**Optimal energy management for a flywheel assisted battery electric vehicle**

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**Abstract**

Battery electric vehicles (BEV) are crucial to the reduction of dependence on fossil fuels and for moving towards a zero emission transport system. Though BEV technology has been rapidly improving, the limited driving range and high cost are significant impediments to the popularity of electric vehicles. The battery is the main element which affects the range and cost of the vehicle. The batteries can provide either high power or high energy but not both. Hybridization of the energy source is one of the methods to improve the energy efficiency of the vehicle, which would involve combining a high energy battery with a high power source. High speed flywheels (FW) have attractive properties and low cost potential which makes them excellent secondary energy storage devices to be used in hybrid and electric vehicles. They are utilized to load level the battery so as to protect it from peak loads and enhance its capacity and life. The flywheel is coupled to the drive line with a continuous variable transmission (CVT). This paper presents the optimal energy management strategy (EMS) for a mechanically connected flywheel assisted BEV (FWBEV) powertrain. The optimization problem is complex due to factors such as the small storage capacity of flywheel, kinematic constraints and slipping of clutches. Dynamic programming is used to calculate the optimal control strategy for torque distribution during operation in real world driving cycles. The results show significant potential for reduction of energy consumption in extra-urban and highway cycles, while reducing battery peak loads during all cycles. The results give a benchmark of the energy saving potential for such a powertrain and insights into how a real sub-optimal controller can be designed.

Keywords: flywheel, battery electric vehicle, energy management, dynamic programming, hybrid vehicle

## 1. Introduction

BEVs offer a promising solution to the problem of reducing carbon dioxide emission from automobiles. Since the last few years a small number of mass produced BEVs such as the Nissan Leaf, the Mitsubishi iMiEV and the Tesla Roadster have been introduced in the markets worldwide

and many more are in the pipeline. However their high cost and limited range, relative to conventional vehicles, are still issues that impede their popularity[1]. The most important element in the BEV is the battery. Though current battery technology offers significant improvement over previously used ones, it is still the most important bottleneck in BEVs and strongly affects the range and cost of the BEV. The batteries offer either high specific power or high specific energy but not both. To provide the BEVs with the characteristic to compete with conventional vehicles it is beneficial to hybridize the energy storage[2]. The typical strategy would be to combine a high energy battery with another high power source. This would shield the battery from peak currents and improve its capacity and life. The challenge of keeping the battery within its preferred operating range would also be greatly reduced. Chau and Wong[3] have discussed the concept of hybridization of the energy source in electric vehicles.

Flywheels are excellent secondary energy storage devices and several applications in road vehicles are under development[4]. High speed flywheels have the characteristics of high specific power, high specific energy, long cycle life, high energy efficiency, quick recharge, low cost and environmental friendliness. They do not suffer from temperature dependence and their state of energy (SOE) is most easily determined. The FW is the only energy storage device that keeps the energy stored in the same form as the moving vehicle i.e. mechanical energy. Dhand and Pullen[5] have discussed in great detail the concept, layouts and advantages of such a hybrid energy storage (HES) comprising of battery and high speed FW for BEV. The main characteristics to define the FW as secondary storage for BEV have been discussed by Dhand and Pullen[6]. As the flywheel usually gains speed when the vehicle is slowing down and loses speed when the vehicle is accelerating, a CVT is used to connect the FW to the driveline. The requirements of the CVT for flywheel energy storage system (FESS) are quite different from those in a conventional vehicle and have been discussed in detail by Dhand and Pullen[7].

The main benefits of the FW in the HES with battery are as follows:

- Improve energy efficiency of the battery by taking care of the peak loads, which would reduce losses in the battery and improve range of the BEV
- Increase life of the battery
- Allow the optimization of battery as pure energy source
- Reduce the cooling requirements of the battery at high temperature and protecting the battery and associated electronics during vehicle start-up in cold conditions when the battery resistance is high
- Allow the powertrain to achieve better regenerative braking efficiency by avoiding energy conversion
- Potential downsizing of the main electric machine in case the FW is connected via a mechanical transmission

In this paper the design of an optimal EMS for a mechanical CVT connected FW assisted BEV powertrain is presented. It will be referred as the hybrid vehicle (HV) since it has two sources of energy. It is based on a C-segment hatchback passenger car as this is the one of most common cars used in private transport especially in Europe. The powertrain is shown in Fig. 1. It consists of a main electric machine (EM) connected to the driveline via clutch B and a fixed reduction gear. The clutch B is used to disengage the drive line to enable the EM to charge the FW when the vehicle is stationary.

The FW is connected to a CVT which is connected to the drive shaft via clutch A. The clutch A is used to provide the CVT with the gear neutral ability. The paper[8] shows the baseline BEV for the presented HV. Besides the FW, CVT and clutches, the one main difference is that the EM in the HV is a downsized version (by about 37%) of the one in the base BEV. Also due to these differences the HV is slightly heavier (by about 40 kg) than the base BEV (kerb mass: 1445 kg) and also has higher rotating inertia. In this paper, the design and sizing of components will not be discussed and the emphasis would be on the design of optimal energy management strategy for such HVs. Fig. 2 shows the general power flow of the drivetrain.

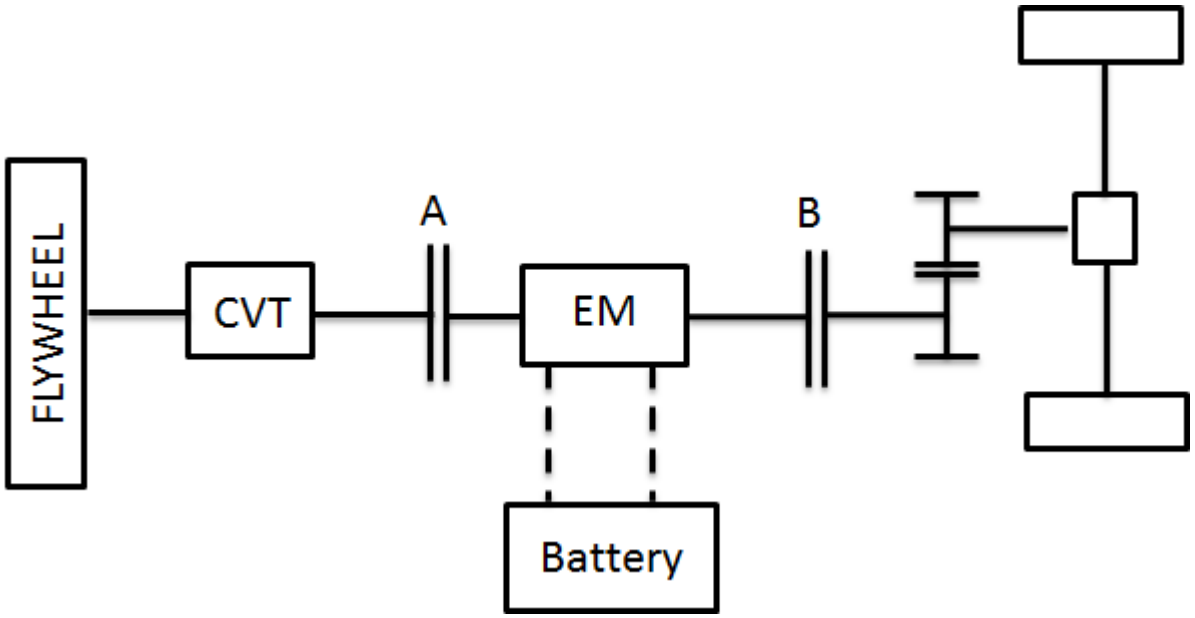


**Figure 1 Schematic of the HV**

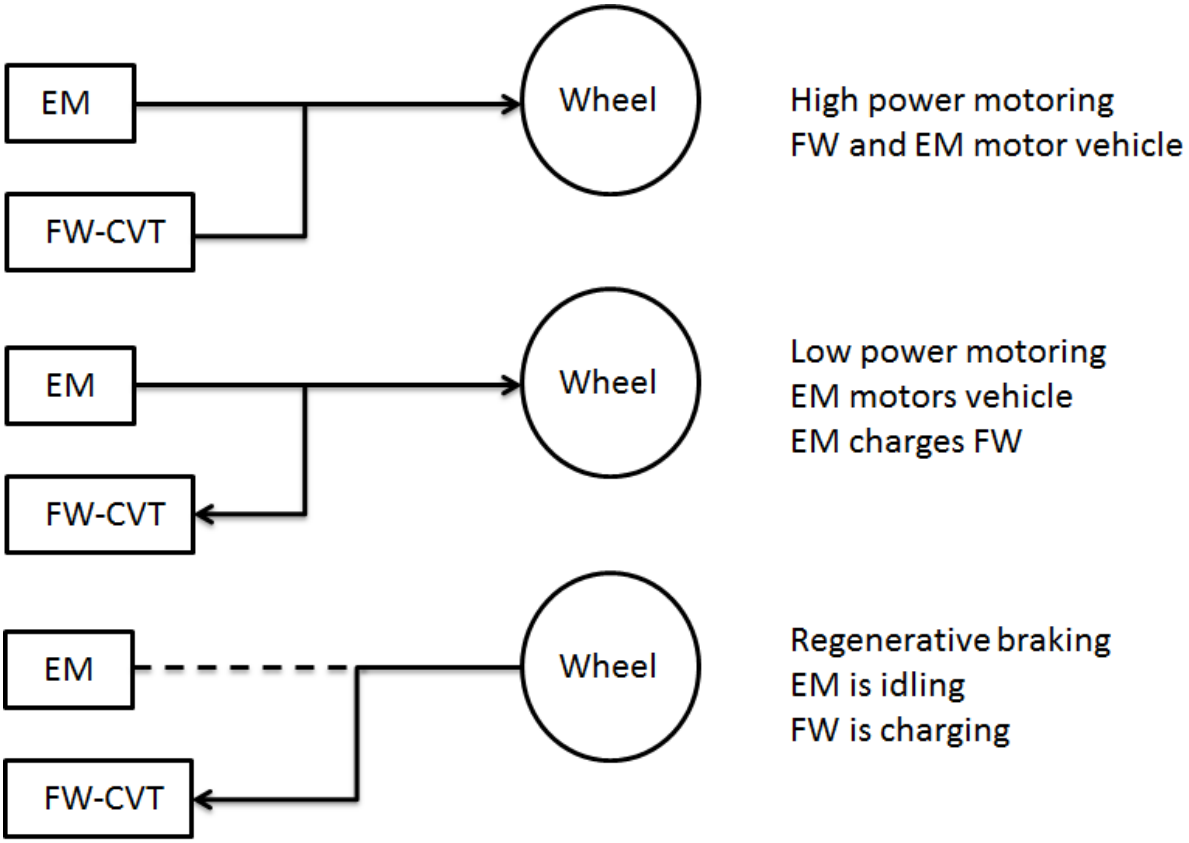


**Figure 2 Power flow of the drivetrain**

## 1.1 Energy Management Strategy

In general the control strategy is one of the most important elements, which decides the energy consumption of the HV. The primary purpose of the supervisory controller is to specify the power distribution between the two sources of energy in the system in order to maximize energy economy while achieving the driver demand. There might be additional requirements such as maintaining state of charge (SOC) of the energy storage and driveability of the vehicle. There are various types of control strategies which can be classified by various means. One type classifies them based on their dependency of the knowledge of future driving conditions as causal and non-causal[9]. Another classification broadly groups them into two categories; Heuristic and Optimal.

- Heuristic control: These are rule based strategies which are generally intuitive in nature. These provide the instantaneous operation of the system depending upon the information of the current or future states. These are easy to implement and are most commonly used in prototypes and production hybrids.
- Optimal control: The objective of optimal control is to provide a set of control parameters which will cause the system to satisfy certain constraints while minimizing or maximizing certain performance criteria. There are two approaches, of which one is based on the work of Richard Bellman[10] called dynamic programming (DP) and the other based on the work of Lev Pontryagin[11] called Pontryagin's minimum principle.

DP is a very useful tool to find out the optimal solution to non-linear problem with given boundary conditions. It results in global optimum, though it is non-causal in nature and requires that the drive cycle to be known in advance. It generally takes a lot of computing power and time, which increases with the number of state variables and length of drive cycle. Due to these facts it does not offer an online implementable solution, though it can be used to set a benchmark for the performance of other sub optimal control strategies. It also provides insight on how the realistic sub optimal controller should be designed.

DP has been extensively applied for the optimization of energy management of hybrid electric vehicles (HEV) to achieve maximum fuel economy over pre-defined drive cycles[12-18]. However, there are only few cases of DP being applied for FW based mechanical hybrid vehicles and all these cases are exclusively for FW based internal combustion engine hybrid vehicles (FWICEHV) [19-21]. Jamzadeh[19] applied DP to find the optimal control policy for a FWICEHV on the federal urban drive cycle (FUDC). In this case the internal combustion engine (ICE) and FW are coupled to the CVT and the driver controls the vehicle torque by the CVT and has no control over the engine operation. The only control decision to be made is whether the ICE should be on or off. This simplifies the system control to a great extent. Van Berkel[20] used DP to optimize the fuel economy for a FWICEHV. In this case the ICE and the FW are connected using clutches and the CVT is downstream. The speeds of ICE and FW are linked and there is no mode of operation where the FW and ICE simultaneously motor the vehicle. Dingel[21] used DP to benchmark and compare the fuel savings for an HEV and a FWICEHV. It has been recognised by Van Berkel[20] and Dingel[21] that unlike for an HEV, there is no univocal approach for applying DP to a FW based mechanical HV and the process is more complex than for an HEV due to many factors including the relatively many kinematic constraints, small energy capacity of the FW and slipping clutches. In case of a mechanically connected FWBEV, the optimization using DP is further complicated due to the fact that both the battery and the FW have state variables associated with them and there are additional options for achieving specific functions. There is no example in literature showing the application of DP to find optimal EMS for FWBEV.

This paper presents the optimal EMS for the mechanical transmission based flywheel assisted BEV powertrain. The main criterion is the minimizing of energy consumption during the pre-defined drive cycle and system constraints are defined. DP is used to calculate the optimal torque distribution over the cycle. Additional options such as vehicle pull away using slipping clutch and using EM as well as variation of initial FW SOE are explored. For the implementation of DP, the model of the hybrid vehicle needs to be defined as discrete step using the backward power flow approach[22].

The paper is organised as follows. The following section 2 describes the various component models and the vehicle model. The various HV modes are explained in section 3. Section 4 defines the optimization problem and DP implementation. The results and discussion are presented in section 5. Finally section 6 presents the conclusions.

## 2. Component models

### 2.1 Battery

The battery here is modelled simply as consisting of an internal resistance and an open source voltage. The primary reason for choosing this model is to reduce complexity and save on computation time. $V_{OC}$ is the open circuit voltage of the battery and $r_{\text{int}}$ is the internal resistance which depend on the SOC of the battery. $P_{bat}$ and $C_{bat}$ denote the power and the capacity of the battery. The simulation time step is denoted as $\Delta t$. Battery current and voltage are denoted by $i_{bat}$ and $V_{bat}$ respectively.

$$V_{bat} = V_{OC} - r_{\text{int}} \times i_{bat} \quad [1]$$

$$P_{bat} = V_{OC} \times i_{bat} - r_{\text{int}} \times i_{bat}^2 \quad [2]$$

$$SOC_t = \frac{SOC_{t-1} \times C_{bat} - i_{bat,t} \times \Delta t}{C_{bat}} \quad [3]$$

## 2.2 Electric Machine

The EM is modelled by using a characteristic map specifying efficiency as a function of torque and speed. $P_{EM}$, $T_{EM}$, $\omega_{EM}$ and $\eta_{EM}$ describe the power, torque, speed and efficiency of the EM respectively. The auxiliary power ($P_{aux}$) includes which includes the power required for vehicle housekeeping and pump losses of the transmission.

$$P_{bat} = P_{EM} + P_{aux} \quad [4]$$

$$P_{EM} = \frac{T_{EM} \times \omega_{EM}}{\eta_{EM}} \quad [5]$$

## 2.3 CVT and FW

The CVT is modelled by its efficiency and lumped input and output inertias. The idling losses of the CVT are neglected. The FW and the input inertia of the CVT are modelled as a single inertia. The following equations describe the torque acting on the FW depending on whether it is providing energy or absorbing it. $T_{CVT}$, $r_{CVT}$ and $\eta_{CVT}$ give the CVT torque, speed ratio and efficiency of the transmission. $T_{FW}$, $T_{loss}$, $\omega_{FW}$, $J_{FW}$, $SOE$ and $E_{FW}$ are defined as the net flywheel torque, flywheel loss torque, flywheel speed, flywheel inertia, state of energy and energy capacity of FW.

In case the FW is providing energy, the following applies

$$T_{FW} = \frac{T_{CVT} \times r_{CVT}}{\eta_{CVT}} + T_{loss} \quad [6]$$

In the case the FW is absorbing the energy

$$T_{FW} = T_{CVT} \times r_{CVT} \times \eta_{CVT} + T_{loss} \quad [7]$$

The loss of the FW is defined as the energy loss of 2% per minute.

$$T_{loss} = 0.5 \times J_{FW} \times \omega_{FW} \times \frac{\log(0.98)}{60} \quad [8]$$

$$\omega_{FW,t} = \Delta t \times \frac{T_{FW,t}}{J_{FW}} + \omega_{FW,t-1} \quad [9]$$

$$SOE = \frac{1}{2} \times J_{FW} \times \frac{(\omega^2{}_{FW} - \omega^2{}_{FW,\min})}{E_{FW}} \quad [10]$$

### 2.4 Driveshaft and Vehicle

The torque required at the drive shaft ($T_{drs}$) is the summation of $T_{EM}$ and $T_{CVT}$ .

$$T_{drs} = T_{EM} + T_{CVT} \quad [11]$$

The driveshaft torque is simply derived from the torque at the wheel ($T_{whl}$) by using the various vehicle resistances, final drive ratio ($r_{FD}$) and efficiency ($\eta_{FD}$). In case of the motoring vehicle, the following applies

$$T_{drs} = \frac{T_{whl}}{r_{FD} \times \eta_{FD}} \quad [12]$$

In case of braking vehicle, the following applies

$$T_{drs} = \frac{T_{whl} \times \eta_{FD}}{r_{FD}} \quad [13]$$

$$T_{whl} = \left(m \times A_{veh} + 0.5 \times C_d \times Ar_{veh} \times \rho_{air} \times V_{veh}^2 + m \times g \times f_r\right) \times r_{dyn} \quad [14]$$

where $A_{veh}$ is the acceleration of the vehicle, $m$ the equivalent mass of the vehicle including the rotating inertias, $C_d$ the discharge coefficient, $Ar_{veh}$ the frontal area, $\rho_{air}$ density, $V_{veh}$ velocity of vehicle, $g$ acceleration due to gravity, $f_r$ rolling resistance and $r_{dyn}$ the rolling radius.

## 3. Hybrid Vehicle Operation

The torque at the drive shaft is calculated in advance for the pre-defined drive cycle. It is assumed that the FW has an initial speed. Following are the various modes of operation.

3.1 $V_{veh} = 0$

The vehicle is stationary and the clutch A is not engaged. During this mode the speed of the FW is decreasing due to the friction losses of the FW and the CVT is at its minimum speed ratio. The only electric load is the auxiliary load.

3.2 $T_{whl} > 0$ and $\omega_{EM} < (\omega_{FW} \times r_{CVT,\min})$

The vehicle is motoring and the clutch A is not engaged. In this mode there are two options, either the EM motors the vehicle till the speed difference between the input and output sides of the clutch is overcome or the FW motors the vehicle while the clutch is slipping till the speed difference is overcome. In the former case, the FW is idling and in the latter case the EM is idling. The CVT ratio is maintained at its minimum value. The electric load on the battery is the sum of the auxiliary load and EM load, in case it is used to motor the vehicle.

3.3 $T_{whl} > 0$ and $\omega_{EM} \geq (\omega_{FW} \times r_{CVT,\min})$

The vehicle is motoring and the clutch A is engaged. During this mode, the torque is split between the EM and the FW, though the option of the EM providing torque to the vehicle and charging the FW is also there. The CVT ratio changes accordingly. Theoretically the FW could also be used to charge the battery via EM, but that option is not used as that would negate the primary purpose of using the FW as the secondary storage device.

3.4 $T_{whl} < 0$ and $\omega_{EM} \geq (\omega_{FW} \times r_{CVT,\min})$

The vehicle is braking and clutch A is engaged. In this case the FW is performing regenerative braking and EM is idling. The CVT ratio varies as needed.

3.5 $T_{whl} < 0$ and $\omega_{EM} < (\omega_{FW} \times r_{CVT,\min})$

The vehicle is braking and clutch A is not engaged. In this case the mechanical brakes are used. This case as well any other case where the clutch A is not engaged, would usually take place at vehicle speeds below 10-15 kph. This situation is similar to the case with the base BEV[8] where the EM does not do brake energy recuperation below 10 kph.

## 4. Optimization

The target of the optimization process is to reduce the energy consumed during a drive cycle. For this purpose the control objective taken is to minimize the total charge removed from the battery which is a direct indicator of the energy consumed during the cycle. Thus the formal problem statement can be written as “to find a control which causes the system to follow a trajectory that minimizes the total charge consumed from the battery during a drive cycle”.

$$\min \sum_{t=1}^{N} i_{bat} \times \Delta t \qquad [15]$$

The boundary condition is that the FW state of energy (SOE) at the end should be the same as that in the beginning so that there is no net energy stored in the FW.

$$SOE(N) = SOE(1) \quad [16]$$

The two state variables are FW SOE and battery SOC. The EM torque is the control variable and is used to derive the FW state. The constraints are applied on the FW speed, CVT ratio, EM torque and battery current. The FW and CVT ratio have to be within their minimum and maximum limits. The transmission design limits have been considered during its design, so they are not applied here. The rate of change of CVT ratio is also observed. Further DP requires gridding of the state and control variables. The important thing to make sure is that this grid should be balanced, in other words the action of the control variable on the state variable should change its state from one grid point to another one which is as close as possible to a grid point. This has an important effect on the computation time. A more balanced grid significantly reduces the computation time. Fig. 3 shows the DP procedure to calculate $SOC_t$, $SOE_t$ and $i_{bat,t}$ at time $t$ when $SOC_{t+1}$ and $SOE_{t+1}$ are known at time $t+1$ during the vehicle motoring operation when the clutch A is engaged. The rest of the process is the usual one.

$$\omega_{FW\min} \le \omega_{FW,t} \le \omega_{FW\max} \quad [17]$$

$$r_{CVT\min} \le r_{CVT,t} \le r_{CVT\max} \quad [18]$$

$$T_{EM,t} \le T_{EM\max} \quad [19]$$

$$i_{bat,t} \le i_{bat\max} \quad [20]$$

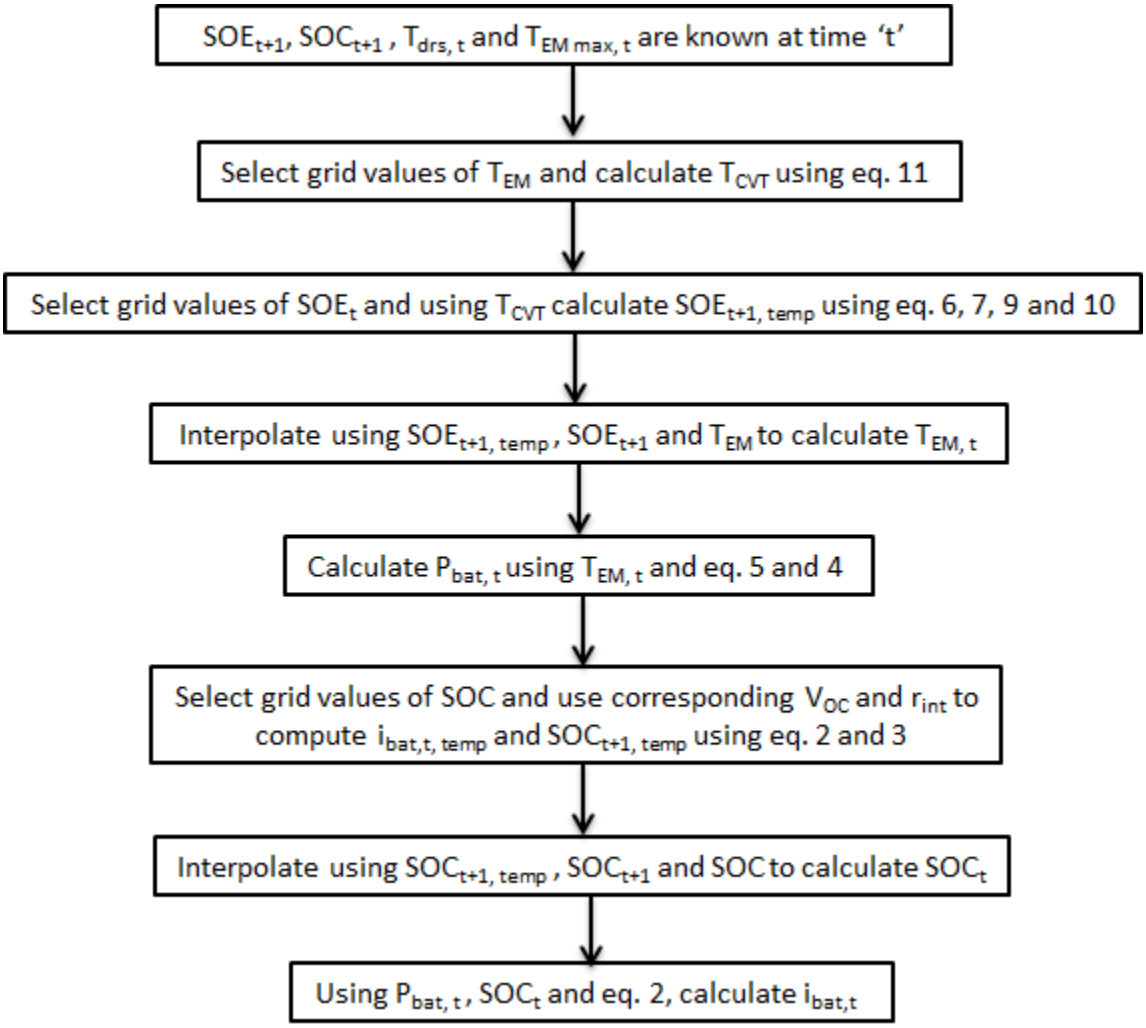


**Figure 3 DP process during the mode when vehicle is motoring and clutch A is engaged**

The DP is performed on three real world driving cycles which cover urban, extra-urban and highway driving. The chosen cycles are Artemis urban (AU), LA92 and US06 cycles. These have been chosen rather than the homologation cycles, which are frequently used by others, as they provide realistic driving situations. Since in case of flywheel hybrid the start SOE of the FW can be controlled, the DP is run at intervals of 10 % SOE for the three cycles. Since the SOE is to be balanced, the DP is run and the control trajectory which gives the SOE balance with the minimum charge consumption for the drive cycle is selected. The following Fig. 4 shows the result for the Artemis Urban cycle. It can be clearly seen that the smallest charge consumption is achieved when the process is started with the smallest initial FW SOE. This is expected since the FW has to be returned to the same SOE as that at the beginning. Further the same process is repeated for the US06 cycle and LA92 cycle and lowest possible initial SOE is the one with least charge consumption. As expected, it is seen that the LA92 and US06 cycles, require a higher initial SOE than the AU cycle sue to the fact that they are relatively higher power cycles. All further DP runs would be done at the initial SOE decided at the previous step for the three concerned cycles.

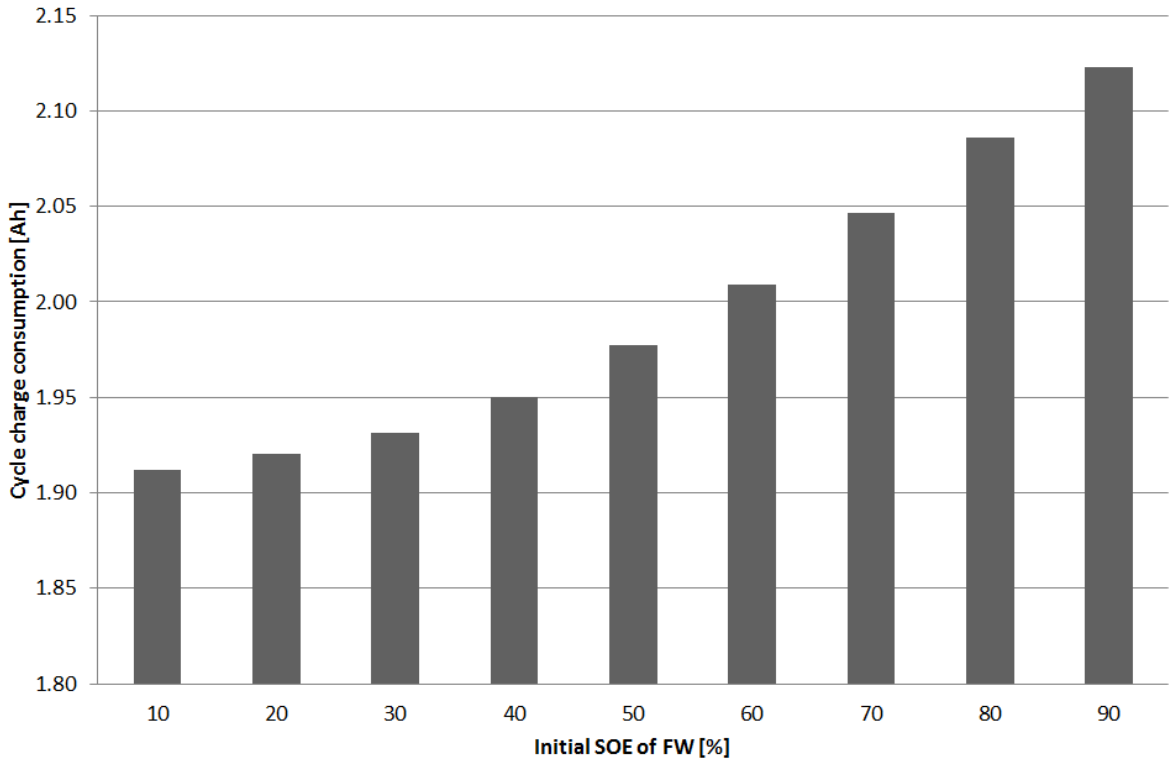


**Figure 4 Cycle charge consumption [Ah] for AU cycle for different initial FW SOE**

As mentioned previously there are two ways of pulling away the hybrid vehicle. It can be done via the EM or via the FW by slipping the clutch A. To test which one to select, DP is run for both the options. Fig 5 shows the increase in cycle charge consumption while going from the option of EM based pull away to FW based pull away. The pull away via EM is more favourable since it consumes less charge. This is due to the energy lost while the clutch is slipping and the vehicle is pulling away. The power lost in the clutch can be calculated by multiplying the torque passed through the clutch and the speed difference across it. Another thing to observe is that the increase in cycle charge consumption is highest for the AU cycle and lowest for the US06 cycle which is expected since the AU is a relatively low powered cycle as compared to US06 cycle so the EM power required to pull away the vehicle is lower which in turn gives lower battery losses. Again for all further DP runs, it is decided based on the result that the vehicle is to be pulled away by using the EM.

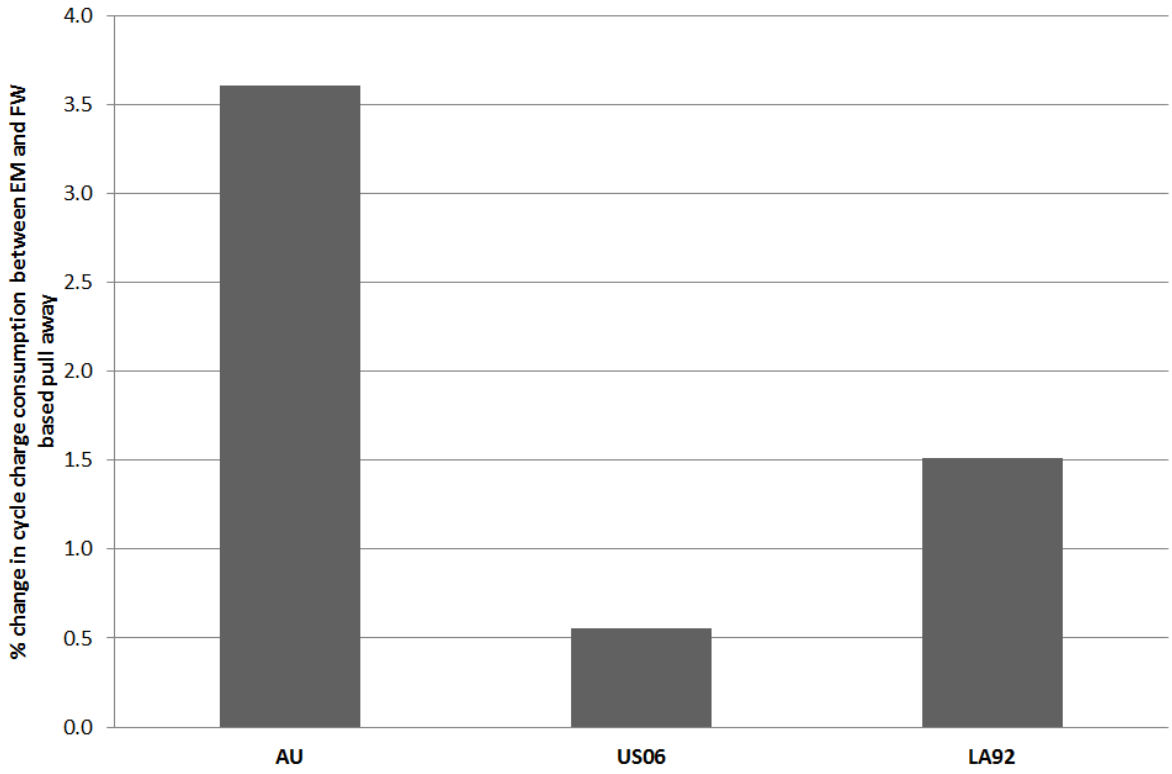


**Figure 5 Difference [%] cycle charge consumption between EM and FW vehicle pull away**

The calculations up to now have been performed on a time step of 0.5 s so as to save computation time. The next step involves comparing the energy consumption of the hybrid vehicle with that of the base BEV on the three cycles. The original base vehicle model[8] was a forward simulation model with variable time step built in AVL Cruise[23]. However for this comparison a backward simulation model with fixed time step is created and that is used to compute the energy consumption to compare it with the hybrid vehicle. Further a suitable time step needs to be chosen which would be a compromise between computation time and accuracy. For this process a smaller time step of 0.1 s is chosen. Applying an average speed of 30 kph for the drive cycles and a torque resolution of 2 Nm for EM torque, the energy handled by the CVT is around 44 J which is roughly equivalent to 0.005 % SOE of the FW if losses are neglected. For the further process, FW SOE resolution of 0.005% and 2 Nm torque increment are used.

## 5. Results and Discussion

In this section the results between the base BEV and the HV are presented and analysed for the three cycles. The start SOC of the battery for the simulations is taken between 85-90% since above 90% regenerative braking is not allowed in the base BEV[8]. The simulations are run steady state at temperature of 25° C. These conditions also represent the best case for BEV since at higher/lower temperatures or at lower SOC; the performance of the battery is expected to be poorer. The Fig. 6 shows the three drive cycles and the variation of FW SOE. It can be seen that due to the end constraint in the DP the final SOE converges to the initial value. Fig. 7 shows the torque split in the

three drive cycles. It can be seen that the EM torque is much reduced during high driveshaft demand torque and the brake torque demand is provided entirely by the FW. Further Fig. 8 shows the energy economy comparison between the base BEV and the HV on the three cycles. It can be seen that the HV has a higher cycle energy economy by 11% and 3.2% as compared to the base vehicle in US06 and LA92 cycles respectively, though it has reduced energy economy by 2% in the AU cycle. Further table 1 provides the comparison of individual component efficiencies of the drive train and other important results.

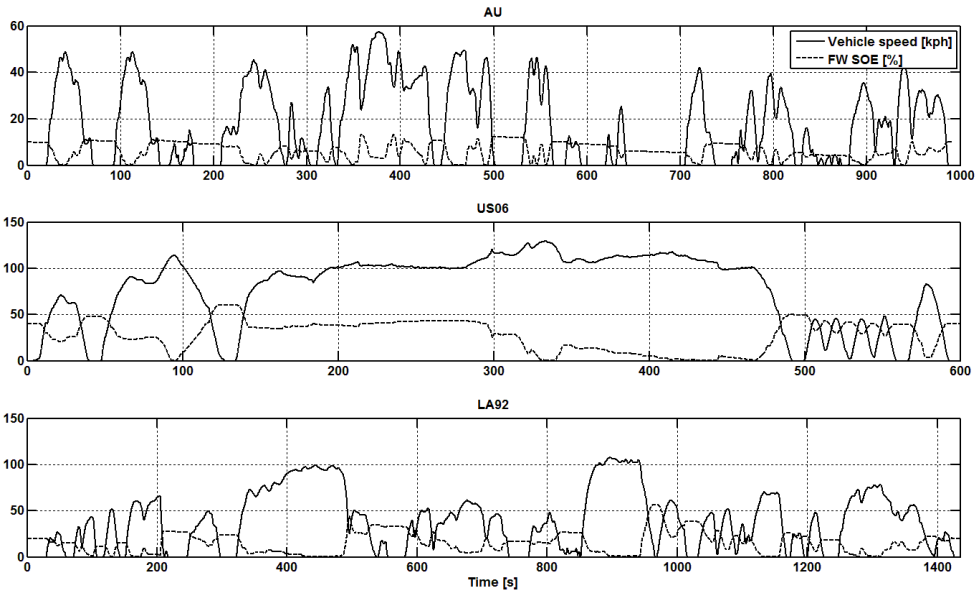


**Figure 6 Vehicle speed [kph] and FW SOE [%] during drive cycle**

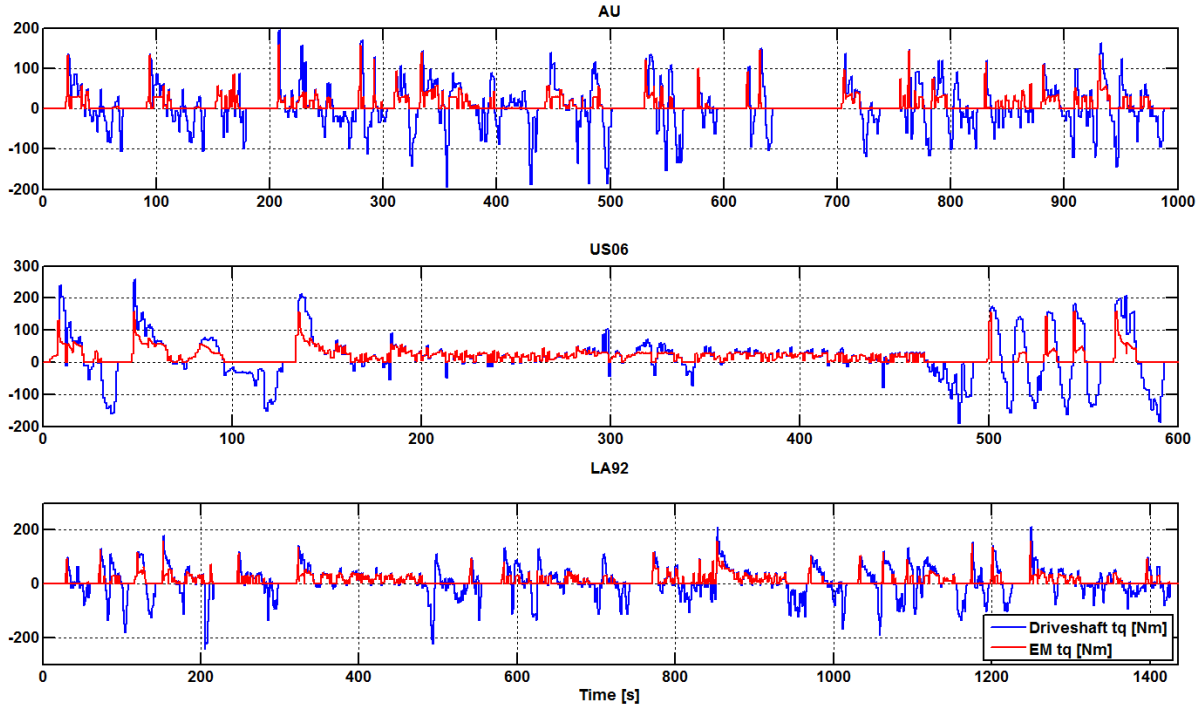


**Figure 7 Driveshaft torque [Nm] and EM torque [Nm] during drive cycles**

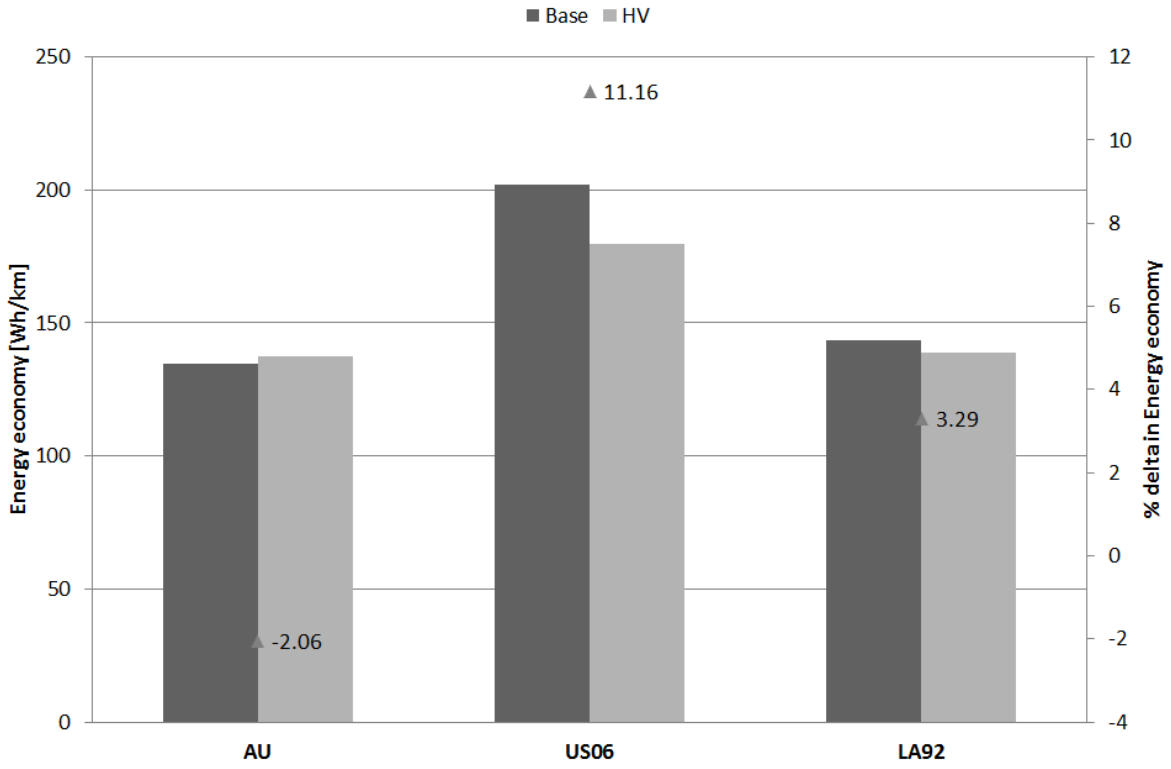


**Figure 8 Energy economy [Wh/km] for the drive cycles and difference [%] in energy economy between BEV and HV**

**Table 1 Comparison between BEV and HV**

| Quantity | AU | | US06 | | LA92 | |
|---|---|---|---|---|---|---|
| | Base | HV | Base | HV | Base | HV |
| Start SOC battery [%] | 88.25 | | 86.30 | | 85.77 | |
| Start FW SOE [%] | - | 10 | - | 40 | - | 20 |
| Motoring energy cycle [Wh] | 839.88 | 891.59 | 2356.16 | 2447.37 | 2486.89 | 2607.06 |
| Cycle energy consumption [Wh] | 655.33 | 668.80 | 2603.93 | 2313.22 | 2264.75 | 2190.27 |
| Battery losses [Wh] | 103.14 | 21.04 | 601.2 | 211.51 | 394.54 | 150.84 |
| Round trip battery efficiency [%] | 84.26 | 96.85 | 76.90 | 90.85 | 82.57 | 93.11 |
| EM losses [Wh] | 159.3 | 48.4 | 313.8 | 157.4 | 379.8 | 148.2 |
| EM efficiency [%] | 89.17 | 89.96 | 90.55 | 92.13 | 90.19 | 91.76 |
| CVT losses [%] | - | 95.6 | - | 148.7 | - | 212.1 |
| CVT efficiency [%] | - | 91.21 | - | 89.99 | - | 90.71 |
| FW losses [Wh] | - | 33.9 | - | 31.6 | - | 60.2 |
| Auxiliary losses [Wh] | 82.75 | 165.5 | 50 | 100 | 119.5 | 239.1 |
| Peak current [A] | 109 | 64 | 261 | 123 | 177 | 118 |
| Peak battery power [kW] | 46.44 | 23.17 | 69.95 | 40.29 | 66.6 | 38.36 |
| Average battery power [kW] | 2.0 | 2.34 | 12.01 | 12.61 | 4.69 | 5.11 |
| % of FW energy gained from EM [%] | - | 0.06 | - | 2.08 | - | 0.42 |

The first thing to observe in table 1 is that the motoring energy required in the cycle for the HV is higher than that of the base. This is primarily due to the higher weight and inertia in the HV. As has already been shown in Fig. 8, the HV has higher energy economy than the base vehicle in the US06 and LA92 cycles and has a corresponding lower value in the AU cycle. The primary aim of the FW in the HES is to protect the battery from high currents to improve its efficiency and life. The improvements in efficiency occur mainly because of the lower battery losses. As can be seen from the table 1 that the battery losses are significantly reduced for the HV as compared to the base vehicle which results in impressive improvements in the battery round trip efficiency.

Further since the EM in the HV is a downsized version of the one in the base vehicle, the operating points on the EM in the HV occur in the higher efficiency regions as compared to the base vehicle. The paper[8] explored the effect of downsizing the EM on the BEV energy consumption. Due to this fact, a slight improvement in the EM cycle efficiency is observed in the HV as compared to the base. Besides the efficiency improvement, additional benefit is expected in terms of cost due to the downsizing. The CVT efficiency in the cycle is around 90%. Another important advantage of the FW in the HV is the drastic reduction of the peak current and peak electric power of the battery as compared to the base vehicle. This reduces the stress on the battery significantly and is expected to improve the life of the battery and lower operating costs. The average battery power is slightly higher in the HV than the base vehicle to take into account the losses in the FW and CVT system.

Although there are significant benefits obtained in the HV in regards to lower battery peak current and power, the energy consumption in the AU cycle is higher than the base vehicle. It is important to note that though the consumption is higher by 2.06%, the absolute value (13.4 Wh) is quite small. The main reason for this is that the AU cycle is relatively lower power cycle than the US06 and LA92 cycles, and the lower battery losses in the HV are negated by higher required cycle motoring energy and auxiliary losses. The higher auxiliary losses in the HV are due to the fact that it includes the

vehicle housekeeping electric loads and transmission pump losses. In the other cycles, these factors have a much lower impact due to the relatively higher power required in these cycles. A check was done to find out if the energy consumption of the HV in AU cycle would improve if it was run as a pure BEV. In this simulation, the clutch A was kept open and the auxiliary power was reduced to the level of the base BEV. It was found that the energy consumption of the HV actually increases by 1.08%.

Another interesting point to note is that although there is an option of the FW being charged by the EM during the vehicle motoring, this option is almost always avoided by the optimal EMS. The percentage of FW energy which is input by the EM is quite low. The bulk of the FW energy is gained only by regenerative braking. The highest is about 2% in the US06 cycle, which reflects the fact that it is most high power cycle among the three drive cycles.

## 6. Conclusions

The BEV is an important technology to reduce the dependence on fossil fuels and though significantly improved over the years, it still has significant challenges in terms of cost and range. One of the methods to improve the BEV is to hybridize the energy storage. FW is an excellent secondary energy storage system which can be used to complement the battery in HES. This would reduce the stress on the battery and improve its efficiency and life. This paper presents the optimal energy management strategy for a mechanical CVT connected FW assisted BEV powertrain. Dynamic programming has been used to find the optimal EMS, which is the first instance of its implementation for a FW assisted BEV application, in three real world driving cycles and the results have been compared to the base BEV. Detailed analysis of the energy saving contribution and efficiency for all the components has been conducted. The simulations show significant potential for reduction of energy consumption in extra-urban and highway cycles, while reducing battery peak loads during all cycles.

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Notation

| | |
|---|---|
| A, B | clutches |
| $Ar_{veh}$ | Frontal area of vehicle |
| $A_{veh}$ | Vehicle acceleration |
| AU | Artemis Urban |
| BEV | Battery electric vehicle |
| $C_{bat}$ | Battery capacity |
| $C_d$ | Coefficient of discharge |
| CVT | Continuously variable transmission |
| DP | Dynamic programming |
| $E_{FW}$ | Energy capacity of FW |

| | |
|---|---|
| EM | Electric machine |
| EMS | Energy management system |
| FESS | Flywheel energy storage system |
| $f_r$ | Rolling resistance coefficient |
| FUDC | Federal urban drive cycle |
| FW | Flywheel |
| FWBEV | Flywheel assisted battery electric vehicle |
| FWICEHV | Flywheel internal combustion engine hybrid vehicle |
| g | Acceleration due to gravity |
| HES | Hybrid energy storage |
| HEV | Hybrid electric vehicle |
| HV | Hybrid vehicle |
| $i_{bat}$ | Battery current |
| ICE | Internal combustion engine |
| J | Moment of inertia |
| m | Mass of vehicle |
| $\eta$ | Efficiency |
| $\eta_{FD}$ | Efficiency of FD |
| P | Power |
| $P_{aux}$ | Auxiliary power |
| $P_{bat}$ | Battery power |
| $\rho_{air}$ | Density of air |
| $r_{CVT}$ | CVT speed ratio |
| $r_{dyn}$ | Dynamic radius of wheel |
| $r_{FD}$ | Final drive ratio |

| | |
|---|---|
| $r_{int}$ | Internal resistance |
| SOC | State of charge |
| SOE | State of energy |
| $\Delta t$ | Time step |
| t | Time |
| T | Torque |
| $T_{loss}$ | Loss torque |
| $T_{drs}$ | Driveshaft torque |
| $T_{whl}$ | Wheel torque |
| $V_{bat}$ | Battery voltage |
| $V_{OC}$ | Open circuit voltage |
| $V_{veh}$ | Velocity of vehicle |
| $\omega$ | Rotational speed |