chapter 6 discusses some of the inadequacies of financial economic theories. Actuaries' concerns about the relevance of these theories seem to be justified because of a number of persistent problems. However, financial economics appears to provide a promising framework for understanding economic phenomena and traditional actuarial techniques do not provide a demonstrably better approach. The difficulties associated with economic modelling are used to support the approach used by, amongst others, Haberman (1994) of initially considering simple tractable stochastic economic models.

Chapter 7 considers econometric methodology and emphasises the particular problems associated with developing econometric models. It examines the theory-directed approach advocated by Darnell and Evans (1990), the vector autoregressive approach recommended by Sims (1980), and the general-to-specific approach developed by, amongst others, Hendry (1995). The general-to-specific approach is recommended as the most promising econometric methodology and it is used to broadly evaluate the main actuarial models that are described in Chapters 3 and 4.

Chapter 8 provides a detailed empirical review of Wilkie's model using the criteria of the general-to-specific approach. Although this model appears to be broadly satisfactory, some potential problems are revealed. In particular the model does not appear to have had constant parameters historically.

Chapter 9 concludes by summarising the main arguments and by suggesting promising areas for future research.

The SAS System (SAS Institute Inc. 1988) and PcFiml (Doornik and Hendry 1994) were used to perform all the calculations.

Chapter 2

SETTING THE ECONOMIC ASSUMPTIONS

2.1 Introduction

Economic assumptions, such as the future interest and inflation rates, constitute a significant part of most actuarial bases. For example, if a retirement fund actuary consistently uses an interest rate assumption that is only 1% less than the actual rate, then this is likely to result in surpluses of the order of 60% of payroll (Thornton and Wilson 1992: 259). In addition, when using the discounted cash flow method to value assets, a change of 1% in the real dividend growth assumption changes the assessed values of the assets by between 20% and 25% (Dyson and Exley 1995: 485). However, these variables are exceptionally difficult to forecast because they are influenced by a wide range of factors in complicated ways. Furthermore, there is no single generally accepted 'best' method for setting these long term economic assumptions. Consequently: "There can be no uniquely correct assumptions in most cases" (Institute of Actuaries 1996: B17.7). This chapter considers how actuaries have responded to the problems associated with economic forecasting. The methods used by actuaries to motivate their economic assumptions are discussed and compared with the methods used in financial economics.

As a result of the uncertainty associated with long term economic forecasts, the nature of actuarial economic assumptions depends on the context in which they are required. The profitability of contracts with fixed conditions, such as general insurance or non-profit life assurance contracts, depends solely on whether the actuarial basis used to calculate the premiums is 'better' or 'worse' than the actual outcome. As a result, a cautious approach is usually adopted in setting these bases; competitive pressures permitting. Relatively cautious or prudent assumptions are also usually required to determine the statutory solvency reserves required by insurance companies or retirement funds.

More realistic, or best estimate, assumptions can generally be used for contracts with relatively flexible conditions, such as defined benefit retirement funds or with-profit life insurance contracts. This is because the actuary is able to adjust the benefits or future contributions of these contracts if the assumptions prove to be inaccurate. The justification for using best estimates or realistic bases in retirement fund valuations is provided by Thornton and Wilson (1992). Thornton and Wilson also attempt to quantify the terms 'best estimate', 'prudent', and 'cautious'. They suggest that these terms should refer to bases that are expected to result in more favourable long term outcomes 50%, 60%, and 70% of the time, respectively. The actuarial assumptions stated in this chapter are usually best estimates; although more prudent or cautious assumptions may be used in practice. Mehta (1992) and Smith (1996) discuss techniques for adjusting these best estimates to allow for risk. These adjustments are beyond the scope of this chapter, which only considers methods for establishing best estimates.

Section 2.2 outlines the professional duties of UK actuaries relating to the setting of the economic assumptions. Section 2.3 examines the traditional actuarial techniques for determining these assumptions. The methods used by actuaries to allow for economic uncertainty are considered and the use of actuarial judgement is discussed. The traditional actuarial approach is then compared with the methods used in financial economics. Section 2.4 briefly describes the principal asset pricing models developed in financial economics and considers how actuaries have responded to these models. Section 2.5 discusses stochastic methods of setting the economic assumptions. Section 2.6 summarises the main arguments.

2.2 Professional Duties of UK Actuaries

In the UK, the responsibility for setting the economic assumptions has in many circumstances rested entirely with the actuary who is required by the profession and by law to exercise appropriate judgement. This individual accountability is reflected in many of the Guidance Notes issued by the Councils of the Institute and Faculty of Actuaries. For example, when conducting actuarial investigations into long term insurance business:

> The Appointed Actuary must decide the rates of interest to be used in the valuation of the liabilities. These are affected by the Appointed Actuary's estimates of the likely future proceeds of the existing assets and the rate at which future proceeds of the existing assets and of the rate at which future investment will be possible. ... Judgement on these factors rests with the Appointed Actuary who must decide the basis of the valuation of the liabilities. (Institute of Actuaries 1996: B1.9)

> The Appointed Actuary must be satisfied that, in each of the assumptions, the margins in any published valuation of the liabilities, including any margins required by statute, are adequate having regard to the Appointed Actuary's own assessment of the risks inherent in the nature and conduct of the company's business. (Institute of Actuaries 1996: B1.11)

Similar principles apply in the context of retirement benefit schemes. However, for SSAP 24 purposes the actuary is instructed to consult the client:

> In signing the certificate, the actuary certifies that in his opinion the resources of the scheme are likely in the normal course of events to be sufficient (Institute of Actuaries 1996: B3.2)

> The selection of actuarial assumptions to be used in assessing pension cost for SSAP 24 purposes is a matter of judgement for the actuary in consultation with the client. (Institute of Actuaries 1996: B17.6)

Individual responsibility for the assumptions also extends to actuaries instructed as expert witness's to the courts or other tribunals:

> The actuary is normally responsible for ... the actuarial assumptions ... that are used in an actuarial analysis given in evidence. (Institute of Actuaries 1996: B24.5)

Thus, the actuarial profession has generally given actuaries the freedom to choose their own assumptions. This decision appears to have been motivated by the belief that any set basis could unduly prejudice particular results and would be arbitrary because there is no single infallible method for establishing these assumptions. Nevertheless, the profession has attempted to bound this freedom by a number of loose directives and recommendations. For example, Appointed Actuaries of insurance companies are required to "use prudent bases determined according to actuarial principles" (Institute of Actuaries 1996: B8.3). For retirement fund SSAP 24 purposes: "The financial and demographic assumptions should meet the requirement of providing a best estimate" (Institute of Actuaries 1996: B17.7).

Moreover, because all assumptions are fallible, actuaries are required to conduct sensitivity analyses to ensure that the financial security of the funds under investigation are sufficiently robust. Consequently, actuaries of retirement benefit schemes are required to express an opinion about whether the scheme's resources are likely to be adequate 'in the normal course of events', which is qualified as:

> a prudent view of the future without taking into account every conceivable unfavourable development. The actuary should regard this as excluding the possibilities of events—including those external to the scheme—which he cannot reasonably be expected to have allowed for in a conservative approach to the matter. (Institute of Actuaries 1996: B9.6)

Appointed Actuaries are also advised to "consider the resilience of the valuation to changes in circumstances, with special reference to more extreme changes to which the office may be vulnerable, and provide appropriate margins in the valuation basis" (Institute of Actuaries 1996: B8.4). In particular, Appointed Actuaries are required to make prudent provision against the consequences of possible fluctuations in the value of the assets. This requirement is characteristically imprecise because in determining the range of possible fluctuations: "the actuary must use professional judgement as an experienced financial practitioner" (Institute of Actuaries 1996: B8.8).

The freedom to select actuarial bases is more strongly limited by the profession in certain cases, such as when actuaries of retirement benefit schemes calculate transfer values, but even in this situation there is still scope for individual actuarial judgement:

> Such actuarial value should be assessed having regard to market rates of interest. One of the ways in which a market value assessment may be made is on the basis of market redemption yields on British Government Stocks of appropriate duration and type at the time of transfer with allowance for investment of future interest receipts at such rates as the actuary considers reasonable. In valuing benefits which are subject to revaluation in accordance with the general index of retail prices, yields on index-linked gilts will be an appropriate criterion. (Institute of Actuaries 1996: B11.2)

This freedom has also been limited by various statutory requirements, including the minimum criteria for setting valuation bases in the Insurance Company Regulations 1994, the Personal Investment Authority rules for calculating projections of future benefits of life assurance contracts (see Institute of Actuaries 1996: B22), and the basis for calculating the minimum funding requirement for retirement funds (see Institute of Actuaries 1996: B27).

Therefore, within limits, actuaries are generally given the discretion to choose their own particular forecasts of the economic variables required in actuarial calculations. They are required to test the sensitivity of their results to these assumptions and to be prudent. Moreover, setting the economic basis is generally regarded as a matter of mature and experienced judgement, which cannot be easily or quickly acquired. Hence, a large number of relatively ill-defined methods for setting economic assumptions exist and there is much scope for disagreement. Nevertheless, the following section attempts broadly to characterise and examine the methods that are traditionally used by actuaries to set the economic assumptions. This description is inevitably a stereotype, but it attempts to relate the essence of the so-called traditional actuarial approach.

2.3 Traditional Actuarial Methods

2.3.1 Description

Traditionally actuaries have only used expected values to depict the relevant economic variables. Furthermore, they have normally taken advantage of their freedom to base these forecasts on subjective factors referred to as actuarial judgement. Thus, economic variables have conventionally been assumed to be constant over time and have been essentially subjectively determined. These traditional methods of setting the economic assumptions have been described by, amongst others, Lee (1986) and discussed more recently by Thornton and Wilson (1992), Mehta (1992), Jones (1993), Wilkie (1995a), and Dyson and Exley (1995). This section attempts to broadly characterise these methods.

The minimum economic information that is typically required in actuarial calculations includes the expected long term future annual average rate of price inflation and of return on the investments of the fund. To determine the expected return on the fund and to value the assets of the fund, actuaries additionally require the expected long term future annual average: equity dividend yield, equity dividend growth rate, and interest rate on long-term fixed-interest securities.

These basic variables are first transformed to reflect quantities that are believed to be of the greatest importance and the most stable over time. Real rates of return are commonly used and price inflation is generally left untransformed. Transformations to real returns are justified by the belief that the preservation of spending power is the fundamental motivation for long term savings and that the required real returns are reasonably stable over time. For defined benefit retirement schemes, real rates are more important than nominal rates because the liabilities are generally linked to inflation. Before retirement, benefits are linked to salary inflation and after retirement, benefits are often linked to price inflation, possibly with an upper limit.

Mehta (1992) transformed the asset returns into risk premiums over Treasury Bill yields and considered the real return on Treasury Bills and price inflation separately. Risk premiums were motivated by the belief, obtained from financial economics (see Section 2.4), that: "investors compare equity risk and return with the risks and returns of alternative investment opportunities" (Mehta 1992: 393). Jones (1993) considered the equity risk premium relative to long-term fixed-interest securities. Wilkie (1995a: 272) analysed the yield gap between the equity dividend yield and the real redemption yield on index-linked securities because: "It is reasonable to argue that index-linked government stocks have taken the place that used to be held by conventional gilt-edged stocks in the non-inflationary era."

The average historical values of these transformed variables are used as initial rough assumptions. Lee (1986) examined the most recent 15 year period for this purpose, whilst Thornton and Wilson (1992) recommend using 20 year moving averages over at least the past 30 years. These averages are then visually compared with plots of data to obtain a feel for how significant the variations in the past have been and to estimate confidence intervals for the variables. If any large structural changes are perceived, then averages over more recent intervals are taken or the exceptional intervals are excluded. For example, Jones (1993) suggested that the relationship between gilts and equities changed fundamentally during the late 1950s and early 1960s.

An illustration of the above actuarial approach is provided by Thornton and Wilson (1992: 237), who describe their method for setting the long term assumption for the future yield on equity investments. Note that they appear to believe that structural changes occurred in the mid 1960s, mid 1970s, and early 1980s and, as a result, assume that the average yield will be slightly lower in the future than it has been in the past. Their argument for a future yield of 4.75% is as follows:

> From the graph [see Figure 2.3.1, up to mid 1991] it will be seen that the yield on the All-Share Index rarely strays beyond the range 4-6%, and when it does stray it does not do so for long. The average yield over the 29 years is just under 5%, and there is little evidence of a trend in either direction away from this figure. Rather, there is evidence of a random step function, with discrete changes in the mid 1960s, mid 1970s and early 1980s. It is perhaps too early to say whether there has been a step upwards to 5%, or whether the market will revert to the lower yield found in the latter half of the 1980s. In consequence, it is possible to suggest that for the future average yields might lie anywhere in the region of 4¼-5%, and our preference is to assume 4¾%.

These historical averages, or 'objective', estimates are not blindly used because it is assumed that economic, political, technological, and social changes imply that the past is not necessarily an adequate description of the future. For example, Wilkie (1995a) attempted to demonstrate how historical averages of the equity risk premium can provide a misleading guide to the future. Hence, the averages are subjectively adjusted after comparing them with other relevant information, including consensus forecasts,

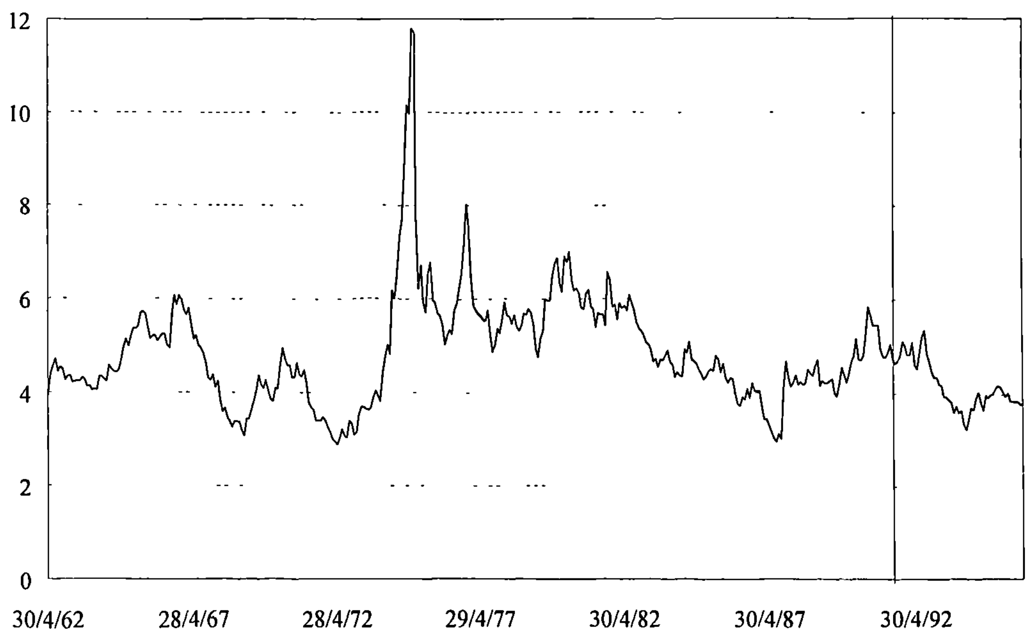

Figure 2.3.1 The dividend yield on the FTSE-Actuaries All-Share Index, 1962-95

implied market forecasts, historical levels of dividend cover, and GDP growth rates. Implied market forecasts tend to be influential, but they are not generally considered to be decisive (see Section 2.3.4). Moreover, the market does not unambiguously provide all the necessary information, such as future salary inflation or long term future equity dividend growth rates and yields. Nevertheless, available market forecasts include: the term structure of interest rates, and the yield gap between conventional fixed-interest and index-linked securities, which: "can be taken as representing 'the market's' view of future inflation, or the implied inflation forecast" (Wilkie 1995a: 272).

In certain circumstances, the prevailing market conditions are taken into account by adjusting the long term average assumptions over a limited future interval. This approach is adopted when the assets of the fund are valued using market values. To ensure consistency with this asset valuation method, the rates of return used to value the liabilities are then obtained from current market information. "Although more complicated, it introduces the discipline of requiring the actuary to reconcile his valuation assumptions with the returns implied by the market prices at each valuation" (Thornton and Wilson 1992: 273). As noted in Section 2.2, this approach is recommended when calculating transfer values for retirement funds.

Forecasts of GDP growth are frequently used to rationalise dividend growth rate assumptions. For instance, Thornton and Wilson (1992) assumed that dividends, allowing for rights issues and new issues, will increase in the long term in line with GDP growth. It was recognised that this hypothesis is only partially valid because a large proportion of company earnings is derived from overseas operations. Still, this hypothesis is roughly supported by historical data, which shows that company profits have been a fairly stable proportion of GDP (Dyson and Exley 1995: 486). Thornton and Wilson predicted a long term real GDP growth rate of 2.5% per annum and assumed that rights and new issues will account for between 1% and 2% of this growth; resulting in a prediction for real dividend growth of between 0.5% and 1.5%. Mehta (1992: 428) provides a similar analysis by examining the equity returns attained by a typical company that grows in real terms by 1%, or which achieves a return on capital employed of 8%. However, in Mehta's analysis a 1% real growth rate is geared up to provide a 4% real dividend growth rate (see Section 2.3.3).

Further examples of the use of general economic information are provided by Jones (1993: 275) who argued that: "Given the high levels of debt in the economy and the low levels of dividend cover, it seems unlikely that [real dividend growth] is going to be much greater than [zero] for some while." Consequently, he suggested that the equity risk premium will be lower in future than it has been over the previous 70 years. In estimating the future real rate of dividend growth, Wilkie (1995a: 288) rationalised his recommended range of between –1% and 2% and a point estimate of 1% as follows:

> One could argue for the continuation of the 1% real growth that has occurred since 1923; or that we are just over the peak of a long upswing in real dividends [see Figure 2.3.2, up to 1994] and that the next swing will be downwards, in which case a real dividend growth of –1% would be possible; or one could take an intermediate position and go for 0%. One might, instead, take into account a possible economic recovery, and take a more optimistic view of inflation, which could allow companies to keep a high rate of real dividend growth, say 2%; but these might both be only medium-term features.

Factors that are even more subjective are also taken into account; for example, when determining a central range for real salary inflation Lee (1986: 262) observed that 2.5% per annum "would probably be thought optimistic in the current climate of opinion." Jones (1993: 254) suggested that the then current low rate of inflation has "an air of permanence about it." Another illustration of the use of subjective factors is provided by Ross in the discussion of Thornton and Wilson (1992: 296). After discussing a number

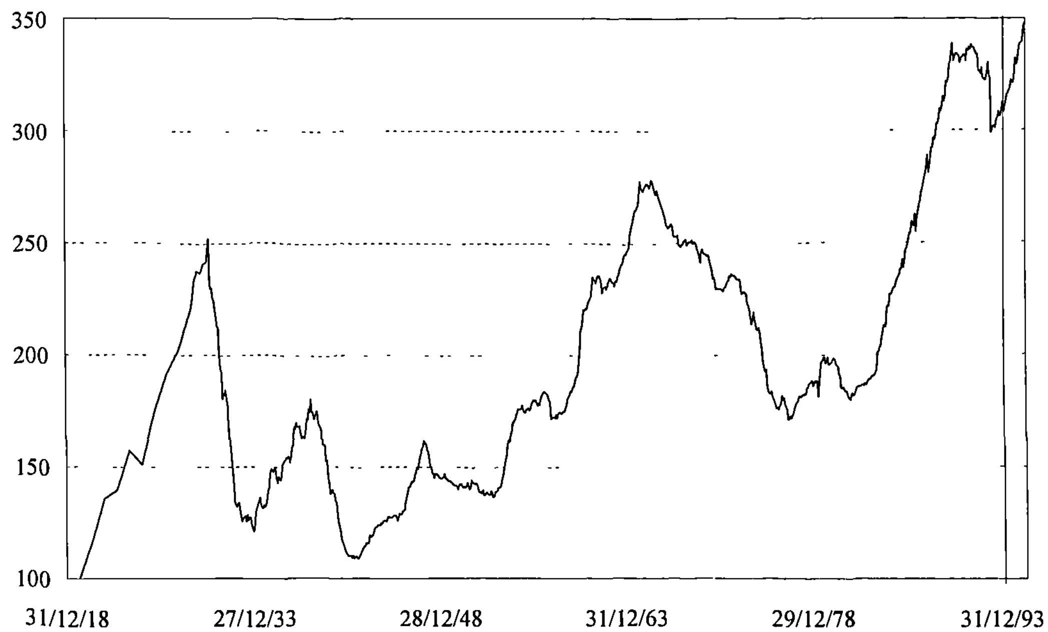

Figure 2.3.2 Real equity dividends, 1919-95

of economic issues such as the ageing population and European integration, he stated that: "I sense that we may be embarking on an era of economic change on a scale which has not been seen for many years." He did not indicate how much this judgement would influence his assumed basis.

Whatever method is adopted, the actuary makes a central forecast of the transformed variables taking all the relevant information into account and usually quotes a confidence interval around these values. These central forecasts are then transformed back into the basic variables required for the actuarial calculations. Allowances are made for dealing expenses and the specific asset mix of the fund.

The difficulties involved in forecasting future economic variables are generally accepted. Lee (1986: 258) admitted that the assumptions "represent no more than the actuary's judgement of what constitutes a reasonable basis." Thornton and Wilson (1992: 287) stated that: "there is nothing immutable about these assumptions." An illustration of the indeterminacy of the economic assumptions is provided by comparing the recommendations of recent authors. Although Thornton and Wilson (1992) and Wilkie (1995a) suggested similar ranges for long term average real dividend growth of 0.5% to 2% and –1% to 2% respectively, Lee (1986) recommended between –3% and 0%. Moreover, Wilkie's point estimate for real dividend growth of 1% was 3% higher than Lee's estimate of –2%. These assumptions contrast with the 'cautious projection' made by Pratten (1993: 190), an economist, of a real dividend growth rate of 2.5% over 10 years. These differences influence the real equity return assumptions, which have ranged from 9% recommended by Mehta (1992), to between 5.5% and 6.5% recommended by Thornton and Wilson, to less than 6% required in determining the minimum funding requirement for retirement funds, to 3% recommended by Lee. For price inflation, Lee suggested a range of between 6% and 8% and a point estimate of 6%, whereas Mehta suggested 5% and the basis for the minimum funding requirement specifies 4%. Given the sensitivity of the actuarial calculations to these assumptions, the effect of these differences is vast.

2.3.2 Actuarial judgement

The previous section illustrates that actuarial judgement and subjective factors are extensively used in setting the economic assumptions. But ideally these factors should not be used because actuaries have "an obligation in the public interest to provide the best possible service and advice" (Institute of Actuaries 1996: A1.1). Ideally the best possible service and advice is free of personal bias and thus completely objective. Consequently, the actuarial profession directs its members to avoid conflicts of interest and asserts:

> For a member in a particular situation to describe the advice he offers as independent he must be free, and must be seen to be free, of any influence which might affect his advice or limit his scope. (Institute of Actuaries 1996: A1.3)

However, subjective judgement cannot be avoided because knowledge is fallible: completely objective methods do not exist (see Chapter 5). Judgement is required to determine what to include in a theory or model and whether the model is adequate. It is particularly important when an adequate model does not exist. This includes situations in which the model does not accommodate relevant peripheral information and situations in which a structural change is assumed to have invalidated the model. Hence, the pertinent issue is not whether judgement is appropriate, but to what extent judgement should be used and how it should be used. Resolving this issue requires an investigation into the characteristics of human judgement.

Human judgement has been found to depend on a limited number of elementary heuristic principles that are usually effective, but are potentially subject to a number of logical and statistical errors (see Kahneman *et al.* 1982). These heuristics include representativeness, availability, and anchoring (Tversky and Kahneman 1974). Representativeness describes the tendency for people to assess the probability that a particular event belongs to a class of events by solely considering the similarity of the event to the class of events; availability describes the inclination to evaluate the probability of an event by the relative ease with which examples of it can be thought of; and, anchoring describes the tendency to estimate values by making adjustments from an initial value or anchor. The fallacies that have been found to be associated with these heuristics include: that material information, such as events that are difficult to imagine, the sample size, the base-rate frequencies and predictability of events, tend to be ignored; that spurious regressions are held (see Section 7.4); that recent events are given undue weight; and that subjects do not adjust their anchors sufficiently. However, many of these errors and inconsistencies are to be expected because the principles of statistics and logic usually need to be taught; they are not naturally understood.

Furthermore, Evans (1987) asserted that judgement is prone to confirmation and belief bias: where confirmation bias describes the inclination for people to test their hypotheses using tests that maximise the chance of a successful result and then to attach undue significance to this result (see Section 5.2.4); and belief bias refers to the tendency for people to be excessively influenced by their prior beliefs. These biases may result in overconfident judgements. It has also been found that a regression model of an expert's judgement policy tends to be better than the expert's actual judgements possibly because the model eliminates inconsistencies in application (see Dawes 1979). Moreover, Makridakis *et al.* (1993) found that in a real-time forecasting competition of various financial and economic time series, simple quantitative methods, such as exponential smoothing, generally outperformed more sophisticated methods using expert judgement. Thus, Brehmer (1987: 205) concluded that: "Whatever advantages human judgement may have, they are not to be found in any ability to use huge amounts of data, or in any particularly complex form of processing." This research into human judgement emphasises that it is fallible and should only be used with considerable caution.

However, this evidence against the use of judgement is not conclusive (see Beach *et al.* 1987). Bunn and Wright (1991) documented a number of studies that illustrate the value of the use of judgement in forecasting. In particular, Turner (1990) reported that macroeconomic forecasts are frequently and extensively adjusted using judgement and that these adjustments tend to improve the results. Bunn and Wright (1991) investigated the situations in which judgement was most effective. They suggested that it is valuable when used in conjunction with mathematical models, when experts in probability and in the specific subject area under investigation are involved, when a formal coherent structure is used, and when evidential support is provided. They warned against using judgement informally because of its vulnerability when challenged. This undermines the credibility of process and exposes the forecaster to greater personal risk. They suggested that forecasters should at least record an audit trail of the information and reasoning used in making the subjective adjustment. Hence, Bunn and Wright (1991: 512) concluded that: "when experts are used in their real world context and the judgmental process is made explicit through a form of decomposition or audit trail, empirical studies and surveys of its practice give a general endorsement of its value."

Actuaries appear to be aware of these problems associated with the use of professional judgement. Thornton and Wilson (1992: 234) warned that: "it is remarkably easy to delude oneself into believing in the reasonableness of the conclusions reached." They suggested that: "this subjectivity has led to bases being further out of line with what would be justified by a more statistical approach than is desirable" (Thornton and Wilson 1992: 235). It was also accepted that there "is a serious danger that too much weight is put on the events of the immediate past" (Thornton and Wilson 1992: 234). Consequently, they argued that: "professional judgement does need to be applied carefully to steer the pension funds through what are likely to be turbulent conditions in the short and medium term, but professional judgement must be based on a proper scientific approach" (Thornton and Wilson 1992: 311). Furthermore, actuaries are experts in long term economic modelling, they have had training in probability, and they usually provide evidential support for their assumptions. This suggests that actuaries are broadly justified in using professional judgement, especially if it is believed that the economic system is highly irregular.

2.3.3 The strength of actuarial justifications

The extent to which a 'proper scientific approach' has been followed by actuaries can be examined by analysing and comparing the justifications, or audit trails, for the equity return assumption suggested by Thornton and Wilson (1992) and Mehta (1992). Thornton and Wilson examined historical equity real returns and found that they were approximately 6%. Projecting this result into the future and assuming a future inflation rate of 5% results in a forecast of 11% for future equity returns. Whereas, Mehta considered the equity risk premium over Treasury Bills and reported that the average historical risk premium was approximately 6%. Using this result and assuming a real return on Treasury Bills of 3% and an inflation rate of 5% results in a forecast of 14%

for future equity returns. The difference between these forecasts is accounted for by the assumption of a future real return on Treasury Bills of 3%, when historically the average real return was 0%. This assumption was justified by the lifting of UK exchange controls in 1979: "the absence of exchange controls has led to an expectation that real returns in the U.K. will match those in other countries, ... The average real return on short-term instruments for the leading industrialised nations in the period 1979 to 1990 was 3%" (Mehta 1992: 426). Therefore, this difference reflects conflicting opinions on which relationships are temporally stable or robust and whether structural changes have occurred. Mehta's view on this particular issue is largely subjective because relatively little evidence is furnished to support it.

Nevertheless, Mehta (1992) did provide additional evidence to substantiate his equity return assumption by comparing it to likely future economic growth. He argued that, if the economy grows at a real rate of 1% per annum then dividends will grow at a real rate of 4% per annum. This hypothesis rested on the following analysis: assume that a company has a debt to equity ratio of b and grows at a nominal rate of g per annum, then the total capital employed at time $t+1$ is equal to $(1+g)\cdot(1+b)\cdot Q(t)$, where $Q(t)$ represents the equity capital employed at time t. Assuming that the amount of debt remains constant, the equity capital employed at time $t+1$: $Q(t+1) = [(1+g)\cdot(1+b) - b]\cdot Q(t)$, which represents a growth rate of $(1+b)\cdot g$. Hence, assuming that average future inflation is 5%, $b = 0.5$, $g = 6\%$ (real economic growth rate of 1%), and that dividends are a constant proportion of equity capital (assumed to be 5%); the real dividend growth rate will be 4% per annum and equity returns will be 14% per annum.

This analysis demonstrates that real equity returns could be higher than real economic growth rates because of gearing. This weakens Thornton and Wilson's (1992) argument that real dividends will only grow in line with GDP growth after allowing for rights and new issues. Moreover, it illustrates the potential problems associated with arguments based on simplified situations. These explanations may omit relevant information and, consequently, they do not provide strong support by themselves. Hypotheses need to be extensively tested before they can be considered to be corroborated (see Chapter 5). Mehta's (1992) argument is also subject to these potential problems, even though it is more detailed than Thornton and Wilson's (1992); it merely provides a promising hypothesis that deserves further consideration. Furthermore, these justifications rely on the debatable assumption that reliable forecasts of GDP growth rates and debt to equity ratios are available. If GDP growth rate forecasts are just as unreliable as dividend growth rate forecasts, then little is achieved by conditioning dividend growth rate forecasts on GDP growth rate forecasts.

Wilkie (1995a) attempted to explain the above difference between an ex post equity risk premium of roughly 6% and the orthodox actuarial assumption of a premium between 1% and 2%. He claimed that this difference is largely because inflation has been higher than was expected. Over the interval 1923 to 1993 inflation averaged 4.5% and Wilkie argued that this was 'wholly unexpected'. As it is inappropriate to assume that investors will continue to underestimate future inflation, Wilkie claimed that the ex post risk premium provides a distorted measure of the future and argued in favour of the orthodox actuarial view. Although this hypothesis is intuitively appealing, it is difficult to test because it is not possible to precisely determine what investors were expecting. Consequently, Wilkie's justification of this hypothesis is not demonstrative.

Wilkie justified his position by firstly showing that inflation averaged approximately zero over the interval 1600 to 1914. This motivated his assumption that investors in 1923 expected that future inflation would be zero. This assumption was also supported by quotations from two actuarial investment text books published in 1949 and 1965. Furthermore, he assumed, without any supporting evidence, that investors in 1923 did not anticipate that dividends would grow. These two assumptions imply that the yield gap between equity securities and long-term fixed-interest securities, which was approximately 2% in 1923, provides a measure of investors' expectations of the future equity risk premium. However, over the intervals 1923 to 1958 and 1923 to 1993 the equity risk premium averaged roughly 5% and 5.5%, respectively. Wilkie accounted for most of the difference between his assumption of investors' expectations and the actual outcomes of 3% and 3.5% by observing that equity dividends had unexpectedly grown by 2.5% and 5.5% per annum, respectively. Moreover, he assumed that equity dividends grew in line with inflation, which had averaged 2% and 4.5% over these intervals. Thus, the differences could be approximately explained by the higher than expected inflation.

This justification is not particularly cogent because it only considers two arbitrary time intervals and does not fully explain the differences. Moreover, the interpretation of the evidence provided by the, somewhat selective, historical texts seems to be biased in favour of the chosen hypothesis and could equally support a range of alternative hypotheses. The assumption that dividends grew in line with inflation may also be inaccurate because companies may have been unable to fully recover their inflationary cost increases by increasing their product prices. As a result, in the discussion of Wilkie (1995a: 323), Dimson, an economist, observed that: "reconfiguring the historical evidence, in order to come up with a figure which we regard as more acceptable, is a poor way to forecast the future." To this, Wilkie (1995a: 330) responded that: "there is much evidence that equity investors often do not expect more than quite a modest extra return to compensate them for the risks." But the details of this supporting evidence were not provided, possibly suggesting that it was subjective information. Furthermore, as Mehta, in the discussion of Wilkie (1995a: 315), pointed out, this opinion appears to contradict one of the major motivations for the paper: that companies employ 'too high' a rate of return when assessing capital projects.

Nevertheless, it is conceivable that unexpected inflation was an important factor that contributed to the seemingly higher than expected equity risk premium over the above intervals. However, this hypothesis does not explain the ex post risk premiums over the other intervals that were examined because Wilkie does not provide any evidence about investors' expectations at the start of these intervals. Over both the intervals 1958 to 1993 and 1978 to 1993 the average equity risk premiums (relative to Consols) were roughly 6% and the average inflation rates were roughly 7%. The corresponding real dividend growth rates were 1.5% and 3.5%, respectively. It would be difficult to explain these results, especially the latter result, in terms of unexpected inflation. Consequently, Wilkie's analysis only provides limited support for the orthodox actuarial assumption. According to Dimson: "we will not be able to play with the numbers to make the high *ex-post* equity risk premium go away" (Wilkie 1995a: 323).

2.3.4 The use of market information

The above examples illustrate that actuarial justifications are not particularly rigorous. This is partially explained by the difficulty associated with economic forecasting, but it also indicates a potential weakness in actuarial methods. As a result, Dyson and Exley (1995) are highly critical of subjective methods for setting the economic assumptions, especially in the context of retirement fund valuations that are used for statutory or public reporting purposes, termed compliance valuations. They asserted that actuaries are not justified in using forecasts that are different from those implied by market information. Adopting non-market forecasts implies that actuaries are taking an investment view that certain assets are cheap relative to others and therefore switching assets will have a material impact on the valuation result (Arthur and Randall 1990). In addition, Dyson and Exley (1995: 489) argued that the long term real dividend growth rate "is not a stable entity and our degree of confidence in a single long-term average is very low indeed for out-of-sample forecasts." This view was motivated by an analysis of historical real dividend growth rates (see Figure 2.3.2) and by arguing that the traditional economic arguments that suggest that dividends will grow in line with inflation are only roughly accurate. As a result, they claimed that it is inappropriate to determine an asset valuation using a 'subjective' real dividend growth rate assumption. They suggested that an 'objective' and 'prudent' method is to use the market values to value the assets and to use the spot real and nominal discount rates implied by index-linked and conventional fixed-interest securities to value the liabilities. These recommendations originate from financial economics (see Section 2.4). Hence, Dyson and Exley (1995: 500) held the view that: "the market gives rational prospective expectations at any point in time" and recommended that these forecasts should be used whenever possible.

Dyson and Exley (1995) also suggested that the economic assumptions that cannot be derived from market information should be prescribed for compliance valuations. McLeish and Stewart (1993: 79) held a similar view by suggesting that the only 'objective' solvency test for retirement funds is whether the market value of their assets is sufficient, after winding-up expenses, to purchase appropriate matching contracts from life insurance companies and, where appropriate, to meet the payment of the premiums to the National Insurance Fund. In replying to the discussion, McLeish then considered alternative solvency tests and asked the questions: "Do we need a statutory basis? ... do we need some kind of standard basis set by the profession? Certainly, I suggest that it should not be left to the individual actuarial opinion of the Scheme

Actuary" (McLeish and Stewart 1993: 123). These questions have now been answered with the introduction of a prescribed basis for calculating the minimum funding requirements of retirement funds. These suggestions also accord with the view expressed in the study commissioned by the Institute and Faculty of Actuaries on the future of the Actuarial Profession (Nowell *et al.* 1995: 35) that: "There may well be a move to more prescriptive SSAP24 bases". Furthermore, this study predicted a continuing trend towards increased consumer awareness and simple products, which is unlikely to be satisfied by subjective bases that depend on personal judgements.

Implied market forecasts and prescribed bases reduce inconsistencies in actuarial valuations, but they are not necessarily any less subjective or more accurate than individually determined estimates. It remains to be seen how appropriate the current basis for calculating the minimum funding requirement is and how frequently it will need to be altered. Prescriptive bases and 'objective' methods also restrict the freedom that has traditionally been granted to the actuary (see Section 2.2). As a result, in the discussion of Dyson and Exley (1995), a number of actuaries defended the traditional 'subjective' methods of setting economic assumptions. In particular, Wise illustrated when he considered actuarial judgement to be appropriate:

> When I consider that I have nothing to add to the combined knowledge of the market, then my assumptions will be strongly influenced by market valuation. When I consider that the current market is particularly influenced by investors with short time horizons, then my assumptions will be more influenced by long-term economic factors. I am justified in taking such views from time to time, for reasons of long practical experience and investment analysis. (Dyson and Exley 1995: 545)

This issue of whether the market provides 'rational prospective expectations' or whether it can be dominated by investors with short term objectives has been examined in financial economics and is discussed in Section 2.4 and Chapter 6.

2.3.5 Economic uncertainty and actuarial methods

As suggested in the previous sections, establishing long term forecasts of economic variables is a hazardous task because the economy has been notoriously unstable in the past, especially over the last 50 years. A number of authors have even questioned whether it is a legitimate activity: Redington (1982: 89) stated that: "to estimate the long-term rate of interest is not science: it is science-fiction"; Keynes (1936: 149) claimed that: "Our knowledge of the factors which govern the yield of an investment some years hence is usually very slight and often negligible." Furthermore, historical economic forecasts have often been inaccurate (see Chapters 5 and 7).

Hence, Redington (1983) argued that many actuarial methods, in particular the gross-premium valuation method, are incongruous because they rely on realistic economic forecasts, which are impracticable. He suggested that actuaries should rather aim to develop flexible methods and contracts that explicitly recognise the impossibility of prediction. The aim of these methods should be to enhance understanding rather than to provide more realistic predictions. Consequently, he suggested that long term non-profit contracts were unsuitable and argued in favour of the artificial net-premium valuation method and the flexible funding methods used by retirement funds. Furthermore, he recommended that actuaries should consider the flow of funds as these elements are known with relative certainty. However, even this primitive concept is problematical because dividend growth is highly uncertain and it tends to ignore future asset sales. As actuarial methods are unavoidably associated with the future, actuaries are unlikely to be able to refrain from making some unreliable economic predictions. Still, actuaries appear to have largely recognised and responded to the concerns articulated by Redington. In particular, actuaries have attempted to manage future economic uncertainty by examining, and attempting to reduce, the sensitivity of their methods and contracts to the economic assumptions.

A common approach for managing economic uncertainty in life insurance is "to build up reserves to cushion the impact of events" (Redington 1952: 304). These reserves may be either explicitly established as a distinct reserve or may be implicitly established by using a 'strong' valuation basis or by not giving immediate credit to policyholders for capital appreciation or by including penalties for early withdrawal from the fund. Part of these reserves is usually returned to policyholders in the form of terminal bonuses. This approach considerably reduces the policyholder's flexibility and it may result in a reduction in the rate of return, which is used to set up and maintain the reserve. The reserves required may also be prohibitively large, such as in the case of maturity guarantees (see Ford *et al.* 1980). Hence, actuaries have been cautious in providing guarantees or options related to economic variables.

Moreover, the disadvantages of setting up reserves have largely contributed towards the introduction and growth of unit-linked contracts, deposit administration retirement funds, managed retirement funds, and defined contribution retirement funds. These contracts all transfer investment risk away from insurance companies to policyholders or to sponsoring employers in the case of defined benefit retirement funds. In return, the policyholders or sponsoring employers gain greater flexibility and full credit for the actual returns earned on their premiums. In addition, these contracts are more transparent because their rates of return are clearly defined in terms of the value of the underlying assets rather than determined by the actuary on some complicated, possibly subjective, basis.

Other methods for managing economic uncertainty include matching (see Wise 1984a,b), immunisation (Redington 1952), and mismatching (Wise 1987a,b). The first two of these techniques involve devising an asset portfolio with similar economic characteristics to the liabilities so that the contracts are immune to general changes in the economy. Likewise, Mehta (1992: 385) proposed that: "an investor can examine the individual cash flows which make up the aggregate net revenue of the office and can consider the returns available on traded securities in order to obtain an assessment of comparative value." These methods are difficult and usually impossible to implement precisely, but they provide a useful theoretical benchmark. Mismatching involves quantifying the effects of adopting a portfolio that is different to the 'matching portfolio', but still 'efficient' in some sense. The importance of the principles underlying these techniques is illustrated by their inclusion in the Guidance Notes; for retirement benefit schemes the actuary is required to comment on "the compatibility of the basis of valuing the assets with that of valuing the liabilities" (Institute of Actuaries 1996: B9.3); for long term insurance business:

> The Appointed Actuary must also pay regard to the relationship between the term of the assets and that of the corresponding liabilities. The importance of this will vary widely from one situation to another, but experience suggests that this can be an area of particular danger. (Institute of Actuaries 1996: B1.10)

Hence, although there is a great deal of uncertainty about future economic events, actuaries have managed this uncertainty by reducing their exposure to it or by building up reserves. As a result, the economic assumptions are not as critical as they would otherwise have been and actuaries have tended to avoid referring to setting actuarial bases as forecasting. Nevertheless most actuarial methods, such as gross-premium valuations and the projected unit method of retirement funding, attempt to model the future realistically.

2.3.6 Summary

The traditional actuarial approach uses information from a variety of sources and often borrows individual theories from other disciplines, including financial economics. Individual intuition, or actuarial judgement, is usually relied on to select the relevant information that is used to motivate a particular set of economics assumptions. Actuaries have not developed a unified body of formal theories. This informal approach is motivated by the view that: "the synthesis of wide judgement is a better guide than the analysis of narrow mathematics" (Redington 1952: 310).

The traditional approach is potentially susceptible to the numerous fallacies associated with human judgement and it tends to be vulnerable when challenged. Consequently, actuaries should keep audit trails of the motivations for their particular decisions and should use more rigorous methods whenever possible. Furthermore, actuaries should attempt to develop contracts that minimise the negative consequences of making incorrect forecasts because any set of economic assumptions is likely to be unreliable. The following sections consider techniques that may improve the traditional method of setting realistic economic assumptions.

2.4 Financial Economic Methods

2.4.1 The nature of financial economics

Financial economics and actuarial science are both concerned with the valuation of financial contracts under uncertainty. However, whereas actuaries have tended to rely on informal methods, financial economists have developed a set of formal mathematical theories. These theories are often used by actuaries in setting the economic assumptions (see Section 2.3.4), but few actuaries recommend their exclusive use. UK actuaries have generally been cautious about unreservedly accepting financial economic theories and certain of these theories have even been completely rejected (Clarkson and Plymen 1988). This section broadly describes the methodological approach followed by financial economists and later sections consider whether financial economics should be more extensively used in setting the economic assumptions.

Financial economists have generally investigated financial systems using axiomatic deductive models. The axioms, or fundamental assumptions, on which these models are grounded are similar to those used in neo-classical economics and include that the economy is composed of distinct agents whose only interactions are the voluntary exchange of goods and services. These agents are further assumed to be knowledgeable, self-serving, utility maximisers and to have complete and consistent preferences. From these and other assumptions, an extensive range of theories has been developed. Hence, financial economists have attempted to explain financial phenomena in terms of rational agents acting in their own interests. To make this task tractable, numerous simplifying assumptions are initially made, which may be weakened or modified over time in response to empirical evidence. This has involved a considerable investment in theoretical and empirical research.

An important assumption required by financial economic models concerns agent's expectations. Most financial decisions relate to the future and are thus dependent on expectations, but expectations are inherently unobservable. An influential assumption has been that agents have rational expectations (see Vercelli 1991). The strong version of this assumption implies that agent's expectations are consistent with the underlying mechanism that produces the investment data, or the data generating process (see Section 7.4). This assumption implies that agents do not make systematic ex post forecasting errors. This version of the rational expectation hypothesis is frequently weakened to imply that agent's expectations are equivalent to the predictions of relevant economic theory. This assumption merely implies that agents do not knowingly make ex ante forecasting errors. It aims to establish consistency within economic models so that they do not permit arbitrage opportunities. Arbitrage opportunities imply that knowledgeable agents are irrationally foregoing profitable trading opportunities.

An example of the type of models produced by financial economists is the fundamental valuation equation, which is the basic discrete time intertemporal asset pricing model (see Constantinides 1989). This model assumes that the price of a security at time t can be expressed as the expected value of the product of the payoff at time $t+1$ with the marginal rate of substitution of consumption at time t for consumption at time $t+1$ of the numeraire good and is given by:

$$P(t) = \mathrm{E}_t\left[\frac{u'_{t+1}(C(t+1))}{u'_t(C(t))}\cdot(P(t+1)+D(t+1))\right] \tag{2.4.1}$$

where E_t represents the expectations operator conditional on all information at time t, denoted H(t), $u_t(\cdot)$ represents the representative investor's von Neumann-Morgenstern utility function at time t, $P(t)$ represents the ex dividend price of the security at time t, $D(t)$ represents the dividends or cash flows paid in year t, and $C(t)$ represents real aggregate consumption at time t. Most of the quantities in this chapter, including $P(\cdot)$, $D(\cdot)$, $C(\cdot)$, are denominated in a non-depreciating numeraire.

The above model can be derived and generalised using a number of methods and assumptions. For example, the approach adopted by Constantinides (1989) was to consider an exchange economy under uncertainty with a single perishable consumption good. Further, it is assumed that investors receive no exogenous income, that there is a complete market, and that investors have rational expectations and state-independent time-additive utility functions. Under these conditions aggregate consumption is equal to the aggregate dividends paid, an efficient allocation of goods is achieved, and security prices can be determined as if a representative investor existed. The expected utility of the representative investor's total consumption over the interval t to T is given by:

$$u_t(C(t)) + \mathrm{E}_t\left[\sum_{k=1}^{T-t} u_{t+k}(C(t+k))\right] \tag{2.4.2}$$

In equilibrium, the price of any security at time t, $P(t)$, must satisfy the conditions that the representative investor holds all the shares and is indifferent to buying or selling a fraction of these shares; that is, that following function is maximised at $\eta = 0$:

$$\max_{\eta}\left\{u_t(C(t) - \eta \cdot P(t)) + \mathrm{E}_t\left[\sum_{k=1}^{T-t} u_{t+k}(C(t+k) + \eta \cdot D(t+k))\right]\right\} \tag{2.4.3}$$

After some minor manipulation, this condition results in the fundamental valuation equation 2.4.1. This equation can be rewritten in many different forms, in particular:

$$\mathrm{E}_t\left[\frac{u'_{t+1}(C(t+1))}{u'_t(C(t))} \cdot (1 + \rho_j(t+1))\right] = 1 \tag{2.4.4}$$

where $\rho_j(t) = (P_j(t) - P_j(t-1) + D_j(t)) / P_j(t-1)$ represents the rate of return on the jth security in year t.

Furthermore, if a risk-free security, f, exists then it must satisfy equation 2.4.4 and the fundamental valuation equation, conditional on $\mathrm{H}(t)$, can be expressed as:

$$\mathrm{E}_t[u'_{t+1}(C(t+1)) \cdot (\rho_j(t+1) - \rho_f(t+1))] = 0 \tag{2.4.5}$$

The fundamental valuation equation can be used to derive most of the discrete time asset pricing models, including the present value model, the capital asset pricing model (CAPM), and the arbitrage pricing theory (APT) (see Appendix 2A). Appendix 2A is largely based on Huang and Litzenberger (1988) and Bhattacharya and Constantinides (1989). These books and the references contained therein provide a more comprehensive treatment of these standard models. This appendix merely attempts to convey the broad nature of the models used in financial economics in an actuarial context. These models attempt to explain financial phenomena, but as they depend on inexact simplifying assumptions they are not necessarily appropriate. Before these models can be reliably used they need to be empirically tested (see Chapter 6).

2.4.2 The actuarial response

The concerns that actuaries have about these asset pricing models were discussed in a debate, organised by the Institute of Actuaries, on whether the work of actuaries could be enhanced by financial economics (Wilkie *et al.* 1993). In this debate, much of the criticism of financial economics was directed at the unrealistic simplifying assumptions typically made. For example, Wilkie (Wilkie *et al.* 1993: 400) claimed that the random walk model of equity prices was an invalid model when used over a long time interval. Clarkson argued that these assumptions inferred that the methods were of "no relevance to the financial world in which we actually live" (Wilkie *et al.* 1993: 404). In particular, it was asserted that the methods used by financial economists ignore some of the essential features of the financial system and rely too heavily on inductive inferences. For instance, Arthur (Wilkie *et al.* 1993: 401) argued that the equity risk premiums received in the past are not a good reflection of the future risk premiums because this ignores the effect of the historical changes in dividend cover. Furthermore, it was alleged that the power of quantitative methods in economics is generally limited because of the absence of consistent regularities within the economy. Actuarial methods, with their reliance on professional judgement, were perceived by many to be more flexible and better suited to studying the economy. Hence, this debate raised important concerns about the empirical adequacy of financial economic models. Although the motion that actuarial work could benefit from financial economics was accepted, many actuaries felt that their assumptions were too restrictive for them to be practically useful. This issue is discussed further in Chapter 6.

In addition to these concerns, there are important contextual differences between actuarial and finance applications. In particular, an important assumption used by financial economists is the absence of arbitrage opportunities. This assumption depends on the existence of a highly liquid market. However, actuarial liabilities are relatively illiquid, which reduces the scope for arbitrage and speculative buying.

Despite these reservations, actuaries have been attracted to financial economics by its ability to tackle problems that are beyond the capability of traditional actuarial methods, such as valuing conditional liabilities. This has also been the principal motivation for the stochastic models discussed in the following section. Examples of the use of financial economic theories by actuaries include Mehta (1992), who proposed that the CAPM (see Appendix 2A.3) should be used to determine the interest rates required to value life office liabilities. Wilkie (1987b) illustrated how option pricing techniques could be used to assess bonus policies and Sherris (1992) demonstrated how these techniques could be used to calculate the reserves for deferred unrealised capital gains tax. Smith (1996) illustrated how various financial economic theories could be used by actuaries. The fundamental valuation equation is also familiar to actuaries. This equation equates the value of security to the expected value of the product of its future cash flow and the marginal rate of substitution, which is a more general version of the present value model used in actuarial valuations. However, actuaries have traditionally derived this model using less abstract arguments and thus interpret it slightly differently.

Furthermore, practising actuaries should be obliged to give financial economic theories serious consideration as they have a professional commitment to ensure the financial soundness of insurance companies and retirement funds. Moreover, actuaries are required to provide the 'best possible service and advice'. Financial economics has been widely accepted as representing the orthodox expert knowledge; actuaries would be unwise to ignore it completely. However, actuaries should also be agnostic about any particular finance theory because they are neither true nor false; they are provisional ideas that are subject to future vindication or disproof (see Chapter 5). Practising actuaries need to consider all the available theories, but ultimately they need to make a prudent decision based on their judgement. Therefore, financial economics potentially provides a rigorous approach that could be used to strengthen traditional actuarial methods of setting the economic assumptions. Another method for determining more rigorous and more detailed assumptions, which is examined in the following section, is to develop a stochastic asset model. These models may incorporate restrictions suggested by financial economics or they may be purely descriptive statistical models.

2.5 Stochastic Methods

2.5.1 Introduction

Stochastic asset models provide a more complete description of the relevant economic variables than deterministic actuarial bases do. Not only do stochastic models forecast the long term expected values of these variables, but they also quantify the extent to which these variables are likely to deviate from their expected values over time. This additional information is essential for certain actuarial applications; such as valuing insurance contracts with benefits that are conditional on the economic variables or assessing the risks inherent in various strategies. Stochastic models are also potentially less subjective than deterministic bases because they tend to rely less on personal beliefs and more on historical data and economic theory. They could thus be used, in conjunction with actuarial judgement, to motivate particular economic bases.

However, stochastic models have generally not been used to forecast the expected values required for actuarial bases; they have rather aimed to forecast the variability of and the interactions between the economic variables (see Wilkie 1995b: 785). Furthermore, empirically adequate stochastic models have proved difficult to construct because of the complex nature of the economy and the consequent absence of a reliable and detailed theoretical foundation. Some have even questioned the existence of an adequate model because of the apparent lack of empirical regularities in the economy. This section considers the objectives and intended applications of actuarial stochastic asset models.

2.5.2 Objectives

Wilkie (1987a: 65) suggested the following objectives for an actuarial stochastic asset model:

- it should demonstrate a reasonable long term structure, but need not be concerned with very short term forecasts,
- it should be economically realistic and conform to the reasonable intuition of investment experts,
- it should be sufficiently comprehensive to model, albeit in a simplified way, all the assets of an insurance company.

The first objective summarises a significant feature that distinguishes actuarial models from other econometric models. As insurance contracts usually extend over long time intervals, it is particularly important that actuarial models 'demonstrate a reasonable long term structure'. Although it is usually beneficial if models are able to produce accurate short term forecasts, the longer term forecasting ability of actuarial models is decisive. This objective is vaguely phrased because of the diversity of actuarial applications (see Section 2.5.3) and in recognition of the difficulty associated with developing an adequate model. Hence, the long term structure is only required to be 'reasonable'. The time intervals over which actuarial models are generally used range from 10 to 50 years. Wilkie (1984) examined the properties of his model over a 100 year interval.

The second objective reflects the standard criteria for evaluating econometric models, which include that models should be consistent with the historical data and prior economic theory (see Chapter 7). Consistency with the historical data, or empirical adequacy, is established by performing a number of goodness-of-fit tests. These tests include assessments of the structural stability of the model and assessments of whether its error terms are independent and identically distributed. An important characteristic of actuarial asset models is that they only attempt to model the long term features of the data. As a result, these models should be evaluated taking this into account. Econometric models also need to be interpretable or consistent with prior economic theory because of the potential problems associated with data mining, or data sample dependency (see Section 7.2.2). Hence, Wilkie required that a model should 'conform to the reasonable intuition of investment experts'. It appears that this particular phrase was used because actuaries generally do not wholly accept financial economic theory (see Section 2.4.2). In addition, 'reasonable intuition' appears to be similar to the concept of actuarial judgement.

An additional econometric criterion, that was not specifically mentioned in the above quotation, is that the model should be parsimonious. This criterion is dependent on the application of the model because it implies that the features of the model that are not significant for that particular application should be excluded. Consequently, certain features of short term forecasting models may be excluded from actuarial models.

The third objective specifies the variables required for actuarial investigations. The major asset classes of insurance companies are equity securities, government fixed-interest securities, corporate fixed-interest securities, property, and index-linked securities. These asset classes are usually further subdivided by industries and geographical location. The economic variables required include price and wage inflation. However, due to data shortages, the primary variables that have been examined include: price inflation, equity prices, equity dividend yields, and long-term government fixed-interest securities. Auxiliary variables, such as GDP growth rates, are generally not included in actuarial models. These variables are not specifically required for actuarial investigations and their influence is assumed to be 'subsumed' in the model (Wilkie 1987a: 69). Although the inclusion of auxiliary variables, in particular the lagged values of these variables, may improve a model's short term forecasts, they do not necessarily improve its long term forecasts. This is because, to obtain long term forecasts, all the variables usually need to be projected and the projected values of the auxiliary variables may be unreliable.

Therefore, actuarial stochastic asset models aim to parsimoniously describe the expected long term behaviour of inflation and the returns on the major asset classes. These models are evaluated using the criteria of empirical adequacy and consistency with prior theory.

Although these objectives are widely accepted, they are rarely all satisfied in practice and there is considerable disagreement about the relative importance of the individual objectives. In particular, certain theoretical features are sometimes included in models even though they do not appear to be empirically significant (see Section 3.2.2). This debate is considered further in Chapter 7.

As all models are controversial, an alternative approach is to select the most general mathematically tractable model that is broadly consistent with financial economic theory. Until an empirically adequate and theoretically consistent model is discovered, these hypothetical models are often the most pragmatic alternative. Many of the suggested actuarial stochastic models do not attempt to satisfy the criterion of mathematical tractability; simulation exercises are normally required. Hence, relatively simple autoregressive models have been used to examine the effects of stochastic asset returns on retirement funding (see Gerrard and Haberman 1996; Haberman 1994; Dufresne 1988). Similarly, in financial economics, continuous time Wiener processes are widely used to model rates of return.

2.5.3 Applications

Stochastic asset models can be used in place of deterministic actuarial bases in almost all actuarial applications. The advantage of stochastic models is that they describe the complete distribution of the economic variables rather than only providing point estimates of these variables. This additional information enables actuaries to measure the robustness of their results and to determine the likelihood of specific events occurring, such as insolvency. It also enables actuaries to value a wider range of insurance contracts, such as contracts with benefits that are conditional on the future values of the economic variables. Hence, stochastic models provide information that is required in a broad range of financial planning and risk analysis applications. However, as actuarial stochastic models are primarily concerned with describing the long term features of the economic variables, they may be unsuitable for certain investment applications, such as valuing short term derivative securities. This section provides examples of some specific applications of actuarial stochastic asset models and discusses the potential problems associated with the use of these models. For a more comprehensive treatment of the actuarial applications of stochastic models see Daykin *et al.* (1994).

A notable early application of stochastic models was the valuation of unit linked life assurance contracts with maturity guarantees (see Ford *et al.* 1980). Traditional valuation methods cannot be used to value maturity guarantees because the value of these guarantees is dependant on the rate of return on the underlying units. If expected rates of return are used then the value of the maturity guarantees will usually be underestimated and will often be zero. Ford *et al.* (1980) recommended that maturity guarantees should be valued using Monte Carlo simulation techniques and developed a stochastic model for simulating the future equity returns. Other life office applications include: assessing the risk of insolvency (Limb *et al.* 1986; Hardy 1993); investigating the effectiveness of 'resilience' tests (Purchase *et al.* 1989); comparing the likely effects of various investment and bonus strategies (Ross 1989; Ong 1996); and determining the appropriate level of risk-based capital for a with-profits fund (Needleman and Roff 1995).

Retirement fund applications of stochastic models include: determining appropriate investment strategies (Wise 1987b) and analysing alternative valuation methods (Dyson and Exley 1995). Hypothetical stochastic models have also been used to analyse the funding of retirement schemes (Haberman 1994; Dufresne 1988).

Stochastic models have been used in general insurance to assess the solvency of general insurance companies and to obtain a variety of management information (see Daykin and Hey 1990). This information includes: the impact of writing a specific class of business on the emergence of profit or loss and on the balance sheet; the suitability of various investment strategies; and the likely future variability of the company's solvency margins and profits.

Other suggested applications of stochastic models include: the assessment of capital projects (Lewin *et al.* 1995), the assessment of methods for calculating unit trust expenses (Wilkie 1987a), and the investigation of dynamic investment strategies (Wilkie 1986b).

The above applications illustrate the potential value of stochastic asset models over deterministic bases. Stochastic methods quantify economic uncertainty so that its influence on financial institutions can be measured. Furthermore, the adequacy of this measure can generally be independently evaluated using empirical tests. However, the use of stochastic methods also involves a substantial increase in cost due to the large number of simulations that are required. In some cases, detailed stochastic investigations currently seem to be impracticable (Geoghegan *et al.* 1992). Consequently, Geoghegan *et al.* (1992) suggested that in certain applications it may be adequate to only consider a relatively small number of representative scenarios rather than a large number of simulations.

These representative scenarios could be developed using the scenario approach (see Schoemaker 1993). This approach has been successfully employed in corporate planning (Wack 1985a,b) and long-term forecasting (Schwartz 1991). The scenario approach is similar to pattern modelling (see Section 5.6.3). It seeks to tackle fundamental uncertainty by constructing a number of coherent narratives describing the possible evolution of a particular system from a multi-disciplined perspective. These narratives attempt to identify the major stakeholders, trends or predetermined events, and key uncertainties in the system. They describe the interrelationships between these elements and stress the holistic and systemic nature of the system. Particular scenarios are checked for their internal consistency and plausibility. However, they cannot be rigorously tested because they do not use a set of formal theories. The scenario approach does not generally aim to discover the most likely future paths of events and consequently it does not attempt to attach probabilities to individual scenarios. It is assumed that fundamental uncertainty cannot be predicted or modelled. The aim of the scenario approach is generally to emphasise the range of possible future developments within a system so that users can prepare for, and respond quickly and appropriately to, events when they occur. Alternatively, individual scenarios could be obtained from a stochastic asset model and would be equivalent to a set of non-uniform deterministic bases.

The scenario method of analysing the effects of uncertainty has also been used by the UK Government Actuary to assess the resilience of life offices' solvency to changes in financial conditions (see Hardy 1993; Purchase *et al.* 1989). However, Hardy (1993) rejected the view that scenario testing can adequately replace stochastic methods. Hardy conducted solvency tests on a number of hypothetical life offices using both individual scenarios and a stochastic model. She found that scenario testing appears to underestimate the risk of insolvency. Furthermore, Hardy found that the scenarios did not provide an accurate method for assessing the relative riskiness of the life offices.

These conclusions cannot necessarily be generalised; they are dependant on a specific stochastic asset model and hypothetical life offices. This emphasises another potential problem with stochastic methods: their results are dependent on the particular stochastic asset model used and the adequacy of these models generally cannot be proven (see Chapter 7). Hence, stochastic investigations are usually inconclusive and it is important to assess the robustness of the results to alternative model structures. Hardy acknowledged this problem but still asserted that relative insolvency risk can be better assessed using stochastic methods. Hardy (1993: 148) claimed that, for this purpose, "it is not necessary to assume that the model is absolutely accurate, only that the range of results, especially considering the lower 20% tail, is broadly credible." This view was supported by Thomson *et al.* (1995), who claimed that the relative results of solvency tests are insensitive to the asset model. Stochastic methods were generally recommended because these methods consider an extensive range of possible future events and can be used to identify potentially problematic situations. However, they warned that stochastic methods may be unreliable in applications that are more dependant on the particular stochastic asset model, such as assessing the absolute risk of insolvency or assessing various investment strategies. This view has been supported by Harris (1995b) who found that absolute insolvency risk is sensitive to the asset model used. Furthermore, Smith (1996) and Ong (1996) found that optimal asset allocation decisions are highly sensitive to the asset model used.

Another important consideration when using stochastic models to determine optimal investment strategies is whether the model implicitly assumes that markets are efficient. If the model assumes that an asset's returns can be forecast then the optimum investment strategy would exploit this by buying or selling that asset depending on whether the returns are forecast to be above or below their long term average. However, if such a profitable trading strategy existed then it would become common knowledge and would most likely cease to be profitable in future (see Appendix 2A.1). This suggests that historical market inefficiencies should not be extrapolated into the future. Moreover, models that incorporate inefficiencies should generally not be used to identify optimal dynamic investment strategies (Wilkie 1986b; Smith 1996).

2.6 Summary

Actuaries are frequently required to make certain long term economic assumptions. For statutory and public reporting purposes, these assumptions are often fixed to ensure consistency. For other purposes, actuaries are individually responsible for setting these assumptions and, in so doing, are required to provide the best possible service and advice. The problem is that there is no single best method for determining these economic assumptions. Actuaries have tended to use relatively informal methods that rely heavily on actuarial judgement. These methods are not particularly robust and could be supported by financial economic and stochastic methods. Financial economics aims to provide formal explanations of financial phenomena in terms of the behaviour of individual rational agents. This has resulted in the development of numerous mathematical models. However, actuaries have been reluctant to embrace financial economic models because they appear to depend on simplifying assumptions that do not provide sufficiently realistic descriptions of reality. This concern is considered further in Chapter 6.

Stochastic methods potentially provide a more realistic description of economic variables and can be used to quantify the degree of uncertainty associated with deterministic point estimates. These models attempt to discover structure in the economic system. They thereby seek to expand the set of relatively certain future events and to bound the uncertainty of the other events. Stochastic methods tend to be more rigorous than traditional actuarial methods because they require their models to be extensively tested.

However, financial economic and stochastic methods depend on there being sufficient structure, that can be identified, in the financial system. This assumption has been rejected by Redington (1983: 531), amongst others, who stated: "The conditions never repeat themselves; there are no parameters; there are no foundations even for probability theory." If the existence of regularities is denied then it seems that the best an actuary can do is to minimise their exposure to the economic assumptions and to develop simple robust methods, including analysing the sensitivity of the results to the uncertain elements. For these minimal objectives deterministic models of the economic variables seem to be adequate; any further sophistication would be falsely scientific. In addition, due to their simplicity, deterministic models are comparatively inexpensive to implement and are relatively transparent and flexible. They can easily incorporate possible future events such as short term trends, steps, or spikes. They have also proved to be an effective means for understanding most actuarial problems. However, deterministic models cannot be used to analyse more complicated contracts with conditional benefits, such as limited price indexation or maturity guarantees. The remainder of this thesis considers whether there is sufficient structure in the financial system for financial economic and stochastic methods to be of value.

Appendix 2A: The Derivation of Asset Pricing Models

2A.1 Present value models

The present value model implies that the price of a security is equal to the discounted value of its expected future cash flows, using a constant discount factor, v. This discounted amount is often referred to as the security's fundamental value. The standard present value model, also known as the martingale model, is given by:

$$P(t) = \sum_{k=1}^{\infty} v^k \cdot \mathrm{E}_t[D(t+k)] \tag{2A.1}$$

where E_t represents the expectations operator conditional on all information at time t, denoted $\mathrm{H}(t)$, $P(t)$ represents the price of the security at time t, $D(t)$ represents the dividends or cash flows paid in year t.

Assuming that dividend growth rates are constant in future, this model approximates the Gordon (1962) model. The Gordon model states that in a static, steady growth world the dividend-price ratio is equal to the discount rate less the dividend growth rate. Present value models are also used ex post to calculate the realised return on securities over finite historical intervals in both real and nominal terms. In these calculations the present value model is merely an accounting identity with no economic theory content. This appears to be the intuitive actuarial justification for using these models to value streams of expected future asset and liability cash flows.

Present value models were first derived in financial economics from the constraint that there are no arbitrage opportunities (see LeRoy 1989). This condition implies that expected real returns, less a constant, are a fair game or, equivalently, that the discounted total return index of a security is a martingale. Assuming that expected rates of return, less a constant $\rho = (1 - v) / v$, are a fair game implies:

$$\mathrm{E}_t\left[\frac{P(t+1) + D(t+1)}{P(t)}\right] = 1 + \rho \tag{2A.2}$$

Assuming $v^k \cdot E_t[P(t+k)] \rightarrow 0$ as $k \rightarrow \infty$ (which disallows speculative bubbles), equation 2A.2 can be solved by recursive substitution using the law of iterated expectations to obtain the present value model.

Hence the present value model implies that rates of return cannot be forecast. This assumption is at variance with the view held by some actuaries that markets do not constantly reflect fundamental value (see Section 2.3.4).

The no arbitrage assumption is motivated by the competitive nature of investment markets in equilibrium. If a profitable trading, arbitrage, opportunity existed on a security then it is likely that rational investors would bid its price to a value at which no further profitable trading is possible. Therefore, relevant information that is universally available should not be able to be used to develop profitable trading rules; this is known as the efficient market hypothesis. A comparative advantage in financial markets can only be obtained by investors having differences in information. As a result, investors should evaluate whether their information is common knowledge before acting on it, which suggests that the value of information depends on its dispersion. Furthermore, this implies that security prices respond to information about events when the information becomes known, rather than when the events actually occur. Assuming that the amount of publicly available information is large, it should not be possible for individual investors to gain a comparative advantage and hence it should not be possible to forecast rates of returns. This assumption was strengthened by a number of earlier empirical studies, including Cowels (1933) and Kendall (1953), which suggested that it was difficult to outperform the market. If markets are informationally efficient then investors should merely aim to diversify their portfolios to minimise their specific risk and should select these portfolios solely on the basis of their level of risk tolerance.

The standard present value model can also be derived from the fundamental valuation equation (see Section 2.4.1) by assuming investors have risk-neutral preferences and use a constant discount factor, v. These assumptions are not unique. Ohlson (1977) obtained the present value model by assuming that dividend growth rates are serially independent, that investors have constant relative risk aversion, and that there is no lending or borrowing of the risk-free asset in equilibrium. Furthermore, the present value model can be derived if expectations are taken relative to a risk-neutral probability measure, rather than investors' actual probability measure (see Duffie 1992: 5). This is a useful theoretical result, which enables asset prices to be examined assuming investors are risk-neutral. However, assumptions concerning both investors' actual expectations and their risk tolerances are usually required to interpret empirical evidence.

The standard present value model is limited by the assumption that the discount rate is constant over time. To explore the effects of variations in the discount rate in a general log-linear asset pricing model, Campbell and Shiller (1989) developed the dividend-price ratio model. Other advantages of this model are that it uses logarithms and that the growth adjusted discount rate ($r(t) - d(t)$) does not depend on the price index used, which reduces its vulnerability to measurement errors in the price deflator. Log-linear models of economic data are more suitable than linear models on theoretical grounds (Banerjee *et al.* 1993). This model implies that the logarithm of the dividend-price ratio of a security is equal to the discounted value of the expected growth adjusted discount rate plus a constant and is given (in real terms) by:

$$y(t) = \sum_{k=0}^{\infty} v^k \cdot \mathrm{E}_t[r(t+k+1) - d(t+k+1)] + \frac{\kappa - \varpi}{1 - v} \tag{2A.3}$$

where $y(t)$ represents the logarithm of the dividend-price ratio on the security at time t, v represents a discount factor obtained from the linearization, $r(t)$ represents the force of return in year t, $d(t)$ represents the force of dividends growth in year t, κ represents the risk premium over $r(t)$, and $\varpi = -v \cdot \log_e v - (1 - v) \cdot \log_e(1 - v)$.

Campbell and Shiller (1989) suggested four versions of this model depending on the measure of the discount rate, $r(t)$, used. These four versions approximate most other asset pricing models. The first version assumes that the discount rate is constant, which approximates the standard present value model. The second version assumes that the discount rate is the short-term interest rate. The third version assumes that the discount rate is a multiple of the growth in real aggregate consumption per capita, which approximates the fundamental valuation equation. The fourth version assumes that the discount rate is a multiple of the volatility of the security's rate of return. The dividend-price ratio model (in nominal terms) also approximates the expectations theory of the term structure model (see Appendix 2A.2) as fixed-interest securities have constant nominal cash flows. Assuming that both discount rates and dividend growth rates are constant in future, this model approximates the Gordon (1962) model. Therefore, the dividend-price ratio model further generalises the Gordon model and has been referred to as dynamic Gordon model (Campbell and Shiller 1989).

The dividend-price ratio model can be derived by taking rational expectations of an approximation to the logarithm of the dividend yield of a security. This approximation is obtained from the actual force of return in year $t + 1$, which is given by:

$$r^{*}(t+1) = \log_e\left(\frac{P(t+1)+D(t+1)}{P(t)}\right) = d(t+1) + y(t) + \log_e\left(1+\frac{P(t+1)}{D(t+1)}\right) \qquad (2A.4)$$

Using a first order Taylor series expansion around the point $1 / v$, the following linearization is obtained:

$$\begin{aligned} \log_e\left(1+\frac{P(t+1)}{D(t+1)}\right) &\approx -\log_e v + v\cdot\left(1-\frac{1}{v}+\frac{P(t+1)}{D(t+1)}\right) \\ &= -\log_e v + v\cdot\left(\frac{P(t+1)}{D(t+1)}-1\right) + (1-v)\cdot\left(\frac{v}{1-v}-1\right) \\ &\approx -\log_e v - v\cdot y(t+1) - (1-v)\cdot\log_e\left(\frac{1-v}{v}\right) \end{aligned} \qquad (2A.5)$$

Assuming $\log_e(1 + x) \approx x$.

Substituting this result into equation 2A.4 yields:

$$r^{*}(t+1) \approx \varpi + y(t) - v\cdot y(t+1) + d(t+1) \qquad (2A.6)$$

The accuracy of this approximation is discussed in Campbell and Shiller (1989). Solving equation 2A.6 forward and assuming $v^k \cdot y(t + k) \to 0$ as $k \to \infty$:

$$y(t) \approx \sum_{k=0}^{\infty} v^k \cdot (r^{*}(t+k+1) - d(t+k+1)) - \frac{\varpi}{1-v} \qquad (2A.7)$$

Taking expectations of this approximation, conditional on information available at time t, and assuming the expected force of return on the security is equal to the some other measure of the force of return plus a constant risk premium ($E_t[r^*(t+k)] = E_t[r(t+k)] + \kappa$), yields the dividend-price ratio model. This assumption implies that expected discount rates can be measured by some variable that is not necessarily directly related to the asset class under investigation and the difference between the return on the security and the discount rate is not predictable.

Further, to estimate the relative importance of expectations of future dividend growth, interest rates, and excess returns to unexpected excess returns Campbell and Ammer (1993) adapt the dividend-ratio model into a dynamic accounting framework. This was achieved by allowing the risk premium over short-term interest rates, $\kappa_{b1}(t)$, to vary over time. Therefore, equations 2A.3 and 2A.6 become:

$$y(t) = \sum_{k=0}^{\infty} v^k \cdot E_t[r_{b1}(t+k+1) + \kappa_{b1}(t+k+1) - d(t+k+1)] - \frac{\varpi}{1-v} \qquad (2A.8)$$

$$\kappa_{b1}(t+1) = \omega + y(t) - v \cdot y(t+1) + d(t+1) - r_{b1}(t+1) \qquad (2A.9)$$

where $r_{b1}(t)$ represents the force of return on 1-year fixed-interest securities in year t.

Using these equations, unexpected excess equity returns can be related to changes in rational expectations of future dividend growth, future real interest rates, and future excess equity returns as follows:

$$\begin{aligned}&\kappa_{b1}(t+1) - E_t[\kappa_{b1}(t+1)] \\ &= (E_{t+1} - E_t)\left[\sum_{k=0}^{\infty} v^k \cdot d(t+k+1) - \sum_{k=0}^{\infty} v^k \cdot r_{b1}(t+k+1) - \sum_{k=1}^{\infty} v^k \cdot \kappa_{b1}(t+k+1)\right]\end{aligned} \qquad (2A.10)$$

As equations 2A.8 and 2A.9 are approximate, equation 2A.10 only holds approximately. Campbell and Ammer (1993) also state the logarithm of the excess real return on n-year zero-coupon fixed-interest securities, the gross redemption yield on n-year zero-coupon fixed-interest securities, and the yield spread between the yield on n-year and 1-year zero-coupon fixed-interest securities as similar functions of revisions in investors' expectations of future dividends, real interest rates, inflation rates, and excess asset returns. The yield spread was used to eliminate the apparent unit root in the yields. These functions hold exactly.

2A.2 Term structure models

Although the above asset pricing models are applicable for any security, they have been primarily used for examining equities. Fixed-interest securities have been analysed using term structure models. Term structure models attempt to measure the relationship between default free fixed-interest securities with different terms to maturity and thereby represent the market's expectation of future interest rates. Most of these models are based on the belief that investors view fixed-interest securities with similar characteristics as potential substitutes for one another (see Hull 1993).

The expectations hypothesis assumes that fixed-interest securities are priced so that the forward rates implied by the current term structure equal the expected future spot interest rates. This hypothesis is based on the view that the expected return over any future interval is the same for securities of all maturities. The liquidity preference hypothesis assumes that securities with longer maturities have higher risk because uncertainty increases with time. As a result this hypothesis asserts that investors have a preference for liquidity and a term premium is required to induce investors to hold longer term securities. These term premiums are assumed to increase with increasing maturities so that the yield curve is 'naturally' upward sloping. The market segmentation hypothesis states that investors have strong maturity preferences so that securities of different maturities are not typically seen as potential substitutes. Hence, interest rates at particular maturities are assumed to be determined largely independently of interest rates at other maturities. This hypothesis rejects the assumption that interest rates of different maturities are necessarily related in a consistent manner and is thus a heterodox hypothesis.

The expectations and liquidity preference hypotheses imply that long term yields are a weighted moving average of the expected future yields on short-term fixed-interest securities plus a constant term premium, which is given by (see Shiller 1990):

$$i(t,\infty) = (1-v)\cdot\sum_{k=0}^{\infty} v^k \cdot \mathrm{E}_t[i(t+k,1)] + \kappa \tag{2A.11}$$

where $i(t,\infty)$ represents the par redemption yield on a perpetuity at time t, $i(t,\tau)$ represents the par redemption yield on τ-year fixed-interest securities at time t, κ is a term premium, and v is a parameter of linearization.

The term structure present value model can be derived from the definition of the term premium which is defined as the difference between the yield on a fixed-interest security maturing at time $t+n$ and the conditional rational expected holding period return from rolling over a sequence of 1-year fixed-interest securities for n periods (Shiller 1990):

$$\kappa_{bn}(t) = \log_e(1+i(t,n)) - \mathrm{E}_t\left[\log_e\left(\prod_{k=0}^{n-1}(1+i(t+k,1))\right)^{\frac{1}{n}}\right] \qquad (2A.12)$$

where $\kappa_n(t)$ represents the term premium at time t on n-year fixed-interest securities. Therefore:

$$\log_e(1+i(t,n)) = \mathrm{E}_t\left[\frac{1}{n}\cdot\sum_{k=0}^{n-1}\log_e(1+i(t+k,1))\right] + \kappa_{bn}(t) \qquad (2A.13)$$

Linearizing around r, using the approximations $e^x \approx (1+x)$ and $\log_e(1+x) \approx x$, yields:

$$i(t,n) \approx \mathrm{E}_t\left[\sum_{k=0}^{n-1}\left(\frac{e^{-r\cdot k} - e^{-r\cdot(k+1)}}{1-e^{-r\cdot n}}\right)\cdot i(t+k,1)\right] + \kappa_{bn}(t) \qquad (2A.14)$$

The present value model is obtained by letting n tend to infinity, assuming the term premium is constant through time ($\kappa_{b\infty}(t) = \kappa$, for all t), and setting $v = e^{-r}$.

Another interest rate model is the Fisher relation (see Fisher 1930), which states that expected inflation is fully reflected in nominal interest rates, that is:

$$i^*(t,n) = i(t,n) - \mathrm{E}_t\left[\left(\prod_{k=t+1}^{t+n}(1+\rho_q(k))\right)^{\frac{1}{n}}\right] \qquad (2A.15)$$

where $i^*(t,n)$ represents the ex ante real redemption yield at time t on an n-year fixed-interest security, and $\rho_q(t)$ represents the rate of inflation in year t.

This relation is not precisely correct under uncertainty, but this has largely been ignored in the empirical literature. The correct version of the Fisher relation for a 1-year fixed-interest security can be derived from the fundamental valuation equation (see Section 2.4.1), as follows (see Shome, Smith, and Pinkerton 1988):

$$1 = \mathrm{E}_t\left[\frac{u'_{t+1}(C(t+1))}{u'_t(C(t))} \cdot \left(\frac{1+i(t,1)}{1+\rho_q(t+1)}\right)\right] \tag{2A.16}$$

Assuming that investors are risk-neutral and use a constant discount factor, $v = 1/(1+\rho)$, then the Fisher relation approximates the following relation with $\rho = i^*(t,1)$:

$$1+\rho = (1+i(t,1)) \cdot \mathrm{E}_t\left[\frac{1}{1+\rho_q(t+1)}\right] \tag{2A.17}$$

2A.3 The capital asset pricing model

The CAPM states that a security's expected excess return is proportional to the covariance of the security's return with the market portfolio of all risky securities. The standard version of this model is given by (see Huang and Litzenberger 1988):

$$E[\rho_j] - \rho_f = \beta_{j,\Omega} \cdot (\mathrm{E}[\rho_\Omega] - \rho_f) \tag{2A.18}$$

where ρ_j represents the rate of return on security j, $\beta_{j,k}$ represents covariance between the returns of securities j and k divided by the variance of security k's returns, f represents the risk-free asset, and Ω represents the market portfolio of assets.

Thus the CAPM implies that securities with rates of return that are positively correlated with the market portfolio have positive risk premiums. An intuitive justification for this is that a payment when the economy is in a relatively prosperous state has a lower utility than the same payment when the economy is in a poor state (Breeden 1979). Hence a security whose payments are positively correlated with the market portfolio should have a lower price than a security whose payments have the same expected value but are negatively correlated with the market portfolio. Having a lower price implies that the rate of return is expected to be higher than it would otherwise have been. Similarly, securities that are negatively correlated with the market portfolio should have lower expected rates of return.

The CAPM can be derived from the two fund separation theorem (see Huang and Litzenberger 1988), which assumes that there exists two portfolios of assets k and l such that for any given portfolio j there exists a scalar λ such that:

$$E[u(\lambda \cdot \rho_k + (1-\lambda) \cdot \rho_l)] \geq E[u(\rho_j)] \tag{2A.19}$$

When two fund separation holds, the separating portfolios, k and l, are on the mean-variance frontier. This result follows from the properties of portfolios that exhibit stochastic dominance. Hence, it follows that individual investors' optimal portfolios are frontier portfolios. Further, if a risk-free security is assumed to exist, then the rate of return on any feasible portfolio j can be represented as a linear combination of the return on the risk-free security and any frontier portfolio m other than the risk-free security, that is:

$$\rho_j = (1-\beta_{j,m}) \cdot \rho_f + \beta_{j,m} \cdot \rho_m + \varepsilon_{j,m} \tag{2A.20}$$

where $\text{cov}(\rho_m, \varepsilon_{j,m}) = E[\varepsilon_{j,m}] = 0$.

The market portfolio is a linear combination of individual investors' portfolios because in equilibrium markets must clear. As linear combinations of frontier portfolios are on the frontier, the market portfolio must be a frontier portfolio in equilibrium. The CAPM follows directly from equation 2A.20 by replacing the frontier portfolio, m, with the market portfolio.

The above derivation of the CAPM depends on the mean-variance model of security choice, which assumes that investors have a preference for expected return and an aversion to variance. This does not represent a general model of security choice, but can be motivated by assuming that investors have quadratic utility functions or by assuming that the distribution of the rates of return on securities satisfies the conditions for two fund separation. Quadratic utility functions are generally inappropriate because they have the unrealistic property of increasing absolute risk aversion, which implies that risky securities are inferior goods. The conditions for fund separation are given by Ross

(1978). The multivariate normal distribution is a particular example of a rate of return distribution that satisfies these conditions. Although the multivariate normal distribution has not been found to be particularly suitable, in a continuous time framework multivariate normal distributions can generally be replaced by lognormal diffusion processes, which are reasonably appropriate.

The CAPM can also be derived from the fundamental valuation equation (see Section 2.4.1) in a single period setting (see Constantinides 1989). If it is assumed that firms pay a liquidating dividend at the end of the interval, $t = 1$, then consumption equals aggregate wealth: $W(1) = C(1)$. Further, if it is assumed that $W(1)$ and $\rho_j(1)$ are bivariate normally distributed and that $u(\cdot)$ satisfies certain regularity conditions, then using Stein's lemma the fundamental valuation equation is equivalent to:

$$\mathrm{E}_0[\rho_j(1) - \rho_f(1)] = -\frac{\mathrm{E}_0[u_1''(W(1))]}{\mathrm{E}_0[u_1'(W(1))]} \cdot \mathrm{cov}_0(\rho_j(1) - \rho_f(1), W(1)) \qquad (2A.21)$$

As aggregate wealth is a linear function of the market portfolio return: $W(1) = W(0)\cdot(\rho_\Omega(1) + 1)$. Further, assuming that the risk-free rate of return is known at the start of the interval, equation 2A.21 becomes (taking expectations at $t = 0$):

$$\mathrm{E}[\rho_j] - \rho_f = -\frac{\mathrm{E}[u''(W(1))]}{\mathrm{E}[u'(W(1))]} \cdot W(0) \cdot \mathrm{cov}(\rho_j, \rho_\Omega) \qquad (2A.22)$$

Since equation 2A.22 holds for any security, it must hold for the market portfolio. The CAPM is then obtained by substituting equation 2A.22 with j replaced by Ω into the original equation 2A.22 and simplifying. Using a similar approach, the CAPM can be derived by assuming a quadratic utility function.

Furthermore, similar arguments can be used to derive the consumption based CAPM (CCAPM), which relates the expected excess return on a security to covariance of its excess return with aggregate consumption. Although the standard CAPM is only a single period cross-sectional model, it has also been extended into an intertemporal setting, termed the ICAPM (see Constantinides 1989).

2A.4 Arbitrage pricing theory

The APT states that there is an approximate linear relation between expected returns on risky securities and the factor betas for most securities when the economy is large. Formally the APT relation is that there exists λ_j and an ε such that (see Huberman 1982):

$$\sum_{j=1}^{l}\left(\mathrm{E}[\rho_j]-\rho_f-\sum_{k=1}^{K}\beta_{j,k}\cdot\lambda_k\right)^2\leq\varepsilon \tag{2A.23}$$

where $\beta_{j,k}$ represents the weights for the jth security and the kth factor in a K factor model, K represents the number of factors, and l represents the number of risky securities.

To avoid degenerate cases, the number of factors is assumed to be small relative to the number of risky securities. Huberman (1982) provided a simple derivation of the APT by assuming that there are no arbitrage opportunities in the limit and by assuming that a generalised version of equation 2A.20 applies. An arbitrage opportunity is defined as a sequence of portfolios whose expected rates of return is bounded below above zero and whose variances converges to zero. Equation 2A.20 was generalised into the following K-factor model:

$$\rho_j=\mathrm{E}[\rho_j]+\sum_{k=1}^{K}\beta_{j,k}\cdot\phi_k+\varepsilon_j \tag{2A.24}$$

where $\mathrm{E}[\phi_k]=0$ for all k, $\mathrm{E}[\varepsilon_j]=0$ for all j, $\mathrm{E}[\varepsilon_j\cdot\varepsilon_l]=0$ for $j\neq l$, and the variances of ε_j are bounded.

The APT can also be broadly motivated by substituting equation 2A.24 into the fundamental valuation equation (see Section 2.4.1) and simplifying to obtain (see Constantinides 1989):

$$\mathrm{E}[\rho_j-\rho_f]=-\sum_{k=1}^{K}\frac{\mathrm{E}[u'(C(1))\cdot\phi_k]}{\mathrm{E}[u'(C(1))]}\cdot\beta_{j,k}-\frac{\mathrm{E}[u'(C(1))\cdot\varepsilon_j]}{\mathrm{E}[u'(C(1))]} \tag{2A.25}$$

If the ε_j are assumed to be virtually zero because they represent specific diversifiable risk, then the second term on the right hand side of equation 2A.25 is zero and each security's excess return is a linear function of the weights $\beta_{j,k}$. Although the APT states that the linear relationship only holds for most assets, bounds on the deviations from a this relationship have been established.

The above derivation of the original APT is based solely on the assumptions of no arbitrage and a linear factor structure of rates of return. A similar result can be obtained within the equilibrium framework of the CAPM if it is assumed that the market portfolio is well diversified with respect to a given set of factors (see Shanken 1985). These factors are assumed to be economic variables. Hence, the market return, and as a result the market betas, can be represented as a linear combination of the factors. This derivation of the APT implies an exact linear relationship between expected returns and the factors and is known as the equilibrium APT or the multi-beta CAPM.

The APT attempts to address the criticism of the CAPM: that the market portfolio is practically unobservable. However, empirical tests of the APT are problematic (see Shanken 1985, 1992; Dybvig and Ross 1985). The APT does not pre-identify the factors so that a negative test result may merely imply that the 'correct' factors were not used. In addition, Shanken (1992) illustrated that the APT relationship will hold for proxy variables that are related to the factors in a specific manner. Consequently, a positive test result does not necessarily indicate that the correct fundamental factors have been identified. Moreover, the APT is only an approximate relationship and it assumes that there are infinitely many assets. These characteristics mean that the APT cannot be unambiguously tested. The bounds on deviations from the exact linear relationship seem to suggest a precise empirical test, but these bounds have been shown to be mathematical tautologies for finite sets of linearly independent assets (Shanken 1992). Nevertheless, the equilibrium APT does imply an exact testable relationship. However, this version of the APT is derived from more restrictive assumptions and depends on the market portfolio.

Chapter 3

ECONOMETRIC MODELS

3.1 Introduction

Stochastic models potentially provide more detailed and more rigorous actuarial economic bases. These models are usually derived from the probabilistic structure of the data and from theoretical considerations, which are primarily obtained from financial economics. Although both of these considerations play a part in all models, the available models have tended to emphasise either the historical data or economic theory. This chapter describes the main comprehensive actuarial stochastic asset models that have been developed principally from data considerations, especially Wilkie's (1986a, 1995b) model. Chapter 4 describes the main theoretical stochastic asset models. These models are initially defined and their theoretical and statistical properties are then discussed. They are then reviewed from a methodological perspective in Chapter 7. Chapter 8 examines the empirical adequacy of Wilkie's model.

Section 3.2 discusses Wilkie's (1986a, 1995b) model and its related literature. Wilkie's model is considered in detail because it was the first comprehensive actuarial stochastic model and because it has been the most influential and widely used model in the UK. Section 3.3 briefly describes some of the other actuarial stochastic models, including the Finnish model developed by Pentikäinen *et al.* (1995), the Australian models developed by Carter (1991), FitzHerbert (1992), and Harris (1994, 1995a), and the South African model developed by Thomson (1994). Section 3.4 summarises the main arguments. Appendix 3A provides proofs for the equations used to examine the statistical properties of Wilkie's model.

3.2 Wilkie's Model

3.2.1 Introduction

Wilkie's (1984, 1986a) original model was the first comprehensive UK actuarial stochastic asset model to be published. Its purpose "is to provide a realistic variance and covariance structure for many years ahead" (Wilkie 1995b: 783); rather than to provide accurate short term forecasts. This purpose reflects the long term nature of its intended applications. Wilkie's model was initially used by a Faculty of Actuaries working party to investigate criteria for assessing the solvency of life offices (Limb *et al.* 1986). Since then, it has been extensively used in a range of applications and appears to have become the standard UK stochastic asset model (see references in Section 2.5.3).

The development of Wilkie's model started in the 'Report of the Maturity Guarantees Working Party' (Ford *et al.* 1980), which proposed a model for simulating equity returns. Other preliminary work for the model included Wilkie (1981) and an unpublished report prepared by Gwilym Jenkins and Partners. The original model is made up of four interconnected models, namely: a price inflation model, an equity dividend yield model, an equity dividend model, and a long-term interest rate model. These models are essentially conventional ARIMA transfer function models that were developed from UK data over the interval 1919-82; earlier data was considered for the price inflation and long-term interest rate models. Wilkie (1992, 1995a, 1995b) updated the original model and extended it to include: alternative ARCH and VAR price inflation models, a wage inflation model, a short-term interest rate model, a property yield model, a property income model, an index-linked yield model, and an exchange rate model. These extensions left the original model's structure virtually unaltered. Furthermore, these models were fitted to data from numerous countries, data collected at monthly intervals were examined, cointegration relationships were considered, and numerous other ARCH models were studied. The results of these latter investigations are not specifically reported in this section because they are generally inconclusive or incomplete (see Wilkie 1995b for this information). Section 3.2.3 only reports the main UK models that appeared to have been specifically recommended for actuarial use.

Wilkie's model was first reviewed by Kitts (1988, 1990). Kitts only considered the empirical adequacy of the price inflation model and reported that its residuals failed tests of normality and independence. In particular, these residuals were found to contain unusually long runs of the same sign. Furthermore, Kitts questioned the long term validity of the model because it does not accommodate structural changes, such as the apparent structural change in price inflation that occurred in the early 1900s (see Wilkie 1981). Kitts' review prompted the Institute and Faculty of Actuaries to establish a joint working party to review Wilkie's model and its applications (Geoghegan *et al.* 1992). Geoghegan *et al.* (1992: 179) also focused on the price inflation model and expressed concern that it was unable to account for:

(1) the existence of bursts of inflation, indicating that once an upward trend in inflation is established, there is a tendency for it to continue ...
(2) the existence of large, irregular shocks, such as those of the mid-1970s ...
(3) the possible non-normality of residuals, through asymmetry, etc. ...

Nevertheless: "The Working Party agreed that there was little evidence to suggest that a better fitting parsimonious model could be estimated using standard Box-Jenkins methodology" (Geoghegan *et al.* 1992: 179). Geoghegan *et al.* (1992) discussed a number of possible alternative models, but did not make any specific recommendations. A member of the Working Party, Clarkson (see Clarkson 1991), and Wilkie suggested alternative inflation models. Geoghegan *et al.* (1992: 186) concluded that "considerably more research is required in this area."

Other comments on Wilkie's model have been made by, amongst others, Daykin and Hey (1990), Ludvik (1993), Daykin *et al.* (1994), and Smith (1996). Daykin and Hey (1990) noted that the price inflation model generates a much higher proportion of years with negative inflation than has been observed over the post-war interval 1951-88. The price inflation model implies that the probability of negative inflation is roughly 20%: negative annual inflation has not occurred since the 1960s. Daykin and Hey (1990) suggested a few relatively minor modifications to the model's parameter values. Daykin *et al.* (1994) suggested that a skew distribution should be used to describe the price inflation model's residuals (see equation 3.3.1). Ludvik (1993) reported that the historical correlations, between the total returns on UK equity and fixed-interest securities, were far greater than the correlations implied by Wilkie's model. Smith (1996) noted that, by exploiting the market inefficiencies incorporated in Wilkie's model, investors should be able to achieve additional returns of roughly 3% for no extra risk (this was first reported in Wilkie 1986b). Furthermore, the model implies that extremely large returns with very low levels of risk can be realised if short positions are permitted.

3.2.2 Model derivation

The objective of Wilkie's model is to parsimoniously describe the long term behaviour of inflation and the returns on the major asset classes (see Section 2.5.2). Wilkie (1995b) assessed this objective using the criteria of empirical adequacy and theoretical consistency. The model appears to have been initially formulated using economic theory and its detailed structure was largely determined on empirical grounds using the Box and Jenkins (1970) methodology. Hence, "a great variety of alternatives" (Wilkie 1986a: 345) were considered and the best fitting parsimonious model was chosen. The criterion of parsimony is dependent on the intended applications of the model: features that were not significant for actuarial applications were excluded. Moreover, models that could be rationalised in terms of economic theory were favoured (see Geoghegan *et al.* 1992: 178).

Wilkie (1995b: 926) assessed the empirical adequacy of the models by testing whether their residuals were independent and normally distributed. If a model failed any of these diagnostic tests, then more elaborate models seem to have been considered. For example, Wilkie (1995b: 926) recommended that if a model's residuals are found to be autocorrelated, then "a higher order AR(p) or MA(q) model should be tried."

The criteria used to assess the theoretical consistency of the models is less clear. Certain economic theories, such as the efficient market hypothesis, were rejected on empirical grounds, whereas other theories, such as the purchasing power parity hypothesis, were included regardless of the empirical evidence (see Wilkie 1995b: 890).

3.2.3 Description

3.2.3.1 Inflation models

Wilkie (1995b) suggested three price inflation models; a univariate AR(1) model, a univariate AR(1) model with an ARCH effect, and a VAR(1) model with the force of wage inflation. These models can be represented by the following equations (for $t > 0$):

$$r_q(t) = \mu_q + \alpha_q \cdot (r_q(t-1) - \mu_q) + \alpha_{qw} \cdot (r_w(t-1) - \mu_w) + \upsilon_q(t) \cdot z_q(t) \tag{3.2.1}$$

$$\upsilon_q(t)^2 = \sigma_q^2 + \lambda_q \cdot (r_q(t-1) - \nu_q)^2 \tag{3.2.2}$$

where $r_q(t)$ represents the force of price inflation in year t and $z_j(t)$ represent sequences of independently distributed unit normal random variables.

Two wage inflation models were suggested; an AR(1) transfer function model and a VAR(1) model with the force of price inflation. These models can be represented by the following equation (for $t > 0$):

$$\begin{aligned} r_w(t) = {} & \mu_w + \omega_w \cdot r_q(t) + \omega_{w1} \cdot r_q(t-1) \\ & + \alpha_w \cdot (r_w(t-1) - \mu_w - \omega_w \cdot r_q(t-1) - \omega_{w1} \cdot r_q(t-2)) \\ & + \alpha_{wq} \cdot (r_q(t-1) - \mu_q) + \sigma_w \cdot (\varphi_w \cdot z_q(t) + \sqrt{1-\varphi_w^2} \cdot z_w(t)) \end{aligned} \tag{3.2.3}$$

where $r_w(t)$ represents the force of wage, or earnings, inflation in year t.

Recommended parameter values are given in Table 3.2.1. The full and reduced standard bases represent the original parameter values recommended by Wilkie (1984, 1986a). These values were based on estimates obtained using data over the interval 1919-82. The reduced basis appeared to have been initially recommended for most applications. The Wilkie (1995b), ARCH, and the VAR values were based on estimates obtained using data over the interval 1923-94. The interval 1919-22 was excluded from the latter investigations because it was considered to an exceptional post-war period and was found to have an unduly large influence on the parameter estimates (see Wilkie 1995a: 256).

Table 3.2.1 Parameter values for Wilkie's inflation models

Parameter	*Full Std.*	*Reduced Std.*	*Wilkie (1995b)*	*ARCH*	*VAR*
μ_q	0.05	0.05	0.047	0.04	0.0359
α_q	0.6	0.6	0.58	0.62	0.1817
α_{qw}	-	-	-	-	0.5927
σ_q	0.05	0.05	0.0425	0.0256	0.0408
λ_q	-	-	-	0.55	-
ν_q	-	-	-	0.04	-
μ_w	-	-	0.021	0.021	0.0509
ω_w	-	-	0.6	0.6	-
ω_{w1}	-	-	0.27	0.27	-
α_w	-	-	0	0	0.5618
α_{wq}	-	-	-	-	0.2315
φ_w	-	-	-	-	0.7139
σ_w	-	-	0.0233	0.0233	0.0335

Wilkie (1995b) did not specifically recommend parameter values for the VAR model. The VAR model's parameter values reported in Table 3.2.1 were obtained from the full model estimated over the interval 1923-94 (see Wilkie 1995b: 813).

'Neutral' initial conditions are: $r_q(0) = r_q(-1) = \mu_q$, $r_w(0) = \mu_w + (\omega_{w1} + \omega_{w2}) \cdot \mu_q$.

3.2.3.2 Equity and property models

The equity dividend and property yield models describe the logarithm of the yields less a multiple of price inflation as AR(1) models. They can be represented by the following equation (for $t > 0, j =$ s and p):

$$
\begin{aligned}
y_j(t) = \log_e \mu_{yj} + \omega_{yj} \cdot r_q(t) + \alpha_{yj} \cdot (y_j(t-1) - \log_e \mu_{yj} - \omega_{yj} \cdot r_q(t-1)) \\
+ \sigma_{yj} \cdot z_{yj}(t)
\end{aligned}
\tag{3.2.4}
$$

where $y_s(t)$ represents the logarithm of the equity dividend yield and $y_p(t)$ represents the logarithm of the property income yield at time t.

The equity dividend and property income models describe cash flow growth less a moving average of price inflation and a multiple of the yield error in the previous year as MA(1) models with errors that are correlated with the yield models' errors. They can be represented by the following equations (for $t > 0, j = \mathrm{s}$ and p):

$$d_j(t) = \mu_{\mathrm{d}j} + \Theta_j(t) + \omega_{\mathrm{d}j} \cdot \sigma_{\mathrm{y}j} \cdot z_{\mathrm{y}j}(t-1) \\ + (1 + \phi_{\mathrm{d}j} \cdot \mathrm{L}) \cdot (\varphi_{\mathrm{d}j} \cdot \sigma_{\mathrm{y}j} \cdot z_{\mathrm{y}j}(t) + \sigma_{\mathrm{d}j} \cdot z_{\mathrm{d}j}(t)) \tag{3.2.5}$$

$$\Theta_k(t) = \beta_k \cdot \left(\frac{\theta_k}{1-(1-\theta_k)\cdot \mathrm{L}} \right) \cdot r_{\mathrm{q}}(t) + \beta_{k1} \cdot r_{\mathrm{q}}(t) \tag{3.2.6}$$

where $d_{\mathrm{s}}(t)$ represents the force of growth of equity dividends in year t, $d_{\mathrm{p}}(t)$ represents the force of growth of property income in year t, and L represents the lag operator.

The terms $\Theta_j(t)$ represent the inflationary component of equity dividend and property income growth and the remainder of equation 3.2.5 represents the real growth component. If $\beta_j + \beta_{j1} = 1$ then cash flows will grow in line with inflation, so that a 1% increase in inflation will eventually result in a 1% increase in dividends or property income. Wilkie (1995b: 840) suggested that the moving average component ϕ_{ds} could be justified by a tendency for boards of directors to smooth dividend payments over time.

Recommended parameter values are given in Table 3.2.2. 'Neutral' initial conditions are

Table 3.2.2 Parameter values for Wilkie's equity and property models

		Equity		*Property*
Parameter	*Full Std.*	*Reduced Std.*	*Wilkie (1995b)*	*Wilkie (1995b)*
$\mu_{\mathrm{y}j}$	0.04	0.04	0.0375	0.074
$\omega_{\mathrm{y}j}$	1.35	1.35	1.8	-
$\alpha_{\mathrm{y}j}$	0.6	0.6	0.55	0.91
$\sigma_{\mathrm{y}j}$	0.175	0.175	0.155	0.12
$\mu_{\mathrm{d}j}$	0	0	0.016	0.003
β_j	0.8	0.8	0.58	1
β_{j1}	0.2	0.2	$1-\beta_{\mathrm{ds}}$	$1-\beta_{\mathrm{dp}}$
θ_j	0.2	0.2	0.13	0.11
$\omega_{\mathrm{d}j}$	−0.2	−0.3	−0.175	-
$\phi_{\mathrm{d}j}$	0.375	0	0.57	-
$\varphi_{\mathrm{d}j}$	-	-	-	0.24
$\sigma_{\mathrm{d}j}$	0.075	0.1	0.07	0.06

(for $j = \mathrm{s}$ and p): $y_j(0) = \log_e \mu_{yj} + \omega_{yj} \cdot \mu_q$, $r_q(0) = \mu_q$, $z_{yj}(0) = z_{dj}(0) = 0$, $\Theta_{dj}(0) = (\beta_{dj} + \beta_{dj1}) \cdot \mu_q$.

3.2.3.3 Interest rate models

The long-term interest rate model is made up of a real interest rate component $Y_b(t) - \Theta_b(t)$ and a component representing investors' inflationary expectations $\Theta_b(t)$. This structure is intended to represent the Fisher relation (Fisher 1930), which implies that expectations about future inflation are fully reflected in security prices (see Appendix 2A.2). The logarithm of the real component is an AR(3) model with errors that are correlated with the equity dividend yield model's errors. The inflationary expectations component is obtained from the equity dividend and property income models (see equation 3.2.6). The model can be represented by the following equation (for $t > 0$):

$$\begin{aligned} \log_e(Y_b(t) - \Theta_b(t)) &= \log_e \mu_b + \varphi_b \cdot \sigma_{ys} \cdot z_{ys}(t) + \sigma_b \cdot z_b(t) \\ &+ (\alpha_b + \alpha_{b1} \cdot \mathrm{L} + \alpha_{b2} \cdot \mathrm{L}^2) \cdot (\log_e(Y_b(t-1) - \Theta_b(t-1)) - \log_e \mu_b) \end{aligned} \tag{3.2.7}$$

where $Y_b(t)$ represents the long-term interest rate at time t.

The short-term interest rate model describes the logarithm of the short-term interest rates less long-term interest rates as an AR(1) model. It can be represented by the following equation (for $t > 0$):

$$y_m(t) = \mu_m + y_b(t) + \alpha_m \cdot (y_m(t-1) - \mu_m - y_b(t-1)) + \sigma_m \cdot z_m(t) \tag{3.2.8}$$

where $y_m(t)$ represents the logarithm of the short-term interest rate at time t and $y_b(t)$ represents the logarithm of the long-term interest rate at time t.

The index-linked yield model is an AR(1) model with errors that are correlated with the long-term interest rate model's errors. It can be represented by the following equation (for $t > 0$):

$$y_r(t) = \log_e \mu_r + \alpha_r \cdot (y_r(t-1) - \log_e \mu_r) + \varphi_r \cdot \sigma_b \cdot z_b(t) + \sigma_r \cdot z_r(t) \tag{3.2.9}$$

where $y_r(t)$ represents the logarithm of the real yield on index-linked securities at time t.

Table 3.2.3 Parameter values for Wilkie's interest rate models

	Long-term interest rates			*Short-term*	*Index-linked*
Parameter	*Full Std.*	*Reduced Std.*	*Wilkie (1995b)*	*Wilkie (1995b)*	*Wilkie (1995b)*
μ_j	0.035	0.035	0.0305	−0.23	0.04
β_b	1	1	1	-	-
β_{b1}	-	-	-	-	-
θ_b	0.045	0.05	0.045	-	-
α_j	1.2	0.91	0.9	0.74	0.55
α_{b1}	−0.48	0	-	-	-
α_{b2}	0.2	0	-	-	-
φ_j	0.06	0	0.34	-	0.22
σ_j	0.14	0.165	0.185	0.18	0.05

The transformations used in the long-term interest rate model appear to be perverse because they allow negative nominal yields but prevent negative real yields. In practice, negative real yields are possible but negative nominal yields are not. It would be more consistent to use $\log_e Y_b(t) - \Theta_b(t)$ rather than $\log_e(Y_b(t) - \Theta_b(t))$ in equation 3.2.7.

Recommended parameter values are given in Table 3.2.3. A minimum value of 0.005 is postulated for $Y_b(t)$ to prevent negative nominal yields from occuring. 'Neutral' initial conditions are: $\Theta_b(0) = (\beta_b + \beta_{b1})\cdot\mu_q$, $y_m(0) - y_b(0) = \mu_m$, $y_r(0) = \log_e \mu_r$, $\log_e(Y_b(0) - \Theta_b(0)) = \log_e(Y_b(-1) - \Theta_b(-1)) = \log_e(Y_b(-2) - \Theta_b(-2)) = \log_e \mu_b$.

3.2.3.4 Currency exchange rate model

The currency exchange rate model is defined by the following equation (for $t > 0$):

$$\begin{aligned}\log_e R_{xj}(t) &= \mu_{xj} + \log_e R_{qj}(t) - \log_e R_q(t) + \sigma_{xj}\cdot z_{xj}(t) \\ &\quad + \alpha_{xj}\cdot(\log_e R_{xj}(t-1) - \mu_{xj} - \log_e R_{qj}(t-1) + \log_e R_q(t-1))\end{aligned} \tag{3.2.10}$$

where $R_{xj}(t)$ represents the pound sterling currency j exchange rate at time t, $R_{qj}(t)$ represents the price inflation index for country j at time t.

This model reflects the purchasing power parity hypothesis, which states that in the long term the exchange rate between two currencies is determined by the relative price levels in the two countries. This hypothesis was used even though it did not appear to be supported by the historical data.

Although Wilkie (1995b) fitted this model to numerous pound sterling exchange rates over the interval 1972-94, specific parameter values were not recommended and the results of goodness-of-fit tests were not reported. Furthermore, this model requires joint price inflation models for both countries, which were not specifically provided by Wilkie (1995b). Hence, there do not appear to be specific recommended parameter values for this model.

3.2.4 Theoretical properties

3.2.4.1 Fisher relation

The Fisher relation was explicitly included in the long-term interest rate model. This model assumes that the average future real return required by investors is given by $Y_b(t) - \Theta_b(t)$ and that investors' expectation of average future inflation is given by $\Theta_b(t)$. Figure 3.2.1 shows the values of these two components, over the interval 1923-1994, calculated using Wilkie's (1995b) long-term interest rate model. Thus, Wilkie's model implies that investors required average future real returns of over 10% in 1974, and returns of over 5% during most of the interval 1969-82. These returns appear to be

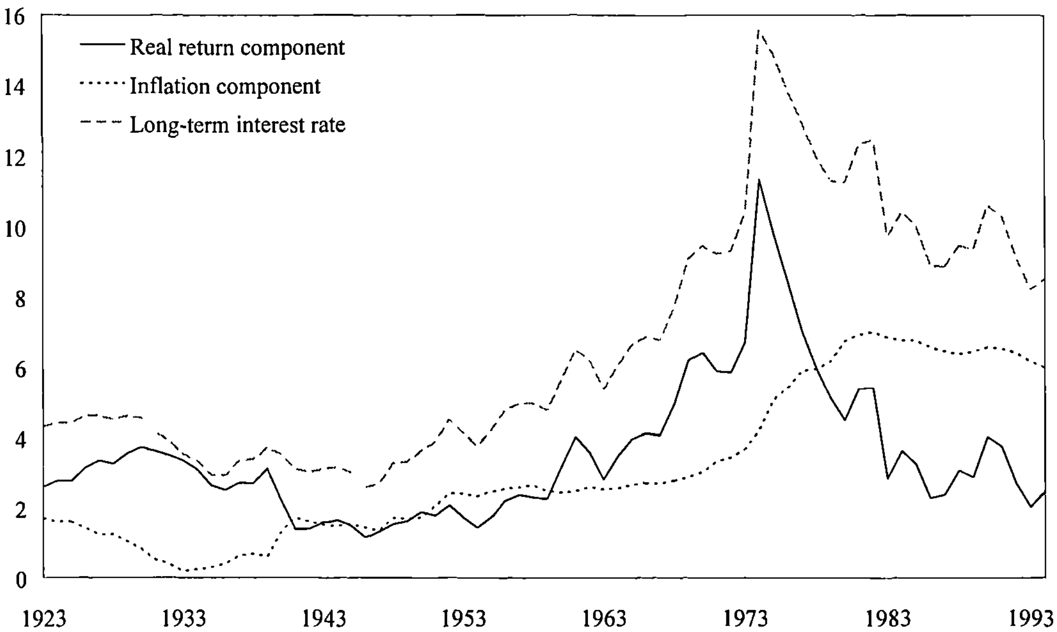

Figure 3.2.1 Expected price inflation and real returns, 1923-1994

high by historical standards. Moreover, this appears to contradict Wilkie's (1995a: 267) assertion that over the interval 1968-78: "the concept of 'negative real returns' was widely spoken of."

Furthermore, $\Theta_b(t)$ can be compared with the optimal estimate of average future inflation, which is given by:

$$\lim_{n\to\infty}\left\{E_t\left[\frac{1}{n}\cdot\sum_{r=1}^{n} r_q(t+r)\right]\right\} = \mu_q \tag{3.2.11}$$

where E_t represents the expectations operator conditional on all information available at time t.

This comparison is illustrated in Figure 3.2.2, which shows that Wilkie's model implies that investors consistently underestimated inflation over the interval 1923-75 and overestimated inflation since 1975. Furthermore, if Wilkie's model is true, then investors will continue to overestimate average future inflation by at least 0.5% until 2012. This contradicts the rational expectations hypothesis, which states that investors do not knowingly make systematic ex ante forecasting errors (see Section 2.4.1).

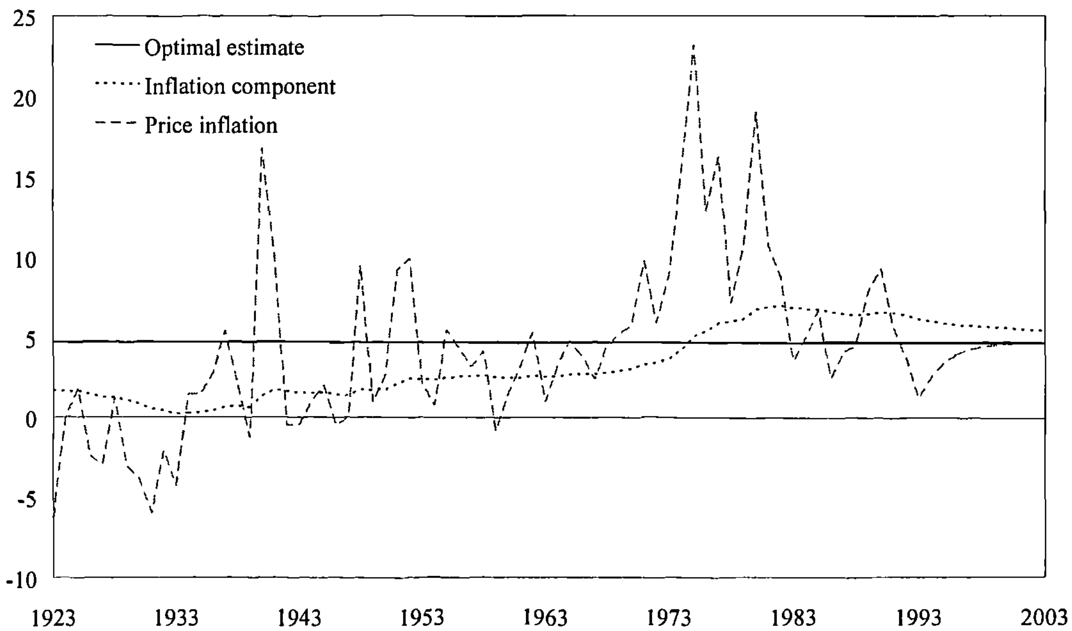

Figure 3.2.2 Expected price inflation, 1923-2003

If a stochastic asset model were to incorporate the rational expectations hypothesis, then the inflation expectation implicit in the long-term interest rate model would need to be constrained to be consistent with the price inflation model. This can be accomplished in a number of ways. For example, if Wilkie's long-term interest rate model were left unchanged then the following price inflation model would incorporate the rational expectations hypothesis:

$$r_q(t) = \Theta_b(t-1) + \upsilon_q(t) \cdot z_q(t) \tag{3.2.12}$$

Multiplying both sides of equation 3.2.12 by $(1 - (1 - \theta_b) \cdot L)$ and rearranging gives (see equation 3.2.6):

$$\begin{aligned} r_q(t) &= (1 - \theta_b + \beta_b \cdot \theta_b) \cdot r_q(t-1) \\ &\quad + (1 - (1 - \theta_b) \cdot L) \cdot (\beta_{b1} \cdot r_q(t-1) + \upsilon_q(t) \cdot z_q(t)) \end{aligned} \tag{3.2.13}$$

If $\beta_b = 1$ and $\beta_{b1} = 0$, as assumed by Wilkie, then equation 3.2.13 implies that price inflation is a non-stationary (integrated of order one) variable. This accords with the view that price inflation is largely determined by individuals' expectations and that these expectations are not necessarily consistent over time (Black 1986: 540). Numerous other authors have also suggested integrated price inflation models, including Osborn (1990), Franses and Paap (1994), and Clare *et al.* (1994). These suggestions were supported by the results of unit-root tests on the UK General Index of Retail Prices, which was first published in 1956 (see Appendix 8A). Moreover, Wilkie (1995a) argued that long term average price inflation has increased from roughly 0% over the interval 1660-1900 to roughly 5% since the 1950s. However, an integrated model implies that the variance of the predicted force of price inflation in year $t + k$ tends to infinity as k tends to infinity (see Appendix 3A.1). Consequently, the use of an integrated model implies that (Wilkie 1986a: 361):

> at term 100 the average mean rate of inflation is 7.69% with a standard deviation of 32.39%. The range is enormous, and includes both "hyper-inflations" and "hyper-deflations". These latter are a consequence of the model, but, unlike the former, which have in fact occurred, they seem to me to be economically unrealistic.

Furthermore, over the extended interval 1923-95, the Dickey-Fuller unit-root test (see Doornik and Hendry 1994) suggests that price inflation is not integrated. Hence, a univariate integrated ARMA price inflation model is probably not a reasonable long term model.

Another method of incorporating both the Fisher relation and the rational expectations hypothesis into the price inflation model is to set:

$$E_t[r_q(t+1)] = Y_b^*(t) - \mu_b + \alpha_b \cdot (r_q(t) - Y_b^*(t-1) + \mu_b) \tag{3.2.14}$$

where $Y_b^*(t)$ represents the long-term force of interest, μ_b is the mean real return, and α_b represents a possible real return autoregressive effect.

Alternatively, the following relationship would make the long-term interest rate model consistent with the inflation expectations implied by the price inflation model:

$$E_t[Y_b^*(t+1)] = \mu_q + \mu_b + \alpha_b \cdot (Y_b^*(t) - \mu_q - \mu_b) \tag{3.2.15}$$

The above relationships illustrate the types of asset models that are consistent with the rational expectations hypothesis (see also Chapter 4). They do not represent complete alternative models. The empirical adequacy of these theoretical relationships would need to be examined before they are included in a stochastic asset model.

3.2.4.2 Efficient market hypothesis

The efficient market hypothesis was considered in the development of the equity models (see Appendix 2A.1). However, the main implication of this hypothesis, that security returns cannot be forecast, was rejected on empirical grounds (see Ford *et al.* 1980). Thus, Wilkie's model assumes that security returns can be partially forecast (see Appendix 3A.1). But, over short time horizons Wilkie's model is virtually identical to a random walk, or martingale, model, which is consistent with the efficient market hypothesis (Wilkie 1995b: 826). This assumption is partially supported by other empirical tests of the efficient market hypothesis (see Section 6.2.1) and it was also suggested by Shiller (1981: 294).

However, whereas in practice it has been difficult for individual investors to consistently achieve above average returns (Malkiel 1990), Wilkie's model assumes that it is possible to achieve excess returns of roughly 3% for virtually no extra risk by switching between equity and fixed-interest securities (Wilkie 1986b; Smith 1996). This seems to be an unrealistic assumption. Consequently, Wilkie (1986b: 41) warned that his model should not be used to develop dynamic trading strategies. If Wilkie's (1986b) strategy of investing in the asset class with the highest expected return in the following year calculated using Wilkie's (1986a) model had been followed over the interval 1983-95, then an average return of 2% less than the return on equities would have been achieved.

This consequence of the model could be avoided if the expected real returns for each asset class are assumed to be constant over time. Alternatively, it would at least be necessary to assume that the expected risk premiums are consistent over time. These risk premiums could be assumed to be constant or a positive increasing function of the expected real returns. The latter assumption reflects the view that a payment has a lower utility when the economy is in a poorer state (Breeden 1979; see Appendix 2A.3). Hence, a higher risk premium is assumed to be required when real returns are expected to be relatively high. This assumes that the relative riskiness of securities is independent of the level of real returns. This view is implicit in the short-term interest rate model, which implies that the expected term premium or spread, $Y_{\mathrm{b}}(t) - Y_{\mathrm{m}}(t)$, is an increasing function of interest rates.

Another implication of the efficient market hypothesis is that prices respond to information about events when this information becomes known rather than when the events occur. As a result, equity price changes are likely to anticipate future changes in equity dividends because information affecting equity dividends is often available before the dividends are declared. Thus, Wilkie's model, by incorporating the term $\omega_{\mathrm{ds}} \cdot \sigma_{\mathrm{ys}} \cdot z_{\mathrm{ys}}(t-1)$ in equation 3.2.5, assumes that equity prices anticipate future changes in equity dividend growth rates.

3.2.4.3 Unit gain

Another hypothesis that was considered and included in the share dividend and property income models is that dividends and property income respond to inflation with 'unit gain'; so that a 1% increase in prices will eventually lead to a 1% increase in dividends and property income. This hypothesis is intuitively appealing, but it is not essential because "there is also a case for arguing that dividends 'in real terms' do better in times of stable prices than in periods of high and uncertain inflation" (Wilkie 1995b: 840).

3.2.4.4 Portfolio theory

Portfolio theory broadly states that investors select investments on the basis of their expected risk and return: investors will only include a security in their portfolio if its inclusion either increases the expected return or decreases the expected risk of the portfolio (see Huang and Litzenberger 1988). A common, though controversial, measure of risk is the standard deviation of return. Hence, assets with a lower expected return should either have a lower expected standard deviation of return or be sufficiently negatively correlated with other assets.

However, Wilkie's model assumes that the property asset class has a higher expected total return, a lower standard deviation of return, and a similar covariance structure compared to the equity asset class (see Tables 3.2.4 and 3.2.5). Furthermore, the index-linked asset class has a higher expected real return, a lower standard deviation, and a roughly similar covariance structure compared to the long-term fixed-interest asset class. Hence, there appears to be little incentive to invest in either the equity or the long-term fixed-interest asset classes (Smith 1996).

This possible weakness can be overcome by changing the mean or standard deviation parameters of the model. This accords with Wilkie's (1995b: 785) recommendation that users of his model "should form their own opinions about the choice of appropriate mean values." Specific values for these parameters could be obtained using Smith's (1996) 'equilibrium' method (see Section 4.3).

3.2.5 Statistical properties

3.2.5.1 Inflation models

Wilkie's original price inflation model and transfer function wage inflation model imply that the force of price inflation in year *t* is normally distributed (see Appendix 3A.1). From a neutral starting position and using the Wilkie (1995b) parameter values, the means of the force of price and wage inflation are constant and equal to 4.70% and

6.19%. The standard deviation of the force of price inflation in year 1 and year 20 is 4.25% and 5.22%. The standard deviation of the force of wage inflation in year 1 and year 20 is 3.45% and 4.73%. For any initial values, these means and standard deviations tend to 4.70% and 5.22% for price inflation and 6.19% and 4.73% for wage inflation. The rate of price and wage inflation in year t has a lognormal distribution.

Similar results would be obtained for the VAR model. The standard deviations for the ARCH model are far greater than those reported above (see Wilkie 1995b: 906).

3.2.5.2 Equity and property models

Using the original price inflation model, Wilkie's equity and property yield models imply that the force of price inflation in year t is normally distributed (see Appendix 3A.1). From a neutral starting position and using the Wilkie (1995b) parameter values, the means of the logarithm of the equity and property yields are constant and equal to $\log_e(0.0408)$ and $\log_e(0.0740)$. The standard deviation of the logarithm of the equity yield in year 1 and year 20 is 0.1729 and 0.2080. The standard deviation of the logarithm of the property yield in year 1 and year 20 is 0.1200 and 0.2861. For any initial values, these means and standard deviations tend to $\log_e(0.0408)$ and 0.2080 for equity yields and $\log_e(0.0740)$ and 0.2894 for property yields. Equity and property yields in year t have a lognormal distribution.

Furthermore, the force of equity dividend and property income growth in year t is normally distributed. The means of the force of equity dividend and property income growth are constant and equal to 6.30% and 5.00%. The standard deviation of the force of equity dividend growth in year 1 and year 20 is 7.31% and 9.08%. The standard deviation of the force of property income growth in year 1 and year 20 is 6.67% and 7.01%. For any initial values, these means and standard deviations tend to 6.30% and 9.08% for equity dividends and 5.00% and 7.02% for property income. Equity dividend and property income growth rates in year t have a lognormal distribution.

3.2.5.3 Interest rate models

Using the original price inflation model, Wilkie's long-term interest rate model implies that the distribution of long-term interest rate in year t is the sum of a normal and lognormal distribution (see Appendix 3A.1). From a neutral starting position and using the Wilkie (1995b) parameter values, the mean and standard deviation of the long-term interest rate in year 1 are 7.81% and 0.63% and in year 20 are 8.06% and 2.03%. For any initial values, these means and standard deviations tend to 8.06% and 2.15%.

As a result of the non-linear transformations used in the long-term interest rate model, it appears to be virtually impossible to analytically determine the distribution of the short-term interest rate in year t. Hence, it seems preferable to analyse the properties of this model using Monte Carlo simulation techniques (see Section 3.2.5.5).

Wilkie's index-linked yield model implies that the logarithm of the yield on index-linked securities in year t is normally distributed (see Appendix 3A.1). From a neutral starting position and using the Wilkie (1995b) parameter values, the mean of the logarithm of the yield on index-linked securities is constant and is equal to $\log_e(0.0400)$. The standard deviation of the logarithm of the index-linked yield in year 1 and year 20 is 0.0645 and 0.0772. For any initial values, these means and standard deviations tend to $\log_e(0.0400)$ and 0.0772. The yield on index-linked securities in year t has a lognormal distribution.

3.2.5.4 Currency exchange rate model

Wilkie's currency exchange rate model implies that the logarithm of the pound sterling currency j exchange rate in year t is normally distributed (see Appendix 3A.1). The pound sterling currency j exchange rate in year t has a lognormal distribution.

3.2.5.5 Averaged total returns

Using the original price inflation model and the Wilkie (1995b) parameter values, Tables 3.2.4 to 3.2.7 illustrate the means, standard deviations, and correlations of nominal and real averaged total returns over various intervals. These tables were obtained from Wilkie (1995b: 905-6) who derived them using Monte Carlo simulation techniques. The means and standard deviations of averaged price inflation could have been calculated analytically and tend to 4.81% and zero as the averaging interval tends to infinity (see Appendix 3A.2).

Table 3.2.4 Means and standard deviations of nominal averaged total returns

Asset Class	*Statistic*	1	2	5	10	20	50
				Term			
Prices	*mean*	5.00	4.97	4.85	4.74	4.77	4.80
	std dev	4.45	4.14	3.71	2.99	2.28	1.47
Short-term	*mean*	6.16	6.22	6.34	6.42	6.48	6.53
	std dev	0.00	0.62	1.07	1.28	1.32	1.16
Long-term	*mean*	8.03	7.86	7.74	7.89	7.92	7.94
	std dev	7.92	5.47	2.92	1.70	1.05	1.09
Index-linked	*mean*	9.45	9.46	9.01	8.89	8.97	8.99
	std dev	8.19	5.78	4.15	3.22	2.39	1.53
Equity	*mean*	13.20	11.90	11.04	10.91	10.75	10.79
	std dev	19.47	12.71	7.41	4.80	3.48	2.31
Property	*mean*	13.97	13.66	13.22	13.16	13.07	13.16
	std dev	14.76	8.92	4.59	3.21	2.80	2.31

Source: Wilkie (1995b: 904)

Table 3.2.5 Correlations between nominal averaged total returns

Asset Class					*Term*		
j	*l*	1	2	5	10	20	50
Short-term	*Prices*	0.00	0.08	0.17	0.33	0.45	0.56
Long-term	*Prices*	−0.32	−0.39	−0.55	−0.55	−0.16	0.46
	Short-term	0.00	−0.19	−0.28	−0.25	0.24	0.77
Index-linked	*Prices*	0.56	0.75	0.93	0.97	0.99	0.99
	Short-term	0.00	0.02	0.14	0.32	0.45	0.58
	Long-term	0.30	0.06	−0.34	−0.43	−0.09	0.49
Equity	*Prices*	−0.26	−0.06	0.17	0.34	0.52	0.62
	Short-term	0.00	−0.01	−0.00	0.09	0.25	0.35
	Long-term	0.30	0.27	0.05	−0.06	0.07	0.33
	Index-linked	−0.14	−0.00	0.15	0.33	0.52	0.61
Property	*Prices*	0.06	0.09	0.25	0.49	0.61	0.59
	Short-term	0.00	0.01	0.08	0.16	0.27	0.39
	Long-term	0.02	−0.03	−0.13	−0.27	−0.01	0.35
	Index-linked	0.11	0.09	0.23	0.47	0.60	0.59
	Equity	−0.00	0.04	0.07	0.21	0.35	0.38

Source: Wilkie (1995b: 904)

Table 3.2.6 Means and standard deviations of real averaged total returns

		Term					
Asset class	*Statistic*	1	2	5	10	20	50
Short-term	*mean*	1.29	1.35	1.54	1.67	1.67	1.66
	std dev	4.29	3.99	3.57	2.76	1.99	1.22
Long-term	*mean*	3.18	2.99	2.94	3.11	3.06	3.01
	std dev	9.89	7.79	5.68	4.08	2.60	1.34
Index-linked	*mean*	4.24	4.28	3.96	3.96	4.01	4.00
	std dev	6.47	3.64	1.46	0.76	0.38	0.17
Equity	*mean*	8.21	6.80	5.99	5.93	5.72	5.72
	std dev	20.25	13.06	7.41	4.57	2.89	1.73
Property	*mean*	8.70	8.42	8.07	8.08	7.94	7.99
	std dev	14.55	9.14	5.03	3.12	2.24	1.79

Source: Wilkie (1995b: 905)

Table 3.2.7 Correlations between real averaged total returns

Asset class		Term					
j	*l*	1	2	5	10	20	50
Short-term	*Prices*	−1.00	−0.99	−0.96	−0.91	−0.82	−0.66
Long-term	*Prices*	−0.68	−0.78	−0.91	−0.94	−0.92	−0.72
	Short-term	0.69	0.76	0.84	0.84	0.84	0.84
Index-linked	*Prices*	−0.01	0.00	0.00	0.01	−0.03	−0.04
	Short-term	0.01	−0.01	−0.02	−0.00	0.08	0.19
	Long-term	0.45	0.36	0.23	0.15	0.17	0.25
Equity	*Prices*	−0.46	−0.37	−0.34	−0.32	−0.19	−0.07
	Short-term	0.46	0.37	0.32	0.29	0.17	0.05
	Long-term	0.47	0.44	0.38	0.36	0.25	0.09
	Index-linked	0.01	0.06	−0.02	0.00	0.05	−0.07
Property	*Prices*	−0.25	−0.38	−0.54	−0.51	−0.32	−0.11
	Short-term	0.25	0.37	0.52	0.46	0.26	0.14
	Long-term	0.20	0.30	0.49	0.48	0.34	0.15
	Index-linked	0.08	0.03	−0.00	−0.02	0.02	0.02
	Equity	0.12	0.18	0.21	0.21	0.10	0.02

Source: Wilkie (1995b: 905)

For the other asset classes the means and standard deviations of nominal and real averaged returns generally decrease over increasing intervals. The decrease in the standard deviations is more noticeable than the decrease in the means.

The correlations between nominal averaged returns all generally increase over increasing intervals except for the correlations with long-term fixed-interest and index-linked security returns, which decrease before increasing. Over a 20 year interval the correlations between all the asset class returns and the return on long-term fixed-interest securities are negative. Over a 50 year interval, all the correlations are positive. The correlations between price inflation and security returns are initially negative for all asset classes except property. The correlations between equity and index-linked returns are also initially negative.

The correlations between real averaged returns all generally decrease over increasing intervals except for the correlation between the returns on short-term and long-term fixed-interest securities and for the correlations with property, which increase initially. Over a 50 year interval there is little correlation between any of the returns except for between short-term and long-term fixed-interest securities and between price inflation and the return on short-term and long-term fixed-interest securities. There is little correlation between index-linked real returns and the returns on the other asset classes over all the intervals. The correlations between price inflation and the real returns of all the asset classes are generally negative.

3.2.6 Extensions to Wilkie's model

To address the perceived inability of Wilkie's price inflation model to adequately allow for extended periods of high inflation and inflation shocks Clarkson (1991) developed a non-linear inflation model. This model explicitly models shocks using a Bernoulli random variable and periods of high inflation using a positively biased trend term. This model is given by (for $t > 0$):

$$r_q(t) = \mu_q + \alpha_q \cdot (r_q(t-1) - \mu_q) + \vartheta_q \cdot \mathrm{T}^+\{r_q(t)\} + \varpi_q \cdot \zeta_q(t) + \varsigma_q(t) \cdot z_q(t) \qquad (3.2.16)$$

where $T^{+}\{r_q(t)\}$ represents the trend of the force of inflation series if it is positive and zero otherwise: this trend is calculated using a geometrically weighted least squares regression line with a recommended parameter of 0.5; $\varsigma_q(t)$ represents 50% of this average, subject to a minimum of 0.015; and, $\zeta_q(t)$ is zero if it was equal to one in the previous four time periods otherwise it is a Bernoulli random variable that takes a value of 1 with probability 0.06. Recommended parameter values are: $\mu_q = 0.04$, $\alpha_q = 0.5$, $\vartheta_q = 1$, $\varpi_q = 0.1$.

Clarkson's model is a non-standard econometric model and it is thus difficult to investigate its empirical adequacy (see Wilkie 1995b: 803). As a result, the recommended parameter values were largely determined using subjective judgement. Nevertheless, the model appears to capture some of the important features of price inflation.

Ong (1994) proposed the following short-term interest rate model as an extension to Wilkie's original model (for $t > 0$):

$$Y_m(t) = Y_b(t) + \alpha_m \cdot (Y_m(t-1) - Y_b(t-1)) + \sigma_m \cdot z_m(t) \qquad (3.2.17)$$

This model is similar to Wilkie's short-term interest rate model (see equation 3.2.8) except that yields are modelled rather than the logarithm of the yields. This model was fitted to UK Treasury Bill data over the interval 1955-93 and suitable parameter values were found to be $\alpha_m = 0.4$ and $\sigma_m = 0.02$. This particular data interval was chosen to avoid the period during and after the Second World War in which short-term interest rates were fixed by the Government.

3.3 Other Models

3.3.1 Introduction

This section briefly defines a number of other stochastic asset models that have been developed primarily from data considerations. The structure of most of these models is broadly comparable with the structure of Wilkie's model. These models are not examined in detail mainly because they were not specifically developed for UK actuarial use.

3.3.2 The Finnish insurance modelling group model

This model has been developed in a number of publications, including Pentikäinen *et al.* (1994, 1995), Pukkila *et al.* (1994), and Daykin *et al.* (1994). The version of this model that was reported in Pentikäinen *et al.* (1995) is briefly described in this section. The short-term interest rate model and its parameters were obtained from Pentikäinen *et al.* (1994). The model was primarily developed for the Finnish economy, but data from other developed economies were considered. It is broadly comparable with Wilkie's model and consists of a price inflation model, a long-term and short-term interest rate model, an equity price and dividend yield model, and a property price and yield model. These models appear to have been developed by first selecting appropriate transformations and transfer functions and then fitting the 'best' autoregressive model using the Bayesian Information Criterion (see Pukkila *et al.* 1994).

Following Wilkie, the price inflation model is independent of the other models and is defined by the following equation (for $t > 0$):

$$r_{\mathrm{q}}(t) = \mu_{\mathrm{q}} + \alpha_{\mathrm{q}} \cdot (r_{\mathrm{q}}(t-1) - \mu_{\mathrm{q}}) + \eta_{\mathrm{q}}(t) \tag{3.3.1}$$

where $r_{\mathrm{q}}(t)$ represents the force of price inflation in year t, $\eta_j(t)$ are sequences of independently distributed shifted gamma random variables with mean zero, standard deviation σ_j, and skewness γ_j (see Daykin *et al.* 1994: 84). Recommended parameter values for all the models are given in Table 3.3.1.

The long-term interest rate model uses Wilkie's inflation expectations component (see equation 3.2.6) and is defined by the following equation (for $t > 0$):

$$Y_{\mathrm{b}}(t) = \mu_{\mathrm{b}} + \Theta_{\mathrm{b}}(t) + \alpha_{\mathrm{b}} \cdot (Y_{\mathrm{b}}(t-1) - \mu_{\mathrm{b}} - \Theta_{\mathrm{b}}(t-1)) + \eta_{\mathrm{b}}(t) \tag{3.3.2}$$

where $Y_{\mathrm{b}}(t)$ represents the long-term interest rate at time t.

Table 3.3.1 Parameter values for the Finnish insurance modelling group model

Parameter	k = q	k = m	k = b	k = ps	k = ys	k = pp	k = yp
μ_k	0.04	0	0.03	0.035	0.04	0.035	0.04
α_k	0.8	-	0.7	0.85	0.75	1.1	0.75
α_{k1}	-	-	-	0	-	−0.4	-
ω_k	-	0.751	-	0	-	0	-
ω_{k1}	-	-	-	−5	-	−3	-
ω_{k2}	-	-	-	−5	-	−7	-
ψ_k	-	-	-	-	0.3	-	0.2
θ_k	-	-	0.15	-	-	-	-
β_k	-	-	1	-	-	-	-
β_{k1}	-	-	0	-	-	-	-
σ_k	0.01	0.012	0.008	0.10	0.003	0.07	0.004
γ_k	0.5	0	0.7	0	0	0.4	0

The short-term interest rate model assumes that real short-term and long-term interest rates are contemporaneously correlated and is similar to the model suggested by Ong (1994). It is defined by the following equation (for $t > 0$):

$$Y_{\mathrm{m}}(t) = \mu_{\mathrm{m}} + r_{\mathrm{q}}(t) + \omega_{\mathrm{m}} \cdot (Y_{\mathrm{b}}(t) - r_{\mathrm{q}}(t)) + \eta_{\mathrm{m}}(t) \tag{3.3.3}$$

where $Y_{\mathrm{b}}(t)$ represents the short-term interest rate at time t.

The equity and property price index models assume that real equity and property prices increase by a constant trend and a variable growth rate related to long-term interest rates. They are defined by the following equations (for $t > 0, j$ = s and p):

$$\log_e P_j(t) = t \cdot \log_e(1 + \mu_{\mathrm{p}j}) + \log_e R_{\mathrm{q}}(t) + \log_e \Pi_{\mathrm{p}j}(t) \tag{3.3.4}$$

$$\begin{aligned} \log_e \Pi_k(t) &= (\omega_k + \omega_{k1}\mathrm{L} + \omega_{k2}\mathrm{L}^2) \cdot (Y_{\mathrm{b}}(t) - Y_{\mathrm{b}}(0)) \\ &+ (\alpha_k + \alpha_{k1}\mathrm{L}) \cdot (\log_e \Pi_k(t-1) - (\omega_k + \omega_{k1}\mathrm{L} + \omega_{k2}\mathrm{L}^2) \cdot (Y_{\mathrm{b}}(t-1) - Y_{\mathrm{b}}(0))) \\ &+ \eta_k(t) \end{aligned} \tag{3.3.5}$$

where $P_j(t)$ represents the price index of asset class j at time t and $R_{\mathrm{q}}(t)$ represents the price inflation index at time t.

The equity and property yield models assume that expected equity and property yields are related to their respective trend adjusted real price indices. They are defined by the following equations (for $t > 0, j$ = s and p):

$$Y_j(t) = \mu_{yj} \cdot \left(\frac{\Psi_{yj}(t)}{\Pi_{pj}(t)} \right) + \alpha_{yj} \cdot \left(Y_j(t-1) - \mu_{yj} \cdot \left(\frac{\Psi_{yj}(t-1)}{\Pi_{pj}(t-1)} \right) \right) + \eta_{yj}(t) \qquad (3.3.6)$$

where:

$$\Psi_{yj}(t) = \left(\frac{\psi_{yj}}{1 - (1 - \psi_{yj}) \cdot \mathrm{L}} \right) \cdot \Pi_{pj}(t) \qquad (3.3.7)$$

where $Y_j(t)$ represents the yield on asset class j at time t.

3.3.3 Australian models

Australian stochastic asset models have been developed by, amongst others, Carter (1991), FitzHerbert (1992), and Harris (1994, 1995a, 1995b). Carter (1991) initially attempted to fit Wilkie's (1986a) model to quarterly Australian data, but found that it was unsuitable. Using the Box-Jenkins transfer function methodology, Carter (1991) developed an alternative Australian model. The series' were transformed into force of return or force of interest rate series. The force of price inflation, the force of short-term interest rate, and the force of long-term interest rate series' were differenced; which implies that these models are non-stationary. Seasonal terms were not specifically included: Wilkie (1995b: 792) commented that this may have led Carter (1991) to over-difference the price inflation series. The only transfer functions included in this model were a short-term interest rate transfer function in the long-term interest rate model and price inflation transfer functions in the short-term interest rate model, the equity dividend yield model, and the property total return model. The equity price model is independent of all the other models.

FitzHerbert (1992) suggested that trend and mean adjusted real price indices should be modelled as stationary autoregressive models. The general form of these models is given by the following equation (for $t > 0$):

$$\log_e P_j(t) = \mu_j + \lambda_j \cdot t + \omega_j \cdot \log_e R_q(t) + \zeta_j(t) \qquad (3.3.8)$$

where $P_j(t)$ represents the price index for asset class j at time t, $R_q(t)$ represents the price inflation index at time t, and $\zeta_j(t)$ represents an autoregressive process at time t. For commodities: $\lambda_j = 0$ and $\omega_j = 1$. For property: $\lambda_p < 0$, to allow for the depreciation of buildings over time, and $\omega_p = 1$. Equity prices were assumed to have an allowance for retained profits represented by λ_s and a partial allowance for inflation, represented by ω_s. It was argued that equity prices do not move in line with inflation, as was assumed by Wilkie (1995b), because company assets are generally treated in money terms under historical cost accounting. FitzHerbert (1992) argued that equity prices have only increased at the rate of retention of company profits.

Harris (1995b) developed an exponential ARCH model, termed an ERCH model. This model was fitted to Australian data and consists of a real GDP growth rate model, a price inflation model, a short-term interest rate model, a model for 2 and 10 year interest rates, and an equity price and dividend yield model. These models were found to fit the Australian data better than Wilkie's model. Harris (1996) has also considered regime switching models and has found them to be useful. These regime switching models are not reported here because, at this stage, only univariate models have been fitted.

Harris' ERCH models can be represented by the following equations (for $t > 0$):

$$\boldsymbol{X}(t) = \mathbf{M} + \mathbf{A} \cdot \mathbf{Y}(t-1) + \mathbf{x}(t) \tag{3.3.9}$$

$$\mathbf{x}(t) = \mathbf{F}(t-1) \cdot \boldsymbol{Z}(t) \tag{3.3.10}$$

where:

$$\mathbf{E} \le \log_e \mathbf{F}(t) = \operatorname{diag}\{\mathbf{B} + \mathbf{K} \cdot \mathbf{W}(t-1)\} \le \mathbf{H} \tag{3.3.11}$$

$$\boldsymbol{Z}(t) \sim \mathrm{N}(\mathbf{0}, \mathbf{S} = \mathbf{L} \cdot \mathbf{L}^{\mathrm{T}}) \tag{3.3.12}$$

$$\boldsymbol{X}(t)^{\mathrm{T}} = [r_{\mathrm{g}}(t) \quad r_{\mathrm{q}}(t) \quad p_{\mathrm{s}}(t) \quad y_{\mathrm{s}}(t) \quad y_{\mathrm{m}}(t) \quad y_{\mathrm{b2}}(t) \quad y_{\mathrm{b10}}(t)] \tag{3.3.13}$$

$$\mathbf{x}(t)^{\mathrm{T}} = [\xi_{\mathrm{g}}(t) \quad \xi_{\mathrm{q}}(t) \quad \xi_{\mathrm{ps}}(t) \quad \xi_{\mathrm{ys}}(t) \quad \xi_{\mathrm{m}}(t) \quad \xi_{\mathrm{b2}}(t) \quad \xi_{\mathrm{b10}}(t)] \tag{3.3.14}$$

$$\mathbf{F}(t)^{\mathrm{T}} = [r_{\mathrm{g}}(t) \quad r_{\mathrm{q}}(t) \quad \log_e|\xi_{\mathrm{m}}(t)| \quad \log_e|\xi_{\mathrm{b2}}(t)| \quad \log_e|\xi_{\mathrm{b10}}(t)| \quad p_{\mathrm{s}}(t-1)] \tag{3.3.15}$$

$$
\mathbf{Y}(t) = \begin{bmatrix} r_{\mathrm{g}}(t) - \mathrm{M}_{\mathrm{g}} \\ r_{\mathrm{q}}(t) - \mathrm{M}_{\mathrm{q}} \\ p_{\mathrm{s}}(t) - \mathrm{M}_{\mathrm{ps}} \\ y_{\mathrm{s}}(t) - \mathrm{M}_{\mathrm{ys}} \\ y_{\mathrm{m}}(t) - \mathrm{M}_{\mathrm{m}} \\ y_{\mathrm{b2}}(t) - \mathrm{M}_{\mathrm{b2}} \\ y_{\mathrm{b10}}(t) - \mathrm{M}_{\mathrm{b10}} \\ \xi_{\mathrm{g}}(t) \\ r_{\mathrm{g}}(t) \cdot p_{\mathrm{s}}(t) - r_{\mathrm{g}}(t-1) \cdot p_{\mathrm{s}}(t-1) \\ y_{\mathrm{m}}(t) - y_{\mathrm{m}}(t-1) \\ r_{\mathrm{g}}(t-1) - \mathrm{M}_{\mathrm{g}} \\ p_{\mathrm{s}}(t-1) - \mathrm{M}_{\mathrm{ps}} \end{bmatrix} \tag{3.3.16}
$$

where $r_{\mathrm{g}}(t)$ represents the force of real GDP growth in year t, $r_{\mathrm{q}}(t)$ represents the force of price inflation in year t, $p_{\mathrm{s}}(t)$ represents the force of equity price growth in year t, $y_{\mathrm{s}}(t)$ represents the logarithm of the equity dividend yield at time t, $y_{\mathrm{m}}(t)$ represents the logarithm of the short-term interest rate at time t, $y_{\mathrm{b2}}(t)$ and $y_{\mathrm{b10}}(t)$ represent the logarithm of the 2 and 10 year interest rate at time t, $|\cdot|$ denotes the absolute value, and $\mathrm{X}_{i,j}$ denotes the (i,j)th element of matrix $\mathbf{X}$. Recommended parameter values are given in Tables 3.3.2 and 3.3.3.

Table 3.3.2 Parameter values for Harris' (1995b) ERCH model

Parameter	k = g	k = q	k = ps	k = ys	k = m	k = b2	k = b10
M_k	0.038	0.061	0.071	−3.05	−2.87	−2.72	−2.615
B_k	−3.98	−4.49	−5.65	−2.29	−0.885	−1.36	−1.44
E_k	−3.98	−4.57	−3.00	−3.15	−9.62	−5.53	−10.04
H_k	−3.98	−2.50	−1.576	−1.14	−1.01	−1.47	−1.76
$\mathrm{A}_{k,1}$	0	0.43	0	0	0	0	0
$\mathrm{A}_{k,2}$	−0.28	0.86	0	0	0	0	0.24
$\mathrm{A}_{k,3}$	0.075	0	0	0	0	0	0
$\mathrm{A}_{k,4}$	0	0	0.256	0.5	0	0	0
$\mathrm{A}_{k,5}$	0	0	0	0	0.86	0	0
$\mathrm{A}_{k,6}$	0	0	0	0	0	0.92	0
$\mathrm{A}_{k,7}$	0	0	0	0	0	0	0.92
$\mathrm{A}_{k,8}$	0.38	0	0	0	0	0	0
$\mathrm{A}_{k,9}$	0	0	3.59	0	0	0	0
$\mathrm{A}_{k,10}$	0	0	0	0.32	0	0	0
$\mathrm{A}_{k,11}$	0	0	−1.20	0	0	0	0
$\mathrm{A}_{k,12}$	0	0	0	0	0.44	0.35	0.23

Table 3.3.3 Parameter values for Harris' (1995b) ERCH model

Parameter	k = g	k = q	k = ps	k = ys	k = m	k = b2	k = b10
$K_{k,1}$	0	0	127	0	0	0	0
$K_{k,2}$	0	10.65	0	0	0	0	0
$K_{k,3}$	0	0	0	0	0.35	0	0
$K_{k,4}$	0	0	0	0	0	0.17	0
$K_{k,5}$	0	0	0	0	0	0	0.27
$K_{k,6}$	0	0	0	2.2	0	0	0
$\Lambda_{k,1}$	1	0.35	0	0	0	0	0
$\Lambda_{k,2}$	0	0.93675	0	0	0	0.320256	0.427008
$\Lambda_{k,3}$	0	0	1	−0.7	0	0	−0.3
$\Lambda_{k,4}$	0	0	0	0.714143	0.490098	0.420084	0.196039
$\Lambda_{k,5}$	0	0	0	0	0.871667	0.624226	0.520751
$\Lambda_{k,6}$	0	0	0	0	0	0.575593	0.357594
$\Lambda_{k,7}$	0	0	0	0	0	0	0.538681

3.3.4 A South African model

Thomson (1994) developed a stochastic asset model based on South African data using the Box-Jenkins transfer function methodology. Data was only available from 1961 for price inflation, equity dividend yields and growth rates, and short-term and long-term interest rates. For property, data was only available from 1968. Consequently, the model may be unreliable for long term projections. Furthermore, the socio-political changes that have recently occurred in South Africa may have influenced the stability of the historical relationships between the variables.

The equity dividend and dividend yield models are independent of the other models and are defined by the following equations (for $t > 0$):

$$d_s(t) = \mu_{ds} + (1 - \phi_{ds} \cdot L) \cdot \sigma_{ds} \cdot \varepsilon_{ds}(t) \tag{3.3.17}$$

$$y_s(t) = \mu_{ys} + \alpha_{ys} \cdot (y_s(t) - \mu_{ys}) + \sigma_{ys} \cdot z_{ys}(t) \tag{3.3.18}$$

where $d_s(t)$ represents the force of growth of equity dividends in year t, $y_s(t)$ represents the logarithm of the equity dividend yield at time t, $z_j(t)$ are sequences of independently distributed unit normal random variables, and $\varepsilon_{ds}(t)$ is a sequence of translated beta distributed error terms (see Thomson 1994: 31). Recommended parameter values for all the models are given in Table 3.3.4. The probability density function of $\varepsilon_{ds}(t)$ is given by (for $-2.1 \leq w \leq 6$):

$$f_{ds}(w) = 2.22215 \cdot 10^{-6} \cdot (2.1 + w)^{1.6} \cdot (6 - w)^{6} \tag{3.3.19}$$

The price inflation model is defined by the following equation (for $t > 0$):

$$\begin{aligned} r_q(t) &= \mu_q + \alpha_q \cdot (r_q(t-1) - \mu_q) \\ &\quad + (\omega_q - (\omega_{q1} + \alpha_q \cdot \omega_q) \cdot \mathrm{L} + \alpha_q \cdot \omega_{q1} \cdot \mathrm{L}^2) \cdot (d_s(t) - \mu_{ds}) + \sigma_q \cdot z_q(t) \end{aligned} \tag{3.3.20}$$

where $r_q(t)$ represents the force of price inflation in year t.

The long-term and short-term interest rate models use Wilkie's inflation expectation component (see equation 3.2.6) and can be represented by (for $t > 0$):

$$\log_e(1 + Y_b(t)) = \mu_b + \Theta_b(t) + (1 - \phi_b \cdot \mathrm{L}) \cdot \sigma_b \cdot \varepsilon_b(t) \tag{3.3.21}$$

$$\begin{aligned} \log_e(1 + Y_m(t)) &= \mu_m + \Theta_m(t) + \omega_m \cdot (d_s(t) - \mu_{ds}) + (1 - \phi_m \cdot \mathrm{L}) \cdot \sigma_m \cdot z_m(t) \\ &\quad + \omega_{ml} \cdot (\log_e(1 + Y_b(t)) - \mu_b - \Theta_b(t-1)) \end{aligned} \tag{3.3.22}$$

where $Y_b(t)$ and $Y_m(t)$ represent the long-term and short-term interest rates at time t, and $\varepsilon_b(t)$ is a sequence of Pearson type VII distributed error terms (see Thomson 1994: 31). The probability density function of $\varepsilon_b(t)$ is given by:

$$f_b(w) = 0.4365 \cdot (1 + 0.1406 \cdot w^2)^{-5} \tag{3.3.23}$$

Table 3.3.4 Parameter values for Thomson's (1994) model

Parameter	k = q	k = m	k = b	k = ds	k = ys	k = dp	k = yp
μ_k	0.092	0.028	0.041	0.093	1.634	1.87	0.096
α_k	0.899	-	-	-	0.810	0.680	-
ϕ_k	-	−0.531	−0.618	−0.651	-	-	−0.606
ω_k	0.088	−0.091	-	-	-	-	0.545
ω_{k1}	−0.077	0.885	-	-	-	-	-
θ_k	-	0.15	0.15	-	-	0.26	-
β_k	-	0.94	0.84	-	-	2.15	-
β_{k1}	-	0	0	-	-	0	-
σ_k	0.020	0.019	0.010	0.116	0.198	0.061	0.068

The direct property model can be represented by the following equations (for $t > 0$):

$$y_{\mathrm{p}}(t) = \mu_{\mathrm{yp}} + \Theta_{\mathrm{yp}}(t) + \alpha_{\mathrm{yp}} \cdot (y_{\mathrm{p}}(t-1) - \mu_{\mathrm{yp}} - \Theta_{\mathrm{yp}}(t-1)) + \sigma_{\mathrm{yp}} \cdot z_{\mathrm{yp}}(t) \qquad (3.3.24)$$

$$d_{\mathrm{p}}(t) = \mu_{\mathrm{dp}} + \omega_{\mathrm{dp}} \cdot (y_{\mathrm{p}}(t) - y_{\mathrm{p}}(t-1)) + (1 - \phi_{\mathrm{dp}}) \cdot \sigma_{\mathrm{dp}} \cdot z_{\mathrm{dp}}(t) \qquad (3.3.25)$$

where $y_{\mathrm{p}}(t)$ represents the logarithm of the yield on direct property at time t and $d_{\mathrm{p}}(t)$ represents the force of growth of property income on direct property in year t. Thomson (1994) also developed a property unit trust model, which is not reported here.

3.4 Summary

The stochastic models described in this chapter were developed and motivated primarily on the grounds of the probabilistic structure of the historical data. Economic theory was considered, but these models were not specifically designed to be consistent with a particular theoretical framework. Hence, these models can only be partially interpreted. Chapters 5, 6, and 7 examine the significance of this and review this approach from a methodological perspective.

The principle UK actuarial stochastic model, Wilkie's (1995b) model, was examined in more detail. Its long-term interest rate model implies that investors' expectations of average future price inflation are not consistent with the price inflation models. Hence, the model does not incorporate the rational expectations hypothesis. The long-term interest rate model's transformations are illogical because they prevent negative real yields but permit negative nominal yields. Furthermore, the model is not consistent with the efficient market hypothesis nor does it appear to be consistent with certain aspects of portfolio theory. The importance of these findings is discussed in Chapter 6 and Chapter 8 reviews the empirical adequacy of Wilkie's model.

Appendix 3A: Statistical Properties of Wilkie's Model

3A.1 Distribution of the predicted values

3A.1.1 Transfer function inflation models

As shown by Kitts (1988), Hürlimann (1992), and Wilkie (1995b) the distribution of the force of price inflation in year t, for the original price inflation model, is given by (for $\alpha_q \neq \pm 1$ and $t > 0$):

$$r_q(t) \sim \mathrm{N}\left(\mu_q + \alpha_q^t \cdot (r_q(0) - \mu_q), \frac{\sigma_q^2 \cdot (1 - \alpha_q^{2 \cdot t})}{1 - \alpha_q^2}\right) \tag{3A.1}$$

where $\mathrm{N}(\mu, \sigma^2)$ represents a normal distribution with mean μ and standard deviation σ.

For $\alpha_q = 1$ and $t > 0$:

$$r_q(t) \sim \mathrm{N}(r_q(0), t \cdot \sigma_q^2) \tag{3A.2}$$

The distribution of the predicted values of the force of wage inflation in year t, for the transfer function wage inflation model and the original price inflation model, is given by (for $\alpha_q \neq \pm 1$ and $t > 0$):

$$r_w(t) \sim \mathrm{N}(\mu_w(t), \sigma_w^2(t)) \tag{3A.3}$$

where:

$$\mu_w(t) = \mu_w + (\omega_w + \omega_{w1}) \cdot \mu_q + \alpha_q^{t-1} \cdot (\omega_w \cdot \alpha_q + \omega_{w1}) \cdot (r_q(0) - \mu_q) \tag{3A.4}$$

$$\sigma_w^2(1) = \sigma_w^2 + \omega_w^2 \cdot \sigma_q^2 \tag{3A.5}$$

$$\sigma_w^2(t+1) = \sigma_w^2 + \sigma_q^2 \cdot \left(\omega_w^2 + (\omega_w \cdot \alpha_q + \omega_{w1})^2 \cdot \left(\frac{1 - \alpha_q^{2 \cdot (t-1)}}{1 - \alpha_q^2}\right)\right) \tag{3A.6}$$

The distribution of the inflationary component in year t, for the original price inflation model, is given by (for $j =$ s, p, b, $\alpha_q \neq \pm 1$, $(1 - \theta_j) \neq \pm 1$, $\alpha_q \cdot (1 - \theta_j) \neq \pm 1$, $\alpha_q - (1 - \theta_j) \neq 0$, and $t > 0$):

$$\Theta_j(t) \sim \mathrm{N}(\mu_{\Theta j}(t), \sigma_{\Theta j}^2(t)) \tag{3A.7}$$

where:

$$\mu_{\Theta j}(t) = \mu_q \cdot (\beta_j + \beta_{j1}) + (1-\theta_j)^t \cdot (\Theta_j(0) - \beta_{j1} \cdot r_q(0) - \beta_j \cdot \mu_q) \\ + (r_q(0) - \mu_q) \cdot (\beta_{j1} \cdot \alpha_q^t + (\pi_j - \beta_{j1}) \cdot (\alpha_q^t - (1-\theta_j)^t)) \tag{3A.8}$$

$$\sigma_{\Theta j}^2(t) = \sigma_q^2 \cdot \left(\pi_j^2 \cdot \left(\frac{1-\alpha_q^{2\cdot t}}{1-\alpha_q^2} \right) - 2 \cdot \pi_j \cdot \psi_j \cdot \left(\frac{1-(\alpha_q \cdot (1-\theta_j))^t}{1-\alpha_q \cdot (1-\theta_j)} \right) \right. \\ \left. + \psi_j^2 \cdot \left(\frac{1-(1-\theta_j)^{2\cdot t}}{1-(1-\theta_j)^2} \right) \right) \tag{3A.9}$$

$$\pi_j = \frac{\beta_j \cdot \theta_j \cdot \alpha_q}{\alpha_q - (1-\theta_j)} + \beta_{j1} \text{ and } \psi_j = \frac{\beta_j \cdot \theta_j \cdot (1-\theta_j)}{\alpha_q - (1-\theta_j)} \tag{3A.10}$$

3A.1.2 Equity and property models

The distribution of the logarithm of the equity dividend and property yield in year t, for the original price inflation model, is given by (for j = s, p, $\alpha_q \neq \pm 1$, $\alpha_{yj} \neq \pm 1$, and $t > 0$):

$$y_j(t) \sim \mathrm{N}(\mu_{yj}(t), \sigma_{yj}^2(t)) \tag{3A.11}$$

where:

$$\mu_{yj}(t) = \log_e \mu_{yj} + \omega_{yj} \cdot \mu_q + \alpha_q^t \cdot \omega_{yj} \cdot (r_q(0) - \mu_q) \\ + \alpha_{yj}^t \cdot (y_j(0) - \log_e \mu_{yj} - \omega_{yj} \cdot r_q(0)) \tag{3A.12}$$

$$\sigma_{yj}^2(t) = \frac{\sigma_{yj}^2 \cdot (1-\alpha_{yj}^{2\cdot t})}{1-\alpha_{yj}^2} + \frac{\omega_{yj}^2 \cdot \sigma_q^2 \cdot (1-\alpha_q^{2\cdot t})}{1-\alpha_q^2} \tag{3A.13}$$

Note that equation (3A.12) implies that equity dividend and property yields, and hence equity and property returns, can be partially forecast because it includes terms involving $r_q(0)$ and $y_j(0)$. However, if $\alpha_{yj} = \alpha_q$, then $r_q(0)$ has no influence on the mean, $\mu_{yj}(t)$.

The distribution of the force of the equity dividend and property income growth in year t, for the original price inflation model, is given by (for j = s, p, $\alpha_q \neq \pm 1$, $(1-\theta_j) \neq \pm 1$, $\alpha_q \cdot (1-\theta_j) \neq \pm 1$, $\alpha_q - (1-\theta_j) \neq \pm 1$, and $t > 0$):

$$d_j(t) \sim \mathrm{N}(\mu_{dj}(t), \sigma_{dj}^2(t)) \tag{3A.14}$$

where:

$$\mu_{dj}(1) = \mu_{dj} + \mu_{\Theta j}(1) + \omega_{dj} \cdot \sigma_{yj} \cdot z_{yj}(0) + \phi_{dj} \cdot (\omega_{dj} \cdot \sigma_{yj} \cdot z_{yj}(0) + \sigma_{dj} \cdot z_{dj}(0)) \tag{3A.15}$$

$$\sigma^2_{dj}(1) = \sigma^2_{dj} + \varphi^2_{dj} \cdot \sigma^2_{yj} + \sigma^2_{\Theta j}(1) \tag{3A.16}$$

$$\mu_{dj}(t+1) = \mu_{dj} + \mu_{\Theta j}(t+1) \tag{3A.17}$$

$$\sigma^2_{dj}(t+1) = \sigma^2_{dj} \cdot (1 + \phi^2_{dj}) + \sigma^2_{yj} \cdot (\varphi^2_{dj} + (\phi_{dj} \cdot \varphi_{dj} + \omega_{dj})^2) + \sigma^2_{\Theta j}(t+1) \tag{3A.18}$$

3A.1.3 Interest rate models

The distribution of the logarithm of the real yield on long-term fixed-interest securities in year t is given by (for $t > 0$):

$$\log_e(Y_b(t) - \Theta_b(t)) \sim \mathrm{N}(\mu_b(t), \sigma^2_b(t)) \tag{3A.19}$$

where:

$$\mu_b(t) = \log_e \mu_b + \sum_{k=1}^{3} \kappa_k(t) \cdot (\log_e(Y_b(1-k) - \Theta_b(1-k)) - \log_e \mu_b) \tag{3A.20}$$

$$\sigma^2_b(t) = (\sigma^2_b + \varphi^2_b \cdot \sigma^2_{ys}) \cdot \sum_{k=0}^{t-1} (\kappa_1(k))^2 \tag{3A.21}$$

For j = 1, 2, and 3:

$$\kappa_j(t) = \alpha_b \cdot \kappa_j(t-1) + \alpha_{b1} \cdot \kappa_j(t-2) + \alpha_{b2} \cdot \kappa_j(t-3) \tag{3A.22}$$

where $\kappa_1(0) = \kappa_2(-1) = \kappa_3(-2) = 1$, $\kappa_1(-1) = \kappa_1(-2) = 0$, $\kappa_2(0) = \kappa_2(-2) = 0$, $\kappa_3(0) = 0$, and $\kappa_3(-1) = 0$.

For the full standard basis (for $t > 0$ and j = 1, 2, and 3):

$$\kappa_j(t) = \eta_j \cdot \lambda^t + \gamma^t \cdot (\rho_j \cdot \cos(\iota \cdot (t-1)) + \pi_j \cdot \sin(\iota \cdot (t-1))) \tag{3A.23}$$

where $\lambda = 0.9143$, $\gamma = 0.4677$, $\iota = 1.2604$, $\eta_1 = 1.0536$, $\rho_1 = 0.5062$, $\pi_1 = 0.2185$, $\eta_2 = -0.3010$, $\rho_2 = -0.4378$, $\pi_2 = -0.4566$, $\eta_3 = 0.2305$, $\rho_3 = -0.0229$, and $\pi_3 = 0.2347$.

For the other models (for $t > 0$ and $\alpha_b \neq \pm 1$):

$$\kappa_1(t) = \alpha_b^t, \text{ and } \kappa_2(t) = \kappa_3(t) = 0 \tag{3A.24}$$

The mean and variance of the nominal yield on long-term fixed-interest securities in year t is given by (for $t > 0$):

$$\mu_{\Theta b}(t) + \exp[\mu_b(t) + \tfrac{1}{2} \cdot \sigma_b^2(t)] \tag{3A.25}$$

$$\sigma_{\Theta b}^2(t) + \exp[2 \cdot \mu_b(t) + \sigma_b^2(t)] \cdot (\exp[\sigma_b^2(t)] - 1) \tag{3A.26}$$

where $\exp[\cdot]$ represents the exponential function.

The distribution of the logarithm of the yield on index-linked securities in year t is given by (for $\alpha_r \neq \pm 1$ and $t > 0$):

$$y_r(t) \sim \mathrm{N}\left(\log_e \mu_r + \alpha_r^t \cdot (y_r(0) - \log_e \mu_r), (\sigma_r^2 + \varphi_r^2 \cdot \sigma_b^2) \cdot \left(\frac{1 - \alpha_r^{2 \cdot t}}{1 - \alpha_r^2} \right) \right) \tag{3A.27}$$

3A.1.4 Currency exchange rate model

The distribution of the logarithm of the exchange rate between pound sterling and currency j in year t, for the original price inflation model, is given by (for $\alpha_q \neq \pm 1$, $\alpha_{qj} \neq \pm 1$, $\alpha_{xj} \neq \pm 1$, and $t > 0$):

$$\log_e R_{xj}(t) \sim \mathrm{N}(\mu_{xj}(t), \sigma_{xj}^2(t)) \tag{3A.28}$$

where:

$$\begin{aligned} \mu_{xj}(t) = \mu_{xj} &+ \alpha_{xj}^t \cdot (\log_e R_{xj}(0) - \mu_{xj}) + t \cdot (\mu_{qj} - \mu_q) \\ &+ (1 - \alpha_{xj}^t) \cdot (r_{qj}(0) - r_q(0)) + \mathrm{A}_j(t) - \mathrm{A}_x(t) \end{aligned} \tag{3A.29}$$

$$\sigma_{xj}^2(t) = \frac{\sigma_{xj}^2 \cdot (1 - \alpha_{xj}^{2 \cdot t})}{1 - \alpha_{xj}^2} + \mathrm{B}_{xx}(t) + \mathrm{B}_{jj}(t) - 2 \cdot \mathrm{B}_{xj}(t) \tag{3A.30}$$

$$\mathrm{A}_j(t) = (r_{qj}(0) - \mu_{qj}) \cdot \left(\frac{\alpha_{qj} \cdot (1 - \alpha_{qj}^t)}{1 - \alpha_{qj}} \right) \tag{3A.31}$$

$$\mathrm{B}_{jl}(t) = \left(\frac{\sigma_{qjl} \cdot \sigma_{qj} \cdot \sigma_{ql}}{(1-\alpha_{qj}) \cdot (1-\alpha_{ql})} \right) \cdot$$

$$\left(t - \frac{\alpha_{qj} \cdot (1-\alpha_{qj}^{t})}{1-\alpha_{qj}} - \frac{\alpha_{ql} \cdot (1-\alpha_{ql}^{t})}{1-\alpha_{ql}} + \frac{\alpha_{qj} \cdot \alpha_{ql} \cdot (1-\alpha_{qj}^{t} \cdot \alpha_{ql}^{t})}{1-\alpha_{qj} \cdot \alpha_{ql}} \right) \tag{3A.32}$$

where $r_{qx}(t) \equiv r_q(t)$, $\mu_{qx} \equiv \mu_q$, $\alpha_{qx} \equiv \alpha_q$, $\sigma_{qx} \equiv \sigma_q$, $r_{qj}(t)$ represents the force of price inflation in country j at time t, μ_{qj}, α_{qj}, and σ_{qj} are the parameters for country j's price inflation model, and σ_{qjl} represents the correlation between $z_{qj}(t)$ and $z_{ql}(t)$.

3A.2 Averaged price inflation

The mean and variance of averaged price inflation, for the original price inflation model, over the interval 0 to t are given by (for $t > 0$):

$$\mathrm{E}[\overline{R}_q(t)] = \exp[\overline{\mu}_q(t) + \tfrac{1}{2} \cdot \overline{\sigma}_q^2(t)] - 1 \tag{3A.33}$$

$$\mathrm{var}[\overline{R}_q(t)] = \exp[2 \cdot \overline{\mu}_q(t) + \overline{\sigma}_q^2(t)] \cdot (\exp[\overline{\sigma}_q^2(t)] - 1) \tag{3A.34}$$

where:

$$\overline{R}_q(t) = \left(\frac{R_q(t)}{R_q(0)} \right)^{\frac{1}{t}} - 1 \tag{3A.35}$$

$$\overline{\mu}_q(t) = \mu_q + \frac{\alpha_q \cdot (1-\alpha_q^{t}) \cdot (r_q(0) - \mu_q)}{t \cdot (1-\alpha_q)} \tag{3A.36}$$

$$\overline{\sigma}_q^2(t) = \left(\frac{\sigma_q^2}{t^2 \cdot (1-\alpha_q^2)} \right) \cdot \left(t - \frac{2 \cdot \alpha_q \cdot (1-\alpha_q^{t})}{1-\alpha_q} + \frac{\alpha_q^2 \cdot (1-\alpha_q^{2 \cdot t})}{1-\alpha_q^2} \right) \tag{3A.37}$$

Chapter 4

THEORETICAL MODELS

4.1 Introduction

This chapter describes the main UK actuarial stochastic asset models that have been specifically developed to comply with financial economic theories. Although these models were 'calibrated' to the data, their empirical adequacy was not examined. The apparent reasons for not considering the empirical adequacy of these models include that they merely represent initial models that require further development and that historical asset return data may be misleading because past inefficiencies are unlikely to be repeated in future (see Appendix 2A.1). The models examined include those proposed by Dyson and Exley (1995) and Smith (1996). Dyson and Exley (1995) suggested that the expectations hypothesis of the term structure of interest rates should be incorporated in a stochastic asset model. This idea was elaborated by Smith (1996) who developed and fitted a comprehensive stochastic asset model that was termed Dyson and Exley's model. Smith (1996) used the ideas behind Dyson and Exley's model to produce the jump-equilibrium model.

Section 4.2 discusses the model proposed by Dyson and Exley (1995) and Section 4.3 examines the jump-equilibrium model developed by Smith (1996). Section 4.4 concludes. Appendix 4A and 4B provide proofs for the equations used to examine the statistical properties of Dyson and Exley's model and the jump-equilibrium model.

4.2 Dyson and Exley's Model

4.2.1 Model derivation

This model was derived from the following relationship, which represents the expectations hypothesis of the term structure of interest rates (for j = x ≡ sterling, q ≡ consumer goods or retail prices, s ≡ equity securities, p ≡ property, $t \geq 0$, and $\tau > 0$):

$$\delta_j(t,\tau) = \mathrm{E}_t\left[\frac{1}{\tau}\cdot\sum_{k=0}^{\tau-1}\delta_j(t+k,1)\right] \qquad (4.2.1)$$

where E_t represents the expectations operator conditional on all information available at time t, $\delta_j(t, \tau)$ represents the τ-year spot force of interest, or the τ-year zero-coupon yield, for asset class j at time t (see Hull 1993: 81). Hence, $\delta_{\mathrm{x}}(t, \tau)$ represents the fixed-interest zero-coupon yield curve and $\delta_{\mathrm{q}}(t, \tau)$ represents the index-linked zero-coupon yield curve. For equity securities and property, $\delta_{\mathrm{s}}(t, \tau)$ and $\delta_{\mathrm{p}}(t, \tau)$ represent the rates of interest on notional zero-coupon bonds that are backed by equity securities or property. These latter interest rates are not observable, but this does not affect the final model.

Furthermore, it was assumed that expectations of real interest rates in each year represent unbiased estimators of real interest rates; that the difference between nominal and real interest rate expectations represent unbiased estimators of price inflation; and that the difference between real interest rate expectations and implied dividend, and property income, growth rates represent unbiased estimators of real dividend, and property income, growth. These assumptions imply the following relationships, (for $t > 0$ and $k \geq 0$):

$$\mathrm{E}_t[\delta_{\mathrm{q}}(t+k,1)] = \mathrm{E}_{t-1}[\delta_{\mathrm{q}}(t+k,1)] + \varepsilon_{\mathrm{q}}(t) \qquad (4.2.2)$$

$$\begin{aligned}\mathrm{E}_t[r_{\mathrm{q}}(t+k)] &\equiv \mathrm{E}_t[\delta_{\mathrm{x}}(t-1+k,1) - \delta_{\mathrm{q}}(t-1+k,1)] \\ &= \mathrm{E}_{t-1}[\delta_{\mathrm{x}}(t-1+k,1) - \delta_{\mathrm{q}}(t-1+k,1)] + \varepsilon_{\mathrm{x}}(t)\end{aligned} \qquad (4.2.3)$$

$$\begin{aligned}\mathrm{E}_t[d_{\mathrm{s}}(t+k) - r_{\mathrm{q}}(t+k)] &\equiv \mathrm{E}_t[\delta_{\mathrm{q}}(t-1+k,1) - \delta_{\mathrm{s}}(t-1+k,1)] \\ &= \mathrm{E}_{t-1}[\delta_{\mathrm{q}}(t-1+k,1) - \delta_{\mathrm{s}}(t-1+k,1)] + \varepsilon_{\mathrm{s}}(t)\end{aligned} \qquad (4.2.4)$$

$$\begin{aligned}\mathrm{E}_t[d_{\mathrm{p}}(t+k) - r_{\mathrm{q}}(t+k)] &\equiv \mathrm{E}_t[\delta_{\mathrm{q}}(t-1+k,1) - \delta_{\mathrm{p}}(t-1+k,1)] \\ &= \mathrm{E}_{t-1}[\delta_{\mathrm{q}}(t-1+k,1) - \delta_{\mathrm{p}}(t-1+k,1)] + \varepsilon_{\mathrm{p}}(t)\end{aligned} \qquad (4.2.5)$$

where $\varepsilon_j(t)$ represents a normally distributed error term at time t that has a mean of zero and is independent over time but dependent on the other $\varepsilon_k(t)$, $r_{\mathrm{q}}(t)$ represents the force of price inflation in year t, and $d_j(t)$ represents the force of cash flow growth for asset class j in year t.

The assumptions reflected in equations 4.2.1 to 4.2.5 can then be used to determine the values at time t of τ-year asset class j zero-coupon bonds, which are defined by the following equations (see Appendix 4A.1, for $t > 0$):

$$B_{\mathrm{b}}(t, \tau_{\mathrm{b}}) = \exp[-\tau_{\mathrm{b}} \cdot \delta_{\mathrm{x}}(t, \tau_{\mathrm{b}})] \tag{4.2.6}$$

$$B_{\mathrm{r}}(t, \tau_{\mathrm{r}}) = \exp\left[-\tau_{\mathrm{r}} \cdot \delta_{\mathrm{q}}(t, \tau_{\mathrm{r}}) + \sum_{k=1}^{t} r_{\mathrm{q}}(k)\right] \tag{4.2.7}$$

$$B_{\mathrm{s}}(t, \tau_{\mathrm{s}}) = \exp\left[-\tau_{\mathrm{s}} \cdot \delta_{\mathrm{s}}(t, \tau_{\mathrm{s}}) + \sum_{k=1}^{t} d_{\mathrm{s}}(k)\right] \tag{4.2.8}$$

$$B_{\mathrm{p}}(t, \tau_{\mathrm{p}}) = \exp\left[-\tau_{\mathrm{p}} \cdot \delta_{\mathrm{p}}(t, \tau_{\mathrm{p}}) + \sum_{k=1}^{t} d_{\mathrm{p}}(k)\right] \tag{4.2.9}$$

where $\exp[\cdot]$ represents the exponential function.

These equations can be used to calculate the total return on portfolios of constantly re-balanced τ-year zero-coupon securities, which is defined by the following equation (for $j = \mathrm{b} \equiv$ fixed-interest securities, $\mathrm{r} \equiv$ index-linked securities, $\mathrm{s} \equiv$ equity securities, and $\mathrm{p} \equiv$ property, $t > 0$):

$$r_j(t) = \log_e\left(\frac{B_j(t, \tau_j - 1)}{B_j(t-1, \tau_j)}\right) \tag{4.2.10}$$

where $r_j(t)$ represents the force of return on asset class j in year t.

Dyson and Exley's model can then be derived from equations 4.2.1 to 4.2.10 (see Appendix 4A.1). The price inflation model is derived from equation 4.2.3. The short-term interest rate model is derived from equation 4.2.10 with $\tau_{\mathrm{b}} = 1$.

4.2.2 Description

Dyson and Exley's price inflation model can be represented by the following equation (for $t > 0$):

$$r_{\mathrm{q}}(t) = t \cdot (\delta_{\mathrm{x}}(0,t) - \delta_{\mathrm{q}}(0,t)) - (t-1) \cdot (\delta_{\mathrm{x}}(0,t-1) - \delta_{\mathrm{q}}(0,t-1)) + \sum_{l=1}^{t} \sum_{k=1}^{n} \varphi_{\mathrm{q}k} \cdot z_k(l) \tag{4.2.11}$$

where $n = 4$ is equal to the number of basic asset classes and $z_j(t)$ represent sequences of independent and identically distributed unit normal random variables.

Dyson and Exley's short-term interest rate model can be represented by the following equation (for $t > 0$):

$$r_{\mathrm{m}}(t) = t \cdot \delta_{\mathrm{x}}(0,t) - (t-1) \cdot \delta_{\mathrm{x}}(0,t-1) + \sum_{l=1}^{t} \sum_{k=1}^{n} \varphi_{\mathrm{m}k} \cdot z_k(l) - \sum_{k=1}^{n} \varphi_{\mathrm{m}k} \cdot z_k(t) \tag{4.2.12}$$

These models are consistent with the rational expectations hypothesis (see Section 2.4.1) because mean inflation rates and interest rates are equivalent to those implied by the real and nominal interest rate yield curves. Note that the returns on short-term fixed-interest securities in the first year are deterministic.

Dyson and Exley's long-term interest rate, index-linked, equity, and property models can be represented by the following equations (for $t > 0$):

$$r_{\mathrm{b}}(t) = r_{\mathrm{m}}(t) + (1 - \tau_{\mathrm{b}}) \cdot \sum_{k=1}^{n} \varphi_{\mathrm{m}k} \cdot z_k(t) \tag{4.2.13}$$

$$r_{\mathrm{r}}(t) = r_{\mathrm{m}}(t) + \sum_{k=1}^{n} (\tau_{\mathrm{r}} \cdot \varphi_{\mathrm{q}k} + (1 - \tau_{\mathrm{r}}) \cdot \varphi_{\mathrm{m}k}) \cdot z_k(t) \tag{4.2.14}$$

$$r_{\mathrm{s}}(t) = r_{\mathrm{m}}(t) + \sum_{k=1}^{n} \varphi_{\mathrm{s}k} \cdot z_k(t) \tag{4.2.15}$$

$$r_{\mathrm{p}}(t) = r_{\mathrm{m}}(t) + \sum_{k=1}^{n} \varphi_{\mathrm{p}k} \cdot z_k(t) \tag{4.2.16}$$

These equations imply that the expected force of return on all asset classes are equal, which implies that the model is consistent with the efficient market hypothesis. This does not appear to be an appropriate assumption (Singleton 1990); particularly because the variances of the expected returns are different for each asset class (see Section 4.2.3). This appears to have been one of the important reasons why Smith (1996)

Table 4.2.1 Parameter values for Dyson and Exley's model

Parameter	$k = 1$	$k = 2$	$k = 3$	$k = 4$
$\varphi_{\mathrm{m}k}$	−0.00795	0	0	0
$\varphi_{\mathrm{s}k}$	0.0890	0.1704	0	0
$\varphi_{\mathrm{p}k}$	0.0084	0.0426	0.1402	0
$\varphi_{\mathrm{q}k}$	−0.003445	0.00037	0.00051	0.00611

rejected this model and developed the alternative jump-equilibrium model. Note that equations 4.2.15 and 4.2.16 illustrate that specific values do not need to be assigned to τ_{s} and τ_{p} to obtain the equity and property models: their values are subsumed in the estimates of $\varphi_{\mathrm{s}k}$ and $\varphi_{\mathrm{p}k}$.

This representation was used by Smith (1996: 78) to determine the model's parameter values (see Table 4.2.1). Recommended durations for general applications were $\tau_{\mathrm{b}} = 15$ and $\tau_{\mathrm{r}} = 10$.

4.2.3 *Statistical properties*

4.2.3.1 *Price inflation*

The mean and variance of the force of price inflation in year t, implied by Dyson and Exley's model, are given by the following equations (for $t > 0$):

$$\mu_{\mathrm{q}}(t) = t \cdot (\delta_{\mathrm{x}}(0, t) - \delta_{\mathrm{q}}(0, t)) - (t - 1) \cdot (\delta_{\mathrm{x}}(0, t - 1) - \delta_{\mathrm{q}}(0, t - 1)) \qquad (4.2.17)$$

$$\sigma_{\mathrm{q}}^2(t) = t \cdot \sum_{k=1}^{n} \varphi_{\mathrm{q}k}^2 \qquad (4.2.18)$$

Assuming that the initial yield curves satisfy general regularity conditions and tend to finite limits a faster rate than t, $\mu_{\mathrm{q}}(t) \to \delta_{\mathrm{x}}(0, \infty) - \delta_{\mathrm{q}}(0, \infty)$ as $t \to \infty$. The variance of price inflation in year t tends to positive infinity over time. Hence, Dyson and Exley's price inflation model is non-stationary; the error terms have a permanent effect on inflation rates. Using the parameters in Table 4.2.1 and assuming that $\delta_{\mathrm{x}}(0, \tau) = \log_e(1.0375^2)$ and $\delta_{\mathrm{q}}(0, \tau) = \log_e(1.0175^2)$ for all τ, as Smith (1996: 96) does, $\mu_{\mathrm{q}}(1) = \mu_{\mathrm{q}}(20) = 3.89\%$, $\sigma_{\mathrm{q}}(1) = 0.70\%$, and $\sigma_{\mathrm{q}}(20) = 3.15\%$.

Table 4.2.2 Standard deviations of predicted total nominal return

Year	*Short-term*	*Long-term*	*Index-Linked*	*Equity*	*Property*
1	0.00	11.13	7.18	19.22	14.68
20	3.47	11.66	7.97	19.53	15.08

4.2.3.2 Asset class returns

Dyson and Exley's model assumes that the force of return on the other asset classes are non-stationary. However, the risk premiums over short-term fixed-interest securities are stationary over time, which implies that the model is cointegrated. Using the parameters in Table 4.2.1 and assuming that $\delta_x(0, \tau) = \log_e(1.0375^2)$ for all τ, and $\tau_b = 15$ and $\tau_r = 10$, Table 4.2.2 illustrates the standard deviations of the nominal force of return in years 1 and 20 for all the asset classes. The means, for all the asset classes, are 7.36%, which can generally be derived from the following equation (for j = m, b, r, s, p, $t > 0$):

$$\mu_j(t) = t \cdot \delta_x(0, t) - (t-1) \cdot \delta_x(0, t-1) \qquad (4.2.19)$$

The standard deviations were calculated in a similar manner to the standard deviations of price inflation (see equation 4.2.18).

4.2.3.3 Averaged total returns

Assuming that $\tau_b = 15$, $\tau_r = 10$, $\delta_x(0, \tau) = \log_e(1.0375^2)$, $\delta_q(0, \tau) = \log_e(1.0175^2)$ for all τ, and using the parameters in Table 4.2.1, Tables 4.2.3 to 4.2.6 illustrate the means, standard deviations and correlations of nominal and real averaged total returns over various intervals (see Appendix 4A.2). These tables can be compared with the corresponding tables for Wilkie's model (see Tables 3.2.4 to 3.2.7).

Over increasing intervals, the means and standard deviations of nominal and real averaged returns generally decrease initially and then increase; except for price inflation and the returns on short-term fixed-interest securities, whose means and standard deviations increase consistently over increasing intervals. The increase in the standard deviations over very long intervals contrasts with Wilkie's model, which assumes that the standard deviations decrease over corresponding intervals.

Table 4.2.3 Means and standard deviations of nominal averaged total returns

		Term						
Asset Class	*Statistic*	1	2	5	10	20	50	100
Prices	*mean*	3.97	3.97	3.98	3.98	3.99	4.01	4.06
	std dev	0.73	0.82	1.09	1.44	1.96	3.04	4.26
Short-term	*mean*	7.64	7.64	7.64	7.65	7.66	7.70	7.75
	std dev	0.00	0.43	0.94	1.44	2.13	3.44	4.91
Long-term	*mean*	8.31	7.95	7.74	7.67	7.65	7.66	7.71
	std dev	12.09	8.21	4.63	2.69	1.40	2.16	3.92
Index-linked	*mean*	7.92	7.77	7.69	7.66	7.66	7.69	7.74
	std dev	7.75	5.33	3.18	2.23	2.06	3.11	4.62
Equity	*mean*	9.65	8.62	8.01	7.82	7.72	7.70	7.73
	std dev	21.28	14.70	8.99	6.14	4.26	3.52	4.50
Property	*mean*	8.81	8.22	7.87	7.76	7.72	7.72	7.76
	std dev	16.06	11.25	7.11	5.14	4.04	4.01	5.09

Table 4.2.4 Correlations between nominal averaged total returns

Asset Class		Term						
j	*l*	1	2	5	10	20	50	100
Short-term	*Prices*	-	0.44	0.48	0.49	0.49	0.49	0.49
Long-term	*Prices*	−0.49	−0.46	−0.41	−0.35	−0.08	0.45	0.48
	Short-term	-	−0.68	−0.74	−0.65	−0.12	0.92	0.99
Index-linked	*Prices*	0.49	0.52	0.63	0.79	0.87	0.69	0.59
	Short-term	-	−0.29	−0.16	0.17	0.66	0.95	0.99
	Long-term	0.52	0.48	0.34	0.12	0.15	0.91	0.98
Equity	*Prices*	−0.18	−0.16	−0.12	−0.06	0.07	0.35	0.46
	Short-term	-	−0.30	−0.29	−0.18	0.07	0.65	0.91
	Long-term	0.46	0.45	0.40	0.32	0.25	0.67	0.91
	Index-linked	0.28	0.26	0.19	0.10	0.15	0.64	0.90
Property	*Prices*	0.06	0.07	0.11	0.19	0.30	0.45	0.49
	Short-term	-	0.00	0.09	0.23	0.48	0.83	0.95
	Long-term	0.06	0.03	−0.04	−0.13	−0.03	0.77	0.94
	Index-linked	0.11	0.10	0.09	0.14	0.39	0.81	0.94
	Equity	0.28	0.27	0.25	0.23	0.29	0.66	0.90

Table 4.2.5 Means and standard deviations of real averaged total returns

		Term						
Asset class	*Statistic*	1	2	5	10	20	50	100
Short-term	*mean*	3.53	3.53	3.54	3.54	3.55	3.58	3.63
	std dev	0.73	0.74	1.02	1.43	2.02	3.21	4.55
Long-term	*mean*	4.22	3.86	3.65	3.59	3.56	3.57	3.61
	std dev	12.02	8.31	5.00	3.36	2.46	2.80	4.10
Index-linked	*mean*	3.77	3.64	3.56	3.54	3.54	3.56	3.60
	std dev	7.12	4.76	2.52	1.33	1.02	2.36	3.93
Equity	*mean*	5.49	4.49	3.90	3.71	3.62	3.60	3.63
	std dev	20.62	14.29	8.84	6.16	4.41	3.67	4.45
Property	*mean*	4.65	4.09	3.75	3.65	3.60	3.60	3.64
	std dev	15.42	10.80	6.80	4.89	3.79	3.69	4.66

Table 4.2.6 Correlations between real averaged total returns

Asset class		Term						
j	*l*	1	2	5	10	20	50	100
Short-term	*Prices*	−1.00	−0.86	−0.63	−0.53	−0.47	−0.44	−0.42
Long-term	*Prices*	−0.53	−0.53	−0.59	−0.69	−0.84	−0.75	−0.59
	Short-term	0.53	0.21	−0.06	0.01	0.35	0.86	0.97
Index-linked	*Prices*	0.41	0.38	0.33	0.21	−0.23	−0.40	−0.41
	Short-term	−0.41	−0.65	−0.71	−0.45	0.58	0.97	1.00
	Long-term	0.55	0.52	0.45	0.35	0.42	0.86	0.97
Equity	*Prices*	−0.21	−0.21	−0.24	−0.29	−0.38	−0.50	−0.50
	Short-term	0.21	0.06	−0.05	0.01	0.21	0.67	0.90
	Long-term	0.47	0.47	0.44	0.42	0.45	0.73	0.92
	Index-linked	0.30	0.28	0.23	0.17	0.24	0.67	0.90
Property	*Prices*	0.01	−0.01	−0.04	−0.11	−0.21	−0.35	−0.40
	Short-term	−0.01	−0.01	0.06	0.20	0.45	0.81	0.95
	Long	0.08	0.06	0.03	0.02	0.17	0.70	0.92
	Index-linked	0.09	0.07	0.00	−0.04	0.30	0.80	0.94
	Equity	0.29	0.28	0.27	0.27	0.33	0.66	0.89

The correlations between the nominal returns generally decrease initially and then increase over increasing intervals, except for the correlations with price inflation and short-term interest rates, which generally increase constantly. Over a 50 and 100 year interval all the returns are highly positively correlated. The correlations between price inflation and the nominal returns on short-term and long-term fixed-interest securities and on equity securities are initially negative. Furthermore, the correlations between the nominal returns on short-term fixed-interest securities and the nominal returns on the other asset classes, except property, are initially negative.

The correlations between price inflation and the real returns on all the asset classes are generally negative. Over a 50 and 100 year interval all the returns are highly positively correlated. This contrasts with Wilkie's model (see Table 3.2.7). The correlations between the asset class returns generally decrease and then increase over increasing intervals. The correlations between the real returns on short-term fixed-interest securities and the real returns on index-linked securities and property are initially negative.

4.3 The Jump-Equilibrium Model

4.3.1 Introduction

In addition to the general objectives of stochastic asset models (see Section 2.5.2), Smith (1996: 46) required a model that describes the fixed-interest and index-linked yield curves, allows for occasional price 'jumps', incorporates a risk-neutral law, and uses the same mathematical structure to describe all the asset classes or is symmetrical. The risk-neutral law was required so that the model could be used to solve certain optimisation problems and to price derivative securities. Furthermore, Smith (1996: 33) argued that, for most actuarial applications, it is prudent to assume that markets are efficient. This is because historical inefficiencies are likely to be exploited by rational investors in the future (see Appendix 2A.1). Although Smith (1996) did not specifically recommend that the jump-equilibrium model should be used for long term actuarial applications, it could be used in some of these applications in place of other econometric models, such as Wilkie's (1995b) model (but see Wilkie's discussion of Smith 1996).

The jump-equilibrium model does not attempt to closely fit the data. In particular, Smith (1996: 47) reported some of the features of the jump-equilibrium model that appear to be inconsistent with historical information:

> it is often observed that markets tend to have bursts of high volatility alternating with more stable periods, sometimes referred to as an ARCH effect, which I have ignored. Neither have I allowed for mean reverting or error-correction effects. I have little doubt that the gamma distribution can be shown to be a poor fit to the various series where I have used it. Over a short time period, I only allow the yield curves to make parallel shifts ... The model also ... allow[s] negative yields as a possibility for all asset classes ... Finally, many of the fitting techniques I have used are of questionable validity.

This section aims to further explore the theoretical and statistical properties of the jump-equilibrium model.

4.3.2 Model derivation

The jump-equilibrium model was derived from the following hypothetical equation for asset class j (for $j = \mathrm{x} \equiv$ sterling, $\mathrm{q} \equiv$ consumer goods, $\mathrm{s} \equiv$ equity securities with dividends reinvested, and $\mathrm{p} \equiv$ property with income reinvested, $t > 0$):

$$X_j(t) = f_j(t) \cdot \prod_{k=1}^{n} \exp\left[\beta_{jk} \cdot G_k(t) - \lambda_{jk} \cdot \int_0^t G_k(w)dw \right] \qquad (4.3.1)$$

where $X_j(t)$ represent the capital value of a unit of asset class j denominated in a notional non-depreciating risk-neutral currency at time t, $f_j(t)$ are deterministic functions of t, $n = 4$ represents the number of basic asset classes, and $G_j(t)$ represent independent compound Poisson processes so that $G_j(t+\tau) - G_j(t) \sim \Gamma(\tau \cdot \theta_j, 1)$ and $G_j(0) = 0$. The parameters β_{jk} and λ_{jk} are constrained so that $\beta_{jk} < 1$ and $\lambda_{jk} \geq 0$. These constraints ensure that bond prices are finite and positive (see equation 4.3.3). Note that the equity and property asset classes include reinvested cash flows so that $X_\mathrm{s}(t)$ and $X_\mathrm{p}(t)$ represent total return indices denominated in the notional currency. This differs from the equity and property asset classes used in Dyson and Exley's model (see Section 4.2.1).

The $X_j(t)$ series' are not observable because they are denominated in a notional currency. However, as each asset class is denominated in the same notional currency, observable quantities can be derived by taking ratios. For example (for $t > 0$):

$$R_q(t) = \frac{X_q(t)}{X_x(t)} \tag{4.3.2}$$

where $R_q(t)$ represents the actual numerical value of the retail price index at time t.

Bond prices can be derived from equation 4.3.1 by taking expectations because the notional currency is assumed to be risk-neutral. Therefore, at time t, the price in units of asset class j of a τ-year zero-coupon bond paying one unit of asset class j is given by (see Appendix 4B.1, for $t > 0$ and $\tau > 0$):

$$B_j(t, \tau) = \frac{\mathrm{E}_t[X_j(t+\tau)]}{X_j(t)}$$

$$= \left(\frac{f_j(t+\tau)}{f_j(t)}\right) \cdot \prod_{k=1}^{n} \left\{ \exp[\tau \cdot (\theta_k - \lambda_{jk} \cdot G_k(t))] \cdot \left(\frac{(1-\beta_{jk})^{1-\beta_{jk}}}{(1-\beta_{jk} + \lambda_{jk} \cdot \tau)^{1-\beta_{jk}+\lambda_{jk}\cdot\tau}} \right)^{\frac{\theta_k}{\lambda_{jk}}} \right\} \tag{4.3.3}$$

If $\lambda_{jk} = 0$ then the limiting form of the above equation is used, which involves replacing the term inside the product with $(1 - \beta_{jk})^{-\theta_k \cdot \tau}$.

The functions $f_j(t)$, for $t > 0$, can then be derived from equation 4.3.3 using the following identity (for $\tau > 0$):

$$B_j(0, \tau) = \exp[-\tau \cdot \delta_j(0, \tau)] \tag{4.3.4}$$

where $\delta_j(t, \tau)$ represents the τ-year spot force of interest for asset class j at time t. For equity securities and property $\delta_j(t, \tau)$ is not measurable as these asset classes do not consist of securities with fixed durations.

Furthermore, from equation 4.3.1, $f_j(0) = X_j(0)$. Numerical values of $X_j(0)$ can be obtained by arbitrarily setting $X_x(0) = 1$, then, from equation 4.3.2, $X_q(0) = R_q(0)$. For equity securities and property $X_j(0)$ is equal to the value of an appropriate total return index at time 0. Therefore, the functions $f_j(t)$ are given by (for $t \geq 0$):

$$f_j(t) = X_j(0) \cdot \exp[-t \cdot \delta_j(0, t)]$$
$$\cdot \prod_{k=1}^{n} \left\{ \exp[-t \cdot \theta_k] \cdot \left(\frac{(1-\beta_{jk} + \lambda_{jk} \cdot t)^{1-\beta_{jk}+\lambda_{jk} \cdot t}}{(1-\beta_{jk})^{1-\beta_{jk}}} \right)^{\frac{\theta_k}{\lambda_{jk}}} \right\} \tag{4.3.5}$$

The limiting form of the above equation is used if $\lambda_{jk} = 0$ (see equation 4.3.3).

Equation 4.3.3 can also be used to derive total return indices, which are given by (see Appendix 4B.2, for $t > 0$ and $\tau > 0$):

$$RX_j(t, \tau) = RX_j(0, \tau) \cdot \prod_{k=1}^{n} \exp[(\beta_{jk} - \lambda_{jk} \cdot \tau) \cdot G_k(t)] \cdot (1 - \beta_{jk} + \lambda_{jk} \cdot \tau)^{\theta_k \cdot t} \tag{4.3.6}$$

where $RX_j(t, \tau)$ represents the total return index of a constantly rebalanced portfolio of τ-year zero-coupon asset class j bonds denominated in the notional currency.

Note that the equity and property asset classes include reinvested cash flows so that $X_s(t)$ and $X_p(t)$ already represent total return indices denominated in the notional currency. Hence, $RX_s(t, 0) = X_s(t)$ and $RX_p(t, 0) = X_p(t)$, which implies that $\lambda_{sk} = \lambda_{pk} = 0$ for all k and that $\delta_s(0, t) = \delta_p(0, t) = 0$ for all $t > 0$.

The observed sterling total return indices are then given by (for j = m ≡ short-term fixed-interest securities or cash, b ≡ long-term fixed-interest securities, r ≡ index-linked securities, s ≡ equity securities, and p ≡ property, $t > 0$):

$$R_j(t) = \frac{RX_j(t, \tau_j)}{X_x(t)} \tag{4.3.7}$$

where $R_j(t)$ represents the observed sterling total return index for asset class j at time t, $RX_m(t, \tau) \equiv RX_b(t, \tau) \equiv RX_x(t, \tau)$, $RX_r(t, \tau) \equiv RX_q(t, \tau)$, and $\tau_m = \tau_s = \tau_p = 0$.

The jump-equilibrium model can then be derived from equations 4.3.2 and 4.3.7.

4.3.3 Description

The price inflation jump-equilibrium model can be represented in discrete time by the following equations (for $t > 0$):

$$r_q(t) = \mu_q(t) + \zeta_q(t) \tag{4.3.8}$$

$$\begin{aligned}\mu_q(t) &= t \cdot (\delta_x(0,t) - \delta_q(0,t)) - (t-1) \cdot (\delta_x(0,t-1) - \delta_q(0,t-1)) \\ &+ \sum_{k=1}^{n} \theta_k \cdot \left(\beta_{qk} - \beta_{xk} - (\lambda_{qk} - \lambda_{xk}) \cdot (t - \tfrac{1}{2}) + \log_e \left(\frac{\psi_{qk}(t,1)}{\psi_{xk}(t,1)} \right) \right)\end{aligned} \tag{4.3.9}$$

$$\begin{aligned}\zeta_q(t) &= \sum_{k=1}^{n} (\beta_{qk} - \beta_{xk}) \cdot (G_k(t) - G_k(t-1) - \theta_k) \\ &\quad - (\lambda_{qk} - \lambda_{xk}) \cdot \left(\int_{t-1}^{t} G_k(w)dw - \theta_k \cdot (t - \tfrac{1}{2}) \right)\end{aligned} \tag{4.3.10}$$

where: $$\psi_{jk}(t,\tau) = \left(\frac{(1 - \beta_{jk} + \lambda_{jk} \cdot t)^{1-\beta_{jk}+\lambda_{jk}\cdot t}}{(1 - \beta_{jk} + \lambda_{jk} \cdot (t-\tau))^{1-\beta_{jk}+\lambda_{jk}\cdot(t-\tau)}} \right)^{\frac{1}{\lambda_{jk}}} \tag{4.3.11}$$

where $r_j(t) = \log_e R_j(t) - \log_e R_j(t-1)$ represents the force of return on asset class j over the interval $t-1$ to t. The limiting form of $\psi_{jk}(t,\tau)$ at $\lambda_{jk} = 0$ is: $\exp[\tau] \cdot (1 - \beta_{jk})^{\tau}$.

The jump-equilibrium model for the return on asset class j can be represented in discrete time by the following equations (for j = m, b, r, s, and p, $t > 0$):

$$r_j(t) = \mu_j(t) + \zeta_j(t) \tag{4.3.12}$$

$$\begin{aligned}\mu_j(t) &= t \cdot \delta_x(0,t) - (t-1) \cdot \delta_x(0,t-1) \\ &+ \sum_{k=1}^{n} \theta_k \cdot \left(\beta_{jk} - \beta_{xk} - \lambda_{jk} \cdot \tau_j + \lambda_{xk} \cdot (t - \tfrac{1}{2}) + 1 + \log_e \left(\frac{1 - \beta_{jk} + \lambda_{jk} \cdot \tau_j}{\psi_{xk}(t,1)} \right) \right)\end{aligned} \tag{4.3.13}$$

$$\begin{aligned}\zeta_j(t) &= \sum_{k=1}^{n} (\beta_{jk} - \beta_{xk} - \lambda_{jk} \cdot \tau_j) \cdot (G_k(t) - G_k(t-1) - \theta_k) \\ &\quad + \lambda_{xk} \cdot \left(\int_{t-1}^{t} G_k(w)dw - \theta_k \cdot (t - \tfrac{1}{2}) \right)\end{aligned} \tag{4.3.14}$$

where, as in equation 4.3.7, $\beta_{mk} = \beta_{bk} = \beta_{xk}$, $\lambda_{mk} = \lambda_{bk} = \lambda_{xk}$, and $\beta_{rk} = \beta_{qk}$, $\lambda_{rk} = \lambda_{qk}$.

From these equations it can be seen that in discrete time the jump-equilibrium models do not necessarily allow for 'occasional price jumps' (see Smith 1996: 46). The models merely use a complex combination of gamma distributions in their error terms. This feature of the model is only strictly relevant in continuous time.

The model's recommended parameter values are given in Table 4.3.1. Recommended durations for general applications were $\tau_b = 15$ and $\tau_r = 10$. Note that $\beta_{xk} = \beta_{qk}$ for all k, which implies that the price inflation model (equations 4.3.9 and 4.3.10) can be simplified. The model's parameters appear to have been determined by equating the means of the model's risk premiums to values implied by theoretical considerations and by equating the other moments of the model's risk premiums to values implied by historical data (see Smith 1996: 74-83). These risk premiums were calculated relative to the force of return on short-term fixed-interest securities. The means and covariances, or variances if $j = l$, of these risk premiums are independent of time and are given by (for $j, l =$ b, r, s, and p):

$$\mu_j(t) - \mu_{\mathrm{m}}(t) = \sum_{k=1}^{n} \theta_k \cdot \left(\beta_{jk} - \beta_{xk} - \lambda_{jk} \cdot \tau_j + \log_e \left(\frac{1 - \beta_{jk} + \lambda_{jk} \cdot \tau_j}{1 - \beta_{xk}} \right) \right) \tag{4.3.15}$$

$$\begin{aligned} &\operatorname{cov}[r_j(t) - r_{\mathrm{m}}(t), r_l(t) - r_{\mathrm{m}}(t)] \\ &\qquad = \sum_{k=1}^{n} \theta_k \cdot (\beta_{jk} - \beta_{xk} - \lambda_{jk} \cdot \tau_j) \cdot (\beta_{lk} - \beta_{xk} - \lambda_{lk} \cdot \tau_l) \end{aligned} \tag{4.3.16}$$

where cov[·,·] represents the covariance function.

Table 4.3.1 Parameter values for the jump-equilibrium model

Parameter	$k = 1$	$k = 2$	$k = 3$	$k = 4$
β_{xk}	0.0623	0.1185	0.0326	0.0029
β_{qk}	0.0623	0.1185	0.0326	0.0029
β_{sk}	0.0410	0.0596	0.0326	0.0029
β_{pk}	0.0630	0.1048	−0.0212	0.0029
λ_{xk}	0.0033	0.0005	0	0
λ_{qk}	0.0016	0.0004	0.0002	0.0019
λ_{sk}	0	0	0	0
λ_{pk}	0	0	0	0
θ_k	4.9284	10	6.7772	10

As these risk premiums are independent of time, the model is consistent with the efficient market hypothesis. Hence, it is generally not possible to use this model to determine dynamic investment strategies that result in above average returns (see Smith 1996: 35).

The assumptions used to determine the theoretical risk premiums included that a representative investor has a power utility function and optimally holds a portfolio of 15% long-term fixed-interest securities, 5% index-linked securities, 60% equity securities, and 20% property. The returns for this portfolio were measured relative to the returns on short-term fixed-interest securities. This is a novel method for determining risk premiums and could be used to parameterise other asset models (see Section 3.2.4.4). It has been termed the 'equilibrium' method because it ensures that a representative investor would optimally hold the market portfolio assuming mean-variance portfolio theory.

4.3.4 Statistical properties

4.3.4.1 Price inflation

The price inflation model's mean terms, $\mu_q(t)$, generally imply that investors require a risk premium on fixed-interest securities over index-linked securities with the same duration, if rational expectations are assumed (see Section 2.4.1). If no risk premium was required, as assumed by Wilkie (1995a: 273) and Dyson and Exley (1995: 498), then investors' expectations of the rate of price inflation in year t would be equal to:

$$t\cdot(\delta_x(0,t)-\delta_q(0,t))-(t-1)\cdot(\delta_x(0,t-1)-\delta_q(0,t-1)) \qquad (4.3.17)$$

Therefore, the summation term in equation 4.3.9 can be interpreted as an allowance for a risk premium on fixed-interest securities over index-linked securities. Using the parameters in Table 4.3.1, this summation term decreases continuously over time and tends to negative infinity. This suggests that the fixed-interest risk premium is assumed to increase with increasing duration.

This feature of the jump-equilibrium model implies that the initial yield curves need to be chosen with caution. If it is assumed that $\delta_x(0,\tau)=\log_e(1.0375^2)$ and

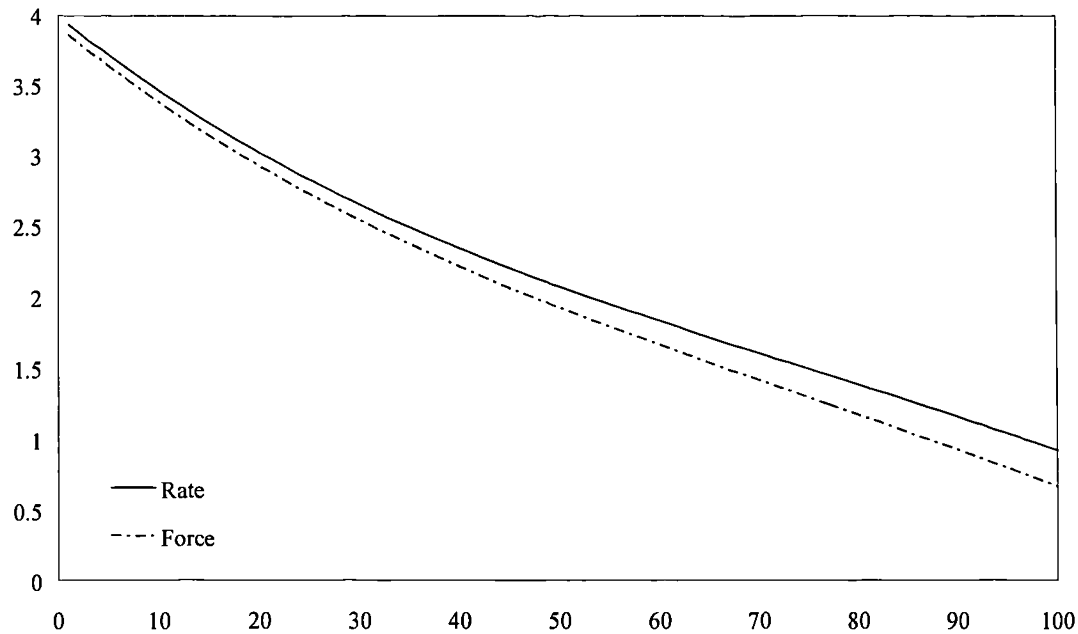

Figure 4.3.1 Expected annual rate and force of price inflation in year t

$\delta_q(0, \tau) = \log_e(1.0175^2)$ for all τ, as Smith (1996: 96) did, and using the parameters in Table 4.3.1, then the expected rate of price inflation decreases over time from 3.86% in the first year to negative infinity (see Figure 4.3.1 and Appendix 4B.3). The expected rate of price inflation becomes negative after year 122. Similar results are achieved if the annual rate, rather than the force, of price inflation is considered (see Figure 4.3.1 and Appendix 4B.5). However, this decrease in expected price inflation over time is not necessarily problematic. It merely reflects the assumptions that the initial yield curves are parallel to one another.

The variance of the rate of price inflation in year t is given by (see Appendix 4B.4, for $t > 0$):

$$\sigma_q^2(t) = \sum_{k=1}^{n} \theta_k \cdot ((t - \tfrac{11}{12}) \cdot (\lambda_{qk} - \lambda_{xk})^2 + (\beta_{qk} - \beta_{xk} - \tfrac{1}{2} \cdot (\lambda_{qk} - \lambda_{xk}))^2) \qquad (4.3.18)$$

As $\theta_k > 0$, $\sigma_q^2(t) \to +\infty$ as $t \to \infty$ unless $\lambda_{xk} = \lambda_{qk}$ for all k. Hence, the jump-equilibrium model assumes that next year's rate of price inflation is known with relative certainty and that virtually no information exists on the rate of price inflation at infinity. The assumption, that the variance of price inflation tends to infinity is also incorporated in integrated econometric price inflation models (see, for example, Clare *et al.* 1994). This

Table 4.3.2 Distribution of the annual rate of price inflation in year *t*

	Year						
Statistic	1	2	5	10	20	50	100
mean	3.94	3.88	3.72	3.46	3.02	2.08	0.92
std dev	0.43	0.85	1.54	2.25	3.22	5.10	7.17
skewness	−0.30	−0.17	−0.07	−0.01	0.04	0.12	0.19
kurtosis	3.70	3.25	3.07	3.03	3.02	3.03	3.06

implies that the degree of confidence in price inflation forecasts decreases the further into the future these forecasts are made: the funnel of doubt continues to increase. This was rejected by Wilkie (1995b) (see Section 3.2.4).

Using the parameters in Table 4.3.1 and assuming that $\delta_x(0,\tau) = \log_e(1.0375^2)$ and $\delta_q(0,\tau) = \log_e(1.0175^2)$ for all τ, Table 4.3.2 reports the mean, standard deviation, skewness, and kurtosis of the annual rate of price inflation in various years (see Appendix 4B.5). The means and standard deviations are similar to those calculated using the force of return. The decrease in the mean rate of price inflation reflects the assumptions that the initial yield curves are parallel. The skewness and kurtosis coefficients do not depend on the initial yield curve assumptions. The skewness coefficient is negative for the first 11 years and positive thereafter. This is inconsistent with the assumption made by Pentikäinen *et al.* (1995) that the price inflation distribution should be positively skewed (see Section 3.3.2). A positively skewed price inflation distribution reflects the view that large positive price inflation 'jumps' are more likely than large negative price inflation 'jumps'. The price inflation distribution is only mildly leptokurtic. This illustrates that in discrete time the jump-equilibrium model does not necessarily allow for price inflation 'shocks' over annual intervals, such as those in Clarkson's (1991) model (see Section 3.2.6).

Price inflation rates are highly positively autocorrelated (see Appendix 4B.5). Using the parameters in Table 4.3.1, the correlation between the annual rate of price inflation in year 1 and year 2 is 0.75.

4.3.4.2 Asset class returns

The asset class return models are broadly similar to the price inflation model. The summation term in equation 4.3.13 can be interpreted as an allowance for a liquidity premium on different duration fixed-interest securities. Furthermore, the expected real rate of return on asset class j in year t is given by (for j = m, b, r, s, and p, $t > 0$):

$$\begin{aligned}\mu_j(t) - \mu_q(t) &= t \cdot \delta_q(0,t) - (t-1) \cdot \delta_q(0,t-1) \\ &+ \sum_{k=1}^{n} \theta_k \cdot \left(\beta_{jk} - \beta_{qk} - \lambda_{jk} \cdot \tau_j + \lambda_{qk} \cdot (t - \tfrac{1}{2}) + 1 + \log_e\left(\frac{1 - \beta_{jk} + \lambda_{jk} \cdot \tau_j}{\psi_{qk}(t)} \right) \right)\end{aligned} \quad (4.3.19)$$

This suggests that summation term in equation 4.3.19 can be interpreted as an allowance for a liquidity premium on different duration index-linked securities.

Equations 4.3.13 and 4.3.19 can be used to determine 'neutral' initial zero-coupon yield curves, which imply constant expected nominal and real rates of return. Using the parameters in Table 4.3.1, Figure 4.3.2 plots the 'neutral' initial zero-coupon yield curves that would result in a constant expected nominal and real rate of return on short-term fixed-interest securities of $\log_e(1.0375^2)$ and $\log_e(1.0175^2)$, respectively. The 'neutral' fixed-interest yield curve increases from 7.36% for $\tau = 0$ to 9.29% for $\tau = 46$ and decreases thereafter, becoming negative after duration 158. The 'neutral' index-

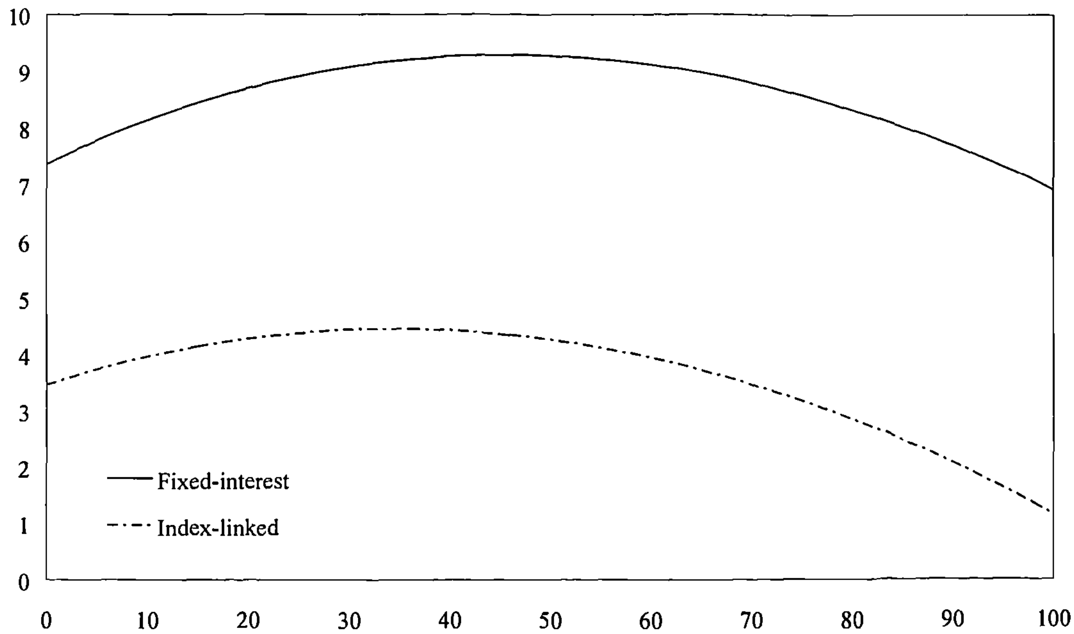

Figure 4.3.2 'Neutral' initial zero-coupon yield curves, $\delta_x(0, \tau)$ and $\delta_q(0, \tau)$

linked yield curve increases from 3.47% for $\tau = 0$ to 4.45% for $\tau = 34$ and decreases thereafter, becoming negative after duration 111. These 'neutral' yield curves also diverge with increasing duration, which reflects the increasing fixed-interest risk premium required to obtain constant expected price inflation (see Section 4.3.4.1).

These 'neutral' yield curves appear to be difficult to justify. In particular, they are not consistent with the liquidity preference hypothesis (see Appendix 2A.2). If initial yield curves that continually increased with increasing duration were used, then the model would imply that expected nominal and real returns increase to positive infinity over time. However, this feature of the model is only significant for applications with very long time horizons. If the model is used for these applications then it is important that the initial yield curves are carefully chosen.

If it is assumed that $\tau_b = 15$, $\tau_r = 10$, $\delta_x(0, \tau) = \log_e(1.0375^2)$ for all τ, as Smith (1996: 96) did, and using the parameters in Table 4.3.1, the expected nominal rate of return on all assets, $\mu_j(t)$, decreases for the first 31 years by a total of 2.5% and increases thereafter to infinity (see Appendix 4B.3). This is illustrated in Figure 4.3.3, which plots the expected force of return on short-term fixed-interest securities over a 100 year interval. These results concur with Lee's remark in the discussion of Smith (1996).

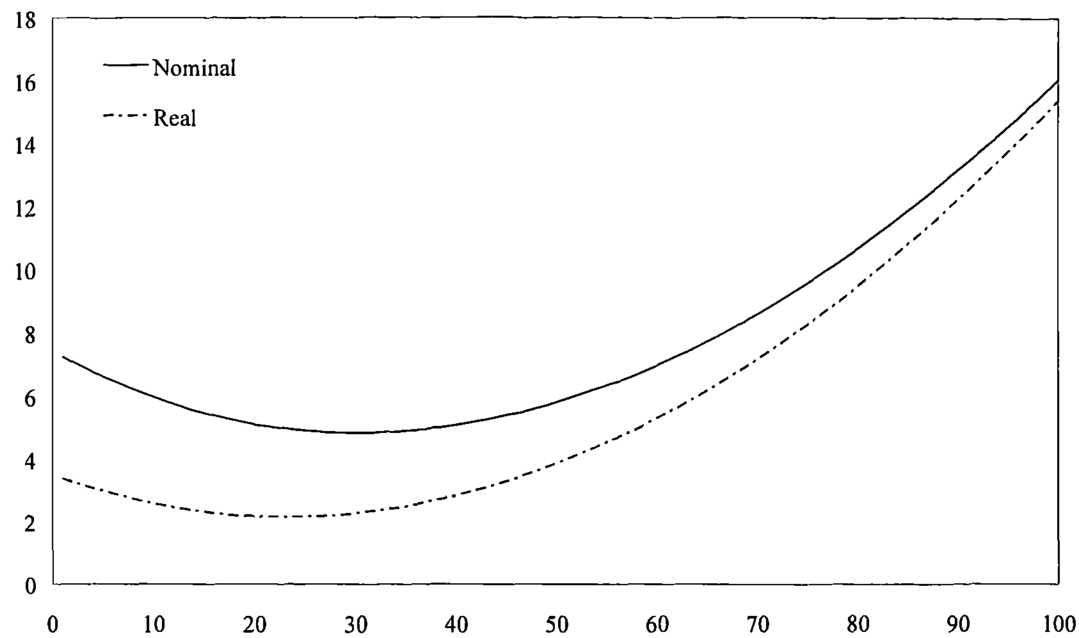

Figure 4.3.3 Nominal and real expected force of return on short-term fixed-interest securities in year t

Similar graphs would be obtained for the other asset classes, because the risk premiums over short-term fixed-interest securities are constant over time (see equation 4.3.15). Assuming that $\delta_q(0, \tau) = \log_e(1.0175^2)$ for all τ and using the parameters in Table 4.3.1, the expected real force of return on all asset classes decreases for the first 23 years by a total of 1% and increases thereafter to infinity (see Figure 4.3.3). This pattern of decreasing and then increasing returns can be accounted for by the difference between the assumed initial yield curves and the 'neutral' initial yield curves.

The variance of the nominal rate of return on asset class j in year t is given by (see Appendix 4B.4, for j = b, r, s, p, and $t > 0$):

$$\sigma_j^2(t) = \sum_{k=1}^{n} \theta_k \cdot ((t - \tfrac{11}{12}) \cdot \lambda_{xk}^2 + (\beta_{jk} - \beta_{xk} - \lambda_{jk} \cdot \tau_j + \tfrac{1}{2} \cdot \lambda_{xk})^2) \qquad (4.3.20)$$

As $\theta_k > 0$, $\sigma_j^2(t) \to +\infty$ as $t \to \infty$ unless $\lambda_{xk} = 0$ for all k. Thus, the funnel of doubt concerning the rate of return on all securities continues to increase. Similarly, the variance of the real rate of return on all asset classes increases to positive infinity over time. This assumption is not included in Wilkie's (1995b) model. Wilkie (1995b: 779) stressed that, for long term applications: "interest rates, or at least real interest rates, must be modelled as statistically stationary series."

Using the parameters in Table 4.3.1 and assuming that $\delta_x(0, \tau) = \log_e(1.0375^2)$ and $\delta_q(0, \tau) = \log_e(1.0175^2)$ for all τ, Table 4.3.3 reports the mean, standard deviation, skewness, and kurtosis of the annual nominal rate of return on all the asset classes in various years (see Appendix 4B.5). Broadly similar statistics are obtained if real rates of return are considered. The means and standard deviations are similar to those calculated using the force of return transformation. The means reflect the initial yield curve assumptions. The distributions of the returns on all the asset classes, except short-term fixed-interest securities, are initially negatively skewed. This implies that large negative asset returns are generally more likely than large positive returns. It also appears to suggest that large increases in interest rates are more likely that large decreases; this concurs with the assumption made by Pentikäinen *et al.* (1995) (see Section 3.3.2). The skewness in equity returns is not substantial. The distribution of returns on short-term and long-term fixed-interest securities and index-linked securities are leptokurtic. The

Table 4.3.3 Distribution of the annual nominal rate of return in year *t*

		Year						
Asset Class	*Statistic*	1	2	5	10	20	50	100
Short-term	*mean*	7.55	7.37	6.88	6.19	5.31	6.10	17.76
	std dev	0.47	0.93	1.67	2.44	3.48	5.61	8.84
	skewness	1.12	0.72	0.45	0.35	0.29	0.28	0.31
	kurtosis	5.07	3.83	3.33	3.20	3.15	3.15	3.18
Long-term	*mean*	10.27	10.09	9.58	8.88	7.97	8.79	20.75
	std dev	11.50	11.51	11.54	11.62	11.81	12.72	15.53
	skewness	−0.50	−0.49	−0.47	−0.43	−0.37	−0.21	−0.03
	kurtosis	3.17	3.17	3.16	3.14	3.11	3.07	3.06
Index-linked	*mean*	8.78	8.60	8.10	7.41	6.51	7.31	19.11
	std dev	7.41	7.45	7.55	7.71	8.06	9.25	12.08
	skewness	−0.29	−0.28	−0.25	−0.20	−0.12	0.04	0.19
	kurtosis	3.05	3.05	3.04	3.04	3.03	3.05	3.11
Equity	*mean*	16.62	16.43	15.90	15.15	14.19	15.05	27.70
	std dev	21.22	21.20	21.16	21.12	21.12	21.82	25.19
	skewness	−0.04	−0.03	−0.03	−0.02	0.00	0.06	0.14
	kurtosis	2.80	2.80	2.80	2.81	2.82	2.86	2.94
Property	*mean*	10.77	10.59	10.08	9.38	8.46	9.28	21.29
	std dev	15.54	15.53	15.53	15.54	15.63	16.39	19.33
	skewness	−0.24	−0.23	−0.22	−0.21	−0.17	−0.09	0.03
	kurtosis	2.90	2.90	2.90	2.90	2.91	2.93	2.98

distributions of the returns on equity and property securities are initially platykurtic. This emphasises the point that in discrete time the jump-equilibrium model does not necessarily produce 'occasional price jumps'.

However, over both shorter and longer averaging intervals, equity returns are leptokurtic (see Table 4.3.4). Broadly similar statistics are obtained for the other asset classes, except for the skewness and kurtosis of the returns on short-term fixed-interest securities which are far more extreme over the shorter intervals. The statistics in Table 4.3.4 suggest that the model generates large negative equity returns over relatively short time intervals, but these large infrequent negative returns cancel out with more typical positive returns over annual intervals. This pattern seems to be appropriate, but the kurtosis of the distributions of daily and annual returns appear to be, compared to historical evidence, too high and too low respectively (see Anderson and Breedon 1996).

Table 4.3.4 Distribution of the nominal rate of return on equities over the first interval

	Daily	*Weekly*	*Monthly*	*Half-Yearly*	*Yearly*	*5-Yearly*
mean	0.04	0.30	1.30	8.02	16.62	111.19
std dev	0.95	2.53	5.32	13.90	21.22	85.99
skewness	−10.34	−3.84	−1.73	−0.41	−0.04	0.94
kurtosis	158.90	24.31	7.14	3.01	2.80	4.38

The kurtosis of the distribution of monthly returns appears to be roughly appropriate. The skewness of the return distributions at shorter intervals appears to be too low. These conclusions are only tentative because it is impossible to precisely determine appropriate skewness and kurtosis coefficients from historical data; these statistics are highly sensitive to outliers. Over longer averaging intervals, the equity return distribution is positively skewed and leptokurtic. Hence, the large negative equity returns only have a slight impact on long term rates of return.

Using the parameters in Table 4.3.1, rates of return on short-term fixed-interest securities are highly positively autocorrelated (see Appendix 4B.5). The correlation between the annual nominal rate of return on short-term fixed-interest securities in year 1 and year 2 is 0.75. Rates of return on the other asset classes are initially negatively autocorrelated, but these autocorrelations increase over time and tend to one. The correlation between the annual nominal rate of return on long-term fixed-interest, index-linked, equity, and property securities in year 1 and year 2 are −0.072, −0.053, −0.018, and −0.001, respectively.

Appendix 4B.7 reports the means and standard deviations of the annual rate of return on different duration fixed-interest and index-linked zero-coupon securities in year one. This shows that virtually every index-linked security is mean-variance dominated by a fixed-interest security. For example, the mean and standard deviation of a 9-year fixed-interest security in the first year, calculated using nominal rates of return, are 9.20% and 6.79%. These values dominate the corresponding mean and standard deviation of a 10-year index-linked security, which are 8.78% and 7.41%. Moreover, property is mean-variance dominated by fixed-interest securities with durations of between 18 and 20 years (see Table 4.3.3). Fixed-interest securities with durations of between 28 and 52 years are mean-variance dominated by equity securities (see Table 4.3.3). Similar results are obtained if real returns are considered. Hence, according to mean-variance capital market theory (see Huang and Litzenberger 1988), there appears to be little incentive for investors to purchase virtually every zero-coupon index-linked security, property, and zero-coupon fixed-interest securities with durations of between 28 and 52 years. However, these findings are not conclusive because these securities may provide diversification benefits when considered in the context of a portfolio. In addition, these results reflect the relative supply of index-linked securities and property (see Section 4.3.3). Nevertheless, this suggests a possible difficulty with the strict implementation of the 'equilibrium' method in term structure models.

4.3.4.3 Average total returns

Assuming that $\tau_b = 15$, $\tau_r = 10$, $\delta_x(0, \tau) = \log_e(1.0375^2)$, $\delta_q(0, \tau) = \log_e(1.0175^2)$ for all τ, and using the parameters in Table 4.3.1, Tables 4.3.5 to 4.3.8 report the means, standard deviations and correlations of nominal and real averaged total returns over various intervals (see Appendix 4B.6). These tables can be compared with the corresponding tables for Wilkie's model and Dyson and Exley's model (see Sections 3.2.5 and 4.2.3).

Over increasing intervals, the means and standard deviations of nominal and real averaged returns generally decrease initially and then increase; except for expected price inflation, which decreases constantly, and the standard deviation of price inflation and the returns on short-term fixed-interest securities, which increase constantly. These results reflect the assumption that the initial yield curves are flat. The constant increase in the standard deviation of price inflation and the returns on short-term fixed-interest securities is related to their high autocorrelation coefficients (see Sections 4.3.4.1 and 4.3.4.2). Mean averaged price inflation tends to −100% and mean averaged real and nominal returns on all asset classes tend to positive infinity as the averaging interval tends to infinity (see Appendix 4B.6). This contrasts with Wilkie's model, which generally assumes that the means of averaged price inflation and asset returns tend to finite limits and that their standard deviations tend to zero.

Table 4.3.5 Means and standard deviations of nominal averaged total returns

		Term						
Asset Class	*Statistic*	1	2	5	10	20	50	100
Prices	*mean*	3.94	3.91	3.83	3.69	3.45	2.86	2.14
	std dev	0.43	0.60	0.95	1.35	1.90	2.99	4.20
Short-term	*mean*	7.55	7.46	7.20	6.82	6.23	5.65	8.23
	std dev	0.47	0.66	1.04	1.47	2.06	3.24	4.70
Long-term	*mean*	10.27	9.85	9.38	8.93	8.29	7.68	10.29
	std dev	11.50	7.99	4.58	2.68	1.38	2.01	3.76
Index-linked	*mean*	8.78	8.55	8.21	7.80	7.19	6.60	9.19
	std dev	7.41	5.15	3.10	2.19	2.00	2.94	4.44
Equity	*mean*	16.62	15.51	14.62	14.00	13.26	12.59	15.31
	std dev	21.22	15.08	9.39	6.46	4.49	3.62	4.62
Property	*mean*	10.77	10.12	9.51	9.01	8.34	7.72	10.33
	std dev	15.54	11.16	7.15	5.18	4.04	3.90	4.98

Table 4.3.6 Correlations between nominal averaged total returns

Asset Class		Term						
j	*l*	1	2	5	10	20	50	100
Short-term	*Prices*	0.53	0.53	0.53	0.53	0.53	0.53	0.53
Long-term	*Prices*	−0.45	−0.45	−0.42	−0.37	−0.10	0.48	0.52
	Short-term	−0.85	−0.84	−0.80	−0.69	−0.19	0.90	0.99
Index-linked	*Prices*	0.43	0.47	0.60	0.78	0.88	0.73	0.63
	Short-term	−0.41	−0.35	−0.17	0.17	0.65	0.95	0.99
	Long-term	0.49	0.45	0.31	0.09	0.10	0.90	0.98
Equity	*Prices*	−0.14	−0.13	−0.09	−0.03	0.10	0.38	0.50
	Short-term	−0.37	−0.35	−0.28	−0.17	0.07	0.63	0.90
	Long-term	0.44	0.42	0.37	0.29	0.22	0.65	0.90
	Index-linked	0.28	0.25	0.19	0.11	0.16	0.63	0.90
Property	*Prices*	0.07	0.09	0.13	0.20	0.32	0.48	0.52
	Short-term	−0.02	0.01	0.10	0.24	0.48	0.82	0.95
	Long-term	0.03	0.00	−0.07	−0.15	−0.07	0.75	0.93
	Index-linked	0.11	0.10	0.09	0.15	0.39	0.80	0.94
	Equity	0.28	0.27	0.25	0.24	0.29	0.64	0.89

Table 4.3.7 Means and standard deviations of real averaged total returns

		Term						
Asset class	*Statistic*	1	2	5	10	20	50	100
Short-term	*mean*	3.47	3.42	3.26	3.03	2.71	2.76	6.04
	std dev	0.43	0.60	0.95	1.34	1.89	2.99	4.36
Long-term	*mean*	6.12	5.74	5.38	5.08	4.72	4.75	8.09
	std dev	11.25	7.97	4.89	3.33	2.45	2.73	4.04
Index-linked	*mean*	4.64	4.46	4.22	3.96	3.62	3.67	6.97
	std dev	6.97	4.70	2.53	1.35	0.95	2.17	3.76
Equity	*mean*	12.22	11.18	10.41	9.97	9.52	9.52	12.99
	std dev	20.49	14.61	9.20	6.44	4.61	3.76	4.60
Property	*mean*	6.57	5.97	5.47	5.13	4.75	4.76	8.10
	std dev	14.93	10.70	6.83	4.91	3.76	3.56	4.57

Table 4.3.8 Correlations between real averaged total returns

Asset class		Term						
j	*l*	1	2	5	10	20	50	100
Short-term	*Prices*	−0.44	−0.44	−0.44	−0.44	−0.44	−0.44	−0.44
Long-term	*Prices*	−0.48	−0.51	−0.58	−0.69	−0.84	−0.78	−0.63
	Short-term	−0.42	−0.39	−0.29	−0.10	0.30	0.84	0.96
Index-linked	*Prices*	0.38	0.37	0.34	0.22	−0.22	−0.43	−0.44
	Short-term	−0.85	−0.83	−0.76	−0.50	0.50	0.97	0.99
	Long-term	0.50	0.48	0.41	0.30	0.39	0.85	0.97
Equity	*Prices*	−0.16	−0.17	−0.20	−0.25	−0.35	−0.50	−0.52
	Short-term	−0.24	−0.21	−0.15	−0.04	0.18	0.64	0.89
	Long-term	0.45	0.43	0.41	0.39	0.41	0.70	0.90
	Index-linked	0.28	0.26	0.21	0.15	0.22	0.65	0.89
Property	*Prices*	0.04	0.03	−0.01	−0.07	−0.18	−0.35	−0.41
	Short-term	−0.07	−0.04	0.04	0.18	0.42	0.79	0.94
	Long	0.04	0.03	0.00	−0.01	0.13	0.67	0.90
	Index-linked	0.09	0.07	0.01	−0.04	0.26	0.78	0.94
	Equity	0.28	0.27	0.26	0.26	0.31	0.64	0.88

The nominal returns on long-term fixed-interest securities have a higher mean and a lower standard deviation than the corresponding returns on short-term and index-linked securities over a 20 year interval. This suggests that short-term fixed-interest securities and index-linked securities are relatively unattractive long term investments. However, for index-linked securities, this is not the case if real returns are considered. Property does not appear to be a particularly attractive long term investment compared to long-term fixed-interest securities.

The correlations between the nominal returns generally decrease initially and then increase over increasing intervals, except for the correlations with the returns on short-term fixed-interest securities, which increase constantly. Over a 50 and 100 year interval all the nominal returns are highly positively correlated. The correlations between price inflation and the nominal returns on long-term fixed-interest securities and on equity securities are initially negative. In addition, the correlations between the nominal returns on short-term fixed-interest securities and the nominal returns on all the other asset classes are initially negative. These findings are broadly similar to those for Dyson and Exley's model (see Table 4.2.4). These correlations differ from the corresponding correlations for Wilkie's model in many respects. In particular, Wilkie's model assumes that there is little initial correlation between equity and property returns and between short-term fixed-interest security returns and the returns on all the other asset classes, including price inflation.

The correlations between price inflation and the real returns on all the asset classes are generally negative. Over a 50 and 100 year interval all the real returns are highly positively correlated, which contrasts with Wilkie's model (see Table 3.2.7), but is similar to Dyson and Exley's model (see Table 4.2.6). The correlations between the asset class real returns, except for short-term fixed-interest securities, generally decrease and then increase over increasing intervals. The correlations between the real returns on short-term fixed-interest securities and the real returns on all the other securities are initially negative and increase constantly.

4.3.4.4 *Yield curves*

The spot rate yield curve for asset class j implied by the jump-equilibrium model in year t is given by (see equation 4.3.3, for j = x, q, $\tau > 0$, and $t \geq 0$):

$$\delta_j(t,\tau) = \frac{(t+\tau)\cdot\delta_j(0,t+\tau) - t\cdot\delta_j(0,t)}{\tau} + \sum_{k=1}^{n}\theta_k\cdot\left(\frac{1}{\tau}\cdot\log_e\left(\frac{\psi_{jk}(\tau,\tau)}{\psi_{jk}(t+\tau,\tau)}\right) + \lambda_{jk}\cdot t\right) + \sum_{k=1}^{n}\lambda_{jk}\cdot\left(G_k(t) - \theta_k\cdot t\right) \tag{4.3.21}$$

where $\psi_{jk}(t, \tau)$ is defined in equation 4.3.11.

This equation implies that spot interest rates are calculated from the initial forward interest rates allowing for a liquidity premium and a stochastic term. These liquidity premiums are the same as those implied by the 'neutral' initial yield curves (see Figure 4.3.2). After allowing for a liquidity premium, as noted by Smith (1996: 47), the jump-equilibrium model only permits 'parallel' shifts in the yield curves. This is because the stochastic term, represented by the second summation term in equation 4.3.21, is independent of duration.

4.4 Summary

Dyson and Exley's expectations model and the jump-equilibrium model are elegant models with many attractive theoretical features. Their principal advantage over other econometric stochastic asset models is that they describe the complete fixed-interest and index-linked yield curves. For many financial applications this information is essential. This appears to be one of the fundamental motivations for these models. Another pragmatic advantage of these models, over models such as Wilkie's (1995b) model, is that they can often be used to derive analytical, rather than simulated, solutions to many applications. However, for applications that do not require yield curves or analytical solutions, the benefits derived from using these models are less obvious.

Neither Dyson and Exley (1995) nor Smith (1996) assessed the empirical adequacy of these theoretical models. Smith (1996: 47) merely stated that the jump-equilibrium model's error terms do not seem to accurately model the relevant economic variables in that they ignore certain features of the data, such as autoregressive and ARCH effects. In this respect, Smith has sacrificed empirical adequacy for the criteria of mathematical tractability and parsimony. Moreover, the complicated nature of the jump-equilibrium model makes it difficult to empirically test its predictions. This is an important reason why financial economists and econometricians generally use normally distributed random variables. Chapter 5 considers how these models can be justified given that they have not been empirically assessed.

Nevertheless, some of the properties of these theoretical models that were examined in this chapter suggest possible weaknesses. Dyson and Exley's model assumes that all the asset classes have virtually identical expected returns. In addition, when securities of various durations are considered, the model permits arbitrage opportunities (Smith 1996: 35).

The properties of the jump-equilibrium model include that the model's mean terms are largely dependant on the initial yield curves. For long term actuarial applications of the model, considerable care is required in selecting these initial yield curves to ensure that expected price inflation and asset returns do not tend to infinite limits. The model is similar to integrated econometric models in that the variance of price inflation and asset returns tends to positive infinity over time. The model implies that the distributions of annual price inflation and the annual returns on all the asset classes, except short-term fixed-interest securities, are initially negatively skewed. The distributions of the annual returns on equity securities and property are initially platykurtic. Furthermore, the model only permits 'parallel' shifts in the yield curves. This limitation significantly weakens the attraction of having a model that describes the full yield curves.

Appendix 4A: Statistical Properties of Dyson and Exley's Model

4A.1 Model derivation

This section provides a more detailed description of the derivation of Dyson and Exley's model. The derivation of the property model is not reported in this section because it is identical to the derivation of the equity model. The following relationships can be derived from the assumptions represented in equations 4.2.1 to 4.2.4 (for $t > 0$ and $k \geq 0$):

$$\mathrm{E}_t[\delta_q(t+k,1)] = (t+k+1)\cdot\delta_q(0,t+k+1) - (t+k)\cdot\delta_q(0,t+k) + \sum_{l=1}^{t}\varepsilon_q(l) \qquad (4A.1)$$

$$\begin{aligned}\mathrm{E}_t[\delta_x(t-1+k,1) - \delta_q(t-1+k,1)] &= (t+k)\cdot(\delta_x(0,t+k) - \delta_q(0,t+k)) \\ &\quad -(t+k-1)\cdot(\delta_x(0,t+k-1) - \delta_q(0,t+k-1)) + \sum_{l=1}^{t}\varepsilon_x(l)\end{aligned} \qquad (4A.2)$$

$$\begin{aligned}\mathrm{E}_t[\delta_q(t-1+k,1) - \delta_s(t-1+k,1)] &= (t+k)\cdot(\delta_q(0,t+k) - \delta_s(0,t+k)) \\ &\quad -(t+k-1)\cdot(\delta_q(0,t+k-1) - \delta_s(0,t+k-1)) + \sum_{l=1}^{t}\varepsilon_s(l)\end{aligned} \qquad (4A.3)$$

Furthermore, as expected price inflation is assumed to be equivalent to the expected difference between nominal and real interest rates, equation 4A.2 with $k = 0$ implies that (for $t > 0$):

$$r_q(t) = t\cdot(\delta_x(0,t) - \delta_q(0,t)) - (t-1)\cdot(\delta_x(0,t-1) - \delta_q(0,t-1)) + \sum_{l=1}^{t}\varepsilon_x(l) \qquad (4A.4)$$

Similarly, as expected real dividend growth is assumed to be equivalent to the expected difference between real interest rates and dividend interest rates, equation 4A.3 with $k = 0$ implies that (for $t > 0$):

$$\begin{aligned}d_s(t) - r_q(t) &= t\cdot(\delta_q(0,t) - \delta_s(0,t)) \\ &\quad -(t-1)\cdot(\delta_q(0,t-1) - \delta_s(0,t-1)) + \sum_{l=1}^{t}\varepsilon_s(l)\end{aligned} \qquad (4A.5)$$

From equations 4.2.1 and 4A.1 to 4A.5, bond prices are given by ($t > 0$):

$$B_{\mathrm{b}}(t,\tau_{\mathrm{b}}) = \exp[t\cdot\delta_{\mathrm{x}}(0,t) - (t+\tau_{\mathrm{b}})\cdot\delta_{\mathrm{x}}(0,t+\tau_{\mathrm{b}}) - \tau_{\mathrm{b}}\cdot\sum_{k=1}^{t}(\varepsilon_{\mathrm{x}}(k)+\varepsilon_{\mathrm{q}}(k))] \tag{4A.6}$$

$$\begin{aligned} B_{\mathrm{r}}(t,\tau_{\mathrm{b}}) = \exp[&t\cdot\delta_{\mathrm{x}}(0,t) - (t+\tau_{\mathrm{r}})\cdot\delta_{\mathrm{q}}(0,t+\tau_{\mathrm{r}}) \\ &+\sum_{k=1}^{t}((t-k+1)\cdot\varepsilon_{\mathrm{x}}(k) - \tau_{\mathrm{r}}\cdot\varepsilon_{\mathrm{q}}(k))] \end{aligned} \tag{4A.7}$$

$$\begin{aligned} B_{\mathrm{s}}(t,\tau_{\mathrm{s}}) = \exp[&t\cdot\delta_{\mathrm{x}}(0,t) - (t+\tau_{\mathrm{s}})\cdot\delta_{\mathrm{s}}(0,t+\tau_{\mathrm{s}}) \\ &+\sum_{k=1}^{t}((t-k+1+\tau_{\mathrm{s}})\cdot\varepsilon_{\mathrm{s}}(k) + (t-k+1)\cdot\varepsilon_{\mathrm{x}}(k) - \tau_{\mathrm{s}}\cdot\varepsilon_{\mathrm{q}}(k))] \end{aligned} \tag{4A.8}$$

Lastly, from equations 4.2.10 and 4A.6 to 4A.8, total returns are given by (for $t > 0$):

$$r_{\mathrm{b}}(t) = t\cdot\delta_{\mathrm{x}}(0,t) - (t-1)\cdot\delta_{\mathrm{x}}(0,t-1) - \tau_{\mathrm{b}}\cdot(\varepsilon_{\mathrm{q}}(t)+\varepsilon_{\mathrm{x}}(t)) + \sum_{k=1}^{t}(\varepsilon_{\mathrm{q}}(k)+\varepsilon_{\mathrm{x}}(k)) \tag{4A.9}$$

$$r_{\mathrm{r}}(t) = t\cdot\delta_{\mathrm{x}}(0,t) - (t-1)\cdot\delta_{\mathrm{x}}(0,t-1) - \tau_{\mathrm{r}}\cdot\varepsilon_{\mathrm{q}}(t) + \sum_{k=1}^{t}(\varepsilon_{\mathrm{q}}(k)+\varepsilon_{\mathrm{x}}(k)) \tag{4A.10}$$

$$r_{\mathrm{s}}(t) = t\cdot\delta_{\mathrm{x}}(0,t) - (t-1)\cdot\delta_{\mathrm{x}}(0,t-1) + \tau_{\mathrm{s}}\cdot(\varepsilon_{\mathrm{s}}(t)-\varepsilon_{\mathrm{q}}(t)) + \sum_{k=1}^{t}(\varepsilon_{\mathrm{q}}(k)+\varepsilon_{\mathrm{x}}(k)) \tag{4A.11}$$

4A.2 Averaged total returns

Expected averaged price inflation over the interval 0 to t is given by (for $t > 0$):

$$\begin{aligned} \mathrm{E}[\overline{R}_{\mathrm{q}}(t)] &= \mathrm{E}\left[\exp\left[\delta_{\mathrm{x}}(0,t) - \delta_{\mathrm{q}}(0,t) + \sum_{l=1}^{t}\sum_{k=1}^{n}\varphi_{\mathrm{q}k}\cdot\left(\frac{t-l+1}{t}\right)\cdot z_k(l)\right]\right] - 1 \\ &= \exp[\delta_{\mathrm{x}}(0,t) - \delta_{\mathrm{q}}(0,t) + \tfrac{1}{2}\cdot\Sigma_{\mathrm{q}}(t)] - 1 \end{aligned} \tag{4A.12}$$

where:

$$\overline{R}_j(t) = \left(\frac{R_j(t)}{R_j(0)}\right)^{\frac{1}{t}} - 1 \tag{4A.13}$$

$$\Sigma_j(t) = \sum_{l=1}^{t}\sum_{k=1}^{n}\left(\frac{\phi_{jk}(l)}{t}\right)^2 \tag{4A.14}$$

$$\phi_{\mathrm{q}lk}(t) = \varphi_{\mathrm{q}k}\cdot l \tag{4A.15}$$

The variance of averaged price inflation over the interval 0 to t is given by (for $t > 0$):

$$\mathrm{var}[\overline{R}_{\mathrm{q}}(t)] = \exp[2\cdot(\delta_{\mathrm{x}}(0,t) - \delta_{\mathrm{q}}(0,t)) + \Sigma_{\mathrm{q}}(t)]\cdot(\exp[\Sigma_{\mathrm{q}}(t)] - 1) \tag{4A.16}$$

Similarly, expected averaged nominal returns on asset class j over the interval 0 to t are given by (In this section, the equations for the property model are identical to those for the equity model. The equations for the short-term fixed-interest model can be obtained from the long-term fixed-interest model by setting $\tau_{\mathrm{b}} = 1$):

$$\mathrm{E}[\overline{R}_j(t)] = \exp[\delta_{\mathrm{x}}(0,t) + \tfrac{1}{2}\cdot\Sigma_j(t)] - 1 \tag{4A.17}$$

where:

$$\phi_{\mathrm{b}k}(l) = \varphi_{\mathrm{m}k}\cdot(l - \tau_{\mathrm{b}}) \tag{4A.18}$$

$$\phi_{\mathrm{r}k}(l) = \varphi_{\mathrm{m}k}\cdot(l - \tau_{\mathrm{r}}) + \varphi_{\mathrm{q}k}\cdot\tau_{\mathrm{r}} \tag{4A.19}$$

$$\phi_{\mathrm{s}k}(t) = \varphi_{\mathrm{m}k}\cdot(l - 1) + \varphi_{\mathrm{s}k} \tag{4A.20}$$

Expected averaged real returns on asset class j over the interval 0 to t are given by (for $t > 0$):

$$\mathrm{E}[\overline{R}_j(t)] = \exp[\delta_{\mathrm{q}}(0,t) + \tfrac{1}{2}\cdot\Sigma_{\mathrm{r}j}(t)] - 1 \tag{4A.21}$$

where:

$$\phi_{\mathrm{r}jk}(l) = \phi_{jk}(l) - \phi_{\mathrm{q}k}(l) \tag{4A.22}$$

The covariance between averaged price inflation and the nominal returns on asset class j over the interval 0 to t is given by (for $t > 0$):

$$\begin{aligned}\mathrm{cov}[\overline{R}_{\mathrm{q}}(t), \overline{R}_j(t)] = {} & \exp[2\cdot\delta_{\mathrm{x}}(0,t) - \delta_{\mathrm{q}}(0,t)] \\ & \cdot(\exp[\tfrac{1}{2}\cdot\Sigma_{\mathrm{q}j}(t)] - \exp[\tfrac{1}{2}\cdot(\Sigma_{\mathrm{q}}(t) + \Sigma_j(t))])\end{aligned} \tag{4A.23}$$

where:

$$\phi_{\mathrm{q}jk}(l) = \phi_{\mathrm{q}k}(l) + \phi_{jk}(l) \tag{4A.24}$$

The covariance between averaged price inflation and the real returns on asset class j over the interval 0 to t is given by (for $t > 0$):

$$\begin{aligned}\mathrm{cov}[\overline{R}_{\mathrm{q}}(t), \overline{R}_{\mathrm{r}j}(t)] = {} & \exp[\delta_{\mathrm{x}}(0,t)] \\ & \cdot(\exp[\tfrac{1}{2}\cdot\Sigma_j(t)] - \exp[\tfrac{1}{2}\cdot(\Sigma_{\mathrm{q}}(t) + \Sigma_{\mathrm{r}j}(t))])\end{aligned} \tag{4A.25}$$

The covariance, or variance if $i = j$, between the averaged nominal returns on asset classes i and j over the interval 0 to t is given by (for $t > 0$):

$$\operatorname{cov}[\overline{R}_i(t), \overline{R}_j(t)] = \exp[2 \cdot \delta_x(0, t)] \cdot (\exp[\tfrac{1}{2} \cdot \Sigma_{ij}(t)] - \exp[\tfrac{1}{2} \cdot (\Sigma_i(t) + \Sigma_j(t))]) \tag{4A.26}$$

where: $$\phi_{ijk}(l) = \phi_{ik}(l) + \phi_{jk}(l) \tag{4A.27}$$

The covariance, or variance if $i = j$, between the averaged real returns on asset classes i and j over the interval 0 to t is given by (for $t > 0$):

$$\operatorname{cov}[\overline{R}_{ri}(t), \overline{R}_{rj}(t)] = \exp[2 \cdot \delta_q(0, t)] \cdot (\exp[\tfrac{1}{2} \cdot \Sigma_{rij}(t)] - \exp[\tfrac{1}{2} \cdot (\Sigma_{ri}(t) + \Sigma_{rj}(t))]) \tag{4A.28}$$

where: $$\phi_{rijk}(l) = \phi_{rik}(l) + \phi_{rjk}(l) \tag{4A.29}$$

Appendix 4B: Statistical Properties of the Jump-Equilibrium Model

4B.1 Expected bond prices

Equation 4.3.3 can be derived from equation 4.3.1 as follows:

$$\frac{\mathrm{E}_t[X_j(t+\tau)]}{X_j(t)} = \left(\frac{f_j(t+\tau)}{f_j(t)}\right) \cdot \prod_{k=1}^{n} \exp[-\lambda_{jk} \cdot \tau \cdot G_k(t)] \\ \cdot \prod_{k=1}^{n} \mathrm{E}_t\left[\exp\left[\beta_{jk} \cdot (G_k(t+\tau) - G_k(t)) - \lambda_{jk} \cdot \int_t^{t+\tau} (G_k(w) - G_k(t))dw\right]\right] \tag{4B.1}$$

Now:

$$\begin{aligned}
&\mathrm{E}_t\left[\exp\left[\beta_{jk} \cdot (G_k(t+\tau) - G_k(t)) - \lambda_{jk} \cdot \int_t^{t+\tau} (G_k(w) - G_k(t))dw\right]\right] \\
&= \mathrm{E}_t\left[\lim_{h\to 0}\left\{\prod_{l=1}^{N=\frac{\tau}{h}} \exp[(\beta_{jk} - \lambda_{jk} \cdot h \cdot (N-l+1)) \cdot (G_k(t+h\cdot l) - G_k(t+h\cdot(l-1)))]\right\}\right] \\
&= \lim_{h\to 0}\left\{\prod_{l=1}^{N} (1-\beta_{jk} + \lambda_{jk} \cdot h \cdot (N-l+1))^{-\theta_k \cdot h}\right\} \\
&= \exp\left[\lim_{h\to 0}\left\{-\theta_k \cdot \sum_{l=1}^{N} h \cdot \log_e(1-\beta_{jk} + \lambda_{jk} \cdot h \cdot (N-l+1))\right\}\right] \\
&= \exp\left[-\theta_k \cdot \int_t^{t+\tau} \log_e(1-\beta_{jk} + \lambda_{jk} \cdot (t+\tau-w))dw\right] \\
&= \exp[\theta_k \cdot \tau] \cdot \left(\frac{(1-\beta_{jk})^{1-\beta_{jk}}}{(1-\beta_{jk} + \lambda_{jk} \cdot \tau)^{1-\beta_{jk}+\lambda_{jk}\cdot\tau}}\right)^{\frac{\theta_k}{\lambda_{jk}}}
\end{aligned}$$

Substituting this result into equation 4B.1 gives equation 4.3.3.

4B.2 Total return indices

Equation 4.3.6 can be derived as follows:

$$\frac{RX_j(t,\tau)}{RX_j(0,\tau)} = \lim_{h\to 0}\left\{\prod_{l=1}^{N=\frac{t}{h}} \frac{E_{l\cdot h}[X_j((l-1)\cdot h+\tau)]}{E_{(l-1)\cdot h}[X_j((l-1)\cdot h+\tau)]}\right\} \tag{4B.2}$$

Using equations 4.3.1 and 4.3.3:

$$\frac{\mathrm{E}_{l\cdot h}[X_j((l-1)\cdot h+\tau)]}{\mathrm{E}_{(l-1)\cdot h}[X_j((l-1)\cdot h+\tau)]}$$

$$\begin{aligned}&=\prod_{k=1}^{n}\{\exp[-h\cdot\theta_k+(\beta_{jk}-\lambda_{jk}\cdot(\tau-h))\cdot(G_k(l\cdot h)-G_k((l-1)\cdot h))]\\&\quad\cdot\exp\left[-\lambda_{jk}\cdot\left(\int_{(l-1)\cdot h}^{l\cdot h}G_k(w)dw-h\cdot G_k((l-1)\cdot h)\right)\right]\\&\quad\cdot(1-\beta_{jk}+\lambda_{jk}\cdot(\tau-h))^{\theta_k\cdot h}\cdot\left[1+\frac{-\lambda_{jk}\cdot h}{1-\beta_{jk}+\lambda_{jk}\cdot\tau}\right]^{\theta_k\cdot h\cdot\left(\frac{1-\beta_{jk}+\lambda_{jk}\cdot\tau}{-\lambda_{jk}\cdot h}\right)}\}\end{aligned}\tag{4B.3}$$

Substituting equation 4B.3 into equation 4B.2 gives:

$$\begin{aligned}\frac{RX_j(t,\tau)}{RX_j(0,\tau)}&=\lim_{h\to 0}\prod_{k=1}^{n}\{\exp[(\beta_{jk}-\lambda_{jk}\cdot(\tau-h))\cdot(G_k(t)-G_k(0))]\\&\quad\cdot(1-\beta_{jk}+\lambda_{jk}\cdot(\tau-h))^{\theta_k\cdot t}\\&\quad\cdot\exp\left[-\lambda_{jk}\cdot\left(\int_0^t G_k(w)dw-\sum_{l=1}^{N}h\cdot G_k((l-1)\cdot h)\right)\right]\\&\quad\cdot\exp[-\theta_k\cdot t]\cdot\left[1+\frac{-\lambda_{jk}\cdot h}{1-\beta_{jk}+\lambda_{jk}\cdot\tau}\right]^{\theta_k\cdot t\cdot\left(\frac{1-\beta_{jk}+\lambda_{jk}\cdot\tau}{-\lambda_{jk}\cdot h}\right)}\}\\&=\prod_{k=1}^{n}\exp[(\beta_{jk}-\lambda_{jk}\cdot\tau)\cdot G_k(t)]\cdot(1-\beta_{jk}+\lambda_{jk}\cdot\tau)^{\theta_k\cdot t}\end{aligned}\tag{4B.4}$$

4B.3 Limits of the expected force of price inflation and asset returns

The limit of $\mu_q(t)$ as t tends to infinity can be derived as follows:

$$\begin{aligned}\lim_{t\to\infty}\mu_q(t)&=\lim_{t\to\infty}\{t\cdot(\delta_x(0,t)-\delta_q(0,t))-(t-1)\cdot(\delta_x(0,t-1)-\delta_q(0,t-1))\}\\&\quad+\sum_{k=1}^{n}\theta_k\cdot(\beta_{qk}-\beta_{xk}+\tfrac{1}{2}\cdot(\lambda_{qk}-\lambda_{xk}))+\lim_{t\to\infty}\left\{t\cdot\sum_{k=1}^{n}\theta_k\cdot(\lambda_{xk}-\lambda_{qk})\right\}\\&\quad+\lim_{t\to\infty}\left\{\sum_{k=1}^{n}\theta_k\cdot\log_e\left(\frac{\psi_{qk}(t,1)}{\psi_{xk}(t,1)}\right)\right\}\end{aligned}\tag{4B.5}$$

Assuming that the initial yield curves satisfy general regularity conditions and tend to finite limits 'at a faster rate' than the function t:

$$\begin{aligned}&\lim_{t\to\infty}\{t\cdot(\delta_x(0,t)-\delta_q(0,t))-(t-1)\cdot(\delta_x(0,t-1)-\delta_q(0,t-1))\}\\&\quad=\delta_x(0,\infty)-\delta_q(0,\infty)\end{aligned}\tag{4B.6}$$

Furthermore:

$$\begin{aligned}
&\lim_{t\to\infty}\{\log_e \psi_{jk}(t,1)\} \\
&= \lim_{t\to\infty}\left\{\log_e(1-\beta_{jk}+\lambda_{jk}\cdot(t-1)) + \log_e\left(1+\frac{-\lambda_{jk}}{1-\beta_{jk}+\lambda_{jk}\cdot t}\right)^{\frac{1-\beta_{jk}+\lambda_{jk}\cdot t}{-\lambda_{jk}}}\right\} \\
&= 1+\lim_{t\to\infty}\{\log_e(1-\beta_{jk}+\lambda_{jk}\cdot(t-1))\}
\end{aligned} \tag{4B.7}$$

Using equations 4B.6 and 4B.7 equation 4B.5 becomes:

$$\begin{aligned}
\lim_{t\to\infty}\mu_{\mathrm{q}}(t) &= \delta_{\mathrm{x}}(0,\infty)-\delta_{\mathrm{q}}(0,\infty)+\sum_{k=1}^{n}\theta_k\cdot(\beta_{\mathrm{q}k}-\beta_{\mathrm{x}k}+\tfrac{1}{2}\cdot(\lambda_{\mathrm{q}k}-\lambda_{\mathrm{x}k})) \\
&\quad+\lim_{t\to\infty}\left\{t\cdot\sum_{k=1}^{n}\theta_k\cdot\left(\lambda_{\mathrm{x}k}-\lambda_{\mathrm{q}k}+\frac{1}{t}\cdot\log_e\left(\frac{1-\beta_{\mathrm{q}k}+\lambda_{\mathrm{q}k}\cdot(t-1)}{1-\beta_{\mathrm{x}k}+\lambda_{\mathrm{x}k}\cdot(t-1)}\right)\right)\right\}
\end{aligned} \tag{4B.8}$$

Therefore, the limit of the expected force of price inflation depends on the values of $\lambda_{\mathrm{q}k}$ and $\lambda_{\mathrm{x}k}$. Using Smith's (1996) recommended parameter values:

$$\begin{aligned}
&\lim_{t\to\infty}\left\{\sum_{k=1}^{n}\theta_k\cdot\left(\lambda_{\mathrm{x}k}-\lambda_{\mathrm{q}k}+\frac{1}{t}\cdot\log_e\left(\frac{1-\beta_{\mathrm{q}k}+\lambda_{\mathrm{q}k}\cdot(t-1)}{1-\beta_{\mathrm{x}k}+\lambda_{\mathrm{x}k}\cdot(t-1)}\right)\right)\right\} \\
&= \sum_{k=1}^{n}\theta_k\cdot(\lambda_{\mathrm{x}k}-\lambda_{\mathrm{q}k})<0
\end{aligned} \tag{4B.9}$$

Therefore, from equations 4B.8 and 4B.9, Smith's recommended parameters and the assumptions concerning the initial yield curves imply that the expected force of price inflation tends to negative infinity as t tends to infinity.

Similarly for $\mu_j(t)$, assuming that the initial fixed-interest yield curve satisfies general regularity conditions and tends to a finite limit 'at a faster rate' than the function t (for j = m, b, r, s, and p):

$$\begin{aligned}
\lim_{t\to\infty}\mu_j(t) &= \delta_{\mathrm{x}}(0,\infty)+\lim_{t\to\infty}\left\{t\cdot\sum_{k=1}^{n}\theta_k\cdot\left(\lambda_{\mathrm{x}k}-\frac{\log_e\psi_{\mathrm{x}k}(t,1)}{t}\right)\right\} \\
&\quad+\sum_{k=1}^{n}\theta_k\cdot(\beta_{jk}-\beta_{\mathrm{x}k}-\lambda_{jk}\cdot\tau-\tfrac{1}{2}\cdot\lambda_{\mathrm{x}k}+1+\log_e(1-\beta_{jk}+\lambda_{jk}\cdot\tau))
\end{aligned} \tag{4B.10}$$

From equation 4B.7 this quantity is only finite if $\lambda_{\mathrm{x}k} = 0$ for all k.

For the real force of return on asset class j, assuming that the initial index-linked yield curve satisfies general regularity conditions and tends to a finite limit 'at a faster rate' than the function t (for j = m, b, r, s, and p):

$$\lim_{t\to\infty}\{\mu_j(t)-\mu_q(t)\} = \delta_q(0,\infty)+\lim_{t\to\infty}\left\{t\cdot\sum_{k=1}^{n}\theta_k\cdot\left(\lambda_{qk}-\frac{\log_e\psi_{qk}(t,1)}{t}\right)\right\}$$
$$+\sum_{k=1}^{n}\theta_k\cdot(\beta_{jk}-\beta_{qk}-\lambda_{jk}\cdot\tau-\tfrac{1}{2}\cdot\lambda_{qk}+1+\log_e(1-\beta_{jk}+\lambda_{jk}\cdot\tau)) \qquad (4B.11)$$

From equation 4B.7, this quantity is only finite if $\lambda_{qk}=0$ for all k.

4B.4 Variance of the expected force of price inflation and asset returns

The variance of price inflation can be obtained from the moment generating function of $\zeta_q(t)$, which is given by, using equation 4B.1:

$$m_{\zeta_q(t)}(u) = \mathrm{E}_t[\exp[u\cdot\zeta_q(t)]$$
$$=\prod_{k=1}^{n}\{\exp[u\cdot\theta_k\cdot(\lambda_k\cdot(t-\tfrac{1}{2})-\beta_k)]\cdot(1+u\cdot\lambda_k)^{-\theta_k\cdot(t-1)}\cdot(1+u\cdot(\lambda_k-\beta_k))^{-\theta_k} \qquad (4B.12)$$
$$\cdot\exp[\theta_k]\cdot\left(1+\frac{u\cdot\lambda_k}{1-u\cdot\beta_k}\right)^{-\theta_k\cdot\left(\frac{1-u\cdot\beta_k}{u\cdot\lambda_k}\right)}\}$$

where $\lambda_k\equiv\lambda_{qk}-\lambda_{xk}$ and $\beta_k\equiv\beta_{qk}-\beta_{xk}$.

Now:

$$\frac{\partial}{\partial u}m_{\zeta_q(t)}(u) = m_{\zeta_q(t)}(u)\cdot\sum_{k=1}^{n}\theta_k\cdot\left(\lambda_k\cdot(t-\tfrac{1}{2})-\beta_k-\frac{\lambda_k\cdot(t-1)}{1+u\cdot\lambda_k}\right.$$
$$\left.-\frac{\lambda_k-\beta_k}{1+u\cdot(\lambda_k-\beta_k)}+\left(\frac{1}{u^2\cdot\lambda_k}\right)\cdot\left(\log_e\left(1+\frac{u\cdot\lambda_k}{1-u\cdot\beta_k}\right)-\frac{u\cdot\lambda_k}{1+u\cdot(\lambda_k-\beta_k)}\right)\right) \qquad (4B.13)$$
$$= m_{\zeta_q(t)}(u)\cdot g(u)$$

From equation 4B.13:

$$\left.\frac{\partial}{\partial u}m_{\zeta_q(t)}(u)\right|_{u=0} = 0 \qquad (4B.14)$$

This result can be obtained by representing the terms in equation 4B.13 of the form $\log_e(1+x)$ and $(1-x)^{-1}$ as infinite sequences and simplifying. This result confirms that the mean of $\zeta_q(t)$ is zero.

Therefore, from equation 4B.13 and using equation 4B.14:

$$\frac{\partial^2}{\partial u^2} m_{\zeta_q(t)}(u)\bigg|_{u=0} = \frac{\partial}{\partial u} g(u)\bigg|_{u=0}$$

$$= \sum_{k=1}^{n} \theta_k \cdot \left(\frac{\lambda_k^2 \cdot (t-1)}{(1+u\cdot\lambda_k)^2} + \frac{(\lambda_k - \beta_k)^2}{(1+u\cdot(\lambda_k-\beta_k))^2} - \left(\frac{1}{u^3\cdot\lambda_k}\right) \right.$$

$$\left.\cdot\left(2\cdot\log_e\left(1+\frac{u\cdot\lambda_k}{1-u\cdot\beta_k}\right) - \frac{u\cdot(2\cdot\lambda_k-\beta_k)}{1+u\cdot(\lambda_k-\beta_k)} - \frac{u\cdot\beta_k}{1-u\cdot\beta_k} - \frac{u^2\cdot\lambda_k\cdot(\lambda_k-\beta_k)}{(1+u\cdot(\lambda_k-\beta_k))^2}\right)\right)\Bigg|_{u=0}$$

$$= \sum_{k=1}^{n} \theta_k \cdot (\lambda_k^2 \cdot (t - \tfrac{2}{3}) + \beta_k^2 - \lambda_k \cdot \beta_k)$$

This quantity is also obtained by representing terms of the form $\log_e(1+x)$ and $(1-x)^{-1}$ as infinite sequences and simplifying.

The variance of the continuously compounded nominal rate of return on asset class j can be obtained using the above results with $\lambda_k \equiv -\lambda_{xk}$ and $\beta_k \equiv \beta_{jk} - \beta_{xk} - \lambda_{jk}\cdot\tau_j$. The variance of the real rate of return on asset class j can also be obtained using the above results with $\lambda_k \equiv -\lambda_{qk}$ and $\beta_k \equiv \beta_{jk} - \beta_{qk} - \lambda_{jk}\cdot\tau_j$.

4B.5 Annual rates of return

The expected annual rate of price inflation over the interval $t-1$ to t is given by (for $t > 0$):

$$\mathrm{E}\left[\frac{R_q(t)}{R_q(t-1)} - 1\right] = \left(\frac{f_x(t-1)\cdot f_q(t)}{f_x(t)\cdot f_q(t-1)}\right)$$

$$\cdot\prod_{k=1}^{n}\mathrm{E}\left[\exp\left[(\beta_{qk}-\beta_{xk})\cdot(G_k(t)-G_k(t-1)) - (\lambda_{qk}-\lambda_{xk})\cdot\int_{t-1}^{t} G_k(w)dw\right]\right] - 1 \qquad (4B.15)$$

$$= \left(\frac{f_x(t-1)\cdot f_q(t)}{f_x(t)\cdot f_q(t-1)}\right)\cdot \mathrm{M}_{qx}(1,1)\cdot\prod_{k=1}^{n}(1+\lambda_{qxk})^{-\theta_k\cdot(t-1)} - 1$$

where (for $1 - t\cdot\beta_{jk} > 0$ and $1 - t\cdot(\beta_{jk} - \lambda_{jk}\cdot\tau) > 0$):

$$\mathrm{M}_j(t,\tau) = \prod_{k=1}^{n} \exp[\theta \cdot \tau] \cdot \left(\frac{(1 - t \cdot \beta_{jk})^{1-t \cdot \beta_{jk}}}{(1 - t \cdot (\beta_{jk} - \lambda_{jk} \cdot \tau))^{1-t \cdot (\beta_{jk} - \lambda_{jk} \cdot \tau)}} \right)^{\frac{\theta_k}{t \cdot \lambda_{jk}}} \tag{4B.16}$$

$$\beta_{qxk} = \beta_{qk} - \beta_{xk} \text{ and } \lambda_{qxk} = \lambda_{qk} - \lambda_{xk} \tag{4B.17}$$

This result follows from the proof in Appendix 4B.1.

The variance of the annual rate of price inflation over the interval $t - 1$ to t is given by (for $t > 0$):

$$\begin{aligned} \operatorname{var}\left[\frac{R_q(t)}{R_q(t-1)} - 1 \right] &= \left(\frac{f_x(t-1) \cdot f_q(t)}{f_x(t) \cdot f_q(t-1)} \right)^2 \\ &\cdot \left(\mathrm{M}_{qx}(2,1) \cdot \prod_{k=1}^{n} (1 + 2 \cdot \lambda_{qxk})^{-\theta_k \cdot (t-1)} - \left(\mathrm{M}_{qx}(1,1) \cdot \prod_{k=1}^{n} (1 + \lambda_{qxk})^{-\theta_k \cdot (t-1)} \right)^2 \right) \end{aligned} \tag{4B.18}$$

The autocovariance between the annual rate of price inflation in year $t + u$ and year t is given by (for $u, t > 0$):

$$\begin{aligned} \operatorname{cov}\left[\frac{R_q(t+u)}{R_q(t+u-1)} - 1, \frac{R_q(t)}{R_q(t-1)} - 1 \right] &= \left(\frac{f_x(t+u-1) \cdot f_q(t+u)}{f_x(t+u) \cdot f_q(t+u-1)} \right) \\ &\cdot \left(\frac{f_x(t-1) \cdot f_q(t)}{f_x(t) \cdot f_q(t-1)} \right) \cdot \left(\mathrm{M}_{qx}(1,2) \cdot \prod_{k=1}^{n} (1 + \lambda_{qxk})^{-\theta_k \cdot (u-1)} \cdot (1 + 2 \cdot \lambda_{qxk})^{-\theta_k \cdot (t-1)} \right. \\ &\left. - (\mathrm{M}_{qx}(1,1))^2 \cdot \prod_{k=1}^{n} (1 + \lambda_{qxk})^{-\theta_k \cdot (2 \cdot t + u - 2)} \right) \end{aligned} \tag{4B.19}$$

The expected annual nominal rate of return on asset class j over the interval $t - 1$ to t is given by (for j = m, b, r, s, and p and $t > 0$):

$$\begin{aligned} &\mathrm{E}\left[\frac{R_j(t)}{R_j(t-1)} - 1 \right] \\ &= \left(\frac{f_x(t-1)}{f_x(t)} \right) \cdot \mathrm{M}_{xj}(1,1) \cdot \prod_{k=1}^{n} (1 - \beta_{jk} + \lambda_{jk} \cdot \tau_j)^{\theta_k} \cdot (1 - \lambda_{xk})^{-\theta_k \cdot (t-1)} - 1 \end{aligned} \tag{4B.20}$$

where: $$\beta_{xjk} = \beta_{jk} - \lambda_{jk} \cdot \tau_j - \beta_{xk} \text{ and } \lambda_{xjk} = -\lambda_{xk} \tag{4B.21}$$

The variance of the annual nominal rate of return on asset class j over the interval $t-1$ to t is given by (for j = m, b, r, s, and p and $t > 0$):

$$\operatorname{var}\left[\frac{R_j(t)}{R_j(t-1)}-1\right]=\left(\frac{f_{\mathrm{x}}(t-1)}{f_{\mathrm{x}}(t)}\right)^2\cdot\left(\prod_{k=1}^{n}(1-\beta_{\mathrm{x}jk}+\lambda_{\mathrm{x}jk}\cdot\tau_j)^{2\cdot\theta_k}\right)$$
$$\cdot\left(\mathrm{M}_{\mathrm{x}j}(2,1)\cdot\prod_{k=1}^{n}(1+2\cdot\lambda_{\mathrm{x}jk})^{-\theta_k\cdot(t-1)}-\left(\mathrm{M}_{\mathrm{x}j}(1,1)\cdot\prod_{k=1}^{n}(1+\lambda_{\mathrm{x}jk})^{-\theta_k\cdot(t-1)}\right)^2\right) \tag{4B.22}$$

The autocovariance between the annual nominal rate of return on asset class j in year $t+u$ and year t is given by (for j = m, b, r, s, and p and for u, $t > 0$):

$$\operatorname{cov}\left[\frac{R_j(t+u)}{R_j(t+u-1)}-1,\frac{R_j(t)}{R_j(t-1)}-1\right]=\left(\frac{f_{\mathrm{x}}(t+u-1)}{f_{\mathrm{x}}(t+u)}\right)\cdot\left(\frac{f_{\mathrm{x}}(t-1)}{f_{\mathrm{x}}(t)}\right)$$
$$\cdot\left(\prod_{k=1}^{n}(1-\beta_{\mathrm{x}jk}+\lambda_{\mathrm{x}jk}\cdot\tau_j)^{2\cdot\theta_k}\right)\cdot\left(-(\mathrm{M}_{\mathrm{x}j}(1,1))^2\cdot\prod_{k=1}^{n}(1+\lambda_{\mathrm{x}jk})^{-\theta_k\cdot(2\cdot t+u-2)}\right. \tag{4B.23}$$
$$\left.+\mathrm{M}_{\mathrm{x}j}(2,1)\cdot\prod_{k=1}^{n}(1+\lambda_{\mathrm{x}jk})^{-\theta_k\cdot(u-1)}\cdot(1+2\cdot\lambda_{\mathrm{x}jk})^{-\theta_k\cdot(t-1)}\right)$$

4B.6 Averaged total returns

Expected averaged annual price inflation over the interval 0 to t is given by (for $t > 0$):

$$\mathrm{E}[\overline{R}_{\mathrm{q}}(t)]$$
$$=\left(\frac{f_{\mathrm{q}}(t)\cdot f_{\mathrm{x}}(0)}{f_{\mathrm{x}}(t)\cdot f_{\mathrm{q}}(0)}\right)^{\frac{1}{t}}\cdot\prod_{k=1}^{n}\mathrm{E}\left[\exp\left[\frac{1}{t}\cdot\left(\beta_{\mathrm{qx}k}\cdot G_k(t)-\lambda_{\mathrm{qx}k}\cdot\int_0^t G_k(w)dw\right)\right]\right]-1 \tag{4B.24}$$
$$=\left(\frac{f_{\mathrm{q}}(t)\cdot f_{\mathrm{x}}(0)}{f_{\mathrm{x}}(t)\cdot f_{\mathrm{q}}(0)}\right)^{\frac{1}{t}}\cdot\mathrm{M}_{\mathrm{qx}}(\tfrac{1}{t},1)-1$$

where:
$$\overline{R}_j(t)=\left(\frac{R_j(t)}{R_j(0)}\right)^{\frac{1}{t}}-1 \tag{4B.25}$$

The variance of averaged annual price inflation over the interval 0 to t is given by (for $t > 0$):

$$\operatorname{var}[\overline{R}_{\mathrm{q}}(t)] = \left(\frac{f_{\mathrm{q}}(t)\cdot f_{\mathrm{x}}(0)}{f_{\mathrm{x}}(t)\cdot f_{\mathrm{q}}(0)}\right)^{\frac{2}{t}} \cdot (\mathrm{M}_{\mathrm{qx}}(\tfrac{2}{t},1) - (\mathrm{M}_{\mathrm{qx}}(\tfrac{1}{t},1))^2) \tag{4B.26}$$

Expected averaged annual nominal returns on asset class j over the interval 0 to t is given by (for j = m, b, r, s, and p and $t > 0$):

$$\mathrm{E}[\overline{R}_j(t)] = \left(\frac{f_{\mathrm{x}}(0)}{f_{\mathrm{x}}(t)}\right)^{\frac{1}{t}} \cdot \left(\prod_{k=1}^{n}(1-\beta_{jk}+\lambda_{jk}\cdot\tau_j)^{\theta_k}\right) \cdot \mathrm{M}_{\mathrm{x}j}(\tfrac{1}{t},1) - 1 \tag{4B.27}$$

The covariance between averaged annual price inflation and the annual nominal returns on asset class j over the interval 0 to t is given by (for $t > 0$):

$$\begin{aligned}\operatorname{cov}[\overline{R}_{\mathrm{q}}(t), \overline{R}_j(t)] = {} & \left(\frac{f_{\mathrm{q}}(t)\cdot(f_{\mathrm{x}}(0))^2}{(f_{\mathrm{x}}(t))^2\cdot f_{\mathrm{q}}(0)}\right)^{\frac{1}{t}} \cdot \left(\prod_{k=1}^{n}(1-\beta_{jk}+\lambda_{jk}\cdot\tau_j)^{\theta_k}\right) \\ & \cdot (\mathrm{M}_{\mathrm{qx}j}(\tfrac{1}{t},1) - \mathrm{M}_{\mathrm{qx}}(\tfrac{1}{t},1)\cdot\mathrm{M}_{\mathrm{x}j}(\tfrac{1}{t},1))\end{aligned} \tag{4B.28}$$

where: $$\beta_{\mathrm{qx}jk} = \beta_{\mathrm{qx}k} + \beta_{\mathrm{x}jk} \quad \text{and} \quad \lambda_{\mathrm{qx}jk} = \lambda_{\mathrm{qx}k} + \lambda_{\mathrm{x}jk} \tag{4B.29}$$

The covariance, or variance if $j = l$, between the averaged annual nominal returns on asset classes j and l over the interval 0 to t is given by (for $t > 0$):

$$\begin{aligned}\operatorname{cov}[\overline{R}_j(t), \overline{R}_l(t)] = {} & \left(\frac{f_{\mathrm{x}}(0)}{f_{\mathrm{x}}(t)}\right)^{\frac{2}{t}} \cdot \left(\prod_{k=1}^{n}(1-\beta_{jk}+\lambda_{jk}\cdot\tau_j)^{\theta_k}\right) \\ & \cdot \left(\prod_{k=1}^{n}(1-\beta_{lk}+\lambda_{lk}\cdot\tau_l)^{\theta_k}\right) \cdot (\mathrm{M}_{\mathrm{x}jl}(\tfrac{1}{t},1) - \mathrm{M}_{\mathrm{x}j}(\tfrac{1}{t},1)\cdot\mathrm{M}_{\mathrm{x}l}(\tfrac{1}{t},1))\end{aligned} \tag{4B.30}$$

where: $$\beta_{\mathrm{x}jlk} = \beta_{\mathrm{x}jk} + \beta_{\mathrm{x}lk} \quad \text{and} \quad \lambda_{\mathrm{x}jlk} = \lambda_{\mathrm{x}jk} + \lambda_{\mathrm{x}lk} \tag{4B.31}$$

The means, variances, autocovariances, and covariances of the real rates of return are similar to the above equations and can generally be obtained by replacing the subscript x with the subscript q. Higher moments and moments calculated over other intervals can also be obtained using similar methods.

4B.7 Returns on zero-coupon fixed-interest and index-linked securities

Tables 4B.1 and 4B.2 report the means and standard deviations of the annual nominal and real rate of return on different duration fixed-interest and index-linked securities.

Table 4B.1 Returns on zero-coupon fixed-interest and index-linked securities

	Nominal				*Real*			
	Fixed-interest		*Index-linked*		*Fixed-interest*		*Index-linked*	
Duration	*mean*	*std dev*	*mean*	*std dev*	*mean*	*std dev*	*mean*	*std dev*
0	7.55	0.47	7.55	0.47	3.47	0.43	3.47	0.43
1	7.73	0.47	7.67	0.69	3.65	0.69	3.59	0.42
2	7.92	1.23	7.80	1.38	3.83	1.41	3.71	1.12
3	8.10	2.02	7.92	2.12	4.01	2.16	3.82	1.85
4	8.29	2.82	8.04	2.87	4.19	2.92	3.94	2.58
5	8.47	3.61	8.16	3.63	4.37	3.68	4.06	3.31
6	8.65	4.41	8.29	4.38	4.54	4.44	4.18	4.04
7	8.83	5.20	8.41	5.14	4.72	5.20	4.29	4.77
8	9.02	5.99	8.53	5.90	4.90	5.96	4.41	5.50
9	9.20	6.79	8.66	6.66	5.07	6.72	4.53	6.24
10	9.38	7.57	8.78	7.41	5.25	7.48	4.64	6.97
11	9.56	8.36	8.90	8.17	5.42	8.24	4.76	7.69
12	9.74	9.15	9.02	8.93	5.60	8.99	4.88	8.42
13	9.92	9.93	9.14	9.69	5.77	9.75	4.99	9.15
14	10.09	10.71	9.27	10.44	5.94	10.50	5.11	9.88
15	10.27	11.50	9.39	11.20	6.12	11.25	5.23	10.61
16	10.45	12.28	9.51	11.96	6.29	12.00	5.34	11.34
17	10.63	13.06	9.63	12.71	6.46	12.75	5.46	12.07
18	10.80	13.83	9.75	13.47	6.63	13.50	5.57	12.80
19	10.98	14.61	9.88	14.23	6.80	14.25	5.69	13.53
20	11.16	15.39	10.00	14.99	6.97	14.99	5.80	14.26
21	11.33	16.16	10.12	15.74	7.14	15.74	5.92	14.98
22	11.51	16.94	10.24	16.50	7.31	16.48	6.04	15.71
23	11.68	17.71	10.36	17.26	7.48	17.23	6.15	16.44
24	11.86	18.49	10.48	18.02	7.65	17.97	6.27	17.17
25	12.03	19.26	10.60	18.78	7.82	18.72	6.38	17.90
26	12.20	20.03	10.72	19.54	7.99	19.46	6.50	18.63
27	12.38	20.80	10.84	20.30	8.15	20.20	6.61	19.36
28	12.55	21.57	10.96	21.06	8.32	20.94	6.73	20.10
29	12.72	22.34	11.08	21.82	8.49	21.68	6.84	20.83

Table 4B.2 Returns on zero-coupon fixed-interest and index-linked securities

	Nominal				*Real*			
	Fixed-interest		*Index-linked*		*Fixed-interest*		*Index-linked*	
Duration	*mean*	*std dev*	*mean*	*std dev*	*mean*	*std dev*	*mean*	*std dev*
30	12.89	23.11	11.20	22.58	8.65	22.42	6.96	21.56
31	13.06	23.88	11.32	23.34	8.82	23.16	7.07	22.29
32	13.23	24.65	11.44	24.10	8.99	23.90	7.18	23.03
33	13.40	25.42	11.56	24.87	9.15	24.64	7.30	23.76
34	13.57	26.19	11.68	25.63	9.32	25.38	7.41	24.50
35	13.74	26.95	11.80	26.40	9.48	26.12	7.53	25.23
36	13.91	27.72	11.92	27.16	9.64	26.86	7.64	25.97
37	14.08	28.49	12.04	27.93	9.81	27.60	7.76	26.70
38	14.25	29.26	12.16	28.70	9.97	28.34	7.87	27.44
39	14.42	30.03	12.28	29.46	10.13	29.08	7.98	28.18
40	14.58	30.79	12.40	30.23	10.30	29.82	8.10	28.92
41	14.75	31.56	12.52	31.00	10.46	30.55	8.21	29.66
42	14.92	32.33	12.64	31.78	10.62	31.29	8.32	30.40
43	15.08	33.10	12.76	32.55	10.78	32.03	8.44	31.15
44	15.25	33.86	12.88	33.32	10.94	32.77	8.55	31.89
45	15.41	34.63	13.00	34.10	11.10	33.51	8.66	32.64

Chapter 5

ECONOMIC METHODOLOGY

5.1 Introduction

The previous chapters considered various models of the economic assumptions required for actuarial calculations. These models ranged from deterministic point estimates that are established using professional judgement to complicated stochastic models that are developed using advanced statistical techniques. None of these models have acquired universal support. This partly reflects differences in applications, but it also reflects fundamental philosophical and methodological differences. Although some actuaries, including Redington (1983), maintain that stochastic models are not justified because significant statistical economic regularities cannot be detected, other actuaries have attempted to construct stochastic models.

Actuaries are also divided over whether their models should conform to orthodox financial economic theory. Wilkie (1995b) explicitly rejected the efficient market hypothesis and developed a model that permits substantial arbitrage opportunities. Actuarial deterministic bases also often assume that the markets are incorrectly valued. In addition, Wilkie's model assumes that investors do not have rational expectations concerning future levels of inflation. These contrary assumptions are defended on the grounds of professional judgement and empirical evidence. Moreover, financial economic theory is often criticised for being based on unrealistic simplifying assumptions (see Section 2.4.2). Other actuarial models, including Dyson and Exley's (1995) model and the jump-equilibrium model, reject these arguments and conform to financial economic theory without being too concerned with historical data. Which approach is the most appropriate? Are actuaries justified in rejecting orthodox financial economic theory? Are all stochastic models pseudo-scientific?

This chapter attempts to partially resolve these fundamental questions by considering the literature on the philosophy of science and economic methodology. This literature is concerned with the development of scientific knowledge and central to this is how this knowledge can be rationally justified. It emphasises the limitations of knowledge, the importance and difficulty of empirical testing, and the role of a theoretical framework. As this literature does not appear to have been comprehensively discussed by actuaries, a relatively wide ranging review is provided. The insights provided by this review for actuarial economic modelling are then discussed and this discussion is continued and expanded in subsequent chapters.

Chapter 5 briefly reviews some of the philosophical and methodological issues that are relevant to economics. This review is not comprehensive and is primarily based on the more detailed reviews provided by Blaug (1992), Caldwell (1994a), Hausman (1992), and Honderich (1995). These sources, especially Blaug (1992), provide more detailed lists of references on this literature. Section 5.2 discusses some of the relevant background material from the philosophy of science. Few direct references are given in this section because the information discussed was principally obtained from the above mentioned sources. Section 5.3 introduces economic methodology and considers the predominant economic methodological positions. Friedman's (1953) influential methodology is then discussed in Section 5.4. The limitations of economic prediction are considered in Section 5.5. Section 5.6 considers Hausman's (1992) defence of the methodological practice of economists. The specific implications of this discussion on actuarial economic modelling are then explored in Section 5.7.

5.2 Philosophical Background

5.2.1 Logical positivism

Logical positivism was an influential philosophical movement that started in the 1920s. The logical positivists developed a program for the study of science that combined empiricism with logical analysis. Science was viewed as having a hierarchical structure that was built up from observations, measurements, generalisations, and theories, to scientific disciplines and theoretical schemes. Thus, sense-data, or information that is directly perceived, were believed to provide foundational knowledge. The logical positivists sought to use logical analysis to precisely determine the manner in which scientific claims relate to sense-data. They thereby aimed to develop a rigorous method for assessing the empirical justification of scientific claims. Formal symbolic logic was favoured because it provided an unambiguous language that could be used in all scientific disciplines.

The logical positivists were only interested in clarifying the scientific status of existing knowledge, known as the context of justification. They were not concerned with how this knowledge was discovered, termed the context of discovery. The context of discovery may be subject to irrational influences, but the context of justification should depend on strict rational rules. The logical positivists also attempted to precisely define other fundamental scientific concepts, such as explanation and prediction. Moreover, they intended to demonstrate that significant past scientific achievements, especially in mathematics and mathematical physics, were consistent with their proposed methods. Therefore, the logical positivists aimed to formalise scientific methods and thereby illustrate how objective knowledge can be established.

The logical positivists believed that the scientific status of individual statements should be assessed using the verification principle, which requires that scientific statements should be either analytic or synthetic. Analytic statements are self contradictions or tautologies, such as 'triangles have three sides', that can be assessed by the meaning of the words used and their grammatical structure. Synthetic statements are factual statements that can be empirically tested against sense-data. Moreover, the strict version of the verification principle requires that synthetic statements should be capable, at least in principle, of complete verification by observational evidence. This requirement was based on the general idea that scientific knowledge should enhance our ability to predict empirical events. Statements that are neither analytic nor synthetic were considered to be logically meaningless, implying that they are neither true nor false. It was argued that meaningless statements should not be used in scientific theories. The relationship between sense-data and scientific theories should always be precise and transparent.

The verification principle has significant difficulties. It implies that metaphysical, religious, aesthetic, and ethical statements do not constitute valid scientific knowledge because they generally cannot be empirically assessed. Hence, statements with theoretical terms that refer to unobservable phenomena, such as electrons, are classified as meaningless. This implies that significant work in many 'scientific' disciplines, including mathematical physics, is meaningless.

Furthermore, the verification principle suggests that statements of universal form, such as 'all ravens are black', are meaningless because it cannot be guaranteed that an exception to these statements will never be found. Particular premises are insufficient to prove that a universal statement is true. This problem is related to Hume's problem of induction, which broadly states that there is no empirical justification for assuming the future will be like the past. Past observations cannot form the sole basis of generalisations because they are specific to a historical period and location. As a result, inductive reasoning is unable to provide conclusive material proof for a hypothesis or theory. Inductive evidence does provide support for generalisations, but it is difficult to establish the precise extent to which a generalisation is confirmed by this evidence. This difficulty is illustrated by the paradox of the ravens. The statement 'all ravens are black' is confirmed by every observation of a black raven. However, this statement is also logically equivalent to the statement 'anything that is not black is not a raven', which is confirmed by every observation of a non-black non-raven. Hence, if it is assumed that all statements that are logically equivalent are confirmed by the same observations then a white handkerchief would confirm that 'all ravens are black'.

An additional difficulty with the verification principle is that is not possible to observe the world from a neutral perspective. Observation is theory laden because prior theoretical knowledge influences what is observed and how it is interpreted. For instance, the colour of a white piece of paper observed under a red light depends on the previous experience of the observer. Moreover, what is observable is dependent on the development of theory and consequently is subject to change over time. For example, microscopes and telescopes have significantly increased the types of phenomena that are observable to humans. Thus, it is generally not possible to test individual claims in isolation. This conclusion is more generally known as the Duhem-Quine thesis, which states that factual statements only have meaning within systems of statements so that they can only be tested holistically. When individual claims are tested within systems of statements then the analytic-synthetic distinction is largely irrelevant as theory is essential for descriptive purposes. Any individual claim can also be made to be true if it is conjoined with other suitably chosen statements. In the extreme, any observational statement can be maintained by claiming that they represent hallucinations. Therefore, empirical evidence is not necessarily unambiguous because perception is relative to prior theoretical knowledge.

The above difficulties suggest that observations, or sense-data, cannot be relied on to provide an objective foundation to scientific knowledge. According to Neurath (see Cartwright 1995), accumulating knowledge is like rebuilding a boat while still having to stay afloat in it on the open sea. There are no permanent foundations to knowledge: we are usually forced to rely on what we think we know.

Additional problems with logical positivism include its attempt to formalise science. Gödel proved that mathematics cannot be shown to be a comprehensive and provably consistent system (see Giaquinto 1995). Hence, it is impossible to completely formalise mathematics, which was regarded as providing one of the best examples of objective knowledge. Formal logic has also proved to be a difficult and awkward language to use. Lastly, the sharp distinction between the context of discovery and the context of justification has been questioned. In econometrics, for example, it is difficult to assess models without knowledge of how they were discovered and there exist rule-based methods for developing econometric models (see Section 7.2.1). The context of discovery is not necessarily an irrational process that is irrelevant to the context of justification. Therefore, the ambitious program developed by the logical positivists has met with significant difficulties and most of its theses have proved to be untenable. Nevertheless, this work emphasised the limitations of knowledge and it is often used as a starting point for discussions in the philosophy of science.

5.2.2 Scientific explanation

Hempel, who was originally a logical positivist, developed a definition of scientific explanation that proved to be more durable than logical positivism (see Hempel and Oppenheim 1948). He defined scientific explanation in terms of the covering-law models, which require that the phenomena to be explained must be able to be logically deduced, or inferred with a high inductive probability from a set of true statements that include at least one general law. The covering-law models imply that the operations of explanation and prediction use the same rules of logical inference, the only difference between them is temporal. This implication is the symmetry thesis.

The covering-law models show the events to be explained as being part of a more general theoretical system or an instance of a general law. They describe why the events occurred and why they could have been anticipated. Moreover, they illustrate that explanations expose the causes of the events, which are given by the set of initial conditions that make up the premise. The covering-law models emphasise that explanations require true statements and must involve at least one general law as opposed to an accidental generalisation.

The controversial claim that every legitimate explanation can be characterised by the covering-law models has been criticised as it disallows certain seemingly scientific explanations in the social and historical sciences because they cannot provide reliable predictions. It has also been noted that statistical premises are often used to describe events that have a low probability of occurring. Counterexamples have been suggested of explanations that fit the covering-law models but appear to be illegitimate. One such example is where the height of a building can be determined from the general law of trigonometry and the initial conditions of the length of its shadow and the angle from the top of the building to the end of the shadow. However, this 'explanation' does not explain why the building has this height. Explanations seem to require premises that include relevant causal influences. The building explains its shadow, but its shadow does not explain the building. Moreover, the building's shadow is not a relevant factor that determined its height. Constructive empiricism, due to van Fraassen, emphasises the context dependency of explanations (see Boylan and O'Gorman 1995). Explanations usually depend on the specific contrast being made and are evaluated relative to this contrast. For example the explanation of 'why security X was purchased' depends on whether the context is 'why security X rather than Y was purchased' or whether it is 'why security X was purchased at a specific time'. Hence, explanations have an important pragmatic element.

Although the covering-law models describe a wide range of scientific explanations, they appear to be too restrictive to describe all legitimate explanations. Broader definitions of explanation have been sought, but none has managed to attract widespread support.

5.2.3 Theory assessment

The logical positivists asserted that individual statements should be assessed using the verification principle. However, this approach was found to have significant difficulties and was ultimately abandoned. This led to the development of a more sophisticated positivist methodology known as logical empiricism. The logical empiricists accommodated theoretical terms by assuming that cognitive significance should be attributed to theories, rather than individual statements. Theories acquire meaningfulness if they can be tested by comparing their empirical consequences with the available data. Hence, theoretical terms were assumed to gain cognitive significance indirectly from the theory in which they are embedded.

The logical empiricists asserted that scientific theories should be tested using the hypothetico-deductive method. In essence, this method consists of the following four steps. Firstly, formulate some hypothesis or theory. Secondly, deduce some testable implication from the theory and other premises including initial conditions and ceteris paribus clauses. Thirdly, test the testable implication. Lastly, judge whether the theory is confirmed or disconfirmed based on the test result. The hypothesis is not proven or disproven by the test alone because false hypotheses may have true implications and a negative test result may be due to the other premises used to derive the testable implication. For example, the law of supply and demand is not necessarily disconfirmed if evidence of an increase in both price and quantity demanded is discovered.

The hypothetico-deductive method is commonly used, but it has some major difficulties that relate to the Duhem-Quine thesis and the notion of evidential relevance. In particular, the hypothetico-deductive method does not provide a method for establishing which parts of the theory are relevant to the test result. It is unhelpful to simply reject entire theories on the basis of a single test result. The hypothetico-deductive method leaves it up to the user to determine which particular hypothesis should account for the test result. The Duhem-Quine thesis is particularly relevant in economics because of the large number of factors that potentially influence economic phenomena and the inability to marginalise these factors using controlled experiments. Moreover, it presents a serious practical problem if the other premises that are required to derive the testable implications are unreliable. If this is the case then little can be learnt from empirical evidence. It is possible to never reject a hypothesis by always claiming that some other premise is at fault. Furthermore, the hypothetico-deductive method does not rule out the possibility of using irrelevant evidence to defend a hypothesis (see Hausman 1992: 306).

Bayesian methods provide an approach that overcomes some of the difficulties of the hypothetico-deductive method. Bayesians assume that individuals assign prior subjective probabilities to propositions and update these probabilities in response to new evidence using Bayes' theorem. Bayes' theorem states:

$$\mathrm{P}(H|E\&B) = \frac{\mathrm{P}(E|H\&B)\cdot\mathrm{P}(H|B)}{\mathrm{P}(E|B)} \tag{5.2.1}$$

where $\mathrm{P}(x|y)$ represents the probability of x given y, H represents the hypothesis being tested, E represents the evidence used to test the hypothesis, and B represents the individual's background beliefs or theory. Hence, $\mathrm{P}(H|B)$ represents the individual's prior degree of confidence in the hypothesis, $\mathrm{P}(H|E\&B)$ represents the individual's posterior degree of confidence in the hypothesis given the new evidence, $\mathrm{P}(E|B)$ represents the prior expectation that E will occur regardless of whether H is true, and $\mathrm{P}(E|H\&B)$ represents the prior expectation that E will occur if H is true.

The Bayesian approach has many attractions. It quantifies the degree of confirmation a hypothesis receives from a test, given by $\mathrm{P}(H|E\&B) - \mathrm{P}(H|B)$. It explains why tests that have a low prior probability of being true strongly confirm a hypothesis if a positive test result is recorded. It also prevents irrelevant evidence from being used to support a hypothesis because if E was irrelevant to H then $\mathrm{P}(E|H\&B) = \mathrm{P}(E|B)$. However, the Bayesian approach is not without difficulties. It assumes that the individual's background beliefs, B, are not affected by the test result. The assignment of numbers to degrees of confidence is generally arbitrary and this makes it difficult to implement Bayesian methods. Moreover, the Bayesian approach implies that old evidence is worthless. If E is known to be true then the unconditional probability $P(E) = 1$, so that E cannot be used to confirm H.

The Bayesian approach, like the hypothetico-deductive method, does not provide any information on the relevance of evidence to specific parts of the theory. Another method of theory assessment that attempts to overcome this problem is known as bootstrapping. The idea behind this method is that specific hypotheses, which belong to a theory, are confirmed by evidence if the hypothesis can be deduced from the evidence using the theory. This method is termed bootstrapping as the theory is used to test parts of the theory.

The above methods only account for empiricist approaches to theory assessment. These methods are the most popular but they are not universally accepted. For example, modern Austrian economists believe that the fundamental postulates of economics are synthetic a priori truths (see Caldwell 1994a). The above methods are also unable to completely account for the process of theory assessment and discipline-specific knowledge is usually required to implement them. Nevertheless, they provide a useful sketch of theory assessment and its associated problems.

5.2.4 Falsificationism

The logical positivists believed that incontrovertible, unambiguous observations provide the inspiration for scientific theories and that scientists aim to discover theories with high inductive probabilities. This belief was implicit in their adoption of the verification principle because it attempts to measure the relative strength of inductive arguments. As a result, an inductive logic was considered to be a fundamental part of science. However, Hume's problem of induction (see Section 5.2.1) demonstrates that there are no decisive arguments with only basic statements as premises and universal statements as conclusions. Moreover, a theory's degree of confirmation can be influenced more by accident than by its truth status and confirming examples of theories are relatively easy to find. An illustration of how seeking to verify theories can be misleading is provided by Jevons' failed attempt, in the 1870s, to prove that business cycles are caused by sunspot cycles (see Morgan 1990). After becoming entranced by this hypothesis, Jevons searched for evidence to support it. Although he was derided by his colleagues, he based his belief primarily on his inductive finding that the lengths of sunspot cycles were similar to the length of commercial credit cycles. However, relatively little significance should have been attached to this evidence alone because, as Jevons admitted:

> I am free to confess that in this search I have been thoroughly biased in favour of a theory, and that the evidence which I have so far found would have no weight, if standing by itself. (Jevons 1878; quoted in Morgan 1990: 21)

To avoid the problem of induction, Karl Popper (see Popper 1959) adopted a different position on the problem of distinguishing between scientific and non-scientific theories. Popper exploited the logical asymmetry between the verifiability and falsifiability of universal statements. Although universal statements cannot be established or verified with absolute certainty, they can be refuted or falsified by a single exception. For example, the discovery of a single non-black raven would falsify the statement 'all ravens are black'. This illustrates that it is possible to provide valid deductive arguments against universal statements. These insights led Popper to suggest that theories should only be considered to be scientific if they are, at least in principle, falsifiable. If a theory is incapable of being falsified, then nothing can be learnt from testing it. Popper's demarcation criterion emphasises that science does not accumulate proven truths. Scientific theories are fallible. Falsificationism avoids the problem of induction by accepting that there are no decisive arguments in support of a theory, but it claims that there can be good arguments for refuting a theory. Justification has no part to play in scientific activities.

However, as illustrated by the logical positivists, empirical evidence is not unambiguous so it is generally impossible to conclusively falsify theories. As a result, Popper clarified his demarcation criterion for scientific theories by defining a category of 'basic statements' that are considered to be true by convention. Basic statements are not infallible, but are easily tested and are readily accepted as being true by the community of scientists. Popper defined a theory to be falsifiable if and only if it is capable of being contradicted by a finite set of basic statements. Hence, theories are not judged to be false in an absolute sense, but according to a conventional notion of truth.

Strict falsificationism is also impracticable for statistical inference because probability statements do not preclude any events and are consequently inherently not falsifiable. Nevertheless, conventions can also be established for testing statistical hypotheses and for determining the conditions that would result in their rejection. The Neyman-Pearson theory of hypothesis testing (see Silvey 1975) represents such a set of conventions. This theory specifies that hypotheses are falsified if the probability that they are true, for a given sample of observations, is less than some small predetermined value. An implicit assumption of this theory is that the hypothesis of interest (null hypothesis) is accepted until proven to be false. These conventions are unavoidable, but can potentially lead to irresolvable debates about whether the conventional criteria are appropriate. Moreover, if conventions are permitted, then the 'verification' of theories is also possible.

The Duhem-Quine thesis, which emphasises that tests generally involve multiple auxiliary hypotheses, poses another problem for falsificationism. Most individual theories are not by themselves inconsistent with sets of basic statements. They usually require non-basic auxiliary hypotheses to produce empirical predictions. Only whole systems of theories are logically falsifiable. Thus, the criterion of logical falsifiablity applies to systems of theories rather than to individual theories. This demarcation criterion is not particularly helpful when assessing theories because it requires that the whole system of theories being tested be rejected if a negative test result occurs. Falsificationism does not provide a role for justification and consequently it does not permit one to judge which parts of the system of theories are falsified by the evidence. To test individual theories it is necessary to rely on other auxiliary theories, but this requires one to regard these auxiliary theories as well established. In addition, this demarcation criterion hardly rules out any theories because most theories are falsifiable if they are combined with other theories.

The above difficulties illustrate that falsifiability is not an absolute or demanding criterion. However, Popper was not solely concerned with demarcating scientific theories; he also stressed that the problem of demarcation includes distinguishing scientific practices from non-scientific practices. In this regard, Popper argued that scientists should have a critical attitude. This requirement included that scientists should only consider falsifiable theories and they should prefer theories that prohibit more (have a higher empirical content) because they can be more severely tested. Hence, Popper recommended that theories with the highest degree of universality and precision (or simplicity) should be preferred. In this context, level of universality refers to the domain of the variables included in the theory and degree of precision refers to the form of the hypothesised relationships between the variables. As theories that forbid more are usually not the most probable, Popper was opposed to the logical empiricist's search for theories with high inductive probabilities. More can be learnt from theories that produce unexpected results than from theories whose results have a high ex ante probability of being true.

Having a critical attitude also requires that scientists should try to falsify their theories using harsh tests rather than try to confirm them. When a theory is falsified then scientists must reject the theory and seek alternatives unless there is a rational reason for the test result that can be easily tested. Popper noted that falsified theories can be saved by immunising, or conventionalist, stratagems, such as: ad hoc auxiliary hypotheses, modifications to definitions, and unreasonable questioning of the test used. To limit these defensive tactics a number of conventions were suggested, including that auxiliary hypotheses should only be used if they increase the degree of falsifiability of the theory, that scientists should state their methodological rules clearly, and that they should state the conditions that falsify them before their theories are tested. As this procedure is biased in favour of rejecting theories, Popper admits to situations where dogmatism may be permitted. In particular, concessions may be allowed if a theory has not yet been properly formulated. If a theory survives repeated attempts at falsification then they are merely regarded as worthy of further critical examination. Theories are fallible and are not accepted as representing the truth.

Hausman (1992) questioned Popper's insistence that scientists should only attempt to falsify theories and should only regard theories that have not been falsified as worthy of criticism. As discussed above, individual theories usually cannot be either completely verified or falsified. Only systems of theories can be tested. To meaningfully interpret test results it is often sensible to regard theories that have passed harsh tests as well established background knowledge. This does not necessarily imply that these theories are infallible: they may be both fallible and well established. Moreover, it is not necessarily misguided or dogmatic to try to confirm rather than falsify theories. Although confirming evidence is easily obtainable, substantial confirming evidence is just as difficult to obtain as substantial falsifying evidence. In Bayesian terms, a good test requires an improbable prediction, a low $P(E|B)$, and a high prior likelihood given the theory is true, $P(E|H\&B)$ (see equation 5.2.1). It is generally accepted that scientists should be concerned with testing their theories using harsh tests and taking the test results seriously. But it is not generally accepted that a sceptical anti-inductivist position is always appropriate.

The above conventional interpretation of Popper's views is generally termed falsificationism. This interpretation is widely, but not universally, accepted. Boland (1994) suggested that critical debate, rather than falsificationism, is central to Popper's philosophy of science. According to this view, falsificationism is merely used to demonstrate that knowledge cannot be conclusively justified, it is conjectural and fallible. Hence, science is a never-ending trial and error process that is motivated by inter-subjective criticism rather than justification: learning occurs by discovering errors. The goal of the methodologist is then to stimulate constructive criticism; there are no certain methods for discovering truths or errors. A recommended approach towards achieving this goal is that methodologists should understand solutions in terms of the problem that motivated them, termed the problem situation (Popper 1959: 22). Situational analysis, which is similar to the assumption of rational choice in economics and describes the assumption that scientists aim to solve problems subject to various constraints, is then used to interpret the behaviour of scientist. To promote constructive criticism, the disputed solution should initially be favourably presented. Thus, situational analysis is employed as a tool to explain the effectiveness of criticism taking into account the problem under consideration.

Therefore, Popper and the logical positivists were both concerned with the problem of demarcation, but Popper did not pursue it from an analysis of language or meaning and he placed less emphasis on formal reformulations of scientific theories. Popper did not maintain that non-scientific activities were meaningless. What he considered to be disreputable was pseudo-science: when seemingly scientific theories are proposed that never make predictions and the practice of dogmatically clinging to disconfirmed theories. Popper required scientists to adopt a critical attitude and argued that theories are never justified by evidence. Although it is generally accepted that scientists should take testing seriously and that theories are fallible, it is not generally accepted that there is no role for justification in science and that falsified theories should always be discarded. Theories are frequently problematic in some respect, but theoretical systems are generally too important to completely discard and alternatives are usually difficult to find. If theoretical systems are not totally abandoned when negative test results are recorded then a measure of confirmation is required to interpret these results.

5.2.5 The growth of knowledge

The logical positivists aimed to establish universal definitions of legitimate scientific practice, but experienced numerous difficulties. In particular, their definitions were undermined by the demonstration that objective facts and decisive tests are virtually non-existent. Moreover, theories are usually tested against each other rather than against empirical evidence because they only tend to be rejected if a suitable alternative exists. Theories also often have different domains making any comparison between them difficult. These findings imply that there is no infallible empirical knowledge or completely reliable method for ensuring that our fallible knowledge is optimal. The absence of decisive tests also makes it difficult to sharply distinguish between scientific and non-scientific practices. As knowledge is fallible, new information can change the assessments of theories and, as a result, the appraisal of theories inevitably evolves over time. Hence, the post-positivists, from the 1960s, considered the growth of knowledge over time. However, no single generally accepted approach has arisen in this post-positivist era.

Kuhn (1970) transformed the philosophy of science from abstractly analysing scientific theories towards considering the broader context in which these theories evolved. He achieved this by employing a historical descriptive approach to the problem of the growth of knowledge. Kuhn questioned the appropriateness of prescriptive methodologies that did not accord with the history of science. He argued that science is characterised by relatively long intervals of normal science that are punctuated by scientific revolutions that cause sharp breaks in the development of science.

Normal science is described as a puzzle-solving activity that occurs within a theoretical framework (paradigm or, more specifically, disciplinary matrix) that is based on past scientific achievements. In normal science, researchers do not confirm or falsify theories against data. They attempt to solve the puzzles that result from fitting their theoretical framework to reality. Extraordinary research occurs in response to an accumulation of anomalies within the current paradigm. This research may result in a scientific revolution if it discovers a new paradigm that accommodates the anomalies or that changes the perceptions of the relevance of problems. Hence, theories are only rejected in favour of other theories. A characteristic of the interval in which extraordinary research takes place is the prevalence of methodological debates. During intervals of normal science, the shared paradigm directs research and methodological rules are rarely disputed. As the new paradigm often has methodological differences to the old paradigm and redefines the meanings of some terms, it is usually not possible to logically prove that the new paradigm is superior. Hence, although rational arguments are used to motivate the new paradigm, conversions to it are ultimately similar to a *Gestalt* switch. The overriding criterion for theory choice is the proven ability to structure and solve relevant puzzles.

This view has been challenged for being exaggerated and for not being able to incorporate: intervals in which more than one paradigm exists, intervals in which productive and critical research takes place, and gradual shifts in paradigms. There is also disagreement about whether science should progress according to Kuhn's description. As this portrayal of scientific growth does not guarantee that revolutions will occur, it is possible that scientific research could become trapped in intervals where unsuitable theories are dogmatically retained. Popper rejected the claim that rational choice between theories is not possible and recommended permanent revolution, or extraordinary research, as the norm. It has also been suggested that having a number of theories at any one time is a better way of guaranteeing that revolutions take place because revolutions require promising alternatives (Feyerabend 1988). Nevertheless, Kuhn's work inspired other philosophers to consider the historical development of theories and the wider sociological factors that may influence science.

Imre Lakatos (see Lakatos 1978) extended the work of Popper and adopted many of Kuhn's ideas by proposing a methodology that evaluates the development of theories within the broader context of a dynamic system or scientific research program. This methodology attempts to provide a rational reconstruction of the development of science.

A scientific research program is defined by a set of methodological rules that regulate research within the program. These rules consist of two sorts, a negative and a positive heuristic. The negative heuristic indicates the research paths that are improper and disallows investigation of certain assumptions that are considered to be irrefutable, known as the 'hard core'. Hard core assumptions that have been suggested for neo-classical economics include that agents have complete and relevant knowledge. To apply the hard core to specific problems, additional, refutable, assumptions and procedures are required; termed the 'protective belt'. The positive heuristic indicates legitimate research paths and is a set of partially formed ideas on how this protective belt should evolve. Although the hard core is relatively more rigid, both the hard core and the protective belt change over time.

The aim of a methodological study is to compare theories by assessing whether these changes cause the program to progress or to degenerate. A program is considered to be theoretically progressive if its new theories, or theoretical problemshifts, predict some previously unknown facts. It is empirically progressive if these theories are corroborated. If theories continue to be corroborated then it is assumed that they are moving towards the truth: they have increasing verisimilitude or truth likeness. The concept of verisimilitude permits scientists to aim to discover true theories even though this goal is recognised to be unattainable. If modifications to the program are introduced solely to explain anomalies, then the program is considered to be degenerating and should ultimately be rejected. Therefore, the fundamental empirical criterion for a scientific research program is the discovery of novel facts over time, rather than the degree of confirmation. Consequently, theories are appraised by only assessing their excess content in relation to other theories rather than by testing individual theories. Theoretical changes are judged by whether they result in improvements but, in keeping with Popper, there is no role for justifying individual theories.

Lakatos's methodology of scientific research programs provides an alternative method for recovering practice that does not completely abandon appraisal (see Backhouse 1994). This methodology is both positive, it aims to understand practice in terms of methodological rules, and normative, it evaluates programs in terms of prediction of novel facts. It suggests that the proliferation of theories is beneficial as science is assumed to advance by programs that are judged by whether they produce facts. It also suggests that programs should not be discarded too quickly because they can only be evaluated over relatively long periods of time.

However, as the long run cannot be clearly defined in a non-arbitrary manner, programs may never be abandoned. This conclusion, together with the use of the problematical notions of verisimilitude and novel facts, implies that this methodology does not resolve the demarcation problem or the problem of establishing unambiguous criteria for theory choice. Lakatos's methodology also assumes that research programs are self-contained entities, but in practice they are often interdependent. This methodology does not allow for new theories that do not only produce novel facts but also result in some loss of content. Furthermore, apart from the concept of verisimilitude, Lakatos does not allow for questions of justification in theoretical science. But some measure of justification is required to effectively apply theories to practical problems and to efficiently assess test results. These problems make it difficult to appraise programs using Lakatos's methodology. In particular, considerable difficulty has been experienced in attempting to use this methodology to describe the development of economic thought (Hausman 1992). However, it does attempt to limit the potential for error and it has been found to provide a valuable description of how some disciplines have evolved.

5.2.6 Summary

The philosophy of science has not resulted in a well-founded general philosophical system that is able to solve every methodological difficulty. On the contrary, it has demonstrated that such a system is unfeasible and as a result it has mainly produced criticism rather than universal methodological rules. However, methodology is unavoidable in science and it is useful to appreciate what philosophers have learnt so that similar mistakes are not made. Although there is widespread disagreement amongst philosophers, there is broad support for some basic methodological principles. In particular, scientists should always adopt a critical attitude and their theories must be exposed to empirical assessment. Exacting tests and explanatory success are required to effectively assess theories. Disconfirming evidence needs to be taken seriously. Knowledge is fallible. The following sections consider how economists have reacted to the philosophy of science and the specific philosophical problems that result from studying economic systems.

5.3 The Scientific Status of Economics

Economic methodology is concerned with the philosophical issues that arise in economics. These issues have tended to be motivated by the "need to understand what sort of an intellectual discipline economics is and what sort of credence its claims merit" (Hausman 1994a: 1). Interest in this problem has been heightened by recent developments within both economics and the philosophy of science. In economics, theoretical disputes have proved difficult to resolve and economists have been unable to provide solutions to many important economic problems. This has led some economists to critically examine the fundamentals of their discipline. Furthermore, many of the recent advances in the philosophy of science, discussed in the previous section, have influenced economic methodology. The importance of these developments are not purely philosophical. Science commands considerable authority and influence in society, which makes a discipline's perceived scientific status practically relevant. The rest of this chapter explores various methodological positions that have been proposed for economics.

5.3.1 The method a priori

Economic methodology is especially problematic because of the nature of the economy. Economic events tend to be influenced, in a complex way, by an extensive and diverse range of factors and the apparent relationships between economic phenomena are often not homogeneous over time. These characteristics make it virtually impossible to learn about the economy by merely observing it directly. In the natural sciences, knowledge about complex systems is often obtained by experimentation. Experiments enable researchers to isolate the phenomena of interest and to observe them under various controlled conditions. Experimentation is an efficient means of gaining and testing knowledge because it limits the number of causal factors that could have affected the observed result. However, it is usually not possible to conduct economic experiments. Economists are unable to control economic variables, such as interest rates. This makes it difficult to establish causality amongst economic variables (see Freeman 1991). As economic data is usually collected through sampling, it is only possible to observe a particular temporal ordering of events. It is not possible to conclusively conclude that a specific event caused other events because there is no way of knowing what would have happened had that event not occurred.

The method a priori provides an alternative means of discovering and testing economic knowledge claims to pure induction and experimentation. This methodological approach has a long history and was developed by, amongst others, John Stuart Mill in the 1830s. It was commonly accepted by economists in the nineteenth and early twentieth century and Hausman (1992: 124) claimed that it is "the view to which most economists (regardless of what they *say* in methodological discussion) still *apparently* subscribe."

The method a priori asserts that the fundamental causal factors of economic phenomena are relatively well known and can be determined by induction using a combination of empirical observation and introspection. Introspection is regarded as a valuable source of information that is not available in the natural sciences. Examples of fundamental causal factors, or basic postulates, include that agents prefer more wealth to less and that agents can and do order their preferences. These postulates are assumed to represent true descriptions of the predominant causal factors, but they are inexact rather than universal generalisations. They only apply exactly in the absence of other influences. For example, the claim that an agent's preferences are transitive assumes that the agent's tastes remain constant. However, due to the complexity of the economy, it is usually not possible to fully specify the conditions in which the basic postulates apply exactly. Hence, they are inevitably qualified with vague ceteris paribus clauses and consequently they merely represent statements of tendencies.

Having established these fundamental economic causal factors, the method a priori asserts that they should then be used to deduce theories concerning the more complicated phenomena of interest. To achieve this, the basic postulates are combined with auxiliary postulates and expository devices. Auxiliary assumptions usually reflect the contemporary economic environment, such as current legislation. Expository devices are approximately realistic assumptions that are employed to make the problem tractable. As these assumptions are necessarily incomplete and the basic postulates are inexact, theories are assumed to represent hypothetical rather than positive truths. Hence, the method a priori assumes that economics only studies the consequences of a limited number of established postulates. Although economists aim for exactness, the nature of the economy suggests that this goal will only be rarely achieved. As a result, the predictions of deduced theories need to be tested to assess their reliability in various circumstances. Therefore, the method a priori recommends that economic phenomena should be studied by deducing theories from more fundamental causal factors that are inductively established and by testing the resulting predictions.

There is nothing controversial in this method of studying the economy. Falsificationists would not even consider it to be within the ambit of science because it is primarily concerned with discovering hypotheses. However, the method a priori is not only a method for developing theories, it also proposes a controversial method for justifying theories. The standard hypothetico-deductive method maintains that theories are assessed solely on the grounds of their predictive success (see Section 5.2.3). If a negative test result is observed then it asserts that one needs to judge which particular hypotheses should account for the failure. It is usually unnecessarily destructive to holistically reject falsified theories. However, the hypothetico-deductive method is difficult to implement in the context of economic theories because of the inexactness of the basic postulates and the large number of potential causal factors. If economic theory is inexact, and thus frequently falsified, then what grounds do economists have for continuing to use it? The method a priori attempts to partially resolve these important practical problems.

The method a priori claims that, although the basic postulates are inexact, they are proven truths that are never contradicted by empirical test results. Hence, economic theories are assumed to be valid but incomplete. Economic theories start with well-established 'laws' rather than hypotheses that need to be tested. Consequently, if a negative test result is obtained then economists only need to determine whether the deduction was logically valid, which interferences influenced the result, and whether the basic postulates reflected all the important causal factors. This significantly narrows the range of the enquiry. It may also provide constructive information on the directions in which the current theory should be extended. Moreover, if a positive test result is recorded then the theory is taken seriously not only because its implications have been empirically confirmed but, what is more important, because it has been deduced from the fundamental causal factors. Economists derive their confidence in inexact economic theory primarily from the validity of its basic postulates rather than from the accuracy of its predictions.

The claim that the validity of the basic postulates is virtually beyond dispute is highly controversial because these postulates only represent inexact 'laws' that are qualified with vague ceteris paribus clauses. A reliable universal law cannot even be proven to be absolutely true. The ceteris paribus clauses make the problem worse because they can be used as immunising stratagems that insulate the basic postulates from empirical criticism. However, Hausman (1992) argued that inexact generalisations can be rationally defended if they are lawlike, reliable, refinable, and excusable. They must be capable, at least in principle, of being made more reliable, or reliable in a larger domain, in a non-ad hoc manner and the reasons for test failures should be understood. If these conditions are met then Hausman claimed that economists are justified in being committed to the truth of inexact generalisations even though they are apparently falsified. This defence does not justify the assertion that the basic postulates are infallible, it merely claims that it can be rational to have confidence in inexact laws.

The method a priori is also controversial because it is only a partial deductive method in that it only considers the predominant causal factors. There is a risk that significant causal factors will be overlooked. It has also been suggested that this method produces theories that are of little practical value because they only describe tendencies rather than accurate predictions. Policy makers are not interested in what would happen 'other things being equal', they need to know what will happen. Nevertheless, the method a priori has been defended on pragmatic grounds as providing the only practical method of learning about a complex system such as the economy.

5.3.2 Falsificationism in economics

Popper's falsificationism provided a serious challenge to the method a priori. A notable early argument for this methodological view, rather than the method a priori, in economics was made by Hutchison (1938). He claimed that much of theoretical economics was without empirical content. It produces either elaborate tautologies or theories with illegitimate ceteris paribus clauses that make them impossible to test. Hutchison asserted that ceteris paribus clauses should only be permitted if the theory is almost always true and the infrequent exceptions are of a clearly definable type. These conditions are similar to but are far stricter than Hausman's (1992) (see Section 5.3.1). They suggest that most economic generalisations are inadequate because economic generalisations frequently fail and economists generally cannot and do not precisely classify the factors that cause the failures. Nevertheless, Hutchison did not completely reject the use of tendency laws. Tendency laws provide potential foundations for falsifiable exact laws, which are attained by specifying and enumerating the ceteris paribus clauses over time. Falsificationists also consider the method of introspection to be an illegitimate means of justifying economic claims because it does not provide an objective empirical test. Moreover, Hutchison criticised the use of abstract idealised theoretical models. These models rely on false assumptions, such as agents have perfect knowledge, and these assumptions have not been progressively replaced by assumptions that are closer to the truth. Therefore, Hutchison claimed that economics does not produce empirically testable theories and consequently does not satisfy Popper's demarcation criterion for an empirical science. He recommended that economists should adopt falsificationism rather than the method a priori, but he was unable to specify how it could be implemented.

Another notable argument in favour of falsificationism was provided by Blaug (1992). This book presents a comprehensive survey of economic methodology that unequivocally endorses falsificationism, or more precisely Lakatos' methodology of scientific research programs. It is promoted on both prescriptive grounds, that it should be used, and on historical grounds, that it has been successfully applied. However, Blaug admits that falsificationism is difficult to implement in economics and is frequently disregarded by economists. Nevertheless, he argued that falsificationism provides the best method for establishing the truth, or more accurately the falsity of theories, and that this information is essential for what he regarded as the ultimate function of economics: to provide meaningful advice to policy makers. Blaug (1992: 248) summarised his beliefs by suggesting that the ultimate question for any economic research program is: "what events, if they materialised, would lead us to reject that program?"

These recommendations for a strict falsificationist methodology are revolutionary because they suggest that a substantial part of economics is unscientific and should be rejected (see Caldwell 1994b). Hausman (1992: 158) claimed that it has resulted in methodological schizophrenia because economists have been reluctant to dispute contemporary philosophy of science but have been unable to implement it. Falsificationism is particularly difficult to practice in economics because of a number of previously discussed factors, including that controlled experiments are generally not possible, economists cannot detach themselves from their subject matter, the available economic data are often only a crude proxy of the required data, and there are no well substantiated universal economic laws (see Section 5.2.4). As these factors are to some extent present in all subject areas, they do not by themselves imply that falsificationism is inappropriate for economics. However, they at least suggest that a greater tolerance of economic theories is required. At most, it has been asserted that they make prediction, and thus falsificationism, impracticable in economics. The latter assertion has been substantiated by the sceptical claim that economic theory and econometrics have not been able to substantially improve the precision of their predictions (Rosenberg 1992).

The falsificationist critiques have been disquieting because they question the scientific status of economics and they emphasise genuine difficulties with economic theory. However, falsificationism is not flawless and it is impossible to unambiguously distinguish between science and pseudo-science (see Section 5.2). Rather than rejecting most economic theory, some economic methodologists have chosen to analyse the practice of economists and have suggested that Popper's critical debate and situational analysis provide useful techniques for these purposes (Caldwell 1994b; see Section 5.2.4).

An alternative response to the difficulties associated with developing adequate economic predictions is to distinguish between the science of economics and the art of economics, or applied economics (Keynes 1891). Applied economics is concerned with applying the insights of abstract theories obtained from the science of economics to specific practical problems taking into account real world considerations, especially the goals and ideals determined in normative economics. Colander (1994) suggested six rules for applied economics, which are intended as guidelines that are only to be broken after giving them sufficient consideration. These rules include:

1 Calculations should not be carried out and results should not be reported to more significant digits than the least significant variable used. Further precision may be misleading.
2 Applied analysis cannot escape employing judgements. However, when judgements are used, economists should attempt to maintain objectivity as far as possible and should consider how a reasonable representative person would respond to the proposals. This includes identifying the groups that are likely to be advantaged or disadvantaged by the proposals.
3 Although applied economists are unlikely to contribute to theoretical developments, they should be aware of them and always use the best available theories.
4 It is important that applied economists are guided by the problem under consideration rather than by the available, generally mathematically tractable, techniques or theories.
5 In applied analysis, the role of empirical work is to try to understand the data. Thus, data mining (see Section 7.2.1) may be considered to be acceptable, provided the results are not justified by standard statistical tests; applied economists must rely on their judgement.
6 As a consequence of the fifth rule, applied economists must not be falsely scientific.

Most of these rules are relevant for actuarial science, which could be considered to be a branch of applied economics. However, relatively little significance should be attached to applied economics claims: otherwise there would not be a need for the distinction between the science and the art of economics. Hence, Blaug (1992) urged economists to try harder to increase and improve the predictive content of their work.

5.4 Friedman's Instrumentalism

5.4.1 The realism of assumptions issue

Concerns about the empirical adequacy of economic theory intensified in the 1940s when some economists attempted to test the fundamental assumptions of economic theory, such as whether companies aim to maximise their profits. Many of these assumptions are clearly inaccurate so how can their continued use be defended? Friedman (1953) replied to these concerns by claiming that the aim of economics is to develop theories that provide reliable predictions. As a result, theories should be virtually exclusively judged on how well they perform this principal task. If theories have similar predictive abilities then secondary selection criteria should be used, including their simplicity and their research potential. Friedman asserted that the truth or falsity of a theory's assumptions is largely irrelevant to its assessment. Theories necessarily abstract from a more complex reality so their assumptions must be descriptively false. Therefore, Friedman made the influential assertion that economic theories should be judged by the accuracy of their predictions rather than the realism of their assumptions. As economics was assumed to be predictively accurate, the use of unrealistic assumptions is warranted. This view is notably contrary to the method a priori, which assumes that confidence in economic theories is derived from their established fundamental postulates or assumptions.

Friedman substantiated his claims by stating that it is often useful to approximate motivational assumptions by inaccurate as-if assumptions. For example, it may be fruitful but unrealistic to assume that individuals behave as-if they consciously optimise the utility of their wealth and that companies maximise their profits. Friedman claimed that this maximisation-of-returns assumption is appropriate because the dynamic process of competition ensures that similar results are achieved regardless of whether it is true and because of its continued successful use in numerous applications.

This argument supporting the maximisation-of-returns assumption is not necessarily true and it has been criticised for being defensive. Friedman's reference to numerous confirming examples has been interpreted as an attempt to protect the assumption from critical examination because he did not specify any events that would conceivably contradict it. Furthermore, competition will only ensure that surviving companies maximise their profits if the economic environment is in a state of equilibrium. Maximising behaviour may only result in success over relatively long time intervals and the proximity of certain behaviour to maximising behaviour generally depends on the changeable economic environment. Thus, competition does not necessarily guarantee that survivors, in any particular time interval, behave as-if they maximise utility or profit in that interval. Furthermore, survival can be achieved by merely producing positive profits obtained by relative efficiency rather than by producing maximised profits. The only inference that can safely be drawn is that surviving companies were better adapted to the economic environment than companies that failed. However, the issue is not whether this particular assumption is true, but whether its truth or falsity is relevant to the assessment of economic theories. Should theories only be judged on the accuracy of their predictions?

Friedman's methodology is broadly similar to instrumentalism. Instrumentalists similarly argue that theories are merely useful instruments and do not claim that they are true representations of reality. This philosophical position is often justified by the sceptical argument that all past theories have proved to be false and there is no reason to believe that current or future theories are true. However, the distinction between a theory's assumptions and its predictions is ambiguous because it depends on how the theory is formulated. Theories can generally be reformulated so their 'assumptions' become predictions and 'assumptions' are often themselves predictions. The maximisation-of-returns assumption can be both an assumption and a prediction. Moreover, false assumptions are likely to produce false predictions. Consequently, standard instrumentalists generally consider all the observable implications of a theory to be significant to its assessment.

However, Friedman avoids these difficulties by making the additional claim that a theory's predictive success should be determined by only considering the events that it was designed to explain (Hausman 1992: 164). Useful theories do not need to be versatile. Economic theory is not designed to explain the behaviour of individual firms so it is irrelevant that the maximisation-of-returns assumption is inaccurate. Assumptions are assumed to merely fulfil the subordinate roles of depicting the hypothesised key elements in a particular situation and determining the likely environmental conditions in which the theory may be appropriate.

Friedman's position, as it is articulated above, is difficult to justify as a general methodological approach. Predictive success in a limited domain of application does not provide a complete assessment of a theory's future performance. Valuable information can be obtained by testing a theory's assumptions or component parts in the same way that a comprehensive mechanic's report is useful when purchasing a second-hand car (Hausman 1992: 166). This information is especially relevant when the theory fails or if the theory is used in situations in which it has not been comprehensively tested. Knowledge about the relative strength of a theory's assumptions is required to meaningfully interpret test results and to suggest appropriate avenues for the theory's future development (see Section 5.2.3). If the predictions from a black-box theory are found to be inadequate, then the entire theory usually has to be discarded. If they are adequate then it is not usually possible to tell whether this was achieved by accident or by design. Moreover, if a theory is to be used in new situations, which is often the main purpose of a theory, then it is essential to know how well each of its component parts describes reality. Therefore: "Wide, not narrow predictive success, constitutes the grounds for judging whether a theory's assumptions are adequate approximations" (Hausman 1992: 167).

However, as Friedman (1953) stated, the use of unrealistic assumptions is unavoidable. Mäki (1994) asserted that the relevant issue is not whether unrealistic assumptions are legitimate, but whether specific types of unrealistic assumptions are legitimate in specific contexts. He distinguished between core assumptions and peripheral assumptions, which include auxiliary assumptions and expository devices. Mäki claimed that expository devices can be legitimate in certain situations, including when factors that are assumed to have a negligible effect are ignored in the initial formulation of the problem. On the other hand, core assumptions are generally chosen to be as realistic as possible. Mäki argued that the realism of assumptions issue is generally motivated by disagreements about which assumptions are fundamental and whether the chosen assumptions capture the essential features of the problem. For example, the claim made by actuaries that the assumptions of financial economic models are unrealistic could be construed as reflecting the belief that market inefficiencies are an essential feature of market behaviour.

5.4.2 The goals of science

Along with the above problems, Friedman's view that the goal of economics is solely to develop theories that provide adequate predictions has been challenged. Scientific realists argue that although predictive success should be an important goal of science, as it enables us to order and to anticipate events, it should not be its only goal. Realists assert that theories should also aim to discover the truth. If a theory is found to be well supported, then realists generally believe that its assertions are true. Consequently, scientific realism is antithetic to instrumentalism. However, realists do not completely reject 'false' theories; they recognise their practical value in certain situations. But realists seek more from scientific theories. As a result, realists are more concerned with developing consistent universal theories. Realists may sacrifice predictive accuracy for other factors, such as coherence, degree of explanatory power, or degree of unification. Instrumentalists are more likely to use unrealistic assumptions to achieve predictive success in limited domains of application.

Realists do not necessarily believe that the goal of discovering the truth is attainable. Fallibilists, including Popper, acknowledge that it is impossible to attain perfect knowledge. Nevertheless, this does not necessarily imply that the pursuit of truth is an unworthy ideal. It encourages scientists to continually endeavour to improve their knowledge and it is not content with just predictive success. The sole criterion of predictive success is criticised because it is possible to derive a true, or approximately true, conclusion from a false premise.

An instrumentalist or realist position can also be taken on how unobservable things, such as electrons, should be treated. Instrumentalists suspend their judgement about the existence of unobservables whereas realists argue that they represent the truth if the theory that postulates them is well supported. However, an instrumentalist position on unobservables does not necessarily imply an instrumentalist position on the goals of science. For example, constructive empiricists and causal holists believe that theoretical science aims to discover the truth about observable phenomena only (see Boylan and

O'Gorman 1995). They seek to establish empirically, or descriptively, adequate theories; empirical adequacy is determined by comparing a theory's factual statements with observable phenomena. The truth status of theoretical statements that refer to unobservable events is considered to be indeterminate. Theories are accepted if they provide accurate descriptions of the observable world.

Nevertheless, neither instrumentalism nor realism has been unanimously endorsed. Both positions have been defended and adopted. However, even if an instrumentalist position on the goals of economics is adopted, the assertion that the realism of a theory's assumptions is irrelevant is still problematic. Rosenberg (1992) argued that the subject of realism of assumptions only arose because economic models were unable to produce satisfactory predictions. Inaccurate assumptions will inevitably bedevil predictions, but they only become an issue if these predictions are not sufficiently accurate. Hence, Friedman's methodology has been interpreted as being defensive and undermined by its central assumption: that economic theory is predictively successful. Instrumentalism is impracticable if it is impossible to construct any theories with adequate predictions.

5.5 The Limitations of Economic Prediction

A theory's predictive and explanatory power is generally improved by either sharpening the measurements of its initial conditions or by refining the theory itself. In particular, the predictions of the economic theories can be precisely quantified, as suggested by Hutchison (1938), if their ceteris paribus clauses are fully specified. This is likely to be possible if the theory is widely applicable, if specific qualifications in the past have made it more reliable, and if, in the situations in which it failed, the disturbing factors were identified. On the other hand, this goal is unrealisable unless the conditions subsumed by the ceteris paribus clauses are finite in number, measurable, and manageable. Rosenberg (1992) argued that, for much of economic theory, it is implausible that such a set of manageable conditions exist; implying that economic predictions are constrained to being generic. Generic predictions merely predict the existence of phenomena or events rather than predicting detailed or quantitative descriptions about their character. For instance, Samuelson (1983) required that a theory should at least be able to predict the algebraic sign of a change in certain economic variables caused by changes in other variables.

Rosenberg's (1992) argument was based on the characteristics of certain fundamental economic exogenous variables, specifically agents' expectations and preferences. These variables depend on beliefs, not just reality. Traditionally economists avoid having to make assumptions about what agents believe by assuming that they have perfect knowledge, rational expectations, and that they reveal their preferences by their choices, which are transitive. These assumptions would be sufficient if the resulting economic theory produced adequate, or at least improving, predictions. As this does not appear to be the case (see Chapter 6), it is possible that the problem lies in the appropriateness of these assumptions and thus the solution is to obtain better measurements of these initial conditions.

However, Rosenberg argued that this is virtually impossible because expectations and preferences are intentional variables. They cannot be independently revealed because actions can only reveal preferences if expectations are known and, conversely, actions can only reveal expectations if preferences are known. Even beliefs themselves are generally dependent on one another, so that it is impossible to establish individual beliefs without knowledge of other beliefs. To make matters worse, there are infinitely many possible combinations of expectations and preferences that could conceivably result in particular behaviour. This observation also implies that direct questions on expectations and preferences may result in misleading answers. Moreover, questioning is limited to artificial environments and thus may not reveal realistic beliefs or desires. Neuroscience may ultimately be expected to be able to establish individual's preferences and expectations, but even this assumption has difficulties. Philosophical and psychological arguments suggest that the brain cannot be organised in such a way that it contains specific statements (see Rosenberg 1992: 140-148). Thus, preferences and expectations do not represent physically measurable variables. These arguments suggest that it is generally not possible to independently establish specific individual beliefs.

Even so, this constraint does not prevent economists from constructing and continually refining utility functions. However, this line of research is likely to be unproductive, because there are infinitely many factors that potentially affect utility functions. Without a general theory about how additional information and choices influence utility, which is in effect another utility function, this research can only suggest ad hoc changes to utility functions. Therefore, Rosenberg (1992: 129) stated that: "The upshot of the intentional character of the explanatory variables of economic theory is obvious. We cannot expect the theory's predictions and explanations of the choices of individuals to exceed the precision and accuracy of the common-sense explanations and predictions with which we have all been familiar since prehistory." Not being able to improve on explanations and predictions of the choices of individuals suggests that macroeconomic predictions are bound by the same fate.

Lawson (1994: 281) came to a similar conclusion and stated: "If the predictive goal in question is the successful forecasting of events then clearly an implication is that such a goal is likely to be only rarely if ever attainable, at least in an unqualified form." This conclusion was motivated by a transcendental realist theory. Constant event regularities usually only exist in closed systems, or experimental situations, that marginalise certain influences. In open systems, such as the economy, these regularities may not apply due to the infinite number of possible disturbing factors that exist. In these systems, causal laws only indicate tendencies rather than universal laws. Transcendental realists maintain that the primary aim of economics should be to identify and to understand these tendencies. Lawson's optimism that these underlying invariant mechanisms exist and can be discovered, is not shared by all, including modern Austrian economists and some institutionalists.

Rosenberg (1992) suggested that economics should be viewed as a branch of applied mathematics, similar to Euclidean geometry, rather than as an empirical science. Interpreted in this way, economics is an a priori discipline that examines the formal properties of a set of assumptions or axioms. This interpretation does not necessarily deny economics of any practical relevance. Euclidean geometry is able to provide accurate approximations for calculations required by engineers and surveyors even though it is based on the false assumption that light travels in straight lines. The major difference between geometry and economics is that there exists another discipline, namely physics, that is able to explain the limitations and improve on the predictions of

Euclidean geometry. In economics, it is not possible to precisely determine where and why its predictions fail because the economy is influenced by infinitely many variables that are not all reducible to physical mechanisms. Nevertheless, economics can still be used to motivate broad policy guidelines without providing precise predictions. For example, general equilibrium theory demonstrates that an economy, controlled by a large number of individuals who are motivated by self-interest, is compatible with an efficient allocation of scarce resources.

However, economics is not purely abstract and mathematical. A substantial proportion of economic work is empirical and reliable economic predictions are required by policy makers. Consequently, economics is not and is unlikely to ever be only applied mathematics. But, if economics is unable to provide precise economic predictions then how can its methods be justified? Falsificationists, including Blaug (1992), recognise the empirical difficulties associated with economics but they do not regard them to be incapacitating. Instead, they urge economists to try harder and to increase and improve the empirical content of their work. Hutchison (1994) remarked that economics has resulted in the creation of numerous economic statistical series that are used in conjunction with economic theory to make tentative predictions. These predictions are arguably less inaccurate and less unreliable than other forms of prediction and may help to avoid politico-economic catastrophes, such as that in 1929-33. Thus, economics should not be severely criticised if its predictions are inaccurate and economic predictions should always be qualified. This suggests that expectations or actions based on economic predictions should always consider the possibility of completely unexpected events. An alternative justification of economic methods, known as the inexact deductive method, has been provided by Hausman (1992).

5.6 The Inexact Deductive Method

5.6.1 Theoretical hypotheses and models

One part of Hausman's (1992) attempt to rationalise the practice of economics was his distinction between theoretical hypotheses and models. He defined a model as a definition of a concept or type of system. For example, rationality is defined by the model that consists of the assumptions that agents' preferences are complete, continuous, and transitive and that agents maximise their utility (see Section 2.4.1). Models do not make empirical claims or provide predictions. They are either trivially true or neither true nor false. Consequently, theoretical economists do not necessarily need to always be committed to the truth of their assumptions and it is inappropriate to assess models empirically. The model of rationality merely defines rationality, it does not state that people behave rationally.

The principal aim of models is to provide the means for studying the conceptual, logical, and mathematical implications of sets of assumptions. This guides research activities and enables the development of internally consistent systems. For instance, Smith (1996) initially analysed some of the implications of Dyson and Exley's (1995) pure expectations model (see Chapter 4). This information was then employed in the development of the jump-equilibrium model, whose properties are examined in Chapter 4. Models have heuristic and pedagogic value in that they provide the concepts that are essential for explaining and comprehending actual phenomena. Without models, knowledge would be restricted to blindly discovering correlations between events. Hence, the development of new concepts, or models, is an important part of economics.

However, constructing models is only one part of scientific research. Another vital part is formulating theoretical hypotheses. Theoretical hypotheses are lawlike statements that claim that a model is true of some actual system. Although models are assumed to be interpreted and not simply syntactic objects, they do not explicitly specify their domains. Theoretical hypotheses state the domain of application of models and thereby create testable empirical predictions. They claim that the model is true, at least to some degree of approximation, of an actual system. For example, it may be asserted that people behave rationally when making financial decisions. A scientific theory is defined as the combination of a model and a theoretical hypothesis.

The distinction between theories, theoretical hypotheses, and models is controversial because it is not clear-cut. Theoretical and empirical research is often inextricably interlinked and it may not be helpful to separate them. These definitions also differ from other interpretations of scientific theories, including the syntactic and semantic views (see Hausman 1992). Nevertheless, this distinction is useful for understanding the theoretical practice of economists, because it is generally not possible to conduct 'closed' economic experiments. Models generally make systems comprehensible by excluding the complications of reality. In the natural sciences, these 'closed' systems can often be artificially created in laboratory experiments, which makes model building appear to be similar to empirical investigations. Economic models are not able to precisely define economy, which causes economic model building and empirical investigations to be more distinct. Thus, in economics there is inevitably a partial separation of theoretical and empirical activities.

This definition of a model partially explains why economists do not generally view their theories as universal laws; they are often merely models. It also attempts to deflect superficial criticisms of economic practice. However, economics needs to provide more than just elegant models if it is to be of practical benefit. If these models are to be useful then it is important to know whether they represent a good guide. As a result, researchers need to be either committed to the truth of their assumptions or satisfied that the conclusions would not be materially different if true assumptions were used. Hence, Hausman rejected Friedman's methodology (see Section 5.4). Hausman asserted that scientists should aim to discover generalisations with broad scope rather than models with limited domains.

5.6.2 The deductive method

The distinction between theoretical hypotheses and models is intended to emphasise the importance of developing models. But models ultimately need theoretical hypotheses if they are to be useful. Determining the adequacy of the resulting theories is a central problem in economic methodology. Hausman (1992) rejected Popper's and Lakatos' approaches towards theory assessment mainly because they do not permit scientific justification (see Sections 5.2.4 and 5.2.5). Hausman also argued that the methodological rule of the method a priori, that test results never disconfirm the basic postulates, is unjustified. This rule would retard progress because it prevents the discovery of deficiencies in the basic postulates. Nevertheless, Hausman argued that economists are often justified in following the method a priori in practice.

The practical problems associated with testing economic theory (see Section 5.4) make it difficult to interpret test results. Economists cope with these difficulties using the 'weak-link principle': "When a false conclusion depends on a number of uncertain premises, attribute the mistake to the most uncertain of the premises" (Hausman 1992: 207). This principle invariably implies that the basic postulates will not be questioned. Although the weak-link principle is not always appropriate, it represents a rational response to a complex problem. Hence, in principle economists may follow the hypothetico-deductive or the Bayesian method of theory assessment, but in practice they may appear to follow the method a priori. This is due to practical difficulties rather than a dogmatic methodological rule.

Another consequence of the problems associated with assessing economic tests is that a theory's pragmatic qualities are likely to be more important than they would otherwise have been. As it is difficult to establish that a particular theory is clearly superior, economists prefer theories that are mathematically tractable and provide broadly adequate approximations. If a more complicated theory was shown beyond doubt to be better, then it is likely that economists would use it. But, if the alternative only appeared to be marginally better and the evidence supporting this view was ambiguous then it is unlikely that economists would relinquish a theory with substantial pragmatic benefits. Hence, economists have rational empirical and pragmatic grounds for retaining their basic postulates even though the empirical evidence supporting them is equivocal. These grounds could cause economists to become unreasonably dogmatic and to cling to inadequate theories, but this is not a necessary consequence.

The above considerations were used by Hausman in the development of an alternative method of theory assessment, known as the deductive method. The deductive method is an adaptation of both the method a priori and the hypothetico-deductive method. It asserts that economists start with credible and convenient generalisations that describe relevant causal factors. These generalisations are then used to deduce the required predictions and these predictions are tested. If these predictions are incorrect then economists should: "compare alternative accounts on the basis of explanatory success, empirical progress, and pragmatic usefulness" (Hausman 1992: 222). The deductive method does not assume the basic postulates are infallible, but it allows their initial credibility to be used in assessing test failures. Pragmatic benefits are also given an important role because of the acuteness of the Duhem-Quine problem in economics.

5.6.3 The separate science of economics

One feature of economic practice that Hausman criticised was the commitment to the vision of economics as a separate science. Hausman (1992: 90) argued that neoclassical economics is governed by the following theses:

1 Economics is defined in terms of the causal factors with which it is concerned, not in terms of a domain.
2 Economics has a distinct domain, in which its causal factors predominate.
3 The "laws" of the predominating causal factors are already reasonably well-known.
4 Thus, economic theory, which employs these laws, provides a unified, complete, but inexact account of its domain.

These causal factors are similar to the basic postulates and include the pursuit of wealth and their 'laws' include notions such as agents can and do order their preferences. The economic domain is defined as the social phenomena that are primarily driven by these causal factors. Economics studies a particular aspect of human behaviour rather than a particular domain. Moreover, economics aims to provide a complete mathematical description of an abstract economy consisting of rational agents. Consequently, abstract general equilibrium theory is important even though it does not provide any specific predictions because it shows that voluntary exchanges by individuals, who are motivated purely by self-interest, can result in a coherent and efficient economy. This provides theoretical reassurance. Thus, Hausman (1992: 95) asserted that: "economics studies the consequences of rational greed." It is acknowledged that other 'irrational' motives influence human behaviour and market phenomena, but these motives are generally not considered unless they have a clearly defined significant systematic influence, such as aversion to labour. An implication of this interpretation of economics is that economic explanations must relate to the rational choices of individual agents. Therefore, Hausman asserted that economists are committed to the goal of developing a single theory that broadly accounts for the whole economic domain.

This vision discourages economists from exploring non-rational choice explanations and places a barrier between economics and the other social sciences. This response is partially justified because of the difficulties associated with economics and because unification is an important goal of scientific explanation. But, neoclassical economics has not been overwhelmingly successful so theories with a narrower scope, such as those offered by institutionalists (see Wilber and Harrison 1978; Wisman and Rozansky 1991), should not be rejected outright. Institutionalists attempt to understand socio-economic systems using pattern models, which are networks of significant recurrent themes that are similar to scenarios (see Section 2.5.3). These themes are developed from a wide variety of sources and tend to emphasise the holistic, systemic, and evolutionary characteristics of the system under investigation. Pattern models do not usually produce a body of formal theories or universal laws that are location and time invariant. They attempt to capture the unique features of their subject matter, which may include irrational behaviour. Nevertheless, the construction of pattern models is usually guided and organised by a few universal preconceptions. Examples of these preconceptions include that technological change is a significant causal factor of social change and that human behaviour, which is culturally determined, tends to resist this change. Institutionalists usually strongly discount historical information. They do not believe that it is feasible to develop structural models that are valid over long time intervals.

Therefore, Hausman (1992) argued that economists should be more willing to consider theories with 'non-economic' causal factors, but he acknowledged that this is unlikely to happen unless economists are able to produce better data and improved statistical methods of analysing this data. Hausman (1992: 280) concluded that: "Although some theorists should keep pushing the current strategy as hard as they can, I would urge economists to be more eclectic, more opportunistic, more willing to gather data, more willing to work with generalisations with narrow scope, and more willing to collaborate with other social scientists."

5.7 The Importance of Methodology

As most methodological advice has been found to be flawed in some respect, it may be asserted that methodological investigations are pointless. Economists are best placed to assess theories and philosophers can add little to economic practice. This extreme view is partially supported by Feyerabend (1988) who argued that all universal standards and rigid traditions are likely to hinder scientific progress. The complexity of the world "defies analysis on the basis of rules which have been set up in advance and without regard to the ever-changing conditions of history" (Feyerabend 1988: 11).

Feyerabend claimed that, although certain rules may encourage progress in specific situations, the only universally defensible principle is methodological anarchism: 'anything goes'. There will be situations in which any rule is inappropriate. For example, it may be productive to improve rather than discard a falsified theory or degenerating research program. No theory can be decisively rejected and, theories can often only be assessed and new discoveries made, if incompatible alternatives exist. Hence, it may be beneficial to introduce and develop numerous contradictory hypotheses. This argument suggests that it is unreasonable to insist that new theories must be consistent with well-established theories. As theories are never perfect, the criterion of consistency gives precedence to the first adequate theory, which preserves the old and the familiar. Feyerabend rejected the claim that his views would lead to chaos or to increased 'intellectual pollution'. He suggested that charlatans can never be completely excluded by strict rules. Furthermore, Feyerabend did not argue against rules or standards in general, his main attack was directed towards the imposition of universal laws on third parties.

McCloskey (1986) also argued against traditional empiricist methodology. He asserted that empiricism is impracticable in economics because economic prediction is both impossible and unnecessary. He argued that economics is a historical, rather than a predictive, science. He suggested that economists should merely study how they are persuaded by arguments using literary criticism, termed rhetorical or discourse analysis. This provides a method for understanding the development of the economics literature rather than evaluating its validity. Arguments are usually unable to decisively determine the validity of an issue. Their validity is ultimately a matter of judgement. Thus, knowledge is assumed to be negotiated rather than empirically established; a good argument is one that persuades the majority of economists (see Black 1986: 537). There are no other standards of truth to aim at. Traditional empiricist methodology is considered to be inadequate for the task of understanding arguments because it excludes a number of factors that commonly persuade in practice, such as arguments by analogy and the authority of the author.

The rhetorical approach has been criticised for ranking rigour and precision above relevance. In particular, Rosenberg (1992: 37) argued that: "It is a doctrine according to which the produce with the most effective advertising campaign is the best purchase." Rhetorical analysis is useful for improving communication, but artful persuasion does not imply justification, assuming this can be established. Furthermore, McCloskey's claim that prediction is unnecessary has been challenged because economic predictions are required by policy makers. Predictions of some sort are also required to obtain certified knowledge about a system, such as the economy, whose existence does not depend on theory. Although economics has not been able to produce quantitative predictions, generic predictions have been successfully made. Hence, Boylan and O'Gorman (1995) argued that it is not appropriate to universally replace traditional methodology with rhetoric. They suggested that each approach is useful within specific domains and that: "despite the fragility and fallibility of economic facts and economic testing, empirical evidence is the final arbiter for the science of economics" (Boylan and O'Gorman 1995: 57). Moreover, economists need to adopt, or at least be aware of, universal standards of knowledge if they wish to have any influence amongst non-economists. Although all methodological rules have their weakness, there are no grounds for concluding that all methodological investigations are misconceived. Methodological rules, or norms, are unavoidable and a study of their limitations provides practical information.

5.8 Implications for Actuarial Economic Modelling

Methodological discussions help to clarify fundamental issues and provide a wealth of ideas, but they do not offer universal rules or definitive answers to these fundamental issues. Logical positivism unintentionally demonstrated that judgement is unavoidable in science and that knowledge is fallible. This suggests that policy makers, including actuaries, should always maintain a degree of scepticism about scientific claims. Financial economic theory cannot be proven to be absolutely true. Nevertheless, the philosophy of science also provides constructive suggestions as to how various positions can be rationally defended. It stresses that empirical testing is essential for checking knowledge claims even though this evidence is usually ambiguous. These tests should consider all the predictions made by a theory and negative test results should be taken particularly seriously. Positive test results do not necessarily convey meaningful information: harsh tests are required. Alternative theoretical frameworks should be encouraged because they enable comparative tests to be carried out and alternatives are usually required before existing theoretical frameworks are surrendered. These alternatives are likely to take time to develop and they should be treated generously in their initial stages of development. Moreover, researchers should be aware of all the rhetorical considerations that may influence theory choice.

The emphasis placed on empirical testing by economic methodologists suggests a possible difficulty with the aims of actuarial economic models. Actuarial models are generally only concerned with the long term, but little empirical evidence exists on the long term. This makes it virtually impossible to empirically assess actuarial models and this aim could easily be interpreted as an immunising stratagem. According to Keynes (1923: 65):

In the long run we are all dead. Economists set themselves too easy, too useless a task if in tempestuous seasons they can only tell us that when a storm is long past the ocean is flat again.

Moreover, models with limited domains are not as satisfactory as more universal models. Hence, although actuaries need to be aware of longer term influences, they should possibly aim to develop adequate short term models.

This aim is extremely demanding, given the nature of the economy. Economic predictions have rarely been precise and accurate and this chapter has presented arguments suggesting that economic predictive success is always likely to be limited. Moreover, Redington's (1983) sceptical view on the utility of stochastic models is also held by some Austrian and institutionalist economists. Falsificationists maintain that it is pseudo-scientific to retain models that have been falsified or are not subjected to harsh tests. These arguments suggest that applications of stochastic models should not demand too much from the stochastic model. Precise calculations and strong conclusions are possibly unjustified. Sensitivity testing is vital and professional judgement will always play an important part. Actuaries need to develop systems that are flexible and not completely dependent on specific economic models or assumptions. Thus, applications such as determining precise tactical asset allocation strategies are possibly less suitable than applications such as comparing the relative riskiness of two portfolios of insurance policies.

The difficulties associated with developing adequate economic predictions appear to explain why actuaries tend to refrain from referring to their bases, or stochastic models, as providing predictions. Actuaries tend to claim that stochastic asset models provide 'projections' rather than forecasts (see Wilkie 1995b: 803; and Daykin *et al.* 1994: 230). However, the applications of these models generally suggest that they are predictions. Actuaries are only concerned with probable events, purely hypothetical scenarios are of little practical value. Actuarial predictions are however often less precise. For solvency purposes, actuaries need realistic predictions of disaster scenarios, but they do not necessarily need to know the exact timing of these events. More precise predictions would be more useful, but for many applications generic predictions are adequate.

However, these sceptical conclusions concerning the limitations of economic predictions are not proven and it is possible that future economic theory will be able to produce significantly improved predictions. Actuaries and economists should not give up on trying to improve their models and theories, but current applications should not be too dependent on them.

When attempting to improve a model, the philosophy of science emphasises the value of having a theoretical framework. Kuhn argued that research generally takes place within a disciplinary matrix, Lakatos suggested that research programs have a hard core and a protective belt, and Mill claimed that a deductive method founded on basic postulates was essential for economics. Theoretical frameworks enable one to meaningfully interpret test results and they suggest potential avenues for future research if problems emerge. They provide essential information on which hypotheses are approximate and which are fundamental. Theories do not need to be perfect, but they must be checked for reliability, refinability, and excusability. Thus, although actuaries are broadly right to criticise the unrealistic assumptions used in financial economics, they should acknowledge that unrealistic assumptions are unavoidable and should attempt to demonstrate in specific cases whether the above conditions are met. This task is attempted in Chapter 6, which considers various finance theories, especially the efficient market hypothesis. In addition, the heuristic and pedagogic value of models (see Section 5.6.1) should not underestimated.

Models developed without a theoretical framework have similar problems to black-box models. If they fail, they can only be holistically rejected so that alternatives need to be completely redeveloped. What is more important, little significance can generally be given to positive test results of black-box models unless they have been extensively tested in a variety of conditions. Black-box models are usually pure inductive claims that are not supported by other more fundamental studies. These problems are particularly acute for actuarial economic black-box models because of the relative paucity of data that is available to test them. This problem is associated with the more general problem of data mining, which is discussed in Chapter 7. As predictive success is difficult to establish, a theory's pragmatic qualities become more important than they would otherwise have been. These arguments suggest that Smith's (1996) approach towards developing a stochastic model is more suitable than Wilkie's (1995b).

The jump-equilibrium model derives its main support from financial economic theory, which also enables it to be easily interpreted. This theory could be viewed as providing the hard core of the model, which is usually retained even if the model is disconfirmed. The empirical information presented in Chapter 4 suggests that Smith's use of the gamma distribution appears to be problematic, but it is unlikely to suggest that the use of the efficient market hypothesis is inappropriate. The jump-equilibrium model is also able to provide mathematically tractable solutions for many applications. Wilkie's (1995b) model is less tractable and only employs economic theory in a seemingly ad hoc manner. Equilibrium arguments are used to support the use of the purchasing power parity assumption in the exchange rate model, but similar equilibrium arguments are rejected in the equity models, which permit arbitrage opportunities. Thus, it seems as though Wilkie's model can only be justified on the basis of its predictive success, which is given detailed consideration in Chapter 8.

A problem with theoretical stochastic asset models is that their theoretical 'hard core' is usually relatively sparse. Finance theory provides no information on what error distributions should be used and it is unable to suggest specific utility functions. It appears that this information can only be established using an ad hoc trial and error process in which researchers simply try out their favourite hypotheses. This process is bound to be arbitrary. This further emphasises the limitations of stochastic models and the importance of their pragmatic characteristics. A further potential problem with theoretical models is that they depend on fallible theories. The following chapter considers the support for some of the fundamental finance theories.

Chapter 6

ASSET PRICING THEORY

6.1 Introduction

A fundamental issue in actuarial economic modelling is whether actuarial models should conform to financial economic theory. Actuaries have generally been reluctant to wholly embrace financial economics because its abstract models do not appear to adequately describe economic phenomena. The assumptions of asset pricing theories are frequently chosen for their mathematical tractability rather than strictly on the basis of their degree of realism. For example, the convenient assumption that agents have time-additive utility functions is unrealistic because it implies that the utility of consumption at a specific point in time is independent of consumption at any other time. Allais' paradox demonstrates that a fundamental axiom of expected utility theory, the independence axiom, appears to be inadequate (see Huang and Litzenberger 1989). Furthermore, many of the assumptions of the CAPM are unrealistic, including: that agents can borrow and lend at the same riskless interest rate and that agents can easily sell risky assets short (Stiglitz 1989).

However, as Friedman (1953) stressed, unrealistic assumptions are inevitable in abstract models. Economic theories provide internally consistent systems that only abstract the important features of the economy. Hence, they are never completely realistic. Moreover, the main alternatives to developing abstract theories, which involve determining generalisations by experimentation or direct observation, are problematic in finance because of the nature of the economy. As a result, actuaries are unlikely to be able to avoid making some unrealistic assumptions. The more important issues are whether financial economic theories incorporate the fundamental causal factors, whether they produce reliable predictions that are capable of being refined, and whether their inadequacies can be explained. Stiglitz (1989: 353) stated, in the context of the mutual fund separation theory:

The parameterizations that give rise to these results are extremely convenient, particularly for obtaining closed-form results. Their widespread use within finance is, accordingly, hardly surprising. The problem is, what credence can we give to the generality of the results derived?

This chapter aims to address this last question by examining the development of asset pricing theory. A comprehensive review of this literature is beyond the scope of this thesis. This review merely attempts to illustrate the significance of having a theoretical framework and to rationalise the commitment of financial economists to asset pricing theory. It uses and builds on the ideas presented in the previous chapter on economic methodology. Economic methodology, and the Duhem-Quine thesis in particular, emphasises that the evaluation of theoretical developments is a complex procedure because theories are made up of a collection of hypotheses. For example, asset pricing theories are generally expressed in ex ante terms and are thus not directly observable. To test these theories, they need to be combined with a hypothesis about agents' expectations, such as the rational expectations hypothesis. Moreover, the asset pricing models themselves are made up of a number of individual hypotheses or assumptions. Hence, tests of specific asset pricing theories are usually ambiguous and considerable care is required when assessing theoretical developments.

Section 6.2 is largely based on LeRoy (1989) and reviews the results of various tests of the present value models that are described in Appendix 2A.1. These tests suggest that present value models are unable to adequately account for certain features of the market. Section 6.3 examines possible explanations for these test results and their implications for finance theory. However, none of these explanations has attracted widespread support and this suggests that asset pricing theory is not empirically adequate. Given these difficulties, Section 6.4 considers how asset pricing theory may be justified and whether actuarial economic models should conform to asset pricing theories. Section 6.5 concludes.

6.2 Tests of the Present Value Model

6.2.1 Volatility tests

Direct tests of the efficient market hypothesis or the present value model, such as those conducted by Ford *et al.* (1980) and Wilkie (1986a), that attempt to detect profitable trading rules are inconclusive. This is because these tests usually assume that agents had complete knowledge. The discovery of a profitable relationship in past data may merely suggest that investors were not aware of the relationship rather than suggesting that they irrationally forfeited the opportunity to profit from the relationship. If the relationship can be shown to be genuine then investors will exploit it soon after it is discovered so that it will cease to exist in future. Therefore, the existence of arbitrage opportunities in retrospect does not necessarily refute the efficient market hypothesis.

To avoid these information related difficulties, LeRoy and Porter (1981) developed a volatility test of the present value model that is independent of investors' information sets (see LeRoy 1989). This test is based on the implication of the present value model that the volatility of security prices should be low relative to the volatility of discounted dividends. This implication was derived by observing that the discounted future dividends, $P^*(t)$, of a security can be represented as the sum of its current price $P(t)$, and an error term $\varepsilon(t)$: $P^*(t) = P(t) + \varepsilon(t)$. If the present value model holds then: $E_t[P^*(t)] = P(t)$, which implies that $P(t)$ is a forecast of $P^*(t)$. Hence, the error term has an expected value of zero and must be uncorrelated with $P(t)$. Therefore, the variance of $P^*(t)$ must be greater than or equal to the variance of $P(t)$. LeRoy and Porter (1981) and Shiller (1981) tested this inequality and found that it was grossly violated.

The strength of this conclusion was subsequently tempered by the detection of technical econometric problems associated with the tests used. Flavin (1983) revealed that the tests were subject to small sample biases. Kleidon (1986, 1988) showed that the excess volatility test results could be accounted for if security prices are nonstationary. Moreover, Marsh and Merton (1986) claimed that the excess volatility results were probably caused by the dividend model used in the tests. They showed that the inequalities reverse if it is assumed that managers smooth dividend payments, rather than assuming that dividends are serially uncorrelated. These defences illustrate the difficulties associated with testing individual theories and show that the straightforward interpretation of test failures is not necessarily appropriate. The difficulties arise because these volatility tests need to make an assumption about how investors expect dividends to behave in future. However, a generally accepted robust model of dividend behaviour does not exist. This makes it relatively easy to dismiss these negative test results by questioning the dividend model assumption. Nevertheless, Mankiw *et al.* (1985) and Campbell and Shiller (1989) responded to these defences by developing alternative 'unbiased' excess volatility tests for security prices. These tests also suggested that the volatility of equity prices is excessive.

Similar volatility tests have been developed for the term structure present value model (Shiller 1979). These tests suggest that, if the model is valid then long term interest rates should be far less volatile than short rates. This implication was rejected by Shiller (1979). As in the above volatility tests, the original term structure volatility tests were unable to distinguish between a hypothesis that short rates are non-stationary and one of excess volatility (Flavin 1983). To avoid these problems, Campbell and Shiller (1987) tested and rejected the term structure present value model using the spread between long and short term interest rates.

In an attempt to explain the above volatility test results, it has been noted that $\varepsilon(t) = P^*(t) - P(t)$ is approximately equal to a weighted average of future returns (see Campbell and Shiller 1988: 668). Moreover, by definition, $P(t)$ represents a weighted average of past returns. Thus, if security prices are excessively volatile then it is likely that their rates of return are correlated over time. Fama and French (1988a,b) tested this implication by analysing the correlation of returns averaged over various intervals. Although returns calculated over short intervals and very long intervals were found to be uncorrelated, considerable evidence of negative correlation in three to five year returns was found. They stated that this pattern was consistent with a mean reverting model for equity prices. This also supports Wilkie's (1995b) assertion that the random walk, or martingale model, while being adequate over the short term, is inappropriate over longer time horizons. In addition, using variance ratios of multi-period returns, Poterba and Summers (1988) found evidence of a mean reverting component in equity prices. These results appear to corroborate the excess volatility findings, but there is some dispute about their statistical validity (Richardson and Stock 1989). Nevertheless, after considering the effects of small sample biases, Hodrick (1992) concluded that there is considerable evidence of predictability of one month ahead returns. He found that changes in dividend yields seem to precede significant persistent changes in expected stock returns. Moreover, Lo and MacKinlay (1988) reported the results of volatility based tests that appeared to reject the random walk hypothesis for weekly equity prices. But Lo and MacKinlay also rejected the hypothesis that equity prices can be adequately described using a stationary mean reverting model.

Kleidon (1986), amongst others, has suggested that the empirical problems of the present value model could be partly due to the assumption of constant future discount rates. The effect of assuming a variable discount rate was analysed by Campbell and Shiller (1989). They used the dividend-price ratio model (see Appendix 2A.1) on equity data and found that, although there was some evidence that the logarithm of the dividend-price ratio moved with rationally expected future dividend growth, the model was unable to account for a substantial part of the variation in the logarithm of the dividend-price ratio.

Furthermore, Shiller and Beltratti (1992) used the dividend-price ratio model to examine the relationship between changes in equity prices and changes in long term fixed-interest security yields. They noted that, if the theoretical model holds then these variables should generally be slightly negatively correlated. However, the actual correlation was found to more negative than the model implied, which suggests that these markets over-react to one another. Further, no evidence of excessive correlation of either the equity or fixed-interest security markets with changes in inflation rates was found. As in previous studies, Shiller and Beltratti (1992) rejected the restrictions of unpredictability of excess returns for both the equity and fixed-interest security markets.

Campbell and Ammer (1993) attempted to account for these volatility results using a dynamic accounting framework derived from the dividend-price ratio model (see Appendix 2A.1). This framework was used to estimate the relative importance of various components to the historical behaviour of security returns. The results of this analysis suggested that the most important component of the variance of excess equity and fixed-interest security returns is changing expectations of future excess equity returns and inflation rates, respectively. The real interest rate component was found to contribute little to the variance of excess equity and fixed-interest security returns. However, changes in expectations of future real interest rates were found to be an important component of the variance of excess short term interest rates. In addition to this study, a number of papers, summarised by Shiller (1990), have attempted to explain time varying term premiums in terms of a wide range of variables, such as measures of variability, volume of trade, and business confidence.

Although the volatility test results are not decisive, they emphasise that present value models are unable to produce reliable predictions. This supports the claim made by actuaries that asset pricing theories are not empirically adequate. Additional evidence that appears to suggest that the present value model is empirical inadequate is presented in the following section.

6.2.2 Other anomalies

Security prices have also been found to display a number of other anomalies, possibly indicating that the market is inefficient (see Dimson 1988). For example, the January effect is the anomaly that rates of return in January have been significantly higher than the returns in the rest of the year. Additional anomalies include: the price-earnings ratio anomaly, the losers anomaly, the weekend effect, and the Wednesday effect. These anomalies are similar to direct tests of the fair game assumption, which attempt to forecast equity returns using a range of financial and economic variables (see Granger 1992). For instance, Pesaren and Timmermann (1994) demonstrated that a regression model was able to produce portfolios that mean-variance dominated (see Appendix 2A.3) the market portfolio after allowing for transaction costs. The variables used in this model include dividend yields, inflation rates, discount rates on short term fixed-interest securities, and the change in industrial production. Other examples of direct tests include Clare *et al.* (1994) who showed that the ratio of long term fixed-interest security yields to equity dividend yields could be used to develop a profitable trading rule and Campbell and Shiller (1988) who found that a moving average of earnings helped to predict future real dividends. However, these tests are subject to the problems discussed earlier and some of these anomalies may be due to spurious data mining (see Chapter 7).

Furthermore, the volume of trade generally observed in markets appears to be too large to be justified by the assumptions underlying the present value model (see LeRoy 1989). According to this model, profitable trading is only possible if an investor has information that is not fully reflected in security prices. However, even if superior information was assumed to be held, it is unlikely that speculative trading would occur (Milgron and Stokey 1982; Tirole 1982). This is because rational investors would allow for the superior information presumably held by the other party to the trade. As trading is a negative sum game, allowing for transaction costs, it should not compensate investors for the risk undertaken.

Other empirical contradictions of the present value model include (see LeRoy 1989): that only a small part of ex post equity returns can be explained by fundamental factors, stock market crashes cannot be fully explained by new information, and that excessive amounts of money seem to be spent on obtaining investment advice.

In response to the above evidence, LeRoy (1989: 1615) stated that: "it is extraordinarily difficult to formulate nontrivial and falsifiable implications of capital market efficiency that are not in fact falsified." This contrasts with Marsh and Merton's (1986: 483) assertion that: "the majority of empirical studies report results that are consistent with stock market rationality." Marsh and Merton appear to attribute all the contradictory results to problems with the other hypotheses being tested or to statistical problems, including data mining. In addition, confirming evidence is assumed to have less chance of being published. These arguments illustrate how difficult it is to learn from empirical evidence in finance. The auxiliary assumptions on which most tests are based do not represent well-established background knowledge and can therefore be legitimately challenged. Nevertheless, the above evidence is problematic because it shows that finance theory has been unable to produce reliable predictions. The following section considers whether these predictions have been successfully refined.

6.3 The Response to the Test Results

6.3.1 The CAPM

Present value models should be expected to have some empirical problems because they are based on unrealistic assumptions. In particular, they generally assume that investors are risk neutral, which implies that all risky securities have equivalent expected real returns in equilibrium. This implication does not appear to be satisfied by equity data (Singleton 1990). If it is assumed that investors are risk averse, then it is possible that returns can be partly forecast because trading rules that provide higher expected returns may not be acted upon if they are utility decreasing. For example, the CAPM (see Appendix 2A.3) implies that securities could have persistently lower than average expected returns if they are negatively correlated with the market portfolio.

The development of the CAPM provides an example of the role of a theoretical framework. Scientific theories generally evolve over time. The most promising tractable theories are first explored and then refined in response to empirical tests. The initial theories are often highly abstract and oversimplified because they merely aim to capture the fundamental features of the economy. These initial theories are closer to models than theories (see Section 5.6.1) and they provide the essential concepts for explaining phenomena. Hence, the CAPM could be interpreted as a model that provides a valuable pedagogic device for understanding portfolio decision making. A considerable investment in time and effort is generally required before realistic theories are established. A theoretical framework makes this process more efficient because it enables test results to be interpreted and it usually suggests possible refinements to particular theories. Although these refinements may appear to be intuitively appealing in the context of the theoretical framework, their implications need to be tested. Policy makers need to know how good their models are.

The CAPM suggests that the dividend policy of a company should be irrelevant to its value because investors simply purchase all risky assets (Stiglitz 1989). This implication is similar to the Modigliani and Miller (1958) propositions on the irrelevance of dividend policy to the value of the company. Thus, there is no reason to expect managers to continue to adopt a consistent dividend policy. Consequently, doubts have been expressed about whether it is sensible to model dividends using linear time series models (Campbell and Ammer 1993). In the extreme, it is possible for a company that does not ever intend to pay dividends to have a positive value. However, this argument has less force in practice because debt to equity ratios are relevant for tax and other reasons. Even so, changes in taxation and other legislative requirements may be irregular and, as a result, difficult to describe using mathematical models. This highlights the difficulty with the volatility tests that were discussed earlier because these tests generally require a mathematical model of dividends.

There are also significant problems associated with testing the CAPM including the fact that the market portfolio is not observable and that the market betas are potentially unstable (see Huang and Litzenberger 1989). The market portfolio includes assets, such as human capital and other forms of nontraded assets, whose returns are practically unobservable. Consequently, empirical tests use proxies of the market portfolio so that the results of these tests reflect the joint hypothesis of the CAPM and of the suitability of the proxy (Roll 1977). Furthermore, econometric problems arise in testing the CAPM if the market betas are not stationary over time, but these technical difficulties have been largely overcome.

Notwithstanding the above difficulties, empirical tests of the CAPM have produced mixed results at best. Its main empirical implications are that the expected returns on securities have a positive linear relationship with their market betas and that these betas are sufficient to describe the expected returns. The results of early studies, such as Black *et al.* (1972) and Fama and MacBeth (1973), supported the prediction of a positive linear relation between average returns and market betas. However, the results of more recent studies, such as Gibbons (1982) and Fama and French (1992), contradict this evidence. Fama and French (1992) reported that this positive linear relationship is insignificant over the interval 1963-90. They concluded that the cross-sectional variations in average returns are adequately described by company size and the ratio of book to market value. This evidence is contrary to the CAPM, which implies that the market betas fully explain expected returns. Other potential risk factors that have been found to add to the explanation of security returns include leverage and earnings yield;

but Fama and French did not find these to be significant. These results suggest that the CAPM is empirically inadequate.

However, using an alternative data source and betas calculated using annual rather than monthly returns, Kothari *et al.* (1995) disputed Fama and French's (1992) findings. Furthermore, Kothari *et al.* (1995) warned that evidence about size and other effects may be caused by spurious data mining because of the large number of variables that could potentially have been examined. They found that annual betas were significant and cast doubt on the significance of the book to market value effect. Nevertheless, they also found evidence of a size effect.

Further, Stiglitz (1989) argued that the mutual fund separation theorem, which implies the CAPM, has additional false implications: including that all investors hold widely diversified portfolios and that a company's debt to equity ratio or dividend policy is inconsequential. Thus, Stiglitz (1989: 353) concluded that: "the major use of the mutual funds theorems has been a cautionary one."

6.3.2 The fundamental valuation equation

The fundamental valuation equation (see Section 2.4.1) is another refinement of the present value model that has had limited empirical success. Mehra and Prescott (1985) found that it was unable to generate a plausible equity risk premium. The observed equity risk premium was found to be too large to be justified by the fundamental valuation equation, assuming constant relative risk aversion. This suggests that differences in mean returns cannot be explained by differences in consumption risk. Using a volatility-type test, Grossman and Shiller (1981) also reported that the fundamental valuation equation requires unreasonably high degrees of risk aversion.

The lower than observed equity risk premium implied by the fundamental valuation equation could conceivably be used to support the intuitive actuarial view that the historically high risk premium will not persist in future (see Section 2.3). However, the actuarial justification of this view that was presented in Section 2.3.3 is inconsistent with financial economics. Actuaries assumed that investors underestimated inflation in the past and, as this is unlikely to continue, they forecast a relatively low risk premium.

But, financial economists are committed to the rational expectations hypothesis, which implies that investors' expectations are ordinarily fulfilled ex post. Hence, financial economists would generally not assume that investors significantly underestimated inflation, unless there was a clear rational justification for the discrepancy.

The problems with the fundamental valuation equation have persisted in more sophisticated specifications, including models with cash and stochastic inflation (Labadie 1989), models that relax the assumption of time additive utility (Weil 1989), and those incorporating durable goods (Dunn and Singleton 1986). Labadie (1989) noted that the size of the equity risk premium is sensitive to the volatility of consumption growth and increases in both consumption growth and inflation volatility increase the equity risk premium. Weil (1989) reported that relaxing the restriction of time additive utility results in an additional puzzle: the riskless rate appears to have been too low. Furthermore, Rose (1988) found evidence that the ex ante real returns are nonstationary, or affected by permanent shocks, but rejected the hypothesis that consumption growth rates are subject to permanent shocks. This result is also inconsistent with the fundamental valuation equation. Hence, it appears that allowing for risk aversion using the fundamental valuation equation does not solve the empirical problems of the present value model. In the context of term structure models derived from the fundamental valuation equation, Singleton (1989: 156) concluded: "In sum, the evidence does not support these representative agent models of the term structure of real bond returns."

However, possible resolutions of these puzzles include: that consumers are heterogeneous and the market is incomplete so that individual investors have undiversifiable consumption risk (see Scheinkman 1989), and that investors' utilities exhibit habit persistence (Constantinides 1990). Constantinides (1990) argued that investors have a subsistence level of consumption, which is a weighted average of past consumption. Hence, small falls in consumption translate into large falls in surplus consumption, where surplus consumption is consumption above the subsistence level. Campbell (1993) suggests that the apparent difficulties with the fundamental valuation equation could be attributed to the use of aggregate consumption data because it may be a poor proxy for individual investors' consumption and it is subject to measurement error. He derived approximate relationships, which accommodated risk aversion, but do not depend on consumption data. These relationships were not tested.

These potential problems and resolutions illustrate the difficulties facing financial economists. Their theories are abstract and a wide range of phenomena could account for the problems. Moreover, they need to work with incomplete and sometimes unreliable data. To test their fundamental theories robust models of dividends and agents' expectations are required but are not available. Falsificationists may argue that asset pricing theories have been falsified and should be rejected. However, Kuhnians may reply that this requires an alternative theoretical framework that accommodates most of the above problems. Until such an alternative is available, financial economists will continue to try to solve these puzzles within the current disciplinary matrix. Lakatosians could argue that some of the fundamental theory is within the hard core of the finance research program and should not be questioned. However, asset pricing theories have shown little empirical progress and should possibly be categorised as empirically degenerating. Although asset pricing theories can be refined, their refinements have not produced reliable predictions and the reasons for the test failures are not well understood. Nevertheless, asset pricing theories are relatively immature and appear to represent the most promising current paradigm. The following sections consider some other responses to the empirical problems of asset pricing theory.

6.3.3 Structural changes

A further possible partial explanation of these negative test results is that they are biased because of structural changes (Shiller 1990) or infrequent disasters (Reitz 1988), sometimes called the peso problem (Flavin 1983). Mankiw and Miron (1986) found an abrupt, but appropriate, change in the relationship between long and short term interest rates when the US Federal Reserve Bank was founded in 1915. The expectations theory was only found to have substantial predictive power before 1915. Hence, Mankiw and Miron suggested that the failure of the expectations theory after 1915 could be attributed to the Federal Reserve's commitment to stabilising interest rates. Huizinga and Mishkin (1984) found a statistically significant shift in the stochastic process of real interest rates around October 1979, when the Federal Reserve shifted their policy from interest rate targets towards monetary aggregate targets.

Pesaren and Timmermann (1995) conducted a simulated real time forecasting study to assess whether equity returns were predictable given the historical information available at the time. They reported that the predictive power of the explanatory variables used, such as inflation, changed over time. The timing of the inclusion of variables tended to be associated with significant macroeconomic events, such as the oil price shocks and the change in the Federal Reserve's operating procedures in 1979. The only variable that was included over the *entire* interval was the *one month* US Treasury Bill rate. There was no evidence to suggest that excess returns, after allowing for transaction costs, could have been obtained in the 1960s, but in the more volatile 1970s significant excess returns could have been achieved. There was only marginal evidence that excess returns could have been obtained in the 1980s. Pesaren and Timmermann suggested that these results could be explained by the hypothesis of incomplete learning by investors after a shock or regime shift.

Furthermore, Evans and Lewis (1995) accounted for the apparent negative correlation between ex post real returns and expected inflation (see Marshall 1992), which contradicts the Fisher relation, by allowing for infrequent shifts in the inflation process. This was achieved using a Markov switching process to describe permanent shocks in the inflation and nominal interest rate processes. Evans and Lewis (1995) concluded that the Fisher relation holds in the long run. Thus, the ex ante real interest rate was found to be stationary after allowing for rationally anticipated infrequent shifts in the inflation process.

Brown *et al.* (1995) argued that empirical studies of security returns are generally biased because they are implicitly conditional on the survival of the market from which the data is obtained. As a result, long term studies are only usually conducted on US and UK markets and may underestimate the true risks involved. Similar issues arise in the construction of certain financial market indices. In addition, conditioning on survival biases tests of long term dependence towards rejecting the random walk hypothesis. However, accounting for catastrophic events in empirical theories is difficult because these events are irregular and infrequent so that little empirical information is available.

6.3.4 Economic uncertainty and noise

Another explanation of the volatility of security prices is that it reflects the extreme precariousness of our knowledge about the long term future (Keynes 1936). There is no reasonable method for estimating the future because there is no basis for determining the probabilities that are required by the financial economic theories. This accords with Redington's (1983) view that there are few material regularities in the economy. Keynes (1936) suggested that investors respond to this uncertainty by generally basing their decisions on extrapolations of existing information about fundamental factors unless there are specific reasons for expecting a change. This implies that investors assume the market valuation is correct and that variations in security prices are caused by the arrival of new information or by changes in the confidence that investors have in the maintenance of the status quo. This behaviour causes security prices to be reasonably stable over certain intervals as predicted by the present value model.

However, because it is impossible to prove that the market valuation is correct, security prices are always susceptible to violent and unpredictable fluctuations. If the confidence that investors have in the maintenance of the status quo is weak, then prices may be affected by a number of seemingly irrational factors. These factors may appear to be irrational in retrospect, but may be rational at the time taking into account investors' lack of knowledge about the long term future. As O'Donnell (1989: 248) noted, Keynes was suggesting that investors "behave as rationally as their circumstance permit." Thus, Keynes (1936: 163) stated that: "[we choose] between the alternatives as best we are able, calculating where we can, but often falling back for our motive on whim or sentiment or chance."

Keynes (1936: 163) also asserted that investors are "largely concerned, not with making superior long term forecasts of the probable yield of an investment over its whole life, but with foreseeing changes in the conventional basis of valuation a short time ahead of the general public." This is because market liquidity allows investors to continually reassess their commitment to their investments. As a result, investors only need to anticipate changes in fundamental factors and investor confidence over relatively short term future time horizons. Investors are assumed to adopt this method because it is easier to form a reasonable opinion about likely changes over the short term future than about the fundamental value of securities. Pratten (1993) conducted a survey of fund managers to assess the contemporary applicability of Keynes's views and concluded that they were broadly relevant.

Keynes' views are broadly similar to Black's (1986). Black introduced the term noise to describe a diverse set of factors that are difficult to quantify. In financial markets, noise represents information that motivates trading but is illusory, possibly because it is already reflected in market prices. For example, as it is generally impossible for investors to assimilate all the relevant information, they may choose to use security price changes as an indirect source of information (Pratten 1993). Moreover, DeLong *et al.* (1990) conjectured that noise trading may be partially caused by inadequacies associated with human judgement, especially overconfidence (see Section 2.3.3). Noise trading could account for the excessively volatile security prices, the high volume of trade that has occurred, and the equity premium puzzle (DeLong *et al.* 1990). However, noise trading is excluded from orthodox asset pricing theories because it is irrational and should be eliminated by rational investors. Black (1986) explains this inconsistency by asserting that it is usually extremely difficult to distinguish between noise and genuine information. Like Keynes, Black suggested that investors are unable to precisely determine a security's fundamental value. As a result, investors cannot usually confidently identify arbitrage opportunities resulting from noise trading so that it is able to persist. However, the further market prices deviate from fundamental value, the easier it becomes to identify arbitrage opportunities. Thus, Black proposed that security prices will almost always be between half and double their fundamental value. This concession is a major departure from the orthodox view and suggests that asset pricing theories are unlikely to be able to produce precise predictions.

Furthermore, DeLong *et al.* (1990) argued that if investors have relatively short time horizons then it is not always possible to take advantage of arbitrage opportunities. Investment managers' time horizons are generally determined by how frequently their performance is measured, which implies that their time horizons tend to be relatively short. Hence, the additional risk created by the unpredictability of the noise traders reduces the attractiveness of arbitrage. This implies that genuine traders will tend to have limited influence. Moreover, the noise traders will be compensated for the risks they create with higher average returns so that they are virtually indistinguishable from genuine investors with low levels of risk aversion. This argument also suggests that investors with longer time horizons will be more likely to gain from arbitrage than those with short horizons and thus can afford to be more aggressive.

In econometrics and macroeconomics, Black asserted that noise makes it difficult to interpret empirical evidence. For example, investors' expectations are difficult to quantify because they are practically unobservable. Expectations cannot necessarily be inferred from past data either because they are liable to change in an erratic and possibly irrational manner over time. Hence, Black (1986: 530) predicted that: "research will be seen as a process leading to reliable and relevant conclusions only very rarely, because of the noise that creeps in at every step."

6.4 The Justification of Asset Pricing Theories

Noise poses a serious practical problem for finance theory. If finance theory is incapable of producing exact reliable predictions because of a diverse range of factors that cannot be precisely identified then what grounds do financial economists have for remaining committed their theories. Is noise not just an immunising stratagem that prevents economists from exploring the real behaviour of financial markets?

These methodological issues can be addressed using Hausman's (1992) explication of economic methodological practice (see Section 5.6). The economy is too complex to study it solely by direct observation, or pure induction, and it is generally not possible to conduct economic experiments. The appropriate method for studying economic phenomena is to start with a set of fundamental postulates that can be established using introspection and empirical observation. These postulates represent the main features of the economy and are used to deduce relevant theories. Confidence in these theories is derived from the basic postulates, rather than from the accuracy of their predictions. Predictive accuracy is difficult to determine because the auxiliary hypotheses are generally substantial and unreliable. For example, the efficient market hypothesis is supported by the basic postulate that agents prefer more wealth to less. Agents are unlikely to knowingly allow others to benefit at their expense from arbitrage opportunities. Moreover, agents have a financial incentive to discover and exploit arbitrage opportunities. This suggests that inefficiencies are likely to be ephemeral. Finance theory does not attempt to explain these inefficiencies because it is only concerned with robust explanations of economic phenomena. Similar arbitrage arguments can be used to defend the rational expectations hypothesis.

These more fundamental arguments are used to interpret empirical test results. For example, Ross (1989: 93) asserted that: "what is being tested in the empirical work on the consumption beta theory is not so much a theory of asset pricing as it is a theory of individual choice." Furthermore, Ross (1989: 93) argued that:

> the success of financial asset pricing theories comes from their appeal to the stronger force of arbitrage rather than from a neoclassical demand and supply equilibrium. The most empirically successful theories in finance succeed by emphasising the relative pricing of assets in terms of close substitutes.

Ross appears to be suggesting that the negative test results of asset pricing theories are due to inadequacies in the economic models of individual choice. The assumption of no arbitrage is assumed to be valid because of the above more fundamental arbitrage argument.

Arbitrage arguments suggest that actuaries are not justified in assuming that markets are, and will continue to be, inefficient in some specific respect. Everyone faces the same difficulties when attempting to assess fundamental value and there is no reason to believe that actuaries have access to superior information. Even though markets may only be partially efficient, market prices still provide the best measure of fundamental value. Actuarial methods are intuitive and are potentially subject to cognitive psychological biases, especially overconfidence (see Section 2.3.3). Moreover, actuaries are not confident enough to allow their methods to influence the investment policy of the fund, even though these funds often have long investment time horizons (see Dyson and Exley 1995).

Nevertheless, the actuarial method of determining the asset value of a fund can be defended on pragmatic grounds. An important principle underlying actuarial valuations is that consistent methods should be used to value both the assets and liabilities of a fund. Therefore, if market values are used to value the assets, then the assumptions used to determine these values should be used to value the liabilities. However, financial economics has illustrated that there does not exist a generally accepted, empirically adequate explanation for how market values are determined, especially in the case of equity securities. In particular, market values seem to be too volatile. Consequently, actuaries are forced to use either the market values of the assets and relatively volatile long term economic assumptions to value the liabilities; or to use relatively stable long term assumptions to value both the assets and the liabilities. This should be a practical decision because there is no theoretically correct or 'objective' answer. Furthermore, if the latter more orthodox actuarial method is chosen, then this does not necessarily imply that the actuary believes that the market is incorrectly valued. It may simply be seen as a more practical method for calculating the surplus of the fund. Nevertheless, given the communication problems associated with the orthodox method, the market value method may be preferred.

The assumptions included in Wilkie's (1995b) model, that markets are inefficient and that investors do not have rational expectations, are more difficult to defend (see Smith 1996). If Wilkie's model could be shown to be true, then the inefficiencies would most likely be exploited and the model would eventually cease to be valid. This suggests that Wilkie's model is unstable and consequently unsuitable as a long term model.

However, a significant problem with neoclassical economics and finance theory is that their theories are inexact. The above mentioned arbitrage arguments are inexact because they assume that investors have sufficient funds and are willing to take sufficient risks to eliminate inefficiencies (see Pepper 1994). In addition, little confidence can be placed in many of the auxiliary hypotheses that are required to test finance theories. This makes it difficult to construct decisive tests and to interpret test results.

The inexact deductive method is defended on pragmatic grounds as the only practical method for studying the economy. If this is the case, then the predictions of actuarial economic models should also be expected to be imprecise and unreliable. This would be especially true because actuarial models are usually only concerned with describing the behaviour of the major asset classes and these asset classes are not 'close substitutes'. These conclusions suggest that a model's pragmatic characteristics should be more important than they would otherwise have been. This supports the approach adopted by

Haberman (1994) and others of initially only considering simple tractable stochastic models. When and if financial economists resolve the current empirical difficulties with their models, then the resulting more complicated models will be considered. This is a pragmatic response to the significant limitations of financial economics. However, the inexact deductive method is not the only method for developing actuarial economic models. Econometrics provides an alternative method that potentially avoids the above sceptical conclusions. This approach is considered in the following chapter.

6.5 Summary

Finance theory is based on abstract models that attempt to capture the important features of the relevant economic environment. As these models have a clear interpretation, they provide researchers with valuable information on which assumptions are realistic and which assumptions need to be refined. This provides a framework in which theories can develop over time. The initial models, such as efficient markets, rational expectations, and utility theory, provide the essential tools for explaining phenomena. These models are supported by the basic postulates of neoclassical economics. However, these models only provide inexact generalisations and for many applications inexact generalisations are insufficient.

Specific empirical features of markets that finance theories have been unable to adequately account for include: the volatility of asset prices relative to their cash flows, the volatility of long-term interest rates relative to short-term interest rates, the relatively high volume of trade in securities, and the relatively high historical equity risk premium. These are significant weaknesses and they support the actuarial view that financial economics is empirically inadequate.

However, it is debatable whether any other method can produce significantly improved predictions. Substantial resources have been devoted to financial economics and it currently represents the orthodox view. This suggests that actuaries should take financial economics seriously and only reject its implications after giving them due consideration. Nevertheless, as financial economics has not been overwhelmingly successful, alternative frameworks should be encouraged and given time to develop.

Chapter 7

ECONOMETRIC METHODOLOGY

7.1 Introduction

Econometrics is a relatively young discipline that was only formally established in the 1930s upon the formation of the Econometric Society (see Morgan 1990). Its primary goals were to quantify and to test economic theory. It was also hoped that econometric investigations would result in the discovery of "sustainable and interpretable relationships between observed economic variables" (Hendry 1995: 3). The Phillips curve is an example of such an empirically established relationship. Thus, econometrics aims to provide empirical content to abstract economic theory and to stimulate the development of new theories. This information is ultimately required to provide reliable and relevant advice to economic policy makers.

These goals have proved difficult to achieve. Tests of economic theories were frequently ambiguous (see Chapter 6) and economic data was often insufficient to distinguish between competing theories. The quantity of economic data is limited because it is generally impossible to generate new data by experimentation. Furthermore, the forecasting performance of econometric models was found to be poor relative to simpler time-series models (see Nelson 1972). Econometric models also proved to be vulnerable to regime shifts caused by, amongst other things, changes in government policy, legal changes, and technological innovations. For example, the predictions of econometric models were found to be inadequate for a range of variables after the extreme events in the 1970s, including the oil crisis. More recently, UK macroeconomic forecasts failed to predict the consumer boom in the late 1980s and the depth and duration of the recession in the early 1990s. These failures have been especially damaging for econometrics because its predictions were of critical importance in these exceptional times.

These difficulties have resulted in widespread methodological disagreement on how econometric investigations should be conducted. The focus of much of this controversy has been clarifying the specific roles of economic theory and empirical evidence in econometric modelling. A purely theoretical approach restricts the role of econometrics to measuring and testing prior theory and thus relies on the existence of a detailed theoretical framework. A data-directed approach uses data instigated theories and thus relies on the specific characteristics of the data sample used. Data-directed models do not distinguish between representative features and accidental or transient features and thus are often unable to produce reliable predictions. This methodological debate has reformed econometric practice and it has renewed the optimism that econometrics can achieve its goals (see Hendry and Richard 1982). These methodological issues have been broadly considered in Chapter 5. This chapter examines these issues in more detail and considers their relevance to actuarial economic model building.

Section 7.2 discusses the traditional theory-directed approach to econometrics. The fundamental problem with this approach is that a sufficiently detailed theoretical framework is generally non-existent. The vector autoregressive approach responds to this problem by largely ignoring economic theory and it uses the probabilistic structure of the data to formulate models. This approach and its weaknesses are considered in Section 7.3. The general-to-specific approach is then described in Section 7.4. This approach attempts to unify the above approaches and in many respects it provides the most pragmatic methodology for developing econometric models. Section 7.5 considers various criteria by which actuarial economic models should be assessed and briefly reviews the available actuarial models. These criteria are then used to conduct a detailed review of Wilkie's (1995b) model in Chapter 8. Wilkie's model is considered in detail because it appears to have become the orthodox UK actuarial stochastic asset model (see Chapter 3). Section 7.6 concludes.

7.2 Theory-Directed Econometrics

7.2.1 The textbook approach

The Duhem-Quine thesis, the inexact nature of economic theories, and the inability to experiment present significant practical problems for empirical investigations in economics (see Chapters 5 and 6). Theories can only be tested holistically so that empirical test results reflect the adequacy of the combination of fundamental economic theories and numerous auxiliary hypotheses. Moreover, the phenomena described by the auxiliary hypotheses generally have a substantial effect on tests and these effects cannot be marginalised by experimentation. As the auxiliary hypotheses are also often unreliable, it is difficult to learn from economic data. Important auxiliary hypotheses in many economic and finance applications relate to the time-series and probabilistic characteristics of the data. Economic theory is rarely precise enough to determine these features. It is generally only able to specify the signs of some parameter values and whether certain variables should be significant. Hence, the problem facing the econometrician is to try to translate abstract theoretical models into well-defined statistical models about observable phenomena.

One response to this problem, termed the 'textbook' approach by Spanos (1986, 1988), starts by formulating an initial model comprised of the relevant theoretical relationships and relatively simple auxiliary hypotheses. This is illustrated by the following simplified example. If a short-term interest rate model is required that is based on a version of the Fisher relation (see Appendix 2A), then the initial model may be given by:

$$\delta_m(t) = \mu_m + \omega_m \cdot r_q(t) + \varepsilon_m(t) \tag{7.2.1}$$

where $\delta_m(t)$ is the force of interest on short-term fixed-interest securities at time t and $r_q(t)$ is the force of price inflation in year t. The deterministic part of equation 7.2.1, which is made up of the parameters μ_m and ω_m, aims to describe the important systematic features of the behaviour of short-term interest rates. The error terms $\varepsilon_m(t)$ are commonly assumed to be independent normal random variables. These error terms approximate the unknown dynamics, including: the effects of omitted variables, errors in the functional form selected, and a genuine stochastic element. This stochastic element describes phenomena whose outcomes cannot be predetermined, which may include certain actions by individuals if it is believed that human agents are able to make real choices (see Section 5.5).

The next stage of the 'textbook' approach is to test the empirical adequacy of the initial model. These tests attempt to establish the reliability of the auxiliary hypotheses so that the subsequent tests of the relevant economic theory can be meaningfully assessed. It is generally not appropriate to base inferences on inadequate models (McAleer *et al.*

1985). This practice also accords with the view that all aspects of a theory should be tested (see Section 5.4.1). If the auxiliary hypotheses are found to be inadequate then the error term is respecified until an adequate model is obtained. If more than one adequate model is found then a criterion such as the Akaike Information Criterion is used to select the best fitting model.

Only once the 'best' adequate model has been established can the prior theoretical restrictions be assessed. For example, a restriction such as $\omega_m \approx 1$ may be tested in equation 7.2.1. If the theoretical restrictions are accepted then the model is recommended for use in various applications including providing advice to policy makers. If the theoretical restrictions are rejected then an explanation for the rejection is sought. This frequently entails questioning the validity of the auxiliary hypotheses, which can be rationalised by a version of the 'weak-link principle' (see Section 5.6.2). Whereas the theoretical relationships are derived from established basic postulates, the auxiliary hypotheses are merely derived from a trial and error respecification process. Hence, alternative auxiliary hypotheses may be tested until a 'satisfactory' explanation of the relevant relationship is found. The process of respecifying the auxiliary hypotheses, known as specification searching or data mining (Leamer 1978), may involve fitting a large number of models each having a different combination of possibly relevant variables. Although the above description is to some extent an exaggeration of actual econometric practice, it provides a useful caricature for discussing the limitations of econometrics.

7.2.2 The limitations of econometrics

A potential problem with using econometric models to assess the effects of policy changes is described by the Lucas critique. Lucas (1976) argued that agents' behaviour is dependent on the economic environment and, as a result, agents change their behaviour in response to regime shifts, such as economic policy changes. These changes may invalidate the econometric model that was used to formulate the policy change unless the model accounts for the behavioural plans of agents, including their expectations. This illustrates the need to test the robustness of models to policy changes and other regime shifts (see Section 6.3.3). Models that are significantly influenced by policy changes are unlikely to be of value for predictive purposes unless it is possible to predict future policy changes and their effects. The Lucas critique also motivated economists to explicitly consider the expectations of agents, which has led to the development of rational expectations theory (see Section 2.4.1). For instance, in the above simplified example the correct specification of the short-term interest rate model may be given by:

$$\delta_{\mathrm{m}}(t) = \mu_{\mathrm{m}} + \omega_{\mathrm{m}} \cdot \mathrm{E}_t[r_{\mathrm{q}}(t+1)] + \varepsilon_{\mathrm{m}}(t) \tag{7.2.2}$$

$$r_{\mathrm{q}}(t) = \mu_{\mathrm{q}} + \alpha_{\mathrm{q}} \cdot r_{\mathrm{q}}(t-1) + \varepsilon_{\mathrm{q}}(t) \tag{7.2.3}$$

Hence, assuming rational expectations, equation 7.2.2 can be re-expressed as follows:

$$\delta_{\mathrm{m}}(t) = (\mu_{\mathrm{m}} + \omega_{\mathrm{m}} \cdot \mu_{\mathrm{q}}) + \omega_{\mathrm{m}} \cdot \alpha_{\mathrm{q}} \cdot r_{\mathrm{q}}(t) + \varepsilon'_{\mathrm{m}}(t) \tag{7.2.4}$$

Equation 7.2.4 suggests that the theoretical restriction $\omega_{\mathrm{m}} \approx 1$ in the original model specification (equation 7.2.1) may be inappropriate. This illustrates that considerable caution is required when interpreting models and imposing theoretical restrictions.

The curse of dimensionality poses another problem for econometric modelling. The economy is influenced by an extremely large number of interdependent variables but, due to identification and measurement problems, econometric models are only able to incorporate a subset of all the potentially relevant variables. No matter how many variables are included in a model there are always others that are potentially more important (Black 1986). The importance of this was noted by Hendry (1995: 353): "the most likely cause of predictive failure in applied research is a change in the data properties of a relevant, but omitted, variable." Only considering a subset of the potentially relevant variables also makes it difficult to interpret empirical evidence if the variables are collinear (see Leamer 1983). For example, if short-term interest rates are influenced by both inflation rates and an exchange rate denoted $r_{\mathrm{x}}(t)$ and the exchange rate is influenced by inflation rates such that the following models hold:

$$\delta_{\mathrm{m}}(t) = \mu_{\mathrm{m}} + \omega_{\mathrm{m}} \cdot r_{\mathrm{q}}(t) + \phi_{\mathrm{m}} \cdot r_{\mathrm{x}}(t) + \varepsilon_{\mathrm{m}}(t) \tag{7.2.5}$$

$$r_{\mathrm{x}}(t) = \mu_{\mathrm{x}} + \omega_{\mathrm{x}} \cdot r_{\mathrm{q}}(t) + \varepsilon_{\mathrm{x}}(t) \tag{7.2.6}$$

Then, if exchange rates are omitted from equation 7.2.5 and equation 7.2.1 is estimated, the following relationship will be estimated:

$$\delta_{m}(t) = (\mu_{m} + \phi_{m} \cdot \mu_{x}) + (\omega_{m} + \phi_{m} \cdot \omega_{x}) \cdot r_{q}(t) + \varepsilon'_{m}(t) \qquad (7.2.7)$$

This illustrates how parameter estimates can be biased and interpretations misled if relevant variables are excluded. However, data shortages make it impossible to include all the relevant variables in econometric models.

The above schematic examples illustrate some of the difficulties with interpreting econometric models and they suggest that a dogmatic enforcement of certain a priori theoretical restrictions is unreasonable. Econometricians need to carefully assess test failures using a sophisticated theoretical framework. In the above examples, knowledge of rational expectations theory and of the omitted variables is essential to correctly interpret the fitted parameter values of equation 7.2.1. However, the current theoretical framework is incomplete and the limited available data can often support a number of possible interpretations. Consequently, a wide ranging specification search may be conducted before the final model is reported.

The problem with specification searches is that they invalidate traditional statistical inference because traditional inference assumes that the statistical model is known beforehand (see Draper 1995; Chatfield 1995). Thus, Leamer (1983: 38) stated:

The concepts of unbiasedness, consistency, efficiency, maximum-likelihood estimation, in fact, all the concepts of traditional theory, utterly lose their meaning by the time an applied researcher pulls from the bramble of computer output the one thorn of a model he likes best, the one he chooses to portray as a rose.

When a search is conducted, part of the data is 'spent' on specifying the model and cannot be used to legitimately estimate the model's parameters or to test the model. This practice results in biased parameter estimates, termed model selection biases, and usually in overconfident assessments of the model's suitability or fit, known as the optimism principle (Chatfield 1995). The greater the range of the search, the greater the degree of optimism (see Steerneman and Rorijs 1986). Hence, this problem is especially relevant when the potential number of models considered is large. For example, Ford *et al.* (1980: 136) claimed that over 400 models were considered before a final equity model was selected. Wilkie (1986a: 345) stated: "The particular models have been chosen after consideration of a great variety of alternatives." The framework used to develop Harris' (1995b) ERCH model (see Section 3.3.3) could potentially accommodate 70 variables in the vector $\Psi(t)$ if all products of variables, all error terms, and all lags of up to two periods are considered. This model would have 490 potential parameters in the **A** matrix alone. If all models with say 18 out of the 490 parameters were considered, then this would involve fitting 3×10^{32} different models. Although Harris (1995b) did not consider such a large number of models, this example suggests the magnitude of the potential problem with specification searches. The importance of this problem is further emphasised by Leamer's (1978: 13) comment that it has given rise to "a growing cynicism amongst economists toward empirical work."

The optimism principle is related to the effects of testing multiple hypotheses (see Savin 1984). When multiple hypotheses are tested, the overall level of significance is greater than the levels of significance used in the individual tests. An upper bound on the overall level of significance is: $n \cdot p$, where n represents the number of tests and p represents the individual level of significance. If the tests are independent then the overall level of significance is: $1 - (1 - p)^n$. This suggests that a lower level of significance should be used when multiple tests are conducted.

Specification searches can also be problematic if the statistical test results are used to suggest an alternative model. An example of this is the practice of 'correcting' residual autocorrelation (see Hendry 1995). This occurs when diagnostic tests reveal the presence of autocorrelation in the error terms and the response is, as recommended by Wilkie (1995b: 926), to add appropriately lagged variables to eliminate this autocorrelation (see Section 3.2.2). The problem with this response is that it is not obvious whether it leads to an improved model or simply conceals the real problems. For example, if the following model is the true model for some variable $X(t)$ (for $0 < t < T$):

$$X(t) = \mu(t) + \sigma \cdot \varepsilon(t) \tag{7.2.8}$$

where:

$$\mu(t) = \begin{cases} \mu_1 & \text{for} \quad 0 < t \leq a \\ \mu_2 & \text{for} \quad a < t \leq T \end{cases} \tag{7.2.9}$$

And if the following, constant mean, model is estimated: $X(t) = \mu + \sigma \cdot \varepsilon(t)$, then the autocovariance of its residuals is given by (see Hendry 1995: 574):

$$\begin{aligned}
\operatorname{cov}(\varepsilon(t), \varepsilon(t-1)) &= \left(\frac{1}{T \cdot \sigma^2}\right) \cdot \mathrm{E}\left[\sum_{k=1}^{T} (X(t) - m_1) \cdot (X(t-1) - m_2)\right] \\
&\approx \left(\frac{1}{T \cdot \sigma^2}\right) \cdot \mathrm{E}\left[\sum_{k=1}^{T} (X(t) - m_1) \cdot (X(t-1) - m_1)\right] \\
&= \left(\frac{a \cdot (T - a - 1)}{T^2}\right) \cdot \left(\frac{\mu_1 - \mu_2}{\sigma}\right)^2 \\
&> 0
\end{aligned} \tag{7.2.10}$$

where:

$$m_1 = \frac{1}{T} \cdot \mathrm{E}\left[\sum_{k=1}^{T} X(t)\right] = \frac{1}{T} \cdot (a \cdot \mu_1 + (T - a) \cdot \mu_2) \tag{7.2.11}$$

$$m_2 = \frac{1}{T} \cdot \mathrm{E}\left[\sum_{k=0}^{T-1} X(t)\right] = \frac{1}{T} \cdot ((a + 1) \cdot \mu_1 + (T - a - 1) \cdot \mu_2) \tag{7.2.12}$$

Therefore, an unmodelled change in the mean results in positive autocorrelation. It is not appropriate to 'correct' this autocorrelation by fitting an autoregressive model.

In most circumstances, statistical tests only determine whether there is sufficient evidence to reject the null hypothesis; rejecting the null hypothesis does not necessarily imply that the alternative is appropriate. A misspecified model is likely to fail a number of diagnostic tests and the failure of any one test generally invalidates an elementary interpretation of other tests. This is an illustration of the Duhem-Quine thesis (see Section 5.2.1). Therefore, test failures merely indicate the presence of a problem and individual test results should generally not be used to recommend a specific alternative course of action. These considerations emphasise the potential problems associated with starting from a simple model and generalising it on the basis of statistical test results. Generalising an inadequate model could be used as an immunising stratagem because it reduces the precision of the model, which makes it more difficult to falsify (see Borland 1989).

Furthermore, models can be, either deliberately or inadvertently, designed to satisfy most diagnostic tests (see Hendry 1995: 554). Hence, it is important to distinguish between statistical tests employed as design or specification criteria and genuine misspecification tests, such as tests conducted on data that was not available when the model was fitted (Mizon 1977). It is meaningless to test a model using the same criteria that were used to design the model; "the good fit of a best fitting model should not be surprising!" (Chatfield 1995: 427).

A further technical problem with the 'textbook' approach is that statistical tests are generally conducted using a fixed level of significance (see Leamer 1978). However, as models are inevitably false in some respect, they will be rejected at any fixed significance level if the sample size is sufficiently large. Thus, test results may be more of a reflection of the sample size used than of the adequacy of the model. This suggests that the significance level used should be a decreasing function of the sample size. Hendry (1995: 490) recommends significance levels of approximately: 10%, 5%, 2.5%, 1%, and 0.1% for samples of sizes: 20, 50, 100, 350, and 2000 respectively.

This section has identified some of the potential weaknesses with the 'textbook' approach and econometric investigations in general. In particular, econometric models can only properly assessed if they are tested against 'new' data. But, the time-series nature of most economic data implies that this information is not readily available and it cannot be manufactured by experimentation. The following sections consider various methods that attempt to overcome these difficulties.

7.2.3 Falsificationist econometrics

Darnell and Evans (1990) argued that the fundamental problem with the 'textbook' approach is that it does not attempt to falsify economic theories. The 'textbook' approach is primarily concerned with quantifying economic theories rather than falsifying them. This attitude is considered to be dogmatic because it assumes that the theory is valid, but in practice all theories are fallible. It may discourage the development of improved theories. Moreover, Darnell and Evans were strongly opposed to the practice of using the probabilistic structure of the historical data to specify models. This practice does not attempt to falsify a theory and the resulting model is merely a pure inductively based generalisation. However, inductive generalisations are weak because they are specific to a particular time and location. The problem of induction demonstrates that there is no demonstrative argument for believing that these generalisations will apply at any other time or location: "we have no idea whether we may expect such stability in the future, nor of what might cause it to disappear" (Darnell and Evans 1990: 86. See Section 5.2.1). Furthermore, these inductive generalisations generally cannot be harshly tested in a range of conditions because of the limited availability of economic data. Darnell and Evans suggested a number of relatively minor changes to the 'textbook' approach to place econometrics within a falsificationist framework.

The falsificationist approach restricts the function of econometrics to solely testing economic theories. This is accomplished by firstly stating a main hypothesis and a set of auxiliary hypotheses, which together produce a refutable prediction. The falsificationist approach emphasises that all the auxiliary hypotheses must be explicitly stated. This draws attention to all the assumptions involved in the model so that it can be comprehensively assessed. These assumptions include, amongst others, that the omitted variables do not significantly affect the model and that the chosen functional form is adequate. As in the 'textbook' approach, the next stage is to pre-test the auxiliary hypotheses to determine whether the particular algebraic representation chosen is appropriate. In particular, the auxiliary hypothesis that the error term can be approximated by a white noise stochastic process is tested. If the chosen representation is found to be suitable then the main hypothesis is tested and if it is corroborated then the model is accepted as an appropriate working hypothesis.

The falsificationist and the 'textbook' approaches differ in their responses to test failures. Both approaches accept that the main hypothesis can only be properly tested if the model, represented by the auxiliary hypotheses, is adequate and both approaches allow the auxiliary hypotheses to be respecified in response to test failures. However, the falsificationist approach limits the number of legitimate respecifications and requires that each attempted alteration be explicitly and publicly stated. Furthermore, the falsificationist approach does not permit the statistical test results to influence the nature of the alterations: "Statistical considerations are used to identify the *need* for re-specification, but economic theory dictates the *direction* of re-specification; thus it is economic theory which is the driving force at each stage of re-specification" (Darnell and Evans 1990: 69). These methodological rules attempt to limit the problems associated with specification searches by ensuring that these searches are purposeful rather than based on an ad hoc trial and error process.

A statistical problem with the theory-directed approach, which was identified by Darnell and Evans is associated with the sequential ordering of the tests, known as pre-test bias or model uncertainty. In testing the main hypothesis, it is implicitly assumed that the pre-tests on the model formulation were not falsely accepted. As it is possible that these tests may have been wrongly accepted, the statistical tests on the main hypothesis will be biased. Pre-test bias can only be quantified if the power of statistical tests can be quantified. As this is generally an intractable problem, Darnell and Evans suggest that pre-test bias can only be informally taken into account. This suggestion implies that the standard interpretations of statistical tests are not strictly appropriate. They propose that the methodological norm, that pre-test bias is ignored, should be adopted to make their falsificationist methodology practicable. Consequently, they warn that econometric investigations should be interpreted with caution rather than with confidence.

The fundamental difficulty with implementing the above falsificationist approach is related to the rule that all respecifications are determined using only theoretical considerations. This makes it currently impossible to develop models because a sufficiently detailed theoretical framework does not exist. To this Darnell and Evans (1990: 92) responded that: "Economists must attempt to work their way forward to the point where economic theory *does* provide information about the dynamic specification of economic behaviour." They did not suggest how this might be achieved or what should be done in the meantime. As current economic theory can support a number of different auxiliary hypotheses, the falsificationist approach does not rule out wide ranging specification searches. Furthermore, although the falsificationist approach states that only a limited number of respecifications are permissible, this rule cannot be properly enforced; particularly because failed attempts are often not reported in public (see Sterling 1959; Dawid and Dickey 1977).

Darnell and Evans' approach also inherits the more general problems associated with Popper's falsificationism (see Sections 5.2.4 and 5.3.2). In particular, it does not provide a scientific role for justification and it requires theoretical frameworks to be holistically rejected when falsifying evidence is discovered. These problems are emphasised by the need for the rather arbitrary convention that pre-test bias should be ignored. This convention appears to contradict Popper's view that hypotheses are never justified. However, falsificationism does not appear to be essential for the central elements of Darnell and Evans' methodological position, namely, that all theories need to be harshly tested and that explanations must be based on general laws. These requirements have widespread support in the philosophy of science. They suggest that economic models that are developed using data-directed specification searches are not convincing and do not provide genuine explanations. These models cannot generally be harshly tested because economic data is limited. Moreover, as they are not based on general laws, it is difficult to explain why they 'work' and what might cause them to fail. Therefore, Darnell and Evans (1990) claimed that econometrics should be limited to testing economic theory and that statistical modelling does not provide a legitimate method for overcoming the limitations of economic theory.

7.3 Vector Autoregressions

7.3.1 Unrestricted models

Sims (1980) proposed the vector autoregressive (VAR) approach towards econometric modelling, which is contrary to the approach recommended by Darnell and Evans (1990). Whilst economic theory is central to the falsificationist approach, it only has a negligible role in the VAR approach. Hence, the VAR approach has been termed 'atheoretical'. Sims (1980) argued that the methods used by economists for achieving identification in large scale macroeconomic models are often inappropriate. He stated that the practice of categorising certain variables as exogenous is generally not justified by economic theory and that theoretical restrictions can generally only be imposed in the context of a system rather than on a single equation basis. Moreover, he argued that identification was complicated by the influence of agents' expectations and policy regime changes. Sims maintained that economic variables are interdependent and they should thus all be treated as endogenous in an unrestricted symmetrical system. Hence, the VAR approach attempts to avoid restrictions based on prior economic theory and recommends that the joint temporal structure of economic variables should be analysed using VAR models.

Although the VAR approach attempts to avoid theoretical restrictions, some pragmatic restrictions are essential because of data limitations. These minimal restrictions involve determining the relevant set of variables and selecting an appropriate lag length for the model. Once these restrictions have been chosen the VAR approach proceeds by transforming the selected variables into a stationary format and then fitting the appropriate VAR model. Statistical criteria are then employed to try to simplify the model by reducing the lag length or by imposing arbitrary 'smoothness' restrictions on the parameters. The resulting model is assumed to provide a convenient summary of the data, which can be used to address various issues such as testing economic hypotheses. However, the descriptions provided by VAR models are generally difficult to interpret because they are made up of a relatively large number of correlated parameters. Thus, Sims suggested that VAR models should be analysed by studying the effects of residual shocks to the system under investigation. This is achieved by simulating the response of the modelled variables to a 'shock' of a residual of one standard deviation for each error term. The resulting functions have been termed impulse responses (see Sims 1991). Impulse response functions depend on how the error terms are transformed and Sims (1980) recommended that a triangular orthogonalisation of the residuals should be used.

The VAR approach responds to the limitations of economic theory by allowing the joint probability structure of the data to completely specify the model. This avoids the potential biases caused by the imposition of restrictions implied by inadequate theoretical hypotheses. It also capitalises on the finding that statistical time-series models frequently provide better forecasts than structural macroeconomic models (see Nelson 1972). Furthermore, the VAR approach largely avoids the problems of data-directed specification searches because it does not attempt to construct a parsimonious model that is necessarily consistent with economic theory.

However, VAR models have been found to be sensitive to the set of variables chosen and to the lag length used (see Hafer and Sheenan 1989; Pagan 1987). This raises doubts about their suitability for policy analysis or even for forecasting applications. These doubts are compounded because VAR models merely represent pure inductive generalisations (Darnell and Evans 1990). They are entirely dependent on the data sample used and may depict some accidental generalisations or time-specific rather than structural features. Further, the VAR approach provides little information on whether the system is likely to be invariant to policy changes, which is important for policy analysis. The sensitivity of the VAR approach to the set of variables modelled is also problematic because of the curse of dimensionality. The number of parameters fitted in a VAR model is relatively large and equal to: $l \cdot n^2$ where l is the lag length and n is the number of variables. This suggests that unrestricted VAR models can only be used to describe comparatively small-scale systems. However, these descriptions may be biased because relevant variables are excluded.

Additional difficulties with the VAR approach are associated with the recommended impulse response analysis (see Cooley and LeRoy 1985). Impulse response functions depend on the order in which the variables are arranged when the error terms are orthogonalised. This order suggests a causal structure in which 'shocks' to specific variables only have an immediate influence on the variables that appear lower down the order. Wilkie's (1995b) model has a similar type of triangular structure in which, for example, inflation 'shocks' influence interest rates but interest rate 'shocks' have no effect on inflation. Hence, the ordering of the variables is not inconsequential. To establish an appropriate ordering, theoretical considerations are required, which suggests that the VAR approach is unable to avoid using economic theory. Furthermore, it is difficult to meaningfully interpret impulse responses because orthogonalised errors are generally artificial; they have no material counterpart.

The VAR approach provides a method for developing economic models that largely avoids theoretical restrictions. These models represent summaries of the historical data that can be profitably used in some forecasting and testing applications. However, the justification for individual VAR models is relatively weak because they merely depict a specific data set. No further information is available on whether they are likely to be robust to extensions of the data set. This information can usually only be obtained if a model is embedded in a theoretical framework. Theoretical frameworks provide additional support for models because they are built up from established basic postulates that can usually be more widely tested (see Section 5.6). Furthermore, theoretical frameworks guide research activities (see Chapter 6). If a VAR model is found to be empirically inadequate, then no further analysis is generally possible using the 'atheoretical' VAR approach. Otherwise the VAR approach is likely to degenerate into mindless data mining. However, VAR models can play an important role in the development of theories if they are interpreted as hypotheses that require further explanation (Cooley and LeRoy 1985). They enable researchers to learn from the data and stimulate the development of new hypotheses. Whether this is the most efficient method of gaining knowledge is an open question.

Hall (1995) surveyed the theoretical developments in macroeconomics since the publication of Sims (1980) and argued that many of the difficulties with theoretical macroeconomics that were identified by Sims had been largely overcome. Hall (1995: 975) concluded that a theory-directed approach "remains the most promising approach to understanding macroeconomic behaviour generally and is the most likely approach to provide a really powerful policy tool." However, he acknowledged that the current theoretical framework is still not complete and, in particular, "a very serious outstanding question facing the learning approach is how to choose an appropriate expectations rule" (Hall 1995: 980). Hall argued that a best practice was emerging that combined the 'atheoretical' VAR approach with a theory-directed approach. The resulting approach is generally known as general-to-specific modelling, which is considered in Section 7.4.

7.3.2 Bayesian methods

Unrestricted VAR models tend to have a large number of parameters relative to the available data. This makes it likely that these models will overfit the data by describing some accidental, rather than structural, features. The problems of overfitting have been evidenced by the relatively poor forecasts, especially over longer time horizons, produced by unrestricted models (Todd 1984). Theory-directed approaches manage to limit overfitting by excluding variables that seem, on the basis of prior theory, to be relatively insignificant. However, the approach of using theoretical restrictions and subjective adjustments is regarded by Bayesians as too informal and, as a result, difficult to document and evaluate. An alternative method of using prior information to improve the forecasts of VAR models, which was suggested by Sims (1980), is known as the

Bayesian vector autoregressive (BVAR) approach. The BVAR approach uses Bayes theory (see Section 5.2.3) to explicitly combine prior beliefs with data evidence and thereby imitate the process of learning from the data.

An important practical difficulty with implementing Bayesian methods is that they require prior beliefs to be formulated in terms of multivariate statistical distributions. But researchers frequently do not have sufficiently well defined priors to completely specify these distributions. A method of dealing with this difficulty is to use the Minnesota prior (see Todd 1984). The main features of the Minnesota prior are that it represents a random walk hypothesis and that the prior variances of the lagged coefficients are scaled so that they decrease with increasing lag length. This exploits the finding that the random walk model often provides a reasonable approximation to many economic variables. It also reflects the view that the degree of confidence that lagged coefficients are insignificant increases with increasing lag length.

More fundamental difficulties with Bayesian methods are briefly discussed in Section 5.2.3. In particular, Bayesian methods assume that an individual's background beliefs are not affected by the test result and that old evidence is worthless. These difficulties are especially relevant to economic modelling because of the time-series nature of economic data. Completely 'new' evidence takes time to accumulate and if 'old' evidence is used then it is virtually impossible to separate genuine prior beliefs from data evidence.

Furthermore, Bayesian models are only relevant to individuals with similar prior beliefs, which emphasises the subjective nature of Bayesian methods. However, classical statistical methods are also unable to avoid subjective prior beliefs. For example, Neyman-Pearson hypothesis tests tend to be biased towards accepting the null hypothesis. All scientific methods are to some extent subjective (see Chapter 5). To limit the influence of subjective beliefs, Leamer (1978) argued that researchers should assess the robustness of inferences to alternative priors. If the adequacy of specific hypotheses are found to be sensitive to prior beliefs then Leamer suggested that it should be concluded that any inference is too fragile to be believed. Although there are difficulties with Leamer's position (see McAleer *et al.* 1985), it seems to be generally appropriate to assess the sensitivity of inferences to 'doubtful' features, whether classical or Bayesian methods are used (Pagan 1987). Leamer (1985) recommended that sensitivity analyses should be properly organised and should be an essential part of any empirical study. Sensitivity analyses across competing models can also be used to assess model uncertainty (see Draper 1995).

Bayesian methods provide an objective means of accommodating subjective prior beliefs in economic models. However, there are substantial practical problems with implementing them. BVAR models are also similar to unrestricted VAR models in that they are generally difficult to interpret. As a result of these difficulties comparatively few models have been developed using Bayesian methods (Bunn and Wright 1991: 510). Nevertheless, if researchers have clearly defined priors (see Wilkie 1995b: 779), then it seems preferable that the influence of these prior beliefs should be formally allowed for using Bayesian methods. Subjective adjustments to models developed using classical techniques are difficult to justify.

7.4 General-to-Specific Modelling

The forecasting performance of statistical time-series models, such as ARIMA and VAR models, relative to theoretical models suggested a significant weakness in the theory-directed approach. Theoretical considerations are generally insufficient to fully specify the dynamic structure of models, but statistical considerations suggest that these features are important. Moreover, allowing for dynamic features using specification searches is potentially problematic and difficult to justify. A fully specified theoretical approach is a desirable long term goal, but it is currently impracticable. A more relevant issue is whether useful models can be developed from partial knowledge. An influential approach towards addressing this issue has been advocated by, amongst others, Hendry (1995), which has been termed general-to-specific modelling. This approach aims to produce models that are statistically adequate and interpretable. This is achieved by combining aspects of both the theory-directed approach and the classical, rather than Bayesian, statistical approaches. The general-to-specific approach has been developed in numerous publications, including: Mizon (1977), Hendry (1979, 1980), Hendry and

Richard (1982, 1983), Spanos (1986, 1988), and Hendry and Doornik (1994). Gilbert (1986) provides a good description of this methodological view.

The concept of a data generating process is central to the general-to-specific approach. The data generating process is the actual underlying joint probability density function of all the observable economic variables over the sample interval and is given by:

$$D_x[x(1),...,x(T)|X(0);\theta] = \prod_{t=1}^{T} D_x[x(t)|X(t-1);\theta] \tag{7.4.1}$$

where $x(t)$ is the vector of all variables at time t, θ is a vector of parameters, T is the sample size, $D_x[\cdot]$ represents the joint density function of the variables $x(\cdot)$, and $X(t) = \{...x(-r),..., x(t)\}$ is a vector of initial conditions with an unspecified number of values.

The aim of econometrics is to approximate the data generating process. As a result of the curse of dimensionality and the limitations of economic theory and data, this goal is pursued by studying simplifications of the complete data generating process. This is achieved by marginalising the joint density with respect to insignificant variables and conditioning the endogenous variables on exogenous variables. An appropriate representation of the conditioned marginalised data generating process is then sought and fitted to the observed data. The joint density function of this restricted system is given by:

$$\prod_{t=1}^{T} D_y[y(t)|Y(t-1),Z(t);\theta_2] \tag{7.4.2}$$

where:

$$\begin{aligned} D_x[x(t)|X(t-1);\theta] = {} & D_w[w(t)|X(t-1),y(t),z(t);\theta_1] \\ & \cdot D_y[y(t)|Y(t-1),Z(t);\theta_2] \cdot D_z[z(t)|Y(t-1),Z(t-1);\theta_3] \end{aligned} \tag{7.4.3}$$

And $w(\cdot)$ represents the variables that are irrelevant to the variables of interest $y(\cdot)$, $z(\cdot)$ represents the weakly exogenous variables.

It is not possible to establish whether the conditioning in equation 7.4.3 is valid because the data generating process is unknown. The appropriateness of this reduction can only be broadly assessed if an empirically adequate and sufficiently comprehensive statistical model is available. Furthermore, it is efficient to initially consider more general models because they can be used to determine whether an entire class of sub-models is likely to be adequate. This limits the need for intensive specification searches. For these reasons, the general-to-specific approach starts by specifying an intentionally overparameterised general model. This stage is similar to the unrestricted VAR approach. Economic theory is only broadly used to select the functional form and the variables that appear to be appropriate for the application under consideration. The initial model should not usually be constrained to only representing one specific theory, it should be general enough to accommodate most competing theories. Vector autoregressions, or autoregressive distributed lag models, are usually chosen as the functional form because most linear models are special cases of VAR models and because they are relatively easy to estimate and comprehend. However, the general-to-specific approach does not exclude non-linear functional forms. ARIMA models are excluded from this approach because general ARIMA models are likely to contain redundant parameters. Hence, Box and Jenkins (1970) suggested that ARIMA models should be developed by starting with a parsimonious model and using a data-directed specification search to deal with potential inadequacies (see Wilkie 1995b: 926). Excluding moving average terms is unlikely to cause major problems, but it may result in additional autoregressive parameters (Hendry 1995: 565).

Once the general model has been formulated, it is fitted to the data and then tested. These tests attempt to establish whether the model adequately approximates the data generating process. The general-to-specific approach emphasises the need for harsh tests of all aspects of models. The residuals are tested for independence using a range of tests (see Doornik and Hendry 1994) and normality, if a normal distribution was used to formulate the model. These tests determine whether the model is data coherent. The transformations are assessed to determine whether they are able to produce inadmissible values, such as negative interest rates, or whether the model is data admissible. The constancy of the model's parameters is also tested and a subset of data may be set aside for this purpose. If the model fails to satisfy any of the above criteria then the general model is completely respecified and the testing process repeated. These respecifications should not be ad hoc because all the possibly relevant variables should have been included in the general model. If the model satisfies all the above criteria then it is interpreted as an empirically adequate summary of the data. However, as in the case of unrestricted VAR models, the justification for the resulting model is relatively weak.

The second stage of the general-to-specific approach aims to simplify the statistical model to establish an interpretable empirically adequate econometric model. This stage tests prior economic theories and, if they are corroborated, imposes the relevant theoretical restrictions. These theoretical restrictions usually relate to the long term features of the model. Other 'theoretical' tests include assessing whether certain variables can be treated as exogenous. This stage also involves eliminating the 'statistical' coefficients that are insignificant to reduce the problems of overfitting.

Notable sets of simplifying tests are unit-root tests and tests of cointegration (see Banerjee *et al.* 1993). These tests are carried out to try to avoid the problems of nonsense, or spurious, regression without neglecting possible long term equilibrium relationships between the variables. Nonsense regressions arise when the residuals of a regression equation are integrated; this problem is not solved by detrending the original data. To facilitate conventional statistical analysis an attempt is usually made to transform the original variables into stationary variables. The most straightforward method of achieving this is to difference each variable until they are integrated of order zero. However, this procedure effectively destroys any evidence of long term relationships between the original variables. This can be avoided by first determining whether there exist any linear combinations of the integrated variables that are integrated of order zero. If such a linear combination is found then the variables are said to be cointegrated and the model is reparameterised to reflect these relationships. Although cointegrating vectors can be established using only statistical considerations, Hendry warned against such an approach because of data limitations (see Hargreaves 1994: 4). Cointegrating vectors should usually be formulated using theoretical considerations and then tested against the available empirical evidence.

The resulting simplified model is then retested using the criteria of: data coherency, data admissibility, and parameter constancy. Furthermore, the simplified model is required to satisfy the additional criteria of: theory consistency, robustness, parsimony, and encompassing. Hence, the model must be interpretable in terms of some economic theory. Where possible, this theory should be stated in advance to limit ex post rationalisations, which provide little additional information. This requirement is emphasised by the claim that the general-to-specific approach provides a method of establishing Lakatosian progressive research strategies (see Hendry and Richard 1982; Section 5.2.5). Progressive research strategies require the prediction of novel facts, which must be formulated beforehand using theoretical considerations. However, the general-to-specific approach appears to be only tenuously linked to Lakatos' methodology of scientific research programs because it does not require a hard-core or a protective-belt. Moreover, Lakatos only appraised theories on their excess content, whereas the general-to-specific approach is also concerned with testing individual theories in isolation.

The criterion of robustness requires that the model's coefficients should preferably be near-orthogonal so that they can be individually interpreted. The criterion of parsimony requires that all the insignificant coefficients should be eliminated. This increases the precision of the model so that it can be more harshly tested in future. Parsimonious models are also easier to comprehend. The final model should also encompass competing models by providing more accurate forecasts and by being able to explain the coefficients of rival specifications. The criterion of encompassing is consistent with Lakatos' requirement that theories should be judged on the basis of their excess content (see Section 5.2.5). This limits the proliferation of models and ensures that knowledge accumulates in an orderly manner. If the model satisfies all these criteria then it is considered to be a tentatively adequate econometric model. Hendry (1995: 546) summarised the criteria for progress within the general-to-specific approach as:

> When all results are encompassed by a parsimonious, data coherent, and interpretable empirical model which has constant parameters historically, then that model constitutes a useful addition to empirical understanding of the economic mechanism under investigation, as part of a progressive accumulation of knowledge. The cycle is completed by testing the model against new data, and consolidating the resulting knowledge in a theoretical framework which also accounts for other phenomena of interest.

The general-to-specific approach can be interpreted as a pragmatic response to the difficulties associated with the theory-directed and VAR approaches. The major weakness of the theory-directed approach is the need for substantial auxiliary hypotheses. This is dealt with by initially establishing an adequate statistical model that provides the necessary framework for testing the relevant theories. An important limitation of the VAR approach is that the resulting models are difficult to interpret and they are only supported by limited inductive evidence. This weakness is addressed in the general-to-specific approach by requiring that models must be consistent with prior economic theory. However, neither the theory-directed approach nor the VAR approach rule out the general-to-specific approach. The theory-directed approach merely emphasises the need for a more detailed theoretical framework to guide research and to provide fundamental support for theories. The VAR approach does not exclude the imposition of theoretical restrictions to improve the efficiency of models (Sims 1980); it represents the sceptical view that these restrictions are rarely adequate. In this respect, the general-to-specific approach provides an optimistic account of how econometric models should develop over time.

The main problem with the general-to-specific approach is the ability to establish a sufficiently universal initial model because of data limitations and the curse of dimensionality. Hence, most applications of general-to-specific modelling have only been concerned with relatively few variables and relatively simple functional forms, such as VAR models. There is usually insufficient data to fit more general initial models, such as threshold models (Tong 1990) or state-dependent models (Priestly 1980). This limitation suggests that the initial models may not be able to be formulated so that they accommodate all the possibly relevant prior theoretical information. If this is the case, then the general-to-specific approach is unable to prevent wide ranging specification searches if the initial model is found to be inadequate.

The process of simplifying a general model also frequently involves a specification search and, to enable the simplified models to be properly assessed, Pagan (1987) and Spanos (1986), amongst others, argued that this process should be comprehensively documented. However, Hendry and Mizon (1985) argued that this is unnecessary because the context of discovery is irrelevant to the context of justification (see Section 5.2.1). But this view appears to ignore the fact that econometric models are generally assessed and developed using the same data set. As a result of the effects of multiple hypothesis testing, all tests conducted on the same data are interdependent (Pagan

1987). The context of justification is only independent of the context of discovery if a completely new data set is available to test the model. This appears to be the justification for setting aside a subset of the data for diagnostic testing. However, if the fit of the model over the set aside data is used as a model selection criterion in a specification search, then tests carried out on this data are no longer independent. Moreover, this practice has been questioned because it results in a loss of efficiency for the fitted parameters (Roeker 1991).

Another feature of the general-to-specific approach that has been questioned is the criteria of encompassing. Although encompassing ensures that no significant loss of information occurs, it may limit theoretical progress because it tends to maintain the status quo. Hausman (1994b: 202) stated that: "modifications of theories in science usually come with some loss [of information]." According to Kuhn (1970), theories in different paradigms may be incommensurable. These considerations suggest that encompassing should not be used as an absolute criterion in all circumstances. Moreover, encompassing ensures that only one model is considered, which tends to disregard the issue of model uncertainty and rules out forecast combination (Diebold 1989). However, encompassing can be interpreted as part of a long term strategy that aims to discover the true data generating process. In this context, model uncertainty is a temporary phenomenon that is of secondary importance.

Notwithstanding these difficulties, the general-to-specific approach appears to provide the most promising, non-Bayesian, methodology for establishing econometric models. It recognises that valid inferences can only be derived from empirically adequate models by initially requiring that an appropriate statistical model be developed. Further, it recognises that statistical models tend to be inefficient and only provide weak knowledge claims by requiring that models must be interpreted. Moreover, the general-to-specific approach emphasises that models must be harshly tested and are fallible.

7.5 Actuarial Economic Model Evaluation Criteria

Chapter 5 emphasised the need for demanding tests of all aspects of a theory and this requirement was reiterated by the general-to-specific approach. In addition, Chapter 5 illustrated the need for a theoretical framework to direct research activities and to enable models to be more widely tested. Theoretical frameworks are essential for establishing scientific explanations and thereby gaining a profound understanding of the operation of the system under investigation. This information enables researchers and decision makers to make considered judgements about future events. However, as discussed in Chapter 6, financial economic theory is inexact and substantial auxiliary hypotheses are required to test them and to develop comprehensive models. These auxiliary hypotheses are generally established using various statistical, or econometric, techniques and, as a result, they only represent pure inductive inferences that are specific to a particular data set. Furthermore, as these auxiliary hypotheses are formulated and tested using the same data set and as the available data sets are small relative to the complexity of the economy, these auxiliary hypotheses usually only have weak support. This makes it difficult to decisively interpret tests of the more fundamental theories.

These considerations illustrate the difficulties with developing and evaluating actuarial economic models. Nevertheless, they stress the need for both a comprehensive theoretical framework and adequate auxiliary hypotheses. These needs are recognised by the general-to-specific approach, which appears to provide the best methodology for establishing econometric models, including actuarial models. Hence, actuarial models should be rigorously tested using a range of statistical tests and should be required to be interpreted in terms of established economic theory. In particular, actuarial models should be evaluated against the criteria of: data coherency, data admissibility, parameter constancy, theory consistency, robustness, parsimony, and encompassing. Models satisfying all these criteria would provide a valuable actuarial tool and a useful addition to our knowledge.

The econometric actuarial models discussed in Chapter 3 broadly attempt to satisfy most of the above criteria. They were mainly justified using a range of statistical test results and some attempts were made to rationalise them in terms of intuitive economic concepts. However, these models were generally not tested for parameter constancy and little attempt was made to systematically test various financial economic theories. Theoretical consistency appeared to have been generally established via ex post rationalisations rather than deliberate attempts to test specific theoretical frameworks.

Moreover, little attempt was generally made to quantify the effects of specification searches. This is important because specification searches make it difficult to assess the reported statistical test results and consequently they undermine the primary justification for these models.

Contrary to this approach, the theoretical actuarial models discussed in Chapter 4 were developed to conform to specific orthodox financial economic theories and relatively little attention was paid to establishing realistic auxiliary hypotheses. These models only attempted to satisfy the criteria of parsimony and theory consistency. Their empirical adequacy was not formally tested; they were merely calibrated to the data. Nevertheless, as discussed in Section 5.6.1, these theoretical actuarial models can be motivated by interpreting them as 'models' as opposed to 'theories'. They attempt to explore the implications of abstract concepts in order to stimulate future theoretical developments.

Although these theoretical models are clearly unrealistic, they have important pedagogic value. They provide decision makers with information about an understandable idealised environment. This establishes a basis from which they can make their own subjective adjustments if they wish. This illustrates the importance of being able to interpret actuarial economic models. Economic models cannot be thoroughly tested, especially over long time horizons because of data limitations. Consequently, no model can be confidently, or mechanically, used in actuarial applications. Hence, actuaries need a conceptual basis for evaluating the results of these applications and this can only be obtained from interpreted models. These considerations support the initial use of relatively simple economic models, such as those used by Haberman (1994) and deterministic scenarios (see Institute of Actuaries 1996: B2). These models do not produce final answers for applications; they merely assist actuaries in understanding the relevant issues.

However, data limitations are less acute for models with shorter time horizons. In these cases, it may be feasible to adopt an instrumentalist view (see Section 5.4.2) and to use purely empirical methods to develop models, such as neural networks or state-space reconstruction methods (see Weigend and Gershenfeld 1994). These methods may uncover significant short term regularities. But it is important that these regularities are eventually incorporated into a theoretical framework if they are to be demonstrably useful for actuarial applications with longer time horizons. Theoretical considerations may originate from a number of diverse sources so actuaries should not ignore models developed using purely data-directed methods.

Having several models that are developed using different techniques is also beneficial when constructing sensitivity analyses, including evaluating model uncertainty. Sensitivity analyses help to determine whether inferences are sensitive to particular doubtful assumptions and this information is essential for establishing whether models are useful. Detailed sensitivity analyses are especially important for applications using actuarial economic models because none of the currently available models clearly satisfy all the criteria for an adequate econometric model. Furthermore, model averaging, in a Bayesian context (see Draper 1995) and in a non-Bayesian context (see Clemen 1989; Palm and Zellner 1992), has been found to result in models that produce more accurate forecasts. However, it is difficult to interpret averaged forecasts and in the long term researchers should attempt to identify an encompassing interpretable model.

7.6 Summary

Econometrics potentially provides a method for establishing robust auxiliary hypotheses that enable inexact economic theories to be tested and incorporated in comprehensive models. This task has been difficult to accomplish because of data limitations and the apparent complexity of the economy. Economic phenomena are influenced by a large number of interdependent variables, which complicates the interpretation of reduced economic models. Some of these variables are also not stable over time and this instability has proved to be difficult to describe using standard mathematical models. These difficulties have frequently led researchers to fit numerous models before the final model is reported. However, these specification searches bias statistical tests of the model and generally make models appear to describe the data better than they actually do.

The general-to-specific approach attempts to limit these potential problems by initially formulating a sufficiently general statistical model and by requiring that the final simplified model be consistent with economic theory. A theoretical framework is essential for providing scientific explanations and thereby for gaining an in-depth understanding of economic phenomena. In addition, the general-to-specific approach stresses the need to test all aspects of models. This approach appears to provide the best method of establishing models as it incorporates aspects of the other main approaches, namely the theory-directed approach and the VAR approach. The major weakness of the theory-directed approach is the lack of a sufficiently detailed theoretical framework. The VAR approach only establishes statistical models, which tend to be inefficient because the available data is limited. Bayesian methods can also be used to reduce these inefficiencies, but it is usually difficult to establish appropriate, generally acceptable, prior distributions.

None of the currently available actuarial models appears to satisfy all the demanding criteria of the general-to-specific approach. This suggests that it is important for actuaries to conduct detailed sensitivity analyses to assess the robustness of their inferences. It also suggests that it is preferable to have interpreted models so that decision makers can use their judgement to allow for the deficiencies in the models. The following chapter provides a more detailed review of Wilkie's (1995b) model, which is the most widely used UK actuarial stochastic asset model.

Chapter 8

A REVIEW OF WILKIE'S MODEL

8.1 Introduction

Wilkie's (1986a, 1995b) influential stochastic asset model (see Section 3.2) does not appear to satisfy the criteria of data admissibility and theory consistency. The long-term interest rate model permits negative yields (see Section 3.2.3). Moreover, the model is inconsistent with the rational expectations hypothesis, the efficient markets hypothesis, and aspects of portfolio theory (see Section 3.2.4).

However, these theories are not proven and Wilkie (1995b) argued that his model provided a sufficiently realistic description of the long term behaviour of the relevant economic variables. This claim was supported by empirical tests showing that the model satisfied the criteria of parsimony and data coherency. The model's parameters were shown to be statistically significant and the model's residuals were shown to satisfy tests of independence and normality. However, as the model appears to have been partially developed using the Box-Jenkins methodology, it is possible that some data mining occurred (see Section 7.2.2). If this was the case, then the model's goodness-of-fit tests may have been biased. This chapter re-examines the empirical adequacy of Wilkie's model and, in particular, it considers the stability of the model's parameters over time. Tests of parameter constancy are likely to constitute genuine misspecification tests because they did not appear to have been considered when the model was developed.

Sections 8.2 to 8.5 review Wilkie's inflation, equity, interest rate, and property models. Section 8.6 concludes by discussing the overall adequacy of Wilkie's model. Appendix 8A graphically illustrates and briefly discusses the data used by Wilkie (1986a, 1995b).

8.2 Inflation Models

8.2.1 Price inflation

Wilkie (1995b: 781) reported that the mean of the residuals from his original price inflation model over the out-of-sample interval 1983-94 was not significantly different from its expected value, but that the variance of these residuals was significantly less than its expected value. In addition, Wilkie (1995b: 785) reported that the residuals from his updated original model appeared to be independent, but that they did not appear to be normally distributed. He suggested that the significantly low variance of the residuals over the out-of-sample interval and the apparent non-normality of the updated model's residuals could have been due to an unmodelled ARCH effect. Consequently, he fitted an ARCH model and indicated that it appeared to be empirically adequate.

In addition to the above tests, the parameter constancy of the original price inflation model can be examined by recursively estimating its parameters on incrementally larger data sets (see Spanos 1986; Hendry 1995). Figures 8.2.1 and 8.2.2 present recursive estimates and approximate 95% confidence intervals of α_q and μ_q, respectively, calculated using data sets from 1923 to the years on the x-axes. These graphs suggest that α_q and μ_q may not be constant over the interval 1923-94. In Figure 8.2.1, the

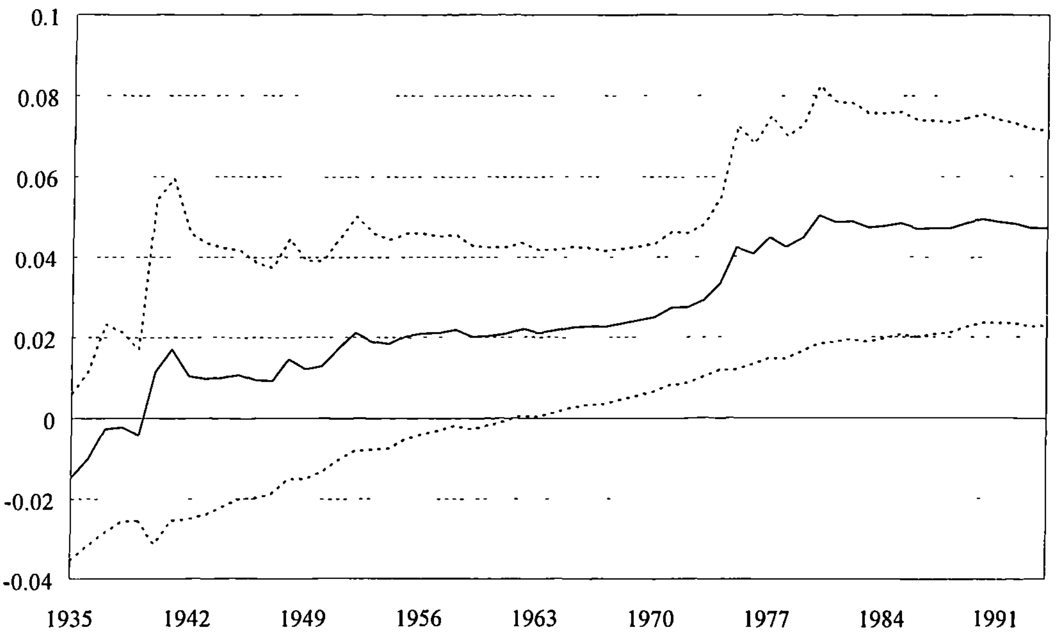

Figure 8.2.1 Recursive estimates of μ_q with approximate 95% confidence intervals, from 1923

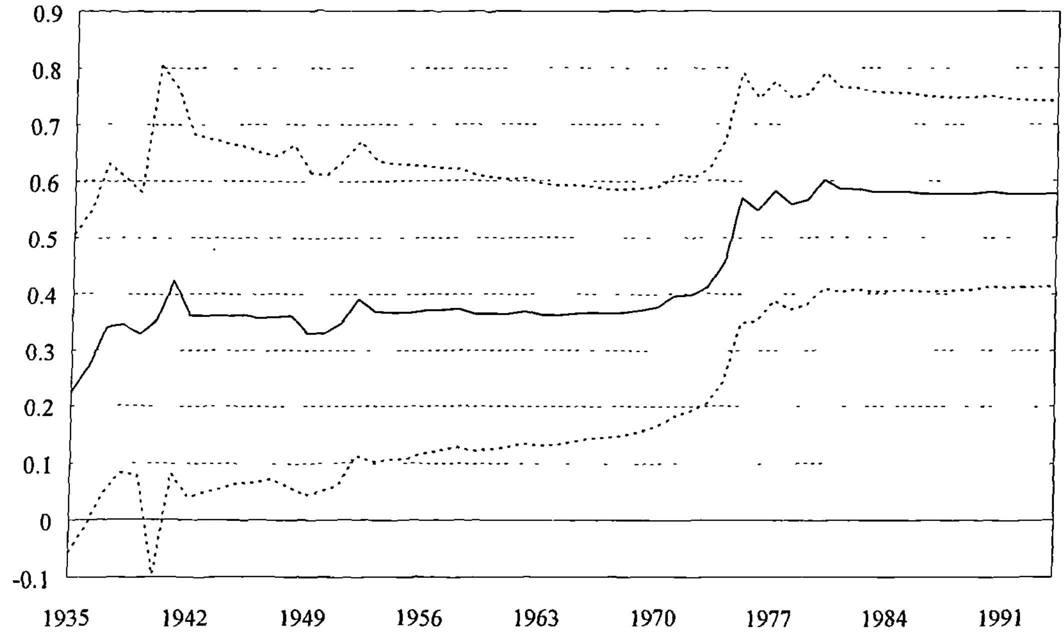

Figure 8.2.2 Recursive estimates of α_q with approximate 95% confidence intervals, from 1923

recursive estimates of μ_q tend to increase over most of the period and only become significantly different from zero after 1960. The recursive estimates of μ_q after 1980 are also significantly larger than those calculated over earlier intervals. This supports Wilkie's (1986a: 346) comment that there is "considerable uncertainty about the value to use for [μ_q]." In Figure 8.2.2, the recursive estimates of α_q jump, in the mid-1970s, from a value of approximately 0.37 to a value of approximately 0.58. This contradicts Wilkie's (1986a: 346) remark that: "There is fairly little uncertainty about the appropriate [value] for [α_q]."

The Chow test (Spanos 1986: 483-5) can be used to test whether a model's parameters are constant. This test is made up of two parts. The first part tests the null hypothesis that the variances of the residuals are equal over both sub-periods against the alternative that they are different. If the model satisfies the first test, the second part tests the null hypothesis of parameter constancy against the alternative of non-constancy. Table 8.2.1 shows the parameter estimates obtained from fitting the model over the two equally sized, and arbitrarily chosen, intervals 1923-58 and 1959-94. Over these sub-periods, the variance of the residuals appears to be unchanged ($F(34,34) = 1.17$, $p = 0.3238$), but the null hypothesis of parameter constancy is rejected at the 5% level ($F(2,68) = 3.16$, $p = 0.0487$).

Table 8.2.1 Estimated parameters for the original price inflation model

Interval	μ_q	α_q	σ_q
1923-94†	0.0473	0.5773	0.0427
	(0.0120)	(0.0798)	
1923-58	0.0218	0.3744	0.0424
	(0.0118)	(0.1230)	
1959-94	0.0674	0.6584	0.0392
	(0.0197)	(0.1304)	

†*Source:* Table 2.3 of Wilkie (1995b)

As discussed in Section 7.2.2, it is difficult to interpret these results. They may simply be due to the non-normality of the residuals or they could be due to the change in the calculation of the official UK price index (see Appendix 8A.1). Over the interval 1923-47 a cost-of-living index was calculated whereas over the interval 1947-94 a general index of retail prices was calculated (see Wilkie 1995b: 942). Alternatively, as suggested in Section 3.2.4, the parameter non-constancy could be due to changes in the mean rate of inflation (see Figure 3.2.2). Moreover, a change in the mean rate of inflation may have biased the recursive estimates of α_q and caused them to increase in the mid-1970s (see Section 7.2.2 and Figure 8.2.2).

A non-parametric test that can be used to assess whether the mean rate of inflation was not constant is the rank-sum test. This test has a broadly similar intention to the test used by Kitts (1990). If the mean rate of inflation is not constant then the sums of the ranks of the model's residuals are likely to be lower (higher) than expected over intervals where the mean is lower (higher) than average. After ranking the residuals from the model fitted over the interval 1923-94, the sums of the ranks over the intervals 1923-58 and 1959-94 are 1487 and 1141, respectively. These are marginally not significantly different from the expected sum 1314 at the 5% level ($z = 1.95$, $p = 0.0514$). This result is inconclusive, but it suggests a potential area of weakness in the price inflation model.

Wilkie's ARCH model is able to effectively deal with the problem of non-normality and heteroskedasticity. However, it is unlikely to be able to accommodate possible changes in the mean rate of inflation. This is supported by a rank-sum test. The sums of the ranks of the ARCH model's residuals over the intervals 1923-58 and 1959-94 are 1496 and

1132, respectively. These are significantly different from the expected sum 1314 at the 5% level ($z = 2.05$, $p = 0.0404$). This result suggests that the ARCH model's residuals may not be independent and identically distributed. Nevertheless, on the whole, the ARCH model appears to describe the data better than the original model. Thus, it should generally be used in applications of the model, unless the ARCH effect is not significant for those particular applications.

8.2.2 Wage inflation

Wilkie (1995b: 810) reported that the transfer function wage inflation model's residuals over the interval 1923-94 appeared to be normally distributed, but were significantly correlated with the price inflation model's residuals. Consequently, he fitted a VAR model with price and wage inflation as input variables (see Section 8.2.3). Out-of-sample residuals are not yet available for the wage inflation model because it was first reported in Wilkie (1995b).

Further tests on the wage inflation model reveal that their residuals do not appear to be independent and identically distributed. The model's residuals only have 25 runs, which is significantly low at the 5% level ($z = -2.83$, $p = 0.0046$). Moreover, the sums of the ranks of the model's residuals over the intervals 1923-58 and 1959-94 are 1518 and 1110, respectively. These are significantly different from the expected sum 1314 at the 5% level ($z = 2.30$, $p = 0.0216$). These results suggest that the transfer function wage inflation model is not empirically adequate.

8.2.3 VAR inflation model

Wilkie (1995b: 813) reported that the VAR inflation model's residuals failed tests of normality at the 5% level. No other test results were reported.

In fitting the VAR model, Wilkie did not use lagged data for the initial values as he did in fitting all the other models. The reason for this inconsistency appears to be the unusually low values of price and wage inflation in 1922 of –20% and –31% respectively. These starting values would have had an undue influence on the estimates of the model's parameter values. However, it is not possible to determine the model's residuals, and to conduct further tests, without knowing the actual starting values that were used. Hence, to further evaluate the VAR model, it has been refitted over the interval 1925-94 using lagged data for the initial values (see Table 8.2.2). These estimates are similar to Wilkie's (1995b: 813), except that the mean values are larger: Wilkie's estimates of μ_q and μ_w were 0.0359 and 0.0509 for the full model and 0.0205 and 0.0344 for the reduced model. The mean values reported in Table 8.2.2 are more consistent with the other inflation model's mean values (see Table 3.2.1). Table 8.2.2 suggests that the parameters α_q and α_{wq} are not significantly different from zero because the log likelihood of the reduced model is not significantly greater than the log likelihood of the full model ($\chi^2_2 = 2.15$, $p = 0.3405$). Hence, the reduced model appears to be the most suitable VAR model and will be evaluated further.

The reduced model's residuals satisfy the runs test (for prices $z = -0.42$, $p = 0.3362$, for wages $z = -0.91$, $p = 0.1831$). The sums of the ranks of the model's residuals over the intervals 1925-59 and 1960-94 are 1371 and 1114 for prices, and 1406 and 1079 for wages. These are not significantly different from the expected sum 1242.5 at the 5% level (for prices $z = 1.51$, $p = 0.1312$, for wages $z = 1.92$, $p = 0.0548$). Note that the

Table 8.2.2 Estimated parameters for the VAR inflation model

Parameter	1925-94	1925-94
μ_q	0.0457	0.0455
	(0.0133)	(0.0135)
α_q	0.1484	-
	(0.1682)	
α_{qw}	0.6134	0.7533
	(0.1863)	(0.0981)
μ_w	0.0609	0.0607
	(0.0146)	(0.0166)
α_w	0.5906	0.7692
	(0.1459)	(0.0770)
α_{wq}	0.1896	-
	(0.1318)	
φ_w	0.6919	0.6961
σ_q	0.0399	0.0402
σ_w	0.0313	0.0318

result for wages is only marginally not significant, which suggests that the VAR model may not have been able to fully overcome the problems associated with the other inflation models. The model's residuals fail tests of normality at the 5% level (for prices skewness = 1.52, z = 5.20, p = 0.0000 and kurtosis = 6.74, z = 6.39, p = 0.0000, for wages skewness = 1.35, z = 4.60, p = 0.0000 and kurtosis = 5.04, z = 3.48, p = 0.0005). Nevertheless, despite these results, this model appears to provide the most promising price and wage inflation models.

8.3 Equity Models

8.3.1 Share dividend yields

Wilkie (1995b: 822) reported that the dividend yield model was 'satisfactory'. Over the out-of-sample interval 1983-94, the mean and variance of the model's residuals were not significantly different from their expected values. Over the interval 1923-94, the model's residuals were found to be independent and normally distributed. All the model's parameters were found to be significant.

However, Wilkie (1984: 58) reported that: "The values of [ω_{ys}] vary considerably according to the period chosen." Over the intervals 1919-82, 1933-82, and 1946-82 Wilkie (1984) estimated ω_{ys} as 1.35, 2.41, and 1.77 respectively. Furthermore, Wilkie (1995b: 831) found that the estimates of ω_{ys} for various other countries were noticeably variable and ranged from 0.5 for the US to 1.8 for the UK. This suggests that ω_{ys} may not be constant over time.

The suitability of ω_{ys} can be examined by re-expressing the share dividend yield model as follows (for $t > 0$) and plotting the resulting regression (see Figure 8.3.1):

$$(1-\alpha_{ys}\cdot \mathrm{L})\cdot(y_s(t)-\log_e \mu_{ys}) = \omega_{ys}\cdot(1-\alpha_{ys}\cdot \mathrm{L})\cdot r_q(t)+\sigma_{ys}\cdot z_{ys}(t) \qquad (8.3.1)$$

Figure 8.3.1 illustrates the sensitivity of ω_{ys} to the years 1940 and 1974. These years correspond to the years in which the greatest increases in prices and yields occurred. If they are excluded from the regression then ω_{ys} becomes insignificantly different from

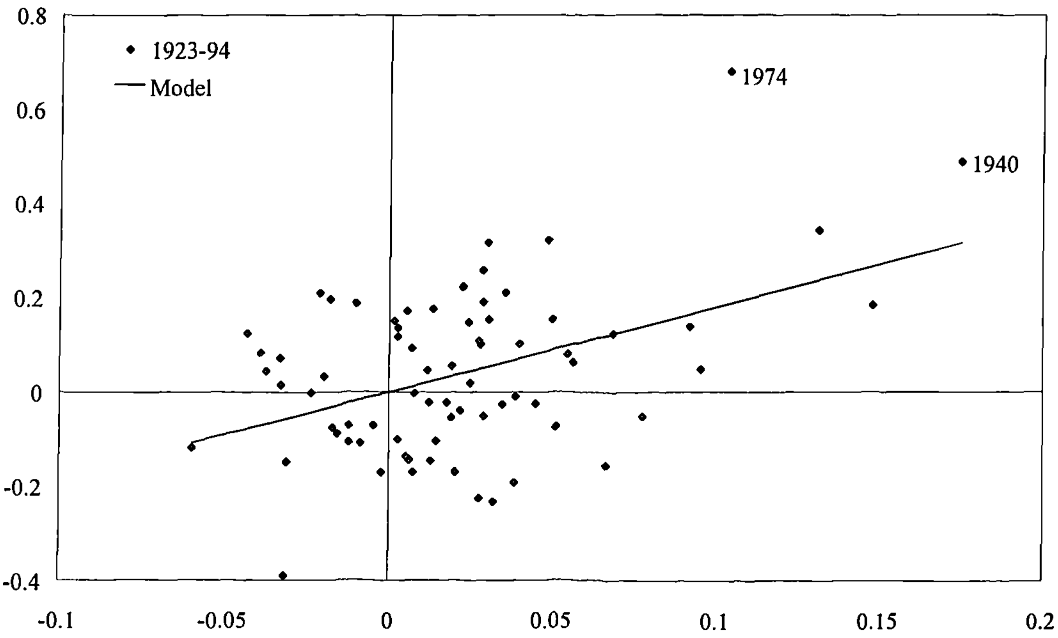

Figure 8.3.1 $(1-\alpha_{ys}\cdot L)\cdot(y_s(t)-\log_e\mu_{ys})$ plotted against $(1-\alpha_{ys}\cdot L)\cdot r_q(t)$

zero (t-value of ω_{ys} with intervention variables in 1940 and 1974 is 1.67, $p = 0.0990$). Wilkie (1995b: 822) appeared to acknowledge this finding but nevertheless concluded that ω_{ys} was justified because its estimate was significantly greater than zero. The problem with including ω_{ys} is that it results in a general tendency for changes in yields to be correlated with changes in inflation, but this correlation only seems to be appropriate for large increases in yields and inflation. Other than this possible weakness, the share dividend yield model appears to fit the data reasonably well.

8.3.2 Share dividend growth

Wilkie (1995b: 840) reported that the mean and variance of the share dividend model's residuals over the out-of-sample interval 1983-94 were not significantly different from their expected values. However, Wilkie reported that the model's residuals over the interval 1923-94 failed tests of independence and normality at the 5% level. These residuals had too many runs of the same sign, they were negatively skewed, and they were leptokurtic. Wilkie (1995b: 844) suggested that the latter results may have been due to the large falls in dividends in 1925, 1928, 1931, 1932, and 1941. Furthermore, the estimate of θ_s appeared to be insignificantly different from zero. These findings suggest that the share dividend model is not empirically adequate.

In addition to the above tests, the correlation between the out-of-sample residuals from this model and the price inflation model is 0.76, which is significant at the 5% level (standard error of the correlation coefficient ≈ 0.29). The model was also fitted over the intervals 1923-58 and 1959-94 (see Table 8.3.1). The Chow test suggests that the variance of the model's residuals is significantly lower over the latter interval ($F(31,31) = 2.52$, $p = 0.0060$). Hence, the share dividend model does not appear to have had constant parameters historically. This result may have been caused by the change in the dividend index used (see Appendix 8A.2). Wilkie (1995b: 943) mainly used the Actuaries Indices up to 1962 and the FTSEA All-Share Index thereafter. The Actuaries Indices were based on far fewer securities than the FTSEA All-Share Index, which suggests that the Actuaries Indices are likely to have been more variable than the FTSEA All-Share Index.

In addition, θ_s and β_s are far less significant over the latter interval 1959-94. The significance of θ_s can be examined by refitting the model with β_{s1} included and θ_s and β_s excluded (see Table 8.3.1). This shows that the model may have been over-parameterised because the variance of the model's residuals does not increase significantly after replacing θ_s and β_s with β_{s1} ($F(1,67) = 3.57$, $p = 0.0631$). However, this result is not decisive. Excluding θ_s and β_s results in an optimal value of β_{s1} that is less than one, which indicates that the 'unit gain' effect may not be appropriate (see Section 3.2.4.3).

Table 8.3.1 Estimated parameters for the dividend model

Interval	μ_{ds}	θ_s	β_s	β_{s1}	ω_{ds}	ϕ_{ds}	σ_{ds}
1923-94†	0.0157	0.1344	0.5793	$1-\beta_s$	−0.1761	0.5733	0.0671
	(0.0124)	(0.0800)	(0.2157)		(0.0439)	(0.1295)	
1923-58	0.0001	0.1492	1.0343	$1-\beta_s$	−0.3404	0.6075	0.0751
	(0.0219)	(0.1747)	(0.4207)		(0.0927)	(0.1579)	
1959-94	0.0251	0.0818	0.3540	$1-\beta_s$	−0.1307	0.4712	0.0473
	(0.0185)	(0.2260)	(0.2542)		(0.0464)	(0.1647)	
1923-94	0.0296	-	-	0.6513	−0.1711	0.6000	0.0689
	(0.0152)			(0.1786)	(0.0433)	(0.0985)	

†*Source:* Table 5.3 of Wilkie (1995b)

8.4 Interest Rate Models

8.4.1 Long-term interest rates

Wilkie (1995b: 858) reported that the mean of the long-term interest rate model's residuals over the out-of-sample interval 1983-94 was not significantly different from its expected value, but that the variance of these residuals was highly significantly greater than its expected value. Wilkie (1995b: 861) also reported that the model's residuals over the interval 1923-94 appeared to be normally distributed and uncorrelated with one another. However, these residuals were found to be significantly correlated with the residuals from the price inflation model and the share dividend yield model. Wilkie suggested that it may be appropriate to consider alternative values of θ_b or β_b to alleviate this model's empirical problems.

Further tests emphasise the empirical inadequacy of this model. The correlation between the out-of-sample residuals from this model and the price inflation model is 0.59, which is significant at the 5% level (standard error of the correlation coefficient ≈ 0.29). The Chow test rejects the hypothesis that variances of the residuals are equal over the intervals 1923-58 and 1959-94 at the 5% level (see Table 8.4.1. $F(33,33) = 2.15$, $p = 0.0156$). Therefore, there is considerable evidence to suggest that the parameter σ_b is not constant. An important event that may have influenced this result is that during and after World War II the government set minimum prices for government fixed-interest securities.

This model can be further examined by considering the estimation procedure used by Wilkie (1984). The model's parameters were estimated by setting β_b to one, β_{b1} to zero, and θ_b to a 'plausible' value, estimating the other parameters to minimise σ_b, and repeating this process, after adjusting θ_b, until σ_b was minimised (Wilkie, 1984: 99). To check whether Wilkie's estimates are optimal and to obtain estimates of the standard errors of θ_b, β_b, and β_{b1}, the long-term interest rate model was refitted including θ_b, β_b, and β_{b1}. However, as noted by Wilkie (1984: 98), it is not possible to obtain a set of reasonable parameters for the long-term interest rate model without constraining some of the parameter values. Appropriate constraints appear to be: $\Theta_b(t) < Y_b(t)$, $0 < \theta_b \leq 1$, $\beta_b \geq 0$, and $\beta_{b1} \geq 0$. The first constraint is required because of the log transformation

Table 8.4.1 Estimated parameters for the long-term interest rate model

Interval	μ_b	α_b	θ_b	β_b	β_{b1}	φ_b	σ_b
1923-94†	0.0305	0.8974	0.0450	1.0000	-	0.3371	0.1853
	(0.0065)	(0.0442)				(0.1436)	
1923-58	0.0237	0.8918	0.0450	1.0000	-	0.1862	0.1421
	(0.0056)	(0.0742)				(0.1680)	
1959-94	0.0392	0.8200	0.0450	1.0000	-	0.4406	0.2083
	(0.0082)	(0.0870)				(0.2375)	
1923-94	0.0710	0.9650	1.0000	0.0000	0.0198	0.2205	0.0919
	(0.0249)	(0.0232)			(0.0099)	(0.0760)	
1923-94	0.0745	0.9652	-	-	-	0.1941	0.0945
	(0.0276)	(0.0236)				(0.0766)	

†*Source:* Table 6.3 of Wilkie (1995b)

used (see equation 3.2.7). The other two constraints ensure that inflation expectations are a positive sum of historical price inflation.

Table 8.4.1 shows the optimal parameter estimates obtained using these constraints: the optimal estimates of θ_b and β_b were notably 1 and 0. These results illustrate that Wilkie's estimates are not optimal because they result in a higher residual standard deviation than the alternative estimates. Table 8.4.1 also shows the parameter estimates of the model with β_{b1} excluded. This suggests that β_{b1}, and consequently the entire price inflation transfer function, is only just significant ($F(1,68) = 3.86$, $p = 0.0535$). Excluding this transfer function from the long-term interest rate model is theoretically feasible because it results in a model that is consistent with the Fisher relation and the rational expectations hypothesis (see equation 3.2.15).

In addition, Table 8.4.1 shows that the optimal estimate of α_b is 0.9650 with a standard error of 0.0232. This suggests that an integrated model could be appropriate (see Figure 8A.5). However, as stressed by Wilkie (1995b: 779) an integrated model is probably inappropriate for real rates of return.

Lastly, Wilkie (1995b: 860) noted that φ_b becomes insignificantly different from zero when an intervention variable for 1974 was included. Hence, φ_b appears to have a similar problem to ω_{ys} (see Section 8.3.1). The parameter φ_b seems to mainly describe the event that the largest increase in interest rates coincided with the largest residual from the share dividend yield model. Note that φ_b is not significant over the interval 1923-58 (see Table 8.4.1). However, if φ_b is set to zero, then Wilkie's model implies that there is no relationship between equity returns and real interest rates. As this does not appear to be a reasonable assumption, it may explain why Wilkie included φ_b in the model. Nevertheless, the relationship described by φ_b does not appear to be particularly robust.

8.4.2 Short-term interest rates

Wilkie (1995b) reported that the short-term interest rate model's residuals over the interval 1923-94 appeared to be independent and normally distributed.

These findings are confirmed by additional tests. The recursive estimates of μ_m and α_m do not change significantly over the interval 1923-94. The parameter estimates do not appear to have been significantly affected by outliers. The runs test is also satisfactory. Hence, the short-term interest rate model appears to be empirically adequate.

8.4.3 Index-linked interest rates

The index-linked yield model was only fitted over the interval 1981-94. This is insufficient data to carry out a full empirical appraisal. Consequently, this model should be used with caution in long-term studies. Nevertheless, certain tests can be conducted to obtain a broad view of the model's suitability. These tests should use a higher level of significance, 10% say, to reflect the relative shortage of data (see Section 7.2.2). Wilkie (1995b) conducted these tests and reported that the index-linked model's residuals appeared to be independent and normally distributed. Thus, based on the limited evidence available, this model appears to be satisfactory.

8.5 Property Models

8.5.1 Property income yields

Data for the property models was only available over the interval 1967-94. Hence, a limited empirical appraisal of these models can only be conducted and a higher level of significance, 10% say, should be used in empirical tests (see Section 7.2.2). Wilkie (1995b) reported that this model's residuals appeared to be independent and normally distributed.

Further evidence suggests that α_{yp} may not be constant over time. Figure 8A.8 presents the property yield data used by Wilkie (1995b) and shows that property yields changed substantially in the late 1960's and in the 1990's. As in the original price inflation model, these changes may have biased the estimate of α_{yp} (see Section 7.2.2). Over the interval 1970-90 the estimate of α_{yp} is 0.3435 with a standard error of 0.2133, which compares with Wilkie's (1995b: 877) estimate of 0.9115 with a standard error of 0.1007. However, there is insufficient data to draw any definitive conclusions and given the available evidence, Wilkie's estimates are optimal.

8.5.2 Property income growth

Wilkie (1995b) reported that the property income model's residuals over the interval 1967-94 appeared to be independent and normally distributed. However, as in the share dividend model, θ_p was found to be not significantly different from zero.

The significance of θ_p can be examined by refitting the model with θ_p and β_p excluded (see Table 8.5.1). This shows that the model may have been over-parameterised because

Table 8.5.1 Estimated parameters for the property income model

Interval	θ_p	β_p	μ_{dp}	φ_{dp}	σ_{dp}
1968-94†	0.1289	1	0.0032	0.2363	0.0599
	(0.0689)		(0.0132)	(0.0974)	
1968-94	-	-	0.0797	0.2695	0.0637
			(0.0129)	(0.1069)	

†*Source:* Table 8.2 of Wilkie (1995b)

the increase in the variance of the model's residuals after excluding θ_p and β_p is only just significant ($F(1,24) = 3.19$, $p = 0.0865$). This finding is not decisive and is similar to that for the share dividend model (Section 8.3.2).

8.6 Summary

Adequate stochastic asset models are extremely difficult to construct because of the complicated nature of the economy. These difficulties are compounded by data shortages, the absence of a reliable and detailed theoretical foundation, and possible regime shifts caused by a number of factors including changes in government policy and technological innovations. A further consequence of these difficulties is that econometric investigations are especially susceptible to the problems of data mining. These considerations make it difficult to evaluate models and make it important that a range of evaluation criteria is used, including theory consistency, data admissibility, data coherency, parameter constancy, parsimony, robustness, and encompassing.

Chapters 3 and 6 illustrated that Wilkie's model is not consistent with financial economic theory. However, Wilkie's model does incorporate the intuitive concept of 'unit gain'. Furthermore, the long-term interest rate model does not satisfy the criterion of data admissibility. These apparent weaknesses are significant because they suggest that the model is unstable (see Section 6.4) and that the model can only be partially interpreted (see Section 7.5). However, uninterpreted models can provide useful forecasting tools if they fit the data sufficiently well.

This chapter examined the empirical adequacy of Wilkie's model. The model appears to describe the historical data reasonably well, but possible empirical weaknesses include that the inflation models do not appear to adequately represent the apparent changes in the mean rate of inflation. The parameters ω_{ys} and φ_b seem to have been unduly affected by outliers. The 'unit gain' effect may not be appropriate in both the share dividend and property income models. The inflationary expectations component of the long-term interest rate model does not appear to be appropriate. A more significant empirical problem with Wilkie's original model is that it did not provide an adequate variance and covariance structure for the out-of-sample residuals. The variances of the out-of-sample residuals from the price inflation model and the long-term interest rate model were significantly less than and significantly greater than the respective values implied by the original model. There was also a significant cross-correlation between the out-of-sample residuals from the price inflation model and the share dividend and long-term interest rate models.

Due to the problems associated with data mining, these empirical weaknesses should not generally be used to suggest an alternative model structure. A complete re-evaluation of economic theory and the data is required before an alternative can be suggested. In particular, detailed consideration should be given to incorporating theories, such as the rational expectations hypothesis and the efficient market hypothesis, in an alternative stochastic asset model.

Appendix 8A: The Data

8A.1 Inflation data

The price inflation index, used in Wilkie (1984), was constructed by linking the Schumpeter-Gilboy Consumers' Goods Index A and B (1661-1790), the Gayer, Rostow and Schwarz Domestic and Imported Commodities Index (1790-1850), the Rousseaux Overall Price Index (1850-1871), the Board of Trade Wholesale Price Index (1871-1914), the Cost of Living Index (1914-1947), the Interim Index of Retail Prices (1947-1956), and the General Index of Retail Prices (1956-1982).

The data for the earlier indices, the Cost of Living Index and the Retail Prices Index over the interval 1947-61, was obtained from Mitchell (1962) and Mitchell and Jones (1971). This data represents the annual average values of these indices (not June values as was intended), which induces a spurious moving average effect (Working 1960). From 1962 the inflation index was constructed from the June Retail Prices Index values. This index can be obtained from the publication *Labour Market Trends*.

Wilkie (1995b) updated and adjusted this index by extending it to 1994, including the data over the interval 1264-1661 that was obtained from Phelps *et al.* (1956), and by

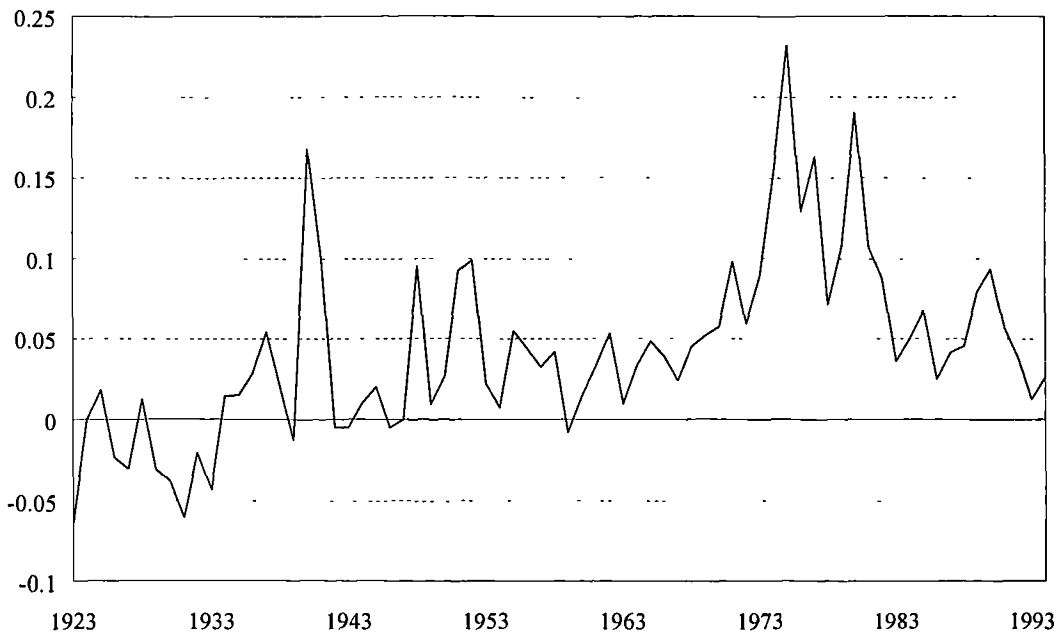

Figure 8A.1 The force of price inflation, 1923-94 using the June values of the Cost of Living Index and the Retail Prices Index rather than annual averages. The price inflation data used in the construction of the model is displayed in Figure 8A.1.

Most of the earlier indices are of doubtful relevance to the modelling of future inflation rates because they do not measure changes in the general level of retail prices. The Gayer, Rostow and Schwarz index is based on the prices of commodities and the Board of Trade index is based on the prices of wholesale goods. These indices and the Cost of Living Index also have a very narrow coverage of goods compared to the General Index of Retail Prices. Furthermore, Allen (1948) questioned the appropriateness of the Cost of Living Index, over the period 1938-47, as it tended to concentrate on items that were subsidised during World War II. Using adjusted weights, Allen estimated that the index would have increased by approximately 60%, over the period 1938-47, compared with an increase of approximately 30% in the official figures. These considerations suggest that investigations conducted on the earlier indices should to be treated with caution.

The wage inflation index, used in Wilkie (1995b), was constructed by linking together the index of basic weekly wage rates (1920-1967) and the index of average earnings: all employees; Great Britain (1967-1994). This data can be obtained from the publications

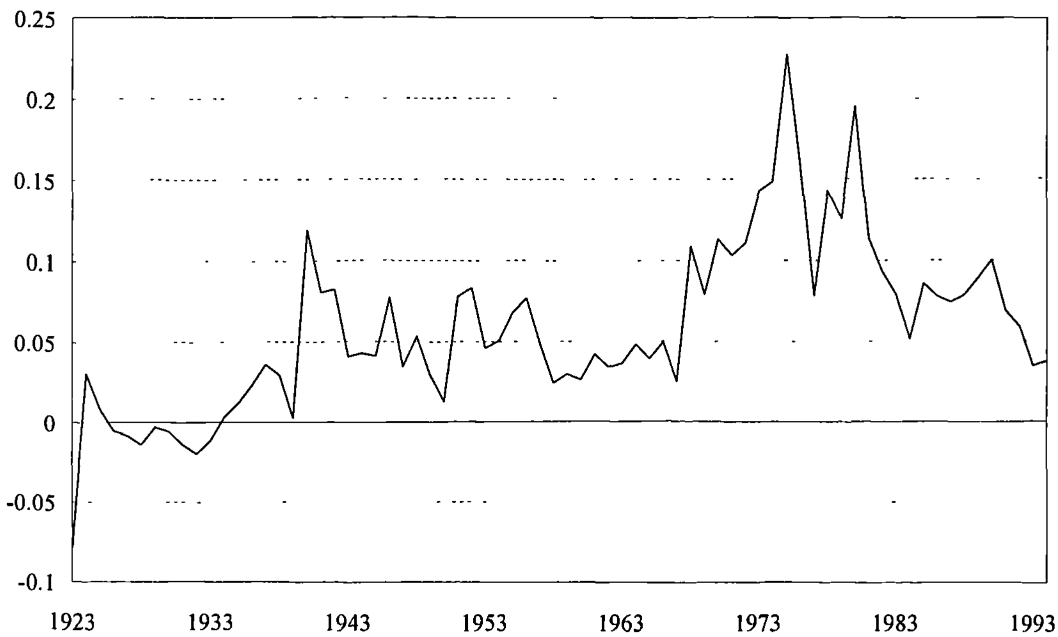

Figure 8A.2 The force of wage inflation, 1923-94

Labour Market Trends and *British labour statistics-historical abstract 1886-1986* (1971). Earlier indices obtained from Mitchell (1962) and Mitchell and Jones (1971) were also considered. The wage inflation data used in the construction of the model is displayed in Figure 8A.2.

8A.2 Equity data

The equity data, used in Wilkie (1984), was obtained from the BZW equity index (1919-30), the Actuaries Industrials (All Classes Combined) Index (1931-53), the Second Series Actuaries Industrials (All Classes Combined) Index (1954-61), and the FTSE-Actuaries All-Share Index (1962-82). The data from the BZW index is only calculated at the end of every year (not in June as was assumed). This data can be obtained from the *BZW Equity & Gilt Study* (1992), *The Actuaries' Investment Index*, the *FT-Actuaries Share Indices*, the *FT-SE Actuaries Share Indices*, and the *Financial Times*.

The dividend yield series' were linked together by taking the FTSE-Actuaries dividend yields at face value, by multiplying the Actuaries dividend yield by 0.7226 (which is equal to the FTSE-Actuaries yield on 30 April 1962 divided by the Actuaries yield on 24 April 1962), and by multiplying the BZW income yield by 0.7237 (which is equal to 0.7226 multiplied by the Actuaries yield on 24 June 1930 divided by the BZW yield on 31 December 1930).

Over the period 1931-1982, the Actuaries and FTSE-Actuaries dividend index series' were constructed by multiplying the above adjusted dividend yield series to a linked price index series. The respective Actuaries price indices' were linked on 28 May 1946, 28 July 1953 and 31 December 1957. The Actuaries price index was linked to the FTSE-Actuaries price index using values on 24 April 1962 and 30 April 1962 respectively. The dividend index series' were linked by multiplying BZW income index by 0.02225 and the Actuaries and FTSE-Actuaries series by 0.4707 (which is equal to the BZW dividend index on 31 December 1930 divided by the Actuaries dividend index on 24 June 1930).

The problem with this method of linking the equity indices is that it reduces the total returns on the earlier indices. This is because the yields on the earlier indices are reduced without making a corresponding adjustment to the equity dividend index data.

Wilkie (1995b) extended this data to 1994, replaced the BZW data with interpolated June values rather than December values, and replaced the data over the interval 1924-

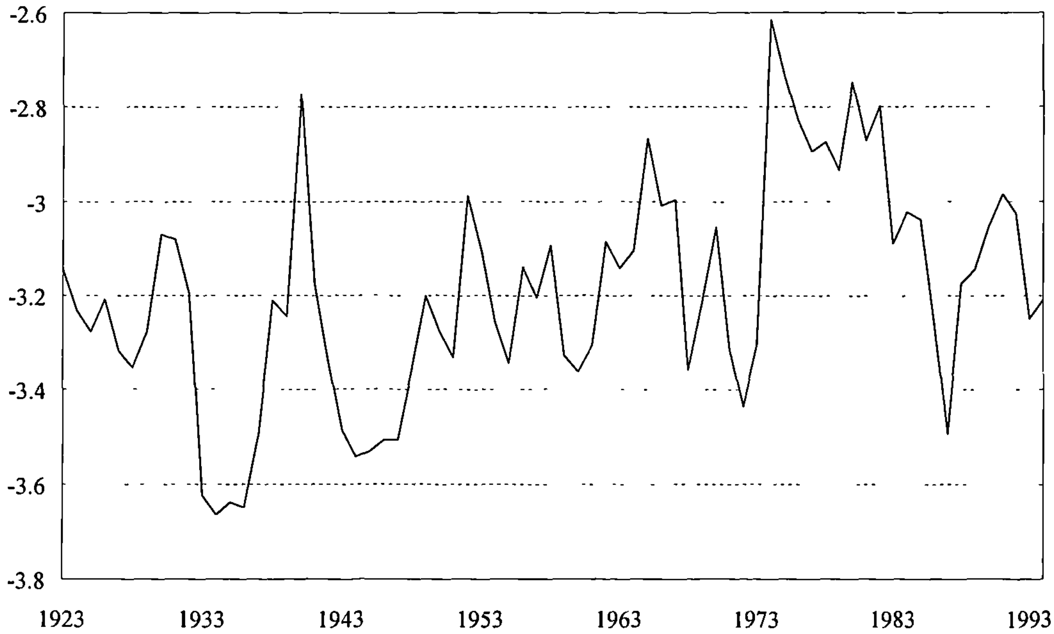

Figure 8A.3 The logarithm of the equity dividend yield, 1923-94

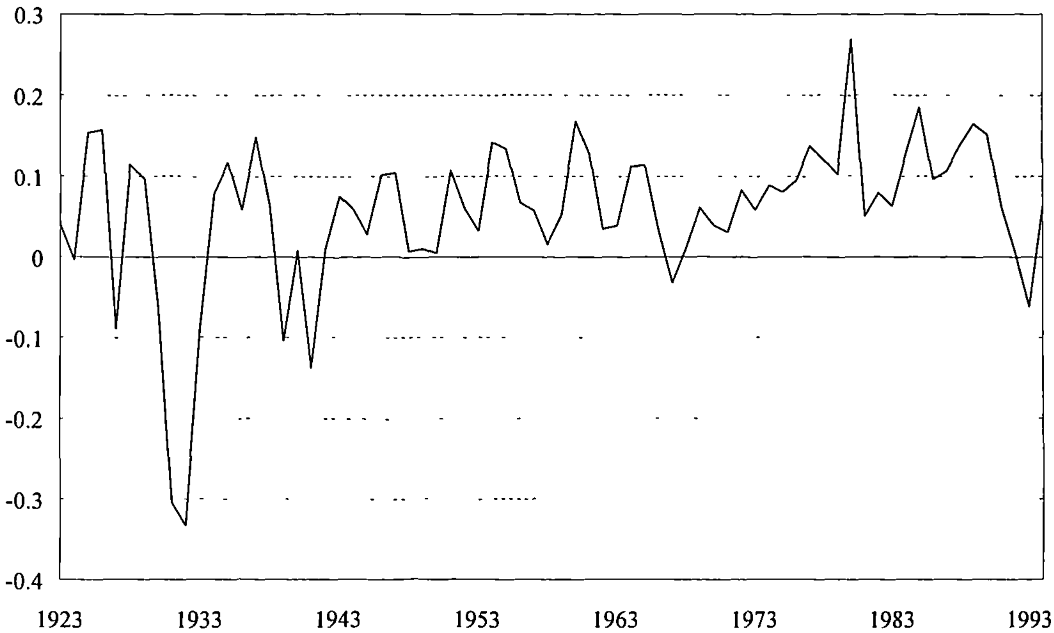

Figure 8A.4 The force of equity dividend growth, 1923-94

28 with data obtained from Douglas (1930). The equity data used in the construction of the model is displayed in Figures 8A.3 and 8A.4.

There are a number of significant differences between the various equity indices used by Wilkie that may influence empirical investigations. The FTSE-Actuaries index includes shares from all types of companies, whereas the other indices exclude financial company shares. The Actuaries indices are geometrically averaged (see Haycocks and Plymen 1956), whereas the other indices are arithmetically averaged. The BZW index was based on 30 shares, the Actuaries indices on roughly 150 shares, and the FTSE-Actuaries index is currently based on roughly 900 shares (594 in 1962).

Another possibly problematic feature of the equity data is that the Actuaries price indices performed poorly relative to most other contemporary indices. Over the intervals 1930-49, 1940-50, 1950-60, the Actuaries price index increased by –48%, 96%, and 152%, respectively. Over similar intervals, the *Investors Chronicle* equity price index and the BZW price index increased by –35%, 115%, and 224%, and 21%, 46%, and 183%, respectively. The *Investors Chronicle* Index was an equal weighted arithmetically averaged index based on roughly 100 shares. Haycocks and Plymen (1964) suggest that the underperformance of the Actuaries indices was possibly due to the downward bias caused by geometric averaging, the inclusion of railway shares until 1948 and the heavy weighting given to the tobacco group.

8A.3 Interest rate data

The long-term interest rate series used in Wilkie (1984) was the 2.5% Consolidated Stock yield obtained from Mitchell (1962) over the interval 1756-1929, *The Actuaries' Investment Index* over the interval 1930-61, the *FT-SE Actuaries Share Indices* over the interval 1962-80, and the *Financial Times* over the interval 1981-82. This information can also be obtained from the *Stock Exchange Daily Official List*.

The yields in Mitchell (1962) appear to represent the coupon divided by the annual average of the daily prices of the stock (not the running yield at the end of June as was intended).

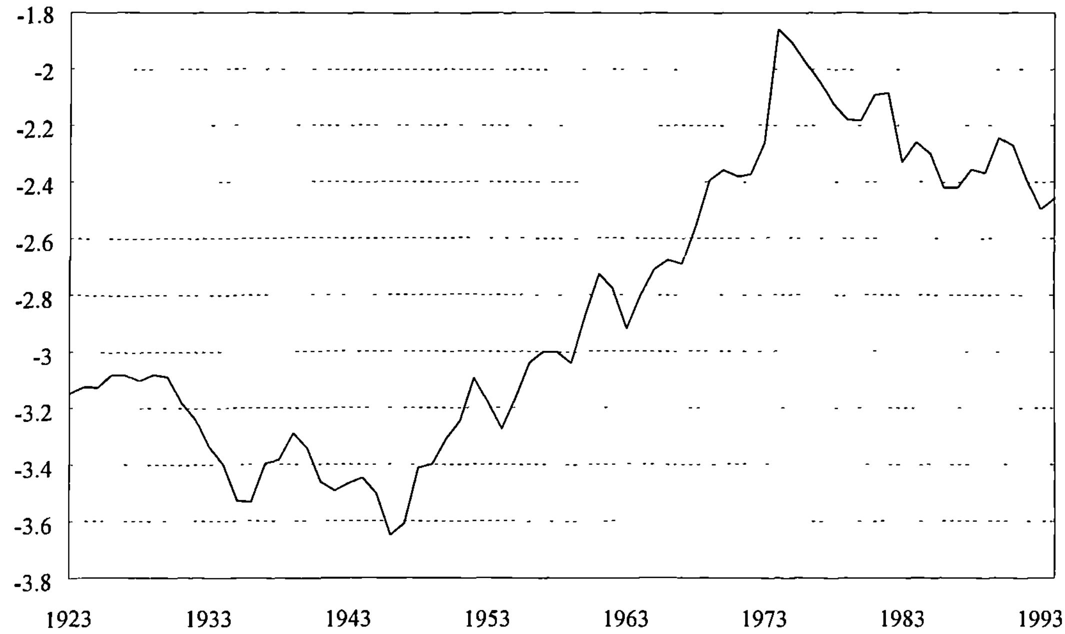

Figure 8A.5 The logarithm of the long-term interest rate, 1923-94

Wilkie (1995b) replaced the above Consols data with data obtained from Mitchell (1962) over the interval 1797-1900, the BZW Gilts book over the interval 1900-29, *The Actuaries' Investment Index* over the interval 1930-62, the *FT-SE Actuaries Share Indices* over the interval 1963-77. Over the interval 1978-94 the FTA BGS Irredeemables index was used rather than the yield on 2.5% Consols. This index was obtained from the *Financial Times*. The long-term interest data used in the construction of the model is displayed in Figure 8A.5.

The short-term interest rate data, used in Wilkie (1995b), was the Bank rate. This data can be obtained from Mitchell (1962) and the publication *Financial Statistics*. Over the interval 1972-1981 the minimum lending rate was used. The short-term interest rate data used in the construction of the model is displayed in Figure 8A.6. An alternative short-term interest rate series that was not considered by Wilkie (1995b) is the average rate of discount on allotment of 91 day Treasury bills. This series is available in *Bankers' Magazine*, *Bank of England Quarterly Bulletin*, and *Financial Statistics*.

The index-linked interest rate data, used in Wilkie (1995b), was the FTA Index-Linked All Stocks Index (1981-1985) and the FTA Index-Linked over 5 years index (1985-94).

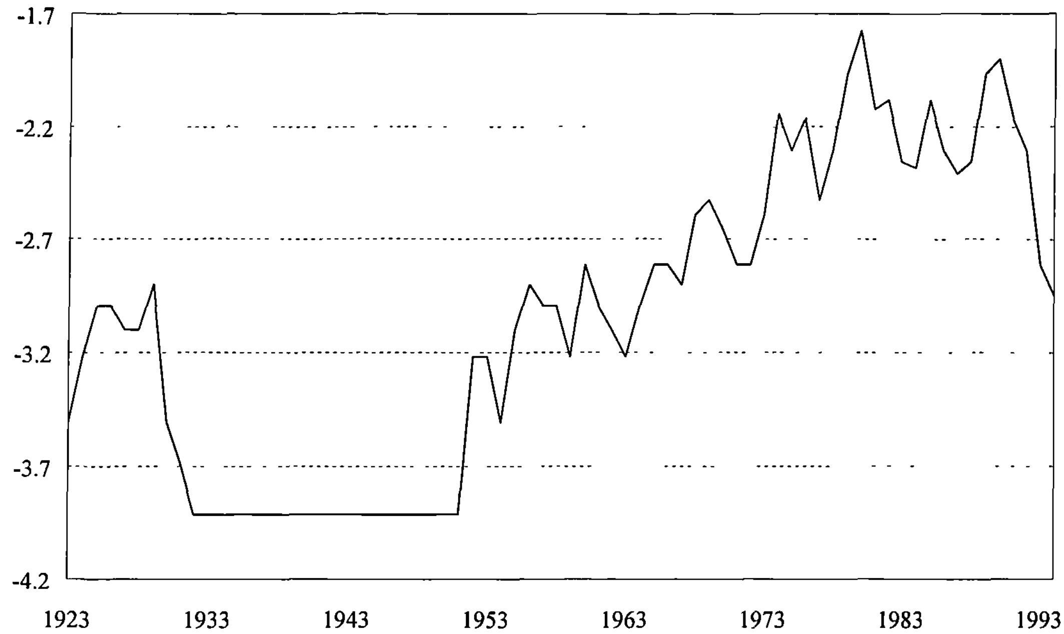

Figure 8A.6 The logarithm of the short-term interest rate, 1923-94

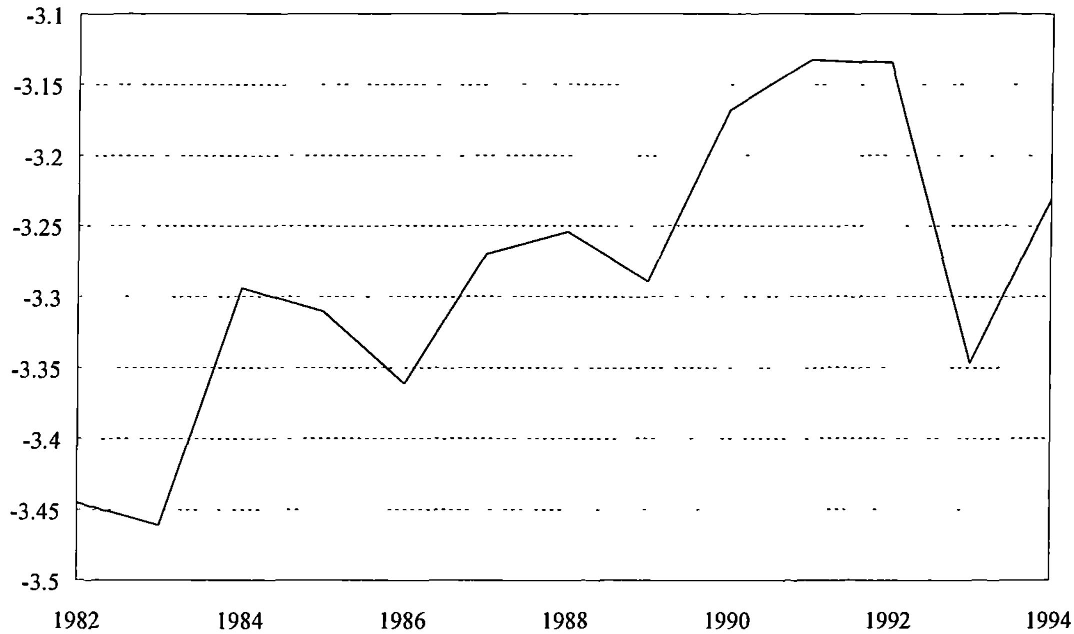

Figure 8A.7 The logarithm of the index-linked interest rate, 1982-94

Both indices assumed 5% inflation. This data can be obtained from the *Financial Times*. The index-linked data used in the construction of the model is displayed in Figure 8A.7.

8A.4 Property data

The property data, used in Wilkie (1995b), were the Jones Lang Wootton indices of net income and income yield. The property data used in the construction of the model is displayed in Figures 8A.8 and 8A.9.

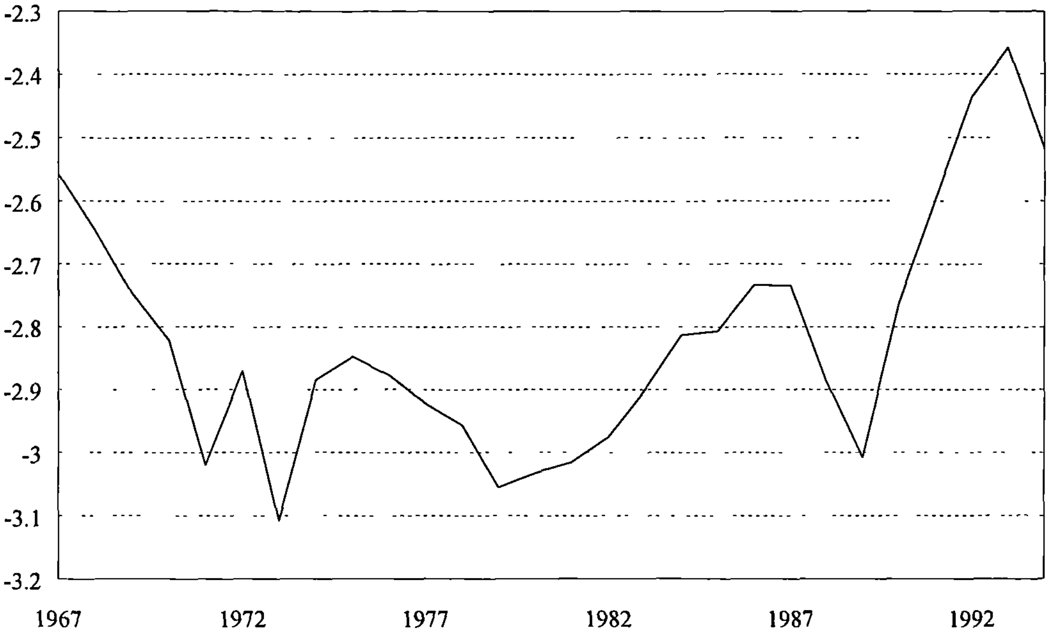

Figure 8A.8 The logarithm of the property yield, 1967-94

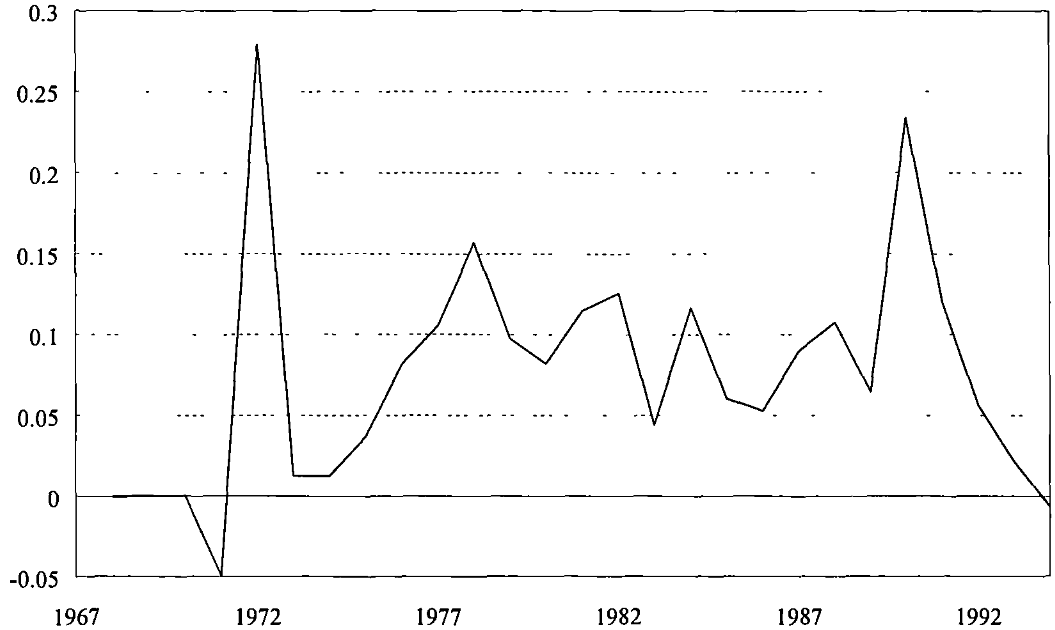

Figure 8A.9 The force of property income growth, 1967-94

Chapter 9

CONCLUSION

9.1 Actuarial Economic Models

Actuarial economic models constitute a significant part of the set of assumptions required for most actuarial calculations. Traditionally actuaries have only estimated the average future value of the relevant economic variables using relatively informal techniques that depend on actuarial judgement. These techniques usually focus on a critical analysis of the historical data, but a variety of other information is also taken into account. In particular, actuaries often consider the information implicit in the current fixed-interest and index-linked yield curves when setting their inflation and interest rate assumptions. However, actuaries do not generally commit themselves completely to a theoretical framework, such as that offered by financial economics. Further, they may adopt assumptions that differ from those suggested by market information.

A significant weakness with traditional techniques is their informal nature, which is associated with their reliance on human judgement. Human judgement is susceptible to numerous biases and it tends to be vulnerable when challenged. Moreover, each actuary is likely to arrive at a different result when confronted with the same problem. These weaknesses are compounded by the sensitivity of actuarial calculations to the economic assumptions and the difficulty of economic forecasting. However, actuaries have responded to these potential problems by analysing the sensitivity of their calculations to these assumptions and reducing this sensitivity where possible. This has been achieved by redesigning the contracts under investigation or by restructuring the asset portfolio supporting these contracts. In addition, to reduce inconsistencies, the actuarial profession has prescribed standard economic bases for certain calculations. The potential for these problems can also be reduced if a more structured, formal approach is used. At a minimum, actuaries should record audit trails of their reasoning so that it can be scrutinised by others.

Financial economics provides a more formal approach towards investigating financial phenomena. It has resulted in the development of numerous models and theories that attempt to explain these phenomena in terms of the behaviour of rational agents. These models are inevitably founded on unrealistic simplifying assumptions and consequently they are not necessarily always applicable. As a result, actuaries have generally been reluctant to wholly embrace financial economics. Although this response is justifiable, actuaries should not dismiss financial economics without giving it serious consideration. Financial economics is widely accepted as providing the best possible method for analysing the financial economy.

Another formal method for studying financial phenomena is to develop a stochastic asset model. These models attempt to summarise the significant durable features of the economy using precise mathematical relationships. They describe the complete statistical distribution of the relevant variables, rather than just the mean values of these variables as traditional methods do. This additional information is essential for certain actuarial applications and it enables actuaries to quantify economic uncertainty. The available actuarial stochastic asset models have been primarily developed and motivated either using the historical data or using financial economic theory. The main UK model that has been developed from data considerations is known as Wilkie's (1986a, 1995b) model and the main UK theoretical models are Dyson and Exley's (1995) expectations model and Smith's (1996) jump-equilibrium model. These theoretical models were calibrated to the historical data, but their empirical adequacy was not specifically examined.

Wilkie's (1995b) model appears to have become the standard UK actuarial stochastic asset model. It only aims to describe the long term features of the returns and yields on the major asset classes, and the rates of price and wage inflation. Although intuitive considerations, such as 'unit gain', were used in the development of the model, it does not incorporate important financial economic theories, including the efficient market hypothesis and the rational expectations hypothesis. Other financial economic theories that were included in the model are the Fisher relation and the purchasing power parity hypothesis. The models were primarily developed using the Box-Jenkins (1970) ARIMA transfer function methodology and they were generally motivated by demonstrating that they satisfied various statistical misspecification tests. The model assumes that inflation rates and rates of return on all asset classes are stationary variables. Wilkie's model has been criticised in other reviews for not accommodating structural changes in the price inflation model, for producing a seemingly unrealistically high proportion of years with negative price inflation, for producing a low correlation between the returns on equity and long-term fixed-interest securities, and for permitting significant arbitrage opportunities. The long-term interest rate model also permits negative yields. Further potential weaknesses of Wilkie's model are summarised in Section 9.2.

Dyson and Exley's (1995) expectations model was developed from the pure expectations hypothesis of the term structure of interest rates. This model is generally consistent with the rational expectations and efficient market hypothesis. It also describes the complete fixed-interest and index-linked yield curves. However, it assumes that the expected returns for all asset classes are equal and, if a number of different duration securities are considered, it permits significant arbitrage opportunities. Contrary to Wilkie's model, Dyson and Exley's model assumes that price inflation and the returns on all the asset classes are integrated of order one.

Smith (1996) appeared to use the ideas behind Dyson and Exley's model to develop the jump-equilibrium model. This model aimed to be symmetrical, to describe the full yield curves, to incorporate a risk-neutral law, to allow for occasional price jumps, and to incorporate the efficient market hypothesis. Furthermore, Smith used a novel 'equilibrium' method to determine the parameters representing the mean values of the asset classes. This method ensures that the model is consistent with capital market portfolio theory. The model did not aim to closely describe the historical data as it ignores possibly significant time-series features, such as those incorporated in Wilkie's model. However, the jump-equilibrium model does not seem to produce price 'jumps' for annual rates of return; the distribution of the annual rates of return on equity and property securities is platykurtic. Other properties of the jump-equilibrium model include that the distribution of annual rates of price inflation is mildly leptokurtic and negatively skewed. The model assumes that nominal and real rates of return are non-stationary. The 'neutral' yield curves required to produce constant expected rates of return do not satisfy the liquidity preference hypothesis. The model only allows for 'parallel' shifts in the yield curves and it permits negative yields.

The above, initial, reviews of the available actuarial stochastic asset models suggest that none of these models are perfect. This has led some actuaries to completely reject the use of all mathematical models to describe the economy. This position is supported by the belief that the economy does not have sufficient structure that can be adequately modelled. If this view is held, then financial economics and stochastic models are only likely to be of limited value and the best possible information is likely to be obtained by using simpler more tractable models. These models may be either stochastic or deterministic. A possible method of developing sophisticated deterministic models is provided by the scenario approach, or by the use of pattern models, which are generally founded on narrative rather than mathematical techniques. These types of models do not generally aim to produce final answers, they merely aim to enhance understanding of the relevant issues so that an informed final decision can be taken.

Therefore, there exist a number of approaches towards setting the economic assumptions. Each approach is motivated using different fundamental assumptions and each approach appears to have difficulties. The following section considers how actuarial models should be appraised and how the available models could be justified.

9.2 The Appraisal of Economic Models

Issues relating to the development of and justification for scientific theories have been extensively explored in the philosophy of science. This literature provides a foundation for assessing actuarial economic models. It emphasises the difficulties of empirical testing. Hume's problem of induction demonstrates that inductive evidence cannot be used to establish the truth of universal statements. Thus, models based solely on inductive evidence, or data considerations, are not persuasive. The Duhem-Quine thesis illustrates that individual theories or statements can only be tested within systems of statements. This implies that tests of individual theories, such as the efficient market hypothesis, are inconclusive. Moreover, Popper argued that it is relatively easy to find supporting evidence for a theory. Consequently, he asserted that researchers should adopt a critical attitude and they should attempt to falsify, rather than verify, their theories.

However, there are significant disagreements within the philosophy of science and it has not produced any universal methodological rules. In particular, it has not been possible to resolve the demarcation problem or the problem of theory choice and many conflicting views have been held. The logical positivists required that individual statements should be capable of complete verification by direct observational evidence, but this proved untenable mainly because of the Duhem-Quine thesis. Falsificationists required that theories should be able to be falsified by a set of basic statements, but this position does not allow for questions of justification. Kuhn suggested that theories are chosen by their ability to solve relevant puzzles and that theoretical frameworks are not discarded until a suitable alternative is found. Lakatos suggested that theories should be assessed on their excess content and by their ability to predict novel facts. Despite these controversies, it is generally accepted that theories must be exposed to extensive empirical assessments using harsh tests, that test failures should be taken seriously, and that knowledge is fallible.

Economic methodology explores the specific methodological issues that relate to the discipline of economics. Economics is a particularly difficult subject area because of the apparent complexity of economic relationships, the large number of factors that potentially influence economic events, and the general inability to conduct controlled economic experiments. These difficulties make it especially difficult to practise a demanding methodology, such as falsificationism. They also suggest that economic predictive success is always likely to be limited. As a result, the method a priori and the deductive method suggest that economists should investigate the consequences of the fundamental causal factors that can be easily established by introspection or observation. However, as these basic postulates are generally inexact and they do not describe all the relevant causal factors, the resulting economic theories will only represent statements of tendencies rather than precise laws. Hausman (1992) asserted that these tendency laws or inexact generalisations can be defended if they are lawlike, reliable, refinable, and excusable. This assertion is controversial, but it appears to represent a rational response to the acute difficulties associated with economics.

The limitations of economics are further emphasised by econometric methodology. Econometrics aims to provide empirical content to inexact economic theories. But, econometric models have often proved to be inadequate and they have been found to be particularly sensitive to regime shifts. These problems appear to be partially related to the shortage of economic data relative to the large number of potentially relevant variables and possible model structures that can generally account for economic theories. If numerous models are fitted before the final model is reported then it is likely to appear to fit the data better than it actually does. As suggested by the philosophy of science, test failures need to be properly investigated and individual test failures cannot be interpreted in isolation. However, in econometrics the Duhem-Quine thesis poses a particularly serious problem because of the large number of factors that can generally account for the test failures of econometric models. The most promising method of developing econometric models that deals with these difficulties appears to be the general-to-specific approach. It stresses the need for both extensive empirical tests and a theoretical framework to interpret the resulting models. In particular, models are required to satisfy the criteria of: data coherency, data admissibility, parameter constancy, theory consistency, robustness, parsimony, and encompassing.

The above considerations suggest that adequate actuarial economic models are likely to be difficult to develop. Nevertheless, they provide criteria for assessing the available models and they suggest how these models may be justified.

The informal traditional approach is particularly vulnerable because it does not produce theories that can be extensively tested. Moreover, actuarial explanations do not usually employ general laws and true statements, which are required for scientific explanations. Nevertheless, this approach is necessary if a sufficiently adequate theoretical framework does not exist. If this is the case, then the best actuaries can do is to consider the available theories, but their final decisions must ultimately be based on mature professional judgement. Given the problems associated with financial economic theory, this approach appears to be justifiable. However, actuaries should recognise the relatively fragile nature of their assumptions and should attempt to develop more robust arguments whenever possible.

Wilkie's model appears to be broadly satisfactory. It was developed from some theoretical considerations and it satisfied various empirical tests. However, there appear to be numerous problems with Wilkie's approach. The objective of only providing long term predictions can be interpreted as an immunising stratagem because it is impossible to empirically test the long term performance of a model. Wilkie's use of financial economic theory is inconsistent. Whereas both the efficient market hypothesis and the purchasing power parity hypothesis are based on equilibrium arguments, Wilkie rejected the former but included the latter. Moreover, by rejecting the efficient market hypothesis and the rational expectations hypothesis, Wilkie created a model that is inherently unstable in the long term. The Box-Jenkins methodology used by Wilkie is also susceptible to the problems of data mining. This suggests that the misspecification test results reported by Wilkie may have been biased. Further tests suggest that the model does not appear to have had constant parameters historically. In particular, the model's out-of-sample residuals do not appear to be independently distributed. Nevertheless, Wilkie's model appears to capture some important features of the relevant economic variables and it represents the best available UK comprehensive actuarial econometric model.

Dyson and Exley's expectations model and Smith's jump-equilibrium model can be justified by interpreting them, using Hausman's definition, as initial models as opposed to theories. They represent specific financial economic theories and can be used to explore the consequences of those theories. This provides users of these models with valuable information when they are attempting to interpret the results produced by these models. The theoretical frameworks place the results of applications in an understandable context, they provide fundamental support for the models, and they provide heuristics that suggest how these models might be improved. However, applications usually require more than initial models and it is important that these models are further developed and tested. In particular, the previous section suggested some weaknesses in these models, which need to be addressed. Furthermore, these models depend on financial economic theories that appear to be inadequate in many respects.

The difficulties associated with developing an adequate stochastic model suggest that it is important to assess the sensitivity of the results produced to the doubtful features of the model. It is possible that certain applications may be too sensitive to the economic model structure to enable any meaningful inferences to be made. These difficulties also imply that a model's pragmatic qualities are likely to be more important than they would otherwise have been. This is because it is difficult to clearly demonstrate that a more complicated model is significantly better. Hence, the use of simple models is justifiable until a clearly adequate superior model is discovered. The results produced by applications of simple models are also easier to interpret and explain. But simple models cannot be employed in a mechanical way and their effective use requires substantial professional judgement.

9.3 Conclusions and Areas for Future Research

The philosophy of science demonstrates that there is no uniquely correct or most efficient way of acquiring scientific knowledge and that all knowledge is fallible. These important considerations appear to be especially relevant to actuarial economic modelling. They imply that the actuarial profession is broadly right to leave it up to individual actuaries to set the economic assumptions, unless the requirement of consistency between actuaries is paramount. No single approach is clearly superior and the alternatives are virtually incommensurable because they are founded on different fundamental assumptions about the existence of sufficient quantifiable structure in the economic system. To provide the best possible service and advice, actuaries should be familiar with but remain agnostic towards all the principle theories and approaches. Researchers involved in the development of fundamental theories are likely to be highly committed to their particular approach and actuaries should be aware of this rhetorical consideration by remaining detached. Thus, the major focus of actuarial research should possibly be towards the application of various theories and approaches rather than towards conducting fundamental research. The UK actuarial profession has limited resources and it would be difficult to compete with the finance or economics professions.

Although actuaries should consider a variety of approaches, they ultimately need to make a decision. This decision must be based on actuarial judgement because no single approach can be demonstrated to be correct. Actuarial judgement is unavoidable. However, to limit the possibility of biases, actuaries should record audit trails, they should consider how a reasonable representative person would respond to their chosen decision, and they should not be falsely scientific. Moreover, the necessity of judgements emphasises the value of pedagogic devices, or models, such as the concept of immunisation or utility theory. These devices help to structure the relevant problem and they enable one to comprehend the important issues. They do not intend to provide mechanistic tools for decision making.

There appear to be two main approaches towards developing future actuarial economic models, depending on whether the existence of a measurable economic structure is accepted. The first approach rejects the existence of such a structure and strongly discounts historical data. This view is held by institutionalists and modern Austrian economists. It assumes that econometric investigations are of limited value and that these investigations are only able to measure short term relationships and trends. Pattern modelling, or the scenario approach, appear to be most suited to this view. These techniques focus less on predicting economic events and more on developing flexible systems that can cope with likely future events. There exists a substantial literature on these approaches that actuaries do not appear to have specifically considered.

The more orthodox approaches assume that economic phenomena can be described and predicted using precise mathematical models. The most promising method of developing these models appears to be the general-to-specific approach. However, there are likely to be substantial difficulties involved in finding a suitable initial general model. In particular, univariate autoregressive models do not appear to be able to adequately describe the price inflation series. Nevertheless, it seems appropriate that long term models incorporate financial economic theories such as the efficient market hypothesis and the rational expectations hypothesis. These theories ensure consistency and long term model stability. The efficient market hypothesis could be incorporated by assuming constant risk premiums or risk premiums could be assumed to vary depending on the level and volatility of interest rates. There also appears to be a need to examine models that allow for regime shifts, but this may be empirically intractable if these shifts are highly irregular. Furthermore, actuarial models would benefit from having more sophisticated yield curve models.

However, given the difficulties of economic modelling, developing an adequate long term model is likely to take a considerable amount of time and effort. This suggests that strong conclusions on the validity of specific hypotheses are probably unwarranted and that actuaries should possibly consider using more tractable models or models with shorter time horizons. Black-box models may also provide useful insights if they are treated as initial hypotheses that require further explanation.

Therefore, actuarial economic modelling is a particularly demanding subject area and none of the available models appears to be adequate. Actuaries should recognise these difficulties and they should attempt to use more formal structured methods whenever possible. A variety of approaches suggested by economics and financial economics should be considered, but actuaries ultimately need to rely on their judgement.

References

The Actuaries' Investment Index, December 1928 to December 1962. Institute of Actuaries, London, and Faculty of Actuaries, Edinburgh. (Also known as: *The Actuaries' Investment Index (Second Series)*, August 1953 to December 1957; *The Actuaries' Investment Index (Second Series-Revised)*, January 1958 to December 1962.)

Allen, R.G.D. (1948). Retail prices. London and Cambridge Economic Service Bulletin, 26: 18.

Anderson, N. and Breedon, F. (1996). UK asset price volatility over the last 50 years. *Bank of England Working Paper*, 51.

Arthur, T.G. and Randall, P.A. (1990). Actuaries, pension funds and investment. *Journal of the Institute of Actuaries*, 117: 1-49.

Backhouse, R.E. (1994). The Lakatosian legacy in economic methodology, in Backhouse, R.E., ed., *New directions in economic methodology*. Routledge, London, UK.

Banerjee, A., Dolado, J., Galbraith, J.W., and Hendry, D.F. (1993). *Co-integration, error-correction, and the econometric analysis of non-stationary data*. Oxford University Press, Oxford, UK.

Bank of England Quarterly Bulletin, December 1960 to July 1994. Bank of England, London, UK.

The Bankers' Magazine, October 1921 to June 1961. Waterlow and Sons Limited, London, UK. (Previously known as: *The Bankers' Insurance Managers' and Agents' Magazine*, October 1921 to September 1954.)

Beach, L.R., Christensen-Szalanski, J., and Barnes, V. (1987). Assessing human judgment: has it been done, can it be done, should it be done? in Wright, G. and Ayton, P., eds., *Judgmental forecasting*. John Wiley and Sons, Chichester, UK.

Bhattacharya, S. and Constantinides, G.M., eds. (1989). *Theory of valuation: frontiers of modern financial theory*. Rowman & Littlefield, Savage, MD.

Black, F. (1986). Noise. *The Journal of Finance*, 41:529-543.

Black, F., Jensen, M.C., and Scholes, M. (1972). The capital asset pricing model: some empirical tests, in Jensen, M.C., ed., *Studies in the theory of capital markets*. Praeger, New York, NY.

Blaug, M. (1992). *The methodology of economics, or, How economists explain*. Second Edition. Cambridge University Press, Cambridge, UK.

Boland, L.A. (1989). *The methodology of economic model building*. Routledge, London, UK.

Boland, L.A. (1994). Scientific thinking without scientific method: two views of Popper, in Backhouse, R.E., ed., *New directions in economic methodology*. Routledge, London, UK.

Box, G.E.P. and Jenkins, G.M. (1970). *Time series analysis forecasting and control*. Holden-Day, San Francisco, CA.

Boylan, T.A. and O'Gorman, P.F. (1995). *Beyond rhetoric and realism in economics: towards a reformulation of economic methodology*. Routledge, London, UK.

Breeden, D.T. (1979). An intertemporal asset pricing model with stochastic consumption and investment opportunities. *Journal of Financial Economics*, 7: 265-296.

Brehmer, B. (1987). Social judgment theory and forecasting, in Wright, G. and Ayton, P., eds., *Judgmental forecasting*. John Wiley and Sons, Chichester, UK.

British labour statistics-historical abstract 1886-1968. (1971). Department of Employment and Productivity. H.M. Stationery Office, London, UK.

Brown, S.J., Goetzmann, W.N., and Ross, S.A. (1995). Survival. *Journal of Finance*, 50: 853-873.

Bunn, D. and Wright, G. (1991). Interaction of judgemental and statistical forecasting methods: issues & analysis. *Management Science*, 37: 501-518.

BZW equity and gilt study. (1992). 38th Edition. Barclays de Zoete Wedd Research Limited, London, UK.

Caldwell, B.J. (1994a). *Beyond positivism: economic methodology in the twentieth century.* Revised Edition. Routledge, London, UK.

Caldwell, B.J. (1994b). Two proposals for the recovery of economic practice, in Backhouse, R.E., ed., *New directions in economic methodology*. Routledge, London, UK.

Campbell, J.Y. (1993). Intertemporal asset pricing without consumption data. *American Economic Review*, 83: 487-512.

Campbell, J.Y. and Ammer, J. (1993). What moves the stock and bond markets? A variance decomposition for long-term asset returns. *Journal of Finance*, 48: 3-37.

Campbell, J.Y. and Shiller, R.J. (1987). Cointegration and tests of present value models. *Journal of Political Economy*, 95: 1062-1088.

Campbell, J.Y. and Shiller, R.J. (1988). Stock prices, earnings, and expected dividends. *Journal of Finance*, 43: 661-676.

Campbell, J.Y. and Shiller, R.J. (1989). The dividend-price ratio and expectations of future dividends and discount factors. *The Review of Financial Studies*, 1: 195-228.

Carter, J. (1991). The derivation and application of an Australian stochastic investment model. *Transactions of the Institute of Actuaries of Australia* 1991: 315-428.

Cartwright, N. (1995). Neurath's boat, in Honderich, T. ed. *The Oxford companion to philosophy.* Oxford University Press, Oxford, UK.

Chatfield, C. (1995). Model uncertainty, data mining and statistical inference. *Journal of the Royal Statistical Society, Series A*, 158: 419-466.

Clare, A.D., Thomas, S.H. and Wickens, M.R. (1994). Is the gilt-equity yield ratio useful for predicting UK stock returns? *Economic Journal*, 104: 303-15.

Clarkson, R.S. (1991). A non-linear stochastic model for inflation. *Proceedings of the 2nd AFIR International Colloquium*, 3: 233-253.

Clarkson, R.S. and Plymen, J. (1988). Improving the performance of equity portfolios. *Journal of the Institute of Actuaries*, 115: 631-691.

Clemen, R.T. (1989). Combining forecasts: a review and annotated bibliography. *International Journal of Forecasting*, 5: 559-583.

Colander, D. (1994). The art of economics by the numbers, in Backhouse, R.E., ed., *New directions in economic methodology*. Routledge, London, UK.

Constantinides, G.M. (1989). Theory of valuation: overview and recent developments, in Bhattacharya, S. and Constantinides, G.M., eds., *Theory of valuation: frontiers of modern financial theory*. Rowman & Littlefield, Savage, MD.

Constantinides, G.M. (1990). Habit formation: a resolution of the equity premium puzzle. *Journal of Political Economy*, 98: 519-543.

Cooley, T.F. and LeRoy, S.F. (1985). Atheoretical macroeconomics: a critique. *Journal of Monetary Economics*, 16: 283-308.

Cowels, A. (1933). Can stock market forecasters forecast? *Econometrica*, 1: 309-324.

Darnell, A.C. and Evans, J.L. (1990). *The limits of econometrics*. Edward Elgar, Aldershot, UK.

Dawes, R.M. (1979). The robust beauty of improper linear models in decision making. *American Psychologist*, 34: 571-582. Reprinted in Kahneman, D., Slovic, P., and Tversky, A., eds., 1982, *Judgment under uncertainty: heuristics and biases*. Cambridge University Press, New York, US.

Dawid, A.P. and Dickey, J.M. (1977). Likelihood and Bayesian inference from selectively reported data. *Journal of the American Statistical Association*, 72: 845-850.

Daykin, C.D. and Hey, G.B. (1990). Managing uncertainty in a general insurance company. *Journal of the Institute of Actuaries*, 117: 173-277.

Daykin, C.D., Pentikäinen, T., and Pesonen, M. (1994). *Practical risk theory for actuaries*. Chapman and Hall, London, UK.

DeLong, J.B., Shleifer, A., Summers, L.H., and Waldmann, R.J. (1990). Noise trader risk in financial markets. *Journal of Political Economy*, 98: 703-738.

Diebold, F.X. (1989). Forecast combination and encompassing: reconciling two divergent literatures. *International Journal of Forecasting*, 5: 589-592.

Dimson, E., ed. (1988). *Stock market anomalies*. Cambridge University Press, Cambridge, UK.

Doornik, J.A. and Hendry, D.F. (1994). *PcFiml 8.0: Interactive econometric modelling of dynamic systems*. International Thomson Publishing, London.

Douglas, C.M. (1930). The statistical groundwork of investment policy. *Transactions of the Faculty of Actuaries*, 12: 173-228.

Draper, D. (1995). Assessment and propagation of model uncertainty. *Journal of the Royal Statistical Society, Series B*, 57: 45-97.

Duffie, D. (1992). *Dynamic asset pricing theory*. Princeton University Press, Princeton, NJ.

Dufresne, D. (1988). Moments of pension contributions and fund levels when rates of return are random. *Journal of the Institute of Actuaries*, 115: 535-543.

Dunn, K.B. and Singleton, K.J. (1986). Modelling the term structure of interest rates under non-separable utility and durability of goods. *Journal of Financial Economics*, 17: 27-55.

Dybvig, P.H. and Ross, S.A. (1985). Yes, the APT is testable. *Journal of Finance*, 40: 1173-1188.

Dyson, A.C.L. and Exley, C.J. (1995). Pension fund asset valuation and investment. *British Actuarial Journal*, 1: 471-557.

Evans, J.St.B.T. (1987). Beliefs and expectations as causes of judgmental bias, in Wright, G. and Ayton, P., eds., *Judgmental forecasting*. John Wiley and Sons, Chichester, UK.

Evans, M.D. and Lewis, K.K. (1995). Do expected shifts in inflation affect estimates of the long-run Fisher relation. *Journal of Finance*, 50: 225-253.

Fama, E.F. and French, K.R. (1988a). Permanent and temporary components of stock prices. *Journal of Political Economy*, 96: 246-273.

Fama, E.F. and French, K.R. (1988b). Dividend yields and expected stock returns. *Journal of Financial Economics*, 22: 3-25.

Fama, E.F. and French, K.R. (1992). The cross-section of expected stock returns. *Journal of Finance*, 47: 427-465.

Fama, E.F. and MacBeth, J.D. (1973). Risk, return, and equilibrium: empirical tests. *Journal of Political Economy*, 81: 607-636.

Feyerabend, P. (1988). *Against method.* Revised Edition. Verso, London, UK.

Financial Statistics, July 1978 to July 1994. Central Statistical Office, H.M. Stationary Office, London, UK.

Financial Times, various issues 30 July 1914 to 1 August 1994. Financial Times Limited, London, UK.

Fisher, I. (1930). *The theory of interest.* Reprinted 1967. Augustus M. Kelly, New York, NY.

FitzHerbert, R.M. (1992). Stochastic investment models. *Transactions of the Institute of Actuaries of Australia* 1992: 197-255.

Flavin, M.A. (1983). Excess volatility in the financial markets: a reassessment of the empirical evidence. *Journal of Political Economy*, 91: 929-956.

Ford, A., Benjamin, S., Gillespie, R.G., Hager, D.P., Loades, D.H., Rowe, B.N., Ryan, J.P., Smith, P., and Wilkie, A.D. (1980). Report of the Maturity Guarantees Working Party. *Journal of the Institute of Actuaries*, 107: 103-212.

Franses, P.H. and Paap, R. (1994). Model selection in periodic autoregressions. *Oxford Bulletin of Economics and Statistics*, 56: 421-439.

Freeman, D.A. (1991). Statistical models and shoe leather. *Sociological Methodology*, 21: 291-313.

Friedman, M. (1953). The methodology of positive economics, in *Essays in positive economics.* The University of Chicago Press, Chicago, IL.

FT-Actuaries Share Indices, 1976 to 1990. Financial Times Limited, London, UK.

FT-SE Actuaries Share Indices, January 1969 to February 1994. Financial Times Limited, London, UK. (Previously known as: *FT-Actuaries Share Indices-Monthly UK Stock Indices,* January 1969 to December 1989; *FT-Actuaries Share Indices-Finstat*, January 1990 to December 1993.)

Geoghegan, T.J., Clarkson, R.S., Feldman, K.S., Green, S.J., Kitts, A., Lavecky, J.P., Ross, F.J.M., Smith, W.J., and Toutounchi, A. (1992). Report on the Wilkie stochastic investment model. *Journal of the Institute of Actuaries*, 119: 173-228.

Gerrard, R. and Haberman, S. (1996). Stability of pension systems when gains/losses are amortized and rates of return are autoregressive. *Insurance: Mathematics and Economics*, 18: 59-71.

Giaquinto, M.D. (1995). Gödel's theorem, in Honderich, T. ed. *The Oxford companion to philosophy.* Oxford University Press, Oxford, UK.

Gibbons, M.R. (1982). Multivariate tests of financial models: a new approach. *Journal of Financial Economics*, 10: 3-27.

Gilbert, C.L. (1986). Professor Hendry's economic methodology. *Oxford Bulletin of Economics and Statistics*, 48: 283-307. Reprinted in Granger, C.W.J., ed., 1990, *Modelling economic series*. Oxford University Press, Oxford, UK.

Gordon, M.J. (1962). *The investment, financing, and valuation of the corporation*. Richard D. Irwin, Homewood, IL.

Granger, C.W.J. (1992). Forecasting stock market prices: lessons for forecasters. *International Journal of Forecasting*, 8: 3-13.

Grossman, S.J. and Shiller, R.J. (1981). The determinants of the variability of stock market prices. *American Economic Review*, 71: 222-227.

Haberman, S. (1994). Pension funding modelling and stochastic investment returns. Actuarial Research Report No. 62, The City University, London, UK.

Hafer, R.W. and Sheenan, R.G. (1989). The sensitivity of VAR forecasts to alternative lag structures. *International Journal of Forecasting*, 5: 399-408.

Hall, S. (1995). Macroeconomics and a bit more reality. *Economic Journal*, 105: 974-988.

Hardy, M.R. (1993). Stochastic simulation in life office solvency assessment. *Journal of the Institute of Actuaries*, 120: 131-151.

Hargreaves, C.P., ed. (1994). *Nonstationary time series analysis and cointegration*. Oxford University Press, Oxford, UK.

Harris, G. (1994). On Australian stochastic share return models for actuarial use. *The Institute of Actuaries of Australia Quarterly Journal* September 1994: 34-54.

Harris, G. (1995a). Statistical data analysis and stochastic asset model validation. *Transactions of the 25th International Congress of Actuaries*, 3: 311-331.

Harris, G. (1995b). A comparison of stochastic asset models for long term studies. Unpublished paper.

Harris, G. (1996). Market phases and cycles? A regime switching approach. Unpublished paper.

Hausman, D. (1992). *The inexact and separate science of economics*. Cambridge University Press, Cambridge, UK.

Hausman, D., ed. (1994a). *The philosophy of economics: an anthology*. Second Edition. Cambridge University Press, Cambridge, UK.

Hausman, D. (1994b). Kuhn, Lakatos and the character of economics, in Backhouse, R.E., ed., *New directions in economic methodology*. Routledge, London, UK.

Haycocks, H.W. and Plymen, J. (1956). Investment policy and index numbers. *Journal of the Institute of Actuaries*, 82: 333-371.

Haycocks, H.W. and Plymen, J. (1964). The design, application and future development of the *Financial Times*-Actuaries Index. *Journal of the Institute of Actuaries*, 90: 267-304.

Hemple, C.G. and Oppenheim, P. (1948). Studies in the logic of explanation. *Philosophy of Science*, 15: 135-75. Reprinted in Hemple, C.G. 1965. *Aspects of scientific explanation and other essays in the philosophy of science*. The Free Press, New York, NY.

Hendry, D.F. (1979). Predictive failure and econometric modelling in macroeconomics: the transactions demand for money, in Ormerod, P., ed., *Economic modelling: current issues and problems in macroeconomic modelling in the UK and the US*. Heinemann, London, UK.

Hendry, D.F. (1980). Econometrics - Alchemy or science. *Economica*, 47: 387-406.

Hendry, D.F. (1995). *Dynamic econometrics*. Oxford University Press, Oxford, UK.

Hendry, D.F. and Doornik, J.A. (1994). Modelling linear dynamic econometric systems. *Scottish Journal of Political Economy*, 41: 1-33.

Hendry, D.F. and Mizon, G.E. (1985). Procrustean econometrics: or streching and squeezing data. Discussion paper no. 68, Centre for Economic Policy Research. Reprinted in Granger, C.W.J., ed., 1990, *Modelling economic series*. Oxford University Press, Oxford, UK.

Hendry, D.F. and Richard, J.F. (1982). On the formulation of empirical models in dynamic econometrics. *Journal of Econometrics*, 20: 3-33. Reprinted in Granger, C.W.J., ed., 1990, *Modelling economic series*. Oxford University Press, Oxford, UK.

Hendry, D.F. and Richard, J.F. (1983). The econometric analysis of economic time series. *International Statistical Review*, 51: 111-163.

Hodrick, R.J. (1992). Dividend yields and expected stock returns: alternative procedures for inference and measurement. *Review of Financial Studies*, 5: 357-386.

Honderich, T. ed. (1995). *The Oxford companion to philosophy*. Oxford University Press, Oxford, UK.

Huang, C.-F. and Litzenberger, R.H. (1988). *Foundations for financial economics*. Prentice-Hall, Englewood Cliffs, NJ.

Huber, P.P. (1995a). A review of Wilkie's stochastic investment model. Actuarial Research Report No. 70, The City University, London, UK.

Huber, P.P. (1995b). A review of Wilkie's stochastic investment model. *Transactions of the 25th International Congress of Actuaries*, 3: 333-364.

Huber, P.P. (1996). A note on the jump-equilibrium model. Actuarial Research Report No. 87, The City University, London, UK.

Huberman, G. (1982). A simple approach to arbitrage pricing theory. *Journal of Economic Theory*, 28: 183-191.

Huizinga, J. and Mishkin, F.S. (1984). Inflation and real interest rates on assets with different risk characteristics. *Journal of Finance*, 39: 699-714.

Hull, J.C. (1993). *Options, futures, and other derivative securities*. Second Edition. Prentice-Hall, Englewood Cliffs, NJ.

Hürlimann, W. (1992). Numerical evaluation of the Wilkie inflation model. *Insurance: Mathematics and Economics*, 11: 311-314.

Hutchison, T.W. (1938). *The significance and basic postulates of economic theory*. Reprinted 1965. Augustus M. Kelly, New York, NY.

Hutchison, T.W. (1994). Ends and means in the methodology of economics, in Backhouse, R.E., ed., *New directions in economic methodology*. Routledge, London, UK.

Institute of Actuaries (1996). *Manual of actuarial practice*. Supplement 3. Institute of Actuaries, London, and Faculty of Actuaries, Edinburgh, UK.

Jones, P.D. (1993). The expected return on United Kingdom equities and an implication for pension fund valuations. *Journal of the Institute of Actuaries*, 120: 253-309.

Kahneman, D., Slovic, P., and Tversky, A., eds. (1982). *Judgment under uncertainty: heuristics and biases*. Cambridge University Press, New York, US.

Kendall, M.G. (1953). The analysis of economic time-series—part I: prices. *Journal of the Royal Statistical Society, Series A*, 116: 11-34.

Keynes, J.M. (1923). *A tract on monetary reform.* Reprinted 1971 in The collected writings of John Maynard Keynes, Volume 4. Macmillan for the Royal Economic Society, London, UK.

Keynes, J.M. (1936). *The general theory of employment, interest and money.* Reprinted 1973 in The collected writings of John Maynard Keynes, Volume 7. Macmillan for the Royal Economic Society, London, UK.

Keynes, J.N. (1891). *The scope and method of political economy.* Fourth Edition 1917. Macmillan, London, UK.

Kitts, A. (1988). Applications of stochastic financial models: a review. Department of Social Statistics, University of Southampton, UK.

Kitts, A. (1990). Comments on a model of retail price inflation. *Journal of the Institute of Actuaries*, 117: 407-412.

Kleidon, A.W. (1986). Variance bounds tests and stock price valuation models. *Journal of Political Economy*, 94: 953-1001.

Kleidon, A.W. (1988). The probability of gross violations of a present value variance inequality: reply. *Journal of Political Economy*, 96: 1093-1096.

Kothari, S.P., Shanken, J., and Sloan, R.G. (1995). Another look at the cross-section of expected stock returns. *Journal of Finance*, 50: 185-224.

Kuhn, T.S. (1970). *The structure of scientific revolutions.* Second Edition, enlarged. University of Chicago Press, Chicago, IL.

Labadie, P. (1989). Stochastic inflation and the equity premium. *Journal of Monetary Economics*, 24: 277-298.

Labour Market Trends, July 1914 to November 1995. Central Statistical Office, London, UK. (Previously known as: *The Board of Trade Labour Gazette*, July 1914 to June 1917; *The Labour Gazette*, July 1917 to May 1922; *The Ministry of Labour Gazette*, June 1922 to March 1949; *Ministry of Labour Gazette*, April 1949 to May 1968; *Employment and Productivity Gazette*, June 1968 to December 1970; *Department of Employment Gazette*, January 1971 to December 1978; *Employment Gazette*, January 1979 to October 1995.)

Lakatos, I. (1978). *The methodology of scientific research programs: philosophical papers.* Volume 1. Edited by Worrall, J. and Currie, G. Cambridge University Press, Cambridge, UK.

Lawson, T. (1994). A realist theory for economics, in Backhouse, R.E., ed., *New directions in economic methodology*. Routledge, London, UK.

Leamer, E.E. (1978). *Specification searches: ad hoc inference with nonexperimental data.* John Wiley and Sons, New York, NY.

Leamer, E.E. (1983). Let's take the con out of econometrics. *American Economic Review*, 73: 31-43. Reprinted in Granger, C.W.J., ed., 1990, *Modelling economic series.* Oxford University Press, Oxford, UK.

Leamer, E.E. (1985). Sensitivity analyses would help. *American Economic Review*, 75: 308-13. Reprinted in Granger, C.W.J., ed., 1990, *Modelling economic series*. Oxford University Press, Oxford, UK.

Lee, E.M. (1986). *An introduction to pension schemes*. Institute of Actuaries, London, and Faculty of Actuaries, Edinburgh.

LeRoy, S.F. (1989). Efficient capital markets and martingales. *Journal of Economic Literature*, 27: 1583-1621.

LeRoy, S.F. and Porter, R.D. (1981). The present-value relation: tests based on implied variance bounds. *Econometrica*, 49: 555-574.

Lewin, C.G., Carne, S.A., de Rivaz, N.F.C., Hall, R.E.G., McKelvey, K.J., and Wilkie, A.D. (1995). Capital Projects. *British Actuarial Journal*, 1: 155-330.

Limb, A.P., Hardie, A.C., Loades, D.H., Lumsden, I.C., Mason, D.C., Pollock, G., Robertson, E.S., Scott, W.F., and Wilkie, A.D. (1986). The solvency of life assurance companies. *Transactions of the Faculty of Actuaries*, 39: 251-340.

Lo, A.W. and MacKinlay, A.C. (1988). Stock market prices do not follow random walks: evidence from a simple specification test. *Review of Financial Studies*, 1: 41-66.

Lucas, R.E. (1976). Econometric policy evaluation: a critique, in Brunner, K. and Meltzer, A.H., eds., *The Phillips curve and labour markets*. North-Holland, Amsterdam.

Ludvik, P.M. (1993). The Wilkie model revisited. *Proceedings of the 3rd AFIR International Colloquium*, 2: 717-724.

McAleer, M., Pagan, A.R., Volker, P.A. (1985). What will take the con out of econometrics? *American Economic Review*, 75: 293-307. Reprinted in Granger, C.W.J., ed., 1990, *Modelling economic series*. Oxford University Press, Oxford, UK.

McCloskey, D.N. (1986). *The rhetoric of economics*. Harvester Wheatsheaf, Brighton, UK.

McLeish, D.J.D. and Stewart, C.M. (1993). The supervision and control of pension funding in the United Kingdom. *Journal of the Institute of Actuaries*, 120: 67-129.

Mäki, U. (1994). Reorienting the assumptions issue, in Backhouse, R.E., ed., *New directions in economic methodology*. Routledge, London, UK.

Makridakis, S., Chatfield, C., Hibon, M., Lawrence, M., Mills, T.C., Ord, J.K., and Simmons, L.F. (1993). The M2-Competition: a real-time judgmentally based forecasting study. *International Journal of Forecasting*, 9: 5-29.

Malkiel, B.G. (1990). *A random walk down Wall Street*. W.W. Norton & Company, New York, NY.

Mankiw, N.G. and Miron, J.A. (1986). The changing behaviour of the term structure of interest rates. *Quarterly Journal of Economics*, 101: 211-228.

Mankiw, N.G., Romer, D., and Shapiro, M.D. (1985). An unbiased reexamination of stock market volatility. *Journal of Finance*, 40: 677-689.

Marsh, T.A. and Merton, R.C. (1986). Dividend volatility and variance bounds tests for the rationality of stock market prices. *American Economic Review*, 76: 483-498.

Marshall, D.A. (1992). Inflation and asset returns in a monetary economy. *Journal of Finance*, 47: 1315-1342.

Mehra, R. and Prescott, E.C. (1985). The equity premium: a puzzle. *Journal of Monetary Economics*, 15: 145-161.

Mehta, S.J.B. (1992). Allowing for asset, liability and business risk in the valuation of a life office. *Journal of the Institute of Actuaries*, 119: 385-455.

Milgrom, P. and Stokey, N. (1982). Information, trade and common knowledge. *Journal of Economic Theory*, 26: 17-27.

Mitchell, B.R. (1962). *Abstract of British historical statistics.* Cambridge University Press, Cambridge, UK.

Mitchell, B.R. and Jones, H.G. (1971). *Second abstract of British historical statistics.* Cambridge University Press, Cambridge, UK.

Mizon, G.E. (1977). Inferential procedures in nonlinear models: an application in a UK industrial cross section study of factor substitution and returns to scale. *Econometrica*, 45: 1221-1242.

Modigliani, F. and Miller, M.H. (1958). The cost of capital, corporation finance and the theory of investment. *American Economic Review*, 48: 261-297.

Morgan, M.S. (1990). *The history of econometric ideas.* Cambridge University Press, Cambridge, UK.

Needleman, P.D. and Roff, T.A. (1995). Asset shares and their use in the financial management of a with-profits fund. *British Actuarial Journal*, 1: 603-688.

Nelson, C.R. (1972). The predictive performance of the FRB-MIT-PENN model of the U.S. economy. *American Economic Review*, 62: 902-917.

Nowell, P., McCutcheon, J., Tompkins, P., Middlemiss, K., Scott, P., Hitchcox, A., Goford, J., Dash, A., Battams, E., Milne, R., Goodwin, E., and Campbell, G. (1995). Future of the Profession: a study looking out to 2005 to identify challenges for the Profession. Institute of Actuaries, London, and Faculty of Actuaries, Edinburgh.

O'Donnell, R.M. (1989). *Keynes: philosophy, economics and politics: the philosophical foundations of Keynes's thought and their influence on his economics and politics*. Macmillan, London, UK.

Ohlson, J.A. (1977). Risk-aversion and the martingale property of stock prices: comments. *International Economic Review*, 18: 229-234.

Ong, A.S.K. (1994). A stochastic model for treasury-bills: an extension to Wilkie's model. Actuarial Research Report No. 68, Department of Actuarial Science and Statistics, The City University, London, UK.

Ong, A.S.K. (1996). Asset allocation decision models in life assurance. Unpublished PhD thesis, Department of Actuarial Science and Statistics, The City University, London, UK.

Osborn, D.R. (1990). A survey of seasonality in UK macroeconomic variables. *International Journal of Forecasting*, 6: 327-336.

Pagan, A.R. (1987). Three econometric methodologies: a critical appraisal. *Journal of Economic Surveys*, 1: 3-24. Reprinted in Granger, C.W.J., ed., 1990, *Modelling economic series.* Oxford University Press, Oxford, UK.

Palm, F.C. and Zellner, A. (1992). To combine or not to combine? Issues of combining forecasts. *Journal of Forecasting*, 11: 687-701.

Pentikäinen, T., Bonsdorff, H., Pesonen, M., Pukkila, T., Ranne, A., Rantala, J., Ruohonen, M., and Sarvamaa, S. (1994). On the asset models as a part of all-company insurance analysis. *Proceedings of the 4th AFIR International Colloquium*, 3: 1471-1501.

Pentikäinen, T., Bonsdorff, H., Pesonen, M., Pukkila, T., Ranne, A., Rantala, J., Ruohonen, M., and Sarvamaa, S. (1995). On stochastic insurance company models. *Transactions of the 25th International Congress of Actuaries*, 3: 217-241.

Pepper, G. (1994). *Money, credit and asset prices*. Macmillan, London, UK.

Pesaren, M.H. and Timmermann, A. (1994). Forecasting stock returns: an examination of stock market trading in the presence of transaction costs. *Journal of Forecasting*, 13: 335-367.

Pesaren, M.H. and Timmermann, A. (1995). Predictability of stock returns: robustness and economic significance. *Journal of Finance*, 50: 1201-1228.

Phelps Brown, E.H. and Hopkins, S.V. (1956). Seven centuries of the prices of consumables, compared with builder's wages. *Economica*, 23: 296-312.

Popper, K.R. (1959). *The logic of scientific discovery*. Reprinted 1992. Routledge, New York, NY.

Poterba, J.M. and Summers, L.H. (1988). Mean reversion in stock prices: evidence and implications. *Journal of Financial Economics*, 22: 27-59.

Pratten, C.F. (1993). *The stock market*. Cambridge University Press, Cambridge, UK.

Priestley, M.B. (1980). State-dependent models: a general approach to non-linear time series analysis. *Journal of Time Series Analysis*, 1: 47-71.

Pukkila, T., Ranne, A., and Sarvamaa, S. (1994). On stochastic modeling of inflation. *Proceedings of the 4th AFIR International Colloquium*, 2: 589-608.

Purchase, D.E., Fine, A.E.M., Headdon, C.P., Hewitson, T.W., Johnson, C.M., Lumsden, I.C., Maple, M.H., O'Keeffe, P.J.L., Pook, P.J., and Robinson, D.G. (1989). Reflections on resilience: some considerations of mismatching tests, with particular reference to non-linked long-term insurance business. *Journal of the Institute of Actuaries*, 116: 347-452.

Redington, F.M. (1952). Review of the principles of life-office valuations. *Journal of the Institute of Actuaries*, 78: 286-340.

Redington, F.M. (1982). The phase of transition: an historical essay. *Journal of the Institute of Actuaries*, 109: 83-96.

Redington, F.M. (1983). Prescience & nescience, in *A ramble through the actuarial countryside: the collected papers, essays and speeches of Frank Mitchell Redington*, edited by Chamberlin, G. 1986. Staple Inn Institute of Actuaries Students' Society, London, UK.

Richardson, M. and Stock, J.H. (1989). Drawing inferences from statistics based on multiyear asset returns. *Journal of Financial Economics*, 25: 323-348.

Rietz, T.A. (1988). The equity risk premium: a solution. *Journal of Monetary Economics*, 22: 117-131.

Roecker, E.B. (1991). Prediction error and its estimation for subset-selected models. *Technometrics*, 33: 459-468.

Roll, R. (1977). A critique of the asset pricing theory's tests: part I: on past and potential testability of the theory. *Journal of Financial Economics*, 4: 129-176.

Rose, A.K. (1988). Is the real interest rate stable? *Journal of Finance*, 43: 1095-1112.

Rosenberg, A. (1992). *Economics - mathematical politics or science of diminishing returns?* University of Chicago Press, Chicago, IL.

Ross, M.D. (1989). Modelling a with-profits life office. *Journal of the Institute of Actuaries*, 116: 691-715.

Ross, S.A. (1978). Mutual fund separation in financial theory—the separating distributions. *Journal of Economic Theory*, 17: 254-286.

Ross, S.A. (1989). Intertemporal asset pricing, in Bhattacharya, S. and Constantinides, G.M., eds., *Theory of valuation: frontiers of modern financial theory*. Rowman & Littlefield, Savage, MD.

Samuelson, P.A. (1983). *Foundations of economic analysis*. Enlarged Edition. Harvard University Press, Cambridge, MA.

SAS Institute Inc. (1988). *SAS/ETS® User's guide*. Version 6, First Edition. SAS Institute Inc., Cary, NC.

Savin, N.E. (1984). Multiple hypothesis testing, in Griliches, Z. and Intriligator, M.D., eds., *Handbook of econometrics*. Volume 2. North-Holland, Amsterdam.

Scheinkman, J.A. (1989). Market incompleteness and the equilibrium valuation of assets, in Bhattacharya, S. and Constantinides, G.M., eds., *Theory of valuation: frontiers of modern financial theory*. Rowman & Littlefield, Savage, MD.

Schoemaker, P.J.H. (1993). Multiple scenario development: its conceptual and behavioural foundation. *Strategic Management Journal*, 14: 193-213.

Schwartz, P. (1991). *The art of the long view*. Doubleday, New York, NY.

Shanken, J. (1985). Multi-beta CAPM or equilibrium-APT?: a reply. *Journal of Finance*, 40: 1189-1196.

Shanken, J. (1992). The current state of the arbitrage pricing theory. *Journal of Finance*, 47: 1569-1574.

Sherris, M. (1992). Reserving for deferred capital gains tax (an application of option pricing theory). *Journal of the Institute of Actuaries*, 119: 45-67.

Shiller, R.J. (1979). The volatility of long-term interest rates and expectations models of the term structure. *Journal of Political Economy*, 87: 1190-1219.

Shiller, R.J. (1981). Do stock prices move too much to be justified by subsequent changes in dividends? *American Economic Review*, 71: 421-436.

Shiller, R.J. (1990). The term structure of interest rates, in Friedman, B.M. and Hahn, F.H., eds., *Handbook of monetary economics*. Volume 1. North-Holland, Amsterdam.

Shiller, R.J. and Beltratti, A.E. (1992). Stock prices and bond yields: can their comovements be explained in terms of present value models? *Journal of Monetary Economics*, 30: 25-46.

Shome, D.K., Smith, S.D., and Pinkerton, J.M. (1988). The purchasing power of money and nominal interest rates: a re-examination. *Journal of Finance*, 43: 1113-1125.

Silvey, S.D. (1975). *Statistical inference*. Chapman and Hall, London, UK.

Sims, C.A. (1980). Macroeconomics and reality. *Econometrica*, 48: 1-48.

Sims, C.A. (1991). Comments: 'Empirical analysis of macroeconomic time series: VAR and structural models' by Clements, M.P. and Mizon, G.E. *European Economic Review*, 35: 922-932.

Singleton, K.J. (1989). Modeling the term structure of interest rates in general equilibrium, in Bhattacharya, S. and Constantinides, G.M., eds., *Theory of valuation: frontiers of modern financial theory*. Rowman & Littlefield, Savage, MD.

Singleton, K.J. (1990). Specification and estimation of intertemporal asset pricing models, in Friedman, B.M. and Hahn, F.H., eds., *Handbook of monetary economics*. Volume 1. North-Holland, Amsterdam.

Smith, A.D. (1996). How actuaries can use financial economics. *British Actuarial Journal*. (forthcoming)

Spanos, A. (1986). *Statistical foundations of econometric modelling*. Cambridge University Press, Cambridge, UK.

Spanos, A. (1988). Towards a unifying methodological framework for econometric modelling. *Economic Notes, Sienna*, 1: 1-28. Reprinted in Granger, C.W.J., ed., 1990, *Modelling economic series*. Oxford University Press, Oxford, UK.

Statistical Abstract for the United Kingdom. In each of the last fifteen years from 1910-1924. (1926). Number 69. H.M. Stationary Office, London, UK.

Statistical Summary, January 1932 to December 1945. Bank of England, London, UK.

Steerneman, T. and Rorijs, G. (1986). Pitfalls for forecasters, in Dijkstra, T.K., ed., *On model uncertainty and its statistical implications*. Workshop proceedings. Springer-Verlag, Berlin.

Sterling, T.D. (1959). Publication decisions and their possible effects on inferences drawn from tests of significance - or vice versa. *Journal of the American Statistical Association*, 54: 30-34.

Stiglitz, J.E. (1989). Mutual funds, capital structure, and economic efficiency, in Bhattacharya, S. and Constantinides, G.M., eds., *Theory of valuation: frontiers of modern financial theory*. Rowman & Littlefield, Savage, MD.

The Stock Exchange Daily Official List, various issues 30 July 1914 to 30 April 1994. The Stock Exchange, London, UK.

Thomson, C., Baron, I., Dann, M., Hardy, M., Hardwick, S., MacDonald, A., Sanders, A., and Wilson, D. (1995). Stochastic modelling for life offices. Institute of Actuaries, London, and Faculty of Actuaries, Edinburgh.

Thomson, R.J. (1994). A stochastic investment model for actuarial use in South Africa. *Proceedings of the 1994 Convention of the Actuarial Society of South Africa*: 1-62.

Thornton, P.N. and Wilson, A.F. (1992). A realistic approach to pension funding. *Journal of the Institute of Actuaries*, 119: 229-312.

Tirole, J. (1982). On the possibility of speculation under rational expectations. *Econometrica*, 50: 1163-1181.

Todd, R.M. (1984). Improving economic forecasting with Bayesian vector autoregression. *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall 1984: 18-29. Reprinted in Granger, C.W.J., ed., 1990, *Modelling economic series*. Oxford University Press, Oxford, UK.

Tong, H. (1990). *Non-linear time series: a dynamical system approach*. Oxford University Press, Oxford, UK.

Turner, D.S. (1990). The role of judgement in macroeconomic forecasting. *Journal of Forecasting*, 9: 315-345.

Tversky, A. and Kahneman, D. (1974). Judgment under uncertainty: heuristics and biases. *Science*, 185: 1124-1131. Reprinted in Kahneman, D., Slovic, P., and Tversky, A., eds., 1982, *Judgment under uncertainty: heuristics and biases*. Cambridge University Press, New York, US.

Vercelli, A. (1991). *Methodological foundations of macroeconomics: Keynes and Lucas*. Cambridge University Press, Cambridge, UK.

Wack, P. (1985a). Scenarios: uncharted waters ahead. *Harvard Business Review*, 63: 73-89.

Wack, P. (1985b). Scenarios: shooting the rapids. *Harvard Business Review*, 63: 139-150.

Weigend, A.S. and Gershenfeld, N.A. eds. (1994). *Time series prediction: forecasting the future and understanding the past*. Sante Fe Institute Studies in the Sciences of Complexity, Proccedings Volume XV, Addison-Wesley, Reading, MA.

Weil, P. (1989). The equity premium puzzle and the risk-free rate puzzle. *Journal of Monetary Economics*, 24: 401-421.

Wilber, C.K. and Harrison, R.S. (1978). The methodological basis of institutional economics: pattern model, storytelling, and holism. *Journal of Economic Issues*, 12: 61-89.

Wilkie, A.D. (1981). Indexing long-term financial contracts. *Journal of the Institute of Actuaries*, 108: 299-360.

Wilkie, A.D. (1984). Steps towards a comprehensive stochastic investment model. *Occasional Actuarial Research Discussion Paper*, 36. Institute of Actuaries, London, UK.

Wilkie, A.D. (1986a). A stochastic investment model for actuarial use. *Transactions of the Faculty of Actuaries*, 39: 341-403.

Wilkie, A.D. (1986b). Some applications of stochastic investment models. *Journal of the Institute of Actuaries Students' Society*, 29: 25-51.

Wilkie, A.D. (1987a). Stochastic investment models - theory and applications. *Insurance: Mathematics and Economics*, 6: 65-83.

Wilkie, A.D. (1987b). An option pricing approach to bonus policy. *Journal of the Institute of Actuaries*, 114: 21-90.

Wilkie, A.D. (1992). Stochastic investment models for XXIst century actuaries. *Transactions of the 24th International Congress of Actuaries*, 5: 119-137.

Wilkie, A.D. (1995a). The risk premium on ordinary shares. *British Actuarial Journal*, 1: 251-330.

Wilkie, A.D. (1995b). More on a stochastic asset model for actuarial use. *British Actuarial Journal*, 1: 777-964.

Wilkie, A.D., Tilley, J.A., Arthur, T.G., and Clarkson, R.S. (1993). This house believes that the contribution of actuaries to investment could be enhanced by the work of financial economists: a debate. *Journal of the Institute of Actuaries*, 120: 393-414.

Wise, A.J. (1984a). A theoretical analysis of the matching of assets to liabilities. *Journal of the Institute of Actuaries*, 111: 375-402.

Wise, A.J. (1984b). The matching of assets to liabilities. *Journal of the Institute of Actuaries*, 111: 445-501.

Wise, A.J. (1987a). Matching and portfolio selection: part 1. *Journal of the Institute of Actuaries*, 114: 113-133.

Wise, A.J. (1987b). Matching and portfolio selection: part 2. *Journal of the Institute of Actuaries*, 114: 551-568.

Wisman, J.D. and Rozansky, J. (1991). The methodology of institutionalism revisited. *Journal of Economic Issues*, 25: 709-737.

Working, H. (1960). Note on the correlation of first differences of averages in a random chain. *Econometrica*, 28: 916-918.