Determination of design spectrum compatible evolutionary spectra via Monte Carlo peak factor estimation

Agathoklis Giaralis[1], Pol D Spanos[2]
[1]School of Engineering and Mathematical Sciences, City University, London, UK
[2]George R. Brown School of Engineering, Rice University, Houston, TX, USA

Abstract: The problem of generating ensembles of artificial non-stationary earthquake accelerograms compatible with a given (target) response/design spectrum is cast on a stochastic basis. The design spectrum of the European aseismic code provisions (EC8) for various soil conditions and damping ratios is used as a paradigm of a design/target spectrum. The generated accelerograms are construed as realizations of a non-stationary random process; they are characterized in the frequency domain by a parametrically defined evolutionary power spectrum (EPS). An appropriate least squared optimization problem is formulated for the determination of the parameters of the EPS. The solution of this problem involves the incorporation of a "peak factor" which is used to relate the target spectrum to the EPS in a probabilistic context. To this end, a comprehensive Monte Carlo study is undertaken to estimate numerically the statistical properties of the peak factor from appropriately computed populations, and to derive polynomial expressions for the median frequency-dependent peak factors (peak factor spectra). These expressions are used in conjunction with the herein adopted optimization problem to determine EPSs compatible with the EC8 design spectrum. The derived median peak factor spectra yield an excellent level of agreement between the EC8 spectrum and the ensemble average and median response spectra of simulated EPS-compatible ensembles of accelerograms.

1 Introduction

Typical dynamic loads applied to various structured facilities by the natural environment such as those associated with the action of earthquakes, winds and sea waves exhibit an inherent level of uncertainty. This uncertainty can be readily incorporated in structural design procedures by representing the induced dynamic loads in a statistical manner via the concept of a stochastic process.

Note that, contemporary code provisions regulating the aseismic design of structures represent the input seismic loads by means of analytically defined response/design spectra (e.g. [4]). This practice allows for considering linear dynamic response-spectrum based types of analyses which significantly facilitates the design of ordinary structures (e.g. [5]). Nevertheless, by its definition, a design spectrum neither corresponds to an individual earthquake record associated with a certain seismic event nor to a specific stochastic process. In this respect, design spectra cannot be straightforwardly considered in conjunction with dynamic time-history analyses which involve the complete knowledge of the acceleration trace (accelerogram) of the motion of the ground. The incorporation of these kinds of analyses is commonly mandated by regulatory agencies in the design of special structures and facilities of critical importance. In this context, aseismic code provisions require the representation of the seismic severity by means of suites of design spectrum compatible accelerograms. That is signals whose average response spectrum is in agreement with a prescribed (elastic) design spectrum satisfying specific "compatibility criteria". Such accelerograms can be derived either by manipulation of field recorded ground motions, or by generation of time-histories belonging to an appropriately defined stochastic process which is consistent with the given design spectrum (see e.g. [7], [25] and references therein).

To this end, various researchers have proposed methods to relate a response/design spectrum to a power spectrum characterizing a stationary stochastic process (e.g. [8]; [17]; [16]). This relation necessitates the consideration of the so-called "peak factor" which is closely associated with the first passage problem of the response of stochastically excited linear single-degree-of-freedom (SDOF) systems (see e.g. [26]). Obviously, assigning a stationary process to a response/design spectrum involves a rather restrictive limitation in dealing with an inherently non-stationary phenomenon (i.e. the strong ground motion during a seismic event). Nevertheless, a very limited number of research studies have considered the case of relating an evolutionary power spectra (EPS) characterizing a non-stationary random process to a given response/design spectrum. The main difficulty in this case, is that no reliable expressions for the peak factor exists. Certain previous studies (e.g. [10]; [27]; [20]; [11]; [12]) have provided numerical results associated with the peak response and the first passage problem of linear SDOF systems excited by non-stationary input processes. However, the considered input EPSs have been somewhat arbitrarily selected as either modulated white noise, or colored noise having a boxcar envelop function. Obviously, these forms of EPSs have limitations in modelling seismic acceleration processes.

In this study, a formulation originally proposed by SPANOS and VARGAS LOLI [23] and recently extended by GIARALIS and SPANOS [7] and SPANOS et al. [25] is adopted to relate a parametrically defined EPS to the design spectrum prescribed by the European code provisions (EC8) [4]. It involves the solution of an inverse stochastic dynamics problem and necessitates the consideration of a peak factor. The adopted parametric form of the EPS is used as a mathematical instrument to facilitate the solution of the aforementioned problem and to capture adequately the main features of the EC8 spectrum. Upon determining of the EPS, an efficient filtering technique is employed to generate EC8 design spectrum compatible non-stationary artificial accelerograms. Further, a comprehensive Monte Carlo analysis is undertaken to estimate the statistical features of the peak factor and of other quantities of engineering interest associated with the first passage problem for non-stationary excitations compatible with the EC8 design spectrum. Special attention is focused on deriving frequency and damping dependent median peak factors compatible with the EC8 spectrum. These are incorporated in the aforementioned formulation to achieve enhanced level of compatibility of the derived artificial accelerograms with the EC8 spectrum compared to the previous study of GIARALIS and SPANOS [7] which have assumed a constant peak factor value. The level of compatibility is assessed by comparison of the average and the median response spectra of ensembles of the generated artificial signals with the EC8 design spectrum.

2 Theoretical Background

2.1 Relation of an evolutionary power spectrum to a design spectrum

Let the acceleration process $u_g(t)$ of the ground motion due to a seismic event be modelled as a uniformly modulated non-stationary stochastic process. That is,

$$u_g(t) = A(t)\,y(t), \tag{1}$$

where $A(t)$ is a deterministic time dependent envelop function and $y(t)$ is a zero-mean stationary stochastic process. The exponential modulated function [1]

$$A(t) = Ct\exp\left(-\frac{b}{2}t\right), \tag{2}$$

is adopted herein to model the non-stationary features of the strong ground motion intensity exhibited by typical field recorded seismic accelerograms. The parameter C>0 is proportional to the intensity of the ground acceleration process while the parameter b>0 controls the shape of the envelope. Assuming that $A(t)$ varies slowly with time, the process $u_g(t)$ can be characterized in the frequency domain via a two-sided evolutionary power spectrum (EPS) $G(t,\omega)$ given by the expression [18]

$$G(t,\omega) = |A(t)|^2\,Y(\omega). \tag{3}$$

In the latter equation $Y(\omega)$ denotes the power spectrum of the stationary stochastic process $y(t)$. In this study, the Clough-Penzien (CP) spectrum given by the expression [6]

$$Y(\omega) = \frac{1+4\zeta_g^2\left(\frac{\omega}{\omega_g}\right)^2}{\left(1-\left(\frac{\omega}{\omega_g}\right)^2\right)^2+4\zeta_g^2\left(\frac{\omega}{\omega_g}\right)^2}\frac{\left(\frac{\omega}{\omega_f}\right)^4}{\left(1-\left(\frac{\omega}{\omega_f}\right)^2\right)^2+4\zeta_f^2\left(\frac{\omega}{\omega_f}\right)^2} \tag{4}$$

with $|\omega| \le \omega_b$.

is considered, where ω_b is a cut-off frequency beyond which $Y(\omega)$ attain negligible values of no engineering interest. This is a widely used phenomenological model which accounts for the influence of the surface soil deposits on the frequency content of the propagating seismic waves via the effective stiffness (ω_g) and damping (ζ_g) parameters. The parameters ω_f and ζ_f control the properties of the high-pass filter incorporated by the model to suppress the low frequencies (see also [7]).

Furthermore, consider a linear quiescent unit-mass SDOF system, with ratio of critical viscous damping ζ_n and natural frequency ω_n, base-excited by the acceleration process $u_g(t)$. The relative displacement response process $x(t)$ of this system with respect to the motion of its base is governed by the equation

$$\ddot{x}(t)+2\zeta_n\omega_n\dot{x}(t)+\omega_n^2x(t)=-u_g(t) \quad ; \quad x(0)=\dot{x}(0)=0\,, \tag{5}$$

in which a dot over a symbol denotes time differentiation. Focusing on lightly damped systems (i.e. ζ_n<0.1), the response $x(t)$ is assumed to be a narrow-band process. In this case the time-evolving variance of the response process $x(t)$ can be replaced by the variance $\sigma_a^2(t)$ of its amplitude [21]. A reliable expression for the latter quantity reads [22]

$$\sigma_a^2(t)=\frac{\pi}{\omega_n^2}\exp(-2\zeta_n\omega_n t)\int_0^t \exp(2\zeta_n\omega_n\tau)G(\tau,\omega_n)\,d\tau\,. \tag{6}$$

In the practice of aseismic design of structures often only a relative displacement design spectrum $S_d(\omega_n, \zeta_n)$ is provided to the designer for the definition of the input seismic severity. This design spectrum is not related to any specific EPS. Clearly, an "inverse" stochastic dynamics problem must be considered to relate an EPS $G(t,\omega)$ defined by Eqs. (2)-(4) to a given design spectrum $S_d(\omega_n, \zeta_n)$. Following SPANOS and VARGAS LOLI [23], this problem can be formulated by relying on the equation (see also [7])

$$S_d(\omega_n,\zeta_n)=r(\omega_n,\zeta_n,G,p)\max_t\left\{\sigma_a(t,\omega_n,\zeta_n,G)\right\}. \tag{7}$$

In Eq. (7) the so-called "peak factor" r is the critical parameter establishing the equivalence between the given design spectrum S_d and the EPS G to be determined in a statistical manner (see e.g. [26]). Essentially, it corresponds to the scalar by which one needs to multiply the peak standard deviation of the response amplitude (assumed to be equal to the peak standard deviation of the response process x) attained at some time instant $t_{max\ var}$ to reach a certain peak response level S_d with probability p. In general, the peak factor depends on the properties ζ_n and ω_n of the considered SDOF system and on the EPS of the input strong ground motion. Further discussion on the peak factor is included in sections 3.2 and 3.3 in light of numerical data obtained from Monte Carlo simulations for input EPSs associated with the design spectrum of the European aseismic code regulations [4].

An approximate point-wise solution of the inverse problem of Eq. (7) can be obtained by minimizing the error [7]

$$e=\sum_{j=1}^{2M}\left(S_j-q_j\right)^2 \tag{8}$$

with $$S_j = \begin{cases} S_d^2\left(\omega_{n(j)}, \zeta_n\right) & , \quad j = 1, ..., M \\ 0 & , \quad j = M+1, ..., 2M \end{cases}$$

$$q_j = \begin{cases} \dfrac{r^2 \pi C^2 t_j^{*2} \exp\left(-b t_j^*\right)}{2\zeta \omega_{n(j)}^3} Y\left(\omega_{n(j)}\right) & , \quad j = 1, ..., M \\ \gamma_{j-M}^2 \left(2t_{j-M}^* - b t_{j-M}^{*2}\right) - 2\gamma_{j-M}\left(1 - b t_{j-M}^*\right) - \\ -2b + 4\zeta_n \omega_{n(j-M)} \exp\left(-\gamma_{j-M} t_{j-M}^*\right) & , \quad j = M+1, ..., 2M \end{cases}$$

$$\gamma_j = 2\zeta_n \omega_{n(j)} - b$$

at a certain set of M natural frequencies $\{\omega_{n(j)}\}$ for $j= 1,\ldots,M$, where the symbol t_j^* denotes the time instant at which the variance $\sigma_a^2(t)$ corresponding to the linear SDOF system with natural frequency $\omega_{n(j)}$ is maximized. In all ensuing numerical results, a Levenberg-Marquardt algorithm with line search (see e.g. [14]) is used to solve the over-determined nonlinear least-square fit optimization problem of Eq. (8) for the unknown parameters C, b, ω_g, ζ_g, ω_f, ζ_f of the herein adopted EPS form.

2.2 Simulation of design spectrum compatible seismic accelerograms

Clearly, an EPS $G(t,\omega)$ obtained as discussed in the previous section defines a non-stationary process $u_g(t)$ whose realizations achieve a certain level of compatibility with the given design spectrum S_d. In particular, it is expected that generated non-stationary time-histories compatible with such an EPS will constitute seismic accelerograms whose response spectra will lie close to the design spectrum S_d. Such accelerograms can be numerically generated by first synthesizing stationary discrete-time signals as sampled versions of the continuous-time stochastic process $y(t)$ appearing in Eq. (1). That is,

$$y[s] = y\left(sT_s\right) \quad , \quad s = 0, 1, ..., N, \tag{9}$$

with $T_s \le \dfrac{\pi}{\omega_b}$ sampling interval

where N should be selected appropriately so that $A(NT_s)$ is negligible. Next, these stationary records are multiplied individually by the corresponding discrete/sampled version of the envelop function defined in Eq. (2) to obtain the final artificial records with non-stationary intensity as Eq. (1) suggests.

In this study, stationary discrete-time signals $\tilde{y}[s]$ are synthesized by filtering arrays of discrete-time Gaussian white noise $w[s]$ with a two-sided unit-intensity power spectrum band-limited to ω_b through an autoregressive-moving-average (ARMA) filter of order (m,n). In a practical numerical implementation setting these arrays comprise pseudorandom numbers belonging to a Gaussian distribution with zero mean and variance equal to $\sqrt{2\omega_b}$. The aforementioned filtering operation is governed by the difference equation

$$\tilde{y}[s] = -\sum_{k=1}^{m} d_k \tilde{y}[s-k] + \sum_{l=0}^{n} c_l w[r-l], \tag{10}$$

in which c_l (l=0,1,…,n) and d_k (k= 1,…,m) are the ARMA filter coefficients. Herein, the auto/cross-correlation matching (ACM) method is adopted to determine these coefficients so that the power spectrum of the process $\tilde{y}[s]$ matches the CP spectrum $Y(\omega)$ of the process $y[s]$. In this manner, the process $\tilde{y}[s]$ can reliably model the process $y[s]$. The mathematical details of the ACM method can be found in [24].

It is further noted that the non-stationary artificial records generated as discussed above are further processed to address the issue of baseline correction (e.g. [2]). This is accomplished efficiently by appropriate zero-padding and forward/backward filtering of the records using a standard Butterworth high-pass filter of order 4 and cut-off frequency 0.10Hz [3].

3 Numerical results pertaining to the EC8 design spectrum

For the purposes of this study, a Monte Carlo analysis to estimate the statistical properties of the peak factor r appearing in Eq. (7) associated with the EC8 design spectrum is performed [4]. Specifically, CP evolutionary power spectra (EPSs) compatible with the EC8 design spectrum for peak ground acceleration (PGA) of 0.36g (g= 9.81 m/sec^2), for three different damping ratios ζ_n= 2%, 5%, and 8% and for all five soil conditions prescribed by the EC8 are considered: a total of 15 EPSs. These have been obtained as discussed in section 2.1 assuming a constant peak factor r= $\sqrt{3\pi/4}$ (see also [7]).

For each of the thus obtained EPSs a suite of 10000 spectrum-compatible non-stationary artificial accelerograms has been generated and base-line adjusted as described in section 2.2. Next, each suite is “fed” to a series of 200 linear SDOF systems with natural periods ranging from 0.02sec to 6sec. The damping ratio of these systems is set to coincide with the value of ζ_n considered in deriving each of the EPS from the corresponding EC8 spectrum. For every such system defined by the properties T_n= $2\pi/\omega_n$ and ζ_n and excited by a specific suite of accelerograms the response ensembles ($x^{(k)}(t)$; k=1,2,…,10000) are calculated via numerical integration of Eq. (5) [13]. Finally, populations of peak factors ($r^{(k)}$; k=1,2,…,10000) are computed from the above ensembles as the ratio of the population of peak responses over the maximum standard deviation of the response ensemble. That is,

$$r^{(k)}(T_n,\zeta_n,G) = \frac{\max_t\left\{\left|x^{(k)}(t,T_n,\zeta_n,G)\right|\right\}}{\max_t\left\{\sqrt{E\left\{\left(x^{(k)}(t,T_n,\zeta_n,G)\right)^2\right\}}\right\}}. \tag{11}$$

It is important to note that these peak factor populations are independent of the intensity of the excitation. Thus, they are neither influenced by the adopted PGA value assumed in the derivation of the 15 considered EPSs nor by the value of the constant peak factor $\sqrt{3\pi/4}$ involved in this derivation. However, they do reflect the different spectral contents and effective durations (as controlled by the *b* parameter of Eq. (2)) of the various *G* EPSs; certainly they depend strongly on the dynamical properties (T_n, ζ_n) of the SDOF system considered.

3.1 Monte Carlo analysis of peak response time instants

In the course of computing the denominator of Eq. (11) two quantities need to be considered. First, is the peak value of the mean square of the response ensembles (equal to the variance since the simulated signals are base-line corrected to have zero mean value). Second is the time $t = t_{max\ var}$ at which this value is attained. Fig. 1 provides plots of both of these quantities as functions of the natural period of the SDOF systems considered for damping ratio ζ_n= 5% for the five EC8 soil types. The spectral shapes of the variance (Fig. 1(a)) are similar to the EC8 displacement design spectrum (see [4]). And, as more flexible oscillators are considered the maximum response variance is reached at later times.

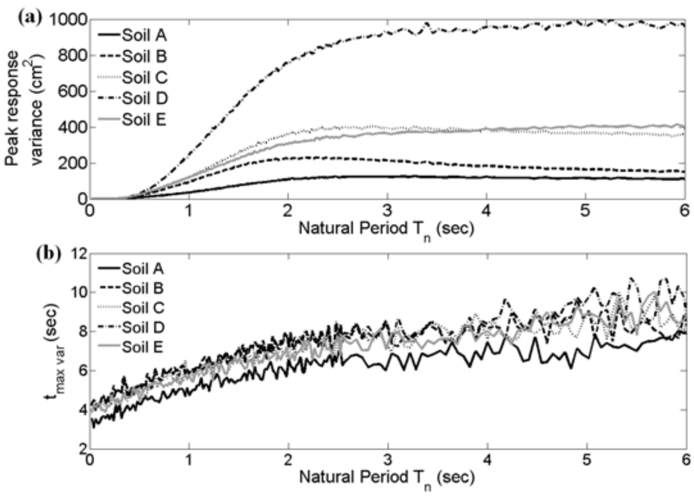

Fig. 1: Peak variances and time instants $t_{max\ var}$ at which these peak values are attained for the response ensembles pertaining to EC8 spectrum compatible EPSs for PGA= 0.36g reported in [7] (ζ_n= 5%). Ensemble includes population: 10000.

Furthermore, the computation of the numerator of Eq. (11) involves the calculation of the time instants $t_{max|x|}$ at which the peak value of each response time-history is attained. Fig. 2 shows certain plots associated with the statistical properties of the $t_{max|x|}$ populations normalized by the $t_{max\ var}$ time instants for ζ_n=5%. Specifically, Fig. 2(a) and 2(b) plots the average and standard deviation, respectively, of these populations for all EC8 soil conditions as a function of natural period. The average spectra fluctuate around unity with small dispersion for all soil types, although a noticeable trend of linear decrease towards the longer periods exists. This result agrees with the intuition which suggests that the time instants at which the peak response and the peak response variance are obtained should be in a close agreement, on the average. Nevertheless, the standard deviation spectra reveal that there is a significant dispersion in the population of the samples (10000 for each oscillator). To further elucidate this point, six histograms of such populations related to certain oscillators and the corresponding fitted gamma distributions (solid lines) are shown in Fig. 2. It is reported that in general the gamma distribution yielded the best parametric fitting results based on a standard maximum likelihood estimation algorithm. The gamma distribution of a random variable z reads [15]

$$f\left(z/\kappa,\theta\right)=\frac{1}{\theta^{\kappa}\Gamma\left(\kappa\right)}z^{\kappa-1}\exp\left(-\frac{z}{\theta}\right), z\geq 0 \tag{12}$$

where $\Gamma\left(\kappa\right)=\int_{0}^{\infty}\exp\left(-t\right)t^{\kappa-1}dt$,

and κ, θ are the "shape" and "scale" parameters, respectively. Similar results as the above have been observed for response ensembles corresponding to ζ_n=2% and 8% not presented here for brevity.

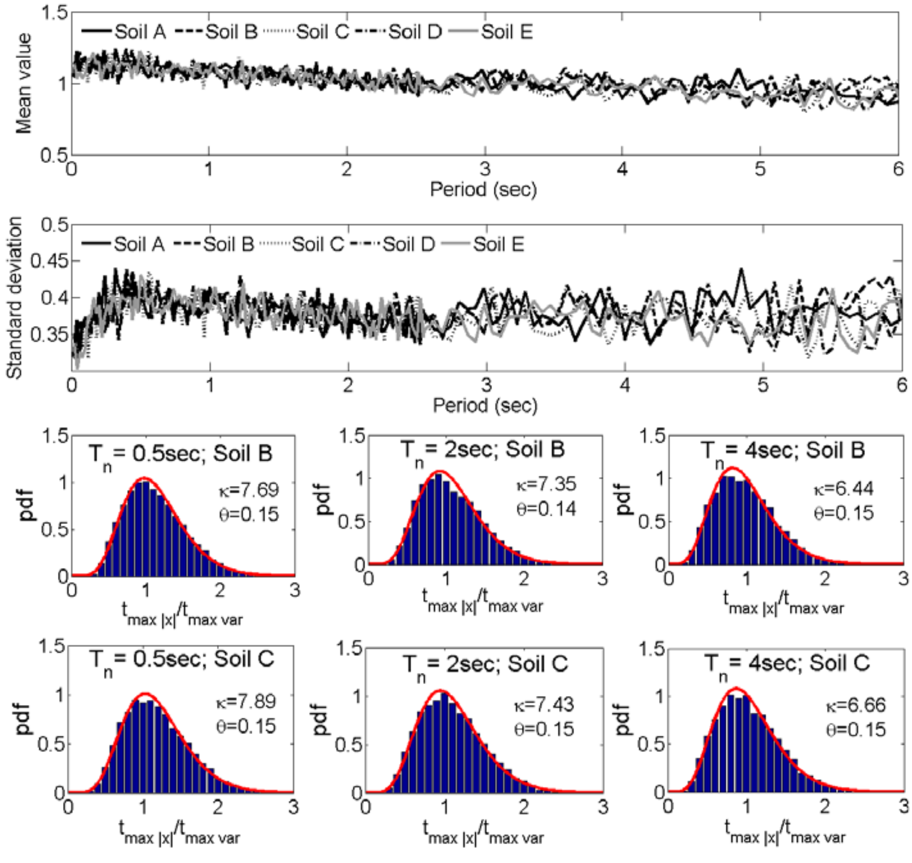

Fig. 2: Mean value spectra, standard deviation spectra, and histograms of populations of time instants $t_{max\,|x|}$ at which the peak response is observed normalized by the time instants $t_{max\,var}$ at which the peak response variance is attained. The considered response ensembles are the same as in Fig. 1.

3.2 Monte Carlo analysis of peak factors

Fig. 3 collects similar results as in Fig. 2 corresponding to peak factor populations calculated by Eq. (11). In particular, the median of the peak factors plotted against the natural period (median peak factor spectra) for all the EC8 soil conditions and for ζ_n= 5% are shown in Fig. 3(a). These spectra possess a significant merit for the purposes of this study. Specifically, when substituted in Eq. (7) the following criterion between a design/response spectrum S_d and an EPS G is established: considering an ensemble of non-stationary samples compatible with G (i.e. generated as described in section 2.2), half of the population of their response spectra will lie below S_d (i.e. S_d is the median response spectrum). Consequently, the probability of exceedance p appearing in Eq. (7) becomes equal to 0.5. Evidently, the median peak factor possesses a complicated dependence with the natural period of linear SDOF oscillators. Interestingly, similar trends have been previously reported in the literature (see Fig. 8.17 and 8.18 in [26]). From a practical viewpoint, the most important conclusion drawn from Fig. 3(a) is that the various shapes of the underlying evolutionary power spectrum corresponding to the different EC8 shapes of the design spectrum depending on the soil conditions has a minor effect on the median peak factor spectrum. In other words, the five curves of Fig. 3(a) lie very close to each other. The significance of this observation in practical terms will become clear in the following section.

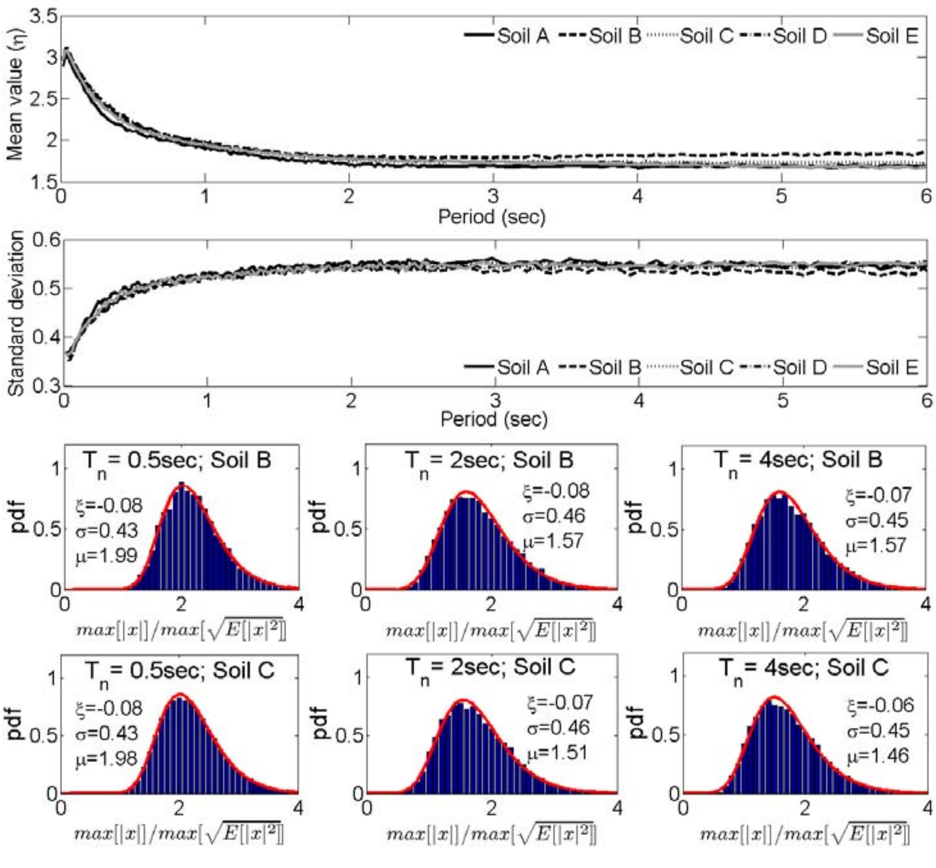

Fig. 3: Median spectra, standard deviation spectra, and histograms of populations of peak responses normalized by the peak ensemble standard deviation. The considered response ensembles are the same as in Fig. 1.

It is further noted, that the standard deviation of the peak factor populations is certainly non-negligible and it also varies for natural periods up to 1sec approximately. Then it attains a practically constant value. Moreover, histograms of peak factor populations have been included in Fig. 3 related to certain oscillators and input EPSs. Generalized extreme value distributions given by the equation,

$$f\left(z/\xi,\sigma,\mu\right)=\frac{1}{\sigma}\exp\left(-\left(1+\xi\frac{z-\mu}{\sigma}\right)^{-1/\xi}\right)\left(1+\xi\frac{z-\mu}{\sigma}\right)^{-1-1/\xi} \tag{13}$$

where $1+\xi\dfrac{z-\mu}{\sigma}>0$,

have been fitted to these histograms (solid lines). In the above equation and in Fig. 3 μ corresponds to the "center of mass" of the population, σ is a "spread" factor and ξ is the "shape" factor. Note that in all cases examined the value of parameter ξ is negative. This corresponds to a "type III" extreme value distribution of the Weibull kind [9].

3.3 EC8 compatible peak factor and evolutionary power spectra

The shape of the median peak factor spectrum is relatively insensitive with respect to the shape of the corresponding evolutionary power spectrum and thus to the shape of the EC8 design spectrum. Thus, it is reasonable to consider the average of these spectral curves for the various soil conditions of EC8 and for each value of the damping ratio herein considered. Further, polynomial curve fitting can be applied to the above averaged median peak factor spectra to obtain an analytical expression to approximate the numerically derived median peak factors. The eigth-order polynomials plotted in Fig. 4 and expressed by the equation

$$\hat{r}\left(T\right)=\sum_{j=0}^{8}p_jT^j\ ,\ 0.02\leq T\leq 6\sec . \tag{14}$$

approximate reasonably well the averaged median peak factor spectra. The coefficients p_j of these polynomials are given in Tab. 1.

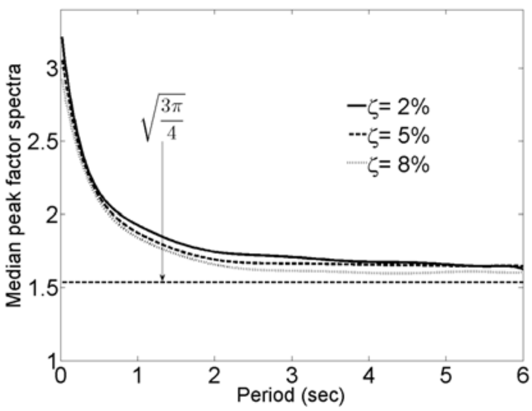

Fig. 4: Polynomial fit to the median peak factors from all soil types as defined in EC8.

Tab. 1: Coefficients of the fitted polynomials to the averaged numerically obtained median peak factor spectra from all EC8 defined soil types.

	p_0	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
ζ= 2%	3.3079	-4.9375	8.3621	-8.3368	5.0420	-1.8983	0.4469	-0.0639	0.0051
ζ= 5%	3.1439	-3.9836	5.9247	-5.3470	2.9794	-1.0439	0.2305	-0.0311	0.0023
ζ= 8%	2.9806	-3.2070	4.1190	-3.1733	1.4746	-0.4144	0.0689	-0.0062	0.0002

Table 2 shows CP evolutionary power spectra compatible with the EC8 design spectrum for PGA= 0.36g, damping ratio 5% and for all soil conditions obtained by solving the optimization problem of Eq. (7) using the frequency-dependent averaged median peak factor spectrum $\hat{r}$ of Eq. (14). In Fig. 5 median pseudo-acceleration response spectra for ensembles of 100 baseline-corrected artificial accelerograms compatible with the CP spectra of Tab. 2 are plotted along with the corresponding target design spectrum. For comparison, average pseudo-acceleration response spectra for ensembles of 100 baseline-corrected artificial accelerograms compatible with CP spectra derived by assuming a constant peak factor equal to $\sqrt{3\pi/4}$, are also included. Clearly, the influence of the peak factor incorporated on the matching of the average response spectra of the artificial signals obtained via solving the stochastic problem of Eq. (7) is quite strong. Evidently, the use of the peak factor of Eq. (14) improves significantly the quality of this matching compared to that achieved via the constant peak factor (see also [7]). Furthermore, in terms of pseudo-acceleration spectral ordinates and for the range of natural periods that most of the ordinary structures are designed to withstand seismically induced forces a satisfactory matching is attained for the frequency-dependent peak factor. More importantly, it is noted that the shapes of the prescribed EC8 design spectra for the various soil types exhibit considerable variations. Thus, it is reasonable to argue that the average EC8 peak factor spectrum of Eq. (14) can yield results of similar quality as those of Fig. 5 for target design spectra prescribed by other contemporary aseismic code provisions, as well. Obviously, the latter claim warrants further numerical investigation.

Incidentally, the significant discrepancy of the average response spectra obtained under the assumption of a constant peak factor η= $\sqrt{3\pi/4}$ from the target spectrum can be readily justified by considering the deviation of the averaged median peak factor spectrum from the constant level of $\sqrt{3\pi/4}$ shown in Fig. 4.

Tab. 2: Parameters for the definition of CP evolutionary power spectra compatible with various EC8 design spectra for the frequency-dependent peak factor of Eq. (14).

Peak ground acceleration	**Soil type**	**CP power spectrum parameters [T_{min}= 0.02,T_{max}= 10] (sec)**					
		C (cm/sec$^{2.5}$)	***b*** (1/sec)	ζ_g	ω_g (rad/sec)	ζ_f	ω_f (rad/sec)
α_g= 0.36*g* (*g*= 981 cm/sec^2)	**A**	8.08	0.47	0.54	17.57	0.78	2.22
	B	17.76	0.58	0.78	10.73	0.90	2.33
	C	19.58	0.50	0.84	7.49	1.15	2.14
	D	30.47	0.50	0.88	5.34	1.17	2.12
	E	20.33	0.55	0.77	10.76	1.07	2.03

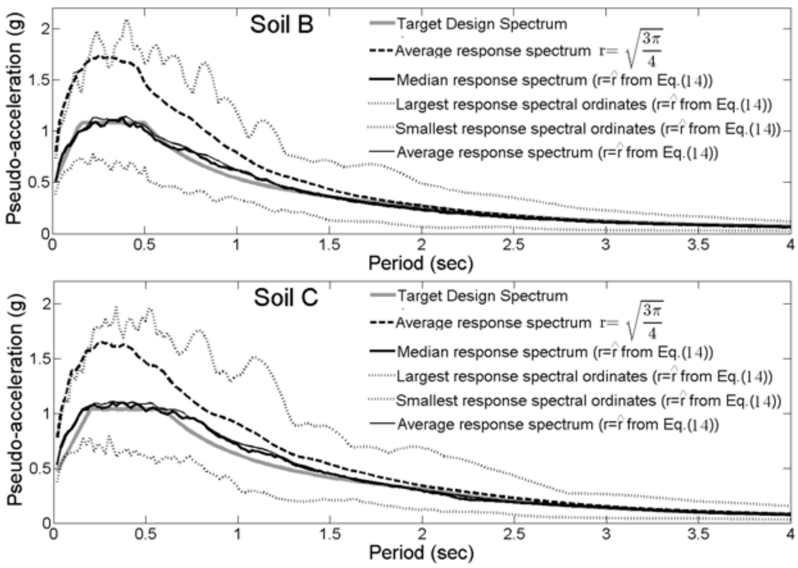

Fig. 5: Pseudo-acceleration response spectra of ensembles of 100 simulated accelerograms compatible with CP evolutionary power spectra derived by assuming a constant peak factor of $\sqrt{3\pi/4}$ and the frequency-dependent peak factor of Eq. (14).

4 Concluding Remarks

In this work, an inverse stochastic dynamics approach has been adopted to relate an evolutionary power spectrum (EPS) to a given seismic design spectrum. A solution of this problem in an approximate point-wise least-squared sense has allowed for defining a design spectrum consistent EPS characterizing a non-stationary stochastic process. Subsequently, an efficient simulation technique for producing EPS compatible non-stationary time-histories has been employed along with acausal high-pass filtering for adjusting the baseline of the simulated signals. Ensembles of thus generated signals constitute artificial seismic accelerograms whose response spectra achieve a certain level of agreement with the design spectrum.

Special attention has been given to the peak factor: a quantity which governs the relation of the EPS with a given design spectrum in a statistical manner. Specifically, using the design spectrum prescribed by the European aseismic code provisions (EC8) as a paradigm, and assuming a uniformly modulated Clough-Penzien type of EPS, a Monte Carlo analysis has been undertaken to estimate numerically median peak factor spectra pertaining to all soil conditions and various damping levels of the EC8 design spectrum. This need has been dictated by the fact that no convenient expression of the peak factor in the non-stationary case considered exists in the open literature. The consistency of the derived median peak factor spectra with the EC8 has been established by utilizing appropriately defined EC8 compatible EPSs as input spectra in the Monte Carlo analysis. Additional numerical data derived as by-products of the above analysis have been also reported to elucidate certain aspects of the response of linear SDOF systems driven by uniformly modulated colored noise processes.

The thus derived median peak factor spectra have been further incorporated in the solution of the aforementioned inverse stochastic problem to yield EC8 consistent EPSs. The achieved level of consistency has been assessed by comparing the average and median populations of response spectra of ensembles of EPS compatible artificial accelerograms. Compared with similar data from a previous study incorporating constant peak factors in the derivation of EC8 compatible EPSs [7], the average and median of the herein generated signals lie significantly closer to the EC8 spectrum. This result establishes the usefulness and practical merit of the reported EC8 compatible median peak factor and evolutionary power spectra for simulating EC8 design spectrum consistent non-stationary accelerograms to be used as input for time-history elastic and inelastic dynamic analyses for the aseismic design of critical structured facilities.

As a final note, and given the enhanced level of consistency of the derived EPSs with the EC8 design spectrum, it is suggested that these EPSs can be further considered as input spectra to perform non-stationary linear and non-linear random vibration analyses for the design of structures regulated by the EC8 provisions. In the non-linear case, the method of statistical linearization can perhaps be employed [19].

5 References

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