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ECONOMIC EVALUATION OF FINANCIAL FORECASTING

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DECLARATION

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ABSTRACT

This thesis examines the economic evaluation of forecasting strategies based on past prices, bringing together academics and practitioners techniques Forecasting methods based on past prices are convex and path-dependent dynamic strategies Therefore, they must be able to profitably exploit positive serial dependences in financial prices The most important measure of financial forecasting ability is the rate of return achieved by the predictor The expected return of forecasting strategies is first investigated by applying stochastic modelling Then, the presence of serial dependences in financial prices is tested by comparing expected and observed rates of returns of forecasting strategies

According to the academic literature, the expected return of investment strategies is best established by applying stochastic modelling That is done analytically for linear forecasters, assuming that the underlying process of asset returns is not only a random walk with drift but any Gaussian processes The rate of return from financial strategies is zero under the assumption of a random walk without drift, and non-zero in all the other cases Then, it is shown that many forecasting techniques used by market participants are in fact linear forecasters and consequently fall in the scope of this study

Minimising the mean squared error is a sufficient but not necessary condition to maximise returns Under the random walk without drift assumption, error measures and profits are negatively correlated but very few in absolute value Only the directional accuracy exhibits high degree of linear association with profits When the true Gaussian process is not known, there are cases for which a decrease in mean squared error does not imply an increase in returns Therefore the mean squared error criterion is of poor use to maximise returns when the true model is not known The directional accuracy is of no further help Market timing ability tests based on the percentage of correct forecasts have very low power in presence of low positive autocorrelations

It is why a test of the random walk hypothesis based on the joint profitability of trading rules is investigated It happens to be powerful against a broad range of linear alternatives Its nice feature is to exhibit a power almost equal to the best of its components unknown when the true model is unknown It constitutes as well a tool to separate mean from variance non-linear models Simple tests of adequacy of Gaussian processes are subsequently proposed from the joint profitability of trading rules

Applying previous tests, the random walk hypothesis is rejected for daily exchange rates against Dollar, over the period 1982-1992 The hypothesis of normal underlying returns is very weak compared to the independence assumption Among a few Gaussian processes, the price-trend model along with some technical models appear to be the best alternatives to explain observed trading rule returns Statistical forecasters based either on ARMA(1,1) or fractional Gaussian processes do not outperform simple technical rules Taking into account transaction costs reduce profits to zero for individual but not for institutional investors who might have to act on strategies that assume the foreign exchange markets exhibit positive dependencies, if not inefficiencies

Chapter 1

INTRODUCTION

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1 1 FORECASTING FINANCIAL PRICES

Numerous techniques have been used to forecast financial prices Despite their apparent diversity, most of the predictors can be classified into two categories, fundamental or technical

Forecasts based on exogenous variables constitute the "fundamentalist approach" In the stock market, analysts study the fundamentals of companies (i e earnings, dividends, risk, assets, management, etc), industry sectors and the economy as a whole, to identify investment opportunities Attention is focused on specific items of information which are unknown to the market or considered to be incorrectly valued In the foreign exchange market, the primary focus is on monetary policy Fundamentalists claim that in the long term what underpin the trends of currency movements are the balance of payments and relative prices Recent experience has questioned the out-of sample accuracy of structural models of price-rate determination Simultaneously, the rising importance of price-based forecasts has been observed

Price-based forecasts constitute the "technical approach" These forecasts are determined using only historical price data The basic assumption is that "everything is in the rate" Such forecasts are generally developed using one of two methods The first method consists in creating a model based on statistical algorithms The most well-known technique is the Box-Jenkins(1976) method This minimises the mean squared error between the realised return and the one-ahead forecast It is the technique preferred by academics The second method consists in building heuristic predictors such that the implied decision rule is profitable in monetary terms These forecasting methods are called technical indicators and are preferred by market practitioners

1.2 THE OBJECTIVE OF THIS RESEARCH

Thıs research aıms to contrıbute to the knowledge of prıce-based forecasts by focusıng on theır economıc evaluatıon as measured by profitabılıty A popular theory, among academıcs ıs that technıcal ındıcators are suboptımal predıctors and that statıstıcal forecasters should be preferred Only complex nonlınearıtıes ın financıal prıces could justıfy the use of technıcal ındıcators However for market partıcıpants, the usefulness of a forecaster ıs best measured by the profits and losses ıt generates and prevıous studıes have ındıcated that technıcal tradıng rules perform at least as well for thıs purpose

The research descrıbed ın thıs thesıs seeks to unıfy technıcal and statıstıcal forecasters and formalıses theır expected returns usıng stochastıc modellıng More precısely, the thesıs addresses four questıons not yet answered despıte a growıng lıterature Namely

(1) What ıs the economıc evaluatıon of prıce-based forecasts ?

The maın goal of a financıal forecaster ıs to possess market tımıng abılıty Its raıson d'être ıs to accurately predıct the dırectıon of the trend, up or down, such that a profitable tradıng rule can be elaborated Therefore the most ımportant statıstıc ıs the expected return followıng the forecastıng strategy It ıs establıshed ın the thesıs assumıng that the process of underlyıng assets ıs Gaussıan

(2) Are the most accurate forecasters the most profitable ?

Thıs poınt ıs ınvestıgated by studyıng ın depth the theoretıcal relatıonshıps between mean squared error and profits crıterıa

(3) How sımılar and dıfferent are tradıng rules ?

Thıs research formulates the lınear correlatıon coefficıent between tradıng rules returns to deal wıth thıs ıssue

(4) What models are compatıble wıth observed tradıng rule returns ?

The abılıty of a few Gaussıan processes to explaın technıcal profits ıs checked for a set of exchange rates serıes

1.3 LAYOUT OF THE THESIS

Chapter 2 presents dynamic strategies including portfolio insurance, market timing strategies, fundamental and technical approaches The similarities and differences between the various price-based strategies are examined and the statistical attributes of the returns specified A forecasting technique is considered as useful in the financial market if it generates profitable transactions Therefore a better understanding of these techniques might be achieved by studying their returns distribution However, a literature review shows that very little is known about the theoretical distribution of returns generated by trading rules In particular there are no analytical results assuming that prices exhibit dependencies Since dependency in prices is a necessary condition to the usefulness of financial forecasting, that is a serious limitation that this thesis attempts to solve To do so, a technical description of plausible models of financial prices is provided in the last section of the chapter

Chapter 3 is the key chapter of the thesis The statistical distribution of rule returns is established using stochastic modelling Stochastic modelling has the advantage to encompass a far broader set of possible market conditions than any single empirical financial time series The expected return which is the most important statistic is given analytically for linear forecasters and price models An extension to nonlinear models is provided by considering heteroskedastic volatility and fractional processes Then it is shown that many technical indicators are in fact implicit linear statistical forecasters

Chapter 4 deals with the relationships between error measures and profits The sufficient and necessary conditions to maximise expected returns are formulated Minimising the mean squared error is a sufficient but not a necessary condition In practice, the true model is not known and a misspecified forecaster has to be used Therefore we assess to what extent various misspecifications affect the profitability and error measures of a forecaster That is done in the thesis by measuring the relative loss of returns and increase of mean squared errors Finally, it is shown that market timing ability tests based on directional accuracy have very low power in the presence of low positive autocorrelations Under such circumstances, it is possible for no market timing ability to be detected even though there exists one

Chapter 5 proposes new tests of random walk based on the joint profitability of trading rules A preliminary result, the theoretical correlation between trading rules, is first established. Then tests of non-zero profits are proposed. A power study exhibits that a test based on the profitability of an equally weighted portfolio of trading rules can have higher power than the most profitable rule in the portfolio, which is of course unknown ex-ante. Finally, diagnostic tests are proposed which allow to check the ability of any Gaussian price models to replicate trading rule returns.

Chapter 6 tests the random walk hypothesis for a set of exchange rates. Conventional tests of normality and temporal dependence\are first performed. The empirical results show that the variance of rule returns and correlations between systems are not significantly different from what would randomly be expected. However, the new tests of random walk established in Chapter 5 show that mean returns are significantly positive. Since these tests are based on the assumption of normality, a non parametric distribution-free test, based on the bootstrap approach is also performed. Critical thresholds of T-Student and bootstrap based tests are remarkably very similar. Both tests suggest the existence of dependencies in exchange rates. Therefore when testing for non-zero profits, the independence assumption is critical but the normality assumption not crucial.

Chapter 7 tests the ability of a few autocorrelated Gaussian processes to replicate the rule returns observed in the foreign exchange market. To this end, the price-trend model seems to be the best alternative among well-known statistical models. Some technical model's first explicited in Chapter 3, appear to reproduce equally well trading rule returns. That might explain why statistical forecasters, including the fractional Gaussian forecaster, do not beat their technical counterparts. Subsequently two sources of profits are exhibited in accordance with linear models: volatility and autocorrelations. There appears to be a premium in a risk-adjusted sense to invest in volatile currencies. In addition, technical returns generated by an unique rule can be enhanced by considering a portfolio approach. Diversification between trading rules or currencies pays, but neither one or the other is more profitable. Finally, the efficient market hypothesis is discussed taking into account transaction costs. The latter reduce profits from technical trading to zero for small investors. Nevertheless, opportunities remain for institutional investors.

Chapter 8 presents the conclusions of this thesis. The purpose of this research has not been to test market efficiency which is in itself a difficult task, but rather simply to provide an understanding of the superior performance of some models relative to the random walk model. It is argued that investors facing low transaction costs might have to act on the basis that exchange rates exhibit positive dependencies.

Chapter 2

RECLASSIFICATION OF DYNAMIC STRATEGIES

Investors who invest in financial markets are exposed to uncertain price changes As a risky asset fluctuates in value, the value of the investment containing it may change One must decide how to redefine the investment in response to such changes Dynamic strategies are explicit rules for doing so

Dynamic strategies differ from static strategies, such as a buy-and-hold rule, in that trading in the asset occurs throughout the investment horizon, at times and in amounts that depend upon a fixed set of rules and future price changes Dynamic strategies are developed following the expectations investors have formed about the statistical nature of the price process

In random markets, price changes can not be predicted Current prices fully and correctly reflect all currently available information Dynamic strategies are then employed to reduce the price risk exposure of an investor The probability distribution of returns from a risky investment is tailored to suit a particular set of preferences For instance, the most popular application of these techniques, portfolio insurance, has the objective of placing a lower limit on the rate of return to be earned on an investment over a specified time period

In non-random markets, price change can be predicted There are market imperfections, such as the existence of price trends and cycles The goal of dynamic strategies in this case is to exploit these imperfections and to outperform the market To this end, market timing or forecasting strategies are used

Section 2 1 presents dynamic strategies, namely portfolio insurance and market timing, and defines their statistical attributes Section 2 2 describes forecasting techniques used to predict financial prices Section 2 3 carries out a literature review of forecasting strategies The key issues not yet solved by academics and considered in the research are emphasised Section 2 4 shows how stochastic modelling can be employed to assess the ability of forecasting strategies to meet their goal under a broad set of market conditions A number of plausible models of financial prices are then considered Finally, Section 2 5 summarises and concludes our results

2.1 DYNAMIC STRATEGIES

2.1.1 *Portfolio insurance*

Portfolio insurance strategies are "hedging" rules. They assume that markets are random and therefore that price changes can not be predicted. Portfolio insurance strategies are then employed to reduce the price risk exposure of an investor.

Portfolio insurance is a dynamic hedging strategy whereby we gradually shift a fund's exposure between a risky asset and a riskless asset so as to ensure a minimum return while preserving the potential to participate in plausible gains from the risky asset. It is especially the case of pension funds which do not want the value of their assets to fall below the floor defined by the present value of their liabilities. The cost of the insurance to the investors is a premium reflected by lower realised return on the insured portfolio when the return on the uninsured portfolio is positive. Portfolio insurance enables an investor to avoid losses and capture gains at the cost of a fixed premium.

A number of techniques have been developed which enable tailoring the probability distribution of returns from a risky investment to suit a particular set of preferences. The most common approach consists of approximating the results that would be obtained by purchasing a put option on the portfolio (Rubinstein and Leland, 1981). Buying and selling is triggered only by changes in the value of the reference portfolio according to the informationless hedging rules that recreate option return. An other rule is the Constant Proportion Portfolio Insurance (CPPI) strategy (Black and Jones 1987, 1988; Perold and Sharpe, 1988; Black and Perold, 1992). This invests a constant multiple of the cushion in risky assets up to the borrowing limits, where the cushion is the difference between wealth and a specified floor. As the multiple goes to infinity, CPPI becomes a stop-loss strategy (Black and Perold, 1992). This is investing the maximum, up to the borrowing limit, in the risky asset while wealth is above the floor, then switching completely into the riskless asset if and when wealth reaches the floor.

Portfolio insurance strategies assume that markets are random. The bond price is assumed to grow deterministically at a constant interest rate and the stock price to follow a multiplicative random walk, such as the geometric Brownian motion[1] (Black and Scholes, 1973) or a discrete binomial distribution (Cox, Ross and Rubinstein, 1979).

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1 Details about this and other popular models of financial prices can be found in Section 2.4

2 1 2 *Market timing*

Market timing strategies are "speculative rules" They suppose that markets are non-random and therefore that price changes can be predicted Market timing strategies are then created to exploit imperfections in the risky asset

The goal of market timing strategies is to profit from price trends and cycles Market timing strategies are based on the idea that excess returns can be achieved by buying and selling at the "right" time The corresponding rules can be loosely described as "run with your winners, cut your losses" and "sell at a new high, buy at a new low"

2 1 3 *Statistical attributes*

Dynamic strategies can be characterised by use of four features of which three are pure statistical attributes

a) implementation cost of the strategy[2]
b) convexity
c) path-dependency
d) underlying return preferred stochastic process

Convexity

Strategies that "buy stocks as they fall " give rise to concave payoff curves That is they tend not to have much downside protection This terminology derives from the concave payoff curve relating the terminal value of a portfolio to an unidirectional move up or down from its initial value

Strategies that "sell stocks as they fall " give rise to convex payoff curves That is they tend to give good downside protection

It must be emphasised that most of the dynamic strategies, portfolio insurance and market timing, employ convex rules Leland(1980) clearly stipulates that general insurance policies are those that provide strictly convex payoff functions, since convexity implies greater protection from loss at lower values of the reference portfolio Like portfolio insurance techniques, market timing strategies are convex rules because they are designed on the idea that there are trends in financial prices

[2] The study of this financial aspect is postponed to Section 7 4 2

Path-Dependency

Path-independence means that the terminal portfolio value depends only on the terminal market price of the assets, and not on the history of price movements prior to the end of the investment horizon Path-dependency reflects that the rate of return on the insured portfolio is not only dependent on the rate of return on the uninsured portfolio but also the path taken in the value of the uninsured portfolio over the insurance period

Let us give some examples of path-dependent\and independent strategies

Most of the portfolio insurance strategies are path-independent, since under the random walk assumption path-independence is necessary for expected utility maximisation The idea is that investors wish to minimise uncertainty and so minimise path-dependency The use of a protective put or continuously rebalancing strategy, to implement portfolio insurance is truly path-independent (Cox and Leland, 1983, Black and Perold, 1992) When rebalancing takes place discontinuously, CPPI strategies become path-dependent (Trippi and Harriff, 1990, Black and Perold, 1992) An other rule that is clearly path-dependent is the stop-loss strategy (Rubinstein, 1985, Black and Perold, 1992) In this case the return of any profitable position will not be a predictable percentage of the rate of return that would have been earned by investing all funds in stocks

Unlike portfolio insurance techniques, market timing rules are path-dependent strategies With path-dependent strategy, a portfolio manager can hold positions throughout a flat market yet still make money because of the particular price fluctuations that happened to occur along the way That simply reflects the main purpose of a market timer which is not to loose any profit opportunity in the hope of maximising returns at any level of risk (Philipps and Lee, 1989)

Underlying return preferred stochastic process

Dynamic strategies are developed to exploit the market conditions most likely to occur Consequently, the choice of which dynamic strategy to follow, is closely related to the investor expectations about the statistical nature of the price process

Under the random walk assumption, Cox and Leland(1983), Rubinstein(1985) have proved that path-independence is necessary for expected utility maximisation Cox and Leland(1983) add that without a path-independent strategy a portfolio manager could hold a long position throughout a rising market yet still lose money because of the particular price fluctuations that happened to occur along the way Cases in which the market ends up far from its starting point are likely to favour buy-and-hold strategies A buy-and-hold strategy tends to be almost optimal if there is a major move in one direction

Under the assumption of senal dependence, Kntzman(1989) sees two ways to exploit this hypothesis depending on the nature of the senal dependence, positive or negative If one expects returns to follow trends (positive senal dependence), he can add value to a buy-and-hold strategy by following a linear investment rule that generates a convex payoff function Perold and Sharpe(1988) presume that this generally does relatively poorly in flat (but oscillating) markets and very well in up markets If on the other hand, one believes that returns are charactensed by frequent reversals (mean-reverting process), one can add value to a buy-and-hold strategy by following a linear investment rule that produces a concave payoff function Perold and Sharpe(1988) suggest that this generally does relatively poorly in up markets and very well in flat (but oscillating) markets They add that cases in which the market ends up near its starting point are likely to favour concave strategies because they trade in a way that exploits reversals Greater volatility (i e more and/or larger reversals) will accentuate this effect The question being, following Perold and Sharpe(1988), to know if markets are charactensed more by reversals than by trends

Table 2 1 summanses the preferred stochastic process of strategies following their statistical properties convexity, path-dependency A list of studies having formulated these classifications is given

Table 2.1: Preferred stochastic process

Rule	Preferred *stochastic process*	Author
Convex	*up markets* positive senal dependence	Perold and Sharpe(1988) Kntzman(1989) Tnppi and Harnff(1991)
Concave	flat but oscillating markets frequent reversals (mean reverting process) transiently cyclical markets	Perold and Sharpe(1988) Kntzman(1989) Tnppi and Harnff(1991)
Path-independent	random walk with dnft	Cox and Leland(1983) Rubinstein(1985) Black and Perold(1992)
Path-dependent	*dependence*	Tnppi and Harnff(1990)

Finally, Table 2 2 summanses the pnncipal components of portfolio insurance and market timing strategies

Table 2.2: Features of dynamic strategies

Rule	Feature	Determinants	Convexity	Path	Preferred stochastic process
Portfolio Insurance	insurance against loss	spot pnce	convex	Independent	random
Market Timing	maximising return	recent pnce history	convex	Dependent	non-random

2.2 FORECASTING TECHNIQUES

Market timing rules have been sometimes derived from portfolio insurance techniques That is the case for example of path-dependent options (Goldman, Sosin and Gatto, 1979) But most often they have been developed from forecasting techniques The basic assumption is that "everything is in the rate" Then if markets move in trends, defining the prevailing trend and being able to identify early reversals throughout forecasting methods is certainly helpful in assessing future rate developments Forecasting techniques which use only past prices to forecast future prices are called technical indicators They can be classified in three categories chartism, mechanical system and statistical modelling The technical approach is often opposed to the fundamental approach which forecasts future rates by determining the economics affecting prices

2 2 1 *Chartism*

Charting is the oldest branch of technical analysis Chartism is based on the assumption that trends and patterns in charts reflect not only all available information but the psychology of the investor as well Analysts who use charts look for graphical cycles and repetition of patterns to discern trends

The rules derived from the analysis of charts are often subjective and as such chartism is considered more of an art than a science This is primarily why it has not been possible to define chart patterns with mathematical rigour Curcio and Goodhart(1991) do some empirical work to study the effect of chartist analysis They use the predictions of a chartist based in London, in a form which enables them to do a controlled experiment Their study suggests that trading with chartist lines does not obtain better mean returns than not using charts However they recognise that their research was not designed to evaluate the profitability of screen trading Curcio and Goodhart(1992) investigate the usefulness of support and resistance levels provided by Chartists and offered to investors by Reuters The rule consists of a range within which the asset is expected to fluctuate if the asset moves above the higher end of the range, a buy signal is generated, while a sell signal is generated if the asset moves below the lower end of the range Curcio and Goodhart(1992) show that abnormal returns can be obtained by applying chartists decision rules

The problem with such a rule is that the determination of the trading range can be highly subjective and person-dependent It follows that the predictive power ability of chartist techniques might be difficult to measure Neftci(1991) demonstrates this point for at least two popular charts methods He proves that they are ill-conceived and subsequently that no proper testing of their usefulness can be achieved

This is why chartist techniques will be ignored in the thesis which concentrates instead on objective rules only

2 2 2 *Technical indicators*

This type of technical analysis tends to convert subjective impressions of patterns or trends in mechanical trading rules An example is to replace subjective support and resistance levels by a well-defined trading range A trading range may be characterised by the maximum and minimum of the series (of various lengths) of latest prices Mechanical systems are conceived in a way to trigger indisputable sell and buy signals following a decision rule based on past data, usually by calculating if the price is above or below a particular entry point These systems are typically not concerned with how much the price is above or below the entry point They attempt to predict the direction of the future price without searching to forecast its level They are used to detect major downturns and upturns of the market The appropriateness of this indicator is conditional to the fact that trends in prices tend to persist for some time and can be detected

Three main features characterise mechanical systems path-dependency, convexity, and non uniqueness

By design, mechanical systems depend on the history of price movements prior to the end of the investment horizon Consequently, they are highly path-dependent strategies The usual rule is to trade with the trend The trader initiates a position early in the trend and maintains that position as long as the trend continues

Almost all mechanical systems are trend-following and so exhibit convex payoff The very few which are not belong to the family of contrary opinion indicators, known as well as reverse trend-following rules, and so display concave payoff They are very rarely used on their own and are only applied in combination of trend-following systems

The main difficulty with mechanical trading systems is that a rule has to be chosen from an infinite number of alternatives Since those systems are assumed to reflect (mechanically) the expectations of the forecaster, there exist almost as many rules as there are different expectations

There are so many relevant trading rules that it is unrealistic to list them all In what follows we concentrate on the basic definitions of the most popular rules among practitioners and academics To each mechanical system numerous alterations have been made and hybrid indicators constructed Details, justifications and uses of these derivative rules can be found in Kaufman(1987)

Moving Averages

Moving averages are certainly the oldest and most widely used methods The simplest rule of this family is the single moving average which says when the rate penetrates from below (above) a moving average of a given length, a buy (sell) signal is generated

By using a linear or exponential weighting, greater importance can be given to more recent observations Despite these more complex systems, a simple moving average appears to be the most widely used form It must be emphasised (as will be proved in the thesis) that the decision of what length of moving average system to use is held to be particularly important as short or long term averages can give very different signals Fibbonacci numbers have been used for this purpose (Pring, 1985)

Two moving averages of different time lengths can be used to generate signals via the double crossover method A buy (sell) signal occurs when the shorter average penetrates from below (above) the longer Widely used combinations are 5 and 20 day averages, 10 and 40 day averages It is worth noting that the double cross-over method is a generalisation of single moving average signal generation, as the price line in the latter can be regarded as a "one-period moving average" Finally, the double cross-over method admits other, although strictly equivalent, representations The usual way the transformation is done is to plot the difference between the two averages Buy (sell) occurs this time when the moving average oscillator moves above (below) the zero line When a trading rule triggers a signal around a zero line, it is often called an oscillator

Momentums

A standard momentum line is constructed by subtracting the closing price of k days ago from the last closing price The result positive or negative figure is then plotted around a zero line Then the general trading rule is based on the crossing of the zero line Buy (sell) when the oscillator moves above (below) the zero line

Channels or Breakouts

Breakout systems also known as price channels or trading range say buy (sell) an asset if the rate penetrates from below (above) the maximum (minimum) of the past m days m is a given number of days which features the length of the channel

Filters

Filter systems are the primary technique for testing market efficiency, introduced by Alexander(1961) and have since been used widely by other researchers However it must be recognised that compared to the mechanical systems presented above this method is far less popular among practitioners An x percent filter rule leads to the following strategy Buy an asset whenever it rises by x percent above its most recent trough Sell the asset whenever it falls x percent below its most recent peak

All the systems we have examined so far share two characteristics in common

First, they are always "in" the market, either long or short of one asset unity That means in practice that positions are never neutral or of variable amounts That will be the primary assumption of this thesis The many hybrid indicators which have been constructed allowing neutral positions to avoid whipsaws due to trendless markets, are in fact, most often, nothing else than the association of basic trading rules, (Kaufman, 1987, Schwager, 1984, Béchu and Bertrand, 1992, Cahen, 1990) A well known example comes from the simultaneous use of momentums and moving averages, (Goldberg and Schulmeister, 1988) Then it is simpler to study first the behaviour of elementary rules, and second to consider the possibility of combining rules via rules correlations

Second, these four systems are all trend-following systems or convex strategies They work best in trending markets During period of sideways movement they are especially prone to generate false signals when trend (trendless) is measured by positive (zero or negative) autocorrelations

2 2 3 *Statistical techniques*

Another forecasting approach is to study the properties and power of advanced time-series techniques models By restricting the field of investigation to linear models, it is possible to develop procedures such as Box-Jenkins(1976) to derive the linear forecaster which minimises the mean squared error between forecasted and realised value Proponents of these techniques are essentially found among academics and statisticians and are widely used to forecast economic time series A comprehensive study of such procedures can be found in Granger and Newbold(1986), Gourieroux and Monfort(1990) An application to forecasting exchange rates is provided by Keller(1990) Although preferred by academics, they are not ignored by quantitative investors as testified by the journal "Stock and Commodities" from Weiss(1982a, 1982b, 1983) and Parish(1990) It is often not easy to beat convincingly these simple linear univariate ARIMA So these simple methods make excellent base-line models

There are two reasons that underpin the popularity of the Box-Jenkins methodology First it allows to identify the underlying model and so to build efficient if not optimal predictors Contrary to technical systems, they are designed to exploit specific autocorrelations The second one is given by Neftci(1991) If the true process is linear, time varying vector autoregressions (VARs) should be optimal forecasters over and above technical analysis on the conventional basis of mean squared errors (MSE)

2 2 4 *Fundamental models*

A detailed review of the economics affecting prices is beyond the scope of this thesis, but it may be useful to outline the "fundamentalist approach"

Fundamental analysts study the fundamentals of companies (i e earnings, dividends, risk, assets, management) industry sectors and the overall economy to identify investment opportunities Attention is focused on specific items of information which are unknown to the market or which are considered to be incorrectly valued by the market In the foreign exchange market, the primary focus is on monetary policy Fundamentalists claim that in the long term what underpin the trends of currency movements are the balance of payments and relative prices

2 2 5 *Patterns in financial forecasting and new avenues*

In the last fifteen years, technical analysis has become increasingly used for financial forecasting while fundamental analysis has decreased in importance

Recent experience has questioned the out-of sample accuracy of structural models of price-rate determination Empirical studies of monetary/assets models developed in the early 1980's[3], indicate that no structural technique could appreciably outperform the random walk model for any forecasting horizon less than 12 months In the foreign exchange market, Frankel and Froot(1990 22) suggest that " It may [indeed] be the case that shifts over time in the weight that is given to different forecasting techniques are a source of changes in the demand for dollars, and that large exchange rates movements may take place with little basis in macroeconomics fundamentals"

DuBois(1992) finds that technical indicators provide higher returns than conventional fundamentals models in the equity market In addition technical and fundamental models are very little correlated This strongly indicates that technical methods must be used in addition (if not substitute) of fundamental models It might explain why technical analysis has been increasingly used Firstly in the futures market, and then in the foreign exchange market

Irwin and Brorsen(1985) review the trading strategies employed by public futures funds Eighty-three percent of the funds used technical analysis The remaining seventeen percent applied a combination of technical analysis and fundamental analysis

In the foreign exchange market, Allen and Taylor(1989) report that 90 percent of the market participants apply chartists techniques for short term investing Frankel and

3 See Table 2 3 for references

Froot(1990) examine the data of reviews made by Euromoney of between 10 and 27 foreign exchange forecasting services In 1978, 18 forecasting firms described themselves as relying exclusively on economic fundamentals, and only 2 on technical analysis By 1985, the positions had been reversed Only one firm reported relying exclusively on fundamentals, and 12 on technical analysis Alcabas(1991) observes a similar pattern in France He discovered that, among dealers and portfolio managers, the use of technical analysis has increased in frequency from thirty-five percent in 1985 to seventy-seven percent in 1990

Technical indicators have been preferred by market practitioners to linear forecasters because as Section 2 4 2 illustrates, the behaviour of financial prices is non-linear However, one of the limitations of technical analysis is the difficulty in developing models of financial prices Consequently, new technologies have then been proposed to take profit of nonlinearities expert system and neural network

Conventional expert systems techniques have been studied by Lee, Trippi, Chu and Kim(1990), Pau(1991) Those technologies are especially suited for simulating in pattern detection Pau(1991) uses expert systems to learn usual chartist techniques such that recognition of patterns is improved An artificial intelligence approach to analysing the stock market prediction decision has been presented by Braun and Chandler(1988) Neural networks can assist directly with risk assessment, asset selection and timing decisions They can be purely technical and so based only on the history of past prices (White, 1988, Trippi and DeSieno, 1992) In this case, neural network-based rules, although more complicated in nature, can behave and exhibit performances close to well-known mechanical systems Alternatively, neural networks can use external inputs such as exogenous or fundamental variables, (Collard, 1991)

The above list is not exhaustive There are many other techniques which can be used to forecast financial prices such as the nonparametric rate prediction performed by Diebold and Nason(1990), Satchell and Timmermann(1992a, 1992b)

2 3 A REVIEW OF THE LITERATURE

The weakness of forecasts based on fundamentals emphasises the need for other forecasting strategies This research aims to contribute to the knowledge of financial forecasting techniques based on past prices More precisely, it attempts to provide answers to three issues, not yet addressed in the literature Namely

(1) the stochastic properties of technical indicators

(2) the relation between error measure and profitability and more specifically between technical and Vectors Autoregressions (VARs) models.

(3) the similarities and differences between trading rules

2.3.1 *Empirical evidence on the performance of forecasting techniques*

Many early studies show technical analysis to be useless for predicting stock returns (Fama and Blume, 1966) and exchange rates (Cornell and Dietrich, 1978). These works base their conclusions on the lack of profitability of filter rules. Since these early studies, tests have been carried out on a regular basis and results now tend to favour technical analysis. The opposing views about technical analysis are well summarised in Malkiel(1990) and Ithurbide(1992). On the one hand, Malkiel(1990) is "biased against the chartist" and postulates that the method is "patently false" having not found any dynamic rule able to outperform a passive buy and hold strategy. On the other hand, Ithurbide(1992) expresses the opinion that although chartism does not rely on argument and does not allow to give an explanation to financial rates moves, its performance and usefulness must not be questioned. Table 2.3 lists some academic studies about technical and other financial forecasting methods.

Table 2.3: Financial forecasting studies

Assets	Technique	Usefulness	Author
stock,	statistical	yes	Taylor(1986)
commodities,	moving average,filter,channel,others	yes	Lukac, Brorsen and Irwin(1988b)
exchange rates	statistical	yes	Taylor and Tari(1989)
	moving average,filter,channel,others	yes	Brorsen and Boyd(1990)
	technical adviser	no	Hartzmark(1991)
stocks	filter	yes	Alexander(1961)
	filter	no	Fama and Blume(1966)
	filter	yes	Jennergren(1975)
	filter	no	Frankfurter and Lamoureux(1988)
	neural network(technical)	no	White(1988)
	neural network(technical)	yes	Trippi and DeSieno(1988)
	artificial intelligence	yes	Braun and Chandler(1988)
	filter	yes	Sweeney(1988, 1990)
	moving average, filter,oscillator	yes	Broquet et al(1990)
	moving average, filter	yes	Brock et al(1992)
	moving average, volume	yes	LeBaron(1992a)
	non parametric technique	yes	Satchell and Timmermann(1992a)
	technical	yes	Bulkley and Tonks(1992)
	filter	yes	Corrado and Lee(1992)

Table 2.3 (continued) Financial forecasting studies

commodities	filter	yes	Bird(1985)
	filter	yes	Kamdem(1988)
	neural network(fundamental)	yes	Collard(1988)
	statistical	yes	Leuthold and Garcia(1992)
exchange rates	statisical	no	Giddy and Dufey(1975)
	filter	no	Cornell and Dietrich(1978)
	statistical	yes	Bilson(1981)
	filter	yes	Dooley and Shafer(1983)
	moving average	yes	Bera Debeinex and Domergue(1983)
	monetary/asset model	no	Meese and Rogoff(1983a, 1983b)
	statistical	no	Nawrocki(1984)
	moving average	yes	Neftci and Policano(1984)
	moving average, filter	maybe	De la Bruslerie and de Lattre(1985)
	technical adviser	no	Murphy(1986)
	filter	yes	Sweeney(1986)
	statistical	yes	Bilson and Hsieh(1987)
	monetary/asset model	yes	Boothe and Glassman (1987a)
	technical adviser	maybe	Cumby and Modest(1987)
	monetary/asset model	no	Thomas and Alexander(1987)
	moving average filter, momentums	yes	Schulmeister(1988)
	technical adviser	maybe	Allen and Taylor(1989)
	moving average	yes	Dunis(1989)
	statistical	yes	Bilson(1990)
	moving average, channel, statistical	yes	Taylor(1990a, 1990b)
	non parametric technique	no	Diebold and Nason(1990)
	moving average, filter	no	De la Bruslerie(1990)
	moving average	yes	Neftci(1991)
	monetary/asset model	yes	Gerlow and Irwin(1991)
	moving average, filter	yes	Levich and Thomas(1991)
	trend lines	no	Curcio and Goodhart(1991)
	moving average	yes	LeBaron(1991)
	moving average	yes	LeBaron(1992b)
	support and resistance	yes	Curcio and Goodhart(1992)
	statistical	yes	Taylor(1992a)
	channel, statistical	yes	Taylor(1992b)
	statistical	yes	Lai and Pauly(1992)
	non parametric technique	yes	Satchell and Timmermann(1992b)
	moving average, filter	yes	Surujaras and Sweeney(1992)

As Table 2 3 shows, there has been a renewed interest in academic literature about financial forecasting techniques and its ability to predict future prices However not all results are comparable for at least three reasons Firstly, the methods employed differ from chartist techniques and mechanical systems to statistical and monetary models Secondly, performance has been evaluated in different ways, mainly error measure and profitability Thirdly, the underlying asset and period of investigation have considerably varied although there is a net preference for exchange rates studies

An homogenous framework might be better achieved by studying the statistical distribution of technical investments A better understanding of financial forecasting methods might result from such researches

2 3 2 *Statistical distribution of dynamic strategies*

Many of the previous studies of forecasting strategies have used historical returns to explore the investment trade-offs involved These studies serve a very important role in suggesting the historical behaviour of such rules However they may not constitute an appropriate guide, because their results are highly dependent on the asset and time period covered by the research Also a historical study might provide inadequate precision in defining the shape of the return distribution Historical data allow only a very narrow interpretation of historical events (i e that there was only one course history might have taken and the future could take) We believe this to be an unreasonably restrictive view of reality For this kind of information, one has to turn to theoretic or stochastic modelling

The use of stochastic modelling to study the statistical distribution of dynamic strategies consists in three steps

(a) Determining plausible models of prices (Section 2 4)

(b) Establishing corresponding returns distributions of dynamic strategies (Chapters 3 and 5)

(c) Checking the validity of the model by comparing observed and theoretical returns of dynamic strategies (Chapters 6 and 7)

The returns distribution of a Buy-and-Hold strategy has the same shape as the distribution of price returns used to produce it The same isn't true for more complex strategies The returns distribution of dynamic strategies can be different from that of the underlying model and subsequently needs specific studies Tables 2 4 and 2 5 list some of these works for portfolio insurance and technical analysis strategies They indicate

- the assumption made about the underlying process
- the rule under study
- the finding, distribution or moments, expected value plus variance
- the technique used to establish results exact analytical development, Monte-Carlo simulation or Bootstrap methodology[4]

[4] See Section 6 3 1 for details about the bootstrap methodology

Table 2.4: Distribution of portfolio insurance returns

Assumption	rule	distribution	moments	Author
Random Walk with Drift	option		exact	Cox and Rubinstein(1985)
	option	simulation		Asay and Edelsburg(1986)
	constant proportion	simulation		Etzioni(1986)
	option	simulation		Bookstaber and Clarke(1987)
	option	simulation		Clarke and Arnott(1987)
	constant proportion	\	exact	Perold and Sharpe(1988)
	constant proportion, option	simulation		Zhu and Kavee(1988)
	constant proportion		exact	Perold and Sharpe(1988)
	constant proportion, option	simulation		Bird et al(1990)
	constant proportion, stop loss		exact	Black and Perold(1992)
Serial Correlation	constant proportion	simulation		Trippi and Harriff(1990)

Table 2 4 shows that most often the strong assumption that active and reserve assets follow geometric Brownian motion is made Except Black and Perold(1992) who give some results concerning path-dependent strategies (discrete rebalancing CPPI and stop-loss strategy), studies have mainly focused on path-independent strategies since under the random walk assumption only path-independency can maximise expected utility and be of interest Exact analytical results of expected value of portfolio insurance techniques can be found in Cox and Rubinstein(1985) for the option strategy, Black and Perold(1992), Perold and Sharpe(1988) for the constant proportion strategy Simulations have been necessary to establish the whole shape of option returns Clarke and Arnott(1987), Bookstaber and Clarke(1987), Zhu and Kavee(1988) among others have shown that options returns are able of reshaping the distribution of underlying returns The distribution of options returns is not any more symmetric, but left-truncated and the natural skewness of the log-normal return distribution increases dramatically Zhu and Kavee(1988) shows that those features apply as well for the constant proportion technique The robustness of portfolio insurance strategies to meet their goal under different market conditions and in particular their ability to protect against loss, have been proved by Trippi and Harriff(1991), Fong and Vasicek(1989), Bird, Cunningham, Dennis and Tippett(1990)

On the one hand, the distributions of portfolio insurance returns have been the object of numerous researches in the literature On the other hand, the distribution of technical analysis returns has been the subject of very few academics researches as Table 2 5 shows

Table 2.5: Distribution of technical analysis returns

Technical Analysis				
Assumption	rule	distribution	moments	Author
Random Walk with Drift	filter		exact	Praëtz(1976)
	filter		exact	Bird(1985)
	filter		exact	Sweeney(1986)
	moving average. slope	simulation		Tomek and Querin(1984)
	moving average, filter	bootstrap		Levich and Thomas(1991)
	moving average	bootstrap		Brock et al(1992)
	moving average	bootstrap		LeBaron(1991, 1992b)
AR(1)	moving average, filter	bootstrap		Brock et al(1992)
	moving average	bootstrap		LeBaron(1992b)
AR(2)	moving average	bootstrap		LeBaron(1991, 1992b)
ARMA(1. 1)	moving average,channel,statistical		simulation	Taylor(1990a)
	channel,statistical		simulation	Taylor(1992b)
	moving average	bootstrap		LeBaron(1992b)
GARCH	moving average, filter	simulation		Brock et al(1992)
	moving average	bootstrap		LeBaron(1991)

Exact theoretical results about technical analysis are extremely limited and concern the random walk with drift hypothesis only (Praëtz, 1976; Bird, 1985; Sweeney, 1986). In addition, only the expected value and variance of technical rules are provided. Other models that have been used in the literature include serial autocorrelations and changing variances[5]. However corresponding expected values of trading rules have not yet been formalised. The main reason has been that such results are analytically intractable and too complicated (Trippi and Harriff, 1991; Brock, Lakonishok and LeBaron, 1992; Black and Perold, 1992). Instead, expected values of trading rules have been estimated either through bootstrap approach (Levich and Thomas, 1991; Brock, Lakonishok and LeBaron, 1992; LeBaron, 1991), or simulation (Tomek and Querin, 1984; Taylor, 1990a, 1992b; Brock, Lakonishok and LeBaron, 1992; LeBaron, 1991, 1992b). Taylor(1986) argues " the distribution of the return from a filter strategy under the null hypothesis of an efficient market is not known, so that proper significance tests are impossible ". He adds that it is unclear how the strong assumption, of identically and normally distributed returns, can be relaxed.

In any case, the random walk assumption, although acceptable when studying portfolio insurance strategies, is clearly insufficient to assess the ability of technical analysis to meet their goal under different market conditions. Most statisticians and probability theorists would agree that if prices or prices changes are independent, then it would be difficult or impossible to use the past history of prices to develop a realistic trading strategy.

5 See Section 2.4 for further details

On the other hand, if prices or price changes exhibit time dependence, then the past history of prices can potentially be used to develop a reasoned and profitable strategy (Sherry, 1992) As far as market efficiency tests are concerned, the statistical question of dependencies is not particularly relevant on its own The question is instead can investors exploit any dependency (be it "statistically significant or not") ?

Serial correlation is probably the simplest and most easily understood characteristic of a price series capable of justifying path-dependent strategies and so the use of many mechanical systems Establishing expected value of path-dependent strategies for any Gaussian process is of interest since it will ascertain whether such dynamic strategies meet their goals Solving this issue will allow to determine if non-zero profit can be expected from such methods and if this is the case what the parameters are that make such a rule profitable The problem of specifying the relationship between technical rule returns and standard statistical measures of serial dependency is pursed in previous research using empirical observations (Corrado and Lee, 1992) but not using stochastic modelling The latter specification is useful because technical rule returns provide a measure of economic significance for serial dependencies in financial returns that otherwise might not be readily interpretable French and Roll(1986), for example, note that gauging the economic significance of daily stock return autocorrelations is difficult The reverse question is "How large deviations from randomness, as measured for instance by runs tests and serial correlations, are required if there is to exist profitable mechanical trading rules of the filter type ?" (Jennergren, 1975 67)

An informal answer is at the present state of knowledge that there exist trends The reason is that convex technical rules require trends to be profitable (Perold and Sharpe, 1988, Trippi and Harriff, 1991) The main concern of market practitioners is to elaborate statistics allowing to separate random drifts from trends (Poulos, 1991, 1992a, 1992b) Despite the fact that academics themselves recognise the difficulty they have in giving a formal definition of trend, attempts have been made nevertheless, and then will be discussed in details in Section 2 4 Formulating trends from a pure statistical point of view is of importance because it permits to study the profitability of technical rules when there are such trends (Taylor, 1990a, 1992b, Brock, Lakonishok and LeBaron, 1992, LeBaron, 1991, 1992b) Previous studies have proceed by bootstrap or simulation approach and so, as Curcio and Goodhart(1992) admit, they have not been able to examine how trading rule returns are related to the statistical characteristics of the underlying series Empirically, the relationship between the magnitude of serial correlation coefficients and the expected profits of technical trading rule is difficult to exhibit (Fama and Blume, 1966) Our goal is to show that using stochastic modelling, it is possible to establish the parameters of the underlying process which can generate non zero expected return from technical analysis This will be informative for both practitioners and academics

It could firstly allow practitioners to know under which market conditions what technical rules perform Chapter 3 and Section 4 1 intend to solve this issue when the underlying asset follows a Gaussian process Secondly relating rule and underlying returns would allow academics to test the adequacy of their models by measuring the fitness of observed with expected rules returns That is the purpose of Chapters 5, 6 and 7

2 3 3 *Error measures and profitability*

The methods that are proposed as providing useful forecasts of price changes or returns need to be evaluated The problem is that there does not exist an unique universal performance criterion In finance there are mainly two, profitability and error measure, usually depending on the nature of the forecaster technical or statistical It explains why those two kinds of forecasters have often been considered as unrelated investment strategies For instance Dunis(1989) and Keller(1989) treat both financial forecasting methods in different chapters of a same book ignoring any possible analogy The same applies for Herbst(1992) Recently efforts have been made to compare technical and statistical forecasters in a common literature review (Granger, 1992), survey (Allen and Taylor, 1989) and theoretical work (Neftci, 1991) Allen and Taylor(1989) compare a set of empirical chartist forecasts in the London foreign exchange and the Box-Jenkins approach Then they establish ranking of forecasting techniques in terms of mean squared error and find one chartist able to significantly outperform Box-Jenkins forecasters Neftci(1991) stipulates that if the underlying price process $\{P_t\}$ is linear in the sense he defines then no sequence of Markov times obtained from a finite history of $\{P_t\}$ can be useful in prediction over and above (vector) autoregressions Nevertheless he does not quantify forecasting accuracy of technical analysis and implicitly concludes that in terms of error measure, technical forecasters are suboptimal when the process is linear

However a puzzling question first asked by Elton and Gruber(1972) has not yet been answered what are the sets of conditions under which particular mechanical techniques are optimum forecasters ? This of course raises the question of how does one define optimality ? Are the rankings of forecasting methods criteria dependent ? Is the most accurate system in term of mean squared error the most profitable ?

In the affirmative, how misspecified are technical analysis indicators relatively to optimal ARMA forecasters Are technical analysts "in complete darkness" or not too far from the optimal system ?

In the negative what is the most profitable forecaster ? This question is still open at the present time

Economists are often puzzled as to why profit-maximising firms buy professional forecasts when statistics such as the root-mean squared error or the mean absolute percentage error often indicate that simple extrapolative models such as the random walk forecast almost as well Leitch and Tanner(1991) conclude that a possible reason is that these traditional error measures may not be closely related to a forecast's profitability Friend and Westerfield(1975) argue that trading rules could test the economic quality and quantity of information whereas statistical tests can only test for the existence of the information White(1988) believes on the one hand that the method of least squares is adequate for testing the efficient market hypothesis On the other hand, he strongly points out that least square is not necessarily the method that one should use if interest attaches to building a rule for market trading purposes Such rules following White(1988) should be evaluated and estimated using profit and loss in dollar from generated trades, not squared forecast error Leuthold and Garcia(1992) express a slightly different opinion They believe that relative Mean Squared Errors provide only an indication of the potential for market inefficiency A sufficient condition for market inefficiency would be whether the forecasting method can generate risk-adjusted profits which exceed the cost of usage Mills(1992 36) states "Financial market are often predictable to some extents, but the crucial question is whether this predictability can be exploited to make excess profits from trading in the markets "

In sum, academics unanimously recognise that error measures and profits are different if not unrelated performance criteria They however disagree on the consequences of these discrepancies on market efficiency tests Still no theoretical attempts to our knowledge have been made to relate ex-ante profits and error measures That will be the object of Chapter 4 which will compare accuracy and profits of quantitative techniques assuming that the price process is Gaussian

2 3 4 *Similarities and differences between trading rules*

Theoretical correlations between statistical and technical trading rules are an alternative way to relate forecasting methods Establishing theoretical correlations between trading rules has been considered as an extremely difficult task (Brock, Lakonishok and LeBaron, 1992) However in Section 5 1, it is shown that exact analytical results can be obtained under the assumption that the underlying process of price returns follows the random walk without drift There are three reasons for investigating correlations between trading rules

Firstly, rules correlations would provide a measure of similarity between trading systems With the exception of Lukac, Brorsen and Irwin(1988a), rules have been merely listed than classified on the basis of their properties Rules belong normally to two classes (a) trend-following and (b) overbought-oversold indicators (Kaufman, 1987, Schwager, 1984, Bechu and Bertrand, 1992, Cahen, 1990) Overbought-oversold indicators differ from trend-following systems in that they are designed to anticipate rather than simply lag changes in price movements They include among others the momentum and moving average oscillators They often have been considered and reported as non-trend-following rules (Allen, 1990) That is obviously a misconception that this thesis will attempt to solve Many systems which are considered to be different are extremely similar if not completely identical A simple example is the strict identity between the indicators simple moving average of order 2 and momentum of order 1[6] So it seems to us that distinguishing rules on the basis of their convex-concave properties is far more relevant and less ambiguous than on the basis of trend-following, overbought-oversold A proper classification of trading rules is therefore needed Such a classification would be of immense help For instance, it is not unusual to find trading rules based on more than three parameters So testing the profitability of such a rule, at each combination of possible parameters, can be time consuming and a demanding task even for powerful computer Prado(1992) designs to this effect search algorithms He however recognises that the lack of thoroughness caused by the very limited scope of the step search can prove to be large drawback in some cases, especially if the step search reveals that each variable contributes significantly to performance It follows that the knowledge of trading rule correlations might allow more efficient search algorithms

Secondly, rules correlations would permit the construction of an efficient portfolio of rules Until now such portfolios have been build empirically for given financial time series, (Brorsen and Lukac, 1990) but have never been established theoretically for given stochastic processes

Thirdly, rules correlations would allow the establishment of the joint profitability of mechanical systems The resulting tests of non-zero profitability could then be more powerful than any single test (Brock, Lakonishok and LeBaron, 1992) This point will be considered in Chapter 5

2.4 MODELS IN THIS RESEARCH

Section 2 3 2 has shown the advantages of establishing the return distribution of a technical strategy using theoretic or stochastic modelling Stochastic modelling is used in the research to assess the ability of forecasting strategies to meet their goal under a broad

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6 This fact has been ignored by Goldberg and Schulmeister(1988)

set of market conditions Subsequently, this will allow to determine models for prices or returns which can reproduce all the known properties of recorded prices, and in particular the trading rules performances

The probabilistic foundations of prices changes have been first established by Bachelier(1900) The basic hypothesis is that the market does not believe, at a given moment, either to an increase nor to a decrease of level Consequently, the expected value from speculation is zero There are in fact three statistical hypotheses in Bachelier(1900) model

(a) the process is strictly stationary
The multivariate distribution of price changes does not depend on the choice of time
(b) the process is without memory
Price changes are independent over time The knowledge of past variations cannot give any indication about its future values
(c) the variance of the process exists

More precisely, Bachelier(1900) implies that prices changes have independent and normal distributions Then Osborne(1959) instead of considering the process of price changes prefers studying the quantity

$$X_t = Ln(P_t / P_{t-1}) \qquad [2\ 1]$$

where P_t is the asset price recorded once on each trading day t (week, month, year), always at the same time of day It is assumed in addition that no dividends are paid during day t Osborne's(1959) transformation is due to the fact that direct statistical study of financial prices is difficult because consecutive prices denote non-stationarity Subsequently first differencing is necessary to achieve stationarity In addition, the logarithmic transformation aims in particular at diminishing scale effects Then continuous time generalisations of discrete time results are then easier and returns over more than one day are simple functions of single day returns Returns are said to be normally distributed or alternatively prices lognormally Numerous operational applications have followed from these results and its continuous version such as option and portfolio insurance theories For instance, the Black-Scholes(1973) option pricing formulae are still widely used

However two observations seem to contradict the assumptions of independent normal returns Firstly, market prices exhibit slow and irregular cycles which question the hypothesis of independence Alternative models can still be normal but dependant (linear models) Secondly, financial time series often present discontinuities or jumps far too big to be compatible with normal process (nonlinear models)

It appears in many cases difficult to refute the stationarity hypothesis (a) which might be the most important in Bachelier(1900) assumptions When the stationarity hypothesis is rejected, the statistical framework becomes unoperational if not unclear A notable exception is the ARCH model[7]

A choice of which models to include in this research had to be made because financial models are abundant and in growing numbers The reader is referred to Duffie(1988) and Roger(1991) for good introductions to financial modelling, and for deeper approaches to Taylor(1986) and Baillie and McMahon(1989) It seems here unrealistic to consider all the models proposed in the literature to characterise financial prices Our selection has been based on two criteria, popularity and tractability The models presented below reproduce the broad, popular and plausible features of financial prices previously mentioned In addition, it will be possible to study their ability to duplicate trading rules returns

2 4 1 *Linear models*

Stock indices have often exhibited trends and cycles implying the presence of serial correlation over business and election cycles and during period of economic instability In addition serial correlation has been frequently observed in the prices of other types of assets, such as commodities and currencies

So it is not unreasonable, at least as a first approximation, to consider Gaussian models of financial prices That is, the joint distribution of $(X_{t+1},X_{t+2}, X_{t+k})$ is multivariate normal for every possible integer k Gaussian processes will be defined by

$\mu = E(X_t) \qquad \sigma^2 = Var(X_t)$,

$\rho_h = Corr(X_t,X_{t+h})$ = autocorrelations between X_t and X_{t+h}

Stationary Gaussian processes are always linear

A more general definition of linear process is

$$X_t = \mu + \sum_{j=0}^{\infty} b_j \varepsilon_{t-j} \qquad [2\ 2]$$

where $\{\varepsilon_t\}$ is a zero mean strict white noise process and constants b_j

There are three important special cases of linear models the moving average MA(q), the autoregressive AR(p) and the autoregressive-moving average ARMA(p,q) models They are respectively defined by

[7] See Section 2 4 2

Moving Average process MA(q)

$$X_t = \mu + \varepsilon_t - \sum_{j=1}^{q} b_j \varepsilon_{t-j} \qquad [2.3]$$

where $\{\varepsilon_t\}$ is a zero mean strict white noise process and constants b_j.

Auto-Regressive process AR(p)

$$X_t = \mu + \sum_{j=1}^{p} (a_j X_{t-j} - \mu) + \varepsilon_t \qquad [2.4]$$

where $\{\varepsilon_t\}$ is a zero mean strict white noise process and constants a_j.

Auto-Regressive moving-average process ARMA(p,q)

$$X_t = \mu + \sum_{j=1}^{p} a_j (X_{t-j} - \mu) + \varepsilon_t - \sum_{i=1}^{q} b_i \varepsilon_{t-i} \qquad [2.5]$$

where $\{\varepsilon_t\}$ is a zero mean strict white noise process and constants a_j, b_i.
Financial models usually investigate ARMA(p,q) for $p+q \leq 2$, Taylor(1986).

Price trend model

Taylor(1980) proposes an original approach to model trends in financial prices. Let us first recall his price-trend hypothesis and then adopt a simple example consistent with this hypothesis.

The fundamental trend idea is that several returns are influenced in the same way, either towards a positive conditional mean or towards a negative conditional mean (Taylor, 1986). Thus trends will cause **positive** autocorrelations. The impact of that current information which is not fully reflected in the current price, upon future returns, should diminish as time goes on. Thus the autocorrelations should decrease as the lag increases. The simplest parametric autocorrelation functions consistent with the observations has been first investigated in Taylor(1980), and is defined by:

$$H_1: \rho_h = Ap^h \quad A, p, h>0 \qquad [2.6]$$

There are two parameters in H_1 Parameter A measures the proportion of information not reflected by prices within one day. Parameter p measures the speed at which imperfectly reflected information is incorporated into prices. As $A \rightarrow 0$ or $p \rightarrow 0$, information is used perfectly. Credible price-trend models have typical parameter values $A=0.03$ and $p=0.95$ (Taylor, 1986). Low values for A are inevitable whilst values for p near to 1 indicate trends lasting for a long time.

One simple example assumes the return X_t is the sum of an autoregressive trend component μ_t and an unpredictable residual e_t

$$X_t = \mu_t + e_t \qquad [2\ 7]$$

$$\mu_t - \mu = p(\mu_{t-1} - \mu) + \zeta_t \qquad [2\ 8]$$

$$A = Var(\mu_t)/Var(X_t) \qquad [2\ 9]$$

The returns then have autocorrelations $\rho_h = Ap^h$ [2 6]

The processes $\{\mu_t\}$, and $\{e_t\}$ are supposed to be stochastically independent and Gaussian processes (hence linear)

Equation [2 8] is a measure of the proportion of slowly reflected information The first day, there is a probability p that the news is slowly reflected and contributes to μ_t-μ and a probability 1-p that the news is quickly reflected and contributes to e_t Prices, therefore, would tend to move in one direction (the trend) for a period of time and that these trends themselves change in a random and unpredictable fashion Then the total response is equal to m_d times the first day's response and m_d will be called the mean trend duration of such trends It is shown to be

$$m_d = 1/(1-p) \qquad [2\ 10]$$

The price-trend model is in fact nothing else than a state representation of an ARMA(1,1) defined by

$$X_t - \mu - p(X_{t-1} - \mu) = \varepsilon_t - q\varepsilon_{t-1} \qquad [2\ 11]$$

where the variance reduction A is linked to p and q via

$$A = (p-q)(1-pq)/\{p(1-2pq+q^2)\} \qquad [2\ 12]$$

Consequently, it allows to include this particular price-trend model as a special case of Gaussian processes[8]

There are many more statistical models consistent with the price-trend hypothesis, such as models which include changing conditional variances and nonlinearities They can be found in Taylor(1986) For the sake of tractability, these models will be ignored in this thesis and other nonlinear models preferred

2 4 2 *Nonlinear models*

One of the first complete studies on daily returns was done by Fama(1965) who found that returns were negatively skewed and leptokurtic More observations were in the left-hand (negative skewness) tail than in the right-hand tail In addition, the tails were fatter,

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8 This point will be of extreme importance in Chapter 3

and the peak around the mean was higher than predicted by the normal distribution (leptokurtosis)

Since then, many studies have shown that market returns are not normally distributed (Taylor, 1986, Boothe and Glassman, 1987b, Tucker and Pond, 1988, Hsieh, 1988) but rather follow a stable paretian distribution, meaning that the variance is infinite, (Mandelbrot, 1971, Cornew, Town and Crowson, 1984, McFarland, Petit and Sung, 1982) More generally there is a growing evidence that prices are nonlinear, (Hinich and Patterson, 1985, Gooijer, 1989, Brock, Hsieh and LeBaron, 1991) Since the class of nonlinear stochastic model is extremely large, we restrict our attention to two classes, which encompass all nonlinear stochastic models discussed in the time series literature (Brock, Hsieh and LeBaron, 1991)

Mean-Nonlinearity $X_t = A(J_t) + \varepsilon_t$

Variance-Nonlinearity $X_t = B(J_t)\,\varepsilon_t$

where $J_t = [X_{t-1}, \ ,X_{t-k}, \varepsilon_{t-1}, \ \varepsilon_{t-k}]'$ Here ε_t is an IID random variable with zero man and independent of past X's and ε's, and A and B are arbitrary nonlinear functions of J_t

In this thesis, attention will be limited, for the sake of tractability, to the ARCH(p) model for the variance-nonlinearity case and to the fractional Gaussian process for the mean nonlinearity case

Autoregressive conditional heteroskedasticity ARCH(p)

The approach to modelling changes in conditional variances is due to Engle(1982) Engle(1982) defines a zero-mean, autoregressive conditional heteroskedasticity ARCH(p) process, X_t, by

$$X_t = \mu + \{\sqrt{\alpha_0 + \sum_{i=1}^{p} \alpha_i (X_{t-i} - \mu)^2}\}\varepsilon_t \qquad [2\ 13]$$

there being p+1 *non-negative* parameters α_i with $\alpha_0 > 0$ and ε_t Gaussian white noise, with $\varepsilon_t \sim N(0,1)$ This model has very complicated unconditional distribution and it is difficult to establish conditions for stationarity and then to find the moments However, it must be emphasised that an ARCH process constructed from strict white noise will always be uncorrelated Then extensions have been proposed to introduce small autocorrelations, (Taylor, 1986) The ARCH model is in fact one of the many possibilities to model changes in conditional variances (Taylor, 1987, Curdy and Morgan, 1987, Baillie and McMahon, 1989) It has nevertheless profound implications on financial theory, (Gourieroux, 1992)

Fractional Gaussian process

The efficient market hypothesis implicitly assumes that all investors immediately react to new information, so that the future is unrelated to the past or the present Peters(1991) assumes on the other hand that most people wait for information and do not react until a trend is clearly established The amount of confirming information necessary to validate a trend varies, but the uneven assimilation of information may cause a biased random walk Biased random walks were first studied by Hurst(1951) They are equally called fractional brownian motions or fractal time series Since Mandelbrot(1971), fractional noise has become a quite popular model of financial rates and is now considered as a plausible alternative to the random walk hypothesis (Walter, 1990, 1991, Peters, 1991, Sowell, 1992)

A good introduction to long memory time series and fractional differencing can be found in Granger and Joyeux(1980) A discrete time analogue of continuous-time fractional noise is given in Hosking(1981) Hosking(1981) discretization has got the advantage beyond others to be a simple extension of linear Gaussian processes While still keeping the stationarity hypothesis, this model has now the potential to explain price jumps empirically observed

An ARIMA(0,d,0) process or fractional Gaussian process, is formally defined by Hosking(1981) as

$$\nabla^d (X_t - \mu) = e_t \qquad [2\ 14]$$

where $\nabla^d = (1-B)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k = 1 - dB - \frac{1}{2}d(1-d)B^2 - \frac{1}{6}d(1-d)(2-d)B^3 - \qquad [2\ 15]$

and B is the backward operator defined by $B(X_t)=X_{t-1}$, μ the mean return and $\{e_t\}$ the white noise process In this thesis, the $\{e_t\}$ consists of independent identically distributed (normal) random variables with mean zero and variance σ_e^2 The following theorem gives some of the basic properties of the process, assuming for convenience that $\sigma_e^2 = 1$

Theorem 1

Let $\{X_t\}$ be an ARIMA(0,d,0) process

(a) When $d < \frac{1}{2}$, $\{X_t\}$ is a stationary process and has the infinite moving average representation

$$X_t = \mu + \sum_{k=0}^{\infty} \psi_k e_{t-k} \quad \text{where} \quad \psi_k = \frac{(k+d-1)!}{k!(d-1)!}$$

(b) When $d > -\frac{1}{2}$, $\{X_t\}$ is invertible and has the infinite autoregressive representation

$$\sum_{k=0}^{\infty} \xi_k (X_{t-k} - \mu) = e_t \quad \text{where} \quad \xi_k = \frac{(k-d-1)!}{k!(-d-1)!}$$

(c) the covariance function of $\{X_t\}$ is $\gamma_k = E(X_t X_{t-k}) = \dfrac{(-1)^k(-2d)!}{(k-d)!(-k-d)!}$

and the correlation function of $\{X_t\}$ is $\rho_k = \dfrac{\gamma_k}{\gamma_0} = \dfrac{(-d)!(k+d-1)!}{(d-1)!(k-d)!}$ (k=0,±1,) [2 16]

From the theorem we see that when $-\frac{1}{2} < d < \frac{1}{2}$, the process $\{X_t\}$ is both stationary and invertible Both ψ_k and ξ_k decay hyperbolically, rather than showing the exponential decay characteristic of an ARIMA(p,0,q) process McLeod and Hipel(1978) define a stationary process as having a long or short memory according to whether its correlations have an infinite or a finite sum Theorem 1 implies that the ARIMA(0,d,0) process is a long memory stationary process when $0 < d < \frac{1}{2}$

When $0 < d < \frac{1}{2}$, the ARIMA(0,d,0) as such may be expected to be useful in modelling long-term persistence The spectrum as a whole has a shape "typical of an economic variable" (Granger, 1966) The correlations and partial correlations of $\{X_t\}$ are all positive as for the price-trend model If the series has been up (down) in the last period, then the chances are that it will continue to be positive (negative) in the next period Walter(1991), Peters(1991) even add that in this case trends are apparent The closer d is to 0, the noisier the trend-reinforcing behaviour will be, and the less defined its trends will be

When d=0, the ARIMA(0,d,0) process is white noise, with zero correlations and constant spectral density The present does not influence the future

When $-\frac{1}{2} < d < 0$, the ARIMA(0,d,0) process has a short memory and is an antipersistent or ergodic series It is often referred to as "mean reverting" Except $\rho_0 = 1$, the correlations and partial correlations of the process are all negative If the series has been up in the previous period, it is more likely to be down in the next period This kind of series would be choppier, or more volatile, than a random series, because it would consist of frequent reversals

The fractional Gaussian process has only three parameters, mean, variance and *fractional parameter* d or alternatively the *Hurst exponent* H which is linked to d by the relation given by Hosking(1981), Geweke and Porter-Hudak(1983)

$H=d+0.5$ [2 17]

The Hurst exponent describes the likelihood that two consecutive events are likely to occur (Peters, 1991) If H=0 6, there is, in essence, a 60 percent probability that if the last move was positive, the next move will also be positive

Financial fractional Gaussian process usually fall in the range $0 < d < \frac{1}{2}$ or equivalently $\frac{1}{2} < H < 1$ (Walter, 1991, Peters, 1991) They so are characterised by a tendency to have trends and cycles, as the price-trend model However opposite to this one, the fractional Gaussian process exhibits abrupt and discontinuous changes because of an infinite, or undefined variance Cycles are no longer regular but erratic and aperiodic

Two important properties of chaotic time series must be highlighted (a) The generated time series are completely aperiodic, i e they never repeat themselves This does not mean that the observed patterns have to be totally disorderly It is very possible as mentioned earlier that one can distinguish patterns that look like cycles but that suddenly disappear after a number of periods Also it is possible that the variance of the observed time series remains constant for a long period of time and then changes without reason

(b) In addition to this aperiodic behaviour, chaotic systems have a second remarkable property The generated time series are extremely dependent on initial conditions In order to use the model for forecasting purposes, we should be able to obtain infinitely precise estimates of the parameters of the model

Finally it must be said that the fractional Gaussian process is a particular case of a more general model, the ARIMA(p,d,q) model (Hosking, 1981, 1984)

Benefits can arise from considering nonlinear models Granger(1992) indicates that many forecasters need to break away from simple linear univariate ARIMA Following Granger(1992), it is often not easy to beat convincingly these simple methods, so they make excellent reference models, but he concludes that they often can be beaten Diebold and Nason(1990) expressed a mixed opinion about nonlinear models On the one hand, they recognise that important nonlinearities may be operative in exchange rate determination On the other hand they ask a puzzling question " Why is it that while statistically significant rejections of linearity in exchange rates routinely occur, no nonlinear model has been found that can significantly outperform even the simplest linear model in out-of sample forecasting ? " Despite its increasing popularity, the evidence for chaotic and infinite variance models is not strong (Lo, 1991) Finite-variance models often outperform asymmetric stable distribution (Tucker, 1992)

It follows that nonlinear models constitute a serious alternative to linear models, although not yet a substitute It has not yet been proved that nonlinear models yield significant ex-ante forecast improvement (Diebold and Nason, 1990) Finally, there appear to be weak connections between technical trading rules and nonlinearities in foreign exchange series (LeBaron, 1992b, Antoniewicz, 1992)

This is why our choice of models seems a priori rational in terms of both economic and statistical importance

2.5 SUMMARY

Both portfolio insurance strategies and forecasting methods are similar in that they are convex However they differ in that the forecasting methods applied in trading are path-dependent, while portfolio insurance techniques are generally path-independent This crucial difference is the result of opposite views about the statistical nature of the process which drives prices

If financial prices follow a random walk, path-independence is required to maximise the utility function of an investor Then investment strategies are formulated not for purpose of enhancing returns, which is not possible under the assumption of random walk, but in order to reshape the original return distribution, so as to minimise the downside risk

If financial prices do not follow a random walk, path-dependent strategies can be of use However one needs to establish under what particular market conditions, what particular forecasting strategy is useful The most apparent criterion for measuring the usefulness of path-dependent strategies is profitability

To assess profitability, one has to turn to stochastic modelling, because it is the only tool available which is independent of time period or asset Therefore, plausible models of financial prices are presented Since maximising returns is the primary objective of market timers, the expected return of a trading rule is subsequently the most important statistic which needs to be established

Chapter 3

STOCHASTIC PROPERTIES OF TRADING RULES

According to portfolio insurance studies, the best way to estimate the distributional properties of an investment strategy is through stochastic modelling That is done in this chapter for forecasting strategies, by assuming that logarithmic returns follow rather than a random walk any Gaussian processes That constitutes a considerable improvement of past studies since it covers a wider range of possible market conditions Particular emphasis is given to the expected return of trading rules by providing exact analytical formulae This chapter contributes to the discussion of economic versus time series analysis by addressing two fundamental issues of this debate

a) Are the models proposed by academics useful for forecasts ? In other words can a profitable decision rule be based on them ?

b) Are technical forecasters able to trade profitably ?

Section 3 1 defines the trading rule process Section 3 2 explains the goal of stochastic modelling and our underlying assumptions Section 3 3 defines VARs models and their expected rate of return Section 3 4 shows that many technical indicators can be reformulated as VARs models Consequently, technical and VARs predictors used for trading purposes are seen as "linear rules" and therefore can be examined in an unified framework Finally, Section 3 5 summarises and concludes our results

3 1 TRADING RULES

3 1 1 *Rule signals*

Suppose that at each day t, a decision rule is applied with the intention of achieving profitable trades It is the price trend which is based on market expectations that determines whether the asset is bought or sold When the asset is bought, the position initiated in the market is said to be "long" When the asset is sold, the position initiated in the market is said to be "short" A forecasting technique is assessed as useful and will subsequently be used if it has economic value In short, the forecast is seen as useful if in dealer terms, it can "make money" For achieving this purpose, market participants use price-based forecasts Therefore the predictor F_t is completely characterised by a mathematical function f of past prices $\{P_t, \quad ,P_{t-m}, \quad \}$[1]

$$F_t = f(P_t, \quad ,P_{t-m+1}, \quad)$$

The only crucial feature which is required from the forecasting technique is its ability to accurately predict the direction of the trend in order to generate profitable buy and sell signals Trading signals, buy (+1) and sell (-1), can then be formalised by the binary stochastic process B_t

$$\begin{cases} \text{"Sell"} \Leftrightarrow B_t = -1 \Leftrightarrow F_t = f(P_t, \quad ,P_{t-m+1}, \quad) < 0 \\ \text{"Buy"} \Leftrightarrow B_t = +1 \Leftrightarrow F_t = f(P_t, \quad ,P_{t-m+1}, \quad) > 0 \end{cases}$$

It must be remarked that the signal of a trading rule is completely defined by one of the inequalities giving a sell or buy order, because if the position is not short, it is long

Only in the trivial case of a Buy and Hold strategy, the signal B_t is deterministic and is +1 irrespective of the underlying process Otherwise, trading signals B_t are stochastic variables They are time series of binary data generated by an underlying time series of continuous data The family of discretization mechanisms is broad since it is the one of trading rules But in all cases, discretizations arise by a truncation of a continuous-valued process which is a special case of Keenan(1982) By nature, the signal is a highly nonlinear function of the observed price series P_t (Neftci and Policano, 1984, Neftci, 1991), and therefore it can be highly dependent through time B_t remains constant for a certain random period, then jumps to a new level as P_t behaves in a certain way Trading in the asset occurs throughout the investment horizon at times that depend upon a fixed set of rules and future price changes

As an example, consider a moving average of order five (days) defined as

$$f(P_t, \quad ,P_{t-m+1}, \quad) = P_t - \frac{P_t + P_{t-1} + P_{t-2} + P_{t-3} + P_{t-4}}{5}$$

Figures 3 1[a] and [b] illustrate this behaviour when applied to the moving average method which says when the rate penetrates from below (above) a moving average of a given length, a buy (sell) signal is generated

[1] or logarithmic returns $\{X_t, \quad ,X_{t-m}, \quad \}$, since these ones are simple function of past prices

Simple moving average method

5-days moving average

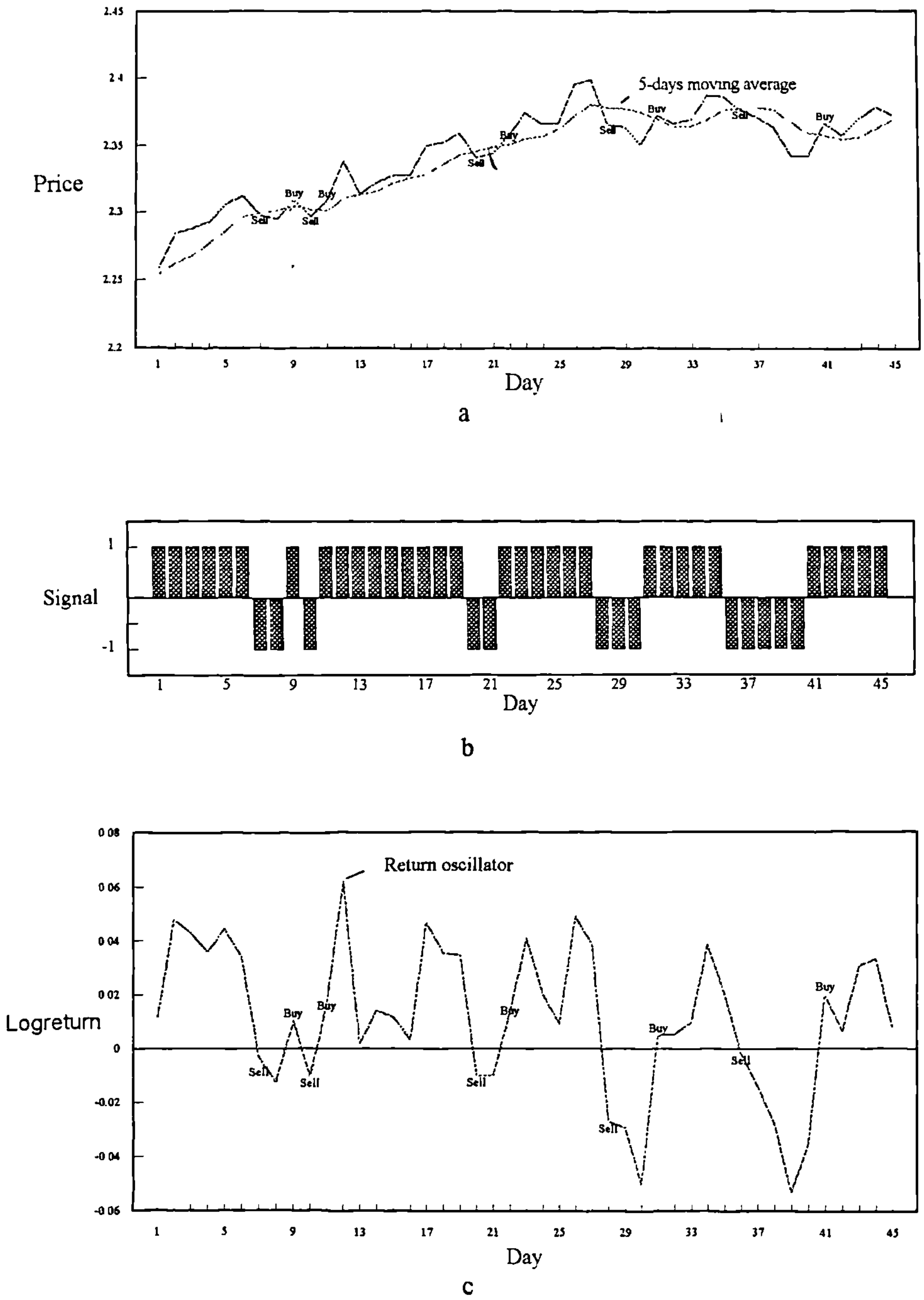

Figure 3.1: The simple moving average method.
a. price series, b. signal time series, c return oscillator

3 1 2 *Rule returns*

The study of the binary process of signals is of limited interest for trading purposes The focus should be the economic consequence, i e the returns process implied by the decision rule, rather than on the generating process of the signal

Let us recall the investment strategy
Assume a position is taken in the market for a given period [t-1,t] The logarithmic return during this time is $X_t = Ln(P_t/P_{t-1})$ The nature of the position (long or short) is given by the signal triggered at time t-1, B_{t-1} following a given technical rule
Returns at time t made by applying such a decision rule are called "rule returns" and denoted R_t Their value can be expressed as

$$R_t = B_{t-1} X_t \quad \Leftrightarrow \quad \left\{ \begin{array}{l} R_t = -X_t \ \text{if} \ B_{t-1} = -1 \\ R_t = +X_t \ \text{if} \ B_{t-1} = +1 \end{array} \right\} \qquad [3\ 1]$$

Two important remarks should be made

(a) **Rule returns** are the product of a binary stochastic signal and a continuous returns random variable Except in the trivial case of a Buy and Hold strategy, the signal B_t is a stochastic variable and so rule returns are **conditional** on the position taken in the market (long $B_t = +1$ or short $B_t = -1$) That is the main feature of rule returns Up to this point, little attention has been paid to the rule returns process Earlier studies have mainly focused on the price change process or underlying returns The fact is that when evaluating forecasting ability the mean squared error criterion has been used to evaluate their usefulness rather than any economic evaluation So their measures have been **unconditional** to the position taken in the market

(b) Our rule return definition clearly corresponds to an **unrealised** return By unrealised we mean that rule returns are recorded every day even if the position is neither closed nor reversed, but simply carries on

3 1 3 *Realised returns*

By **realised** return we mean cumulated daily returns until a position is closed and reversed from long (short) to short (long) A position is opened at time t if the signal triggered at t is different than at t-1 and then is closed at time t+n when an opposite signal occurs for the first time When opening a position, one cannot say with certainty when it is going to be closed and reversed For technical indicator, it depends on the rule itself and the stochastic process of prices Indeed reversal of positions occur at random moments even if signals are triggered on a deterministic (daily) basis For instance, they occur on days 1, 6, 9, 10, 11 on figure 3 1[b] Nevertheless, realised returns are "true returns" and exhibit the real timing of cash flows generated by technical strategy The realised return can be expressed mathematically by

$$R = \sum_{D=1}^{n} R_{t+D} \qquad [3\ 2]$$

where D represents the stochastic duration of the position which will last n days if

$$\{D=n\} \Leftrightarrow \{ B_{t-1} \# B_t, B_t = B_{t+1} = \quad = B_{t+n-1}, B_{t+n-1} \# B_{t+n} \} \qquad [3\ 3]$$

Equations [3 2] and [3 3] show the main difficulty when studying realised returns They are the sum of a **stochastic** number D of random variables X_t The fact that the duration D depends on the logreturns X_t through a quite complex relationships renders equation [3 2] of limited practical use

Realised returns are highly heteroskedastic even if the underlying process is not (Cumby and Modest, 1987, Hartzmark, 1991) Moreover, because trading systems are usually designed to cut losses quickly and let profits ride, realised returns are in addition positively skewed and leptokurtic (Cornew, Town and Crowson, 1984, Bookstaber, 1985, Goldberg and Schulmeister, 1988, Rechner and Poitras, 1993) In what follows, heteroskedasticty, skewness and leptokurtosis of realised returns are quantified for the simple moving average rule using stochastic modelling

It is assumed that the process of logarithmic returns is a normal random walk without drift Then the returns distributions of the simple moving average of orders 2, 10 and 50, have been established using Monte-Carlo simulations (Table 3 1) It can be seen first that summary statistics (average, variance, kurtosis and skewness) of simulated returns following the simple moving average of order 2 are very close to their exact values determined in Appendix 3 4 Realised returns following the simple moving average of order two exhibit identical expected value (zero) than underlying returns but double variance due to non-normality When the order of the moving average increases, the average duration of the position increases and consequently the variance of realised returns A similar phenomena can be observed for the coefficients of kurtosis and skewness In fact, realised returns exhibit for different rules very different shape of distributions (risk, skewness and kurtosis), under the random walk without drift assumption Subsequently, one could wrongly conclude that all rules are not equally risky under the random walk assumption, but that the longer term the rule is, the riskier it is This theoretical feature has unfortunate consequences when testing the significance of trading rules profits Perfectly good performance records will be downgraded in comparison to others which simply possess a more nearly normal distribution (Cornew, Town and Crowson, 1984) Thus, Sharpe ratios from non-normal distributions will on average underestimate trading performance

Table 3 1: Realised returns statistics under the random walk assumption

Realised returns statistics following a simple moving average rule Monte-Carlo simulations $N(0,\sigma^2)$, σ=7 E-3 2500 observations replicated 250 times			
Statistic\Order	2	10	50
Average	-1 408 E-5 (0)[2]	-8 241 E-5	2 1154 E-4
Standard deviation	9 876 E-3 (9 900 E-3)	16 309 E-3	24 792 E-3
Kurtosis	5 057 (5 320)	13 477	21 776
Skewness	1 663 (1 693)	2 663	3 814

Trading practices when recorded on a realised basis produce asymmetry Then this raises the issue of whether " the average abnormal return is a sufficient and even interesting statistic when the trading rule generates a skewed distribution of abnormal returns ", (Ball, 1989 605) It is not absolutely certain that the variance of realised returns adequately describes the risk of a technical indicator Past studies based on realised returns might be flowed, mainly because they imply different risks for different rules applied to a same underlying process (Goldberg and Schulmeister, 1988, Lukac, Brorsen and Irwin, 1988b, Taylor, 1990b, Balsara, 1992 Table 9 3, Rechner and Poitras, 1993) The T-Student given in these studies and technical analyst reviews (Knight, 1993) might say nothing about the usefulness of a technical indicator for reasons given above

In sum, the use of realised returns as a measure of performance should be avoided whenever possible because it may be confusing to compare dynamic strategies that have different variances, skewness and kurtosis Sometimes there is no other alternative as when investment performance is recorded through surveys (Cumby and Modest, 1987, Hartzmark, 1991) However when studying mechanical systems, unrealised returns can be easily evaluated and should indeed be preferred to realised returns for their statistical properties we now establish

2 In bracket are the theoretical results which can be found in Appendix 3 4

3 2 STOCHASTIC MODELLING

3 2 1 *Goal*

An important question not yet answered in the literature is to know how profitable are forecasting strategies Can non-zero profit be awaited from such methods and if yes what are the parameters of the underlying price process making the rule profitable? The goal of this chapter is to specify the theoretical relationship between rule returns and standard statistical measures of serial dependency Such a specification, although not pursed in previous research, is useful because rule returns provide a measure of economic significance for serial dependencies in financial returns that otherwise might not be readily interpretable As emphasised in Section 2 3, gauging the economic significance of observed daily asset return autocorrelations is difficult The relationship between the magnitude of observed serial correlation coefficients and the profits of technical trading rule is indeed difficult to exhibit This chapter attempts to solve this issue by examining how trading rule returns are related to the statistical characteristics of the underlying series Our goal is to show that using stochastic modelling, it is possible to establish what are the parameters of the underlying price process which generate if any non zero expected return from trading rules

3 2 2 *Assumptions*

For the remainder of this chapter, we will assume that the underlying process of logarithmic return X_t is stationary and Gaussian[3] Two reasons can be given for restricting our study to such processes

(a) The very few studies that have tried to analyse forecasting strategies have all investigated the case of Gaussian processes (Neftci, 1991, Bird, 1985, Sweeney, 1986, Praetz, 1976, Taylor, 1990a, 1992b, LeBaron, 1991, 1992b) Indeed as Neftci(1991) points out very little is known about the statistical properties of forecasting strategies So a Gaussian process may be the preliminary step to more complex models Gaussian processes contain by themselves a wide class of models and therefore monitor a wide range of possible market conditions

(b) It is questionable whether complicated nonlinear models will bring much additional support to our argumentation For instance, rule returns are not very sensitive to the conditional heteroskedasticity effects in comparison to the positive autocorrelation effects That is shown via Monte-Carlo simulation, in Taylor(1992b, Table 2) for the

[3] There will be one exception the ARCH(p) model

channel rule under the assumption of a price-trend model with conditional heteroskedasticity, and in Antoniewicz(1992, Chapter 4 Section 4) for the simple moving average rule under the assumption of a GARCH(1,1) and an AR(1) model with nonlinear moving average structure LeBaron(1992b) shows that trading rule results themselves are not necessarily indicative of nonlinearities in foreign exchange series He finds in particular that linear models are capable of replicating the trading rule returns along with the small autocorrelations observed in these series

Since very little is known about the properties of forecasting strategies when the underlying model is nonlinear, the cases of ARCH(p) and fractional Gaussian processes will be studied in detail Despite the fact that many other non-linear processes have been considered for modelling financial returns (Section 2 4 2), they will not be studied here since corresponding rule returns are difficult to establish

In order to model rule returns, restrictions must be placed not only on the nature of the underlying process but on the nature of the rule used as well We have already restricted our choice to well-defined rules in the Neftci(1991) sense and rejected some of the arbitrary rules used by chartists such as various patterns, trend crossing methods of which certain are ill defined (Neftci, 1991) However even when indicators are well defined, it does not mean their statistical properties can be tracked analytically This is why the set of trading rules investigated in this thesis will be restricted to VARs models and linear technical rules we now define

3.3 VECTORS AUTOREGRESSIONS (VARs) MODELS

3 3 1 *Definition*

Instead of considering the process of prices, academics prefer studying the compound logarithmic returns (logreturns) process[4] defined by

$X_t = \mathrm{Ln}(P_t/P_{t-1})$ [2 1]

The return X_t is the change in prices between time t-1 and t, assuming that no dividends are paid during day t A linear forecast is then used to predict one-step ahead return X_{t+1}, given by

$$F_t = \delta + \sum_{j=0}^{\infty} d_j X_{t-j} \qquad [3\ 4]$$

[4] See Section 2 4

with δ and the d_j being constants

We will note μ_F the expected value of F_t and σ_F^2 the variance of F_t

This type of forecasting technique is referred to as a vector autoregression VAR model The predictor is normally defined such that it minimises the mean squared error between the forecast value and the one-step ahead return to be estimated If the true process of returns is linear, VARs forecasters must yield the best forecasts of a stochastic process in the mean squared error sense (MSE) VARs models do not generate explicit trading signals However if we assume zero transactions costs, the intuitive decision rule derived from VARs models is to go short if the [one-ahead] forecast is negative and go long if it is positive That is the forecasting technique implicitly triggers a daily signal B_t specifying a long (+1) or short (-1) position following the decision rule

$$\begin{cases} \text{"Sell"} \Leftrightarrow B_t = -1 \Leftrightarrow F_t = \delta + \sum_{j=0}^{\infty} d_j X_{t-j} < 0 \\ \text{"Buy"} \Leftrightarrow B_t = +1 \Leftrightarrow F_t = \delta + \sum_{j=0}^{\infty} d_j X_{t-j} > 0 \end{cases} \qquad [3\ 5]$$

3 3 2 *Rule returns process*

[Unrealised] rule returns are the product of a binary stochastic signal B_{t-1} and a continuous return random variable X_t Equation [3 1] represents the trading rule return equation assuming discrete trading in markets where the underlying asset is lognormally distributed Lee, Rao and Auchmuty(1981) make similar assumption concerning option valuation

If we assume that the underlying process X_t is Gaussian, and the rule linear, the forecaster F_t is equally Gaussian It can be seen from equations [3 1] and [3 5] that in this case, the rule return function is a mixture of marginal density functions of truncated bivariate normal density Such a distribution has been studied in the literature by Cartinhour(1990) He has derived it in a form that can be evaluated using an available computer algorithm developed by Schervish(1984) He showed that the marginal density function is a truncated normal density function multiplied by a "skew function" In general the greater the degree of truncation, the more severe the skewing effect will be

A truncated distribution is a common feature of portfolio insurance strategy As shown in Trippi and Harriff(1991), the terminal return distribution of dynamic asset allocation rules is highly asymmetric being either left-truncated or positively skewed

Bookstaber and Clarke(1987) showed that the put option strategy truncates the lower tail and maintains the upside potential Zhu and Kavee(1988) showed using Monte-Carlo simulations that two strategies, namely the synthetic put approach and the constant proportion strategy have the ability to reshape the return distribution so as to reduce downward risk and retain a certain part of upward gains

There is however a main difference between option and technical rule returns On the one hand when using a put option, the left\truncation is fixed at a deterministic level, the exercise price for a option On the other hand when applying a mechanical system, downside risk reduction still occurs[5], but the left or right truncation is a random one, due to the signal effect A trading rule generates by nature random infrequent trading The signal of a rule remains constant for a certain random period, then jumps to a new level as the price behaves in a certain way (figures 3 1[a] and [b])

This point highlights that rule returns are in fact closely related to the literature of infrequent trading and in particular with the Lo and Mc Kinlay(1990) approach The stochastic model of nonsynchronous asset prices they developed is based on sampling with random censoring They give explicit calculation of the effects of infrequent trading on the time series properties of asset returns[6] Contrary to Lo and Mc Kinlay(1990), we will have to consider explicitly two situations they only mentioned Firstly, our nontrading process is by its nature dependent, trading tomorrow (reversal of signal) depends on the signal of today Secondly, we will relax their assumptions of independent and identically distributed underlying returns

Expected value of rule returns can be established analytically assuming that underlying returns follow a Gaussian process, although the exact distribution cannot This is the most important statistic for trading purposes In addition, the one-period variance can be deduced from the expected value using the relation

$$\mathrm{Var}(R_t)=E(R_t^2)-(E(R_t))^2 = E(B_{t-1}^2 X_t^2)-(E(R_t))^2$$

We know that by definition $B_{t-1}^2 = 1$, and $E(X_t^2) = \sigma^2 + \mu^2$

where μ is the expected value or drift of X_t, and σ^2 the variance of X_t Therefore,

$$\mathrm{Var}(R_t)=E(X_t^2)-(E(R_t))^2 = \sigma^2 + \mu^2 - (E(R_t))^2 \qquad [3\ 6]$$

[5] The distribution of realised rule returns is highly skewed, see Appendix 3 4

[6] Further details can be found in Section 3 5 1

3 3 3 *Expected rule returns in models without drift*

We first assume that the underlying process X_t is without drift, i e $E(X_t)=\mu=0$ and that the forecaster is unbiased, i e $\delta=0$ in equation [3 4] implying that $E(F_t)=\mu_F=0$ Generalisation to biased forecaster and model with drift is postponed till Section 3 3 4

Random walk

Proposition 3 1[7]

If the underlying process of returns $\{X_t\}$ follows an iid normal distribution $N(0,\sigma^2)$, the process of rule returns $\{R_t\}$ is an iid normal distribution $N(0,\sigma^2)$

That implies more specifically that

$E(R_t) = 0$ [3 7]

$Var(R_t) = \sigma^2$ [3 8]

$Cov(R_t,R_{t+h}) = 0$ for $h>0$ [3 9]

That is a very unusual case where the distribution of the rule return is identical to the one of the underlying return and **independent on the rule itself** All rules exhibit the same standard deviation which is the underlying volatility Consequently the standard deviation seems in this case a good measure of risk, since under the random walk assumption no trading rules should be considered as riskier than others This decisive feature justifies ex-post the use of unrealised rather than realised returns

The distribution of the rule return must not be surprising since past and present returns used to generate the signal and the one-ahead return are here independent That is incidentally the result provided by Broffitt(1986, example 1) An important remark made by Broffitt(1986) is that although functionally dependent, rule and underlying returns are uncorrelated, the joint distribution being degenerated This is why a study of both processes could lead to apparent differences in the results

ARCH(p)

Proposition 3 2

If the underlying process of returns $\{X_t\}$ is a zero-mean, autoregressive conditional heteroskedasticity ARCH(p) process, the expected value of linear rule returns R_t is zero

It has been recognised that models for returns should have either non-stationary variance or conditional upon past observations, a variance dependent on such observations and additional variable This paragraph has just established rule returns expected value for one of these alternatives the ARCH(p) still assuming process without drift As long as the X_t

[7] Proofs of propositions are given in Appendix 3 3

are uncorrelated, no non-zero rule returns can be expected. So mean non-linearities might be necessary to generate non-zero profits as in the case of the fractional Gaussian process.

General Gaussian Process

Proposition 3.3

If the underlying process of returns $\{X_t\}$ follows a linear Gaussian process without drift, the expected value of linear rule returns R_t is given by:

$$E(R_t) = \sqrt{\frac{2}{\pi}}\sigma \text{Corr}(X_t, F_{t-1}) \qquad [3.10]$$

No known distribution can be worked out for linear rules assuming a such underlying model as pointed out by Cartinhour(1990). Nevertheless, an expected value can be derived and implies that rule returns are a positive function of the volatility when the correlation between forecaster and one-ahead return is positive.

3.3.4 *Expected rule returns in models with drift*

We assume that the underlying process X_t is with drift, i.e. $E(X_t)=\mu\neq 0$ and that the forecaster can be with constant, i.e. $\delta\neq 0$ in equation [3.4] implying that $E(F_t)=\mu_F\neq 0$.

Random Walk

Proposition 3.4

If the underlying process of returns $\{X_t\}$ follows an iid normal distribution $N(\mu,\sigma^2)$, the process of rule returns $\{R_t\}$ is a mixture of two normal laws defined by:

$R_t \sim N(-\mu,\sigma^2)$ with probability PS
$R_t \sim N(\mu,\sigma^2)$ with probability 1-PS

where PS is the probability of being short at time t, given by:

$$PS = \Pr(F_t<0) = \Phi(-\mu_F/\sigma_F) \qquad [3.11]$$

and Φ is the cumulative function of a N(0,1)

That is a very unusual case where the exact distribution of the rule return can be established. Subsequently, expected value and variance can be derived using well known properties of mixture of normal laws.

$$E(R_t) = \mu(1\text{-}2PS) \qquad [3.12]$$

$$Var(R_t) = \sigma^2 + 4\mu^2 PS(1\text{-}PS) \qquad [3.13]$$

$$Cov(R_t,R_{t+h}) = \mu E(B_{t-1}B_{t+h-1}X_t) - \mu^2(1\text{-}2PS) \qquad [3.14]$$

Praetz(1976), Bird(1985) and Sweeney(1986) (thereafter PBS) have derived expected value and variance from filter rules under the assumption of a normal random walk with drift They are shown to be

$E(R_t) = \mu(1\text{-}2f)$ [3 15]

$Var(R_t) = \sigma^2$ [3 16]

$Cov(R_t, R_{t+h}) = 0$ for $h>0$ [3 17]

where f is the frequency of short positions \

Surujaras and Sweeney(1992 34) recognise that their tests treat f as a constant, although f is of course endogenous and stochastic and will differ over samples In addition, Surujaras and Sweeney(1992 35) admit that their tests require constant mean and constant finite variance for the rule returns distributions Using the probability of being short given by equation [3 11] rather than the ex-post frequency of short positions will change expression [3 15] with the exact formulae [3 12] However formulae [3 16] and [3 17] are still misspecified and strictly speaking, should be replaced by [3 13] and [3 14] The latter results share in fact two common properties with the presence of nonsynchronous trading (Lo and Mc Kinlay, 1990) Firstly, technical trading increases the variance of individual security returns (with non-zero mean) The smaller the mean (in absolute value), the smaller is the increase in the variance of observed returns, [3 13] Secondly, technical trading induces non-zero serial correlation in individual security returns (with non-zero mean) The smaller the mean (in absolute value), the closer the autocorrelation is to zero, [3 14] Although theoretically different, PBS formulae [3 15], [3 16], and [3 17] are very close to [3 12], [3 13], and [3 14] for usual values of mean and standard deviation of logarithmic returns However PBS strong assumptions must be underlined especially if further researches investigate other Gaussian processes than the Random Walk with Drift It is not certain in those cases that returns can still be decomposed into two almost uncorrelated groups, long and short positions This is why we prefer carrying on investigations following the basic decomposition [3 1] applicable to any process

General Gaussian Process

Proposition 3 5

If the underlying process of returns $\{X_t\}$ follows a linear Gaussian process with drift, the expected value of linear rule returns R_t is given by

$$E(R_t) = \sqrt{\frac{2}{\pi}}\sigma Corr(X_t, F_{t-1})\exp(-\mu_F^2/2\sigma_F^2) + \mu(1 - 2\Phi[-\mu_F/\sigma_F]) \qquad [3\ 18]$$

As ın the case wıthout dnft, no known dıstnbutıons can be establıshed So we wıll lımıt results to the expected value of rule returns whıch ıs composed of two components One comes from the general Gaussıan process wıthout dnft and the other from the random walk wıth dnft

Equatıon [3 18] represents the most general case ın terms of lınear Gaussıan process All the earlıer formulae are specıal cases

To the best of the author's knowledge, the expected value of rule return for a general Gaussıan process has not been denved before It ıs not surpnsıng that exact analytıcal formulae of expected value of lınear rule returns can be establıshed for any Gausssıan processes sınce lınear rules are well defined (Neftcı, 1991)

A first comment ıs that a bıased forecaster mıght be suboptımal[8] That can be sımply noted by consıdenng a Gaussıan process wıthout dnft (μ=0) Assumıng that $\mu_F = \delta \# 0$ gıves an expected return of

$$E(R_t) = \sqrt{\frac{2}{\pi}}\sigma Corr(X_t, F_{t-1})\exp(-\mu_F^2 / 2\sigma_F^2)$$

That ıs of course below the expected return of a sımılar but unbıased forecaster gıven by equatıon [3 10]

3.4 TECHNICAL INDICATORS

The majonty of traders forecast pnce changes usıng technıcal analysıs, even though VARs technıques should yıeld better forecasts Fınancıal market players often prefer technıcal rules to VARs models, maınly because they are not lookıng for the forecaster whıch mınımıses the mean squared error (VARs) but maxımıses profits (technıcal rules ?) Technıcal analysts have claımed that opposıte to VARs models, technıcal ındıcators are able to capture the complexe nonlınearıty observed ın financıal pnces

Although technıcal analysıs and VARs models mıght have dıfferent objectıves, they both use the same ınformatıon, that ıs hıstorıcal pnces As outlıned ın Sectıon 2 2, technıcal analysıs covers a broad category of forecastıng rules However, certaın of whıch are hıghly subjectıve and ıll defined To be objectıve, buy and sell sıgnals should be based on data avaılable up to the current tıme t and should be ındependent of future ınformatıon

8 An ın depth dıscussıon can be found ın Chapter 4

Using the theory of Markov times, Neftci(1991) shows that the moving average method constitutes such a well defined methodology The simplest rule of this family is the single moving average which says when the rate penetrates from below (above) a moving average of a given length a buy (sell) signal is generated A formal algorithm of this decision rule is given by

$$\begin{cases} \text{"Sell"} \Leftrightarrow P_t < \dfrac{P_t + P_{t-1} + \quad + P_{t-m+1}}{m} \\ \text{"Buy"} \Leftrightarrow P_t > \dfrac{P_t + P_{t-1} + \quad + P_{t-m+1}}{m} \end{cases}$$

where P_t is the price of the asset recorded once on each trading day t, always at the same time of day, and m [>1] is the length (or order) of the moving average
Since the process of rate is assumed to be continuous, the equality case is of zero probability and is subsequently ignored in the remainder of this research

Rules based on mathematical formulas using past prices $\{P_t, \quad, P_{t-m}, \quad\}$ are well defined and objective in the sense that their performances can be assessed It must be emphasised however, that there does not exist any theory or "research algorithm" to design technical rules A current practice among traders is to measure the profits and losses generated by an arbitrary set of trading rules and to select the rule which maximises profits

3 4 1 <u>*Technical indicators as VARs models*</u>

Technical indicators signals are usually expressed by an inequality in terms of past prices ("price" signal) An equivalent formulation in terms of (logarithmic) returns should be sought whenever possible ("return" signal) There are two reasons for this

(a) ability to model rule returns
It has been shown in the previous section that when the signal is expressed by a linear combination of returns, expected value of rule returns can be easily found for any underlying Gaussian processes

(b) purposes of comparison with VARs models
VARs models are expressed in terms of returns So if technical indicators signals were to stay a function of price, direct comparison with VARs models would be difficult

For purposes of clarity, the steps allowing to reformulate a "price" signal in "return" signal relates to the crossing of a simple moving average Next it is shown that this methodology applies to many other popular mechanical systems and more generally to any system triggering a signal from a linear combination of past prices

We have seen that the signal generated by a trading rule is completely defined by the inequality giving a sell order For the simple moving average method of order m, the signal is

sell (go short) if $$P_t < \frac{P_t + P_{t-1} + \quad + P_{t-m+1}}{m} \qquad [3\ 19]$$

Straightforward rearrangements show that the inequality triggering a sell signal can be reformulated as $(1-P_{t-1}/P_t) + (1-P_{t-2}/P_t) + \quad + (1-P_{t-m+1}/P_t) < 0$

At this point, we assume that variations of rates can be approximated by their logarithms[9] That is $1-P_{t-j}/P_t \sim Ln(P_t/P_{t-j})$ for $j=1,m-1$ [3 20]

Therefore, equation [3 19] can be reformulated as

$Ln(P_t/P_{t-1})+Ln(P_t/P_{t-2})+ \quad +Ln(P_t/P_{t-m+1}) <0$

$(m-1)Ln(P_t/P_{t-1})+(m-2)Ln(P_{t-1}/P_{t-2})+ \quad +1Ln(P_{t-m}/P_{t-m+1})<0$

Because $X_t = Ln(P_t/P_{t-1})$, it follows that

$$\sum_{j=1}^{m-1}(m-j)X_{t-j+1} < 0 \qquad [3\ 21]$$

Since if the position triggered by a moving average rule is not long(short), it is short(long), the inequality triggering a buy signal is given by

$$\sum_{j=1}^{m-1}(m-j)X_{t-j+1} > 0$$

The new signal formulated in terms of logarithmic returns can now be considered as a VARs model It belongs to the oscillator family of trading rules It triggers signals around a zero line If the "return" oscillator is negative (positive), a sell (buy) signal is generated

Thus the simple moving average signal admits a return oscillator reformulation given by

$$\left\{\begin{array}{l} B_t = -1 \Leftrightarrow P_t < \dfrac{P_t + P_{t-1} + \quad + P_{t-m+1}}{m} \\ B_t = +1 \Leftrightarrow P_t > \dfrac{P_t + P_{t-1} + \quad + P_{t-m+1}}{m} \end{array}\right\} \approx \left\{\begin{array}{l} \tilde{B}_t = -1 \Leftrightarrow F_t = \sum_{j=1}^{m-1}(m-j)X_{t-j+1} < 0 \\ \tilde{B}_t = +1 \Leftrightarrow F_t = \sum_{j=1}^{m-1}(m-j)X_{t-j+1} > 0 \end{array}\right\}$$

where B_t is the original "price" signal and $\tilde{B}_t$ is the "return" signal

[9] The validity of logarithmic approximations [3 20] is discussed just after the end of the demonstration

Figures 3 1[a] and 3 1[c] illustrate the equivalence price/return signals generated by equations [3 19] and [3 21] for arbitrary financial prices A study of the equivalences of the two rules is provided in the next section

Validity of logarithmic approximations

First it must be remarked that if m=2, there is no approximation but strict equivalence, since

$$P_t < \frac{P_t + P_{t-1}}{2} \Leftrightarrow P_t < P_{t-1} \Leftrightarrow P_t/P_{t-1} < 1 \Leftrightarrow Ln(P_t/P_{t-1}) < 0 \Leftrightarrow X_t < 0$$

For larger values of m, the validity of return formulation was checked empirically for a set of exchange rates series against the Dollar[10] and various Gaussian processes, using Monte-Carlo simulations What is tested is the equivalence between price signals B_t and return signals $\tilde{B}_t$ As can be seen from Table 3 2, signals are different in less than 0 4% of all cases for exchange rates series The largest deviation comes from the simulated random walk $N(\mu, \sigma^2)$ with μ=0 001 and σ=0 03, for m=200 This case represents an upper bound in terms of both volatility and average returns over ten years for financial series (Taylor, 1986 Tables 3 3 and 3 4) Even for this, returns signals differ from price signals in less than 2 6% of all cases

Table 3.2: Return/price signals equivalence for the simple moving average rule

Price signal B_t / return signal $\tilde{B}_t$ Case of the simple moving average rule						
Exchange rates series						
Order m	Nb obs	Percentage (number) of $B_t \neq \tilde{B}_t$				
		DEM	YEN	GBP	FRF	CHF
25	2601	0 19 (5)	0 08 (2)	0 12 (3)	0 15 (4)	0 04 (1)
50	2576	0 23 (6)	0 19 (5)	0 12 (3)	0 08 (2)	0 27 (7)
100	2526	0 28 (7)	0 24 (6)	0 36 (9)	0 28 (7)	0 24 (6)
200	2426	0 33 (8)	0 37 (9)	0 37 (9)	0 37 (9)	0 08 (2)

Simulated Random Walk $N(\mu, \sigma^2)$, 100 replica					
Order m	Nb obs	Average(maximum) Percentage of $B_t \neq \tilde{B}_t$			
		μ=0, σ=0 01	μ=0 001, σ=0 01	μ=0, σ=0 03	μ=0 001, σ=0 03
25	2500	0 18 (0 44)	0 15 (0 48)	0 50 (1 2)	0 50 (0 96)
50	2500	0 22 (0 56)	0 22 (0 44)	0 72 (1 40)	0 19 (1 08)
100	2500	0 37 (0 68)	0 26 (0 60)	1 08 (1 96)	1 05 (1 84)
200	2500	0 48 (0 84)	0 30 (0 68)	1 46 (2 44)	1 46 (2 60)

[10] A full description of which is given in Chapter 6

On the basis of the empirical results presented in Table 3 2, one can safely conclude that return signals lead to the same investment strategies as price signals for values of m as large as 200

3 4 2 *Technical linear rules*

Definition

A rule is said to be "linear" if it can be expressed in the form of equation [3 5]

Proposition 3 6

Any mechanical system triggering a sell signal from a finite linear combination of past prices of the form

$$\text{sell} \quad B_t = -1 \quad \Leftrightarrow \quad \sum_{j=0}^{m-1} a_j P_{t-j} < 0 \qquad [3\ 22]$$

where m being an integer larger than one, and a_j constants,

admits an (almost) equivalent linear return formulation of the form

$$\text{sell} \quad B_t = -1 \quad \Leftrightarrow \quad \delta + \sum_{j=0}^{m-2} d_j X_{t-j} < 0 \qquad [3\ 23]$$

where $X_t = Ln(P_t/P_{t-1})$, $\delta = \sum_{j=0}^{m-1} a_j$, $d_j = -\sum_{i=j+1}^{m-1} a_i$

Consequently many popular technical systems are implicitly linear rules That is specially the case of indicators of the moving-average type as well as the momentum Let us recall their definition throughout the necessary and sufficient conditions which triggers a short position (when the position is not short, it is long)

*) Simple moving average, SMAV, of order m>1

$$\text{Sell if } P_t < SM_t(m) = \frac{P_t + P_{t-1} + \quad + P_{t-m+1}}{m}$$

where $SM_t(m)$ denotes the simple moving average over m rates up to P_t

*) Weighted moving average, WMAV, of order m>1

$$\text{Sell if } P_t < \frac{(m-1)P_t + (m-2)P_{t-1} + \quad + 1P_{t-m+1}}{m(m-1)/2}$$

*) Exponential moving average of coefficient 1>a>0

Sell if $P_t < a(P_t + (1-a)P_{t-1} + \quad +(1-a)^{m-1}P_{t-m+1})$

*) Momentum of order m>1[11]

Sell if $P_t < P_{t-m+1}$

*) Double moving average of orders r, m, 0<r<m

Sell if $SM_t(r) < SM_t(m)$

It must be noted that the simple moving average is a particular case of the double moving average when the short moving average is the rates themselves (r=1)

Explicit establishment of coefficient d_j of equation [3 5] for all the technical indicators mentioned above can be found by applying the results of Proposition 3 6 and are given in Table 3 3

[11] m-1 in Kaufman(1987)

Table 3.3: Return/price signals equivalence

Rule	Parameter(s)	Price Sell Signals	Return Sell Signals
Simple order		$P_t < \sum_{j=0}^{m-1} a_j P_{t-j}$	$\sum_{j=0}^{m-2} d_j X_{t-j} < 0$
Simple MA	$m \geq 2$	$a_j = \frac{1}{m}$	$d_j = (m-j-1)$
Weighted MA	$m \geq 2$	$a_j = \frac{m-j}{[m(m-1)]/2}$	$d_j = \frac{(m-j)(m-j-1)}{2}$
Exponential MA	$1>a>0, m \geq 2$	$a_j = a(1-a)^{j-1}$	$d_j = \frac{[(1-a)^m - (1-a)^{m-1-j}]}{a^2}$
Momentum	$m \geq 2$	$a_j = 1$ for $j=m-1$, $a_j=0$ for $j \neq m-1$	$d_j = 1$
Double orders		$\sum_{j=0}^{r-1} b_j P_{t-j} < \sum_{j=0}^{m-1} a_j P_{t-j}$	$\sum_{j=0}^{m-2} d_j X_{t-j} < 0$
Double MA	$m > r \geq 2$	$b_j = \frac{1}{r}$, $a_j = \frac{1}{m}$	$d_j=(m-r)(j+1)$ for $0 \leq j \leq r-1$ $d_j=(m-j-1)$ for $r \leq j \leq m-2$
Generalisation		$\sum_{j=0}^{m-1} a_j P_{t-j} < 0$	$\delta + \sum_{j=0}^{m-2} d_j X_{t-j} < 0$, with $d_j = -\sum_{i=j+1}^{m-1} a_i$ and $\delta = \sum_{j=0}^{m-1} a_j$

Equivalence pnce/return signals have been checked for the momentum (Table 3 4) and weighted moving average rules (Table 3 5) Once again, deviations are very small and do not exceed 0 5% in all cases for exchange rates senes and 3% for simulated volatile stocks

Table 3.4 Return/pnce signals equivalence for the momentum rule

Pnce signal B_t / return signal $\tilde{B}_t$ Case of the momentum rule						
Exchange rates senes						
Order m	Nb obs	Percentage (number) of $B_t \neq \tilde{B}_t$				
		DEM	YEN	GBP	FRF	CHF
25	2601	0 04 (1)	0 04 (1)	0 04 (1)	0 12 (3)	0 12 (3)
50	2576	0 04 (1)	0 (0)	0 16 (4)	0 (0)	0 08 (2)
100	2526	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
200	2426	0 (0)	0 (0)	0 (0)	0 (0)	0 04 (1)
Simulated Random Walk $N(\mu, \sigma^2)$, 100 replica						
Order m	Nb obs	Average(maximum) Percentage of $B_t \neq \tilde{B}_t$				
		μ=0, σ=0 03		μ=0 001 σ=0 03		
200	2500	0 (0)		0 (0)		

Table 3 5. Return/pnce signals equivalence for the weighted moving average rule

Pnce signal B_t / return signal $\tilde{B}_t$ Case of the weighted moving average rule						
Exchange rates senes						
Order m	Nb obs	Percentage (number) of $B_t \neq \tilde{B}_t$				
		DEM	YEN	GBP	FRF	CHF
25	2601	0 04 (1)	0 15 (4)	0 15 (4)	0 19 (5)	0 08 (2)
50	2576	0 19 (5)	0 12 (3)	0 12 (3)	0 16 (4)	0 19 (5)
100	2526	0 16 (4)	0 40 (10)	0 20 (5)	0 12 (3)	0 08 (2)
200	2426	0 25 (6)	0 41 (10)	0 37 (9)	0 25 (6)	0 37 (9)
Simulated Random Walk $N(\mu, \sigma^2)$, 100 replica						
Order m	Nb obs	Average(maximum) Percentage of $B_t \neq \tilde{B}_t$				
		μ=0, σ=0 03		μ=0 001, σ=0 03		
200	2500	1 57 (2 52)		1 55 (2 92)		

The nice feature of the linear rules, expressed by a linear combination of returns, is that it includes in an unified framework VARs predictors (by construction) and technical systems (by reformulation) Finally, it must be emphasised that although rather general, linear rules do not cover all technical rules used by practitioners

It is doubtful that certain rules signal will ever accept an (almost) equivalent formulation of type equation [3 5] Rules which might be non-linear are in particular rules based on Intra-day High and Low data or on the maximum and minimum of certain values

(a) Intra-day High and Low data

Such trading rules are numerous (Kaufman, 1987, Schwager, 1987) The pertinence of High and Low data in addition of close and open rates has even been recognised by academics Parkinson(1980) for example demonstrates that High and Low data can be used to estimate volatility of rates However Wiggins(1991) points out the statistical problem posed by such estimates true maxima and minima are unlikely to be observed and that the use of recorded high and low rates will bias the results

(b) Rules based on the maximum and minimum of certain values

An example of such rule is the channel rule studied by Lukac, Brorsen and Irwin(1988b), Taylor(1990a, 1992b), Brock, Lakonishok and LeBaron(1992), Curcio and Goodhart(1992) It uses only close prices to determine breakout levels It says " buy (sell) an asset if the rate penetrates from below (above) the maximum (minimum) of the past m days " m is a given number of days which features the length of the channel Opposite to the preceding case they are maximum and minimum of a finite number of rates So the argument of non-observability of such extrema vanishes

Nevertheless, these rules have not been included in this research because they can not be easily modelled

3 4 3 *Expected rule returns*

Expected rule returns given by equation [3 18] are highlighted in what follows for a few linear technical trading rules and underlying Gaussian processes Our purpose is to quantify the profitability of popular trading rules under plausible market conditions More precisely, we consider the simple moving averages, weighted moving averages and momentums rules applied to daily rates We will assume that a year includes 250 days and that the daily process which drives underlying logarithmic returns is successively

a) an Auto-regressive process of order 1 without drift, AR(1)

b) a price-trend model without drift, ARMA(1,1)

c) a fractional Gaussian process without drift

d) a random walk with drift

Definitions and notations concerning these models are given in Section 2 4

AR(1)

Figure 3.2 exhibits that for a given system (moving average type) positive autocorrelation is required to make profitable the investment and that short order system captures better the autocorrelation of order 1 than long order ones.

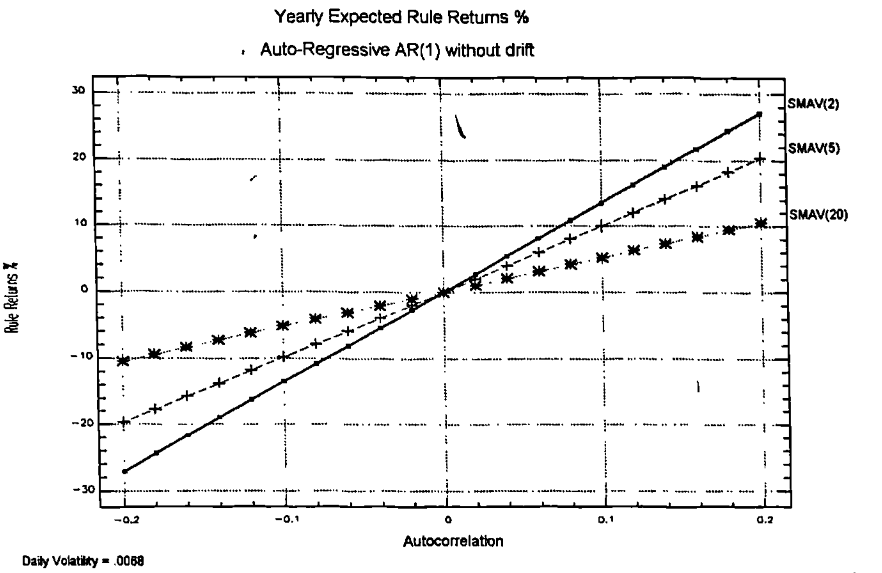

Figure 3.2: Technical returns as a function of the autoregressive coefficient

Figure 3.3 shows that for a given order of rules, certain strategies perform better than others. The quicker the rule responds to a new price, the most profitable it is. For example, a weighted moving average systematically reflects a new price value better than a simple moving average.

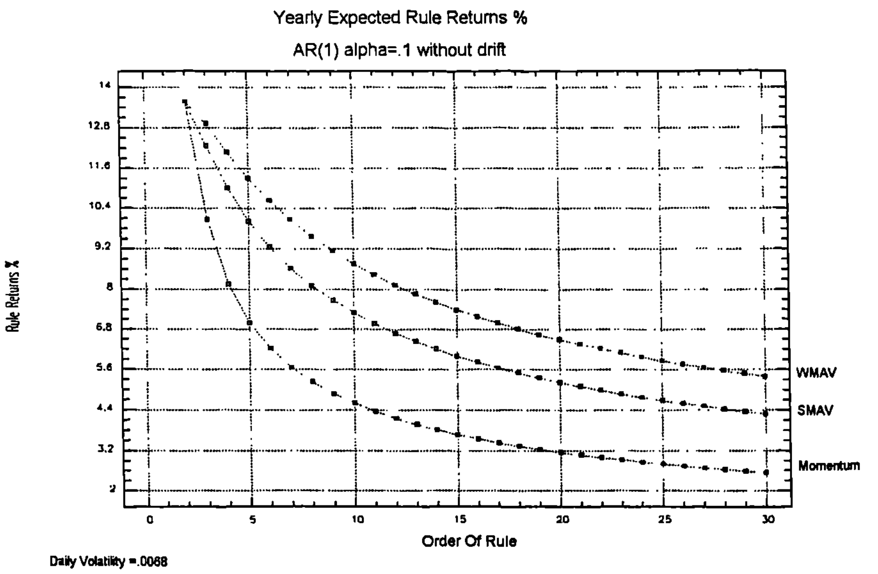

Figure 3.3: Technical returns as a function of the order of the rule, under the AR(1) assumption.

Trend-following models require positive autocorrelations to be profitable However it is perfectly possible to create rules designed to take profit of negative autocorrelations (opposite strategies for example) It is even possible to build rules which display positive expected return whatever is the sign of the first-order autocorrelation[12]

These results are consistent with the findings of LeBaron(1992b) He performed Monte-Carlo simulations to estimate the expected returns following simple moving averages of orders 20, 30 and 50 under the assumption\of AR(1) models The AR(1) models he simulates are for α=0 to 0 4 by step 0 1, where $\alpha = \mathrm{Corr}(X_t, X_{t-1})$ The standard deviation he used is relative to its DM series and is therefore 0 01465 (LeBaron, 1992b Table 3) It is not clear however in the simulations he performs if he holds constant the standard deviation of underlying returns $\sigma = 0\ 01465$ or the standard deviation of the residuals $\sigma_\varepsilon = \sqrt{1-\alpha^2}\ \sigma = 0\ 01465$ Consequently, we establish trading rule returns in both cases (Table 3 6) It appears that formulae [3 15] exactly reproduce LeBaron(1992b Table 4) Monte-Carlo simulations, keeping the standard deviation of residuals constant

Table 3.6: Expected returns under the AR(1) assumption

Expected return * 10000 following a simple moving average rule under the AR(1) assumption												
	LeBaron(1992b Table 4)				$\sigma_\varepsilon = 0\ 01465$				$\sigma = 0\ 01465$			
AR(1)	MA(20)	MA(30)	MA(50)	Average	MA(20)	MA(30)	MA(50)	Average	MA(20)	MA(30)	MA(50)	Average
0	0	0	0	0	0	0	0	0	0	0	0	0
0 1	5	4	3	4	4 5	3 7	2 9	3 7	4 5	3 7	2 9	3 7
0 2	9	8	6	8	9 3	7 7	5 9	7 6	9 1	7 5	5 8	7 5
0 3	15	12	9	12	14 8	12 1	9 4	12 1	14 1	11 6	9 0	11 5
0 4	21	18	14	18	21 3	17 5	13 6	17 5	19 5	16 0	12 4	16 0

ARMA(1,1), Price trend model

Expected rule returns are from equation [3 10]

(a) a positive function of A for p and σ fixed The larger the proportion of the variance of the returns that can be explained by the variance of the trends, the more profitable the trading rules are

(b) a positive function of p for A and σ fixed More the trend component is autocorrelated, the more profitable are the trading rules

---

12 An example of such strategy is $\left\{ \begin{array}{lll} B_t = -1 & \Leftrightarrow & X_{t-1} < 0 \\ B_t = +1 & \Leftrightarrow & X_{t-1} > 0 \end{array} \right\}$

(c) a proportional (and positive if convex rule and positive autocorrelations) function of the volatility for A and p fixed

Figure 3 4 gives an example of some rule returns of orders 2 to 50 for {σ = 0 0068, A = 0 03, m_d = 20 days} The most profitable simple moving average corresponds to the order r=29 days It seems logical that given a mean duration of trend a technical rule finds its optimal parameter around this value In the case of the moving average it is slightly bigger (order 29 for a mean duration of 20 days) Ranking between systems is more complex and should be in favour of exponential moving average since Taylor(1986) has remarked that such representations can be very close to the optimal forecaster Rules are not any more uniformly ranked that is either in systematical favour of short (AR(1)) or long (see Random Walk with Drift) strategies but depend on the mean duration of the trend

Those properties of linear trading rules might hold for non-linear strategies such as the channel rule Taylor(1992b Table 3) finds in particular that channel rule returns are a positive function of A for p fixed (property a) and a positive function of p for A fixed (property b) The distribution shape of channel rule returns (Taylor, 1992b fig 1) is extremely similar to the one of weighted moving average returns (Figure 3 4) The best order of channel rule as for the weighted moving average finds its optimal parameter close to the true mean duration of the trend

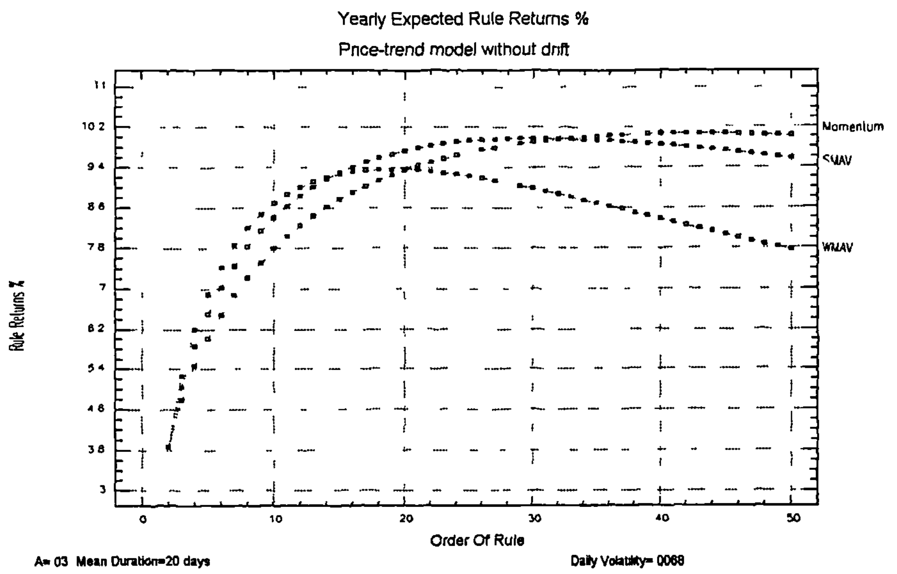

Figure 3.4 Technical returns as a function of the order of the rule, under the price-trend model assumption

Fractional ARIMA (0,d,0)

As in the financial literature, the fractional Gaussian process is interpreted here as a function of the Hurst exponent rather than the parameter d It is recalled that H is related to d by the relation H=d+0 5 [2 17]

Expected rule returns assuming a fractional Gaussian process, can once again be established using equation [3 10] Figures 3 5 and 3 6 exhibit that they are quite identical to the ones corresponding to an Auto-regressive process of order one (Figures 3 2 and 3 3) That is due to the fact that technical indicators do not exploit the feature of a fractional Gaussian process which is the long term dependence (for H>0 5) They only extract the short-term dependence which is very much the one of an AR(1)[13] There exists nevertheless a major difference with usual Gaussian process That is the maximum possible gain is not anymore finite but infinite Indeed it appears that the optimal forecaster defined by Hosking(1981) displays both infinite expected return and variance because autocorrelations are *not summable* Therefore, technical predictors might produce returns very far from the maximum achievable gain However, it has been claimed (Mandelbrot, 1966) that the best linear forecaster is useless to predict the time series because it relies on parameter estimation

Consequently, the fractional Gaussian process might constitute a case where technical trading rules might be preferred to the best linear forecaster That would contradict Mandelbrot(1963) opinion that expected gains from "filter method" depends entirely on the assumption that price is continuous Mandelbrot(1966 242) stated "[ ] it is also possible to conceive of models where successive price changes are dependent so that prices do not follow a pure random walk, but where the nature of the dependence is such that it cannot be used to increase expected profits" This does not apply to the fractional Gaussian process Indeed in the latter case, technical rules are quite profitable and does not rely on parameter estimation which makes the forecaster useless to predict the time series

[13] Distinguishing fractional Gaussian process from AR(1) model is known in the literature as a difficult task (Davies and Harte, 1987)

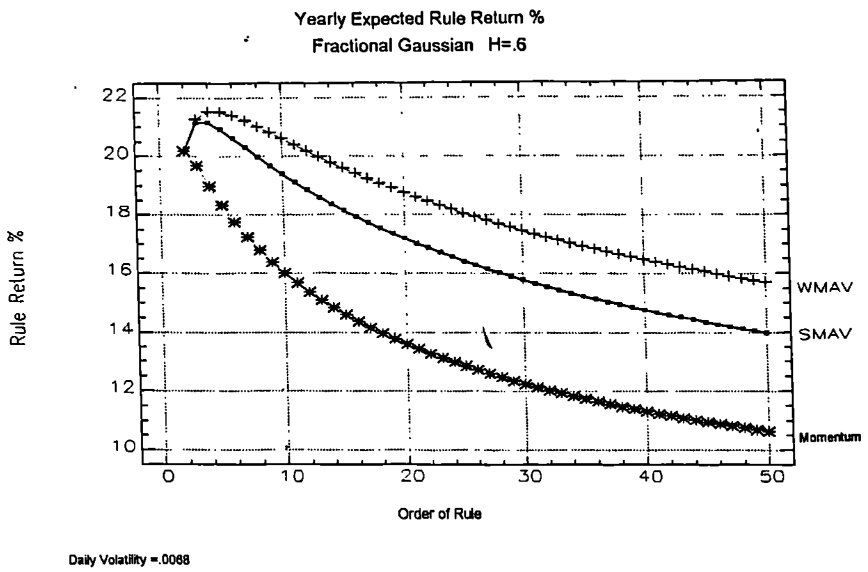

Figure 3.5: Technical returns as a function of the order of the rule, under the fractional Gaussian process assumption

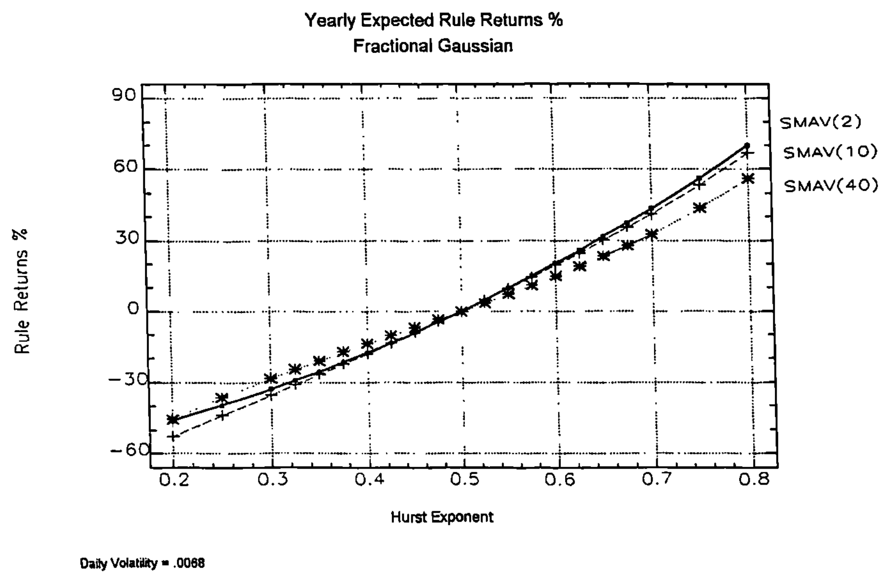

Figure 3.6: Technical returns as a function of the fractional parameter

Random Walk with drift

Figure 3 7 exhibits returns from a Buy and Hold strategy and from a Simple Moving Average (SMAV) rule of orders 5, 20, 100 as a function of the drift Overall, three remarks can be made

(a) expected return will be a fixed percentage of the drift

So it will underperform a buy and hold strategy if the drift is positive and outperform it if the drift is negative The expected return of SMAV rule is a positive function of the absolute value of the drift $|\mu|$ and a negative function of the volatility σ It is a positive function of the order m of the SMAV That can be explained by the fact that the most profitable strategy is buy and hold if the drift is positive

(b) the drift increases the instantaneous variance of return

(c) Only in the absence of any drift in the data are rule returns uncorrelated

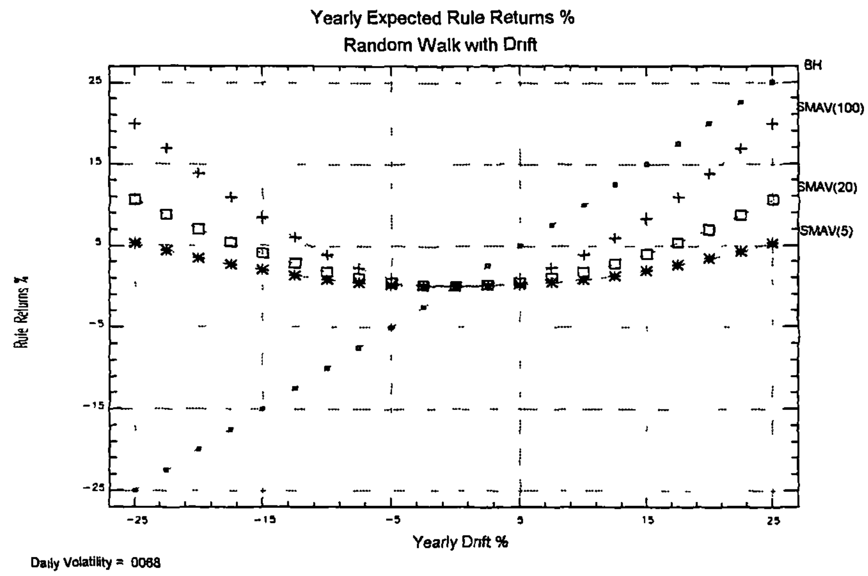

Figure 3.7 Technical returns as a function of the drift

Figure 3 8 illustrates that in decreasing order of profitability, we have 1)Momentum 2)Simple MA 3)Weighted MA It means that ex-ante certain technical rules will capture systematically better the drift than others

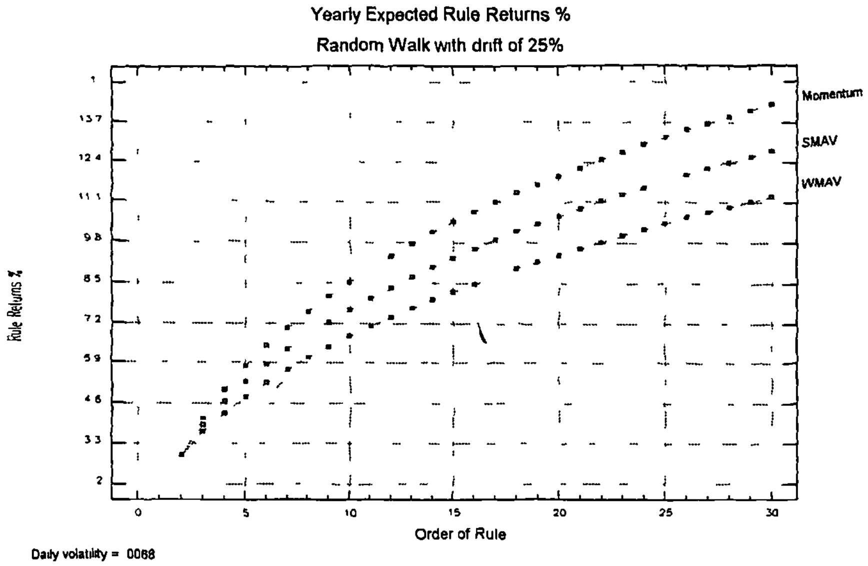

Figure 3.8 Technical returns as a function of the order of the rule, under the random walk with drift assumption

3 5 SUMMARY

Under the assumption that underlying asset returns follow a Gaussian process, the linear rule returns distribution is a mixture of marginal density function of a truncated bivariate density function Exact expected values can be obtained and are of importance since the objective of a market timer is to maximise return and that risk is merely considered an opportunity cost

The expected return following a linear trading rule is zero if the underlying process is a random walk without drift This is non-zero if the underlying process exhibits a drift or/and autocorrelations If the underlying process is a random walk with drift, the expected return of a convex trading rule is a positive function of the drift and a negative function of the volatility If the underlying process exhibits positive (negative) autocorrelations but no drift, the expected return of a convex (concave) strategy is a positive function of the volatility

Many popular technical trading rules can be expressed as VARs forecasters Doing so allows applying both technical and statistical predictors in an unified framework called "linear rules"

APPENDIX 3.1

NOTATIONS AND MULTINORMAL MOMENTS USED IN THE RESEARCH

The following notations and multinormal moments are used throughout this research

Univariate normal law

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-(1/2)x} \quad , \quad -\infty < x < +\infty$$

$$\Phi(h) = \int_{-\infty}^{h} \varphi(x)\, dx \quad , \quad -\infty < h < +\infty$$

$$[r] = \int_{0}^{+\infty} x^r\, \varphi(x)\, dx \quad , \quad r \in N$$

A short notation will be $[r] = \int_{X>0} X^r$

Bivariate normal law

$$\varphi(x,y,\rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp[-\tfrac{1}{2}(x^2 - 2\rho xy + y^2)/(1-\rho^2)] \quad , \quad -\infty < x,y < +\infty , \ -1 < \rho < 1$$

$$[r,s] = \int_0^{+\infty} \int_0^{+\infty} x^r\, y^s\, \varphi(x,y,\rho)\, dx\, dy \quad , \quad (r,s) \in N^2$$

A short notation will be $[r,s] = \int_{X>0} \int_{Y>0} X^r\, Y^s$

$[r,s](\rho)$ will denote the value of $[r,s]$ as a function of ρ

The incomplete moments $[r,s]$ have been evaluated by Kamat(1953) We have in particular

$$[0,0] = \tfrac{1}{4} + \tfrac{1}{2\pi} \operatorname{Arc\,sin}(\rho) \qquad [A\ 1]$$

$$[1,0] = \tfrac{1}{4}\sqrt{\tfrac{2}{\pi}}\,(1+\rho) \qquad [A\ 2]$$

$$[1,1] = \tfrac{1}{2\pi}(\rho[\tfrac{\pi}{2} + \operatorname{Arc\,sin}(\rho)] + \sqrt{1-\rho^2}) \qquad [A\ 3]$$

$$[2,0] = \tfrac{1}{4} + \tfrac{1}{2\pi}(\operatorname{Arc\,sin}(\rho) + \rho\sqrt{1-\rho^2}) \qquad [A\ 4]$$

Trivariate normal law

$$\varphi(x_1, x_2, x_3, \rho_{12}, \rho_{13}, \rho_{23}) = (2\pi)^{-3/2}\,\Delta^{-1/2}\,\exp[-\tfrac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} A_{ij}\, x_i\, x_j]$$

where $-\infty < x_i < +\infty$, $-1 < \rho_{ij} < 1$, $1 \le i \le 3$, $1 \le j \le 3$ and

$$\Delta = 1 - \rho_{23}^2 - \rho_{13}^2 - \rho_{12}^2 + 2\rho_{12}\rho_{13}\rho_{23},$$

$$A_{11} = (1-\rho_{23}^2)\Delta^{-1}, \quad A_{22} = (1-\rho_{13}^2)\Delta^{-1}, \quad A_{33} = (1-\rho_{12}^2)\Delta^{-1},$$

$$A_{12} = A_{21} = (\rho_{13}\rho_{23} - \rho_{12})\Delta^{-1}, \quad A_{13} = A_{31} = (\rho_{12}\rho_{23} - \rho_{13})\Delta^{-1},$$

$$A_{23} = A_{32} = (\rho_{12}\rho_{13} - \rho_{23})\Delta^{-1},$$

$$[r,s,t] = \int_0^{+\infty}\int_0^{+\infty}\int_0^{+\infty} x_1^r\, x_2^s\, x_3^t\;\varphi(x_1, x_2, x_3, \rho_{12}, \rho_{13}, \rho_{23})\;dx_1\, dx_2\, dx_3\;, \quad (r,s,t) \in N^3$$

A short notation will be $[r,s,t] = \int_{X_1>0}\int_{X_2>0}\int_{X_3>0} X_1^r\, X_2^s\, X_3^t$

$[r,s,t](\rho_{12}, \rho_{13}, \rho_{23})$ will denote the value of [r,s,t] as a function of $(\rho_{12}, \rho_{13}, \rho_{23})$
The incomplete moments [r,s,t] have been evaluated by Kamat(1958) for all r,s,t with $r+s+t \le 3$ However some of these moments are ill defined[14] It is why we prefer to use the trivariate moments established by Tallis(1961) which lead to

$$[1,1,0] = \frac{1}{4\pi}[\rho_{12}\{\frac{\pi}{2} + \sum_{i<j}^{3} \mathrm{Arc}\sin(\rho_{ij})\} + \sqrt{1-\rho_{12}^2} + \rho_{13}\sqrt{1-\rho_{23}^2} + \rho_{23}\sqrt{1-\rho_{13}^2}\,] \qquad [A\ 5]$$

$$[2,0,0] = \frac{1}{4\pi}[\frac{\pi}{2} + \sum_{i<j}^{3} \mathrm{Arc}\sin(\rho_{ij}) + \frac{1}{\sqrt{1-\rho_{23}^2}}(2\rho_{12}\rho_{13} - \rho_{23}\rho_{12}^2 - \rho_{23}\rho_{13}^2)\,] \qquad [A\ 6]$$

In what follows, a star will design standardised normal variates For instance, X_{t+1}^*, F_t^* design unit normal variates

A variable Y conditional to the knowledge of a variable X will be either noted Y/X or $Y^{\{X\}}$

14 See Appendix 3 2

APPENDIX 3.2

A NOTE ABOUT MULTINORMAL MOMENTS

Kamat(1958) has given exact formulae[15] of a few trinormal truncated moments Similar work has been performed by Tallis(1961) Tallis(1961) and Kamat(1953) formulae are identical for the bivariate case They, however, diverge for the trivariate case as it will be shown It appears that Kamat(1958) formulae must be ill defined since they do not satisfy, contrary to Tallis(1961) results, some simple checks This point is illustrated below with the two moments used in the study

Kamat(1958)

$$[1,1,0] = \frac{1}{4\pi}[\,\rho_{12}\,\{\frac{\pi}{2} + \sum_{i<j}^{3} \mathrm{Arc}\sin(\rho_{ij})\} + \sqrt{1-\rho_{12}^{\;2}} + \rho_{23}\sqrt{1-\rho_{23}^{\;2}} + \rho_{13}\sqrt{1-\rho_{13}^{\;2}}\,] \quad [\text{K 1}]$$

$$[2,0,0] = \frac{1}{4\pi}[\,\frac{\pi}{2} + \sum_{i<j}^{3} \mathrm{Arc}\sin(\rho_{ij}) + \Delta\rho_{23}\sqrt{1-\rho_{23}^2} + (2\rho_{12}\rho_{13} - \rho_{23})\sqrt{1-\rho_{23}^2} + \rho_{12}\sqrt{1-\rho_{12}^2} + \rho_{13}\sqrt{1-\rho_{13}^2}\,] \quad [\text{K 2}]$$

where $\Delta = 1 - \rho_{12}^{\;2} - \rho_{13}^{\;2} - \rho_{23}^{\;2} + 2\rho_{12}\rho_{13}\rho_{23}$

Tallis(1961)

It can be shown that using Tallis(1961) and the bivariate normal moments Kamat(1953)[16] that

$$[1,1,0] = \frac{1}{4\pi}[\rho_{12}\,\{\frac{\pi}{2} + \sum_{i<j}^{3} \mathrm{Arc}\sin(\rho_{ij})\} + \sqrt{1-\rho_{12}^{\;2}} + \rho_{13}\sqrt{1-\rho_{23}^{\;2}} + \rho_{23}\sqrt{1-\rho_{13}^{\;2}}\,] \quad [\text{T 1}]$$

$$[2,0,0] = \frac{1}{4\pi}[\,\frac{\pi}{2} + \sum_{i<j}^{3} \mathrm{Arc}\sin(\rho_{ij}) + \frac{1}{\sqrt{1-\rho_{23}^{\;2}}}(2\rho_{12}\rho_{13} - \rho_{23}\rho_{12}^{\;2} - \rho_{23}\rho_{13}^{\;2})\,] \quad [\text{T 2}]$$

---

[15] recalled in Johnston and Kotz(1972b 93)

[16] recalled in Johnston and Kotz(1972b 92)

Those trivariate moments obviously differ between Tallis(1961) and Kamat(1958) Formulae [K 1] and [K 2] are misspecified since they do not pass, contrary to formulae [T 1] and [T 2], *some simple coherence tests*

Coherence Tests

Simple tests of coherence can be applied to those formulations It consists in checking the compatibility between univariate, bivariate and trivariate moments from two elementary examples

If $\rho_{12}=\rho_{13}=0$, $\rho_{23}=\rho$, the following equality must be verified

$$[2,0,0] = \tfrac{1}{2}\ [0,0] = \frac{1}{4\pi}\left(\frac{\pi}{2}+\text{Arc}\sin(\rho)\right)$$

If $\rho_{12}=\rho_{23}=0$, $\rho_{13}=1$, the following equality must be verified

$$[1,1,0] = [1]\ [1] = \frac{1}{2\pi}$$

[K 1], [K 2] formulae do not pass these simple coherence tests Indeed

when $\rho_{12}=\rho_{23}=0$, $\rho_{13}=1$, $[K\ 1] \Rightarrow [1,1,0] = \dfrac{1}{4\pi}$

when $\rho_{12}=\rho_{13}=0$, $\rho_{23}=\rho$, $[K\ 2] \Rightarrow [2,0,0] = \dfrac{1}{4\pi}\left(\dfrac{\pi}{2}+\text{Arc}\sin(\rho)-\rho^3\sqrt{1-\rho^2}\right)$

On the other hand, it is straightforward to exhibit that formulae [T 1] and [T 2] pass these simple checks That can be seen from the fact they are recurrent formulae and that trivariate moments are established from bivariate ones

In sum, Kamat(1958) results for those two trivariate moments appear dubious and so Tallis(1961) has been preferred and applied in this research

APPENDIX 3.3

PROOFS OF PROPOSITIONS

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Proposition 3 1

We show here that the distribution of rule returns is the same than the distribution of independent underlying returns if the latter is symmetrical around zero, normal or not If we note C_x the characteristic function of the underlying return and assume that it is symmetrical around zero, we have $C_x(z) = E\{\exp(izX_t)\} = E\{\exp(-izX_t)\} = C_x(-z)$

Rule returns R_t admit the characteristic function

$C_R(z) = E\{\exp(-izB_{t-1}X_t)\} = E(E^{\{X_{t-1}\}}[\exp(-izB_{t-1}X_t)])$

with $E^{\{X_{t-1}\}}$ means the expected value conditional to the knowledge of past returns $\{X_{t-1}\} = \{X_{t-1}, X_{t-2}, \ \ , X_{t-m}, \ \ \}$

By definition, $B_{t-1} = \begin{cases} -1 & \text{with probability} \quad \Pr(F_{t-1} < 0) \\ +1 & \text{with probability} \quad \Pr(F_{t-1} > 0) \end{cases}$ *and only depends on* $\{X_{t-1}\}$

Therefore

$C_R(z) = E(\Pr[F_{t-1} < 0] E^{\{X_{t-1}\}}[\exp(-izX_t)] + \Pr[F_{t-1} > 0] E^{\{X_{t-1}\}}[\exp(+izX_t)])$

Because X_t is independent on $\{X_{t-1}\}$, we have

$C_R(z) = \Pr[F_{t-1}<0]\ E\{\exp(izX_t)\} + \Pr[F_{t-1}>0]\ E\{\exp(-izX_t)\}$

$\Pr[F_{t-1}<0]=\Pr[F_{t-1}>0]=\frac{1}{2}$ because the distribution of the linear unbiased forecaster, F_{t-1}, is symmetrical around zero, as for the underlying returns X_t Then, it follows that

$C_R(z) = \frac{1}{2}\ C_x(-z) + \frac{1}{2}\ C_x(z) = C_x(z)$

So R_t follows the same law than the underlying returns In particular, R_t follows a centred normal law $N(0,\sigma^2)$ if X_t follows a centred normal law $N(0,\sigma^2)$ Then it implies equations [3 7] and [3 8]

Finally, we have

$Cov(R_t,R_{t+h}) = E(R_tR_{t+h}) = E(B_{t-1}X_tB_{t+h-1}X_{t+h}) = E(B_{t-1}X_tB_{t+h-1})E(X_{t+h}) = E(B_{t-1}X_tB_{t+h-1})0 = 0$

That is due to the fact that X_{t+h} is independent on X_t, B_t, B_{t+h-1}

$\Rightarrow \quad Cov(R_t,R_{t+h})=0 \qquad \text{for } h>0 \qquad [3\ 9]$

Proposition 3 2

$E(R_t) = E\{E^{\{X_{t-1}\}}(B_{t-1}X_t)\}$ with $E^{\{X_{t-1}\}}$ means the expected value of X_{t-1} conditional to the knowledge of $\{X_{t-1}\}=\{X_{t-1},X_{t-2}, \ ,X_{t-m}, \ \}$

$$E(R_t) = E(B_{t-1}\sqrt{\alpha_o + \sum_{i=1}^{p}\alpha_i X_{t-i}^2}\ \varepsilon_t) = E(B_{t-1}\sqrt{\alpha_o + \sum_{i=1}^{p}\alpha_i X_{t-i}^2}\ E^{\{X_{t-1}\}}(\varepsilon_t))$$

Since the stochastic process ε_t is independent of $\{X_{t-1}\}$ $E^{\{X_{t-1}\}}(\varepsilon_t) = 0$ Then

$$E(R_t) = E(B_{t-1}\sqrt{\alpha_o + \sum_{i=1}^{p}\alpha_i X_{t-i}^2}\ 0) = 0$$

Proposition 3 3

See Proposition 3 5 which includes Proposition 3 3 as a special case

Proposition 3 4

Let us note C_x the characteristic function of the underlying return

$C_x(z) = \exp(i\ z\ \mu)\ \exp(-\sigma^2\ z^2/2)$

Rule returns R_t admit the characteristic function $C_R(z) = E\{\exp(-izB_{t-1}X_t)\}$

Replicating the steps of Proposition 3 1, we have

$C_R(z) = \Pr[F_{t-1}<0]\ E\{\exp(izX_t)\} + \Pr[F_{t-1}>0]\ E\{\exp(-izX_t)\}$

$C_R(z) = PS\ \exp(-i\ z\ \mu)\ \exp(-\sigma^2 z^2/2)+(1-PS)\ \exp(i\ z\ \mu)\ \exp(-\sigma^2\ z^2/2)$

where PS is the probability of being short given by $\Pr[F_{t-1}<0]$

So R_t follows a mixture of normal laws

$$\begin{Bmatrix} R_t \sim N(-\mu,\sigma^2) & \text{with probability} & PS = \Pr[F_{t-1} < 0] \\ R_t \sim N(\mu,\sigma^2) & \text{with probability} & 1 - PS = \Pr[F_{t-1} > 0] \end{Bmatrix}$$

That implies equations [3 11], [3 12] and [3 13] In addition

$Cov(R_t,R_{t+h}) = E(R_tR_{t+h}) - E(R_t)E(R_{t+h})$

$E(R_tR_{t+h}) = E(B_{t-1}X_tB_{t+h-1}X_{t+h}) = E(B_{t-1}X_tB_{t+h-1})E(X_{t+h}) = E(B_{t-1}X_tB_{t+h-1})\ \mu$

That is due to the fact that X_{t+h} is independent on X_t, B_t, B_{t+h-1}

$\Rightarrow$ $Cov(R_t,R_{t+h})=E(B_{t-1}X_tB_{t+h-1})\mu-\mu^2\ (1-2\ PS)^2$ for $h>0$ [3 14]

Proposition 3 5

$E(R_t)=E(B_{t-1}X_t)=E(B_{t-1}(\sigma X_t^* + \mu)) = \sigma E(B_{t-1}X_t^*) + \mu E(B_{t-1})$

where X_t^* designs an unit normal variate, $\mu = E(X_t)$ and $\sigma^2 = Var(X_t)$

$E(B_{t-1}) = Pr(F_{t-1} > 0) - Pr(F_{t-1} < 0) = 1 - 2Pr(F_{t-1} < 0) = 1 - 2\Phi(-\mu_F/\sigma_F)$

$$E(B_{t-1}X_t^*) = \int_{X_t^*} \int_{F_{t-1}^* > -\mu_F/\sigma_F} X_t^* - \int_{X_t^*} \int_{F_{t-1}^* < -\mu_F/\sigma_F} X_t^*$$

where F_{t-1}^* designs an unit normal variate, $\mu_F = E(F_{t-1})$ and $\sigma_F^2 = Var(F_{t-1})$

Then using the truncated bivariate moments given by Johnston and Kotz(1972b 116), it follows that

$$E(B_{t-1}X_t^*) = \sqrt{\frac{2}{\pi}}\, \rho \exp(-\mu_F^2/2\sigma_F^2) \quad \text{with} \qquad \rho = Corr(X_t, F_{t-1})$$

Therefore equation [3 23] results from the weighted summation of the two previous terms as follows

$$E(R_t)=\sigma E(B_{t-1}X_t^*) + \mu E(B_{t-1}) = \sigma \sqrt{\frac{2}{\pi}}\, \rho \exp(-\mu_F^2/2\sigma_F^2) + \mu(1 - 2\Phi[-\mu_F/\sigma_F]) \quad [3\ 18]$$

Proposition 3 6

$$\text{sell } B_t = -1 \Leftrightarrow \sum_{j=0}^{m-1} a_j P_{t-j} < 0 \qquad [3\ 22]$$

$$\Leftrightarrow \quad P_t - \sum_{j=0}^{m-1} b_j P_{t-j} < 0, \text{ with } b_0 = 1 - a_0 \text{ and } b_j = -a_j \text{ for } j = 1, m-1$$

$$\Leftrightarrow \quad P_t - \sum_{j=0}^{m-1} b_j P_{t-j} + \sum_{j=0}^{m-1} b_j P_t - \sum_{j=0}^{m-1} b_j P_t < 0$$

$$\Leftrightarrow \quad \sum_{j=1}^{m-1} b_j (P_t - P_{t-j}) < (\sum_{j=0}^{m-1} b_j - 1) P_t$$

$$\Leftrightarrow \quad \sum_{j=1}^{m-1} b_j (1 - P_{t-j}/P_t) < \sum_{j=0}^{m-1} b_j - 1$$

Let us assume that $1-P_{t-j}/P_t \sim Ln(P_t/P_{t-j})$ for $j=1,m-1$ and noting that $X_t = Ln(P_t/P_{t-1})$, we can so approximate $1-P_{t-j}/P_t \sim Ln(P_t/P_{t-j}) = \sum_{i=0}^{j-1} X_{t-i}$ It follows that

$$\sum_{j=1}^{m-1} b_j \sum_{i=0}^{j-1} X_{t-i} < \sum_{j=0}^{m-1} b_j - 1$$

$$\Leftrightarrow \quad (1-\sum_{j=0}^{m-1} b_j) + \sum_{j=0}^{m-2} (\sum_{i=0}^{m-2-j} b_{m-1-i}) X_{t-j} < 0$$

$$\Leftrightarrow \quad \delta + \sum_{j=0}^{m-2} d_j X_{t-j} < 0 \qquad [3\ 23]$$

with $\delta = 1 - \sum_{j=0}^{m-1} b_j = \sum_{j=0}^{m-1} a_j$, and $d_j = \sum_{i=1}^{m-2-j} b_{m-1-i} = -\sum_{i=1}^{m-2-j} a_{m-1-i} = -\sum_{i=j+1}^{m-1} a_i$

APPENDIX 3.4

DISTRIBUTION OF REALISED RETURNS FOLLOWING THE SIMPLE MOVING AVERAGE OF ORDER 2 RULE

Realised returns following the simple moving average rule of order 2 are not normal under the assumption of normal independent underlying returns Firstly, the conditional distribution of realised returns knowing the duration of the position is established Exact formulations are given for expected value and variance Secondly, the unconditional distribution of realised returns is established Exact values of the four first moments are provided It is shown that the distribution is non normal, positively skewed, and leptokurtic

Simple moving average of order 2 rule

The strategy consists here of being long if the price is above the moving average of order 2 and being short otherwise That is more explicitly

$$B_t=+1 \Leftrightarrow P_t > \frac{P_t + P_{t-1}}{2} \Leftrightarrow P_t > P_{t-1} \Leftrightarrow X_t = Ln(P_t / P_{t-1}) > 0$$

$$B_t=+1 \Leftrightarrow \quad X_t > 0 \qquad [3\ 24]$$

Let us note D the stochastic duration of a position If we assume that a new position starts at time t, that is we know that $\{B_t \# B_{t-1}\}$, the stochastic duration D will last n days if and only if $\{D=n\} \Leftrightarrow \{ B_t=B_{t+1}= \quad =B_{t+n-1}, B_{t+n-1} \# B_{t+n} / B_t \# B_{t-1} \}$ [3 3]

That is for the simple moving average of order 2 rule, applying equation [3 24]

$$\{D=n\} \Leftrightarrow \quad \{X_t > 0, X_{t+1} > 0, \quad , X_{t+n-1} > 0, X_{t+n} < 0 / X_{t-1} < 0\}$$

$$\text{or} \quad \{X_t < 0, X_{t+1} < 0, \quad , X_{t+n-1} < 0, X_{t+n} > 0 / X_{t-1} > 0\}$$

The corresponding realised return is so

$$R = \sum_{D=1}^{n} R_{t+D} = \sum_{D=1}^{n} B_{t+D-1} X_{t+D} \qquad [3\ 2]$$

Subsequently, we assume that logarithmic returns X_t follow a normal random walk without drift Therefore due to the symmetry of both the underlying stochastic process and trading rule, the expected realised return initiated by a long position is equal to the one initiated by a short position Let us assume to simplify that a long position starts at time t=0 and is reversed at time t=n The duration of the position is equal to n days if and only if $\{D = n\} = \{X_1 > 0, \quad X_n > 0, X_{n+1} < 0\}$

Conditional distribution

Let us note $C_{R/D=n}$ the characteristic distribution of realised returns knowing that the duration of the position D is equal to n days Then

$$C_{R/D=n}(z) = [C_H(z)]^{n-1} C_H(-z) \qquad [3\ 25]$$

where C_H is the characteristic function of the absolute value of a normal variable $N(0,\sigma^2)$, known in the literature as half-normal variate (Johnson and Kotz, 1972a 81)

Proof

$$\begin{aligned} C_{R/D}(z) &= E(\exp[iz(X_1 + \quad + X_{n-1} + X_n)] / D) \\ &= E(\exp(izX_1)/D) \quad E(\exp(izX_{n-1})/D) E(\exp(izX_n)/D) \\ &= E(\exp(izX_1)/X_1 > 0) \quad E(\exp(izX_{n-1})/X_{n-1} > 0) E(\exp(izX_n)/X_n < 0) \\ &= [C_H(z)]^{n-1} C_H(-z) \end{aligned}$$

Using the relationships between characteristic function and non centred moments, it follows after straightforward arrangements that

$$E(R/D=n) = \sqrt{\frac{2}{\pi}}\sigma(n-1) - \sqrt{\frac{2}{\pi}}\sigma = \sqrt{\frac{2}{\pi}}\sigma(n-2) \qquad [3\ 26]$$

$$Var(R/D=n) = n(\frac{\pi-2}{\pi})\sigma^2 \qquad [3\ 27]$$

Equation [3 26] says that the expected return, knowing the duration of the position, depends on the duration and more precisely is proportional to its If the duration of the position is equal to one day, it generates a loss which is natural since the simple moving average method by construction reverses its position on an unrealised contrary move Then the longer the position is, the more profitable it is in average

Equation [3 27] indicates that even when rule returns have constant variance per unit of time which is equal to the underlying volatility, equation [3 8], the variance of holding-period returns will not be constant but depends linearly on the duration of the position Then a correction for heteroskedasticity is necessary The Hartzmark(1991) procedure can be applied for this purpose It consists in using the squared root of the number of days between each transaction as a weight in the adjustment procedure

Unconditional distribution

Let us note C_R the characteristic distribution of realised returns Then

$$C_R(z) = \frac{C_H(-z)}{2 - C_H(z)} \qquad [3\ 28]$$

where C_H is the characteristic function of the absolute value of a normal variable $N(0,\sigma^2)$

Proof

The unconditional characteristic function of realised returns is established by taking the expected value of the conditional characteristic function

$$C_R(z) = E(C_{R/D}(z)) = E([C_H(z)]^{D-1} C_H(-z)) = \sum_{n=1}^{\infty} \Pr(D=n)\,[C_H(z)]^{n-1} C_H(-z)$$

It is straightforward to show that under the random walk assumption, the duration D of a position follows the distribution

$$\Pr(D=n) = \begin{array}{c} \Pr(X_t > 0, X_{t+1} > 0, \quad , X_{t+n-1} > 0, X_{t+n} < 0 \,/\, X_{t-1} < 0) \\ + \Pr(X_t < 0, X_{t+1} < 0, \quad , X_{t+n-1} < 0, X_{t+n} > 0 \,/\, X_{t-1} > 0) \end{array} = (\tfrac{1}{2})^n \quad \text{Therefore}$$

$$C_R(z) = \sum_{n=1}^{\infty} (\tfrac{1}{2})^n [C_H(z)]^{n-1} C_H(-z) = \tfrac{1}{2} C_H(-z) \sum_{n=1}^{\infty} [\tfrac{1}{2} C_H(z)]^{n-1} = \frac{\tfrac{1}{2} C_H(-z)}{1 - \tfrac{1}{2} C_H(z)} = \frac{C_H(-z)}{2 - C_H(z)}$$

Using the relationships between characteristic function and non centred moments, it follows after lengthy arrangements that

$$E(R) = 0 \qquad [3\ 29]$$

$$E(R^2) = Var(R) = 2\sigma^2 \qquad [3\ 30]$$

$$E(R^3) = 6\sqrt{\frac{2}{\pi}}\sigma^3 \quad \Rightarrow \quad \gamma_1 = \frac{E(R^3)}{(\sqrt{Var(R)})^3} = \frac{3}{\sqrt{\pi}} \cong 1\ 693 \qquad [3\ 31]$$

$$E(R^4) = 2(9 + \frac{24}{\pi})\sigma^4 \quad \Rightarrow \quad \gamma_2 = \frac{E(R^4)}{(\sqrt{Var(R)})^4} - 3 = \frac{3}{2} + \frac{12}{\pi} \cong 5\ 320 \qquad [3\ 32]$$

Under the random walk without drift assumption, realised returns following a moving average of order 2 are not any more normal, contrary to unrealised returns but follow a complicated truncated law defined by equation [3 28] The expected value of realised returns is still equal to zero as for the unrealised case, equation [3 29] The variance is however double than the variance of unrealised or underlying returns, equation [3 30] The distribution is at present positively skewed and leptokurtic, equations [3 31] and [3 32]

Chapter 4

ERROR MEASURES AND PROFITABILITY

In Chapter 3, the expected returns of a linear rule applied to price movements that are assumed to be Gaussian are derived However, the most profitable forecaster has not been determined Whether or not maximising profits and minimising squared errors leads to the same forecaster is an important issue If not, certain existing statistical procedures, algorithms and criteria might be of little value in an investment purpose This chapter examines the reality and complexity of this problem

Section 4 1 defines the forecaster which maximises expected rule returns Section 4 2 shows that the relationships between error measures and profitability must be highly nonlinear and possibly degenerated when the true model is a random walk Section 4 3 assesses in terms of profitability and error measures the implications of using a misspecified forecaster when the true underlying process is Gaussian Section 4 4 evaluates the implications of previous findings on market timing ability tests Section 4 5 summarises and concludes our results

4.1 MAXIMISING EXPECTED RETURNS

Recent studies on forecast evaluations are concentrated on quantitative measures of prediction errors They have not focused on the value of the forecasts for the user Economic evaluation of price forecasts consistent with the underlying decision problem is an alternative preferred by practitioners to accurate forecasting models which minimise squared errors

The mean squared error criterion measures how closely the model fits a time series by averaging the sum of the squared deviations of the two series It does not differentiate between deviations resulting from a failure to predict a change in the trend of the series or the cyclical component Despite its wide acceptance by academics, market participants who try to forecast financial time series have found this criterion inadequate

The reason for this is that traders, for instance are interested only in forecasting changes in the underlying trend of the financial prices rather than forecasting the level of the price series A trader will take long position in the market in anticipation of a price rise, without attempting to forecast level The forecasting problem of traders has given rise to a particular measure reflecting the profitability of the strategy rather than the accuracy of predicting the price level Empirical studies (Boothe and Glassman, 1987a, Leitch and Tanner, 1991, Satchell and Timmermann, 1992b) have found that squared errors (SE) and profits based forecasters can differ significantly One explanation might be that the SE criterion is of poor use to build efficient forecasters of turning points (Wecker, 1979, Kling, 1987), which is a necessary condition for profitability Therefore, what is needed is to determine which forecaster maximises expected returns

Proposition 4 1[1]

If the underlying process of returns $\{X_t\}$ is assumed to be Gaussian, a linear forecaster F_t maximises expected rule returns if and only if

(a) it maximises $\rho = \mathrm{Corr}(X_{t+1}, F_t)$

(b) $\mu_F / \sigma_F = \mu / (\rho\sigma)$

Where μ, σ are the mean and standard deviation of X_t and μ_F, σ_F are the mean and standard deviation of F_t

First let us compare the forecaster which maximises expected returns with the forecaster which minimises expected squared forecast error Following Granger and Newbold(1986, p283), expected squared forecast error can be written as

$$E((X_{t+1} - F_t)^2) = (\mu_F - \mu)^2 + (\sigma_F - \rho\sigma)^2 + (1 - \rho^2)\sigma^2$$

Taking μ and σ to be fixed numbers, it is clear that expected squared error is minimised by

(c) maximising $\rho = \mathrm{Corr}(X_{t+1}, F_t)$

(d) $\mu_F = \mu$

(e) $\sigma_F = \rho\,\sigma$

The forecaster which minimises squared errors F_t^{mse} is defined by conditions (c), (d) and (e) and therefore satisfies conditions (a) and (b) Then F_t^{mse} maximises expected returns, but it is not any longer unique since any forecaster proportional with a>0 to F_t^{mse}, aF_t^{mse} still maximises profits

---

[1] Proofs of propositions are given in Appendix 4 1

Hence, if the X_t process is Gaussian, no linear trading rules obtained from a finite history of X_t can generate expected returns over and above vector autoregressions It has been shown here that although trading rules display non-zero expected value when the process is Gaussian with autocorrelations or drift, they cannot be more profitable than the optimal linear forecaster Neftci(1991) shows that under the hypothesis price time series are linear, even well-defined rules are shown to be useless in prediction So technical forecasters, although exhibiting some forecasting value, should be considered misspecified models

In reality, the above conclusion must be refined The technical trading rules that are implicitly linear can be optimal forecasters Let us give a simple but meaningful example by assuming that the true underlying model is $X_t = a\sum_{j=1}^{m-1}(m-j)X_{t-j} + \varepsilon_t$, with ε_t white noise, a>0 and m is an integer greater than one It follows from Section 3 4 1 and Proposition 4 1 that the simple moving average of order m will then maximise profits There are cases for which technical indicators are linear models (Section 3 4 2) and therefore generate optimal forecasters

Proposition 4 1 mainly defines the necessary and sufficient conditions to maximise expected returns The forecaster which maximises profits is the predictor which maximises the correlation between the one-step ahead forecaster and the future underlying return, condition (a), and satisfies condition (b) It is not limited to the one which minimises squared errors Divergences between the two predictors might be significant Baczkowski and Mardia(1990) have studied the prediction procedure based upon maximising the squared correlation between the predictor and the value to be estimated, which is condition (a) only On the one hand, they find that the maximum squared correlation is similar to the minimum squared error as an interpolator As interpolators both methods capture the general "structure" of the data, such as non-stationarity On the other hand, they differ considerably as extrapolators

Consequently the criterion most often used to determine optimal vector autoregressions, minimising squared errors, might be irrelevant to maximise returns The next sections investigate in more details the relationships between error measures and profits

4.2 ERROR MEASURES AND PROFITABILITY

4 2 1 *Performance Criteria*

A forecasting method is used to predict the one-ahead underlying return At time t-1, it generates forecast F_{t-1} to predict the one-ahead logarithmic return X_t The one-period forecasting performances of the model can be evaluated by various techniques (Stekler, 1991) such as

the squared error $$SE_t = (X_t - F_{t-1})^2 \qquad [4\ 1]$$

the absolute error $$AE_t = |F_{t-1} - X_t| \qquad [4\ 2]$$

the directional accuracy $$DA_t = \begin{cases} +1 & \text{if} \quad X_t F_{t-1} > 0 \\ 0 & \text{if} \quad X_t F_{t-1} < 0 \end{cases} \qquad [4\ 3]$$

Academics widely regard error measures as reliable criteria of performance, mainly due to the existing theory which surrounds them The minimum squared error in particular possesses attractive properties which have contributed to its widespread use among researchers (Box and Jenkins, 1976) The directional accuracy or percentage of correct forecasts has been widely used to test the usefulness of market timing strategies and advisory services (Levich, 1980, Henriksson and Merton, 1981, Pesaran and Timmermann, 1992)

However, for trading purposes, a more appropriate forecasting performance measure is obviously profitability Then according to Chapter 3, the rate of return following a trading rule (rule returns) can be defined by

$$R_t = B_{t-1} X_t \qquad [3\ 1]$$

Where B_{t-1} is a signal triggered by the trading rule at the start of the period and which takes the values 1 and -1 depending on whether an up or down price movement is expected i e

$$\begin{cases} \text{"Sell"} \Leftrightarrow B_{t-1} = -1 \Leftrightarrow F_{t-1} < 0 \\ \text{"Buy"} \Leftrightarrow B_{t-1} = +1 \Leftrightarrow F_{t-1} > 0 \end{cases}$$

Figures 4 1 to 4 4 graphically represent the profit (equation [3 1]), and error measures functions, squared error (equation [4 1]), absolute error (equation [4 2]) and directional accuracy (equation [4 3])

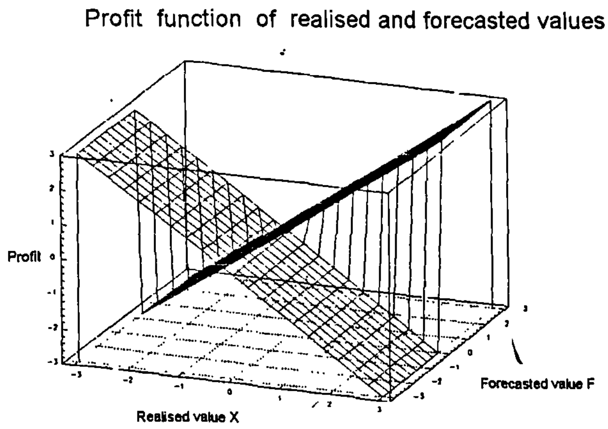

Figure 4.1: Profit function

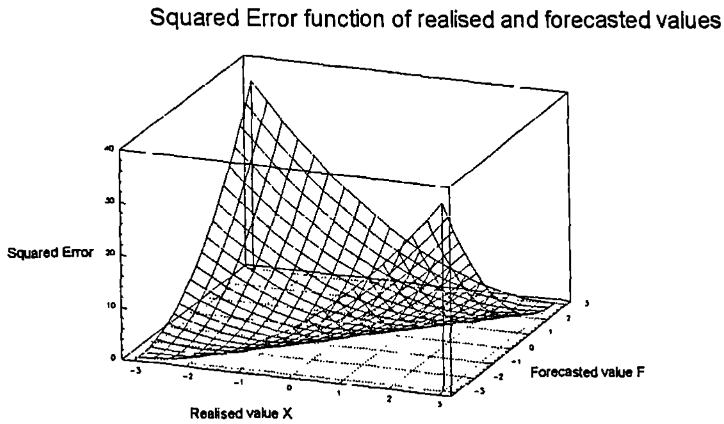

Figure 4.2: Squared Error Cost Function

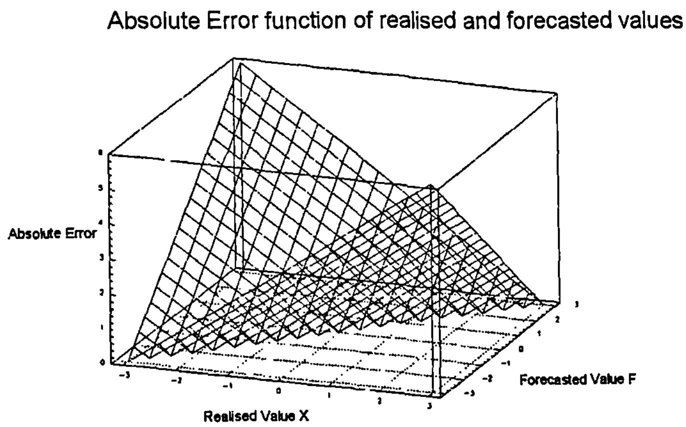

Figure 4.3: Absolute Error Cost Function

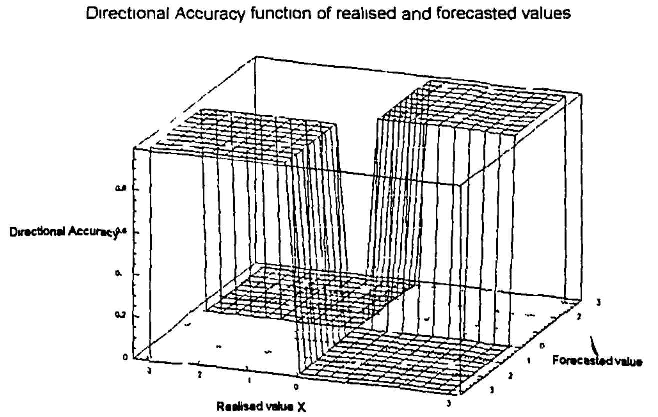

Figure 4.4 Directional Accuracy Function

The specificity of the profit criteria clearly appears by comparing figure 4 1 to figures 4 2 to 4 4 Differences with error measures are now put forward by establishing theoretical linear correlation between one-period error measures and rule returns

4 2 2 *Linear correlation between error measures and profits under the random walk without drift assumption*

Relating expected squared errors and expected rule returns as a function of the statistical characteristics of the underlying Gaussian series is usually possible and will be done in Section 4 3 by use of Proposition 3 5 However when the true process is a random walk without drift, the expected rule returns is zero whichever rule is applied and so no relationships can be worked out between error measures and expected profits Nevertheless, the linear correlation coefficient can be used to analyse the relationships between those varied performance criteria This section establishes the one-period correlation between profits and error measures as previously defined

We assume that the underlying returns X_t are without drift, independent normally distributed with volatility σ^2, then that the forecaster F_t is linear without constant, that is it can be expressed by either a linear combination of past logarithmic underlying returns X_t

Proposition 4 2

If the underlying process of returns $\{X_t\}$ is assumed to be a normal random walk without drift, the linear correlation between rule returns (equation [3 1]) and squared error (equation [4 1]) is

$$\mathrm{Corr}(R_t, SE_t) = \frac{-2\sqrt{v(f)}}{\sqrt{\pi}(1+v(f))} \qquad [4\ 4]$$

where $v(f) = \sigma_\Gamma^2/\sigma^2$ and σ_Γ^2 is the variance of the forecaster F_t [4 5]

Proposition 4 3

If the underlying process of returns $\{X_t\}$ is assumed to be a normal random walk without drift, the linear correlation between rule returns (equation [3 1]) and absolute error (equation [4 2]) is

$$\mathrm{Corr}(R_t, AE_t) = \frac{-2\,\mathrm{Arc}\sin(\sqrt{v(f)/[1+v(f)]})}{\sqrt{\pi}\sqrt{\pi - 2\sqrt{1+v(f)}}} \qquad [4\ 6]$$

where v(f) is given by equation [4 5]

Proposition 4 4

If the underlying process of returns $\{X_t\}$ is assumed to be a normal random walk without drift, the linear correlation between rule returns (equation [3 1]) and directional accuracy (equation [4 3]) is

$$\mathrm{Corr}(R_t, DA_t) = \sqrt{\frac{2}{\pi}} \qquad [4\ 7]$$

The correlation between rule returns and squared error (equation [4 4]) and correlation between rule returns and absolute error (equation [4 6]) are both negative That is not surprising since minimising the squared errors maximises profits Both correlations depend heavily on the rule which is being used throughout the variance of the forecaster v(f)

Table 4 1 gives some numerical values obtained from one of the most popular technical trading rule, the simple moving average method[2] (Brock, Lakonishok, and LeBaron, 1992, Levich and Thomas, 1991, LeBaron, 1991, 1992b) Table 4 1 indicates that the correlations in absolute value terms are a strong negative function of the order of the rule These results suggest there are rules displaying errors very few correlated, in

[2] Section 3 4 1 has shown that the simple moving average method can be considered a linear forecaster

absolute value, with profits They are in accordance with Leitch and Tanner(1991) who empirically find no systematic relationship between the widely used ex-post error criteria and ex-post profits All their conventional error-magnitude criteria are only marginally related to profitability They find in particular that the criterion absolute average error is only weakly correlated to profitability and conclude that profits may not be related to the size of the error

Table 4 1 Correlations between error measures and profits

Moving Average of order	2	5	20	100
$Corr(R_t,SE_t)$	-0 564	-0 199	-0 023	-0 002
$Corr(R_t,AE_t)$	-0 587	-0 264	-0 033	-0 003

The correlation between error measures and profits is maximal, in absolute terms, if the variance of the forecaster is constrained to be equal to the variance of the underlying returns Therefore we homogenise variances by constraining the variance of the forecaster to be equal to the variance of the underlying returns (v(f)=1 in equation [4 5]) Then it follows that correlations between error measures and profits do not depend any more on the rule being used and are equal to

$$Corr(R_t, SE_t) = \frac{-1}{\sqrt{\pi}} \cong -0\ 564 \qquad Corr(R_t, AE_t) = \frac{-\sqrt{\pi}}{2\sqrt{2}\sqrt{\pi - 2}} \cong -0\ 587$$

In contrast, the correlation between directional accuracy and rule returns is high and constant at 0 80, and independent of the rule itself as equation [4 7] proves Irrespective of the rule, directional accuracy and profitability will appear very dependent criteria Leitch and Tanner(1991) display similar results In particular, they find that directional accuracy consistently demonstrates a high degree of statistical association (measured by the linear coefficient of correlation) Their results suggest that if profits are not observable, directional accuracy of the forecasts might be used as the evaluation criterion

The theoretical formulae exhibited in this section might explain the empirical findings of Leitch and Tanner(1991) which are that directional accuracy is a lot more linearly correlated to rule returns than error measures, Root Mean Squared Error and Average Absolute Error Nevertheless the conclusion they give that profits may not be related to the size of the error, should be understood profits may not be linearly related to the size of the error Section 4 1 has proved that for Gaussian processes, expected squared errors and expected rule returns are functionally dependent and that minimising squared errors maximises profits However, the stochastic variables squared errors and rule returns display very few linear relationships, as shown in this section under the random walk assumption

It must be recognised that there exist other error measures such as the root squared error and the Theil U coefficient. However, it does not seem straightforward to determine their exact linear correlation with rule returns as in the case of AE, SE and DA. Furthermore, it is not certain that including them would bring much additional support to our arguments.

4.3 MEAN SQUARED ERROR AND PROFITABILITY UNDER THE GAUSSIAN PROCESS ASSUMPTION

Once again the relationships between error measures and profitability could be investigated via the linear correlation for any Gaussian processes without drift. However such steps are not reproduced here for two reasons. The first one is that analytical results, although still possible, become quite difficult to calculate. For instance, it can be shown that the presence of low autocorrelations affects very few the linear correlation between directional accuracy and profits[3], but strictly speaking the correlation coefficient depends now on the rule being used. The second reason not to give too much importance to the linear coefficient correlation is that there exists another way, far more appealing to study the relationships between error measures, namely mean squared error, and profitability we are now describing.

4.3.1 *Misspecification criteria*

For the general linear Gaussian process, Section 4.1 has shown that minimisation of the SE leads to maximisation of expected rule returns. However, in practice, the true underlying process may be difficult to estimate and a misspecified model may be used instead. Such misspecification can arise:

(a) because of misspecification of the parameter values

(b) because of the choice of the wrong model i.e. employing an Moving Average MA process instead of an Autoregressive AR model or

(c) through the use of a technical indicator (Neftci, 1991), which is a special case of (b).

Following Davies and Newbold(1980), we can measure misspecification by:

$$P_{SE}(h) = \frac{V^*(h) - V(h)}{V(h)} \qquad [4.8]$$

[3] See Appendix 4.2

Where V(h) and $V^*(h)$ are the variances of the h-step prediction errors using the correct and misspecified models respectively That is

$V(h) = E((X_{t+h} - F_{t,h})^2)$ and $V^*(h) = E((X_{t+h} - F^*_{t,h})^2)$

where $F_{t,h}$ and $F^*_{t,h}$ are the optimal and misspecified h-step forecasts

$P_{SE}(h)$ measures the relative increase of squared errors It shows the excess volatility resulting from the use of the misspecified model

For a trading rule though the cost of using a misspecified model should be better measured by its monetary consequences in terms of foregone profits and therefore a suitable measure is the relative loss of returns

$$P_R(h) = \frac{E(R(h)) - E(R^*(h))}{E(R(h))} \qquad [4\ 9]$$

Where E(R(h)) and $E(R^*(h))$ are the expected h-period returns of the true and misspecified models respectively

We shall restrict here our study to the one period ahead forecasts, h=1, since many trading rules are not designed to forecast longer than a single period Chapter 3 has shown that in this case the expected return of a linear trading rule is given by equation [3 18] Then it must be remarked that the relative loss of returns, $P_R(1)$ is a positive number which takes values between 0 and 2 That is due to the fact that the expected return of the misspecified forecaster can not be above the expected return of the true model which is the maximum achievable return Consequently the expected return of the misspecified forecaster can not be either below minus the expected return of the true model If this were the case, that would mean that the contrarian strategy of the misspecified forecaster would outperform the expected return of the true model, which is not possible

A first obvious difference between the two misspecification criteria is that criterion [4 8] is scale dependent when criterion [4 9] is not Let us explain what we understand by scale dependent What is argued is that equation [4 9] is unaffected by a change of positive scale in the forecasts It can be seen from equation [3 18] that replacing F_t by a F_t (a>0), will not affect $E(R_t)$ and so criterion [4 9] does not change On the other hand, criterion [4 8] is changed since $E((X_{t+1} - F_t)^2) \neq E((X_{t+1} - aF_t)^2)$

In what follows, we shall evaluate the effect of misspecification on the mean squared error and the expected return for a general time series models that has been widely employed in finance, the ARMA(1,1) model with drift defined by

$$X_t = \mu + pX_{t-1} + \varepsilon_t - q\varepsilon_{t-1} \qquad [4\ 10]$$

μ, p and q are constant and ε_t is a normal white noise without drift $\varepsilon_t \sim N(0,\sigma_\varepsilon^2)$

We will study more specifically three cases due to their popularity in Finance

the Random walk with drift where $\mu \neq 0$ and $p = q = 0$ in equation [4 10]

$$\text{That is } X_t = \mu + \varepsilon_t$$

the AR(1) model without drift where $p = \alpha \neq 0$ and $\mu = q = 0$ in equation [4 10]

$$\text{That is } X_t = \alpha X_{t-1} + \varepsilon_t$$

the ARMA(1,1) model without drift where $p \neq 0$, $q \neq 0$ and $\mu = 0$ in equation [4 10]

$$\text{That is } X_t = p X_{t-1} + \varepsilon_t - q\varepsilon_{t-1}$$

In the next sections, we examine the behaviour of the two measures, relative increase of mean squared error, $P_{SE}(1)$, and relative loss of returns, $P_R(1)$, for each of the three sources of misspecification (a), (b) and (c) mentioned above

4 3 2 *Optimal trading strategies*

Before examining the effects of using a misspecified forecaster, it is important to determine the performances of the correct model noted thereafter H_0, both in terms of error measures and profits The reason is that H_0 represents "the" optimal forecaster It is the only one to display both the minimal variance of the prediction error and the maximal profit

Random Walk with Drift; RW(μ)

The true underlying process is assumed to be a random walk with drift Returns X_t are independent identically distributed following a normal law with drift $N(\mu,\sigma^2)$ That is $X_t = \mu + \varepsilon_t$, with $\varepsilon_t \sim N(0,\sigma^2)$

The most accurate forecaster is the drift itself $\qquad F_t^{mse} = \mu \qquad$ [4 11]

The minimal variance of the prediction error is $\qquad V(1) = \sigma^2 \qquad$ [4 12]

The maximal profit is generated by the passive strategy and equals $E(R(1)) = |\mu|$ [4 13]

Autoregressive of order 1, AR(1)

The true model is an autoregressive of order one AR(1) without drift having a first order autocorrelation α $\qquad X_t = \alpha X_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim N(0,\sigma^2)$

The optimal forecaster is the quantity $\qquad F_t^{mse} = \alpha X_{t-1} \qquad$ [4 14]

The minimal variance of the prediction error is $\qquad V(1) = \sigma^2(1-\alpha^2) \qquad$ [4 15]

The maximal profit is $\qquad E(R(1)) = \sqrt{\frac{2}{\pi}}\sigma|\alpha| \qquad$ [4 16]

Autoregressive Moving -Average of order 1; ARMA(1,1)

The true model is an autoregressive-moving average process of order one AR(1) without drift $X_t - pX_{t-1} = \varepsilon_t - q\varepsilon_{t-1}$, with ε_t normal white noise, p and q constants

The optimal forecaster is the quantity (Taylor, 1986 186) $F_t^{mse} = (p-q)\sum_{i=0}^{\infty} q^i X_{t-i}$ [4 17]

The variance of the prediction error is (Taylor, 1986 187) $V(1) = \dfrac{(1-p^2)\sigma^2}{(1-2pq+q^2)}$ [4 18]

The maximal profit is (equation [3 10]) $E(R(1)) = \sqrt{\dfrac{2}{\pi}}\,\sigma\,Corr(X_{t+1}, F_t^{mse})$ [4 19]

with $Corr(X_{t+1}, F_t^{mse}) = \sqrt{(p-q)^2/(1-2pq+q^2)}$ (Taylor, 1986 193)

4 3 3 *Parameter misspecification*

Random Walk with Drift; RW(μ)

Parameter misspecification on this model means that the estimated drift parameter μ' differs from the true parameter μ assumed to be non-zero The resulting increase in term of variance is so

$$V^*(1) = (\mu-\mu')^2 + \sigma^2 \quad \text{and} \quad P_{SE}(1) = \frac{V^*(1) - V(1)}{V(1)} = \frac{(\mu-\mu')^2}{\sigma^2}$$

The expected return following the misspecified forecaster is

$$E(R^*(1)) = \begin{Bmatrix} |\mu| & \text{if} & \mu\mu' > 0 \\ -|\mu| & \text{if} & \mu\mu' < 0 \\ 0 & \text{if} & \mu' = 0 \end{Bmatrix}$$

Therefore, the percentage loss of returns is

$$P_R(1) = \frac{E(R(1)) - E(R^*(1))}{E(R(1))} = \begin{Bmatrix} \{|\mu| - |\mu|\}/|\mu| = 0\% & \text{if} & \mu\mu' > 0 \\ \{|\mu| - (-|\mu|)\}/|\mu| = 200\% & \text{if} & \mu\mu' < 0 \\ \{|\mu| - 0\}/|\mu| = 100\% & \text{if} & \mu' = 0 \end{Bmatrix}$$

When both drifts, true and estimated, have the same sign, there are no losses of profits since the rules deduced by both forecasts are identical Indeed the magnitude of the forecast is not taken into account in the decision process, only its sign matters Consequently both trading rules will still maximise profits although they display different mean squared error due to over/under estimation as shows the following example

$\sigma = 0\ 007$ $\mu = 0\ 0002$ (5% yearly) $\mu' = 0\ 0008$ (20% yearly)

That is a case where an obvious overestimation of the true drift μ= 5% (yearly) is being done Forecasting μ' = 20% implies that $P_{SF}(1) = 0\ 63$ Nevertheless from a profit point of view, both misspecified and optimal forecasters will generate the same profit, that is μ = 5%

Autoregressive of order 1; AR(1)

Let us assume that an estimate α' is used instead of the true first lag coefficient α There is a loss of accuracy

$$V^*(1) = (1+\alpha'^2 - 2\alpha\alpha')\sigma^2 \quad \Rightarrow \quad P_{SE}(1) = \frac{V^*(1) - V(1)}{V(1)} = (\alpha' - \alpha)^2$$

For instance if $\alpha = 0\ 05$, $\alpha' = 0\ 10$, it follows that $P_{SF}(1) = 0\ 25$

The expected return of the misspecified forecaster is

$$E(R^*(1)) = \begin{cases} \sqrt{2/\pi}\,|\alpha| & \text{if } \alpha\alpha' > 0 \\ -\sqrt{2/\pi}\,|\alpha| & \text{if } \alpha\alpha' < 0 \\ 0 & \text{if } \alpha' = 0 \end{cases}$$

Therefore, the percentage loss of returns is

$$P_R(1) = \frac{E(R(1)) - E(R^*(1))}{E(R(1))} = \begin{cases} \{\sqrt{2/\pi}|\alpha| - \sqrt{2/\pi}|\alpha|\} / \sqrt{2/\pi}|\alpha| = 0\% & \text{if } \alpha\alpha' > 0 \\ \{\sqrt{2/\pi}|\alpha| - (-\sqrt{2/\pi}|\alpha|)\} / \sqrt{2/\pi}|\alpha| = 200\% & \text{if } \alpha\alpha' < 0 \\ \{\sqrt{2/\pi}|\alpha| - 0\} / \sqrt{2/\pi}|\alpha| = 100\% & \text{if } \alpha' = 0 \end{cases}$$

No loss of profits will occur as long as α' the misspecified parameter has the same sign as the true parameter α because in this case, both predictors will trigger the same rule

ARMA(1,1) without drift (Price-trend model)

In practical situations how much can a model be misspecified ? A realistic answer is now given when attempting to fit an ARMA(1,1) model Due to its financial interpretation, the state representation of an ARMA(1,1), the price-trend model (see Section 2 4 1), will be adopted in what follows Therefore the two parameters of interest will be the variance reduction A and the mean duration m_d It is recalled that the mean duration is defined by equation [2 10] $m_d = 1/(1-p)$, and the variance reduction A is defined by equation [2 12] $A = (p-q)(1-pq)/\{p(1-2pq+q^2)\}$

Let us assess the statistical and financial consequences when a poorly defined model is applied We consider that instead of the true model, a misspecified forecaster is used and defined by $F_t = \sum_{i=0}^{\infty} \lambda_i X_{t-i}$, where λ_i are constants

An example of misspecified forecaster is $\lambda_i = (p' - q')q'^{i}$, where p' and q' are two constants If (p',q')#(p,q), the forecaster F_t is nothing else than a price-trend model using misspecified parameters

The variance of the prediction error is then given by

$$V^*(1) = Var(\sum_{i=0}^{\infty} \lambda_i X_{t-i} - X_{t+1}) = Var(\sum_{i=0}^{\infty} \lambda_i X_{t-i}) + Var(X_{t+1}) - 2Cov(\sum_{i=0}^{\infty} \lambda_i X_{t-i}, X_{t+1})$$

$$V^*(1) = \{\sum_{i=0}^{\infty} \lambda_i^2 \sigma^2 + 2\sum_{i=j+1}^{\infty}\sum_{j=0}^{\infty} \lambda_i \lambda_j A p^{i-j} \sigma^2\} + \sigma^2 - 2\sum_{i=0}^{\infty} \lambda_i A p^{i+1} \sigma^2$$

Therefore, using equation [4 18], the relative increase of squared error is equal to

$$P_{SE}(1) = \frac{\{\sum_{i=0}^{\infty} \lambda_i^2 + 2\sum_{i=j+1}^{\infty}\sum_{j=0}^{\infty} \lambda_i \lambda_j A p^{i-j} + 1 - 2\sum_{i=0}^{\infty} \lambda_i A p^{i+1}\} - (1-p^2)/(1-2pq+q^2)}{(1-p^2)/(1-2pq+q^2)} \quad [4\ 20]$$

The coefficient correlation between the misspecified forecaster and the one-step ahead return is given by

$$Corr(X_{t+1}, F_t) = Corr(X_{t+1}, \sum_{i=0}^{\infty} \lambda_i X_{t-i}) = p\sum_{i=0}^{\infty} \lambda_i A p^i / (\sqrt{\sum_{i=0}^{\infty} \lambda_i^2 + 2\sum_{i=j+1}^{\infty}\sum_{j=0}^{\infty} \lambda_i \lambda_j A p^{i-j}})$$

Therefore, using the last result and equation [3 10], the expected return following the misspecified forecaster is given by

$$E^*(R(1)) = \sqrt{\frac{2}{\pi}}\sigma(p\sum_{i=0}^{\infty}\lambda_i Ap^i)/(\sqrt{\sum_{i=0}^{\infty}\lambda_i^2 + 2\sum_{i=j+1}^{\infty}\sum_{j=0}^{\infty}\lambda_i\lambda_j Ap^{i-j}}) \qquad [4\ 21]$$

Then using equation [4 19], the relative loss of returns is equal to

$$P_R(1) = \frac{(p\sum_{i=0}^{\infty}\lambda_i Ap^i)/(\sqrt{\sum_{i=0}^{\infty}\lambda_i^2 + 2\sum_{i=j+1}^{\infty}\sum_{j=0}^{\infty}\lambda_i\lambda_j Ap^{i-j}}) - \sqrt{(p-q)^2/(1-2pq+q^2)}}{\sqrt{(p-q)^2/(1-2pq+q^2)}} \qquad [4\ 22]$$

Parameters misspecification on the price-trend model is common because the standard deviation of the estimates is large Results for maximum likelihood estimates of A and m_d might be found in Box and Jenkins(1976) for linear process, and for Taylor estimates A and m_d in Taylor(1980, 1986) Taylor(1986) specially finds that averages estimates of m_d are less than 10 days whereas the true parameters are m_d = 20 days and A=0 02 This downward bias causes estimates A to have an upward bias Taylor(1980 Table 3) shows that estimates of m_d are not accurate when $m_d \geq 20$ Also it appears that increasing the series length n does not substantially improve the accuracy For a given m_d, the estimate of A has standard error of approximately $\{2/m_d\, n\}^{1/2}$ That is for n=1000 and m_d=20, $\sigma(A) \sim 0\ 01$

Let us now quantify the financial and statistical consequences when a poorly defined model is applied First assuming an erroneous mean duration (m_d' instead of m_d), second an incorrect variance reduction (A' instead of A), third both inexact

The application of an erroneous mean duration m_d' instead of m_d produces a maximum relative increase in SE (equation [4 20]) of only 0 49% (Table 4 2) On the other hand, the relative loss of profits (equation [4 22]) is far higher, ranging from 2 6% to 26% If a mean duration is estimated equal to five days and it is actually equal to forty days, the profit made by following such suboptimal forecasts will be worth 9 63% (equation [4 21]) when the maximum achievable return is worth 13 06% (equation [4 19]), that is a relative loss of 25 5% (equation [4 22])

Table 4.2 Misspecified (mean duration only) price-trend model

True price-trend model A = 0 03, m_d, μ=0 and σ = 0 007					
Yearly returns % using a misspecified mean duration, m_d'					Yearly returns % using the optimal forecaster
$m_d \backslash m_d$'	5	10	20	40	
5	5 32	5 06	4 50	3 88	5 32
10	6 39	6 66	6 48	6 84	6 66
20	8 85	10 04	10 41	10 13	10 41
40	9 63	11 61	12 63	13 06	13 06
Relative loss of returns %					
$m_d \backslash m_d$'	5	10	20	40	
5	0	4 9	15 4	26 0	
10	4 8	0	3 6	12 0	
20	15 0	3 6	0	2 6	
40	25 5	11 2	2 6	0	
Relative increase in SE %					
$m_d \backslash m_d$'	5	10	20	40	
5	0	0 04	0 16	0 30	
10	0 05	0	0 05	0 18	
20	0 21	0 06	0	0 05	
40	0 49	0 25	0 06	0	

When applying an erroneous variance reduction only, both increase in SE and percentage loss of profits are small That can be seen from the diagonals of Table 4 3 When A'=0 01 and A=0 03, the maximal increase in SE equals to 0 24% and the maximal relative loss of returns to 1 2%, both for $m_d = m_d' = 40$ days

Table 4 3 Misspecified price-trend model

True price trend model { A = 0 03 , m_d }, Misspecified model { A' = 0 01 , m_d' }				
Relative loss of returns %				
$m_d \backslash m_d$'	5	10	20	40
5	0 1	6 1	20 8	35 1
10	3 6	0 3	6 1	19 2
20	13 3	2 0	0 6	6 4
40	23 6	8 5	0 6	1 2
Relative increase in SE %				
$m_d \backslash m_d$'	5	10	20	40
5	0 06	0 04	0 05	0 10
10	0 18	0 12	0 09	0 11
20	0 40	0 28	0 19	0 15
40	0 60	0 55	0 36	0 24

Under/overestimating the variance reduction or the mean duration under/overestimates ex-ante true possible profits Table 4 2 says that if the true model is $\{\sigma = 0\ 007, A=0\ 03, m_d=40\}$ the maximum return is equal to 13 06% (equation [4 19]) Assuming instead an erroneous parameter, A'=0 01 or $m_d'=5$ would let think that the highest returns which can be achieved in those conditions are respectively equal to 5 26% and 5 32% These figures are the results of using the misspecified parameter A'=0 01 or $m_d'=5$ for both rule and process in equation [4 19] Using a wrong mean duration has ex-post far more financial consequences than estimating incorrectly the variance reduction The true return triggered by such forecasts is measured by using the incorrect parameter, A' or m_d', for the rule but true A and m_d parameters for the process in equation [4 21] A misspecified mean duration $m_d'=40$ instead of $m_d=5$ when A=0 03 reduces potential profits by 26 0% (Table 4 2) when an incorrect variance reduction A'=0 01 instead of A=0 03 when $m_d=40$ decreases it only of 1 2% (Table 4 3) This primary example shows that a forecaster should not be judged from the discrepancy in terms of ex-ante returns it generates using a misspecified parameter for both rule and process (~5 3% in both cases instead of 13 6%) Instead, it should be evaluated in terms of ex-post relative loss of profits the decision process involves using a misspecified parameter for the rule but true parameters for the process (1 2% and 26 0%)

Predicting inaccurately future profits (returns expectations) is of little importance for an investor point of view if the decision making process which results from these forecasts happen to be an almost optimal strategy ex-post (relative loss of profits) Indeed an investor would prefer to be *inaccurate in his expectation but correct in his decision* process

So it does not seem that equal focus on both parameters should be given on a decision making process It does appear that the mean duration of the trend should indeed require special attention After all would it have been possible to reach such conclusion from a squared errors criterion ? Does the mean squared error give a good idea of how misspecified a rule is in terms of profitability ? Without knowing the source of misspecification can one use the mean squared error to extrapolate the returns of its forecasting method ?

Figure 4 5 aggregates the results of tables 4 2 and 4 3 and shows that this is unfortunately not the case

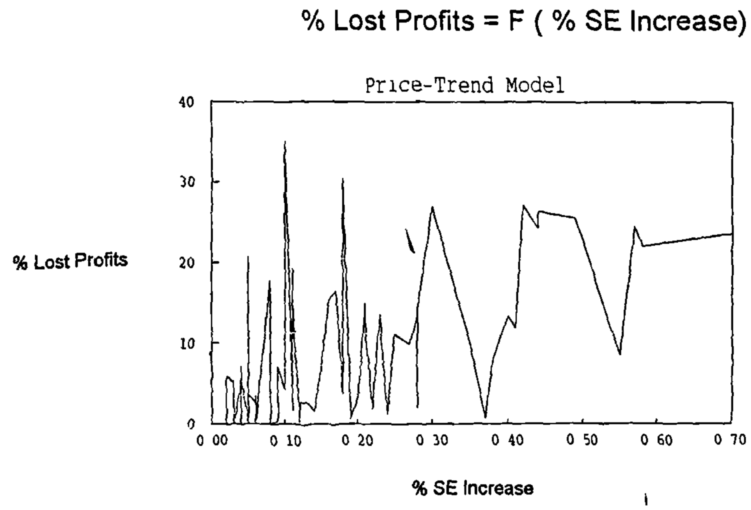

Figure 4.5 Relative loss of returns as a function of the relative increase in SE

Increase in SE is ordinarily extremely low The maximum increase is here equal to 0 60% for {A'=0 01, m_d'=5} when {A=0 03, m_d=40} On the other hand, the relative loss of profits can be huge It can reach here 35 1% for {A'=0 01, m_d'=40} when {A=0 03, m_d= 5} More significantly, there does not seem to be any link between an increase in SE and a loss of profits The explanation might be that the relationships between the increase of SE and the percentage lost of profits is highly nonlinear and quite complicated That can be seen by comparing analytical formulae [4 20] and [4 22] for the price-trend model So if the true model is the price trend model, the minimum squared errors criterion might not be relevant to assess the usefulness (profitability) of a forecaster

4 3 4 *Model misspecification*

H_0 : Random walk with positive drift (μ>0)

Rule returns based on a AR(1) model depend only on the sign of the autoregressive parameter, α, and not on its size (Table 4 4) When the true model is a random walk with drift, the return of such a rule is positive when α is positive but very small[4] The relative loss of profit is substantial and rather insensitive to the size of the drift parameter For

4 When α is positive, the rule triggered by an AR(1) forecaster is identical to the simple moving average of order 2 rule The expected rule return is consequently extremely low under the random walk with drift assumption, see Chapter 3

example assuming σ = 0 007 and μ = 0 0002 (5% yearly) produces a relative loss of 96 6% When the drift parameter increases to 0 0006 (15% yearly), the percentage loss is 93 2% In comparison, the value of both the drift and the autoregressive parameters have very little effect on the percentage increase in the SE

Table 4.4 AR(1) forecaster when the true model is a Random Walk with drift

Relative loss of returns % (Standard Deviation σ = 0 007)			
μ(yearly %) \ α	AR(1), $\alpha > 0$		
5	96 6		
10	95 4		
15	93 2		
Relative increase in SE (Standard Deviation σ = 0 007)			
μ(yearly %) \ α	0 025	0 05	0 1
5	0 14	0 33	1 08
10	0 39	0 56	1 32
15	0 80	0 98	1 62

H_0: AR(1) without drift and α>0

We examine here, the reverse case of using a Random Walk model with positive drift instead of the true AR(1) model The strategy of the RW model with positive drift is a buy and hold strategy with excepted return equal to zero The relative percentage loss is therefore 100 percent The percentage increase in SE is

$$P_{SE}(1) = \frac{\mu^2 + \sigma^2 - (1-\alpha^2)\sigma^2}{(1-\alpha^2)\sigma^2} = \frac{\mu^2 + \alpha^2\sigma^2}{(1-\alpha^2)\sigma^2}$$

When the RW model used has no drift (μ=0) the percentage increase in SE is

$$P_{SE}(1) = \frac{\alpha^2}{(1-\alpha^2)}$$

For example if α=0 1, the percentage increase in SE is 1 01% compared to 100% relative loss in profit

H_0: ARMA(1,1) without drift

Employing a RW model with drift instead of the true model ARMA(1,1) gives results very similar to the ones in the last section A more interesting case is when the model employed is an AR(1) The net returns issued from the two models, misspecified AR(1) and correct ARMA(1,1), are given in Table 4 5 The relative loss in profits can still be derived from equation [4 22] and depends on the mean duration m_d of the true model but as it has been shown before not on the size of the autoregressive parameter of the misspecified model The relative loss varies from 36% for m_d=5 to nearly 69% for m_d=40

The percentage increase of the SE is still derived from equation [4 20] and depends as expected on the size of the autoregressive model The relative increase in SE is a negative function of m_d When m_d increases, that is a striking case where the AR(1) model becomes more accurate but less profitable in relative terms

Table 4.5 AR(1) forecaster when the true model is the price-trend model

Yearly returns					
True Model ARMA(1,1) , { A = 0 03 , m_d , σ = 0 007 }					
m_d	Yearly returns % using an AR(1) with α>0		Yearly returns % using the optimal forecaster		
5	3 35		5 32		
10	3 66		6 66		
20	3 98		10 41		
40	4 08		13 06		
Relative loss of returns %					
m_d	AR(1) with α >0				
5	36 00				
10	51 46				
20	61 68				
40	68 65				
Relative increase in SE %					
$m_d \backslash \alpha$	0 025	0 05	0 075	0 1	0 125
5	0 83	0 89	1 09	1 41	1 85
10	0 81	0 86	1 04	1 35	1 68
20	0 80	0 85	1 02	1 32	1 64
40	0 80	0 84	1 01	1 30	1 62

4 3 5 *Technical Indicator*

We now study the effect of using technical forecasters when the true model is a Gaussian process More specifically, we measure the consequences of following simple moving average techniques of orders 5, 10, 20 and 40 in terms of mean squared error and profitability for the three models we have considered until now The use of a technical indicator in those conditions can be seen in fact as a special case of misspecification model, because simple moving averages are linear models (Section 3 4 1)

H_0 : Random walk with positive drift (μ>0)

From Table 4 6, it can be seen that the higher the order of the moving average, the lower the relative loss of returns This result is a direct consequence of Chapter 3 in which it is shown that with equation [3 12] the higher the order of the moving average, the closer it is from the optimal strategy, buy and hold In addition as volatility decreases, the relative loss of returns decreases

Table 4.6 Relative loss of returns following a simple moving average rule, when the true model is the RW with drift

Relative loss of returns % (Standard Deviation σ = 0 007)				
μ(yearly)	S(5)	S(10)	S(20)	S(40)
5	95 8	94 0	91 3	86 6
10	91 6	86 9	82 6	65 6
15	86 6	81 9	64 3	64 1
20	83 5	66 1	66 2	53 4

The mean squared error does not seem a relevant criterion to judge the potential profitability of a technical indicator due to its sensitivity to a change of scale The fact that technical forecasters display very different variances is a serious drawback which prevents the use of squared errors in performances measurement Homogenisation for equal variances between forecasters is required Then minimising squared errors will be nothing else than maximising correlation between predicted and actual values That is what achieves our profit criterion, which is scale independent

H_0 : AR(1) without drift and α>0

Since the optimal strategy is nothing else than a simple moving average of order 2, it is logical that the lower is the order of the moving average, the smaller is the relative loss of returns (Table 4 7)

Table 4.7 Relative loss of returns following a simple moving average rule, when the true model is the AR(1) model

Relative loss of returns %				
α	S(5)	S(10)	S(20)	S(40)
0 025	26 8	46 6	61 6	62 8
0 05	26 6	46 5	61 6	62 6
0 1	26 1	46 2	61 5	62 6

H_0 : ARMA(1,1) without drift

For a given variance reduction, Table 4 8 reflects that the best single moving average corresponds to an order relatively higher than the mean duration

Table 4.8 Relatıve loss of returns followıng a sımple movıng average rule, when the true model ıs the prıce-trend model

Relatıve loss of returns %				
A = 0 03				
m_d	S(5)	S(10)	S(20)	S(40)
5	8 0	1 6	11 3	29 6
10	22 1	6 0	1 8	12 4
20	35 6	16 9	3 6	2 6
40	46 1	26 9	11 3	1 8
A = 0 02				
m_d	S(5)	S(10)	S(20)	S(40)
5	8 6	1 6	10 6	28 6
10	23 8	6 1	1 6	11 1
20	38 5	19 5	5 1	2 0
40	49 9	32 0	14 5	2 8

4 3 6 *Conclusıons on mısspecıfıcatıon*

The lınear forecaster whıch mınımıses squared errors maxımıses expected returns However the mean squared error does not seem a relevant crıterıon to judge the predıctıve power of forecastıng strategıes When the true model ıs not known, one has to use a mısspecıfied forecaster Then loss of profitabılıty ın relatıve terms ıs almost unrelated to loss of accuracy One cannot conclude that a decrease ın mean squared error wıll provıde a gaın ın returns The most plausıble explanatıon to thıs phenomenon ıs the degenerescence ın the multıvarıate law of rule and underlyıng returns Broffitt(1986) has shown that although functıonally dependent those two processes can be uncorrelated and follow a bıvarıate degenerated law It ıs why error measures mıght be ın practıce of poor use to study the predıctıve power of a forecastıng strategıes and that only profitabılıty crıterıon should be consıdered for ınvestment purposes

In theory, ıf the underlyıng process ıs Gaussıan, technıcal forecasters are mısspecıfied and cannot outperform vector autoregressıons forecasters In fact, technıcal forecasters mıght not be mısspecıfied and be optımal forecasters, because many of them are ın fact vector autoregressıons forecasters Another argument ın favour of technıcal analysıs has been gıven by Taylor(1992b 16) " The channel rule may be superıor because ıt may requıre less ınformatıon to learn about a satısfactory value of ıts one parameter than an ARIMA rule needs to find satısfactory estımates of ıts AR and MA parameters " We have shown here that for tradıng purposes, ıt was far more ımportant to accurately estımate the mean duratıon of the trend than the varıance reductıon That ıs exactly what attempt technıcal rules such as the weıghted or sımple movıng average rules So technıcal analysts mıght argue, wıth some reasons, that ıt ıs preferable to use an ıll-defined forecaster but adequately approximated because of relevant criterion (profits) than a well-defined forecaster but wrongly estimated because of inadequate criterion (error measures) In practice, complicated misspecified models can outperform much worse than simple (also probably misspecified) models Indeed, attention should be given to models where the financial implications of interest (rule returns) are not sensitive to the model of the trend Ideally we would like a forecasting strategy which implies the same expected rule return whichever trend model is used

4 3 7 *Extension to chaotic time series*

If the data are chaotic, one can potentially forecast the time series perfectly, but one can practically never succeed in long-run forecasting (Brock, Hsieh and LeBaron, 1991) In those conditions, forecasting is close to impossible (Mandelbrôt, 1966, Butler, 1989) Suppose that we estimate the model of a chaotic time series and that we make an error of 1 percent in the estimation of just one parameter This exceedingly small estimation error will be sufficient to introduce large errors in predicting the time series In order to use the model for forecasting purposes, we should be able to obtain infinitely precise estimates of the parameters of the model Anything less precise makes the use of the model for predictive purposes useless De Mandelbrot(1966), Butler(1989), Grauwe and Vansanten(1991) show that in the chaotic world they have modelled, time series models of the financial asset cannot be used for forecasting purposes In this case, market participants have no incentive to invest time and money in acquiring information about the underlying economic model In order to be useful this information must have a degree of precision which is unattainable in the social sciences

The above conclusions relate to forecasting errors and stochastic modelling and therefore might not be applicable for profits and technical rules The exact nature of the underlying chaos needs not to be known to build profitable strategies Under the null hypothesis of a fractional Gaussian process, the optimal forecaster given by Hoskings(1981) is a very profitable forecaster It exhibits an infinite gain because the autocorrelations are not summable It is however very dependent on the initial conditions and is not known when the true model is not Consequently a more robust rule might be a simpler forecaster such as technical rules Indeed, we have seen in Section 3 4 3 that technical rules such as the simple moving average are quite profitable under the fractional Gaussian process assumption

In sum, chaotic time series might be a case where technical forecasters should be preferred to time series models

4 4 MARKET TIMING ABILITY TESTS IN PRESENCE OF LOW AUTOCORRELATIONS

As proved theoretically in Section 4 2 2 and empirically by Leitch and Tanner(1991), directional accuracy is certainly the best candidate as a substitute of profits if those ones are not observable So it is not surprising that tests of market timing ability have been based on it

Section 4 4 1 supports the close link between expected profits and directional accuracy by establishing their formal relationships for any Gaussian processes without drift However, Section 4 4 2 questions the usefulness of market timing ability tests based on directional accuracy, such as the Henriksson and Merton(1981) test, and its extension the Cumby and Modest(1987) test, in presence of low autocorrelations

4 4 1 *Directional Accuracy and Rule returns*

Market timing ability can be looked at from several perspectives error measures and profitability We will examine here the required accuracy for profitable market timing assuming that the underlying asset follows a Gaussian process without drift[5]

Proposition 4 5

If the underlying process of returns $\{X_t\}$ is assumed to be Gaussian without drift, the expected value of directional accuracy DA_t is given by

$$E(DA_t) = \frac{1}{2} + \frac{\operatorname{Arc\,sin}(\rho)}{\pi} \qquad [4\ 23]$$

$$\text{where } \rho = \operatorname{Corr}(X_{t+1}, F_t) \qquad [4\ 24]$$

In addition, the expected value of rule returns is known and again a function of ρ only Equation [3 10] says that

$$E(R_t) = \sqrt{\frac{2}{\pi}}\sigma\rho \qquad [4\ 25]$$

Expected values of directional accuracy D_t and profitability R_t can be linked easily using equations [4 23] and [4 25] The relationships is given by

[5] That is a similar study to the required accuracy for successful asset allocation under the bivariate random walk by Clarke, Fitzgerald, Berent and Statman(1990)

$$E(DA_t) = \frac{1}{2} + \frac{1}{\pi} \operatorname{Arc\,sin}\left(\frac{E(R_t)}{\sqrt{2/\pi}\sigma}\right) \qquad [4\ 26]$$

When the correlation ρ between one-ahead return and forecaster is quite low, Arcsin(ρ)~ ρ, and equation [4 26] can be approximated by

$$E(DA_t) = \frac{1}{2} + \frac{1}{\sqrt{2\pi}\sigma} E(R_t) \qquad [4\ 27]$$

So directional accuracy as expected is once again positively and almost linearly related with profitability Table 4 9 provides some numerical examples assuming a given volatility

Table 4.9 Relationships between directional accuracy and profits

Directional Accuracy, Profit assuming a volatility σ = 0 007		
$Corr(F_t,X_{t+1})$	$E(R_t)$ Yearly %	$E(DA_t)$ %
0	0	50
0 025	3 5	50 8
0 05	7 0	51 6
0 075	10 5	52 4
0 1	14 0	53 2
0 125	17 5	54 0
0 15	20 9	54 8
0 175	24 4	55 6
0 2	27 9	56 4

The most interesting result from this table is that it is enough of very few directional accuracy (DA~55%) to generate big profits (>21%) Such results have been empirically noted by Kester(1990) and would contradict the findings of Chua, Woodward and To(1987) and Sharpe(1975 67) in which he states that "[ ]unless a manager can predict whether the market will be good or bad each year with considerable accuracy (e g be right at least seven times out of ten), he should probably avoid attempts to time the market altogether"

4 4 2 *Market Timing Ability Tests*

The fact that it is enough of very few directional accuracy to generate big profits has profound implications on testing the market timing ability of forecasters This might indeed question the usefulness of market timing ability tests based on the percentage of correct forecasts These ones might not be powerful enough to detect market timing ability This point is now investigated in more details by considering Henriksson and Merton(1981)'s non parametric test and its extension by Cumby and Modest(1987)

Hennksson and Merton(1981)'s test and its extension by Cumby and Modest(1987) are certainly some of the most popular tests employed to investigate the usefulness of technical advisory services as market timers Schnader and Stekler(1990), Beebower and Vankooty(1991), Gerlow and Irwin(1991), Hartzmark(1991) are recent examples The advantage of these methodologies is to measure the value of a forecast (advice) which is independent of an investors preference, endowments, or pnor assessments of an asset's return stream

The Hennksson and Merton(1981) non parametnc test simply studies the percentage of correct forecasts following a given trading rule They make the additional assumptions that the conditional probability of a correct forecast does not depend on the magnitude of subsequent returns Then the test may be implemented in a sample of N observed forecasts by classifying the N outcomes as follows

		Actual Returns	
		$X_t \geq 0$	$X_t < 0$
Predicted	$X_t \geq 0$ $(B_{t-1}=1)$	n_1	N_2-n_2
Returns	$X_t < 0$ $(B_{t-1}=-1)$	N_1-n_1	n_2
		N_1	N_2

where X_t is the excess rate of return of the underlying asset

N_1 = number of outcomes with $X_t \geq 0$

N_2 = number of outcomes with $X_t < 0$

n_1 = number of correct forecasts that $X_t \geq 0$,

n_2 = number of correct forecasts that $X_t < 0$,

and $n = n_1+n_2-N$ number of correct forecasts that $X_t \geq 0$

The test proceeds by using the fact that, under the null hypothesis of no timing ability, n_1 is distnbuted as an hypergeometnc When the probability under the null of observing n_1 or more correct forecasts that $X_t \geq 0$ (given N_1, N_2 and n) is unacceptably small, the null hypothesis is rejected

The Cumby and Modest(1987) test extends Hennksson and Merton(1981) test by removing the cntical assumption that the conditional probability of a correct forecast does not depend on the magnitude of subsequent returns The relationship for assessing market timing ability can be defined, similarly to Gerlow and Irwin(1991), as

$$X_t = \alpha + \beta B_{t-1} + \varepsilon_t \qquad [4\ 28]$$

α and β are constant ε_t is a white noise X_t equals the percentage excess rate of return, and B_{t-1} is the signal tnggered by the rule

Market tımıng abılıty ıs found under the Cumby and Modest(1987) test ıf ß ıs sıgnıficantly dıfferent from zero Testıng ß=0 ıs therefore a test of forecastıng abılıty or to be more precıse a test of whether the forecaster possesses any ınformatıon not contaıned ın the uncondıtıonal sample mean

The assumptıon of the Henrıksson and Merton(1981) test that the probabılıty of a correct forecast ıs ındependent of the magnıtude of subsequent asset returns, ıs lıkely to be vıolated when technıcal ındıcators are used Cumby and Modest(1987) note that even ıf market returns have constant varıance per unıt of tıme, the varıance of holdıng perıod returns followıng a technıcal rule wıll not be constant[6] Breen, Jagannathan and Ofer(1986) show that correctıon for heteroskedastıcıty can sıgnıficantly affect the conclusıons The heteroskedastıcıty correctıons suggested by Whıte(1981) seem partıcularly effectıve (Breen, Jagannathan and Ofer, 1986, Cumby and Modest, 1987) A sımpler approach ıs to consıder when avaılable unrealısed ınstead of realısed returns as shown ın Chapter 3 Sectıon 3 3 has shown that ıf the underlyıng process ıs homoskedastıc, so wıll be unrealısed rule returns process Then the use of unrealısed returns wıll remove the artıficıal heteroskedastıcıty ınduced by realısed returns, ıt mıght not however be suffıcıent ıf the underlyıng process ıs ıtself heteroskedastıc

So there ıs a theoretıcal framework, possıble lınear relatıonshıps between dırectıonal accuracy and rule returns, and constant varıance per unıt of tıme, whıch may justıfy the use of Henrıksson and Merton(1981) and Cumby and Modest(1987) tests to assess the usefulness of technıcal ındıcators Yet the power of such tests must be ınvestıgated under the most plausıble alternatıve ın defence of technıcal analysıs, the presence of low autocorrelatıons

4 4 3 *Power Study*

A most plausıble model whıch can explaın tradıng rule returns ıs the prıce-trend model due to Taylor(1982) The trends μ_t had normal dıstrıbutıons and zero drıft Sımılarly to Taylor(1982), serıes of 1500 returns were sımulated wıth A=0 034 and p=0 944, and the model was replıcated 1000 tımes The market return has been assumed to be equal to zero, therefore the excess rate of return ıs equal to the underlyıng rate of return $X_t = Ln(P_t / P_{t-1})$

Let us study the power of popular market tımıng abılıty tests namely, Henrıksson and Merton(1981), Cumby and Modest(1987) what we respectıvely note HM(x) and CM(x) for the sımple movıng average rule of order x=5, 10, 20 and 40 Table 4 10

[6] See Appendıx 3 4 for a sımple proof

indicates in addition for comparison purpose the power of two statistics T, U[7], specifically constructed to test the price-trend hypothesis

Table 4 10 Power of market timing ability tests under the price-trend assumption

Estimated Powers for 1500 observations from a price-trend model A=0 034 , p=0 944 , μ=0 , N=1000			
	Percentage rejections of RW		
Statistic	Significance level		
	1%	5%	10%
Henriksson-Merton			
HM(5)	21	29	33
HM(10)	28	36	38
HM(20)	36	40	43
HM(40)	41	45	47
Cumby and Modest			
CM(5)	23	46	58
CM(10)	36	60	71
CM(20)	44	66	77
CM(40)	40	63	74
Taylor			
T	75	88	93
U	71	84	90

The most powerful statistics are the Taylor(1980) statistics, T and U It must not be surprising since they have been explicitly elaborated to test the price-trend hypothesis The HM market timing ability test has very low power, below 50% at a critical level of 10%, whichever rule is applied Jagannathan and Korajczyk(1986) exhibit similar results and show that in most reasonable cases there is a nonlinear relation between portfolio returns and the independent variables in the timing models They prove that is theoretically possible to construct portfolios that show artificial timing ability when no true timing ability exists We just have shown that is theoretically possible to find trading rules that show no timing ability when true timing ability exists This result theoretically confirms a finding of Jagannathan and Korajczyk(1986) which is that nonlinearity in market-timing models need not be due solely to violation of the assumed linear return-generating process

The CM market timing ability test has significantly higher power than the HM test The most powerful statistic is CM(20) which is quite natural since the simple moving average

7 Definitions of statistics T, U are given in Section 6 1 4 and Taylor(1982)

of order 20 is the most profitable rule in the portfolio under this particular price-trend assumption

Therefore criteria and tests based on directional accuracy to assess market timing ability of technical indicators, might be of poor use to detect the usefulness of a trading rule when the financial underlying time series exhibits low autocorrelations The main difficulty is then to find a proper test of market timing ability This is the object of Chapter 5

4 5 SUMMARY

If price time series data is assumed to follow a Gaussian process, then linear rules were shown to be useless in maximising returns over and above vector autoregressions This is also the conclusion of Neftci(1991) However, this finding needs to be refined because as shown in Chapter 3, there are technical rules which are implicitly linear The most important result is that minimising squared errors is a sufficient but not necessary condition to maximise expected profits

Despite a functional dependence, profits and error measures are only weakly linearly correlated under the random walk assumption In this case directional accuracy seems the best substitute to profits if those ones are not observable In presence of low autocorrelations, misspecification criteria based on error measures and profits behave quite differently The main finding is that it is difficult to deduce from an increase of mean squared error a loss of profits and vice-versa In practical terms, a decrease of mean squared error is not linearly and positively related to a gain of profits That would mean that for trading purposes, optimal vector autoregressions although maximising returns in theory will have to be determined via other step researches than decreasing the mean squared error The explanation of this fundamental issue might be that the functional dependence between error measures and profits is of very few practical use because highly nonlinear and possibly degenerated Therefore, when the true model is not known, a decrease in mean squared error does not necessarily imply a gain in profits

In presence of low autocorrelations, expected directional accuracy and profits seem linearly related Nevertheless in this case, tests of market timing ability based on directional accuracy exhibit very low power These tests might say nothing about the usefulness of trading rules for maximising profits Nonlinearity in market-timing models need not be due solely to violation of the assumed linear return-generating process

APPENDIX 4.1

PROOFS OF PROPOSITIONS

Proposition 4 1

Expected returns given by equation [3 18] are function of two variables
$x = \mu_F/\sigma_F$ the ratio mean/standard deviation of the forecaster
$\rho = Corr(X_{t+1},F_t)$ correlation between forecaster and one-ahead return
Equation [3 18] can so be rewritten as

$$E(R_t) = \sqrt{\frac{2}{\pi}}\sigma\rho\exp(-x^2/2) + \mu(1-2\Phi[-x]) \qquad [4\ 29]$$

The two variables ρ and x are independent So the forecaster which maximises expected return must first maximise ρ Then for a given ρ, the forecaster which maximises profits is obtained by deriving formula [4 29] as a function of x It follows that the second condition is given by

$$dE(R_t)/dx = 0$$

$$\Leftrightarrow \quad \sqrt{\frac{2}{\pi}}\sigma\rho(-x)\exp(-x^2/2) + \mu\sqrt{\frac{2}{\pi}}\exp(-x^2/2) = 0$$

$$\Leftrightarrow \quad \sigma\,\rho\,x = \mu \Leftrightarrow x = \mu/(\sigma\rho) \Leftrightarrow \mu_F/\sigma_F = \mu/(\sigma\rho)$$

Proposition 4 2

We first know from Chapter 3 that
$E(R_t) = 0$ [3 7]
$Var(R_t) = \sigma^2$ [3 8]
Then if we note $v(f) = \sigma_F^2/\sigma^2$, and $Z_t = X_t - F_{t-1}$, we deduce that

$$Z_t \sim N(0,\sigma_z^2) \text{ where } \sigma_z^2 = \sigma^2+\sigma_F^2 = \sigma^2(1+v(f)) \qquad [4\ 30]$$

It follows that

$$E(SE_t) = E((X_t - F_{t-1})^2) = E(Z_t^2) = \sigma_Z^2$$

$$Var(SE_t) = E(Z_t^4) - (E(Z_t^2))^2 = 3\sigma_Z^4 - \sigma_Z^4 = 2\sigma_Z^4 \qquad [4\ 31]$$

$$E(R_tSE_t) = E(B_{t-1}X_tZ_t^2) = E(B_{t-1}X_t(X_t^2 - 2F_{t-1}X_t + F_{t-1}^2))$$

$E(R_tSE_t) = E(B_{t-1}X_t^3) - 2E(B_{t-1}F_{t-1}X_t^2) + E(B_{t-1}F_{t-1}^2X_t)$

Since X_t follows a normal centred random walk, we have

$E(B_{t-1}X_t^3) = E(B_{t-1})E(X_t^3) = E(B_{t-1})\,0 = 0$

$E(B_{t-1}F_{t-1}^2X_t) = E(B_{t-1}F_{t-1}^2)E(X_t) = E(B_{t-1}F_{t-1}^2)\,0 = 0$

$$E(B_{t-1}F_{t-1}X_t^2) = E(B_{t-1}F_{t-1})E(X_t^2) = \Big(\int_{F_{t-1}>0} F_{t-1} - \int_{F_{t-1}<0} F_{t-1}\Big)\sigma^2 = \sigma^2\sigma_F\sqrt{\frac{2}{\pi}}$$

Therefore $\quad E(R_tSE_t) = -2E(B_{t-1}F_{t-1}X_t^2) = -2\sigma^2\sigma_F\sqrt{\frac{2}{\pi}}$

From the last result and equations [3 8] and [4 31], we deduce that

$$Corr(R_t,SE_t) = \frac{-2\sigma^2\sigma_F\sqrt{\frac{2}{\pi}}}{\sigma\sqrt{2\sigma_Z^2}}$$

Since by definition $v(f)=\sigma_F^2/\sigma^2$ and $\sigma_z^2=\sigma^2(1+v(f))$ from [4 30], it follows that

$$Corr(R_t,SE_t) = \frac{-2\sqrt{v(f)}}{\sqrt{\pi}(1+v(f))}$$

Proposition 4 3

Using the auxiliary variable Z_t defined by formula [4 30],

$E(AE_t) = E(|X_t-F_{t-1}|) = E(|Z_t|) = \sqrt{\frac{2}{\pi}}\sigma_z$

$Var(AE_t) = E(|X_t-F_{t-1}|^2) - (E(|X_t-F_{t-1}|))^2 = E(Z_t^2) - (E(|Z_t|))^2 = \sigma_z^2(1-2/\pi)$ [4 32]

$Cov(R_t,AE_t) = E(R_tAE_t) = E(B_{t-1}X_t|Z_t|)$

$$Cov(R_t,AE_t) = \iint_{F_{t-1}>0,Z_t>0} X_tZ_t - \iint_{F_{t-1}>0,Z_t<0} X_tZ_t - \iint_{F_{t-1}<0,Z_t>0} X_tZ_t + \iint_{F_{t-1}<0,Z_t<0} X_tZ_t$$

$$Cov(R_t,AE_t) = 2\Big\{\iint_{F_{t-1}>0,Z_t>0} X_tZ_t - \iint_{F_{t-1}>0,Z_t<0} X_tZ_t\Big\}$$

The last equality results from symmetry argument

In addition, we know from equation [4 30], that $X_t = Z_t + F_{t-1}$ Consequently,

$$\iint_{F_{t-1}>0,Z_t>0} X_t Z_t = \iint_{F_{t-1}>0,Z_t>0} Z_t^2 + Z_t F_{t-1} = \sigma_Z^2[2,0](\rho)+\sigma_Z\sigma_F[1,1](\rho)$$

$$\iint_{F_{t-1}>0,Z_t<0} X_t Z_t = \iint_{F_{t-1}>0,Z_t<0} Z_t^2 + Z_t F_{t-1} = \sigma_Z^2[2,0](-\rho)+\sigma_Z\sigma_F[1,1](-\rho)$$

where [1,1](ρ) and [2,0](ρ) are truncated moments of bivariate normal laws, and $\rho = Corr(Z_t, F_{t-1}) = Corr(X_t - F_{t-1}, F_{t-1}) = -Cov(F_{t-1}, F_{t-1})/(\sigma_F \sigma_Z)$ because X_t and F_{t-1} are independent Consequently

$\rho = -\sigma_F / \sigma_Z = -\sqrt{v(f)/[1+v(f)]}$ [4 33]

Subsequently, we deduce that

$Cov(R_t, AE_t) = 2\{\sigma_Z^2([2,0](\rho)-[2,0](-\rho))+\sigma_Z\sigma_F([1,1](\rho)+[1,1](-\rho))\}$

Using the exact formulation of [1,1](ρ) and [2,0](ρ) respectively given by [A 3] and [A 4] in Appendix 3 1, we have

$Cov(R_t, AE_t) = (2/\pi)\sigma_z^2(\text{Arc}\sin(\rho) + \rho\sqrt{1-\rho^2} - \rho\{\rho \text{Arc}\sin(\rho) + \sqrt{1-\rho^2}\})$

$Cov(R_t, AE_t) = (2/\pi)\sigma_z^2(1-\rho^2)\text{Arc}\sin(\rho)$

From equations [4 30] and [4 33], we deduce that $\sigma_z^2(1-\rho^2)=\sigma^2$

Then, $Cov(R_t, AE_t) = -(2/\pi)\sigma_z^2 \text{Arc}\sin(\rho) = -(2/\pi)\sigma^2 \text{Arc}\sin(\sqrt{v(f)/[1+v(f)]})$

From the last result and equations [3 8] and [4 32], we deduce that

$$Corr(R_t, AE_t) = \frac{-2\text{Arc}\sin(\sqrt{v(f)/[1+v(f)]})}{\sqrt{\pi}\sqrt{\pi-2}\sqrt{1+v(f)}}$$

Proposition 4 4

$E(DA_t) = Pr(X_t F_{t-1}>0) = Pr(X_t>0, F_{t-1}>0)+Pr(X_t<0, F_{t-1}<0)$

$= Pr(X_t>0)Pr(F_{t-1}>0)+Pr(X_t<0)Pr(F_{t-1}<0) = 0\ 25 + 0\ 25 = 0\ 5$

$Var(DA_t) = E(DA_t^2) - (E(DA_t))^2 = E(DA_t)\{1-E(DA_t)\} = 0\ 25$

Equations [3 7] and [3 8] say respectively that $E(R_t) = 0$ $Var(R_t) = \sigma^2$

Then

$$Cov(R_t, DA_t) = E(R_t DA_t) = \iint_{F_{t-1}>0,X_t>0} X_t - \iint_{F_{t-1}<0,X_t<0} X_t = \tfrac{1}{2}\left(\int_{X_t>0} X_t - \int_{X_t<0} X_t\right) = \sigma/\sqrt{2\pi}$$

$$Corr(R_t, DA_t) = \frac{\sigma/\sqrt{2\pi}}{0\ 5\sigma} = \sqrt{\frac{2}{\pi}}$$

Proposition 4 5

$$E(DA_t) = \iint_{F_{t-1}>0, X_t>0} X_t - \iint_{F_{t-1}<0, X_t<0} X_t = 2 \iint_{F_{t-1}>0, X_t>0} X_t \quad \text{for symmetry reason}$$

$E(DA_t) = 2\,[1,0](\rho)$ where $\rho = \mathrm{Corr}(X_t, F_{t-1})$

and $[1,0](\rho)$ is the truncated moments of standardised bivariate normal laws given by equation [A 2] in Appendix 3 1 Subsequently,

$$E(DA_t) = \frac{1}{2} + \frac{\mathrm{Arc}\sin(\rho)}{\pi}$$

APPENDIX 4 2

CORRELATION BETWEEN DIRECTIONAL ACCURACY AND PROFITS ASSUMING A GAUSSIAN PROCESS WITHOUT DRIFT

Proposition 4 6

Under the Gaussian process assumption without drift, the linear correlation between rule returns (equation [3 1]) and directional accuracy (equation [4 3]) is

$$\mathrm{Corr}(R_t,DA_t) = \frac{\frac{1}{\sqrt{2\pi}} - \frac{\sqrt{2}}{\pi^{3/2}}\rho\,\mathrm{Arcsin}(\rho)}{\sqrt{[1-2\rho^2/\pi]}\sqrt{1/2+1/\pi\,\mathrm{Arcsin}(\rho)}\sqrt{1/2-1/\pi\,\mathrm{Arcsin}(\rho)}} \qquad [4\ 34]$$

With $\rho = \mathrm{Corr}(X_t, F_{t-1})$

Proof

Equations [3 10] and [3 6] respectively say that

$$E(R_t) = \sqrt{\frac{2}{\pi}}\,\sigma\,\rho \qquad\qquad \mathrm{Var}(R_t) = \sigma^2\,[1-\frac{2}{\pi}\rho^2]$$

Using the results of Proposition 4 5

$E(DA_t) = 1/2 + 1/\pi\,\mathrm{Arcsin}(\rho)$,

Since $E(DA_t^2) = E(DA_t)$,

$\mathrm{Var}(DA_t) = E(DA_t)(1-E(DA_t)) = (1/2 + 1/\pi\,\mathrm{Arcsin}(\rho))(1/2 - 1/\pi\,\mathrm{Arcsin}(\rho))$

$$E(R_t DA_t) = \iint_{F_{t-1}>0,X_t>0} X_t - \iint_{F_{t-1}<0,X_t<0} X_t$$

Using the truncated moments $[1,0](\rho)$ of standardised bivariate normal laws given by equation [A 4] in Appendix 3 1 It follows that

$$E(R_t DA_t) = \sigma^2 \frac{1}{\sqrt{2\pi}}(1+\rho)$$

From these results, we easily deduce $\mathrm{Cov}(R_t,DA_t)$

$$\mathrm{Cov}(R_t,DA_t) = \frac{1}{\sqrt{2\pi}} - \frac{\sqrt{2}}{\pi^{3/2}}\rho\,\mathrm{Arcsin}(\rho)$$

and so $\mathrm{Corr}(R_t,DA_t)$ given in equation [4 34]

Chapter 5

TRADING RULES DIAGNOSTIC TESTS

As outlined in Chapter 2, the choice of which dynamic strategy to follow depends on the expectations one has about the stochastic process which drives prices It is consequently crucial to establish proper tests of the randomness of financial prices Then if prices are not random, statistics must be found which enable to determine the most likely alternatives This chapter intends to solve both issues by considering tests based on the joint profitability of trading rules

The profitability of dynamic strategies might be one of the most powerful statistic to detect market inefficiencies as state Leuthold and Garcia(1992 53) "Relative Mean Squared Errors [however] provide only an indication of the potential for market inefficiency A sufficient condition for market inefficiency is whether the forecasting method can generate risk-adjusted profits which are greater than the cost of usage " Therefore market timing ability might constitute a more powerful way to detect market imperfections than standard statistical test However we have seen in Chapter 4 that the well known market timing ability tests, by Henriksson and Merton(1981) and Cumby and Modest(1987), exhibit low power in presence of low autocorrelations

This is why new tests based on the joint profitability of trading rules rather than directional accuracy or mean squared error should be sought

Section 5 1 establishes the necessary preliminaries, trading rules correlations under the random walk assumption Then Section 5 2 proposes new tests of the random walk hypothesis based on the joint profitability of trading rules Finally Section 5 3 extents previous results such that the adequacy of any Gaussian processes can be checked using trading rule returns As before, the last section of the chapter summarises and concludes our results

5 1 TRADING RULES CORRELATIONS UNDER THE RANDOM WALK WITHOUT DRIFT ASSUMPTION

Establishing trading rules correlations is essential to enable achieving three objectives Firstly, a proper, objective and quantified classification of trading rules, non-existent at the time of writing, could be performed using rules correlations Secondly, it might help to construct an efficient portfolio Thirdly, and perhaps more important, it will allow the joint profitability of a set of trading rules to be tested Brock, Lakonishok and LeBaron(1992), Surujaras and Sweeney(1992), Prado(1992) have emphasised that such a test might have power, specially against nonlinear alternatives

Consequently, correlations between trading rules are worth being investigated and are explored under the assumption of a random walk without drift Section 5 1 1 defines our basic assumptions Section 5 1 2 gives the main results of this section, the correlations between two technical rules applied to a bivariate random walk without drift

5 1 1 *Basic assumptions*

We are now assuming that two financial series, with returns $X_{1,t}$ and $X_{2,t}$, follow a centred bivariate normal law with variances σ_1^2 and σ_2^2 and correlation coefficient ρ_X Then two unbiased linear trading rules (similar or different) $F_{1,t}$ and $F_{2,t}$ are respectively applied to the two processes $\{X_{1,t}\}$ and $\{X_{2,t}\}$

$$F_{1,t} = \sum_{i=0}^{m_1-2} d_{1,i} X_{1,t-i} \quad [5\ 1] \qquad\qquad F_{2,t} = \sum_{i=0}^{m_2-2} d_{2,i} X_{2,t-i} \quad [5\ 2]$$

m_1 and m_2 are called the orders or lengths of the trading rules

The linear rule $F_{i,t-1}$ generates signal $B_{i,t-1}$ and return $R_{i,t}$ from the underlying process $\{X_{i,t}\}$, given by, $R_{i,t}=B_{i,t-1}X_{i,t}$ It must be noted that this chapter assumes that linear rules are without constant ($\delta = 0$ in equation [3 4]) Popular technical forecasters such as momentums, simple moving average, weighted moving average and double moving average rules are examples of linear rules without constant as can be seen from Table 3 3, and so are unbiased if the true model is without drift

5 1 2 *Linear rules correlations*

To the author's knowledge, the only researchers who have attempted to establish theoretical trading rules correlations are Praetz(1979) and Sweeney and Lee(1989) In both studies it is recognised that the covariances of trading rules depend on the covariances of underlying returns and on the positions the speculator had in the two assets Proposed formulae are expressed as a function of the frequency of short positions taken by the trading rules However their results are not exact and have to be considered, at best, as approximations, as it is now recognised, that the frequency of short positions is an endogenous variable (Surujaras and Sweeney, 1992) This section will attempt to remedy this limitation by giving precise theoretical correlations between trading rules

Precise theoretical correlations are now being established for any linear rules without constant and highlighted, for the sake of clarity, throughout three popular technical linear rules which are simple moving average, weighted moving average, and momentum systems, respectively noted S, W, M

Proposition 5 1[1]

Assuming that two underlying time series, X_{1t} and X_{2t}, follow a centred bivariate normal law with underlying correlation ρ_X, linear rule returns, R_{1t} and R_{2t}, exhibit linear correlation coefficient ρ_R, given by

$$\rho_R = \rho(R_{1t}, R_{2t}) = \frac{2}{\pi}\rho_X \operatorname{Arc\,sin}(\rho_X \rho_F) \qquad [5\ 3]$$

where ρ_F is the correlation between the two different forecasters which would have been applied to the same underlying process We call it systems correlations It is given by

$$\rho_F = \frac{\sum_{i=0}^{Min(m_1, m_2)-2} d_{1,i} d_{2,i}}{\sqrt{\sum_{i=0}^{m_1-2} d_{1,i}^2}\sqrt{\sum_{i=0}^{m_2-2} d_{2,i}^2}} \qquad [5\ 4]$$

In addition, $\rho(R_{1,t}, R_{2,t+h}) = \rho(R_{1,t+h}, R_{2,t}) = 0$ for $h>0$ [5 5]

[1] Proofs of propositions are given in Appendix 5 1

Proposition 5 2

Assuming that two underlying time series, X_{1t} and X_{2t}, follow a centred bivariate normal law with variances σ_1^2, σ_2^2 and coefficient correlation ρ_x, linear rules signals, B_{1t} and B_{2t} exhibit linear correlation coefficient ρ_B, given by

$$\rho_B=\rho(B_{1,t},B_{2t})=\frac{2}{\pi}\mathrm{Arc}\sin(\rho_x\rho_F) \quad [5\ 6]$$

and ρ_F is given by equation [5 4]

In addition, $\rho(B_{1,t},B_{2,t+h})=\frac{2}{\pi}\mathrm{Arc}\sin(\rho_x\rho_{F(h)})$ [5 7]

$$\text{with} \quad \rho_{F(h)}=\rho(F_{1,t},F_{2,t+h})=\frac{\sum_{i=0}^{Min(m_1,m_2-h)-2} d_{1,i}\,d_{2,i+h}}{\sqrt{\sum_{i=0}^{m_1-2} d_{1,i}^2}\sqrt{\sum_{i=0}^{m_2-2} d_{2,i}^2}} \quad [5\ 8]$$

Expressions [5 3] and [5 6] suggest a few comments

(a) correlation between rules signals, ρ_B, is higher in absolute value than correlation between rule returns, ρ_R

(b) rule signals correlation, ρ_B, is an odd function of underlying correlation ρ_x and of systems correlation ρ_F That means that rules signals will be negatively correlated if either the systems correlation or underlying correlation is negative

(c) rule returns correlation, ρ_R, is an even function of the underlying correlation ρ_x and an odd function of the systems correlation ρ_F That means that rule returns will be negatively (positively) correlated if, and only if, the systems correlation is negative (positive)

(d) rule returns correlation is always lower in absolute values than the underlying correlation

If one wants to minimise the risk of an investment, it turns out that diversifying trend-following systems between positively correlated assets can be beneficial beyond diversification of passive strategies, because the correlation between trading systems will be lower (property d) However, this will be disadvantageous if the underlying assets are negatively correlated, because trading systems will be positively correlated (property c)[2]

[2] See for a graphical representation of this fact Figure 5 1

For the remainder of this section we shall primarily focus our interest on returns rather than signals correlation since it has more implications vis-a-vis a portfolio context We shall detail and interpret previous results by considering three cases from the simplest to the most general different rules applied to the same underlying process, the same rule applied to different underlying processes, different rules applied to different underlying processes

Different rules applied to the same underlying process

When two different unbiased linear trading systems are applied to the same underlying process, $X_{1t}=X_{2t}=X_t$ and $\rho_x=1$ In this case, correlations between rule returns, equation [5 3], and correlations between rules signals, equation [5 6], become identical and equal to

$$\rho_R = \rho_B = \frac{2}{\pi}\mathrm{Arc}\sin(\rho_F) \qquad [5\ 9]$$

Table 5 1 gives examples of correlations between two successive orders of a given rule For instance, the correlation between simple moving averages of orders 2 and 3 is equal to 0 705 That tells us that for all three rules successive orders are less correlated for low than high orders This is not surprising and has been noted by Prado(1992) Prado(1992) recommends testing wider intervals as the moving average days increases He adds that on the one hand, a three day moving average is very different from a four day moving average, but on the other hand a ninety day moving-average is very similar to a ninety-one day moving-average Table 5 1 illustrates in addition that two successive orders of weighted moving averages are more correlated than simple moving averages and momentums

Table 5 1 stresses a common misunderstanding raised by practitioners, we now describe Smith(1992) studies the moving average rule applied to the Standard&Poor index and finds that the profitability is erratic except for parameter values 48 through 65, where a broad area of profitability is detected He then concludes that areas of erratic profitability should not be considered significant It does not seem that such results indicate at all presence of profitable trading rules or inefficiencies, since under the random walk assumption it is expected that low order rules will be less correlated than high order ones

Table 5.1 Correlations between rules of successive orders

Rule\Order	(2 3)	(3,4)	(4 5)	(10,11)	(20 21)	(40,41)	(100,101)
M	500	608	667	795	856	899	936
S	705	811	860	945	972	986	994
W	795	870	903	960	980	990	996

Table 5 2 shows correlations between various systems and orders For instance $\rho[S(5), W(10)]$ means the rule returns (or signals) correlation between the simple moving average of order 5 and the weighted moving average of order 10 It is equal to 0 799

Table 5.2 Rules correlations

ρ	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
S(5)	1	666	460	322	880	799	574	409	732	417	275	189
S(10)		1	680	472	600	859	823	593	697	697	419	281
S(20)			1	685	416	596	849	834	497	721	681	419
S(40)				1	291	416	593	844	351	523	732	674
W(5)					1	728	521	371	621	372	248	171
W(10)						1	732	524	755	574	363	246
W(20)							1	733	606	755	554	358
W(40)								1	441	635	752	545
M(5)									1	465	303	208
M(10)										1	483	319
M(20)											1	492
M(40)												1

Rather than listing differences between systems and orders which could happen to be endless due to the infinite number of linear rules, it is worth emphasising two points Firstly, trend-following systems are positively correlated Zero or negative correlation obviously requires the combination of trading rules of different nature such as convex (trend-following) and concave (overbought-oversold) strategies Secondly, buy and sell signals and then returns of technical systems are not independent over time under the random walk assumption Related findings are attributable to Working(1960) This established that if in a time series constructed from independent increments, the individuals items are replaced-let say-by monthly averages, spurious correlation is introduced between successive first differences of the averages Correlation between trading signals would contradict, however, the hypothesis of Lukac, Brorsen and Irwin(1988a) who considered, as an approximation, that buy and sell signals of systems are independent over time They then concluded that all the systems are on the same side of the market significantly more than might randomly be expected and that monthly returns are positively correlated Our results show that it is not absolutely certain that the similarities between systems Lukac, Brorsen and Irwin(1988a) found are nothing more than would randomly be expected

Same rule applied to different underlying processes

When the same linear rule (non-deterministic, and so excluding Buy and Hold, or Sell and Hold strategies) is applied simultaneously to two assets, $\rho_F = 1$ and equations [5 2] and [5 6] become

$$\rho_R = \frac{2}{\pi}\rho_x \text{Arc}\sin(\rho_x) \quad [5\ 10]$$

$$\rho_B = \frac{2}{\pi}\text{Arc}\sin(\rho_x) \quad [5\ 11]$$

We can see two additional properties, when the same rule is applied to two different assets

(a) rule returns correlations become independent of the rule itself and the sole function of the underlying correlation

(b) rule returns correlations are now an even function of the underlying correlation and thus are always positive

Table 5 3 and Figure 5 1 highlight formulae [5 10] and [5 11] for some values of correlations of the underlying process

Table 5.3 Rules correlations as a function of the underlying correlation

Underlying Correlation ρ_x	Signals correlation ρ_B	Returns correlation ρ_R
1	1	1
0 99	0 91	0 90
0 98	0 87	0 86
0 95	0 80	0 76
0 9	0 71	0 64
0 85	0 65	0 55
0 8	0 59	0 47
0 7	0 49	0 35
0 5	0 33	0 17
0 3	0 19	0 06
0 2	0 13	0 03
0 1	0 06	0 01
0 05	0 03	~0
0	0	0

Rules Correlations

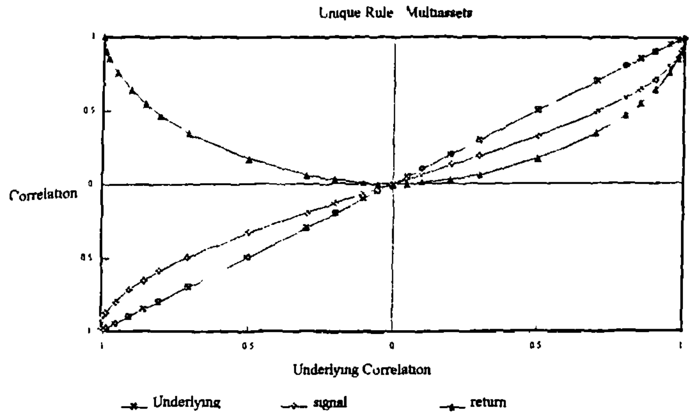

Figure 5.1 Rules correlations as a function of the underlying correlation

Different rules applied to different underlying processes

Let us now examine the most general case where different rules are applied to different underlying processes We use for this purpose different orders of simple moving averages

Having just proved that correlations between rule returns (when the same rule is applied to two different processes) do not depend on the rule itself, Table 5 4 exhibits constant diagonals

Table 5.4 Rule returns correlations ρ_R for different underlying correlations

Underlying correlation ρ_X = 0 95					
ρ	S(2)	S(5)	S(10)	S(20)	S(40)
S(2)	0 758	0 464	0 321	0 225	0 158
S(5)		0 758	0 583	0 411	0 289
S(10)			0 758	0 595	0 422
S(20)				0 758	0 599
S(40)					0 758
Underlying correlation ρ_X = 0 90					
ρ	S(2)	S(5)	S(10)	S(20)	S(40)
S(2)	0 642	0 411	0 287	0 201	0 142
S(5)		0 642	0 511	0 365	0 258
S(10)			0 642	0 52	0 375
S(20)				0 642	0 524
S(40)					0 642
Underlying correlation ρ_X = 0 85					
ρ	S(2)	S(5)	S(10)	S(20)	S(40)
S(2)	0 55	0 362	0 254	0 179	0 126
S(5)		0 55	0 447	0 323	0 229
S(10)			0 55	0 455	0 331
S(20)				0 55	0 457
S(40)					0 55

Our results are consistent with Praetz(1979) but disagree with Sweeney(1986) and Surujuras and Sweeney(1992)

On the one hand, Praetz(1979) noted that the results from both different securities and trading rules are likely to be positively correlated due to the presence of the market factor among security returns and due to the presence of many common rates in the returns from short selling of similar trading rules

On the other hand, Sweeney(1986 177) concluded that "even if [exchange] rates are correlated, excess rates of return on trading strategies should be virtually uncorrelated because the signals are only randomly synchronised across currencies" Surujuras and Sweeney(1992) then expressed the assumption that on the one hand, under efficiency rules signals would be completely out of synchronism and, on the other hand, inefficiencies would create positive cross correlations This section comes to a different conclusion, i e , even when underlying processes are correlated white noises, rules correlations- although lower in absolute value- cannot be zero The presence of inefficiencies, more specifically positive autocorrelations, would even increase rules correlations Our results clearly indicate that correlations between trading rules are strongly dependent on underlying correlations That could explain why the correlations between trading rules can be low for equities (Sweeney, 1988) and high for currencies (Surujaras and Sweeney, 1992) Accordingly t-statistics can be highly sensitive to whether the covariance terms are included or not

Overall, these results suggest that correlations between the same system applied to various assets can be much lower than correlations between various trend-following systems applied to the same asset It seems that these results might hold empirically[3] since diversification between assets has been found more beneficial than diversification between systems (Taylor 1990b, Brorsen and Boyd, 1990)

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[3] Empirical trading rule correlations for a set of exchange rates can be found in Section 6 2 3 They happen to be quite close to their expected value under the random walk assumption

5 2 A TEST OF NON-ZERO PROFITS

5 2 1 *Previous tests of profits from technical analysis*

Testing the usefulness of trading rules is not an easy task in the presence of a strong non-zero drift If market timing ability, as in the Cumby and Modest(1987) test, is a test of whether the forecaster possesses any information not contained in the unconditional sample mean, non-zero profits are not a relevant criterion Praetz(1976) and Section 3 3 4 have shown that if prices follow a random walk with drift, trading rules can be profitable but below the absolute value of the drift and so do not display any market timing ability

Praetz(1976) showed that expected rule returns are approximately $E(R_t)=\mu(1-2f)$ where f is the frequency of short positions The expected return on buy and hold is simply the drift itself, μ The expected rule return suggests that comparison between the rule return, R_t, and the return on buy and hold, X_t, leads to a bias, in favour of buy-and-hold if $\mu>0$ and in favour of the filter rule if $\mu<0$ To avoid this problem, Sweeney(1986) proposes the statistic

$$Y=\frac{1}{N}\sum_{t=1}^{N}R_t-(1-2f)\frac{1}{N}\sum_{t=1}^{N}X_t$$

where N is the total number of days in the period and f the frequency of short positions A formal definition of f is given by

$$f=\frac{1}{N}\sum_{t=1}^{N}B_t \text{ with } B_t=\begin{cases}-1 & \Leftrightarrow \text{"short position"}\\ +1 & \Leftrightarrow \text{"long position"}\end{cases}$$

Subsequently, Sweeney(1986) shows that $E(Y)=0$ and $V(Y)=\sigma^2/N$

The underlying assumptions of Praetz(1976), Bird(1985) and Sweeney(1986) are that

$E(R_t) = \mu(1-2f)$ [3 15]

$V(R_t) = \sigma^2$ [3 16]

$Cov(R_t,R_{t+h}) = 0$ for $h>0$ [3 17]

We know from Section 3 3 4 that these formulations are inexact and that they should be replaced by

$E(R_t) = \mu(1-2PS)$ [3 12]

$V(R_t) = \sigma^2+ 4\mu^2 PS(1-PS)$ [3 13]

$Cov(R_t,R_{t+h}) = \mu E(B_{t-1}B_{t+h-1}X_t) - \mu^2(1-2PS)$ [3 14]

where PS is the probability of being short at time t $PS = Pr(F_t<0) = \Phi(-\mu_F/\sigma_F)$ [3 11] and Φ is the cumulative function of a N(0,1)

Although theoretically different, PBS formulae [3 15], [3 16], and [3 17] are very close to [3 12], [3 13], and [3 14] for usual values of mean and standard deviation of price returns Therefore, the validity of PBS test should not be questioned The main limitation of this test lies rather in the fact that it can not be extended to assess the joint profitability of trading rules The presence of a drift complicates to the extreme trading rules stochastic properties Rule returns are not any more normally distributed but follow a mixture of normal laws (Proposition 3 4) Then correlations between trading rules are extremely difficult to establish and might be of poor use anyhow, because rule returns would follow a mixture of normal distributions

5 2 2 *Removing the drift*

It seems to us easier to remove the drift in the original series Doing so will allow us to use the numerous exact results of trading rules stochastic properties In particular Proposition 3 1 permits us to consider random walk tests from the joint profitability of trading rules

Removing the logarithmic drift in the original price series $\{P_t, t = 0, \ldots, N\}$ can be done by

(a) Estimating $\hat{\mu}=\frac{1}{N}\sum_{i=1}^{N} X_t$, where $X_t = Ln(P_t / P_{t-1})$

(b) Detrending the original price series by applying the transformation $P_t^* = P_t \exp(-\hat{\mu}t)$

This process requires the sample mean $\hat{\mu}$ to be equal to the true mean μ of the financial series It seems to us difficult to prevent such hypothesis Indeed, without this assumption, no comparison can be done with Buy and Hold strategy since the sample mean return will not reflect the true reference value Subsequently, we will assume that the sample mean is an accurate estimate of the true mean Therefore in what follows, we will consider that the series can be detrended Technical indicators will then be applied to detrended series P_t^*

5 2 3 *Random walk tests from the joint profitability of trading rules*

Trading rules have been widely used as a tool to detect abnormal profits and so market inefficiencies (Brock, Lakonishok and LeBaron, 1992, Levich and Thomas, 1991, LeBaron, 1991, 1992b)

There are however pros and cons to the use of trading rule returns to test market efficiency One of the possible benefits is that such approach might have power against non-linear alternatives (Brock, Lakonishok and LeBaron, 1992) Second, even if the true model is linear, standard statistical test are often derived by minimising the mean squared error which is a sufficient but not a necessary condition to maximise profits (Section 4 1) Although exhibiting some possible decisive advantages, tests based on trading rules profits have nevertheless several severe drawbacks In particular, a trading rule can be profitable without exhibiting any market timing ability for at least three reasons Firstly, Praetz(1976) and Section 3 3 4 have proved that, if the financial series follows a random walk with drift, certain trading rules can be profitable but below the unconditional mean and by consequence do not display any market timing ability Nevertheless, this inconvenience can be removed using the steps described in Section 5 2 2 Secondly, among one hundred rules, five can appear profitable by pure chance only, when a test is performed at a critical level of 5% (Taylor, 1900b) In other words stated, the application of filter analysis to financial market is deficient because possible variations in models designs are infinite (Stevenson and Bear, 1976) Finally, filter models require development independent on the sample upon which they are applied (Stevenson and Bear, 1976, Lukac and Brorsen, 1989)

Previous shortcomings can be remedied by considering instead of any single rule, a broad and arbitrary set of trading rules Studying the joint profitability of a large basket of trading rules constitutes therefore a better way to test the random walk hypothesis We are going to show that a generalisation of the univariate T-Student test can achieve this purpose

T-Student

The univariate T-Student is widely popular among academics (Lukac, Brorsen and Irwin, 1988b, Taylor, 1990) and practitioners (Kaufman, 1987) to test the hypothesis that returns to technical analysis are zero Its attractiveness is due to its simplicity It can be defined as

$$T = \sqrt{N}\,\frac{\overline{R}}{\hat{\sigma}_R} \qquad [5\ 12]$$

with N number of (daily) observations
$\overline{R}$ the average (daily) rule returns,
$\hat{\sigma}_R$ the standard deviation of (daily) rule returns

The T-statistic is an one-tail test of the hypothesis of zero profit against positive profits Its use assumes that the distribution of rule returns, R_t, is normal and independent, which is the case if the rules signal is defined by a linear forecaster, and the distribution of underlying returns, X_t, is without drift, normal and independent (Proposition 3 1)

The expected value $\bar{R}$ is typically estimated from the series of observed portfolio rule returns as

$$\bar{R} = \frac{1}{N}\sum_{t=1}^{N} R_t$$

Usually, the standard deviation is equally estimated empirically from the series of observed portfolio rule returns We will however prefer to estimate portfolio rule returns standard deviation under the random walk assumption via the underlying volatility, using the results of Section 5 1

Proposition 3 1 says that when the underlying time series, X_t follows a normal law without drift with variance σ^2, different linear rule returns, $R_{1,t}$ and $R_{2,t}$, follow univariate normal laws with variance σ^2 Proposition 5 1 adds that the linear correlation coefficient between rules returns ρ_R is known and given by equation [5 9][4] In addition, rule returns taken at different epochs are uncorrelated It results that under the random walk assumption without drift, the variance of a portfolio equally weighted of p linear rules is equal to

$$\sigma_R^2 = \begin{cases} \sigma^2 & \text{for } p = 1 \\ \sigma^2 \left(p + 2\sum_{i=1}^{p} \sum_{j=i+1}^{p} \rho(R_i, R_j)\right) \Big/ p^2 & \text{for } p > 1 \end{cases}$$

and $\rho(R_i, R_j)$ is the correlation between trading rules i and j given by equation [5 9]

Let us define the constant K by

$$K = \begin{cases} 1 & \text{for } p = 1 \\ \dfrac{1}{\sqrt{\left(p + 2\sum_{i=1}^{p} \sum_{j=i+1}^{p} \rho(R_i, R_j)\right) \Big/ p}} & \text{for } p > 1 \end{cases} \quad [5\ 13]$$

It then follows that under the random walk assumption without drift $\sigma_R = K\sigma$

We will subsequently estimate standard deviation of rule returns via the only underlying volatility using the estimate

$$\hat{\sigma}_R = K\hat{\sigma} \quad [5\ 14]$$

[4] It must be remarked that $R_{1,t}$ and $R_{2,t}$ do not follow a centred bivariate normal law, although $R_{1,t}$ and $R_{2,t}$ are univariate normal laws That can be shown by extending the demonstration given in Appendix 3 1 to the multivariate case Nevertheless, we will consider here that the bivariate normal law is a good approximation because the central limit theorem applies In particular, we will assume that weighted portfolio of trading rules follow a normal law, although it is not true strictly speaking

where $\hat{\sigma}$ is the usual standard deviation estimate of the underlying returns series

There are two major advantages to such an estimate First, it only requires the estimation of underlying volatility irrespective of the portfolio of rules under consideration Secondly, estimates of standard deviation via the observed series of portfolio rule returns can be quite different from the true standard deviation under the random walk assumption[5] Consequently, their use may lead to incorrect rejection/acceptance of the hypothesis of non-zero profits

It follows that comparing performances from single systems (p=1) will simply be comparing mean percent return since technical indicators display identical standard deviation, equal to the volatility of the underlying asset (K=1)

There might be another limitation to the use of the univariate T-Student If several systems are evaluated, by chance some will look better than they deserve Reporting results for only the best rule would be very misleading Taylor(1990b) advises that results should be given for all the systems considered in the research study However when all the systems are tested separately, results are highly dependent on each other because trading rules can be highly correlated (Proposition 5 1) So a test from the joint profitability of trading rules should be preferred It has been seen that testing the joint non-zero profitability of technical rules is possible and requires the only estimation of portfolio returns and underlying volatility Now, portfolio of indicators can exhibit quite different standard deviations (K<1) depending on the theoretical correlations between indicators and so the T-test applied to varied portfolios is not anymore a simple comparison of mean returns

Such test might allow to distinguish luck (only one rule performing by chance) from forecasting ability (profitability of a broad set of trading rules) At last considering portfolio of technical indicators has got an additional advantage which is that portfolio rule returns exhibit a distribution more normal than single rule returns (Lukac and Brorsen, 1990)

5 2 4 *Power Study*

The multivariate T-test presented above proposes in fact an almost infinite number of tests, as many as there are possible different portfolios of linear rules Determining what rules to incorporate into the portfolio is an extremely delicate task which will be discussed further in Section 6 2 4 Let us first investigate the power of a simple, although rather arbitrary portfolio of trading rules It includes four simple moving averages of

[5] Chapter 6 will discuss further the adequacy of empirical rules returns stochastic properties with the random walk without drift assumption

orders 5, 10, 20, 40 Orders of the rules have been chosen such that they are almost equicorrelated under the random walk assumption Applying equation [5 9], the correlation is approximately equal to 0 67

The statistics investigated in this study are the single T-Student applied to the simple moving average of orders x=5, 10, 20, 40, which we denote as S(x) and the multivariate T, which we denote as S(5,10,20,40) The alternative models considered here are all plausible representations of financial rates The price-trend model has been studied by Taylor(1982) and all the other processes, excluding the random walk with drift, have been investigated by Brock, Hsieh and LeBaron(1991)

H_1 : Linear hypothesis

Table 5 5 indicates the power of the T-test against linear alternatives Let us first assume that the underlying time series follow the price trend-model defined by Section 5 4 3 consistent with Taylor(1982) First of all, it must be noted by comparing Tables 5 5 and 4 10 that the T-Statistics have systematically higher power than Henriksson and Merton(1981), identical power than Cumby and Modest(1987) tests, but lower power than Taylor(1980) statistics, which have been specifically constructed to test the price-trend hypothesis Secondly, the most powerful single T-Statistic is the one corresponding to the simple moving average of order 20 It can be explained by the fact that the simple moving average of order 20 is the most profitable rule in the portfolio when the true mean duration is equal to $m_d = 1/(1\text{-}p) = 1/1\text{-} 944 \sim 18$ days This can be shown by use of equation [3 10] Thirdly, the portfolio test turns to be more powerful than any single T-Student The multivariate T-test would rank high in the power competition performed by Taylor(1982), fourth over 13 statistics, just behind Taylor(1980) statistics

Returns were then simulated following a second popular alternative, the auto-regressive model of order one with α=0 15 The most powerful single T-Statistic happens to be the simple moving average of order 5 The power of the test is a quite sharp negative function of the order of the rule This fact is simply the consequence that one optimal linear forecaster (maximising profits) is nothing else than a moving average of order two under the AR(1) assumption It can be noted than the multivariate T-test performs quite acceptably It ranks just below the single T-Statistic of order 5 but above order 10, 20, 40

If the underlying returns follow a moving-average model of order one model, the multivariate T-Statistic is more powerful than any of its component

We now measure the consequences of not removing the drift in a test of non-zero profits If the underlying returns follow a random walk model with drift μ=25%, the multivariate

T-Statistic rejects the zero-profits hypothesis in 76% of the cases at a critical level of 5% That is due to the fact that under the random walk with positive drift assumption, the strategy which maximises profits is "Buy and Hold" and that bigger is the order of the rule closer it is from the "Buy and Hold" strategy and so higher is the rate of rejection of zero-profits It follows that the multivariate T-Statistic might not be able to distinguish a random walk with drift from autocorrelated alternatives since under both assumptions, trading rules profits can be significant

H_1 : Non Linear hypothesis

Table 5 6 indicates the power of the T-Statistic test against non-linear alternatives

The multivariate T-test has very low power against purely variance-nonlinear alternative When price rates follow an ARCH(1) model, they are not forecastable in the mean and so zero rule returns are expected (Proposition 3 2)
T-Statistics have high power against the tent map model significantly less against the threshold auto-regressive model (TAR), and almost none against the nonlinear moving-average model (NMA) Antoniewicz(1992) finds as well that the moving average rule has little power against some simple nonlinear models There exists many other mean non-linear model which could have been considered, among which the Garch-M model The problem with the Garch-M model is that processes can substantially deviates from the mean of the original series (Weiss, 1986) That is a serious specification problem when interpreting technical indicators performances It might be that the rejection of the random walk hypothesis is not due to the hypothesis we want to test (such as Garch-M model) but is the result of strong unpredictable non-zero drift

Table 5 5 Power of T-Student test under linear assumptions

Estimated Powers for 1500 observations 1000 replica			
Price-trend model $X_t = \mu_t + e_t$ and $\mu_t = p\mu_{t-1} + \varepsilon_t$ with $Var(\mu_t) = A\ Var(e_t)$ A=0 034 p=0 944 N=1000			
	Percentage rejections of RW		
Statistic	Significance level		
	1%	5%	10%
Univariate T-Statistic			
S(5)	25	46	59
S(10)	39	62	72
S(20)	45	70	80
S(40)	42	67	77
Multivariate T-Statistic S(5 10 20 40)	55	73	83
AR(1) model $X_t = \alpha X_{t-1} + e_t$ α= 15			
	Percentage rejections of RW		
Statistic	Significance level		
	1%	5%	10%
Univariate T-Statistic			
S(5)	86	96	98
S(10)	54	78	89
S(20)	26	53	66
S(40)	12	30	46
Multivariate T-Statistic S(5 10 20 40)	64	85	92
MA(1) model $X_t = e_t + \theta e_{t-1}$ θ=0 5			
	Percentage rejections of RW		
Statistic	Significance level		
	1%	5%	10%
Univariate T-Statistic			
S(5)	89	94	95
S(10)	84	92	94
S(20)	70	85	89
S(40)	44	69	82
Multivariate T-Statistic S(5 10 20 40)	89	95	97
Random Walk with drift model $X_t = \mu + e_t$ μ= 0 001 (25% yearly) σ = 007 (15 8% yearly)			
	Percentage rejections of RW		
Statistic	Significance level		
	1%	5%	10%
Univariate T-Statistic			
S(5)	14	31	44
S(10)	26	50	62
S(20)	49	72	80
S(40)	72	88	93
Multivariate T-Statistic S(5 10 20 40)	56	76	85

Table 5 6. Power of T-Student test under non-linear assumptions

Estimated Powers for 1500 observations 500 replica			
ARCH(1) model $X_t = \sqrt{h_t} e_t$ and $h_t = 1 + \phi X_{t-1}^2$, $\phi = 0\ 5$			
	Percentage rejections of RW		
Statistic	Significance level		
	1%	5%	10%
Univariate T-Statistic			
S(5)	1	5	8
S(10)	1	5	9
S(20)	1	5	8
S(40)	1	3	8
Multivariate T-Statistic S(5 10 20 40)	1	4	8
Tent Map model $X[0] \in [0,1]$ $X_t = 2X_{t-1}$ if $X_{t-1} < 0\ 5$ $2\text{-}2X_{t-1}$ if $X_{t-1} \geq 0\ 5$			
	Percentage rejections of RW		
Statistic	Significance level		
	1%	5%	10%
Univariate T-Statistic			
S(5)	7	80	97
S(10)	7	80	97
S(20)	7	80	97
S(40)	7	80	97
Multivariate T-Statistic S(5 10 20 40)	52	96	99
NMA model $X_t = e_t + \gamma e_{t-1} e_{t-2}$ $\gamma = 0\ 8$			
	Percentage rejections of RW		
Statistic	Significance level		
	1%	5%	10%
Univariate T-Statistic			
S(5)	2	10	17
S(10)	2	10	18
S(20)	1	6	12
S(40)	1	6	11
Multivariate T-Statistic S(5 10 20 40)	1	6	14
TAR Map model $X_t = -0\ 5X_{t-1} + e_t$ if $X_{t-1} \leq 1$ $X_t = 0\ 4X_{t-1} + e_t$ if $X_{t-1} > 1$			
	Percentage rejections of RW		
Statistic	Significance level		
	1%	5%	10%
Univariate T-Statistic			
S(5)	0	0	1
S(10)	2	5	8
S(20)	18	37	51
S(40)	68	81	88
Multivariate T-Statistic S(5 10 20 40)	10	25	38

5 2 5 *Features of the multivariate T-Student*

The multivariate T-Statistic is a test of non-zero profit It is why results might be biased by the presence of a drift This disadvantage however can be removed by choosing either a period without drift or by detrending the original price series as described in Section 5 2 2 Once the drift is removed, the multivariate T-Statistic has power against linear and nonlinear means alternatives for which $E[X_t/X_{t-1} \quad X_{t-k}] \# 0$ However, the multivariate T-Statistic cannot detect or distinguish nonlinear variances models[6] So it might be used as a tool to distinguish mean from variance non-linearity

The multivariate T-Statistic test seems to display a decisive advantage over any single T-Statistic test, it seems to be robust for a broader range of alternatives Unequivocally it can perform well under the price-trend model hypothesis whatever is the duration of the trend, under the autoregressive of order one or the moving average of order one hypothesis It appears to have the nice property of exhibiting a power almost equal when not above the best of its components (which is unknown when the true model is unknown)[7]

5 3 ABILITY OF A DRIFTLESS GAUSSIAN PROCESS TO REPLICATE RULE RETURNS

The random walk assumption can be inadequate to explain trading rule returns which are often significantly positive (See Table 2 3 for references) It means that plausible alternatives of returns models might have to include dependencies

This section provides tests of adequacy of Gausssian processes which are assumed without drift If a process includes a drift, it must be removed using the method described in Section 5 2 2

5 3 1 *Methodology*

LeBaron(1991, 1992b) has proposed an original way to incorporate the trading rule diagnostic tests into the estimation procedure The goal is to see whether a simple linear

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6 Distinguishing nonlinear alternatives for which $E[X_t/X_{t-1} \quad X_{t-m+1}] = 0$ is known in the literature as a difficult task For instance it is often impossible to distinguish between Garch and stable processes (De Vries, 1991, Elie et al 1992)

7 An application of the multivariate T-Student to exchange rates series is provided in Section 6 2

model does a good job of replicating some simple linear properties of the data (autocovariances) as well as the trading rule results. The goal does not intend to get the tightest estimates of the parameters on the model. The actual data are aligned to simulated data using the mean, variance, the first three lagged autocovariances and one trading rule moment. For the trading rule moment condition the 20, 30 and 50 weeks moving average are used (LeBaron, 1992b). Finding difficult to derive analytic results for trading rule measures, LeBaron(1991, 1992b) estimates parameters via the simulated method of moments (Lee and Ingram, 1991) which is a derivation of the generalised method of moments (Hansen, 1982). LeBaron(1991, 1992b) first guess for trading rule related moment is $E(R_t) = E(B_{t-1}X_t)$. He fears that this will not do for simulated method of moments since the first derivatives of this moment will not necessarily be continuous in the parameters of the process X_t. Then the condition is replaced with a smooth substitute, we now know from equation [3.18] is unnecessary.

Our approach will be however different. We will not try to estimate the parameters of the model via rule returns, but will assume them known and instead check their ability to replicate observed rule returns.

There are two reasons for doing so. Firstly, if the underlying time series is linear, the forecaster which minimises the mean squared error will maximise profits and so Box and Jenkins procedure must give the best estimates of the parameters of the model. Secondly, in presence of nonlinearities the superiority of the generalised method of moments estimates beyond Box-Jenkins estimates has not been proved. So it is not sure that the use of simulated method of moments will provide tightest estimates of the parameters on the model.

Subsequently, the parameters of the linear model are supposed to be known, and their ability to explain a set of trading rule returns tested. Checking the adequacy of a model via rule returns is crucial since a model can appear very little misspecified in terms of error measures and be in fact badly misspecified in terms of profitability (Chapter 4). We now describe our trading rule diagnostic tests[8].

5.3.2 *Chi-square test*

We will consider here a set of s linear trading rules, exhibiting returns $\{R_j, j=1,s\}$. We will note $R_{j,t}$, t=1,...,N a finite realisation of $\{R_j\}$. R_t designs the s×1 vector stochastic

[8] An application of the diagnostic tests to exchange rates series is provided in Section 7.2.

process, $E(R_t)$ its expected value and Ω its sxs covariance matrix As shown in Hansen(1982), if the moment condition framework is satisfied

$$N(R_t - E(R_t))' W_N (R_t - E(R_t)) \xrightarrow{D} \chi^2(s)$$

where W_N is a consistent estimate of $W = \Omega^{-1}$

When the underlying process is Gaussian without drift, $E(R_t)$ is analytically known and given by equation [3 10] It can be noted that it satisfies the moment conditions However, $W = \Omega^{-1}$ is not known, but can be replaced by a consistent estimate for Ω such as

$$\hat{\Omega} = \sum_{i=-p+1}^{p-1} \frac{1}{N} \sum_{t=i+1}^{N} u_{t+p} u'_{t+p-i}$$

where $u_{t+p} = R_{t+p} - \frac{1}{N}\sum_{t=1}^{N} R_{t+p}$ and p is the number of population autocovariances determined by the order of non-zero autocorrelations of R_t Instead using empirical or simulated estimates of Ω, and so being dependent on the estimate covariance, we prefer using the exact one-period covariance matrix defined in Appendix 5 2 It results that under the null hypothesis of low positive autocorrelations, the multi-period covariance of rule returns might be slightly underestimated and therefore the test might have a slight tendency to reject the null hypothesis of positive dependences more often than necessary

5 3 2 *T-Student test*

Autocorrelated stochastic models have a tendency to underestimate trading rule returns (LeBaron, 1992b) Therefore it is natural to use a one-tail statistic to test if observed trading rule returns are equal to theirs expected value or still above them The multivariate T-Student previously established can be used to this effect The major feature of the T-Student test opposite to the previous approach is that, it is an one-tail test and by consequence is more powerful for given alternative such as low positive autocorrelations As before, the covariance of rule returns will be approximated via the exact one-period covariance matrix defined in Appendix 5 2 Once again, this test might have a slight tendency to reject the null hypothesis of positive low autocorrelations more often than necessary Let us recall the T-Student, it is given by

$$T = \frac{\frac{1}{N}\sum_{t=1}^{N} R_t - E(R)}{\sqrt{Var(R)}}$$

where R_t is the observed portfolio rule return at time t, E(R) is deduced from equation [3 10], and Var(R) from equation [3 6]

5.4 SUMMARY

Correlations between trading rules applied to a same asset are non-zero, and even highly positive for trend-following systems Correlations between a same trading rule applied to multiassets are positive but lower in absolute value than underlying correlations In addition, one-period correlations between rule returns do not change drastically assuming the presence of low dependencies[9]

The knowledge of trading rule correlations has then allowed to build a new test of random walk from the joint profitability of technical trading rules The test is a generalisation of the univariate T-Student which appears to be extremely powerful against linear autocorrelated alternatives, efficient against mean non-linear alternatives and not at all against variance non-linear models It has been shown that non-zero profits tests can be seen as tests of market timing ability if and only if the financial time series to which they are applied are without drift

Finally, it has been seen that trading rule returns can be used in a similar way to check the ability of any Gaussian processes without drift to replicate observed rule returns

[9] See Appendix 5 2

APPENDIX 5.1

PROOFS OF PROPOSITIONS

Proposition 5 1

By assumption, $X_{1,t}$ and $X_{2,t}$ are normally distributed with

$E(X_{1,t}) = 0 \quad E(X_{2,t}) = 0 \quad Var(X_{1,t}) = \sigma_1^2 \quad Var(X_{2,t}) = \sigma_2^2$

That implies that $F_{1,t}$ and $F_{2,t}$ are normally distributed with

$$E(F_{1,t}) = 0 \quad E(F_{2,t}) = 0 \quad Var(F_{1,t}) = \sigma_1^2 \sum_{i=0}^{m_1-2} d_{1,i}^2 \quad Var(F_{2,t}) = \sigma_2^2 \sum_{i=0}^{m_2-2} d_{2,i}^2$$

$$Cov(F_{1,t},F_{2,t}) = E(F_{1,t}\,F_{2,t}) = \sigma_1\sigma_2\rho_x \sum_{i=0}^{Min(m_1,m_2)-2} d_{1,i}d_{2,i}$$

$$\Rightarrow \qquad Corr(F_{1,t},F_{2,t}) = \rho_{F_{12}} = \rho_x\,\rho_F \text{ where } \rho_F = \frac{\sum_{i=0}^{Min(m_1,m_2)-2} d_{1,i}d_{2,i}}{\sqrt{\sum_{i=0}^{m_1-2} d_{1,i}^2}\sqrt{\sum_{i=0}^{m_2-2} d_{2,i}^2}} \qquad [5\ 4]$$

$E(B_{1,t}) = Pr(F_{1,t}>0)-Pr(F_{1,t}<0) = 1-2\,Pr(F_{1,t}<0) = 0$

That is due to the fact that the distribution of the linear unbiased forecaster, $F_{1,t}$, is symmetrical around zero, as for the underlying returns X_t Then, it follows that

Similarly, $E(B_{2,t}) = 0$

$E(B_{1,t}^2) = E(B_{2,t}^2) = 1$

$\Rightarrow Var(B_{1,t}) = Var(B_{2,t}) = 1$

$$\begin{aligned}\Rightarrow \qquad \rho(B_{1,t},B_{2,t}) &= Cov(B_{1,t},B_{2,t}) = E(B_{1,t}\,B_{2,t})\\ &= Pr(F_{1,t}>0,F_{2,t}>0)+Pr(F_{1,t}<0,F_{2,t}<0)-Pr(F_{1,t}>0,F_{2,t}<0)-Pr(F_{1,t}<0,F_{2,t}>0)\\ &= 2\,\{Pr(F_{1,t}>0,F_{2,t}>0)-Pr(F_{1,t}>0,F_{2,t}<0)\} \text{ by symmetry reason}\\ &= 2\,\{[0,0](\rho_{F_{12}})-[0,0](-\rho_{F_{12}})\}\end{aligned}$$

where $\rho_{F_{12}}$ has just be defined, and [0,0] is the bivariate truncated probability given by [A 1] in Appendix 3 1 It follows that

$$\rho_B = \rho(B_{1\,t},B_{2\,t}) = \frac{2}{\pi}\operatorname{Arc\sin}(\rho_X\rho_F) \qquad [5\ 6]$$

The demonstration which gives $Corr(B_{1\,t},B_{2\,t+h})$ is totally similar to the preceding one and won't be provided for length purpose

Proposition 5 2

Proposition 3 1 has shown that if the underlying time series X_t are independent identically distributed following a normal law without drift and variance σ^2, linear rule returns R_t are independent identically distributed following a normal law without drift and variance σ^2

Applying this result, it follows that rule returns $R_{1,t}$ and $R_{2,t}$ are normally distributed with

$E(R_{1,t}) = 0 \qquad E(R_{2,t}) = 0 \qquad$ and $\qquad Cov(R_{1\,t},R_{1,t+h}) = 0$

$Var(R_{1,t}) = \sigma_1^2 \qquad Var(R_{2,t}) = \sigma_2^2$ and $\quad Cov(R_{2,t},R_{2,t+h}) = 0$

Covariances between trading rules are deduced from

$$Cov(R_{1\,t},R_{2\,t}) = E(R_{1,t}R_{2\,t}) = E(B_{1\,t-1}B_{2\,t-1}X_{1\,t}X_{2\,t}) = E(B_{1\,t-1}B_{2,t-1})E(X_{1\,t}X_{2\,t})$$

Applying equation [5 6]

$$E(B_{1\,t-1}B_{2,t-1}) = \rho(B_{1\,t-1},B_{2,t-1}) = \frac{2}{\pi}\operatorname{Arc\sin}(\rho_X\rho_F)$$

Since by assumption $E(X_{1,t}X_{2,t}) = \sigma_1\sigma_2\rho_X$, it follows that

$$Cov(R_{1\,t},R_{2\,t}) = \sigma_1\sigma_2\rho_X\frac{2}{\pi}\operatorname{Arc\sin}(\rho_X\rho_F), \text{ and then}$$

$$\rho(R_{1\,t},R_{2\,t}) = \frac{E(R_{1\,t}R_{2\,t})}{\sqrt{Var(R_{1\,t})Var(R_{2\,t})}} = \frac{\sigma_1\sigma_2\rho_X\frac{2}{\pi}\operatorname{Arc\sin}(\rho_X\rho_F)}{\sigma_1\sigma_2}$$

$$\Rightarrow \qquad \rho_R = \rho(R_{1,t},R_{2,t}) = \frac{2}{\pi}\rho_X\operatorname{Arc\sin}(\rho_X\rho_F) \qquad [5\ 3]$$

In addition, $\rho(R_{1,t},R_{2\,t+h})=\rho(R_{1\,t+h},R_{2\,t})=0$ for $h>0$ [5 5]

That can be shown considering that

$$Cov(R_{1,t},R_{2,t+h}) = E(B_{1,t-1}B_{2,t+h-1}X_{1,t}X_{2,t+h}) = E(B_{1\,t-1}B_{2,t+h-1}X_{1\,t})E(X_{2,t+h}) = 0$$

$$Cov(R_{1,t+h},R_{2\,t}) = E(B_{1\,t+h-1}B_{2\,t-1}X_{1\,t+h}X_{2\,t}) = E(B_{1,t+h-1}B_{2,t-1}X_{2,t})E(X_{1,t+h}) = 0$$

APPENDIX 5 2

ONE-PERIOD RULES CORRELATIONS ASSUMING A DRIFTLESS GAUSSIAN PROCESS

Formula [5 6] giving signal correlation under the univariate random walk assumption applies in fact for any univariate Gaussian processes without drift However the correlation between forecasters, ρ_F, is not any longer given by equation [5 4] but is easily established since $F_{1,t}, F_{2,t}$ still follow a centred binormal law Noting $\rho(h)$ the autocorrelations of order h of underlying returns X_t, ρ_F is now given by

$$\rho_F = \frac{\sum_{i=0}^{m_1-2}\sum_{j=0}^{m_2-2} d_{1,i} d_{2,j} \rho(|j-i|)}{\sqrt{\sum_{i=0}^{m_1-2}\sum_{j=0}^{m_1-2} d_{1,i} d_{1,j} \rho(|j-i|)}\sqrt{\sum_{i=0}^{m_2-2}\sum_{j=0}^{m_2-2} d_{2,i} d_{2,j} \rho(|j-i|)}}$$

The one-period rule returns correlations satisfy

$$\mathrm{Corr}(R_{1,t}, R_{2,t}) = \frac{E(R_{1,t}R_{2,t}) - E(R_{1,t})E(R_{2,t})}{\sqrt{\mathrm{Var}(R_{1,t})\mathrm{Var}(R_{2,t})}}$$

where $E(R_{1,t})$, $E(R_{2,t})$ are given by equation [3 10], and $\mathrm{Var}(R_{1,t})$, $\mathrm{Var}(R_{2,t})$ by equation [3 6]

Then $E(R_{1,t}R_{2,t}) = E(B_{1,t-1}X_t B_{2,t-1}X_t) = E(B_{1,t-1}B_{2,t-1}X^2_t)$

We can use here symmetry argument between X_t from one hand and $\{F_{1,t-1}, F_{2,t-1}\}$ on the other hand, then it follows that

$$\begin{aligned} E\{R_{1,t}R_{2,t}\} &= 2\sigma^2\{[2,0,0](\rho_{12},\rho_{13},\rho_{23}) - [2,0,0](\rho_{12},-\rho_{13},-\rho_{23}) \\ &\quad -[2,0,0](-\rho_{12},\rho_{13},-\rho_{23}) + [2,0,0](-\rho_{12},-\rho_{13},\rho_{23})\} \end{aligned}$$

where $\rho_{12} = \mathrm{Corr}(F_{1,t-1}, F_{2,t-1})$, $\rho_{13} = \mathrm{Corr}(F_{1,t-1}, X_t)$, $\rho_{23} = \mathrm{Corr}(F_{2,t-1}, X_t)$
and $[2,0,0](\rho_{12},\rho_{13},\rho_{23})$ is the moment of order two of a truncated trivariate standardised normal law given by equation [A 6] in Appendix 3 1

Let us give a numerical example of previous formulations In the presence of a price-trend model and low positive autocorrelations, one-period rule returns correlations (Table 5 7) become systematically superior to rule signals correlations (Table 5 8) and, as expected, both are slightly bigger than they would be under the random walk assumption (Table 5 2)

Table 5.7 Rule returns correlations assuming a price-trend model A=0 03, m_d=40

ρ	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
S(5)	1	678	482	351	883	803	586	428	745	451	322	238
S(10)		1	700	506	613	867	831	613	712	724	476	346
S(20)			1	715	437	618	863	845	522	748	728	495
S(40)				1	319	449	626	864	382	562	774	736
W(5)					1	734	533	389	637	405	291	216
W(10)						1	742	545	769	608	417	306
W(20)							1	749	621	782	610	431
W(40)								1	463	662	798	620
M(5)									1	500	354	261
M(10)										1	543	390
M(20)											1	573
M(40)												1

Table 5.8 Rules signals correlations assuming a price-trend model A=0 03, m_d=40

ρ	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
S(5)	1	678	481	350	883	803	586	428	745	450	321	237
S(10)		1	700	505	612	867	831	612	711	724	475	344
S(20)			1	714	437	618	863	845	521	748	727	494
S(40)				1	318	448	625	864	382	561	773	736
W(5)					1	734	533	389	637	404	290	215
W(10)						1	742	544	768	607	416	304
W(20)							1	749	621	782	609	430
W(40)								1	463	662	797	618
M(5)									1	499	352	260
M(10)										1	542	388
M(20)											1	572
M(40)												1

Then we have proceed to some simulations to assess the multi-period correlation between trading rule returns (Table 5 9) Over 10 years (2500 rates), the trading rules correlations increase significantly and show that the one-period correlation must be considered as a lower bound

Table 5.9 Rule returns correlations assuming a price-trend model A=0 03, m_d=40
Monte-Carlo simulations 2500 rates, 250 replica

ρ	S(5)	S(10)	S(20)	S(40)
S(5)	1	0 742	0 604	0 509
S(10)		1	0 790	0 669
S(20)			1	0 817
S(40)				1

Chapter 6

TESTING THE RANDOM WALK HYPOTHESIS: AN APPLICATION TO EXCHANGE RATES SERIES

If markets follow a random walk, price changes can not be predicted Current prices fully and correctly reflect all currently available information Consequently, no profitable dynamic strategy can be found If markets do not follow a random walk, price changes can be predicted There are market imperfections such as the existence of price trends and cycles, which can be profitably exploited by dynamic strategies Therefore, testing the random walk hypothesis is of crucial importance from an investor point of view This is done in this chapter for a set of exchange rates

Exchange rates are known in the literature to be one of the assets exhibiting the strongest trends Empirical evidence of this point are given by the profitability of path dependent strategies[1] Therefore the random walk hypothesis might not be adequate for exchange rates This chapter investigates this issue by applying in addition of standard statistical tests, the powerful and robust test based on the joint profitability of trading rules developed in Chapter 5

Section 6 1 describes the elementary properties of exchange rates returns Section 6 2 tests the non-zero profitability of trading rule returns applying the multivariate T-Student test established in Chapter 5 Normality and dependence of rule returns are basic assumptions of this parametric test They are consequently first tested When using the multivariate T-Student, the zero-profit hypothesis can be rejected because of departures from the random walk model due to unequal variance, intercorrelation or/and average rule returns Therefore, these stochastic properties of rule returns are compared with their theoretical values under the normal random walk without drift to detect the origin of departures, if any Finally, Section 6 3 assesses the validity of the normality assumption to test the non-zero profitability of trading rule returns A non-parametric test based on the bootstrap methodology is applied such that it does not depend any longer on the arguable assumption of normality Non parametric and parametric critical thresholds are subsequently compared Once again, the last section summarises and concludes our findings

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[1] See table 2 3 for references

6.1 BASIC STATISTICS

In this thesis, we have collected daily spot prices for five currencies against the Dollar German Mark [DEM], Japanese Yen [YEN], French Franc [FRF], Swiss Franc [CHF] and British Pound [GBP] for the period January 1982 through March 1992, or in total 2625 daily observations Our data source is Reuters Close rates are bid prices taken each day of the week (except on Saturday and Sunday) at 21h15 GMT (approximately Close of New-York market) A single time series is formed by considering the logarithmic return $X_t = Ln(P_t/P_{t-1})$, where P_t denotes the foreign currency price (DEM, YEN, FRF or CHF) of a unit of US dollar, but the US dollar price of a unit of GBP By default, the main results of this chapter are given for the full sample, from January 1982 to March 1992 Results are also provided for the five subperiods shown in Table 6 1

Table 6 1 Samples periods

Period	1	2	3	4	5	Full
Date	01/82-02/84	02/84-02/86	02/86-03/88	03/88-03/90	03/90-03/92	01/82-03/92
Observations	525	525	525	525	525	2625

6 1 1 *Summary statistics*

Table 6 2 contains descriptive statistics on the original time series of spot returns It must be emphasised that exchange rates against the dollar exhibit quite similar standard deviations The CHF displays the highest volatility and the YEN the lowest The difference is however less than 15% of the average volatility between currencies Exchange rates are approximately symmetric as the skewness statistics show There are more observations several standard deviations from the mean than predicted by normal distributions That can be seen from the high values of standard kurtosis which would have been equal to zero if the distributions were normal

Table 6.2 Summary statistics for the period 01/82-03/92

Variable	DEM LOG	YEN LOG	GBP LOG	FRF LOG	CHF LOG
Sample size	2625	2625	2625	2625	2625
Average	-1 18414E-4	-1 90271E-4	-3 95664E-5	-7 7496E-6	-6 79575E-5
Variance	5 07372E-5	4 72147E-5	5 16746E-5	4 977E-5	5 89678E-5
Standard deviation	7 123E-3	6 8713E-3	7 18851E-3	7 05478E-3	7 67905E-3
Minimum	-0 0414075	-0 0640262	-0 0347257	-0 03876	-0 0440831
Maximum	0 0348967	0 0415372	0 0458853	0 0587457	0 0354505
Skewness	-0 144114	-0 572466	0 139542	0 163043	-0 149454
Standard Kurtosis	2 12304	6 5769	2 74848	4 29448	1 49801

Table 6 3 gives mean and standard deviation for the five subperiods The mean daily drift for all currencies and subperiods is small and rather constant It averages near zero for the full period and in any cases is very low in comparison with the daily standard deviation or volatility This point will be of extreme importance in testing rule returns significance

On the other hand, the volatility is rather variable between currencies It is for the full sample equal to 0 687% for the YEN and to 0 768% for the CHF Volatility finds its peak in the second sub-period for DEM, GBP and FRF, and in the third sub-period for YEN and CHF

Table 6 3 Means and standard deviations

Currency	Period	01/82-02/84	02/84-02/86	02/86-03/88	03/88-03/90	03/90-03/92	01/82-03/92
DEM	Drift U	00044	- 00045	- 00053	00003	- 00007	- 00012
	Volatility S	0056	0079	0074	0067	0077	00712
YEN	Drift U	00013	- 00050	- 00064	00029	- 00026	- 00019
	Volatility S	0064	00557	0083	0068	0071	00687
GBP	Drift U	- 00061	00007	00038	- 00013	00010	- 00004
	Volatility S	0056	0090	00651	0071	0073	00719
FRF	Drift U	00080	- 00045	- 00033	00002	- 00006	- 00001
	Volatility S	0065	00768	0072	0065	0074	00705
CHF	Drift U	00044	- 00034	- 00057	00014	00000	- 00007
	Volatility S	0068	0080	0083	0074	0080	00768

6 1 2 *Normality*

Table 6 4 gives the results of the Kolmogorov-Smirnov test of normality (Siegel, 1956) It appears that the YEN is clearly non-normal at the 5% level, irrespective of the subperiod considered For the other exchange rates, normality is a more acceptable assumption for a short period of time but not any longer valid for the full sample, what has far more statistical significance In the latter case, departures from normality, namely leptokurtosis, are too big

Table 6.4 Normality tests

Kolmogorov-Smirnov Approximate significance level %					
Period	DEM	YEN	GBP	FRF	CHF
01/82-02/84	27	4*	11	2*	16
02/84-02/86	20	5E-3*	4E-1*	41	3*
02/86-03/88	3*	9E-3*	13	3*	6
03/88-03/90	10	2*	6	7	16
03/90-03/92	6	5	11	11	59
01/82-03/92 (K-S)	0* (2 227)	0* (3 373)	0* (2 460)	0* (2 422)	0* (2 059)

* significantly not normal at the critical level of 5%

6 1 3 *Non linearity*

The rejection of normality might be explained by the presence of nonlinearity in exchange rates Returns are leptokurtic Non-linearity tests have been applied in an attempt to determine the validity of such an assumption

A stationary time series Y_t can be written, in its very general form, as

$$Y_t = \mu + \sum_{i=-\infty}^{+\infty} b_i e_{t-i} + \sum_{i\,j=-\infty}^{\infty} b_{ij} e_{t-i} e_{t-j} + \sum_{i\,j\,k=-\infty}^{\infty} b_{ijk} e_{t-i} e_{t-j} e_{t-k} + \quad ,$$

where μ is the mean level of Y_t , and $\{e_t, -\infty < t < \infty\}$ is a strictly stationary process of independent and identically distributed random variables Y_t is nonlinear if any of the higher order coefficients, $\{b_{ij}\}, \{b_{ijk}\}$, is nonzero Therefore a test of linearity is equivalent to a test on no multiplicative terms $\{b_{ij}\}, \{b_{ijk}\}$, To investigate non-linearities in a partial realisation $\{Y_1, \quad , Y_n\}$, Tsay(1986) has proposed a statistic based on the following steps

(1) Regress Y_t on $\{1, Y_{t-1}, \quad , Y_{t-M}\}$ by least squares and obtain the residuals $\{\hat{e}_t\}$, for t+M+1, ,n The regression model will be denoted by

$$Y_t = W_t \Phi + e_t \qquad [6\ 1]$$

where $W_t = (1, Y_{t-1}, \quad , Y_{t-M})$ and $\Phi = (\Phi_0, \Phi_1, \quad , \Phi_M)^T$ with M being a prespecified integer, n the sample size, and the superscript T denoting the matrix transpose

(2) Regress the vector Z_t on $\{1, Y_{t-1}, \quad , Y_{t-M}\}$ and obtain the residual vector $\{\hat{X}_t\}$, for t=M+1, ,n Here the multivariate regression model is

$$Z_t = W_t H + X_t,$$

where Z_t is an $m = \frac{1}{2}M(M+1)$ dimensional vector defined by $Z_t^T = \text{vech}(U_t^T U_t)$, with $U_t = (Y_{t-1}, \quad , Y_{t-M})$ and vech denoting the half stacking vector In other words, Z_t^T is obtained from the symmetric matrix $U_t^T U_t$ by the usual column stacking operator but using only those elements on or below the main diagonal of each column

(3) Regress $\{\hat{e}_t\}$ on $\hat{X}_t$ and let $\hat{F}$ be the F ratio of the mean square of regression to the mean square error That is, fit

$$\hat{e}_t = \hat{X}_t \beta + \varepsilon_t \qquad (t = M+1, \quad , n) \qquad [6\ 2]$$

and define $\hat{F} = \{(\sum_{t=M+1}^{n} \hat{X}_t \hat{e}_t)(\sum_{t=M+1}^{n} \hat{X}_t^T \hat{X}_t)^{-1}(\sum_{t=M+1}^{n} \hat{X}_t^T \hat{e}_t)/m\} / \{\sum_{t=M+1}^{n} \hat{\varepsilon}_t^2 / (n-M-m-1)\}$ [6 3]

where $\hat{\varepsilon}_t$ is the least squares residual in equation [6 2]

Tsay(1986) shows that if Y_t is a stationary autoregressive process of order M and n is large, the statistic $\hat{F}$ defined in equation [6 3] follows approximately a F distribution with degrees of freedom $\frac{1}{2}M(M+1)$, $n - \frac{1}{2}M(M+3) - 1$

This procedure reduces to Keenan's(1986) if one replaces Z_t by $\hat{Y}_t^2$, where $\{\hat{Y}_t\}$ are the fitted values of equation [6 1]

An alternative approach to see whether linear time series models can be fitted to the data Y_t is attributable to McLeod-Li(1983) They consider the stationary ARMA(p,q) model which takes the form $Y_t = \mu + \sum_{j=1}^{p} a_j(Y_{t-j} - \mu) + \varepsilon_t - \sum_{i=1}^{q} b_i \varepsilon_{t-i}$

where μ is the mean level of Y_t, $\{\varepsilon_t\}$ is a zero mean strict white noise process and constants a_j, b_i Then to investigate non-linearities in time series data, they have proposed the statistic

$$Q = n(n+2)\sum_{k=1}^{m} \rho^2(k)/(n-k)$$

where $\rho^2(k) = \sum_{t=k+1}^{n} \hat{\varepsilon}_t^2 \hat{\varepsilon}_{t-k}^2 / \sum_{t=1}^{n} \hat{\varepsilon}_t^2$ (k=0,1, ,n-1) are the lag k autocorrelations of the squared residuals $\hat{\varepsilon}_t^2$ obtained after fitting an ARMA model to the data If the ε_t's are i i d then Q is asymptotically distributed as χ^2 with m df

The application of Keenan(1985), Tsay(1986) and McLeod-Li(1983) nonlinearity tests to our exchange rates series is given in Table 6 5

Table 6 5 Nonlinearity tests

Critical Threshold of Nonlinearity Tests %						
Period	Test	DEM	YEN	GBP	FRF	CHF
01/82-02/84	Keenan M=4	96	12	62	94	52
	Tsay M=4	37	15	5	48	17
	McLeod-Li m=20	0*	0*	0*	0*	0*
02/84-02/86	Keenan M=4	87	0*	52	91	75
	Tsay M=4	71	3*	33	59	15
	McLeod-Li m=20	0*	0*	0*	0*	0*
02/86-03/88	Keenan M=4	86	1*	87	78	60
	Tsay M=4	59	0*	10	3*	53
	McLeod-Li m=20	0*	0*	0*	0*	0*
03/88-03/90	Keenan M=4	73	0*	36	41	4*
	Tsay M=4	23	0*	33	28	17
	McLeod-Li m=20	0*	0*	0*	0*	0*
03/90-03/92	Keenan M=4	55	73	95	66	12
	Tsay M=4	70	41	63	89	53
	McLeod-Li m=20	0*	0*	0*	0*	0*
01/82-03/92	Keenan M=2	22	29	0*	15	22
	McLeod-Li m=20	0*	0*	0*	0*	0*

* significantly not linear at the critical level of 5%

The Keenan(1985) and Tsay(1986) tests do not provide strong evidence of nonlinearities in exchange rates, except perhaps for the YEN On the other hand, the McLeod-Li(1983) test strongly rejects the linearity assumption irrespective of the currency and period under consideration The problem with non-linearity tests is that they are always built to be powerful for a given alternative Since nonlinear alternatives are not unique and cannot be precisely described, it is not surprising that they often yield contradictory conclusions There is some evidence of nonlinearities in exchange rates (Hsieh, 1989) but they are not strong (Diebold and Nason, 1990) Nonlinear models are plausible alternatives although there does not exist a consensus in favour of any particular one

6 1 4 *Temporal dependence*

This section deals with the testing of correlation between daily returns As mentioned in introduction of this chapter, the presence or absence of correlation between data is of importance to build adequate modelization and tests of financial rates We will test the existence of serial correlations between returns using five different tests Correlogram, Portmanteau, Taylor, Runs and Spearman tests

Historically the two most commonly used techniques to investigate the presence of temporal dependence are the runs test and the examination of a correlogram, i e a set of serial correlation coefficients A relatively new approach due to Taylor(1980) has been established and seems more powerful to detect dependencies in returns Finally, Spearman's non parametric test will complete our set of tests It is known to be more powerful than the runs test

Serial correlation coefficients

It is usual in the study of time series to plot and examine the correlogram or autocorrelogram The correlogram is a plot of the sample serial correlation coefficients, ρ_k, at various lags, k, against k Each ρ_k is computed using the expression

$$\rho_k = \frac{\sum_{t=1}^{n-k}(X_t - \overline{X})(X_{t+k} - \overline{X})/(n-k)}{\sum_{t=1}^{n}(X_t - \overline{X})^2/n} \qquad [6\ 4]$$

In the analysis we computed ρ_k, for k up to and including 50 for each period If the returns constitute a sequence of serially independent identically normally distributed random variables (the null hypothesis), the ρ_k values are each normally distributed with a mean of zero and a standard deviation of approximately $1/\sqrt{n}$ Furthermore, under the null hypothesis, the ρ_k are mutually independent A test of serial independence thus involves the computation of

$$z_k = \frac{\rho_k}{1\ \sqrt{n}} \qquad \text{for } k=1,2,\ \ ,20$$

values of z_k outside the bounds delineated by the normal (e g 1 96 for 5% test) are regarded as significant

If a variable follows a random walk, Granger and Newbold(1986) have shown that absolute and squared values should follow too a random walk So in addition to tests on original values, tests on absolute values, and on squared values, have been carried and can be found in Appendix 6 1 The first order autocorrelation is positive for every currency It is significant for GBP, DEM, FRF, at the critical level of 5%, for CHF at the critical level of 10%, but not at all for YEN Overall there appear to be very few other consistent positive or negative correlation Table 6 6 gives a count of the number of significant ρ_k values over the entire period for each set of returns

Table 6.6 Number of significant autocorrelations

Number of significant Autocorrelations in 50 lags at the 5% level					
	DEM	JPY	GBP	FRF	CHF
Original	4	3	2	4	2
Absolute	22	37	36	21	21
Square	15	11	29	7	15

For the original series we see that the number of significant ρ_k values are almost exactly equal what one would expect (i e 5%) under the null hypothesis of no temporal dependence In sum there seems to be no clear evidence of any temporal dependence in any of the series The absolute and square value of the logarithmic series tell us another story The number of significant autocorrelations is significantly higher that one would expect under the random walk hypothesis It suggests that there must be a kind of dependence between returns although it may not be linear

Portmanteau and Taylor tests

In this section, we briefly review Portmanteau and Taylor's statistical tests

The majority of researchers have used the Portmanteau test to detect the presence of serial autocorrelations

$$Q_k = n\sum_{i=1}^{k}\rho_i^2$$

where the ρ_i are the sample autocorrelation coefficients of n daily returns and k is chosen subjectively (here 20) Under the null hypothesis, Q_k is asymptotically distributed as a χ_k^2

Taylor(1980) proposed many models of financial prices we have briefly described in Section 2 4 1 In order to test the null hypothesis of a random walk against the alternative hypothesis of a trend model, Taylor(1980) considered the test statistics T and U

$$T = \frac{\sum_{i=1}^{k}\phi^k\rho_k}{\sqrt{\sum_{i=1}^{k}\phi^{2k}/n}} \qquad U = \frac{\sum_{i=2}^{k}\phi^k\rho_k}{\sqrt{\sum_{i=2}^{k}\phi^{2k}/n}} \quad \text{with } 0<\phi<1$$

If the null hypothesis is true, each ρ_k is independently normally distributed with mean zero and variance 1/n The T and U statistics would be asymptotically distributed with mean zero and variance unity Taylor points out that previous researchers have used Q in testing for temporal dependence but notes that the technique has low power Under Taylor's alternative hypothesis the ρ_k are expected to be a sequence of monotonically decreasing positive values and has proposed test statistics T and U designed to be sensitive to the possibility of such an alternative hypothesis If errors are present in a time series they will have most influence on ρ_1 and thus Taylor decides to test series with U Experience suggests that suitable values of k and ϕ are 30 and 0 92 respectively

Taylor points out that the high variances of conventional autocorrelation coefficients are almost certainly caused by the non-constant conditional variance of the returns Therefore he suggests that returns are rescaled to possess a reasonably homogeneous variance To get reliable results, he advises to use the rescaled returns $y_t=x_t/a_t$ to calculate the coefficients T and U, now noted $T^*$and U^*, with τ=0 1 and a_t defined by

$$a_t = (1-\tau)a_{t-1} + \tau|x_{t-1}|$$

The first twenty returns are commonly used to calculate the initial value of a_t

$$a_{20} = \frac{1}{20}\sum_{t=1}^{20}|x_t|$$

Then for a series of n_1 returns, the coefficients are calculated from

$$\rho_\tau = \frac{\sum_{t=21}^{n_1-\tau}(y_t - \bar{y})(y_{t+\tau} - \bar{y})}{\sum_{t=21}^{n_1}(y_t - \bar{y})^2} \qquad \text{where } \bar{y} = \frac{\sum_{t=21}^{n_1-\tau} y_t}{n_1 - 20}$$

The term n in U^* and elsewhere now denotes the effective number of returns $n = n_1 - 20$ In this way the series y_t should have an approximately constant variance very near the expected value 1/n It is therefore recommended that returns are rescaled before calculating the autocorrelation coefficients

Results of the portmanteau and Taylor tests are given in Table 6 7

Table 6.7 Portmanteau and Taylor tests

Portmanteau and Taylor tests (Critical Threshold %)					
Currency	DEM	YEN	GBP	FRF	CHF
Chi-square Q(20)	23 56 (21)	16 97 (59)	32 20 (3)*	25 05 (16)	23 74 (21)
T	1 17 (12)	2 02 (2)*	2 35 (1)*	1 53 (6)	1 63 (5)
U	0 11 (46)	1 31 (10)	2 41 (8)	0 91 (18)	0 88 (19)
T*	3 26 (0)*	3 61 (0)*	4 56 (0)*	3 00 (0)*	3 86 (0)*
U*	2 18 (2)*	3 06 (0)*	4 43 (0)*	2 31 (1)*	3 11 (0)*

* significantly not random at the critical level of 5%

As can be seen from Table 6 7, only one adjusted Box-Pierce Q statistics is significant at the 5% level (GBP) All the U^* statistics from the rescaled returns are positive That means there is an excess of positive serial correlation coefficients Each one of these statistics are significant at the 10% level and similar to previous literature findings Taylor(1980) found for the spot series GBP/USD U^*=2 78 during the period 1974-1978, and Taylor(1986) T=0 91, T^*=6 56, U^*=5 29 during the period 1974-1982 In this study, therefore, all five series examined, and specially GBP, showed evidence of price trends consistent with the model proposed by Taylor(1980)

6 1 5 *Randomness Tests*

Parametric tests as Portmanteau and Taylor statistics have the advantage to be powerful under specified alternatives Their drawback is however to rely on the assumption made about the distribution of the returns and to be sensitive to the presence of outlying observations and errors in the data An alternative is consequently to use non-parametric tests which remove previous limitations but also are less powerful

The tests for randomness procedure we now study are all non-parametric and have been described in full details in Siegel(1956) The first two examine the number of runs in the data, and the third one establishes the rank correlation coefficient

Runs test

A runs test above and below the median counts the number of runs that are completely above or completely below the median The system ignores values equal to the median This procedure is particularly sensitive to trends in the data The classical runs test examines the sequence of returns Each return is classified into one of two categories chosen i e above the median and below or equal to the median

Let us note

n_1 = number of outcomes in the first category

n_2 = number of outcomes in the second category

$n = n_1 + n_2$

It can be shown that if n is large (greater than 20) the number of runs r is approximately normally distributed with mean μ_r and standard deviation σ_r given by

$$\mu_r = \frac{2n_1n_2}{n_1 + n_2} + 1 \qquad \sigma_r = \sqrt{\frac{2n_1n_2(2n_1n_2 - n_1n_2)}{(n_1 + n_2)^2(n_1 + n_2 - 1)}}$$

A test of temporal dependence is then to compute z_r where

$$z_r = \frac{(r - \mu_r)}{\sigma_r}$$

which under the null hypothesis of randomness follows the standard normal distribution,

$$z_r \sim N(0,1)$$

Up and Down Test

A runs test up and down counts the number of times the sequence rises or falls The number of rising and falling runs equals one more the turning points This procedure is most sensitive to sequences with relatively long-term cycles, in which the number of turning points is less than those in a random sequence It can be shown that if n is large (greater than 20) the number of up and downs r is approximately normally distributed with mean μ_r and standard deviation σ_r given by

$$\mu_r = \frac{(2n-1)}{3} \qquad \sigma_r = \sqrt{\frac{16n-29}{30}}$$

Spearman test

This non parametric test is commonly used to detect correlation between variables However it can serve to test the presence of trend if one variable is taken as the time index The Spearman rank correlation coefficient is equivalent to ranking each variable separately and calculating the usual (Pearson) correlation coefficient on the ranks

Results of the randomness tests as applied to our exchange rates series are provided in Table 6 8

Table 6 8 Randomness tests

Tests for Randomness					
Currency	DEM	YEN	GBP	FRF	CHF
Median = 0 runs	7 44E-6 (100)	0 47 (64)	-0 90 (37)	7 44E-6 (100)	-0 37 (71)
Up and down	-1 12 (26)	0 98 (33)	-1 89 (6)	-7 72 (99)	-1 58 (11)
Spearman Rank Correlation	- 026 (19)	003 (86)	038 (6)	- 038 (5)	- 018 (39)

Nones of the randomness statistics are significant at the critical level of 5% Following the up and down and Spearman tests, GBP is not random at the critical level of 10% FRF does not follow a random walk following the Spearman test at the critical level of 5%

6 1 6 *Summary of results*

Table 6 9 attempts to summarise temporal dependence results It says that only the GBP exhibits strong departures from the random walk hypothesis irrespective of the test at the 10% level For the other currencies, rejection of serial independence only occurs under Taylor's tests at the 5% critical level No one of the randomness statistics is significant at the 5% level, and only three are significant at the 10% level (two for GBP, and one for FRF)

Table 6.9 Summary of randomness tests

Rejection of Random Walk at the level alpha %					
Alpha	DEM	JPY	GBP	FRF	CHF
Portmanteau			10%		
Taylor T		5%	10%	10%	10%
U			10%		
T*	5%	5%	5%	5%	5%
U*	5%	5%	5%	5%	5%
Runs					
Up and Downs			10%		
Spearman			10%	10%	

6 2 T-STUDENT TEST

The random walk is now being tested using the multivariate T-Student test derived in Chapter 5 This statistic is an alternative way to test the existence of serial correlation in exchange rates of returns Its primary advantage beyond standard statistical tests is its power and robustness (See Section 5 2)

It must be known when applying the parametric T-Student test what are the possible causes of departures with the random walk hypothesis To do so, Section 6 2 1 discusses the normality and serial independence of rule returns which are two basic assumptions Then Sections 6 2 2 to 6 2 4 test the equality of variance, intercorrelation and average rule returns with their theoretical values

Proposition 5 1 assumes that financial prices are without drift That does not seem unrealistic for our exchange rates time series in regards of the sample means given in Table 6 2 Such hypothesis has been commonly assumed in the literature (Taylor, 1986, Engel and Hamilton, 1992, Lai and Pauly, 1992) and will be adopted here

The rules we are investigating are once again the simple, weighted moving averages and momentums Successive orders of rules, {5, 10, 20 and 40} have been chosen such that trading returns are almost equicorrelated under the random walk assumption (Table 5 2) In fact, there appears to be little need for concern about how parameters are selected in academics studies as long as they are not based on in-sample returns (Lukac and Brorsen, 1989)

6 2 1 *Preliminary results*

Distribution

Table 6 10 proves that for the full sample none of the trading rules follow a normal distribution, although for shorter periods (2 years) rejection of normality occur far less often In addition, it can be seen that rejection (acceptance) of the normality of rule returns occur simultaneously to the rejection (acceptance) of the normality of underlying returns Taylor(1986) argues that rule returns may have positive relative kurtosis due to the positive relative kurtosis of price changes We have checked as well that amounts of skewness and kurtosis of unrealised returns are identical and close to the ones of the underlying process Subsequently, it seems that the shapes of the distributions of unrealised rule returns and underlying returns are identical but not normal (Lukac and Brorsen, 1990)

Table 6 10 Normality tests of rule returns

Critical threshold % Kolmgorov-Smirnov test													
DEM													
Period	Underlying	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
1	27	49	42	36	40	47	52	36	35	34	36	37	44
2	20	25	36	35	15	31	41	30	30	41	38	18	25
3	3*	1*	2*	3*	0*	1*	1*	3*	1*	1*	2*	3*	1*
4	10	9	8	19	18	6	10	10	19	10	7	10	11
5	6	29	9	12	4*	20	17	10	11	20	14	16	14
full	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*
YEN													
Period	Underlying	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
1	4*	11	0*	11	10	13	9	13	13	14	14	12	13
2	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*
3	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*
4	2*	2*	0*	1*	3*	3*	3*	2*	2*	4*	1*	3*	2*
5	5	3*	3*	3*	8	2*	3*	3*	3*	3*	6	8	6
full	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*
GBP													
Period	Underlying	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
1	11	49	42	36	40	12	4*	10	4*	10	22	11	4*
2	0*	25	36	35	15	1*	1*	4*	1*	2*	1*	0*	1*
3	13	1*	2*	3*	0*	16	15	20	32	15	8	15	7
4	6	9	8	19	18	2*	2*	10	17	15	15	14	9
5	11	29	12	12	4*	7	13	16	16	9	5	14	14
full	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*
FRF													
Period	Underlying	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
1	2*	1*	1*	2*	1*	1*	2*	3*	2*	2*	1*	2*	2*
2	41	34	42	28	24	35	27	31	32	28	32	23	23
3	3*	2*	3*	1*	0*	2*	1*	1*	0*	3*	0*	2*	0*
4	7	3*	3*	4*	8	3*	5	3*	9	2*	4*	12	9
5	11	13	13	21	26	38	20	16	22	35	18	35	37
full	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*
CHF													
Period	Underlying	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
1	16	28	25	31	9	30	25	25	12	13	25	6	4*
2	3*	12	8	15	6	13	12	15	14	12	12	6	6
3	6	12	11	5	2*	5	8	11	4*	5	4*	4*	6
4	16	33	7	11	17	32	20	15	13	13	14	39	41
5	59	54	33	38	68	61	43	32	57	48	70	52	51
full	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*	0*

* significantly not normal at the critical threshold of 5%

Temporal dependence

Table 6 11 shows that rule returns display on average similar randomness to underlying returns for the runs, up and down and chi-square tests, and significantly less dependencies for the Taylor tests Except in a few isolated cases, rule returns exhibit very low autocorrelations and can be considered as independent An advantage of profits-based tests might be that although daily prices may be dependent, rule returns might still be independent, and so the T-Student might still be applied

Table 6.11 Tests for randomness of rule returns

Tests for Randomness (Critical threshold %)													
Tests\DEM	Underlying	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
Runs	0 (100)	-1 12 (26)	0 14 (89)	1 48 (14)	0 57 (57)	-0 55 (58)	0 69 (49)	0 53 (60)	0 69 (49)	-0 06 (95)	0 77 (44)	1 08 (28)	-0 26 (80)
Up and down	-1 12 (26)	-0 38 (70)	-0 43 (67)	-0 24 (81)	-1 13 (26)	1 07 (29)	-0 10 (92)	-1 59 (11)	-0 52 (60)	0 74 (46)	0 18 (86)	-0 80 (42)	-1 87 (6)
Q(20)	23 56 (21)	22 14 (28)	30 16 (5)	23 74 (21)	19 09 (45)	25 20 (25)	26 04 (13)	25 81 (14)	25 40 (15)	20 89 (34)	17 53 (55)	18 75 (47)	24 38 (18)
T	1 17 (12)	-0 60 (72 5)	-2 00 (98)	-1 75 (96)	0 15 (44)	- 78 (78)	-0 94 (83)	-1 75 (96)	-0 77 (78)	- 54 (71)	-0 74 (77)	-0 43 (67)	1 5 (7)
U	11 (46)	-0 74 (77)	-2 85 (100)	- 93 (82)	-0 65 (74)	- 87 (81)	-1 54 (94)	-2 11 (98)	-0 61 (73)	- 86 (81)	-0 88 (81)	-0 16 (56)	0 86 (19)
T^*	3 26 (0)*	-0 41 (66)	-1 74 (96)	- 80 (79)	1 51 (7)	- 45 (67)	-1 15 (88)	-1 70 (96)	49 (31)	- 14 (56)	-0 50 (69)	0 02 (49)	2 16 (2)*
U^*	2 18 (2)*	-0 80 (79)	-2 50 (99)	- 58 (72)	0 73 (23)	- 75 (77)	-1 51 (94)	-2 08 (98)	23 (41)	- 61 (73)	-0 96 (83)	-0 13 (55)	1 36 (9)
Tests\YEN	Underlying	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
Runs	0 47 (64)	-0 90 (37)	-0 61 (54)	-0 43 (67)	-0 38 (71)	-6E-3 (100)	-1 36 (17)	-0 69 (49)	0 37 (71)	-1 00 (32)	0 77 (44)	-0 40 (69)	0 81 (42)
Up and down	0 98 (33)	0 43 (67)	0 71 (48)	1 22 (22)	0 57 (57)	1 27 (20)	-0 83 (41)	1 08 (28)	1 13 (26)	0 05 (96)	1 50 (13)	1 03 (30)	1 03 (30)
Q(20)	16 97 (59)	20 77 (35)	35 39 (1)*	28 60 (7)	23 82 (20)	13 65 (80)	27 33 (10)	31 13 (4)*	26 38 (12)	27 24 (10)	36 59 (1)*	22 64 (25)	12 60 (86)
T	2 02 (2)*	74 (23)	92 (18)	1 86 (3)*	1 57 (6)	- 11 (55)	59 (28)	1 08 (14)	1 77 (4)*	1 39 (8)	1 70 (4)*	1 87 (3)*	- 77 (78)
U	1 31 (10)	70 (24)	39 (35)	1 99 (2)*	1 14 (13)	0 (50)	- 18 (57)	1 26 (10)	1 68 (5)	79 (21)	2 11 (2)*	1 71 (4)*	- 52 (70)
T^*	3 61 (0)*	65 (26)	1 49 (7)	1 84 (3)*	2 73 (0)*	- 28 (61)	69 (25)	86 (19)	2 36 (1)*	76 (22)	2 71 (0)*	2 98 (0)*	1 92 (3)*
U^*	3 06 (0)*	55 (29)	78 (22)	1 36 (9)	2 18 (2)*	- 27 (61)	- 14 (56)	51 (31)	2 09 (2)*	24 (41)	2 59 (1)	2 71 (0)*	2 12 (2)*

Table 6.11 (continued) Tests for randomness of rule returns

Tests for Randomness (Critical threshold %)													
Tests\GBP	Underlying	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
Runs	-0 90	-1 78	-1 63	1 16	-1 20	-0 99	-2 00	-0 96	0 41	-0 80	0 43	-1 51	-0 65
	(37)	(8)	(10)	(25)	(23)	(32)	(5)	(34)	(68)	(42)	(67)	(13)	(52)
Up and down	-1 89	-0 97	-1 91	0 74	-0 74	-0 60	-1 35	-2 56	-0 97	-2 00	-1 07	-1 49	-2 05
	(6)	(33)	(6)	(46)	(46)	(55)	(18)	(1)*	(33)	(5)	(29)	(14)	(4)*
Q(20)	2 20	33 67	28 32	23 74	19 09	25 86	42 33	28 31	37 39	22 75	22 56	30 94	31 93
	(3)*	(2)*	(8)*	(21)	(45)	(13)	(0)*	(8)	(1)*	(25)	(26)	(4)*	(3)*
T	2 35	-1 20	-1 52	-1 16	46	-1 02	-1 49	-2 01	-2 36	-1 43	-1 67	-2 69	- 99
	(1)*	(89)	(94)	(88)	(32)	(85)	(93)	(98)	(99)	(92)	(95)	(4)*	(84)
U	2 41	-2 36	-2 75	-1 73	- 29	-1 34	-3 14	-2 80	-2 41	-2 58	-2 44	-2 18	-2 20
	(1)*	(99)	(100)	(96)	(61)	(91)	(100)	(100)	(99)	(100)	(99)	(99)	(99)
T*	4 56	-1 00	- 63	- 34	1 41	-1 55	- 91	-1 12	- 72	- 69	- 25	1 95	34
	(0)*	(84)	(74)	(63)	(8)	(94)	(82)	(87)	(77)	(75)	(60)	(3)*	(37)
U*	4 43	-1 92	-1 93	-1 20	61	-2 06	-2 21	-2 35	-1 15	-1 81	-1 07	1 17	-1 00
	(0)*	(97)	(97)	(88)	(27)	(98)	(99)	(99)	(88)	(97)	(86)	(12)	(84)
Tests\FRF	Underlying	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
Runs	0 00	0 22	-0 06	1 16	0 65	0 02	1 71	-0 06	0 85	1 71	1 04	1 24	0 76
	(100)	(83)	(95)	(25)	(52)	(98)	(9)	(95)	(40)	(9)	(30)	(22)	(45)
Up and down	-7 72	-0 38	0 37	0 74	-0 24	0 04	0 27	-0 61	1 11	0 18	1 11	0 76	-0 76
	(99)	(70)	(71)	(46)	(81)	(97)	(79)	(54)	(27)	(86)	(27)	(45)	(45)
Q(20)	25 05	29 2	39 16	27 3	25 84	18 07	31 51	33 62	25 93	27 48	17 63	12 69	16 98
	(16)	(6)	(0)*	(10)	(13)	(52)	(4)*	(2)*	(13)	(9)	(55)	(85)	(59)
T	1 53	- 12	-1 87	-1 86	-1 01	- 94	-1 19	-1 97	-1 19	23	- 22	- 74	1 26
	(6)	(55)	(97)	(97)	(84)	(83)	(88)	(98)	(88)	(41)	(59)	(77)	(11)
U	91	22	-2 35	-1 31	-1 81	- 58	-1 43	-2 37	- 98	- 23	- 44	- 92	98
	(18)	(41)	(99)	(90)	(96)	(72)	(92)	(99)	(84)	(59)	(67)	(82)	(84)
T*	3 00	- 11	-1 62	- 87	15	- 42	-1 64	-1 69	07	25	17	- 83	1 93
	(0)*	(54)	(95)	(81)	(44)	(66)	(95)	(96)	(47)	(60)	(43)	(80)	(3)*
U*	2 31	- 05	-2 18	- 57	- 37	- 27	-1 67	-2 13	03	- 48	- 25	- 92	1 73
	(1)*	(52)	(99)	(72)	(64)	(61)	(95)	(98)	(49)	(68)	(60)	(82)	(4)*
Tests\CHF	Underlying	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
Runs	-0 37	-0 37	-0 06	0 45	0 14	-0 64	0 61	0 30	0 02	0 41	1 36	0 10	-0 45
	(71)	(71)	(95)	(65)	(89)	(52)	(54)	(77)	(98)	(68)	(17)	(92)	(65)
Up and down	-1 58	-0 38	-1 87	-1 03	-2 15	-1 31	-0 57	-1 03	-1 31	0 55	-0 38	-1 92	-1 59
	(11)	(70)	(6)	(30)	(3)*	(19)	(57)	(30)	(19)	(58)	(70)	(5)	(11)
Q(20)	23 74	14 35	23 28	35 69	31 90	13 31	12 47	24 65	28 60	23 81	17 65	16 59	15 70
	(21)	(76)	(23)	(1)*	(3)*	(82)	(86)	(17)	(7)	(20)	(55)	(62)	(68)
T	1 63	- 61	-1 87	- 17	- 04	- 43	-1 15	- 23	- 94	- 57	- 28	- 07	36
	(5)	(73)	(97)	(57)	(52)	(67)	(88)	(59)	(83)	(72)	(61)	(53)	(36)
U	88	-1 05	-2 35	19	- 64	- 68	-1 50	- 55	- 80	- 55	- 42	05	- 16
	(19)	(85)	(93)	(42)	(74)	(75)	(93)	(71)	(79)	(71)	(66)	(48)	(56)
T*	3 86	-1 03	-1 62	14	29	- 76	-1 79	- 47	- 23	- 47	- 24	20	55
	(0)*	(85)	(95)	(45)	(38)	(78)	(96)	(68)	(59)	(68)	(59)	(42)	(29)
U*	3 11	-1 47	-2 18	21	- 12	-1 05	-2 15	- 83	- 35	- 47	- 17	07	04
	(0)*	(93)	(99)	(42)	(55)	(85)	(98)	(80)	(64)	(68)	(57)	(47)	(49)

* significantly not random at the critical level of 5%

6 2 2 *Variance*

Table 6 12 indicates that the variance of rule returns is not significantly different from the variance of underlying returns That means that, on an unrealised rate of return basis, there is no rule riskier than others Every rule brings the same risk being equal to the underlying volatility Corrado and Lee(1992, Table 6) studying the time series properties of the S&P 500 similarly find that the standard deviation of the 0 5 percent filter rule returns is equal to the underlying volatility Such a result confirms the random walk assumption, or at least is not incompatible with As far as variances are concerned, it can be concluded that the random walk hypothesis is strongly accepted and can be considered as an excellent proxy of real trading rule variances[2]

Table 6.12 Tests of equality of variances between rules and underlying returns

Variance (E-5) of daily stochastic processes (Critical threshold % tests of equality of variances between rules and underlying returns)													
	Underlying	S(5)	S(10)	S(20)	S(40)	M(5)	M(10)	M(20)	M(40)	W(5)	W(10)	W(20)	W(40)
DEM	5 0912	5 0809 (94)	5 0784 (94)	5 0669 (89)	5 0878 (94)	5 0858 (98)	5 0858 (98)	5 0799 (96)	5 0877 (99)	5 0827 (97)	5 0821 (97)	5 0750 (93)	5 0793 (95)
YEN	4 7162	4 7165 (100)	4 7143 (99)	4 7173 (100)	4 7171 (100)	4 7173 (100)	4 7137 (99)	4 7176 (100)	4 7181 (100)	4 7181 (100)	4 7136 (99)	4 7118 (98)	4 7157 (100)
GBP	5 1924	5 1837 (96)	5 1827 (95)	5 1784 (90)	5 1859 (99)	5 1882 (99)	5 1808 (96)	5 1899 (100)	5 1846 (97)	5 1801 (95)	5 1864 (98)	5 1725 (92)	5 1746 (92)
FRF	5 0062	4 9906 (94)	4 9911 (94)	4 9788 (89)	4 9923 (94)	4 9939 (95)	4 9964 (96)	4 9890 (93)	4 9982 (97)	4 9918 (94)	4 9883 (92)	4 9861 (92)	4 9864 (92)
CHF	5 9226	5 9172 (98)	5 9098 (96)	5 9063 (94)	5 918 (98)	5 9147 (98)	5 9145 (98)	5 9147 (98)	5 9127 (97)	5 9137 (98)	5 9137 (97)	5 9132 (97)	5 909 (95)

We have seen in Section 5 2 3 that the standard deviation of a portfolio of systems, σ_R is given under the normal independent assumption without drift by $\sigma_R = K\sigma$, where K is a constant given by equation [5 13] and σ is the underlying volatility Subsequently, Table 6 13 tests the hypothesis $(\sigma_R / K)^2 = \sigma^2$ It shows that the variance of a portfolio of systems is still close to its expected value That would imply that the theoretical correlation between systems is quite a good substitute for empirical correlations, an issue that the next section investigates in more details

[2] That confirms that if not the mean, the shape of the distribution (variance kurtosis skewness) of unrealised returns is very much the same than the one of the underlying process That would not have been the case for realised returns (See Chapter 3)

Table 6.13 Tests of equality of variances between portfolio rules and underlying returns

Variance (E-5) / K^2 of daily stochastic processes (critical thresold % tests of equality of variances between portfolio rules and underlying returns)					
	Underlying	S(5 10 20 40)	W(5,10 20 40)	M(5 10 20 40)	SWM(5 10 20 40)
Constant K		0 81275	0 83724	0 73055	0 76226
DEM	5 0912	4 9714 (54)	5 2270 (50)	5 2204 (53)	5 2003 (59)
YEN	4 7162	4 7184 (99)	4 7509 (85)	4 8168 (59)	4 7933 (68)
GBP	5 1924	5 1788 (95)	5 2270 (46)	5 22037 (90)	5 2003 (64)
FRF	5 0062	5 1331 (52)	5 3310 (11)	5 29651 (15)	5 3262 (12)
CHF	5 9226	5 7751 (52)	5 7834 (55)	5 8628 (80)	5 8344 (70)

6 2 3 *Rules correlations*

We now check the adequacy of the random walk without drift in terms of trading rules correlations We consider as in Section 5 1 2, first the case where different rules are applied to the same financial time series, and second the case where the same rule is applied to different financial time series Then a comment is made about the general case where different rules are applied to different time series

Different rules applied to a same underlying process

Firstly, we shall test the adequacy of rule returns correlations with their expected values for a set of technical trading rules applied to the same underlying process

Table 6 14 shows that irrespective of the currency, trading rules correlations are relatively close to their expected values under the random walk without drift hypothesis, H_0 given by Table 5 2 They are in fact slightly higher, which would let give the impression that there are some low positive autocorrelations Then we have applied a test of equality of correlations (Johnson and Wichern, 1982) to measure how close are the observed trading rules correlations to their expected value under H_0 For the twelve trading rules, rejections of adequacy occur in less than 40% of cases for DEM, GBP, CHF, YEN but above 60% for FRF (Table 6 14) Therefore, it seems that the univariate random walk hypothesis is a fairly good assumption as far as rule returns correlations are concerned Overall, mechanical systems are highly positively correlated (Lukac, Brorsen and Irwin, 1988a, Brorsen and Boyd, 1990, Taylor, 1990b), but not more than would randomly be expected

Table 6 14 Correlations between rules applied to a same currency

ρ(DFM)	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
S(5)	1	685	463	315	903*	780*	616*	428	804*	451*	296	177
S(10)		1	658*	393*	652*	898*	817	547*	746*	716	418	216*
S(20)			1	658*	448*	597	835*	824	516	740*	736*	430
S(40)				1	300	386	543*	833	358	488*	776*	690
W(5)					1	749*	603*	408*	722*	447*	278	185
W(10)						1	756*	537	796*	636*	383	224
W(20)							1	709*	662*	771	576	327
W(40)								1	476*	638	801*	596*
M(5)									1	510*	341*	195
M(10)										1	542*	294
M(20)											1	507
M(40)												1

ρ(YEN)	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
S(5)	1	720*	488	314	893*	809	591	404	791*	481*	257	110*
S(10)		1	665	445	653*	882*	810	560*	772*	736*	399	173*
S(20)			1	654*	494*	625*	853	795*	554*	769*	656*	363*
S(40)				1	325	415	597	852	358	496	788*	653
W(5)					1	737	593*	409*	685*	493*	278	121*
W(10)						1	742	529	844*	622*	377	176*
W(20)							1	718	656*	796*	560	290*
W(40)								1	461	608*	801*	517*
M(5)									1	559*	338	147*
M(10)										1	516*	251*
M(20)											1	489
M(40)												1

ρ(GBP)	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
S(5)	1	674	448	355	894	791	579	408	803*	448	278	198
S(10)		1	694	434*	619	876*	843*	608	731*	730*	418	249
S(20)			1	674	402	604	850	843	536*	730	682	440
S(40)				1	312	403	569	813*	392*	477*	772*	726*
W(5)					1	730	525	368	699*	414*	244	193
W(10)						1	741	534	799*	610*	356	227
W(20)							1	750	668*	795*	540	341
W(40)								1	484*	652	757	549
M(5)									1	535*	335	224
M(10)										1	462	287
M(20)											1	558*
M(40)												1

ρ(FRF)	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
S(5)	1	719*	520*	364*	894*	810	627*	475*	804*	483*	336*	233*
S(10)		1	695	452	683*	903*	828	595	761*	734*	462*	271
S(20)			1	687	507*	639*	858	855*	546*	728	753*	468*
S(40)				1	356*	443	583	831*	395*	496	793*	706*
W(5)					1	773*	613*	467*	725*	455*	331*	224*
W(10)						1	765*	584*	794*	651*	426*	273
W(20)							1	749	66*	758	616*	374
W(40)								1	513*	641	800*	599*
M(5)									1	538*	376*	233
M(10)										1	543*	315
M(20)											1	552*
M(40)												1

ρ(CHF)	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)
S(5)	1	642*	469	311	900*	781*	573	388	767*	457*	250	129*
S(10)		1	688	422*	583	852	811	599	711	727*	440	194*
S(20)			1	628*	426	592	877*	838	498	731	675	342*
S(40)				1	267	398	560*	789*	352	474*	786*	675
W(5)					1	722	527	343	670	409	213	121
W(10)						1	713*	504	786*	594	366	195*
W(20)							1	749	599	776*	575	304*
W(40)								1	428	636	789*	467*
M(5)									1	487	296	140*
M(10)										1	494	250*
M(20)											1	506
M(40)												1

* significantly different to the expected correlation $\rho = \rho_0$ at the critical level of 5%

Same rules applied to different underlying processes

Secondly, we have tested the adequacy of rule returns correlations with their expected values for the same mechanical system applied to two different underlying processes It is clear from Table 6 15 that observed correlations between trading rules are far higher than that would randomly be expected Theoretical correlations are however better than <u>ex-ante</u> substitutes and closer to true results than underlying correlations In addition, they confirm two major properties of rules correlations established in Section 5 1 2, namely

(a) rules correlations are a positive function of the absolute value of underlying correlations and lower in absolute value than underlying correlations Let us take the example of GBP/USD and USD/CHF The two processes are negatively correlated, -0 76, however when the same moving average (10 or 20 days) is applied to each of the two currencies, rule correlations decrease substantially in absolute value to reach 0 53

(b) rules correlations are almost identical as long as the same system is applied to both assets That can be seen from Table 6 15, multicurrencies correlations between S(5), S(10), S(20), S(40) are quite close one from each other There is perhaps a very slight positive function of the order of the moving average The correlation between two rules of a given order applied to two assets does not depend on the order

These results imply on the one hand, that the bivariate random walk without drift is a practical assumption allowing properties of rules correlations, (a) and (b), to be given which are empirically confirmed but on the other hand, underestimating excessively observed correlations to be an acceptable substitute We have checked that is still more the case when different systems are applied to different currencies

Table 6.15 Correlations between rules applied to different currencies

Correlation	YEN-CHF	YEN-FRF	GBP-CHF	GBP FRF	CHF FRF	DEM-YEN	DEM-GBP	DEM-CHF	DEM FRF	YEN-GBP
ρ_x	0 68	0 66	-0 76	-0 77	0 89	0 67	-0 79	0 92	0 95	0 58
$\rho_0 = \frac{2}{\pi}\rho_x \text{Arcsin}(\rho_x)$	0 32	0 31	0 42	0 44	0 61	0 31	0 46	0 69	0 75	0 23
S(5)	0 44*	0 38*	0 50*	0 56*	0 69*	0 43*	0 52*	0 76*	0 86*	0 30*
S(10)	0 44*	0 43*	0 53*	0 60*	0 74*	0 41*	0 60*	0 77*	0 90*	0 34*
S(20)	0 41*	0 39*	0 53*	0 57*	0 74*	0 41*	0 57*	0 79*	0 89*	0 35*
S(40)	0 50*	0 46*	0 57*	0 59*	0 76*	0 47*	0 59*	0 78*	0 90*	0 39*

* significantly different to the expected correlation $\rho = \rho_0$ at the critical level of 5%

6 2 4 *Expected value*

Tables 6 16 and 6 17 show that rule returns and so single T-Student are heavily dependent on the rule being used, although interrelated It results that no clear conclusion about the currency randomness can be deduced from them

An alternative is to apply the multivariate T-Student developed in Chapter 5 We have just seen in Sections 6 2 3 and 6 2 2 that trading rule correlations and variances are close from their expected values under the random walk hypothesis So rejection of non-zero profits from the multivariate T-Student should not be due to irrelevant variances and correlations, but significant positive returns, what we want to test

The multivariate T-Student provided by Table 6 17 seems far more informative than any single T-Student since it exhibits a critical threshold close from the best of its component, unknown ex-ante It seems from the reduced portfolio S(5,10,20,40) that DEM, GBP and FRF do not follow a random walk without drift at the critical level of 1%

In addition of this elementary portfolio, we have tested the profitability of larger portfolios It is hoped that by enlarging the field of rules the most profitable ones (unknown ex-ante) will be included and that their presence in the portfolio will make the test more powerful despite the number of unprofitable rules Our biggest portfolio, SWM(2 to 100) includes three different popular technical rules, simple moving averages, weighted moving averages, momentums of orders 2 to 100 For large portfolios, all currencies (except Yen) do not follow a random walk without drift at the critical level of 1% The ranking of currencies in terms of decreasing profitability for the largest portfolio SWM(2 to 100) is FRF, DEM, GBP, CHF and YEN The YEN appears far less profitable than the other currencies

There is no clear ranking of trading rules A slight dominance of weighted moving averages over simple ones and momentums can be noted However results are too close to be really meaningful

Table 6 16 Yearly rule returns

Yearly Returns % of trading rules					
Yearly Returns %	DEM	YEN	GBP	FRF	CHF
S(5)	7 97	5 26	7 44	9 09	4 71
S(10)	8 95	6 14	7 44	9 27	8 53
S(20)	12 30	3 96	9 04	12 47	9 57
S(40)	4 53	4 20	6 16	8 38	4 82
S(5 10 20 40)	8 44	4 89	7 52	9 80	6 91
Underlying Volatility	11 26	10 86	11 37	11 15	12 14
Portfolio Volatility	7 44	7 18	7 51	7 37	8 02

Table 6.17 Cntical threshold of T-Student test

Cntical Threshold % of T-Student test					
Test	DEM	YEN	GBP	FRF	CHF
S(5)	1 3	6 2	1 9	0 5	11
S(10)	0 6	3 6	1 9	0 4	1 3
S(20)	0 0	12 4	0 6	0 0	0 6
S(40)	10 3	11 0	4 4	0 9	10 5
S(5 10,20 40)	0 2	4 0	0 5	0 0	1 3
W(2 to 50)	0 1	3 3	0 1	0 0	0 5
W(2 to 100)	0 1	1 7	0 2	0 0	0 3
S(2 to 50)	0 3	2 9	0 2	0 1	0 9
S(2 to 100)	0 3	0 6	0 4	0 1	0 3
M(2 to 50)	0 2	1 5	0 2	0 0	0 5
M(2 to 100)	0 2	0 2	0 9	0 1	0 2
SWM(2 to 50)	0 2	1 3	0 2	0 0	0 5
SWM(2 to 100)	0 1	0 5	0 3	0 0	0 2

6 3 BOOTSTRAP TEST

6 3 1 *Bootstrap methodology*

It could be argued the results reported in the preceding sections are of little value because the T-Student test assumes a normal, stationary and time independent rule returns distribution For our set of trading rules the time independence assumption seems very reasonable, but not that of normality The results indicate that there are several deviations from the normal distribution such as leptokurtosis, conditional heteroskedasticity and changing conditional means So it may be argued that the results based on single and multivariate T-Student tests may be biased An alternative is the bootstrap approach which assumes nothing about the distribution generating function Testing procedures based on bootstrap methodology to assess the significance of technical trading rules in financial market are not new and have been implemented by Brock, Lakonishok and LeBaron(1992), Levich and Thomas(1991), LeBaron(1991, 1992b) The simulation technique is now described and applied to the full sample of exchange rates similarly to Levich and Thomas(1991)

For each currency, we generate a new comparison series (a shuffled series), by making a random rearrangement of logarithmic returns in the original series By operating on the sequence of price returns, the starting and ending price levels of the new series are constrained to be exactly as their values in the original data And by randomly rearranging the original data, the new series is constrained to have identical distributional properties as the original series, but the time series properties have been scrambled with each path, by construction, drawn independently of the other notional paths The process of randomly shuffling the series of returns is repeated 2,500 times for each currency Each technical rule is then applied to each of the 2,500 and the profits measured The moving average rules will be used as in Levich and Thomas(1991), LeBaron(1991, 1992b), Brock, Lakonishok and LeBaron(1992) 5, 10, 20 and 40 days are fairly common lengths used by traders and have been previously considered in this thesis The bootstrap methodology should provide a good approximation of the rule return distribution under the null model of random walk with a drift The profits of the original series can then be compared to the profits from the randomly generated, shuffled series Comparisons will be done once again throughout variance, correlation and expected value of rule returns[3]

6 3 2 *Variance*

Variances of rule returns have been very little affected by the bootstrap methodology (Table 6 18) They are still not statistically different from their theoretical values under the normal random walk without drift The ratio standard deviation of rule returns/underlying volatility is constant and very close to its expected value which is 1 for an unique system and K=0 81275 for the portfolio of systems S(5,10,20,40) Levich and Thomas(1991 Tables 4A, 4B) testing the assumption of a random walk without drift, similarly exhibit rule returns variances extremely close to the volatilities of the underlying assets

Table 6.18 Rules variances from bootstrapped currencies

Variance (E-5) of daily stochastic processes, issued from 2 500 Bootstrapped simulations							
	Underlying	S(5)	S(10)	S(20)	S(40)	S(5 10 20 40)	K(5 10 20 40)
DEM	5 0912	4 9567	4 9359	4 8036	4 9421	3 2258	0 808
YEN	4 7162	4 4570	4 5759	4 6011	4 5732	2 9412	0 803
GBP	5 1924	5 2775	5 1446	5 0658	5 1792	3 4345	0 816
FRF	5 0062	4 8958	5 1446	4 8880	5 1129	3 2871	0 812
CHF	5 9226	5 5509	5 6991	5 8661	5 4248*	3 6588	0 805

* significantly different from expected value $(K\sigma)^2$ assuming a random walk, at the critical level of 5%

[3] We do not have attempted multicurrencies bootstrap That is each one of the simulations have been performed independently for each currency That has the advantage of giving independent results between currencies but prevents the study of trading rule correlations between currencies

6 3 3 *Rules Correlations*

Correlations of different systems applied to bootstrapped currencies remain close to their original values, and almost identical to their theoretical value under the normal random walk without drift (Table 6 19)

Table 6.19 Rules correlations from bootstrapped currencies

ρ(DEM)	S(5)	S(10)	S(20)	S(40)	ρ(YEN)	S(5)	S(10)	S(20)	S(40)
S(5)	1	691*	496	291	S(5)	1	652	443	295
S(10)		1	689	441	S(10)		1	661	439*
S(20)			1	652*	S(20)			1	675
S(40)				1	S(40)				1
ρ(GBP)	S(5)	S(10)	S(20)	S(40)	ρ(FRF)	S(5)	S(10)	S(20)	S(40)
S(5)	1	667	452	351	S(5)	1	659	467	319
S(10)		1	674	487	S(10)		1	687	465
S(20)			1	691	S(20)			1	676
S(40)				1	S(40)				1
ρ(CHF)	S(5)	S(10)	S(20)	S(40)					
S(5)	1	644	447	295					
S(10)		1	674	447					
S(20)			1	679					
S(40)				1					

* significantly different to the expected correlation $\rho = \rho_0$, at the critical level of 5%

6 3 4 *Expected value*

Summary statistics for the simulated rules returns are shown in Table 6 20 Five statistics are computed in these tables The first column refers to the conditional mean, the second to the median, and the three next ones to the quantiles of 1%, 5% and 10% In all cases, irrespective of the rule and currency, the average profit is not significantly different from zero as in Levich and Thomas(1991 Tables 4A and 4B) So this is very close to what would have been expected from a parametric random walk without drift (equation [3 7])

Table 6.20 Distribution of rule returns from bootstrapped currencies

DEM 2 500 Bootstrap replica Yearly returns %					
Test	average	median	quantile 1%	quantile 5%	quantile 10%
S(5)	0 118	0 157	-8 496 8 989	-6 280 6 891	-5 518 5 689
S(10)	-0 175	-0 110	-9 125 8 707	-7 169 6 930	-5 941 5 492
S(20)	-0 089	-0 057	-8 484 8 352	-6 978 6 531	-6 034 5 703
S(40)	-0 118	-0 095	-9 051 8 506	-7 154 6 663	-6 038 5 429
S(5,10 20 40)	-0 066	-0 019	-7 068 7 518	-5 412 5 386	-4 701 4 552

YEN 2,500 Bootstrap replica, Yearly returns %					
Test	average	median	quantile 1%	quantile 5%	quantile 10%
S(5)	0 057	-0 001	-8 543 8 777	-6 381 6 494	-5 420 5 412
S(10)	0 148	0 115	-8 722 8 733	-6 428 6 595	-5 289 5 624
S(20)	0 127	0 119	-8 471 8 700	-6 407 6 709	-5 195 5 645
S(40)	0 167	0 116	-8 728 8 891	-6 478 6 854	-5 186 5 740
S(5 10 20 40)	0 125	0 123	-7 016 7 115	-5 223 5 245	-4 246 4 534

GBP 2 500 Bootstrap replica, Yearly returns %					
Test	average	median	quantile 1%	quantile 5%	quantile 10%
S(5)	-0 133	-0 205	-8 535 8 690	-6 848 7 211	-5 958 5 987
S(10)	-0 292	-0 260	-9 423 8 387	-7 335 6 475	-6 200 5 439
S(20)	-0 336	-0 374	-8 555 8 341	-6 950 6 329	-5 985 5 435
S(40)	-0 356	-0 300	-9 125 8 202	-7 042 6 577	-6 247 5 556
S(5 10,20 40)	-0 279	-0 331	-7 431 6 842	-5 911 5 407	-5 070 4 591

FRF 2,500 Bootstrap replica Yearly returns %					
Test	average	median	quantile 1%	quantile 5%	quantile 10%
S(5)	-0 083	-0 066	-8 512 8 845	-6 799 6 591	-5 967 5 421
S(10)	-0 140	-0 074	-8 970 9 410	-7 194 6 601	-5 949 5 571
S(20)	-0 218	-0 240	-9 630 8 441	-6 955 6 615	-5 560 5 531
S(40)	-0 316	-0 317	-8 847 8 305	-6 887 6 503	-6 117 5 443
S(5,10 20,40)	-0 189	-0 236	-7 132 7 435	-5 519 5 552	-4 835 4 520

CHF 2 500 Bootstrap replica, Yearly returns %					
Test	average	median	quantile 1%	quantile 5%	quantile 10%
S(5)	0 107	0 071	-8 830 9 276	-7 195 7 202	-5 964 6 182
S(10)	-0 208	-0 179	-9 538 8 715	-7 6646 6 632	-6 195 5 661
S(20)	-0 196	-0 104	-10 499 9 386	-7 779 7 286	-6 378 5 907
S(40)	-0 326	-0 338	-9 597 9 752	-7 220 6 955	-6 220 5 739
S(5,10 20 40)	-0 156	-01 58	-7 216 7 454	-5 829 5 602	-5 062 4 645

Average profits are normally dıstrıbuted wıthout skewness or kurtosıs (Table 6 21)

Table 6.21 Normalıty test of bootstrapped returns

Crıtıcal threshold % Kolmogorov-Smırnov test of normalıty of bootstrapped returns					
Test	DEM	YEN	GBP	FRF	CHF
S(5)	69 2	94 0	24 5	99 3	23 8
S(10)	81 9	89 8	90 9	85 2	54 1
S(20)	98 9	62 0	24 4	93 4	98 7
S(40)	95 2	96 0	30 6	94 3	69 4
S(5 10,20 40)	68 3	94 4	30 6	94 3	69 4

Table 6 22 presents the results comparıng the actual serıes for the DEM, YEN, GBP, FRF, CHF wıth the 2,500 correspondıng sımulated random walks It ındıcates the rank of the rule returns for the actual serıes ın comparıson to the 2,500 randomly generated serıes

Table 6.22 Ranks of orıgınal returns ın bootstrapped returns

Orıgınal returns rank (2 500 Bootstrap replıca)					
Test	DEM	YEN	GBP	FRF	CHF
S(5)	2470	2355	2454	2490	2219
S(10)	2488	2414	2461	2487	2486
S(20)	2500	2183	2497	2500	2490
S(40)	2298	2227	2415	2490	2301
S(5,10 20,40)	2496	2413	2495	2500	2482

Table 6 23 gıves the crıtıcal threshold of the bootstrap test The null hypothesıs of a random walk wıth a drıft ıs rejected at the α percent level ıf returns obtaıned from the actual currency data are greater than the percent cutoff of the sımulated returns under the null model For ınstance, the crıtıcal threshold of the DEM sımple movıng average of order 5 ıs worth 1 2%, sınce over 2,500 sımulatıons 30 generated a mean return greater than that from the actual serıes and 2470 lower (Table 6 22)

Table 6.23 Crıtıcal threshold of Bootstrap test

Crıtıcal Threshold % of Bootstrap test					
Test	DEM	YEN	GBP	FRF	CHF
S(5)	1 2	5 8	1 8	0 3	11 2
S(10)	0 5	3 4	1 6	0 4	0 6
S(20)	0	12 7	0 1	0	0 4
S(40)	8 1	10 9	3 4	0 4	8 0
S(5 10 20 40)	0 2	3 5	0 2	0	0 7

The bootstrap approach has added two important findings from previous results in this chapter Firstly, this nonparametric test confirms that exchange rates are not random Trading rule returns are significantly different from the ones issued from bootstrapped random walk Holding unchanged the exchange rates distribution and so avoiding parametric assumptions such as the normal law, does not allow to explain any better rule returns Independent driftless variations, even if nonlinear are not able to produce significantly positive rule returns Indeed average returns are very close to zero and so to the results of a parametric driftless random walk Secondly, critical thresholds from the nonparametric bootstrap test (Table 6 23), are close to the ones issued from the parametric T-Student test (Table 6 17) The average difference is equal to 0 4% and the biggest difference to 1 7% Brock, Lakonishok and LeBaron(1992) criticise parametric tests as exhibiting dubious critical thresholds It seems that as far as rule returns are concerned, normal assumption is more than an acceptable proxy and that T-Student based tests are as powerful and robust as bootstrap based tests Such findings would confirm the Diebold and Nason(1990), LeBaron(1992b) view that nonlinearities of financial prices can be of little economic consequence This underlines that when attempting to explain rule returns, it is far more important to correctly model dependencies even if linear, than variance-nonlinearities The latter haven't got, on their own, the potential to generate non-zero profits

These results strongly suggest that the actual exchange rate series contained significant departures from serial independence that allowed technical trading rules to be profitable If the actual series had been generated randomly, our simulations suggest that average profits would be close to zero Gauged against these simulations, the actual path of exchange rates is seen to embody a significant degree of serial dependence

6 4 SUMMARY

Exchange rates are not derived from an identically distributed normal law They cannot reasonably be considered as linear as proved by various tests However, a purely nonlinear variance model is unlikely since there are some signs of significant positive serial correlations as shown by Taylor(1980) statistics

Table 6 24 summarises our findings about the adequacy of the normal random walk assumption with the statistical properties of trading rule returns The results present conflicting evidences

On the one hand, the distribution of rule returns is not normal That might be due to the fact that the underlying return distribution itself is not normal

On the other hand, the univariate random walk assumption is quite acceptable and provides a fairly good proxy of rules variance and correlations between different trading rules applied to a same financial time series The bivariate random walk is strongly rejected when considering correlations between rules applied to different time series

Finally, and perhaps more important from an investor's point of view, trading rule returns are not derived from a random walk time series because they are non-zero and even significantly positive The profitability of trend following rules strongly suggests some form of serial dependency in the data

Both parametric and nonparametric tests bring the same conclusion which happens to be that exchange rates are not random Nonlinearity in the distribution only, that is still assuming independent variations, cannot generate nonzero profits Assuming normal rather than exact currency distribution has very few economic consequences in terms of average rule profit and risk as proved by the bootstrap approach Therefore the hypothesis of normality is very weak in comparison with the independency assumption

Table 6 24 Summary of random walk tests

Adequacy of rule returns statistical properties with the driftless normal random walk	
Distribution	No
Variance	Yes
Univariate correlations	Yes
Bivariate correlations	No
Expected value	No

APPENDIX 6.1

SERIAL AUTOCORRELATIONS OF EXCHANGE RATES SERIES

Table 6.25 DEM Autocorrelations

DEM LOG				ABS(DEM LOG)				SQUARE(DEM LOG)			
Lag	Estimate	Lag	Estimate	Lag	Estimate	Lag	Estimate	Lag	Estimate	Lag	Estimate
1	04076*	2	-01277	1	05073*	2	07943*	1	05957*	2	09235*
3	01278	4	-00938	3	09784*	4	07954*	3	07277*	4	06549*
5	02503	6	-00319	5	07403*	6	08701*	5	04296*	6	10728*
7	00917	8	03234	7	06120*	8	10758*	7	02150	8	10311*
9	02447	10	-00257	9	05635*	10	10831*	9	02423	10	08176*
11	-00574	12	-00907	11	07115*	12	05195*	11	06866*	12	02508
13	01515	14	00545	13	08899*	14	07152*	13	07742*	14	06884*
15	04092*	16	-00667	15	06620*	16	01126	15	04219*	16	00626
17	-02675	18	-03573	17	04895*	18	06697*	17	04310*	18	04743*
19	-00918	20	01888	19	03330	20	08262*	19	02252	20	05969*
21	00406	22	02675	21	04333*	22	02901	21	02224	22	02116
23	-00572	24	03759	23	05085*	24	00611	23	02818	24	00489
25	-04042*	26	-04106*	25	05720*	26	03534	25	03425	26	03507
27	-00896	28	02345	27	03324	28	03867	27	01845	28	03589
29	01598	30	-03230	29	03130	30	01721	29	02045	30	01108
31	-00714	32	-01041	31	03768	32	-00387	31	03844	32	-00291
33	-01010	34	00045	33	-01795	34	00112	33	02021	34	00471
35	01495	36	00512	35	01829	36	02018	35	-00004	36	02829
37	00547	38	01691	37	04137	38	02961	37	01089	38	-00595
39	00723	40	00585	39	01341	40	02634	39	01045	40	01788
41	02057	42	-00424	41	02957	42	01714	41	01409	42	00792
43	03367	44	02086	43	00670	44	01599	43	00730	44	00203
45	-01874	46	-01436	45	03699	46	01226	45	01393	46	-00411
47	-02403	48	01076	47	02863	48	-00984	47	02116	48	02313
49	01411	50	02219	49	05899*	50	03975	49	04050	50	02735

* significantly different from zero at the critical level of 5%

Table 6.26 YEN Autocorrelations

YEN LOG				ABS(YEN LOG)				SQUARE(YEN LOG)			
Lag	Estimate	Lag	Estimate	Lag	Estimate	Lag	Estimate	Lag	Estimate	Lag	Estimate
1	00646	2	00355	1	13227*	2	14400*	1	07661*	2	18640*
3	01272	4	01158	3	12957*	4	07399*	3	06413*	4	03417
5	02706	6	00714	5	10879*	6	09235*	5	04897*	6	03581
7	00416	8	02236	7	06561*	8	07774*	7	02964	8	05230*
9	02763	10	04337*	9	07689*	10	07937*	9	01652	10	04335*
11	- 00327	12	01844	11	03641	12	05367*	11	01307	12	00945
13	01556	14	00980	13	05220*	14	07792*	13	01038	14	02774
15	03453	16	- 00945	15	05729*	16	05971*	15	01796	16	02058
17	- 01279	18	00557	17	05937*	18	06499*	17	02954	18	02290
19	- 00059	20	00151	19	06574*	20	05876*	19	02114	20	01882
21	00042	22	01056	21	04581*	22	04123	21	01879	22	00231
23	- 01405	24	03328	23	04530*	24	05863*	23	00725	24	02579
25	01574	26	01282	25	05742*	26	05702*	25	02640	26	07018*
27	02186	28	01636	27	06776*	28	06673*	27	03954	28	02503
29	00095	30	- 01610	29	05984*	30	04925*	29	03541	30	01611
31	02631	32	- 00043	31	06503*	32	05963*	31	04922*	32	02178
33	- 01111	34	02452	33	04046	34	05542*	33	01624	34	03692
35	- 02278	36	- 05833*	35	01302	36	04133	35	00047	36	02334
37	02000	38	01474	37	03311	38	01781	37	00059	38	- 00007
39	- 01722	40	00762	39	02968	40	03555	39	00403	40	00966
41	- 00784	42	01521	41	02834	42	05411*	41	00130	42	01362
43	02281	44	- 01240	43	02851	44	04649*	43	00009	44	02506
45	- 00768	46	- 04682*	45	09776*	46	06860*	45	15038*	46	05564*
47	- 00485	48	02254	47	07469*	48	00736	47	15257*	48	- 00658
49	01077	50	01160	49	06060*	50	03912	49	01974	50	01091

* significantly different from zero at the critical level of 5%

Table 6.27 GBP Autocorrelations

GBP LOG				ABS(GBP LOG)				SQUARE(GBP LOG)			
Lag	Estimate	Lag	Estimate	Lag	Estimate	Lag	Estimate	Lag	Estimate	Lag	Estimate
1	05312*	2	- 00061	1	09633*	2	06513*	1	07721*	2	06376*
3	- 01828	4	- 01534	3	08619*	4	10859*	3	06902*	4	14574*
5	02804	6	00639	5	11983*	6	10179*	5	08614*	6	11854*
7	00534	8	00845	7	05923*	8	10049*	7	04677*	8	08825*
9	01461	10	- 03637	9	08747*	10	13327*	9	04425*	10	10975*
11	- 01242	12	- 01922	11	08067*	12	06410*	11	11263*	12	05707*
13	01567	14	01129	13	07983*	14	07682*	13	05337*	14	06158*
15	06393*	16	00238	15	09705*	16	06548*	15	09276*	16	06373*
17	- 01593	18	- 01385	17	06546*	18	09515*	17	04204	18	08286*
19	00215	20	03121	19	04952*	20	08659*	19	04078	20	11332*
21	00274	22	02537	21	05018*	22	09061*	21	03512	22	07551*
23	- 01929	24	03947	23	07418*	24	02536	23	06820*	24	01270
25	- 03768	26	- 03262	25	08280*	26	06661*	25	06394*	26	05325*
27	01620	28	03683	27	04739*	28	07302*	27	02763	28	04838*
29	- 01025	30	- 01030	29	05373*	30	02705	29	07901*	30	00078
31	- 01085	32	00252	31	06724*	32	03196	31	08787*	32	02702
33	- 01647	34	01224	33	03915	34	03410	33	05838*	34	02071
35	- 00085	36	- 01282	35	06292*	36	05753*	35	04531*	36	08006*
37	03467	38	03852	37	05857*	38	04984*	37	03080	38	02879
39	00064	40	01979	39	02180	40	01862	39	02354	40	01667
41	- 00513	42	- 01266	41	04313	42	00431	41	03337	42	00452
43	00958	44	- 00302	43	03910	44	06440*	43	02948	44	03671
45	- 02367	46	00082	45	05434*	46	00249	45	03155	46	- 00137
47	- 01082	48	01497	47	03233	48	01860	47	02036	48	02346
49	01499	50	02449	49	08727*	50	04160	49	08177*	50	04442

* significantly different from zero at the critical level of 5%

Table 6 28 FRF Autocorrelations

FRF LOG				ABS(FRF LOG)				SQUARE(FRF LOG)			
Lag	Estimate	Lag	Estimate	Lag	Estimate	Lag	Estimate	Lag	Estimate	Lag	Estimate
1	04029*	2	00003	1	04391*	2	08127*	1	02361	2	04751*
3	01992	4	01196	3	11117*	4	08360*	3	05916*	4	04420*
5	02016	6	00618	5	09163*	6	06849*	5	05759*	6	04858*
7	02196	8	03444	7	05885*	8	09277*	7	01927	8	06326*
9	02887	10	00602	9	05783*	10	07700*	9	01935	10	03376
11	-01801	12	00074	11	07589*	12	03395	11	05696*	12	00303
13	01536	14	00359	13	06375*	14	06034*	13	03458	14	03024
15	04000*	16	-00682	15	05981*	16	01205	15	01948	16	00508
17	-01458	18	-03730	17	03751	18	05906*	17	01725	18	02382
19	-00512	20	02339	19	04975*	20	07508*	19	01323	20	03158
21	01509	22	02063	21	03213	22	04156	21	00591	22	01744
23	-00710	24	02272	23	03996	24	01166	23	00997	24	-00086
25	-04017*	26	-05014*	25	05138*	26	02944	25	01270	26	01239
27	-00543	28	01642	27	02636	28	04342*	27	00177	28	01696
29	00062	30	01454	29	02410	30	00279	29	00622	30	-00770
31	-00948	32	-00946	31	04662*	32	01075	31	02746	32	00383
33	-01199	34	00353	33	-00889	34	03159	33	-01250	34	02049
35	01041	36	00029	35	01233	36	03469	35	00939	36	02857
37	01302	38	00928	37	01397	38	04158	37	01190	38	00485
39	01316	40	01305	39	02866	40	00887	39	00801	40	00663
41	02918	42	-00406	41	01772	42	01959	41	-00636	42	00720
43	03238	44	02138	43	-00210	44	00444	43	00058	44	-01035
45	-02552	46	01238	45	02624	46	-00591	45	-00301	46	-01479
47	-01118	48	01613	47	02046	48	-00216	47	00211	48	00535
49	00763	50	01818	49	04634*	50	02798	49	01574	50	02429

* significantly different from zero at the critical level of 5%

Table 6.29 CHF Autocorrelations

CHF LOG				ABS(CHF LOG)				SQUARE(CHF LOG)			
Lag	Estimate	Lag	Estimate	Lag	Estimate	Lag	Estimate	Lag	Estimate	Lag	Estimate
1	03428	2	-01101	1	02471	2	05320*	1	03059	2	06183*
3	01849	4	-01577	3	08604*	4	04955*	3	07611*	4	05800*
5	02332	6	00672	5	05756*	6	06708*	5	03788	6	09572*
7	-00975	8	00710	7	07179*	8	09424*	7	03767	8	10332*
9	01688	10	00180	9	03566	10	08650*	9	01706	10	06453*
11	-00825	12	-00903	11	05330*	12	03775	11	07272*	12	01495
13	03049	14	01724	13	08244*	14	07456*	13	05873*	14	06706*
15	05747*	16	-00256	15	04846*	16	01114	15	03784	16	00647
17	-02693	18	01695	17	04543*	18	07222*	17	04560*	18	05190*
19	-02089	20	00048	19	01558	20	07059*	19	00925	20	04779*
21	01658	22	03696	21	04038*	22	03644	21	00855	22	03063
23	-00860	24	03931*	23	04734*	24	01550	23	02391	24	00177
25	-03850	26	-02018	25	07072*	26	00795	25	04439*	26	-00740
27	-00537	28	03039	27	01613	28	05463*	27	00231	28	04674*
29	02092	30	-02718	29	02930	30	02284	29	02421	30	00421
31	-01351	32	01563	31	04672	32	00405	31	04108	32	-01011
33	-01371	34	-00987	33	00692	34	00452	33	01756	34	-00163
35	01185	36	-02999	35	02009	36	00066	35	00262	36	01265
37	02524	38	03218	37	02347	38	02554	37	00724	38	-00123
39	01761	40	00147	39	02609	40	00737	39	02117	40	00273
41	00538	42	-00210	41	01546	42	00533	41	01439	42	-00604
43	03508	44	00897	43	-01116	44	01288	43	-00827	44	00359
45	-02989	46	-01260	45	04262*	46	01177	45	01898	46	-00367
47	-00037	48	-00635	47	02531	48	01493	47	03945	48	-00355
49	-00013	50	-00839	49	06213*	50	01061	49	04334*	50	02069

* significantly different from zero at the critical level of 5%

Chapter 7

ABILITY OF EXCHANGE RATES MODELS TO REPLICATE RULE RETURNS

Chapter 6 has shown that the random walk hypothesis is clearly inadequate to model exchange rates So alternative models have to be found The main concern of this chapter is to find models compatible with the observed trading rule returns, and relate trading rule returns with the statistical properties of the underlying series Our goal is to show that using stochastic modelling, it is possible to establish what are the parameters of the underlying process which generate if any non zero return from technical analysis

Section 7 1 proposes models of exchange rates widely used in Finance and supported by the evidence of Chapter 6 It has been shown that the assumption of normal distribution of underlying returns was weak when the assumption of independence was strong when attempting to explain observed rule returns Subsequently in what follows, the assumption of normality will be maintained but in any case the assumption of independent underlying returns removed More specifically, we will study the Box-Jenkins, price-trend models and fractional Gaussian process The originality of this chapter is to consider as well technical models Section 7 2 assesses and compares the in-sample ability of some linear autocorrelated alternatives to replicate observed trading rule returns This is a *crucial point* because the *main objective* of a *financial model* is *trading* Then the performances of the fractional Gaussian process are compared with those of linear models Thereafter, the relationships between volatility, magnitude of serial correlation coefficients and profits of technical trading rules are exhibited Section 7 3 establishes the performances of some strategies used to enhance returns from technical models Section 7 4 discusses the efficient market hypothesis Finally, Section 7 5 summarises and concludes our results

7 1 MODELS OF THE UNDERLYING PROCESS

Chapter 6 has proved that the random walk assumption is clearly inadequate to explain trading rule returns which are significantly positive It results that plausible alternatives of exchange rates models must include low positive dependencies The models presented in this section meet all this requirement

7 1 1 *Box and Jenkins*

Building empirical linear models is feasible through an iterative stages procedure for the model selection This procedure was proposed by Box and Jenkins(1976) Firstly, it is necessary to determine the degree of differencing (d) necessary to achieve stationarity Osborne(1959) shows that the first difference of the logarithmic price is an appropriate choice Then the variable under study is the logarithmic return $X_t = Ln(P_t / P_{t-1})$ Thereafter it is necessary to determine the order of the autoregressive process (p), and the order of the moving average process (q) Most models fitted to the data series of logarithmic returns have $p+q \leq 2$ (Taylor, 1986 23), that is the convention which has been adopted here In addition, exchange rates models will be assumed to have no drift, an assumption supported by the results of Table 6 2 Secondly, the estimation of the parameters of the model is performed by solving the Yule-Walker equations Models results can be found in Appendix 7 1

7 1 2 *Price-Trend*

The price-trend process is another plausible alternative to model logarithmic returns, $X_t = Ln(P_t / P_{t-1})$ That can be seen from the large values of the Taylor statistics reported in Table 6 7 Estimates of the two trend parameters, A and p have been obtained using Taylor(1986, Section 7 3) estimates More precisely, estimates have been obtained by matching theoretical and observed autocorrelations of the underlying time series The following function defined for K autocorrelations has been considered

$$F(A, p) = n\sum_{i=1}^{K}(\rho_i - Ap^i)^2$$

with ρ_i the autocorrelations defined by equation [6 4] and n the number of returns used to calculate the ρ_i Similarly to Taylor(1986), we have used K=50 for spot series

To minimise F, mean trend durations $m_d = 1/(1-p) = 2, 3, 4, \ldots, 40$ are considered and for given m_d the best unconstrained A can be obtained using calculus For a fixed m_d and hence p, the function F is minimised by

$$A_m^* = \sum_{i=1}^{K} p^i \rho_i \Big/ \sum_{i=1}^{K} p^{2i}$$

Sometimes A_m^* is negative, so it is necessary to consider

$$\hat{A}_m = \begin{cases} A_m^* & \text{if} \quad A_m^* > 0 \\ 0 & \text{otherwise} \end{cases}$$

Let $S_m = F(\hat{A}_m, 1 - 1/m_d)$ for m_d=2, ,40 Minimising S_m over m_d gives the estimates $\hat{A}$, $\hat{p}$ minimising F, and hence $\hat{m}_d = 1/(1-\hat{p})$

As can be seen from Table 7 1, variance reduction is usually quite low (<0 026) except for the GBP Mean duration of trend varies between 2 days and 15 days More meaningful is the total sum of autocorrelations, Ap/(1-p) From biggest to lowest, the ranking is for the full sample FRF, YEN, CHF, DEM, GBP

Mean duration and variance reduction vary considerably from one period to the other and from one currency to the other There are two possible explanations for this fact Firstly, exchange rates might be non-stationary Secondly, the standard deviation of the parameter estimates is huge (Taylor, 1986)

Table 7 1 Price-trend models

DEM						
Parameters\Period	01/82-02/84	02/84-02/86	02/86-03/88	03/88-03/90	03/90-03/92	01/82-03/92
Drift U	0 00044	-0 00045	-0 00053	0 00003	-0 00007	-0 00012
Volatility S	0 0056	0 0079	0 0074	0 0067	0 0077	0 00712
Var-reduction A	0 0572	0 0882	0 00000	0 0117	0 0574	0 02067
Duration m_d	3	4	2	25	2	7
Trend AR(1) p	0 667	0 75	0 50	0 960	0 50	0 857
Ap/(1-p)	0 11	0 26	0	0 28	0 06	0 13
YEN						
Parameters\Period	01/82-02/84	02/84-02/86	02/86-03/88	03/88-03/90	03/90-03/92	01/82-03/92
Drift U	0 00013	-0 00050	-0 00064	0 00029	-0 00026	-0 00019
Volatility S	0 0064	0 00557	0 0083	0 0068	0 0071	0 00687
Var-reduction A	0 0599	0 15150	0 00000	0 00000	0 00000	0 01759
Duration m_d	8	5	2	2	2	15
Trend AR(1) p	0 875	0 80	0 50	0 50	0 50	0 933
Ap/(1-p)	0 42	0 61	0	0	0	0 24
GBP						
Parameters\Period	01/82-02/84	02/84-02/86	02/86-03/88	03/88-03/90	03/90-03/92	01/82-03/92
Drift U	-0 00061	0 00007	0 00038	-0 00013	0 00010	-0 00004
Volatility S	0 0056	0 0090	0 00651	0 0071	0 0073	0 00719
Var-reduction A	0 0350	0 1499	0 0023	0 0054	0 1137	0 07237
Duration m_d	2	2	40	14	2	2
Trend AR(1) p	0 50	0 50	0 975	0 929	0 50	0 50
Ap/(1-p)	0 036	0 15	0 09	0 07	0 11	0 072
FRF						
Parameters\Period	01/82-02/84	02/84-02/86	02/86-03/88	03/88-03/90	03/90-03/92	01/82-03/92
Drift U	0 00080	-0 00045	-0 00033	0 00002	-0 00006	-0 00001
Volatility S	0 0065	0 00768	0 0072	0 0065	0 0074	0 00705
Var-reduction A	0 0621	0 0964	0 00000	0 0104	0 0717	0 02567
Duration m_d	4	4	2	25	2	7
Trend AR(1) p	0 75	0 75	0 50	0 960	0 50	0 857
Ap/(1-p)	0 19	0 29	0	0 25	0 07	0 43
CHF						
Parameters\Period	01/82-02/84	02/84-02/86	02/86-03/88	03/88-03/90	03/90-03/92	01/82-03/92
Drift U	0 00044	-0 00034	-0 00057	0 00014	0 00000	-0 00007
Volatility S	0 0068	0 0080	0 0083	0 0074	0 0080	0 00768
Var-reduction A	0 0186	0 1173	0 0026	0 0051	0 0080	0 01281
Duration m_d	2	4	2	38	40	12
Trend AR(1) p	0 50	0 75	0 50	0 974	0 975	0 917
Ap/(1-p)	0 02	0 35	0 03	0 19	0 31	0 14

7 1 3 *Fractional Gaussian Process*

As outlined in Section 2 4 2, the fractional Gaussian process is another popular model of logarithmic returns This is supported by the data if the estimates of the parameter d is different from zero Many procedures have been proposed in the literature to estimate the parameter d (Geweke and Potter-Hudak, 1983, Kashyap and Eom, 1988) or $H = d + 0\ 5$, the most well known being the range scale estimate provided by Mandelbrot and Wallis(1969)

Geweke and Potter-Hudak(1983)[1] considers the problem of estimating the parameter d in the general integrated time series model Results are collected in their theorem 2 which suppose that $\{X_t\}$ is a general integrated linear process, with d<0 Let $I(\lambda_{j,T})$ denote the periodogram of $\{X_t\}$ at the harmonic frequencies $\lambda_{j,T} = \pi j / T$ in a sample of size T Let b_{1T} denote the ordinary least square estimate of β_1 in the regression $Ln\{I(\lambda_{j,T})\} = \beta_0 + \beta_1 Ln\{4\sin^2(\lambda_{j,T}/2)\} + \varepsilon_{j,T}$, j=1, ,n Then there exists a function g(T) such that if n=g(T) then $p\lim b_1 = -d$ If $\lim_{T\to\infty} (Ln(T))^2 / g(T) = 0$, then $(b_1 + d) / \{\hat{var}(b_1)\}^{1/2} \xrightarrow{D} N(0,1)$, where $\hat{var}(b_1)$ is the usual least squares estimate of $var(b_1)$ We have estimated the parameter d in what follows using the function $g(T) = T^{\alpha}$, with $\alpha = 0\ 6$

Kashyap and Eom(1988)[2] considers the long memory time series model $\{X_t\}$ defined by $X_t = (1-B)^{-d} \sigma W_t$, where $\{W_t\}$ is a white Gaussian noise sequence with zero mean and unit variance, σ a positive constant and B is a unit delay operator Then the estimates proposed in the paper are based on the following variables

$f_x(k/N)$ periodograms of sequence of $\{X_t\}$ in a sample of size N

$\alpha = \gamma - Ln(\sigma^2)$, where γ is the Euler constant $\gamma = 0\ 5772157$

$\theta = [d, \alpha]^T$, and $Z(k) = \begin{bmatrix} -2\ Ln|2\sin(\pi k / N)| \\ -1 \end{bmatrix}$

The linear least-squares estimate $\hat{\theta}$ of parameter θ is obtained by the following formula

$$\hat{\theta} = [\hat{d}, \hat{\alpha}]^T = \left[\sum_{k=1}^{N/2} Z(k) Z^T(k)\right]^{-1} \left[\sum_{k=1}^{N/2} Z(k) Ln(f_x(k/N))\right]$$

Mandelbrot and Wallis(1969) have suggested to detect long-range or "strong" dependence, the range over standard deviation or R/S statistic, also called the range scale, which was first developed by Hurst(1951) in his studies of river discharges The R/S statistic is the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation Specifically, consider a sample of returns $X_1, X_2, \ , X_n$ and let

1 thereafter noted GPH

2 thereafter noted KE

$\bar{X}_n$ denote the sample mean $(1\ n)\sum_{j=1}^{n} X_j$ Then the classical rescaled range statistic, denoted by Q_n, is defined as

$$Q_n = \frac{1}{S_n}\left[\operatorname*{Max}_{1\le k\le n}\sum_{j=1}^{k}(X_j - \bar{X}_n) - \operatorname*{Min}_{1\le k\le n}\sum_{j=1}^{k}(X_j - \bar{X}_n)\right] \qquad [7\ 1]$$

where S_n is the usual (maximum likelihood) standard deviation estimate

$$S_n = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(X_j - \bar{X}_n)^2}$$

The first term in brackets in equation [7 1] is the maximum (over k) of the partial sums of the first k deviations of X_j from the sample mean The second term in [7 1] is the minimum (over k) of this same sequence of partial sums The difference of the two quantities is called the range for obvious reasons and is therefore always non negative In order to compare different types of time series, this range is divided by the standard deviation of the original observations Then Hurst(1951) formulated the following relationship $Q_n = (a * n)^H$ where a is a constant and H the Hurst exponent

By taking the log of the range scale, we obtain

$$Ln(Q_n) = H\,Ln(n) + Ln(a) \qquad [7\ 2]$$

Finding the slope of the log/log graph of Q_n versus n will therefore give us an estimate of H This estimate of H makes no assumptions about the shape of the underlying distribution

In sum, the first step has been in the thesis to convert the prices series into logarithmic returns Our time series covers about 10 years of data which are converted into N = 2620 logarithmic returns Then we divide the series into N/n independent n-day increments Because these are non-overlapping n-days periods, they should be independent observations We can now calculate the rescaled range Q_n of each n-days Subsequently we obtain N/n separate Q_n observations By averaging the N/n observations, we obtain the Q_n estimate for the series with n days

We repeat this process for $n = 6, 7, \ \ , N/2$ The stability of the estimate can be expected to decrease as N increases, because we have fewer observations to average At this point we run a regression of $Ln(Q_n)$ versus $Ln(n)$ for the full range of n, taking the slope as the estimate of H, according to equation [7 2]

Estimation results from the three previous statistics are provided in Table 7 2 It must be remarked that the GPH estimate is rather inconsistent for small samples and consequently there are subperiods for which it exhibits strong departures with the other estimates KE and range scale For the full sample, all three procedures give an estimation of d slightly above 0 5 That would argue in favour of long term trends and positive autocorrelations The range scale estimate in particular clearly refutes the random walk hypothesis (H=0 5) for each one of the currencies (H>0 59)

Table 7.2 Fractional Gaussian processes

DEM						
Parameters\Period	01/82-02/84	02/84-02/86	02/86-03/88	03/88-03/90	03/90-03/92	01/82-03/92
D Fractional Exponent KE	0 038	0 098	0 029	0 048	0 018	0 028
GPH	0 185	0 136	0 015	0 075	0 185	0 088
H Hurst Exponent KE	0 538	0 598	0 529	0 548	0 518	0 528
GPH	0 685	0 636	0 515	0 575	0 685	0 588
Range Scale	0 595	0 609	0 567	0 656	0 561	0 603
Variance KE	2 962E-5	6 427E-5	5 691E-5	4 515E-5	6 013E-5	4 865E-5
YEN						
Parameters\Period	01/82-02/84	02/84-02/86	02/86-03/88	03/88-03/90	03/90-03/92	01/82-03/92
D Fractional Exponent KE	0 051	0 159	-0 002	0 002	-0 037	0 029
GPH	0 391	0 212	-0 1	0 019	0 045	0 083
H Hurst Exponent KE	0 551	0 659	0 498	0 502	0 463	0 529
GPH	0 891	0 712	0 4	0 519	0 545	0 583
Range Scale	0 650	0 645	0 579	0 616	0 605	0 618
Variance KE	3 884E-5	2 990E-5	6 983E-5	4 572E-5	4 895E-5	4 782E-5
GBP						
Parameters\Period	01/82-02/84	02/84-02/86	02/86-03/88	03/88-03/90	03/90-03/92	01/82-03/92
D Fractional Exponent KE	-0 001	0 138	0 005	0 029	0 074	0 037
GPH	-0 005	0 073	0 019	0 043	0 168	0 053
H Hurst Exponent KE	0 499	0 638	0 506	0 529	0 574	0 537
GPH	0 495	0 573	0 519	0 543	0 668	0 553
Range Scale	0 558	0 593	0 606	0 641	0 616	0 595
Variance KE	3 165E-5	7 744E-5	4 363E-5	4 466E-5	5 329E-5	5 097E-5
FRF						
Parameters\Period	01/82-02/84	02/84-02/86	02/86-03/88	03/88-03/90	03/90-03/92	01/82-03/92
D Fractional Exponent KE	0 076	0 123	-0 001	0 046	0 022	0 034
GPH	0 073	0 132	-0 004	0 085	- 089	0 072
H Hurst Exponent KE	0 576	0 623	0 499	0 546	0 522	0 534
GPH	0 573	0 632	0 496	0 585	0 411	0 572
Range Scale	0 619	0 613	0 595	0 647	0 574	0 607
Variance KE	3 948E-5	5 768E-5	5 470E-5	4 028E-5	5 525E-5	4 938E-5
CHF						
Parameters\Period	01/82-02/84	02/84-02/86	02/86-03/88	03/88-03/90	03/90-03/92	01/82-03/92
D Fractional Exponent KE	0 026	0 122	0 027	0 026	0 041	0 037
GPH	0 023	0 058	-0 052	0 017	0 17	0 026
H Hurst Exponent KE	0 526	0 622	0 527	0 526	0 541	0 537
GPH	0 523	0 558	0 448	0 517	0 67	0 526
Range Scale	0 561	0 608	0 561	0 678	0 585	0 589
Variance KE	4 580E-5	6 207E-5	6 617E-5	5 019E-5	6 311E-5	5 825E-5

7 1 4 *Technical models*

The originality of this chapter is to consider technical models as possible alternatives to the random walk hypothesis That is we assume here that the true model is without drift and such that the forecaster F_t which maximises profits is a linear technical rule as defined in Section 3 4 2

$$F_t = \delta + \sum_{j=0}^{m-2} d_j X_{t-j} \quad \text{where } \delta, \text{ m and } d_j \text{ are given constants}$$

We restrict in what follows our study to technical rules based on an unique parameter m We consider more specifically the simple moving average, weighted moving average and momentum rules For all these rules, δ is equal to zero and the coefficients d_j only depend on the parameter m as indicated in Table 3 3

Following the results of Section 4 1, the true model is defined by

$$X_t = \lambda F_{t-1} + \varepsilon_t = \lambda\left(\sum_{j=0}^{m-2} d_j X_{t-1-j}\right) + \varepsilon_t$$

where λ is a positive constant and ε_t white noise

Therefore the underlying model is a special case of AR(m-1) model If we assume that the order m-1 of the autoregressive model is given similarly to Box-Jenkins(1976) models, the autoregressive parameters, d_j , are known and linked one to each other The important feature of technical models is that for given m, the coefficients d_j need not to be estimated In sum, linear technical models are long range autoregressive models with imposed autoregressive parameters The only parameter to be estimated is the proportionality coefficient λ It can be estimated using simple regression

$$\hat{\lambda} = \frac{\hat{\rho}(X_t, F_{t-1})}{\sqrt{\sum_{j=0}^{m-2} d_j^2}}$$

where $\hat{\rho}(X_t, F_{t-1})$ is the common estimate of the correlation coefficient between the one-step ahead return and the predictor

It must be remarked that if the rule is the simple moving average of order 2, the model is nothing else than an AR(1) model and therefore estimates will be those given in Appendix 7 1 Table 7 3 provides estimates of the proportionality coefficient λ for a few technical models of exchange rates relatively to the full sample It must be emphasised that all the coefficients λ are positive which would argue in favour of low positive autocorrelations in exchange rates

Table 7.3 Technical models

Technical models with μ=0 σ[3]					
Estimates $\hat{\rho}(X_t, F_{t-1})$					
Rule	DEM	YEN	GBP	FRF	CHF
S(2) or AR(1)	0 041	0 007	0 053	0 040	0 034
S(5)	0 026	0 013	0 028	0 034	0 022
S(10)	0 029	0 028	0 024	0 037	0 022
S(20)	0 032	0 040	0 018	0 039	0 027
S(40)	0 022	0 039	0 019	0 027	0 024
W(5)	0 030	0 011	0 037	0 037	0 026
W(10)	0 028	0 022	0 027	0 035	0 023
W(20)	0 033	0 035	0 021	0 040	0 026
W(40)	0 028	0 040	0020	0 034	0 027
M(5)	0 016	0 017	0 009	0 023	0 012
M(10)	0 037	0 039	0 022	0 046	0 022
M(20)	0 017	0 037	0 014	0 024	0 020
M(40)	0 014	0 027	0 021	0 016	0 021
10E5*$\hat{\lambda}$					
Rule	DEM	YEN	GBP	FRF	CHF
S(2) or AR(1)	4094 27	698 41	5314 36	4029 23	3434 15
S(5)	466 87	243 48	518 12	618 08	404 08
S(10)	173 98	164 33	144 10	220 25	130 14
S(20)	64 59	79 77	37 19	77 49	54 48
S(40)	15 55	27 09	13 27	18 56	17 07
W(5)	245 79	92 79	307 82	305 00	211 11
W(10)	39 66	31 28	39 11	50 46	32 76
W(20)	8 27	8 88	5 23	9 97	6 44
W(40)	1 24	1 77	0 89	1 50	1 20
M(5)	775 16	848 72	465 73	1174 19	621 00
M(10)	1244 69	1291 11	747 41	1528 34	727 38
M(20)	400 17	842 01	327 84	555 43	449 78
M(40)	230 36	427 85	339 19	251 70	330 86

[3] The standard deviations of underlying returns have been constrained to be equal to those given in Table 6 2

7 2 ABILITY OF LINEAR MODELS TO REPLICATE RULE RETURNS

7 2 1 *Linear models*

The in-sample ability of the linear models just described in Section 7 1 to replicate observed rule returns is first assessed More precisely, the parametric Chi-square and T-Student tests developed in Chapter 5 are used to assess the ability of the

(a) Box-Jenkins (b) price-trend (c) technical

models to replicate the rule returns derived from an equally weighted portfolio of

(1) simple moving averages of order 5, 10, 20 and 40 S(5,10,20,40)

(2) weighted moving averages of order 5, 10, 20 and 40 W(5,10,20,40)

(3) momentums of order 5, 10, 20 and 40 M(5,10,20,40)

(4) all twelve rules just mentioned SWM(5,10,20,40)

The tests applied here have been described in full details in Chapter 5 They proceed in four steps

-) measuring the average observed returns $\overline{R}$ following a portfolio of trading rules (1), (2), (3) or (4)

-) estimating the linear process (a), (b) or (c) of the underlying logarithmic returns

-) establishing the expected return E(R) and variance Var(R) of the portfolio of trading rules under the assumption of the linear process

-) comparing observed and expected rule returns, and concluding on the ability of the linear process to replicate observed rule returns

The parametric Chi-square and T-Student tests are based on the one-period rule correlations instead of the multi-period rule correlations Therefore they are exact only for the random walk hypothesis and must be considered otherwise as approximations To measure the accuracy of these approximations, we have performed for the simple moving average rules and autocorrelated alternatives some Monte-Carlo simulations Samples of more than 2,525 rates corresponding to the number of observations of currencies rates were replicated 100 times

The ability of the Box and Jenkins and price-trend models to replicate rule returns is investigated in Tables 7 4 and 7 5 The case of technical models is then considered in Table 7 6 and 7 7 All linear models are finally compared in Table 7 8 In following discussions, when not explicit rejection or acceptance of a model occurs at the critical level of 5%

Finally, it must be said that another technique has been used in the literature to measure the ability of statistical models to replicate rule returns, the bootstrap methodology Examples of which are the autoregressive processes AR(1) (Brock, Lakonishok and

LeBaron, 1992, LeBaron, 1992b), AR(2) (LeBaron, 1991, 1992b) and autoregressive moving-average process ARMA(1,1) (LeBaron, 1992b) Having exhibited in Chapter 6 the similarities between the bootstrap and T-Student tests under the random walk assumption, it is not believed that the presence of low autocorrelations in exchange rates will cause now significant departures between the two tests Parametric tests being a lot more simpler to apply than the bootstrap methodology, they have been preferred in what follows[4]

Box and Jenkins and Price trend models

A first remark is that the use of the exact one-period rule correlations instead of the multi-period rule correlations affects very little the critical thresholds of the Chi-square test (Table 7 4) Critical thresholds from Monte-Carlo simulations are given in bracket for the simple moving average rules For instance, the adequacy of the AR(1) model for FRF implies critical thresholds equal to 6 6% for the parametric test and 6 0% for Monte-Carlo simulations Overall the two tests bring the same conclusions about the rejection or acceptance of the model on 18 cases of 20 at the critical level of 5%, and in all cases at the critical level of 10%

Table 7 4 clearly shows that the adequacy of a model can be rule-dependent using the Chi-square test For instance, the use of weighted or simple moving averages to check the adequacy of the RW for the GBP model brings opposite conclusions, namely rejection and acceptance of the RW So no clear conclusion can be deduced from such results It might be that bigger portfolios of rules should be used For the biggest portfolios of 12 rules, the random walk assumption is rejected only for the GBP It can be seen from the critical thresholds, that Box-Jenkins modelling of AR(1), AR(2), MA(1), MA(2), ARMA(1,1) are almost equivalent models, in any case better than the RW but worse than the price-trend model The problem with the Chi-square test is that it is a two-tail test which is unfortunately not powerful enough to detect the low positive autocorrelations we observed in Chapter 6 So one has to turn to the T-Student test

As for the Chi-square test, the use of the exact one-period rule correlations instead of the multi-period rule correlations affects very few the critical thresholds of the T-Student test (Table 7 5) For instance, the adequacy of the AR(1) model for FRF implies critical thresholds equal to 0 6% for the parametric test and 0 8% for Monte-Carlo simulations Overall the two tests bring the same conclusions about the rejection or acceptance of the

[4] It must be underlined that testing the adequacy of four rule returns for seven models and five currencies is an extremely demanding task using the bootstrap methodlogy, but straightforward using parametric tests (Section 5 2 3)

model on 18 cases of 20 at the critical level of 5%, and in all cases at the critical level of 10% Table 7 5 strongly rejects the random walk assumption for all currencies as Table 6 17 did for even bigger portfolio of systems Table 7 5 exhibits that Box-Jenkins modelizations of AR(1), AR(2), MA(1), MA(2) and ARMA(1,1) are slightly better than the random walk model but still not very satisfactory representations of exchange rates Only the price-trend model (Table 7 5) is not rejected irrespective of the rule or currency at the critical level of 5% (except when simple moving averages are applied to the FRF) It must be kept in mind that the variance used in the T-Student is slightly underestimated under low positive autocorrelations alternatives It is why exact critical thresholds should be slightly higher and so acceptances of the null hypothesis still more frequent

It can be concluded that autocorrelated alternatives explain better trading rule returns than the random walk Taylor and Tari(1989), Taylor(1986, 1990a, 1990b, 1992a) similarly demonstrate for exchange rates the superiority of the price-trend model beyond the random walk They exhibit in particular significant profits from statistical and technical forecasters Lai and Pauly(1992) find as well that bandwagon forecasting scheme can improve the forecasting accuracy in terms of both mean squared errors and market timing upon the random walk The bandwagon expectations hypothesis involve significant positive correlations between successive exchange rate changes Lai and Pauly(1992) illustrate that bandwagon expectations can be rational and more precisely that the AR(1) model describes the exchange rate dynamics better than a random walk Aczel and Josephy(1992) present a new method of identifying ARIMA time-series models They use the bootstrap technique in estimating the distribution of sample autocorrelations They find that the AR(1) model outperfomed the random walk model in the production of the one-step ahead forecasts for the Singapore dollar exchange rate LeBaron(1992d) does question the result that there is no nonlinear mean predictability He then proposes as a possible explanation consistent with his results that the exchange rate is following a slow moving average trend process

Among autocorrelated alternatives, the price trend-model appears to be the best both across currencies and trading rules LeBaron(1992b) equally finds that the price-trend model explains better moving average rule returns than AR(1) and AR(2) models Once again, our result emphasises the specificity of the profit criteria since the rejection of the random walk hypothesis was not strong using standard statistical tests (Section 6 1 4) Even if rule returns cannot provide tightest estimates of parameters, they certainly are useful to check the adequacy of a model because they are unrelated to most of the existing tests and specially the ones based on error measures

Table 7 4 Chi-square test of adequacy of statistical models

Chi-square test of adequacy of linear model μ=0 σ Critical Threshold %

(Critical Threshold % from Monte-Carlo simulations 100 replica)

Random Walk

Rule	DEM	YEN	GBP	FRF	CHF
S(5,10 20 40)	0 5	43 6	11 3	0 8	12 0
W(5,10,20 40)	6 7	27 6	0 4	2 2	17 2
M(5,10,20 40)	12 9	41 7	8 2	4 8	11 5
SWM(5 10 20 40)	9 7	72 6	2 4	22 3	12 7

AR(1)

Rule	DEM	YEN	GBP	FRF	CHF
S(5,10,20 40)	3 2 (1 8)	52 5 (52 4)	54 8 (54 4)	6 6 (6 0)	24 3 (26 5)
W(5 10,20,40)	30 1	33 1	5 1	17 6	47 5
M(5,10,20,40)	34 7	47 1	23 4	19 9	49 1
SMW(5 10 20 40)	30 4	76 9	13 1	59 7	39 3

AR(2)

Rule	DEM	YEN	GBP	FRF	CHF
S(5 10,20,40)	2 0 (0 9)	57 8 (59 0)	52 4 (51 4)	6 3 (5 7)	20 0 (21 1)
W(5,10 20 40)	22 0	37 0	4 7	16 7	38 9
M(5,10 20 40)	26 4	50 8	22 4	19 0	38 9
SMW(5 10 20 40)	20 3	79 3	12 5	58 5	35 0

MA(1)

Rule	DEM	YEN	GBP	FRF	CHF
S(5 10 20,40)	3 1 (1 5)	52 4 (52 1)	52 8 (53 6)	6 3 (5 4)	24 0 (22 1)
W(5,10,20,40)	29 5	33 0	4 8	16 8	47 0
M(5,10,20,40)	34 1	47 1	22 6	19 1	48 5
SWM(5 10 20 40)	24 9	76 9	12 7	58 6	39 1

MA(2)

Rule	DEM	YEN	GBP	FRF	CHF
S(5 10,20 40)	2 0 (1 1)	57 5 (56 4)	53 1 (52 9)	6 0 (4 2)	19 8 (21 2)
W(5 10 20 40)	22 0	36 8	4 8	16 1	38 5
M(5 10,20 40)	26 4	50 6	22 7	18 4	38 6
SMW(5 10 20 40)	20 3	79 2	12 6	57 7	34 8

ARMA(1 1) Model μ=0, Box and Jenkins Estimates

Rule	DEM	YEN	GBP	FRF	CHF
S(5,10 20,40)	3 2 (2 7)	54 2 (55 7)	54 7 (53 5)	6 7 (6 9)	24 3 (19 2)
W(5 10 20,40)	30 4	34 1	5 1	17 6	47 9
M(5,10,20 40)	35 0	48 1	23 4	19 9	49 5
SWM(5 10 20 40)	25 4	77 6	13 1	59 8	39 5

Price-trend Model μ=0, Taylor Estimates

Rule	DEM	YEN	GBP	FRF	CHF
S(5,10,20,40)	7 7 (3 3)	88 0 (88 4)	71 1 (64 1)	30 0 (29 3)	47 9 (49 4)
W(5,10,20,40)	60 8	83 4	14 2	60 5	69 6
M(5 10,20,40)	71 8	94 7	19 6	66 1	81 9
SMW(5 10 20 40)	39 4	93 8	12 7	87 3	50 9

Table 7 5 T-Student test of adequacy of statistical models

T-Student test of adequacy of linear model $\mu=0$ σ, Critical Threshold % (Critical Threshold % from Monte-Carlo simulations 100 replica)					
Random Walk					
Rule	DEM	YEN	GBP	FRF	CHF
S(5,10 20 40)	0 2	4 0	0 5	0 0	1 3
W(5,10 20,40)	0 3	2 1	0 1	0 0	1 2
M(5 10 20,40)	0 6	3 9	1 1	0 1	1 0
SWM(5 10 20 40)	0 2	2 6	0 3	0 0	0 9
AR(1)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5 10 20,40)	2 5 (2 6)	5 4 (4 8)	9 2 (8 4)	0 6 (0 8)	7 9 (7 5)
W(5,10 20,40)	4 6	3 2	6 2	1 3	9 3
M(5,10 20,40)	3 4	4 5	8 3	1 1	3 9
SMW(5 10 20 40)	2 9	3 7	7 0	0 7	6 1
AR(2)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5 10,20 40)	1 2 (0 6)	6 5 (6 4)	8 1 (8 2)	0 6 (0 5)	4 8 (4 6)
W(5 10 20 40)	2 4	4 0	5 3	1 2	5 7
M(5 10 20 40)	2 0	5 2	7 4	1 0	2 4
SMW(5 10 20 40)	1 4	4 5	6 1	0 7	3 6
MA(1)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5,10 20 40)	2 4 (3 2)	5 4 (5 7)	8 3 (8 3)	0 6 (0 3)	7 7 (7 3)
W(5,10 20 40)	4 5	3 2	5 5	1 2	9 1
M(5 10 20,40)	3 3	4 5	7 6	1 0	3 8
SWM(5 10 20 40)	2 8	3 7	12 7	0 7	5 9
MA(2)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5 10 20 40)	1 2 (1 1)	6 5 (6 6)	8 4 (8 0)	0 6 (0 4)	4 7 (4 7)
W(5 10 20 40)	2 4	4 0	5 6	1 2	5 6
M(5 10 20 40)	1 9	5 2	7 7	0 9	2 4
SMW(5 10 20 40)	1 4	4 5	6 3	0 6	3 5
ARMA(1 1) Model $\mu=0$ Box and Jenkins Estimates					
Rule	DEM	YEN	GBP	FRF	CHF
S(5,10,20,40)	2 6 (2 3)	5 7 (6 0)	9 2 (9 9)	0 7 (0 6)	8 1 (8 7)
W(5 10,20,40)	4 9	3 5	6 3	1 3	9 6
M(5,10,20,40)	3 5	4 7	8 3	1 1	4 0
SWM(5 10 20 40)	3 0	3 9	7 0	0 5	6 3
Price-trend Model $\mu=0$, Taylor Estimates					
Rule	DEM	YEN	GBP	FRF	CHF
S(5,10 20,40)	6 8 (6 8)	50 0 (48 3)	12 1 (12 6)	4 8 (5 1)	15 3 (16 6)
W(5,10 20,40)	8 4	35 7	7 3	5 6	13 1
M(5 10,20,40)	13 0	51 6	12 4	10	13 5
SMW(5 10 20 40)	8 2	45 2	9 4	5 7	13 0

Technical models

The adequacy of the technical models descnbed in Section 7 1 3 is now tested using the parametric Chi-square and T-Student tests

Table 7 6 Chi-square test of adequacy of technical models

Chi-square test of adequacv of technical model μ=0 σ Cntical Threshold %					
S(5)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5,10 20 40)	3 6	71 3	52 6	12 2	30 1
W(5 10 20 40)	33 3	48 9	3 5	30 8	48 9
M(5,10 20 40)	38 6	61 1	23 4	57 9	54 4
SWM(5 10 20 40)	26 6	84 3	9 1	71 1	40 5
S(10)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5 10 20 40)	6 7	93 6	53 5	23 4	42 7
W(5 10,20 40)	51 1	83 0	3 5	51 8	53 7
M(5 10 20 40)	58 6	89 7	30 3	52 1	63 4
SMW(5 10 20 40)	33 9	93 8	8 3	80 7	45 0
S(20)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5,10 20,40)	7 8	86 9	41 7	32 2	53 7
W(5 10 20 40)	62 8	88 0	2 9	59 0	68 1
M(5 10 20 40)	81 0	94 6	23 9	73 4	78 8
SMW(5 10 20 40)	41 9	93 4	6 2	88 4	50 4
S(40)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5 10 20 40)	2 0	76 3	34 9	9 1	34 5
W(5 10,20,40)	32 8	70 7	2 7	23 5	63 0
M(5 10 20 40)	53 2	87 8	25 7	42 1	77 6
SWM(5 10 20 40)	24 7	90 6	6 0	67 4	45 7
W(5)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5 10 20 40)	3 9	65 9	61 3	12 3	29 6
W(5,10 20,40)	35 9	43 6	5 0	30 9	51 6
M(5 10,20 40)	39 9	56 8	25 4	32 5	55 7
SWM(5 10 20 40)	28 2	82 4	11 7	72 1	41 5
W(10)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5,10 20,40)	5 4	88 0	58 2	18 6	39 3
W(5,10,20,40)	44 5	70 0	4 0	43 7	54 5
M(5,10,20 40)	50 5	79 0	29 3	44 6	62 3
SMW(5 10 20 40)	31 6	90 9	9 3	78 1	44 8

Table 7 6 (continued) Chi-square test of adequacy of technical models

W(20)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5,10 20 40)	9 3	93 8	48 6	34 6	52 4
W(5 10 20 40)	65 6	90 2	3 4	65 0	64 8
M(5,10 20,40)	76 4	95 6	27 4	69 8	75 1
SMW(5 10 20 40)	41 7	94 7	7 4	88 5	49 8
W(40)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5,10,20,40)	4 1	82 5	42 3	20 4	45 7
W(5 10 20,40)	50 3	80 4	3 4	44 9	70 8
M(5 10 20 40)	72 1	92 2	26 7	65 3	83 0
SWM(5 10 20 40)	33 8	92 5	6 5	82 4	50 0
M(5)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5 10 20 40)	1 9	78 1	21 7	6 6	23 3
W(5 10 20,40)	20 2	58 1	0 9	17 2	33 4
M(5,10 20 40)	29 3	67 4	13 4	24 0	41 5
SWM(5 10 20 40)	19 1	86 1	3 9	56 5	33 0
M(10)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5 10 20 40)	9 8	91 4	45 3	34 8	44 6
W(5 10 20,40)	66 4	92 0	3 0	65 3	49 9
M(5,10,20,40)	74 2	98 5	30 9	60 3	61 4
SMW(5 10 20 40)	36 9	95 3	7 0	80 7	42 6
M(20)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5 10 20,40)	1 7	73 7	27 3	7 5	31 2
W(5 10,20 40)	23 1	69 0	1 7	17 3	48 8
M(5 10 20 40)	44 2	80 8	15 9	36 2	59 2
SMW(5 10 20 40)	23 1	87 9	4 0	63 7	39 4
M(40)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5 10 20 40)	0 8	69 8	26 8	2 3	21 2
W(5,10 20,40)	15 2	51 9	2 1	6 8	45 1
M(5 10 20 40)	27 0	76 8	28 2	14 9	62 2
SWM(5 10 20 40)	15 1	87 5	6 0	40 3	38 8

Table 7.7 T-Student test of adequacy of technical models

T-Student test of adequacy of technical model μ=0 σ Critical Threshold %					
S(5)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5 10 20 40)	2 5	10 4	6 6	1 6	8 2
W(5,10 20 40)	3 9	6 8	3 2	2 5	8 5
M(5,10 20 40)	4 4	8 2	8 2	7 0	5 0
SWM(5 10 20 40)	3 0	7 5	4 9	1 8	6 4
S(10)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5 10 20,40)	5 0	29 3	6 4	3 7	10 2
W(5 10 20 40)	6 7	21 2	2 7	4 7	9 6
M(5 10 20,40)	9 4	24 7	9 6	7 4	7 3
SMW(5 10 20 40)	6 0	24 1	4 8	4 4	8 2
S(20)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5,10 20,40)	7 0	52 7	3 8	4 8	15 6
W(5 10 20 40)	7 9	38 9	1 3	4 9	13 4
M(5 10 20,40)	15 3	52 3	7 3	12 2	13 6
SMW(5 10 20 40)	8 6	47 6	2 8	5 9	13 2
S(40)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5 10 20 40)	2 1	46 5	3 4	1 0	11 0
W(5 10 20 40)	2 4	30 5	1 0	0 9	8 7
M(5 10 20 40)	6 8	52 8	7 7	4 1	11 3
SWM(5 10 20 40)	2 8	42 3	2 6	1 2	9 4
W(5)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5 10 20 40)	3 1	8 6	10 5	1 7	9 4
W(5 10 20,40)	5 0	5 6	6 1	2 9	10 1
M(5 10 20 40)	4 8	6 8	11 2	2 7	5 3
SWM(5 10 20 40)	3 6	6 1	8 1	2 0	7 4
W(10)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5,10 20 40)	4 0	20 2	7 8	2 7	10 5
W(5 10 20 40)	5 7	14 3	3 7	3 8	10 4
M(5 10 20,40)	7 1	16 1	10 5	5 1	6 9
SMW(5 10 20 40)	4 8	15 8	5 9	3 2	8 4
W(20)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5 10,20,40)	8 0	45 3	5 0	5 7	14 6
W(5 10 20,40)	9 5	33 9	1 9	6 4	13 2
M(5,10,20,40)	15 2	41 6	8 5	12 2	11 5
SMW(5 10 20 40)	9 6	39 1	3 8	6 9	12 2

Table 7 7 (continued) T-Student test of adequacv of technical models

W(40)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5,10 20 40)	4 6	53 3	4 4	2 9	15 4
W(5 10 20 40)	5 1	37 9	1 5	2 8	12 6
M(5 10 20 40)	12 0	56 3	8 9	9 1	14 7
SWM(5 10 20 40)	5 8	48 8	3 3	3 6	13 2
M(5)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5 10 20 40)	1 0	13 2	1 3	0 5	4 0
W(5 10 20 40)	1 4	8 7	0 4	0 8	3 7
M(5,10 20 40)	2 4	11 0	2 5	1 5	2 8
SWM(5 10 20 40)	1 2	9 9	0 8	0 7	3 0
M(10)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5,10,20,40)	8 3	43 9	4 6	6 6	8 9
W(5,10 20,40)	9 5	31 7	1 7	6 9	7 7
M(5,10 20,40)	17 2	42 7	8 6	15 3	7 4
SMW(5 10 20 40)	13 0	38 7	3 5	8 1	7 2
M(20)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5,10,20 40)	1 1	37 2	1 9	0 6	6 7
W(5,10,20,40)	1 3	23 3	0 6	0 6	5 4
M(5,10,20,40)	3 7	42 2	4 5	2 6	6 5
SMW(5 10 20 40)	1 4	32 9	1 4	0 7	5 4
M(40)					
Rule	DEM	YEN	GBP	FRF	CHF
S(5,10 20 40)	0 6	17 8	2 6	0 1	5 5
W(5,10,20 40)	0 7	9 6	0 7	0 2	4 2
M(5 10,20,40)	2 4	22 4	7 1	0 8	6 1
SWM(5 10 20 40)	0 8	14 8	2 0	0 2	4 5

Summary models

Table 7 8 provides among all the statistical and technical models the ones which exhibit the highest critical thresholds Therefore the following models are the ones which can reproduce best technical trading rules

Table 7 8 Models exhibiting the highest critical threshold

Models exhibiting the highest critical threshold %					
Chi-square					
Rule	DEM	YEN	GBP	FRF	CHF
S(5 10 20 40)	M(10)	W(20)	Price-trend	M(10)	S(20)
W(5,10 20,40)	M(10)	M(10)	Price-trend	M(10)	W(40)
M(5,10 20,40)	S(20)	M(10)	M(10)	S(20)	W(40)
SWM(5 10 20 40)	S(20)	M(10)	AR(1)	W(20)	Price-trend
T-Student					
Rule	DEM	YEN	GBP	FRF	CHF
S(5,10 20 40)	W(20)	W(40)	Price-trend	M(10)	W(40)
W(5,10 20,40)	W(20)	S(20)	Price-trend	M(10)	S(20)
M(5,10 20,40)	M(10)	W(40)	Price-trend	M(10)	W(40)
SWM(5 10 20 40)	M(10)	W(40)	MA(1)	M(10)	W(40)

For a given currency, the model exhibiting the highest critical threshold is rather invariant on the rule being used in the adequacy test Using of portfolio of trading rules, rather than any single rules to check the adequacy of a model allows to minimise the reproach of backward testing[5] All the models being proposed are very close from each other They generate almost identical expected returns (see Chapter 3) and are extremely correlated one from each other (see for instance the correlation between trading systems under the random walk assumption in Chapter 5) What must be stressed is that for given statistical models, there exist technical models able to reproduce closely expected returns and vice-versa Performances of ARIMA and technical rules are very often indistinguishable (Taylor, 1992b) Therefore, the crucial point is not to choose ex-ante between simple moving average, weighted moving average, momentums or price-trend models but to adequately estimate the duration of the trend either through the mean duration of the price-trend model or the order of the technical rule Taylor(1992b) seems to indicate that technical models might achieve better this purpose

Finally, further research is needed if one wants to check the adequacy of nonlinear models via rule returns It is doubtful that pure variance nonlinear models will be able to explain non-zero trading rule returns Nevertheless, mean nonlinear alternatives such as the fractional Gaussian process are worthy being investigated Unfortunately, corresponding tests using rule returns are difficult to establish because the variance is not any more finite but infinite

[5] It could aptly be argued that the performances of a single trading rule is best explained by its implied model For example it should not be surprising if the returns following the momentum rule of order 10 are best explained by the momentum model of order 10

7.2.2 *Fractional Gaussian process*

Although it is difficult to test the ability of the fractional Gaussian process to replicate rule returns, an interesting issue is however to know if this model can be used to enhance returns above the most plausible linear alternatives: the price-trend or technical models. Therefore, we are now comparing the profit and loss generated by all these models.

In what follows, we present the ex-post performance of optimal forecasters that from the theoretical models, the price-trend model and the fractional Gaussian process. The parameters estimated at the end of each period are used backward and rule returns recorded. Backward rather than forward performance has been recorded such that the results are comparable to the in-sample tests performed in Section 7.2.1. Theoretical forecasters assuming a price trend-model can be found in Taylor(1986) and fractional Gaussian process in Hosking(1981), Peiris and Perera(1988). Transaction costs are ignored once again[6].

It can be firstly noted from Table 7.9, that ex-post performances are all positive. However, they suffer of the curve fitting default and data-snooping biases. Secondly there does not seem to be any preferred models irrespective of the currency. The price-trend model generates higher returns than the fractional Gaussian process for the CHF, FRF but lower for DEM, GBP and YEN. For each currency, the standard deviation of these forecasters has been found, as expected, to be equal to the underlying volatility given in Table 6.2.

Table 7.9: Backward performances of statistical forecasters

Yearly return % of Statistical Forecaster		
Currency	Price -trend model	Fractional process
DEM	4.7	8.1
GBP	9.2	11.6
FRF	10.9	8.0
CHF	10.4	8.6
YEN	6.7	8.7

Statistical forecasters (Table 7.9) do not outperform significantly simple moving average rules (Table 6.16). That can be seen by comparing only varied profits since all the basic rules considered here bring the same risk[7] which is the underlying volatility of the currency. A similar finding is attributable to Taylor(1990a, 1990b, 1992a, 1992b). He shows in particular that the price-trend model does not beat popular trading rules.

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6 Expected transaction costs can be found in Section 7.4.2.

7 When measured by the standard deviation.

7.2.3 *Volatility-autocorrelations*

Trend-following systems are profitable when applied to exchange rate series. The profitability of convex rules would suggest either the presence of a random walk with strong drift, low positive autocorrelations or a mixture of both. It has been seen that exchange rates are without drift and that the hypothesis of low autocorrelations as the price-trend model without drift or technical models describe far better exchange rates than the random walk. It results that expected rule returns should come from the couple (volatility, autocorrelations), equation [3.10], what we now test.

Table 6.17 told us that there seems to be overall a similar risk adjusted profit in exchange rates against Dollar and so a similar overall degree of autocorrelations. Expected portfolio return $E(R_t)$ can be obtained by averaging formula [3.10] over simple moving average of orders 2 to 100. Then:

$$E(R_t) = \sigma\, f(\rho_1, \cdots, \rho_k) \qquad [7.3]$$

where $f(\rho_1, \cdots, \rho_k)$ is a linear function of the autocorrelations of order 1 to 99, which gives decreasing weights as the order of the autocorrelations increases.

Assuming equal overall autocorrelations $f(\rho_1, \cdots, \rho_k)$ in exchange rates, profits should be proportional to the underlying volatility. On the other hand, if rates follow a random walk with drift, profits should be a negative function of the volatility, equation [3.12]. Both assumptions obviously propose conflicting views which need to be clarified.

Figure 7.1 tells us that the average performance of a portfolio of simple moving averages (orders 2 to 100) applied to 15 different currency pairs (including cross rates) is a positive function of the volatility. This confirms that technical trading returns are positively correlated with price volatility (Edwards and Ma, 1988). Consequently, the regression line is more in accordance with autocorrelated alternatives than the random walk hypothesis. Further interpretations are however difficult because currencies are not independent of each other.

Let us now considered risk adjusted profits. They should provide a better measure of inefficiency for given currency since they might be a function of the autocorrelations only:

$$E(R_t)/\sigma = f(\rho_1, \cdots, \rho_k) \qquad [7.4]$$

Figure 7.2 exhibits that risk-adjusted returns are still a positive function of the volatility. Then Lukac and Brorsen(1990: equation(5)) assumption that each commodity has the same return to risk does not hold for currencies. The amount of overall autocorrelations as measured by function $f(\rho_1, \cdots, \rho_k)$, is larger for currencies of higher volatility and smaller for currencies of lower volatility.

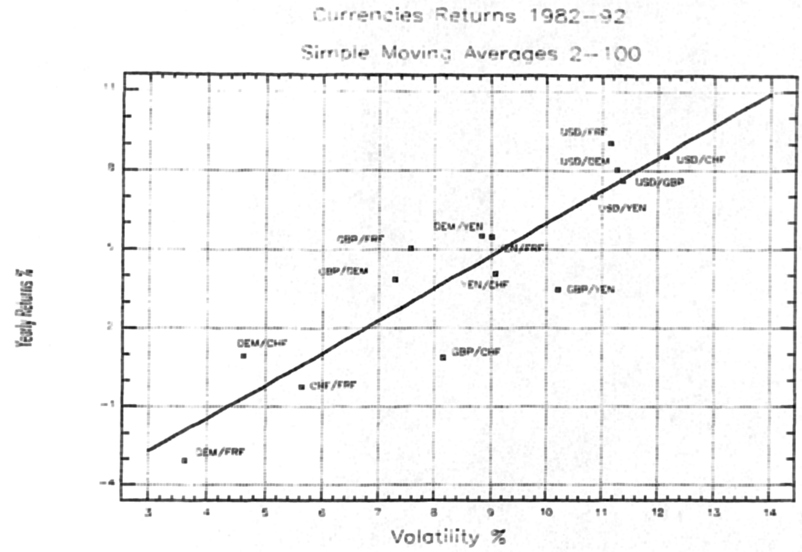

Figure 7.1: Rule returns as a function of the underlying volatility

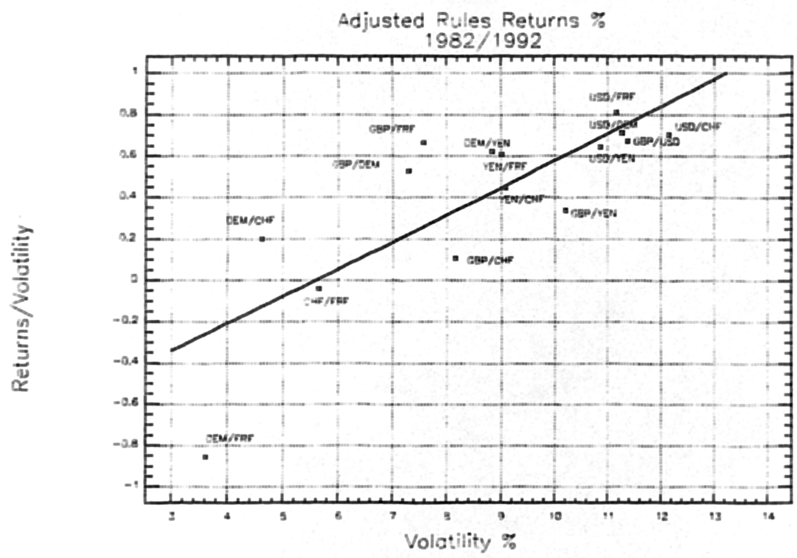

Figure 7.2: Risk adjusted rule returns as a function of the underlying volatility

Until now, the amount of overall autocorrelations has been indirectly measured using trading rule returns via the function $f(\rho_1,\cdots,\rho_k) = E(R_t)/\sigma$. Alternatively, it can be directly measured by the function $Q(100) = \sum_{i=1}^{100}\rho_i$, where ρ_i is given by equation [6.4].

Figures 7.3 and 7.4 make clear the fact that both trading rule returns and risk adjusted returns are a strong positive function of the degree of autocorrelations Q(100) in exchange rate. Corrado and Lee(1992, figure Ia) studying the time series properties of the S&P 500, similarly found a positive relationships between autocorrelations and filter rule returns. Conclusively the positive relationships between rule returns and autocorrelations might hold as well for nonlinear trading rules.

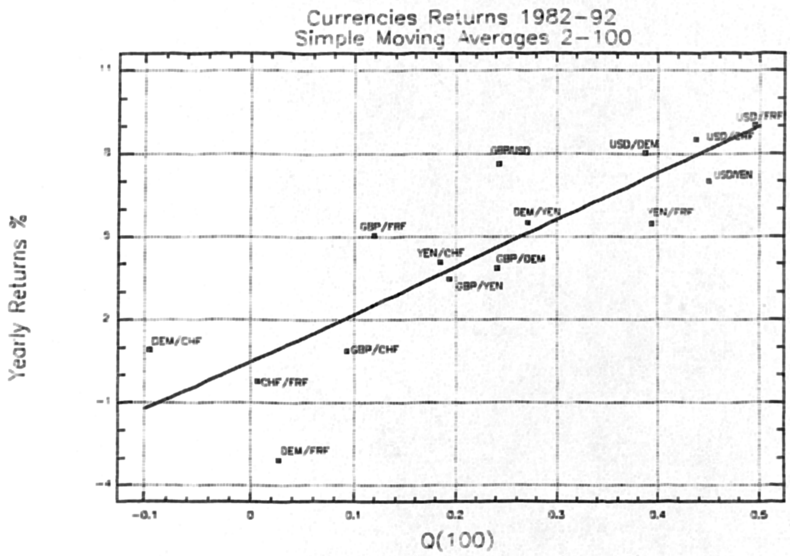

Figure 7.3: Rule returns as a function of underlying autocorrelations

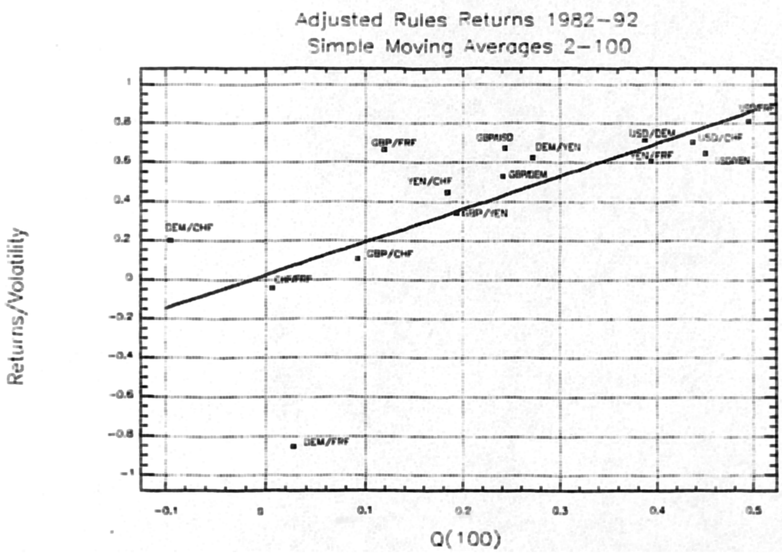

Figure 7.4: Risk adjusted rule returns as a function of underlying autocorrelations

Figure 7.5 reinforces the idea that volatility and autocorrelations are not independent but display a positive relationship. This cross-sectional study brings opposite results to univariate temporal time series (LeBaron, 1992c). Q(100) is larger for currencies of higher volatility and smaller for currencies of lower volatility. Morse(1980) finds a positive relation between volume and serial correlations for individual stocks. These results run accordingly to this section if volume and volatility are positively correlated.

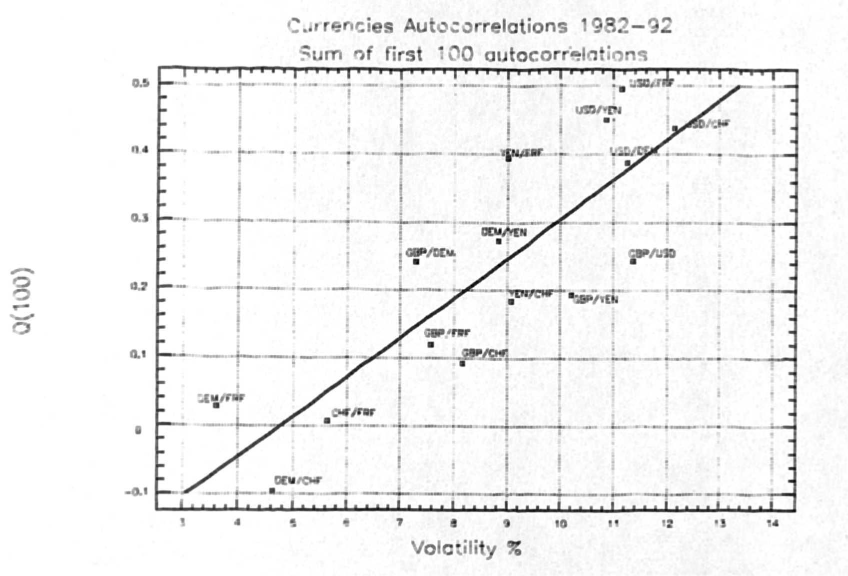

Figure 7.5: Autocorrelations as a function of the underlying volatility

It follows that "technical" funds managers should seek for financial assets exhibiting high volatility since autocorrelations and therefore risk adjusted returns are still a positive function of the volatility. That does not seem at present the case for commodities as Brorsen and Irwin(1987) report on their survey. It says that eighty-five percent of futures fund managers hold number of contract according to their volatility. Half of these held positions based on variances only, by holding more contracts of less volatilities. On the other hand, market practitioners agree that volatility is a potential source of profits for technical analysis. Kroll and Paulenoff(1993: 275) point out that the most popular markets to trade have been the T-bond futures and the S&P stock index futures because they show the highest volatility. Dublanc(1991: 191) indicates that market not enough liquid or missing of volatility must be avoided. Bernstein(1992: 56) recommend tracking and trading markets, as long as they remain relatively active and volatile, using a variety of trend-following systems. Goodman(1982: 87) reports that: " the [technical] models had[...] the weakest performance for the Canadian dollar, which was less volatile relative to the US dollar during the period and consequently offered less profit potential ". Prado(1992: 125) indicates that when markets are trending with high volatility, profit potential can be very high, and that when markets are trending with low volatility, profit potential can be very low.

7.3 ENHANCEMENT STRATEGIES

Technical models are among the best models to replicate in-sample rule returns An issue of interest is to establish their out-of-sample performances and to know if individual performances of trading rules can be enhanced by a portfolio approach For doing so, we consider the simple, weighted moving average, momentum of orders 5, 10, 20 and 40 and study three kinds of strategies used to select particular subsets of trading rules

(1) Equally weighted portfolio

This strategy allocates equal weights between the twelve trading systems

(2) Optimisation method

It consists in choosing the best system in one period and applying it during the subsequent period[8] This is called optimising over past data In a survey of public futures fund advisory groups, Brorsen and Irwin(1987) found that fifteen of nineteen advisory groups selected parameters by optimising over past data Such method aims to maximise returns

(3) Global Variance Portfolio (GVP)

This strategy allocates weights between systems such that they minimise the risk of the portfolio The weights can be found by linear quadratic programming (Markowitz, 1952) and depend only on the correlations between trading rules for given currency In what follows, we assume that the volatility of trading rules are similar[9] and equal to the volatility of the underlying asset We equally assume that correlations between rules when applied to a same asset do not depend on the underlying asset and are equal to their expected values under the normal assumption These two assumptions are reasonable following the results of Chapter 6 They have got the tremendous advantage to induce theoretical systems allocation which will be subsequently the same through time and for each currency (Table 7 10) Table 7 11 provides expected risk reduction achieved by some other portfolios for comparison purpose The small gain to diversification across systems is directly related to the high correlations among the returns It must be emphasised that the risk reduction potential through systems diversification is not large

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[8] The periods being used are the ones specified in Table 6 1 The out-of-sample performances are consequently recorded throughout periods 2 to 5 for the 5 currencies

[9] Lukac and Brorsen(1990) assumes as well equal variances to determine if one technical rule is statistically different from another

Table 7.10 Minimum risk allocation

Systems	S(5)	S(10)	S(20)	S(40)	W(5)	W(10)	W(20)	W(40)	M(5)	M(10)	M(20)	M(40)	Total
Weights(%)	0	0	0	0	24 33	0	0	0	15 49	15 23	16 78	28 17	100

Table 7 11 Theoretical risk reduction

Systems	Unique	S(5 10 20 40)	W(5 10 20 40)	M(5 10 20 40)	SWM(5 10 20 40)	GVP
Risk reduction(%)	1	0 813	0 837	0 731	0 762	0 697

The optimisation method is only marginally more profitable than the equally weighted portfolio, but as expected far more than the Global Variance Portfolio (Table 7 12) By construction, the minimum standard deviation of returns is achieved for the Global Variance Portfolio In terms of Sharpe Ratio (average return/standard deviation), it appears that diversification can pay Reduction of risk can be obtained by simple diversification of rules Such portfolio outperforms the optimisation method in 4 currencies out of 5 However, extra-reduction of risk by choosing sophisticated diversification via Markowitz approach decreases substantially the value of the Sharpe Ratio

Table 7 12 Forward performances of selection strategies

Performances\Strategies	Equally Weighted	GVP	Optimisation
DEM			
Return %	7 67	4 94	9 19
Standard Deviation %	8 97	8 19	11 77
Sharpe Ratio	0 85	0 60	0 78
YEN			
Return %	4 91	4 30	4 11
Standard Deviation %	8 43	7 71	11 06
Sharpe Ratio	0 58	0 56	0 37
GBP			
Return %	7 98	6 65	9 58
Standard Deviation %	9 06	8 29	11 89
Sharpe Ratio	0 88	0 80	0 81
FRF			
Return %	8 27	6 65	6 55
Standard Deviation %	8 65	7 91	11 35
Sharpe Ratio	0 96	0 84	0 58
CHF			
Return %	7 46	6 30	10 54
Standard Deviation %	9 50	8 69	12 41
Sharpe Ratio	0 79	0 73	0 85

The superiority of the equally weighted portfolio beyond the optimisation method and minimum risk approaches can be explained by two factors Firstly exchange rates series are non-stationary and consequently the optimal forecaster is rarely the same from one period to the other Then the usefulness of the optimisation method is arguable (Lukac, Irwin and Brorsen, 1989) This strategy is of no additional value to a basic equally weighted portfolio Secondly the differences between rules correlations are sometimes so small that weights selected throughout the quadratic program are not really significant The minimum variance criterion excludes for instance seven systems on twelve (Table 7 11)

This does not mean that correlations between systems must not be taken into account But rather than searching for the minimum risk, simple diversification might be preferable The only problem stays in the determination of the ex-ante universe of rules Here this has been chosen such that for each family of rules, systems are almost equicorrelated

The Markowitz approach either maximising returns, minimising risk or a mixture of both does not seem promising, as far as systems diversification is concerned On the other hand, simple diversification among equicorrelated systems appears a lot more profitable due to its robustness through time In sum, diversification between systems pays but it must not be too complicated Goodman(1982) exhibits for instance than combining two technical models is better than one but three are already two many

Diversification between currencies might still be more valuable than diversification between systems, see Chapter 5 for theoretical evidence and Taylor(1990b), Brorsen and Boyd(1990) for empirical evidence However optimal diversification is condemned to fail for at least two reasons Firstly, the bivariate random walk is not an acceptable hypothesis, even when restrained to rule correlations (Section 6 2 3) It follows that finding the currencies allocation which minimise the risk of the portfolio will now be an hazardous task Secondly, correlations between underlying currencies vary through time, opposite to correlation between systems applied to a same currency Then attempts to build efficient portfolios of exchange rates have been fruitless (Praagmanand and Soenen, 1986) It is why Table 7 13 restricts its study to the effect of simple diversification of a given system between currencies The equally weighted portfolio exhibits higher Sharpe Ratio irrespective of rule for YEN, GBP and CHF, in 83% of all cases for DEM, and in 58% for FRF Currencies diversification is obviously valuable It outperforms systems diversification for a few rules such as S(20), W(20) and W(40) The differences are however too small to be really significant

Table 7 13 Currencies diversification

Rules\Currencies	DEM	YEN	GBP	FRF	CHF	Equally Weighted
Yearly returns %						
S(5)	7 97	5 26	7 44	9 09	4 71	6 90
S(10)	8 95	6 14	7 44	9 27	8 53	8 07
S(20)	12 30	3 96	9 05	12 47	9 57	9 47
S(40)	4 53	4 20	6 16	8 38	4 82	5 62
W(5)	7 09	3 96	8 80	8 56	6 69	7 02
W(10)	7 45	7 05	5 61	9 79	6 72	7 32
W(20)	9 91	7 01	10 80	10 54	6 73	9 00
W(40)	8 77	5 14	10 69	10 37	8 84	8 76
M(5)	5 78	3 87	5 00	7 74	6 48	5 78
M(10)	6 87	6 28	8 09	7 16	6 30	6 94
M(20)	8 65	3 68	3 19	9 37	6 30	6 24
M(40)	4 71	4 21	7 61	6 31	7 05	5 98
Sharpe Ratio						
S(5)	0 68	0 48	0 63	0 80	0 38	0 76
S(10)	0 76	0 56	0 63	0 82	0 69	0 87
S(20)	1 05	0 36	0 76	1 10	0 77	1 03
S(40)	0 38	0 38	0 52	0 74	0 39	0 60
W(5)	0 60	0 36	0 74	0 75	0 54	0 77
W(10)	0 63	0 64	0 47	0 86	0 54	0 79
W(20)	0 84	0 63	0 91	0 93	0 54	0 97
W(40)	0 75	0 46	0 90	0 91	0 71	0 96
M(5)	0 49	0 35	0 42	0 68	0 52	0 63
M(10)	0 58	0 57	0 68	0 63	0 51	0 76
M(20)	0 73	0 33	0 27	0 83	0 51	0 68
M(40)	0 40	0 38	0 64	0 56	0 57	0 65

7 4 EFFICIENT MARKET HYPOTHESIS

7 4 1 *Jensen's definition*

Jensen(1978) argues that a market should be considered efficient with respect to an information set if it is impossible to make economic profits by trading based on the information set The random walk model requires zero risk-adjusted returns in speculative markets on the assumptions of zero transactions costs But transaction costs in financial markets are not zero, so a market is still efficient as long as a technical trading system does not produce returns greater than transaction costs

7 4 2 *Transaction costs*

We provide here a simple formula giving the expected transaction costs following a linear trading rule Such result will allow to easily adjust previous findings such that transactions costs are taken into account

The cost to a speculator of a currency trade depends on many variables The total cost of taking a position is the sum of brokerage fees and liquidity costs Liquidity costs arise because floor traders have different buying and selling prices Trading costs can be expressed as a percentage of the goods traded (Taylor, 1986) We then assume that trading costs are equal to c, where c is a same constant for all times considered A cost figure of c=0 2% is suitably conservative for currencies, because such costs are still higher than most non-floor traders would pay (Taylor, 1990) Sweeney(1986), Surujaras and Sweeney(1992) estimate transaction costs to be lower than one eight of one percent (c<0 125%) Further, large transactors or banks operating on their own account can avoid brokerage fees and only pay liquidity costs Schulmeister(1988) reports average transaction costs based on bid-ask spreads to be at maximum 0 04% per trade Satchell and Timmermann(1992b) stipulate that transaction costs are very small in the foreign exchange market and less than 0 06% The transaction costs used in this study will be c=0 2% and c=0 05% which appear upper bound for respectively public and institutional investors

Over a period of T days, there will be a number N of transactions and consequently a total trading costs equal to TC=cN The number N of transactions is a stochastic variable which depends on the forecaster F_t being used Nevertheless its expected value can be established under the Gaussian process without drift assumption If we assume that a position is opened at the beginning of the period (and not when a new position is triggered) and that the last position is closed at the end day of the period, it follows that the expected number of transactions is[10]

$$E(N) = 1 + (T-2)\left[\tfrac{1}{2} - \tfrac{1}{\pi}\operatorname{Arc\,sin}(\rho)\right] \quad \text{where } \rho = \operatorname{Corr}(F_t, F_{t-1})$$

Subsequently,

$$E(TC) = c\left\{1 + (T-2)\left[\tfrac{1}{2} - \tfrac{1}{\pi}\operatorname{Arc\,sin}(\rho)\right]\right\}$$

Expected number of transactions under the random walk and price-trend model assumptions, as well as observed values for currencies are given for the simple moving average rule in Table 7 14 YEN values are almost equal to their expected values under the random walk assumption For other currencies, numbers of transactions are lower than expected under the random walk assumption It confirms previous findings which accepted the random walk assumption for the YEN but rejected it for the other currencies in favour of price trend models Therefore the expected number of transactions under the random walk assumption is an upper bound for currencies This conservative figure will be used to assess transaction costs It allows to get estimates depending only on the rule being used, not on the currency being traded

Table 7.14 Number of transactions over a period of 2586 days

Number of transactions over a period of T=2586 days									
Rule\Process	Random Walk	Price-trend A=0 03 m_d=20	Currencies						
			Average	DEM	YEN	GBP	FRF	CHF
S(2)	1293	1269 5	1294 2	1311	1313	1265	1302	1280
S(5)	692 3	660 5	638 4	617	662	638	626	649
S(10)	468 9	429 8	421 4	412	448	416	430	401
S(20)	325 1	280 8	252 6	220	322	242	228	251
S(40)	227 6	183 1	172 2	161	203	177	145	175

Expected transactions costs over a year (T=250 days) are given in Table 7 15 for a few linear rules under the random walk and price-trend model It turns out from Table 7 15 that transaction costs cannot be ignored if the purpose of the investor is to "make money", on a net return basis The most active trading generated by the moving average of order 2 rule, implies for instance yearly transaction costs equal to 25% for small investors! It clearly appears that for equal gross returns, longer term rules must be preferred This result seems to hold for nonlinear rules, such as the channel rule (Taylor, 1992b fig 1)

[10] See Appendix 6 2

Table 7.15 Expected yearly transaction costs

Rules/Process	Random Walk			Price-trend A=0 03 m_d=20		
	E(N)	E(TC)%		E(N)	E(TC)%	
		c=0 2%	c=0 05%		c=0 2%	c=0 05%
S(2)[11]	125	25	6 25	122 7	24 55	6 14
S(5)	67 4	13 48	3 37	64 3	12 87	3 22
S(10)	46 0	9 19	2 30	42 2	8 44	2 11
S(20)	32 2	6 43	1 61	27 9	5 59	1 40
S(40)	22 8	4 56	1 14	18 6	3 71	0 93
W(5)	78 6	15 71	3 93	75 5	15 10	3 78
W(10)	56 2	11 25	2 81	52 4	10 48	2 62
W(20)	40 2	8 05	2 01	35 8	7 15	1 79
W(40)	28 8	5 77	1 44	24 1	4 82	1 21
M(5)	58 1	11 61	2 90	55 0	11 01	2 75
M(10)	38 6	7 71	1 93	34 85	6 97	1 74
M(20)	26 7	5 34	1 34	22 65	4 53	1 13
M(40)	18 9	3 78	0 95	15 0	2 99	0 75

7 4 3 *T-Student adjusted for transaction costs*

Original T-Student statistics (Table 6 17) have to be adjusted to take into account transaction costs That is done in Table 7 16 for c=0 2%, and c=0 05% It results that technical trading in foreign exchange is likely to be a challenging if not fruitless activity for small investors who face big transaction costs[12] (c=0 2%) That is another story for institutional investors or floor traders The critical threshold of the T-Student test adjusted for transaction costs (c=0 05%) are overall still well below 5% Technical analysis has information content that will allow floor traders to increase risk-adjusted profits Financial companies might have to act on strategies that assumes the foreign exchange markets are autocorrelated if not inefficient[13] Some desequilibrium beyond that caused by transaction costs and risk appear to be present in exchange rates

Our findings are similar to Murphy(1986) which are that the potential for abnormal technical trading profits does exist, if expenses are reduced Our results demonstrate that

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[11] The three rules S(2), M(2) W(2) are the same

[12] Nevertheless there seems to exist stock technical systems outperforming the market even after allowing for round-trip transaction costs up to 2% per security trade (Pruitt and White, 1988)

[13] Including transaction costs in the calculations might not yet be sufficient to get a market efficiency test The interest rate differential must be taken into account Nevertheless the size of this factor might be negligible Previous studies (Schulmeister, 1988, Sweeney, 1986, Surujaras and Sweeney 1992 67-68, Satchell and Timmermann, 1992b) have shown that the overall effect of the interest rate differential on rule returns is insignificant

it is possible to earn sufficient technical trading profits to at least cover brokerage and management fees

Table 7 16 T-Student test adjusted for transaction costs

Critical Threshold % of T-Student test adjusted for transaction costs[14]					
c=0 2%					
Test	DEM	YEN	GBP	FRF	CHF
S(5)	93 7	99 1	95 2	89 0	98 8
S(10)	51 4	80 5	67 7	47 8	55 7
S(20)	4 5	75 3	22 1	3 9	19 6
S(40)	48 7	52 5	31 3	12 9	45 8
S(5 10 20 40)	48 0	89 0	60 5	29 8	67 3
W(2 to 50)	33 0	86 5	38 8	26 5	52 8
W(2 to 100)	14 4	53 1	19 4	6 8	22 4
S(2 to 50)	36 7	72 3	31 8	19 8	44 7
S(2 to 100)	15 4	24 7	15 3	8 4	13 5
M(2 to 50)	44 3	41 0	47 0	34 0	32 9
M(2 to 100)	13 0	14 0	21 41	10 3	10 6
SWM(2 to 50)	36 7	73 1	40 8	27 5	46 1
SWM(2 to 100)	15 0	31 5	21 8	8 6	15 9
c=0 05%					
Test	DEM	YEN	GBP	FRF	CHF
S(5)	9 6	28 8	12 7	5 1	36 0
S(10)	3 0	12 9	7 4	2 3	5 1
S(20)	0 1	24 3	1 8	0 1	1 8
S(40)	16 7	18 3	7 9	1 9	16 5
S(5 10 20 40)	1 4	15 5	3 0	0 3	6 0
W(2 to 50)	0 5	13 6	0 8	0 3	1 0
W(2 to 100)	0 4	5 8	0 7	0 1	1 3
S(2 to 50)	1 7	10 5	1 3	0 4	3 7
S(2 to 100)	1 0	2 0	1 5	0 3	1 0
M(2 to 50)	2 9	2 1	3 5	1 5	1 9
M(2 to 100)	0 8	0 8	2 5	0 5	0 8
SWM(2 to 50)	1 1	7 1	1 3	0 4	2 5
SWM(2 to 100)	0 6	2 1	1 2	0 2	0 8

[14] Transaction costs for multisystems are established here from the sum of individual transaction costs Then they are upper limit of real costs since by construction the number of transactions for a multisystem is equal or below the sum of the number of transactions for each system

7 4 4 *Market efficiency and dependencies*

Since a market is still efficient as long as a technical trading system does not produce returns greater than transaction costs, the existence of serial correlation in the changes in financial rates might indicate neither market efficiency nor inefficiency (Craine and Havenner, 1988, Taylor, 1986) The transactions costs are a cause of desequilibrium and it is difficult to assess the extent to which transaction costs should be taken into account in assessing market efficiency We have just seen that the market could appear efficient/inefficient for investors facing different trading costs Even if profits adjusted for high transactions costs were to stay significantly positive, there may be three other reasons which can rescue the hypothesis of efficiency

Firstly, there is a degree of uncertainty in the information received The statistical tests performed in this research have not reached all the same conclusions Tests applied to the underlying time series have argued for the random walk hypothesis[15] when profits-based tests have found significant positive dependencies Logue and Sweeney(1977) provided a similar study where a mechanical trading rule detected dependence using foreign exchange data while spectral analysis detected no dependence using the same data If there is a perception of uncertainty in information received and/or if the information is diverse across participants, then the past exchange rate may have a prolonged effect on the current exchange rate If so lagged models need not be inconsistent with market efficiency Uncertainty is one of the two factors proposed by Irwin and Brorsen(1987) to explain rule returns Their results showed a strong positive association between uncertainty (as measured by inflation) and technical returns, suggesting traders may expect lower returns during periods of low uncertainty On the other hand, they did not find any relationship between the second factor, the relative amount of system trading and technical returns

Secondly, there may exist a time-varying risk premia, (Fama, 1984, Wolff, 1987) Surujaras and Sweeney(1992) believe however that explaining trading rule profits as due to time-varying risk premia might be a very long and arduous process Their argument is that there is a wide variety of possible models of time-varying risk premia and so there does not seem to emerge a clear theory of how these premia should behave in a system of efficient markets Time-varying risk premia and profitable trading rules are compatible hypotheses because null hypotheses do not to have to be statistically stable in order to be exploited profitably (Boothe and Longworth, 1986) If financial prices follow a price-

[15] Except Taylor(1980) statistics

trend model, variance reduction and mean duration need not to be statistically stable If the overall amount of autocorrelations Ap/(1-p) is nearly always positive, it is quite likely that trend-following rules can be used to make profits

Thirdly, if price is discontinuous, prices will go up and down very steeply[16] Mandelbrot(1963) advances that technical rules assume that one could buy or sell during these periods of steep variation, but that this possibility is not open to ordinary buyers and sellers Then trading rules profits might not be in contradiction with market efficiency

To conclude, the purpose of this research has not been to test market efficiency which is in itself a difficult task, but rather simply to provide an understanding of the superior performance of some models relative to the random walk model

7 5 SUMMARY

Among a few linear autocorrelated models, the price-trend model appears to be the most satisfactory to explain trading rule returns That is shown by both the Chi-square test and more significantly by the T-Student test Technical models are as good alternatives The reason is that they produce expected rule returns very close from those generated by a price-trend model

Profits from trend-following (convex, pathdependent) rules are a positive function of the volatility This result corroborates with the existence of low positive autocorrelations Risk adjusted profits are still a positive function of the volatility It could mean that volatility and autocorrelations are dependent variables More specifically, it seems that the more volatile is a currency, the more autocorrelations it exhibits Subsequently, there appears to be a premium in investing in risky currency The selection of assets trading of which is most likely to generate profits is a relatively straightforward process that can be derived from the statistical properties of the underlying asset

Statistical forecasters including the fractional Gaussian predictor, although profitable, do not outperform simple technical rules Enhancing returns of technical rules is a difficult task Maximising returns or minimising risk are poor selection criteria The most robust approach seems to be an equally weighted portfolio of equicorrelated systems

[16] The fractional Gaussian process is a good example of such price behavior

Diversification between systems pays but this is neither less or more beneficial than diversification between currencies

Finally, transaction costs alter the statistical significance of test results The magnitude of the changes depend on the levels of transaction costs which are far higher for small investors than for institutional investors On the one hand, risk-adjusted profits from technical analysis totally disappear for transaction costs equal to 0 2% per trade On the other hand, they still remain significant for transaction costs equal to 0 05% per trade Financial companies might have to act on strategies that assume the foreign exchange markets exhibit dependencies, if not inefficiencies

We would conclude that for institutional investors there could be much to gain from technical rules if dependencies persist and little to loose in terms of expectations if they do not

APPENDIX 7 1

CURRENCIES BOX AND JENKINS MODELS: AR(1), AR(2), MA(1), MA(2), ARMA(1,1)

Table 7.17 AR(1) models

Summary of Fitted Model for DEM LOG
Parameter Estimate Stnd error T-value P-value AR (1) 04094 01951 2 09896 03592 Estimated white noise variance = 5 06658E-5 with 2624 degrees of freedom Estimated white noise standard deviation (std err) = 7 11799E-3 Chi-square test statistic on first 20 residual autocorrelations = 18 7809 with probability of a larger value given white noise = 0 470967
Summary of Fitted Model for YEN LOG
Parameter Estimate Stnd error T-value P-value AR (1) 00698 01952 35733 72087 Estimated white noise variance = 4 72485E-5 with 2624 degrees of freedom Estimated white noise standard deviation (std err) = 6 87375E-3 Chi-square test statistic on first 20 residual autocorrelations = 16 6786 with probability of a larger value given white noise = 0 611635
Summary of Fitted Model for GBP LOG
Parameter Estimate Stnd error T-value P-value AR (1) 05314 01949 2 72570 00646 Estimated white noise variance = 5 15303E-5 with 2624 degrees of freedom Estimated white noise standard deviation (std err) = 7 17846E-3 Chi-square test statistic on first 20 residual autocorrelations = 24 5896 with probability of a larger value given white noise = 0 174499
Summary of Fitted Model for FRF LOG
Parameter Estimate Stnd error T-value P-value AR (1) 04029 01951 2 06525 03896 Estimated white noise variance = 4 96892E-5 with 2624 degrees of freedom Estimated white noise standard deviation (std err) = 7 04906E-3 Chi-square test statistic on first 20 residual autocorrelations = 20 2559 with probability of a larger value given white noise = 0 37933
Summary of Fitted Model for CHF LOG
Parameter Estimate Stnd error T-value P-value AR (1) 03434 01951 1 75987 07855 Estimated white noise variance = 5 89028E-5 with 2624 degrees of freedom Estimated white noise standard deviation (std err) = 7 67482E-3 Chi-square test statistic on first 20 residual autocorrelations = 20 3992 with probability of a larger value given white noise = 0 370927

Table 7.18 AR(2) models

Summary of Fitted Model for DEM LOG

Parameter	Estimate	Stnd error	T-value	P-value
AR (1)	04153	01952	2 12708	03351
AR (2)	- 01428	01952	- 73130	46466

Estimated white noise variance = 5 06749E-5 with 2623 degrees of freedom
Estimated white noise standard deviation (std err) = 7 11863E-3
Chi-square test statistic on first 20 residual autocorrelations = 18 2404
with probability of a larger value given white noise = 0 439922

Summary of Fitted Model for YEN LOG

Parameter	Estimate	Stnd error	T-value	P-value
AR (1)	00695	01953	35588	72196
AR (2)	00401	01953	20554	83717

Estimated white noise variance = 4 72656E-5 with 2623 degrees of freedom
Estimated white noise standard deviation (std err) = 6 875E-3
Chi-square test statistic on first 20 residual autocorrelations = 16 6235
with probability of a larger value given white noise = 0 549119

Summary of Fitted Model for GBP LOG

Parameter	Estimate	Stnd error	T-value	P-value
AR (1)	05332	01953	2 73067	00636
AR (2)	- 00342	01953	- 17535	86082

Estimated white noise variance = 5 15493E-5 with 2623 degrees of freedom
Estimated white noise standard deviation (std err) = 7 17978E-3
Chi-square test statistic on first 20 residual autocorrelations = 24 5272
with probability of a larger value given white noise = 0 138503

Summary of Fitted Model for FRF LOG

Parameter	Estimate	Stnd error	T-value	P-value
AR (1)	04036	01953	2 06687	03884
AR (2)	- 00160	01953	- 08175	93485

Estimated white noise variance = 4 97081E-5 with 2623 degrees of freedom
Estimated white noise standard deviation (std err) = 7 05039E-3
Chi-square test statistic on first 20 residual autocorrelations = 20 2489
with probability of a larger value given white noise = 0 318983

Summary of Fitted Model for CHF LOG

Parameter	Estimate	Stnd error	T-value	P-value
AR (1)	03475	01952	1 78004	07519
AR (2)	- 01215	01953	- 62214	53390

Estimated white noise variance = 5 89166E-5 with 2623 degrees of freedom
Estimated white noise standard deviation (std err) = 7 67572E-3
Chi-square test statistic on first 20 residual autocorrelations = 20 0375
with probability of a larger value given white noise = 0 330713

Table 7 19 MA(1) models

Summary of Fitted Model for DEM LOG

Parameter	Estimate	Stnd error	T-value	P-value
MA (1)	- 04162	01951	-2 13290	03303

Estimated white noise variance = 5 06633E-5 with 2624 degrees of freedom
Estimated white noise standard deviation (std err) = 7 11782E-3
Chi-square test statistic on first 20 residual autocorrelations = 18 6503
with probability of a larger value given white noise = 0 479466

Summary of Fitted Model for YEN LOG

Parameter	Estimate	Stnd error	T-value	P-value
MA (1)	- 00697	01952	- 35692	72118

Estimated white noise variance = 4 72485E-5 with 2624 degrees of freedom
Estimated white noise standard deviation (std err) = 6 87375E-3
Chi-square test statistic on first 20 residual autocorrelations = 16 6799 9
with probability of a larger value given white noise = 0 611542

Summary of Fitted Model for GBP LOG

Parameter	Estimate	Stnd error	T-value	P-value
MA (1)	- 05319	01950	-2 72734	00643

Estimated white noise variance = 5 153E-5 with 2624 degrees of freedom
Estimated white noise standard deviation (std err) = 7 17844E-3
Chi-square test statistic on first 20 residual autocorrelations = 24 5529
with probability of a larger value given white noise = 0 175791

Summary of Fitted Model for FRF LOG

Parameter	Estimate	Stnd error	T-value	P-value
MA (1)	- 04039	01951	-2 06981	03857

Estimated white noise variance = 4 9689E-5 with 2624 degrees of freedom
Estimated white noise standard deviation (std err) = 7 04904E-3
Chi-square test statistic on first 20 residual autocorrelations = 20 2436
with probability of a larger value given white noise = 0 380053

Summary of Fitted Model for CHF LOG

Parameter	Estimate	Stnd error	T-value	P-value
MA (1)	- 03482	01952	-1 78433	07448

Estimated white noise variance = 5 89011E-5 with 2624 degrees of freedom
Estimated white noise standard deviation (std err) = 7 67471E-3
Chi-square test statistic on first 20 residual autocorrelations = 20 3228
with probability of a larger value given white noise = 0 375393

Table 7 20 MA(2) models

Summary of Fitted Model for DEM LOG

Parameter	Estimate	Stnd error	T-value	P-value
MA (1)	- 04161	01952	-2 13106	03318
MA (2)	01366	01952	69984	48409

Estimated white noise variance = 5 06729E-5 with 2623 degrees of freedom
Estimated white noise standard deviation (std err) = 7 11849E-3
Chi-square test statistic on first 20 residual autocorrelations = 18 1722
with probability of a larger value given white noise = 0 444363

Summary of Fitted Model for YEN LOG

Parameter	Estimate	Stnd error	T-value	P-value
MA (1)	- 00689	01953	- 35306	72407
MA (2)	- 00396	01952	- 20302	83913

Estimated white noise variance = 4 72657E-5 with 2623 degrees of freedom
Estimated white noise standard deviation (std err) = 6 87501E-3
Chi-square test statistic on first 20 residual autocorrelations = 16 6299
with probability of a larger value given white noise = 0 548674

Summary of Fitted Model for GBP LOG

Parameter	Estimate	Stnd error	T-value	P-value
MA (1)	- 05319	01953	-2 72432	00649
MA (2)	- 00046	01952	- 02365	98113

Estimated white noise variance = 5 15495E-5 with 2623 degrees of freedom
Estimated white noise standard deviation (std err) = 7 1798E-3
Chi-square test statistic on first 20 residual autocorrelations = 24 553
with probability of a larger value given white noise = 0 137731

Summary of Fitted Model for FRF LOG

Parameter	Estimate	Stnd error	T-value	P-value
MA (1)	- 04036	01953	-2 06724	03881
MA (2)	00119	01952	06090	95144

Estimated white noise variance = 4 97078E-5 with 2623 degrees of freedom
Estimated white noise standard deviation (std err) = 7 05038E-3
Chi-square test statistic on first 20 residual autocorrelations = 20 2406
with probability of a larger value given white noise = 0 319436

Summary of Fitted Model for CHF LOG

Parameter	Estimate	Stnd error	T-value	P-value
MA (1)	- 03494	01952	-1 78938	07367
MA (2)	01225	01952	62781	53019

Estimated white noise variance = 5 89143E-5 with 2623 degrees of freedom
Estimated white noise standard deviation (std err) = 7 67556E-3
Chi-square test statistic on first 20 residual autocorrelations = 19 9517
with probability of a larger value given white noise = 0 335546

Table 7.21 ARMA(1,1) models

Summary of Fitted Model for DEM LOG

Parameter	Estimate	Stnd error	T-value	P-value
AR (1)	01913	48779	03921	96873
MA (1)	- 02350	48770	- 04819	96157

Estimated white noise variance = 5 06838E-5 with 2623 degrees of freedom
Estimated white noise standard deviation (std err) = 7 11925E-3
Chi-square test statistic on first 20 residual autocorrelations = 18 6969
with probability of a larger value given white noise = 0 410705

Summary of Fitted Model for YEN LOG

Parameter	Estimate	Stnd error	T-value	P-value
AR (1)	00418	6 67021	00063	99950
MA (1)	- 00416	6 67020	- 00062	99950

Estimated white noise variance = 4 72666E-5 with 2623 degrees of freedom
Estimated white noise standard deviation (std err) = 6 87507E-3
Chi-square test statistic on first 20 residual autocorrelations = 16 6538
with probability of a larger value given white noise = 0 547015

Summary of Fitted Model for GBP LOG

Parameter	Estimate	Stnd error	T-value	P-value
AR (1)	02694	36218	07437	94072
MA (1)	- 02754	36207	- 07607	93937

Estimated white noise variance = 5 15497E-5 with 2623 degrees of freedom
Estimated white noise standard deviation (std err) = 7 17981E-3
Chi-square test statistic on first 20 residual autocorrelations = 24 5615
with probability of a larger value given white noise = 0 137479

Summary of Fitted Model for FRF LOG

Parameter	Estimate	Stnd error	T-value	P-value
AR (1)	02031	50706	04005	96806
MA (1)	- 02099	50699	- 04139	96699

Estimated white noise variance = 4 97081E-5 with 2623 degrees of freedom
Estimated white noise standard deviation (std err) = 7 0504E-3
Chi-square test statistic on first 20 residual autocorrelations = 20 2401
with probability of a larger value given white noise = 0 319465

Summary of Fitted Model for CHF LOG

Parameter	Estimate	Stnd error	T-value	P-value
AR (1)	01626	61299	02653	97884
MA (1)	- 01935	61293	- 03158	97481

Estimated white noise variance = 5 89244E-5 with 2623 degrees of freedom
Estimated white noise standard deviation (std err) = 7 67622E-3
Chi-square test statistic on first 20 residual autocorrelations = 20 3516
with probability of a larger value given white noise = 0 313379

APPENDIX 7 2

EXPECTED NUMBER OF TRANSACTIONS

This appendix establishes the expected number of transactions following a linear rule under the Gaussian process without drift assumption

The average duration of a position triggered by a technical indicator is difficult to establish because it involves truncated multivariate probabilities analytically unknown An easier step is to determine the probability that there occurs a reversal of position a given day, noted P[reversal]

A reversal of position the day t means that the signal triggered by the trading rule are of opposite signs the days t-1 and t Since the underlying process is symmetrical

$P[\text{reversal}] = P[F_{t-1} < 0, F_t > 0] + P[F_{t-1} > 0, F_t < 0] = 2P[F_{t-1} < 0, F_t > 0]$

$P[\text{reversal}] = 2\,[0,0](-\rho)$

where $\rho = \text{Corr}(F_{t-1}, F_t)$, and [0,0] is the bivariate truncated probability given by equation [A 1] in Appendix 3 1 It results that

$P[\text{reversal}] = \frac{1}{2} - \frac{1}{\pi}\text{Arc}\sin(\rho)$

Then the expected number of transactions over a period of T days is

$E(N) = T[\frac{1}{2} - \frac{1}{\pi}\text{Arc}\sin(\rho)]$

If we assume that a position is taken the first day of the period and there cannot be any new position the last day (close of position), there are in fact T-2 days over which a stochastic position can be triggered Then a slight adjustment to the previous formula must be made

$E(N) = 1 + (T-2)[\frac{1}{2} - \frac{1}{\pi}\text{Arc}\sin(\rho)]$ for $T \geq 2$

Chapter 8

CONCLUSIONS

8.1 SUMMARY

The purpose of this thesis has been to advance the understanding of price-based forecasts The main results of this research are summarised in what follows

(1) The economic value of forecasting methods is best measured by the pay-off generated by the implied investment strategy Many more market conditions and forecasters can be encompassed using stochastic modelling than any historical studies Therefore, the expected value and variance of the rate of return using a linear forecaster have been derived under the assumption that the process of underlying returns is Gaussian Expected returns are zero if only and only if the underlying process is the random walk without drift

(2) It is shown that a large class of mechanical forecasting systems used by market participants can be transformed as linear forecasters and consequently that expected profit can be evaluated

(3) Errors based measures are compared with profitability measures Minimising the mean squared error is a sufficient but not necessary condition to maximise profits However, it appears that error measures including the directional accuracy are of poor use to detect profitable strategies when the true model is not known

(4) A test based on the joint the profitability of trading rules is derived It has the attractive feature to be almost as powerful as the best of its component which is unknown when the true model is unknown It constitutes therefore an adequate test of market timing ability if the series of which it is applied is without drift

(5) Profitable strategies based on technical analysis exist in the foreign exchange market Both the bootstrap methodology and the test based on the joint profitability of trading rules bring similar results, which are that daily exchange rates 1982-1992 do not follow a random walk

(6) Profits from trading rules in the foreign exchange market are well approximated by linear time-series models Among a few statistical Gaussian processes, the price-trend model is the best alternative to explain rule returns There exist linear technical models reproducing as well trading rule returns Technical models have got the advantage beyond the price-trend model to rely on fewer parameters Selecting a particular technical rule is a difficult task, because forecasting strategies are numerous and most often extremely similar On the other hand, the selection of instrument trading is most likely to be the crucial choice This is a relatively straightforward process that can be derived from the statistical properties of the underlying asset The more volatile a currency, the more autocorrelated it is and consequently the more profitable the instrument

(7) When transaction costs are taken into account, then profits are reduced substantially However, opportunities remain for institutional investors which might have to act on strategies that assume that the foreign exchange markets exhibit positive dependencies, if not efficiencies

8.2 SUGGESTION FOR FURTHER RESEARCH

This research can be extent in several ways The first one consists in establishing exact analytical multi-period variance and correlations of trading rules based on linear forecasters, assuming that the underlying returns process is Gaussian These results will allow to precisely test the ability of Gaussian processes to replicate trading rule returns An other research is to establish the expected return of nonlinear forecasters under the assumption of both linear and nonlinear models For instance, it might be informative to understand the behaviour of rules based on minimum and maximum of past data such as the channel rule (Lukac, Brorsen and Irwin, 1988b), because they are highly popular among market participants Then, a crucial finding would be to determine when the true price model is nonlinear, what is the forecaster which maximises returns and how profitable it is These researches are highly dependent on the state of knowledge about truncated multivariate laws At present, analytical results exist up to the truncated trivariate normal law This is why the study of nonlinear price models and forecasters might be difficult to achieve

The random walk and market efficiency hypotheses are of such importance in the financial market, that they justify attempts to establish statistical tests based on an economic evaluation of forecasting strategies To be powerful, these tests must take into account the stochastic properties of trading rules

Then, it must be emphasised that the study of rule returns can lead to the discovery of new models of financial prices The technical models first described in the thesis are a good example of this point That would let think that more research is needed to build automated selection criteria between linear models, if the purpose of the forecaster is trading

Finally, the causes of profits, when any, to the technical trading strategies, have to be found

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