Stochastic process model for timber-concrete composite beam deterioration

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ABSTRACT: The aim of this paper is to present a new stochastic process model that will capture the true nature of deterioration of timber-concrete composite beams. Composite elements such as timber-concrete composite sections are designed to take advantage of compatibility of materials under sustained loading. Applied sustained load on timber-concrete composite structures causes gradual increase of deformation and deflection. In particular for timber-concrete composite beams, component materials will deteriorate at different pace over the life-cycle. In order to enable efficient management of structures in terms of required maintenance, repair and/or replacement, it is essential to be able to capture the uncertain nature of the deterioration process. We focus on modeling the deterioration of mid-span deflection of the timber-concrete composite beam over long term under sustained load. As the increasing deflection of the timber-concrete composite beam over time is generally uncertain and non-decreasing, it can best be regarded as a continuous gamma process. Examples of continuous gamma process representation have been included.

1 INTRODUCTION

Composite structural systems are designed to take advantage of compatibility of materials under loading. The timber-concrete composite (TCC) structure is a structural system in which a timber beam is connected to an upper concrete flange using different types of connectors. The best properties of both materials can be exploited because bending and tensile forces induced by gravity loads are resisted primarily by the timber and compression by the concrete topping systems, while the connection system transmits the shear forces between the two components. In Yeoh et al. (2011) is given a survey on the state-of-the-art of timber-concrete composite research in the recent years.

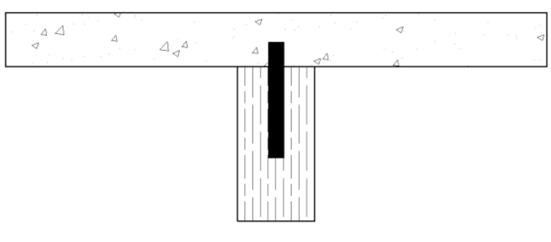

Figure 1. Cross section of timber-concrete composite beam.

In the last decade, timber-concrete composite sections are increasingly used in Europe due to sustainable nature of constituent materials, aesthetics, light weight structures, fast construction times, etc. As the market share for this form of construction is increasing, so is the possibility of costly remedial actions for non-conforming sections. Timber-concrete composite structural systems are successfully used in highway bridges, hangar aprons, wharves, piers, platforms and as well in upgrading and post-strengthening of existing timber floors in residential/office buildings as well as new constructions for buildings.

By connecting a concrete slab to timber beam it is possible to significantly increase the stiffness of the section compared to timber floors while at the same time improve the acoustic separation and increase the thermal mass, important to reduce the energy consumption needed to heat and cool the building. On the other hand, by replacing the lower part of a reinforced concrete section, which is ineffective because of the cracking induced by tensile stresses, with timber joists or a solid timber deck, it is possible to achieve better performing structure. In particular such composite sections enable rapid erection of the timber part, particularly if prefabricated offsite, due to its lower weight, benefits of permanent formwork for the concrete topping, reduced load imposed on the foundation, reduced mass and, hence, reduced seismic effects, etc. (Lukaszewska et al. 2008)

The increase in depth of the cross section of a composite element compared to the cross sections of its individual sub-elements results in a longer internal moment arm. Consequently, the normal strains resulting from an arbitrary external load are reduced within the structure. For a very stiff connection, the concrete slab is mainly exposed to compression, while tensile stresses are concentrated in the wooden part. This means that both materials, concrete with its high compression capacity and timber with its high tensile capacity in the grain direction, are being used effectively.

It is apparent that the critical part of any composite structure is the connection between the constituent materials. Shear connector, apart from being stiff, needs to have a certain strength or shear capacity. A failure of the shear connector in a composite structure could lead to a global collapse of the beam if the individual components are unable to take the full load effects. The balance between the number anf the stiffness of the connectors is a matter of design so in practice for exisitn beams there will be a rather large variability in the composition of the section and therefore its capacity. In addition to a high level of stiffness and strength, a good post-peak behaviour is often a concern for a shear connector. A desirible post-peak behaviour for the connectors is considered to be ductile as it gives users a warning in case of an imminent collapse and it is preferred to the failure of concrete or tensile/bending-failure mode of wood that are sudden and brittle. Inevitably over the long term behavior of shear connectors is uncertain and site specific so sophisticated predictions for the section capacity for an exisitng beam are needed.

In the first timber-concrete composite structures the connection systems used were influenced by traditional timber construction. There are many joints traditionally used in timber structures that have been adapted to be used in timber-concrete structures. Among those, nails are most frequently used connector at present in timber-concrete applications. Other types of fasteners, such as screws, dowels or bolts, have also been used, but they have in most cases the disadvantage of requiring pre-drilling. One attempt to improve the mechanical behaviour of dowels was made by using them combined with epoxy resin. It is useful to note that long-term tests in both uncontrolled and controlled climatic conditions were performed on dowel-type fasteners by Dias (2005).

2 LONG-TERM BEHAVIOUR OF TIMBER-CONCRETE COMPOSITE

As with other structures sustained load on timber-concrete composite structures causes gradual increase of deformation and deflection. Changes in the moisture content of the timber section caused by variations in relative humidity and variations in the environmental temperature will accelerate the deformations and associated reductions in strength, and may lead to excessive deflection in the long-term, Fragiacomo, (2006). Current design code, Eurocode 5, suggests limit values of *l/250* to *l/200* for long-term deflections of simply supported beams where *l* is the span length. For medium to long span beams and/or heavy environmental conditions (e.g. bridges or roof structures), the serviceability limit state of maximum deflection may be the most severe design criterion.

In order to consider fully the long term behaviour time dependent behaviour of constituent materials needs to be considered. Concrete displays creep and shrinkage effects however timber behaviour is sensitive to the environmental effects such as the moisture content. Published records of long-term loading experiments have demonstrated that connections can creep, even more than timber, and the viscous behavior is influenced by timber moisture content changes. The environmental thermo-hygrometric variations affect the behavior of TCC's since they cause inelastic strains in timber and concrete, influence the elastic modulus of timber and the mechano-sorptive creep of all components. Because of the long term processes that occur in the component materials the stress and strain distribution within the timber concrete composite section change in time, Fragiacomo (2006).

Standard four loading conditions are often used to establish the stress and strain distribution in a timber-concrete composite beam. These are dead and live loads, those arising from different response to environmental conditions from both concrete slab and timber beam. As a result the overall deflection of the composite beam will be affected. Hence, it is often recommended that the loading conditions to be considered are:

-permanent and variable actions;

-inelastic strains due to concrete shrinkage,

-annual variations of environmental temperature and relative humidity:

-daily variations of temperature,

For example a decrease in temperature produces a larger shrinkage of concrete slab compared to timber beam, with an overall increase in deflection. In the same manner, an increase in relative humidity leads to timber moistening with swelling and an overall increase in deflection of the beam.

The effects due to various load components cn be superimposed. In practice, sustained load conditions, such as those above that increase the deflection and modify the stress distribution in time, are considered by using the well known "Effective Modulus Method". The inelastic strains due to environmental variations of relative humidity and temperature are calculated by using the elastic solution for composite beams with flexible connection. A simplified approach to evaluate the long-term response of timber-concrete composite beams as proposed by Fragiacomo and Ceccotti (2006) is implemented.

In order to enable efficient management of structures in terms of required maintenance, repair and/or replacement, it is essential to be able to capture the uncertain nature of the deterioration process. In particular for timber-concrete composite beams, component materials will deteriorate at different pace over the life-cycle. Any analytical model designed to estimate the long term behaviour of timber-concrete composite sections has to include such diverse effects that develop at different times in the lifecycle such as timber creep , timber shrinkage and swelling following reductions and increases in its moisture content, and/or temperature, respectively, etc. Equally for concrete effects of creep, drying shrinkage, thermal strains and cracking in tensile zones have to be included. Furthermore the shear connectors are flexible, apart from few exceptions, and exhibit both creep and mechano-sorptive creep. However, given the complexities of the phenomena involved, as mentioned above, closed form solutions will have limited validity in real conditions, and since it is impossible to test sufficiently diverse systems under all combinations of environmental conditions they may encounter, numerical modeling is often applied. Furthermore, in ideall circumstances one would wish to consider as many of the significant variables as possible and equally the measure of uncertainty associated with each variable. Since early seventees, structural reliability methods have emerged as a partial solution as they have enabled inclusion of uncertainties at least in some form and usually by introducing relevant random variables. In recent years, however, it has emerged that for decision making purpose random variable models have limitations as they are unable to capture temporal effects that could be very relevant for long lifecycles such as those in construction. These issues are even further relevant when a composite section such as the one considered here is concerned. Rather slowly, but since the late nineties, continuous stochastic processess have emerged as an alternative approach to random variable models to provide predicted profiles for componental deterioration. As an alternative to continuous stochastic processes discrete stochastic process have also been in use where management of infrastructure was of concern but these models can sometimes result in udully conservative estimates, Ohadi and Micic (2011).

3 CONTINUOUS GAMMA PROCESS MODELLING

The continuous gamma process belongs to a general class of stochastic processes, referred to as the Markov process. Gamma process is a stochastic process with independent non-negative increments that are gamma distributed with identical scale parameter. Abdel-Hameed (1977), was the first to propose the gamma process as a proper model for deterioration occurring random in time. The gamma process is suitable to model gradual damage monotonically accumulating over time, such as wear, fatigue, corrosion, crack growth, erosion, consumption, creep, swell, etc. An advantage of gamma processes is that the required calculations are relatively straightforward, Van Noortwijk (2009).

Firstly, the gamma process is defined in mathematical terms as follows. A random quantity X has a gamma distribution with shape parameter k > 0 and scale parameter $\theta > 0$ if its probability density function is given by:

$$Ga(x|k,\theta) = \frac{1}{\Gamma(k)\cdot\theta^{k}} x^{k-1} \exp\left\{\frac{-x}{\theta}\right\} \quad (1)$$

where

$$\Gamma(a) = \int_{z=0}^{\infty} z^{a-1} e^{-z} dz \quad (2)$$

is the gamma function for $a > 0$. Furthermore, we assume $k(t)$ to be a non-decreasing, right-continuous, real-valued function for $t \geq 0$, with $k(0) \equiv 0$. The gamma process with shape function $k(t) > 0$ and scale parameter $\theta > 0$ is considered a continuous-time stochastic process $\{X(t);\ t \geq 0\}$ with the following properties:

(i) $X(0) = 0$ with probability one;

(ii) $\Delta X(t) = X(t+\Delta t) - X(t) \sim Ga(\Delta k(t), \theta)$,

where $\Delta k(t) = k(t+\Delta t) - \Delta k(t)$ for any $t \geq 0$ and $\Delta t > 0$; (3)

(iii) For any choice of $n \geq 1$ and $0 \leq t_0 < t_1 < \ldots < t_n < \infty$, the random variables $X(t_0)$, $X(t_1) - X(t_0)$, …, $X(t_n) - X(t_{n-1})$ are independent.

Let $X(t)$ denote the deterioration at time t, $t \geq 0$, and let the probability density function of $X(t)$, in conformity with the definition of the gamma process, be given by

$$f_{X(t)}(x) = Ga(x|k(t),\theta) \quad (4)$$

with expectation and variance

$$E(X(t)) = k(t)\cdot\theta,\ \ Var(X(t)) = k(t)\cdot\theta^{2} \quad (5)$$

The coefficient of variation is defined by the ratio of the standard deviation and the mean

$$Cov(X(t)) = \frac{\sqrt{Var(X(t))}}{E(X(t))} = \frac{1}{\sqrt{k(t)}} \quad (6)$$

It is evident here that in comparison with standard random variable representation for deterioration where coefficient of variation is constant stochastic process representation offers improved model of time related effects.

3.1 *Parameter estimation of gamma process models*

For modeling of the temporal variability in the deterioration, key input is the trend of the expected deterioration increasing over time. Empirical studies show that the expected deterioration at time t can often be represented as a power law:

$$E(X(t)) = k(t) \cdot \theta = ct^b \cdot \theta \quad (7)$$

for some physical constants $c > 0$, $b > 0$ and $\theta > 0$. There is often engineering knowledge available about the shape of the expected deterioration in terms of the parameter b, so that this parameter may be assumed constant. Some examples of expected deterioration according to a power law are the expected degradation of concrete due to corrosion of reinforcement (linear: $b = 1$; sulphate attack (parabolic: $b = 2$); diffusion-controlled ageing (square root: $b = 0.5$; creep ($b = 1/8$), and the expected scour-hole depth ($b = 0.4$). The gamma process is called stationary if the expected deterioration is linear in time, i.e., when $b = 1$ and non-stationary when $b \neq 1$, Van Noortwijk (2009).

For the practical consideration of expected deterioration in terms of power law, parameters c and θ have to be assessed using available techniques. These could be expert judgment and/or statistics. Cinlar et al. (1977) demonstrated how a non-stationary gamma process can be transformed into a stationary gamma process and how the parameters of a gamma process can be estimated using the Method of Moments and the Method of Maximum Likelihood.

In order to apply the gamma process model to practical examples, statistical methods for the deterioration parameter estimation are required. We can assume that a typical data set for the structure consists of inspection times t_i, $i = 1, \ldots, n$, where $0 = t_0 < t_1 < t_2 < \cdots < t_n$, and corresponding observations of the cumulative amounts of deterioration x_i, $i = 1, \ldots, n$, where $0 = x_0 \le x_1 \le x_2 \le \cdots \le x_n$. For our gamma process model we require the shape function $k(t) = ct^b$ and scale parameter θ. We can assume that the value of the power b is known, but c and θ are unknown. The two most common methods of parameter estimation are Maximum Likelihood Method and Method of Moments. Here we apply Maximum Likelihood Method. The latter is considered more appropriate and applied here.

The maximum-likelihood estimators for c and θ can be obtained by maximising the logarithm of the likelihood function of the increments. The likelihood function of the observed deterioration increments $\delta_i = x_i - x_{i-1}$, $i = 1, \ldots, n$, is a product of independent gamma densities:

$$\begin{aligned} l(\delta_1,\ldots,\delta_n | c,\theta) &= \prod_{i=1}^{n} f_{X(t_i)-X(t_{i-1})}(\delta_i) = \\ &= \prod_{i=1}^{n} Ga(\delta_i | c \cdot (t_i^b - t_{i-1}^b), \theta) = \\ &= \prod_{i=1}^{n} \frac{1}{\Gamma(c \cdot (t_i^b - t_{i-1}^b)) \cdot \theta^{c \cdot (t_i^b - t_{i-1}^b)}} \cdot \\ &\cdot \delta_i^{c \cdot (t_i^b - t_{i-1}^b) - 1} \exp\left(\frac{-\delta_i}{\theta}\right) \end{aligned} \quad (8)$$

Further, we consider the first partial derivatives of the log-likelihood function of the increments with respect to c and θ

$$\frac{\partial \log l(\delta_1,\ldots,\delta_n | c,\theta)}{\partial c} = 0 \quad (9)$$

$$\frac{\partial \log l(\delta_1,\ldots,\delta_n | c,\theta)}{\partial \theta} = 0 \quad (10)$$

From the last two equations, the maximum-likelihood estimates c and θ can be obtained:

$$\hat{\theta} = \frac{x_n}{\hat{c} \cdot t_n^b} \quad (11)$$

$$\begin{aligned} &\sum_{i=1}^{n} [t_i^b - t_{i-1}^b] \{ \log \delta_i - \psi(\hat{c}[t_i^b - t_{i-1}^b]) \} = \\ &= t_n^b \log\left(\frac{x_n}{\hat{c} t_n^b}\right) \end{aligned} \quad (12)$$

where the function $\psi(\ )$ is digamma function and can be computed numerically.

Given the maximum-likelihood estimator of θ, the expected deterioration at time t can be written as

$$E[X(t)] = x_n \left[\frac{t}{t_n}\right]^b \quad (13)$$

Because cumulative amounts of deterioration are measured, the last inspection (at time t_n) contains the most information and the expected deterioration at the last inspection is:

$$E[X(t_n)] = x_n \quad (14)$$

4 APPLICATION OF GAMMA PROCESS FOR A SAMPLE COMPOSITE SECTION

In this paper we consider a timber-concrete composite section where glued steel bars are used as shear connectors. We focus on modeling the deterioration of mid-span deflection of the timber-concrete composite beam over time and under sustained load. Only if there is sufficient knowledge of the condition of the timber-concrete composite beam, further actions such as maintenance planning can be carried out in the most efficient way. Therefore, the progress of the deterioration of the considered beam and the estimate of its remaining lifetime, have to be quantified. As we mentioned earlier, given the complexities of the phenomena involved in long-term behaviour of timber-concrete composite sections, closed form solutions will have limited validity in real conditions. In practice it is impossible to set up experimental programme for such structures to reflect all permutations of environmental conditions they may encounter and often numerical modeling is applied, with aim to incorporate as many of the significant variables as possible. This is rather cumbersome approach. Our initial approach is to generate the deterioration model at time t from outcomes that can be a result from expert judgement. This is to simulate that the condition of the beam is monitored through periodic inspections and in that way inspections reveal the progress of the deterioration of the beam.

For now we follow the deterioration progress through the increase in mid-span deflection. This deflection measure for the timber-concrete composite beam over time is generally uncertain and non-decreasing, thus it can be regarded as a continuous gamma process. A timber-concrete composite beam will reach serviceability limit state when mid-span deflection reaches assumed limit values u_L that is say *l/250* for long-term deflections according to Eurocode 5. In our case, where a simply supported beam of span 4.2m is considered that limit value on the long term deflection is 16.8 mm. Due to the nature of this composite section the initial deflection is assumed to be the elastic deflection u_{el}. Thus we define mid-span deflection of the beam at time t as:

$$u(t) = u_{el} + X(t) \tag{14}$$

Where the $X(t)$ is the long term change (increase) in deflection. Thus we can identify the critical level of the stochastic process $X(t)$ as ρ

$$\rho = u_L - u_{el} \tag{15}$$

Once the deterioration process $X(t)$ reaches this critical level ρ, a timber-concrete composite beam will reach serviceability limit. Figure 2 shows trend of the expected progress of mid-span deflection of the timber-concrete composite beam over time under sustained load.

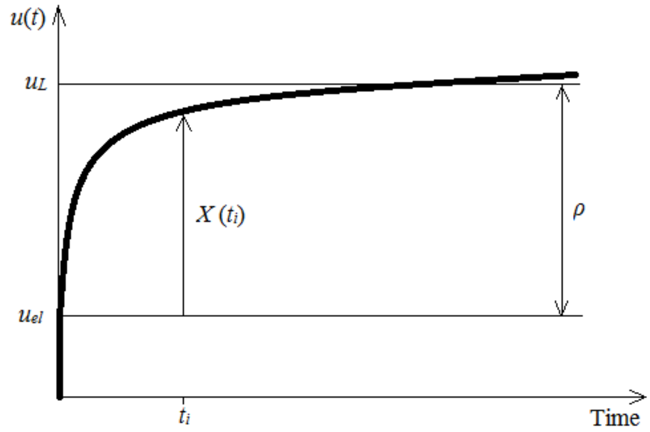

Figure 2. Trend of the expected mid-span deflection increasing over time under sustained load

We recall that the expected deterioration at time t can be described using a power law function and therefore the expected deterioration at time t can be written as ct^b for $c, b > 0$. In order to find the relation between the degradation level of mid-span deflection and time we first fitted a number of deterministic curves that present theoretical models of behaviour of the TCC beam over time. This way it was possible to apply least-square fitting approach to the available data to get power b. The parameter estimate of the power factor used here was $b = 1/8$, that suggests that the deterioration is non-linear in time and we can assume that the expected value of the stochastic gamma process follows a non-linear power function. Shape and scale parameters have been established using the method of maximum likelyhood. Thus with those parameters that define the continuous gamma process representation of deterioration for the generic timber-concrete composite section has been established.

In Table 1 the maximum likelihood estimates of the parameters of the gamma process in various years of observing.

Table 1.Gamma process parameters for timber concrete composite section estimated at different time for the generic structure.

Year	Shape	Scale
	k(t)	*Θ*
10	243.35	0.0372
20	448.67	0.0204
30	632.22	0.0144

Figure 3 demonstrates the use of Gamma process representation to predict expected deterioration $X(t)$ at certain target year from different inspection (information gathering) years.

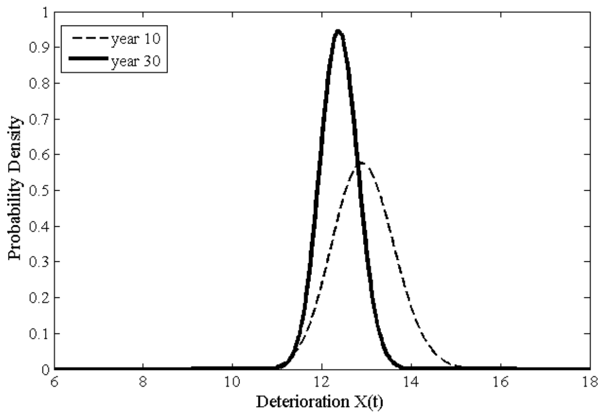

Figure 3. Comparison of expected deterioration $u(t)$ at year 60, using the prediction from years 10 and 30

It is evident that gamma process representation provides a refined prediction for deterioration process. As expected the increased number of inspections (by year 30) are reflected fully.

5 DISTRIBUTION OF SERVICEABILITY LIFETIME

For practice the estimate of the time when the structure would reach serviceability limit of some form would be of interest to owners and stake holders. The serviceability lifetime T can be defined as the first time when the sample path of $X(t)$ exceeds the limit ρ_L. Due to the gamma distributed deterioration, the serviceability lifetime can then be written as:

$$\begin{aligned} F_T(t) &= \Pr(T \le t \mid X(t) \ge \rho_L) = \\ &= \int_{x=\rho_L}^{\infty} f_{X(t)}(x)\,dx = \frac{\Gamma(k(t), \rho_L/\theta)}{\Gamma(k(t))} \end{aligned} \tag{16}$$

where

$$\Gamma(a,x) = \int_{t=x}^{\infty} t^{a-1} e^{-t}\,dt \tag{17}$$

is the incomplete gamma function for $x \ge 0$ and $a > 0$. Using the chain rule for differentiation, the probability density function of the serviceability lifetime T is

$$\begin{aligned} f_T(t) &= \frac{\partial}{\partial t}\left[\frac{\Gamma(k(t), \rho/\theta)}{\Gamma(k(t))}\right] = \\ &= \frac{\partial}{\partial t}\left[\frac{\Gamma(\tilde{k}, \rho/\theta)}{\Gamma(\tilde{k})}\right]\Bigg|_{\tilde{k}=k(t)} k'(t) \end{aligned} \tag{18}$$

under the assumption that the shape function $k(t)$ is differentiable. The partial derivative in Equation 18 can be calculated using numerical techniques. Following the recommendation from Van Noortwijk et al. 2007 using a series expansion and a continued fraction expansion, it is possible to compute the first and second partial derivatives with respect to x and a of the incomplete gamma integral, Equation 19.

$$P(a,x) = \frac{1}{\Gamma(a)} \int_{t=0}^{x} t^{a-1} e^{-t}\,dt = \frac{\Gamma(a) - \Gamma(a,x)}{\Gamma(a)} \tag{19}$$

The first two moments of the serviceability lifetime T are then expressed as

$$E(T) = \int_0^{\infty} (1 - F_T(t))\,dt \tag{20}$$

$$E(T^2) = 2\int_0^{\infty} t \cdot (1 - F_T(t))\,dt \tag{21}$$

which need to be evaluated numerically as well.

5.1 *Distribution of remaining serviceability lifetime*

The distribution of remaining lifetime, the remaining time to the certain critical level ρ_L of the gamma process, given its value at time τ, $X(\tau) = x_\tau < \rho_L$, is readily established using the independent increments property as

$$\begin{aligned} F_T(t|\tau) &= \Pr(T \le t | X(\tau) = x_\tau) = \\ &= \Pr(X(t) - X(\tau) \ge \rho_L - x_\tau) = \\ &= \frac{\Gamma(k(t-\tau), (\rho_L - x_\tau)/\theta)}{\Gamma(k(t-\tau))} \end{aligned} \tag{22}$$

for $t \ge \tau$. As inspection outcomes become available parameters of the gamma process can be updated and, consequently, the remaining serviceability lifetime distribution can be updated.

The probability density function of the remaining serviceability lifetime is

$$\begin{aligned} f_T(t|\tau) &= \frac{\partial}{\partial t}\left[\frac{\Gamma(k(t-\tau), (\rho_L - x_\tau)/\theta)}{\Gamma(k(t-\tau))}\right] = \\ &= \frac{\partial}{\partial t}\left[\frac{\Gamma(\tilde{k}, (\rho_L - x_\tau)/\theta)}{\Gamma(\tilde{k})}\right]\Bigg|_{\tilde{k}=k(t-\tau)} k'(t-\tau) \end{aligned} \tag{23}$$

under the assumption that the shape function $k(t\text{-}\tau)$ is differentiable. Probability density function of remaining serviceability lifetime has no closed form expression but it can be evaluated numerically as already explained.

Figure 4 presents the gamma process representation for the cumulative distribution function of remaining lifetime following the inspection data at certain year.

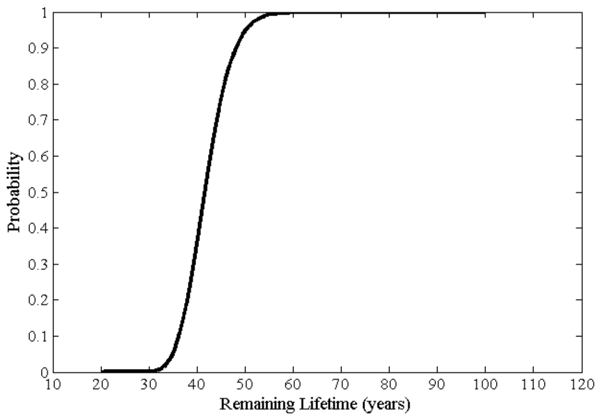

Figure 4. Cumulative distribution function for remaining serviceability lifetime using inspection outcomes in year 20

It is interesting to present remaining serviceability lifetime as the survival curve plotted on a log scale as shown in Figure 5.

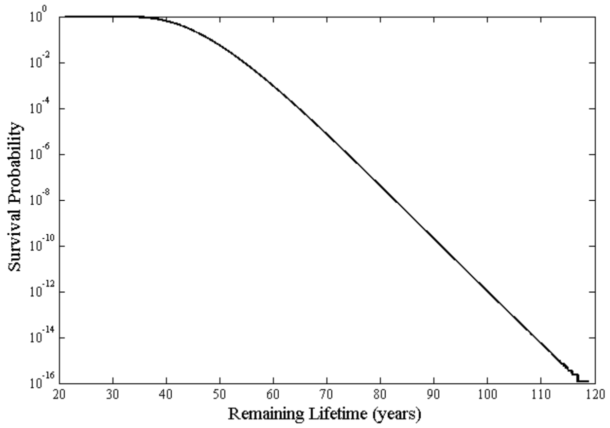

Figure 5. Survival function of remaining serviceability lifetime at year 20

The survival function shown in Figure 5 is obviously very useful for management of structures as it can be established as site specific on the basis of inspection outcomes. It remains to be considered in the future how to model the variability in quality of inspection techniques as it will have major impact on the survival function.

6 CONCLUSIONS

In this paper we have considered a new continous stochastic process model that can capture the true nature of deterioration of timber-concrete composite beams. As an illustrative example the gradual increase in mid-span deflection during application of sustained load on timber-concrete composite structures is considered. An explicit function has been identified for the long term effects of various deterioration mechanisms that act over the lifecycle on timber concrete composite simply supported beam deterioration. It has been identified that, for such beams, component materials will deteriorate at different pace over the life-cycle and the stochastic process approach would be able to capture site specific effects. The modelling is demonstrated in the paper by considering the effect of the variable time of inspection have been carried out. Application of continuous gamma process for deterioration modeling for target lifecycle is included in the paper. As the increasing deflection can lead to violation of serviceability limit state the remaining serviceability lifetime is of interest and therefore considered in this paper. Using the requirement of serviceability limit state the example of gamma process representation for the remaining life distribution function is included. Furthermore, it is demonstrated that it is possible to generate the ‘survival’ curve for the structure on the basis of site specific inspection and the selected serviceability criteria.

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